Living in Infamy: Bad Reputations in Emerging Markets

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Abstract

We present a model of sovereign borrowing in the presence of default risk and asymmetric information. Optimizing sovereigns come in two persistent types with different levels of patience and hence different proclivities to default and borrow. In a stylized model, we construct a pooling equilibrium that is 'infamous' in the sense that the patient sovereign is constrained to borrow like the impatient sovereign on the equilibrium path. We argue that this provides an explanation for the observed lack of private capital markets for many low- and middle-income countries, and use the model to evaluate proposed policy interventions aimed at building a good reputation for such countries.

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1 Introduction

Many sovereign nations have little access to *private* international capital markets, such as bank loans and bond issuance, and instead borrow largely from other governments and the multilateral development banks (MDBs). However, both traditional (Paris Club) and new (China and gulf states) government creditors are scaling back their sovereign lending, while MDBs are close to their lending limits. This has raised concerns in international policy circles about the future availability of development finance, as well as proposals for increasing private lending. For example, based on the presumption that private creditors were deterred by debt overhang and weak institutions in these countries, the Highly Indebted Poor Countries initiative (HIPC) and its successor the Multilateral Debt Relief Initiative (MDRI) both offered debt relief in return for institutional and policy reforms. Unfortunately, the response of private creditors has been modest.

In this paper, we propose an alternative explanation for the lack of access to private capital. We hypothesize that these sovereigns are caught in a bad reputational trap in which good credit risks are unable to borrow enough to establish their ability to repay higher levels of debt. Instead, they are stuck in an 'infamous' pooling equilibrium in which they are constrained to borrow the small amounts available to worse credit risks. What is worse, bad reputations are partially self-fulfilling in that negative investor perceptions are causal in generating worse behavior.

We begin by presenting a stylized model of sovereign borrowing in the presence of default risk as in Arellano (2008), augmented by presence of asymmetric information about a sovereign's type. There are two optimizing types of sovereigns who are assumed to be risk-neutral with different discount factors that are not known to private investors. Everything else

equal, the more impatient (bad) type prefers to borrow more and defaults more frequently; the more patient (good) type prefers to borrow less and defaults less frequently. Both types are more impatient relative to private investors and seek to borrow against their future (constant) endowment in order to consume more in the present. An unobserved shock that affects the desirability of default is symmetric across the types and is the only form of uncertainty. The support of this shock is finite so that equilibrium revelation of types is possible.

In this environment, we construct a pooling Markov Perfect Equilibrium that is 'infamous' in the sense that the good type is constrained to borrow at the maximum level available to the bad type on the equilibrium path. That is, access to private capital markets is severely limited. The good type also defaults with the same frequency (never) as the bad type on the equilibrium path. This infamous equilibrium exists under mild restrictions on out of equilibrium beliefs: They are only assumed to be monotone in borrowing, in that borrowing more than the equilibrium amount cannot improve a sovereign's reputation. The infamous equilibrium is also unique in the class of equilibria wherein both types borrow the same amount at the risk-free rate. This implies that it can only happen that the bad type drags the good type down. The good type can never raise the bad type up, because the bad type always default when they borrow beyond their debt limit regardless of the possible future rewards.

The intuition is the following. When there is perfect information, a powerful feedback loop exists that rewards the good type for its good behavior. The good type defaults less often and so can issue bonds at better prices. These better prices lower debt service costs, which raises the value of repayment relative to default, which fosters even better behavior on the part of the sovereign. Conversely, when types are hidden, investors cannot

condition prices on those types but only on their beliefs regarding them. Conditional on beliefs, the good and bad types face the same budget set. The good type is thus not rewarded for being the good type and, in equilibrium, his behavior worsens to the point where on the equilibrium path he is constrained to borrow as if he were the bad type. Off the equilibrium path his (default) behavior would diverge from the bad type's, but this does nothing to change equilibrium dynamics.

