Reversal Patterns in Risk-Adjusted Returns: Evidence of Excess Volatility in Anomalies*

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Abstract

According to the no-arbitrage condition, risk-adjusted returns should be unpredictable. Using several prominent factor models and a large cross-section of anomalies, we find that past cumulative risk-adjusted returns negatively predict future anomaly returns. We interpret these cumulative returns as deviations of an anomaly price from the value implied by the given factor model, introducing a novel anomaly-specific predictor endogenous to any test asset-factor model combination. The observed reversal pattern in risk-adjusted returns is consistent with a transitory component in prices, suggesting excess volatility in anomaly prices beyond what can be attributed to discount rate dynamics implied by standard factor models.

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1 Introduction

The well-documented phenomenon of excess volatility (Shiller, 1981; LeRoy and Porter, 1981) refers to the observation that prices can exhibit prolonged but temporary deviations from a benchmark fundamental value. These deviations, tied to mean-reverting components in prices, manifest as return reversals (e.g., Fama and French, 1988), motivating the use of reversal patterns as a test for excess price volatility (e.g., De Bondt and Thaler, 1985). However, the presence of reversal patterns alone is insufficient to conclude that prices are excessively volatile. Two key factors must be considered. First, the nature of reversals—whether short-term or long-term—is critical. While short-term reversals provide insight into short-run inefficiencies, they offer limited evidence of persistent divergences between prices and fundamental values, making long-term reversals the central focus for studying excess volatility (e.g., Lehmann, 1990). Second, the "excess volatility puzzle" may be rationalized by time-varying discount rates (Campbell and Shiller, 1988).

In this paper, we extend the concept of excess volatility to equity anomalies, bridging two of the most prominent literatures in asset pricing. Our key innovation is to move beyond the constant discount rate benchmark by incorporating time-varying discount rate dynamics derived from factor-based stochastic discount factors (SDFs) designed to capture the cross-section of expected returns. Traditional excess volatility tests often regress future aggregate equity returns on valuation ratios—such as price-to-dividend or price-to-earnings ratios—which serve as proxies for temporary deviations of prices from fundamentals. In our framework, we construct an anomaly-specific price deviation by cumulating past long-term risk-adjusted returns relative to a benchmark factor model. We then regress future risk-adjusted returns on their price deviations and find strong negative coefficients. While the traditional excess volatility puzzle implies reversal patterns in aggregate equity returns, we document *long-term* reversal patterns in *risk-adjusted* returns across a broad set of equity anomalies. This suggests that, whereas standard evidence for excess volatility challenges constant discount rate models, our findings also challenge the time-varying discount rate dynamics implied by factor models, uncovering a pervasive form of excess volatility that cannot be fully attributed to discount rate adjustments (Giglio and Kelly, 2017).

Our predictive framework is characterized by three main features that allow us to test

for excess volatility. First, unlike approaches that use external data—such as sentiment, accounting signals, or macroeconomic information—our candidate predictor relies exclusively on the dynamics of anomaly and factor returns. Specifically, it is constructed as the difference between the price of a portfolio, proxied by long-term returns (in the spirit of Bondt and Thaler, 1985), and the price implied by the factor model, proxied by long-term factor returns. This approach differs from factor timing methods based on valuation ratios (Haddad, Kozak, and Santosh, 2020), which assume that current dividends or book equity capture fundamental value. By preserving internal consistency with the SDF model under examination, our framework adheres to the Campbell-Shiller present value identity, which defines excess volatility as:

$$\text{Var}\left(E_t \sum_{j=0}^{\infty} \rho^j r_{t+1+j}\right) > 0 \ .$$

As noted by Nagel (2024), the magnitude of this variance depends on the conditioning information considered by the econometrician (e.g., market capitalization rescaled by sales versus earnings). Our framework minimizes discretion by making the predictor endogenous to the model, aligning the conditioning set with the factors driving the SDF.

Second, our framework imposes a sign restriction on the regression loading of anomaly returns on our endogenous predictor. This restriction, derived from the relationship between the SDF model and anomaly return dynamics, mandates a negative loading. This reflects the reversal pattern in returns: when prices rise due to positive news ($r_{t+1} - E_t r_{t+1}$), we expect lower future returns. This aligns with the Campbell-Shiller present value identity, which suggests that price changes driven by expected return news are temporary and will be offset by future returns in the opposite direction:[1]

$$r_{t+1} - E_t r_{t+1} = -\Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j r_{t+1+j} \ .$$

Thus, $\text{Var}\left(E_t \sum_{j=0}^{\infty} \rho^j r_{t+1+j}\right)$ reflects the variance of the transitory component of stock prices (see, e.g., Fama and French, 1988). Consistent with this, we demonstrate that the

[1] The Campbell-Shiller present value identity is: $p_t = \text{const.} + d_t + E_t \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} - E_t \sum_{j=0}^{\infty} \rho^j r_{t+1+j}$. If the price moves in period $t+1$ due to news about future expected returns, with $d_{t+1} = E_t d_{t+1}$ and no cash flow news, the identity holds as shown.

magnitude of the loading is inversely related to the persistence of the temporary component in stock prices. These two features—the endogenous predictor and the sign restriction on loadings—make our documented reversal pattern in anomalies unique, and closely linked to the transitory nature of price level deviations.

Finally, our approach integrates information from both the time-series of returns and cross-sectional factor models (see Karolyi and Van Nieuwerburgh, 2020). This integration allows us to evaluate whether excess volatility extends beyond what an SDF model, defined by a given set of factors, can predict. Our results reveal the limitations of heuristic SDFs, which are designed to explain cross-sectional return differences, in capturing the time-series dynamics of returns. Importantly, the reversal patterns we document are fundamentally different from those previously studied. Unlike cross-sectional long-term reversals (e.g., Bondt and Thaler, 1985) or monthly reversals (e.g., Jegadeesh, 1990), our patterns emerge in a time-series context over long horizons (one to five years) and are based on risk-adjusted past returns. Furthermore, our portfolio-based approach highlights delayed price reactions to common risk factors, contrasting with cross-sectional studies of individual firms, which primarily emphasize underreaction or delayed overreaction to firm-specific news (e.g., Jegadeesh and Titman, 1995).[2]

Our analysis is grounded in the expected return-beta representation, which posits a linear relationship between expected returns of any asset and the expected return on the mean-variance efficient (MVE) portfolio.[3] A time-series representation of this cross-sectional model implies that, when the model accurately captures risk and risk compensation, risk-adjusted returns should be unpredictable.[4] A critical challenge in testing this economic restriction is identifying a meaningful information set, as the selection of return predictors is subject to data snooping (e.g. Lo and MacKinlay, 1990) and publication bias (e.g. McLean and Pontiff, 2016). To mitigate these concerns, we propose constructing a predictor that is

[2]A similar argument regarding momentum is discussed in Lewellen (2002).

[3]Giglio and Kelly (2017) approach excess volatility beyond discount rates by using an affine model under the (risk-neutral) $\mathcal{Q}$-measure, which is a natural framework for term structure analysis. In contrast, our use of the expected return-beta representation is more appropriate for equity markets. The equivalence between the two — risk-neutral and expected return-beta representations—is discussed by Cochrane (2005).

[4]The risk adjustment implied by factor models is consistent with both behavioral and rational explanations. On the behavioral side, sentiment-driven investor demand may align with covariances linked to dominant factors (Kozak, Nagel, and Santosh, 2018). On the rational side, it could reflect technological shocks (Bansal and Yaron, 2004) or time-varying risk aversion (Campbell and Cochrane, 1999).

endogenous to the test asset–factor model combination: the difference between assets and factors long-term returns, referred to as price deviation. This approach leverages the idea that long-term returns can reveal model misspecification more effectively than single-period returns (Chernov, Lochstoer, and Lundeby, 2021), and that they are better suited to detect persistent mispricing driven by the temporary component of prices (see, e.g. Shiller, 1981; Fama and French, 1988).

We study the time variation in expected anomaly returns relative to a heuristic MVE portfolio constructed using several popular factor models. Specifically, we employ various versions of the Fama and French (2015, henceforth FF5) five-factor model. First, we use a standard combination of the FF5 factors based on their unconditional first and second moments. Then, to account for conditioning information about the factors' mean and volatility in the construction of the MVE portfolio, we implement a version with factor timing (Haddad, Kozak, and Santosh, 2020), and another with volatility timing (Moreira and Muir, 2017). Finally, we use the characteristic-efficient factors of Daniel, Mota, Rottke, and Santos (2020) since Kozak and Nagel (2022) show that hedging the unpriced components of heuristic factor returns makes them more likely to span the stochastic discount factor. We further conduct robustness tests by considering the Hou, Xue, and Zhang (2015) q-factor model and its timed version for constructing the MVE portfolio.

Our main analysis examines 90 portfolios comprising the long and short legs of 45 widely studied characteristics-based strategies (e.g., Haddad, Kozak, and Santosh, 2020; Kelly, Kozak, and Giglio, 2020). Regardless of how we construct heuristic MVE portfolios, we demonstrate that anomaly price deviations negatively predict future anomaly returns. Notably, we observe similar results when using the classic 25 portfolios sorted by size and book-to-market, as well as the extensive cross-section proposed by Chen and Zimmermann (2021) as test assets. Overall, our findings provide robust evidence of a pervasive temporary component in anomaly portfolio prices that cannot be attributed to the discount rate dynamics implied by the factor models considered.

The evidence for anomaly return predictability is obtained through an out-of-sample perspective, where price deviations are constructed in real-time to mitigate over-fitting and identify ex-ante mispricing. This approach accounts for potential time-varying exposures of test assets to the MVE portfolio. Specifically, we employ short-window regressions using daily

returns (e.g., Lewellen and Nagel, 2006) and a classic fixed-length rolling window approach (e.g., Fama and French, 1997). Across both methodologies, we consistently document robust evidence of anomaly predictability using price deviations.

Importantly, our price deviations retain their predictive power even after controlling for other asset-specific predictors, including book-to-market ratio, the momentum or reversal effects captured by 1- and 5-year past returns, and aggregate sentiment measures (Baker and Wurgler, 2006; Huang, Jiang, Tu, and Zhou, 2014). Notably, these deviations continue to forecast returns even when we control for portfolio reversals based on long-term (5-year) past returns. Since the price deviations are derived from cumulative past returns relative to cumulative MVE returns, their statistical significance—even after adjusting for absolute 5-year past returns— implies that relative mispricing is more informative for predicting returns than absolute mispricing.

While the out-of-sample R^2 is a standard metric in return predictability, (see, e.g., Rapach and Zhou, 2022), Kelly, Malamud, and Zhou (2023) highlight its limitations in capturing the model's economic value. Thus, we also implement a portfolio exercise to quantify the economic magnitude of the temporary component in anomaly prices. Specifically, we construct a zero-cost portfolio that buys anomalies with high one-year-ahead expected returns and sells anomalies with low one-year-ahead expected returns based on the signal provided by the anomaly-specific price deviations. Such a zero-cost investment strategy generates an out-of-sample annualized Sharpe ratio of 0.8 and 1.0 when the deviations are relative to the Fama-French five-factor model or to the Daniel, Mota, Rottke, and Santos (2020) hedged factors, respectively. These results suggest that the volatility in anomaly returns, beyond what leading SDF models predict, is both statistically and economically significant.

The pervasive negative loading of future portfolio risk-adjusted returns on the current price deviation documented in this paper implies that when asset prices are higher (lower) than the long-run price level implied by the factor model, we expect lower (higher) returns in the next period so that deviations are corrected. This reversal pattern in anomaly return dynamics indicates systematic mispricing that gradually dissipates, resembling models of overreaction and slow price adjustment to news. However, we show that the performance of our price deviation-based portfolio cannot be explained by behavioral factor models that aim to capture temporary, long- and short-horizon deviations of prices from fundamental

values (Daniel, Hirshleifer, and Sun, 2020), nor by models constructed to capture aggregate mispricing (e.g., Stambaugh and Yuan, 2016; Bartram and Grinblatt, 2018). In fact, we find that regressing our zero-cost portfolio returns on the mispricing factor model of Bartram and Grinblatt (2018) or on the behavioral factor model proposed by Daniel, Hirshleifer, and Sun (2020) yields large and statistically significant alphas.

The data-generating process underlying our predictive framework assumes that price deviations will eventually correct, as in, e.g., Dong, Li, Rapach, and Zhou (2021). However, this correction process can unfold over multiple periods. Examining a broad cross-section of assets and averaging across various factor models, we find that when a test asset's value exceeds the target implied by the empirical MVE portfolio, it predicts negative returns over the next two to three years, after which the price deviation gradually dissipates. Notably, the extended period required for anomaly prices to revert toward their target value aligns with the evidence in Daniel, Klos, and Rottke (2022), who show that the beliefs of optimistic agents gradually decay toward rational beliefs over approximately five years.

The mean-reverting behavior of price deviations toward the price implied by the MVE portfolio suggests that factor models provide a meaningful description of the financial system in the long run—a similar interpretation is discussed in Merton (1987)'s presidential address. Related, our documented portfolio-level return predictability can be interpreted in terms of speed of adjustment to the long-run price implied by the factor model. Our predictability disappears when the speed of adjustment is instantaneous while it becomes more apparent when the speed of adjustment to the long-run price reduces because of, e.g., slow adjustment of prices to new information (e.g., Amihud and Mendelson, 1987) or slow moving capital (e.g., Duffie, 2010).

A low speed of adjustment and persistent price deviations can be attributed to market frictions, such as noise trader risk (e.g., De Long, Shleifer, Summers, and Waldmann, 1990), holding costs (Tuckman and Vila, 1992), and idiosyncratic risk and transaction costs (e.g., Pontiff, 1996, 2006). To empirically evaluate the role of these frictions in our findings, we analyze how limits to arbitrage influence price-deviation-based investment strategies. Since limits to arbitrage can stem from various frictions (Gromb and Vayanos, 2010), we employ various proxies, including idiosyncratic volatility (IVOL) and liquidity (Pástor and Stambaugh, 2003). Our results show that while price deviations predict returns under both

high- and low-friction conditions, their predictive power is stronger during periods of elevated volatility and reduced liquidity. Given that IVOL is often associated with a shortage of funds for arbitrageurs (e.g., Pontiff, 2006), and that arbitrage activity is more constrained when liquidity is scarce, these findings suggest that arbitrageurs trade only gradually on the mispricing due of limits to arbitrage. This gradual trading behavior leads to the slow correction of an anomaly's price toward the target value implied by the MVE portfolio.

Related Literature. This study contributes to several strands of the literature by offering new insights into the dynamics of price deviations and their broader economic implications.

First, the analysis builds upon the empirical literature that studies temporary deviations of asset values from fundamentals. In an early contribution, Poterba and Summers (1988) find positive autocorrelation in returns over short horizons and negative autocorrelation over longer horizons attributing this pattern to persistent but transitory divergences between prices and fundamental values. Concurrently, Fama and French (1988) argue that the observed U-shaped regression slope from forward h-period industry returns $r_{t,t+h}$ on past returns $r_{t-h,t}$ reflects a transitory component in prices. Our finding that deviations of a portfolio's price from a given factor model forecast the portfolio returns is consistent with the permanent-transitory decomposition of prices proposed by Fama and French (1988).

Recently, Dong, Li, Rapach, and Zhou (2021) show that returns of the short- and (to a lesser extent) the long-leg of anomaly portfolios are positively related to the next period's market return. To explain this finding, these authors also leverage the permanent-transitory decomposition of an anomaly portfolio price. Differently from them, we show predictability at the individual anomaly level, rather than at the aggregate market level. Moreover, our time-series predictive framework provides direct evidence on the presence or absence of return dynamics consistent with a given SDF. Importantly, the evidence in our paper is that price deviations forecast reversal, not continuation, of returns. Thus, our evidence complements that in Ehsani and Linnainmaa (2021) about positive auto-correlations in anomalies.

Second, this paper contributes to the literature on the relationship between cointegrated variables and error correction models (e.g., Hendry, 1986; Engle and Yoo, 1987; Johansen, 1995; Liu and Timmermann, 2013). A seminal example in macro-finance is Lettau and Ludvigson (2001a), who show that aggregate consumption, asset holdings, and labor income share a common long-term trend, with temporary deviations predicting short- and

medium-term stock returns. While their focus is on economic variables and the aggregate market, our study examines anomaly portfolios and the relationship between asset and factor prices. Bansal, Dittmar, and Kiku (2007) and Bansal and Kiku (2011) emphasize the role of cointegration between dividends and consumption for expected equity returns and portfolio strategies over long horizons. Departing from these studies, our approach posits that portfolio and factor prices share a common stochastic trend. Interestingly, despite the widespread use of factor models in equity return analysis, common trend modeling has been largely absent. This omission has left the transitory component of anomaly prices—and the associated excess volatility—largely unexplored.

Third, the paper intersects with recent advancements in factor modeling and pricing, in particular with the recent literature on "self-destructing" SDF.[5] Chernov, Lochstoer, and Lundeby (2021) show that popular factor pricing models fail to price their own factors across multiple return horizons, attributing this to the conditional dynamics of factor returns. While their focus is on factor pricing over different horizons, we examine the risk-adjusted short-run dynamics of anomaly returns by leveraging information from long-run (cumulative) asset and factor returns. Relatedly, Baba-Yara, Boyer, and Davis (2022) show that cutting-edge asset pricing models cannot explain the average returns of the MVE (MVE) portfolios implied by other models. In doing so, they rely on information external to a given model (but contained in other models) for their conditional asset pricing test. In contrast, we focus exclusively on information endogenous to a specific factor model to assess whether the conditional dynamics of anomaly returns are internally consistent with that model. Distinct from both studies, our predictive framework uniquely quantifies the excess volatility of anomaly prices and the magnitude of their temporary components.

Fourth, our paper contributes to the literature that links the time-series and cross-sectional predictability. Studies such as Maio and Santa-Clara (2012), Boons (2016), and Koijen, Lustig, and Van Nieuwerburgh (2017) explore the consistency between time-series and cross-sectional behavior of factors, while Lettau and Pelger (2020) emphasize the challenges of balancing these objectives in factor model design. Our contribution lies in demonstrating that incorporating information from multi-period returns (i.e., prices) into cross-sectional models can effectively capture the time-series dynamics of portfolio returns. Fur-

[5] We thank Valentin Haddad for suggesting this term.

thermore, we complement research on factor timing (Cohen, Polk, and Vuolteenaho, 2003; Haddad, Kozak, and Santosh, 2020; Baba-Yara et al., 2020) by showing that price deviations predict a vast array of portfolio returns.

Finally, we relate to the recent papers challenging the notion of a trade-off between systematic risk and expected returns. Recently, Lopez-Lira and Roussanov (2022) show how to construct a portfolio that exploits individual stock return predictability while hedging all undiversifiable risk; they document that such portfolio delivers a Sharpe ratio above one. Kim, Korajczyk, and Neuhierl (2020) propose a procedure that gives characteristics maximal explanatory power for risk premiums before attributing any explanatory power to alphas (mispricing). Whereas Lopez-Lira and Roussanov (2022) exploit a wide range of characteristics to forecast stock returns, we instead show how to construct a predictor that is endogenous to the factor model under scrutiny, and link this predictability to the conditional pricing ability of the model. We then show how to exploit this endogenous mispricing to form portfolios that hedge out the systematic risk associated with the MVE factor, in a spirit similar to the arbitrage portfolios of Kim, Korajczyk, and Neuhierl (2020).

2 Mean-variance returns, prices and predictability

2.1 The Intuition

The presence of a transitory component in stock *prices* leads to mean reversion in *returns* (Fama and French, 1988). Our objective is to develop a reversal strategy for a given portfolio return, r_t, that capitalizes on these transitory price changes while adjusting for risk fluctuations. This strategy would quantify the excess volatility in equity anomaly prices.

The simplest and long-established signal for building a reversal strategy is cumulative past returns: $r_{t-N,t} = \sum_{n=0}^{N} r_{t-n}$.[6] The reversal signal is evaluated by predicting future

[6] Jegadeesh (1990) documents profits of about 2% per month over 1934-1987 using a *short-term* reversal strategy that, each month, sorts stocks into deciles on the basis of prior-month returns, and then buys stocks in the bottom decile (losers) and sells stocks in the top decile (winners). Jegadeesh and Titman (1995) investigate how stock prices respond to distinct sources of information, distinguishing between firm-specific factors and common factors. Bondt and Thaler (1985) document *long-run* reversal effect using monthly US stock return data for the period 1926-1982 by focusing on stocks that experienced either extreme capital

returns:

$$r_{t+1} = \delta r_{t-N,t} + \varepsilon_{t+1}$$

and testing whether δ is significantly negative.

Although this reversal signal may represent a meaningful predictor, it is not informative about excess volatility beyond discount rates, as the strategy does not adjust for risk. For instance, it could be the case that asset returns are high and then low during periods when aggregate market returns are similarly high and then low. To address this concern, one can use the abnormal asset return $r_t^{abnormal} = r_t - \beta r_t^{mkt}$ and repeat the same steps.[7] Specifically, one would use long-term lagged abnormal returns, $u_t = \sum_{n=0}^{N} r_{t-n}^{abnormal}$, to predict returns:

$$r_{t+1}^{abnormal} = \delta u_t + \varepsilon_{t+1}$$

and test whether δ is negative.

Starting with Fama and French (1992, 1993), the literature on multifactor models has documented various sources of risk beyond aggregate market fluctuations. To account for these risks, one can use the asset return $r_t^{abnormal} = r_t - \beta_{i,1} f_{1,t} - \ldots - \beta_{i,K} f_{K,t}$ where, e.g., $K = 1, 3$ and 5 in the case of CAPM, Fama–French three- and five-factor models, respectively. Then, one would test whether the signal u_t—which adjusts for the risks in a given model—predicts future returns with a negative sign.

Importantly, the implication of a true K-factor model is that

$$E_t\left[r_{t+1}^{abnormal}\right] = E_t\left[r_{t+1}\right] - E_t\left[\beta_{i,1} f_{1,t} - \ldots - \beta_{i,K} f_{K,t}\right] = 0 \tag{1}$$

essand This means that after removing exposure to risk factors, the abnormal asset return cannot be predicted. Note also that the expectation operator conditions on the agent's information set at time t and our predictive regression is using only past excess returns—

gains or losses over the past three to five years. Differently from these cross-sectional strategies, we evaluate time-series reversal signals adjusted for cross-sectional factor models. Da et al. (2024) document weekly market-wide reversals following high levels of retail attention, which generate temporary price pressure. Differently, we examine low-frequency reversal in anomaly portfolios rather than high-frequency market-wide ones.

