Capital Structure and ESG Integration

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Abstract

We analyze how borrowers' capital structure affects their incentives to integrate ESG. Borrowers may make socially valuable but financially underperforming investments to reduce expected payments to outside investors. These financial gains increase as the expected payoffs of securities issued for borrowers' investments sensitively change with financial profitability. ESG-focused investors may accept financial losses to fund ESG investments, particularly when borrowers have relatively weak social preferences. However, low capital costs can shift borrowers' focus back to financial profits, although investors hold strongly sensitive securities to ensure ESG integration. This incentive misalignment is mitigated when all competing investors bid with equities.

Keywords: capital structure, ESG, greenwashing, moral hazard, security design

JEL Codes: D82, D86, G32, G38

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1 Introduction

Over the past decade, environmental, social, and governance (ESG) has experienced great growth in interest and attention but recently has faced a significant pushback (Bloomberg, 2024). One important reason for this, especially in the United States, is the concern about greenwashing from both corporate issuers and investors (Reuters, 2020; SEC, 2022). Specifically, it is difficult to assess whether firms that receive capital then integrate ESG into their decision-making as they have promised to do. Several factors can impede discerning borrowers' ESG performance: ESG is an amorphous concept (i.e., not clearly defined); there are fundamental complexities and opacity surrounding ESG metrics; and objective and reliable information on companies' ESG performance is not yet fully available (Berg, Koelbel and Rigobon, 2022; Serafeim and Yoon, 2023; Gipper, Ross and Shi, 2022). These features hinder asset allocators' ability to assess companies' follow-through on ESG commitments. The difficulty in enforcing corporate borrowers' ESG integration raises the following important research questions of interest: how can investors, especially the large asset allocators who care about sustainable finance, provide capital to those truly willing to follow through on their ESG promises? Which financial factors crucially influence corporate borrowers' incentive to integrate ESG?

To address these questions, this paper explores the role that corporate borrowers' capital structure plays in their ESG integration and follow-through. Specifically, it develops a model in a security design framework where a firm borrows from outside investors to finance a project. In our model, the firm invests the funds in either a green (G) or brown (B) project. Project G represents a socially valuable investment with low financial profitability; it is likely to yield low financial returns compared to project B but, separate from the financial return, generates a positive social return in the form of an externality. In contrast, project B represents a financially profitable but socially harmful investment; it yields zero social return but is likely to generate high financial returns compared to project G. Investors compete to finance the firm's project in the capital market. Each investor bids with a *security*, which specifies a repayment schedule, contingent on every realized financial return of the firm's project. The firm borrows from the investor offering the best bid and chooses the project that best serves its own interests.

Our model entails several assumptions that directly incorporate the distinctive features of the ESG financial market. First, investors and the firm have heterogeneous preferences for nonfinancial social value. One group of investors is the green type, whose utility function is a

linear sum of the financial payoff and the social return with an equal weight. That is, green investors consider the social impact of their financial investments as heavily as the financial income. Another group of investors is the brown type, whose utility function equals financial income. These investors focus only on financial payoffs, as traditional investors do in the standard corporate finance theory. Lastly, the firm's utility function is a linear sum of the social return from its project and financial income with different weights, where the weight put on the social return represents the firm's preference for social value. To introduce incentive misalignment, we assume that the firm prefers project B if it can finance itself, while every green investor strictly prefers project G if she can undertake the project herself.

A more critical assumption is that the firm's project selection after borrowing from an investor is not contractually enforceable, which reflects the current state of the world (i.e., investors' inability to enforce a firm's follow-through on ESG commitments, which leads to greenwashing). Indeed, in our model, a key determinant of the firm's equilibrium project selection is the *sensitivity* of the security held by an investor selected to finance the firm's project, formally measured by the difference in the expected payment between projects G and B. Since high financial returns are more likely to be realized under project B than G, a security is strongly sensitive if its repayment schedule increases steeply for high financial returns but gradually for low ones. In this context, debt contracts are the least sensitive: the investor secures the full financial return if the firm fails to pay the face value, due to the realization of low financial returns, whereas the investor receives a fixed amount of financial repayment equal to the face value only when the firm's project yields high financial returns. By contrast, levered equity (or call option) contracts effectively turn the firm into a debtholder and thus are the most sensitive.

In any equilibrium, the firm chooses project G if and only if the security issued to raise capital is sufficiently sensitive to project selection. Selecting project G instead of B impacts the firm's payoff in three ways. First, the firm enjoys a higher social return from project G than B. Second, since low financial returns are more likely to be realized under project G, the expected financial return of the firm's project decreases as the firm chooses project G. By the same token, the expected payment to an investor from the firm's project shrinks as the firm chooses project G instead of B. By the definition of sensitivity, only the impact of the firm's project selection on the expected payment to the funding investor is amplified if the security specifying the payment becomes more sensitive, whereas the other impacts remain unchanged. Therefore, the firm's total benefit from choosing project G substantively increases when the security held by a funding investor is strongly sensitive. Furthermore, the minimum

level of security sensitivity incentivizing the investment in project G decreases with the firm's preference for social value. That is, the firm's financial gains from lowering the expected payment need not be very high to induce socially valuable investments when the firm itself strongly prefers social value.

The equilibrium is fully characterized by the firm's preference for social value. First, suppose the firm's preference for social value is weak. In that case, brown investors believe the firm will choose project B and thus competitively bid with securities that give zero expected financial payoff under financially profitable project B. These securities do not have to require large repayments for high financial returns to break even for the bidding investors. Hence, the firm finds the brown investors' security bids insensitive and thus chooses project B. A green investor may want the firm to select project G by direct financing, but she must bid with a strongly sensitive security by charging substantively large shares of financial returns, which the firm will reject outright, due to the high cost of capital. Next, if the firm's preference for social value is strong, brown investors believe the firm will choose project G. Thus, they must charge large repayments for high financial returns to ensure their investments break even under a financially under-profitable project G. Such a repayment structure increases the sensitivity of the securities held by brown investors, incentivizing the firm to choose project G. Green investors have no incentive to compete with brown investors; they enjoy the social return from project G funded by brown investors as free-riding benefits.

Lastly, suppose the firm's preference for social value is neither strong nor weak. In contrast to the previous equilibrium outcomes, a green investor must win the competition to finance the firm's project by taking a financial loss offset by the social return from project G. This green investor's loss-making bid discourages brown investors from bidding, as they do not want to sacrifice their financial payoffs. Furthermore, the green investor's security bid should be sensitive enough for the firm to (weakly) prefer project G, necessarily compensating the winning green investor for the financial loss. Lastly, some investors must place "losing" bids with insensitive securities, which the firm will accept (and then it will choose project B), should the single "winning" green investor request slightly larger shares of financial returns to reduce her financial loss. This equilibrium bidding strategy profile implies a multiplicity of equilibria; the equilibrium is endogenously determined by the winning green investor's belief about the other investors' losing bids and vice versa. In particular, when investors believe the capital market is intensely competitive, the green investor decides to admit a considerably large financial loss to win the lending game, making her winning security bid insensitive. Then, there exists a mixed-strategy equilibrium where the firm becomes indifferent between

project G and B and chooses socially harmful project B with some positive probability.

The equilibrium bidding strategy profile above offers a potential route in the context of corporate capital structure, wherein the firm attracts multiple security bids with varying sensitivities. This prompts an exploration into the potential outcomes if investors opt for conventional securities, such as debt or equity. Under a debt financing scenario, the equilibrium choice for the firm tends toward project B. Here the insensitivity inherent in debt contracts constrains the firm's ability to derive substantial financial benefits from opting for the less profitable project G with lowered expected payments to investors. Conversely, with (levered) equity financing, the firm leans toward project G in equilibrium. Equity contracts that are the most sensitive to project selection incentivize the firm to minimize expected payments to investors, thus favoring project G. Our analysis underscores the notion that equity financing, instead of debt financing, can better facilitate ESG integration. Note that this starkly differs from findings in the literature on security design, which often advocate debt financing as optimal in various asymmetric information scenarios (Myers and Majluf, 1984; Innes, 1990; Nachman and Noe, 1994).

This paper proceeds as follows. Section 2 presents our model and Section 3 characterizes incentive conditions that determine the investment decision. In Section 4, we provide full equilibrium analysis and discuss policy implications. Section 5 discusses related literature and Section 6 concludes. All proofs are deferred to the appendix.

2 Model

2.1 Setup

Consider a static economy with a firm running a financial project and investors competing to fund the firm's project. When the game begins, each investor competitively offers to finance the firm's project, and the firm borrows from the investor who provides the most favorable offer. The firm then chooses a certain type of financial project that maximizes its total payoff. At the end of the game, the firm's project generates returns distributed to the firm and the funding investor according to the payment terms they agreed to. We assume every player in the economy is risk-neutral.

The firm chooses one of two financial projects: green (G) or brown (B). To run its project, the firm must pay a unit initiation cost. Project G yields a stochastic financial

return $x \in [0, \infty)$, a continuous random variable drawn from a density function $f_G(\cdot)$ (and the corresponding distribution function $F_G(\cdot)$). Project G also generates a constant social return $\phi > 0$ in the form of an externality. That is, every investor can enjoy ϕ, once the firm chooses project G, even though she does not directly finance the firm. Note that the main results of our paper endure even if the social return ϕ from project G is a random variable; since every player is risk-neutral, each player will consider the expected value of ϕ when that player values the social return. Like project G, project B also yields the stochastic financial return x, a continuous random variable drawn from a density function $f_B(\cdot)$ (and the corresponding distribution function $F_B(\cdot)$). However, in contrast to project G, project B generates no social return. We assume the monotone likelihood ratio property (MLRP) between project B and G such that $f_B(x)/f_G(x)$ increases with x. In other words, project B (G) is more likely to generate a high (low) financial return than project G (B).[1] For the exposition, we define

$$\mathbb{E}_i[h(x)] := \int_0^\infty h(x) f_i(x) dx$$

for any integrable function $h(\cdot)$ and $i \in \{G, B\}$.

We further assume that the firm's utility is represented by a linear sum of financial income with a unit weight and social return with a different weight $\lambda \in (0, 1)$, which captures the firm's (inherent) preference for social value. This utility representation is consistent with the approaches taken by the theoretical literature on ESG integration in financial markets (Chowdhry, Davies and Waters, 2019; Pástor, Stambaugh and Taylor, 2021; Broccardo, Hart and Zingales, 2022; Goldstein et al., 2022; Oehmke and Opp, 2024). Chaigneau and Sahuguet (2024, pp. 13–15) and Chang, Rhee and Yoon (2024, p. 6) also extensively survey the literature empirically studying that socially responsible investments can enhance long-term firm value in different forms, such as preferential regulatory treatments, increased labor productivity, and improved consumer awareness.

The firm is assumed to be cashless, so it has to finance its project externally from an outside investor. In the capital market, many investors, each endowed with unit cash, compete to invest in the firm's project in Bertrand fashion. Specifically, each investor, if she wants to fund the firm's project, bids with a *security* defined by a continuous function $s : [0, \infty) \to [0, \infty)$ that specifies repayment, conditional on every realized financial return x.

[1] In the security design literature, the MLRP assumption is often introduced for ranking different investment projects based on their stochastic financial returns (Inderst and Mueller, 2006; Axelson, 2007). Furthermore, Milgrom (1981) finds that many standard continuous distributions satisfy the MLRP.

As standard in the literature of security design, we restrict investors to offer securities $s(x)$ satisfying the following conditions for every $x \in [0, \infty)$: *(i)* both $s(x)$ and $x - s(x)$ weakly increase with x; *(ii)* $0 \leq s(x) \leq x$ (Nachman and Noe, 1994; Hart and Moore, 1995; DeMarzo, Kremer and Skrzypacz, 2005). To provide a general analysis, we do not restrict the types of feasible securities that investors bid to fund the firm's project. This flexibility in the design of securities is akin to the cases in which venture capitalists tailor repayment terms in their favor when financing startups, which typically have relatively short credit histories or little collateral and thus have limited access to the retail market (Yang and Zeng, 2019). However, as one can see in Section 4.3, our analysis can be extended to other cases where the firm raises capital by issuing standard securities, such as debt or equity, corresponding to the financing of public firms' projects.

Of all available offers by bidding investors, the firm finances from an investor who makes the most favorable bid by issuing the security to the investor corresponding to her bid. If multiple investors make the most favorable offer, the firm randomly chooses one of the investors with equal likelihood. Formally, if the firm decides to accept a security s and there are $n \in \mathbb{N}$ investors who bid with that same security s, the firm ends up contracting with each of them with probability $1/n$. We interpret this randomization as a move of a chance node rather than the firm's voluntary randomization.

We observe a few key characteristics in the real world with regard to ESG in the capital markets and incorporate them into our model by making the following assumptions. First, investors have heterogeneous preferences for the social return the firm can generate by choosing project G. Specifically, an investor is of the green type to the extent that a linear sum of financial payoff and social return with equal weights represents her utility. This green investor represents a group of market participants recently emerging in the capital market, that is, those who heavily consider the social impacts of their investments. The assumption of a single green investor is made for analytical convenience: the main results of our paper still stand even when we allow multiple green investors. Furthermore, there is more than one brown investor whose utility function is simply equal to the financial payoff only. These brown investors are akin to traditional investors in the corporate finance literature.

Second, and perhaps more importantly, the firm's project choice after funding from an investor cannot be contractible. This assumption is crucial to incorporate corporate borrowers' "greenwashing" behavior in the real world into our model. That is, the firm makes its investment decision to maximize its payoff, even though this decision leads to lower production of the social return, possibly against the preferences of the green investor. Lastly, we assume

the following regularity conditions that incorporate potential incentive misalignment between the green investor and the firm into our model.

Assumption 1. *(i)* $\mathbb{E}_G[x] + \phi > \mathbb{E}_B[x] > \mathbb{E}_G[x] + \lambda\phi$*; (ii)* $\mathbb{E}_B[x] > \mathbb{E}_G[x] > 1$*; (iii)* $\phi \leq 1$.

Assumption 1-(i) means that project G generates higher total surplus than project B from the green investor' perspective ($\mathbb{E}_G[x] + \phi > \mathbb{E}_B[x]$), while the firm will invest in project B if it can self-finance ($\mathbb{E}_B[x] > \mathbb{E}_G[x] + \lambda\phi$). Assumption 1-(ii) means that both project G and B are financially profitable, while project B is more lucrative. This assumption rules out a trivial equilibrium in which project G is never chosen due to a negative net present value. Lastly, Assumption 1-(iii) means that the green investor is unwilling to sacrifice all the financial payoffs, even if the firm invests in project G. Analytically, Assumption 1-(iii) is made to rule out a trivial equilibrium in which a (green) investor charges zero interest for funding the firm's green project.

2.2 Technical Preliminaries: Sensitivity of Securities

In this section, we discuss some mathematical properties of securities that facilitate the understanding of our main analysis. First, we formalize some standard types of securities commonly issued and traded in real-world financial markets.

Definition 1.

(i) A debt contract with face value D is defined as $d(x, D) := \min\{x, D\}$*;*

(ii) A levered equity contract with a strike price S is defined as $c(x, S) := \max\{x - S, 0\}$.

Suppose the firm finances its project with a debt contract $d(x, D) = \min\{x, D\}$. When the firm's project generates a high financial return (i.e., $x \geq D$), a funding investor will always get D whether x is close to but above D or substantially higher than x. Furthermore, the investor's senior claim is fully secured by the project in the event $x < D$; i.e., the firm defaults on servicing its debt. Next suppose the firm finances from an investor with a levered equity contract $c(x, S) = \max\{x - S, 0\}$. Under such financing terms, the investor effectively becomes a residual claimant of the financial return of the underlying project; it is as if the investor owes a fixed amount of S to have a claim of the remaining financial return $x - S$. At the same time, the investor is protected by limited liability if the firm's project yields a low financial return $x < S$.

We next introduce a key characteristic of securities coined "sensitivity (to the project selection)," which is crucial to understanding our main analysis. Formally, the sensitivity of a security $s(\cdot)$ is defined as $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)]$, i.e., the differential of the expected payment to a funding investor between two projects. Note first that, since MLRP implies the first order stochastic dominance (FOSD), we have $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] \geq 0$ for any weakly increasing security $s(\cdot)$, where the inequality strictly holds whenever $s(x)$ is strictly increasing for a positive measure of x.

The sensitivity of a security $s(\cdot)$ is determined by how steeply $s(x)$ changes with x, particularly for high values of x. To provide intuition, consider an example of a binary random variable $x \in \{x_l, x_h\}$ with $x_l < x_h$ and $Pr(x = x_h) = p \in (0, 1)$. If $s(x_h) - s(x_l)$ is small, the expected payoff of $s(x)$ barely changes with p. An extreme case is $s(x) = x_l$ for every x: the expected payoff of $s(x)$ stays at x_l for every $p \in (0, 1)$. However, if $s(x_h) - s(x_l)$ is large, then the expected payoff of $s(x)$ significantly increases with p. For example, suppose $s(x) = x$ for every $x \in \{x_l, x_h\}$. Then the expected value of $s(x)$ is $p(x_h - x_l)$, whose derivative with respect to p is $x_h - x_l$, which is the highest of all other securities.

One question of interest is whether and how securities are ordered by their sensitivities. Since we impose few restrictions on the forms of the securities, infinitely many types of securities are feasible for bidding in our model, inhibiting our ordering them completely by sensitivity. However, as the following lemma states, we can find the least and most sensitive securities under the MLRP of the stochastic financial return x between project B and G.