To understand why the patient sovereign cannot signal its type through its borrowing decisions, suppose they were to borrow more than the equilibrium amount. Since higher levels of borrowing are risky, this causes the price of the sovereign bonds to fall, so that the sovereign receives less resources from borrowing on both the marginal unit of borrowing as well as on all inframarginal borrowing up to that point. This cannot directly improve the reputation of the sovereign given our weak monotonicity assumption on out of equilibrium beliefs. However, with some probability, the unobserved taste shock is realized which gives the patient sovereign the opportunity to earn a better reputation by repaying when the impatient sovereign would default. For low enough risk-free interest rates, this gain is guaranteed to be smaller than the lost resources from borrowing at the lower bond price, regardless of the level of the sovereign's initial reputation. That is, any (strictly positive) level of investor pessimism can sustain an infamous equilibrium given a sufficiently small risk-free rate.

There are some technical conditions on the discount rates and shock process to be satisfied before this risk-free argument applies. We show in Appendix B, though, that such infamous dynamics hold even in calibrated quantitative versions of the model in the spirit of Arellano (2008) that we solve with novel machine learning techniques based on Scheidigger and Bilionis (2019) and Gu and Stangebye (2023). As such, the technical pa-

rameter restrictions we require for analytic tractability are not crucial for the general intuition to maintain.

If a good type is caught in an infamous equilibrium, there is nothing that he can do to escape from it. All deviations from equilibrium borrowing and repayment decisions can only worsen his situation. However, there is room for a third-party to improve his circumstances. In the spirit of the HIPC intitiative, we consider whether a benevolent third-party such as an MDB or foreign government can induce separation and, in doing so, free the country from its bad reputation.

We find that this is possible so long as the third-party is willing to shoulder some expected costs. In particular, they may be wrong in their conjecture that the country is the good type and discover the hard way that the country is in fact the bad type. To mitigate this risk, we show that the third-party can employ a gradualistic approach. First, it must lend a relatively small amounts at sub-market yields. In contrast to the infamous equilibrium, each repayment of this loan improves the country's reputation and reduces expected default risk in the next period. Eventually, it becomes safe enough to lend a larger loan that induces separation.

We characterize the set of reputations for which this strategy can be effectively applied as a function of the losses the third-party is willing to endure: The more expected losses the third-party is able to incur, the worse can be the initial reputation of the country to be 'saved.' Worse reputations are more expensive to address not because the sub-market-yield loans are different in size, but because (1) the third-party is more likely to be wrong in its conjecture that the type is good, and (2) separation takes longer as reputations take longer to build.

1.1 Related Literature

Information frictions have long been known to plague markets for international debt e.g. Cole and Kehoe (1998), Chari and Kehoe (2003), Angeletos and Werning (2006), Cole et al. (2016), Carlson and Hale (2006), Van Nieuwerburgh and Veldkamp (2009), or Gu and Stangebye (2023) to name only a few. Investors typically do not have access to all of the information that they would like to have at the time they make the investment decision, largely because the borrowing country is foreign in nature.

In our work, we present a recursive Markov signaling game between a sovereign government and foreign investors. The extant literature on asymmetric information tends to explore models in which default alone as a signal has been extensively studied, though there has been some variation (see Cole and Kehoe [1998], Sandleris [2008], Phan [2017a, 2017b], Amador and Phelan [2021]), or Morelli and Moretti [2023]).

D'Erasmo (2008) is an important quantitative contribution in which default and borrowing both serve as signals in a reputational model in the vein of Arellano (2008) or Aguiar et al. (2016a). We differ from D'Erasmo (2008) in a number of ways. First, our primary contribution is theoretical rather than quantitative, establishing the existence and uniqueness of the infamous equilibrium. Second, our underlying assumptions regarding the sources of asymmetric information are different in relevant ways. Third, in our quantitative extension in Appendix B we employ continuous rather than discrete methods, which Hatchondo and Martinez (2010) show is relevant for this class of models. And finally, in our quantitative application we select the stationary Markov equilibrium as the limit of a finite-horizon game rather than applying a nested loop iterative algorithm.[1]

[1] Different solutions techniques may yield different equilibria entirely in models which have the capacity to engender multiplicity.