[7] De Bondt and Thaler (1985) uses $r_t - r_{mkt}$, meaning there is no risk adjustment beyond the movements of the overall market and the adjustment is identical for all asset.

rather than some firm-generated accounting variable—which are arguably readily available to the investor.

There are three takeaways from this analysis. First, by relying on reversals, our approach uses a statistically significant and negative δ to detect violations of (1) and uncover mean-reverting price movements. Second, while a transitory component in prices is naturally associated with excess volatility in returns (Shiller, 1981; Nagel, 2024), it can also align with discount rate variation. Because our method focuses on *abnormal* return predictability, it reveals excess volatility *beyond* discount rates, in a spirit similar to Giglio and Kelly (2017). Third, the profitability of a timing strategy which buys when $u_t = \sum_{n=0}^{N} r_{t-n}^{abnormal}$ quantifies the economic value of the statistical findings.

In summary, given a portfolio return r_t and a candidate factor model, we construct a reversal signal that accounts for the model's implied risk and then assess its predictive performance. The next section formalizes our approach from a no-arbitrage perspective and links the return predictability test to temporary deviations in asset price levels.

As a preview, we present results for two standard portfolios, size (small) and book-to-market (value), with risk adjustment based on the Fama-French five-factor model. We find:

$$r_{Small,t+1}^{abnormal} = \underset{(3.64)}{-5.34} - \underset{(0.08)}{0.23}\ \hat{u}_{i,t} + \epsilon_{i,t+1} \quad \text{with} \quad R^2 = 11.7\% \tag{Small}$$

$$r_{Value,t+1}^{abnormal} = \underset{(3.10)}{-6.45} - \underset{(0.11)}{0.26}\ \hat{u}_{i,t} + \epsilon_{i,t+1} \quad \text{with} \quad R^2 = 13.4\% \tag{Value}$$

where $\widehat{u}_{i,t}$ represents cumulative past returns, adjusted for time-varying risk exposures.[8] Figure 1 illustrates the model fit. In addition to in-sample predictions, we include out-of-sample forecasts by recursively estimating the predictive regression in (Small) and (Value) using an expanding window. We use an initial window size of 20 years and make the first out-of-sample forecast for December 1988. The figure highlights two main points. First, cumulative risk-adjusted past returns capture significant variation in future anomaly returns. Second, the close alignment between in-sample and out-of-sample forecasts suggests that this predictability holds out-of-sample as well.[9] These findings demonstrate the robustness and

[8] We use $\widehat{u}_{i,t}$ to indicate that the βs to construct the cumulative risk-adjusted returns are estimated.

[9] In Appendix F.1, we show the out-of-sample performance of predictive regressions compared to the historical mean.

simplicity of our approach with standard data. The remainder of the paper extends these results to a broader set of anomaly portfolios and factor models, demonstrating that the time-series reversal pattern we uncover is ubiquitous and not subsumed by existing predictors.

2.2 The Asset Pricing Framework

Let R^e_{t+1} be the vector collecting the return on asset i in excess of the risk-free rate, R^{ei}_{t+1}. The conditional MVE (CMVE) portfolio is given by[10]

$$R^{mv}_{t+1} = \left(k_t^{-1} V_t\left(R^e_{t+1}\right)^{-1} E_t\left[R^e_{t+1}\right]\right)^{\intercal} R^e_{t+1}, \tag{2}$$

where $V_t\left(R^e_{t+1}\right)$ and $E_t\left[R^e_{t+1}\right]$ are the conditional first and second moments of excess returns, and k_t is a time-varying scalar, known at time t, governing the leverage of the portfolio.

The no-arbitrage condition

$$E_t\left[R^e_{t+1}\right] = -\frac{Cov_t\left(M_{t+1}, R^e_{t+1}\right)}{E_t\left[M_{t+1}\right]}, \tag{3}$$

implies the conditional beta-pricing representation:[11,12]

$$E_t\left[R^e_{t+1}\right] = \beta_{i,t} E_t\left[R^{mv}_{t+1}\right] \tag{4}$$

For any return i included in the portfolio, the validity of Equation (4) requires that in a time series regression of the form:

$$R^{ei}_{t+1} = \beta_{i,t} R^{mv}_{t+1} + \varepsilon_{i,t+1} \tag{5}$$

the error terms should be unpredictable, i.e. $E_t\left[\varepsilon_{i,t+1}\right] = 0$ (see, e.g., Ferson and Harvey,

[10] See Hansen and Richard (1987); Ferson and Siegel (2001); Cochrane (2005); Chernov, Lochstoer, and Lundeby (2021). For completeness, we show the derivations of the CMVE portfolio in Appendix A.1.

[11] Consider the linear SDF $M_t = 1 - b_t\left(R^{mv}_{t+1} - E_t\left[R^{mv}_{t+1}\right]\right)$. Applying the no-arbitrage condition (3) to the MVE portfolio we have: $E_t\left[M_{t+1}\right] = b_t \frac{Var_t\left[R^{mv}_{t+1}\right]}{E_t\left[R^{mv}_{t+1}\right]}$. By substituting this expression and $Cov_t\left(M_{t+1}, R^e_{t+1}\right) = -b_t Cov_t\left(R^{mv}_{t+1}, R^e_{t+1}\right)$ into (3), one obtains the desired expression.

[12] Ferson and Siegel (2001); Chernov, Lochstoer, and Lundeby (2021), e.g., show how to construct the unconditional MVE (UMVE) portfolio, a timed version of (2) which implies the unconditional beta pricing relationship $E\left[R^e_{t+1}\right] = \beta_i E\left[R^{mv}_{t+1}\right]$. In contrast, our analysis focuses on the conditional beta pricing relationship, which is better suited for studying anomaly return predictability.

1991, 1999; Ferson and Korajczyk, 1995). Otherwise, one would buy (sell) the hedged portfolio $R^{ei}_{t+1} - \beta_{i,t} R^{mv}_{t+1}$ when the error is expected to be positive (negative), making a risk-adjusted profit and violating the fact that the SDF prices conditionally the given asset.

To test whether asset prices exhibit excess volatility even after accounting for risk, we propose examining the coefficient δ_i in the following model-implied regression specification

$$r^e_{i,t+1} + \frac{\sigma^2_{i,t}}{2} = \beta_{i,t} r^{mv}_{t+1} + \delta_i u_{i,t} + \varepsilon_{i,t+1} \ . \tag{6}$$

Here, $r^e_{i,t+1} = r_{i,t+1} - r_{f,t+1}$ represents the log excess return, and $\sigma^2_{i,t} \equiv \text{Var}_t r_{i,t+1}$ is the conditional variance of the risky asset return. The use of log returns, as we show below, facilitates the interpretation of our predictor in terms of price deviations.

Since log-linearizing the Euler condition (3) leads to:[13]

$$E_t r^e_{i,t+1} + \frac{\sigma^2_{i,t}}{2} = \beta_{i,t} E_t r^{mv}_{t+1} \tag{7}$$

it becomes clear that if the portfolio is CMVE, then $\delta_i = 0$. In words, risk-adjusted returns $r^e_{i,t+1} - \beta_{i,t} r^{mv}_{t+1}$ should be unpredictable and a nonzero $\delta_i = 0$ would indicate excess volatility beyond what is captured by risk adjustment.

There are, of course, many possible choices for $u_{i,t}$. We construct a predictor that is endogenous to the model, meaning it depends solely on the candidate CMVE, as follows:[14]

$$u_{i,t} = u_{i,t-1} + \underbrace{\left(r^e_{i,t} - \beta_{i,t-1} r^{mv}_t\right)}_{\varepsilon_{i,t}} \tag{8}$$

i.e., our predictor is the cumulative sum of risk-adjusted returns. To interpret $u_{i,t}$, it is convenient to define the log price of asset i as the cumulative log return: $\ln P_{i,t+1} = \ln P_{i,t} + r_{i,t+1}$. Similarly, we have $\ln P_{mv,t+1} = \ln P_{mv,t} + r_{mv,t+1}$ for the CMVE portfolio, and $\ln P_{rf,t+1} = \ln P_{rf,t} + r_{f,t+1}$ for the risk-free asset. Note now that if $\beta_{i,t} \simeq \beta_{i,t-1}$, then

$$u_{i,t} = \ln P_{i,t} - \ln P_{rf,t} - \beta_{i,t} \ln P_{mv,t} \ , \tag{9}$$

[13] Expression (7) holds exactly if the SDF and the asset i returns have a joint conditional lognormal distribution.

[14] See also Bondt and Thaler (1985, p. 795): "If stock prices systematically overshoot, then their reversal should be predictable from past return data alone, with no use of any accounting data."

so that $u_{i,t}$ captures deviations of test asset prices from the price warranted by the CMVE portfolio.[15] The intuition behind our proposed predictor is that persistent mispricing will manifest in the price level (Shiller, 1981). Equation (9) suggests measuring this mispricing by comparing the portfolio's price level to the value implied by the mean-variance portfolio, $\beta_{i,t} \ln P_{mv,t}$. As we will discuss in Section 2.2.1, this approach isolates the transitory component of the price.

The interpretation of $u_{i,t}$ as price deviations rest on the assumption $\beta_{i,t} \simeq \beta_{i,t-1}$, i.e. the portfolios' betas vary slowly and smoothly over time. This assumption is supported by several economic models. E.g., Gomes, Kogan, and Zhang (2003) suggest that betas are a function of productivity shocks, which are often calibrated with an autocorrelation of 0.95 at the quarterly horizon. This translates into a monthly autocorrelation of conditional betas above 0.98. Similarly, in Santos and Veronesi (2006), stock betas change as the ratio of labor income to total consumption changes, which is also a highly persistent variable. Also, many previous empirical studies (see, e.g., Jagannathan and Wang, 1996; Lettau and Ludvigson, 2001b; Petkova and Zhang, 2005; Lewellen and Nagel, 2006; Ang and Chen, 2007; Pelger, 2020; Lopez-Lira and Roussanov, 2022) find that conditional betas are stable within short time window.[16] Our analysis accommodates time-varying exposures but, importantly, our results do not rely on betas evolving slowly over time. We will demonstrate that our findings are robust across various methods of measuring time-varying exposures, whether these methods allow for slower or faster changes in betas.

[15] Practically, by cumulating log excess returns on asset i we abtract from any source of long-run nominal comovement between the asset prices and the MVE prices. To see this, consider for ease of exposition the CAPM model (the market is always included in the factor models studied in this paper) and constant betas. Under the null, we have: $(r_{i,t+1} - r_{f,t+1}) = \beta_i (r_{m,t+1} - r_{f,t+1}) + \varepsilon_{i,t+1}$.
Compounding the left- and right-hand side yields:

$$\begin{aligned} \ln P_{i,t+1} - \ln P_{rf,t+1} &= \beta_i (\ln P_{m,t+1} - \ln P_{rf,t+1}) + u_{i,t+1}, \\ \text{or, equivalently,} & \\ \ln P_{i,t+1} &= \beta_i \ln P_{m,t+1} + (1-\beta_i) \ln P_{rf,t+1} + u_{i,t+1}. \end{aligned}$$

The term $(1-\beta_i) \ln P_{rf,t+1}$ effectively removes inflation-related trends that are common to the market factor and the asset prices.

[16] To account for the time-varying nature of factor betas in (period-by-period) neural networks, Fan, Ke, Liao, and Neuhierl (2023) propose local-PCA which also rely on conditional mispricing and risk premia varying slowly over time. Specifically, they use a 60-months window for estimation which they slide forward by one month, as we do.

Note that testing for $\delta_i = 0$ in equation (6) is equivalent to the null:

$$u_{i,t} = u_{i,t-1} + \varepsilon_{i,t} \tag{H_0}$$

Indeed, when the factor model is CMVE, the error term $\varepsilon_{i,t}$ is unpredictable, and $u_{i,t}$ is a martingale ($E_t[u_{i,t+1} - u_{i,t}] = E_t[\varepsilon_{i,t+1}] = 0$). In this case, the deviation of an asset's price $\ln P_{i,t}$ from the price implied by the mean-variance portfolio (c.f., equation (9)) is permanent and has no predictive power for risk-adjusted excess returns.

The alternative hypothesis, which we entertain in this paper, is that the price deviations are persistent but mean reverting:

$$u_{i,t} = \rho_i u_{i,t-1} + \varepsilon_{i,t} \tag{H_1}$$

which implies $\delta_i = \rho_i - 1 < 0$ in Eq. (6).[17,18] In words, if asset prices are above the target value implied by the mean-variance portfolio, and if these price deviations are mean-reverting (i.e. $\rho_i < 1$), then future expected returns are lower (higher) on a risk-adjusted basis (i.e. after controlling for $\beta_{i,t} r^{mv}_{t+1}$).

Finally, note that under the null (H_0), $u_{i,t}$ is a martingale, i.e. $\rho_i = 1$ and $\delta_i = 0$ in Eq. (6). Thus one can view (H_1) as the unrestricted model, and (H_0) as the restriced model.

2.2.1 Statistical Interpretation of Price deviations: a Data Generating Process

In this section we show that assessing the predictability of risk-adjusted asset returns essentially reduces to identifying the transitory component in prices.

In their seminal work, Fama and French (1988) propose a model for the log of a stock's price, $\ln P_{i,t}$, that consists of two components: a permanent component $q_{i,t}$, modeled as a

[17] The result obtains by first differencing $u_{i,t} = (\ln P_{i,t} - \ln P_{rf,t}) - \beta_{i,t} \ln P_{mv,t}$ (see Eq. (9)), using the autoregressive dynamics for $u_{i,t}$ under (H_1) and our definition of log prices as cumulative log returns, and using the assumption that betas are changing slowly over time, i.e. $\beta_{i,t+1} \simeq \beta_{i,t}$ which implies $\beta_{i,t+1} \ln P_{mv,t+1} - \beta_{i,t} \ln P_{mv,t} \simeq \beta_{i,t} r^{mv}_{t+1}$.

[18] Da, Liu, and Schaumburg (2014) and He and Zhou (2023) sort stocks into portfolios based on $\varepsilon_{i,t}$, i.e. on the difference between raw return and expected returns from a given factor model. Da, Liu, and Schaumburg (2014) also purges the realized return of fundamental cash flow news. Differently, we look at cumulative risk-adjusted returns $u_{i,t}$ and, in the next section, derive a relation between the sign the predictive coefficient δ_i in (6) and the persistence of the transitory componet of prices.

random walk with drift, and a temporary component $u_{i,t}$, modeled as a stationary AR(1) process:

$$\begin{aligned} \ln P_{i,t} &= q_{i,t} + u_{i,t} \\ q_{i,t} &= q_{i,t-1} + \alpha_i + \eta_{i,t} \\ u_{i,t} &= \rho_i u_{i,t-1} + v_{i,t} \end{aligned} \tag{10}$$

where $\eta_{i,t}$ and $v_{i,t}$ are independent processes with zero mean and constant variance and $|\rho_i| < 1$. Fama and French (1988) demonstrate that the slow mean-reversion of the temporary component induces return predictability. This becomes evident when the price equation (10) is differenced:

$$r_{i,t} = \alpha_i + (1 - \rho_i)u_{i,t-1} + v_{i,t} \tag{11}$$

From equation (9), we infer that, for the Fama and French (1988) framework to align with our model of asset expected returns, the permanent component in the log of stock prices must take the form $q_{i,t} = \beta_i \ln P_{mv,t}$. This implies a common permanent component across all assets, with asset-specific loadings β_i. Unlike the original Fama and French (1988) model, where each asset has a distinct permanent component, our approach imposes this restriction to link asset *prices* to a risk-adjusted model of *returns*.

Thus, evidence of *return* predictability based on deviations of asset prices from their common permanent trend, captured by the mean-variance portfolio, is equivalent to identifying a mean-reverting temporary component in asset *prices*.

2.3 Construction of Price Deviations

A large literature (e.g., Goyal and Welch, 2007, Rapach and Zhou, 2013, Martin and Nagel, 2020, Boudoukh, Israel, and Richardson, 2021) emphasizes that out-of-sample tests provide the most rigorous and relevant evidence on stock return predictability. Therefore, to construct our predictor $u_{i,t}$ and to test the null $\delta_i = 0$ in (6), we adopt an out-of-sample perspective. The out-of-sample nature of our exercise is crucial, as our goal is to detect ex ante mispricing. Indeed, we will confirm that the price deviations captured by $u_{i,t}$ can be exploited in real time to predict asset returns.

In our empirical analysis, we also employ conditional betas, as assuming constant betas is not innocuous, as shown by Hansen and Richard (1987).[19] For example, with constant betas, price deviations could simply be a byproduct of time-varying loadings. To this end, we follow Lewellen and Nagel (2006) and we directly estimate conditional betas using short-window regressions. We adopt this approach because it gets around the problem of instrumenting time-varying factor loadings with the "right" state variables (e.g., Shanken, 1990; Jagannathan and Wang, 1996; Lettau and Ludvigson, 2001b). That is, we estimate the following regression separately every month using daily returns:[20]

$$r^e_{d(\tau)} = c_{i,t} + \beta_{i,t} r^{mv}_{d(t)} + \varepsilon_{d(t)}, \ \tau = t-2, t-1, t. \tag{12}$$

where $d(\tau)$ denotes the dth day observation in month τ, and where the constant $c_{i,t}$ accommodates unconditional differences in mean returns and, crucially, prevents the conflation of pure time-series anomaly predictability, which is the focus of this paper, with the sorting of assets based on mean returns (see Huang et al. (2020, Section 5.2) for a discussion).[21] Alternatively, one can demean returns, as in, e.g., Kim, Korajczyk, and Neuhierl (2020).

Then, we construct the risk-adjusted return at time $t+1$ as:

$$\widehat{\varepsilon}_{i,t+1} = r^e_{i,t+1} - \widehat{c}_{i,t} - \widehat{\beta}_{i,t} r^{mv}_{t+1} \tag{13}$$

where the beta and the constant are obtained from the short-window regression (using information up to time t only). We repeat this same steps at time $t+2$ and construct $\widehat{\varepsilon}_{i,t+2}$ based on betas (and constant) from a short-window regression estimated using the daily

[19] Kelly, Moskowitz, and Pruitt (2021) explain momentum as compensation for time-varying covariance risk with five IPCA factors. Gormsen and Jensen (2022) show that the alpha of major equity risk factors can partly be explained by time-varying market betas.

[20] To correct for nonsynchronous price movements, we follow Dimson (1979), and include both current and lagged mean-variance returns in the regressions, estimating beta as the sum of the slopes on all lags. Although we chose a quarterly interval to estimate betas, our results remain robust when using alternative time intervals. Additionally, our findings hold under different methods for estimating conditional betas. See discussion in Section 3.3.1.

[21] Also, the constant captures the Jensen's effect arising from our use of log expected returns.

observations in months $t-1, t, t+1$. Our predictor is given by:

$$\widehat{u}_{i,t} = \sum_{\tau=0}^{t} \widehat{\varepsilon}_{i,\tau} \tag{14}$$

and, importantly, it is obtainable in real time. We then run the predictive regression

$$r^e_{i,t+1} - \widehat{c}_{i,t} - \widehat{\beta}_{i,t} r^{mv}_{t+1} = \delta_i \widehat{u}_{i,t} + \epsilon_{i,t+1} \ . \tag{15}$$

and test the null hypothesis $\delta_i = 0$ in Section 3.2. A rejection of the null, and in particular a negative δ_i, suggests that underpricing ($u_{i,t} < 0$) is followed by positive returns. We exploit this insight, and the fact that $u_{i,t}$ can be obtained in real time, to develop a trading strategy based on mispricing in Section 3.3. As we will see, the price deviations take time to be reabsorbed, which implies that our trading strategy does not require high-frequency rebalancing, reducing possible concerns about trading costs.

The following steps summarize our approach to test for the presence of excess volatility beyond what discount rate changes alone can explain:

1. Start from a chosen factor model.
2. Construct the CMVE portfolio R^{mv}_{t+1} given in equation (2).
3. Using a short-window regression, estimate equation (12) to then construct real-time risk-adjusted returns $\widehat{\varepsilon}_{i,t}$ and price deviations $\widehat{u}_t$.
4. Run the predictive regression (15). A negative $\delta < 0$ indicates the presence of a transitory price component that induces a reversal pattern in risk-adjusted returns.

Finally, if $\delta < 0$, we describe how to engage in anomaly timing in Section 3.3.

3 Main Results

3.1 Data

Our analysis focuses on characteristics-based factors. To pit models in a fair manner, we work with the MVE portfolio implied by the characteristics-based factors (this approach is

also adopted by, e.g., Chernov, Lochstoer, and Lundeby, 2021 and Baba-Yara, Boyer, and Davis, 2022).[22] Specifically, we implement the MVE portfolio using the following factor model representation:

$$R^{mv}_{t+1} = b_t^{\intercal} C_t R^e_{t+1} = b_t^{\intercal} \mathbf{f}_{t+1} \ , \tag{16}$$

where C_t is a $K \times N_t$ matrix of stock-level characteristics which define a set of K factors, $\mathbf{f}_{t+1} = C_t R^e_{t+1}$; and b_t is a $K \times 1$ timing vector that optimally combines these factors over time to get to the minimum variance portfolio (see, e.g., Haddad, Kozak, and Santosh, 2020; Moreira and Muir, 2017). Theoretically, the variation in the minimum variance portfolio weights must be driven by factor and volatility timing: $b_t \propto V_t \left(\mathbf{f}_{t+1}\right)^{-1} E_t \left[\mathbf{f}_{t+1}\right]$. We use the Fama and French (2015, FF5, henceforth) as factors, i.e. $\mathbf{f}'_t = [MKT_t \ SIZE_t \ HML_t \ RMW_t \ CMW_t]$ in equation (16), and entertain a version of FF5 with either factor return (factor-timing, henceforth) or volatility timing (vol-timing, henceforth). Mindful that standard factors are contaminated with unpriced components,[23] we also employ the hedging approach of Daniel, Mota, Rottke, and Santos (2020, DMRS) that aims at removing unpriced risks from the original factors. We call the residualized (with respect to the hedge portfolio returns) FF5 factors, FF5-DMRS.[24] This gives a total of four candidate SDFs. Additionally, in Appendix D, we replicate our main analysis using the market as the sole factor (i.e., CAPM), and by employing the Hou, Xue, and Zhang (2015) q-factors, as well as its volatility-timed version, to construct three additional candidate MVE portfolios.

