Capital Replacement and Innovation Dynamics*

Fabio Bertolotti§ Andrea Lanteri¶

December 2024

Abstract

We analyze the role of endogenous capital-embodied innovation for macroeconomic dynamics. We provide empirical evidence that new-product introduction by capital-goods producers is an engine of long-run technological progress, but drops during recessions, when investment demand is weak. We then develop a model of investment with heterogeneous firms and endogenous capital-embodied innovation. The model features two endogenous state variables: (i) the cross-sectional distribution of firms and (ii) the quality of new capital. The fraction of firms replacing their capital determines the incentives for capital-goods producers to innovate through a market-size effect. In turn, growth in new-capital quality induces final-good producers to replace their old capital. This feedback provides a propagation mechanism for macroeconomic shocks and generates fluctuations in productivity. We analyze efficiency in the model and show that capital-replacement subsidies have stronger stimulus effects than innovation subsidies in the short run, but are less desirable in the long run.

*We thank seminar and conference participants at Bank of Italy, Federal Reserve Bank of Chicago, Society for Economic Dynamics 2023 Annual Meeting, 2023 Congress of the European Economic Association, EIEF Workshop on Macro and Monetary Economics, Duke-UVA-Richmond Fed Workshop, Barcelona School of Economics Summer Forum 2024 "Macro Fluctuations with Micro Frictions," as well as David Berger, Gadi Barlevy, Andrés Blanco, Julieta Caunedo, Diego Comin, Thomas Drechsel, Cosmin Ilut, Aubhik Khan, Giuseppe Fiori, John Leahy, Pietro Peretto, and Ricardo Reis for helpful comments. We are especially grateful to Alessandro Gavazza for insightful conversations. Hyeonsik Yoon and Zhiting Yuan provided excellent research assistance.

§Department of Economics and Statistics, Bank of Italy. Via Nazionale 91, Rome 00184, Italy. Email: fabio.bertolotti@bancaditalia.it.

¶Department of Economics, Duke University. 213 Social Sciences, Durham, NC 27708, United States. Email: andrea.lanteri@duke.edu.

1 Introduction

Many important innovations are embodied in new capital goods. As a consequence, some of the most pressing policy challenges involve the replacement of old capital with new capital, which embodies a higher level of technology.

The energy transition to address climate change is a leading example. Many governments are subsidizing the replacement of energy-inefficient equipment and structures with new, cleaner capital. At the same time, they are stimulating research that aims to increase the energy efficiency of new capital. The global race for Artificial Intelligence is another salient example. Policymakers are designing bold interventions to subsidize investment and innovation embodied in new generations of microprocessors.

The goal of this paper is to analyze the roles of capital replacement and embodied innovation for macroeconomic dynamics and the transmission of these policy interventions. To this end, we develop a quantitative model of the equilibrium interactions between the investment dynamics of heterogeneous firms and the path of technology embodied in capital goods.

A crucial feature of capital-embodied technological advances is that, in order to exploit them in production, firms need to invest in the new vintage of capital. As a result, our main insight is that capital investment decisions and innovation effort play *complementary* roles for the dynamics of aggregate output and productivity. A high level of investment creates a large market for embodied innovations and induces capital-goods producers to accelerate improvements in the quality of capital. In turn, fast improvements in the quality of new capital increase the incentives for firms to invest and replace their obsolete capital.

Using our quantitative model, we show that these feedback effects between the distribution of firms and the level of technology provide a powerful propagation mechanism in response to macroeconomic shocks and are critical to understand the dynamic effects of stimulus policies such as innovation subsidies and investment subsidies.

We begin our analysis by providing empirical evidence on technological progress embodied in capital. We observe positive comovement between aggregate capital expenditures and expenditures on Research and Development (R&D) in manufacturing sectors that produce capital goods. This comovement prompts us to investigate deeper the feedback effects between demand and supply in the market for capital good.

To provide more direct evidence on the dynamics of innovations embodied in capital, we zoom in on the level of technology embodied in new commercial aircraft. This type of capital provides a useful laboratory for our analysis for several reasons. First, the

availability of information on aircraft allows us to assemble a detailed dataset that includes the technical characteristics and volume of sales of new models since the 1970s. Second, aircraft is the subject of a global policy debate on capital replacement and innovation due to its environmental impact.

We show that new-product introduction is a crucial engine for long-run technological progress. However, during recessions—when new-aircraft orders decrease—the volume of new-model introduction drops and new models do not contribute significantly to technological progress. We also discuss similar empirical evidence on the dynamics of new-product introduction and new-product quality for other types of capital, such as vehicles and computers.

These empirical observations motivate us to develop an equilibrium framework that combines two building blocks: (i) a capital-replacement model with heterogeneous firms subject to aggregate and idiosyncratic shocks and (ii) an analytically tractable model of endogenous technological innovations embodied in new capital goods.

Final-good producers make lumpy investments to update their capital stock in response to idiosyncratic productivity shocks. Capital-goods producers invest in R&D to create new products, climb a quality ladder for new capital goods, and obtain temporary monopoly profits from their innovations. Sustained capital-replacement activity from final-good producers determines a market-size effect that promotes innovation. In turn, growth in the quality of new capital vintages renders installed capital obsolescent, further stimulating capital replacement. The equilibrium feedback effects between these two blocks of the model determine the dynamics of investment, output, and productivity.

We calibrate our model to replicate several salient features of firm investment dynamics and innovation activity. We then use the calibrated model to perform a quantitative analysis of the macroeconomic role of complementarities in capital replacement and embodied innovations. In response to an aggregate productivity shock, the model successfully accounts for the comovement of investment, new-capital quality, and output. Moreover, the evolution of the distribution of firm capital produces persistent dynamics in measured aggregate productivity.

We show that the endogenous response of capital quality is critical for our results. A negative productivity shock induces a temporary decrease in investment, which reduces the demand for improvements in the quality of capital. The resulting slowdown in embodied innovations persistently depresses the efficiency of capital used in production, generating a protracted recession. In contrast, a counterfactual model with exogenous capital quality predicts a quick reversal in the dynamics of aggregate investment and capital after the

shock. We also show that the feedback mechanism between investment and innovation is also relevant to understand the response of the economy to other sources of macroeconomic fluctuations. Notably, in our model, an exogenous increase in the interest rate leads to a recession that features an endogenous persistent decline in aggregate productivity.

To leverage our empirical evidence more directly in the model analysis, we also provide a separate calibration of the model that targets specific features of the aircraft market and show that this version of the model quantitatively accounts for our empirical findings on the drop in innovations embodied in new aircraft models during recessions.

We then analyze the efficiency of the dynamics of capital-embodied innovation. In the model, innovators only internalize the short-run profits they obtain while their inventions are protected by competition. However, quality improvements have a positive future social value, because they contribute to future aggregate capital quality. This externality implies that the competitive equilibrium features inefficiently low R&D effort and growth relative to the social planner's allocation. Accordingly, we also find that the pursuit of short-run profits induces inefficiently large fluctuations in embodied innovation in response to business-cycle shocks.

Finally, we use the model to simulate the effects of policy interventions aimed at stimulating demand for new capital goods or the supply of capital-embodied innovations. We consider two alternative stimulative policies: capital-replacement subsidies on the demand side of the market for capital and innovation (R&D) subsidies on the supply side.

Because of the complementarity between investment and innovation, either policy intervention fosters both capital replacement and technological progress in the long run. Quantitatively, R&D subsidies are more effective at generating output growth and increasing welfare, for a given fiscal budget. However, in the short run, R&D subsidies lead to an optimal postponement of investment activity, while firms wait for a higher quality of capital to materialize. As a result, capital-replacement subsidies are a more effective countercylical stimulus policy.

The rest of the paper is organized as follows. Section 2 discusses our contributions to the literature. Section 3 presents the empirical evidence on innovation in capital-goods manufacturing. Section 4 introduces our model of capital replacement and innovation. Section 5 discusses our calibration and analyzes the balanced-growth path. Section 6 presents the quantitative results on the propagation of aggregate shocks. Section 7 analyzes efficiency and the effects of stimulus policies. Section 8 discusses additional quantitative analyses. Section 9 concludes.

2 Related Literature

Our paper contributes to several strands of the literature. First, a growing body of work analyzes models of investment with firm heterogeneity and adjustment frictions (e.g., Cooper, Haltiwanger, and Power, 1999; Cooper and Haltiwanger, 2006; Khan and Thomas, 2008; Lanteri, 2018; Baley and Blanco, 2021; Winberry, 2021). This literature emphasizes the importance of the extensive margin of investment at the micro level for the dynamics of aggregate investment. However, capital producers typically play no role and an exogenous productivity process drives both long-run growth and short-run fluctuations. Fiori and Scoccianti (2021) introduces vintage capital in a quantitative model of investment with heterogeneous firms. Lanteri and Rampini (2023) analyze the role of financing constraints for investment in energy-efficient capital. Ottonello and Winberry (2024) analyze the role of financing constraints for firm investment and innovation. We contribute to this literature by combining heterogeneous-firms investment demand with a model of the *supply of new capital*, which features an endogenous process of product innovation, consistent with the empirical evidence we provide. Specifically, we model capital-embodied innovation as a quality ladder (Grossman and Helpman, 1991; Aghion and Howitt, 1992). This framework allows us to analyze the interplay between investment and technological progress both in the long run and in response to aggregate shocks and investment subsidies.

Second, our paper contributes to the literature on investment-specific technical change and on vintage capital. Several papers emphasize the macroeconomic importance of technological progress in the quality of capital goods (e.g., Greenwood, Hercowitz, and Krusell, 1997; Krusell, 1998; Samaniego, 2010; Jones and Liu, 2022; Caunedo and Keller, 2021; Caunedo, Jaume, and Keller, 2023), as measured for instance in the work of Gordon (1990) and Cummins and Violante (2002) (or Bils and Klenow (2001) for household durable goods). A related body of work analyzes the role of exogenous shocks to the price of new investment goods for investment or macroeconomic fluctuations (e.g., Greenwood, Hercowitz, and Krusell, 2000; Fisher, 2006; Justiniano, Primiceri, and Tambalotti, 2011; Fishman and Jovanovic, 2021; Drechsel, 2023). Building on early contributions on vintage-capital technology (e.g., Solow, 1959; Solow, Tobin, von Weizsäcker, and Yaari, 1966), several papers model capital replacement and aggregate dynamics when technology is embodied in capital (e.g., Boucekkine, Germain, and Licandro, 1997; Cooley, Greenwood, and Yorukoglu, 1997). Relative to this literature, we make two contributions. First, we provide new empirical evidence on the dynamics of the quality of new capital, building on the empirical

approach of Knittel (2011), and emphasize the role of new-product introduction.[1] Second, we endogenize innovations in the quality of capital in a quantitative model of investment with heterogeneous firms.

Third, a large literature on economic growth and technology adoption analyzes the role of complementarities among the decisions of different economic agents. Since the early work of Schmookler (1966), this literature emphasizes the relevance of demand conditions and market size for innovation. In a seminal paper, Shleifer (1986) proposes a model in which innovators coordinate to take advantage of high aggregate demand during expansions. Comin and Hobijn (2007) models innovation and technology adoption and stresses that high adoption effort stimulates innovation. In recent work, Buera, Hopenhayn, Shin, and Trachter (2021) develops a framework in which a high rate of technology adoption facilitates further adoption from other firms, which amplifies the effects of distortions. A related literature on the persistent effects of macroeconomic shocks builds on these insights and introduces endogenous technological progress (e.g., Romer, 1990) in business-cycle models (e.g., Comin and Gertler, 2006; Barlevy, 2007; Benigno and Fornaro, 2018; Anzoategui, Comin, Gertler, and Martinez, 2019; Bianchi, Kung, and Morales, 2019; Vinci and Licandro, 2020; Fornaro and Wolf, 2021).[2] We contribute to this body of work by proposing a model of capital replacement and innovation embodied in new capital goods. Building on the literature, our model also features a market-size effect: A high volume of investment stimulates innovation embodied in new capital. Moreover, our model highlights an additional channel: Because different vintages of capital are heterogeneous in their technological content, innovation decreases the relative value of old capital through an *obsolescence* effect. This mechanism hinges on the durability of capital and gives rise to a feedback loop whereby capital replacement stimulates innovation and innovation reinforces the incentives for replacement. Finally, our model is also consistent with the lumpy nature of investment dynamics at the micro level, which implies that the endogenous distribution of firms over capital vintages determines aggregate productivity.

[1] Relatedly, several papers analyze product innovation and quality dynamics in markets for nondurable consumption goods: Broda and Weinstein (2010) and Argente, Lee, and Moreira (2018) provide evidence of procyclical product reallocation; Jaimovich, Rebelo, and Wong (2019) document that the quality of consumption is also procyclical and embed quality choice in a business-cycle model; Argente, Lee, and Moreira (2024) provide an empirical and quantitative analysis of the life cycle of products.

[2] In a related contribution, Schmitz (2021) analyzes the role of heterogeneity in innovation intensity across firms. Cerra, Fatás, and Saxena (2023) surveys the literature on hysteresis and business cycles.

3 Empirical Dynamics of Embodied Innovation

In this section, we provide empirical evidence that R&D and innovations embodied in capital goods are procyclical. We first analyze expenditures on R&D by capital-goods manufacturers. We then leverage a newly-assembled dataset on commercial aircraft to provide evidence on new-product introduction and estimate the dynamics of new-capital quality. Finally, we discuss related evidence on other types of capital.

3.1 R&D by Capital-Goods Producers

We begin our empirical analysis by documenting the dynamics of R&D expenditures by capital-goods producers, which constitute an *input* in innovation activity. We find that R&D in durable-goods manufacturing is highly procyclical, consistent with the findings of Barlevy (2007), and it contributes importantly to the cyclicality of aggregate R&D expenditures.

We consider US data during 1981-2012 because, during this period, the National Science Foundation dataset on R&D expenditures features a more consistent taxonomy of sectors. To quantify the comovement between aggregate investment in equipment and R&D expenditures by durable-goods manufacturers, we HP-filter these two series and obtain a correlation between their cyclical components equal to 0.52. Appendix A.1 provides evidence on the dynamics of aggregate investment in equipment and R&D in durable-goods manufacturing.

R&D by capital-good producers is the main driver of the pro-cyclical behavior of total R&D expenditures. Durable manufacturing R&D is the largest component of total R&D in our sample, accounting for 46% of the aggregate on average. Moreover, durable manufacturing R&D generates the bulk of the observed positive covariance between GDP and total R&D over the business cycle.[3] The correlation between the cyclical component of R&D in durable-good manufacturing and GDP is 0.53 and this component of R&D expenditures is four times more volatile than GDP over the business cycle. In contrast, R&D expenditures in other sectors have a larger relative volatility but a lower correlation with GDP.

[3]R&D in nondurable manufacturing and nonmanufacturing industries constitute on average 26% and 28% of total R&D between 1981 and 2012.

3.2 Product Introduction and New-Capital Quality: Evidence on Commercial Aircraft

We now analyze the dynamics of new-capital quality, which constitutes the *output* of capital-embodied innovation activity. To this end, we assemble a new dataset on the technical characteristics of commercial aircraft.

Commercial aircraft represents an ideal empirical laboratory for several reasons. First, it is a type of capital for which we can leverage a rich availability of data to measure investment, product introduction, and product characteristics. Second, according to the International Energy Agency, aviation contributes to 2% of global carbon emissions (IEA, 2023), resulting in an active policy debate on the need to replace the stock of aircraft with more energy-efficient equipment (Brueckner, Kahn, and Nickelsburg, 2024).

We provide evidence that new-product introduction, which constitute the main driver of technological progress in capital goods, is procyclical. This pattern induces a procyclical growth rate of new-capital quality.

3.2.1 Aircraft Data Description

We begin by describing our dataset on orders and technical characteristics of all Boeing aircraft models. Appendix A.4 shows that our findings are robust to including in the analysis aircraft models produced by Airbus, the second largest manufacturer of commercial aircraft.[4]

Orders. We obtain data on new orders of Boeing aircraft models from Boeing's website. The dataset includes rich information at the order level on the monthly date, model, manufacturer of the engines with which the aircraft is equipped, customer, and number of delivered units. Data cover the period 1955-2023, over which we observe close to 4,700 monthly orders by 578 customers for 58 distinct models. As aircraft can often operate with different engine variants, the total number of Boeing model-engine manufacturer pairs is over twice as large as the number of models. We define an aircraft model m as in Boeing data.

Model Characteristics. To accurately measure aircraft quality changes over time, we obtain data on each model's characteristics from Wikipedia. Information covers the most important technical attributes for efficiency, including: aircraft size, wings width and area, aircraft weight (maximum weight at take-off and operating empty weight), cruise speed,

[4]We exclude Airbus models from the baseline analysis because for several of them we have less precise information on orders and date of introduction.

fuel capacity, maximum range, and number of engines. When missing, we complement this information with data from detailed technical documents for each aircraft published on Boeing's website for administrative reasons. Moreover, we obtain additional data on actual miles flown and fuel consumed by distinct models over their operating life from the U.S. Bureau of Transportation Statistics.[5] Finally, Wikipedia lists engine models with which different aircraft operate. This is particularly relevant because power (thrust), fuel consumption and, therefore, efficiency directly depend on engines.

Engine Characteristics. We obtain detailed data on engine efficiency from the ICAO (International Civil Aviation Organization) Aircraft Engine Emissions Databank hosted by the European Union Aviation Safety Agency (EASA), henceforth referred to as the ICAO dataset. This administrative source collects information on tests performed by manufacturers to comply with existing regulations on exhaust emissions. Data include information on engine power and fuel consumption over the Landing Take-Off (LTO) cycle for more than 400 engine variants over 1972-2022.

The final dataset includes rich information on orders, new aircraft introduction, and aircraft and engine characteristics for more than 100 model-engine brand combinations over more than 50 years. We merge orders data from Boeing with aircraft characteristics from Wikipedia and the U.S. Bureau of Transportation Statistics by aircraft model m and with ICAO engine attributes by aircraft model and engine manufacturer e. Therefore, the unit of observation is a triplet defined by aircraft model m, such as Boeing 747-100, -200, -300, -400; engine brand e, including BMW-Rolls-Royce, CFM International, General Electrics, Pratt&Whitney, and Rolls-Royce; and order month t. Appendix A.2 provides additional details on data construction.

3.2.2 Product Introduction, New-Capital Quality, and the Business Cycle

We now analyze the volume of new-product introduction across expansions and recession in our dataset and measure the evolution of embodied technology. We show that new products are an important engine of technological progress in the long run, but new-product introduction drops during recessions, which leads to procyclical new-aicraft quality.

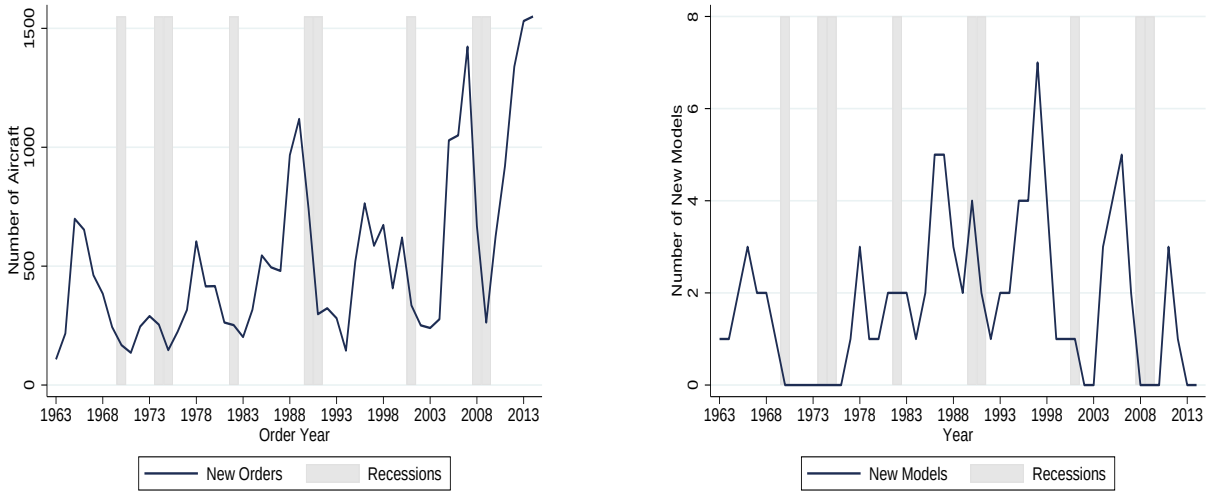
In the market for commercial aircraft, aircraft producers often contact airlines (the demand side) at specialized industry events ("air shows") and gauge their interest in new models. They then complete product development, form an order book with definite orders,

[5] We use the dataset "Air Carrier Summary : T2: U.S. Air Carrier TRAFFIC And Capacity Statistics by Aircraft Type". Brueckner, Kahn, and Nickelsburg (2024) use these data to measure aircraft fuel efficiency in their analysis of decarbonization policy in the aviation industry.

and begin production. We focus on the time at which a new product becomes available for investment. Hence, we measure product introduction based on the month t when the first order for model m is recorded and we consider a model as new for the year following its introduction.

Figure 1 displays the total number of orders of Boeing aircraft (left panel) and the number of newly introduced Boeing aircraft models. Both series are highly procyclical, featuring spikes during economic expansions and drops during recessions, which are highlighted with grey areas in the figure. Most strikingly, product introduction nearly stops both during the 2001 and the 2008-09 recessions.[6]

Figure 1: Orders and New-Model Introduction



(a) Orders

(b) New Models Introduced

Notes: Panel (a) illustrates the yearly number of new orders of Boeing aircraft. Panel (b) plots the yearly number of newly introduced Boeing aircraft models. A model is defined new in the year when the first order is observed and in the subsequent year. We end the analysis in 2014, three years after the last introduction of a new model (Boeing 737 MAX). Grey areas denote NBER recession periods.

This observation motivates us to estimate the effect of procyclical product introduction for the evolution of technology embodied in new aircraft. To this end, we leverage information about aircraft and engine characteristics and estimate the path of aircraft quality building on the approach of Knittel (2011). We consider each aircraft model, combined

[6]In the first part of the sample, the total volume of orders appears to slightly lead product introduction, whereas in the more recent part of the sample product introduction sometimes leads orders. In our analysis we focus on the overall comovement of these two endogenous variables and abstract from this heterogeneity in timing at a high frequency.

with a specific engine type, as a bundle of attributes, such as fuel efficiency, physical dimensions, engine power, etc. We then measure the evolution of the trade-off among these attributes over time.