More recent and related papers include Amador and Phelan (2021) and Fourakis (2023). In the former, the authors demonstrate that when one type (the good type) has perfect commitment and follows a prescribed borrowing rule, the (bad) optimizing type will find it optimal to repay (with some probability) to get better prices right up until the point where he defaults and resets expectations. Their equilibrium features strategic default and pooling at the level of the *good type*. In our environment, where both types are optimizing along the default and borrowing margins, we find the opposite, namely that the equilibrium features pooling via borrowing at the level of the *bad type*. The bad type in Amador and Phelan (2021) will also eventually default in equilibrium, whereas in our framework neither type ever defaults as they never engage in risky borrowing.

Fourakis (2023) proposes a similar default model with two optimizing types differentiated by their discount factors among other things. The environment is fundamentally different in that the author proposes a discrete environment with specialized and hidden i.i.d. taste shocks of infinite support such that all possible actions are chosen with some probability in every period. This obviates the need to specify off-equilibrium beliefs. This strategy is also employed by Chatterjee et al. (2023), who explore similar questions in consumer credit. In contrast, our model is fully continuous with well-specified off-equilibrium beliefs that are important for existence and equilibrium dynamics. In fact, a key result in our work is the set of restrictions on off-equilibrium beliefs required to sustain a perfect Bayesian equilibrium.

The rest of the paper is organized as follows. In Section 2 we describe the benchmark model and define the equilibrium notion. In Section 3 we derive the benchmark results and explore their ramifications. In Section 4 we consider the policy consequences for an altruistic third-party such as an

MDB or foreign government.

2 Analytic Model

2.1 Sovereign Government

Consider an infinitely-lived sovereign country that has a constant endowment $y_t = y$. The country has a risk-neutral flow utility function and discounts the future at rate $\beta_u < 1$. $u \in \{L, H\}$ is a permanent state that is unobserved by the lenders but known to the sovereign. We assume that $\beta_L < \beta_H$ and refer to the former as the 'bad' type and the latter as the 'good' type.

At time t the country enters the period with short-term debt obligations, b_t. It can choose at this time to default on its debt. If it does so, it sets the default variable, $d_t = 1$. Defaulting entails a loss of access to credit markets as well as a permanent output cost, $\phi \geq 0$, which implies a Bellman equation given by

$$V_{D,t}(u) = y - \phi + \beta V_{D,t+1}(u) \tag{1}$$

If the country does not default, it maintains access to credit markets and can issue new debt, b_{t+1}, at an equilibrium price to be discussed momentarily.

There is a single, i.i.d. additive utility shock to defaulting in every period, m_t. Like the sovereign's type, it is observed only by the sovereign and not the lenders. Unlike the sovereign's type, though, it is iid over time. Its transitory nature introduces a plausible signal-noise structure to the investors' problem. Whenever sovereign behavior is observed, investors will attempt to infer from it the extent to which the behavior was driven by the persistent inclinations of the hidden type as opposed to transitory exigencies induced by some realization of a particular m_t.

We assume further that this shock is binary, satisfying, $m_t \in \{0, m\}$, where $m > 0$. The probability distribution of this Bernoulli variable is time and type invariant and characterized by $Pr(\tilde{m}_t = m) = p \in (0, 1)$.[2]

We collect publicly observable variables at time t into a vector, $x_t = \{d_t, b_{t+1}\}$. Lenders' beliefs regarding the country's unobservable type can be summarized by $\rho_t = Pr(\tilde{u} = L|x^{t-1})$, where x^{t-1} is the history of publicly observable states, $x^{t-1} = \{x_{t-1}, x_{t-2}, \dots\}$. Note that because all agents are Bayesian, ρ_t will operate as a sufficient statistic to describe lender beliefs and their evolution over time.[3]

Following Eaton and Gersovitz (1981), we assume that if the country chooses to repay its debt it can subsequently issue new debt at an equilibrium price $q_t(b_{t+1}|\rho_t, b_t)$. We also make the critical assumption that, just as it cannot commit to repaying extant debt, the government *cannot commit* to a particular level of debt issuance at the time it is making its default decision at the beginning of the period. This is an additional intra-period limited commitment friction similar to Cole and Kehoe (1996), though the timing of the debt auction and default decision are reversed here relative to their work.

This structure obviates the need to consider multi-dimensional signaling spaces in a plausible way, as at any given point in time the sovereign is choosing only a single action and taking all future decisions as given. Multi-dimensional signaling structures introduce significant complications (see Battaglini [2002] or Chakraborty and Harbaugh [2007]) that, for our purposes, are unnecessary.