Given our factors (e.g., the FF5), we estimate b such that the single-horizon monthly returns to the factors themselves are priced without error. For the volatility timing version, we follow Moreira and Muir (2017) and use $b_{i,t} = b_i V_t^{-1} \left(f_{i,t+1}\right)$ which is computed using squared realized daily factor returns. For the factor timing version, we follow Haddad, Kozak, and Santosh (2020) and use $b_{i,t} = b_i E_t \left(f_{i,t+1}\right)$ where the out-of-sample expectations for the factors are constructed using each factor's book-to-market ratio. In all cases, we estimate the constant of proportionality b_i for each factor i by matching the in-sample average returns to the timed factors in the model at hand, analogous to how we estimated the vector b in the baseline FF5 models.

[22] See Bessembinder, Burt, and Hrdlicka (2022) for a discussion on the time variation in the number of significant factors and its relation to time-varying economic complexity.

[23] For example, Gerakos and Linnainmaa (2017) find that the HML value factor is contaminated with unpriced components.

[24] We are grateful to Simon Rottke for sharing the up-to-date hedged FF5 factors.

As test assets, we use the top and bottom deciles from a large cross-section of anomaly portfolios based on single-sorts of 45 different characteristics, for a total of 90 portfolios. These test assets, or a subset of those, have been used by Kozak, Nagel, and Santosh (2020), Kelly, Kozak, and Giglio (2020), Haddad, Kozak, and Santosh (2020), and Lettau and Pelger (2020), among others.[25] In Appendix E, we provide robustness using an alternative, even larger, cross-section of portfolios from Chen and Zimmermann (2021).

Our sample period spans 1967 to 2019. For most of our analysis, we use monthly observations, but we focus on 1-year holding-period returns. A long holding period allows returns at time $t+1$ to react to asset price deviations from the mean-variance portfolio prices at time t. Unless stated otherwise, r_{t+1} will denote the one-year-ahead log excess returns (i.e., $r_{t+\frac{1}{12},t+1}$, where we omit the first subscript). This choice is also in line with recent empirical studies on time-variation in anomaly returns (e.g., Lochstoer and Tetlock, 2020) and on the dynamics of equity portfolios (e.g., Kelly, Kozak, and Giglio, 2020). Interestingly, in our sample (see discussion of Figure 2) there is statistical evidence in favor of return predictability for one- up to twenty month ahead, i.e. $\hat{u}_{i,t}$ manifests forecasting ability for $r_{t+h/12}$ with $h = 1, \ldots, 20$.

3.2 Price Deviations and Excess Volatility beyond Discount Rates

To test for excess volatility beyond what a specific factor model would predict, we run the following predictive regression:

$$\widetilde{r}_{i,t+1} = a + \delta \widehat{u}_{i,t} + \epsilon_{i,t+1} \tag{17}$$

where $\widetilde{r}_{i,t+1} = r^e_{i,t+1} - \widehat{c}_{i,t} - \widehat{\beta}_{i,t} r^{mv}_{t+1}$ is the log excess return of test asset i at time $t+1$ net of the exposure to the log return on the MVE portfolio r^{mv}_{t+1}; and $\widehat{u}_{i,t}$ measures the deviations

[25] We thank Serhiy Kozak for making his data available at https://sites.google.com/site/serhiykozak/data?authuser=0. Appendix Table B.1 lists the categories and the portfolios included in each category. Notably, these test assets include the within-industry short-term reversal portfolio of Da, Liu, and Schaumburg (2014) and Hameed and Mian (2015), which exploits cross-sectional variations in stock price (over)reactions to *firm-specific information*. In contrast to those authors, we find that this reversal portfolio also overreacts in the time-series to systematic factors, suggesting that portfolio prices respond sluggishly to *common factors*. Thus, our findings contribute to a deeper understanding of the distinct causes underlying time-series and cross-sectional contrarian strategies (Jegadeesh and Titman, 1995).

of asset i prices from the mean-variance portfolio ones. The null is $H_0 : \delta = 0$ against $H_1 : \delta < 0$.[26,27]

We begin by describing the properties of the price deviations $u_{i,t}$, i.e. our predictor in (17). In particular, Table 1 shows the half-life (Panel A) and volatility (Panel B) of $u_{i,t}$ across the 90 test assets. The price deviations are persistent but mean-reverting, with an average half-life between 2.5 and 3.5 years. Notably, the hedged version of the FF5 model shows the shortest half-life, suggesting greater effectiveness in reducing the persistence of price deviations. Additionally, all models exhibit economically significant price deviations, with an average volatility ranging from 26% to 30%. Panel C reveals positive correlations between price deviations across different SDFs, with the strongest correlations observed between the FF5 SDF and its volatility-timed version, as well as between the factor-timing and hedged FF5 models.

Table 2, column (1), shows the results from the pooled regression (17). Each panel refers to a different candidate SDFs, namely the FF5 model, its factor timed and volatility managed versions, and the FF5 residualized with respect to the DMRS hedge portfolio. Independently from the candidate SDF, we find a negative and statistically significant coefficient on the price deviations. The coefficient is economically large: for example, for the FF5-DMRS we find that a one standard deviation increase of (log) portfolio prices from the model-implied SDF value, implies an expected return that is lower by 7.3% over the next year (on average, across portfolios).[28,29] Also, note that the R^2 associated with the predictability induced by

[26] Note that the factor models considered perform well in pricing *unconditionally* the cross-section of the test assets. For example, the MVE portfolio implied by the FF5 factor model and its hedged version (FF5-DMRS) reduces the mean absolute pricing errors on the 90 test assets from 5.21% per year in a risk-neutral benchmark economy to 2.77% and 2.64%, respectively, per year.

[27] One can view (17) as a test for the conditional validity of a given SDF. Under this interpretation, our test and the one proposed by Chernov, Lochstoer, and Lundeby (2021) are complementary. To see this, consider i.i.d. factor returns. In this case, the SDF prices the factors both conditionally and unconditionally and would pass the novel test of Chernov, Lochstoer, and Lundeby (2021) (which is solely based on factor dynamics). However, we show in Appendix A.2 that this SDF could still admit test asset return predictability ($\delta < 0$), i.e. the SDF does not correctly price excess returns on test assets conditionally.

[28] From Table 1, we know that the average standard deviation is about 28% for FF5-DMRS. Multiplying this value by the coefficient in Panel D of Table 2 we have: $-0.26 \times 0.28 = -7.3\%$. Equivalently, a pooled regression on standardized price deviations from FF5-DMRS yields a coefficient of -4.65%.

[29] A potential concern arises when factors exhibit positive autocorrelation (i.e., factor momentum), while returns themselves are uncorrelated. In that case, the constructed residual $\widetilde{r}_{i,t+1} = r^e_{i,t+1} - \widehat{c}_{i,t} - \widehat{\beta}_{i,t} r^{mv}_{t+1}$ might display reversals simply because it removes the (potentially autocorrelated) mean-variance portfolio

the price-deviations are about 10%, or larger, and thus comparable to the R^2 found in the aggregate market return predictability literature(e.g., Cochrane, 2008, 2011).

Figures 2 reports the asset specific $\widehat{\delta}_i$, along with its standard error, obtained from estimating equation (17) for each top decile portfolio.[30] The figure shows that $\widehat{\delta}_i$ is negative and significantly different from zero for all the portfolios and all the candidate SDFs considered. Hence, the evidence points to an ubiquitous rejection of the null in favor of price deviations that are persistent but mean reverting ($0 < \rho_i < 1$).

Since our regressor is a cumulative difference of past returns (c.f. equations (13)-(14)), which can be interpreted as a lagged price difference, the regression of cumulative one-year ahead returns on log prices is likely subject to small-sample bias (Stambaugh, 1999). Figure 4 presents results from a bootstrap simulation, showing that the bias is smaller than the estimated coefficient (-0.056 vs. -0.196). Appendix A.3 provides further details on the bootstrap exercise and a discussion of the sources of this bias. We also stress that any type of bias will be resolved in our out-of-sample tests.

Thus far, we have used annual returns in equation (17). Next, we discuss the predictive ability of price deviations over alternative horizons. Figure 2 presents the estimates of δ from a pooled regression that predicts monthly returns h-period ahead (i.e., returns are not compounded), with h ranging from 1 to 60. To facilitate comparison with the annual estimates reported in Table 2, we multiply these coefficients by twelve. Across all models, there is statistical evidence in favor of return predictability for each of the future twenty months. Moreover, the magnitude of the coefficient is negative, similar across models, and decaying to zero only slowly as we increase the forecasting horizon. Comparing different SDFs, we observe that the price deviations from the FF5 and FF5-DMRS predict returns quite persistently, for up to forty months. On the other hand, the adoption of factor timing and, to a lesser extent, volatility management make the price deviations more transient as confirmed by a faster decay pattern in the coefficients, which become insignificant between 1.5 and 3 years. In sum, a value of the test asset above the target value implied by the

from i.i.d. returns. To address this issue, Table F.2 repeats our analysis with $r_{i,t+1}$ on the left-hand-side of the panel regression while controlling for $\widehat{\beta}_{i,t} r^{mv}_{t+1}$ on the right-hand side. The negative and significant loading on $u_{i,t}$ persists, indicating that any "mechanical" factor momentum effect is not driving our results. We thank Zhi Da for this observation and test.

[30] Appendix Figure F.2 displays the analogous analysis for each bottom decile portfolio.

mean-portfolio signals future negative returns over the next two to three years (in line with the average half-life reported in Table 1), at which point the price deviation is washed away. Interestingly, the long spell of time it takes for returns to revert toward their target value is in line with the evidence in Daniel, Klos, and Rottke (2022) who show that the beliefs of optimistic agents (who overreact to positive information) decay towards rational beliefs over a roughly 5-year period.

Recall that our price deviations signal is obtained in real time, using only information up-to-time t. Thus, we now evaluate its predictive ability for each portfolio using the out-of-sample (OOS) R^2 metric proposed by Campbell and Thompson (2008). Table 3 shows the results for each test asset.[31] Each panel refers again to a given SDF model. On average (across models), we document positive OOS R^2 for more than 70% of anomaly portfolios. Most importantly, we find significant out-of-sample R^2 for relevant characteristics-sorted portfolios such as value, duration (Weber, 2018; Gormsen and Lazarus, 2020), and investment (Cooper, Gulen, and Schill, 2008). Although the out-of-sample R^2 from a forecasting model is a commonly used metric in the return predictability literature (see, e.g., Rapach and Zhou, 2022), Kelly, Malamud, and Zhou (2023) pointed out that it is an incomplete measure of the model economic value. Thus, in Section 3.3 we implement a portfolio exercise to quantify the economic profits of a market timer that exploits the price deviations implied by a given SDF model. Before doing so, in the next subsection we make sure that the predictive power of the price deviations is not subsumed by well known predictors.

3.2.1 The information content of price deviations

In this section we assess the forecasting ability of price deviations $u_{i,t}$ relative to other portfolio return predictors commonly used in the literature. For this purpose, we estimate the following pooled regression:

$$\widetilde{r}_{i,t+1} = a + \delta \widehat{u}_{i,t} + \gamma \mathrm{X}_{i,t} + \epsilon_{i,t} \tag{18}$$

where $X_{i,t}$ is an alternative candidate predictor for the anomaly portfolio i. Columns (2) to (6) of Table 2 show the estimates when we control for long-term reversal in the portfolio,

[31] For this exercise, we also estimate the mean-variance portfolio weights out-of-sample.

past one-year returns, book-to-market ratio, the aggregate sentiment as measured by the Baker and Wurgler (2006) investor sentiment index, and the short-term pricing error. Each panel refers to a specific SDF. We focus on results from the pooled regression only for ease of exposition, but all our conclusions hold when we run asset-specific individual regressions.

In column (2) we consider the reversal signal based on past 5-year returns (skipping the most recent year) as an additional anomaly portfolio predictor. After all, our price deviations are obtained from the cumulative past returns relative to the cumulative MVE ones. We see that the series of past returns *relative to* the mean-variance portfolio remains statistically significant after controlling for the (absolute) 5-years past return series. Moreover, the loading $\hat{\delta}$ is always negative and of similar magnitude to the value reported in column (1). This result suggests that there is more information content in relative (to a given factor model) mispricing than in absolute mispricing as captured by the stand-alone past return series.

In column (3), we report results for regression (18) when we include the portfolio's performance over the prior year from month $t-12$ to $t-1$ along with the price deviations. Ehsani and Linnainmaa (2021) document that most factors are positively autocorrelated, and propose a factor that bets on the continuation in factor returns. Contrary to their work, our framework focuses on price deviations that forecast reversal, not continuation, of returns. It is then not surprising to see that our price deviations (capturing reversal) continue to be statistically significant after controlling for the portfolio momentum (capturing continuation).

Column (4) reports results for regression (18) when the control variable $X_{i,t}$ is the portfolio' book-to-market ratio. Indeed, valuation ratios are often used in return forecasting regressions (e.g., Cochrane, 2005; Campbell, 2017) as they represent a natural predictor according to the Campbell-Shiller (1988) log-linear present value model. Even after controlling for the book-to-market ratio, the coefficient on the price deviations is statistically significant, and negative: e.g., for the FF5 model, a one standard deviation increase of log portfolio prices from the model-implied MVE portfolio, implies a return that is lower by 4.6% over the next year (c.f., computation in footnote 28).

Shen, Yu, and Zhao (2017) document a negative predictive relation between the returns to portfolios sorted on macro-related risk factors and investor sentiment proxied by Baker and

Wurgler (2006) index. Related, Avramov, Chordia, Jostova, and Philipov (2019) show that mispricing occurs across financial distressed firms during periods of high market sentiment because in these times both retail and institutional investors are overly optimistic about the likelihood and consequences of financial distress. The sluggish investors' response to correct overpricing leads to a wide range of anomalies in the cross-section of stocks and bonds. Column (5) of Table 2 displays the results from a predictive regression that controls for sentiment. Once again, we find that the predictive content of the price deviations is not driven away by aggregate sentiment. This result continue to hold true when we use the improved aggregate sentiment of Huang, Jiang, Tu, and Zhou (2014).

Finally, Da, Liu, and Schaumburg (2014) and He and Zhou (2023) emphasize the predictive power of pricing errors, defined as the difference between short-term realized returns and expected returns from a factor model, denoted by $\widehat{\varepsilon}_{i,t+1}$ in (13). Our approach is complementary, focusing on the predictive ability of *cumulative* risk-adjusted returns for anomaly portfolios, i.e., price deviations, as shown in (14). Column (6) demonstrates that price deviations convey information that is distinct from the one-period difference between realized and expected returns. Our approach is rooted in long-term return differences between anomaly portfolios and factors, directly linking our predictive signal to price deviations. Meanwhile, the papers referenced above focus on short-term reversal patterns that may be detached from the price level, even if they are economically profitable. In contrast, we document *price correction* in assets whose long-term returns deviate from the SDF-implied long-term returns, indicating that portfolio prices anchor to factor prices over extended horizons. Furthermore, those studies examine stock-level pricing errors driven by *firm-specific* information and mispricing, exploiting them cross-sectionally. By comparison, our work emphasizes the delayed reaction of a given portfolio to *common, systematic* factors. Consequently, the reversal strategies in Da, Liu, and Schaumburg (2014) and He and Zhou (2023) differ fundamentally from ours in terms of their profit sources.

Overall, our evidence suggests that price deviations convey information about the time-series dynamics of risk-adjusted returns, $r^e_{i,t+1} - \beta_{i,t} r^{mv}_{t+1}$, for a wide range of portfolios. The predictive informative content of these price deviations is not subsumed by valuation ratios, momentum or reversal in individual factors, or aggregate sentiment.

3.2.2 How Large Is the Transitory Component of Anomaly Prices?

As discussed in Section 2.2.1, the evidence of return predictability based on price deviations aligns with the data-generating process described by Fama and French (1988). This process features a permanent component, represented by the mean-variance portfolio that determines the target price level, and a mean-reverting temporary component in asset prices. In this section, we quantify how large is this transitory compnent of stock prices.

Specifically, we are interested in computing:

$$V_{tran,i} = \text{Var}\left(E_t \sum_{j=0}^{\infty} \rho^j r_{i,t+1+j}\right),$$

which represents the variance of the transitory component of stock prices for anomaly i. To understand why, consider the Campbell-Shiller present value identity, rearranged to isolate p_t on the left-hand side:

$$p_t = \text{const.} + d_t + E_t \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} - E_t \sum_{j=0}^{\infty} \rho^j r_{t+1+j}.$$

If the price change in period $t+1$ arises solely from news about future expected returns, with $d_{t+1} = E_t d_{t+1}$ and no cash flow news, then

$$r_{t+1} - E_t r_{t+1} = -\Delta E_t \sum_{j=1}^{\infty} \rho^j r_{t+1+j}$$

In this case, the entire price movement due to expected return news is temporary. It is subsequently reversed by adjustments in expected returns in the opposite direction, as captured by the sum of future expected return terms on the right-hand side. This highlights why V_{tran} reflects the variance of the transitory component in prices.

Following Cochrane (2011), rather than reporting V_{tran}, we report $\sqrt{V_{tran}}$ scaled by the average return of the anomaly portfolio under consideration. This normalization allows us to assess the economic significance of transitory variations in prices relative to their anomalous return levels. In practice, we compute $\sqrt{V_{tran,i}}$ as the volatility of the fitted value from a

regression of h-period risk-adjusted returns for anomaly i on its price deviations.

Table 4 presents the results. At the 1-year horizon, the scaled transitory volatility, $\frac{\sqrt{V_{tran,i}}}{E[r_{i,t}]}$, averaged across anomaly portfolio is ranges from 0.98 for the Fama-French five-factor (FF5) model to 1.38 when the price deviations are obtained using factor timing. Over a 3-year horizon, this measure ranges from 0.91 for FF5 to 1.36 with factor timing. At the 5-year horizon, the scaled volatility declines slightly, averaging about 0.88 across factor models.

These findings indicate that the transitory component of anomaly prices induces substantial variation in anomaly returns, comparable to or even exceeding their unconditional levels, underscoring the economic relevance of these price fluctuations. Furthermore, since our anomaly returns are already adjusted for SDF-induced risk variation, this volatility exceeds what can be attributed to discount rate fluctuations.

3.3 Return Dynamics, Mispricing, and Trading Strategy

In Section 3.2 we provided evidence against the null that price deviations follow a random walk and that risk-adjusted returns are unpredictable. In this section we evaluate the economic magnitude of this predictability by studying the performance of a strategy that, according to our signal, goes long underpriced portfolios and short those that are overpriced.

We proceed as follows. First, recall that $u_{i,t}$ measures the deviation of the portfolio price i from the mean-variance target price, and $\widehat{u}_{i,t}$ is obtained in real-time using only information up to time t (c.f., Section 2.3).[32] Second, in our framework $\delta_i u_{i,t}$ is a proxy for anomaly expected returns. We estimate δ_i using an expanding window with a burn-in sample the first 20 years of observations (1967–1987). This provides us with an out-of-sample estimate of expected returns $\widehat{\delta}_{i,t}\widehat{u}_{i,t}$ for portfolio i. We go long portfolios with prices below the model-implied target ($u_{i,t} < 0$), and consequent positive expected returns given the ubiquitous negative δ_i (c.f., Section 3.2), and go short those with prices higher than what the mean-variance portfolio would suggest ($u_{i,t} > 0$) and negative expected returns.[33]

[32] The weights of the mean-variance portfolio are also estimated in an out-of-sample fashion.

[33] Estimation of equation (12) introduces measurement noise in the computation of $u_{i,t}$. As discussed in Burt and Hrdlicka (2021), this noise can lead to lower predictability. Thus, the reported performance of the

This timing strategy that increases exposure to assets after they have fallen and decreased exposure to stocks after they have risen in price emerges naturally, e.g., in a world where noise traders' misperception of returns follows a mean-reverting process (c.f. Section IV in De Long, Shleifer, Summers, and Waldmann, 1990).

We focus on a sample of anomalies based on single-sorts of 45 different characteristics (see Kozak, Nagel, and Santosh, 2020), for a total of 90 portfolios. Because our sample has a limited cross-section, we construct our zero-cost long-short portfolios using the entire cross-section of anomaly portfolios to mitigate the influence of outliers following, e.g, Asness, Moskowitz, and Pedersen (2013); Koijen et al. (2018). Specifically, for any anomaly $i = 1, \ldots, N$ at time t, we weight anomalies in proportion to their cross-sectional rank based on $\widehat{\delta}_{i,t}\widehat{u}_{i,t}$ minus the cross-sectional average rank:

$$w_{i,t} = c_t \left(\text{rank}\left(\widehat{\delta}_{i,t}\widehat{u}_{i,t}\right) - \sum_i \text{rank}\left(\widehat{\delta}_{i,t}\widehat{u}_{i,t}\right)/N \right)$$

where c_t is a scaling factor such that the overall portfolio is scaled to one dollar long and one dollar short. The return on the long-short portfolio is then the weighted sum of returns on the individual anomalies: $r_{t+1} = \sum_i w_{i,t} r_{i,t+1}$. We rebalance long-short portfolios once per year while we observe returns monthly.

Figure 3 shows the performance of the long and short sides of our mispricing strategy, along with the aggregate market returns. The top left panel refer to the results obtained when we compute price deviations relative to the FF5 mean-variance portfolio. The next two panels refer to the results for the factor and volatility timed version of the FF5. The bottom right panel refers to DMRS hedged version of FF5. As expected, the long leg which contains underpriced test assets outperforms the market, whereas the short leg with overpriced portfolios underperforms. A strategy that goes long underpriced test assets and short overpriced assets generates an annualized average excess return of 5.2% and 6.6% for the FF5 model and its version that hedges unpriced risks. The associated annualized Sharpe Ratios are 0.80 and 1, respectively. A version of the mean-variance portfolio that times factors' returns obtains even stronger performance with annualized average excess return of 6.8% and a Sharpe ratio of 1.05. The performance of a strategy based on deviations from a

anomaly timing strategy represents a lower bound.

volatility-timed mean-variance portfolio attains an average return of 4% and a Sharpe ratio of 0.73, instead.