We assume that the marginal-cost function is Cobb-Douglas in these attributes and estimate the average level of embodied technology in year t by regressing one attribute—fuel efficiency—on a year fixed effect, controlling for the other attributes. Thus, our empirical specification is

$$\ln(\text{Fuel Eff.}_{m,e,t}) = \gamma_t + \beta_1 \ln(\text{Max. Weight}_{m,e,t}) + \beta_2 \ln(\text{Thrust}_{m,e,t}) + X_{m,e,t}\eta + \varepsilon_{m,e,t}, \quad (1)$$

where Fuel Eff.$_{m,e,t}$ represents fuel-efficiency for model-series m equipped with engine e at time t, Max. Weight$_{m,e,t}$ is the maximum allowed weight at take-off, Thrust$_{m,e,t}$ denotes power, obtained by power of engine e times the number of engine used by aircraft model m, $X_{m,e,t}$ is a vector of controls including: cruise speed, wings aspect-ratio, minimum number of seats, cargo volume, length, take-off distance, fixed effects for number of engines and engine manufacturers.

We stress that this methodology does not depend on a specific choice of left-hand side variable. Rather, it aims to estimate the evolution of the trade-off among many characteristics, one of which is reported on the left-hand side and used to measure the units of the fixed effect. Consistent with this notion, our main findings are robust to using, for instance, aircraft weight as the product attribute on the left-hand side.

Figure 2 displays the estimated year fixed effects and highlights the evolution of embodied technology across expansions and recessions. Table 1 summarizes the main quantitative findings and compares three alternative measures of fuel efficiency as a dependent variable in equation (1).

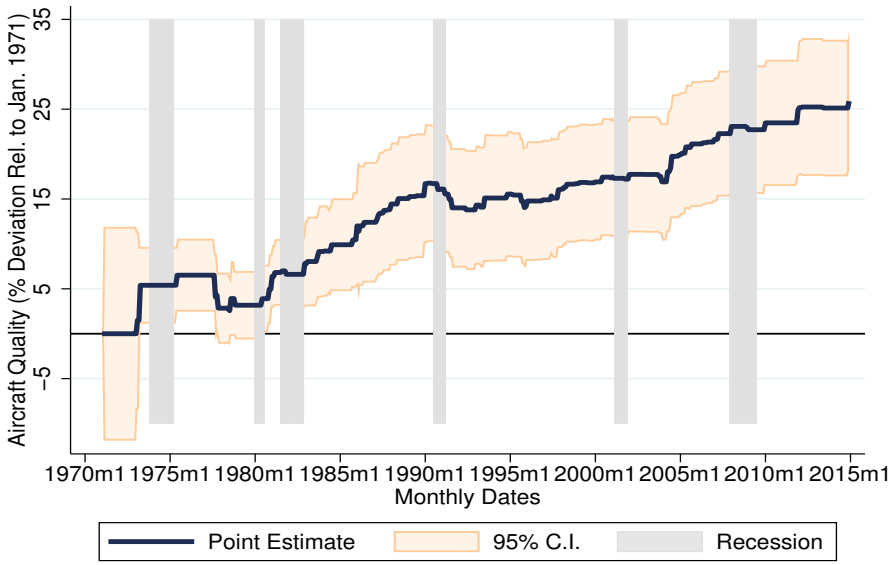
Using our baseline definition of fuel efficiency, we estimate that the quality of new aircraft grows by approximately 0.6% per year during our sample period. When we analyze the contribution of different phases of the business cycle, we find that this technological progress is entirely concentrated in expansion years. In contrast, on average there is no quality growth during recession years, as the second and third columns of the table show.

The fourth column of the table relates this finding with our observation on the cyclicality of new-product introduction. When we include in equation (1) an indicator variable for a model being new—i.e., introduced within the past 12 months—we estimate that new models feature a technological level that is on average 5% higher than previously introduced models that are still on the market. As a result, the drop in new-product introduction accounts

for the lack of embodied technological progress during recessions.

The table shows that our findings on the cyclicality of innovations and on the role of new products are robust with respect to alternative measures of aircraft fuel efficiency. Appendix A.3 provides additional details on our empirical estimates.

Figure 2: Aircraft Quality Growth



Notes: The figure plots the estimated time-fixed effects γ_t from regression (1) having as dependent variable the natural logarithm of the ratio between maximum range and fuel capacity. Estimates represent improvements in the technological quality of aircraft models available for sale during 1971-2014. The orange bands represent 95% confidence intervals based on two-way clustered standard errors by month and model-engine maker combination. Grey vertical bands represent NBER recession dates.

3.3 Evidence on Other Capital Goods

We now discuss related empirical evidence that confirms the same stylized facts on the cyclicality of new-product introduction and technological progress for two other types of capital goods.

Vehicles. Bertolotti, Gavazza, and Lanteri (2023) analyzes a large dataset on new-vehicle transactions and technical characteristics around the US Great Recession of 2008-09. The paper provides evidence that new-product introduction dropped during the recession.[7] Moreover, using a similar econometric approach to this paper, their analysis shows that the

[7] Grieco, Murry, and Yurukoglu (2023) document a pro-cyclical dynamics of new car models introduction between the 1980s and 2020s.

Table 1: Aircraft Quality Growth and Role of New Products

Measure	Quality Growth			Δ% New-Old
	All	Expansion	Recession	
ln(Range/Fuel Capacity)	0.589	0.688	0.047	4.638
	(0.090)	(0.088)	(0.202)	(1.248)
ln(1/Fuel LTO Cycle)	0.405	0.454	0.0946	3.858
	(0.083)	(0.090)	(0.141)	(1.101)
ln(Miles Flown/Fuel Cons.)	0.572	0.674	-0.086	5.613
	(0.093)	(0.105)	(0.176)	(1.284)

Notes: The table reports evidence of the procyclical nature of quality improvements for three distinct measures of aircraft efficiency: (i) fuel efficiency during cruise, computed as the aircraft maximum range divided by fuel capacity; (ii) fuel efficiency during the Landing Take-Off (LTO) cycle, computed as the inverse of total fuel consumption during the LTO cycle; (iii) an alternative measure of fuel efficiency during cruise computed as the ratio total miles flown divided by fuel consumption as compiled by the U.S. Bureau of Transportation Statistics. The first, second, and third columns report the average yearly growth rate of different quality measures in the whole sample, during NBER expansions, and during NBER recessions, respectively. Average growth rates are computed by normalizing the difference in γ_t quality estimates from regression (1) between the end and the start of the same and of each expansion and recession. The fourth column reports the average percent quality difference between newly-introduced models and previously existing models. Quality difference between new and old models is estimated by adding to specification (1) a dummy variable taking value one in the first month an order is observed for model-engine maker combination (m, e) and in the following year. Figures in parentheses report standard errors clustered by model-engine maker combination and monthly date.

quality of new models introduced during the recession was not higher than the quality of previously introduced models. Whereas new products typically push the frontier of technology embodied in vehicles, this driver of technological progress stopped during the Great Recession, leading to persistent effects on the quality of the stock of registered vehicles in the US.

Computers. We analyze a dataset assembled by Danowitz, Kelley, Mao, Stevenson, and Horowitz (2012) on product introduction and product quality in the market for CPUs (Central Processing Units) during 1985-2014. In Appendix A.5, we document that the introduction of new microprocessors and their performance is also procyclical. On average, the number of newly-introduced micro-processors increases at a fast rate of approximately 20% per year, but introduction falls markedly in recessions. In the 2001 recession, when the IT industry was directly affected by the downturn, introduction dropped by 16% relative to 2000, and in the Great Recession there was a 7% decline. Similarly, quality measures based on standardized performance tests reported in the dataset of Danowitz, Kelley, Mao, Stevenson, and Horowitz (2012) show that, on average, new CPUs are 21.5% more efficient

than those introduce in the previous year. However, this growth halts in recessions, with an approximate 20% decline during both the 2001 recession and the Great Recession.

3.4 Evidence on Prices

Our analysis of capital-embodied technological progress based on product introduction and characteristics is complementary to the literature that measures changes in prices of new investment goods (Fisher, 2006; Beaudry, Moura, and Portier, 2015) and provides quality adjustments for these price series (Gordon, 1990; Cummins and Violante, 2002). We stress that measuring technological progress starting from prices is challenging because prices are affected by both technology and markups, as will be the case in our model. Nevertheless, a comparison of PPI indices for equipment goods with the corresponding quality-adjusted versions constructed by Cummins and Violante (2002) confirms that quality adjustment leads to a significantly faster average price decline for new equipment. Furthermore, new-investment prices are typically negatively correlated with GDP (with a correlation coefficient approximately equal to -0.3 for annual HP-filtered data), consistent with procyclical embodied technological progress, although Beaudry, Moura, and Portier (2015) argues that this pattern varies across subsamples.

3.5 Discussion

The empirical evidence analyzed in this section points to a positive comovement between aggregate activity (investment) and innovation in the capital-goods sector. More specifically, we have shown that new-product introduction, which is the main engine of embodied technological progress, slows significantly during recessions.

This observation complements several related empirical findings in the literature. It is consistent with the observed procyclicality of R&D expenditures (Barlevy, 2007), particularly in the capital-goods manufacturing sector, and it complements the analysis of shocks to the efficiency of production of new capital based on the cyclical properties of quality-adjusted equipment price indices.

This empirical evidence motivates us to develop a quantitative model of investment with endogenous technological progress embodied in new capital goods. We will use the model to account for the observed comovement between investment and embodied innovation through product introduction. We will then analyze the role of these equilibrium interactions for the transmission of aggregate shocks and stimulus policies.

4 Model of Capital Replacement and Innovation

In this section, we present our model of capital replacement and innovation. On the demand side of the market for new capital, heterogeneous final-good producers subject to idiosyncratic productivity shocks choose the optimal time of capital replacement. On the supply side, capital-goods producers innovate to improve the quality of new capital. Combining these problems, we define the endogenous balanced-growth path equilibrium of the economy.

4.1 Environment

Time is discrete and infinite, $t = 0, 1, ...$ The economy is populated by a representative household with linear preferences over streams of consumption and discount factor B. The household owns a continuum of final-good producing firms indexed by $i \in \mathcal{I} \equiv [0, 1]$ and a continuum of capital-good producing firms indexed by $j \in \mathcal{J} \equiv [0, 1]$. Final-good firms produce output using capital goods. Capital-good producers' innovations determine the path of technology ("quality") embodied in capital.

4.2 Final-Good Producers: Capital Replacement

We begin by describing the capital-replacement problem of final-good producers, which builds on the model of Cooper, Haltiwanger, and Power (1999). Each final-good firm produces a homogeneous output good with a linear technology that employs capital:

$$y_{i,t} = z_t s_{i,t} K_{i,t-1}, \tag{2}$$

where z_t denotes exogenous aggregate productivity and $s_{i,t}$ denotes an idiosyncratic productivity shock.

Capital $K_{i,t-1}$ is subject to a one-period time to build and depreciates at rate δ. At each date t, firms face a discrete choice on their production capacity for the following date: They can either let their capital depreciate or they can replace it. To focus on the extensive margin of firm investment and preserve the computational tractability of the model, we assume that, upon replacement, old capital is scrapped and the level of efficiency units of new capital is common across firms. This level is given by $\kappa_t \equiv \kappa \bar{q}_t$, where $\kappa > 0$ is a parameter and $\bar{q}_t$ denotes the average quality of capital goods produced at date t.

The evolution of the capital stock is thus given by

$$K_{i,t} = \begin{cases} (1-\delta)K_{i,t-1} & \text{if } \; x_{i,t} = 0 \\ \kappa_t & \text{if } \; x_{i,t} = 1, \end{cases} \tag{3}$$

where $x_{i,t}$ is an indicator function for capital replacement.

Capital replacement requires paying a cost F_t, which we further specify below. Firms solve the following dynamic capital-replacement problem:

$$\max_{\{x_{i,t}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} B^t \left(z_t s_{i,t} K_{i,t-1} - x_{i,t} F_t \right), \tag{4}$$

subject to the law of motion for capital (3).

Upon replacement, the old capital stock is worthless due to irreversibility.[8] The new capital stock is assembled using a continuum of capital goods $j \in \mathcal{J}$. The efficiency of the bundle of capital goods depends on both the quantity $v_{i,j,t}$ and the *quality* $q_{j,t}$ of the capital goods purchased, according the following technology:

$$\kappa_t = \int_{j \in \mathcal{J}} q_{j,t}^{\gamma} v_{i,j,t}^{1-\gamma} dj, \tag{5}$$

with $\gamma \in (0,1)$. Hence, when firm i replaces its capital stock, it solves the following static expenditure minimization problem:

$$\min_{\{v_{i,j,t}\}_{j \in \mathcal{J}}} \int_{j \in \mathcal{J}} p_{j,t} v_{i,j,t} dj, \tag{6}$$

subject to the technological constraint (5).

Because all final-good firms face the same expenditure-minimization problem, the solution is independent of i and the resulting expenditure determines the capital-replacement cost: $F_t = \int_{j \in \mathcal{J}} p_{j,t} v_{j,t} dj$. We denote by λ_t the Lagrange multiplier on constraint (5), which is also common across replacing firms. The cost-minimizing quantity of variety j demanded by each replacing firm is given by

$$v_{j,t} = \lambda_t^{\frac{1}{\gamma}} (1-\gamma)^{\frac{1}{\gamma}} p_{j,t}^{-\frac{1}{\gamma}} q_{j,t}, \tag{7}$$

[8] We leave for future work the analysis of a model that combines endogenous growth in the quality of new goods, which we model in this framework, with reallocation of old and lower-quality goods in secondary markets, which Gavazza and Lanteri (2021) analyzes.

and aggregate demand for variety j is given by $M_t v_{j,t}$, where M_t is the mass of firms replacing their capital at date t: $M_t \equiv \int_{i \in \mathcal{I}} x_{i,t} di$.

4.3 Capital-Goods Producers: Innovation

Next, we describe the profit-maximization and innovation problem of capital-goods producers, which builds on a quality-ladder model of technological progress (Grossman and Helpman, 1991; Aghion and Howitt, 1992). Appendix B.1 provides additional model derivations for this block of the model.

Capital-goods producers invest to improve the quality of their products, which they sell to final-good producers. The success of their innovation effort is stochastic. Upon a successful innovation, the quality of variety j increases by an exogenous factor $\eta > 1$, $q_{j,t} = \eta q_{j,t-1}$, and capital-good producers enjoy profits from monopolistic competition for one period, before it becomes free to imitate the innovation.[9] The technology requires c units of final output to produce one unit of capital, giving the following profit-maximization problem:

$$\max_{v_{j,t}} M_t v_{j,t} \left(p_{j,t} - c \right), \tag{8}$$

subject to the demand function given by (7). The optimality condition implies that capital-goods producers charge a constant markup, $p_{j,t} = \frac{c}{1-\gamma}$, and their optimal quantity produced satisfies

$$v_{j,t} = (1-\gamma)^{\frac{2}{\gamma}} c^{-\frac{1}{\gamma}} \lambda_t^{\frac{1}{\gamma}} q_{j,t}. \tag{9}$$

Thus, profits for successful innovators are linear in quality $q_{j,t}$ and given by

$$\Pi_{j,t} = \gamma (1-\gamma)^{\frac{2-\gamma}{\gamma}} c^{\frac{\gamma-1}{\gamma}} \lambda_t^{\frac{1}{\gamma}} M_t q_{j,t}. \tag{10}$$

In contrast, when innovation is not successful, the quality of variety j remains constant, $q_{j,t} = q_{j,t-1}$, and capital-goods producers sell their output at a price equal to their marginal cost c, because their predetermined quality level can be freely imitated by a competitive fringe.

To innovate, capital-goods producers invest $R_{j,t}$ units of the final good. Critically, this cost does not depend on the volume of sales of a successful product, which gives rise to a market-size effect on innovation.

We define the innovation intensity of firm j as $\psi_{j,t} \equiv \frac{R_{j,t}}{\eta q_{j,t-1}}$. Innovation is successful

[9] In Section 8 we consider an extension of the model with multi-period protection of innovations from imitation, as well as other extensions such as additional time to build in investment and innovation lags.

with probability $\phi(\psi_{j,t})$, with $\phi(0) = 0$, $\phi' > 0$, $\phi'' < 0$, and $\lim_{\psi\to+\infty}\phi(\psi) \geq 1$. The optimal expenditure on $R\&D$ for variety j solves

$$\max_{R_{j,t}} \phi\left(\frac{R_{j,t}}{\eta q_{j,t-1}}\right)\Pi_{j,t} - \left(\frac{\bar{q}_t}{\vec{q}_t}\right)^{\chi_1} R_{j,t}, \tag{11}$$

where $\vec{q}_t \equiv (1+g)^t$ denotes average new-capital quality along a balanced-growth path (BGP) with a constant growth rate of quality equal to g. The term $\left(\frac{\bar{q}_t}{\vec{q}_t}\right)^{\chi_1}$, with $\chi_1 > 0$, scales the cost of $R\&D$ to ensure that the model dynamics are stationary around the BGP.[10]

The first-order condition of this problem implies that the optimal $R\&D$ intensity is constant across varieties and satisfies

$$\phi'(\psi_t)\gamma\left(1-\gamma\right)^{\frac{2-\gamma}{\gamma}} c^{\frac{\gamma-1}{\gamma}} \lambda_t^{\frac{1}{\gamma}} M_t = \left(\frac{\bar{q}_t}{\vec{q}_t}\right)^{\chi_1}. \tag{12}$$

As a result, the average quality of new capital, which accounts for both successful and unsuccessful innovations, evolves according to the following law of motion:

$$\begin{aligned}\bar{q}_t &= \int_{j\in\mathcal{J}} \left(\phi(\psi_t)\eta q_{j,t-1} + (1-\phi(\psi_t))q_{j,t-1}\right) dj \\ &= \bar{q}_{t-1} + \phi(\psi_t)(\eta-1)\bar{q}_{t-1}.\end{aligned} \tag{13}$$

4.4 Balanced Growth

We now focus on a balanced-growth path, along which average capital quality grows at constant rate g and at each date there is a constant mass of final-good firms M replacing their capital. On the balanced-growth path, we can formulate the capital replacement problem of final-good firms recursively as follows:

$$V(k,s) \equiv \max_{x\in\{0,1\}} sk - xf + \beta\mathbb{E}V\left[(k',s')\,|s\right], \tag{14}$$

subject to the transition

$$k' = \begin{cases} \frac{1-\delta}{1+g}k & \text{if } \ x = 0 \\ \frac{\kappa}{1+g} & \text{if } \ x = 1, \end{cases}$$

[10] Absent a response of the marginal cost of innovation to deviations of new-capital quality from its trend, any transitory aggregate shock would have permanent effects on the path of aggregate output.

where $\beta \equiv B(1+g)$ and we use lower-case variables to denote upper-case variables divided by the BGP level of capital quality $(1+g)^t$, such as $k \equiv \frac{K_{t-1}}{(1+g)^t}$ and $f \equiv \frac{F_t}{(1+g)^t}$.

This law of motion for firm capital highlights that both physical depreciation δ and economic obsolescence, through the term g, determine the evolution of the installed capital stock relative to the economy's technological trend.

A **recursive balanced-growth equilibrium** is defined as a value function V with an associated policy function for capital replacement x, a distribution of individual states m with an associated fraction of investing firms $M = \int x(k,s)dm(k,s)$, a growth rate of quality g and investment cost f, such that: The value function and policy function solve the dynamic program (14); the distribution of firms perpetuates itself given the policy function and the distribution of productivity shocks; and, given the constant fraction of investing firms M, the solution to the innovators' problem in Section 4.3 implies that the average new-capital quality (equation (13)) grows at rate g and the replacement cost equals f.

We solve for the balanced-growth path by iterating between the replacement problem and the innovation problem until convergence. Given a guessed level of growth g and replacement cost f, the stationary replacement problem delivers an optimal fraction of replacing firms M for which $x = 1$. In turn, given a mass of replacers M, the innovation problem of capital producers gives the optimal innovation intensity, which in turn determines the fraction of successful innovations, and thus the replacement cost and the growth rate of average quality from equation (13). Appendix B.2 provides details on the solution algorithm.

5 Calibration and Balanced-Growth Path

In this section, we discuss how we quantify our model. We perform two alternative calibrations of the balanced-growth path. As a baseline, we focus on a macro calibration, which targets broad empirical evidence on firm investment and innovation. We use this calibration to assess the macroeconomic implications of endogenous capital-embodied innovation. We then perform an aircraft-specific calibration that leverages more directly our empirical evidence on technological progress embodied in aircraft.

5.1 Calibration

We now describe our choices of functional forms and parameter values, which we report in Table 2.

Macro calibration. We calibrate the capital-replacement problem using a standard strategy and parameter values—e.g., for discount factor and depreciation rate—commonly used in the quantitative literature on investment dynamics with heterogeneous firms. We assume that the idiosyncratic productivity process is AR(1) in logs, with autocorrelation ρ_s and standard deviation of innovations σ_ϵ. We then assign parameter values in the range of existing estimates in the literature (e.g., Khan and Thomas, 2013; Winberry, 2021) and obtain that the constant fraction M of final-good producers replacing their capital stock on the balanced-growth path equals 20%, consistent with the micro empirical evidence of Cooper and Haltiwanger (2006) on the frequency of large investment episodes (spikes).[11]

We set the scale of new capital purchases κ to obtain that the ratio of aggregate investment expenditures ($F_t M_t$) to aggregate output is approximately equal to 0.2 on the balanced-growth path.

We set the marginal cost of producing new capital c consistent with the standard assumption in one-sector macroeconomic models—i.e., one unit of output is needed to produce one unit of new investment goods. Next, we assume that the innovation probability function takes the form $\phi(\psi) \equiv \phi_0 \left(1 - e^{-\phi_1 \psi}\right)$, with $\phi_0 \in [0, 1]$ denoting the upper bound on the innovation probability and $\phi_1 > 0$ governing the curvature of the function.

We then set the four parameters of the innovation problem, namely ϕ_0, ϕ_1, η, and γ to jointly match four empirical targets. Specifically, we target: a growth rate equal to 2%; an elasticity of innovation with respect to R&D intensity equal to 0.9, which is the mid-range of the estimates in Dechezleprêtre, Einiö, Martin, Nguyen, and Van Reenen (2023); an average share of new products—i.e., successful innovations—approximately equal to 15%, consistent with our estimates on the frequency of product innovations in aircraft and vehicles; a gap between the growth rate of quality-adjusted and that of quality-unadjusted new-equipment prices approximately equal to 3%, consistent with the estimates of Gordon (1990).[12] As a baseline calibration, we thus parameterize the innovation side of the model targeting aggregate evidence on growth rates and R&D.

We note that our calibration strategy implies that our model is also broadly consistent with the following empirical moments. The output share of R&D expenditures by capital-

[11] If we interpret the capital varieties j of the model as attributes of a single type of capital with quality $\bar{q}_t$, firms in the model can only invest in the latest vintage. Hence, in mapping the model to the data we implicitly assume that investment spikes in the data are instances of adoption of the latest vintage. To alleviate the concern that spikes may also involve purchases of old capital, we stress that one can also interpret capital varieties j as different types of capital, each of which has its own vintage, corresponding to the date of the last successful innovation for that variety. Under this interpretation, capital replacement involves purchases of capital of different vintages, with an overall average quality $\bar{q}_t$.