Lemma 1 (ranking securities by sensitivity)**.**

(i) $\mathbb{E}_B[d(x, D)] - \mathbb{E}_G[d(x, D)]$ *is increasing in* D*.*

(ii) $\mathbb{E}_B[c(x, S)] - \mathbb{E}_G[c(x, S)]$ *is decreasing in* S*.*

(iii) Fix any security $s(\cdot)$ *that is not a debt contract. For each* $i = G$ *and* B*, there exists* $D_i \geq 0$ *such that* $\mathbb{E}_i[s(x)] = \mathbb{E}_i[d(x, D_i)]$ *and* $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] > \mathbb{E}_B[d(x, D_i)] - \mathbb{E}_G[d(x, D_i)]$*.*

(iv) Fix any security $s(\cdot)$ *that is not a levered equity contract. For each* $i = G$ *and* B*, there exists* $S_i \geq 0$ *such that* $\mathbb{E}_i[s(x)] = \mathbb{E}_i[c(x, S_i)]$ *and* $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] < \mathbb{E}_B[c(x, S_i)] - \mathbb{E}_G[c(x, S_i)]$*.*

We first analyze how the sensitivity of standard securities, such as debt and (levered) equity, is determined. First, the slope of a debt contract $d(x, D) = \min\{x, D\}$ is positive and

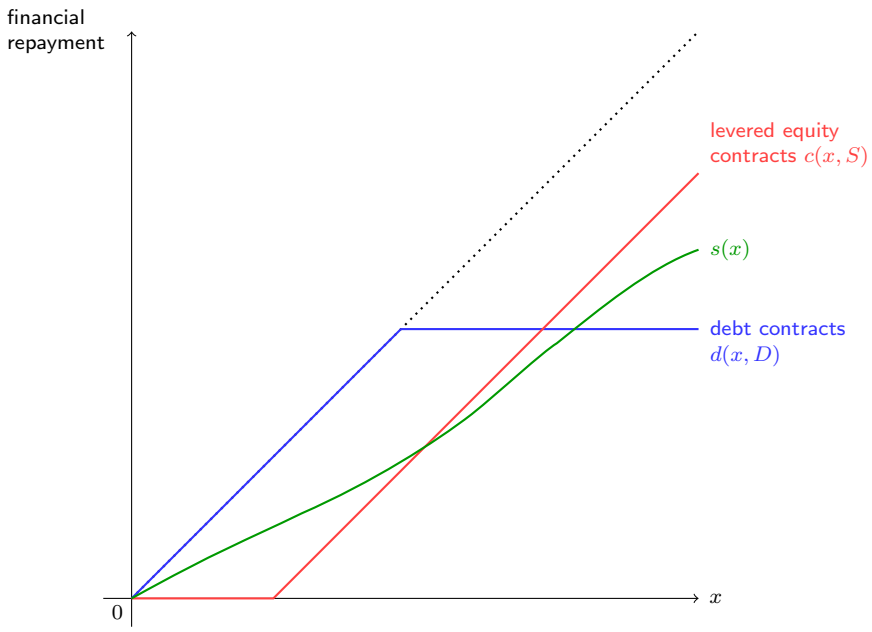


Figure 1 – Sensitivity of securities

steep (indeed equal to 1) if $x \leq D$ but is completely flat (i.e., equal to zero) if $x > D$. Thus, the range of x with which $d(x, D)$ increases expands as face value D increases, making the debt contract more sensitive to the firm's project selection (Lemma 1-(i)). By contrast, the slope of a levered equity contract $c(x, S) = \max\{x - S, 0\}$ is completely flat if $x \leq S$ but is the steepest if $x > S$. Hence, the subset of x with which $c(x, S)$ increases shrinks as the strike price S increases, making the levered equity contract less sensitive (Lemma 1-(ii)). Note that an increase in D also increases both $\mathbb{E}_G[d(x, D)]$ and $\mathbb{E}_B[d(x, D)]$. Similarly, decreasing S increases both $\mathbb{E}_G[c(x, S)]$ and $\mathbb{E}_B[c(x, S)]$. In short, standard securities become more sensitive as the firm gives an investor a larger share of the financial returns.

Lemma 1-(iii) and (iv) are the replication of DeMarzo, Kremer and Skrzypacz (2005). Indeed, the MLRP is useful for ranking securities based on their sensitivity. Figure 1 depicts the repayment schedules of several securities, such as debt contracts, levered equity contracts, and an arbitrary security, which facilitates comparing the sensitivity of the securities. First, the debt contract is the least sensitive of all securities with the same expected payment under a specific project type. The intuition is as follows: Compared to the other securities, the slope of the debt contract is the highest (equal to 1) for relatively low values of x (i.e., $x \leq D$) but is the lowest (equal to zero) for relatively high values of x (i.e., $x > D$). Thus, for any arbitrary security $s(\cdot)$, we have $s(x) < d(x, D)$ for relatively small values of x but have $s(x) > d(x, D)$

for relatively large values of x. The MLRP implies that low (high) values of x are more likely to be realized under project G (B). Hence, the debt contract $d(x, D)$ has a relatively high expected payoff compared to $s(x)$ under project G, while the opposite holds true under project B, implying that debt contracts are less sensitive than any other securities.

By contrast, the levered equity contract is the most sensitive security to project selection. Indeed, compared to the other securities, the slope of a levered equity contract $c(x, S)$ is the lowest (equal to zero) for relatively low values of x (i.e., $x < S$) but is the highest (equal to 1) for relatively high values of x (i.e., $x > S$). Thus, for any security $s(\cdot)$, we have $s(x) > c(x, S)$ for relatively small values of x and $s(x) < c(x, S)$ for relatively large values of x. Again, by the MLRP, the levered equity contract $c(x, S)$ has a relatively high expected payoff compared to $s(x)$ under project B, while the opposite holds true under project G, suggesting that levered equity contracts are more sensitive than any other types of securities.

Lastly, notice that our research focuses on understanding how securities' sensitivity influences the firm's incentive to make green investments, not the roles of specific security types in green financing. In this context, the main purpose of assuming the MLRP is to facilitate the ranking of securities based on their sensitivity. Namely, Lemma 1 shows that debts and equities, standard types of securities issued through public listings, simply represent the least and the most sensitive securities with the MLRP, respectively. The main result of our analysis will be qualitatively unchanged even in the absence of the MLRP once securities eligible for bidding are completely ordered in sensitivity.

3 Ex Post Incentive Conditions for Green Investment

We now proceed to the main analysis. We first analyze how the sensitivity of investors' security bids influences the incentive structure of the firm's project selection, which will eventually feed back into the investors' bidding strategies and thus shape the equilibrium outcome.

Fix a security $s^*(\cdot)$ that the firm accepts from a bidding investor in equilibrium. Then the firm will receive the expected payoff $\mathbb{E}_G[x - s^*(x)] + \lambda\phi$ from project G and $\mathbb{E}_B[x - s^*(x)]$ from project B, respectively. Hence, the firm's optimal project selection is type G if and only if $\mathbb{E}_G[x - s^*(x)] + \lambda\phi \geq \mathbb{E}_B[x - s^*(x)]$. Then doing simple algebra yields the following result:

Lemma 2. *Let $s^*(\cdot)$ denote an equilibrium security offer the firm accepts. Then, the firm*

chooses project G if

$$\mathbb{E}_B[s^*(x)] - \mathbb{E}_G[s^*(x)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi, \tag{1}$$

and the firm chooses project B if

$$\mathbb{E}_B[s^*(x)] - \mathbb{E}_G[s^*(x)] \leq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi. \tag{2}$$

Lemma 2 reveals an important necessary condition for the green investment: the security the firm issues to raise capital must be sufficiently sensitive to incentivize the firm to choose project G. How can the security's sensitivity influence the firm's incentive for project selection? The answer lies in the expected payoff of the security held by a funding investor. By choosing project G instead of B, the firm can reduce the expected payment to the funding investor by $\mathbb{E}_B[s^*(x)] - \mathbb{E}_G[s^*(x)]$. This effect amplifies as the repayment schedule specified by the security $s^*(\cdot)$ becomes more sensitive. Nevertheless, the positive impact of lowering the expected payment to the funding investor itself cannot overcome the lower profitability of project G than B. Indeed the firm suffers a net financial loss if it switches from project B to G, due to the decrease in the expected return of the project; $\mathbb{E}_G[x - s^*(x)] < \mathbb{E}_B[x - s^*(x)]$ since $x - s^*(x)$ is an increasing function of x and F_B is strict FOSD over F_G. However, this net financial loss from project G is partially compensated by its nonfinancial social return ϕ, which the firm adds to its payoff with a discounted weight λ. Furthermore, the decrease in the expected financial return by choosing project G (i.e., $\mathbb{E}_B[x] - \mathbb{E}_G[x]$) does not change with the sensitivity of the security $s^*(\cdot)$. Therefore, the security sensitivity influences the firm's financial gains by lowering the expected payment to the funding investor only. Thus, in addition to the social return, the total benefit of choosing project G can outweigh its cost when the security $s^*(\cdot)$ is strongly sensitive.

Furthermore, the minimum security sensitivity level supporting the green investment decreases with λ, a proxy of the firm's preference for social value. Suppose the firm has a weak preference for social return. In that case, the green investor must offer strongly sensitive securities enough for the firm to pursue financial gains from reducing the expected payment to the funding investor by choosing project G. In this context, we can conjecture that equity financing may facilitate ESG integration compared to debt financing, particularly for a relatively low λ; (levered) equity contracts are the most sensitive securities, while the debt contracts are the least (Lemma 1-(iii) and (iv)). This conjecture will be analytically confirmed in Section 4.3.

However, we should not hastily judge that an investor's bid with a strongly sensitive security always guarantees the green investment in equilibrium. Indeed competitive pressure in the capital market may result in an insensitive security as an equilibrium funding term the firm accepts. Certainly, the cost of capital is positively correlated with the sensitivity of the security held by the funding investor. For example, consider a (green) investor bidding with a (levered) equity contract $c(x, S) = \max\{x - S, 0\}$. Given a low λ, the green investor may have to increase the sensitivity of her (levered) equity contract offer by decreasing S substantively (Lemma 1-(ii)). However, lowering S increases the total repayment the firm owes, no matter what project it chooses afterward. Hence, the firm may want to borrow from someone else who bids with an insensitive security, even though such a security bid may lead to the investment in project B. In other words, a security bid must be strongly sensitive to induce the green investment *ex post*, while the firm will reject such an offer *ex ante* due to the associated high cost of capital.[2] This dilemma indeed hampers the firm's choice of project G, which will be shown in Section 4.3.

From the discussion above, one can find that a key parameter that pins down the equilibrium outcome will be λ, representing the firm's preference for the social return. If λ is high, every security bid that can yield the green investment does not need to be strongly sensitive, making such bid a cheap financing option. However, if λ is low, the firm may find a security bid incentivizing the green investment too costly. In what follows, we will analyze how these countervailing effects of the security sensitivity determine the equilibrium outcome for each value of λ.

4 Equilibrium Analysis

In this section, we analyze how the firm's equilibrium project selection changes with its preference for social return. We continue to analyze how the firm's capital structure supports or hampers green investments.

For the exposition, we first introduce new notations.

[2]For a similar reason, our main results are not qualitatively changed even if the green investor is a *nonconsequentialist*; i.e., she receives an extra utility from funding project G directly (Dangl et al., 2023). Although the green investor can offer cheaper financing to the firm than her competitors by internalizing her extra utility from the direct green investment, she will not exercise her competitive advantage in the funding game. Indeed, securities claiming low expected repayments tend to be insensitive (Lemma 1), which adversely incentivizes the firm to choose project B rather than G and thus removes the green investor's additional utility, a source of cheap financing. See Section 3 of Chang, Rhee and Yoon (2024) for more details.

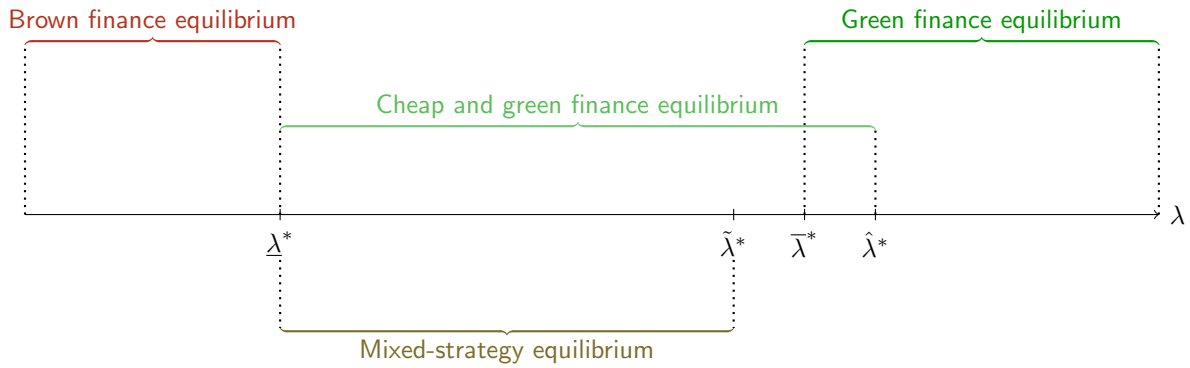


Figure 2 – Equilibrium characterization

Definition 2.

(i) $D_y^i := \min\{D \geq 0 : \mathbb{E}_i[d(x, D)] \geq y\}$ *for each* $i \in \{G, B\}$;

(ii) $S_y^i := \min\{S \geq 0 : \mathbb{E}_i[c(x, S)] \geq y\}$ *for each* $i \in \{G, B\}$.

D_y^i is the face value of a debt contract that yields the expected value y under project $i \in \{G, B\}$. Similarly, S_y^i is the strike price of a levered equity contract that yields the expected value y under project $i \in \{G, B\}$. Note from Definition 1 that D_y^i increases with y, while S_y^i decreases with y. Furthermore, MLRP between $f_B(\cdot)$ and $f_G(\cdot)$ implies that $D_y^G > D_y^B$ and $S_y^G < S_y^B$ for every $y > 0$.

In what follows, we establish a subgame perfect equilibrium, which is referred to as the equilibrium going forward, where all investors employ pure strategies. We hereafter refer to $s^*(\cdot)$ as the equilibrium security bid the firm issues to the funding investor.[3] In equilibrium, the firm accepts a security $s(\cdot)$ only if it maximizes its expected payoff. Furthermore, after accepting the issuance of a security $s(\cdot)$, the firm chooses project G with positive probability only if $s(\cdot)$ satisfies (1) and chooses project B with positive probability only if $s(\cdot)$ satisfies (2). Moreover, investors cannot strictly benefit from deviating from their equilibrium bids. As discussed earlier, a key parameter determining the equilibrium outcome is λ, representing the firm's preference for a social return. Figure 2 depicts how the equilibria presented in Theorem 1 - 4 below are characterized by λ.

[3]The notation $s^*(\cdot)$ implicitly assumes that the firm chooses $s^*(\cdot)$ with probability one among all feasible securities. This is indeed the case in all equilibria discussed in this section. Restricting attention to such equilibria does not result in any loss of generality. Indeed, we allow the firm to randomize over multiple securities in all proofs in the appendix and confirm that all results remain valid.

4.1 Equilibrium with Expensive Financing

We first characterize an equilibrium with *expensive* financing, where the firm's payments to a funding investor specified by the equilibrium security break even in expectation – i.e., $\mathbb{E}_B[s^*(x)] = 1$ ($\mathbb{E}_G[s^*(x)] = 1$, *resp.*) when the firm chooses project B (G, *resp.*). One can guess that such an equilibrium will arise when brown investors dominate the green investor in competition in the capital market; these investors will fund the firm only when the expected payments specified by their securities guarantee a nonnegative financial payoff.

We first analyze equilibrium where the firm chooses project B. The following statement reveals that such an equilibrium arises from relatively low values of λ.

Theorem 1 (brown finance equilibrium)**.** *Define* $\underline{\lambda}^* > 0$ *as*

$$\underline{\lambda}^* := \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)]\right\}\right]. \tag{3}$$

Then there exists an equilibrium in which the firm chooses project B after accepting a security offer $s^*(\cdot)$ such that $\mathbb{E}_B[s^*(x)] = 1$ if and only if $\lambda \leq \underline{\lambda}^*$.

From (3), we have

$$\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] \leq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$$

for every $\lambda \leq \underline{\lambda}^*$. From Lemma 2-(ii), the firm will choose project B if it issues a security $s(\cdot)$ that is less sensitive than the levered equity contract $c(\cdot, S_1^B)$.

The equilibrium in Theorem 1 is called the "brown finance equilibrium" from the firm's choice of project B. Fix any $\lambda \leq \underline{\lambda}^*$ to establish the equilibrium. Then brown investors rationally infer that the firm will eventually choose project B, and they competitively bid with an equilibrium security $s^*(\cdot) = c(\cdot, S_1^B)$. This offer breaks even for the selected funding investor, conditional on project B (i.e., $\mathbb{E}_B[s^*(x)] = 1$). Then, from (2), the security $s^*(\cdot)$ is insensitive enough for the firm to pursue a financial profit by choosing project B.

Furthermore, brown investors cannot profitably deviate from their equilibrium bidding strategy $s^*(\cdot)$. Since $s^*(\cdot)$ breaks even the funding brown investor, a brown investor can profitably deviate only by bidding with a security $s'(\cdot)$ that induces the choice of project G. For such a deviation to make the deviating brown investor better off, we must have $\mathbb{E}_G[s'(x)] \geq \mathbb{E}_B[s^*(x)] = 1$. However, since $\mathbb{E}_G[s^*(x)] < \mathbb{E}_B[s^*(x)]$, the firm will enjoy a lower

cost of capital under $s^*(\cdot)$, even conditional on its choice of project G. Hence, brown investors cannot profitably deviate.

Lastly, there is no feasible security bid with which the green investor can beat the brown investors to fund the firm's project and incentivize the firm to choose project G. Suppose the green investor deviates and bids with a security $s''(\cdot)$ sensitive enough for the firm to choose project G. Since project G generates the social return ϕ, the green investor is willing to take a financial loss up to $-\phi$, once the firm certainly switches to project G. To make the deviating security bid more attractive than the equilibrium security $s^*(\cdot)$, the green investor must charge a lower cost of capital by designing $s''(\cdot)$ to satisfy $\mathbb{E}_G[s''(x)] \leq \mathbb{E}_G[s^*(x)]$; if not, the firm will get a strictly higher payoff even from accepting $s^*(\cdot)$ rather than $s''(\cdot)$, conditional on its choice of project G.