This finding is particularly relevant to the debate on whether contrarian profits arise primarily from overreactions to firm-specific information or from common factors (Lo and MacKinlay, 1990; Jegadeesh and Titman, 1995). Jegadeesh and Titman (1995) show that contrarian strategies on size-sorted portfolios do not generate significant abnormal returns, indicating that most contrarian profits stem from stock prices overreacting to firm-specific news. By contrast, our contrarian strategy on a broad cross-section of characteristic-sorted portfolios is profitable, suggesting that stock prices also react slowly to common factors. A key difference is that we use a much longer window (i.e., the limit difference between compounded log returns) to measure the gap between factors and anomaly portfolios—whereas Jegadeesh and Titman (1995) rely on weekly lead-lag return relations. This extended horizon aligns with the idea that longer intervals are more effective in capturing systematic mispricings in price levels (Shiller, 1981).

Of course, it is important to ascertain that our mispricing zero-cost portfolio displays alphas relative to factor models, in particular those behavioral models aiming at capturing temporary, long- and short-horizon deviations of prices from fundamental values (Daniel, Hirshleifer, and Sun, 2020), as well as models where factors are constructed to capture aggregate mispricing (e.g., Stambaugh and Yuan, 2016; Bartram and Grinblatt, 2018). Table 5 shows the alpha of our zero-cost strategy based on model-implied price deviations. Several observations stand out. First, the average return of our strategy is not captured by standard characteristics-based or behavioral factors. In particular, our price deviations convey different information from that captured by the Daniel, Hirshleifer, and Sun (2020) behavioral factors and from the mispricing factor of Bartram and Grinblatt (2018): In correspondence of these two models (see rightmost two columns of Table 5), we observe that the constant remains large and statistically, for every panel (i.e. independently of the factors used to construct the mean-variance portfolio).

Second, we observe that, for three our of the four candidate SDFs (the exception being the volatility-managed FF5), the lowest alpha obtains in correspondence of the mispricing factor model of Stambaugh and Yuan (2016). Nevertheless, focusing on the FF5-DMRS in Panel (d), the alpha remains large and significant at 4.08% per year. Third, we find that

our strategy has a positive and (often) statistically significant loading (not reported) on the Mgmt and Perf factors of Stambaugh and Yuan (2016), on the Daniel, Hirshleifer, and Sun (2020) behavioral factor FIN which is designed to capture long-horizon mispricing, as well as on the mispricing factor of Bartram and Grinblatt (2018). This suggests that indeed our strategy captures under/over reaction of asset price levels and, to describe accurately such price dynamics, one needs additional mispricing factors outside those included in the candidate SDF model (which, in turn, dictates the target price level). Having said that, none of the mispricing factor proposed in the literature subsumes the performance of our strategy.

3.3.1 Robustness Checks

We provide several robustness checks for the evidence that anomaly returns can be predicted by the deviation of a portfolio price from the mean-variance target, and for the performance of our mispricing portfolio along three main dimensions.

First, we test the robustness of our results using alternative methods for estimating time-varying betas. In Appendix C, we repeat the analysis by estimating conditional betas either with a 12-month window of daily return (see, e.g., Lewellen and Nagel, 2006 and Welch, 2022) or using a 60-month rolling window of monthly returns, as in Fama and French (1997) and Ferson and Harvey (1999).[34] Tables C.1 and C.3 replicate the pooled predictive regressions from Table 2, consistently showing that δ_i is negative and significantly different from zero across all portfolios. Tables C.2 and C.4 further demonstrate that our strategy continues to generate statistically significant alphas when price deviations are computed using these conditional betas.

Second, we assess the impact of using alternative, well-established factor models to construct the mean-variance portfolio. Appendix Table D.1 presents results from pooled predictive regressions (17) and (18), using price deviations relative to the CAPM (Panel A), and the mean-variance portfolios implied by the Hou, Xue, and Zhang (2015, HXZ) q-factors

[34] Using a 60-month rolling window avoids overconditioning bias, as discussed by Boguth, Carlson, Fisher, and Simutin (2011). While the choice of window length is arbitrary, unreported results confirm robustness when using a shorter 24-month window, as in Hasler and Martineau (2023), a longer 120-month window, or an expanding window.

and their volatility-managed versions (Panels B and C). Across all factor models, we find a negative and significant loading on price deviations, even after controlling for well-known predictors. In Table D.2, we examine the performance of our strategy when price deviations are computed relative to these different SDFs. The CAPM-based strategy delivers an annualized return of 5.9% and a Sharpe ratio of 0.86, while the strategy that uses deviations of prices from the q-factor model yields an annualized return of 4.1% and a Sharpe ratio of 0.65. In contrast, using the q-factors results in an average return of 4.1% and a Sharpe ratio of 0.65. The table further shows that these alphas remain significant even after controlling for benchmark models incorporating behavioral or mispricing factors.

A potential concern is that our results may be driven by mismeasurement of the factor weights in the SDF, as DeMiguel, Garlappi, and Uppal (2009) highlight the impact of measurement error in the mean-variance weights on portfolio performance. To address this, we replicate our main results using a mean-variance portfolio constructed from the FF5 model with equal $1/N$ weights. Table F.1 reports that, even in this case, the loading on price deviations remains negative and statistically significant.

Finally, we explore the robustness of our results to different choices of test assets. So far, we have used a cross-section of 45 characteristics and the corresponding 90 portfolios from the long and short sides of these strategies. We first evaluate the FF5 factor model using as test assets the 25 Fama-French book-to-market and size-sorted portfolios, which are built using the same characteristics featured in the FF factor model.[35] Figure E.1 shows that price deviations predict anomaly portfolio returns negatively, providing a conditional rejection of the FF5 model. Table E.1 further demonstrates that a strategy exploiting price deviations in the 25 book-to-market and size-sorted portfolios generates a statistically significant alpha, with magnitudes ranging from 3.12% to 5.04% per year, even after controlling for benchmark models incorporating behavioral or mispricing factors. Lastly, we employ a larger cross-section of anomaly portfolios from Chen and Zimmermann (2021), which exceeds our benchmark set of 90 test assets. A rank-weighted investment strategy using the FF5 factor model with this expanded set generates an annualized return of 7.2% and a Sharpe ratio of 1.13, representing a more than 40% increase compared to the benchmark case. This improvement is likely due to the greater potential to identify mispriced portfolios

[35]The MVE portfolio implied by the FF5 factor model reduces the mean absolute pricing errors on these 25 test assets from 6.87% per year in a risk-neutral benchmark economy to 2.03% per year.

in a larger cross-section.Table E.2 reports results for our strategy with this broader set of test assets, showing significant abnormal returns from buying underpriced anomaly portfolios and selling overpriced ones. Importantly, the alpha remains statistically significant across all panels, even after controlling for the Stambaugh and Yuan (2016) and Bartram and Grinblatt (2018) mispricing factors and the Daniel, Hirshleifer, and Sun (2020) behavioral model. This evidence suggests that expanding the test asset universe enhances the economic gains for investors using price deviations to predict anomaly returns.

Overall, our analysis suggests that deviations of portfolio prices from their long-term mean-variance level contain valuable information for predicting anomaly returns out-of-sample. Our conclusions are robust to different factor models used to construct the SDF (e.g., HXZ or FF5, and their volatility-managed versions), non-parametric methods for estimating time-varying exposures, and varying test asset universes. The evidence strongly supports the view that mean-reverting price deviations ($0 < \rho_i < 1$) onvey predictive information about future anomaly returns.

4 Price Deviations and Predictability: Discussion

4.1 Factor structure in price deviations

A natural question is the extent to which price deviations vary across different factor models. Panel A of Table 7 reveals that the first principal component captures approximately 80% of the total variation, regardless of the factor model used for the SDF. Panel B further demonstrates that pooling price deviations from the four models (FF5, their volatility and factor-timed versions, and the FF5 without the unpriced component) results in the first principal component explaining about 79% of the variation, with the second and third components raising this to 90%. Overall, price deviations exhibit a strong factor structure.

This raises a further question: how has such a strong factor structure in *price* deviations gone unnoticed when developing factor models for *returns*? A common model for excess returns assumes that asset excess returns, $R_{n,t}, i = 1, \ldots, N$, consist of a systematic component driven by $K < N$ factors and an idiosyncratic component capturing asset-specific

risk:

$$R^e_{i,t} = F_t \Lambda_n^\intercal + \varepsilon_{i,t} \tag{19}$$

Principal Component Analysis (PCA) is often used to estimate the loadings Λ_n and the factors F_t.[36] One would expect that increasing the number of principal components would reduce the time-series dependence in the idiosyncratic component, making the hypothesis H_0 more likely to be satisfied. However, as most factor models focus on 3 to 5 factors, it seems that more than 5 principal components are needed to eliminate the predictability based on price deviations. We show this to be the case.

Figure 6 displays the coefficient on price deviations from pooled predictive regressions (17) and (18), using price deviations relative to a statistical factor model based on PCs extracted from test assets. The left panel shows a clear case: using 10 book-to-market sorted portfolios, whose expected returns are well described by a 3-PC model (similar to the Fama and French (1992) three-factor model) we see that at least 6 PCs are needed to eliminate predictability from price deviations. A similar pattern is observed with our benchmark set of 90 test assets, where more than 30 PCs are required to remove this predictability.

These findings are consistent with research highlighting strong factors in cross-sectional variation of expected returns and hidden factors driving the time-series behavior of future returns (Lettau and Pelger, 2020). However, an important point must be noted. The model in (19) is an unrestricted factor model. In our analysis, we have instead used a disciplined approach, constructing a mean-variance portfolio from a restricted combination of benchmark factors. Consistent with our main results in Section 3, even when using 40 PCs, but now combined into a mean-variance portfolio, we continue to find evidence of return predictability from price deviations.[37]

[36] PCA, grounded in Ross (1976) seminal Arbitrage Pricing Theory (APT), remains the most popular technique in finance to analyze latent factor models for returns with key empirical contributions dating back to Chamberlain and Rothschild (1983) and Connor and Korajczyk (1986, 1988). Recently, Kelly, Pruitt, and Su (2019) propose instrumental principal component analysis (IPCA) where the factor loadings are dynamic and can be instrumented with observable portfolio characteristics. Giglio and Xiu (2021) show that using ridge regression instead of PCA for reducing the dimensionality of the returns space yields similar results.

[37] This analysis aligns with the findings of Lewellen (2022), who argues that the benefits of including extra factors in a model may well outweigh the costs, even if the factors are redundant.

4.2 Economic drivers of the price deviations

The strong correlation between price deviations across different SDFs indicates that each model misses a common aspect of price dynamics. Figure 5 explores this common factor by plotting the time-series of the first principal component against the sentiment measure from Baker and Wurgler (2006). The positive and high correlation (40%) between price deviations and sentiment suggests that widespread optimism contributes to overpriced assets.

For optimism to lead to mispricing, it's essential that arbitrageurs do not correct price deviations immediately, or prices would adjust instantly to levels implied by the factor models. Limits to arbitrage cause gradual trading on price deviations, resulting in anomaly return predictability. Table 6 investigates how proxies for these limits to arbitrage relate to the out-of-sample predictive power of price deviations for anomaly returns.[38]

Our first proxy is idiosyncratic volatility (IVOL), which indicates periods when arbitrage capital is limited due to funding shortages (Pontiff, 2006), resulting in slow responsiveness of arbitrage capital to price deviations. We hypothesize that anomaly predictability from price deviations should be stronger during high IVOL periods. The first three columns confirm this: while our investment strategy based on price deviations performs positively and significantly in both high and low IVOL periods, it is more pronounced during high IVOL.[39]

In the remaining columns, we examine the effect of market frictions using the negative of the liquidity measure from Pástor and Stambaugh (2003) (labeled illiquidity) and the VIX as proxies for constraints on the risk-bearing capacity of arbitrageurs (e.g. Nagel, 2012). We find that the predictive power of price deviations is stronger when market frictions are higher and arbitrage activity is more constrained.

Overall, the evidence supports the idea that when limits to arbitrage are high, the perfor-

[38] Besides gradual trading due to limits to arbitrage, the degree of arbitrageur participation is another determinant of persistence in anomaly predictability. E.g., Abreu and Brunnermeier (2002) propose a coordinated arbitrage model in which arbitrageurs may be reluctant to trade on known mispricing because of holding costs and synchronicity risk.

[39] We follow Garcia, Mantilla-García, and Martellini (2014) and use the cross-sectional standard deviation of stock returns as a model-free measure for aggregate IVOL. Garcia, Mantilla-García, and Martellini (2014) find that the correlation between their measure and the model-based idiosyncratic volatility computed as in Ang, Hodrick, Xing, and Zhang (2006) is above 99%.

mance of a strategy based on price deviations improves. This suggests that gradual trading, due to market frictions, allows price deviations to predict anomaly returns over time.

5 Conclusion

In this paper, we provide novel evidence of excess volatility in anomaly portfolios beyond what can be explained by discount rate dynamics implied by standard SDF models. Using a mean-variance framework and long-horizon returns, we show that deviations of portfolio prices from their intrinsic value, as implied by the MVE portfolio, negatively predict future risk-adjusted portfolio returns. This result highlights the existence of a transitory component in anomaly prices—an essential feature for understanding excess volatility in asset prices.

Our findings challenge the no-arbitrage principle that risk-adjusted returns should be unpredictable, revealing instead that model rejection persists even when accounting for time-varying dynamics in risk premia and risk exposures. The negative loading of future returns on price deviations suggests that when asset prices deviate from the long-run level implied by the SDF, they revert over time, implying a correction process that aligns asset prices with their fundamental values. We interpret these price deviations as signals of under- or over-pricing relative to the MVE portfolio's implied prices, suggesting systematic mispricing driven by a transitory component.

In addition to providing statistical evidence of excess volatility, our analysis demonstrates that strategies based on price deviations lead to significant economic gains. A real-time zero-cost portfolio that buys underpriced anomalies and sells overpriced anomalies generates Sharpe ratios ranging from 0.7 to 1.2, highlighting the economic importance of these deviations. Moreover, we investigate the role of limits to arbitrage in shaping the effectiveness of price deviation-based strategies. We find that such strategies perform significantly better during periods of high volatility and low liquidity, consistent with the notion that arbitrageurs are constrained in their ability to trade on mispricing when market frictions are high. This highlights the importance of market frictions, such as noise trader risk and liquidity constraints, in delaying the correction of price deviations. Our results suggest that mispricing gradually dissipates as arbitrage activity becomes more constrained under these conditions.

In sum, our findings provide compelling evidence of excess volatility in asset prices beyond what is explained by standard SDF models, with important implications for both empirical asset pricing and investment strategies. On one hand, the pervasive reversal patterns in risk-adjusted portfolio returns highlight the need to incorporate time-series price dynamics into cross-sectional asset pricing models (see, e.g., Karolyi and Van Nieuwerburgh, 2020). On the other hand, as emphasized in Merton (1987)'s presidential address, the mean-reverting behavior of price deviations toward the level implied by the MVE portfolio suggests that factor models provide a meaningful long-term characterization of the financial system, reinforcing their relevance for long-horizon investors.

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Table 1: Price Deviations: Summary Statistics

This table reports descriptive statistics for price deviations computed using different heuristic mean-variance efficient portfolios. Price deviations $\hat{u}$ are computed as in equation (14). Test assets are 90 top and bottom deciles from the anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020); see Appendix Table B.1 for a description of the anomalies. To compute different heuristic mean-variance efficient portfolios, we employ Fama and French (2015, FF5), its factor-timing and volatility-managed versions, and its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. Panel A reports the half-life (in months) distribution of price deviations for each factor model. Panel B reports the standard deviation (in percentage) distribution of price deviations for each factor model. Panel C reports the average correlation across price deviations for each factor model. Monthly observations. The sample period is 1967 to 2019.

Panel A: Half-Life

Model	Mean	Min	Pctl(25)	Median	Pctl(75)	Max
FF5	38.4	13.5	31.2	36.1	45.9	62.7
FF5 with factor-timing	33.1	17.4	28.5	32.7	36.1	49.2
Volatility-managed FF5	42.7	15.1	34.3	41.7	49.2	69.0
FF5-DMRS	29.8	16.6	24.4	28.0	34.3	49.2

Panel B: Standard Deviation

Model	Mean	Min	Pctl(25)	Median	Pctl(75)	Max
FF5	26.6	14.9	22.1	26.9	30.5	41.0
FF5 with factor-timing	29.6	17.4	26.7	29.6	32.6	40.2
Volatility-managed FF5	30.1	16.7	26.2	30.3	34.0	44.6
FF5-DMRS	28.0	17.0	25.3	28.3	30.0	42.3

Panel C: Correlation Matrix

	FF5	factor-timing	vol-timing	FF5-DMRS
FF5	1	0.430	0.975	0.383
FF5 with factor-timing		1	0.432	0.698
Volatility-managed FF5			1	0.378
FF5-DMRS				1

Table 2: Predicting Anomaly Returns with Price Deviations

This table reports pooled estimates for δ_i from predictive regression (18). Test assets are 90 top and bottom deciles from the anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). Price deviations $\hat{u}$ are computed as in equation (14). We report results for price deviations computed using different heuristic mean-variance efficient portfolios. Panel A reports results for the Fama and French (2015, FF5) factor model, Panels B and C report results for its factor-timing and volatility-managed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. Our panel features: $n = 565$, $T = 90$, $N = 50850$. Values in parenthesis are Driscoll and Kraay (1998) robust standard errors for panel models with cross-sectional and serial correlation. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations of annual returns. The sample period is 1967 to 2019.

Panel A: FF5

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.196^{***}	-0.192^{***}	-0.215^{***}	-0.196^{***}	-0.204^{***}	-0.198^{***}
	(0.033)	(0.041)	(0.035)	(0.032)	(0.036)	(0.033)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.099	0.099	0.113	0.099	0.102	0.100

Panel B: FF5 with factor-timing

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.280^{***}	-0.265^{***}	-0.282^{***}	-0.275^{***}	-0.294^{***}	-0.287^{***}
	(0.034)	(0.039)	(0.038)	(0.032)	(0.042)	(0.033)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.139	0.143	0.139	0.153	0.141	0.146

Panel C: Volatility-managed FF5

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.199^{***}	-0.191^{***}	-0.215^{***}	-0.198^{***}	-0.204^{***}	-0.202^{***}
	(0.034)	(0.040)	(0.036)	(0.033)	(0.035)	(0.034)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.100	0.100	0.108	0.100	0.101	0.103

Panel D: FF5-DMRS

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.261^{***}	-0.197^{***}	-0.246^{***}	-0.249^{***}	-0.248^{***}	-0.265^{***}
	(0.043)	(0.048)	(0.039)	(0.040)	(0.045)	(0.043)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.129	0.169	0.132	0.154	0.132	0.132

Table 3: Out-of-Sample Predictability

This table reports the out-of-sample R^2 (R^2_{OOS}) for the predictive regression $\tilde{r}_{i,t+1} = a_i + b_i \hat{u}_{i,t} + \epsilon_{i,t}$, where $\tilde{r}_{i,t+1}$ is the test asset i log risk-adjusted return at time $t+1$ and price deviations $\hat{u}$ are computed as in equation (14). Test assets are the long legs for the 45 anomalies constructed in Kozak, Nagel, and Santosh (2020). We report results for price deviations computed using different heuristic mean-variance efficient portfolios. Panel A reports results for the Fama and French (2015, FF5) factor model, Panels B and C report results for its factor-timing and volatility-managed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. The R^2_{OOS} is computed as in Campbell and Thompson (2008); p-values for R^2_{OOS} are computed as in Clark and West (2007). The burn-in sample starts in Jan 1967 and ends in Dec 1987, we then use an expanding window for estimating the predictive regressions. The weights of the mean-variance portfolio are also estimated in an out-of-sample fashion. For example, for the baseline FF5, we compute $V^{-1}(\mathbf{f})$ and $E[\mathbf{f}]$ using information up to Dec 1987. Monthly observations of annual returns.

Panel A: FF5

Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}
accruals	12.19***	indmom	12.79***	price	11.19***
age	11.85***	indmomrev	9.68***	prof	9.66***
aturnover	10.77***	indrrev	12.18***	roaa	8.27***
betaarb	11.89***	indrrevlv	9.5***	roea	9.88***
cfp	15.89***	inv	13.6***	season	5.97***
ciss	13.05***	invcap	14.89***	sgrowth	16.9***
divg	14.18***	ivol	11.95***	shvol	14.49***
divp	18.22***	lev	10.83***	size	14.8***
dur	11.77***	lrrev	9.49***	sp	13.48***
ep	16.28***	mom	10.39***	strev	12.91***
exchsw	11.81***	mom12	8.31***	valmom	15.51***
fscore	11.61***	momrev	11.04***	valmomprof	10.18***
gmargins	9.86***	nissa	12.41***	valprof	12.64***
growth	14.85***	nissm	13.77***	value	13.14***
igrowth	16.57***	noa	10.14***	valuem	18.19***

Panel B: FF5 with factor-timing

Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}
accruals	11.43***	indmom	4.84***	price	8.88***
age	12.11***	indmomrev	4.84***	prof	9.08***
aturnover	10.89***	indrrev	10.08***	roaa	7.57***
betaarb	12.06***	indrrevlv	7.82***	roea	8.41***
cfp	13.33***	inv	14.42***	season	3.26***
ciss	10.13***	invcap	12.44***	sgrowth	15.49***
divg	14.82***	ivol	8.85***	shvol	13.58***
divp	15.14***	lev	9.63***	size	13.47***
dur	16.7***	lrrev	9.75***	sp	13.9***
ep	11.72***	mom	7.22***	strev	7.59***
exchsw	10.64***	mom12	5.86***	valmom	12.76***
fscore	11.21***	momrev	10.51***	valmomprof	8.75***
gmargins	9.81***	nissa	12.71***	valprof	12.15***
growth	15.01***	nissm	13.58***	value	12.07***
igrowth	13.59***	noa	8.49***	valuem	18.91***

Panel C: Volatility-managed FF5

Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}
accruals	16.31***	indmom	3.53***	price	10.93***
age	14.46***	indmomrev	13.9***	prof	7.79***
aturnover	11.37***	indrrev	18.79***	roaa	6.66***
betaarb	9.85***	indrrevlv	13***	roea	8.56***
cfp	13.52***	inv	17.46***	season	12.87***
ciss	15.55***	invcap	16.92***	sgrowth	16.57***
divg	13.54***	ivol	10.25***	shvol	16.86***
divp	14.73***	lev	14.78***	size	25.46***
dur	20.89***	lrrev	20.87***	sp	14.59***
ep	14.37***	mom	12.17***	strev	18.66***
exchsw	12.87***	mom12	8.97***	valmom	18.15***
fscore	12.46***	momrev	22.1***	valmomprof	12.92***
gmargins	8.84***	nissa	13.9***	valprof	13.83***
growth	21.22***	nissm	13.64***	value	17.25***
igrowth	17.14***	noa	15.47***	valuem	21.06***

Panel D: FF5-DMRS

Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}	Anomaly	R^2_{OOS}
accruals	15.56***	indmom	16.18***	price	12.63***
age	17.14***	indmomrev	7.05***	prof	12.14***
aturnover	14.69***	indrrev	7.88***	roaa	12.75***
betaarb	17.71***	indrrevlv	4.25***	roea	11.69***
cfp	20.47***	inv	13.83***	season	8***
ciss	16.86***	invcap	25.14***	sgrowth	12.24***
divg	15.18***	ivol	15.7***	shvol	16.29***
divp	10.7***	lev	1.46***	size	17.83***
dur	16.53***	lrrev	17.65***	sp	16.57***
ep	17.04***	mom	14.41***	strev	7.98***
exchsw	15.04***	mom12	13.83***	valmom	17.16***
fscore	14.72***	momrev	14.48***	valmomprof	18.04***
gmargins	10.75***	nissa	15.07***	valprof	15.92***
growth	15.1***	nissm	16.33***	value	10.17***
igrowth	17.67***	noa	12.33***	valuem	15.69***

Table 4: How Large are Anomaly Price Transitory Components?