[12] We provide additional details on the quality-adjusted price measure in our model in the next subsection.

goods manufacturers equals 0.5% in our model and 0.7% in the data. The quality step size of innovations η is similar to the estimates of Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018) and in the range of estimates based on the evidence of Section 3, namely higher than the average innovation in aircraft and lower than the average innovation in computers. Our chosen value for γ produces a markup of 50% for monopolistic varieties (Broda and Weinstein, 2006) and an average markup in the economy of 11% (Foster, Haltiwanger, and Tuttle, 2022).

Aircraft-specific calibration. To compare more directly our quantitative model with our empirical findings, we perform a separate calibration and target the empirical evidence on quality dynamics for aircraft. Specifically, we set the step-size of innovation to match the observed quality gap between new and continuing aircraft models based on our preferred measure of fuel efficiency. We jointly calibrate the parameters of the innovation probability function and the weight of quality in the capital bundle to match three empirical targets: an average quality growth rate equal to 0.6%, as in our empirical estimates; an elasticity of innovation with respect to R&D intensity equal to 0.9, as in the macro calibration; and an average markup rate equal to 7.5%, consistent with empirical estimates for the aircraft manufacturing industry (Irwin and Pavcnik, 2004). We then set the rate of physical depreciation so that the combination of physical depreciation and obsolescence match the Bureau of Economics Analysis estimates of economic depreciation for aircraft. Finally, we calibrate the investment scale parameter to match an average life of aircraft in operation equal to 10.8 years (Gavazza, 2011).[13] In Section 6.5 we provide the quantitative results for this calibration and in Section 8 we extend this analysis to allow for innovation lags.

5.2 Complementarities and Endogenous Growth

We now use the quantitative model to investigate the role of complementarities between capital replacement and embodied innovation for the long-run evolution of the economy. To this end, we focus on our macro calibration.

Figure 3 illustrates the two equilibrium mappings that determine the mass of firms replacing their capital M (which we will also occasionally refer to as the replacement rate) and the growth rate of capital quality g on the balanced-growth path equilibrium. Specifically, the solid line represents the demand for new capital—i.e., the fraction of firms

[13] We also verify that our main results are consistent to an alternative calibration of the AR(1) process for idiosyncratic productivity shocks s. Specifically, we verify that we obtain similar results when we match the persistence and standard deviation implied by the stochastic process for airlines productivity shocks used in Gavazza (2011).

Table 2: Parameters Values

	Parameter	Symbol	Baseline	Aircraft
Capital Replacement	Discount Factor	B	0.96	0.96
	Physical Depreciation	δ	0.08	0.06
	Persistence of s Shocks	ρ_s	0.9	0.9
	St. Dev. of s Shocks	σ_ε	0.12	0.12
	Investment Scale	κ	0.8	13.25
	Aggregate Productivity	z	1	1
Innovation	Maximum Innovation Prob.	ϕ_0	0.55	0.55
	Curvature of Innov. Prob.	ϕ_1	191	4.21
	Innovation Step Size	η	1.135	1.0464
	Quality Weight	γ	0.345	0.373
	Marginal Cost	c	1	1

Notes: The table reports the parameter values used in the analysis.

that optimally choose to replace their capital as a function of quality growth. As the quality of new capital grows at a faster rate, the effective depreciation of old capital due to obsolescence becomes more rapid, inducing more firms to replace. Therefore, aggregate demand for new capital is upward sloping in the growth rate g.[14]

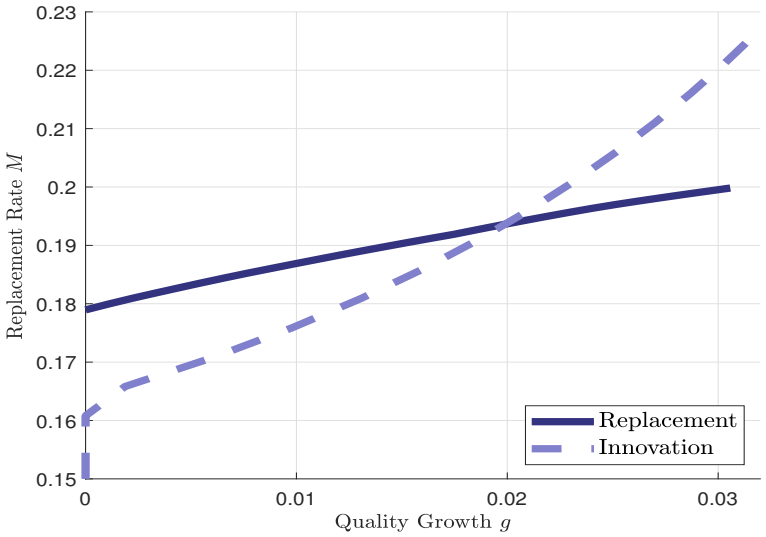
The dashed line refers to the aggregate supply of new-capital quality. It displays the growth rate of capital quality g as a function of the mass of replacing firms M. As more firms decide to replace their capital, total profits for innovators increase due to a market-size effect. Hence, R&D intensity, innovation, and capital quality growth increase, which accounts for the positive slope of the curve in the figure. Moreover, we find that the model features a region where no R&D and growth occur, when the mass of replacing firms is sufficiently small.

The result that both equilibrium mappings are upward sloping highlights the complementarity between demand and supply forces in the determination of the long-run technological path of the economy. In our calibration, the BGP of the model features an equilibrium quality growth of approximately 2% per year and a share of replacing firms around 0.19. At this point, the two equilibrium mappings described above intersect. Furthermore, as the figure shows, the calibrated model suggests that in the (g, M) space the

[14]To generate the solid line in Figure 3, we vary innovation intensity ψ, which determines both quality growth g (on the x-axis) and the replacement cost f, through changes in the average markup on new capital. While the solid line in the figure summarizes the overall effect of innovation on the replacement frequency, in Appendix B.3 we provide an additional decomposition of the separate effects of quality growth and prices on M.

supply-side mapping is steeper. Quantitatively, a counterfactual higher replacement rate of 22% would induce a 3% growth rate through the market-size effect on innovation. However, a counterfactual 3% growth rate would only lead to a 20% replacement rate through its effect on capital obsolescence.

Figure 3: Growth and Replacement in the Balanced-Growth Path



Notes: The figure provides a graphical representation of the two equations that determine the balanced-growth path (BGP) equilibrium of the economy. The solid line represents the aggregate demand for new capital—i.e., the mass M of firms that replace their capital stock as a function of the growth rate g of new capital quality. The dashed line represents the aggregate supply of capital quality—i.e., the growth rate g as a function of M. The BGP equilibrium is identified by the pair (g, M) at the intersection of these two mappings.

Furthermore, in the BGP, the quality-adjusted price index of new capital declines at constant rate, consistently with the literature on investment-specific technical change. To compute the quality-adjusted price index, we consider the cost of replacing the capital stock by purchasing a capital bundle with a constant amount of efficiency units $\kappa_t = \kappa$. This counterfactual replacement cost evolves endogenously at a rate that depends negatively on the growth rate of quality g. Appendix B.4 derives our model counterpart to the quality-adjusted price of new capital.

In our calibration, the quality-adjusted price index declines by approximately 1% per year, in contrast with the unadjusted price F_t, which grows at $g = 2\%$. Quantitatively, this difference is consistent with the findings of Gordon (1990), which documents a 3% difference between the growth rates of unadjusted and quality-adjusted price indices of equipment and durable goods.

6 Macroeconomic Dynamics and Capital Quality

In this section, we analyze the response of the quantitative model to aggregate shocks and show that the model accounts for the comovement of investment and embodied innovation. We provide several decompositions that highlight the role of endogenous capital quality for the propagation of macroeconomic shocks and the dynamics of aggregate productivity. Appendix B.2 discusses our solution algorithm for the equilibrium transitional dynamics.

6.1 Aggregate Productivity Shock

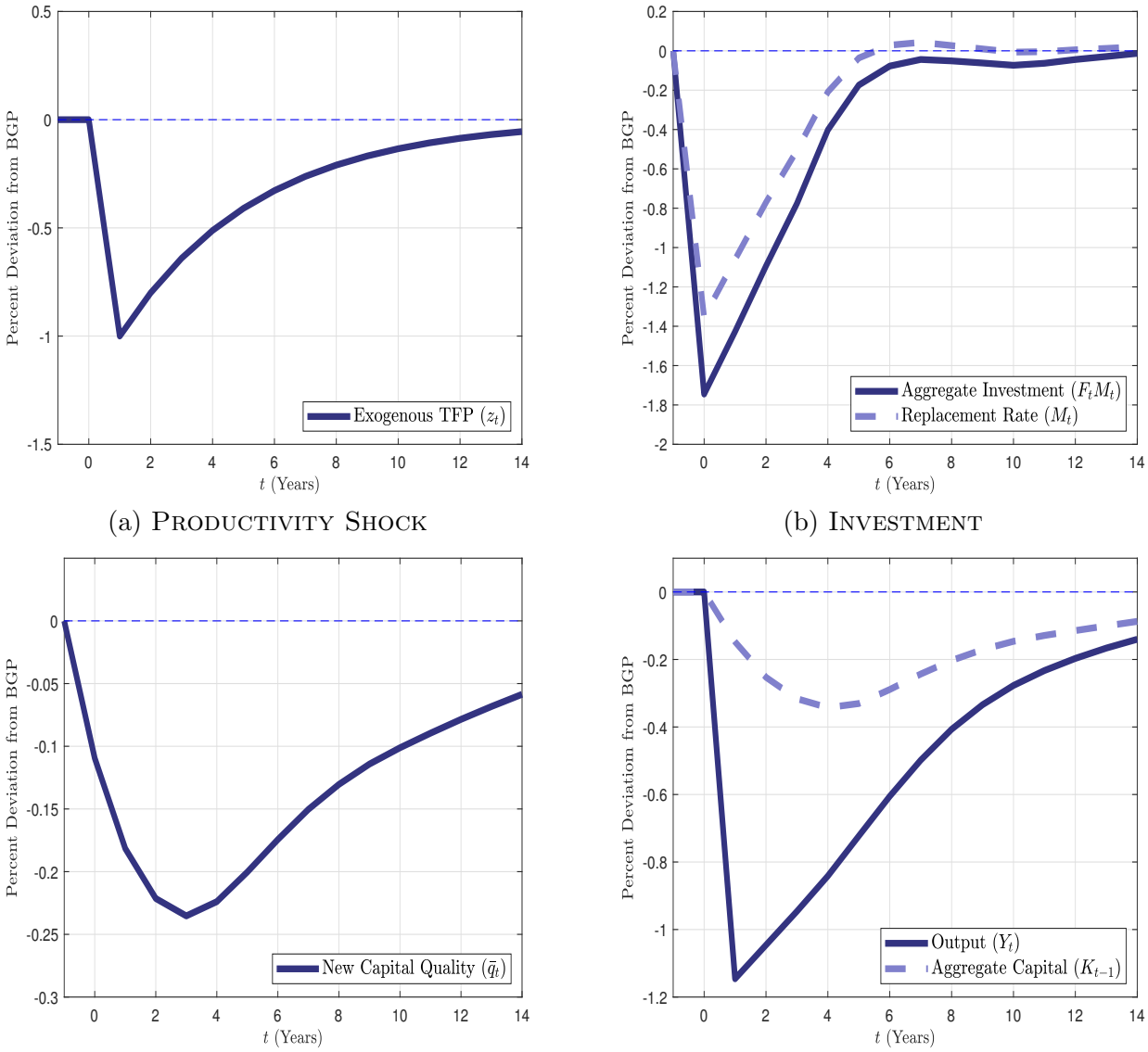
We begin by considering an unexpected negative aggregate productivity shock. We assume that the economy is on its BGP until $t = 0$, at which point firms learn that at $t = 1$ the exogenous component of aggregate total-factor productivity (TFP) z will be 1% lower than its permanent long-run value; after that, z_t gradually reverts to its permanent value.

We parameterize the speed of mean reversion of the shock so that measured TFP, which we discuss in more detail later, has an autocorrelation of approximately 0.85 in logs, close to common calibrations in the literature. Furthermore, to obtain that all aggregate variables revert to trend, we set the elasticity of the marginal cost of innovation with respect to deviations of capital quality from trend, $\chi_1 = 1.5$, which implies that the coefficient of autocorrelation of output in response to the shock equals 0.86, similar to our target for measured TFP.

Figure 4 illustrates the aggregate response of the economy to this shock. Each endogenous variable is displayed in deviation from its BGP trend. The top-left panel displays the path of the shock z_t. The top-right panel shows that the shock induces a drop in investment activity. Both the fraction of firms replacing their capital and aggregate capital expenditures decline and then gradually recover. On impact, aggregate investment drops by approximately 2% (relative to its trend).

Because of this decline in investment activity, incentives for capital-embodied innovation decrease. Thus, the quality of new capital declines persistently, as the bottom-left panel illustrates. In turn, low investment and low capital quality contribute to a recession. Accordingly, the bottom-right panel shows that the aggregate quality-adjusted capital stock and aggregate output display a persistent recession. Output drops by over 1% and the capital stock reaches a trough of approximately -0.35% relative to trend, a few years after the shock. Overall, the model produces a comovement between aggregate investment and new-capital quality, consistent with our empirical findings.

Figure 4: Aggregate Dynamics in Response to a Negative TFP Shock



(a) Productivity Shock

(b) Investment

(c) New-Capital Quality

(d) Output and Capital

Notes: The figure illustrates the response of the economy to a persistent 1% decline in exogenous TFP z_t at time $t = 1$, which agents learn at $t = 0$. Specifically, the panels display: (a) the exogenous path imposed to z_t; (b) the measure M_t of final-good producers replacing their capital and gross investment $F_t M_t$ in percent deviation from the BGP; (c) new capital quality $\bar{q}_t$ in percent deviation from the BGP $\vec{q}_t$; (d) aggregate final output Y_t and aggregate quality-adjusted capital K_{t-1} in percent deviation from the BGP. We assume that the economy is on the BGP at time $t = -1$, that all agents learn about the shock at $t = 0$, and that they have perfect foresight of the evolution of all exogenous and endogenous variables.

6.2 Capital Quality and Measured Aggregate Productivity

We now investigate the role of endogenous capital quality for the dynamics of aggregate productivity in response to the shock. To this end, we measure the aggregate capital stock of the economy assuming that only physical units of capital are observable for each firm, that is, firm capital is $\kappa, (1-\delta)\kappa, (1-\delta)^2\kappa$, depending on the date of the last replacement.[15] In contrast, we assume that the corresponding capital efficiency level, $\bar{q}_{t-1}, \bar{q}_{t-2}, \bar{q}_{t-3}$ is not observed. We define measured aggregate TFP as the ratio of aggregate output and this measure of aggregate physical capital, which is not corrected for quality. Thus, measured TFP grows at rate g on the BGP, but deviates from this growth rate in response to aggregate shocks, as both (i) new-capital quality and (ii) the distribution of firms over capital vintages evolve over time.

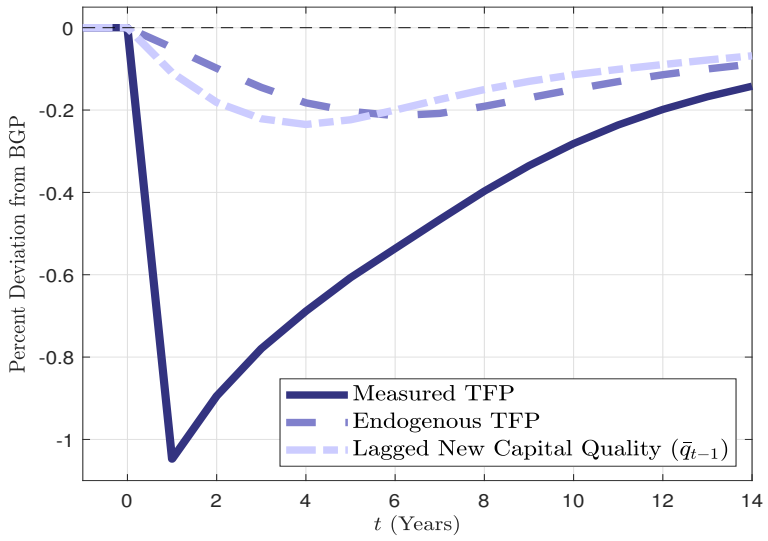
Figure 5 displays the dynamics of measured aggregate TFP (solid line) relative to its trend in response to the negative shock to z_t. Moreover, the figure illustrates a decomposition that highlights the role of capital quality. The dashed line denotes the endogenous component of measured TFP, eliminating the mechanic effect of the the exogenous dynamics of z_t; the dashed-dotted line reproduces the quality of new capital $\bar{q}_t$ illustrated in the bottom-left panel of Figure 4.

The figure highlights that in the immediate aftermath of the shock, measured TFP depends largely on the exogenous component z_t. However, over time, the quality of new capital $\bar{q}_t$ drops persistently. While new vintages of capital embody such low quality, all replacing firms invest in them and the low-quality capital they acquire remains used in production until the same firms replace their capital again. As a result, the average quality of capital in production declines persistently because of the lumpy nature of firm investment.

This mechanism depresses the endogenous component of aggregate TFP, which follows the dynamics of new-capital quality with a lag. As the exogenous component of measured productivity z_t recovers, the endogenous component contributes to an increasing fraction of the gap between measured TFP and its trend. In particular, eight years after the initial shock, this fraction is approximately one half, suggesting that endogenous capital-quality and vintage-distribution dynamics approximately double the medium-run effects of the shock on measured TFP.

[15] Formally, let $h = 1, .., H$ denote the vintage of capital, or distance in time between the current date and the last replacement decision. With a slight abuse of notation, let $m_t(h, s)$ denote the joint distribution of capital vintage and idiosyncratic productivity at date t. We compute aggregate physical capital at date t as $\sum_s \sum_h m_t(h, s)\kappa(1-\delta)^{h-1}$.

Figure 5: Capital Quality and Measured TFP



Notes: The figure illustrates the response of the economy to a persistent 1% decline in exogenous TFP z_t at time $t = 1$, which agents learn at $t = 0$. The figure compares measured TFP (solid line) computed as aggregate output Y_t divided by aggregate capital in physical units against its endogenous component (dashed line), computed as aggregate output divided by exogenous TFP z_t and aggregate capital in physical units. The figure also overlays lagged new capital quality. All variables are plotted in percent deviation from BGP. We assume that the economy is on the BGP at time $t = -1$, that all agents learn about the shock at $t = 0$, and that they have perfect foresight of the evolution of all exogenous and endogenous variables.

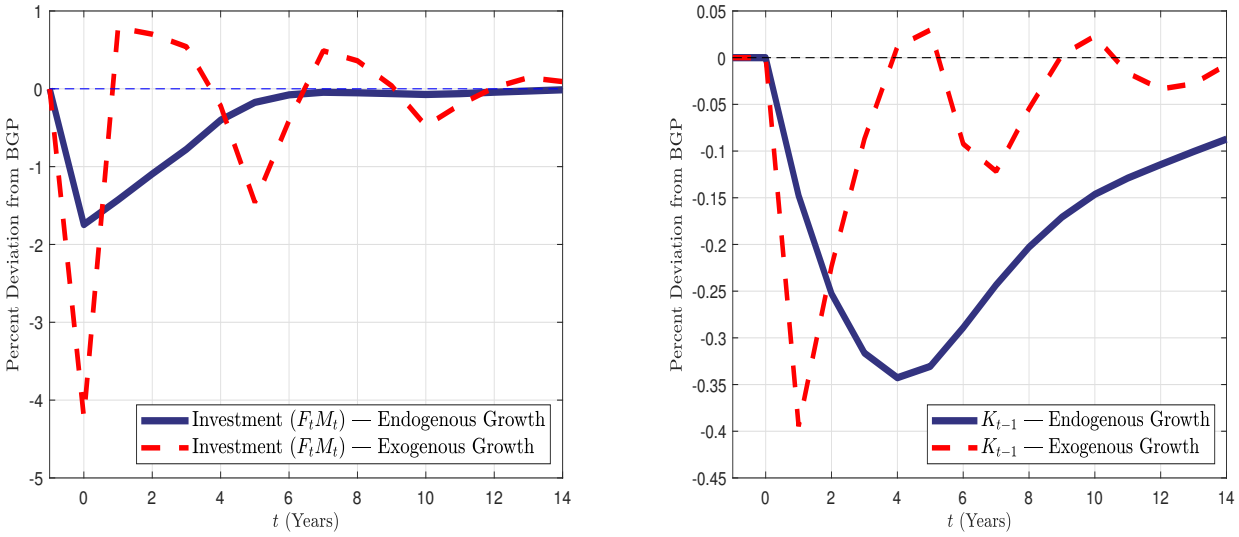
6.3 Roles of Quality and Prices

To further investigate the role of endogenous capital quality for the propagation of the productivity shock, we now compare the dynamics in our model (solid lines) with those in a counterfactual economy in which the growth rate of capital quality is constant and exogenously set to match the BGP growth rate in our calibration. Figure 6 shows that endogenous capital quality plays a crucial role in propagating the effects of the productivity shock over time in our model (solid lines). The initial investment drop induces a decline in the quality of new capital because of the market-size effect. In turn, a persistently low quality of new capital reduces the incentive to invest in new capital for several years, while the economy gradually recovers.

Consistent with the critical role of this feedback between capital demand and supply for the shock propagation, we find that the same shock generates short-lived effects on investment in the counterfactual model with exogenous quality growth (dashed lines). Moreover, the initial drop in investment is quickly reversed as firms that do not replace their capital initially have an incentive to do so within few years. Overall, the aggregate capital stock

recovers quickly when quality is exogenous. These high-frequency endogenous cycles in replacement are consistent with the findings of Cooper, Haltiwanger, and Power (1999), which features exogenous obsolescence as our counterfactual economy, and more broadly with a high intertemporal elasticity of investment featured in most models with non-convex adjustment costs.

Figure 6: Role of Endogenous Capital Quality for Propagation



(a) Investment (b) Aggregate Capital

Notes: The figure illustrates the response of the economy to a persistent 1% decline in exogenous TFP z_t at time $t = 1$, which agents learn at $t = 0$. Specifically, the panels display: (a) gross investment F_tM_t in percent deviation from the BGP; (b) aggregate quality-adjusted capital K_{t-1} in percent deviation from the BGP. Blue solid lines refer to our baseline model with endogenous capital quality and red dashed lines refer to a counterfactual economy in which capital quality grows exogenously at a constant rate equal to the BGP equilibrium growth rate of capital of our model. We assume that the economy is on the BGP at time $t = -1$, that all agents learn about the shock at $t = 0$, and that they have perfect foresight of the evolution of all exogenous and endogenous variables.