The presence of b_{t+1} in the pricing function reflects the assumption that

[2]We will express unrealized random variables using a tilde to distinguish them from their scalar or vector realizations.

[3]Beliefs regarding the realization of m_t will exist, but will serve no purpose in forecasting either β_u or m_{t+1}, and so for brevity we do not assign them any notation.

the sovereign is a monopolist in its own debt market and internalizes the impact of debt issuance on equilibrium prices. The presence of ρ_t implies that beliefs and reputations can influence prices.

The presence of b_t is needed because investors can use it in conjunction with sovereign issuance (b_{t+1}) to infer information about the sovereign's type. It implies that investors can potentially learn about the sovereign's type twice in a period: First through its repayment decision, i.e., d_t, and second through its issuance decision, i.e., b_{t+1}.

We will use the following notation to distinguish between beliefs at these different points in a period. Beliefs at the beginning of period t, i.e., prior to any decisions being made, will be denoted by ρ_t. Beliefs *after* repayment but *before* debt issuance will be denoted by ρ_t^R.

With this notation we can write out the repayment Bellman as follows.

$$
\begin{aligned}
V_{R,t}(b_t, \rho_t^R|u) = \max_{b_{t+1}\in[0,\bar{b}]} \; & y - b_t + q_t(b_{t+1}|\rho_t^R, b_t)b_{t+1} + \qquad (2)\\
& \beta_u \mathbb{E}_t \left[(1 - d_{t+1}(b_{t+1}, \tilde{m}_{t+1}, \rho_{t+1}|u))\, V_{R,t+1}(b_{t+1}, G_{t+1}^R(b_{t+1}, \rho_{t+1})|u) + \right.\\
& \left. d_{t+1}(b_{t+1}, \tilde{m}_{t+1}, \rho_{t+1}|u)\, (V_{D,t+1}(u) + \tilde{m}_{t+1}) \right]\\
\text{where} \quad & \rho_{t+1} = G_t(b_{t+1}, b_t, \rho_t^R)
\end{aligned}
$$

where G_t and G_t^R are belief-updating functions for debt issuance and repayment respectively that will be discussed momentarily. d_{t+1} is the binary default decision that is made in the future to which the sovereign cannot commit today. The sovereign takes all of these as given.

Note that we assume debt levels lie on a continuous interval from $[0, \bar{b}]$, where $\bar{b} < \infty$. This last assumption rules out potential Ponzi schemes, though we will assume that $\bar{b}$ is so large as to never impose a binding constraint in equilibrium debt issuance.

Because the sovereign lacks commitment, he will default on his extant

debt stock whenever the value of doing so is strictly greater than the value of repaying it. This condition defines the default decision rule as follows.

$$d_t(b_t, m_t, \rho_t|u) = \mathbf{1}\{V_{R,t}(b_t, G_t^R(b_t, \rho_t)|u) < V_{D,t}(u) + m_t\} \tag{3}$$

Notice that the sovereign accounts for the fact that repaying his debt will influence lender beliefs, as evidenced by the passing of them through G_t^R. Defaulting would also influence them, but because the sovereign never again interacts with foreign investors, these beliefs are irrelevant to the sovereign's value of defaulting and thus are omitted to simplify notation.

Because investors do not observe m_t directly, it will be notationally useful for us later to define default probabilities that average over these shocks, i.e.,

$$\bar{d}_t(b_t, \rho_t|u) = (1-p)d_t(b_t, 0, \rho_t|u) + pd_t(b_t, m, \rho_t|u) \tag{4}$$

2.2 Foreign Investors

Foreign investors are risk-neutral, deep-pocketed, and have access to an outside option that yields a gross risk-free return, R. Their optimal behavior thus implies an equilibrium pricing condition given by

$$q_t(b_{t+1}|\rho_t^R, b_t) = \frac{\mathbb{E}_t\left[1 - d_{t+1}(b_{t+1}, \tilde{m}_{t+1}, G_t(b_{t+1}, b_t, \rho_t^R))|\tilde{u})\right]}{R} \tag{5}$$

The only thing that distinguishes this condition from its standard counterparts in Arellano (2008) or Aguiar and Gopinath (2006) is that investors must take expectations not only over fundamental shocks (here, $\tilde{m}_{t+1}$), but also over the potential types (here, $\tilde{u}$) according to their belief distribution.