However, lowering the cost of capital makes the deviation offer $s''(\cdot)$ not sensitive enough to incentivize the choice of project G. For example, consider a levered equity contract $c(\cdot, S'')$ such that $\mathbb{E}_G[c(x, S'')] = \mathbb{E}_G[s''(x)]$. Then, since $\mathbb{E}_G[s''(x)] \leq \mathbb{E}_G[s^*(x)]$, the strike price S'' exceeds S_B^1, implying that $c(\cdot, S'')$ is less sensitive than $s^*(\cdot) = c(\cdot, S_1^B)$ (Lemma 1-(ii)), so that incentivizes the firm to choose project B instead of G. Furthermore, $c(\cdot, S'')$ is the most sensitive contract among all securities that yield the same expected payment under project G (Lemma 1-(iv)). In sum, there is no feasible security bid that provides a low cost of capital to win the funding competition and strong enough sensitivity to support the choice of project G.

The infeasibility of securities supporting green investment highlights the green investor's dilemma in the funding competition with brown investors, as we briefly discussed at the end of Section 3. It is necessary for any security bid from the investor to be sufficiently sensitive to incentivize the firm to reduce the expected payment to the funding investor by choosing project G. An increase in sensitivity, however, places a large repayment burden on the firm. In particular, any security inducing the green investment must be strongly sensitive to project selection when the firm has a weak preference for social return. Since the firm prefers lowering the repayment burden, it will borrow from an investor who offers a security that is insensitive to project selection.

We next analyze the equilibrium where the firm chooses project G. In contrast to the brown finance equilibrium, such equilibrium will arise from high values of λ.

Theorem 2 (green finance equilibrium)**.** *Define* $\overline{\lambda}^* > \underline{\lambda}^*$ *as*

$$\overline{\lambda}^* := \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)]\right\}\right]. \tag{4}$$

Then there exists an equilibrium in which the firm chooses project G *after accepting a security offer* $s^*(\cdot)$ *such that* $\mathbb{E}_G[s^*(x)] = 1$ *if and only if* $\lambda \geq \overline{\lambda}^*$.

From (4), we have

$$\mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$$

for every $\lambda \geq \overline{\lambda}^*$. From Lemma 2-(i), the firm will choose project G if it accepts the issuance of any security $s(\cdot)$ more sensitive than the debt contract $d(\cdot, D_1^B)$.

The equilibrium in Theorem 2 is called the "green finance equilibrium" simply because the firm chooses project G. When λ is sufficiently high, (brown) investors rationally infer that the firm will eventually choose project G. Brown investors then competitively bid with the security $s^*(\cdot)$ that breaks even the funding investor, conditional on project G (i.e., $\mathbb{E}_G[s^*(x)] = 1$). Moreover, the firm finds it optimal to choose project G under the security $s^*(\cdot)$. To show this more precisely, consider a debt contract $d(\cdot, D_1^G)$, which is (weakly) less sensitive than $s^*(\cdot)$ (Lemma 1-(iii)) and breaks even for the funding investor under the firm's choice of project G. Since $D_1^G > D_1^B$, even the debt contract $d(\cdot, D_1^G)$ is strictly more sensitive than $d(\cdot, D_1^B)$ (Lemma 1-(i)) and thus satisfies (1). This implies $s^*(\cdot)$ is also sensitive enough to incentivize the firm to reduce the expected payment to the funding investor from $\mathbb{E}_B[s^*(x)]$ to $\mathbb{E}_G[s^*(x)]$ by choosing project G.

Furthermore, no investor can profitably deviate from the equilibrium bidding strategy $s^*(\cdot)$. Since $s^*(\cdot)$ gives zero expected financial payoff to a funding investor, brown investors can profitably deviate only by bidding with an insensitive security $s'(\cdot)$ that incentivizes the firm to choose project B. For such a security bid to be profitable, we must have $\mathbb{E}_B[s'(x)] \geq 1$. However, we know from Lemma 1-(i) and (iii) that such a security is more sensitive than the debt contract $d(\cdot, D_1^B)$ such that $\mathbb{E}_B[d(\cdot, D_1^B)] = 1$. From (4), however, the firm with high $\lambda \geq \overline{\lambda}^*$ will find the deviation security $s'(\cdot)$ sensitive enough to prioritize reducing the expected payment to the funding investor by choosing project G. Hence, no feasible security gives any investor a higher financial payoff and induces the firm's choice of project B. Lastly, the green investor has no incentive to deviate from offering $s^*(\cdot)$: that investor already enjoys a high payoff from the social return ϕ, even if they withdraw from bidding for a project and

leave the capital market; the green investor even finds it strictly suboptimal to bid with any insensitive security that may yield the firm's choice of project B, which strictly decreases the social return from ϕ to zero.

Some features are worth noting. First, the firm's strong preference for social return prevents profitable deviation by any brown investor. Any security bid that breaks even for a funding investor under project B is sensitive enough for the firm to prefer reducing the expected payment to the funding investor by choosing project G. Second, the green investor enjoys free-riding benefits from brown investors. Knowing that the firm chooses project G under the brown investors' security bids, brown investors – who only care about financial payoffs – will charge a large share of the financial return as repayments because of the low profitability of project G. This in turn increases the sensitivity of the equilibrium security, incentivizing the green investment. By contrast, the green investor can withstand a financial loss if the social return outweighs the financial loss. However, the green investor does not have to take the financial loss to fund the firm's project directly; that investor can enjoy the full social return indirectly from the brown investors' equilibrium bid strategy, which leads to the firm's selection of project G. Notice that this free-riding benefit relies on the belief that many brown investors actively compete to finance the firm with the expectation that the firm chooses project G. Such a belief is consistent with the equilibrium behaviors of brown investors and the firm when the firm's inherent preference for social value is sufficiently strong.

4.2 Equilibrium with Cheap Financing

Next we find the equilibrium for intermediate values of $\lambda \in (\underline{\lambda}^*, \overline{\lambda}^*)$, i.e., when the firm has a moderate preference for social value. One important equilibrium characteristic is that any equilibrium where the firm receives expensive financing, like in Theorem 1 or 2, no longer exists. To see why, suppose there is a brown finance equilibrium, where the investors fund project B to get the zero expected financial payoff. Since λ exceeds $\underline{\lambda}^*$, the green investor can deviate by bidding with a security that incurs a financial loss less than $-\phi$ and is sensitive enough to yield the firm's choice of project G. Next suppose a green finance equilibrium exists where the investors fund project G with zero expected financial payoff. Then the condition $\lambda < \overline{\lambda}^*$ implies that a brown investor will profitably deviate by bidding with an insensitive security that weakly breaks even for the deviating investor and yields the firm's selection of project B.

The discussion above implies that, in equilibrium, the firm must enjoy *cheap* financing

to the extent that a funding investor suffers a financial loss – i.e., $\mathbb{E}_i[s^*(x)] < 1$ for some (or every) project type $i \in \{G, B\}$. This equilibrium characterization also implies that the green investor must win the competition to finance the firm's project; brown investors will never accept a funding term that incurs a strictly negative financial payoff. Furthermore, the firm must be incentivized to choose project G to compensate the green investor for her financial loss.

There are two possible equilibrium outcomes that vary with the firm's project selection strategy. Specifically, there is an equilibrium in which the firm chooses one type of project – project G – with certainty. Furthermore, there is another equilibrium in which the firm randomizes its project selection. We first characterize the pure-strategy equilibrium in which the firm chooses project G with probability 1.

Theorem 3 (cheap and green finance equilibrium)**.** *Define* $\hat{\lambda}^* > \overline{\lambda}^*$ *as*

$$\hat{\lambda}^* := \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[d(x, D^G_{1-\phi})] - \mathbb{E}_G[d(x, D^G_{1-\phi})]\right\}\right]. \tag{5}$$

Then there exists an equilibrium in which the firm chooses project G *after accepting a security offer* $s^*(\cdot)$ *such that* $\mathbb{E}_G[s^*(x)] \in [1-\phi, 1)$ *if and only if* $\lambda \in [\underline{\lambda}^*, \hat{\lambda}^*]$.

The equilibrium in Theorem 3 is named the "cheap and green finance equilibrium." In this equilibrium, the green investor provides cheap financing ($\mathbb{E}_G[s^*(x)] < 1$) to the firm, which also induces green financing – i.e., the firm's selection of project G. As discussed earlier, a funding investor must bid with a security that incurs a net financial loss. Since brown investors never accept negative financial payoffs from funding the firm, the investor who finances the firm's project must be the green type, who willingly accepts the financial loss $\mathbb{E}_G[s^*(x)] - 1 < 0$ partially compensated by the social return ϕ from project G. This implies the winning green investor's total payoff, $(\mathbb{E}_G[s^*(x)] - 1) + \phi$, must be nonnegative or equivalently $\mathbb{E}_G[s^*(x)] \in [1-\phi, 1)$. Furthermore, the equilibrium security $s^*(\cdot)$ must be sufficiently sensitive to incentivize the firm to choose project G.

Indeed, the green investor's bid with a low cost of capital to the firm is necessary to discourage the deviation by brown investors. Suppose a brown investor deviates and bids with a security that, if accepted, incentivizes the firm to choose project B. Notice that the deviation security bid must incur a cheaper cost of capital than the equilibrium security $s^*(\cdot)$. This means that the deviation security bid will have to yield a financial loss to the deviating brown investor, discouraging brown investors from placing a counter-bid against $s^*(\cdot)$.

However, the green investor as a single winning bidder would eventually hold market power, empowering her to charge a higher financial repayment to the firm than the equilibrium security $s^*(\cdot)$ specifies. To prevent such a deviation, another investor must place a losing bid with a security different from $s^*(\cdot)$, which will worsen the green investor if she deviates. Specifically, the losing bid can be constructed as a security $\hat{s}(\cdot) \neq s^*(\cdot)$ such that *(i)* the firm prefers project B under the losing bid $\hat{s}(\cdot)$; *(ii)* the firm is indifferent between project B after accepting $\hat{s}(\cdot)$ and project G after accepting $s^*(\cdot)$.[4] The investor placing the losing bid $\hat{s}(\cdot)$ still receives zero expected payoff since her bid is never accepted on the equilibrium path. Nevertheless, the losing bid keeps the green investor from exploiting her market power to ask for a larger financial repayment, even by a tiny margin. If she does so, the firm would rather raise capital from the losing investor and then choose project B instead, substantially decreasing the green investor's payoff from $(\mathbb{E}_G[s^*(x)] - 1) + \phi$ to zero.[5]

The upper threshold $\hat{\lambda}^*$ of λ supporting the cheap and green finance equilibrium concerns the feasibility of the losing security bid. If λ is excessively high, the firm strongly prefers the social return of project G to the extent that it will choose project G under any specified losing bid designed to prevent the green investor from raising her bid.[6] Therefore, any λ supporting the cheap and green finance equilibrium should be bounded above by an upper threshold $\hat{\lambda}^*$, while we relegate the mathematical derivation of $\hat{\lambda}^*$ in (5) to the appendix.

We can find some noteworthy features by comparing the cheap and green finance equilibrium with the previous equilibrium of the green finance type. First, for every $\lambda \in [\overline{\lambda}^*, \hat{\lambda}^*]$, each investor's belief about the competitiveness in the capital market is endogenously formed, naturally leading to a multiple equilibria response. Suppose multiple brown investors

[4]The first condition is equivalent to the argument that $\hat{s}(\cdot)$ satisfies (2). The second condition is mathematically expressed as $\mathbb{E}_B[\hat{s}(x)] - \mathbb{E}_G[s^*(x)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Under these conditions, any deviation of the green investor by claiming even marginally larger shares of financial returns as repayments will incentivize the firm to finance the project from an investor with the losing bid and choose project B. In the appendix, we show that these two conditions are necessary and sufficient for a security $\hat{s}(\cdot)$ to be a losing bid that supports the equilibrium.

[5]There is a single active green investor in our model, and therefore the losing bid should be placed by the brown investor. Alternatively, we can assume the existence of another inactive green investor (in addition to the active one who bids $s^*(\cdot)$), and this green investor places the losing bid. The equilibrium analysis remains valid. Thus, the cheap and green finance equilibrium still exists in this alternative setup.

[6]For example, suppose the green investor offers $s^*(\cdot) = d(\cdot, D^G_{1-\phi})$. Since the firm is indifferent between project G with the winning bid and project B with the losing bid, the losing bid $\hat{s}(\cdot)$ must satisfy $\mathbb{E}_B[x - \hat{s}(x)] = \mathbb{E}_G[x - d(x, D^G_{1-\phi})] + \lambda\phi$ or equivalently $\mathbb{E}_B[\hat{s}(x)] - \mathbb{E}_G[d(x, D^G_{1-\phi})] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. This implies $\mathbb{E}_B[\hat{s}(x)] < \mathbb{E}_B[d(x, D^G_{1-\phi})]$. However, even if $\hat{s}(\cdot)$ is a debt contract $d(\cdot, \hat{D})$, we only have $\hat{D} < D^G_{1-\phi}$. Hence, we must have $\mathbb{E}_B[\hat{s}(x)] - \mathbb{E}_G[\hat{s}(x)] > \mathbb{E}_B[\hat{s}(x)] - \mathbb{E}_G[d(x, D^G_{1-\phi})] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$, leading the firm to choose project G, even with the losing bid. Consequently, there is no feasible losing bid to support the cheap and green finance equilibrium.

are believed to bid to finance the firm competitively. Since $\lambda \geq \overline{\lambda}^*$, the green investor would infer that these bidding brown investors rationally expect the firm to choose project G. Thus, the brown investors' equilibrium bidding strategy $s^*(\cdot)$ must guarantee zero expected financial payoff under project G with low profitability, i.e., $\mathbb{E}_G[s^*(x)] = 1$. This in turn makes $s^*(\cdot)$ sufficiently sensitive enough to induce the choice of project G consistently, leading to the green finance equilibrium in which green investors enjoy the social return as an externality without financing the firm's project directly.

However, in the cheap and green finance equilibrium, no brown investor willingly funds the firm's project by sacrificing her financial payoff. Instead the green investor may need to accept a financial loss to fund the firm's project directly. Despite its small expected payoff, this green investor's security bid can be sufficiently sensitive to induce the firm to choose project G since λ is relatively high $\lambda \geq \overline{\lambda}^*$. Furthermore, since $\lambda \leq \hat{\lambda}^*$, the green investor cannot raise her bid to avoid the financial loss, lest the losing bid with an insensitive security induce the firm's choice of project B. Facing the green investor's overly generous security bid, brown investors refrain from placing any counter-bid that would incur a strict financial loss.

Second, the equilibrium outcomes in our model encompass the previous works on socially responsible investments. For example, Chowdhry, Davies and Waters (2019) show that corporate borrowers are incentivized to pursue nonfinancial social return instead of financial profit under the borrowing condition that lenders claim a substantively large share of financial return. Under these financing terms, the borrowers will see a meager net financial income after repayment, even from financially lucrative but socially harmful investments. Therefore, the borrowers' optimal investment strategy is to maximize social return, which will not be distributed but commonly shared with the lenders, regardless of financing conditions.

The economic insights of Chowdhry, Davies and Waters (2019) for the socially responsible investment are akin to the green finance equilibrium in Theorem 2 of our analysis. In that equilibrium, brown investors finance the firm's project G with zero expected financial payoff. The funding investor's financial payoff exceeds that in the cheap and green finance equilibrium in Theorem 3, where the funding investor suffers a negative financial payoff. Indeed our analysis clarifies a key economic force inducing green investment that Chowdhry, Davies and Waters (2019) found. Specifically, the firm's choice of project G is primarily motivated by lowering the expected payment to the funding investor. To this end, the security held by the funding investor should be sufficiently sensitive (Lemma 2-(i)). The high cost of capital is a byproduct of the strongly sensitive equilibrium security to incentivize green investment.

Contrary to Chowdhry, Davies and Waters (2019), Oehmke and Opp (2024) argues that a green investor necessarily sacrifices her financial payoff by funding the firm's project cheaply in exchange for enforcing the green investment by attaching a binding covenant in the firm's project selection to the financial contract. Their analysis resembles the cheap and green finance equilibrium in Theorem 3 of our paper. A key feature commonly observed in our analysis and Oehmke and Opp (2024) is that the green investor willingly accepts a financial loss to discourage competitive brown investors from making a counter-bid that can lead to more brown investment.

Nevertheless, we stress that the economic force yielding cheap financing in equilibrium in our analysis is substantially different. Oehmke and Opp (2024) allow the green investor to attach a binding covenant that dictates the borrowing firm's project selection. Hence, the green investor can strike a package deal with the firm, giving a discount to the funding rate, conditional on the green investment. The entry of competitive brown investors incentivizes the green investor to provide an even higher discount to the lending rate, encouraging the firm's green investments.

In our analysis, the green investor faces a challenge in inducing the green investment, due to the lack of contractual enforceability of the firm's ex post action. As in Oehmke and Opp (2024), the green investor may have to bid with a loss-making security to finance the firm directly as a condition for the green investment. This condition is necessary to discourage the brown investors from competing. The green investor also must design her security bid, despite its cheap cost of capital, to be strongly sensitive to induce the selection of project G. However, a cheap cost of capital usually lowers the sensitivity of the security, which may adversely incentivize the firm to make socially harmful investments. Since cheap financing terms are necessary only in the presence of brown investors, our analysis suggests that the entry of brown investors into the ESG capital market may not facilitate ESG integration when the borrowers can deviate from their commitments.

The concern with the negative impacts of competition with brown investors is pronounced in the last type of equilibria, where the firm no longer plays pure strategy *ex post* in its project selection. Indeed, there is a mixed-strategy equilibrium where the firm randomizes its project selection by assigning probability $\beta \in (0,1)$ to the selection of project B.

Theorem 4 (mixed-strategy equilibrium)**.** *Define* $\tilde{\lambda}^* \in [\underline{\lambda}^*, \hat{\lambda}^*)$ *as*

$$\tilde{\lambda}^* := \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[c(x, S^G_{1-\phi})] - \mathbb{E}_G[c(x, S^G_{1-\phi})]\right\}\right]. \tag{6}$$

Then there exist mixed-strategy equilibria if and only if $\lambda \in [\underline{\lambda}^*, \tilde{\lambda}^*)$. *In each mixed-strategy equilibrium for a given* λ, *the firm randomizes its project selection by choosing project* B *with some probability* $\beta^* \in (0, \tilde{\beta}^*]$, *where* $\tilde{\beta}^* < 1$ *for any* $\lambda \in [\underline{\lambda}^*, \tilde{\lambda}^*)$ *and* $\tilde{\beta}^*$ *converges to zero as* λ *approaches* $\tilde{\lambda}^*$.