Furthermore, we find that in the counterfactual model with constant growth in capital quality, the initial drop in the replacement rate is larger than in our model. This difference is due to the dampening effect of the endogenous cost of new capital F_t in our model. The decrease in innovation when the negative shock hits implies that fewer varieties of new capital are sold at a markup over their marginal cost. The associated decline in the replacement cost dampens the negative effect of the productivity shock on investment.[16]

[16]In Appendix B.5 we also consider the opposite counterfactual scenario in which the replacement cost F_t does not deviate from its trend, whereas new-capital quality evolves as in our baseline scenario. In this case, the productivity shock has an effect on investment that is approximately twice as large as in

This general-equilibrium mechanism contributes to dampen the contemporaneous response of aggregate investment to macroeconomic shocks, similar to other general-equilibrium effects highlighted in the literature, such as interest-rate changes, or other mechanism that resemble convex adjustment costs at the aggregate level.

In Appendix B.5, we compare the dynamics of the equilibrium replacement cost and of the quality-adjusted replacement cost. The decline in quality during the recession accounts for a decrease in the average markup as well as a decline in average quality. The markup channel accounts for an initial drop in both series, whereas the two series depart from each other in the medium run as capital becomes more expensive in quality-adjusted terms.[17]

6.4 Interest-rate Shock

Our findings on the comovement between investment and capital-embodied innovation do not depend on a specific source of business cycles. To show this, we now consider a different source of aggregate dynamics, namely an exogenous increase in the discount factor that final-good producers use to discount profits. While the model is entirely real, this analysis is informative to approximate the effects on investment of a monetary-policy shock in an economy with nominal rigidities.

Figure 7 displays the results associated with a 1% increase in the real interest rate $r \equiv B^{-1} - 1$, which then gradually returns to its BGP value (top-left panel). The higher discount factor depresses aggregate investment, by approximately 4% (top-right panel). In turn, lower demand for new capital leads to low innovation and low quality of new-capital vintages, which drops by approximately 0.5% relative to its trend. Consistent with the analyses of the previous subsections, low capital quality leads over time to a persistent decrease in measured aggregate TFP as more and more firms replace their capital (bottom-left panel) and contributes to a prolonged recession (bottom-right panel).

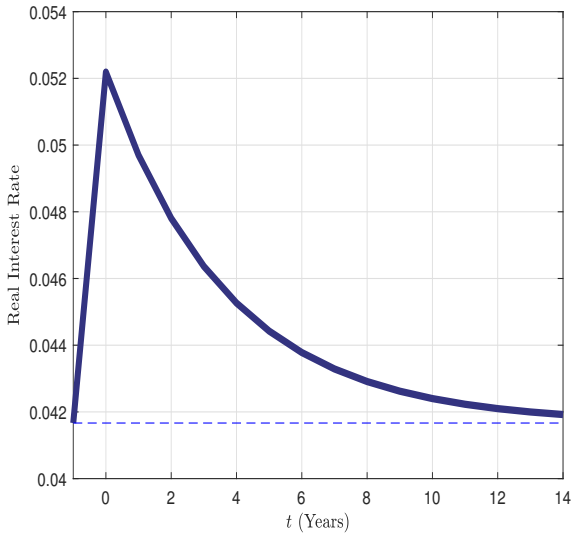
Overall, because of the endogenous dynamics of capital quality, the transmission of the interest-rate shock is broadly consistent with recent empirical evidence that monetary-policy shocks may have persistent effects on capital accumulation and productivity (Jòrda, Singh, and Taylor, 2023). Thus, our analysis provides a new mechanism for supply-side effects of monetary-policy changes, complementary to the one proposed by Baqaee, Fahri, and Sangani (2023), which focuses on reallocation of production across producers with

the equilibrium case, which confirms the important role of new-capital quality for investment and the dampening effect of the replacement cost F_t.

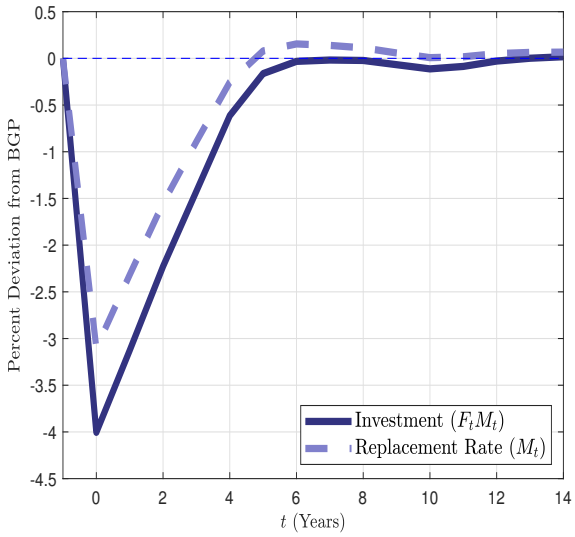
[17] Consistent with the analysis of Hobijn (2001), the price of new capital reflects both new-capital quality and markups in the presence of imperfect competition.

heterogeneous markups.

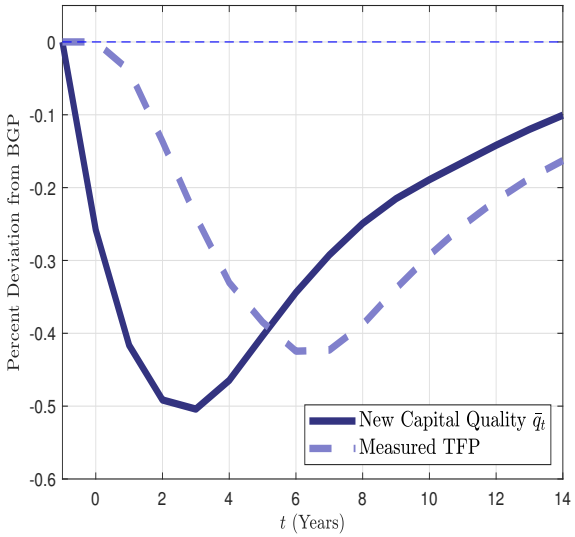
Figure 7: Aggregate Dynamics in Response to a Real Interest Rate Shock



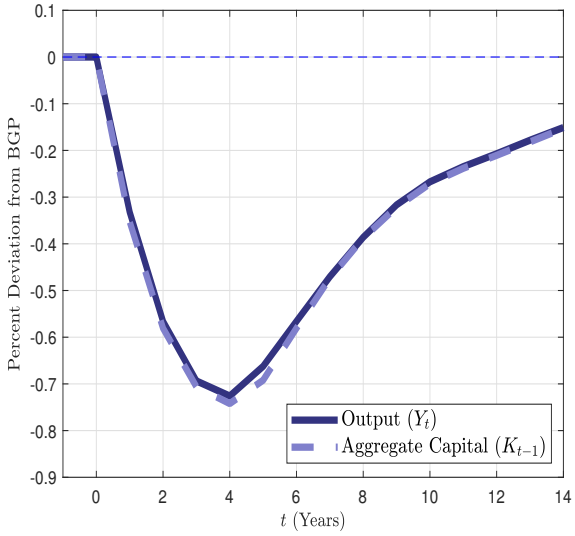
(a) Real Interest Rate



(b) Investment



(c) New-Capital Quality and Measured TFP



(d) Output and Capital Stock

Notes: The figure illustrates the response of the economy to an unexpected and persistent 1% increase in the real interest rate $r = B^{-1} - 1$ at time $t = 0$. Specifically, the panels display: (a) the exogenous path imposed to r; (b) the measure M_t of final-good producers replacing their capital and gross investment $F_t M_t$ in percent deviation from the BGP; (c) new capital quality $\bar{q}_t$ in percent deviation from the BGP $\vec{q}_t$ and measured endogenous TFP, computed as final output divided by z_t and aggregate capital in physical units, in percent deviation from the BGP; and (c) aggregate quality-adjusted capital K_{t-1} and aggregate output Y_t in percent deviation from the BGP. We assume that the economy is on the BGP at time $t = -1$, that all agents learn about the shock at $t = 0$, and that they have perfect foresight of the evolution of all exogenous and endogenous variables.

6.5 Capital Quality Dynamics in the Aircraft Calibration

To compare model results and empirical evidence more directly, we now analyze the impact of a negative productivity shock on capital quality in the aircraft-specific calibration. We set the size of the shock to induce a decline in capital replacement equal to the observed percentage difference in orders across expansions and recessions, which we infer from Boeing orders data over the period 1971-2014.[18] This strategy implies a negative 16.5% shock, consistent with high cylicality of this industry, and we maintain the same shock persistence as in the baseline analysis.

Figure 8 depicts the dynamics of investment and new-capital quality in this alternative scenario. Due to the large drop in replacement and expected profits from innovation, quality growth completely halts for one period. Remarkably, this result is quantitatively consistent with our empirical estimate of average aircraft quality growth during recessions. As a consequence, new-capital quality declines by 0.6% relative to its balanced-growth path on impact. Overall, the model accounts quantitatively for the comovement of investment and capital-embodied innovation.

Furthermore, replacement and growth remain subdued for some time after the shock, but in the subsequent recovery quality growth accelerates averaging 0.67%—close to our empirical estimate of quality growth during expansions. In Section 8 we leverage again the aircraft-specific calibration to analyze the role of innovation lags.

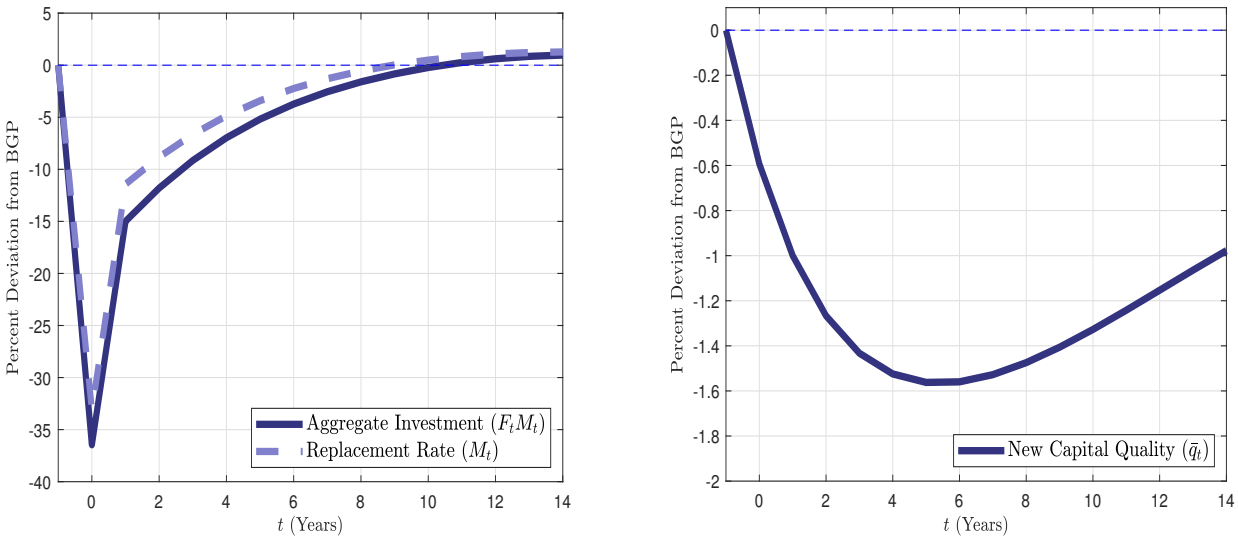
7 Efficiency and Policy Interventions

Having established the positive properties of our model, in this section we turn to a policy analysis. We first leverage a planning formulation of our model to analyze the efficiency of the equilibrium. We then analyze the role of endogenous capital quality for the transmission of policy interventions aimed at stimulating investment and capital-embodied innovation.

In our positive analysis, because of linear preferences of the representative household, it was immaterial for the allocation whether final-good firms and capital-goods firms were all located in the same closed economy, or whether some capital goods were imported. However, in modelling a planner that can affect both demand and supply forces in the model, we now assume that the economy is closed (or, alternatively, there is a global policymaker).

[18] We restrict our attention to the same sample period of our quality growth estimates. During expansions, the average number of new aircraft orders is 49.1 per month. During recessions, this figure drops to 33.4, corresponding to an approximate -32% difference.

Figure 8: Aggregate Dynamics in Response to a Negative TFP Shock — Aircraft-Specific Calibration



(a) Investment (b) New-Capital Quality

Notes: The figure illustrates the response of the economy to a persistent decline in exogenous TFP z_t at time $t = 1$, which agents learn at $t = 0$, under our aircraft-specific calibration. Specifically, the panels display: (a) the measure M_t of final-good producers replacing their capital and gross investment F_tM_t in percent deviation from the BGP; (b) new capital quality $\bar{q}_t$ in percent deviation from the BGP $\vec{q}_t$. We assume that the economy is on the BGP at time $t = -1$, that all agents learn about the shock at $t = 0$, and that they have perfect foresight of the evolution of all exogenous and endogenous variables.

7.1 Efficient Capital-Embodied Innovation

The equilibrium of our model is inefficient because of the presence of externalities in innovation and markups on capital goods. First, capital-good producers do not fully internalize the social benefit of technological progress. When their innovations are successful, they obtain monopoly profits for one period, after which there is free imitation. However, the social value of these innovations is positive even after imitation occurs because future innovations build upon frontier capital quality.[19]

Second, the aggregate intensity of innovation affects aggregate capital replacement, which determines the level of profits for each successful innovator. In turn, profits of successful innovators generate incentives for innovation. Each producer j does not internalize the effect of individual innovation on other producers' profits. Finally, varieties with a successful innovation are sold at a markup over marginal cost, inducing a wedge in the cost

[19] In the extended version of the model with multi-period profits (Section 8), there is also a "business-stealing" externality, because innovation may erode pre-existing monopoly rents.

of capital replacement.

In contrast to these sources of inefficiency on the supply side of the model, conditional on a path of quality and replacement costs, the capital-replacement decision of final-good producers is efficient, in the sense that it maximizes household utility. To analyze the efficient allocation in our model, we thus replace the innovation block of the model with the solution to a planning problem.

A social planner chooses innovation effort on behalf of capital-good producers to maximize household utility $\sum_{t=0}^{\infty} B^t C_t$.[20] The planning problem is subject to the output-good resource constraint,

$$C_t = z_t \kappa \sum_{h=1}^{\infty} m_t(h,s) s \bar{q}_{t-h} (1-\delta)^{h-1} - M_t F_t^* - \left(\frac{\bar{q}_t}{\vec{q}_t}\right)^{\chi_1} \psi_t \eta \bar{q}_{t-1}, \tag{15}$$

which states that consumption equals aggregate output net of capital investment—evaluated at the marginal cost of capital replacement $F_t^* \equiv c\kappa^{\frac{1}{1-\gamma}} \overline{q}_t$—and R&D expenditures.

Furthermore, the planner's allocation is constrained by the law of motion for the average quality of new capital,

$$\bar{q}_t = Q(\bar{q}_{t-1}, \psi_t) \equiv (1 + \phi(\psi_t)(\eta - 1))\bar{q}_{t-1}, \tag{16}$$

which relates new-capital quality to past quality and innovation intensity. We denote by θ_t the Lagrange multiplier associated with this law of motion.

We maintain our assumptions about the equilibrium determination of capital replacement and the evolution of the distribution of firms, given paths for innovation and replacement costs. Because the demand side of the model does not feature inefficiencies, the resulting dynamics are the same as if the planner explicitly chose both innovation and capital replacement.

The optimality conditions that determine the planner's evolution of capital quality are:

$$\theta_t Q_{\psi,t} = \eta \left(\frac{\bar{q}_t}{\vec{q}_t}\right)^{\chi_1} \bar{q}_{t-1} \tag{17}$$

$$\theta_t = B\theta_{t+1} Q_{q,t+1} + \frac{\partial \sum_{s=0}^{\infty} B^s C_{t+s}}{\partial \bar{q}_t}. \tag{18}$$

The optimality condition (17) equates the marginal benefit and marginal cost of innovation

[20] We abstract from other sources of social preferences for capital quality, such as environmental externalities induced by the use of old capital. Adding such externalities to the model may strengthen the incentives for the planner to stimulate innovation.

intensity. The marginal benefit is the product of the shadow value of new-capital quality θ_t and the marginal efficiency of innovation $Q_{\psi,t}$. The marginal cost of innovation effort is due to the associated reduction in output available for consumption.

In turn, equation (18) is an Euler equation for the shadow value of new-capital quality. At the social optimum, the marginal value of capital quality equals the sum of two terms: (i) the value of the marginal contribution of current quality to future quality and (ii) the consumption effect of quality, that is, the effect of current capital quality on the the present discounted value of resources available for consumption.

Quantitatively, focusing on our macro calibration, we find that the planner's internalization of the future social value of capital quality leads to a high growth rate of output. In order to obtain a bounded present discounted value of consumption under the planner's solution, we need to reduce the discount factor B to 0.91. In this case, focusing on balanced growth paths, the socially optimal growth rate is approximately equal to 7%, whereas the decentralized-equilibrium growth rate is less than 1%.

We then use the planning formulation to assess the efficiency of the comovement between investment and innovation. We consider again a negative TFP shock as in Section 6.1. Overall, we find that the complementarity between the timing of physical investment and embodied innovation is present also in the planning problem. However, quantitatively, most of the equilibrium fluctuations in innovation are inefficient and driven by the pursuit of short-run monopoly profits. Under the planner's solution, the decline in new-capital quality in the recession is two orders of magnitude smaller than in the decentralized equilibrium.

7.2 Policies for Long-run Growth

Next, we use our model to analyze the effects of stimulus policy interventions that are commonly used by many governments. We begin this analysis by focusing on the long-run effects of two policies that stimulate output growth. Specifically, we contrast two alternative permanent interventions in our macro calibration: a proportional subsidy to R&D expenditures $\tau^{R\&D} = 1\%$ and a proportional subsidy to capital replacement $\tau^{I} = 1\%$. In the first scenario, capital-goods producer j faces a subsidized R&D cost $R_{j,t}(1-\tau^{R\&D})$. In the second scenario, final-good producers face a subsidized replacement cost $F_t(1-\tau^{I})$. Both subsidies are financed with lump-sum taxes on households.

Because capital quality is endogenous in our model, both of these policies have a positive effect on output growth and welfare—i.e., household utility. The capital-replacement subsidies stimulates investment demand, which in turn expands the market size for innovators,

resulting in faster growth. The innovation subsidy directly stimulates R&D intensity.

Although both policies stimulate innovation, we find that they generate qualitatively different transitional dynamics after their implementation and that their aggregate effects display important quantitative differences in the long run.

To simulate the effects of these policies, we assume that they are implemented starting from the undistorted BGP equilibrium and we compute their effects along the transition and in the new long-run BGP that they induce. Table 3 reports the effects of the subsidies on output and consumption both in the short and in the long run. The R&D subsidy induces larger welfare gains in the long run—and especially so when the size of the government budget is assumed to be the same across the two policies.

The reasons for this finding are that R&D is a significantly smaller fraction of aggregate output than investment and moreover the investment subsidy transfers resources to many inframarginal firms that would replace their capital even absent the policy.

In contrast, the investment subsidy is more successful at stimulating output in the short run. Figure 9 illustrates this finding by displaying the short-run dynamics induced by the subsidies and comparing them with the undistorted competitive equilibrium. In the first five years after the implementation of the policies, output is higher when the stimulus is directed at capital replacement. After that, the output path is higher under the R&D subsidy.

Whereas capital-replacement subsidies reduce the effective replacement cost on impact, and thus increase investment and output in the short term, R&D subsidies *increase* the replacement cost because a larger fraction of capital varieties experience innovations and are sold at a markup. Over time, however, imitation erodes these markups.[21] Furthermore, R&D subsidies generate stronger expectations of cumulative growth in future capital quality. For all of these reasons, final-good firms have an incentive to postpone replacement and wait for higher capital quality few years after the implementation of the innovation subsidy.

Overall, these findings highlight that our analysis of quality and price effects in response to macroeconomic shocks (Section 6) is critical to understand the dynamic effects of policy interventions that stimulate capital-embodied innovation.

Finally, we also find that R&D subsidies are overall more efficient in terms of fiscal burden for a given desired growth target, because R&D is a smaller component of output

[21] This feature of the model is consistent with empirical evidence on the life-cycle of the price of durable goods. Copeland, Dunn, and Hall (2011) and Copeland and Shapiro (2016) document that the price of car and computer models, respectively declines monotonically after product introduction.

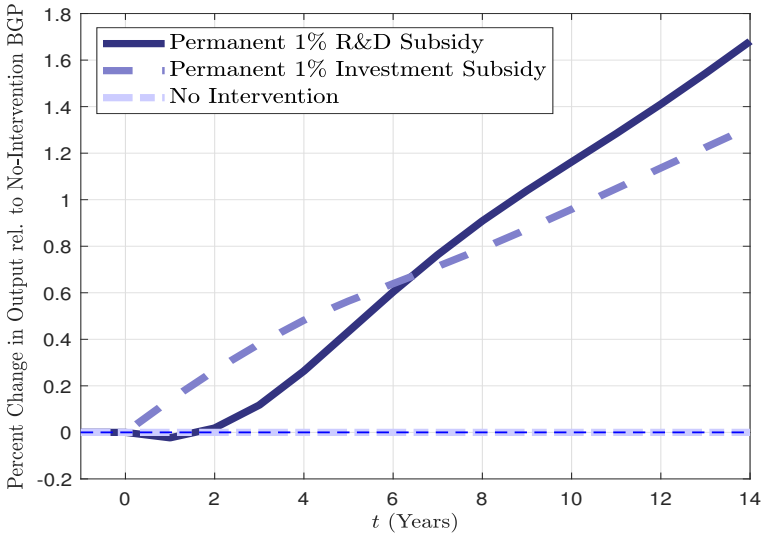
Table 3: Effects of Permanent R&D and Investment Subsidies

	1% $R\&D$ Subsidy	1% I Subsidy	Budget-Eqv. I Subsidy
Y – Long Run	6.321	4.350	0.118
Y – Short Run	0.072	0.248	0.007
C – Long Run	6.012	3.987	0.108
C – Short Run	-0.135	-0.152	-0.003

Notes: The table reports the effects of implementing (i) a permanent 1% R&D subsidy (column 1), (ii) a permanent 1% capital investment subsidy (column 2), or (iii) a permanent capital investment subsidy whose present discounted budget equates that of a permanent 1% R&D subsidy (column 3). The figures refer to the percentage change in the present discounted value of output and consumption relative to the no-intervention BGP. The long-run figures integrate over the infinite horizon, and the short-run figures integrate over $t = 0, ..., 4$.

than investment. To confirm this result, we compute the permanent investment subsidy that has the same fiscal cost (in present discounted value) as a 1% R&D subsidy. We find that the subsidy rate is significantly smaller than 1% and, accordingly, it induces smaller effects on output and consumption.

