Low Interest Rates, Debt Heterogeneity, and Firm Dynamics*

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Abstract

This paper develops a quantitative model of firm dynamics with debt heterogeneity to explore the implications of changes in real interest rates for firm size distribution and firm dynamics. The analysis demonstrates that the decline in long-term real interest rates since the early 1980s can explain significant shifts in employment shares, firm size distribution, and debt composition observed in the U.S. over the same period. In the model, firms endogenously choose between financial intermediaries issuing debt under either earnings-based (EBC) or asset-based (ABC) borrowing constraints by deciding whether to pledge their earnings. A decline in real interest rates disproportionately benefits firms with EBC due to their relatively less reliance on accumulated wealth and the large debt capacity generated by earnings-based financing. As firms with higher earnings optimally choose earnings-based lending, the decline in real interest rates shifts employment shares toward larger firms. Furthermore, the expansion of large firms crowds out smaller firms through general equilibrium effects. The paper empirically tests this mechanism using data on U.S. firms and finds a strong association between the decline in real interest rates and employment changes, particularly in firms with more earnings-based financing.

Keywords: Real Interest Rate, Firm Dynamics, Firm Size Distribution, Debt Heterogeneity

JEL Codes: E24, E43, E44

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1 Introduction

Over the past few decades, the real interest rate in the U.S. has declined sharply, falling from about 6 percent in the early 1980s to nearly zero by 2019. This significant decline raises an important question: how have falling real interest rates affected firm size distribution and dynamics in the U.S. economy? Addressing this question is crucial as the period of declining interest rates has coincided with notable shifts in the U.S. corporate landscape: (i) employment has shifted from smaller to larger firms, increasing the employment shares of large firms; (ii) the distribution of employment size has become more dispersed; (iii) the number of firms per capita has steadily declined; and (iv) large firms have increasingly dominated the corporate credit .[1]

This paper argues that the decline in long-term real interest rates can account for these changes in firm dynamics through its asymmetric effects on small and large firms. The source of these asymmetric effects is debt heterogeneity by firm size. Recent literature has documented that firms face different types of borrowing limits (Lian and Ma, 2021). Specifically, large public firms issue debt against their earnings (earnings-based borrowing limits), while other firms tend to borrow against their physical assets or collateral, also known as asset-based or collateral-based borrowing limits.

In particular, this paper studies the effects of long-term real interest rates on the firm size distribution and firm dynamics within a general equilibrium model, following the approach of Hopenhayn (1992). A key deviation from the Hopenhayn model is that firms must finance the capital used in production by endogenously choosing between two types of financing: earnings-based and asset-based financing. In particular, while all firms have access to asset-based financing, they can decide whether to pledge their earnings, similar to (Diamond et al., 2022). In equilibrium, these differences result in heterogeneous debt financing by firm size, and the variation in pledgeability, in particular, leads to the coexistence of earnings-based and asset-based borrowing limits.

The equilibrium results of the model demonstrate asymmetric effects: the decline in real interest rates can significantly shift employment shares toward large firms and, meanwhile, generate crowding-out effects for smaller firms. Quantitatively, the decline in real interest rates can explain almost the entire change in employment shares, 20% of the decline in firms per capita since the early 1980s. Additionally, the model can explain more than 20% of the change in U.S. debt composition over the same period, particularly the increase in earnings-based financing.

[1] See Davis et al. (2006) and Haltiwanger et al. (2011, 2012), who noted the decline in the startup rate. Fort et al. (2013) documented that employment shares at large/mature firms have risen since the early 1980s.

Debt heterogeneity is crucial for understanding these impacts. The results suggest that the secular trends in firm size distribution and dynamics are closely linked to the decline in real interest rates. This linkage is novel, and the findings are particularly noteworthy because the decline in real interest rates, conventionally considered a positive financial development, can actually generate pervasive and significant negative effects on the employment and profitability of small firms.

Why can the decline in real interest rates generate asymmetric effects? Ultimately, earnings-based firms find it beneficial to expand their size more in a low-rate environment. Firms internalize the impact of production expansions on relaxing financial conditions when determining their optimal size. Asset-based financing relies more heavily on accumulated wealth, which becomes costlier, particularly in an environment with a lower rate of return. In contrast, earnings-based financing depends less on accumulated wealth and, importantly, is often associated with a different and typically higher debt capacity[2], rooted in its close connection to firms' going-concern values. A higher debt capacity implies that firms utilizing earnings-based financing will find it beneficial to expand their size relatively more, as they can relax borrowing limits through additional profits generated by the expansion, so that they can issue more debt at a lower interest rate, provided that the marginal revenue from such expansion is not too low.[3] Additionally, when an interest coverage ratio is applied, a common metric in earnings-based borrowing, a lower rate allows large firms to utilize greater debt capacity, which is amplified through decreased interest expenses. In equilibrium, the expansion of highly productive firms using earnings-based financing increases competition, generating crowding-out effects for other firms.

The model economy consists of three types of agents: entrepreneurs who run the firms, financial intermediaries, and the representative household. Each period, an exogenous number of potential entrants draw their initial productivity and asset levels. After observing these levels, they decide whether to enter. Incumbent firms face an exogenous exit shock at the beginning of each period. Surviving incumbent firms then choose whether to continue operating or exit. All firms can own physical assets and use labor and capital as inputs to production. Firms finance capital by raising debt from financial intermediaries. Entrepreneurs who exit the market retain their assets.[4]

The model features two types of debt financing offered by financial intermediaries from

[2]Debt capacity is defined here as the maximum amount of debt that a given firm can borrow.

[3]The existence of depreciation rates and/or intermediary costs, or more generally, interest spreads, guarantees this condition.

[4]This setting is similar to models of firm dynamics with financial frictions in the literature. See examples in Kiyotaki et al. (2021); Moll (2014); Cooley and Quadrini (2001), among others.

which firms can borrow capital: earnings-based and asset-based financing. As in Diamond et al. (2022), I assume that all firms have access to asset-based financing; however, they have the option to access earnings-based financing by increasing the pledgability of their earnings, which incurs a cost. The pledgability of earnings, particularly future earnings in a dynamic setting, can not only generate the reduced-form earnings-based borrowing limits observed in the real world but also rationalize the sufficient debt capacity of such financing, which is one of the key drivers of the model's results. In the model, differences in cost structures and potential borrowing capacity lead to sorting: firms with greater profits and greater capital needs will endogenously choose earnings-based lending, while others will opt for asset-based debt.

Finally, there is a representative household that resides in the economy. Each period, the household provides labor, earns wages, consumes goods, and saves through financial intermediaries. I assume that the household is sufficiently patient to act as the lender, while entrepreneurs are the borrowers in the economy. Changes in household preferences will drive changes in the real interest rate.[5]

Based on the model, we study the effects of the decline in real interest rates on the firm size distribution and firm dynamics. To explore the mechanism behind the heterogeneous effects of the decline in real interest rates on firms, I first analyze a simplified version of the model by abstracting from firm entry/exit and endogenous debt choices. This simplification allows for the examination of the determinants of a firm's optimal size and provides analytical solutions that elucidate the mechanism.

The simplified model reveals that a decline in real interest rates can have heterogeneous impacts on firms' optimal decisions. In the long run, a firm's optimal size decision involves a trade-off between the marginal benefits of expansion and the associated costs. Lower interest rates reduce the cost of financing, thereby incentivizing both types of firms to pursue expansion. In determining their optimal size, firms account for the impact of expansion on their borrowing constraints. For asset-based firms, a one-dollar investment in capital increases asset holdings, thereby increasing borrowing capacity. However, this necessitates the accumulation of additional wealth, which becomes increasingly costly in a low return environment. In contrast, earnings-based firms benefit from a one-dollar investment in capital by generating higher revenues, which similarly relaxes their borrowing limits. When the debt capacity provided by earnings-based financing is sufficiently large, the relatively lower reliance on accumulated wealth, coupled with such ample debt ca-

[5]This assumption can be interpreted as the decline in the real interest rate resulting from factors exogenous to firm owners, such as the global saving glut (Bernanke, 2005) or increasing wealth inequality among households (Dynan et al., 2004). Similar assumptions are commonly made in other studies (e.g., Liu et al., 2019; Chatterjee and Eyigungor, 2020; Hall, 2016).

pacity, positions these firms to expand their production more aggressively in response to declining interest rates, provided that the marginal revenue from such expansion remains adequate. Moreover, when an interest coverage ratio is applied, a lower interest rate, reflecting a higher discounted value of future cash flows, further increases debt capacity and amplifies this advantage. In equilibrium, such expansion generates crowding-out effects, resulting in a shift in employment shares and changes in firm dynamics.

Reasonable parameterization ensures that these conditions hold: namely, a sufficiently large debt capacity through earnings-based financing and a sufficiently high marginal revenue from production. The debt capacity is rooted in the close connection between earnings-based borrowing limits and firms' going-concern values. The existence of depreciation and interest spreads prevents the marginal revenue from becoming too low.[6] Interestingly, in this simplified setting, changes in capital prices do not alter the results. While an increase in capital prices, for instance, raises the value of collateral, it also indirectly increases the marginal revenue of capital due to the higher cost of capital, leading earnings-based firms, similar to their asset-based counterparts, to adjust production at the same rate when the capital price changes.

The simplified analysis provides a rich and intuitive demonstration of the underlying mechanism. However, the focus on the steady state and the assumption of exogenous debt types, while necessary for this long-term context, could overlook other important channels that may be at work. In particular, lower interest rates might prompt firms at the margin to switch their debt types endogenously. The larger debt capacity and reduced reliance on accumulated wealth associated with earnings-based financing could significantly facilitate the growth of highly productive firms along their transition paths, especially when the demand for debt increases in a lower interest rate environment.

Thus, I use the full model to perform quantitative experiments and analyze the impacts of the decline in long-term real interest rates on firm size distribution and firm dynamics. I calibrate the model to match a rich set of moments from U.S. firm dynamics, size distribution, and debt composition in the 1980s. The model provides an excellent fit for these moments. I then conduct quantitative experiments within the model, specifically subjecting the economy to a decline in real interest rates, as observed in the U.S. from 1980 to 2020. The experiment demonstrates that the calibrated model can explain a significant fraction of long-term changes in the U.S. economy in terms of stationary equilibrium. Specifically, the model explains almost all the shifts in employment shares and more than 20% of the changes in debt composition experienced in the U.S. over the past forty years. It can also account for more than 20% of the decline in firms per capita. A counterfactual analysis,

[6]Note that depreciation also dampens the pledgeable value of collateral.

based on an economy where only asset-based financing is available, shows that debt heterogeneity plays a primary role in generating these effects.

Finally, I empirically test the model's mechanism using a comprehensive dataset of U.S. firms drawn from Compustat and Capital IQ, covering the period from 2000 to 2019. The analysis investigates the relationship between firm-level interest rates and employment, focusing on the differential impact based on firms' reliance on earnings-based financing. By employing an identification strategy that leverages both time and industry or firm-specific fixed effects, I estimate how changes in interest rates influence employment across firms with varying degrees of reliance on earnings-based financing. The results demonstrate that firms with a higher reliance on earnings-based financing exhibit a greater response to interest rate changes, particularly in their employment decisions. Specifically, when interest rates decline, these firms show a more pronounced increase in employment compared to firms that depend more heavily on asset-based financing. These findings are consistent with the model's predictions and underscore the critical role that firm-specific financial structures play in mediating the effects of changes in interest rates on employment.

Related literature. This paper is related to several strands of literature, particularly those examining changes in firm size distribution and firm dynamics, the role of heterogeneous debt financing, and the impact of declining long-term real interest rates on concentration.

Firm size distribution and dynamics. Recent studies have documented significant shifts in the U.S. firm size distribution, characterized by a more dispersed distribution and a shift in employment shares from smaller firms to large and mature firms, alongside a decline in entrepreneurship activities.[7] This trend has prompted a substantial body of literature exploring its causes and consequences. For example, Poschke (2018) investigates the impact of skill-biased technological changes on global firm size distribution, including in the U.S., while Begenau et al. (2018) suggests that big data disproportionately benefits large firms, contributing to these shifts. Explanations for the decline in entrepreneurship include demographic changes, such as population growth and aging (Hathaway and Litan, 2014; Pugsley and Şahin, 2019; Karahan et al., 2019; Bornstein et al., 2018), business consolidation (Hathaway and Litan, 2014), import competition (Pugsley and Şahin, 2019), and technological changes (Poschke, 2018; Salgado, 2020). Additionally, Akcigit and Ates (2021) highlights the decline in knowledge diffusion between frontier and laggard firms as a significant driver of these trends.

Heterogeneous debt financing. My work also intersects with studies on the causes and consequences of heterogeneous debt financing and its implications for firm dynam-

[7] First documented by Haltiwanger et al. (2011) and Haltiwanger et al. (2012).

ics. Lian and Ma (2021) show that firms vary in their borrowing limits, with large firms, particularly those borrowing from the credit , predominantly issuing debt under earnings-based borrowing limits, while smaller firms mainly rely on asset-based borrowing. This analysis was extended to small and medium-sized enterprises by Caglio et al. (2021). Subsequent research has focused on the implications of debt heterogeneity for short-run fluctuations (e.g., Greenwald et al., 2019; Drechsel, 2022). My contribution to this literature lies in theoretically demonstrating the importance of future earnings in distinguishing between these borrowing limits and integrating this framework into a firm dynamics model to explore its effects on secular trends in the U.S. economy. Unlike much of the existing literature, which often focuses on the short-run implications of financial frictions, this paper emphasizes the long-term aggregate effects of debt heterogeneity, highlighting its significant impact on firm size distribution and business dynamics.

Impact of declining long-term real interest rates. Another related strand of literature examines the impact of declining long-term real interest rates on market concentration. Liu et al. (2019) emphasize the strategic effects of lower rates on competition between market leaders and followers, while Chatterjee and Eyigungor (2020) focus on the relationship between firm size and leverage, suggesting that lower volatility in large firms' earnings enables greater leverage, leading to reduced entry rates and increased market concentration through acquisitions. Asriyan et al. (2021) discuss how lower interest rates can result in capital misallocation, favoring lower-productivity firms, and Kroen et al. (2021) provide empirical evidence that falling nominal rates disproportionately benefit industry leaders, particularly when initial rates are already low.

Overall, this paper complements these existing studies but differs in several important ways. It introduces a novel channel, debt financing heterogeneity, as a key mechanism through which the decline in long-term interest rates affects the economy. My work shows that the decline in real interest rates can generate asymmetric effects even without the presence of specific competitive settings or market structures. Furthermore, this paper provides novel empirical validation, offering robust support for the proposed mechanisms and expanding the understanding of how debt heterogeneity influences long-term economic dynamics.