The equilibrium strategy profile resembles the cheap and green finance equilibrium in Theorem 3. First, the green investor places a winning but loss-making security bid $s^*(\cdot)$. Furthermore, another investor places a losing bid $\tilde{s}(\cdot)$, constructed in the same way as the losing bid $\hat{s}(\cdot)$ in the cheap and green finance equilibrium. Like in the cheap and green finance equilibrium, such a losing bid is necessary to prevent the winning green investor from avoiding her financial loss by charging a larger financial repayment than her equilibrium bid.

The mixed-strategy equilibrium is characterized as follows. First, the firm is indifferent between two projects B and G, so the equilibrium security $s^*(\cdot)$ must satisfy

$$\mathbb{E}_B[s^*(x)] - \mathbb{E}_G[s^*(x)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi. \tag{7}$$

In equilibrium, the firm randomizes its project selection by choosing project B with probability $\beta^* \in (0, 1)$ to the extent that the green investor finds it optimal to fund the firm:

$$\beta^*\{\mathbb{E}_B[s^*(x)] - 1\} + (1 - \beta^*)\{\mathbb{E}_G[s^*(x)] - 1 + \phi\} \geq 0.$$

Since $\mathbb{E}_B[s^*(x)] < 1$ and $\mathbb{E}_G[s^*(x)] - 1 + \phi \geq 0$, the green investor strictly prefers the firm to choose project G with a lower probability than β^*.

Furthermore, to ensure the green investor does not deviate, her winning security bid must be a levered equity contract $c(\cdot, \tilde{S}^*)$, the most sensitive security type:

$$\mathbb{E}_B[c(x, \tilde{S}^*)] - \mathbb{E}_G[c(x, \tilde{S}^*)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi. \tag{8}$$

As was seen in the cheap and green finance equilibrium, the green investor will suffer disutility from the brown investment if she deviates and charges a larger financial repayment than her equilibrium security bid. Another possible deviation is to bid with a security that incurs an even larger financial loss but is strictly more sensitive than $c(\cdot, \tilde{S}^*)$ that may induce the firm's choice of project G with probability one. However, if a security has a lower expected payoff under project G than $c(\cdot, \tilde{S}^*)$, its sensitivity is necessarily less than the levered equity contract

$c(\cdot, \tilde{S}^*)$ (Lemma 1-(ii) and (iv)).[7] Thus, such a security will only incentivize the firm to choose project B, instead of G, with probability one.

Notice that the equilibrium contract $c(\cdot, \tilde{S}^*)$ generates the largest financial loss to the green investor among all other securities satisfying the firm's indifference condition (7). Among all securities that have the same sensitivity, levered equity contracts charge the lowest repayment under project G (Lemma 1-(iv)). Thus, any other securities that also make the firm indifferent between the two projects necessarily generate a lower financial loss to the green investor than $c(\cdot, \tilde{S}^*)$ does.

The last observation has an interesting implication: competition in the capital market may decrease the probability of the green investment in the mixed-strategy equilibrium. An intensified competition would push the brown investors to offer more generous financing conditions to the firm; in other words, their losing bid $\hat{s}(\cdot)$ would charge a less financial repayment. If this is what the green investor believes about the brown investors' bidding strategy, she will respond by charging the firm an even smaller share of the financial return, which in turn lowers the sensitivity of the equilibrium security $c(\cdot, \tilde{S}^*)$ (Lemma 1-(i) and (ii)). With this insensitive security, the firm's incentive to make the brown investment is partially recovered.[8]

This result starkly contrasts with that of Oehmke and Opp (2024), who argue that the green investor's offer of cheap financing promotes green investments. Further, assuming the borrowing firm can scale up its investment, the green investment is made even on a larger scale when the green investor intensely competes with brown investors. Their claim critically relies on the assumption that the firm's project selection is contractually enforceable, so that the green investor can explicitly tie green investment to cheap financing. However, with the lack of covenants dictating the firm's ex post behavior, our analysis reveals that an overly competitive ESG capital market can adversely preserve the firm's incentive to pursue financial profit by choosing project B rather than reduce the payment to the funding investor by choosing project G. Our result also resonates with Chang, Rhee and Yoon (2024), pointing out that intensified competition between investors to provide capital to ESG projects generally deters rather than facilitates ESG integration.

[7] To show this formally, fix any security $s'(\cdot)$ such that $\mathbb{E}_G[s'(x)] < \mathbb{E}_G[c(x, \tilde{S}^*)]$. Then there also exists a levered equity contract $c(\cdot, S')$ which has the same expected payoff under project G as $s'(\cdot)$ (i.e., $\mathbb{E}_G[s'(x)] = \mathbb{E}_G[c(x, S')]$) and is more sensitive than $s'(\cdot)$ (Lemma 1-(iv)). Furthermore, since $\mathbb{E}_G[c(x, S')] < \mathbb{E}_G[c(x, \tilde{S}^*)]$, the strike price S' must be strictly greater than $\tilde{S}^*$, implying that $c(\cdot, S')$ and $s'(\cdot)$ are strictly less sensitive than $c(\cdot, \tilde{S}^*)$ (Lemma 1-(ii)).

[8] As indicated in Figure 2, multiple equilibria coexist for $\lambda \in [\underline{\lambda}^*, \tilde{\lambda}^*)$. The validity of the observation here also depends on which equilibrium is selected among them.

4.3 Debt versus Equity for ESG Integration

How can investors who allocate capital eliminate the possibility of brown investments when the firm's preference for social return is neither strong nor weak? One potential point that they could consider when allocating capital is the role that the firm's capital structure can play in its ESG integration decisions. Recall that the mixed-strategy equilibrium in Theorem 4 can exist only when multiple investors bid with varying types of securities with different sensitivities. Therefore, if every investor bids with the same type of standard security, such as a debt or levered equity contract, the mixed-strategy equilibrium for medium values of λ can be replaced by a pure-strategy equilibrium, as in Theorem 1 or 2. Particularly, the equilibrium outcome will dramatically change as investors choose to offer debt or equity; the firm can have an incentive to integrate ESG to lower the expected payments to the funding investors only when the issued securities are sensitive to the firm's project selection (Lemma 2). The following observations show that the firm's investment decision is indeed pinned down by the type of securities the investors can bid with.

Theorem 5.

(i) Suppose investors are restricted to offer debt contracts only. Then there exists only a pure-strategy equilibrium where the firm chooses project G if $\lambda \geq \overline{\lambda}^*$ and chooses project B otherwise.

(ii) Suppose investors are restricted to offer levered equity contracts only. Then there exists only a pure-strategy equilibrium where the firm chooses project G if $\lambda \geq \underline{\lambda}^*$ and chooses project B otherwise.

Theorem 5 presents a simple but important message to asset allocation. It also has policy implications for regulators across the globe considering deterring greenwashing and inducing ESG integration: equity financing facilitates green investment, while debt financing deters it. The intuition is straightforward. First, suppose all investors bid with debt contracts. Focus on the case that the firm's preference for social return can be viewed as relatively weak to the extent that $\lambda \leq \overline{\lambda}^*$. Combined with the structural insensitivity of debt contracts, the face value of a debt contract must be substantially higher if the firm chooses project G (Lemma 1-(i)). However, a high face value means a high cost of capital, so the firm will reject the bid. Rather, the firm will accept a debt contract offer with a low face value, even though this offer does not provide an incentive for reducing the expected payment to the debtholder

by choosing project G. As a result, debt financing lowers the sensitivity of the firm's capital structure, incentivizing the brown investment. Investors, particularly those of the brown type, rationally infer the firm's selection of project B, so they competitively offer the break-even debt contract that gives zero financial payoff to a winning investor under project B.

Conversely, suppose all investors bid with levered equity contracts. Focus on the case that the firm's preference for social return is now perceived as strong to the extent that $\lambda \geq \underline{\lambda}^*$. Due to its structurally strong sensitivity, (levered) equity financing generally incentivizes the firm to choose project G, no matter how high the strike price of the levered equity contract is. Particularly, brown investors find it infeasible to induce the firm to choose project B without suffering a financial loss; any levered equity contract that breaks even for the funding investor under project B is also sensitive enough for the firm to prefer project G. Investors competitively offer the break-even equity contract that gives zero financial payoff to a winning investor under project G.

The desirability of (levered) equity financing to induce green investment starkly contrasts with the findings of the literature that analyzes the superiority of debt financing in various asymmetric information problems. For example, numerous papers, such as Myers and Majluf (1984), Nachman and Noe (1994), DeMarzo and Duffie (1999), and Dang, Gorton and Holmström (2013), argue that informational insensitivity of debt financing minimizes the severity of the adverse selection problem between a borrower and a lender. Under debt financing, a financially unproductive borrower can extract the smallest information rent by mimicking a financially productive borrower; the expected payoff of debt does not significantly change with the profitability of the borrowers.

Furthermore, DeMarzo, Kremer and Skrzypacz (2005) show that a borrower can signal its quality by issuing debt securities. Moreover, Townsend (1979) and Innes (1990) find the optimality of debt contracts for addressing a moral hazard problem between a borrower and a lender. Both results hinge on the insensitivity of debt contracts; the firm will receive a large share of financial return as residual income when the realized financial return is high, while it will gain little when the realized financial return is low. In signaling games, debt financing is more desirable for the financially productive borrower than the unproductive one. In moral hazard problems, debt contracts optimally incentivize firm managers to put in their efforts, which increases the likelihood of high financial returns and thus the managers' expected residual income.

A key ingredient that differentiates our analysis from the literature is the considera-

tion of social benefits in corporate decision-making. As discussed by Lemma 2, the additional social return attached to project G is necessary for the firm to have an incentive to reduce the expected payment to the funding investors, despite the resulting net financial loss. Furthermore, the firm's incentive to lower its average payment strengthens as the securities issued by the firm to finance its investment are sensitive to the investment decision. However, the negative impact of socially valuable investments on the firm's asset value remains unaffected. Consequently, firms with more equity can better incorporate ESG. Further, investors can consider financing the firm through equity to promote socially desirable outcomes. This could maximize corporate borrowers' incentive to integrate ESG by increasing the sensitivity of their capital structure.

We make two final remarks before ending this section. First, testable predictions for empirical research could be drawn from our analysis. Specifically, a researcher could examine whether companies with a higher proportion of equity financing demonstrate a greater commitment to ESG initiatives compared to those primarily financed through debt. Recent related working papers explore a similar theme but from different angles. For example, Kräussl, Rauh and Stefanova (2024) empirically examine the effects of ESG ratings on firms' security issuances and find that higher environmental scores are associated with increases in equity issuance and decreases in debt issuance. Second, considering equity as a means to deter greenwashing is highly relevant in the current context. Recently, there have been concerns about greenwashing among bond issuers. For instance, Aswani and Rajgopal (2022) finds that firms issuing green bonds often exhibit little ESG follow-through and the market fails to see through this. Although greenwashing in equity has been documented (Kim and Yoon, 2023; Raghunandan and Rajgopal, 2022*a*), more recent data reveal a decrease in greenwashing within the equity space (Pástor, Stambaugh and Taylor, 2021, 2023). Moreover, equity markets are generally more transparent and subject to stricter regulation, facilitating monitoring and verification of a company's ESG commitments. Our theory not only provides a foundation for empirically testable hypotheses but also contributes to the broader discourse on enhancing the credibility and effectiveness of ESG integration in corporate finance.

Remark 1. *It is also noteworthy that the firm may find it optimal to issue debt if it can decide the type of securities that it can issue to fund its project. To see this, suppose the firm can choose either debt or equity and its own preference for social value is modest to the extent that* $\lambda \in [\underline{\lambda}^*, \overline{\lambda}^*]$. *Under debt financing, the firm's equilibrium payoff is* $\mathbb{E}_B[x] - 1$. *Under equity financing, the firm will receive the equilibrium payoff* $\mathbb{E}_G[x] - 1 + \lambda\phi$. *Since* $\mathbb{E}_B[x] - 1 > \mathbb{E}_G[x] - 1 + \lambda\phi$ *by Assumption 1-(i), the firm strictly prefers issuing debt,even*

though the debt induces brown investments. This result indeed highlights the necessity of providing corporate borrowers with incentives for equity issuance, particularly when corporate borrowers do not consider ESG values as heavily as pro-ESG outside investors.

5 Related Literature

Recent empirical research underscores a significant disconnect between firms' ESG commitments and their implementation. For instance, Liang, Sun and Teo (2022) scrutinizes US hedge funds that have endorsed the UN PRI initiative, revealing that, while these endorsers enjoy an influx of flows and assets post commitment, they fail to improve ESG performance and returns. On the corporate issuer side, Raghunandan and Rajgopal (2022*b*) find scant evidence of engagement in stakeholder-centric practices among the Business Roundtable member firms that have made such pledges.

One important reason behind this is that there are important challenges in evaluating and auditing firm ESG activities, due to the lack of standards and expertise from assurance providers as well as the opaque nature of ESG raters (Azarmsa and Shapiro, 2022; Berg, Koelbel and Rigobon, 2022; Gipper, Ross and Shi, 2022). The divergence among raters in gauging ESG outcomes also adds layers of complexity in assessing authentic ESG performance by companies and asset managers. Recent papers highlight some benefits of environmental enforcement(Choy et al., 2021) and mandatory disclosure (Christensen et al., 2017; Grewal, Riedl and Serafeim, 2019; Krueger et al., 2021; Fiechter, Hitz and Lehmann, 2022; Aghamolla and An, 2023; Xue, 2023; Manili, 2024). Our first contribution to this literature is to illuminate the role of borrowers' capital structure in ESG integration. This is important because monitoring ESG follow-through is difficult, even in the presence of regulators, given the disagreement on what is ESG and borrowers can often deviate from their promises *ex post*.

Secondly, we acknowledge a recent stream of theory papers that have examined how ESG integration can be achieved. For example, Chowdhry, Davies and Waters (2019) argue that a high cost of borrowing significantly lowers corporate borrowers' net financial income, even from financially lucrative investments, which in turn incentivizes the borrowers to prioritize social returns over financial returns. In addition, Oehmke and Opp (2024) show that ESG-friendly lenders in competition with non-ESG lenders can achieve ESG investments by sacrificing their financial payoffs but attaching an ESG investment covenant to the terms of funding. Furthermore, Feldhütter and Pedersen (2022) adopt the Modigliani-Miller frame-

work and shows that companies' overall cost of capital does not change, even though they issue green bonds in pursuit of ESG values.

Our research differs in the following ways. First, we generalize the previous analysis of Chowdhry, Davies and Waters (2019) by adopting a security design approach. Furthermore, contrary to Oehmke and Opp (2024), we incorporate the greenwashing problems into our model by allowing corporate borrowers to deviate from their promises on ESG integration. Moreover, our analysis shows the sensitivity of securities held by lenders, not the borrowing cost, crucially determines corporate borrowers' incentive to pursue social returns. Our research further shows that excessively cheap financing resulting from intense competition between lenders may revive the borrowers' incentive to pursue financial profit by downsizing the ESG investment. Lastly, our paper shows that, in the presence of moral hazard, companies can significantly lower their cost of capital by prioritizing ESG over financial profits under equity financing, which differs from Feldhütter and Pedersen (2022).

Furthermore, Barbalau and Zeni (2022) and Allen, Barbalau and Zeni (2023) analyze the impacts of several debt instruments specialized for green investments, such as green bonds and sustainability-linked bonds, on the issuers' incentive to pursue ESG values. A key assumption in their models is that some types of bonds can schedule financial repayments based on the borrowers' (reported) ESG performance. Our research extends their analysis by adopting a generalized security design framework in the presence of moral hazard on ESG: the actions on ESG taken by corporate issuers or their outcomes cannot be contractible. Specifically, we show that the corporate issuers' incentive for the pursuit of ESG can be indirectly aligned with green investors by making the securities held by funding investors sensitive to the profitability of the issuers' investments.

Several papers in financial contracting theory point out the potential inefficiency of debt financing in various aspects. For example, Jensen (1986) and Maksimovic and Titman (1991) argue that firms overly relying on debt do not allocate their free cash flow to improving their long-term value in fear of the increased likelihood of bankruptcy. Inderst and Mueller (2006) also shows that debtholders may take excessive risks, due to the insensitivity of debt contracts; debtholders do not substantively change their optimistic view on the financial value of debt in their holding, even when they observe negative information about the underlying assets. Furthermore, Dang, Gorton and Holmström (2013) argue that debt can turn into the most sensitive security, once investors realize that they are likely to default at a time of financial crisis, exposing debtholders to a severe lemons problem. Several other papers also highlight equity as robust financial contracts when security holders cannot accurately specify stochastic

characteristics of financial returns, due to their inherent uncertainty (Admati and Pfleiderer, 1994; Ravid and Spiegel, 1997; Carroll, 2015; Malenko and Tsoy, 2023). Our analysis extends this literature by showing that equity financing strengthens corporate borrowers' incentive to pursue long-term social value to save the expected payments to outside investors.

Relatedly, we acknowledge other theory papers that focus on the optimal design of measuring and disclosing corporate borrowers' ESG activities in the presence of asymmetric information. Friedman, Heinle and Luneva (2021) show that the optimal design of ESG reporting standards to tackle the greenwashing problem depends on the outside lenders' aggregate preferences for ESG. Bonham and Riggs-Cragun (2022) analyze the impacts of various regulatory schemes on ESG and identifies which can motivate regulated firms to integrate ESG. Cartellier, Tankov and Zerbib (2023) and Chaigneau and Sahuguet (2024) point out the corporate executives' can greenwash by inflating ESG scores. To solve this problem, Chaigneau and Sahuguet (2024) further suggest tying executive compensation to multiple measures of ESG performance, impeding executives' gaming of the system. Edmans, Levit and Schneemeier (2022) argue that companies' actual actions taken for ESG integration must be observable to improve efficiency in capital allocation to the firms that "walk the talk." The main results of those papers crucially rely on the assumption that ESG metrics, despite being noisy, are available to measure companies' ESG follow-through. Our paper adds to this stream of research by showing the importance of capital structure in motivating companies to integrate ESG when companies' ESG performance is not yet verifiable.