Figure 9: Permanent R&D and Investment Subsidies



Notes: The figure presents the evolution of aggregate output Y_t in response to a permanent 1% R&D subsidy (blue solid line) and to a permanent 1% capital investment subsidy (red dashed line). Figures are expressed in percent deviation from the no-intervention BGP (grey dash-dot line). We assume that policies are implemented at time $t = 0$ without anticipation, that the economy is on the BGP at $t = -1$, and that all agents have perfect foresight of the evolution of all exogenous and endogenous variables.

7.3 Short-run Stimulus in a Recession

Our findings on the short-run response to permanent policy motivate us to analyze the effects of short-run output stabilization policy during a recession. To this end, we consider again a negative 1% aggregate productivity shock (as in Section 6.1) and we assume that temporary R&D subsidies $\tau_t^{R\&D}$ or investment subsidies τ_t^I are implemented at $t = 0, 1$ to stimulate the recovery.

Consistent with the analysis of permanent policies, we find that investment subsidies are more effective at fostering a sharp recovery in output, whereas R&D subsidy have a delayed effect because of expectations of future growth in capital quality. Figure 10 displays the output effects of the recession with and without the alternative policy interventions. Quantitatively, the investment subsidy reduces the output loss over the first four years by close to 25%, whereas the R&D subsidy has negligible effects on the recovery.

Overall, our analysis highlights that policy interventions on the demand side of the economy tend to have a stronger expansionary effect in the short run, as they affect output both directly by inducing higher investment and indirectly through their equilibrium effect on capital-embodied innovation.

8 Additional Analyses and Sensitivity

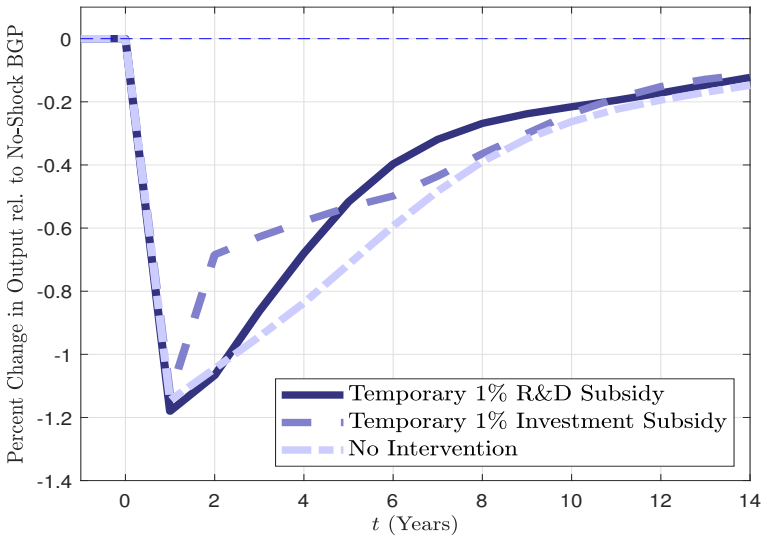
In this section, we discuss additional quantitative analyses that explore the role of alternative calibration strategies and model assumptions. We provide additional details and results for these analyses in Appendix B.

Speed of Obsolescence. In Section 6.5, we analyzed replacement and quality dynamics in response to a negative TFP shock for the aircraft-specific calibration of the model. Aircraft is a long-lived asset with a quality growth that we estimated to be approximately 0.6% per year. We now compare these dynamics with a high-depreciation, fast-obsolescence asset, such as computers. To this end, we modify our baseline macro calibration to consider a larger innovation step-size $\eta = 1.2$ and a rate of physical depreciation $\delta = 0.12$.[22]

In Figure 11 we compare the results for these two calibrations, with a focus on the response of the economy to an aggregate productivity shock. The left panel displays the dynamics of new-capital quality and the right panel the dynamics of measured TFP. The figure shows that both variables reach their trough earlier when the rate of obsolescence is

[22] We adjust the curvature of the innovation probability function $\phi_1 = 85$ to ensure that $\beta = B(1+g) < 1$ in the balanced growth path.

Figure 10: Temporary 1% R&D and Investment Subsidies



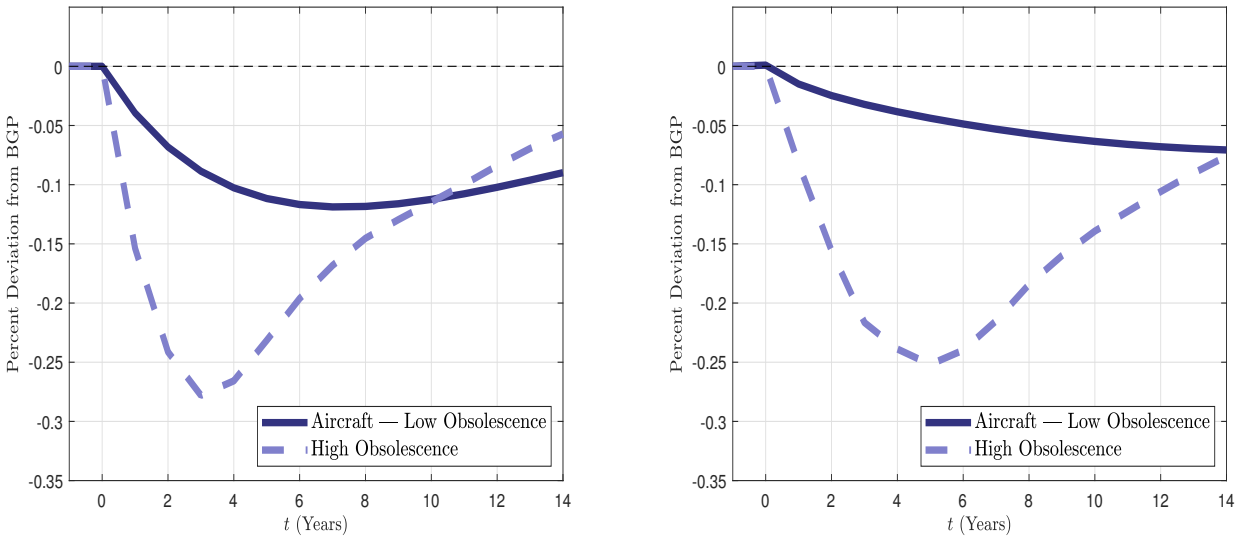
Notes: The figure presents the evolution of aggregate output Y_t in response to a persistent 1% decline in exogenous TFP z at time $t = 1$, with an autocorrelation of 0.85 in logs. The grey dash-dot line refers to the case where no policy intervention occurs; the blue solid line refers to the implementation of a temporary 1% R&D subsidy at time $t = 0$ and $t = 1$; and the red dashed line refers to the implementation of a temporary 1% capital investment subsidy at time $t = 0$ and $t = 1$. Figures are expressed in percent deviation from the no-shock BGP. We assume that the economy is on the BGP at time $t = -1$, that all agents learn about the shocks at $t = 0$, and that they have perfect foresight of the evolution of all exogenous and endogenous variables.

higher. High obsolescence is associated with faster capital replacement. Thus, more firms invest during the recession, when capital quality is low, but then update their capital again within a few years. In contrast, with low obsolescence, the dynamics are slower as firms take time to adopt the new vintage of capital, but then use it in production for a longer time.

Multi-period Monopoly Protection. In our baseline model specification, successful innovators enjoy monopoly protection for one period, after which the quality of their variety can be freely imitated and capital varieties are traded at their marginal cost. This assumption enhances the tractability of our framework by rendering the innovation decision static.

To investigate the sensitivity of our results with respect to this assumption, we extend the model to allow for multi-period monopoly protection. We assume that after a successful innovation, imitation occurs with exogenous probability $\iota \in (0, 1)$. Furthermore, endogenous R&D effort may lead to an innovation that replaces an existing protected variety. Despite the dynamic aspect of the innovation problem, we obtain similar results in

Figure 11: Role of Speed of Obsolescence for Propagation



(a) New-Capital Quality (b) Measured TFP

Notes: The figure illustrates the response of the economy to a persistent 1% decline in exogenous TFP z_t at time $t = 1$, which agents learn at $t = 0$, for two alternative calibrations of the model. Specifically, the solid lines refer to the aircraft calibration of Table 2 and the dashed lines to a "high obsolescence" environment where physical depreciation and quality growth are higher than in the baseline. The panels display: (a) average quality of new capital $\bar{q}_t$ in percent deviation from BGP; (b) the endogenous component of measured TFP, in percent deviation from BGP. We assume that the economy is on the BGP at time $t = -1$ and that all agents have perfect foresight of the evolution of all exogenous and endogenous variables.

response to an aggregate productivity shock. We describe this version of the model and its quantitative analysis in Appendix B.6.

Multi-period Time to Build. Our baseline assumption is that it takes one period for new capital to be installed and become productive, as is standard in macroeconomic models of investment. In the data, there is heterogeneity in the length of time to build across different types of asset. For some types of capital that involve a complex production process, such as aircraft—which we analyze empirically—more than a year can elapse between an order and its delivery. In our sample of Boeing orders, the average delay is between two and three years.

To investigate the sensitivity of our results with respect to the length of time to build, we extend the model and assume that it takes two periods for new capital to become productive. This modification comes at the cost of introducing an additional date t state variable in the capital-replacement problem, namely the replacement decision made at date $t - 2$.

Although this version of the model features a longer lag in the effect of new-capital quality on macroeconomic outcomes, overall we obtain similar results to the one of the baseline model with one period of time to build in response to an aggregate productivity shock. We describe this version of the model and its quantitative analysis in Appendix B.7.

R&D Lags. Our baseline model assumes that, if successful, R&D effort generates quality improvements within the same period. However, R&D projects often involve completing multiple stages, which determine lags between innovation efforts and their outcome. These features may also be relevant in the context of aircraft manufacturing.

To investigate the sensitivity of our results with respect to R&D lags, we extend the model and assume that innovators must successfully complete two subsequent R&D stages before obtaining an innovation that improves the quality of a capital variety. This requires introducing an additional state variable, namely the distribution of varieties over R&D stages. Quality growth not only depends on current R&D effort in the final innovation stage but also on this distribution, which accounts for past R&D investment in the first stage of innovation.

Although this version of the model features richer R&D dynamics, our simulations confirm the comovement between investment and new-capital quality, consistent with our baseline results. We describe this version of the model and its quantitative analysis based on our aircraft calibration in Appendix B.8.

Adjustment Costs. In our baseline economy, firms that invest in new capital need to pay only the replacement cost F_t to capital producers. However, capital replacement may involve additional adjustment costs. Building on Cooper and Haltiwanger (2006), we thus extend our model assuming that, conditional on replacement, firms face an additional adjustment cost equal to a fraction $(1 - \xi) = 0.204$ of output in the adjustment period. Appendix B.9 provides additional details on this version of the model and shows that the response of the economy to a negative TFP shock is qualitatively and quantitatively consistent with the dynamics of our baseline model.

9 Conclusion

When technological advances are embodied in new capital goods, the timing and frequency of firm capital replacement determines the size of the market for innovations. In turn, the path of innovations embodied in capital determines the incentives to upgrade firm capital, and thus increase the level of productivity and output.

This paper provides new evidence on the dynamics of capital-embodied innovation through new-product introduction. It then develops a quantitative framework to analyze the feedback effects between investment dynamics and endogenous technological progress in response to macroeconomic shocks and policy interventions.

The model combines salient features of the quantitative literature on firm investment with a theory of endogenous vertical innovations embodied in new capital goods. In so doing, our framework provides a microfoundation for the important role of investment-specific technical change in macroeconomic dynamics.

Our quantitative analysis shows that the complementarity between investment and capital-embodied innovation is a powerful mechanism in the transmission of aggregate shocks as well as policy changes. Macroeconomic shocks have persistent effects in our model because they generate a feedback loop between the demand for new capital goods and the quality of the new goods supplied. The resulting evolution of the distribution of firms over capital vintages produces endogenous aggregate-productivity dynamics.

The model provides a natural laboratory to analyze policies that stimulate capital replacement and capital-embodied innovation, such as the policy interventions that subsidize the development and adoption of energy-efficient capital. Both interventions on the demand and supply side of capital stimulate long-run growth. However, demand-side stimulus has stronger expansionary effects in the short run, because innovation subsidies create expectations of higher future capital quality, thus inducing a postponement of investment activity.

In developing a number of empirically relevant extensions, such as long-lived innovation rents, additional time to build in investment, innovation lags, and adjustment costs, we demonstrate that our framework is highly tractable. Such tractability will be useful to analyze other important questions in future work, such as the optimal design of subsidies for the replacement of durable goods in the presence of environmental externalities.

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ONLINE APPENDIX

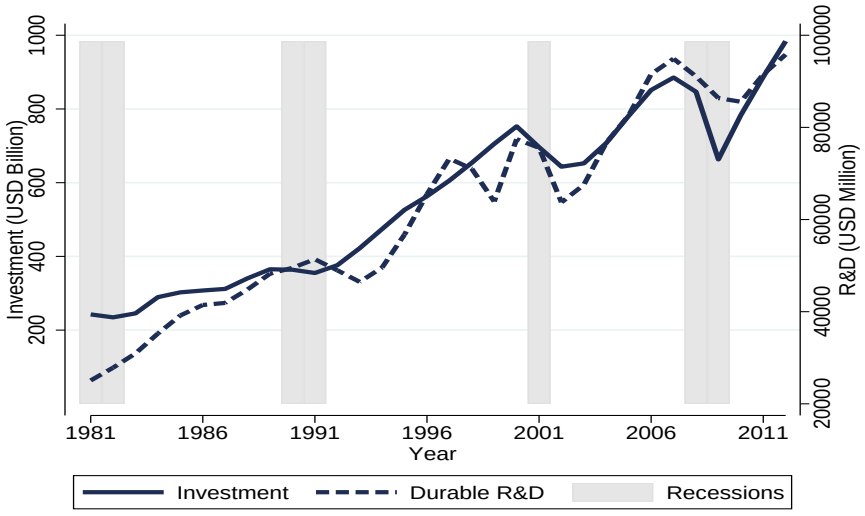
A Additional Empirical Evidence

In this appendix, we provide additional empirical evidence on investment, innovation, and capital quality over the business cycle, leveraging both aggregate data and data on the aircraft industry.

A.1 Investment and R&D in Durable Manufacturing

Figure A1 displays the dynamics of equipment investment and R&D expenditures in durable manufacturing industries during 1981-2012. The figure documents the comovement between capital expenditures and inputs in innovation activity. Both series feature a decline during both the 2001 recession and the 2008 recession.

Figure A1: Equipment Investment and R&D in Durable Manufacturing



Notes: The figure displays the evolution of US private nonresidential investment in equipment from the Bureau of Economic Analysis and US private R&D expenditures in durable manufacturing industries from the National Science Foundation. Horizontal axes report years (1981-2012); gray shaded areas highlight recession periods (1981-1982, 1990-1991, 2001, and 2008-2009).

A.2 Additional Details on Aircraft Dataset

We now provide additional details on the construction of the aircraft dataset.

The definition of engine model is more granular in the ICAO dataset than in Wikipedia. Hence, we assemble our dataset by manually matching engines from these two sources

gathering information from Wikipedia or the Jet Engine Specification Database, available at https://www.jet-engine.net.

We determine a mapping between Boeing's aircraft models and engine variants in the ICAO dataset. Finally, to merge this information to aircraft orders and introduction, we aggregate characteristics of relevant engine variants for each aircraft model at the engine manufacturer level. We obtain Boeing orders data at this level of aggregation.

Similarly, we manually match aircraft model definitions in U.S. Bureau of Transportation Statistics data to Boeing aircraft models. We collapse total miles flown and fuel consumed over aircraft operating life at the model level and compute our alternative fuel efficiency measure as the ratio of these two variables.

We abstract from time-variation in this alternative fuel efficiency measure because it becomes very noisy when miles flown decline close to the retirement of the aircraft model. We verify that our preferred measure of fuel efficiency based on maximum range and fuel capacity is strongly correlated with the alternative fuel efficiency measure based on actual flight data.

In our final dataset, orders vary at the model-engine-month level, engine-related characteristics such as thrust vary by model and engine but are constant over time, and other aircraft attributes from Wikipedia vary across models but remain constant across engine options and over time.

A.3 Additional Evidence on Aircraft

In this subsection, we present additional evidence on the evolution of aircraft quality. Table A1 shows the parameter estimates of regression (1) for alternative measures of efficiency.

A.4 Evidence on Airbus Aircraft

We now proceed to analyze the procyclical behavior of investment and innovations when we extend our sample to include aircraft models produced by Airbus, the second largest manufacturer.

We do not include Airbus models in the main analysis because we do not have direct evidence on product introduction dates, and thus we need to infer them from online sources, such as Wikipedia. Nonetheless, we show in this subsection that our results are robust to this sample extension.

Table A1: Estimated Technological Trade-Offs Parameters

	(1)	(2)	(3)	(4)
ln(Thrust)	-0.202	-0.198	-0.568	-0.239
	(0.099)	(0.099)	(0.110)	(0.145)
ln(Cruise Speed)	-0.288	-0.286	0.176	-0.149
	(0.491)	(0.490)	(0.483)	(0.656)
ln(Number of Seats)	-0.016	-0.015	0.150	-0.090
	(0.112)	(0.111)	(0.101)	(0.103)
ln(Max. Takeoff Weight)	-0.251	-0.256	-0.120	-0.049
	(0.143)	(0.143)	(0.118)	(0.157)
ln(Cargo Volume)	-0.078	-0.077	0.062	-0.056
	(0.103)	(0.102)	(0.089)	(0.101)
ln(Aspect Ratio)	-0.175	-0.197	-0.264	-0.198
	(0.178)	(0.174)	(0.172)	(0.249)
ln(Length)	-0.045	-0.062	-0.125	0.225
	(0.263)	(0.260)	(0.333)	(0.396)
ln(TakeOff Dist.)	-0.253	-0.246	-0.220	-0.407
	(0.146)	(0.145)	(0.049)	(0.071)
Number Engines = 3	-0.213	-0.211	-0.155	-0.218
	(0.069)	(0.068)	(0.039)	(0.045)
Number Engines = 4	-0.216	-0.212	-0.312	-0.450
	(0.074)	(0.074)	(0.053)	(0.066)
New Model		0.046		
		(0.012)		
Observations	9946	9946	9946	9573

Notes: The table reports parameter estimates of regression (1) for several fuel efficiency measures. In the first column, the dependent variable is log fuel efficiency during cruise, computed as aircraft's maximum range over fuel capacity. The second column refers to the same dependent variable when specification (1) includes a dummy variable identifying newly-introduced model-engine maker combinations. A model-engine maker combination is new in the 12 months after the first order. In the third and fourth columns, the dependent variable is, respectively, log fuel efficiency during the Landing Take-Off (LTO) cycle, computed as the inverse of total fuel consumption during the LTO cycle; and the log distance flown divided by fuel consumed by each model based on U.S. Bureau of Transportation Statistics' data. The number of observations is lower than in columns (1), (2), and (3) because two aircraft models (Boeing 737-600 and McDonnel-Douglas 80) are not matched in U.S. Bureau of Transportation Statistics' data. Two-way clustered standard errors by model-engine maker combination and monthly order date are shown in parentheses.

A.4.1 Airbus Data Description

We begin by describing Airbus data on technical characteristics and orders.

Model Characteristics. As for Boeing, we obtain data on each model's characteristics from Wikipedia. Information covers the most important technical attributes for efficiency, including: aircraft size, wings width and area, aircraft weight (maximum weight at take-off and operating empty weight), cruise speed, fuel capacity, maximum range, and number of engines. When missing, we complement this information with data from detailed technical documents for each aircraft published on Airbus' website for administrative reasons. Different from Boeing, we also obtain information on engine-specific power (thrust) directly from Wikipedia. The sample includes 27 distinct models m defined in accordance with Airbus definitions.

Orders. Different from Boeing, Airbus does not provide detailed historical data on aircraft orders by model and customer. Therefore, we obtain this information from Wikipedia. Because the necessary level of detail is not available for all models, we can obtain orders data for 10 out of 27 models for which we have data on characteristics. These models are A220-100, A220-300, A320neo, A321LR, A321XLR, A330-800, A330-900, A350-900, A350-1000, and A380-800.

Introduction Dates. For Airbus models for which we have orders data, we infer aircraft introduction and retirement dates as the month of the first and last firm order, respectively. For the other models, we infer the introduction date based on the date of first flight of the new model, which is more readily available and that we obtain from Wikipedia or other sources. Specifically, using Airbus models for which we have the first order date, we estimate the median lag between the first order and the first flight dates. Then, for all other models we impute as introduction date the first flight date minus the observed median lag.

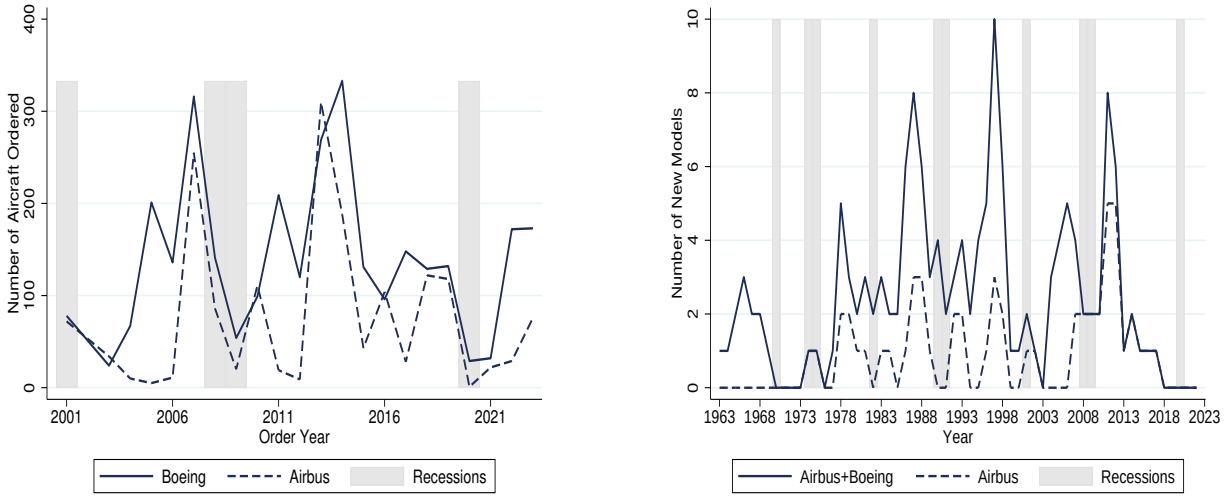
The dataset includes rich information on characteristics and new aircraft introduction for more than 40 model-engine brand combinations over the period 1975-2022. As for Boeing, the unit of observation is a triplet defined by aircraft model m, such as Boeing A220-100, A220-300, etc.; engine brand e; and month t.