This table reports $\frac{\bar{\sqrt{V_{tran,i}}}}{E[r_{i,t}]}$, which represents the scaled volatility of the transitory price component averaged across anomaly portfolios. We compute $\sqrt{V_{tran,i}}$ as the volatility of the fitted value from a regression of h-period risk-adjusted returns for anomaly i on its price deviations. Monthly observations of annual returns. The sample period is 1967 to 2019.

Model	$h=1$	$h=3$	$h=6$	$h=9$	$h=12$	$h=24$	$h=36$	$h=60$
FF5	0.80	0.97	0.96	0.96	0.98	0.95	0.91	0.73
FF5 with Factor-Timing	1.00	1.23	1.34	1.36	1.38	1.37	1.36	1.11
Volatility-Managed FF5	0.82	0.98	1.01	1.04	1.08	1.05	1.01	0.82
FF5-DMRS	1.07	1.20	1.20	1.18	1.19	1.11	1.08	0.84

Table 5: Long-Short Anomaly Portfolio Alphas

This table reports alphas obtained by regressing returns of a zero-cost investment strategy that exploits price deviations on several prominent factor models. We construct zero-cost long-short portfolios using a rank-based weighting scheme for the 90 anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). We rebalance portfolios once per year. Price deviations $\hat{u}$ are computed as in equation (14). We report results for price deviations computed using different heuristic mean-variance efficient portfolios. The weights of the mean-variance portfolio are estimated in an out-of-sample fashion. Panel A reports results for the Fama and French (2015, FF5) factor model, Panels B and C report results for its factor return and volatility timed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. We control for the following factor models: Carhart (1997) (C4), Fama and French (2018) (FF6), Hou, Xue, and Zhang (2015) (q), Stambaugh and Yuan (2016) (SY4), Daniel, Hirshleifer, and Sun (2020) (DHS3), Bartram and Grinblatt (2018) (BG3). Values in parenthesis are Newey and West (1987) robust standard errors. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations. The sample period is 1967 to 2019.

Panel A: FF5

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.57***	0.53***	0.41***	0.41***	0.36***	0.41***	0.59***
	(0.12)	(0.08)	(0.08)	(0.09)	(0.07)	(0.10)	(0.12)
Adjusted R^2	—	0.60	0.66	0.45	0.54	0.46	0.25

Panel B: FF5 with factor-timing

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.56***	0.52***	0.38***	0.38***	0.32***	0.37***	0.61***
	(0.12)	(0.08)	(0.06)	(0.07)	(0.06)	(0.09)	(0.12)
Adjusted R^2	—	0.62	0.71	0.50	0.59	0.57	0.25

Panel C: Volatility-managed FF5

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.62***	0.57***	0.44***	0.43***	0.36***	0.42***	0.66***
	(0.14)	(0.09)	(0.08)	(0.09)	(0.07)	(0.10)	(0.13)
Adjusted R^2	—	0.56	0.63	0.48	0.55	0.47	0.23

Panel D: FF5-DMRS

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.62***	0.56***	0.49***	0.48***	0.38***	0.46***	0.62***
	(0.13)	(0.12)	(0.11)	(0.11)	(0.08)	(0.11)	(0.13)
Adjusted R^2	—	0.37	0.38	0.25	0.37	0.22	0.13

Table 6: Performance in High- and Low-Friction Regimes

This table reports the average return ($E[R]$) of the zero-cost investment strategy based on price deviations over high- and low-friction periods. High-and low-friction periods are defined using the sample median of the variable in the first row. IVOL is calculated as the cross-sectional standard deviation of individual CRSP stock returns; illiquidity is the negative of the aggregate liquidity measure constructed in Pástor and Stambaugh (2003); VIX is the CBOE option-implied equity volatility index. Mean estimates are annualized and in percentage. Values in parenthesis are Newey and West (1987) robust standard errors. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations.

Panel A: FF5

	IVOL			illiquidity			VIX		
	H	L	H-L	H	L	H-L	H	L	H-L
$E[R]$	5.29***	1.54***	3.75**	4.90***	1.93***	2.97***	5.36***	1.44***	3.93**
	(1.55)	(0.47)	(1.75)	(1.13)	(0.66)	(1.06)	(1.57)	(0.51)	(1.70)

Panel B: FF5 with factor-timing

	IVOL			illiquidity			VIX		
	H	L	H-L	H	L	H-L	H	L	H-L
$E[R]$	4.46***	2.25***	2.21	4.59***	2.13***	2.45**	4.94***	1.75***	3.19*
	(1.57)	(0.47)	(1.82)	(1.11)	(0.67)	(1.03)	(1.59)	(0.45)	(1.75)

Panel C: Volatility-managed FF5

	IVOL			illiquidity			VIX		
	H	L	H-L	H	L	H-L	H	L	H-L
$E[R]$	5.90***	1.58***	4.32**	5.07***	2.41***	2.66***	5.63***	1.70***	3.94**
	(1.67)	(0.45)	(1.81)	(1.14)	(0.68)	(1.00)	(1.72)	(0.50)	(1.89)

Panel D: FF5-DMRS

	IVOL			illiquidity			VIX		
	H	L	H-L	H	L	H-L	H	L	H-L
$E[R]$	6.23***	1.21***	5.02***	5.03***	2.40***	2.63**	6.04***	1.39***	4.65***
	(1.60)	(0.35)	(1.73)	(1.15)	(0.63)	(1.04)	(1.56)	(0.50)	(1.66)

Table 7: Factor structure in price deviations

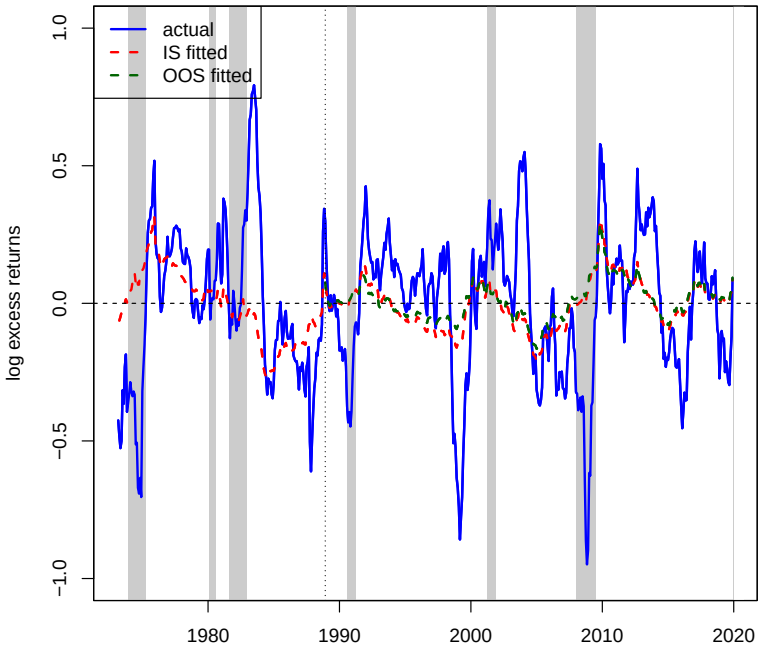
Panel A presents the proportion of variance explained by the first five principal components (PCs) for each factor model. Panel B reports the first ten principal components extracted from the pooled price deviations across the four benchmark factor models (see Table 2). In this case, the analysis includes a total of $360 = 90 \times 4$ price deviations.

Panel A: Principal Component Analysis of price deviations within a given factor model

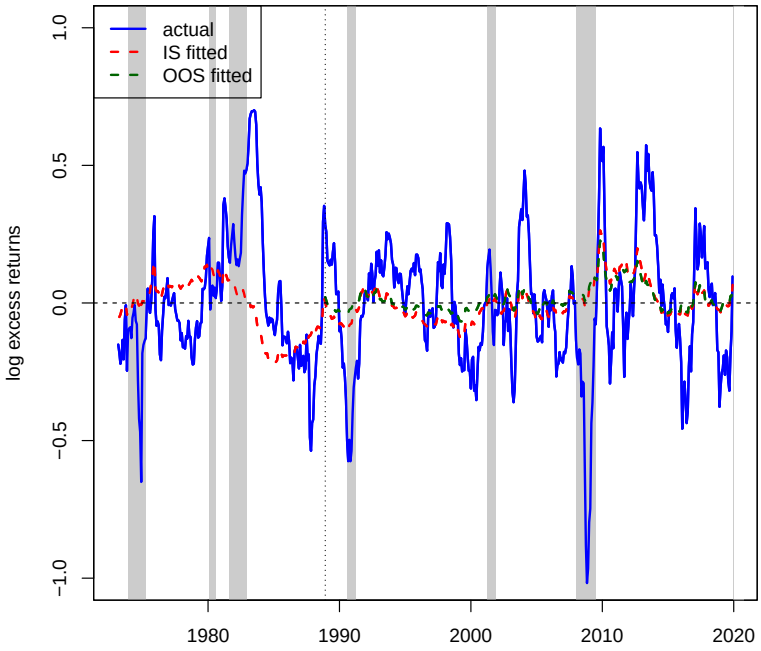
Model	PC1	PC2	PC3	PC4	PC5	cumulative
FF5	0.776	0.069	0.030	0.029	0.016	0.920
FF5 with factor-timing	0.828	0.043	0.030	0.023	0.012	0.936
Volatility-managed FF5	0.816	0.057	0.026	0.019	0.014	0.934
FF5-DMRS	0.789	0.065	0.031	0.025	0.016	0.926

Panel B: Principal Component Analysis of price deviations across factor models

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10
Proportion of Variance	0.789	0.065	0.031	0.025	0.016	0.012	0.010	0.008	0.007	0.006
Cumulative Proportion	0.789	0.854	0.885	0.910	0.926	0.938	0.949	0.957	0.964	0.970



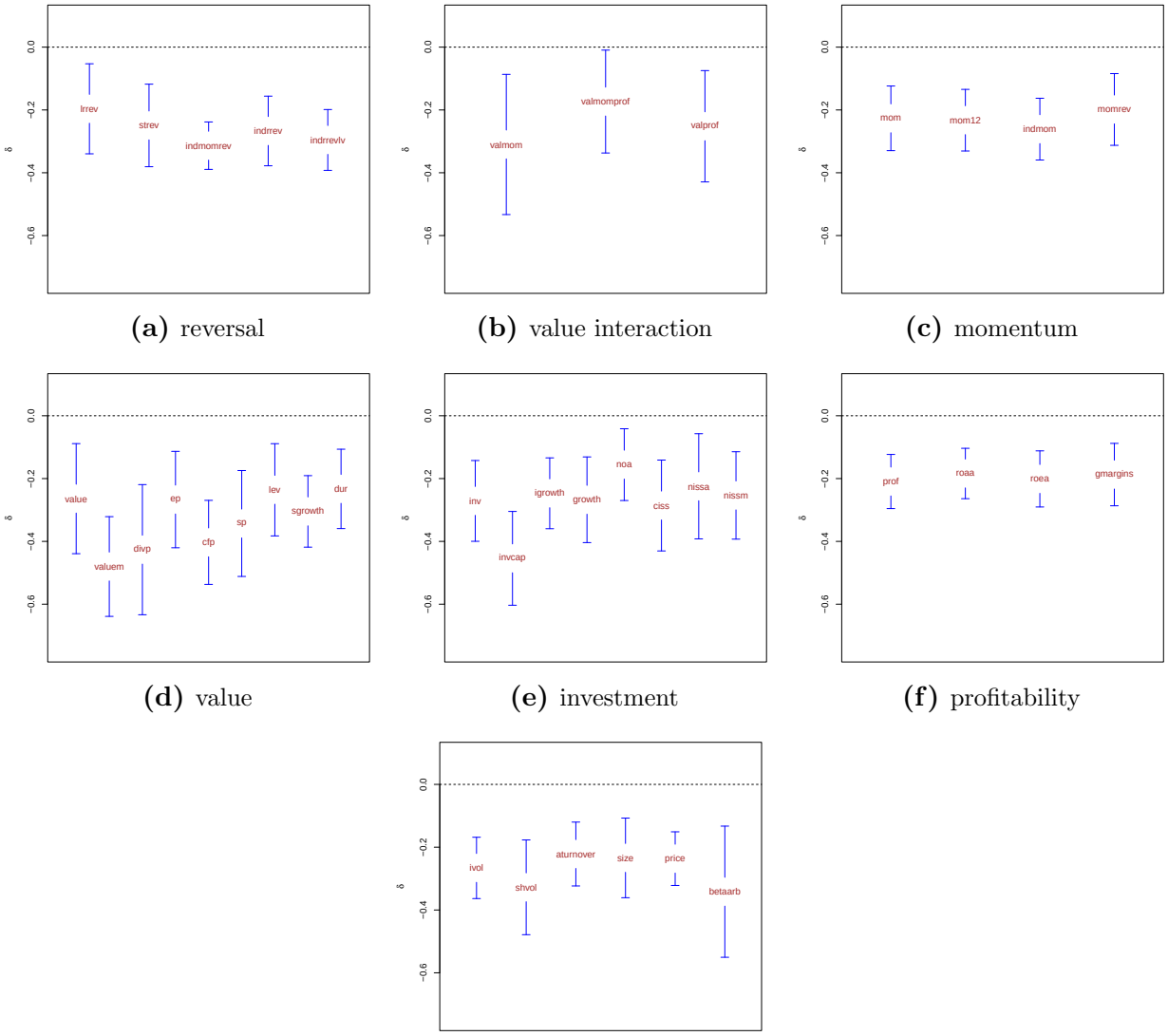
(a) Small



(b) Value

Figure 1: Size and Value Portfolios. The figure shows actual and predicted values for the Fama-French size (small) and book-to-market (value) portfolios using price deviations. Price deviations $\hat{u}$ are computed as in equation (14). The red and green line show the in-sample (IS) and out-of-sample (OOS) fit of predicting one-year ahead portfolio returns. Monthly observations of annual returns. The sample period is 1967 to 2019. The vertical dotted line indicates the beginning of the out-of-sample period in Dec 1988.

Panel A: FF5



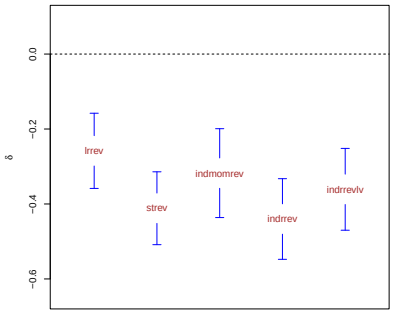
(a) reversal **(b)** value interaction **(c)** momentum

(d) value **(e)** investment **(f)** profitability

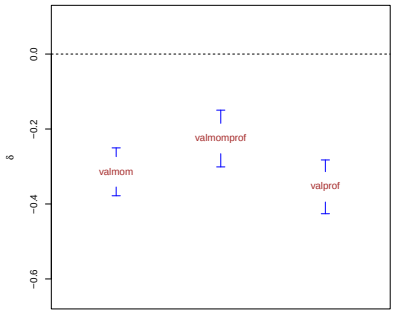
(g) trading frictions

Figure 2: Anomaly Portfolios and Price Deviations. This figure shows estimates for δ_i from regression (17) with respective confidence intervals at 5% level of significance. Test assets are the long legs for the 45 anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). Price deviations $\hat{u}$ are computed as in equation (14). We report results for price deviations computed using different heuristic mean-variance efficient portfolios. Panel A reports results for the Fama and French (2015, FF5) factor model, Panle B and C report results for its factor return and volatility timed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. Standard errors for $\hat{\delta}$ are computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is 1967 to 2019.