Lastly, our paper speaks to the rise of green bonds in the capital market and whether debt financing can induce meaningful follow-through from borrowers. For example, recent papers, such as Flammer (2021) and Tang and Zhang (2020), document that issuing green bonds signals a healthy commitment to ESG by the issuing companies. Anecdotes from the real world confirm the growth in green bonds but present a skeptical viewpoint on bond financing as a means to ESG integration. For example, the Japanese Government Pension Investment Fund (GPIF), the world's largest pension fund, and the World Bank launched one of the largest initiatives in the world to finance companies through green and sustainability bonds.[9] However, their observation is that green bonds have largely failed in generating meaningful outcomes, calling them "a lose-lose product."[10] Such notions are confirmed by more recent academic work (e.g., Aswani and Rajgopal (2022) and Bhagat and Yoon (2023))

[9]For details, see https://www.worldbank.org/en/news/press-release/2017/10/11/world-bank-group-and-gpif-join-forces-to-mobilize-capital-markets-for-sustainable-investments.

[10]For details, see https://www.spglobal.com/marketintelligence/en/news-insights/trending/1ogftjbermtc6a0efqmecg2.

that points out little ESG follow-through from corporate green bond issuers. Our theoretical arguments provide important economic insight into this phenomenon and debate and suggest that equity financing may be more effective than bond financing in inducing borrowing firms to integrate ESG. This would not only have implications for asset allocators who worry about the sustainability of their portfolio but also for regulators who are aware of the importance of better monitoring and follow-through.

6 Conclusion

This paper theoretically dissects the economic interaction between corporate issuers' capital structure and their incentive to pursue ESG investments. The crucial economic friction on ESG integration is that lenders cannot adopt any incentive scheme directly conditional on borrowers' ESG performance, which is hard to verify without credible and transparent ESG metrics that are audited and assured. Given that we do not have such an assurance system at present, it is important to find other ways to incentivize corporate borrowers to follow through on their ESG commitments. This paper illuminates the significant role of companies' capital structure in their incentive for ESG integration.

The key takeaways are as follows. First, the sensitivity of securities issued to finance companies' new projects critically shapes their incentive to integrate ESG. Companies can reduce the expected payments to their funding investors by pursuing ESG projects, which tend to be financially less profitable. The financial gains from the repayment reduction, combined with the social return that ESG projects generate, can outweigh the loss in the value of their assets – the expected financial return of the companies' projects, particularly when the securities held by the funding investors substantially changes with the profitability of the companies' projects specified as the underlying assets.

Second, companies' cost of capital and decisions to integrate ESG are critically determined by their preferences for ESG. If companies strongly prefer the pursuit of ESG, brown investors – who pursue financial payoffs only – charge a premium when financing less profitable ESG projects. This high funding rate in turn incentivizes companies to reduce the average borrowing cost by choosing ESG projects.

However, suppose the companies do not have sufficiently strong preferences for ESG. In that case, brown investors will competitively make a breakeven offer, charging a low funding rate in expectation of financially profitable non-ESG investments and incentivizing the

firm to make non-ESG investments. To achieve ESG investments, the green investor – who heavily considers the social consequences of investments – needs to offer securities that claim a favorably small fraction of financial returns to the companies, even if such an offer incurs a financial loss for the investor. However, cheap financing by the green investor makes the securities in their holdings relatively insensitive, so the companies' incentive to increase its asset value is partially perpetuated, resulting in sizable non-ESG investments. One way to alleviate this incentive misalignment is to let the companies finance their projects by issuing sensitive securities, such as equity. Indeed, equity financing significantly restores companies' incentive to integrate ESG by increasing their financial gains from lowering their expected payments to the funding investors.

The main conclusion drawn from our research is that the impact of informational sensitivity of securities on moral hazard problems in ESG integration is substantially different from those in a standard corporate finance model. We anticipate our research will pave the way for important policy discussions and enhance empirical studies in corporate finance concerning greenwashing.

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A Appendix: Proofs

A.1 Preliminary Observations

Lemma A.1. $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] < \phi$ *for any* $s(\cdot)$ *such that* $x \geq s(x) \geq 0$ *and both* $s(x)$ *and* $x - s(x)$ *are weakly increasing in* x.

Proof. Fix any security $s(\cdot)$ that satisfies the properties stated in the lemma. Let $\varepsilon > 0$ be defined such that $\mathbb{E}_G[x - \varepsilon] = \mathbb{E}_G[s(x)]$. Note that $\mathbb{E}_B[x - s(x)] \geq \mathbb{E}_G[x - s(x)] = \varepsilon$ by the FOSD property, and thus, we also have $\mathbb{E}_B[x - \varepsilon] \geq \mathbb{E}_B[s(x)]$. Then, $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] \leq \mathbb{E}_B[x - \varepsilon] - \mathbb{E}_G[x - \varepsilon] = \mathbb{E}_B[x] - \mathbb{E}_G[x] < \phi$, where the last strict inequality follows from Assumption 1-(i). *Q.E.D.*

A.2 Proof of Lemma 1

It suffices to prove the following lemma.

Lemma A.2. *Fix any two securities* $s(\cdot)$ *and* $t(\cdot) : [0, \infty) \to [0, \infty)$ *such that either* $\mathbb{E}_G[s(x)] = \mathbb{E}_G[t(x)]$ *or* $\mathbb{E}_B[s(x)] = \mathbb{E}_B[t(x)]$ *holds. Then,* $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] \geq \mathbb{E}_B[t(x)] - \mathbb{E}_G[t(x)]$ *if there exists a cutoff* $x^* \in [0, \infty)$ *such that*

$$\begin{aligned} s(x) &\geq t(x) \quad \text{for any } x \geq x^*, \\ s(x) &\leq t(x) \quad \text{for any } x \leq x^*. \end{aligned} \tag{A.1}$$

Furthermore, $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] > \mathbb{E}_B[t(x)] - \mathbb{E}_G[t(x)]$ *if the inequalities in* (A.1) *hold for a positive measure of* $x \in [0, \infty)$.

Proof. In this proof, we focus on the case that $\mathbb{E}_G[s(x)] = \mathbb{E}_G[t(x)]$. The proof for the other case is essentially identical hence omitted here. Indeed, we have the following inequalities:

$$\begin{aligned} \mathbb{E}_B[s(x) - t(x)] &= \int_{x \leq x^*} [s(x) - t(x)] \frac{f_B(x)}{f_G(x)} f_G(x) dx + \int_{x > x^*} [s(x) - t(x)] \frac{f_B(x)}{f_G(x)} f_G(x) dx \\ &\geq \frac{f_B(x^*)}{f_G(x^*)} \int_{x \leq x^*} [s(x) - t(x)] f_G(x) dx + \frac{f_B(x^*)}{f_G(x^*)} \int_{x > x^*} [s(x) - t(x)] f_G(x) dx \end{aligned}$$

where the inequality follows from MLRP and (A.1). Thus,

$$\mathbb{E}_B[s(x) - t(x)] \geq \frac{f_B(x^*)}{f_G(x^*)} \int_0^\infty [s(x) - t(x)] f_G(x) dx = \frac{f_B(x^*)}{f_G(x^*)} \mathbb{E}_G[s(x) - t(x)] = 0,$$

which implies $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] \geq \mathbb{E}_B[t(x)] - \mathbb{E}_G[t(x)]$. The above inequalities hold strictly whenever the inequalities (A.1) hold for a positive measure of $x \in [0, \infty)$. *Q.E.D.*

A.3 Proof of Theorem 1

Note that $\lambda \leq \underline{\lambda}^*$ if and only if

$$\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] \leq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \tag{A.2}$$

where $\mathbb{E}_G[c(x, S_1^B)] < \mathbb{E}_B[c(x, S_1^B)] = 1$ by MLRP and the construction of S_1^B. We first prove the sufficiency of (A.2) and then the necessity of (A.2).

A.3.1 Proof of Sufficiency

Let $s^*(x) := c(x, S_1^B)$ and suppose that all brown investors offer the same contract s^*. The green investor abstains from offering any security acceptable to the firm (e.g., the green investor offers $s(x) \equiv x$). Because the sensitivity of s^* is weakly less than $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$, it is (weakly) optimal for the firm to accept s^* and then choose project B. Thus, it suffices to show that all investors also find it weakly optimal to offer s^*, given that all other investors also offer the same contract. Note that all investors get zero expected payoff from offering s^*.

First, we prove that it is optimal for each brown investor to offer s^*. Suppose that a brown investor deviates and offers s' such that, if accepted, the firm would choose project B. For s' to be a profitable deviation, we must have $\mathbb{E}_B[s'(x)] > 1$. However, such s' will be rejected outright since $\mathbb{E}_B[s'(x)] > 1 = \mathbb{E}_B[s^*(x)]$. Next, suppose that a brown investor deviates and offers s'' such that, if accepted, the firm would choose project G. For s'' to be profitable, we must have $\mathbb{E}_G[s''(x)] > 1$. However, such an offer will be rejected outright since $\mathbb{E}_G[s''(x)] > 1 > \mathbb{E}_G[s^*(x)] = \mathbb{E}_G[c(x, S_1^B)]$.

Next, we prove that the green investor also has no incentive to deviate. Similar to brown investors, the green investor finds it suboptimal to offer s' such that, if accepted, the firm would choose project B. Suppose the green investor deviates and offers $s''(\cdot) \neq s^*(\cdot)$ such

that, if accepted, the firm would choose project G. The firm will find it strictly optimal to choose project G only if

$$\mathbb{E}_B[s''(x)] - \mathbb{E}_G[s''(x)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \geq \mathbb{E}_B[s^*(x)] - \mathbb{E}_G[s^*(x)].$$

Moreover, we must have $\mathbb{E}_G[s''(x)] > \mathbb{E}_G[s^*(x)]$. First, if $s''(\cdot) = c(\cdot, S'')$ for some $S'' > 0$, we must have $S'' < S_1^B$ since $s''(\cdot) \neq s^*(\cdot)$, implying $\mathbb{E}_G[s''(x)] > \mathbb{E}_G[s^*(x)]$. Next, to show $\mathbb{E}_G[s''(x)] > \mathbb{E}_G[s^*(x)]$ when $s''(\cdot)$ is not a call option contract, suppose to the contrary $\mathbb{E}_G[s''(x)] \leq \mathbb{E}_G[s^*(x)]$. Then there must exist a call option contract $c(\cdot, S^G_{\mathbb{E}_G[s''(x)]})$ such that $\mathbb{E}_B[c(x, S^G_{\mathbb{E}_G[s''(x)]})] - \mathbb{E}_G[c(x, S^G_{\mathbb{E}_G[s''(x)]})] \geq \mathbb{E}_B[s''(x)] - \mathbb{E}_G[s''(x)]$. Furthermore, since $\mathbb{E}_G[c(x, S^G_{\mathbb{E}_G[s''(x)]})] = \mathbb{E}_G[s''(x)] \leq \mathbb{E}_G[s^*(x)] = \mathbb{E}_G[c(x, S_1^B)]$, we must have $S^G_{\mathbb{E}_G[s''(x)]} \geq S_1^B$. Since $s''(\cdot)$ is not a call option contract, $S^G_{\mathbb{E}_G[s''(x)]} \geq S_1^B$ implies that $s''(\cdot)$ is strictly less sensitive than $s^*(\cdot)$ by Lemma 1-(iv), a contradiction. However, $\mathbb{E}_G[s''(x)] > \mathbb{E}_G[s^*(x)]$ implies that the firm will reject s'' since $\mathbb{E}_G[x - s''(x)] + \lambda\phi < \mathbb{E}_G[x - s^*(x)] + \lambda\phi \leq \mathbb{E}_B[x - s^*(x)]$. Therefore, there is no feasible deviation offer that induces the firm to choose project G.

A.3.2 Proof of Necessity

In this proof, let $\mathbf{s}^*$ generically denote the set of securities that the firm accepts with positive probability in equilibrium. Suppose to the contrary that

$$\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_B[c(x, S_1^B)] > \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$$

but there exists an equilibrium in which all securities in $\mathbf{s}^*$ induce project B, and furthermore, $\mathbb{E}_B[s(x)] = 1$ for all $s(\cdot) \in \mathbf{s}^*$. Note that in such an equilibrium, every investor earns zero expected payoff. We show that the green investor could strictly benefit by unilaterally deviating and offering $c(x, S_1^B)$, which should not be the case in equilibrium. By supposition, the sensitivity of $c(\cdot, S_1^B)$ exceeds $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Thus, the firm strictly prefers project G to project B once it accepts $c(\cdot, S_1^B)$. Furthermore, we have

$$\mathbb{E}_G[x - c(x, S_1^B)] + \lambda\phi > \mathbb{E}_B[x - c(x, S_1^B)] = \mathbb{E}_B[x - 1] = \mathbb{E}_B[x - s(x)] \quad \forall s \in \mathbf{s}^*, \tag{A.3}$$

and thus, the firm will indeed accept $c(x, S_1^B)$ if offered by the green investor. On the other hand, the green investor will obtain

$$\mathbb{E}_G[c(x, S_1^B)] - 1 + \phi = \mathbb{E}_G[c(x, S_1^B)] - \mathbb{E}_B[c(x, S_1^B)] + \phi > 0,$$

where the last inequality holds because $\mathbb{E}_B[s(x)] - \mathbb{E}_G[s(x)] < \phi$ for any security $s(\cdot)$, as stated in Lemma A.1. Thus, the deviation indeed strictly benefits the green investor, a contradiction.

A.4 Proof of Theorem 2

Note that $\lambda \geq \overline{\lambda}^*$ if and only if

$$\mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \tag{A.4}$$

where $\mathbb{E}_B[d(x, D_1^B)] = 1 > \mathbb{E}_G[d(x, D_1^B)]$ by MLRP and the construction of D_1^B. By Lemma 1, (A.4) also implies

$$\mathbb{E}_B[c(x, S_1^G)] - \mathbb{E}_G[c(x, S_1^G)] \geq \mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi. \tag{A.5}$$

We first prove the sufficiency of (A.4) and then the necessity of (A.4).

A.4.1 Proof of Sufficiency

Let $s^*(\cdot) := c(\cdot, S_1^G)$ such that $\mathbb{E}_B[c(x, S_1^G)] > \mathbb{E}_G[c(x, S_1^G)] = 1$ and suppose that all brown investors offer the same security $s^*(\cdot)$. The green investor withdraws from bidding for a project by offering a security that the firm never finds it profitable to accept. From (A.4), the sensitivity of $s^*(\cdot)$ is weakly larger than $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$, and thus, it is (weakly) optimal for the firm to accept $s^*(\cdot)$ and then choose project G. Thus, it suffices to show that all investors find it suboptimal to deviate from the bidding strategies described above. Note that brown investors and the green investor respectively obtain 0 and ϕ as their expected payoffs from the above bidding strategies.

Suppose that an investor deviates and offers s' such that, if accepted, the firm would choose project G. For s' to be profitable, we must have $\mathbb{E}_G[s'(x)] > 1$. However, such s' will be rejected outright since $\mathbb{E}_G[s'(x)] > 1 = \mathbb{E}_G[s^*(x)] = \mathbb{E}_G[c(x, S_1^G)]$.

Next, suppose that an investor deviates and offers s'' such that, if accepted, the firm

would choose project B. For s'' to be profitable, we must have $\mathbb{E}_B[s''(x)] > 1$. However, by Lemma 1-(iii), there exists a debt contract $d(\cdot, D'')$ such that $\mathbb{E}_B[s''(x)] = \mathbb{E}_B[d(x, D'')]$ and thus $D'' > D_1^B$. By Lemma 1-(i), we must have $\mathbb{E}_B[s''(x)] - \mathbb{E}_G[s''(x)] \geq \mathbb{E}_B[d(x, D'')] - \mathbb{E}_G[d(x, D'')] > \mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)]$, which implies

$$\mathbb{E}_B[s''(x)] - \mathbb{E}_G[s''(x)] > \mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi.$$

Thus, the firm would choose project G once it accepts s'', which contradicts the supposition that s'' would induce project B.

A.4.2 Proof of Necessity

In this proof, let $\mathbf{s}^*$ generically denote the set of securities that the firm accepts with positive probability in equilibrium. Suppose to the contrary that

$$\mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_B[d(x, D_1^B)] < \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$$

but there is an equilibrium in which all securities in $\mathbf{s}^*$ induce project G, and furthermore, $\mathbb{E}_G[s(x)] = 1$ for all $s(\cdot) \in \mathbf{s}^*$. Note that all brown investors obtain zero payoff, whereas the green investor gets the payoff ϕ in such an equilibrium.

Now, we prove that such an equilibrium never exists by showing that a brown investor could strictly benefit by unilaterally deviating and offering $d(x, D_{1+\varepsilon}^B)$, where $\varepsilon > 0$ is a small positive number such that

$$\mathbb{E}_B[d(x, D_{1+\varepsilon}^B)] - \mathbb{E}_B[d(x, D_{1+\varepsilon}^B)] < \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi, \tag{A.6}$$

$$\mathbb{E}_B[x] - 1 - \varepsilon > \mathbb{E}_G[x] - 1 + \lambda\phi, \tag{A.7}$$

where (A.6) and (A.7) hold true for any sufficiently small $\varepsilon > 0$ by Lemma 1 and Assumption 1-(i), respectively. The sensitivity of $d(x, D_{1+\varepsilon}^B)$ is strictly less than $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Thus, the firm will strictly prefer project B to project G once it accepts $d(x, D_{1+\varepsilon}^B)$. Furthermore,

$$\mathbb{E}_B[x - d(x, D_{1+\varepsilon}^B)] = \mathbb{E}_B[x] - 1 - \varepsilon > \mathbb{E}_G[x] - 1 + \lambda\phi = \mathbb{E}_G[x - s(x)] + \lambda\phi \quad \forall s \in \mathbf{s}^*,$$

where the last equality follows from $\mathbb{E}_G[s(x)] = 1$ for all $s \in \mathbf{s}^*$. Hence, the firm will accept

$d(x, D^B_{1+\varepsilon})$ if offered by a brown investor. The deviating brown investor will then obtain

$$\mathbb{E}_B[d(x, D^B_{1+\varepsilon})] - 1 = 1 + \varepsilon - 1 = \varepsilon > 0.$$

Thus, the deviation indeed strictly benefits the deviating brown investor.