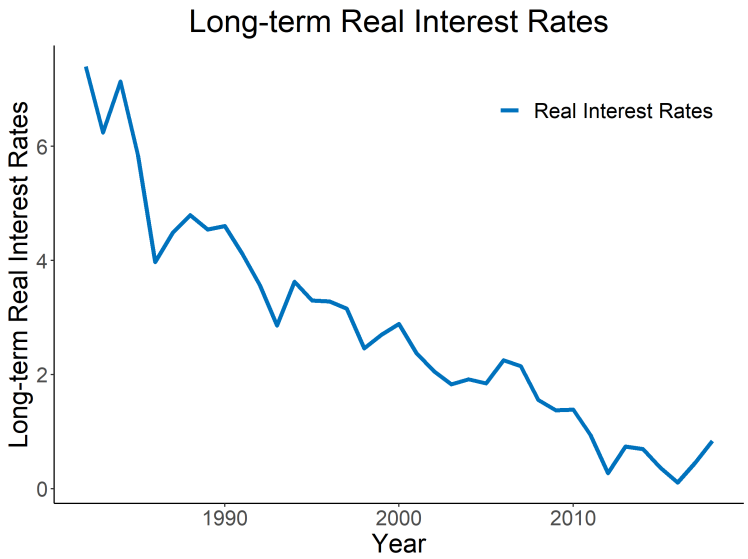
The paper is organized as follows. Section 2 documents the secular trends of the U.S. economy. Section 3 sets up a quantitative model of firm dynamics. Section 4 derives the analytical solutions of a simplified version of the model to provide intuitions. Section 5 reports the results from quantitative experiments. Section 6 provides the empirical evidence to validate the proposed mechanism. Section 7 concludes.

2 Secular Trends of the U.S. Economy

In this section, I present several stylized facts regarding secular trends in the U.S. economy. Specifically, I examine the changes in the long-term real interest rate in the U.S. and provide evidence of heterogeneous debt financing at both the firm and aggregate levels. Additionally, I document the shifts in the U.S. firm size distribution and firm dynamics over time.

2.1 The decline in Real Interest Rates and Debt Heterogeneity

Figure 1: Real Interest Rates over Years



Note: This figure depicts the changes in the long-term real interest rate in the U.S. The plot is created based on Federal Reserve Economic Data (FRED) using the 10-year treasury rates minus the 10-year inflation expectation.

The decline in real interest rates. Figure 1 illustrates the changes in long-term real interest rates in the U.S. over the past 40 years. This figure is based on Federal Reserve Economic Data (FRED), calculated using the 10-year Treasury rate minus the 10-year inflation expectation. It reveals a substantial decrease in the real interest rate, which has fallen from around 6% to below 0% since the early 1980s.

It is helpful to consider some potential explanations for this trend. The debates over the causes of the decline are extensive and remain contentious. Proposed explanations include changes in economic growth, demographic factors, government savings at both national and global levels, and increasing demand for safe assets, particularly in emerging-market economies. Kiley (2020) and Gamber (2020) provide comprehensive reviews of

the empirical evidence and potential causes. For the purposes of this paper, I assume that the decline in the real interest rate is driven solely by changes in lender preferences or foreign factors, rendering the changes in the real interest rate exogenous to the firms in question. Theoretically, the proposed mechanism operates through the real interest rate channel regardless of its underlying causes.[8]

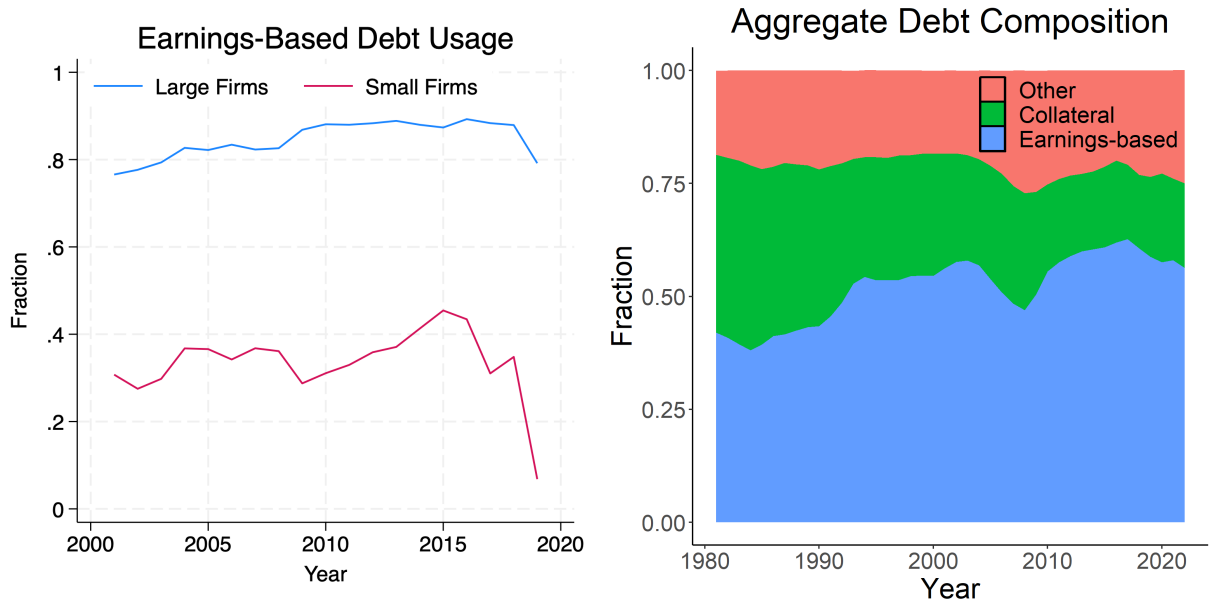
Debt heterogeneity. The existing literature has demonstrated that firms are heterogeneous in terms of their debt types. Colla et al. (2020) provide a comprehensive review of the literature on debt structure, presenting both empirical evidence and potential economic mechanisms that explain the discrepancies between firm debt types and financing costs. As they highlight, most firms tend to specialize in one type of debt. Following the dichotomy proposed by Lian and Ma (2021), debt types can be broadly categorized based on their borrowing limits into asset-based and earnings-based financing.

Figure 2 presents the fraction of different loan types used by Compustat firms, alongside the aggregate composition of debt in the U.S. economy. The left panel replicates the analysis of Lian and Ma (2021), showing the fraction of earnings-based loans used by each Compustat company. I classify large firms as those with employment levels above the median within each 2-digit sector, while the remainder are categorized as small firms. Consistent with Lian and Ma (2021), large firms rely heavily on earnings-based loans, with these loans constituting more than three-quarters of their total debt on average. In contrast, small firms, on average, use only approximately 25% earnings-based loans. Furthermore, these patterns are not limited to Compustat firms. Benmelech et al. (2020) analyze the types of loans used by small businesses over the years and find that about 70% of these loans are secured, which can reasonably be considered asset-based loans. The remainder are primarily lines of credit or credit card debt.

Regarding the aggregate composition, the right panel utilizes data from the Federal Reserve Board to categorize U.S. non-financial corporate debt into earnings-based and asset-based debt. This analysis reveals the heterogeneity of debt types within the U.S. economy, with earnings-based loans comprising more than 40% of total corporate debt. Additionally, the data indicate that firms are increasingly borrowing against earnings, reflecting a shift in debt composition over time.

[8]This assumption is made for both empirical and theoretical reasons. Empirically, as noted by Krueger and Ludwig (2007) and Kiley (2020), global factors, particularly those originating from foreign countries, play a crucial role in the decline of U.S. real interest rates. Furthermore, Del Negro et al. (2017) show that the premium for safety and liquidity, linked to the saving glut in emerging markets first proposed by Bernanke (2005), dominates other factors. Theoretically, this assumption allows for changes in the real interest rate without altering other parameters, thereby isolating the impact of debt heterogeneity from other factors.

Figure 2: Debt Heterogeneity



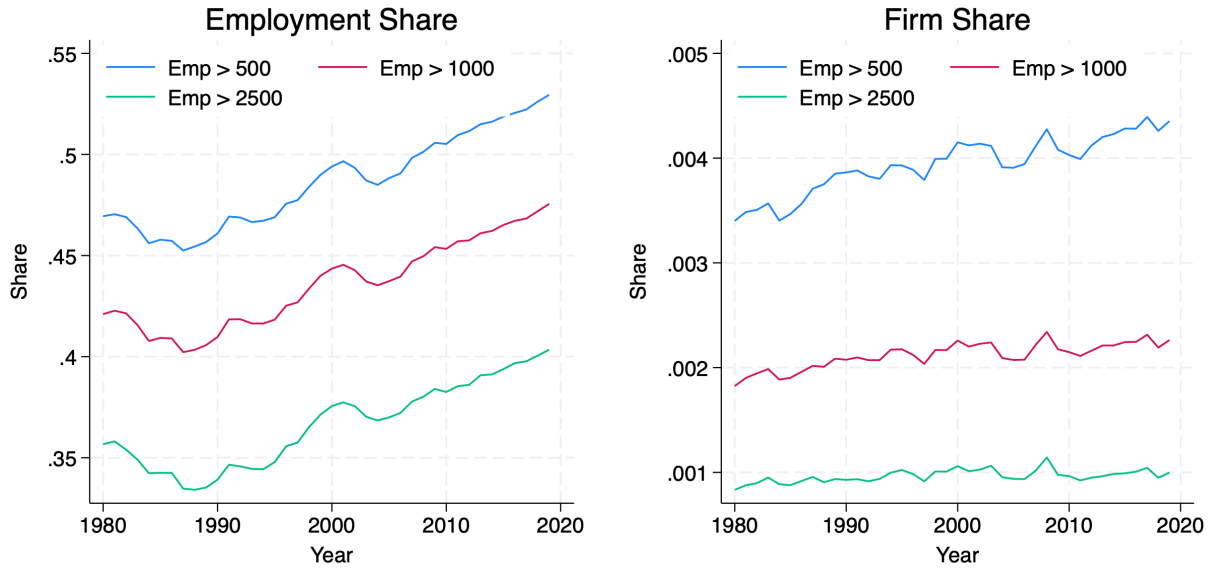
Note: This figure shows the fraction of different loan types used by Compustat firms and the aggregate composition. The left figure plots the median percentage of earnings-based loans of big and small Compustat firms. The percentages of earnings-based loans are calculated based on methods used by Lian and Ma (2021). The right panel categorizes the aggregate composition of different debt types. To keep conservative, I classify all U.S. bonds into earnings-based loans and all bank loans and mortgages into asset-based loans.

2.2 The U.S. Firm Size Distribution and Firm Dynamics over the Years

Shifts in employment shares. The U.S. firm size distribution has changed dramatically over the past four decades, becoming increasingly dispersed, as documented in previous studies (see, for example, Poschke, 2018). As illustrated in Figure 3, employment shares have shifted toward larger firms. The share of firms employing at least 500 workers has increased from 45% to over 50%. Similar trends are observed for firms with at least 1,000 or 2,500 employees, each category experiencing a net increase of approximately 5% in employment share over the past four decades. Notably, the number of large firms relative to the total number of firms has only slightly increased since the 1980s. For instance, the proportion of companies with 500 or more employees was about 0.35% in the early 1980s, rising to 0.4% by the end of 2018—a growth of more than 10%. Similar percentage increases can also be observed for firms with at least 1,000 or 2,500 employees.

In contrast, small firms exhibit a distinct pattern. Figure 4 shows the employment shares and firm shares of small firms. Firms with fewer than 10, 20, or 100 employees have seen a decline in their employment share, even though the proportion of these firms has changed only slightly. For instance, in the early 1980s, firms with fewer than 20 em-

Figure 3: Employment Shares and Firm Shares of Large Firms over the Years



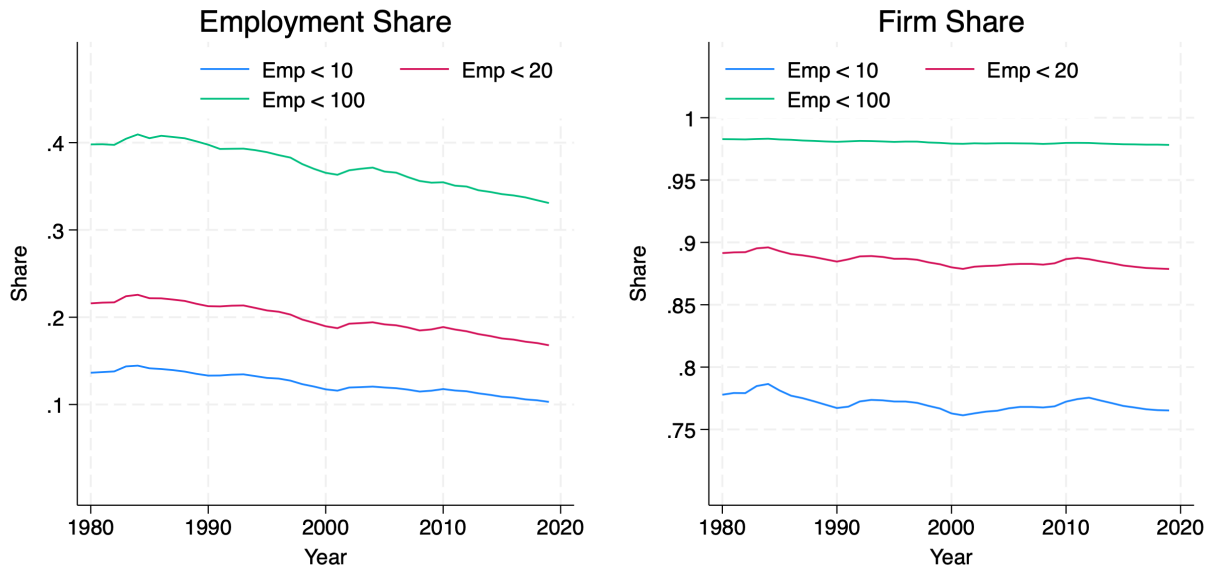
Note: This figure depicts long-term changes in the U.S. firm size distribution. The left panel shows the employment shares of firms with more than 500, 1000, and 2500 employees, respectively. The right panel shows the firm shares of firms with more than 500, 1000, and 2500 employees, respectively. All calculations are based on BDS data.

ployees accounted for approximately 22.5% of total employment. By 2018, this share had declined to below 17.5%, while the percentage of such firms decreased by only about 2%. In other words, small firms are becoming progressively smaller over time.

Changes in the employment size distribution. In addition, consistent with the above patterns, the employment sizes of firms at the top of the distribution grow faster than those of other firms, driven by their stable firm shares and increasing employment shares. The left-hand side of Figure 5 plots the ratio of the average size of Compustat firms to the average size of all firms over the years. The graph clearly shows that public firms, particularly large public firms, have outgrown other firms on average. While the size of all firms has generally increased over the past years, public firms have grown almost twice as fast as the average firm, with those possessing assets above the sector median growing even faster. For comparison, the right-hand side of the figure shows the ratio of the average size of small firms to the average size of all firms. The increase in firm size dispersion is evident, as the size growth patterns for large and small firms differ significantly.