In the following analyses, we use Airbus data either in combination with Boeing data described in Section 3 or on a stand-alone basis.

A.4.2 Product Introduction, New-Capital Quality, and the Business Cycle

We now analyze the volume of new-product introduction across expansions and recession in our dataset. Figure A2a displays the total number of orders of a selected subset of Airbus models (A350-900, A350-1000, and A380-800) for which we have orders data and that we can match to similar models by Boeing (Boeing 747-8, Boeing 767, Boeing 777, Boeing 777X, and Boeing 787). Figure A2b illustrates the number of newly introduced Airbus aircraft models (dashed line) and the total number of new models from both Airbus and Boeing. As in the main analysis, both series are highly procyclical, featuring spikes during economic expansions and drops during recessions, which are highlighted with grey areas in the figure.

Figure A2: Orders and New-Model Introduction – Airbus and Boeing



(a) Orders

(b) New Models Introduced

Notes: Panel (a) plots orders of comparable Airbus and Boeing new aircraft over time. Airbus models include: A350-900, A350-1000, and A380-800. Boeing models include: Boeing 747-8, Boeing 767, Boeing 777, Boeing 777X, and Boeing 787. Panel (b) plots the yearly number of newly introduced Airbus and Boeing aircraft models. A model is defined new in the year when the first firm order is observed and in the subsequent year. Grey areas denote NBER recession periods.

Next, we proceed to estimate the effect of procyclical product introduction for the evolution of technology embodied in new aircraft. To this end, we replicate the analysis of Section 3 based on the methodology of Knittel (2011) and estimate regression (1) in three distinct samples. In the first, we include both Airbus and Boeing models over the original sample period 1971-2014. In the second, we focus only on Airbus models over 1975-2022. In

Table A2: Aircraft Quality Growth and Role of New Products – Airbus and Boeing

Sample	Quality Growth			Δ% New-Old
	All	Expansion	Recession	
Airbus+Boeing 1971-2014	0.621	0.762	-0.188	4.978
	(0.101)	(0.102)	(0.219)	(1.510)
Airbus Only 1975-2022	0.476	0.544	-0.039	3.892
	(0.111)	(0.124)	(0.126)	(1.668)
Airbus+Boeing 1971-2022	0.531	0.637	-0.178	3.836
	(0.090)	(0.090)	(0.213)	(1.491)

Notes: The table reports evidence of the procyclical nature of quality improvements in fuel efficiency, computed as aircraft's maximum range over fuel capacity, for three distinct alternative samples: (i) including both Airbus and Boeing models over the same sample period of the main analysis, namely 1971-2014; (ii) including only Airbus models over the period 1975-2022; (iii) including both Airbus and Boeing models over the extended period 1971-2022.The first, second, and third columns report the average yearly growth rate of different quality measures in the whole sample, during expansions, and during recessions, respectively. Average growth rates are computed by normalizing the difference in γ_t quality estimates from regression (1) between the end and the start of each expansion and recession. The fourth column reports the average percent quality difference between newly-introduced models and previously existing models. Quality difference between new and old models is estimated by adding to specification (1) a dummy variable taking value one in the first month an order is observed for model-engine maker combination (m, e) and in the following year. Figures in parentheses report standard errors clustered by model-engine maker combination and monthly date.

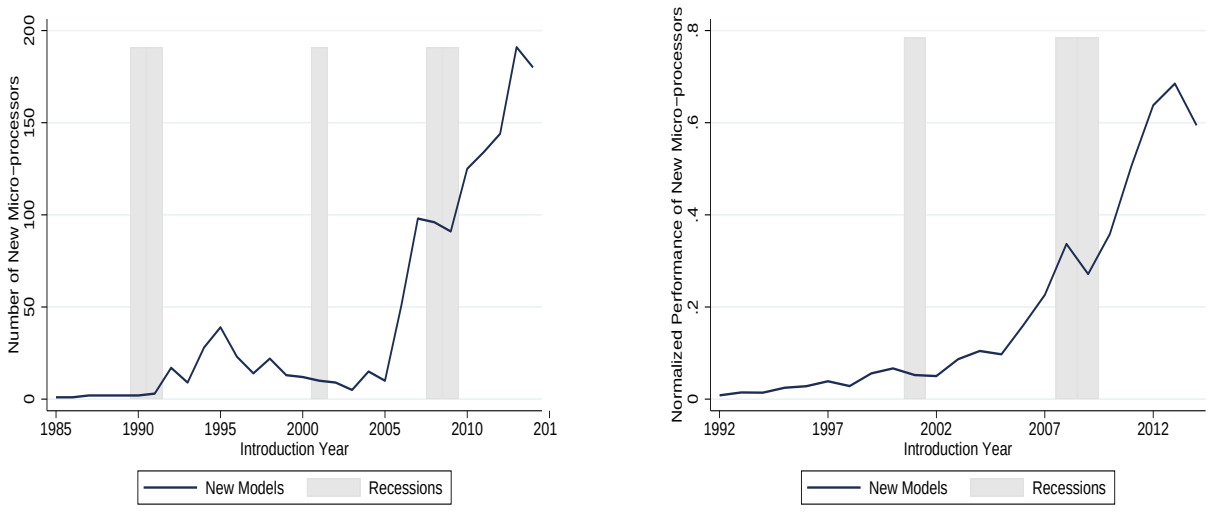
the third, we leverage the maximum coverage of our data and include both aircraft makers over 1971-2022. As in the main analysis, the dependent variable is the natural logarithm of fuel efficiency, computed as the ratio between maximum range and field capacity.

Table A2 summarizes the main quantitative findings on quality growth in expansion and recessions across sample definitions. The results are consistent with the baseline findings of Section 3. Quality growth concentrates during expansion periods and, on average, halts in recessions. As new models feature a technological level that is on average 3.4%-4.6% higher than previously introduced models that are still on the market, the drop in new-product introduction accounts for the lack of embodied technological progress during recessions.

A.5 Evidence on Micro-Processors

In this subsection, we report evidence on the evolution of new product introduction and performance for CPUs. Figure A3 displays the evolution of the number of newly introduced CPUs by release year (left panel) and a normalized performance measure available in Danowitz, Kelley, Mao, Stevenson, and Horowitz (2012) (right panel). Introduction

Figure A3: Introduction and Performance of New CPUs



(a) Introduction (b) Performance

Notes: The figure displays the evolution of the introduction and the performance of new microprocessors by release year. Panel (a) plots the number of newly-introduced microprocessors and panel (b) the time-series of a standardized performance metrics computed by Danowitz, Kelley, Mao, Stevenson, and Horowitz (2012). The sample ends in 2014 and the performance metrics is available after 1992.

and performance accelerate remarkably during expansions but decline in recessions, this showing the same procyclcal behavior observed for new aircraft and vehicles.

Because release date is missing for approximately 35% of CPUs in Danowitz, Kelley, Mao, Stevenson, and Horowitz (2012), we also verify the robustness of the patterns of Figure A3 when we impute release year, where missing, as the introduction year of the first CPU in the same family of micro-processors. This imputation reduces the share of observations with missing release year to 13%.

B Additional Model Analyses

In this appendix, we provide additional analyses related to the model.

B.1 Additional Model Derivations

In this subsection we present additional model derivations. First, we solve for the Lagrange multiplier λ_t of the constraint on the expenditure minimization problem of final good producers that choose to replace their capital. Second, we solve for the replacement cost

F_t as a function of R&D intensity, past quality, and parameters.

Lagrange Multiplier. As a preliminary step, we derive quantity of $v_{j,t}$ of each capital variety j optimally used in the capital bundle. For monopolistic varieties with price $p_{j,t} = \frac{c}{1-\gamma}$, the demand function (7) implies

$$v_{j,t} = \lambda_t^{\frac{1}{\gamma}} (1-\gamma)^{\frac{1}{\gamma}} \left(\frac{c}{1-\gamma}\right)^{-\frac{1}{\gamma}} q_{j,t} \tag{B1}$$

where $q_{j,t} = \eta q_{j,t-1}$ due to successful innovation. In contrast, for competitive varieties, for which the price is driven to the marginal cost $p_{j,t} = c$, the quantity is

$$v_{j,t} = c^{\frac{1}{\gamma}} (1-\gamma)^{\frac{1}{\gamma}} \lambda_t^{\frac{1}{\gamma}} q_{j,t} \tag{B2}$$

where $q_{j,t} = q_{j,t-1}$ because innovation was not successful.

Because varieties become competitive after one period and R&D intensity is symmetric across them, in equilibrium each variety j faces an independent and identical probability $\phi(\psi_t)$ of being monopolistic. Therefore, in each period a fraction $\phi(\psi_t)$ is monopolistic and a fraction $(1-\phi(\psi_t))$ is competitive. Replacing prices and quantities of monopolistic and competitive varieties in equation (5), we can derive

$$\begin{aligned}
\kappa \bar{q}_t &= \int_0^1 q_{j,t}^{\gamma} v_{j,t}^{1-\gamma} dj = \\
&= \int_{j \in \mathcal{J}^{inn}} \underbrace{\eta^{\gamma} q_{j,t-1}^{\gamma}}_{q_{j,t}^{\gamma}} \underbrace{\left((1-\gamma)^{\frac{1}{\gamma}} c^{-\frac{1}{\gamma}} \lambda_t^{\frac{1}{\gamma}} \eta q_{j,t-1}\right)^{1-\gamma}}_{v_{j,t}^{1-\gamma}} dj + \\
&\int_{j \in \mathcal{J}^{no}} \underbrace{q_{j,t-1}^{\gamma}}_{q_{j,t}^{\gamma}} \underbrace{\left((1-\gamma)^{\frac{1}{\gamma}} c^{-\frac{1}{\gamma}} \lambda_t^{\frac{1}{\gamma}} q_{j,t-1}\right)^{1-\gamma}}_{v_{j,t}^{1-\gamma}} dj = \\
&= \lambda_t^{\frac{1-\gamma}{\gamma}} \left(\frac{1-\gamma}{c}\right)^{\frac{1-\gamma}{\gamma}} \left[\phi(\psi_t)\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}} + (1-\phi(\psi_t))\right] \bar{q}_{t-1}
\end{aligned} \tag{B3}$$

where from the first to the second and third lines we distinguish varieties j's based on their innovation outcome ($\mathcal{J}^{inn}$ denotes the set of varieties with a successful innovation and $\mathcal{J}^{no}$ the set of competitive varieties). We further replace equilibrium quality and quantity and finally we leverage the independence of innovation probability with respect to past quality $q_{j,t-1}$ across varieties.

We substitute the law of motion for quality $\bar{q}_t$ (13) on the left hand side of (B3) and

after re-arranging we obtain

$$\lambda_t^{\frac{1}{\gamma}} = \frac{\kappa^{\frac{1}{1-\gamma}}\Big[1+(\eta-1)\phi(\psi_t)\Big]^{\frac{1}{1-\gamma}}}{\Big(\frac{1-\gamma}{c}\Big)^{\frac{1}{\gamma}}\Big[1+\Big(\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}}-1\Big)\phi(\psi_t)\Big]^{\frac{1}{1-\gamma}}} \tag{B4}$$

Replacement Cost. Substituting equilibrium prices and quantities in the objective function of the expenditure minimization (6), we obtain

$$\begin{aligned}
F_t \equiv & \int_0^1 p_{j,t}v_{j,t}dj = \\
= & \int_{j\in\mathcal{J}^{inn}} \underbrace{\Big(\frac{c}{1-\gamma}\Big)}_{p_{j,t}} \underbrace{\frac{(1-\gamma)^{\frac{2}{\gamma}}}{c^{\frac{1}{\gamma}}}\lambda_t^{\frac{1}{\gamma}}\eta q_{j,t-1}}_{v_{j,t}} dj + \int_{j\in\mathcal{J}^{no}} \underbrace{c}_{p_{j,t}} \underbrace{\Big(\frac{1-\gamma}{c}\Big)^{\frac{1}{\gamma}}\lambda_t^{\frac{1}{\gamma}} q_{j,t-1}}_{v_{j,t}} dj \\
= & c^{1-\frac{1}{\gamma}}(1-\gamma)^{\frac{1}{\gamma}}\lambda_t^{\frac{1}{\gamma}}\Big[1+\Big((1-\gamma)^{\frac{1}{\gamma}-1}\eta-1\Big)\phi(\psi_t)\Big]\bar{q}_{t-1} \\
= & c\kappa^{\frac{1}{1-\gamma}}\Big[1+(\eta-1)\phi(\psi_t)\Big]^{\frac{1}{1-\gamma}}\Big[1+\Big((1-\gamma)^{\frac{1}{\gamma}-1}\eta-1\Big)\phi(\psi_t)\Big]^{-\frac{\gamma}{1-\gamma}}\bar{q}_{t-1}
\end{aligned} \tag{B5}$$

where from the first to second line we distinguish between monopolistic and competitive varieties and substitute for equilibrium prices and quantities, from the second to the third line we leverage the independence of innovation probability with respect to past quality $q_{j,t-1}$, and from the third to the fourth line we substitute the expression of $\lambda_t^{\frac{1}{\gamma}}$ from equation (B4).

B.2 Solution Algorithm

This section describes the solution algorithm for the recursive balanced-growth equilibrium and for the transitional dynamics of the model in response to an aggregate shock under the assumption of perfect foresight.

B.2.1 Recursive Balanced-growth Equilibrium

The solution to the recursive balanced-growth equilibrium is determined by the R&D intensity ψ^* and aggregate replacement rate M^* such that

1. Given M^*, the equilibrium R&D intensity ψ^* solves the equation

$$\kappa^{\frac{1}{1-\gamma}}\gamma(1-\gamma)^{\frac{1}{\gamma}-1}cM^*\phi'(\psi^*) = \left[\frac{1+\left(\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}}-1\right)\phi(\psi^*)}{1+(\eta-1)\phi(\psi^*)}\right]^{\frac{1}{1-\gamma}} \tag{B6}$$

which is derived by substituting the Lagrange multiplier λ of the capital-bundle constraint

$$\kappa\vec{q} = \int_0^1 q_j^{\gamma} v_j^{1-\gamma} d_j \tag{B7}$$

in the the optimality condition of the R&D problem (11). Moreover, as we impose a non-negativity constraint on R&D intensity, ψ^* is positive if

$$\kappa^{\frac{1}{1-\gamma}}\gamma(1-\gamma)^{\frac{1}{\gamma}-1}cM^*\phi'(0) > 1 \tag{B8}$$

and zero otherwise.

In turn, λ solves equation (B4) given ψ^*

The optimal R&D intensity determines the endogenous growth rate of the economy as

$$g^* = \phi(\psi^*)(\eta-1) \tag{B9}$$

and the endogenous replacement cost $F/\vec{q}$ in the stationary equilibrium, where $F = \int_0^1 p_j v_j dj$ and therefore

$$f^* = \frac{F}{\vec{q}} = c\kappa^{\frac{1}{1-\gamma}}\left[\frac{1+(\eta-1)\phi(\psi^*)}{1+\left(\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}}-1\right)\phi(\psi^*)}\right]^{\frac{\gamma}{1-\gamma}} \tag{B10}$$

2. Given ψ^*, and hence quality growth g^*, and replacement cost f^*, we solve the optimal replacement decision x^* to the stationary problem (14) for any state (k,s). The aggregation of replacement decisions $x^*(k,s)$ over the stationary joint distribution $m^*(k,s)$ of firms over states determines the aggregate replacement rate $M^* = \int x(k,s)dm^*(k,s)$.

   To derive the solution for the stationary distribution $m^*(k,s)$ given the optimal replacement decisions, we denote by $h = 1,..,H$ the vintage of capital, i.e., the time distance between the current date and the last replacement decision. Moreover, in the stationary equilibrium we can define the following mapping between capital vintage

and the value of capital in efficiency units:

$$k(h) = \kappa \frac{(1-\delta)^{h-1}}{(1+g^*)^h}$$

and with a slight abuse of notation we can also express by $m^*(h,s)$ the joint distribution of capital vintage and idiosyncratic productivity in the stationary equilibrium. Hence, the stationary distribution satisfies the equations

$$\begin{aligned} m^*(1,s') &= \sum_{h=1}^{H} \sum_{s} x^*(h,s)\mathcal{P}(s'|s)m^*(h,s) \\ m^*(h,s') &= \sum_{s} [1 - x^*(h-1,s)]\mathcal{P}(s'|s)m^*(h-1,s) \quad \forall\ h \in [2,H] \end{aligned} \tag{B11}$$

where $\mathcal{P}(s'|s)$ denotes the stationary Markov process of idiosyncratic productivity s and $x^*(h,s)$ is the equilibrium replacement decision of firms with state vector (h,s). These conditions ensure that the distribution of firms over capital vintages and idiosyncratic productivity is constant over time.

The solution is found numerically at the intersection of the $\psi^*(M)$ mapping–obtained by solving for ψ^* for different values of M on a grid (step 1)–and the $M^*(\psi)$ mapping–obtained by solving for M^* for different values of ψ on a grid (step 2).

B.2.2 Transitional Dynamics

We next describe the solution algorithm for the transitional dynamics of the economy in response to an unanticipated aggregate shock. We assume that the economy is on the balanced growth path at time $t = 0$. For concreteness, we consider an unanticipated shock to z inducing the exogenous path $[z_0 = 1, z_1, z_2, z_3, ...z_T = 1]$ as in Section 6.1, of which agents have perfect foresight. Notice that the shock can be persistent but not permanent.

The solution algorithm encompasses the following steps:

1. Guess a path for the mass of replacing firms $\{M_t\}_{t=0}^{T}$ which assumes that by time $t = T = 100$ the economy is back to the stationary equilibrium.

2. Solve for the full path of growth g_t through the following sub-steps

   (a) Given the normalized quality level $\overline{q}_{-1} = 1$ before the shock, we can solve for

R&D intensity ψ_t given M_t through the equation

$$\frac{\kappa^{\frac{1}{1-\gamma}}\gamma(1-\gamma)^{\frac{1}{\gamma}-1}cM_t}{(\bar{q}_t/\vec{q}_t)^{\chi_1}}\phi'(\psi_t) = \left[\frac{1+\left(\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}}-1\right)\phi(\psi_t)}{1+(\eta-1)\phi(\psi_t)}\right]^{\frac{1}{1-\gamma}} \tag{B12}$$

where $\bar{q}_t = [1+\phi(\psi_t)(\eta-1)]\bar{q}_{t-1} = (1+g_t)\bar{q}_{t-1}$ is actual capital quality and $\vec{q}_t = (1+g^*)^t$ represents capital quality in the balanced growth path equilibrium. Note that if the following condition holds,

$$\frac{\kappa^{\frac{1}{1-\gamma}}\gamma(1-\gamma)^{\frac{1}{\gamma}-1}cM_t}{(\bar{q}_t/\vec{q}_t)^{\chi_1}}\phi'(0) > 1 \tag{B13}$$

optimal R&D intensity is zero as R&D investment features full irreversibility.

(b) Given the solution for ψ_t $\forall t = 0, ..., T$ and $\bar{q}_{-1} = 1$, we compute the time-specific price of the new capital bundle, i.e., the replacement cost F_t. To do so, we use its definition $F_t = \int_0^1 p_{j,t}v_{j,t}dj$ and $\bar{q}_t = [1+\phi(\psi_t)(\eta-1)]\bar{q}_{t-1}$. We obtain:

$$\frac{F_t}{\vec{q}_t} = c\kappa^{\frac{1}{1-\gamma}}\frac{\left[1+(\eta-1)\phi(\psi_t)\right]^{\frac{1}{1-\gamma}}}{\left[1+\left(\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}}-1\right)\phi(\psi_t)\right]^{\frac{\gamma}{1-\gamma}}}\frac{\bar{q}_{t-1}}{\vec{q}_t} \tag{B14}$$

3. Given the path of frontier capital quality $\bar{q}_t$ and replacement cost F_t, we can solve backwards for the optimal replacement decisions $x_t(k_t(h), s)$ and the value function $V_t(k_t(h), s)$ where, as before, h denotes capital vintages $h = 1, ..., H$. Depending on the vintage, in the stationary equilibrium the efficiency-units of production capital during the transition are defined as $k_t(h) = \frac{\kappa\bar{q}_{t-h}(1-\delta)^{h-1}}{\vec{q}_t}$. Therefore, the time-$t$ problem along the transition is

$$\begin{aligned} V_t(k_{t-1}(h), s) = \max_{x_t\in\{0,1\}} & z_t s k_{t-1}(h) - x_t\frac{F_t}{\vec{q}_t} + \beta\mathbb{E}_t[V_{t+1}(k_t(h'), s')|s] \\ & \text{subject to} \\ & h' = \begin{cases} h+1 & \text{if } x_t = 0 \\ 1 & \text{if } x_t = 1 \end{cases} \\ & \text{and} \\ & k_t(h') = \begin{cases} \frac{\kappa\bar{q}_{t-h'}(1-\delta)^{h'-1}}{\vec{q}_t} & \text{if } x_t = 0 \\ \frac{\kappa\bar{q}_t}{\vec{q}_t} & \text{if } x_t = 1 \end{cases} \end{aligned} \tag{B15}$$

where $\beta = B(1+g^*)$ and we assume that $V_T(k_T(h), s) = V^*$, where V^* is the solution to the stationary replacement problem (14) in the absence of shocks. Moreover, we compute the expectation $\mathbb{E}_t[V_{t+1}(k_t(h'), s')|s]$ as $\sum_{s'} \mathcal{P}(s'|s)V_{t+1}(k_t(h'), s')$ using the transition matrix $\mathcal{P}(s'|s)$ for the discretized stochastic process governing idiosyncratic productivity s.

4. Given the optimal replacement decisions $x_t(h, s)$ over the transition, we solve forward for the distribution of firms over (h, s) states, where again, with a slight abuse of notation, the vintage $h = 1, ..., H$ replaces the value of capital in efficiency units $k_t(h)$. The law of motion of the distribution satisfies the system

$$\begin{aligned} m_{t+1}(1, s') &= \sum_{h=1}^{H} \sum_{s} x_t(h, s)\mathcal{P}(s'|s)m_t(h, s) \\ m_{t+1}(h, s') &= \sum_{s} [1 - x_t(h-1, s)]\mathcal{P}(s'|s)m_t(h-1, s) \quad \forall\ h \in [2, H] \end{aligned} \tag{B16}$$

5. Finally we update the guess for the replacement rate M_t over the transitional dynamics as

$$M_t = \sum_{h=1}^{H} \sum_{s} x_t(h, s)m_t(h, s) \tag{B17}$$

and we iterate on steps 2 to 5 until convergence of the $\{M_t\}_{t=0}^{T}$ and $\{\psi_t\}_{t=0}^{T}$ sequences, which in turn determine the convergence of equilibrium sequences for quality growth g_t, quality level $\overline{q}_t$, replacement cost F_t, and hence value functions capital values $k_{t-1}(h)$, values $V_t(h, s)$, replacement decisions $x_t(h, s)$, and firms' distribution $m_t(h, s)$.