A.5 Proof of Theorem 3

In this proof, let $\mathbf{s}^*$ generically denote the set of securities that the firm accepts with positive probability in equilibrium. Note that $\lambda \in [\underline{\lambda}^*, \hat{\lambda}^*]$ if and only if

$$\begin{aligned}\mathbb{E}_B[c(x, S^B_1)] - \mathbb{E}_G[c(x, S^B_1)] &\geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \\ &\geq \mathbb{E}_B[d(x, D^G_{1-\phi})] - \mathbb{E}_G[d(x, D^G_{1-\phi})], \end{aligned} \tag{A.8}$$

where $\mathbb{E}_B[c(x, S^B_1)] = 1 > \mathbb{E}_G[c(x, S^B_1)] > 1-\phi$ and $\mathbb{E}_G[d(x, D^G_{1-\phi})] = 1-\phi < \mathbb{E}_B[d(x, D^G_{1-\phi})] < 1$ (Lemma A.1). Hence, we have $S^G_{1-\phi} > S^B_1$ and $D^G_{1-\phi} < D^B_1$, and therefore, at least one of the two inequalities in (A.8) must hold strictly. We first prove the sufficiency of the condition (A.8) and then prove its necessity.

A.5.1 Preliminary Observations

Observation A.1. *Suppose (A.8) holds. Then, there are securities s^* and $\hat{s}$ such that $s^* \neq \hat{s}$, and*

(i) $1 - \phi \leq \mathbb{E}_G[s^*(x)] < 1$ *and* $\mathbb{E}_B[\hat{s}(x)] \leq 1$*;*

(ii) $\mathbb{E}_B[s^*(x)] - \mathbb{E}_G[s^*(x)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$*;*

(iii) $\mathbb{E}_G[x - s^*(x)] + \lambda\phi = \mathbb{E}_B[x - \hat{s}(x)] \geq \mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi$*.*

Proof. Note first that $S^G_{1-\phi} > S^B_1$, which is immediate from Lemma A.1. There are three subcases. First, suppose $\mathbb{E}_B[c(x, S^G_{1-\phi})] - \mathbb{E}_G[c(x, S^G_{1-\phi})] \leq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. In this case,

$$\begin{aligned}E_B[c(x, S^G_{1-\phi})] - \mathbb{E}_G[c(x, S^G_{1-\phi})] &\leq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \\ &\leq \mathbb{E}_B[c(x, S^B_1)] - \mathbb{E}_G[c(x, S^B_1)],\end{aligned}$$

where only one inequality can weakly hold since $S^G_{1-\phi} > S^B_1$. Also, there is $S^* \in [S^B_1, S^G_{1-\phi}]$ such that $\mathbb{E}_B[c(x,S^*)] - \mathbb{E}_G[c(x,S^*)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$ and $1-\phi \leq \mathbb{E}_G[c(x,S^*)] < \mathbb{E}_B[c(x,S^*)] \leq 1$. Now, let $s^*(\cdot) = c(\cdot, S^*)$ and $\hat{s}(\cdot) = d(\cdot, D^B_{\mathbb{E}_B[c(x,S^*)]})$. Then $s^*(\cdot)$ and $\hat{s}(\cdot)$ clearly satisfy (i) and (ii). To show (iii) also holds, note that the sensitivity of $d(\cdot, D^B_{\mathbb{E}_B[c(x,S^*)]})$ is strictly less than the sensitivity of $c(\cdot, S^*)$, while the sensitivity of $c(\cdot, S^*)$ equals $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Thus,

$$\mathbb{E}_G[x - c(x,S^*)] + \lambda\phi = \mathbb{E}_B[x - d(x, D^B_{\mathbb{E}_B[c(x,S^*)]})] \geq \mathbb{E}_G[x - d(x, D^B_{\mathbb{E}_B[c(x,S^*)]})] + \lambda\phi,$$

and hence (iii) holds true.

Second, suppose that

$$\begin{aligned}\mathbb{E}_B[c(x, S^G_{1-\phi})] - \mathbb{E}_G[c(x, S^G_{1-\phi})] &> \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \\ &= \mathbb{E}_B[d(x, D^G_{1-\phi})] - \mathbb{E}_B[d(x, D^G_{1-\phi})].\end{aligned}$$

Define $s^*(\cdot) := \alpha c(\cdot, S^G_{1-\phi}) + (1-\alpha) d(\cdot, D^G_{1-\phi})$, where we choose the weight $\alpha \in (0,1)$ so that the sensitivity of $s^*(\cdot)$ is larger than $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Then,

$$\begin{aligned}&\mathbb{E}_G[s^*(x)] = \alpha \mathbb{E}_G[c(x, S^G_{1-\phi})] + (1-\alpha)\mathbb{E}_G[d(x, D^G_{1-\phi})] = 1 - \phi, \\ &\mathbb{E}_B[s^*(x)] < \mathbb{E}_G[s^*(x)] + \phi = 1 - \phi + \phi = 1,\end{aligned}$$

where the strict inequality follows from Lemma A.1. Thus, $s^*(\cdot)$ satisfies both properties (i) and (ii) above. In addition, let $\hat{s}(\cdot) := d(\cdot, D^G_{1-\phi})$. Note that $s^*(\cdot)$ is not a debt contract (because $\alpha \in (0,1)$), implying $s^*(\cdot) \neq \hat{s}(\cdot)$. Furthermore, $\mathbb{E}_G[\hat{s}(x)] = 1 - \phi$ and $\mathbb{E}_B[\hat{s}(x)] < \mathbb{E}_G[\hat{s}(x)] + \phi \leq 1$ by Lemma A.1, implying $\hat{s}(\cdot)$ satisfies (i). Lastly,

$$\begin{aligned}&\mathbb{E}_G[x - s^*(x)] + \lambda\phi = \mathbb{E}_G[x] - 1 + \phi + \lambda\phi, \\ &\mathbb{E}_B[x - \hat{s}(x)] = \mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi = \mathbb{E}_G[x] - 1 + \phi + \lambda\phi,\end{aligned}$$

where the equality $\mathbb{E}_B[x - \hat{s}(x)] = \mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi$ holds because the sensitivity of $\hat{s}(\cdot)$ equals $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$ by construction. Hence, $s^*(\cdot)$ and $\hat{s}(\cdot)$ also satisfy (iii).

Finally, suppose that

$$\begin{aligned}\mathbb{E}_B[c(x, S^G_{1-\phi})] - \mathbb{E}_G[c(x, S^G_{1-\phi})] &> \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \\ &> \mathbb{E}_B[d(x, D^G_{1-\phi})] - \mathbb{E}_B[d(x, D^G_{1-\phi})].\end{aligned}$$

Define $s^*(\cdot) := \alpha c(\cdot, S^G_{1-\phi}) + (1-\alpha)d(\cdot, D^G_{1-\phi})$, where we choose $\alpha \in (0,1)$ such that the sensitivity of $s^*(\cdot)$ is exactly equal to $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Then,

$$\mathbb{E}_G[s^*(x)] = \alpha\mathbb{E}_G[c(x, S^G_{1-\phi})] + (1-\alpha)\mathbb{E}_G[d(x, D^G_{1-\phi})] = 1-\phi,$$
$$\mathbb{E}_B[s^*(x)] < \mathbb{E}_G[s^*(x)] + \phi = 1 - \phi + \phi = 1,$$

where the strict inequality in the last line follows from Lemma A.1. Thus, $s^*(\cdot)$ satisfies both properties (i) and (ii) above. In addition, let $\hat{s}(\cdot) = d(\cdot, D^B_{\mathbb{E}_B[s^*(x)]})$. Note that $s^*(\cdot)$ is not a debt contract (because $\alpha \in (0,1)$), implying $s^*(\cdot) \neq \hat{s}(\cdot)$. Furthermore, $\mathbb{E}_B[\hat{s}(x)] = \mathbb{E}_B[s^*(x)] < \mathbb{E}_G[s^*(x)] + \phi = 1$ (Lemma A.1), hence $\hat{s}(\cdot)$ satisfies (i). Finally,

$$\mathbb{E}_B[x - s^*(x)] = \mathbb{E}_G[x - s^*(x)] + \lambda\phi = \mathbb{E}_G[x] - 1 + \phi + \lambda\phi,$$
$$\mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi \leq \mathbb{E}_B[x - \hat{s}(x)] = \mathbb{E}_B[x - s^*(x)],$$

where the inequality in the last line holds because $\hat{s}(\cdot)$ is less sensitive than $s^*(\cdot)$ and the sensitivity of $s^*(\cdot)$ equals $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$ by construction and the equality in the last line holds because $\mathbb{E}_B[\hat{s}(x)] = \mathbb{E}_B[s^*(x)]$. Hence, $s^*(\cdot)$ and $\hat{s}(\cdot)$ also satisfy (iii). *Q.E.D.*

Observation A.2. *In any equilibrium such that the firm chooses project G with probability 1 and $\mathbb{E}_G[s(x)] \in [1-\phi, 1)$ for any $s \in \mathbf{s}^*$:*

(i) *$\mathbf{s}^*$ is a singleton (i.e., $\mathbf{s}^* = \{s^*\}$), and only the green investor offers s^*, where $\mathbb{E}_B[s^*(x)] - \mathbb{E}_G[s^*(x)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$.*

(ii) *A brown investor must offer a security $\hat{s} \neq s^*$ (which is not accepted by the firm) that satisfies Observation A.1-(iii). That is,*

$$\mathbb{E}_G[x - s^*(x)] + \lambda\phi = \max\{\mathbb{E}_B[x - \hat{s}(x)], \mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi\} = \mathbb{E}_B[x - \hat{s}(x)].$$

Proof. To show (i), note that brown investors have no preference for the social value of project G, and thus, they will never offer $s \in \mathbf{s}^*$ whose expected return is less than 1. Thus, $\mathbf{s}^* = \{s^*\}$ must be a singleton, and it must be the green investor who offers s^*. In addition, to ensure that s^* induces the firm to choose project G, the sensitivity of s^* must be weakly larger than $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Note that (i) also implies

$$\mathbb{E}_G[x - s^*(x)] + \lambda\phi \geq \max\{\mathbb{E}_B[x - s(x)], \mathbb{E}_G[x - s(x)] + \lambda\phi\}$$

for any security $s(\cdot)$ offered by a brown investor in equilibrium because, otherwise, the green investor's offer s^* is never accepted by the firm, which contradicts (i).

To show (ii), it suffices to show that a brown investor offers $\hat{s}(\cdot)$ such that

$$\mathbb{E}_G[x - s^*(x)] + \lambda\phi = \max\{\mathbb{E}_B[x - \hat{s}(x)], \mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi\} = \mathbb{E}_B[x - \hat{s}(x)]. \tag{A.9}$$

First, suppose the contrary to the first equality in (A.9) that

$$\mathbb{E}_G[x - s^*(x)] + \lambda\phi > \max\{\mathbb{E}_B[x - \hat{s}(x)], \mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi\}$$

for any security $\hat{s}(\cdot)$ a brown investor offers in equilibrium. Suppose that the green investor deviates by offering $s'(\cdot) := c(\cdot, S^G_{\mathbb{E}_G[s^*(x)]+\varepsilon})$, where $\varepsilon > 0$ is a small positive number. Then, by construction, $s'(\cdot)$ is more sensitive than s^* hence also induces the firm to choose project G if accepted (Lemma 1-(iv)). Furthermore, provided that $\varepsilon > 0$ is small, we have

$$\mathbb{E}_G[x - s'(x)] + \lambda\phi = \mathbb{E}_G[x - s^*(x)] + \lambda\phi - \varepsilon > \max\{\mathbb{E}_B[x - \hat{s}(x)], \mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi\}$$

for all $\hat{s}$ offered by brown investors in equilibrium. Hence, the firm will accept $s'(\cdot)$ with probability 1. As a result, the green investor strictly benefits from such a deviation, which should not be possible in equilibrium.

Finally, suppose the contrary to the second equality in (A.9) that

$$\mathbb{E}_G[x - s^*(x)] + \lambda\phi = \mathbb{E}_G[x - \hat{s}(x)] + \lambda\phi > \mathbb{E}_B[x - \hat{s}(x)] \tag{A.10}$$

for any brown investor' offer $\hat{s}(\cdot)$ such that $\mathbb{E}_G[x - s^*(x)] + \lambda\phi = \max\{\mathbb{E}_G[x - s^*(x)] + \lambda\phi, \mathbb{E}_B[x - \hat{s}(x)]\}$. In equilibrium, the green investor earns $\mathbb{E}_G[s^*(x)] - 1 + \phi < \phi$. But if the green investor withdraws her offer $s^*(\cdot)$, the firm will accept one of the brown investors' offers that satisfy (A.10) and then choose project G. Thus, the green investor can get ϕ from withdrawing her offer, a strictly higher payoff than that from offering $s^*(\cdot)$, which should not be possible in equilibrium. *Q.E.D.*

A.5.2 Proof of Sufficiency

Suppose that the green investor offers $s^*(\cdot)$, while all other investors offer $\hat{s}(\cdot)$, where $s^*(\cdot)$ and $\hat{s}(\cdot)$ are the securities as constructed in Observation A.1. The firm employs the following

decision rule: the firm chooses $s^*(\cdot)$ (both on and off the path) if both $s^*(\cdot)$ and $\hat{s}(\cdot)$ are available; the firm will choose project G if it accepts $s^*(\cdot)$; the firm will choose project B if it accepts $\hat{s}(\cdot)$. By Observation A.1-(ii) and (iii), this decision rule is optimal for the firm.

Now we show that, given the other investors' offer strategies and the firm's decision rule as stipulated above, no investor can strictly benefit from any deviation. First, consider a brown investor. In order to win the competition to fund the firm's project and to induce the firm to choose project B, a brown investor must offer a security $s'(\cdot)$ such that $\mathbb{E}_B[s'(x)] \leq \mathbb{E}_B[\hat{s}(x)] \leq 1$. However, such a deviation will never result in a strictly higher payoff for the brown investor and thus is strictly suboptimal. Similarly, in order to win the competition to fund the firm's project and to induce the firm to choose project G, a brown investor must offer a security $s''(\cdot)$ such that $\mathbb{E}_G[s''(x)] \leq \mathbb{E}_G[s^*(x)] < 1$, which would generate a negative payoff to the deviating brown investor. In summary, brown investors have no incentive to deviate from $\hat{s}(\cdot)$, because such a deviation could never generate a strictly positive financial payoff.

Next, consider the green investor. The green investor obtains the payoff $\mathbb{E}_G[s^*(x)] - 1 + \phi \geq 0$ in equilibrium. Similar to the brown investors, the green investor cannot obtain a positive payoff from deviating to any contract that would induce project B. Therefore, we focus on the case that the green investor deviates to $s''(\cdot)$ such that *(a)* the firm finds it optimal to accept $s''(\cdot)$ over $\hat{s}(\cdot)$ and *(b)* $s''(\cdot)$ would induce the firm to choose project G. In order for $s''(\cdot)$ to yield the green investor a strictly higher payoff than $s^*(\cdot)$, we must have $\mathbb{E}_G[s''(x)] > \mathbb{E}_G[s^*(x)]$. However, the firm would find it more profitable to accept $\hat{s}$ over $s''(\cdot)$ because

$$\mathbb{E}_G[x - s''(x)] + \lambda\phi < \mathbb{E}_G[x - s^*(x)] + \lambda\phi = \mathbb{E}_B[x - \hat{s}(x)],$$

where the last equality follows from Observation A.1-(iii).

A.5.3 Proof of Necessity

Suppose to the contrary that, despite (A.8) fails, there is an equilibrium such that *(i)* the firm chooses project G with probability 1 and *(ii)* the firm's expected payment lies in $[1-\phi, 1)$. By Observation A.2, we may assume without loss that $\mathbf{s}^* = \{s^*\}$ is a singleton where $s^*(\cdot)$ is offered by the green investor, $\mathbb{E}_G[s^*(x)] \in [1-\phi, 1)$, and $\mathbb{E}_B[s^*(x)] - \mathbb{E}_G[s^*(x)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Note that all brown investors obtain zero payoff in any such an equilibrium.

Step I: We first show that the first inequality in (A.8) could not fail. In other words, if any

inequality fails in (A.8), it must be the second inequality. Suppose for contradiction

$$\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi > \mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)].$$

Consider the call option $c(\cdot, S^G_{\mathbb{E}_G[s^*(x)]})$. By Lemma 1-(iv), $c(\cdot, S^G_{\mathbb{E}_G[s^*(x)]})$ is more sensitive than $s^*(\cdot)$ and thus also induces the firm to choose project G. Furthermore, the firm's expected payment to an investor is identical under $c(\cdot, S^G_{\mathbb{E}_G[s^*(x)]})$ and $s^*(\cdot)$. Hence, the firm must be indifferent between $s^*(\cdot)$ and $c(\cdot, S^G_{\mathbb{E}_G[s^*(x)]})$.

Now, note that there exists $S^* \in (S^G_{\mathbb{E}_G[s^*(x)]}, S_1^B)$ such that $\mathbb{E}_B[c(x, S^*)] - \mathbb{E}_G[c(x, S^*)] < \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$, and hence, $c(\cdot, S^*)$ would induce the firm to choose project B if accepted by the firm. Suppose that a brown investor deviates by offering $s'(\cdot) = c(\cdot, S^*)$. The striking price S^* is strictly larger than $S^G_{\mathbb{E}_G[s^*(x)]}$, and hence, the firm will find $s'(\cdot)$ strictly more profitable than $s^*(\cdot)$. Hence, the firm would accept $s'(\cdot)$ and then choose project B, which yields the deviating brown investor

$$\mathbb{E}_B[c(x, S^*)] - 1 > \mathbb{E}_B[c(x, S_1^B)] - 1 \geq 0,$$

contradicting the earlier observation that all brown investors earn zero payoff in equilibrium.