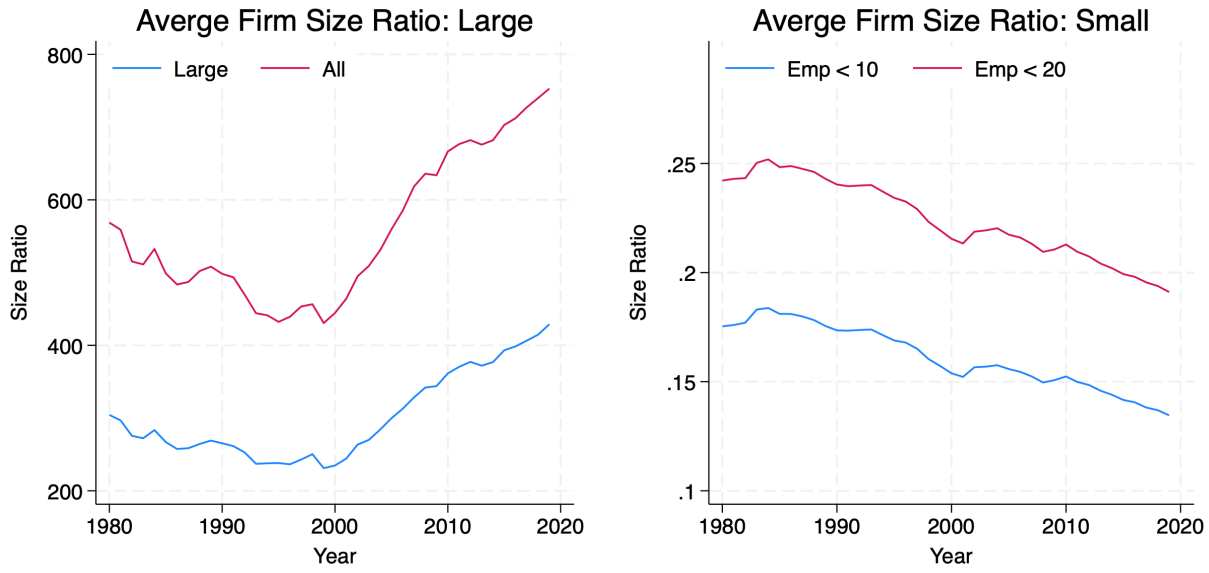
The change in average firm size and entrepreneurship. The left panel of Figure 6 shows that the average firm size has increased dramatically since 1980. Conversely, the number of firms per capita—defined as the total number of firms divided by the total working-age population or the total number of employees—has declined by about 10%

Figure 4: Employment Shares and Firm Shares of Small Firms over the Years



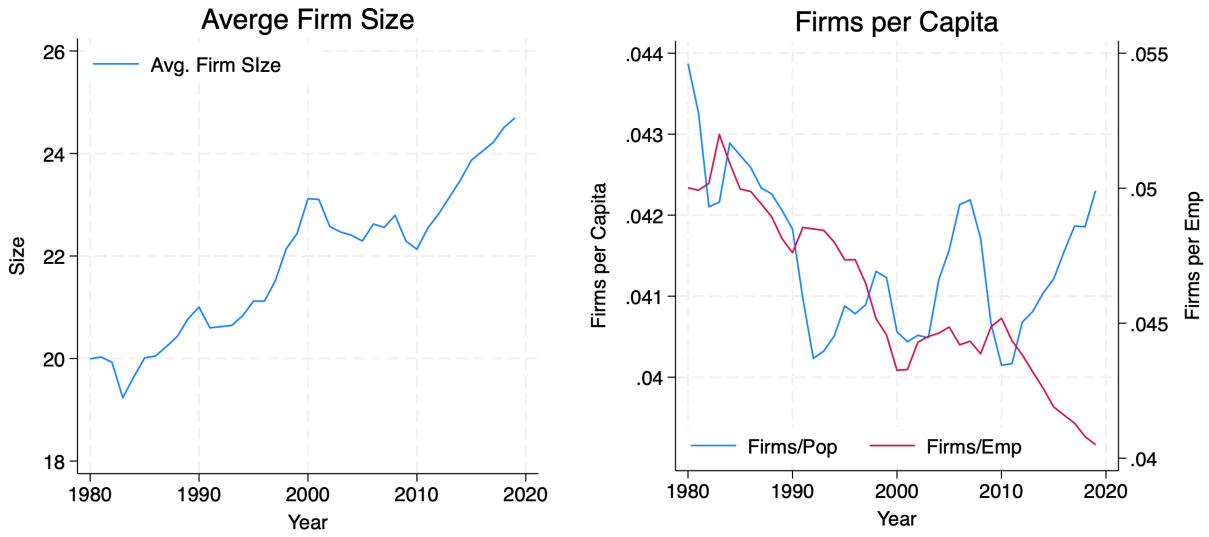
Note: This figure depicts long-term changes in U.S. firm size distribution. The left panel shows the employment shares of firms with less than 10, 20, and 100 employees, respectively. The right panel shows the firm shares of firms with less than 10, 20, and 100 employees, respectively. All calculations are based on BDS data.

Figure 5: Average Firm Sizes on the Top and Bottom over the Years



Note: This figure depicts long-term changes in the average size of firms at the top and bottom of the distribution in the U.S. The left panel shows the average employment of Compustat firms and those assets above the median compared with the average size of all firms. The right panel shows the average employment of firms with less than 10 and 20 employees, compared with the average size of all firms.

Figure 6: The Decline in Firms per Capita



Note: This figure depicts long-term changes in business dynamism in the U.S. The left panel shows the decline in the firm entry rate calculated based on BDS data. The right panel shows firms per capita, defined as the number of total firms divided by the total number of working age population 25 to 54 years old or the total number of employees, respectively.

to 15% from its peak around the 1980s, as shown in the right panel of Figure 6. These patterns indicate a decline in new business activities, with fewer firms being created and fewer people engaged in operating businesses.

3 A Model of Firm Dynamics with Debt Heterogeneity

This section develops a quantitative model of firm dynamics with endogenous firm entry/exit and firm size distribution, building on the framework of Hopenhayn (1992). The model incorporates financial frictions similar to those in Cooley and Quadrini (2001) and Moll (2014). The key distinction of this model, compared to previous literature, is the introduction of earnings-based financing and endogenous choices of debt types.

In the following subsections, I will detail the economic environment, outline the model timing, and describe the problems faced by firms, financial intermediaries, and households. Finally, I will define a stationary equilibrium for the aggregate economy.

3.1 Environment

This subsection describes the model environment. Time is discrete, and the horizon is infinite. The economy consists of three types of agents: entrepreneurs, households, and

financial intermediaries. The economy operates along a balanced growth path.

Entrepreneurs (firms). Entrepreneurs own firms. Each period, there is an exogenous measure of $m_0(1+\eta)^t$ potential entrants and an endogenous measure m_t of incumbent firms. Consistent with the literature, firms are assumed to be heterogeneous in their productivity, which comprises a permanent component z and a stochastic component s_t. Firms operate a decreasing-returns-to-scale (DRS) production technology, $y(z, s_t, k_t, l_t)$, using labor l_t and capital k_t as inputs. The output is sold in a competitive, homogeneous final goods market, with the price of the final goods serving as the numéraire of the economy.

All potential entrants receive an initial injection of equity a_0 from an exogenous asset distribution $\Gamma_{a_0}(a_0)$. They then draw a value of z from the initial distribution $\Gamma_z(z)$ and a value of s_0 from the initial distribution $\Gamma_{s_0}(s_0)$. Based on this draw, they decide whether to enter the market and become incumbents by paying the entry cost c_e.

For each period t, incumbent firms may exit either exogenously or endogenously. With probability ξ, a destruction shock occurs, forcing an incumbent firm to exit. Surviving firms observe their new values of s_t, which are drawn from the conditional distribution $\Gamma_s(ds', s_{t-1})$, and then decide whether to exit or continue production. In the case of either exogenous or endogenous exit, the firm distributes its positive net worth to the entrepreneur.

Firms that continue to produce will determine the optimal levels of labor and capital inputs, as well as select a source for financing their capital. All firms operate within a perfectly competitive and homogeneous labor market. Additionally, firms have the option to accumulate assets, which can be used as collateral and to generate interest revenue. To finance their capital, firms borrow from financial intermediaries

Financial intermediaries and debt choices. Financial intermediaries issue debt to firms, which can be structured under either earnings-based or asset-based constraints, depending on the pledgeability of the firm's earnings. The differences in the pledgeability of earnings lead to two distinct borrowing constraints, arising from the limited enforceability of debt contracts. Firms endogenously decide whether to make their earnings pledgeable. Specifically, firms can borrow against collateral, incurring a lower fixed cost per period. In contrast, to borrow against earnings, firms may opt to pay a higher per-period cost to make their earnings pledgeable. As will be shown, the debt capacity under earnings-based borrowing limits is greater than that under asset-based limits. Consequently, firms must trade off higher debt capacity against the different cost structures associated with these financing options.

Households. There is a measure of $H(1+\eta)^t$ representative households, each con-

sisting of $\bar{L}$ family members. Households are risk-neutral and have a discount factor $\sigma_h \in (0, 1)$. They provide labor and receive wage payments from firms that employ their family members. Additionally, households allocate their savings to financial intermediaries.

Before delving into the details of the firm's problem, it is helpful to outline the timing of events within the model. Each period unfolds in the following sequence: (i) realization of productivity shocks for incumbent firms; (ii) endogenous and exogenous exits of incumbents; (iii) realization of initial productivity and asset levels, followed by the entry decisions of potential entrants; (iv) financing decisions by all firms; (v) production and revenue generation from sales; (vi) payment of wage bills, costs of capital, hiring, and operating expenses; and (vii) saving decisions and household consumption.

3.2 Firm Problem

Exit. At the beginning of each period, incumbent firms face an exogenous shock that may force them to exit. After this shock, firms observe their new productivity levels and decide whether to continue operating and which type of debt to borrow. If a firm decides to exit, it pays out its current net worth. Let $x(z, s, a) \in 0, 1$ represent the exit decision. A firm with a permanent productivity component z, an idiosyncratic component s_t, and equity level a_t has a value V before the destruction shock, which is given by

$$V(z, s_t, a_t) = (1 - \xi) \cdot V^c(z, s_t, a_t) + \xi a_t(1 + r_t),$$

where V^c is the expected value of continuing operations. Firms then choose whether to exit endogenously, and they will do so only if the value offered by the outside option exceeds the value of remaining an incumbent. Thus, the value of continuing operations V^c satisfies

$$V^c(z, s_t, a_t) = \max\{V^i(z, s_t, a_t), a_t(1 + r_t)\},$$

where V^i represents the value of being an incumbent firm.

Entry. In each period, a measure of $m_0(1+\eta)^t$ potential entrants draw their productivity levels and receive equity injections, after which they decide whether to enter the market by paying an entry cost c_e. Let $i_0(z, s, a) \in 0, 1$ denote the entry decision rule, which depends only on the initial productivity draw and equity injection. A firm will decide to enter if its

expected value upon entry exceeds the value of its equity injection.

$$V^i(z, s_0, a_0 - c_e) \geq a_0(1 + r_t)$$

Debt financing choices. An incumbent firm with a permanent productivity component z, idiosyncratic component s_t, and equity level a_t must decide on the source of financing to use. The expected value V^i of the firm is determined by solving

$$V^i(z, s_t, a_t) = \max\{V^a(z, s_t, a_t), V^e(z, s_t, a_t)\},$$

where V^a and V^e are the value functions associated with borrowing under asset-based and earnings-based financing, respectively.

Firm problem. In the spirit of Cooley and Quadrini (2001) and Moll (2014), firms using debt financing of type j ($j \in a, e$) solve the following problem:

$$\begin{aligned} V^j(z, s_t, a_t) &= \max_{d_t, k_t, l_t, a_{t+1}} d_t + \beta_f \mathrm{E_t} V(z, s_{t+1}, a_{t+1}) \\ \text{s.t.} \qquad d_t + a_{t+1} &= f(k_t, l_t; z, s_t) - w_t l_t - \delta q_t k_t - r_{j,t} b_t - f_j + a_t \\ b_t &= q_t k_t - a_t \end{aligned}$$

Firms maximize their value using a discount factor β_f. They determine the next period's equity holding and current dividends by maximizing the revenues from production, net of interest on capital rental, wage payments, depreciation δ, and a fixed cost f_i associated with the chosen type of financing. The interest rate for financing is given by the real interest rate $r_{j,t}$, which equals the saving rate r_t plus a market-specific interest spread ρ_j for positive liabilities.

I further assume that both assets a_t and dividends d_t must be non-negative. Net borrowing, defined as $b_t = q_t k_t - a_t$, is subject to the borrowing constraints specified below. As shown in Moll (2014), this setting is analogous to a scenario where entrepreneurs own and accumulate capital k and can trade in a risk-free bond. I assume perfect elasticity of transformation between capital goods k_t and consumption goods, as the model focuses on long-run relationships. In the next section, I discuss the implications of changes in the price of capital.

3.3 Financial Intermediaries

Asset-based financing. Competitive financial intermediaries issue intraperiod loans to firms to finance the purchase of capital. The debt issuance is subject to financial frictions, with the underlying friction being limited commitment. I assume that there is imperfect enforcement in the event of default, and firms cannot commit to avoiding renegotiation of debt repayment. Specifically, if a firm defaults, the financial intermediary can recover only up to a fraction ϕ_k of the undepreciated physical assets through a fire sale. This recovery constraint determines that the maximum amount of debt a firm can obtain will not exceed the value of its assets, as firms cannot commit to repaying debt if the loan amount surpasses this limit. In other words, the borrowing constraint is given by

$$b_t \leq \phi_k q_t k_t (1-\delta)$$

This equation can be rewritten using $b_t = q_t k_t - a_t$ as follows:

$$q_t k_t \leq \frac{1}{1-\phi_k(1-\delta)} a_t$$

Earnings-based financing. In addition, firms can utilize earnings-based financing by endogenously making their earnings pledgeable through improvements in governance or accounting standards. I assume that firms are subject to two of the most common earnings-based borrowing limits: the debt-to-earnings ratio and the interest coverage ratio. These are expressed as follows.

$$b_t \leq \phi_e^{DE}[f(k_t, l_t; z, s_t) - w_t l_t - f_e]$$

, and

$$r_{j,t} b_t \leq \phi_e^{ICR}[f(k_t, l_t; z, s_t) - w_t l_t - f_e]$$

Here, the measure of earnings is EBITDA, earnings before interest, taxes, depreciation, and amortization.[9] As argued in Drechsel (2022) and Hartman-Glaser et al. (2022), these constraints are closely linked to the firm's going-concern value. In the next section, a microfoundation is provided to show that the above earnings-based borrowing limits at the steady state are the reduced-form representation of the pledgeable component of the firm's

[9] An alternative measure of earnings is EBIT. Our results are robust using this alternative measure.

going-concern values under the limit commitment framework.