B.3 Decomposition of Balanced-Growth Path Mechanism

In this subsection we further investigate the effect of new capital quality growth on replacement in the balanced-growth path equilibrium.

To this end, we exogenously vary R&D intensity on the supply-side of new capital quality and decompose its overall effects on demand-side capital replacement decisions into two channels: (i) obsolescence and (ii) replacement cost. Specifically, an increase in R&D intensity induces: (i) an increase in the growth rate of new capital quality $g^* = (\eta-1)\phi(\psi^*)$, thus fostering economic obsolescence of existing capital; (ii) an increase in the replacement cost of capital according to equation (B10), because more capital varieties are monopolistic and sold at a markup.

Figure B1 provides a graphical decomposition of obsolescence and price effects of R&D-driven quality growth on replacement. While we chart quality growth g on the x-axis for consistency with Figure 3, plotted variation in replacement M (y-axis) is always induced by exogenously varying R&D intensity ψ.

The solid line coincides with the equilibrium mapping plotted in Figure 3 and considers both obsolescence and price effects. The dashed line mutes the price channel by fixing the replacement cost to its BGP equilibrium level but lets economic obsolescence vary with quality growth. The dashed-dotted line mutes the economic obsolescence channel and just considers the effects of variation in the replacement cost.

Isolating the effects of economic obsolescence induces a stronger positive relationship between replacement and quality growth (dashed line). In contrast, price effects alone would discourage replacement as quality growth increases, leading to the negative relationship depicted by the dashed-dotted line. As the obsolescence channel dominates, the balanced growth path equilibrium relationship between replacement and quality growth is positive (solid line).

B.4 Quality-adjusted Price of Capital

In this subsection, we derive an expression for the model counterpart of the quality-adjusted price of capital. To this end, we consider the counterfactual problem of a firm that, in our balanced-growth equilibrium, replaces its capital by purchasing a quantity of capital (in efficiency units) that remains constant over time, i.e., $\kappa_t = \kappa$ for all t, rather than growing with average capital quality as in the benchmark model.

The expenditure minimization problem conditional on replacement becomes

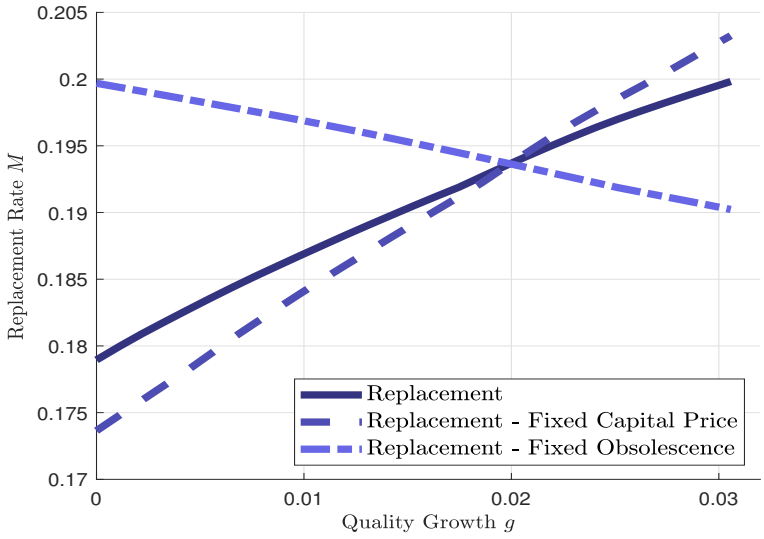
$$\min_{\{v_{i,j,t}\}_j} \int_0^1 p_{j,t} v_{i,j,t} dj \quad \text{subject to} \quad \int_0^1 q_{j,t}^{\gamma} v_{i,j,t}^{1-\gamma} d_j = \kappa$$

The optimal price and quantity for a monopolistic variety j are $p_{j,t} = c/(1-\gamma)$ and $v_{i,j,t} = \left(\frac{(1-\gamma)^2}{c}\right)^{\frac{1}{\gamma}} \lambda_{i,t}^{\frac{1}{\gamma}} q_{j,t}$, respectively. In contrast, for competitive varieties price equals marginal cost $p_{j,t} = c$ and $v_{i,j,t} = \left(\frac{1-\gamma}{c}\right)^{\frac{1}{\gamma}} \lambda_{i,t}^{\frac{1}{\gamma}} q_{j,t}$.

Replacing these expressions in the constraint of the expenditure minimization problem, we obtain an equation that we can solve for $\lambda_{i,t}$:

$$\kappa = \lambda_{i,t}^{\frac{1-\gamma}{\gamma}} \left(\frac{1-\gamma}{c}\right)^{\frac{1-\gamma}{\gamma}} \left[\phi(\psi_t)\eta(1-\gamma)^{\frac{1-\gamma}{\gamma}} + (1-\phi(\psi_t))\right] \bar{q}_{t-1}.$$

Figure B1: Replacement with Fixed Replacement Cost or Obsolescence in the BGP



Notes: The figure provides a graphical decomposition of the effect of new capital quality growth on replacement M (y-axis) in the balanced growth path equilibrium. To produce the plot, we exogenously vary R&D intensity ψ, thus inducing one-to-one variation in capital obsolescence, i.e., in the growth rate of new capital quality $g = (\eta - 1)\phi(\psi)$, and new capital price. While the moving object is R&D intensity ψ for all the lines, we plot g on the x-axis for consistency with Figure 3. The solid line represents the equilibrium demand for new capital–i.e., the fraction of firms that optimally choose to replace their capital–when both obsolescence and replacement cost vary with ψ. The dashed line represents the equilibrium demand for new capital if we fix capital price to its BGP equilibrium level and let vary obsolescence with ψ. The dashed-dotted line represents the equilibrium demand for new capital if we fix economic obsolescence to its BGP equilibrium level and let the replacement cost vary with ψ.

On the balanced-growth path, the right-hand side of this equation must be constant, as κ is a constant parameter. Therefore, we can write the constant growth rate of the multiplier $\lambda_{i,t}$, which we denote g_λ, as a function of quality growth g as follows: $(1+g_\lambda) = (1+g)^{-\frac{\gamma}{1-\gamma}}$.

Moreover, the quality-adjusted replacement cost is

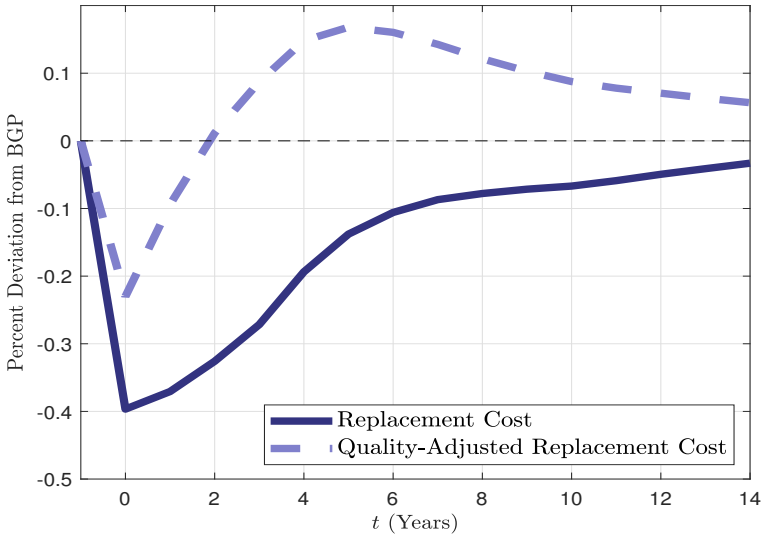
$$F_{i,t}^{noq} = c^{1-\frac{1}{\gamma}}(1-\gamma)^{\frac{1}{\gamma}}\lambda_{i,t}^{\frac{1}{\gamma}}\left[1 + \Big((1-\gamma)^{\frac{1}{\gamma}-1}\eta - 1\Big)\phi(\psi_t)\right]\bar{q}_{t-1}. \tag{B18}$$

The growth rate of this counterfactual replacement cost along the balanced-growth path is $(1+g)^{-\frac{\gamma}{1-\gamma}}$. Thus, the quality-adjusted replacement cost decreases over time because $\gamma \in (0,1)$.

B.5 Replacement Cost and Investment Dampening

Figure B2 displays the dynamics of the detrended replacement cost of capital (solid line) as well as the cost of purchasing a capital bundle of constant quality (dashed line). On impact, both series drop because of the decrease in the fraction of varieties sold at a markup. Over time, however, low quality of new capital implies that the quality-adjusted price of new capital increases and is above its trend for several years. The presence of these two channels that operate in opposite directions may account for the mixed empirical evidence on the cyclicality of quality-adjusted equipment price indices (Beaudry, Moura, and Portier, 2015).

Figure B2: Price Index of New Capital

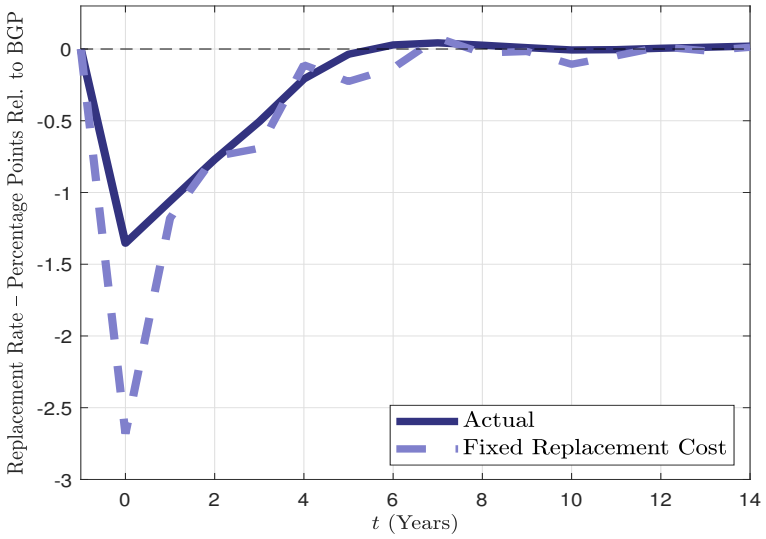


Notes: The figure illustrates the response of the replacement cost F_t (solid line) and the cost of constant-quality capital F_t^{noq} (dashed line), in percent deviation from the BGP, to a persistent -1% shock to exogenous TFP z_t that occurs at time $t = 1$. We normalize $\bar{q}_{-1} = 1$ and assume that the economy is on the BGP at time $t = -1$, that all agents learn about the shock at $t = 0$, and that they have perfect foresight of the evolution of all exogenous and endogenous variables.

Figure B3 illustrates the role of the endogenous replacement cost in dampening the investment response to a negative aggregate productivity shock. The solid line represents the response of replacement M_t to a persistent negative shock to z_t when the replacement cost F_t endogenously adjusts to the shock. The dashed line depicts the evolution of replacement if we hold F_t fixed to its balanced-growth path equilibrium level while imposing that all other variables evolve according to their equilibrium dynamics in response to the same shock. As R&D and innovation decline on impact, so does the replacement cost because fewer varieties are monopolistic. Therefore, new capital available for sale is cheaper, which dampens the decline in investment. In contrast, in the counterfactual scenario where F_t is

fixed, the decline in capital quality amplifies the negative response of replacement.

Figure B3: Replacement with Endogenous or Exogenous Replacement Cost



Notes: The figure illustrates the response of the replacement rate M_t, in percent deviation from the BGP, to a persistent -1% shock to exogenous TFP z_t that occurs at time $t = 1$. We normalize $\bar{q}_{-1} = 1$ and assume that the economy is on the BGP at time $t = -1$, that all agents learn about the shock at $t = 0$, and that they have perfect foresight of the evolution of all exogenous and endogenous variables. The solid line refers to the baseline model where the replacement cost F_t endogenously responds to the shock. The dashed line refers to a counterfactual scenario where new capital quality follows its equilibrium dynamics while F_t is exogenously fixed at its balanced-growth path value.

B.6 Model with Multi-period Monopoly

In this subsection, we develop and analyze a version of our model featuring multi-period monopoly on innovations.

B.6.1 Environment

We extend our model to allow capital-goods producers to retain the monopoly on their innovations for more than one period. To this end, we build on the model assumptions of Aghion and Howitt (1992).

We assume that a unit measure of identical firms conduct R&D to obtain a proportional quality upgrade on a capital variety. Such R&D effort is undirected, i.e., firms cannot target any specific variety. After successfully innovating, a firm's monopoly lasts until either imitation occurs with exogenous probability $\iota \in (0, 1)$ or the variety quality is improved upon by another firm, i.e., it is creatively destroyed by a new innovation. This happens

with endogenous, time-varying probability $\phi(\psi_t)$ in equilibrium. Therefore, the symmetric problem of R&D firms is

$$\max_{R_t} \phi\left(\frac{R_t}{\eta \bar{q}_{t-1}}\right)\mathbb{E}_j W_{j,t} - \left(\frac{\bar{q}_t}{\vec{q}_t}\right)R_t \tag{B19}$$

where, due to undirected nature of R&D effort, the value of a new innovation $\mathbb{E}_j W_{j,t}$ is the average of the expected present value of monopoly on a variety j obtained at t, denoted by $W_{j,t}$, across varieties j. $W_{j,t}$ is

$$W_{j,t} = \Pi_{j,t} + \sum_{l=1}^{\infty}\left(\prod_{h=1}^{l}(1-\phi(\psi_{t+h}))\right)(1-\iota)^l B^l \Pi_{j,t+l} \tag{B20}$$

where $\Pi_{j,t+l}$ are time $t+l$ profits on variety j, discounted accounting for the cumulative probability of being neither creatively destroyed nor imitated until time $t+l$, i.e., $(1-\iota)^l \prod_{h=1}^{l}(1-\phi(\psi_{t+h}))$. Both imitation and creative destruction are taken as given by the firm when choosing how much to invest in R&D. Therefore, the optimal R&D intensity depends on both current and future investment as well as on future innovation. The optimality condition for innovation intensity is as follows:

$$\phi'(\psi_t)c^{1-\frac{1}{\gamma}}(1-\gamma)^{\frac{2}{\gamma}-1}\gamma\left[\lambda_t^{\frac{1}{\gamma}}M_t + \sum_{z=1}^{\infty}\left(\prod_{h=1}^{l}(1-\phi(\psi_{t+h}))\right)(1-\iota)^l B^l \lambda_{t+l}^{\frac{1}{\gamma}} M_{t+l}\right] = \left(\frac{\bar{q}_t}{\vec{q}_t}\right)^{\chi_1}. \tag{B21}$$

The fraction of monopolistic varieties in the new capital bundle also depends on imitation and destruction. Specifically, it evolves according to the law of motion

$$\zeta_t = (1-\iota)(1-\phi(\psi_t))\zeta_{t-1} + \phi(\psi_t), \tag{B22}$$

which emerges from the fact that, at any time t, a measure $\phi(\psi_t)$ of varieties becomes monopolistic due to successful innovation, a measure $(1-\iota)(1-\phi(\psi_t))\zeta_{t-1}$ remains monopolistic even though not improved because it is neither imitated nor creatively destroyed, and a measure $(1-\zeta_{t-1})(1-\phi(\psi_t)) + \zeta_{t-1}(1-\phi(\psi_t))\iota$ is competitive, because imitated at t or in the past and not creatively destroyed.

Therefore, the equilibrium replacement cost is

$$F_t = \frac{c\kappa^{\frac{1}{1-\gamma}}\left[1+(\eta-1)\phi(\psi_t)\right]^{\frac{1}{1-\gamma}}}{\left[1+(\eta(1-\gamma)^{\frac{1}{\gamma}}-1)\phi(\psi_t) + ((1-\gamma)^{\frac{1}{\gamma}}-1)(1-\iota)(1-\phi(\psi_t))\zeta_{t-1}\right]^{\frac{\gamma}{1-\gamma}}} \tag{B23}$$

B.6.2 Quantitative Analysis

For the quantitative analysis, we calibrate the imitation rate $\iota = 0.8$ and we adjust the curvature of the probability of successful innovation to $\phi_0 = 0.48$ to ensure that BGP growth rate and replacement frequency are the same as in the baseline calibration.

Figure B4 illustrates the evolution of investment, quality-adjusted capital and output in response to a 1% decline in exogenous TFP z_t with an autocorrelation coefficient equal to 0.85. The observed response is qualitative and quantitatively similar to that observed in the baseline model. As investment drops due to the shock, capital and innovation decline. The reduction in innovation is smaller than with one-period monopoly, as innovators consider the future recovery in replacement. Nevertheless, quality drops and the recovery of output is sluggish.

B.7 Model with Multi-period Time to Build

Section 4 assumes a one-period time-to-build for capital—i.e., that replacement decisions made at time t affect capital used in production at time $t+1$. In this subsection, we extend the model to a two-period time-to-build setting to reflect the 2.5 years average delivery time of Boeing aircraft. The innovation block of the model remains identical to the baseline economy. In contrast, the replacement problem of final good producers in the balanced growth path features an additional state variable, namely the replacement decision at the previous date x_{-1}, which determines the law of motion of capital.

$$V(k, s, x_{-1}) \equiv \max_{x \in \{0,1\}} sk - xf + \beta \mathbb{E} V\left[(k', s', x) \,|s\right], \tag{B24}$$

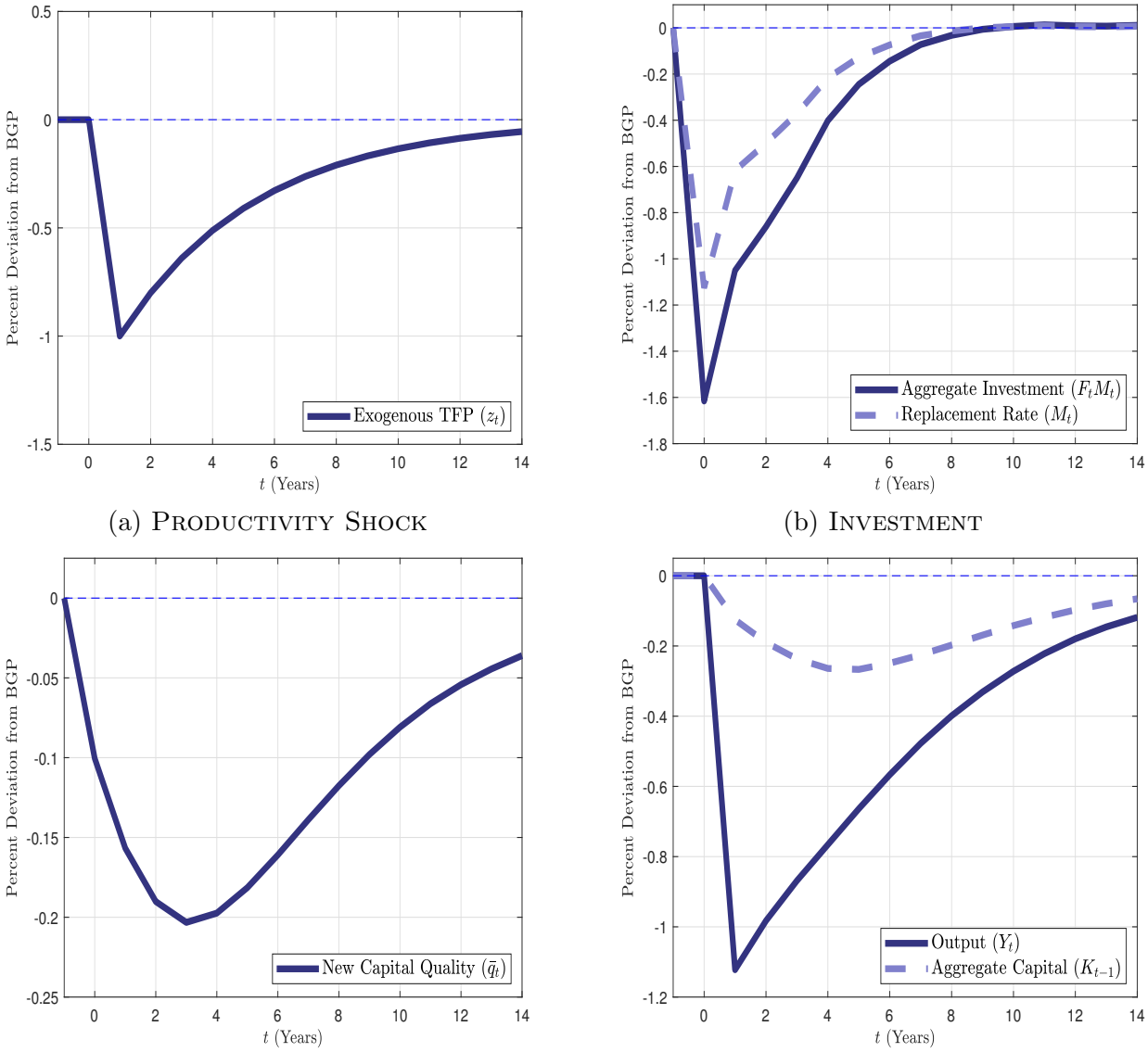
subject to the transition

$$k' = \begin{cases} \frac{1-\delta}{1+g} k & \text{if } \; x_{-1} = 0 \\ \frac{\kappa}{(1+g)} q_{-1} & \text{if } \; x_{-1} = 1, \end{cases}$$

where $\beta \equiv B(1+g)$, we use lower-case variables to denote upper-case variables divided by the growth rate $(1+g)^t$, such as $k \equiv \frac{K_{t-1}}{(1+g)^t}$, and q_{-1} is previous-period average quality of capital, $q_{-1} \equiv \frac{\bar{q}_{t-1}}{(1+g)^{t-1}}$.

As Figure B5 illustrates, the response of the economy to a negative TFP shock is broadly consistent with the findings of the baseline model, although the capital stock responds to the shock with an additional lag.