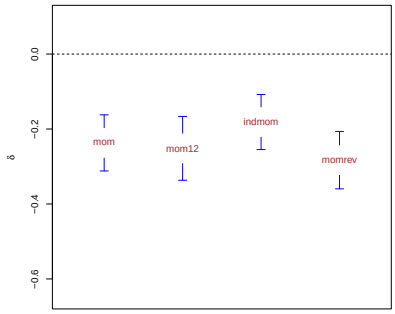
Panel B: FF5 with factor-timing



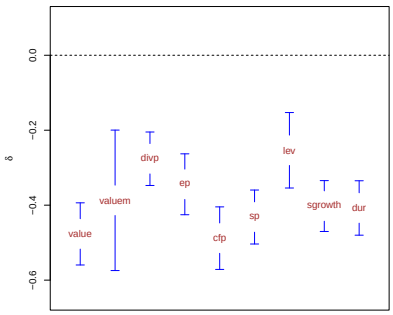
(a) reversal



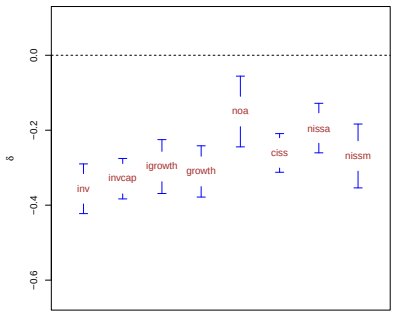
(b) value interaction



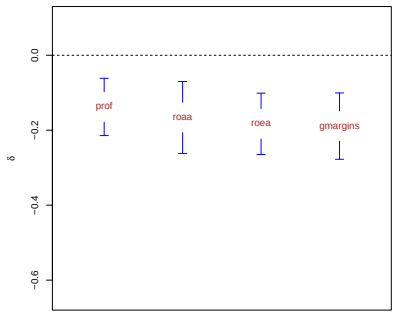
(c) momentum



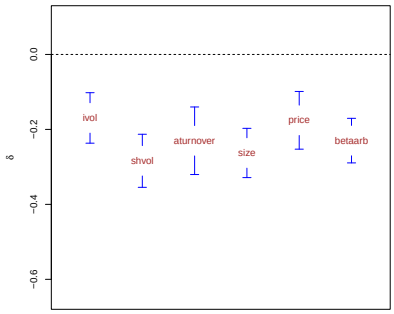
(d) value



(e) investment

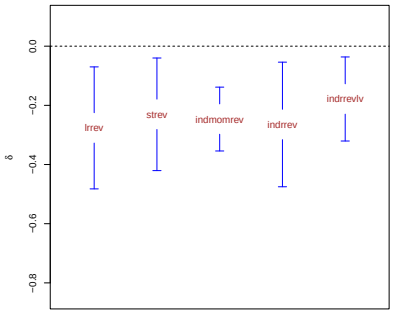


(f) profitability

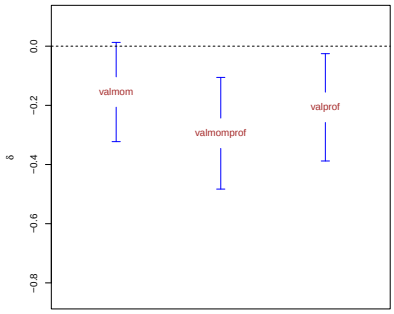


(g) trading frictions

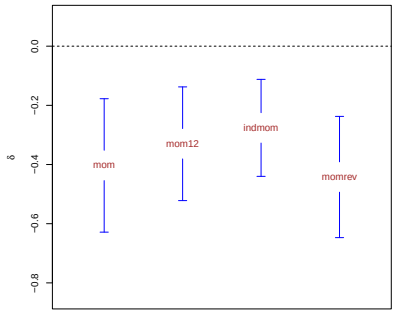
Panel C: Volatility-Managed FF5



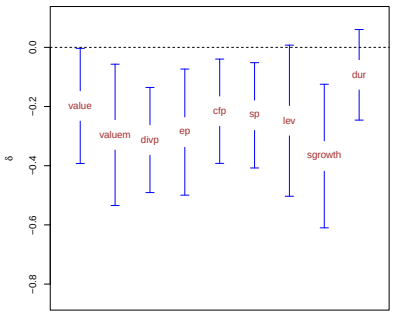
(a) reversal



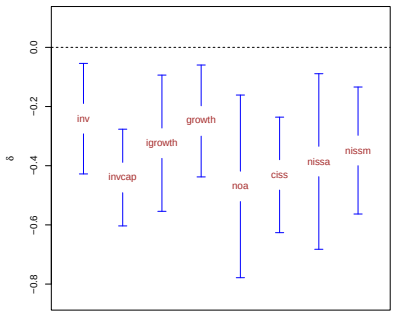
(b) value interaction



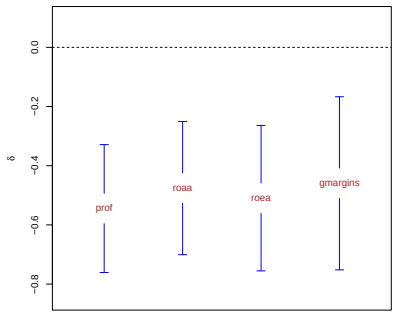
(c) momentum



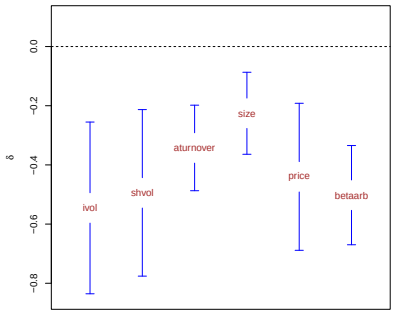
(d) value



(e) investment

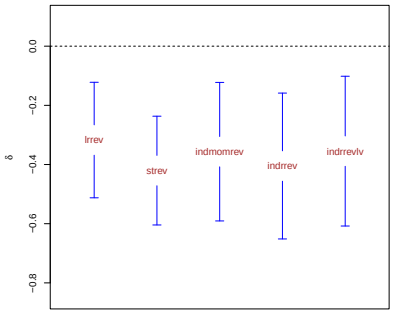


(f) profitability

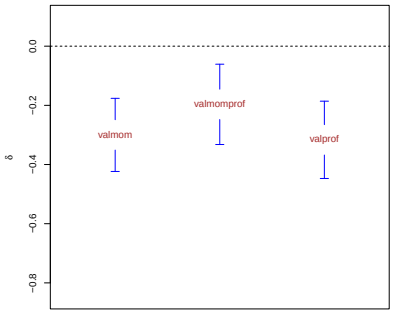


(g) trading frictions

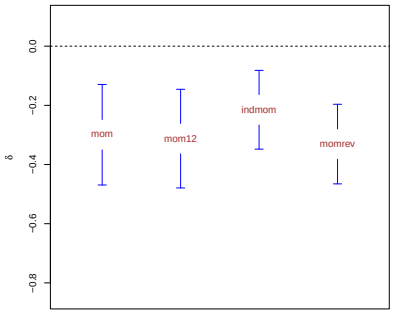
Panel D: FF5-DMRS



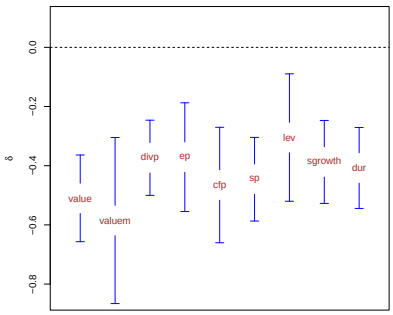
(a) reversal



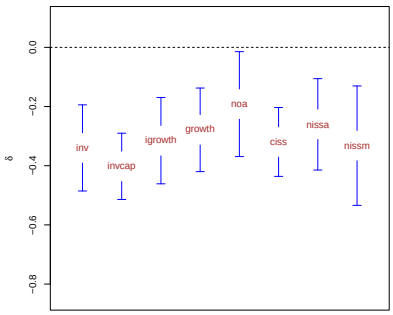
(b) value interaction



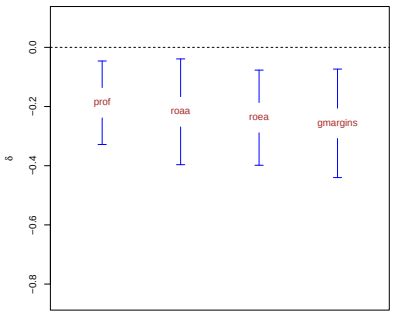
(c) momentum



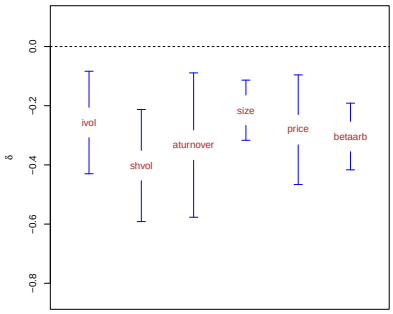
(d) value



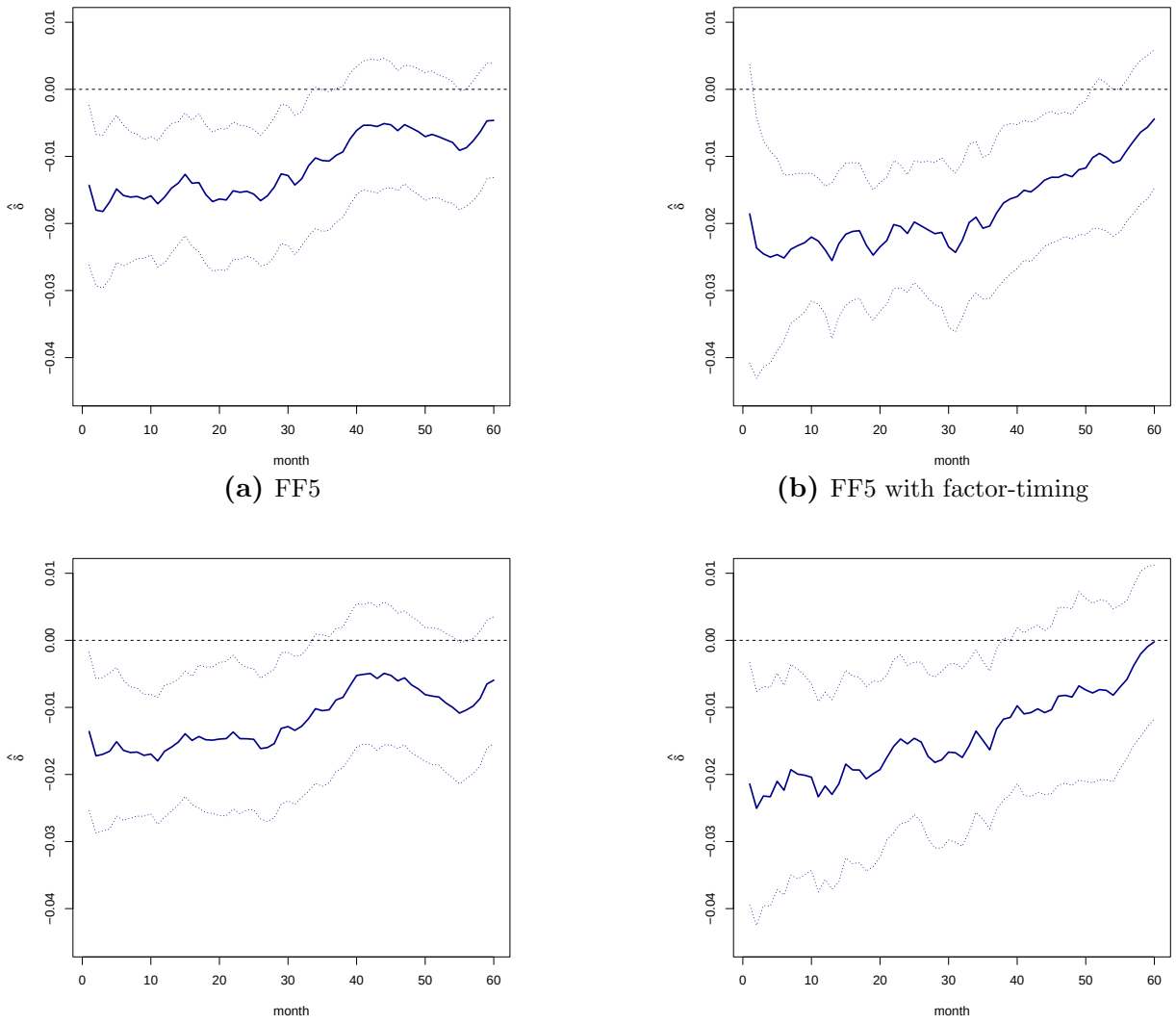
(e) investment



(f) profitability



(g) trading frictions



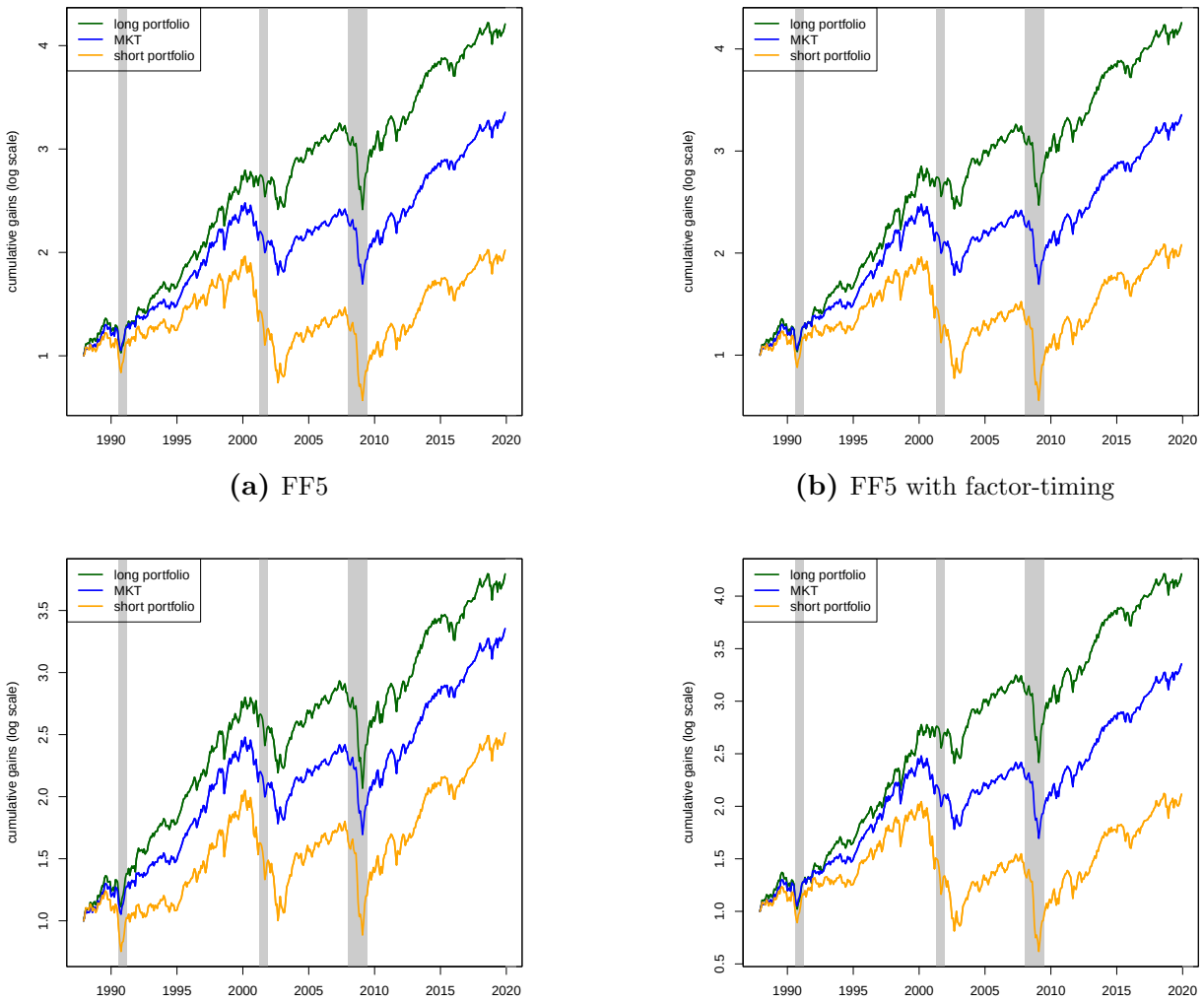
(a) FF5

(b) FF5 with factor-timing

(c) Volatility-managed FF5

(d) FF5-DMRS

Figure 2: Price Deviations and Horizon of Predictability. This figure shows pooled regression estimates of δ_i for equation (17) for h-period ahead monthly returns ($h = 1, \ldots, 60$). Test assets are 90 top and bottom deciles from the anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). Price deviations $\hat{u}$ are computed as in equation (14). We report results for price deviations computed using different heuristic mean-variance efficient portfolios. Panel A reports results for the Fama and French (2015, FF5) factor model, Panle B and C report results for its factor return and volatility timed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. The dashed lines correspond to the 95% confidence intervals; standard errors are computed as in Newey and West (1987) with automatic bandwidth selection procedure as described in Newey and West (1994). Non-overlapping monthly observations. The sample period is 1967 to 2019.



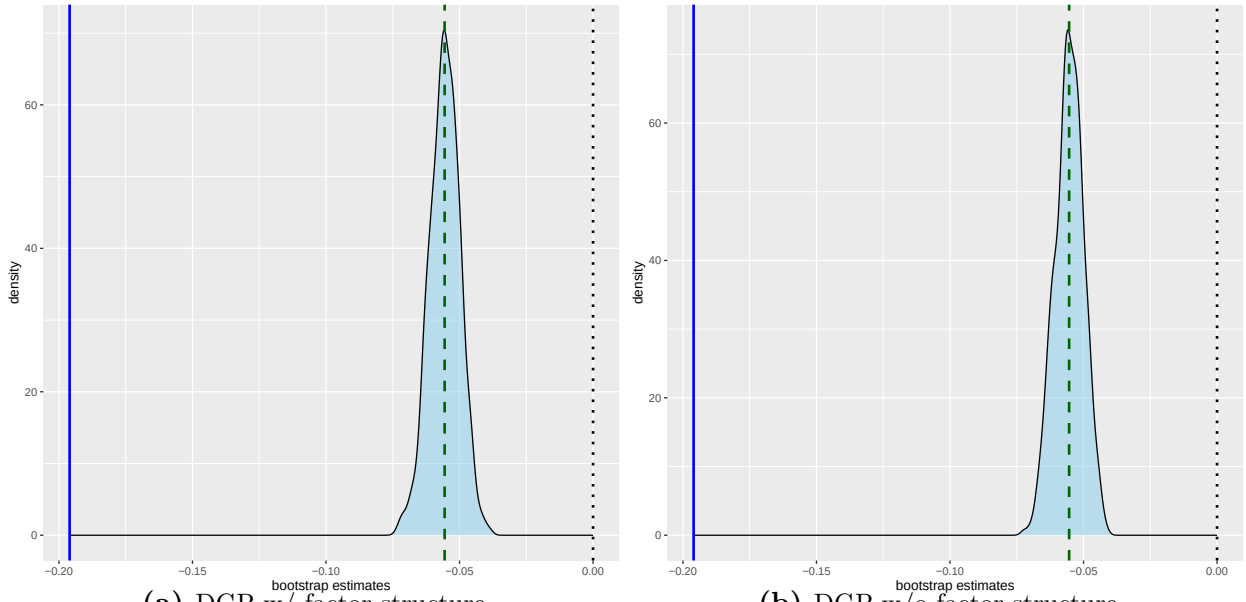
(a) FF5

(b) FF5 with factor-timing

(c) Volatility-Managed FF5

(d) FF5-DMRS

Figure 3: Anomaly Portfolios Rotation using Real-Time Price Deviations. We construct zero-cost long-short portfolios using a rank-based weighting scheme for the 90 top and bottom deciles from the anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). We rebalance portfolios once per year. Long (short) portfolio corresponds to the cumulative gains of a dynamic strategy that goes long on the 45 portfolios associated with the highest (lowest) expected return implied by the portfolio-specific price deviation. MKT is the performance of a static buy-and-hold strategy on the market portfolio in excess of the risk-free rate. Price deviations $\hat{u}$ are computed as in equation (14). We report results for price deviations computed using different heuristic mean-variance efficient portfolios. Panel A reports results for the Fama and French (2015, FF5) factor model, Panle B and C report results for its factor return and volatility timed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. Shaded areas are NBER recessions. Monthly observations.



(a) DGP w/ factor structure **(b)** DGP w/o factor structure

Figure 4: Bootstrap Standard Errors and Bias. This figure shows the estimated δ from Table 2 (solid blue line), together with the bootstrapped distribution. The dashed green line represents the mean of the bootstrapped distribution. The mean-variance efficient portfolio is constructed using the Fama and French (2015) factor model. We simulated 1000 artificial series for the bootstrap simulation, with a sample size of $T = 636$, which matches the size of our data.

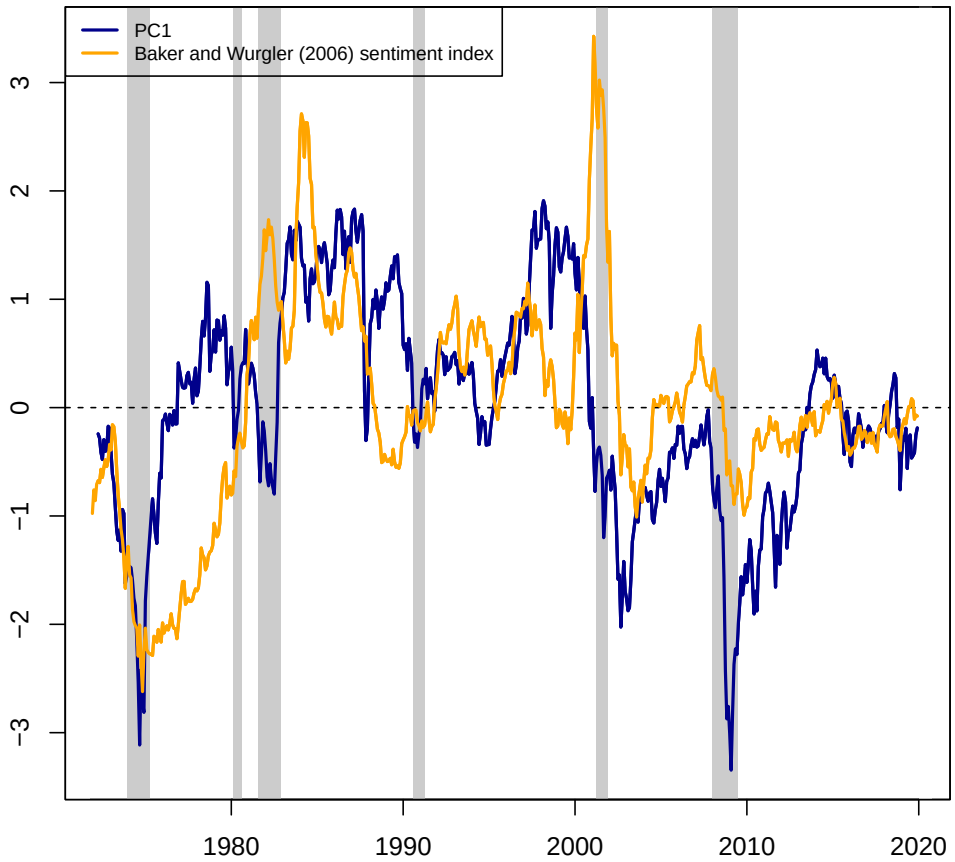
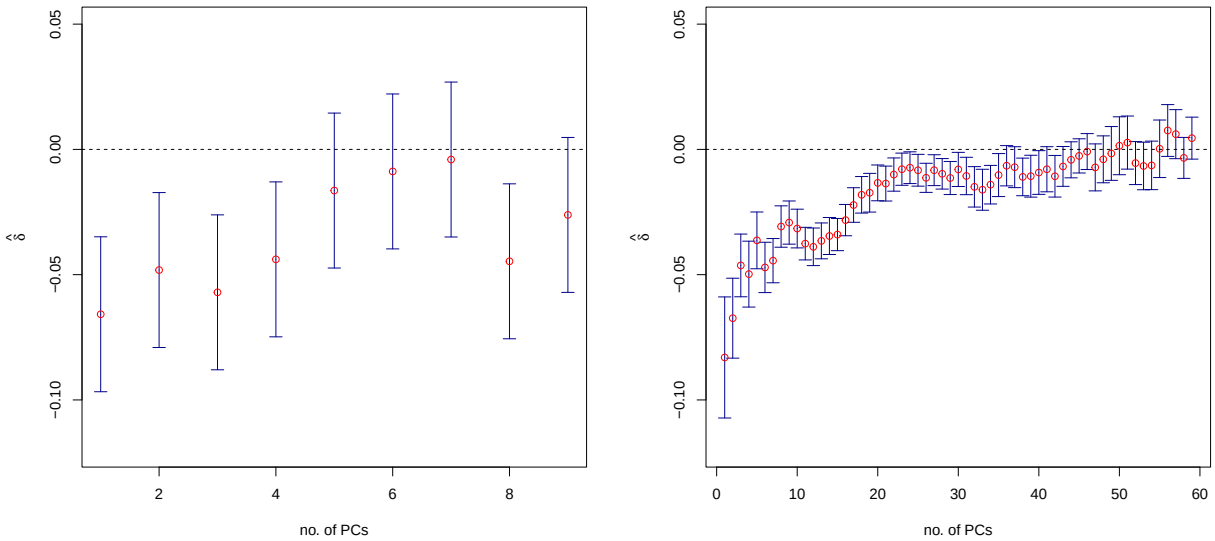


Figure 5: Price Deviations PC1. This figure plots the first principal component (PC1) extracted from the price deviations from the four benchmark factor models (cf. Table 2) together with the Baker and Wurgler (2006) sentiment index. The correlation between PC1 and the sentiment index is 40%. Both variables are demeaned and standardized.



(a) 10 portfolios sorted on B/M **(b)** 90 long and short portfolios

Figure 6: Predictability as a Function of the Number of PCs. This figure plots pooled estimates for δ_i from the predictive regression (18) as a function of the number of principal components (PCs) used to construct price deviations. PCs are extracted from the 90 top and bottom deciles from the anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). The dashed lines correspond to the 95% confidence intervals; standard errors are computed as in Driscoll and Kraay (1998). Monthly observations of annual returns. The sample period is 1967 to 2019.

Internet Appendix

A Technical Details and Derivations

A.1 Mean-variance efficient portfolio and the SDF

Define $\Sigma_t = V_t\left[R^e_{t+1}\right]$ and $\mu_t = E_t\left(R^e_{t+1}\right)$ and consider the following portfolio:

$$R^C_{t+1} = w_t^\intercal R^e_{t+1} \tag{A.1}$$
$$w_t = k_t^{-1}\Sigma_t^{-1}\mu_t \tag{A.2}$$

Next, we show that the SDF

$$\boxed{M^C_{t+1} = 1 - k_t\left(R^C_{t+1} - E_t\left[R^C_{t+1}\right]\right)}$$

prices all assets conditionally:

$$\begin{aligned}
E_t\left[M^C_{t+1}R^e_{t+1}\right] &= E_t\left[1 - k_t\left(R^C_{t+1} - E_t\left[R^C_{t+1}\right]\right)R^e_{t+1}\right] \\
&= E_t\left[R^e_{t+1}\right] - k_t E_t\left[\left(w_t^\intercal R^e_{t+1} - w_t^\intercal E_t\left[R^e_{t+1}\right]\right)R^e_{t+1}\right] \\
&= E_t\left[R^e_{t+1}\right] - k_t w_t^\intercal E_t\left[\left(R^e_{t+1} - E_t\left[R^e_{t+1}\right]\right)R^e_{t+1}\right] \\
&= E_t\left[R^e_{t+1}\right] - k_t k_t^{-1}\mu_t^\intercal\Sigma_t^{-1}\Sigma_t \\
&= 0
\end{aligned} \tag{A.3}$$

The parameter k_t is found by pricing the portfolio R^C_{t+1} itself:

$$\begin{aligned}
E_t\left[M^C_{t+1}R^C_{t+1}\right] &= E_t\left[1 - k_t\left(R^C_{t+1} - E_t\left[R^C_{t+1}\right]\right)R^C_{t+1}\right] \\
&= E_t\left[R^C_{t+1}\right] - k_t E_t\left[\left(R^C_{t+1} - E_t\left[R^C_{t+1}\right]\right)R^C_{t+1}\right] \\
&= E_t\left[R^C_{t+1}\right] - V_t\left[R^C_{t+1}\right]k_t \\
&= 0 \Leftrightarrow k_t = \left(V_t\left[R^C_{t+1}\right]\right)^{-1}E_t\left[R^C_{t+1}\right]
\end{aligned}$$

A.2 Price deviations when factor returns are i.i.d.