What matters in this paper are the implications of the linkage between earnings-based borrowing limits and firms' going-concern values for firms' debt capacity. As illustrated in the next section, the parameters, ϕ_e^{DE} and ϕ_e^{ICR}, that govern the tightness of borrowing limits essentially imply the magnitude of pledgeability of firms' going-concern values, which gives earnings-based firms significantly larger debt capacity without reliance on accumulated wealth.

This implication is crucial for understanding the mechanism behind the asymmetric effects. As demonstrated later, a lower interest rate not only enables large firms to leverage this advantage, but also amplifies the benefit due to less reliance on accumulated assets and reduced interest expenses. This, in turn, leads to the expansion of highly productive firms and generates crowding-out effects.

I further assume that, in general, $f_a < f_e$, so firms will trade off between cost structures and debt capacity. As a result, firms with greater financial needs will opt for earnings-based financing, while firms with sufficient self-financing or less need for external funding will choose asset-based financing. This simple mechanism effectively captures the trade-offs between the two different types of financing. Earnings-based borrowing limits provide advantages in terms of greater borrowing capacities. To generate debt heterogeneity, the higher fixed cost broadly represents the expenses associated with raising pledgeability.

Raising pledgeability, in a narrow sense, might involve improving accounting quality or establishing a stronger board to prevent entrepreneurs from diverting project cash flows for personal use. More broadly, as noted by Rajan (2012), firms can enhance pledgeability through standardization—by simplifying corporate organizational structures and increasing transparency—or by implementing better governance structures, such as appointing a more expert and independent board. Such standardization can not only grant financiers more effective rights over the going-concern surplus through direct control but also make firms more attractive for sale in the secondary market, as discussed by Hartman-Glaser et al. (2022).

It is important to note that the limited commitment framework is not the only approach capable of generating the distinction between asset-based and earnings-based financing. Other commonly used frameworks, such as costly state verification and the presence of second-best users, can also create this distinction through endogenously determined pledgeability.[10]

[10]See examples in Clementi and Hopenhayn (2006) and Hartman-Glaser et al. (2022), among others. Asset- and earnings-based covenants can also be viewed as signals or tripwires for firm performance. Here, as discussed in the following chapter, I emphasize the differing debt capacities implied by various types of financing, where pledgeability provides a clearer interpretation of this distinction.

Therefore, the model here emphasizes the importance and implications of pledgeability in future cash flows rather than focusing on specific underlying frictions. In the quantitative analysis, I will employ both reduced-form borrowing limits. Beyond computational convenience, these reduced-form borrowing limits reflect the actual constraints faced by firms. Previous studies have shown that the tightness of these limits is stable over time, which can be rationalized by the proposition discussed above.

3.4 Household Problem

At time t, the representative household, with a measure of $H(1+\eta)^t$, provides a total of $\bar{L}H(1+\eta)^t$ units of labor and receives wages

$$\begin{aligned} W(B_t) &= \max_{C_t, B_{t+1}} C_t + \beta_h(1+\eta)W(B_{t+1}) \\ \text{s.t.} \quad C_t + B_{t+1} &= w_t\bar{L} + \frac{(1+r_t)}{1+\eta}B_t, \end{aligned}$$

where C_t is household consumption, $\bar{L}$ is the labor supply per household unit, and B_t represents savings deposited with financial intermediaries.

3.5 Stationary Equilibrium

Let $m_t(Z, S, A)$ represent the measure of incumbent firms with permanent productivity $z \in Z$, idiosyncratic productivity $s \in S$, and previous equity holding $a \in A$ at the beginning of period t, following the draw of firm-level productivity but before the exogenous exit shock.

Let $\bar{m_t} = \frac{m_t}{H(1+\eta)^t}$ and $\bar{m_0} = \frac{m_0(1+\eta)^t}{H(1+\eta)^t}$ denote these measures on a per capita basis. Then, the law of motion of the distribution is given by:

$$\begin{aligned} \bar{m_{t+1}}(Z, S, A) = &\iint \frac{(1-\xi)}{1+\eta} Q(z, s_{t+1}, a_{t+1}|z, s_t, a_t)(1 - X(z, a_t, s_t))\bar{m_t}(dz, ds, da) \\ &+ \bar{m_0} \iiint Q_0(z, s_{t+1}, a_{t+1}|z, s_t, a_t)I(z, s_t, a_t)d\Gamma_{a_0}(a_0)d\Gamma_z(z)d\Gamma_{s_0}(s_0), \end{aligned}$$

where $Q_0(z, s_{t+1}, a_{t+1} \mid z, s_t, a_t)$ and $Q(z, s_{t+1}, a_{t+1} \mid z, s_t, a_t)$ are the resulting transition matrices based on the selection of equity holdings and the evolution of productivity shocks for entry and incumbent firms, respectively.

Closing the Model. Finally, the labor market clears such that

$$\bar{L} = \iiint l(z, s_t, a_t)(1 - X(z, a_t, s_t))\bar{m}_t(dz, ds, da) + \iiint l(z, a_t, s_t)I_0(z, s_t, a_t)\bar{m}_0(dz, ds, da),$$

where $l(z, s_t, a_t)$ is the policy function for labor demand for firms with state variables (z, s_t, a_t).

4 Lessons from a Stylized Example

This section employs a simplified version of the full model to illustrate the mechanism linking the decline in the real interest rate to changes in the firm size distribution and the reduction in firm dynamics.

4.1 A Stylized Example

To keep the model tractable and facilitate analysis, I assume, only for this section, that there is no entry or exit and no balanced growth. Additionally, instead of allowing for endogenous choices of debt types, I assume that the economy consists of two types of firms, which are exogenously determined as either earnings-based or asset-based financing firms. There is no productivity difference between these two types of firms.[11]

Therefore, in the economy, an exogenous measure of ν_e earnings-based firms solves the following firm problem:

$$\begin{aligned} &\max_{d_t, k_t, l_t, a_{t+1}} \sum_{t=0}^{\infty} \beta_f^t d_t \\ \text{s.t.} \quad & d_t + a_{t+1} = k_t^\alpha l_t^\gamma - w_t l_t - r_{e,t} b_t - \delta q_t k_t + a_t \\ & r_{e,t} b_t \leq \phi_e \cdot [k_t^\alpha l_t^\gamma - w_t l_t] \\ & 0 \leq a_{t+1}, \end{aligned}$$

where firms aim to maximize each period's dividends, subject to budget and borrowing constraints. Note that the focus here is on the interest coverage ratio, which provides a more comprehensive view of all possible channels. As shown in the appendix and discussed in the following section, similar results can be derived if the debt-to-earnings ratio is used instead. Here, I slightly abuse the notation for simplicity and use ϕ_e to denote the tightness of the borrowing limit.

[11] It will become clear later that the existence of productivity differences does not alter the results.

At the beginning of each period, firms start with an asset level a_t. They produce goods using a Cobb-Douglas production function, $k_t^\alpha l_t^\gamma$, where $\alpha + \gamma < 1$ is assumed to ensure decreasing returns to scale. Firms hire labor l_t and pay a wage w_t. Additionally, firms decide how much to allocate to assets, a_t, and how much to borrow from debt issuers for capital, k_t. After covering the net financing cost $r_t b_t$, firms distribute retained earnings either as current dividends or as savings for the next period, both of which must be non-negative. Net borrowing, b_t, is defined as $b_t = q_t k_t - a_t$.

In contrast, there is a measure of ν_a firms that use asset-based financing and face the following firm problem:

$$\begin{aligned} &\max_{d_t, k_t, l_t, a_{t+1}} \sum_{t=0}^{\infty} \beta_f^t d_t \\ \text{s.t.} \quad & d_t + a_{t+1} = k_t^\alpha l_t^\gamma - w_t l_t - r_{a,t} b_t - \delta q_t k_t + a_t \\ & q_t k_t \leq \frac{1}{1 - \phi_k(1-\delta)} a_t \\ & 0 \leq a_{t+1} \end{aligned}$$

Similar to earnings-based firms, these firms aim to maximize each period's dividends while adhering to budget and borrowing constraints. However, unlike earnings-based firms, asset-based firms are subject to an asset-based borrowing constraint, as shown in the second equation, where the debt level k_t cannot exceed a fraction, $\frac{1}{1-\phi_k(1-\delta)}$, of their savings a_t.

For simplicity, the measures of both types of firms, ν_e and ν_a, are given exogenously, and we focus on an equilibrium in which both types of firms earn positive profits. Additionally, the economy has an inelastic labor supply of $\bar{L}$ units. The market-clearing condition for the labor market at time t is given by

$$\bar{L} = \nu_e \cdot l_{e,t} + \nu_a \cdot l_{a,t},$$

where l_t^e and l_t^a represent the equilibrium labor demand for earnings-based firms and asset-based firms, respectively.

4.2 The Effects of Decline in the Real Interest Rates

First, a statistic should be introduced to assess the effects of the decline in the real interest rate. Using simple algebra, the labor market-clearing condition can be rewritten as:

$$\bar{L} = l_{a,t} \cdot (\nu_e \cdot \frac{l_{e,t}}{l_{a,t}} + \nu_a)$$

In this framework, it is useful to examine the ratio $\frac{l_{e,t}}{l_{a,t}}$ (hereafter referred to as the labor ratio) to gain insights into firm size distribution and dynamics. The labor ratio is directly proportional to employment shares; for instance, if $\frac{l_{e,t}}{l_{a,t}}$ increases, it indicates a shift in employment shares toward earnings-based firms. Simultaneously, the labor demand for asset-based firms will decrease due to the fixed firm measures and inelastic labor supply. Additionally, since labor shares are indicative of firms' profitability, the labor ratio can also be employed to analyze firm dynamics.

Therefore, I will focus on the impact of the decline in real interest rates on the labor ratio in the steady state to reveal their connections and potential mechanisms in the long run. I denote the long-run steady-state labor ratio as $\frac{l_e}{l_a}$. Also, to focus on the implications of borrowing limits, I set $r_t^e = r_t^a = r_t$, and assume the borrowing limits are binding.

The effects of the decline in long-run real interest rates through heterogeneous borrowing limits are summarized in the following proposition.

Proposition 1 *If $\frac{1}{\beta_f} - 1 > r_t$, then:*

- *If $\phi_e \left(1 + \frac{\delta}{1/\beta_f - 1}\right) \geq \phi_k(1-\delta)$, the labor ratio $\frac{l_e}{l_a}$ increases monotonically as r decreases.*
- *If $\phi_e \left(1 + \frac{\delta}{1/\beta_f - 1}\right) < \phi_k(1-\delta)$, the labor ratio $\frac{l_e}{l_a}$ and r exhibit a "U-shaped" relationship: $\exists r^* \in (0, \frac{1}{\beta_f} - 1)$ such that when $r > r^*$, the labor ratio decreases as r decreases, and when $r < r^*$, the labor ratio increases as r decreases.*

Proof. See the appendix. ■

The proposition states that under the interest coverage ratio, the effects of lower interest rates depend on three sets of parameters: (i) ϕ_e and ϕ_k, the debt capacities of the two types of financing, (ii) δ, the depreciation rate, and (iii) β_f, the discount factor.

The intuition behind these results is as follows. In the long term, a firm's optimal size is determined by balancing the marginal benefits and costs of expansion or contraction. A firm considers not only its marginal revenue and cost from such changes, but also internalizes the effects on its borrowing limits. When the interest rate decreases, the lower cost

of capital incentivizes the firm to expand. A firm will increase its size more aggressively if the change has a greater positive effect on its borrowing limit.

For an asset-based firm, a one-dollar increase in capital value results in an increase in the debt limit by $\phi_k(1-\delta)$, which is invariant to r_t. In contrast, for an earnings-based firm, a one-dollar increase in capital value increases the debt limit by $\frac{\phi_e}{r_t}$ multiplied by marginal revenue. When r_t decreases, the debt capacity $\frac{\phi_e}{r_t}$ increases, but the expansion causes marginal revenue to decline. When $\delta = 0$, the debt capacity and marginal revenue change at the same rate but in opposite directions, leading to the following corollary.

Corollary 1 *If $\frac{1}{\beta_f} - 1 > r_t$ and $\delta = 0$, then:*

- *If $\phi_e \geq \phi_k$, the labor ratio $\frac{l_e}{l_a}$ increases monotonically as r decreases.*
- *If $\phi_e < \phi_k$, the labor ratio $\frac{l_e}{l_a}$ and r are "U-shaped": $\exists r^* \in (0, \frac{1}{\beta_f} - 1)$ such that when $r > r^*$, the labor ratio decreases as r decreases, and when $r < r^*$, the labor ratio increases as r decreases.*
- *As $r_t \to 0$, the labor ratio $\frac{l_e}{l_a} \to \infty$.*

If $\delta = 0$, then the change in the labor ratio with respect to r depends solely on ϕ_k and ϕ_e, which govern the leverage ratios of the two types of firms. If $\phi_k \leq \phi_e$, meaning the effective recovery rate of collateral assets is lower than that of earnings, a decline in real interest rates will cause the labor ratio to increase monotonically. In other words, equilibrium employment shares will shift toward firms with earnings-based financing. However, if asset-based firms can achieve sufficient leverage, meaning ϕ_k is higher, the relationship will be "U-shaped." At high real interest rates, a decline in the rate will cause the labor ratio to decrease, while at lower real interest rates, this relationship will reverse.

The "U-shaped" relationship, or more precisely, the reversal at lower interest rates, reflects the advantage of using earnings-based borrowing limits in a low-rate environment—namely, reduced reliance on accumulated assets. Accumulating assets is costly because the return on holding such assets is low, which also drives firms to prefer borrowing over saving. The lower the interest rate, the greater the benefit that earnings-based financing provides by relying less on asset accumulation. Therefore, when $\phi_k > \phi_e$, asset-based financing may facilitate expansion when r_t is relatively high, but this advantage reverses when r_t is lower. Indeed, as indicated by Proposition 1, the lower the discount factor β_f, the stronger the effect that earnings-based borrowing limits can generate. This is because less reliance on assets increasingly benefits earnings-based financing at lower rates. Finally, as r_t approaches zero, the borrowing constraint becomes effectively non-binding, causing the labor ratio to approach infinity.