Figure B4: Negative TFP Shock – Multi-period Monopoly



(a) Productivity Shock

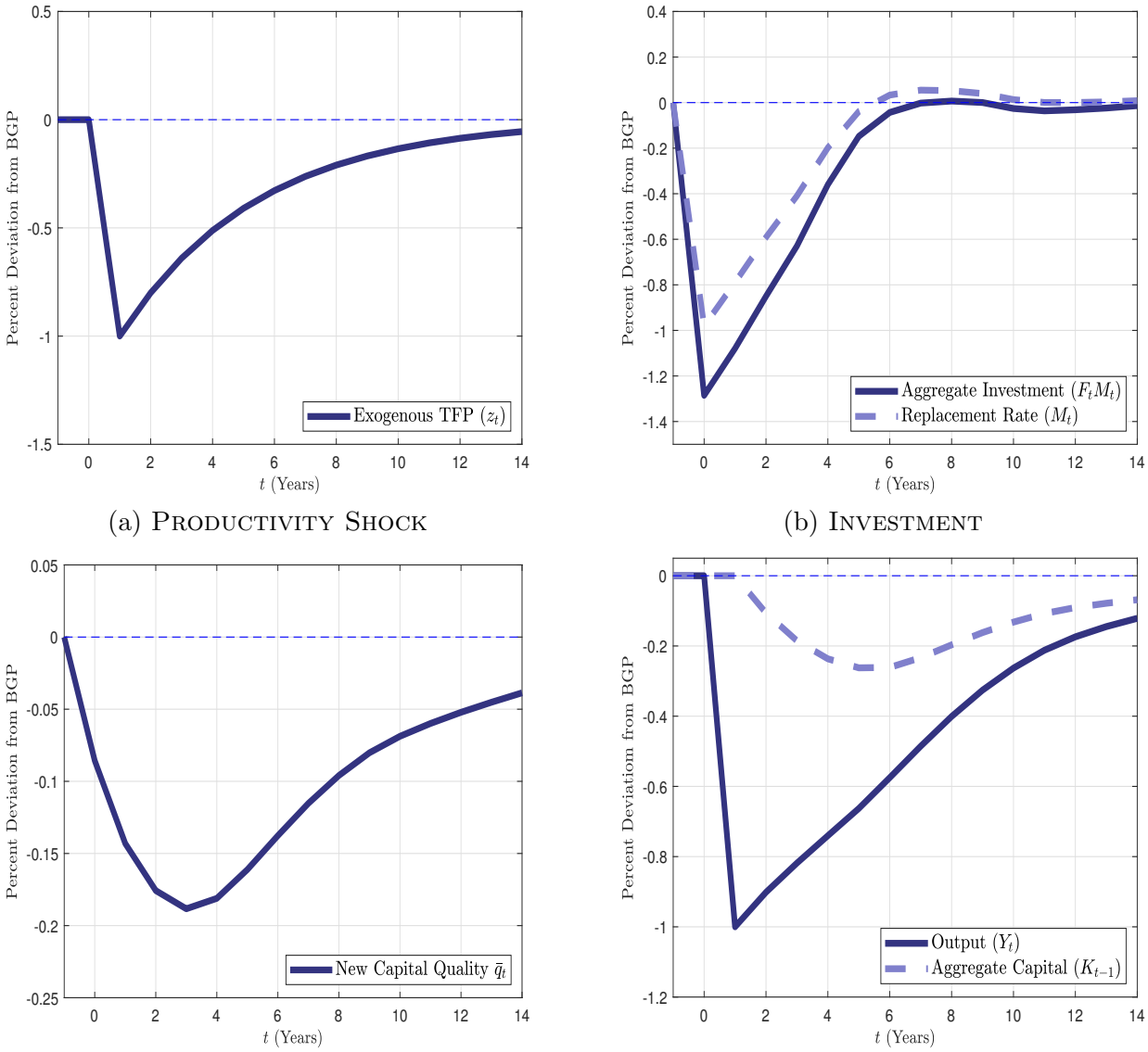
(b) Investment

(c) New-Capital Quality

(d) Output and Capital

Notes: The figure illustrates the response of the economy to a persistent 1% decline in exogenous TFP z_t at time $t = 1$, which agents learn at $t = 0$. Specifically, the panels display: (a) the exogenous path imposed to z_t; (b) the measure M_t of final-good producers replacing their capital and gross investment $F_t M_t$ in percent deviation from the BGP; (c) new-capital quality $\bar{q}_t$ in percent deviation from the BGP $\vec{q}_t$; (d) aggregate final output Y_t and aggregate quality-adjusted capital K_{t-1} in percent deviation from the BGP. We assume that the economy is on the BGP at time $t = -1$ and that all agents have perfect foresight of the evolution of all exogenous and endogenous variables.

Figure B5: Negative TFP Shock – Longer Time to Build



(a) Productivity Shock

(b) Investment

(c) New-Capital Quality

(d) Output and Capital

Notes: The figure illustrates the response of the economy to a persistent 1% decline in exogenous TFP z_t at time $t = 1$, which agents learn at $t = 0$. Specifically, the panels display: (a) the exogenous path imposed to z_t; (b) the measure M_t of final-good producers replacing their capital and gross investment $F_t M_t$ in percent deviation from the BGP; (c) new-capital quality $\bar{q}_t$ in percent deviation from the BGP $\vec{q}_t$; (d) aggregate final output Y_t and aggregate quality-adjusted capital K_{t-1} in percent deviation from the BGP. We assume that the economy is on the BGP at time $t = -1$ and that all agents have perfect foresight of the evolution of all exogenous and endogenous variables.

B.8 Model with Multi-Stage R&D Projects

In this subsection, we analyze a model extension where innovation requires the completion of multiple R&D stages to obtain a quality improvement on a capital variety. Therefore,

we introduce in the baseline model of Section 4 R&D lags as observed in several industries including aerospace.

B.8.1 Environment

We assume that capital-goods producers that seek to innovate on variety j must achieve N quality improvements of step-size $\theta > 1$ relative to $q_{j,t-1}$ in order to obtain an innovation with overall step size η. For simplicity, we consider $N = 2$ and we index each R&D stage by $n = 1, ..., N$.

Therefore, the innovation problem for the incumbent innovator on capital variety j at different stages of the R&D process is described recursively by the following value functions $W_t(n = 1)$ and $W_t(n = 2)$, indexed by n:

$$W_{j,t}(1) = \max_{R_{j,1,t}} \phi\left(\frac{R_{j,1,t}}{\theta q_{j,t-1}}\right) BW_{j,t+1}(2) + \left[1 - \phi\left(\frac{R_{j,1,t}}{\theta q_{j,t-1}}\right)\right] BW_{j,t+1}(1) - \left(\frac{\overline{q}_t}{\vec{q}_t}\right)^{\chi_1} R_{j,1,t} \tag{B25}$$

$$W_{j,t}(2) = \max_{R_{j,2,t}} \phi\left(\frac{R_{j,2,t}}{\theta q_{j,t-1}}\right) \Pi_{j,t} + \left[1 - \phi\left(\frac{R_{j,2,t}}{\theta q_{j,t-1}}\right)\right] BW_{j,t+1}(2) - \left(\frac{\overline{q}_t}{\vec{q}_t}\right)^{\chi_1} R_{j,2,t} \tag{B26}$$

After a successful innovation, out of a fringe of potential innovators on variety j one firm obtains an idea to develop into a new quality upgrade. Equation (B25) represents the value of this firm and its R&D problem in the first innovation stage. Given the first-stage R&D investment $R_{j,1,t}$—where j indexes the variety, the second subscript represents the R&D stage, and t is time—the firm successfully proceeds to the second R&D stage with probability $\phi\left(\frac{R_{j,1,t}}{\theta q_{j,t-1}}\right)$, obtaining the associated value function $W_{t+1}(2)$. With complementary probability $1-\phi\left(\frac{R_{j,1,t}}{\theta q_{j,t-1}}\right)$ first-stage R&D is unsuccessful and the firm tries again next period. As in the baseline model, the R&D marginal cost scales with deviations of current aggregate quality from its long-run trend.

Equation (B26) represents the R&D problem in the second R&D stage. Given the R&D investment of $R_{j,2,t}$ units of final good the firm successfully obtains the innovation and thus the profits $\Pi_{j,t}$ with probability $\phi\left(\frac{R_{j,2,t}}{\theta q_{j,t-1}}\right)$. With complementary probability $1-\phi\left(\frac{R_{j,2,t}}{\theta q_{j,t-1}}\right)$ second-stage R&D is unsuccessful and the firm tries again next period. The final term of the value function represents second-stage R&D cost.

Optimal R&D intensity in the two stages is determined by the conditions

$$\begin{aligned}\phi'(\psi_{j,1,t})\frac{1}{\theta q_{j,t-1}}B\left[W_{j,t+1}(2)-W_{j,t+1}(1)\right]-\left(\frac{\overline{q}_t}{\vec{q}_t}\right)^{\chi_1}=0\\ \phi'(\psi_{j,2,t})\frac{1}{\theta q_{j,t-1}}B\left[\Pi_{j,t}-W_{j,t+1}(2)\right]-\left(\frac{\overline{q}_t}{\vec{q}_t}\right)^{\chi_1}=0\end{aligned} \tag{B27}$$

where $\psi_{j,1,t}\equiv\frac{R_{j,1,t}}{\theta q_{j,t-1}}$ and $\psi_{j,2,t}\equiv\frac{R_{j,2,t}}{\theta q_{j,t-1}}$. We guess and verify that the value functions $W_{j,t}(1)$ and $W_{j,t}(2)$ are linear in $q_{j,t-1}$ and thus R&D intensities $\psi_{1,t}$ and $\psi_{2,t}$ are common across all varieties j characterized by the same R&D stage.

Therefore, we can express the law of motion of new-capital quality as

$$\overline{q}_t=\phi(\psi_{2,t})\Gamma_{t-1}(2)\eta\overline{q}_{t-1}+\left[1-\phi(\psi_{2,t})\Gamma_{t-1}(2)\right]\overline{q}_{t-1}, \tag{B28}$$

where $\phi(\psi_{2,t})\Gamma_{t-1}(2)$ represents the measure of capital varieties on which successful innovation occurs at time t. This measure is the product of the mass of varieties in the second R&D stage at $t-1$, i.e., $\Gamma_{t-1}(2)$, and the homogeneous success probability of second-stage R&D, i.e., $\phi(\psi_{2,t})$.

The distribution of capital varieties over R&D stages $n=1,2$ satisfies the following law of motion

$$\begin{aligned}\Gamma_t(1)&=\Gamma_{t-1}(1)(1-\phi(\psi_{1,t}))+\Gamma_{t-1}(2)\phi(\psi_{2,t})\\ \Gamma_t(2)&=\Gamma_{t-1}(2)(1-\phi(\psi_{2,t}))+\Gamma_{t-1}(1)\phi(\psi_{1,t}).\end{aligned} \tag{B29}$$

The first equation states that the fraction of varieties in the first R&D stage at time t is the sum of those at the same stage at $t-1$ for which R&D has not been successful and those for which second-stage R&D has been successful and re-start the multi-stage innovation process. The second equation expresses the time-t measure of second-stage varieties in a similar fashion.

At any point in time, a fraction $\Gamma_{t-1}(2)\phi(\psi_{2,t})$ of capital varieties are monopolistically priced and improved their quality by a factor η, while the complementary fraction $1-\Gamma_{t-1}(2)\phi(\psi_{2,t})$ is competitive and experiences no quality upgrade. This allows us to use the capital-bundle definition to derive the Lagrange multiplier of the cost-minimization problem of final-good producers that choose to replace their capital. We can also derive the price of new capital in a similar fashion.

B.8.2 Quantitative Analysis

We analyze the effects of a negative TFP shock in this model extension. For the simulation, we focus on our aircraft calibration—for which R&D lags may be particularly relevant—and adjust parameter value of the curvature of the innovation probability function ($\phi_1 = 19.8$) to match a 0.6% quality growth in the balanced growth path of the multi-stage R&D economy.

Figure B6 illustrates the response of the multi-stage R&D economy to the same aggregate productivity shock analyzed in Figure 4. The presence of R&D lags generates similar dynamics to the baseline model. The replacement rate and total investment remain depressed for four years after the shock, inducing a persistent decline in quality and quality-adjusted capital. These dynamics contribute to the sluggish medium-term recovery in aggregate output.

B.9 Model with Adjustment Costs

In this subsection, we analyze a model extension where capital replacement not only requires the payment of the cost F_t but also involves an adjustment cost proportional to output, consistent with Cooper and Haltiwanger (2006). Specifically, we assume that the adjustment cost equals:

$$\begin{cases} 0 & \text{if } \; x_{i,t} = 0 \\ \xi y_{i,t} = \xi z_t s_{i,t} K_{i,t-1} & \text{if } \; x_{i,t} = 1 \end{cases} \tag{B30}$$

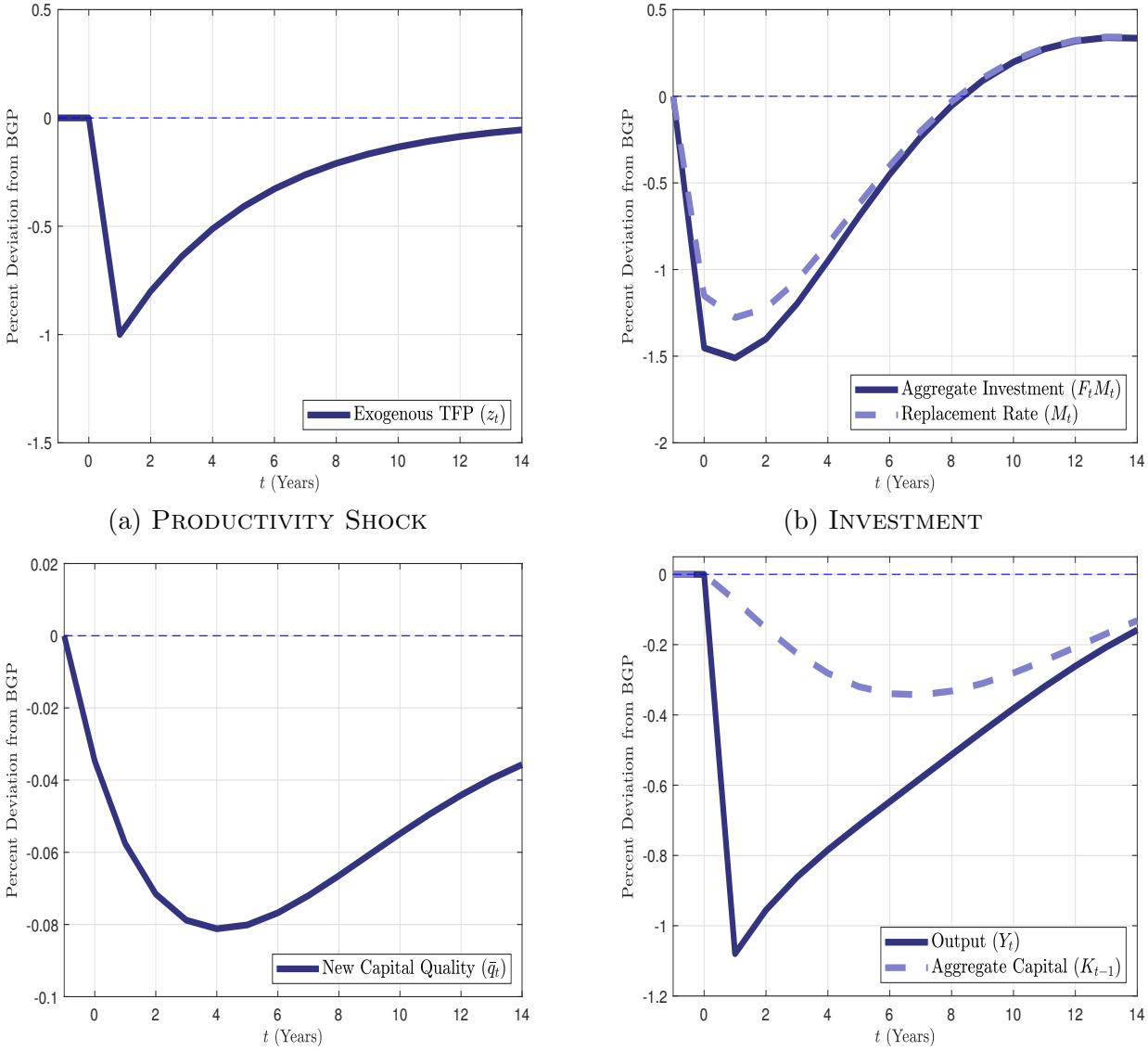
and we calibrate $\xi = 0.204$ (Cooper and Haltiwanger, 2006). Therefore, firms lose approximately 20% of their output in the period when they decide to replace capital.

B.9.1 Quantitative Analysis

We analyze the effects of a negative TFP shock in this model extension. For the simulation, we adjust our baseline calibration of the investment scale parameter to $\kappa = 0.88$ to match a 2% quality growth rate in the balanced growth path equilibrium of the model.

Figure B7 illustrates the response of the economy with adjustment costs to the same aggregate productivity shock analyzed in Figure 4. The presence of adjustment costs proportional to output generates similar dynamics to the baseline model. The replacement rate and total investment remain depressed for four-to-six years after the shock, inducing

Figure B6: Negative TFP Shock – R&D Lags



(a) Productivity Shock

(b) Investment

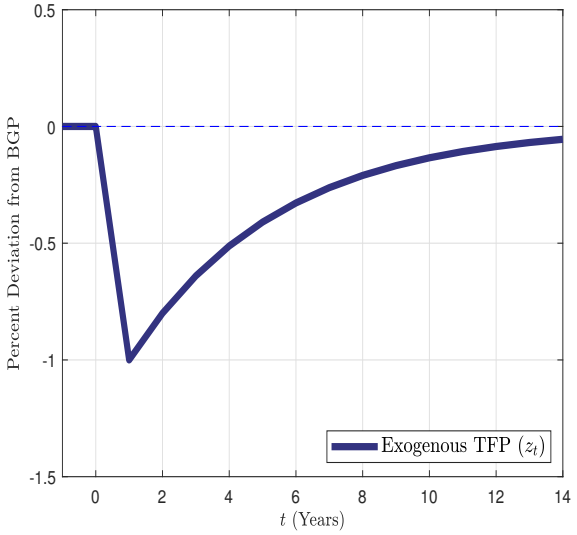
(c) New-Capital Quality

(d) Output and Capital

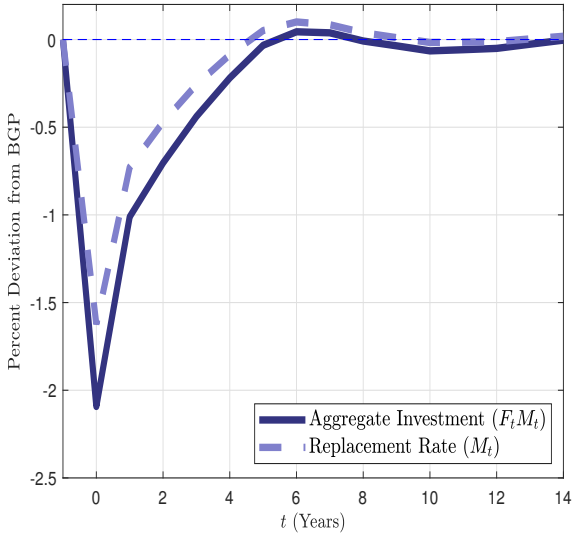
Notes: The figure illustrates the response of the multi-stage R&D economy under the aircraft calibration to a persistent 1% decline in exogenous TFP z_t at time $t = 1$, which agents learn at $t = 0$. Specifically, the panels display: (a) the exogenous path imposed to z_t; (b) the measure M_t of final-good producers replacing their capital and gross investment $F_t M_t$ in percent deviation from the BGP; (c) new capital quality $\bar{q}_t$ in percent deviation from the BGP $\vec{q}_t$; (d) aggregate final output Y_t and aggregate quality-adjusted capital K_{t-1} in percent deviation from the BGP. We assume that the economy is on the BGP at time $t = -1$, that all agents learn about the shock at $t = 0$, and that they have perfect foresight of the evolution of all exogenous and endogenous variables.

a persistent decline in quality and quality-adjusted capital. These dynamics contribute to the sluggish medium-term recovery in aggregate output.

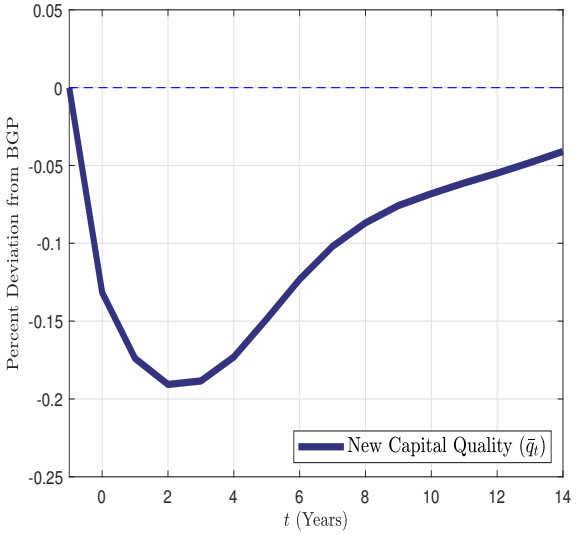
Figure B7: Negative TFP Shock –Adjustment Costs



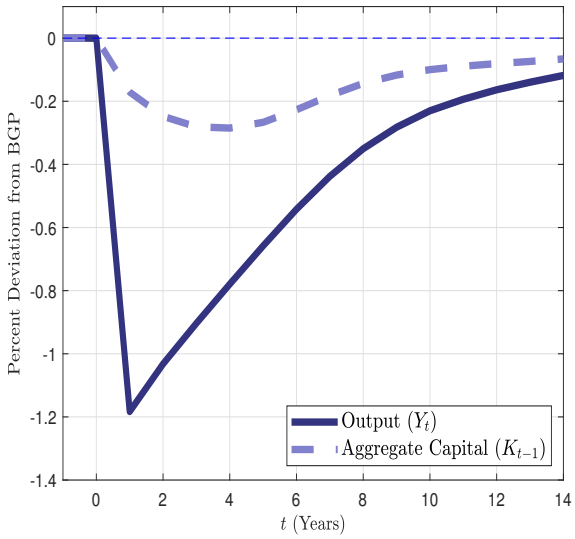
(a) Productivity Shock



(b) Investment



(c) New-Capital Quality



(d) Output and Capital

Notes: The figure illustrates the response of the economy with capital-adjustment costs proportional to output to a persistent 1% decline in exogenous TFP z_t at time $t = 1$, which agents learn at $t = 0$. Specifically, the panels display: (a) the exogenous path imposed to z_t; (b) the measure M_t of final-good producers replacing their capital and gross investment F_tM_t in percent deviation from the BGP; (c) new capital quality $\bar{q}_t$ in percent deviation from the BGP $\vec{q}_t$; (d) aggregate final output Y_t and aggregate quality-adjusted capital K_{t-1} in percent deviation from the BGP. We assume that the economy is on the BGP at time $t = -1$, that all agents learn about the shock at $t = 0$, and that they have perfect foresight of the evolution of all exogenous and endogenous variables.