This example is inspired by Section 2.4 in Chernov, Lochstoer, and Lundeby (2021). Suppose that the true model is given by:

$$M_{t+1} = 1 - \mathbf{b}^{\intercal}\left(\mathbf{f}_{t+1} - E[\mathbf{f}_{t+1}]\right) \ , \quad \mathbf{b} = V\left(\mathbf{f}_{t+1}\right)^{-1} E\left[\mathbf{f}_{t+1}\right]$$

where the factors $\mathbf{f}_{t+1}$ are excess returns to traded portfolios.
Suppose also that the factor returns are i.i.d. Thus, the model prices the factors both conditionally and unconditionally.

Despite the factors being i.i.d., our predictive model (6) implies that test assets' returns are not, since their dynamics feature the (persistent) $u_{i,t}$ term:

$$\begin{aligned} r^e_{i,t+1} &= \beta_i' \mathbf{f}_{t+1} + \underbrace{\Delta u_{i,t+1}}_{\delta_i u_{i,t} + \varepsilon_{i,t+1}} \ . \\ u_{i,t} &= (\ln P_{i,t} - \ln P_{f,t}) - \beta_i \ln P_{f,t}, \\ u_{i,t} &= \rho_i u_{i,t-1} + \varepsilon_{i,t} \end{aligned}$$

where $\delta_i = 1 - \rho_i$ and for simplicity we have omitted the constant.

Note that the SDF prices r^i_{t+1} *unconditionally*:

$$\begin{aligned} E\left[M_{t+1} r^e_{i,t+1}\right] &= E\left[\left(1 - \mathbf{b}^{\intercal}\left(\mathbf{f}_{t+1} - E[\mathbf{f}_{t+1}]\right)\right)\left(\beta_i' \mathbf{f}_{t+1} + \Delta u_{i,t+1}\right)\right] \\ &= (\beta_i' \underbrace{E\left[\left(1 - \mathbf{b}^{\intercal}\left(\mathbf{f}_{t+1} - E[\mathbf{f}_{t+1}]\right)\right)\mathbf{f}_{t+1}\right]}_{=0 \text{ using the definition of } \mathbf{b}} + E\left[\left(1 - \mathbf{b}^{\intercal}\left(\mathbf{f}_{t+1} - E[\mathbf{f}_{t+1}]\right)\right)\Delta u_{i,t+1}\right] \end{aligned}$$

where the last term is zero given our assumption of factors being independent over time and the price deviations $u_{i,t}$ being zero mean. However, the SDF does not prices $r^e_{i,t+1}$

conditionally:

$$\begin{aligned} E_t\left[M_{t+1}r^e_{i,t+1}\right] &= E_t\left[\left(1-\mathbf{b}^\intercal\left(\mathbf{f}_{t+1}-E[f_{t+1}]\right)\right)\left(\beta_i'\mathbf{f}_{t+1}+\Delta u_{i,t+1}\right)\right] \\ &= \beta_i' \underbrace{E_t\left[\left(1-\mathbf{b}^\intercal\left(\mathbf{f}_{t+1}-E[\mathbf{f}_{t+1}]\right)\right)\mathbf{f}_{t+1}\right]}_{=0 \text{ using the definition of } b \text{ and factors being iid}} \\ &+ \underbrace{E_t\left[\left(\mathbf{b}^\intercal\left(\mathbf{f}_{t+1}-E[\mathbf{f}_{t+1}]\right)\right)\Delta u_{i,t+1}\right]}_{=0 \text{ since factors are i.i.d and property of } u_{i,t}} + E_t\left[\Delta u_{i,t+1}\right] \\ &= \delta_i u_{i,t} \end{aligned}$$

where in the last step we exploit the AR(1) dynamics for $u_{i,t}$.
Furthermore, we have that

$$\text{Cov}\left(u_{i,t-1}, u_{i,t}\right) \neq 0$$

In words, through our predictive system we document that test assets feature persistent pricing errors.[40]

A.3 Stambaugh-type bias in regressions of returns on price deviations

We simulate under the null hypothesis of no cointegration between the price of the test asset and the price of the mean-variance portfolio. We consider two scenario. One where there is a factor structure in returns and one where asset returns are i.i.d.

A.3.1 Stambaugh-type bias when returns admit a factor structure

We fix a factor model, $\{r^{mv}, r_{i,t+1}\}$, and implement the following steps:

1. Assume a data-generating process (DGP) with constant betas and run the following regression:

$$r_{i,t+1} = c_i + \beta_i r^{mv}_{t+1} + \varepsilon_{i,t+1}$$

[40] In the notation of Chernov, Lochstoer, and Lundeby (2021), $\delta_i u_{i,t}$ is the conditional pricing error at horizon $h = 1$.

Store the estimated parameters $\widehat{c}_i, \widehat{\beta}_i$ and the volatility of the residuals $\widehat{\sigma}_i$.

2. Given the sample size T, draw shocks from a normal distribution: $\widetilde{\varepsilon}_{i,t+1} = \sigma_i \times \text{randn}(T, 1)$. Repeat this step $B = 1000$ times.

3. Using the parameter estimates, build B artificial series of returns:

$$\widetilde{r}_{i,t+1}^{(b)} = \widehat{c}_i + \widehat{\beta}_i r_{t+1}^{mv} + \widetilde{\varepsilon}_{i,t+1}^{(b)} \; b = 1, ..., 1000$$

Here, we impose a factor structure on returns by using the true (realized) r_{t+1}^{mv}.

These steps are repeated for all assets $i = 1, \ldots, N$.

4. Treat $\left\{ r_{t+1}^{mv}, \widetilde{r}_{i,t+1}^{(b)} \right\}_{b=1}^{B=1000}$ as the true data. Repeat the procedure in Table 2 to obtain $\left\{ \delta^{(b)} \right\}_{b=1}^{B=1000}$ from a pooled regression. In particular,

A.3.2 Stambaugh-type bias when returns are i.i.d.

In this case we replace steps 1 and 3 above with

1. Assume the following data-generating process (DGP) and run the regression:

$$r_{i,t+1} = c_i + \varepsilon_{i,t+1}$$

Store the estimated parameters $\widehat{c}_i$ and the volatility of the residuals $\widehat{\sigma}_i$.

3 Using the parameter estimates, build B artificial series of returns:

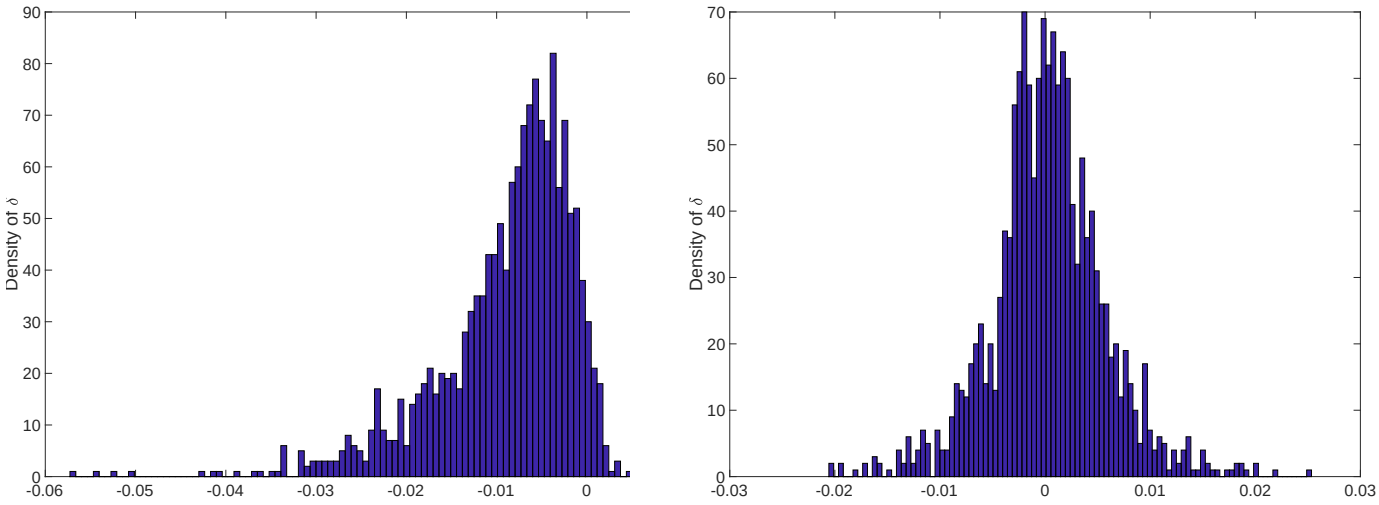
$$\widetilde{r}_{i,t+1}^{(b)} = \widehat{c}_i + \widetilde{\varepsilon}_{i,t+1}^{(b)} \; b = 1, ..., 1000$$

Here, we do not impose a factor structure in returns.

Our simulation under the null of no cointegration reveals that the estimated δ in equation (17) can be negative. Importantly, this estimate is smaller (in absolute value) in absolute value compared to that derived from real data, suggesting that the observed reversal is

unlikely to be spurious. We now investigate the source of this bias and explain why we refer to it as a Stambaugh-type bias.

We start by clarifying an important point.



(a) Returns onto lagged price. **(b)** Returns onto an unrelated random walk.

Figure A.1: Density of $\widehat{\delta}$. This figure shows density of estimated δ when returns are regressed on lagged prices (left panel) and when i.i.d. returns are regressed on a random walk, unrelated to the return process itself (right panel).

Under the null that the model is correct, equation (17) involves regressing a stationary (risk adjusted return) variable onto a non-stationary variable. Indeed, when the factor model is CMVE, the error term $\varepsilon_{i,t}$ is unpredictable, and $u_{i,t}$ is a martingale ($E_t\left[u_{i,t+1} - u_{i,t}\right] = E_t\left[\varepsilon_{i,t+1}\right] = 0$). However, the bias we observe is not due to the general fact that we are regressing a stationary variable onto a non-stationary one but is inherent in regressing (risk-adjusted) returns onto lagged log prices. To understand this bias, consider a simple data-generating process (DGP) where the log-return process is:

$$r_t = \sigma\varepsilon_{i,t} \ , \ \varepsilon_{i,t} \sim N\left(0, \sigma\right)$$

where log prices (normalized $\log P_0 = 0$) are given by

$$\log P_t = \sum_{s=1}^{t} \sigma\varepsilon_s$$

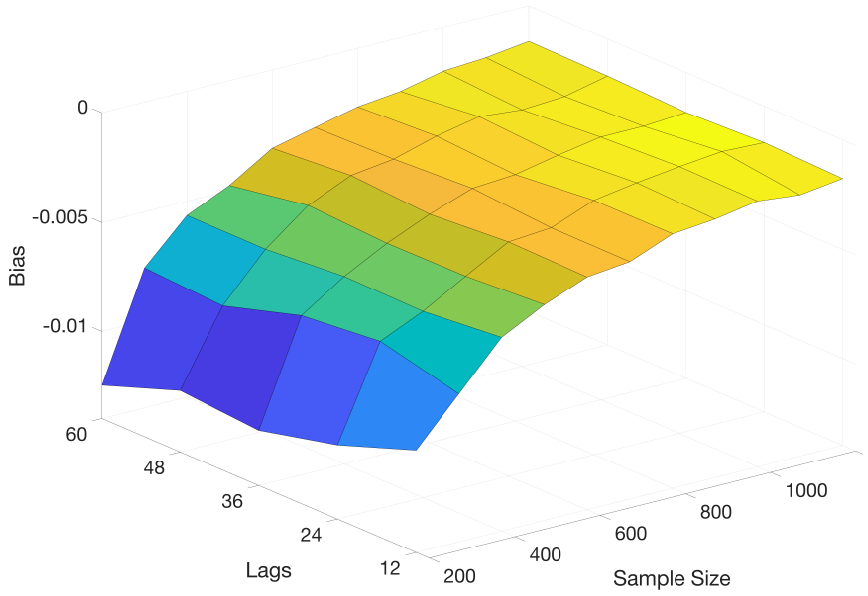


Figure A.2: Bias in OLS estimate of δ in the regression as function of lags and sample size. This figure shows density of estimated δ when returns are regressed on lagged prices (left panel) and when i.i.d. returns are regressed on a random walk, unrelated to the return process itself (right panel).

Next, we demonstrate that regressing an arbitrary i.i.d. process r_t onto an unrelated random walk does not produce the same bias. Indeed Figure A.1 displays the density of the OLS estimator $\widehat{\delta} = \frac{\text{Cov}(r_{t+1}, \log P_t)}{\text{Var}(\log P_t)}$ on the left, and the results from regressing r_{t+1} onto an unrelated price trend $\log P_t = \sum_{s=1}^{t} \sigma u_s$, $u \ N(0, \sigma)$ on the right. The left panel shows that the regression of r_{t+1} on $\log P_t$ exhibits bias in small samples, while this is not observed in the right panel. Therefore, the bias is specific to using asset-specific lagged cumulative returns or log prices (Stambaugh bias) as a regressor, rather than a generic non-stationary trend.[41] Figure A.2 further explores a regression of the form:

$$r_{t+1} = c + \delta r_{t,t-\text{lag}} + \varepsilon_{t+1}$$

and shows that the bias diminishes as the sample size T increases, irrespective of the lag structure. This confirms that the bias is primarily a small-sample phenomenon. Additionally, the use of long-term returns (or prices) only mildly exacerbates the issue, as indicated by the

[41] We thank Bjorn Eraker for pointing this out to us.

relatively flat curves along the lag axis. This suggests that increasing the lag length does not significantly intensify the bias. Comparing this results to our bootstrap simulation in the main text further demonstrates that estimating equations (12) and (13) using the observed mean-variance portfolio return r^{mv} and bootstrapped returns (whether i.i.d. or following a factor structure) amplifies the bias. However, the bias is not severe enough to render our results insignificant.

To further elucidate the source of the bias, we reformulate the regression using continuous time processes (Phillips, 1987). In this framework, the sample average of the price is $\sigma\frac{1}{T}\int_0^T W_s ds$ and the sample average of the dependent variable (returns) is $\sigma\frac{1}{T}\int_0^T dW_s$. We then have:

$$\widehat{\delta} = \frac{\overbrace{\frac{1}{T}\int_0^T W_s dW_s}^{E(xy)} - \overbrace{\frac{1}{T}\int_0^T W_s ds \frac{1}{T}\int_0^T dW_s}^{E(x)E(y)}}{\underbrace{\text{Var}\int_0^T W_s ds}_{Var(x)}}$$

We have that $E\frac{1}{T}\int_0^T W_s dW_s = 0$, but the product of the expectations term is:

$$E\left[\frac{1}{T}\int_0^T W_s ds \frac{1}{T}\int_0^T dW_s\right] = E\left[\frac{1}{T^2}\left(W_T T - \frac{1}{2}T\right)W_T\right] = \frac{1}{2}$$

This indicates that, for a given path, average price and average returns are positively correlated, leading to a negative bias in the sample covariance estimate.

Despite this, our regression on past returns, along with bootstrapped standard errors and bias correction, suggests that the bias is likely much smaller than the estimated δ.

A.4 Exponentially Smoothed Price Deviations

Although the bias in the OLS estimate of δ is much smaller than the estimated loading on price deviations, one might still worry that the difference between the anomaly portfolio's price and the target price implied by the mean-variance portfolio yields a non-stationary forecast. To address this concern, we replicate the analysis in Table 2using an exponentially

weighted (one-sided) moving average of u_i, t, ensuring that the resulting (smoothed) price deviation series remains persistent yet stationary.

As shown in Table A.1, the findings are not only consistent but actually stronger, as evidenced by a higher R^2. This improvement suggests that more recent pricing errors may exert a greater influence than older ones. Determining the optimal weighting scheme for pricing errors when constructing price deviations remains an intriguing topic for future research.

Table A.1: Predicting Anomaly Returns with Exponentially Smoothed Price Deviations Price Deviations

This table reports pooled estimates for δ_i from predictive regression (18). Test assets are 90 top and bottom deciles from the anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). Price deviations $\hat{u}$ are computed as in equation (14), then an exponentially weighted moving average is applied to each price deviation, using a decay factor of $\lambda = 0.94$. We report results for price deviations computed using different heuristic mean-variance efficient portfolios. Panel A reports results for the Fama and French (2015, FF5) factor model, Panels B and C report results for its factor-timing and volatility-managed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. Values in parenthesis are Driscoll and Kraay (1998) robust standard errors for panel models with cross-sectional and serial correlation. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations of annual returns. The sample period is 1967 to 2019.

Panel A: FF5

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.231^{***}	-0.222^{***}	-0.231^{***}	-0.229^{***}	-0.233^{***}	-0.232^{***}
	(0.037)	(0.045)	(0.037)	(0.037)	(0.039)	(0.037)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.104	0.104	0.105	0.104	0.104	0.104

Panel B: FF5 with factor-timing

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.388^{***}	-0.381^{***}	-0.389^{***}	-0.389^{***}	-0.420^{***}	-0.389^{***}
	(0.036)	(0.042)	(0.034)	(0.035)	(0.052)	(0.037)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.171	0.172	0.179	0.192	0.176	0.171

Panel C: Volatility-managed FF5

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.238^{***}	-0.227^{***}	-0.238^{***}	-0.234^{***}	-0.238^{***}	-0.238^{***}
	(0.040)	(0.047)	(0.040)	(0.040)	(0.042)	(0.040)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.107	0.109	0.108	0.109	0.108	0.107

Panel D: FF5-DMRS

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.297^{***}	-0.202^{***}	-0.286^{***}	-0.288^{***}	-0.292^{***}	-0.299^{***}
	(0.049)	(0.057)	(0.045)	(0.045)	(0.052)	(0.049)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.116	0.145	0.140	0.146	0.116	0.116

B Test Assets

Table B.1: Categories

We group anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020) following Lettau and Pelger (2020). This table lists the categories and the portfolios that we include in each category. Differently from Lettau and Pelger (2020), we allocate some of the portfolios in the category "others" across the other categories. In total, we consider 8 categories and 45 anomaly portfolios. Anomalies are defined in Kelly, Kozak, and Giglio (2020), Haddad, Kozak, and Santosh (2020), and Kozak, Nagel, and Santosh (2020).

Category	Anomaly Portfolios
reversal	indmomrev, indrrev, indrrevlv, lrrev, strev
value interaction	valmom, valmomprof, valprof
momentum	indmom, mom, mom12, momrev
value	cfp, divp, dur, ep, lev, sgrowth, sp, value, valuem
investment	ciss, inv, invcap, igrowth, growth, nissa, nissm, noa
profitability	gmargins, prof, roaa, roea
trading frictions	aturnover, betaarb, ivol, price, shvol, size
others	accruals, age, divg, exchsw, fscore, season

Notes: lrrev is long-term reversal calculated as in De Bondt and Thaler (1985). strev is short-term reversal calculated as in Jegadeesh (1990). indmomrev is industry momentum-reversal reversal calculated as in Moskowitz and Grinblatt (1999). indrrev is industry-relative reversal calculated as in Da, Liu, and Schaumburg (2014). indrrevlv is industry-relative reversal low-volatility calculated as in Da, Liu, and Schaumburg (2014). valmom is value-momentum calculated as in Novy-Marx (2013). valmomprof is value-momentum-profitability calculated as in Novy-Marx (2013). valprof is value-profitability calculated as in Novy-Marx (2013). mom is 6-months momentum calculated as in Jegadeesh and Titman (1993). mom12 is 12-months momentum calculated as in Jegadeesh and Titman (1993). indmom is long-term reversal calculated as in Moskowitz and Grinblatt (1999). momrev is momentum-reversal calculated as in Jegadeesh and Titman (1993). value is annual value calculated as in Fama and French (1993). valuem is monthly value calculated as in Asness and Frazzini (2013). divp is dividend yield calculated as in Naranjo, Nimalendran, and Ryngaert (1998). ep is earnings/price calculated as in Basu (1977). cfp is cash-flow/market value of equity calculated as in Lakonishok, Shleifer, and Vishny (1994). sp is sales-to-price calculated as in Barbee Jr, Mukherji, and Raines (1996). lev is leverage calculated as in Bhandari (1988). sgrowth is sales growth calculated as in Lakonishok, Shleifer, and Vishny (1994). inv is investment calculated as in Chen, Novy-Marx, and Zhang (2011). invcap is investment-to-capital calculated as in Xing (2008). igrowth is investment growth calculated as in Xing (2008). growth is asset growth calculated as in Cooper, Gulen, and Schill (2008). noa is net operating asset calculated as in Hirshleifer et al. (2004). ciss is composite issuance calculated as in Daniel and Titman (2006). prof is ross profitability calculated as in Novy-Marx (2013). roaa is annual return on assets calculated as in Chen, Novy-Marx, and Zhang (2011). roea is annual return on equity calculated as in Haugen, Baker et al. (1996). gmargins is gross margins calculated as in Novy-Marx (2013). ivol is idiosyncratic volatility calculated as in Ang et al. (2006). shvol is share volume calculated as in Datar, Naik, and Radcliffe (1998). aturnover is asset turnover calculated as in Soliman (2008). size is size calculated as in Fama and French (1993).

C Alternative Betas

C.1 1-Year Rolling Window using Daily Returns

Table C.1: Predicting Anomaly Returns with Price Deviations

This table replicates Table 2 with betas computed using a 1-year rolling window with daily returns. Monthly observations of annual returns. The sample period is 1967 to 2019.