When $0 < \delta < 1$, the relationship can become monotonic for relatively smaller values of ϕ_e. In other words, the presence of depreciation benefits earnings-based firms when the interest rate declines. There are two primary reasons for this. First, depreciation effectively reduces the debt capacity of asset-based financing by diminishing the value of capital, as demonstrated by the right-hand side of the inequality. More importantly, while marginal revenue decreases with expansion, the presence of δ slows the pace of this decline, preventing marginal revenue from being too low. In fact, the existence of positive interest spreads, $\rho > 0$, can generate the same impact. As a result, earnings-based firms find it more effective to relax their constraints using earnings. The same logic suggests that other factors, such as interest spreads, will generate similar effects.

The analysis here focuses solely on the interest coverage ratio. As shown in the appendix, the same mechanism and similar conditions for monotonicity are found when the earnings-based constraint is expressed as the debt-to-earnings ratio, with the exception that lower rates do not directly relax the limits, so the labor ratio will not reach infinity. In the appendix, I demonstrate that for both types of constraints, a range of reasonable calibrations can indeed generate a monotonic increase in the labor ratio. The following sections perform a more sophisticated calibration and quantitative exercises using the full model, confirming that the results remain robust.

Why should we expect the tightness of two types of constraints, namely ϕ_e and ϕ_k, of similar magnitude? While ϕ_k can be rationalized by the recovery rate in a fire sale, how should we think about ϕ_e? The linkage between earnings-based borrowing limits and firms' going-concern values provides an answer. In particular, if we assume that debt repayment has limited commitment. With limited commitment, the debt amount cannot exceed the maximum that intermediaries can recover in default. Specifically, this is given by:

$$b_t \leq \psi V_t^{end},$$

where V^{end} represents the firm's going-concern value. The following proposition demonstrates that the tightness of earnings-based borrowing limits can be rationalized by the relationship between the two constraints and the pledgeability of a firm's going-concern value in the steady state. For now, we assume no depreciation.[12]

Proposition 2 *At the steady-state equilibrium, if* $b \leq \psi V^{end}$ *and* $\phi_e^{DE} = \frac{\psi}{1-\beta_f+r\psi}$*, then the debt capacity satisfies*

[12]If we consider depreciation, the measure of earnings become Earnings before interest and taxes (EBIT), and the results remain unchanged. In practice, both measures are commonly used, and EBITDA is often used to measures earnings without the effects of financing or accounting decisions.

$$b_t \leq \phi_e^{DE}[f(k,l;z,s) - wl]$$

Furthermore, if $r_{j,t} = \frac{1}{\beta_f} - 1$ *and* $\phi_e^{ICR} = \frac{\psi}{\beta+\psi}$*, then*

$$rb \leq \phi_e^{ICR}[f(k,l;z,s) - wl]$$

Proof. See the appendix. ■

Simple numerical examples can illustrate this point. For interest coverage ratio, $\phi_e^{ICR} = 0.3$, for instance, means firms can borrow up to roughly 40% of firm's going-concern value. Drechsel (2022) documented that the average earnings-to-debt ratio is about 4.6. With $r = 0.05$, this result implies a pledgeability parameter $\psi = 0.36$. This is roughly consistent with empirical findings in Kermani and Ma (2020).[13] In other words, a typical earnings-based borrowing limit implies that the associated firm can borrow up to 35% to 40% of its steady-state going-concern value without reliance on accumulated wealth. This is in a similar, if not larger, magnitude of liquidation value of fixed assets, or ϕ_k, estimated by Kermani and Ma (2020). In fact, as Kermani and Ma (2023) point out, due to the potential asset specificity, companies often have limited liquidation values.

Therefore, the connection between earnings-based financing and firms' going-concern values shapes the interpretation of this paper's results. Being able to borrow against going-concern values gives earnings-based firms advantages, namely a higher debt capacity. The debt capacity advantage grants a comparable tightness of borrowing constraints. More importantly, such an advantage is amplified when a lower interest rate environment triggers a higher demand for credit and when accumulating wealth becomes more costly. When the interest coverage ratio is used in particular, the decline in real interest rates further amplifies such an advantage through the change in the discounted value of the pledgeable component of future cash flows. Firms endogenously choose to make earnings pledgeable (make it general as in Rajan (2012)) to overcome reliance on physical assets, which have limited recovery rates due to irreversibility or asset specificity (Kermani and Ma, 2023).

Finally, how can the price of capital affect the results? In fact, if perfect elasticity of capital supply is assumed, then the steady-state price q will not change with respect to r, and therefore it will not affect the labor ratio. However, the proposition below demonstrates that in this simple model, even without assuming perfect elasticity of capital supply, the steady-state price of capital q does not influence the labor ratio in the steady state.

[13] Kermani and Ma (2020), shows the total debt level is on average about 22% of total book assets, where the going-concern value is on average similar to the value of the total book assets.

Proposition 3 *For a given interest rate r, variations in the capital price q do not influence the steady-state equilibrium labor ratio $\frac{l_e}{l_a}$.*

Proof. See the appendix. ■

In other words, the steady-state equilibrium labor ratio $\frac{l_e}{l_a}$ is independent of the capital price q. The reasoning is as follows: an increase in q raises the cost of capital, causing both types of firms to contract relative to the scenario where q remains unchanged. This change benefits asset-based financing by increasing the collateral value proportionally to q. For firms using earnings-based financing, while they do not directly pledge collateral, the higher capital cost necessitates a proportional increase in marginal revenue to maintain profitability. This increase in marginal revenue, driven by the rise in q, allows these firms to more effectively relax their borrowing limits, as the borrowing capacity is tied to the marginal earnings, which also scales with q. Consequently, a change in the capital price affects the borrowing capacity of both types of firms equally, leading them to adjust their sizes proportionately. As a result, the equilibrium labor ratio remains unaffected by changes in q

It is important to note that the above analyses are based on the long-run steady-state equilibrium. The short-run equilibrium may differ, as the choices of both types of firms are influenced by the levels of their existing assets, which cannot adjust freely. In the long run, however, firms have the flexibility to adjust and accumulate wealth and capital in response to changes in real interest rates.

In addition to borrowing limits, endogenous debt choices, absent in this simple model, can also amplify these effects. As real interest rates decline, marginal firms that previously relied on asset-based financing during periods of higher interest rates may switch to earnings-based financing, enabling them to borrow more and produce more efficiently.

More importantly, when these effects are sufficiently large, the profits of firms using asset-based financing may also decline. Consequently, the equilibrium rate of entrepreneurship may decline.

Another important insight from this analysis is that, in the absence of heterogeneity in debt financing, the labor ratio and equilibrium labor demand would remain unaffected by changes in the real interest rate.

5 Quantitative Experiments

This section presents the results of the quantitative experiments. I calibrate the model to match a comprehensive set of statistics from the U.S. economy. Then, I examine the effects

Table 1: Externally Calibrated Parameters

Parameter	Name	Value	Source
α	capital share	0.3	Poschke (2018)
γ	employment share	0.6	Poschke (2018)
β_f	discounting rate	0.94	Interest Rate at 1980s
δ	depreciation	0.10	Poschke (2018)
ρ_e	interest spread	0.020	Philippon (2015)
ρ_a	interest spread	0.020	Philippon (2015)
η	population growth	0.025	Karahan et al. (2019)
ϕ_e^{ICR}	interest coverage ratio	0.27	Lian and Ma (2021)
ϕ_e^{DE}	debt to earning ratio	3.5	Lian and Ma (2021)
$\bar{L}$	avg employment	19.8	BDS

of exogenous changes in real interest rates on firm dynamics and the firm size distribution at the stationary equilibrium. Additionally, I analyze the transient dynamics in response to a sequence of perfect foresight changes in the real interest rate. Finally, I compare the benchmark economy with a counterfactual scenario that lacks debt heterogeneity, highlighting the role that debt heterogeneity plays.

5.1 Parameterization

Externally calibrated parameters. I begin with the subset of parameters that are calibrated externally. Table 1 presents these parameters. The model is calibrated at an annual frequency. I set both β_f and β_h to 0.94 to replicate the annualized risk-free rate of 6% at the beginning of the 1980s. Since the measure of potential entrants, $\bar{m}_0$, scales $\bar{\mu}_t$, I choose $\bar{m}_0$ to normalize the total measure of incumbent firms to one. The parameters governing the tightness of earnings-based financing, ϕ_e^{ICR} and ϕ_e^{DE}, are calibrated based on values documented in the literature. I also normalize the size of the labor force, $\bar{L}$, so that, given a measure of one for the firms, the average firm size is 19.8, consistent with the Business Dynamics Statistics (BDS) data for the period 1982–1986.

Consistent with empirical evidence and the literature, I calibrate the magnitude of decreasing returns to scale to 0.90 and set the ratio of the output elasticity of capital to labor at 1/2, resulting in values of $\alpha = 0.30$ and $\gamma = 0.60$ (Poschke, 2018). Additionally, I calibrate the interest spread for both types of financing, setting $\rho_e = \rho_c = 0.020$ to match the average cost of financial intermediation.

Finally, the depreciation rate is calibrated to 10%, consistent with values commonly used in the literature. The population growth rate, η, is set at 2.5% to match the U.S. population growth rate, as in Karahan et al. (2019).

Table 2: Internally Calibrated Parameters

Parameter	Name	Value
μ_{s_0}	mean of s_0	0.494
σ_{s_0}	std of log s_0	0.006
μ_{a_0}	mean of a_0	-1.700
σ_{a_0}	std of log a_0	1.699
σ_{z_0}	std of log z_0	0.100
σ_ϵ	std of shocks	0.055
ρ_s	persistence of shocks	0.985
f_b/y	fixed costs / output	0.016
c_e	entry cost	0.002
ξ	shock to exit	0.038
ϕ_k	leverage ratio of asset	0.287
f_m/y	fixed cost / output	0.016

Internally calibrated parameters. The remaining parameters are calibrated internally. Table 2 lists these parameters, which are set by minimizing the weighted distance between an equal number of empirical moments and their equilibrium counterparts in the model. In the following section, I will discuss the specific moments used to calibrate the model and the rationale behind their selection.

The first set of parameters primarily relates to the initial productivity and equity distributions, the productivity shock process, and the shock to exit. The parameter σ_{z_0} governs the initial distribution of permanent productivity z, which is assumed to follow a log-normal distribution with $z_0 \sim N(-\sigma_{z_0}^2/2, \sigma_{z_0})$. The parameters μ_{s_0} and σ_{s_0} govern the initial distribution of the stochastic component of productivity s_0, which is also assumed to follow a log-normal distribution with $s_0 \sim N(\mu_{s_0}, \sigma_{s_0})$. I assume that s_t evolves according to an AR(1) process such that $s_t = (1-\rho_s)s_{ss} + \rho_s s_{t-1} + \sigma_\epsilon \epsilon_t$, where s_{ss} is set so that $E(s_t) = 0$ and $\epsilon_t \sim N(0,1)$. The parameters μ_{a_0} and σ_{a_0} govern the initial distribution of the equity draw a_0, which is assumed to be log-normally distributed with $\log(a_0) \sim N(\mu_{a_0}, \sigma_{a_0})$. These initial distributions and the productivity process are crucial for determining the equilibrium firm size distribution and firm entry/exit dynamics. Additionally, the exogenous exit shock ξ plays a significant role in determining the exit rate of firms, particularly for older firms.

I calibrate these parameters to match key annual statistics of firm size distribution and firm dynamics, averaged over the 1982–1986 period. Specifically, for the firm size distribution, the model is calibrated to match the firm shares and employment shares for firms employing more than 500, 1,000, and 2,500 workers. Additionally, the overall exit rate

Table 3: Targeted Moments

Moment	Model	Data	Source
Firm share ≥ 500	0.0039	0.0034	BDS
Employment share ≥ 500	0.466	0.460	BDS
Employment share ≥ 1000	0.395	0.410	BDS
Employment share ≥ 2500	0.306	0.340	BDS
Exit rate (Entry Rate)	0.100(0.120)	0.097 (0.116)	BDS
S.T.D. of emp growth	0.305	0.420	Gavazza et al. (2018)
Avg. size for large firm	12436	10000	Compustat
Earnings/asset financing ratio	1.140	1.30	Flow of Funds
Debt-to-output ratio	0.339	0.350	Flow of Funds

and firm entry rate are targeted. To better capture firm dynamics, I also match the exit rate by firm age. Furthermore, the model is calibrated to match the standard deviation of employment growth..

Another set of parameters is primarily related to firms' debt choices. The fixed costs, f_e and f_a, play a crucial role in determining firms' financing selections and their characteristics, particularly sizes, within each financing option. The parameter ϕ_k governs the ability of assets to serve as leverage. Given that the majority of firms in the economy rely on asset-based loans, this parameter is calibrated to match the debt-to-output ratio. Additionally, I match the ratio of earnings-based debt to asset-based debt using data from the Flow of Funds.[14] Furthermore, the average size of earnings-based firms is targeted to match those firms with assets above the median in Compustat, as these firms predominantly use earnings-based financing.

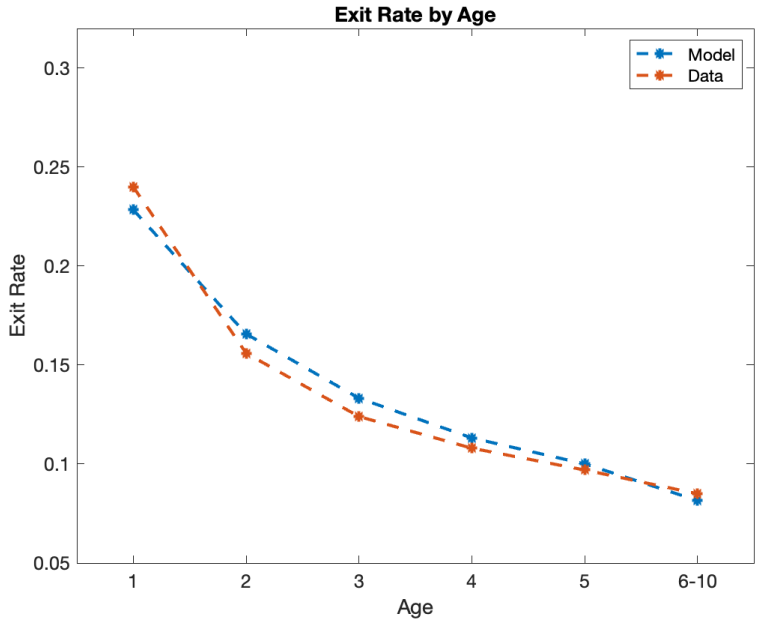
Together, there are 15 moments used to identify 12 parameters. Table 3 presents some of these moments, while Figure 7 displays the rest. Overall, the moments are well-matched, and the model generates a good fit to the data. Additionally, in the appendix, Figure 13 illustrates the moments of the firm age $\times$ size distribution, which were not targeted in the calibration. The figure shows that these non-targeted moments also fit the data well.