Step II: To complete our proof for the necessity of (A.8), we show that the second inequality in (A.8) could not fail either. Suppose for contradiction

$$\begin{aligned}\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] &> \mathbb{E}_B[d(x, D^G_{1-\phi})] - \mathbb{E}_G[d(x, D^G_{1-\phi})] \\ &> \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi.\end{aligned}$$

Consider $\hat{s}(\cdot)$ that is offered by a brown investor and also satisfies Observation A.2-(ii). The sensitivity of $\hat{s}(\cdot)$ must be strictly less than $d(\cdot, D^G_{1-\phi})$, and thus, $\mathbb{E}_G[\hat{s}(x)] < \mathbb{E}_G[d(x, D^G_{1-\phi})] = 1 - \phi$. If $\mathbb{E}_G[\hat{s}(x)] \geq \mathbb{E}_G[d(x, D^G_{1-\phi})]$, there exists a debt contract $d(\cdot, D^G_{\mathbb{E}_G[\hat{s}(x)]})$ such that $D^G_{\mathbb{E}_G[\hat{s}(x)]} \geq D^G_{1-\phi}$, implying $\hat{s}(\cdot)$ is weakly more sensitive than $d(\cdot, D^G_{1-\phi})$. On the other hand, Observation A.2-(ii) implies $\mathbb{E}_G[s^*(x)] \leq \mathbb{E}_G[\hat{s}(x)] < 1 - \phi$. But then, the green investor's expected payoff from offering $s^*(\cdot)$, $\mathbb{E}_G[s^*(x)] - 1 + \phi$, is strictly less than zero, which is impossible in equilibrium.

A.6 Proof of Theorem 4

Note that $\lambda \in [\underline{\lambda}^*, \tilde{\lambda}^*)$ if and only if

$$\begin{aligned}\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] &\geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \\ &> \mathbb{E}_B[c(x, S_{1-\phi}^G)] - \mathbb{E}_G[c(x, S_{1-\phi}^G)]. \end{aligned} \tag{A.11}$$

In this section, we will first prove the necessity of (A.11) for the existence of a mixed-strategy equilibrium, and then we will discuss how $\tilde{\beta}^*$ varies as λ changes between $\underline{\lambda}^*$ and $\tilde{\lambda}^*$. Finally, we will prove the sufficiency of (A.11).

In this proof, let $\mathbf{s}^*$ generically denote the set of securities that the firm accepts with positive probability in equilibrium. For each $k \in \{B, G\}$, $\mathbf{s}_k^* \subset \mathbf{s}^*$ denotes the collection of securities in $\mathbf{s}^*$ that induce the selection of project k with positive probability if accepted by the firm. Note that the firm is indifferent between all securities in $\mathbf{s}^* = \mathbf{s}_B^* \cup \mathbf{s}_G^*$. Thus, whenever both $\mathbf{s}_B^*$ and $\mathbf{s}_G^*$ are non-empty,

$$\mathbb{E}_B[x - s_B(x)] = \mathbb{E}_G[x - s_G(x)] + \lambda\phi \quad \forall (s_B, s_G) \in \mathbf{s}_B^* \times \mathbf{s}_G^*. \tag{A.12}$$

Furthermore, for any $k \in \{B, G\}$, $\mathbb{E}_k[s'(x)] = \mathbb{E}_k[s''(x)]$ for any $s', s'' \in \mathbf{s}_k^*$ because the firm must be indifferent between any two securities in $\mathbf{s}_k^*$.

A.6.1 Preliminary Observation

We first make two preliminary observations, Observation A.3–A.4, which hold regardless of whether (A.11) holds or fails.

Observation A.3. *Consider any equilibrium such that $\mathbf{s}_B^* \neq \emptyset$ and $\mathbf{s}_G^* \neq \emptyset$. For any $s_G \in \mathbf{s}_G^*$ and $s_B \in \mathbf{s}_B^*$, we have the following properties:*

(i) $\mathbb{E}_G[s_G(x)] < \mathbb{E}_B[s_B(x)] \leq 1$.

(ii) $\mathbb{E}_G[s_G(x)] \leq \mathbb{E}_G[s_B(x)] < \mathbb{E}_B[s_B(x)] \leq \mathbb{E}_B[s_G(x)]$.

(iii) $\mathbb{E}_G[s_G(x)] > \mathbb{E}_B[s_G(x)] - \phi \geq \mathbb{E}_B[s_B(x)] - \phi$.

Proof. The first inequality $\mathbb{E}_G[s_G(x)] < \mathbb{E}_B[s_B(x)]$ in (i) directly follows (A.12) and Assumption 1-(i). Note that the first inequality in (i) also implies that the equilibrium payoff of any

brown investor is strictly bounded from above by $r_B - 1$, where $r_B > 0$ is a non-negative number such that $\mathbb{E}_B[s_B(x)] = r_B$ for all $s_B \in \mathbf{s}_B^*$; brown investors' equilibrium payoffs never exactly attain $r_B - 1$ due to the firm's randomization in equilibrium.

To prove the second inequality in (i), suppose to the contrary that $r_B = \mathbb{E}_B[s_B(x)] > 1$ for any $s_B(\cdot) \in \mathbf{s}_B^*$. From the discussion in the previous paragraph, all brown investors earn strictly less than $r_B - 1$ in equilibrium. Now, suppose that a brown investor deviates by offering $\hat{s}(\cdot) = d(\cdot, S^B_{r_B - \varepsilon})$, where $r_B - \varepsilon > 1$. The firm prefers $\hat{s}$ to any $s_B(\cdot) \in \mathbf{s}_B^*$ because $r_B > \mathbb{E}_B[\hat{s}(x)]$. Furthermore, by Lemma 1-(iii), $\hat{s}(\cdot)$ is less sensitive than any security in $\mathbf{s}_B^*$. Hence, the firm will accept $\hat{s}$ and then choose project B under $\hat{s}(\cdot)$. Thus, the deviating brown investor would obtain $r_B - \varepsilon - 1$. This payoff is strictly larger than her equilibrium payoff if we choose $\varepsilon > 0$ sufficiently small, which should not be possible in equilibrium.

To prove (ii), note first that

$$\mathbb{E}_B[x - s_B(x)] = \mathbb{E}_G[x - s_G(x)] + \lambda\phi \geq \mathbb{E}_B[x - s_G(x)] \implies \mathbb{E}_B[s_G(x)] \geq \mathbb{E}_B[s_B(x)],$$
$$\mathbb{E}_G[x - s_B(x)] + \lambda\phi \leq \mathbb{E}_B[x - s_B(x)] = \mathbb{E}_G[x - s_G(x)] + \lambda\phi \implies \mathbb{E}_G[s_G(x)] \leq \mathbb{E}_G[s_B(x)]$$

by construction of $\mathbf{s}_B^*$ and $\mathbf{s}_G^*$. In addition, $\mathbb{E}_G[s_B(x)] < \mathbb{E}_B[s_B(x)]$ by MLRP, which completes the proof of (ii). Finally, the two inequalities in (iii) immediately follow Lemma A.1 and $\mathbb{E}_B[s_G(x)] \leq \mathbb{E}_B[s_B(x)]$ in (ii), respectively. *Q.E.D.*

Observation A.4. *In any equilibrium such that $\mathbf{s}_B^* \neq \emptyset$ and $\mathbf{s}_G^* \neq \emptyset$, we have the following:*

(i) Only the green investor offers a security in $\mathbf{s}_G^*$. Thus, $\mathbf{s}_G^*$ is a singleton.

(ii) The security in $\mathbf{s}_G^*$ is the call option $c(\cdot, S^*)$ such that $\mathbb{E}_B[c(x, S^*)] - \mathbb{E}_G[c(x, S^*)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$.

Proof. Fix any $s(\cdot) \in \mathbf{s}_G^*$. $\mathbb{E}_G[s(x)] < 1$ by Observation A.3-(i). There are two subcases. First, suppose $s(\cdot) \notin \mathbf{s}_B^*$. Then, the firm will choose project G with probability 1 under $s(\cdot)$, and thus, $s(\cdot)$ only generates a strictly negative financial payoff for a funding investor. Next, suppose $s(\cdot) \in \mathbf{s}_B^*$. By Observation A.3-(i), $\mathbb{E}_G[s(x)] < \mathbb{E}_B[s(x)] \leq 1$. Since the firm chooses both projects B and G with positive probabilities, $s(\cdot)$ also generates a strictly negative financial payoff for a funding investor. Hence, any brown investor would find it strictly suboptimal to offer any security in $\mathbf{s}_G^*$, which completes the proof of (i).

To show (ii), let s_G denote the unique security in $\mathbf{s}_G^*$. The green investor must offer s_G in equilibrium. Furthermore, s_G induces project G with positive probability, and hence,

$\mathbb{E}_B[s_G(x)] - \mathbb{E}_G[s_B(x)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Finally, the green investor's expected payoff in equilibrium is

$$V_G := (1-\beta^*)(\mathbb{E}_G[s_G(x)] - 1 + \phi) + \alpha^*(\mathbb{E}_B[s_G(x)] - 1) \tag{A.13}$$

where $1-\beta^* \in (0,1)$ is the probability that the firm accepts s_G and project G in equilibrium, and $\alpha^* \in [0, 1-\beta^*]$ is the probability that the firm accepts s_G and chooses project B. Note that $\alpha^* = 0$ if $s_G \notin \mathbf{s}_B^*$.

Now, suppose for contradiction with (ii) one of the following two fails:

$$\text{(a)}\ \ s_G(\cdot) \text{ is a call option;} \qquad \text{(b)}\ \ \mathbb{E}_B[s_G(x)] - \mathbb{E}_G[s_G(x)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi. \tag{A.14}$$

In either case, suppose that the green investor deviates by offering $s_\varepsilon(\cdot) := c(\cdot, S^G_{\mathbb{E}_G[s_G(x)]-\varepsilon})$, where $\varepsilon > 0$ is sufficiently small such that the sensitivity of $s_\varepsilon(\cdot)$ is strictly larger than $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. By construction, we have $\mathbb{E}_G[s_\varepsilon(x)] < \mathbb{E}_G[s_G(x)]$, and therefore, the firm would strictly prefer $s_\varepsilon(\cdot)$ to $s_G(\cdot)$. Furthermore, the firm will choose project G under $s_\varepsilon(\cdot)$ because the sensitivity of $s_\varepsilon(\cdot)$ exceeds $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Thus, the green investor's expected payoff from the deviation will be

$$V_\varepsilon := \mathbb{E}_G[s_\varepsilon(x)] - 1 + \phi = \mathbb{E}_G[s_G(x)] - \varepsilon - 1 + \phi. \tag{A.15}$$

We will complete the proof by showing that V_ε in (A.15) is strictly larger than V_G in (A.13), provided that $\varepsilon > 0$ is sufficiently small. There are two subcases. First, suppose $\mathbb{E}_B[s_B(x)] = 1$ for any $s_B \in \mathbf{s}_B^*$. In this case,Observation A.3-(ii) also implies $\mathbb{E}_B[s_G(x)] \geq \mathbb{E}_B[s_B(x)] = 1$. If $\mathbb{E}_B[s_G(x)] > \mathbb{E}_B[s_B(x)] = 1$, the firm never accepts $s_G(\cdot)$ to choose project B, and thus, $\alpha^* = 0$. Hence, regardless of whether the inequality $\mathbb{E}_B[s_G(x)] \geq \mathbb{E}_B[s_B(x)]$ holds strictly or not, the second term in (A.13) must be zero. On the other hand, we have $\mathbb{E}_G[s_G(x)] > \mathbb{E}_B[s_B(x)] - \phi \geq 1 - \phi$ from Observation A.3-(iii). Hence,

$$V_G - V_\varepsilon = -\beta^*(\mathbb{E}_G[s_G(x)] - 1 + \phi) + \varepsilon < 0.$$

Next, consider the case $\mathbb{E}_B[s_B(x)] < 1$ for any $s_B \in \mathbf{s}_B^*$. In this case, any brown investor would suffer a negative financial payoff from offering a security in $\mathbf{s}_B^*$, and thus, we necessarily have $\mathbf{s}_G^* = \mathbf{s}_B^* = \{s_G\}$, implying $s_B(\cdot) = s_G(\cdot)$. Hence, $\mathbb{E}_B[s_G(x)] = \mathbb{E}_B[s_B(x)] < 1$, and thus the second term in (A.13) is negative. On the other hand, the green investor must obtain a

non-negative payoff in equilibrium; thus, we also necessarily have $\mathbb{E}_G[s_G(x)] - 1 + \phi > 0$. In summary,

$$\begin{aligned} V_G - V_\varepsilon &= (1-\beta^*)(\mathbb{E}_G[s_G(x)] - 1 + \phi) + \alpha^*(\mathbb{E}_B[s_G(x)] - 1) - (\mathbb{E}_G[s_G(x)] - \varepsilon - 1 + \phi) \\ &< -\beta^*(\mathbb{E}_G[s_G(x)] - 1 + \phi) + \varepsilon < 0 \end{aligned}$$

whenever $\varepsilon > 0$ is sufficiently small. *Q.E.D.*

In what follows, we will denote by s_G^* a unique security in $\mathbf{s}_G^*$.

Observation A.5. *In any equilibrium such that $\mathbf{s}_B^* \neq \emptyset$ and $\mathbf{s}_G^* = \{s_G^*\} \neq \emptyset$, there must be a brown investor who offers $\tilde{s}(\cdot) \neq s_G^*(\cdot)$ (which may not belong to $\mathbf{s}_B^* \cup \mathbf{s}_G^*$) such that*

$$\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \geq \mathbb{E}_B[\tilde{s}(x)] - \mathbb{E}_G[\tilde{s}(x)] \quad \textit{and} \quad \mathbb{E}_B[\tilde{s}(x)] \leq \mathbb{E}_B[s_G^*(x)]. \tag{A.16}$$

Furthermore, if we additionally assume $\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$, there must be a brown investor who offers $\tilde{s}(\cdot) \neq s_G^*(\cdot)$ (which may not belong to $\mathbf{s}_B^* \cup \mathbf{s}_G^*$) such that

$$\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \geq \mathbb{E}_B[\tilde{s}(x)] - \mathbb{E}_G[\tilde{s}(x)] \ \textit{and} \ \mathbb{E}_B[\tilde{s}(x)] = \mathbb{E}_B[s_G^*(x)] \leq 1. \tag{A.17}$$

Proof.

Step I: Suppose for contradiction that (A.16) fails for any security $\tilde{s}(\cdot)$ offered by a brown investor in equilibrium. We claim that the firm strictly prefers $s_G^*(\cdot)$ to any securities offered by the brown investors. First, consider a brown investor's offer $\tilde{s}(\cdot)$ such that $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi < \mathbb{E}_B[\tilde{s}(x)] - \mathbb{E}_G[\tilde{s}(x)]$. The firm will choose project G under any such $\tilde{s}(\cdot)$. On the other hand, $s_G^*(\cdot)$ is the call option contract whose sensitivity is equal to $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$, as identified in Observation A.4-(ii). Thus, by Lemma 1, we have $\mathbb{E}_G[\tilde{s}(x)] > \mathbb{E}_G[s_G^*(x)]$, and hence the firm strictly prefers $s_G^*(\cdot)$ to $\tilde{s}(\cdot)$.

Next, consider a brown investor's offer $\tilde{s}(\cdot)$ such that $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \geq \mathbb{E}_B[\tilde{s}(x)] - \mathbb{E}_G[\tilde{s}(x)]$ and $\mathbb{E}_B[\tilde{s}(x)] > \mathbb{E}_B[s_G^*(x)]$. The firm obtains $\mathbb{E}_B[x - \tilde{s}(x)]$ from accepting any such $\tilde{s}(\cdot)$, while the firm obtains $\mathbb{E}_G[x - s_G^*(x)] + \lambda\phi$ from accepting $s_G^*(\cdot)$. Note that

$$\mathbb{E}_B[x - \tilde{s}(x)] < \mathbb{E}_B[x - s_G^*(x)] \leq \mathbb{E}_G[x - s_G^*(x)] + \lambda\phi$$

where the weak inequality follows from the assumption $s_G^* \in \mathbf{s}_G^*$. Hence, the firm strictly prefers $s_G^*(\cdot)$ to $\tilde{s}(\cdot)$.

Because the firm strictly prefers $s_G^*(\cdot)$ to any other securities offered by the brown investors, the firm would accept $s_G^*(\cdot)$ with probability 1 in equilibrium. The green investor's equilibrium payoff is

$$(1-\beta^*)(\mathbb{E}_G[s_G^*(x)]-1+\phi)+\beta^*(\mathbb{E}_B[s_G^*(x)]-1) \tag{A.18}$$

where $1-\beta^* \in (0,1)$ is the probability that the firm chooses project G under $s_G^*(\cdot)$. By Observation A.3-(iii), this equilibrium payoff is strictly less than $\mathbb{E}_G[s_G^*(x)]-1+\phi$.

Step II: Now, suppose that the green investor deviates by offering $s_\varepsilon(\cdot) := c(\cdot, S^G_{\mathbb{E}_G[s_G^*(x)]+\varepsilon})$, where $\varepsilon > 0$ is sufficiently small so that the firm still strictly prefers $s_\varepsilon(\cdot)$ to all other securities offered by brown investors. $s_\varepsilon(\cdot)$ is more sensitive than s_G^*, and thus, $s_\varepsilon(\cdot)$ would induce the firm to choose project G for sure. Thus, the green investor's payoff from the deviation is $\mathbb{E}_G[s_\varepsilon(x)]-1+\phi = \mathbb{E}_G[s_G^*(x)]+\varepsilon-1+\phi$, which is strictly larger than the green investor's equilibrium payoff in (A.18). Hence, if (A.16) fails for any security $\tilde{s}(\cdot)$ offered by a brown investor, the green investor can strictly benefit from offering $s_\varepsilon(\cdot)$, which should be impossible in equilibrium. This proves Observation A.5-(i).