We assume that investors are more patient than either type of sovereign, i.e., $0 < \beta_L < \beta_R < 1/R < 1$.

2.3 Belief-Updating Functions

In each period, investors have two opportunities to infer payoff-relevant information from sovereign behavior: First at repayment and, conditional on repayment, at issuance.

We begin with the repayment decision. On the equilibrium path, we assume that investors update their beliefs via Bayes' rule. Off the equilibrium path, we will allow for a flexible formulation with the core assumption that repayment improves beliefs. In particular, we assume that off-equilibrium beliefs are dictated by some function $\nu_R(b_t, \rho_t) \in [0, \rho_t]$. This implies that beliefs are always weakly improved following a repayment when equilibrium play dictates default by all types. In the worst case scenario, beliefs remain at the status quo and in the best case scenario investors assign probability one to the sovereign being the good type.

Writing this out explicitly[4] yields

$$G_t^R(b_t, \rho_t) = \tag{6}$$
$$\begin{cases} \frac{\left[1-\bar{d}_t(b_t,\rho_t|L)\right]\rho_t}{\left[1-\bar{d}_t(b_t,\rho_t|L)\right]\rho_t+\left[1-\bar{d}_t(b_t,\rho_t|H)\right](1-\rho_t)}, & d_t(b_t, m_t, \rho_t|u) = 0 \text{ for some } (u, m_t) \\ \nu_R(b_t, \rho_t) & o/w \end{cases}$$

Next is the issuance decision conditional on repayment. We denote equilibrium debt issuance by $a_t(b_t, \rho_t^R|u)$ for a given hidden type u, which is to say that this function satisfies the maximization specified in Equation 2.

On the equilibrium path, beliefs are updated in accordance with Bayes' rule. The country can deviate from the equilibrium path in one of two ways: Over-issuance or under-issuance. As we did with the repayment

[4]Note we do not need to write out the belief-updating function (and its associated off-equilibrium beliefs) for the case when the country defaults, as in this case the country is excluded from credit markets forever and reputation will never again matter. In the quantitative augmentation of the model (Appendix B), this will become relevant because of stochastic re-entry into credit markets.

decision, we model off-equilibrium beliefs in a general way and make only the following intuitive restriction: Over-issuance tends to worsen beliefs while over-issuance tends to improve them. To capture the over-issuance case, we posit a general, decreasing function, $\nu_+ : \mathbb{R}^+ \to [0,1]$ that operates as follows

$$\rho_{t+1} = \nu_+(b_{t+1} - \max_u a_t(b_t, \rho_t|u)) \times \rho_t^R + \Big[1 - \nu_+(b_{t+1} - \max_u a_t(b_t, \rho_t|u))\Big] \times 1$$

which is to say that updated beliefs are a convex combination of the status quo and the worst possible beliefs. The fact that ν_+ is decreasing implies that additional issuance beyond the equilibrium cap can only worsen beliefs; never improve them.

We posit another, general decreasing function for beliefs following under-issuance, $\nu_- : \mathbb{R}^+ \to [0,1]$ that operates similarly, i.e.,

$$\rho_{t+1} = \nu_-(\min_u a_t(b_t, \rho_t|u) - b_{t+1}) \times \rho_t^R + \Big[1 - \nu_+(\min_u a_t(b_t, \rho_t|u) - b_{t+1})\Big] \times 0$$

Here we have a convex combination between the status quo and zero. The decreasing nature of the function implies that beliefs must (weakly) improve as the country issues less and less debt off the equilibrium path.