Panel A: FF5

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.169^{***}	-0.153^{***}	-0.180^{***}	-0.166^{***}	-0.171^{***}	-0.171^{***}
	(0.033)	(0.041)	(0.035)	(0.032)	(0.036)	(0.033)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.081	0.085	0.088	0.084	0.082	0.084

Panel B: FF5 with factor-timing

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.311^{***}	-0.293^{***}	-0.297^{***}	-0.305^{***}	-0.322^{***}	-0.317^{***}
	(0.042)	(0.044)	(0.037)	(0.038)	(0.047)	(0.042)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.154	0.162	0.158	0.176	0.156	0.159

Panel C: Volatility-managed FF5

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.203^{***}	-0.170^{***}	-0.214^{***}	-0.199^{***}	-0.202^{***}	-0.206^{***}
	(0.030)	(0.032)	(0.033)	(0.029)	(0.031)	(0.030)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.098	0.110	0.102	0.107	0.098	0.101

Panel D: FF5-DMRS

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.270^{***}	-0.191^{***}	-0.255^{***}	-0.266^{***}	-0.251^{***}	-0.275^{***}
	(0.041)	(0.046)	(0.034)	(0.038)	(0.043)	(0.041)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.140	0.196	0.142	0.178	0.146	0.143

Table C.2: Long-Short Anomaly Portfolio Alphas

This table replicates Table 5 with betas computed using a 1-year rolling window with daily returns. Monthly observations. The sample period is 1967 to 2019.

Panel A: FF5

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.59***	0.55***	0.44***	0.43***	0.38***	0.41***	0.60***
	(0.12)	(0.09)	(0.08)	(0.09)	(0.08)	(0.10)	(0.12)
Adjusted R^2	—	0.54	0.59	0.40	0.49	0.46	0.23

Panel B: FF5 with factor-timing

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.55***	0.55***	0.43***	0.41***	0.35***	0.40***	0.53***
	(0.13)	(0.10)	(0.11)	(0.10)	(0.09)	(0.11)	(0.12)
Adjusted R^2	—	0.48	0.52	0.33	0.47	0.37	0.26

Panel C: Volatility-managed FF5

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.59***	0.58***	0.48***	0.46***	0.39***	0.43***	0.57***
	(0.13)	(0.11)	(0.11)	(0.10)	(0.09)	(0.12)	(0.13)
Adjusted R^2	—	0.43	0.46	0.30	0.41	0.31	0.21

Panel D: FF5-DMRS

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.50***	0.49***	0.40***	0.36***	0.32***	0.39***	0.45***
	(0.12)	(0.11)	(0.09)	(0.10)	(0.08)	(0.11)	(0.12)
Adjusted R^2	—	0.46	0.49	0.29	0.41	0.29	0.23

C.2 5-Year Rolling Window using Monthly Returns

Table C.3: Predicting Anomaly Returns with Price Deviations

This table replicates Table 2 with betas computed using a 5-year rolling window with monthly returns. Monthly observations of annual returns. The sample period is 1967 to 2019.

Panel A: FF5

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.234^{***}	-0.194^{***}	-0.239^{***}	-0.227^{***}	-0.235^{***}	-0.235^{***}
	(0.032)	(0.031)	(0.036)	(0.030)	(0.031)	(0.032)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.102	0.137	0.103	0.109	0.130	0.103

Panel B: FF5 with factor-timing

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.218^{***}	-0.173^{***}	-0.202^{***}	-0.202^{***}	-0.197^{***}	-0.218^{***}
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.112	0.143	0.116	0.119	0.131	0.112

Panel C: Volatility-managed FF5

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.273^{***}	-0.258^{***}	-0.279^{***}	-0.285^{***}	-0.307^{***}	-0.279^{***}
	(0.059)	(0.053)	(0.056)	(0.058)	(0.052)	(0.059)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.125	0.219	0.125	0.151	0.203	0.130

Panel D: FF5-DMRS

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.265^{***}	-0.188^{***}	-0.267^{***}	-0.252^{***}	-0.245^{***}	-0.268^{***}
	(0.040)	(0.042)	(0.041)	(0.039)	(0.037)	(0.040)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.118	0.173	0.118	0.125	0.169	0.120

Table C.4: Long-Short Anomaly Portfolio Alphas

This table replicates Table 5 with betas computed using a 5-year rolling window with monthly returns. Monthly observations. The sample period is 1967 to 2019.

Panel A: FF5

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.43***	0.39***	0.29***	0.28***	0.23***	0.33***	0.51***
	(0.14)	(0.07)	(0.07)	(0.09)	(0.07)	(0.10)	(0.11)
Adjusted R^2	—	0.62	0.67	0.49	0.55	0.41	0.26

Panel B: FF5 with factor-timing

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.57***	0.50***	0.42***	0.43***	0.33***	0.39***	0.64***
	(0.12)	(0.09)	(0.08)	(0.09)	(0.07)	(0.09)	(0.12)
Adjusted R^2	—	0.54	0.57	0.38	0.46	0.34	0.16

Panel C: Volatility-managed FF5

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.33***	0.32***	0.27***	0.20**	0.25***	0.28***	0.34***
	(0.10)	(0.08)	(0.08)	(0.09)	(0.09)	(0.09)	(0.09)
Adjusted R^2	—	0.40	0.43	0.28	0.32	0.31	0.20

Panel D: FF5-DMRS

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.55***	0.51***	0.41***	0.40***	0.34***	0.42***	0.58***
	(0.14)	(0.11)	(0.10)	(0.10)	(0.08)	(0.11)	(0.13)
Adjusted R^2	—	0.49	0.53	0.37	0.46	0.32	0.18

D Alternative Factor Models

Table D.1: Predicting Anomaly Returns with Price Deviations

This table replicates Table 2 using a mean-variance portfolio constructed using different factor models. Panel A reports results for the CAPM, Panel B reports results for the Hou, Xue, and Zhang (2015, HXZ) factor model, and Panel C reports results for its volatility timed version. Monthly observations of annual returns. The sample period is 1967 to 2019.

Panel A: CAPM

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.334^{***}	-0.231^{***}	-0.358^{***}	-0.317^{***}	-0.305^{***}	-0.342^{***}
	(0.050)	(0.059)	(0.048)	(0.049)	(0.049)	(0.050)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.165	0.226	0.168	0.180	0.198	0.172

Panel B: HXZ

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.347^{***}	-0.300^{***}	-0.352^{***}	-0.332^{***}	-0.340^{***}	-0.351^{***}
	(0.049)	(0.052)	(0.046)	(0.046)	(0.046)	(0.049)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.159	0.185	0.159	0.167	0.183	0.161

Panel C: Volatility-managed HXZ

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.377^{***}	-0.316^{***}	-0.405^{***}	-0.371^{***}	-0.365^{***}	-0.384^{***}
	(0.065)	(0.066)	(0.063)	(0.063)	(0.059)	(0.065)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.176	0.226	0.181	0.198	0.227	0.182

Table D.2: Long-Short Anomaly Portfolio Alphas

This table replicates Table 5 using a mean-variance portfolio constructed using different factor models. Panel A reports results for the CAPM, Panel B reports results for the Hou, Xue, and Zhang (2015, HXZ) factor model, and Panel C reports results for its volatility timed version. Monthly observations. The sample period is 1967 to 2019.

Panel A: CAPM

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.49***	0.48***	0.39***	0.35***	0.31***	0.39***	0.45***
	(0.12)	(0.11)	(0.09)	(0.10)	(0.08)	(0.11)	(0.12)
Adjusted R^2	—	0.44	0.46	0.28	0.42	0.27	0.21

Panel B: HXZ

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.34***	0.36***	0.29***	0.25***	0.24***	0.32***	0.29***
	(0.12)	(0.09)	(0.08)	(0.09)	(0.07)	(0.11)	(0.11)
Adjusted R^2	—	0.54	0.57	0.25	0.46	0.31	0.28

Panel C: Volatility-managed HXZ

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.26**	0.29***	0.25***	0.19**	0.24***	0.27***	0.21*
	(0.10)	(0.09)	(0.07)	(0.09)	(0.08)	(0.10)	(0.11)
Adjusted R^2	—	0.41	0.41	0.14	0.27	0.19	0.17

E Alternative Test Assets

E.1 25 Fama-French Portfolios Sorted on Size and Book-to-Market

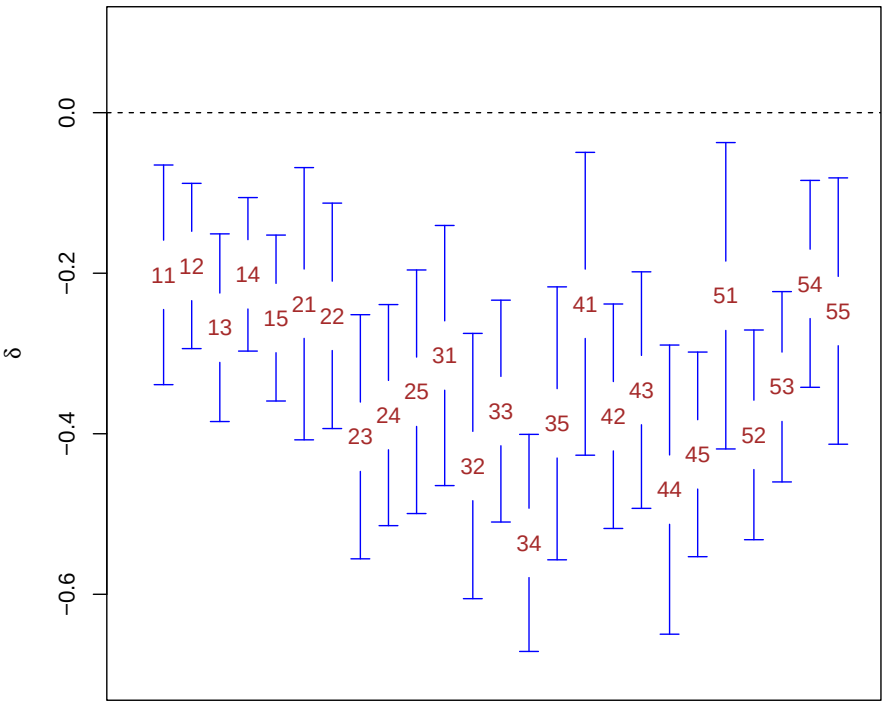


Figure E.1: Anomaly Portfolios and Price Deviations. This figure replicates Figure 2 using the 25 Fama-French portfolios sorted on size and book-to-market. Monthly observations of annual returns. The sample period is 1967 to 2019.

Table E.1: Long-Short Anomaly Portfolio Alphas

This table replicates Table 5 using the 25 Fama-French portfolios sorted on size and book-to-market. Monthly observations. The sample period is 1967 to 2019.

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.44***	0.41***	0.31***	0.30**	0.26***	0.39***	0.42***
	(0.15)	(0.10)	(0.09)	(0.12)	(0.10)	(0.14)	(0.13)
Adjusted R^2	—	0.62	0.65	0.39	0.51	0.20	0.30

E.2 Chen and Zimmermann Open Source Data

Table E.2: Long-Short Anomaly Portfolio Alphas

This table replicates Table 5 using 180 anomaly portfolios from the Chen and Zimmermann (2021) database. Monthly observations. The sample period is 1967 to 2019.

Panel A: FF5

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.60***	0.58***	0.44***	0.45***	0.39***	0.45***	0.70***
	(0.12)	(0.10)	(0.08)	(0.08)	(0.07)	(0.08)	(0.11)
Adjusted R^2	—	0.62	0.70	0.58	0.65	0.55	0.32

Panel B: FF5 with factor-timing

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.69***	0.66***	0.55***	0.57***	0.48***	0.50***	0.79***
	(0.12)	(0.10)	(0.09)	(0.09)	(0.07)	(0.08)	(0.10)
Adjusted R^2	—	0.60	0.66	0.51	0.60	0.51	0.29

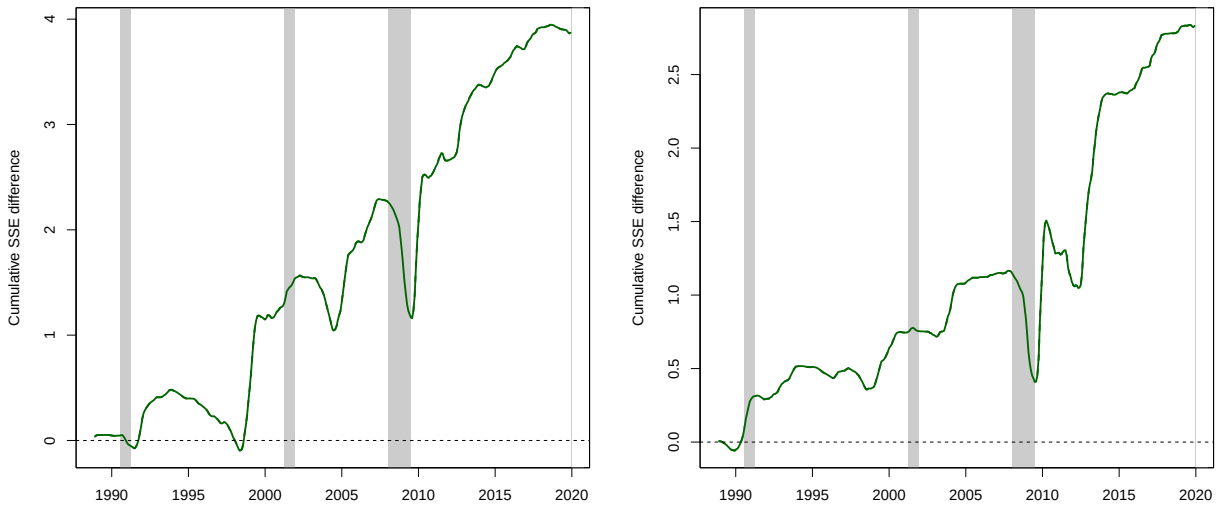
Panel C: Volatility-managed FF5

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.48***	0.48***	0.41***	0.35***	0.37***	0.35***	0.52***
	(0.10)	(0.09)	(0.09)	(0.09)	(0.08)	(0.08)	(0.09)
Adjusted R^2	—	0.48	0.52	0.41	0.45	0.44	0.30

Panel D: FF5-DMRS

	$E[R]$	C4	FF6	q	SY4	DHS3	BG3
Constant	0.61***	0.58***	0.45***	0.43***	0.37***	0.42***	0.68***
	(0.11)	(0.09)	(0.08)	(0.08)	(0.07)	(0.09)	(0.12)
Adjusted R^2	—	0.63	0.70	0.55	0.67	0.55	0.30

F Fama and French (2015): Further Results



(a) Size **(b)** Value

Figure F.1: Size and Value Portfolios. These figures show the OOS performance of predictive regressions for the Fama-French size (small) and value (high) portfolios using price deviations. The graphs show the cumulative squared prediction errors of the historical mean minus the cumulative squared prediction error of the predictive regression using the price deviations. Whenever a line increases, it means that price deviations predict better; whenever it decreases, the prevailing mean predicts better. Price deviations $\hat{u}$ are computed as in equation (14). Monthly observations of annual returns. The sample period is 1967 to 2019; the out-of-sample period starts in 1989.

Table F.1: Predicting Anomaly Returns with Price Deviations: $1/N$ weights

This table replicates Table 2 using a mean-variance portfolio with weights $1/N$. Monthly observations of annual returns. The sample period is 1967 to 2019.

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.307^{***}	-0.220^{***}	-0.328^{***}	-0.290^{***}	-0.280^{***}	-0.312^{***}
	(0.040)	(0.049)	(0.038)	(0.038)	(0.040)	(0.040)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.152	0.194	0.154	0.164	0.167	0.154

Table F.2: Predicting Anomaly Returns with Price Deviations: Decomposing Risk-Adjusted Returns

This table reports pooled estimates for δ_i from predictive regression $r^e_{i,t+1} - \widehat{c}_{i,t} = a + \delta\widehat{u}_{i,t} + \widehat{\beta}_{i,t}r^{mv}_{t+1} + \gamma \mathrm{X}_{i,t} + \epsilon_{i,t}$. Test assets are 90 top and bottom deciles from the anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). Price deviations $\hat{u}$ are computed as in equation (14). We report results for price deviations computed using different heuristic mean-variance efficient portfolios. Panel A reports results for the Fama and French (2015, FF5) factor model, Panels B and C report results for its factor-timing and volatility-managed versions, and Panel D reports results for its characteristics-efficient version computed in Daniel et al. (2020), dubbed FF5-DMRS. Our panel features: $n = 565$, $T = 90$, $N = 50850$. Values in parenthesis are Driscoll and Kraay (1998) robust standard errors for panel models with cross-sectional and serial correlation. ***, **, and * indicates respectively 1%, 5%, and 10% level of significance. Monthly observations of annual returns. The sample period is 1967 to 2019.

Panel A: FF5

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.392^{***}	-0.243^{***}	-0.270^{***}	-0.252^{***}	-0.256^{***}	-0.258^{***}
	(0.041)	(0.038)	(0.034)	(0.030)	(0.032)	(0.031)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.253	0.460	0.465	0.461	0.459	0.460

Panel B: FF5 with factor-timing

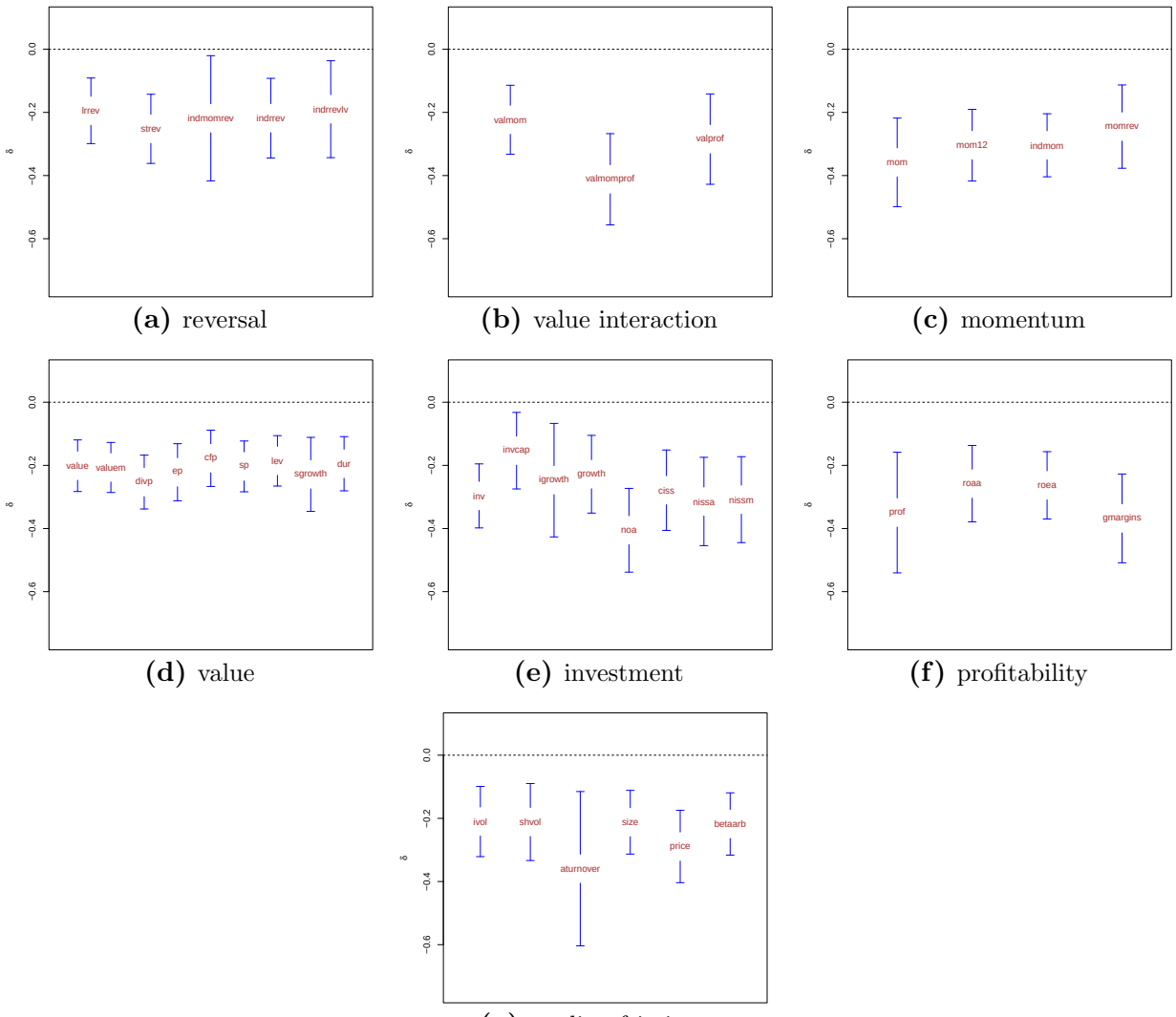
	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.386^{***}	-0.327^{***}	-0.358^{***}	-0.346^{***}	-0.329^{***}	-0.356^{***}
	(0.029)	(0.032)	(0.028)	(0.025)	(0.036)	(0.027)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.274	0.336	0.327	0.343	0.331	0.331

Panel C: Volatility-managed FF5

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.375^{***}	-0.262^{***}	-0.294^{***}	-0.274^{***}	-0.279^{***}	-0.285^{***}
	(0.036)	(0.037)	(0.035)	(0.031)	(0.032)	(0.031)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.275	0.411	0.411	0.412	0.408	0.410

Panel D: FF5-DMRS

	(1)	(2)	(3)	(4)	(5)	(6)
δ	-0.365^{***}	-0.258^{***}	-0.325^{***}	-0.309^{***}	-0.296^{***}	-0.326^{***}
	(0.043)	(0.050)	(0.037)	(0.041)	(0.045)	(0.043)
Controls		Long-Term Reversal	Prior Returns	Book-to-Market	Sentiment	Pricing Errors
Adjusted R^2	0.232	0.304	0.270	0.295	0.284	0.273



(a) reversal **(b)** value interaction **(c)** momentum

(d) value **(e)** investment **(f)** profitability

(g) trading frictions

Figure F.2: Anomaly Portfolios and Price Deviations: Bottom Deciles. This figure replicates Figure 2 using the bottom deciles for the 45 anomaly portfolios constructed in Kozak, Nagel, and Santosh (2020). Monthly observations of annual returns. The sample period is 1967 to 2019.