5.2 Equilibrium Effects of the Decline in Real Interest Rates

This subsection presents the stationary equilibrium outcomes of the economy for different values of the real interest rate. I use the calibrated model to evaluate the effects of declining interest rates on the economy, with a particular focus on the asymmetric effects driven by debt heterogeneity. The results indicate that the decline in the real interest rate can account

[14]To classify earnings-based debt, I include levels of commercial paper, corporate bonds, and other loans from finance companies. I classify all mortgages and bank loans not elsewhere classified as asset-based debt, using the Flow of Funds table L.102 from 1982–1986.

Figure 7: Targeted Firm Exit Rates



Note: This figure shows the targeted firm exit rates. The firm exit rates are calculated as the number of total exiting firms for each age divided by total incumbents at the same age.

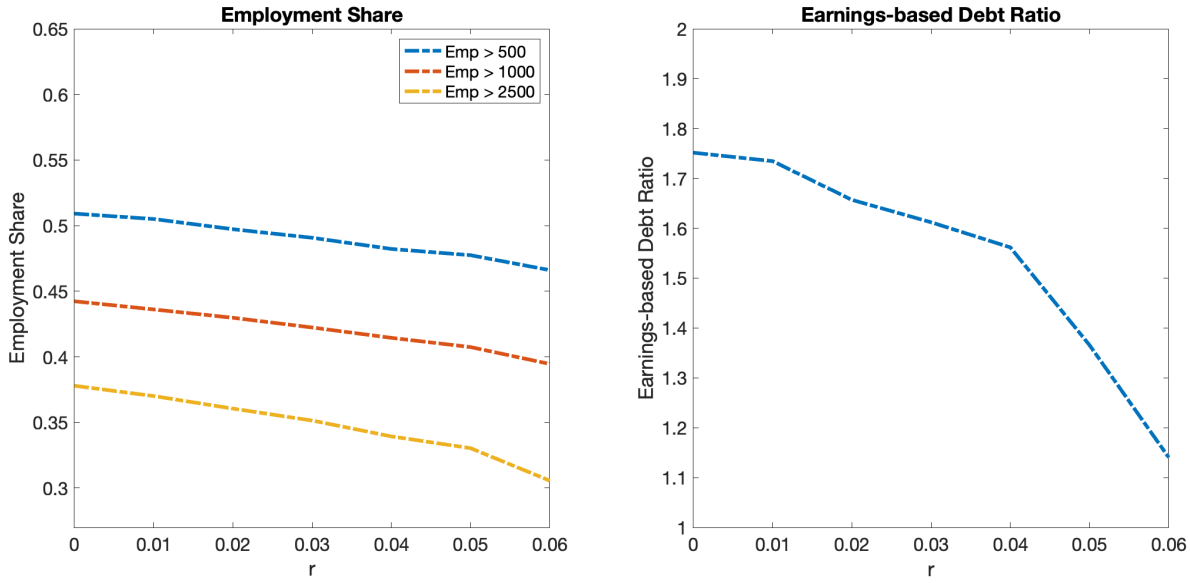
for almost all the changes in the shifts in employment shares and more than 20% of the changes in debt composition observed in the U.S. over the past forty years. The model also explains more than 20% of the decline in firms per capita. Counterfactual analysis further demonstrates that debt heterogeneity plays a crucial role in generating these effects.

Regarding the employment shares of large firms, the left panel of Figure 8 displays the model-predicted employment shares of large firms. Since most firms using earnings-based financing are larger companies in the model, the employment shares of large firms increase. Consistent with empirical patterns, the right panel of Figure 8 shows how debt composition also shifts toward earnings-based debt.

Regarding changes in the employment size distribution, the left panel of Figure 9 shows that the average size of firms at the top of the distribution grows faster than the average size of all firms. In contrast, the average size of smaller firms—those with fewer than 10 or 20 employees—shrinks relative to the average size of all firms, as illustrated in the right panel of Figure 9.

Figure 10 illustrates changes in average firm size and the number of firms per capita. As the interest rate r_t decreases, earnings-based firms—mainly large firms in the model—expand their employment demands. Simultaneously, some firms that initially use asset-based financing switch to earnings-based financing and increase their production. As a result,

Figure 8: Shifts in Employment Shares and Debt Composition



Note: This figure shows the model predicted employment shares and debt compositions at the stationary equilibrium for each level of r.The left panel shows the employment shares of firms with more than 500, 1000, and 2500 employees, respectively. The right panel shows the aggregate earnings-based to asset-based debt ratio.

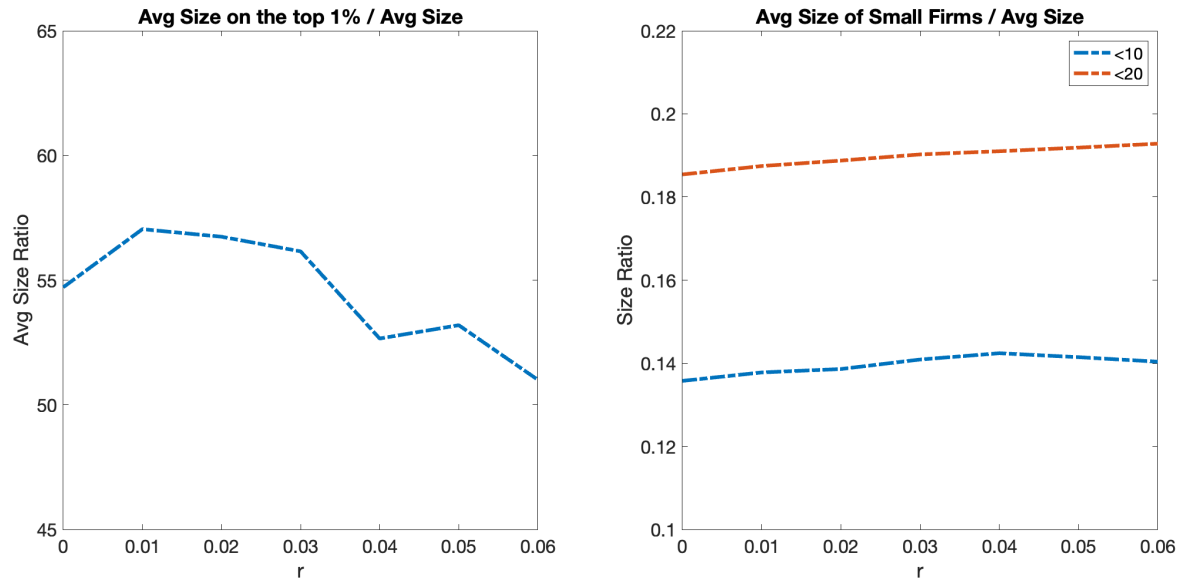
Table 4: Effects of the Real Interest Rate on the Economy

	Emp Share \geq 500		Emp Share \geq 1000		Emp Share \geq 2500		Firms per capita		E/A Financing	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
1982 - 1986	0.458	0.466	0.410	0.395	0.344	0.305	0.051	0.051	1.30	1.14
2013 - 2017	0.518	0.505	0.464	0.436	0.394	0.370	0.042	0.049	4.10	1.73
Changes	0.060	0.039	0.054	0.0415	0.050	0.064	-0.009	-0.002	2.70	0.59

smaller firms, which primarily use asset-based financing, shrink and are crowded out. This leads to a lower number of firms per capita. Less profitable firms exit the market due to rising labor costs driven by the faster growth of earnings-based firms. Consequently, the average firm size increases over time.

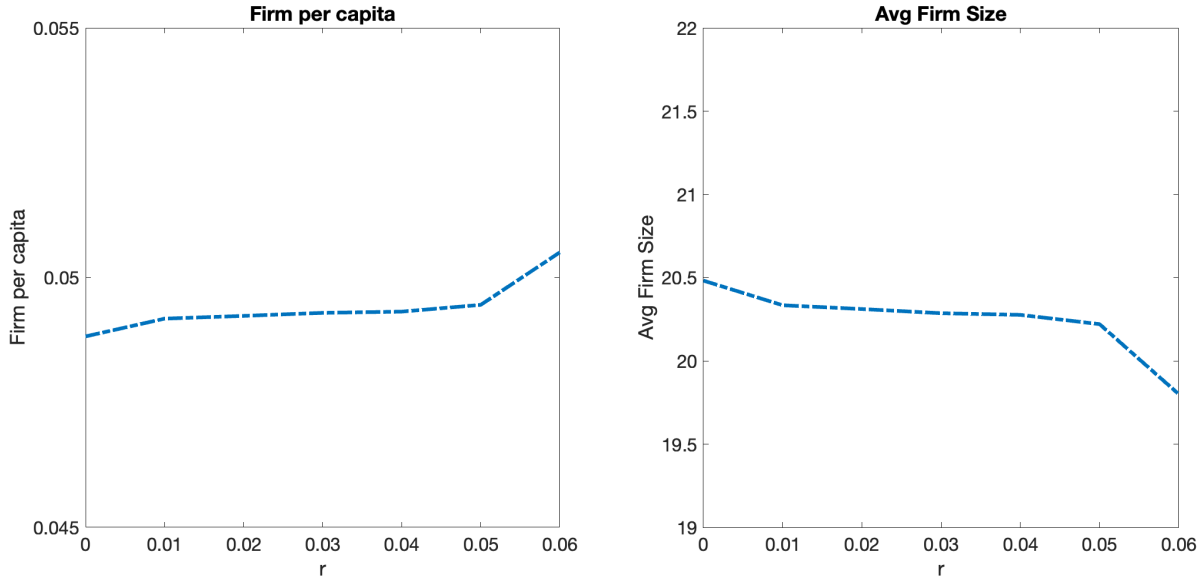
Finally, Table 4 summarizes the comparison between the secular changes observed in the U.S. and the model's predictions. The results indicate that the proposed channel can explain a significant portion of these long-term trends. Specifically, the decline in the real interest rate can account for roughly 80% in employment shares and 22% of the changes in debt composition observed in the U.S. over the past forty years. Additionally, the model explains approximately 20% of the decline in firms per capita.

Figure 9: Employment Size Distribution



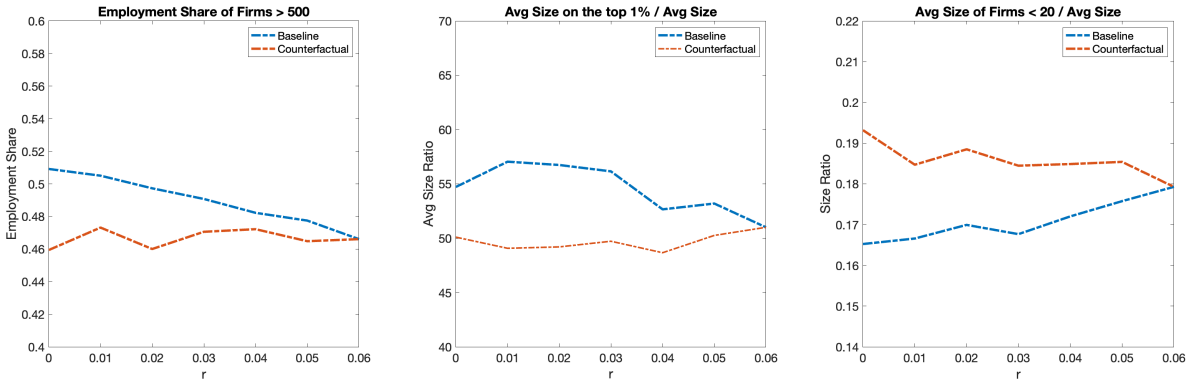
Note: This figure shows the model predicted changes in employment size distribution at the stationary equilibrium for each level of r. The left panel shows the ratio of the average employment size at 1% percentile to the average size of all firms. The right panel shows the ratio of average employment size with less than 10 or 20 employees to the average size of all firms.

Figure 10: The Decline in Firms per capita and the Average Firm Size



Note: This figure shows the model predicted levels of firms per capita and the average firm size at the stationary equilibrium for each level of r. The left panel shows the levels of firms per capita, and the right panel depicts the average firm size.

Figure 11: Changes in the Employment Size Distribution



Note: This figure compares the benchmark economy with a counterfactual economy without debt heterogeneity: all firms use asset-based financing. The results are the outcomes at the stationary equilibrium for each level of r. The left panel compares the shifts in employment shares, and the middle and right panels compare the average employment sizes at the top and the bottom of the distribution.

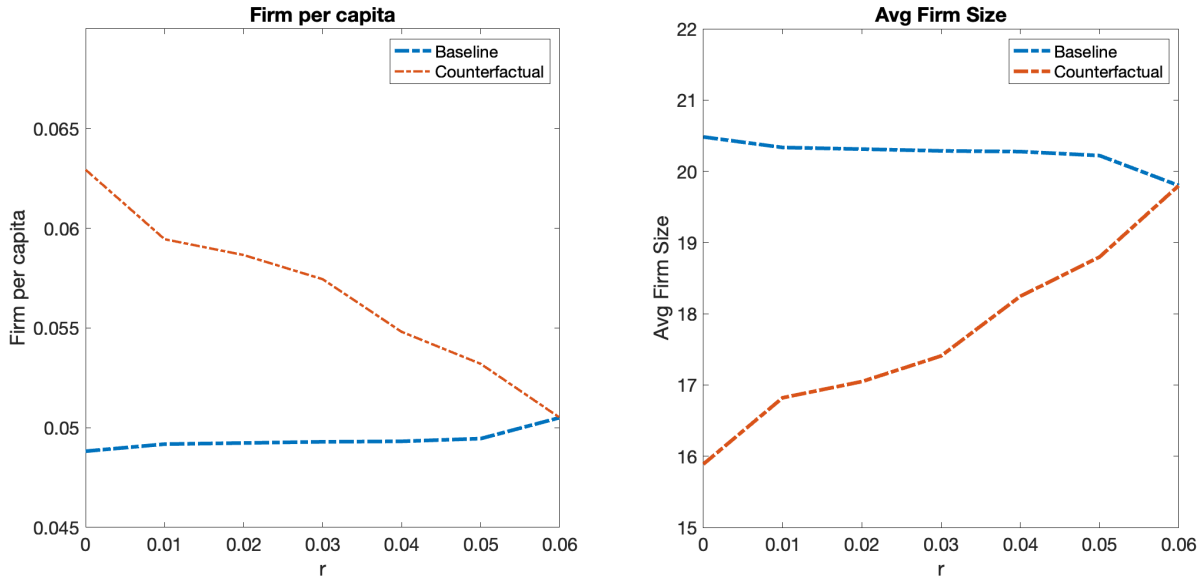
5.3 The Role of Debt Heterogeneity

In this section, I compare the quantitative results from the benchmark model with those from a counterfactual economy. In the counterfactual scenario, I assume there is no debt heterogeneity—meaning all firms rely solely on asset-based financing. I calibrate this model to match the same U.S. statistics as the benchmark model and conduct the same quantitative experiments. The purpose of this exercise is to study the role of debt heterogeneity in shaping the observed economic outcomes.