Spelling this out formally, we get

$$G_t(b_{t+1}, b_t, \rho_t^R) = \tag{7}$$

$$\begin{cases} \frac{\mathbf{1}\{b_{t+1} = a_t(b_t, \rho_t^R|L)\} \times \rho_t^R}{\mathbf{1}\{b_{t+1} = a_t(b_t, \rho_t^R|L)\} \times \rho_t^R + \mathbf{1}\{b_{t+1} = a_t(b_t, \rho_t^R|H)\} \times (1 - \rho_t^R)}, & a_t(b_t, \rho_t^R|u) = b_{t+1} \text{ for some } u \\ 1 - \nu_+(b_{t+1} - \max_u a_t(b_t, \rho_t|u))(1 - \rho_t^R) & b_{t+1} > a_t(b_t, \rho_t^R|u) \ \ \forall u \\ \nu_-(\min_u a_t(b_t, \rho_t|u) - b_{t+1})\rho_t^R & o/w \end{cases}$$

2.4 Equilibrium Definition

We can now define our equilibrium concept. A **Markov Perfect Bayesian Equilibrium** is a collection of functions across time, $\{V_{D,t}, V_{R,t}, d_t, q_t, G_t^R, G_t\}$ for any t such that Equations 1-7 are satisfied.

We assume that if an equilibrium with interior beliefs, i.e., $\rho_t \in (0,1)$ cannot exist, then the economy must coordinate on a full-information equilibrium with $\rho_t \in \{0,1\}$ for all t.[5] These beliefs must be correct in that the good type must be met with $\rho_t = 0$ and the bad type with $\rho_t = 1$. If an alternative equilibrium with interior beliefs exists, then it must be consistent with equilibrium behavior for both types as prescribed in the equilibrium conditions.

3 Analytic Model: Findings

We begin with a few mild and reasonable assumptions.

Assumption 1. $\phi - m(1-\beta_u) \geq 0$ *for* $u \in \{L, H\}$

The first assumption ensures that flow default costs net of the effective flow benefit are large enough to sustain foreign borrowing.

Assumption 2. $\frac{p(\phi - m(1-\beta_u))}{R-1} \geq \frac{(1-p)}{R} m\left(1 - \beta_u R\right)$ *for* $u \in \{L, H\}$

The second assumption will wind up ensuring that the risk of $\tilde{m} = m$ realizing is sufficiently large that, regardless of its type, the sovereign will not borrow into the risky region where default is a possibility. The left-hand-side will wind up being the effective cost of doing this and the right-hand-side will be the benefit. This assumption can be easily satisfied either by setting p to be fairly large, shrinking R toward unity, or both.

Two more assumptions regarding the relative impatiences of the two types is required.

[5] We verify in Appendix A that such an equilibrium always exists.

Assumption 3. $\beta_H \leq .5 \times 1 + .5 \times \beta_L$

Assumption 4. $\left(\frac{1}{1-p} - (1-\rho)\right)(\phi - m(1-\beta_L)) \geq \frac{\beta_H}{1-\beta_H} \times m \times (\beta_H - \beta_L)$

These assumptions dictate that the good type is not too patient relative to the bad type. They help ensure the shape of the infamous equilibrium characterized below and also with precluding certain possible over-issuance deviations.

These assumptions are fairly weak relative to the dynamics they engender, as we'll observe in a simple numerical example soon. While they are necessary for our analytic derivations, they are typically not necessary to engender dynamics like those in the results we will exposit momentarily. We show in Appendix B that many of the 'infamous dynamics' we outline in the section also hold in a quantitative incarnation of the benchmark model in the vein of Arellano (2008) that we solve with novel machine learning techniques based on Scheidigger and Bilionis (2019) and Gu and Stangebye (2023).

With these assumptions in hand, we can begin to lay out the results. Proofs of all results can be found in Appendix A.

Proposition 1 (**Infamous Equilibrium**). *If ν_- does not decrease too quickly and $R > 1$ is sufficiently close to 1, then a time-invariant Markov Perfect Bayesian Equilibrium with interior beliefs of the following kind always exists. The pricing function is decreasing with non-overlapping intervals on the debt issuance domain given by*