Step III: Finally, to prove Observation A.5-(ii), assume that $\mathbb{E}_B[c(x,S_1^B)]-\mathbb{E}_G[c(x,S_1^B)]$ is weakly larger than $\mathbb{E}_B[x]-\mathbb{E}_G[x]-\lambda\phi$, which implies $\mathbb{E}_B[s_G^*(x)] \leq 1$. Suppose for contradiction that

$$\mathbb{E}_B[x]-\mathbb{E}_G[x]-\lambda\phi \geq \mathbb{E}_B[\tilde{s}(x)]-\mathbb{E}_G[\tilde{s}(x)] \quad \text{and} \quad \mathbb{E}_B[\tilde{s}(x)] < \mathbb{E}_B[s_G^*(x)] \leq 1 \tag{A.19}$$

for all $\tilde{s}(\cdot) \neq s_G^*(\cdot)$ offered by a brown investor in equilibrium. Because $\mathbb{E}_B[\tilde{s}(x)] < \mathbb{E}_B[s_G^*(x)]$, the firm strictly prefers $\tilde{s}(\cdot)$ to $s_G^*(\cdot)$ if the firm plans to choose project B. Hence, $s_G^*(\cdot) \notin \mathbf{s}_B^*$, and $\mathbf{s}_B^* \backslash \{s_G^*\}$ must include at least one $\tilde{s}(\cdot)$, offered by a brown investor, that satisfies (A.19). However, since $\mathbb{E}_B[\tilde{s}(x)] < \mathbb{E}_B[s_G^*(x)] \leq 1$ by the supposition, $\tilde{s}(\cdot)$ generates a negative financial payoff to any brown investor who offers $\tilde{s}(\cdot)$. Hence, brown investors will never offer $\tilde{s}(\cdot)$ in equilibrium, a contradiction. *Q.E.D.*

A.6.2 Proof of Necessity

Suppose that, although (A.11) fails, there is an equilibrium such that the firm chooses B with probability $\beta^* \in (0,1)$. Note that we always have $\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] > \mathbb{E}_B[c(x, S_{1-\phi}^G)] - \mathbb{E}_G[c(x, S_{1-\phi}^G)]$ because $\mathbb{E}_B[c(x, S_{1-\phi}^G)] < \mathbb{E}_G[c(x, S_{1-\pi}^G)] + \phi = 1 = \mathbb{E}_B[c(x, S_1^B)]$.

Step I: We first show that the first inequality in (A.11) always holds true. In other words, it must be the second inequality if any inequality in (A.11) fails. Suppose for contradiction that

$$\begin{aligned} \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi &> \mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] \\ &> \mathbb{E}_B[c(x, S_{1-\phi}^G)] - \mathbb{E}_G[c(x, S_{1-\phi}^G)]. \end{aligned} \tag{A.20}$$

From Observation A.4-(i), we must have $\mathbf{s}_G^* = \{s_G^*\}$, where $s_G^*(\cdot) = c(\cdot, S^*)$ and $\mathbb{E}_B[c(x, S^*) - \mathbb{E}_G[c(x, S^*)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. Additionally, from (A.20) and Lemma 1, $s_G^*(\cdot) = c(\cdot, S^*)$ is strictly more sensitive than $c(\cdot, S_1^B)$, implying $S^* < S_1^B$ and $\mathbb{E}_B[s_G^*(x)] > \mathbb{E}_B[c(x, S_1^B)] = 1$. Furthermore, because the sensitivity of $s_G^*(\cdot)$ is equal to $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$, we have

$$\mathbb{E}_G[x - s_G^*(x)] + \lambda\phi = \mathbb{E}_B[x - s_G^*(x)] = \mathbb{E}_B[x - s_B(x)] \quad \Longrightarrow \mathbb{E}_B[s_B(x)] = \mathbb{E}_B[s_G^*(x)] > 1$$

for any $s_B \in \mathbf{s}_B^*$.

Due to the firm's randomization in equilibrium, the brown investors succeed in funding the firm's project with a probability strictly less than 1. Thus, the brown investors' equilibrium payoffs are strictly less than

$$\max_{s_B \in \mathbf{s}_B^*} \mathbb{E}_B[s_B(x)] - 1 = \mathbb{E}_B[s_G^*(x)] - 1 > 0.$$

Now, suppose that a brown investor deviates by offering $s'(\cdot) := d(\cdot, D^B_{\mathbb{E}_B[s_G^*(x)]-\varepsilon})$, where $\varepsilon > 0$ is sufficiently small. For all $s_B(\cdot) \in \mathbf{s}_B^*$, we have $\mathbb{E}_B[s'(x)] < \mathbb{E}_B[s_B(x)] = \mathbb{E}_B[s_G^*(x)]$, and thus, the firm would accept $s'(\cdot)$ for sure. Furthermore, by Lemma 1-(iii), the sensitivity of $s'(\cdot)$ is strictly less than the sensitivity of $s_G^*(\cdot)$, hence the firm will choose project B under $s'(\cdot)$. As a result, the deviating brown investor would earn a strictly positive financial payoff $\mathbb{E}_B[s_G^*(x)] - \varepsilon - 1 > 0$ from offering $s'(\cdot)$. Provided that $\varepsilon > 0$ is sufficiently small, this payoff is strictly larger than the deviating brown investor's payoff from the equilibrium strategy, which should be impossible in equilibrium.

Step II: Next, we show that the second inequality in (A.11) could not fail. Suppose for

contradiction

$$\begin{aligned}\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)] &> \mathbb{E}_B[c(x, S_{1-\phi}^G)] - \mathbb{E}_G[c(x, S_{1-\phi}^G)] \\ &\geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi.\end{aligned} \tag{A.21}$$

By Observation A.4-(i), we have $\mathbf{s}_G^* = \{s_G^*\}$, where $s_G^*(\cdot) = c(\cdot, S^*)$ such that $\mathbb{E}_B[c(x, S^*)] - \mathbb{E}_G[c(x, S^*)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$. However, Lemma 1-(ii) implies that $S^* \geq S_{1-\phi}^G$ and $S^* > S_1^B$, which implies $\mathbb{E}_G[s_G^*(x)] \leq 1 - \phi$ and $\mathbb{E}_B[s_G^*(x)] < 1$. Hence, s_G^* should not be included in $\mathbf{s}_B^*$; otherwise, the green investor's expected payoff is strictly less than zero, which is not possible in equilibrium. Pick any $s_B \in \mathbf{s}_B^*$, which must be offered by a brown investor. However, because the firm is willing to accept $s_B(\cdot)$ only if $\mathbb{E}_B[s_B(x)] \leq \mathbb{E}_B[s_G^*(x)] < 1$, this brown investor would suffer a payoff strictly less than zero, which is also should be impossible in equilibrium.

A.6.3 Identification of $\tilde{\beta}^*$

Suppose (A.11) holds, and consider an equilibrium (if any) such that the firm chooses B with some probability $\beta^* \in (0, 1)$. Let $s_G^*(\cdot)$ denote the unique security in $\mathbf{s}_G^*$. Recall some preliminary observations: first, by Observation A.4-(i), $s_G^*(\cdot) = c(\cdot, S^*)$ and $\mathbb{E}_B[s_G^*(x)] - \mathbb{E}_G[s_G^*(x)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$; next, by (A.11) and Lemma 1, we also have $S_{1-\phi}^G > S^* \geq S_1^B$, which implies

$$1 \geq \mathbb{E}_B[c(x, S^*)] = \mathbb{E}_B[s_G^*(x)] > \mathbb{E}_G[s_G^*(x)] > 1 - \phi. \tag{A.22}$$

Next, we calculate the green investor's payoff in equilibrium. There are two subcases. First, if the firm only accepts $s_G^*(\cdot)$ in equilibrium, the green investor's equilibrium payoff is equal to

$$\beta^*(\mathbb{E}_G[c(x, S^*)] - 1 + \phi) + (1 - \beta^*)(\mathbb{E}_B[c(x, S^*)] - 1). \tag{A.23}$$

Second, consider the case where the firm also accepts an offer $s_B(\cdot) \in \mathbf{s}_B^*$ from a brown investor. Making such a contract $s_B(\cdot)$ is mutually profitable for both the firm and the brown investor only if $\mathbb{E}_B[c(x, S^*)] \geq \mathbb{E}_B[s_B(x)] \geq 1$. Combined with (A.22), this observation implies $\mathbb{E}_B[c(x, S^*)] = 1$, which makes the second term in (A.23) zero and hence irrelevant. Therefore, the green investor's equilibrium payoff is still given by (A.23) in this case, regardless of the

probability that the firm accepts an offer from the brown investors in equilibrium.

In equilibrium, the green investor's payoff must be non-negative, which is the case if and only if

$$0 < \beta^* \leq \tilde{\beta}^* := \frac{\mathbb{E}_G[c(x, S^*)] - 1 + \phi}{\phi - (\mathbb{E}_B[c(x, S^*)] - \mathbb{E}_G[c(x, S^*)])} = \frac{\mathbb{E}_G[c(x, S^*)] - 1 + \phi}{\phi - (\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi)}. \tag{A.24}$$

As λ increases, the sensitivity of $c(\cdot, S^*)$ decreases, and thus, $\mathbb{E}_G[c(x, S^*)]$ decreases. As $\lambda \to \tilde{\lambda}^*$, $\mathbb{E}_G[c(x, S^*)]$ converges to $\mathbb{E}_G[c(x, S^G_{1-\phi})]$ so the upper bound $\tilde{\beta}^*$ converges to zero. On the other hand, $\tilde{\beta}^*$ converges to 1 as $\lambda \to \underline{\lambda}^*$.

A.6.4 Proof of Sufficiency

Suppose (A.11) holds. For any $\beta^* \in (0, \tilde{\beta}^*)$, where $\tilde{\beta}^*$ is the cutoff identified in (A.24), one can easily construct a mixed-strategy equilibrium as follows. (i) The green investor offers the call option $s^*(\cdot) = c(\cdot, S^*)$ constructed in Observation A.4. (ii) All brown investors offer $\tilde{s}(\cdot) \neq s^*(\cdot)$ that satisfies (A.17) in Observation A.5. (iii) If no investor deviates from the above offer strategies, the firm accepts $s^*(\cdot)$ and then chooses projects B and G with probabilities β^* and $1 - \beta^*$, respectively. (iv) If any investor unilaterally deviates, the firm accepts a security $s(\cdot)$ such that $\mathbb{E}_B[s(x)]$ is lowest among all available offers and then chooses project B.

First of all, the sensitivity of $s^*(\cdot) = c(\cdot, S^*)$ is exactly equal to $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi$, and thus, the firm indeed finds projects B and G indifferent under $s^*(\cdot)$. It is straightforward that the other parts of the strategy specified in (iii) and (iv) are also optimal for the firm. In particular, note that Lemma 1-(iii) guarantees that the firm finds project B more profitable under any security $s(\cdot) \neq s^*(\cdot)$ (including $\tilde{s}(\cdot)$) such that $\mathbb{E}_B[s(x)] \leq \mathbb{E}_B[s^*(x)]$.

Next, the green investor obtains non-negative payoff from offering $s^*(\cdot)$ as discussed in Section A.6.3. Furthermore, the green investor has no incentive to deviate from $s^*(\cdot)$ because any deviation would result in the firm selecting project B while generating a weakly negative financial payoff for the green investor. All brown investors also have no incentive to deviate from $\tilde{s}(\cdot)$ for the same reason.

A.7 Proof of Theorem 5

In this proof, let $\mathcal{S}$ denote the set of all securities that satisfy all the regularity conditions we imposed (i.e., $0 \leq s(x) \leq x$ for all $x \geq 0$, and both $s(x)$ and $x - s(x)$ are weakly increasing in

x). Recall from Lemma 1:

(a) The debt contract $d(\cdot)$ is the least sensitive among all securities in $\mathcal{S}$ such that the expected payment under each project $k \in \{B, G\}$ coincides with $\mathbb{E}_k[d(x)]$. Furthermore, between any two debt contracts $d(\cdot)$ and $\tilde{d}(\cdot)$,

$$\begin{aligned} &\mathbb{E}_B[d(x)] - \mathbb{E}_G[d(x)] > \mathbb{E}_B[\tilde{d}(x)] - \mathbb{E}_G[\tilde{d}(x)] \\ \iff &\mathbb{E}_B[d(x)] > \mathbb{E}_B[\tilde{d}(x)] \\ \iff &\mathbb{E}_G[d(x)] > \mathbb{E}_G[\tilde{d}(x)]. \end{aligned}$$

(b) The call option contract $c(\cdot)$ is the most sensitive among all securities in $\mathcal{S}$ such that the expected payment under each project $k \in \{B, G\}$ coincides with $\mathbb{E}_k[c(x)]$. Furthermore, between any two call option contracts $c(\cdot)$ and $\tilde{c}(\cdot)$,

$$\begin{aligned} &\mathbb{E}_B[c(x)] - \mathbb{E}_G[c(x)] > \mathbb{E}_B[\tilde{c}(x)] - \mathbb{E}_G[\tilde{c}(x)] \\ \iff &\mathbb{E}_B[c(x)] > \mathbb{E}_B[\tilde{c}(x)] \\ \iff &\mathbb{E}_G[c(x)] > \mathbb{E}_G[\tilde{c}(x)]. \end{aligned}$$

Proof of (i): In what follows, we consider the case that all investors are restricted to offering debt contracts only, as assumed in Theorem 5-(i). The properties (a) and (b), as stated above, are the only properties we utilized in our proof of Theorem 1 - 4. Therefore, if only debt contracts are available, Theorem 1 and 2 still hold, with modifications to $\underline{\lambda}^*$ and $\overline{\lambda}^*$ as follows:

$$\begin{aligned} \underline{\lambda}^* &= \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)]\right\}\right], \\ \overline{\lambda}^* &= \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)]\right\}\right], \end{aligned}$$

which implies $\underline{\lambda}^* = \overline{\lambda}^*$.

To prove that no other equilibrium exists, first consider an equilibrium in which the firm chooses project G with probability one under the borrowing term $\mathbb{E}_G[s^*(x)] \in [1-\phi, 1)$. Note that Observation A.2 in Section A.5 still hold. Thus, if such an equilibrium exists, we

must have D^* and $\tilde{D} \neq D^*$ such that

$$\mathbb{E}_B[d(x, D^*)] - \mathbb{E}_G[d(x, D^*)] \geq \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \geq \mathbb{E}_B[d(x, \tilde{D})] - \mathbb{E}_G[d(x, \tilde{D})], \tag{A.25}$$

$$\mathbb{E}_G[x - d(x, D^*)] + \lambda\phi = \mathbb{E}_B[x - d(x, \tilde{D})]. \tag{A.26}$$

Note that (A.25) implies $D^* > \tilde{D}$. On the other hand, (A.26) is equivalent to

$$\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi = d(x, \tilde{D}) - d(x, D^*).$$

where $\mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi > 0$ by Assumption 1. Thus, (A.26) implies $\tilde{D} > D^*$. This contradicts the earlier observation that $\tilde{D} > D^*$. Hence, there is no equilibrium such that the firm chooses project G with probability one under the borrowing term $\mathbb{E}_G[s^*(x)] \in [1 - \phi, 1)$.

Next, consider a mixed-strategy equilibrium in which the firm randomizes its project selection by choosing project B with some probability $\beta^* \in (0, 1)$ and project G with probability $1 - \beta^*$. All the arguments in the proof of Theorem 4 remain valid once we replace the call options in each step of the proof with the debt contract with the same expected payment to the funding investor. Thus, an equilibrium of our interest here could exist only if $\lambda \in [\underline{\lambda}^*, \tilde{\lambda}^*)$, where $\underline{\lambda}^*$ and $\tilde{\lambda}^*$ in the statement of Theorem 4 are modified as follows:

$$\underline{\lambda}^* = \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[d(x, D_1^B)] - \mathbb{E}_G[d(x, D_1^B)]\right\}\right],$$

$$\tilde{\lambda}^* = \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[d(x, D_{1-\phi}^G)] - \mathbb{E}_G[d(x, D_{1-\phi}^G)]\right\}\right].$$

Both Observation A.4 and A.5 in Section A.6 must also remain to hold. Thus, whenever such a mixed-strategy equilibrium exists, there are two different debt contracts $d(\cdot, D^*)$ and $d(\cdot, \tilde{D})$ (hence, $D \neq \tilde{D}$) such that (i) both $d(\cdot, D^*)$ and $d(\cdot, \tilde{D})$ are offered in equilibrium, (ii) the firm accepts $d(\cdot, D^*)$ with positive probability, and (iii) the following two conditions are satisfied simultaneously:

$$\mathbb{E}_B[d(x, \tilde{D})] \leq \mathbb{E}_B[d(x, D^*)],$$

$$\mathbb{E}_B[d(x, D^*)] - \mathbb{E}_G[d(x, D^*)] = \mathbb{E}_B[x] - \mathbb{E}_G[x] - \lambda\phi \geq \mathbb{E}_B[d(x, \tilde{D})] - \mathbb{E}_G[d(x, \tilde{D})].$$

However, the condition $\mathbb{E}_B[d(x, \tilde{D})] \leq \mathbb{E}_B[d(x, D^*)]$ holds between two different debt contracts only if $D^* > \tilde{D}$ and $\mathbb{E}_B[d(x, \tilde{D})] < \mathbb{E}_B[d(x, D^*)]$. In that case, the firm strictly prefers $d(\cdot, \tilde{D})$ to $d(\cdot, D^*)$, which contradicts the supposition (ii) above. Therefore, there is no mixed-strategy

equilibrium such that the firm randomizes its project selection.

Proof of (ii): Next, we restrict all investors to offer call options only, as assumed in Theorem 5-(ii). Once again, the above properties (a) and (b) are the only properties we exploited in our proof of Theorem 1 - 4. Thus, if only call options are available, Theorem 1 and 2 remain to hold, with modifications to $\underline{\lambda}^*$ and $\overline{\lambda}^*$ as follows:

$$\underline{\lambda}^* = \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)]\right\}\right],$$
$$\overline{\lambda}^* = \frac{1}{\phi}\left[\left\{\mathbb{E}_B[x] - \mathbb{E}_G[x]\right\} - \left\{\mathbb{E}_B[c(x, S_1^B)] - \mathbb{E}_G[c(x, S_1^B)]\right\}\right].$$

Note that $\underline{\lambda}^* = \overline{\lambda}^*$. Finally, we can also prove that no other equilibrium exists in a way essentially identical to the proof for the case where only debt contracts are available. Therefore, we omit the details.