Figure 11 compares the changes in the firm size distribution between the benchmark model and the counterfactual model. The left panel illustrates that, without debt heterogeneity, the decline in real interest rates does not significantly shift employment shares. The middle panel shows that, in the counterfactual economy, firms at the top grow slightly slower than the average. In contrast, the right panel indicates that smaller firms grow faster than the average in the absence of debt heterogeneity.

Figure 12 compares the changes in average firm size and entrepreneurship between the benchmark model and the counterfactual model. The left panel reveals distinct patterns between the economy with and without debt heterogeneity. In the absence of debt heterogeneity, a decline in real interest rates leads to an increase in firms per capita. This underscores the critical role of debt heterogeneity in shaping the effects of real interest rates on firm dynamics. Additionally, the comparison of average firm sizes shows that, consistent with the firms per capita results, debt heterogeneity is crucial in explaining the differences in average firm size changes between the two economies.

Figure 12: The Change in Average Firm Size and Firms per capita



Note: This figure compares the benchmark economy with a counterfactual economy without debt heterogeneity: all firms use asset-based financing. The results are the outcomes at the stationary equilibrium for each level of r. The left panel compares firms per capita, and the right panel compares the average firm size.

Therefore, it is evident that debt heterogeneity plays a crucial role in linking the decline in long-term real interest rates to the observed changes in firm size distribution and firm dynamics.

6 Evaluating the Mechanism Based on US Firms

In this section, I empirically test the proposed mechanism using data from U.S. firms. The central prediction of the model is that firms with earnings-based financial constraints will experience a more significant change in employment (in percentage) in response to a decrease in interest rates compared to asset-based firms. To investigate this, I develop an identification strategy to analyze the relationship at the firm level. Utilizing linked data from Compustat and Capital IQ, I estimate the relationship between firm-level interest rates and employment, conditional on the degree to which a firm relies on earnings-based financing. The model predicts that when interest rates decline, firms heavily dependent on earnings-based financing will show a more pronounced increase in employment.

Table 5: Summary Statistics

	Full Sample	Large Firms	Small Firms
Assets	5036.0	9747.6	357.0
Employment	11.4	21.3	1.5
Earnings-base Debt (%)	56.2	70.0	42.4
Interest Rates	0.079	0.066	0.093
Physical Asset (% of Assets)	63.0	57.8	68.1
Debt-Assets Ratios	0.65	0.35	0.95
Sales to Assets Ratio	1.11	0.95	1.28
Observations	54,686	27,248	27,438

6.1 Data and Sample Characteristics

For this analysis, I use a comprehensive dataset that includes financial information from Compustat and Capital IQ at annual frequency, covering the period from 2001 to 2019.

The linked dataset contains employment levels, firm-level interest rates, and various firm characteristics. Capital IQ contains loan-level information, which is used to categorize debt types as in Lian and Ma (2021) as well as the firm-level usage of each type of debt. These observations allow me to test how firm-level employment responds to a change in interest rates and how such changes differ for firms using different types of financing. The descriptive statistics for the sample are presented in Table 5.

6.2 Identification Strategy

The identification strategy is designed to isolate the effect of interest rate changes on employment, conditional on a firm's reliance on earnings-based financing. The following equation forms the basis of the empirical model:

$$y_{s,t} = \alpha_s + \gamma_t + i_{s,t} + ratio_{s,t} + i_{s,t} \times ratio_{s,t} + X_{s,t} + \epsilon_{s,t}, \tag{1}$$

where: $y_{s,t}$ represents the log employment level for firm s at time t, α_s captures firm-specific fixed effects, γ_t controls for time-specific effects, $i_{s,t}$is the firm-level interest rate, $ratio_{s,t}$ denotes the fraction of earnings-based financing, $i_{s,t} \times ratio_{s,t}$ is the interaction term of interest, $X_{s,t}$ is firm-level controls, and $\epsilon_{s,t}$ is the error term.

The interaction term $i_{s,t} \times ratio_{s,t}$ allows for the test of the hypothesis that firms with higher earnings-based financing ratios will show a more substantial employment response to interest rate changes.

Table 6: Regression Results

	Log Employment					
	(1)	(2)	(3)	(4)	(5)	(6)
$i_{s,t}$	-0.619***	-0.883***	-0.833***	-0.582**	-0.696***	-0.641***
	(0.101)	(0.109)	(0.112)	(0.098)	(0.100)	(0.101)
$ratio_{s,t}$	0.155***	0.174***	0.135***	0.153***	0.161***	0.122***
	(0.023)	(0.029)	(0.029)	(0.024)	(0.028)	(0.028)
$i_{s,t} \times ratio_{s,t}$	-0.821***	-0.953***	-0.718***	-0.812***	-0.898***	-0.668***
	(0.184)	(0.198)	(0.202)	(0.183)	(0.187)	(0.192)
Sample	Full	Full	Full	Full	Full	Full
Controls	N	N	N	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y
Industry FE	N	Y	N	N	Y	N
Firm FE	N	N	Y	N	N	Y
N	54,686	54,686	54,686	54,686	54,686	54,686

Standard errors in parentheses
p \textless 0.05, p \textless 0.01, p \textless 0.001

6.3 Results

The model predicts that the coefficient on the interaction term $i_{s,t} \times e_r atio_{s,t}$ will be positive and significant. This would indicate that firms relying more on earnings-based financing experience a greater increase in employment when interest rates fall, consistent with theoretical predictions.

The regression analysis presented in Table 6 provides strong empirical support for the proposed relationship between firm-level interest rates, earnings-based financial constraints, and employment. The coefficient on the earnings-based financing ratio (ratio) is consistently positive and statistically significant at the 0.001 level across all specifications. This suggests that firms with a higher reliance on earnings-based financing tend to have higher levels of employment, indicating that larger firms rely more on earnings-based financing. The firm-level interest rate, as expected, exhibits a negative and statistically significant relationship with employment across all models. The size of the coefficient is substantial, demonstrating that higher interest rates are associated with lower employment levels within firms.

Crucially, the interaction term between the interest rate and the earnings-based financing ratio is also negative and statistically significant. This result is in line with the theoretical prediction that firms with a higher earnings-based financing ratio experience a more pronounced reduction in employment when interest rates increase. Conversely, these firms are more likely to increase employment when interest rates decline, relative to firms with lower reliance on earnings-based financing. The significance and direction of

this interaction term emphasize the differential impact of interest rate changes on employment across firms with varying financial structures.

The robustness of these findings is further demonstrated by the consistency of the coefficients across different model specifications. In particular, the inclusion of firm fixed effects in columns (3) and (6) helps control for time-invariant unobserved heterogeneity, while the inclusion of industry fixed effects in columns (2) and (5) controls for industry-specific factors that could influence employment. Despite these additional controls, the coefficients for the ratio, interest rate, and their interaction remain significant.

Overall, these results provide supporting evidence that firms with earnings-based financial constraints are more responsive to changes in interest rates, particularly in their employment decisions.

7 Conclusion

Can the decline in the real interest rate cause the shift in employment shares and the increasing dispersed firm size distribution? This paper shows that the decline can generate asymmetric effects through debt heterogeneity and lead to those secular changes. Specifically, in the model, the decline in real interest rates can explain almost all the changes in the shifts in employment shares and more than 20% of the changes in debt composition experienced in the U.S. for the past forty years. The model can also explain more than 20% of the decline in firms per capita. Debt heterogeneity plays an important role in generating the effects. Through asymmetrically benefiting large firms, debt heterogeneity turns the lower interest rate, conventionally considered a positive financial development, into a detrimental factor that negatively impacts small firms.

Besides its contribution to understanding the effects of long-term real interests on the firm size distribution and firm dynamics, my paper has broader implications for macroeconomics and leaves some open questions. First, theoretically, the linkage of earnings-based financing to pledgeable future cash flows opens the door for possible policy interventions. In particular, one implication of the model is that increasing the transparency of firm information and/or reducing the barrier and costs of restructuring bankruptcy can improve the strength of debt enforceability so that firms can borrow more efficiently. Therefore, it is interesting to look at the empirical effects of those policy interventions.

Secondly, it is worth studying both the empirical and theoretical implications of debt heterogeneity in an economy with multiple sectors. In the model, the economy features only one sector. However, in reality, different sectors may have different accesses to the credit market, which could have possible impacts on structural transformation and the

secular reallocation of employment across sectors (Dent et al., 2016). For example, the decline in employment shares of manufacturing could be the result of the reliance on asset-based debt and difficulties in accessing earnings-based financing due to already large fixed costs to set up plants.

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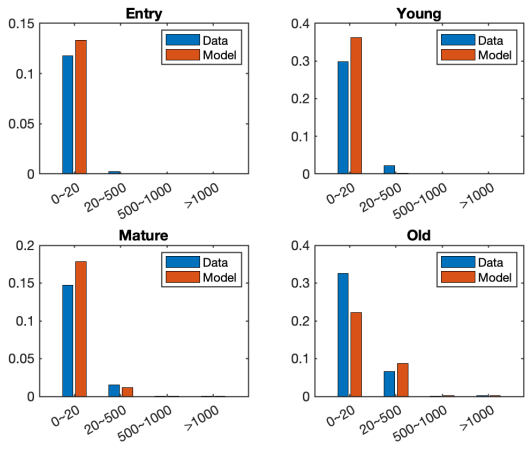
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Appendix A. Additional Tables and Figures

Figures

Figure 13: Non-targeted Moments



Note: This figure shows the non-targeted distributions of size $\times$ age bins. Entry is new firms. Young firms are those ages between 1 and 5. Mature firms are those ages between 6 and 10. Old firms are those with ages older than 10 years. All areas with the same color add up to one.

Appendix B. Proofs of Propositions

Proof of Proposition 1

At the steady state, the earnings-based firm's budget constraint becomes

$$d = f(k, l, z, s) - wl - rb$$

Then, $V^e = \sum_{s=0}^{\infty} \beta_f^s d$.
And,

$$V^e = \frac{1}{1-\beta_f}[f(k, l, z, s) - wl - rb]$$

Therefore, with $b \leq \psi V^e$, we have

$$b \leq \frac{\psi}{1-\beta_f}[f(k, l, z, s) - wl - rb]$$

A little algebra shows

$$b \leq \frac{\psi}{1-\beta_f+r\psi}[f(k, l, z, s) - wl]$$

Further, if

$$1 = \beta_f(1+r)$$

Then,

$$rb \leq \frac{\psi}{\beta_f+\psi}[f(k, l, z, s) - wl]$$

Proof of Proposition 2

Asset-based firms. The problem of asset-based firms is as follows:

$$\begin{aligned} \max_{d_t, k_t, l_t, a_{t+1}} & \sum_{t=0}^{\infty} \beta_f^t d_t \\ \text{s.t.} \quad & d_t + a_{t+1} = k_t^{\alpha} l_t^{\gamma} - w_t l_t - r_t b_t - \delta q_t k_t + a_t \\ & q_t k_t \leq \frac{1}{1-\phi_k(1-\delta)} a_t \\ & 0 \leq a_{t+1} \end{aligned}$$

The first-order conditions of the asset-based firm problem give:

$$\begin{aligned} &[l_t] \quad \gamma k_t^{\alpha} l_t^{\gamma-1} - w_t = 0 \\ &[a_{t+1}] \quad \beta_f[1 + r_{t+1} + \frac{1}{1-\phi_k(1-\delta)}\mu_{a,t+1} + \eta_{a,t+1}] = 1 \\ &[k_t] \quad \alpha k_t^{\alpha-1} l_t^{\gamma} - r_t q_t = q_t \mu_{k,t} \end{aligned}$$

Assume that the economy is at the steady-state equilibrium with interest rate r and wage w, the borrowing constraint is binding, and firms have positive equity holding, then the labor demand for the asset-based firm l_a is

$$l_a = \gamma^{\frac{1}{1-\alpha-\gamma}} w^{-\frac{1-\alpha}{1-\alpha-\gamma}} \{\frac{\alpha}{\gamma} \frac{1}{rq + \mu_a q + \delta q}\}^{\frac{\alpha}{1-\alpha-\gamma}} \tag{2}$$

When $1/\beta_f - 1 > r$, it must be that $\eta_a = 0$. Otherwise, $a = k = 0$ so the firm will not produce.

Earnings-based firms. The problem of earnings-based firms is as follows:

$$\begin{aligned} \max_{d_t, k_t, l_t, a_{t+1}} & \sum_{t=0}^{\infty} \beta_f^t d_t \\ \text{s.t.} \quad & d_t + a_{t+1} = k_t^\alpha l_t^\gamma - w_t l_t - r_t b_t - \delta q_t k_t + a_t \\ & r_t b_t \leq \phi_e \cdot [k_t^\alpha l_t^\gamma - w_t l_t] \\ & 0 \leq a_{t+1} \end{aligned}$$

Replacing $b_t = q_t k_t - a_t$, the first order conditions become:

$$\begin{aligned} &[l_t] \quad (1 + \mu_{e,t}\phi_e) \cdot [\gamma k_t^\alpha l_t^{\gamma-1} - w_t] = 0 \\ &[a_{t+1}] \quad \beta_f[1 + r_{t+1} + r_{t+1}\mu_{e,t+1} + \eta_{e,t+1}] = 1 \\ &[k_t] \quad (1 + \phi_e\mu_{e,t}) \cdot \alpha k_t^{\alpha-1} l_t^\gamma = r_t q_t \mu_{e,t} + r_t q_t + \delta q_t \end{aligned}$$

, where $\mu_{e,t}$ is the Lagrange multiplier of the borrowing constraint, and $\eta_{e,t}$ is the Lagrange multiplier of the non-negative equity holding constraint. If I assume that the economy is at the steady-state equilibrium with interest rate r and wage w, the borrowing constraint is binding, and firms have positive equity holding, then the labor demand for the earnings-based firm l_m is

$$l_e = \gamma^{\frac{1}{1-\alpha-\gamma}} w^{-\frac{1-\alpha}{1-\alpha-\gamma}} (\frac{\alpha}{\gamma} \frac{1 + \phi_e\mu_e}{rq\mu_e + rq + \delta q})^{\frac{\alpha}{1-\alpha-\gamma}} \tag{3}$$

Then, the steady-state equilibrium labor ratio of the two is

$$\frac{l_e}{l_a} = (\frac{(r + \mu_a + \delta)(1 + \phi_e\mu_e)}{r + r\mu_e + \delta})^{\frac{\alpha}{1-\alpha-\gamma}} \tag{4}$$

When $1/\beta - 1 > r$, it is trivial to show that asset-based firms have positive equity so $\eta_a = 0$. Therefore, $\mu_a > 0$ and the borrowing constraint is always binding.

This is not the same for earnings-based firms. When earnings-based firms have positive assets, $\eta_e = 0$. When $\eta_e = 0$, since $1 - \alpha - \gamma > 0$, we need only look at the ratio

$$\frac{(r + \mu_a + \delta)(1 + \phi_e\mu_e)}{r + r\mu_e + \delta}$$

Since $\frac{1}{1-\phi_k(1-\delta)}\mu_a = \frac{1}{\beta_f} - 1 - r = r\mu_e$, replacing r and μ_a with μ_e, one can get

$$\frac{(\frac{1}{\beta_f} - 1 - \phi_k(1-\delta)\frac{\mu_e(\frac{1}{\beta_f}-1)}{1+\mu_e} + \delta)(1+\phi_e\mu_e)}{\frac{1}{\beta_f} - 1 + \delta}$$

Observe that μ_e increases monotonically as r decreases, it is sufficient to look at how the labor ratio changes with respect to μ_e. The derivative with respect to μ_e yields the following.

$$= \frac{A\mu_e^2 + B\mu_e + C}{D}$$

$$A = \phi_e\left((1-\delta)\left(\left(\beta_f - 1\right)\phi_c - \beta_f\right) + 1\right)$$

$$B = 2\phi_e\left((1-\delta)\left(\left(\beta_f - 1\right)\phi_c - \beta_f\right) + 1\right)$$

$$C = \phi_e\left(1 - \beta_f\left(1-\delta\right)\right) - \left(1-\beta_f\right)\left(1-\delta\right)\phi_c$$

$$D = \left(\beta_f\left(\delta - 1\right) + 1\right)\left(\mu_e + 1\right)^2$$

It is easy to see that D > 0. Given that $\beta_f < 1$, $\phi_c < 1$, and $\delta \geq 0$, it is easy to see that A and B > 0. Therefore, the numerator is a quadratic function opening upwards with the vertex at $\mu < 0$. Then, if $C \geq 0$, that is,

$$\phi_e(1 - \beta_f\left(1-\delta\right)) \geq \left(1-\beta_f\right)\left(1-\delta\right)\phi_k,$$

the steady state labor ratio increases monotonically as r decreases.

When $\eta > 0$, the earnings-based firm has zero asset holding. If the borrowing constraint is binding, then the borrowing constraint for earnings-based firms gives

$$\begin{aligned} rk_e &= \phi(k_e^\alpha l_e^\gamma - wl_e) \\ &= \phi(\frac{1}{\gamma} - 1)wl_e \end{aligned}$$

The asset-based firm has the following equation

$$\frac{\gamma k_a}{\alpha} = \frac{wl_a}{r + \delta + \mu_a}$$

Therefore, a little algebra can show

$$\frac{l_e}{l_a} = [(\frac{\gamma\phi(\frac{1}{\gamma} - 1)}{\alpha})\frac{r + \delta + \mu_a}{r}]^{\frac{\alpha}{1-\alpha-\beta}}$$

, and it is trivial that the labor ratio increases monotonically as r decreases.

Finally, with $\delta > 0$, it is possible that the earnings-based firm can be unbinding. In particular, having an unbinding borrowing limit is equivalent to both $\eta = 0$ and $\mu_e = 0$, that is, $a = 0$.

The sufficient and necessary condition is therefore

$$rk_e \leq \phi_e(\frac{1}{\gamma} - 1)\frac{\gamma}{\alpha}(r + \delta)k_e,$$

where the right-hand side combines first-order conditions.

It is trivial that when r gets close to zero, the earnings-based firm will become unbinding. It is again straightforward that the labor ratio will increase monotonically in such a scenario.

When $\delta = 0$, the constraint of the earnings-based firm can be either unbinding or binding. Regardless of the status of the constraint, from the equations shown above, the labor ratio goes to infinity when r reaches zero.

Proof of Debt to Earnings Ratio

Earnings-based firms. The problem of earnings-based firms is as follows:

$$\begin{aligned} &\max_{d_t, k_t, l_t, a_{t+1}} \sum_{t=0}^{\infty} \beta_f^t d_t \\ \text{s.t.} \quad & d_t + a_{t+1} = k_t^\alpha l_t^\gamma - w_t l_t - r_t b_t - \delta q_t k_t + a_t \\ & b_t \leq \phi_e \cdot [k_t^\alpha l_t^\gamma - w_t l_t] \\ & 0 \leq a_{t+1} \end{aligned}$$

First of all, we show that if the constraint is binding when r is close to $\frac{1}{\beta_f} - 1$, then the earnings-based firms must always have positive asset holding for any $r \in [0, \frac{1}{\beta} - 1)$.

For the borrowing limit to hold, it must be that

$$k \leq a + \phi_e(k^\alpha l^\gamma - wl)$$

Applying first-order conditions, that is,

$$\alpha k^{\alpha-1} l^\gamma = \frac{r + \delta + \mu_e}{1 + \phi_e \mu_e},$$

and

$$\gamma k^\alpha l^{\gamma-1} = w$$

, we have

$$k_e(1 - \phi_e \frac{1 - \gamma}{\alpha} \frac{r + \delta + \mu_e}{1 + \phi_e \mu_e}) \leq a$$

Also,

$$1 = \beta_f(1 + r + \mu_e + \eta_e)$$

implies that

$$r + \mu_e \leq \frac{1}{\beta_f} - 1$$

When r is close to $\frac{1}{\beta_f} - 1$, that the constraint is binding implies that

$$\begin{aligned} k_e &= \phi_e \frac{1-\gamma}{\alpha} \frac{r + \delta + \mu_e}{1 + \phi_e \mu_e} k_e + a \\ &= \phi_e \frac{1-\gamma}{\alpha} (\frac{1}{\beta_f} - 1 + \delta) k_e + a \\ &\geq \phi_e \frac{1-\gamma}{\alpha} (\frac{1}{\beta_f} - 1 + \delta) k_e \end{aligned}$$

Therefore,

$$1 \geq \phi_e \frac{1-\gamma}{\alpha} (\frac{1}{\beta_f} - 1 + \delta)$$

Then, for any r, we have

$$k_e(1 - \phi_e \frac{1-\gamma}{\alpha} \frac{r + \delta + \mu_e}{1 + \phi_e \mu_e}) \leq a$$

and,

$$k_e(1 - \phi_e \frac{1-\gamma}{\alpha} (\frac{1}{\beta_f} - 1 + \delta)) \leq k_e(1 - \phi_e \frac{1-\gamma}{\alpha} \frac{r + \delta + \mu_e}{1 + \phi_e \mu_e}) \leq a$$

Since

$$1 \geq \phi_e \frac{1-\gamma}{\alpha} (\frac{1}{\beta_f} - 1 + \delta)$$

a must be greater than zero for k_e to have a positive value. Therefore, the earnings-based firm must have a positive holding of the asset. That is $\eta_e = 0$.

Therefore, the labor ratio is

$$\frac{l_m}{l_b} = (\frac{(r + \mu_c + \delta)(1 + \phi \mu_e)}{r + \mu_e + \delta})^{\frac{\alpha}{1-\alpha-\gamma}} \tag{5}$$

Since $\frac{1}{1-\phi_c(1-\delta)} \mu_c = \frac{1}{\beta_f} - 1 - r = \mu_e$, replacing μ_e and μ_c with r, one can get

$$\frac{\{r + \delta + (\frac{1}{\beta_f} - 1 - r)[1 - \phi_c(1 - \delta)]\}(1 + \phi_e(\frac{1}{\beta_f} - 1 - r))}{\frac{1}{\beta_f} - 1 + \delta}$$

The derivative with respect to r gives

$$\frac{\phi_e \left((\delta - 1) \left(2\phi_c \left(\beta_f \left(r + 1\right) - 1\right) - \beta_f\right) - 1\right) - \beta_f \left(\delta - 1\right) \phi_c}{\beta_f \left(\delta - 1\right) + 1}$$

The labor ratio is increasing when r decreases if

$$\frac{1}{\phi_e} \leq \frac{1}{\phi_c(1-\delta)}(\frac{1}{\beta_f} - 1 + \delta) - 2(\frac{1}{\beta_f} - 1 - r)$$

The labor ratio is monotonically decreasing for $r \in [0, \frac{1}{\beta_f} - 1]$ if and only if

$$\frac{1}{\phi_e} \leq \frac{1}{\phi_c(1-\delta)}(\frac{1}{\beta_f} - 1 + \delta) - 2(\frac{1}{\beta_f} - 1)$$

We reach a similar conclusion from the above equation, that is the labor ratio monotonically decreases when r increases when i) ϕ_e is sufficiently large, ii) ϕ_c is sufficiently small, iii) discount factor β_f is sufficiently small, or iv) δ is large enough. Same logic works here: debt capacity, reliance on asset accumulation, and marginal revenue from production matter for the effects.

Appendix C. Computational Appendix

This section describes how I compute the solution to the balanced growth path. The model is solved globally over a discretized state space using the following procedure.

Step 1: Given the real interes rate and wage, I solve the firm value function over the grid points by value function iteration and find the associated policy function by golden section search. I approximate firm decisions over a grid for permanent productivity z, the stochastic component s, and the level of physical assets a. I use 10 grid points to discretize the distribution of permanent productivity $\Gamma_z(z)$. For z , we use the 0.005 and 0.995 percentiles of the distribution as the grid bounds. For distribution of s, I choose 25 grid points and set the lowest and largest points to correspond to log productivities that are 5 standard deviations below and mean. For the distribution of assets, the lower bound of the grid is 0, and the upper bound is chosen so that the largest customer base in the solution of any version of the model is smaller than the bound. The grid points are chosen evenly across log level of assets.

Step 2: To approximate the distribution of firms, we construct a finer grid with 10, 100 and 100 points in the z, s and a direction, respectively. Then, I solve the firm's problem once on the new grid using the approximation to the value functions from the previous step. I use the grid method to find the stationary distribution. This method approximates the distribution of firms on a histogram based on the finer grid. Since both optimal policies are allowed to vary continuously, we assign values of a that do not fall on points in the grid by using the transition matrix to allocate firms in the histogram based on the proximity of the optimal policy to each point in the finer grid. Finally, the distribution of firms is obtained by iterating until convergence.

Step 3: Finally, I calculate the distance between aggregate labor demand and labor supply. If the distance is small, stop. Otherwise, update wage with a Newton method and go to Step 1.

Appendix D. Calibration of Counterfactual Economy

In the counterfactual economy, I assume all firms borrow from the asset-based financing. I keep externally calibrated parameters the same as the benchmark model. Also, I do not need to calibrate ρ_e and f_m as there is no earnings-based financing. Since there is no earnings-based debt, I set ϕ_k to be similar to the benchmark model. In terms of targeted moments, I follow the benchmark economy as close as possible.

Table 7 and Table 8 show the externally and internally calibrated parameters. Table 9 and Figure 14 show the targeted and non-targeted moments.

Table 7: Externally Calibrated Parameters: Counterfactual

Parameters	Name	Value	Source
α	capital share	0.3	Poschke (2018)
γ	employment share	0.6	Poschke (2018)
β_f	discounting rate	0.94	Interest Rate at 1980s
δ	depreciation	0.10	Poschke (2018)
ρ_a	interest spreads	0.040	Compustat
η	population growth	0.025	Karahan et al. (2019)
$\bar{L}$	avg employment	19.8	BDS

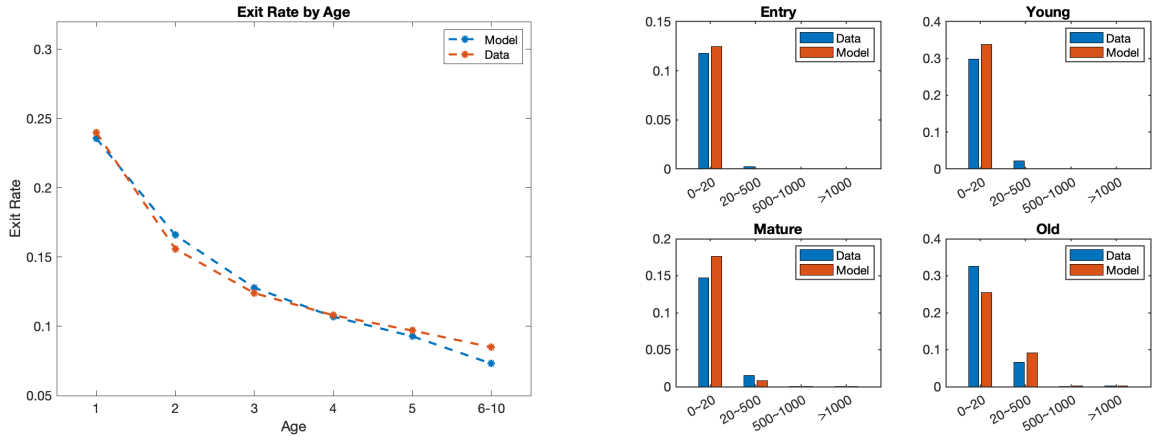
Table 8: Internally Calibrated Parameters: Counterfactual

Parameters	Name	Value
μ_{s_0}	mean of s_0	0.320
σ_{s_0}	std of log s_0	0.200
μ_{a_0}	mean of a_0	0.407
σ_{a_0}	std of log a_0	0.650
σ_{z_0}	std of log z_0	0.100
σ_ϵ	std of shocks	0.049
ρ	persistence of shocks	0.988
f_b/y	fixed costs / output	0.008
c_e	entry costs	0.001
ϕ_k	leverage ratio of assets	0.36728
ϕ_e	leverage ratio of earnings	0.700

Table 9: Targeted Moments: Counterfactual

	Model	Data	Source
Firm share ≥ 500	0.0042	0.0034	BDS
Employment share ≥ 500	0.463	0.460	BDS
Employment share ≥ 1000	0.378	0.410	BDS
Employment share ≥ 2500	0.300	0.340	BDS
Exit rate (Entry Rate)	0.099 (0.118)	0.097 (0.116)	BDS
S.T.D. of emp growth	0.267	0.420	Gavazza et al. (2018)
Debt to output ratio	0.283	0.350	Flow of Funds

Figure 14: Other Targeted and Non-targeted Moments: Counterfactual



Note: This Figure shows the targeted firm exit rates and non-targeted distribution of size $\times$ age bins. Firm exit rates are calculated as the number of total exiting firms for each age divided by total incumbents at the same age. For the right panel, entry is new firms. Young firms are those ages between 1 and 5. Mature firms are those ages between 6 and 10. Old firms are those with ages older than 10 years. All areas with the same color add up to one.