1. $\mathcal{B}_1 = \left[0, \frac{R(\phi - m(1-\beta_L))}{R-1}\right]$, *with price* $q_t(b_{t+1}|\rho_t^R, b_t) = \frac{1}{R}$
2. $\mathcal{B}_2 = \left(\frac{R(\phi - m(1-\beta_L))}{R-1}, \frac{R(\phi - m(1-\beta_L))}{R-1} + m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right)\right]$, *with price* $q_t(b_{t+1}|\rho_t^R, b_t) = \frac{1-p\left[1-\nu_+\left(b_{t+1} - \frac{R(\phi - m(1-\beta_L))}{R-1}\right)(1-\rho_t^R)\right]}{R}$
3. $\mathcal{B}_3 = \left(\frac{R(\phi - m(1-\beta_L))}{R-1} + m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right), \frac{R(\phi - m(1-\beta_L))}{R-1} + m\right]$, *with price* $q_t(b_{t+1}|\rho_t^R, b_t) = \frac{1-p}{R}$

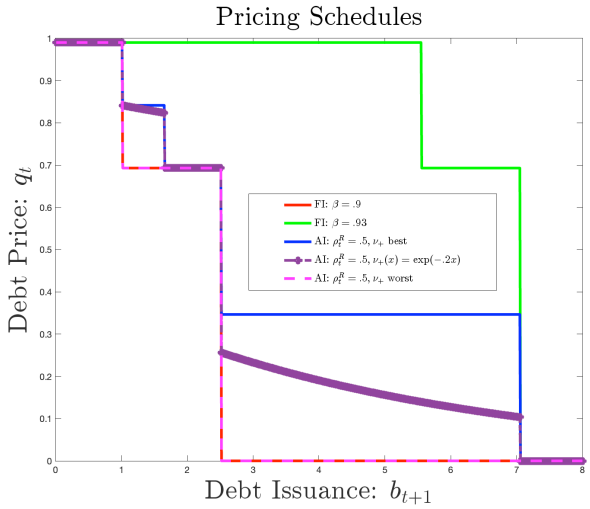


Figure 1: Infamous Equilibrium Examples

4. $\mathcal{B}_4 = \left(\frac{R(\phi - m(1-\beta_L))}{R-1} + m, \frac{R(\phi - m(1-\beta_H))}{R-1} + m\right]$ *with price*
$q_t(b_{t+1}|\rho_t^R, b_t) = \frac{(1-p)\nu_+\left(b_{t+1} - \frac{R(\phi - m(1-\beta_L))}{R-1}\right)(1-\rho_t^R)}{R}$

5. $\mathcal{B}_5 = \left(\frac{R(\phi - m(1-\beta_L))}{R-1} + m, \bar{b}\right]$, *with price* $q_t(b_{t+1}|\rho_t^R, b_t) = 0$

We call this equilibrium the "infamous equilibrium."

There is much to unpack in this key result, but it is perhaps easiest to first observe what the infamous equilibrium looks like visually in a simple example. Consider the following calibration, which satisfies all of the assumptions required for Proposition 1 to hold. We set $R = 1.01$, $p = .3$, $m = 1.5$, $y = 1.0$, and $\phi = .16$. The two values of β are 0.9 and 0.93, both of which are empirically relevant in the quantitative literature (see Aguiar et al. [2016b]).

Figure 1 depicts the full-information pricing schedules[6] as well as a collection of infamous equilibrium pricing schedules differentiated by their specifications for off-equilibrium beliefs. In all cases, we set investors' prior to maximum agnosticism, i.e., $\rho_t^R = 0.5$. The best infamous equilibrium

[6]The full-information equilibrium is unique. See Appendix A for an argument to this effect.

maintains status quo beliefs upon any over-issuance (ν_+ best), the worst infamous equilibrium assumes immediate assignment of beliefs to the bad type upon any over-issuance (ν_+ worst), and the intermediate infamous equilibrium assumes that additional over-issuance gradually worsens beliefs.

We observe, not surprisingly, that the pricing schedule in the worst infamous equilibrium literally overlaps with the full-information schedule of the bad type. The reason is because all over-issuance immediately assigns beliefs irrevocably to the bad type. What is more surprising is just how far the best infamous equilibrium is from the full-information pricing schedule of the good type. Even under these rosy assumptions for off-equilibrium beliefs, i.e., status quo beliefs following over-issuance, there is a significant contraction of the budget set. This is the sense in which the equilibrium is always 'infamous.'

Significant consequences of the infamous equilibrium will be explored in the corollaries that follow.

Corollary 1. *The infamous equilibrium features on-equilibrium pooling along both borrowing and default margins. In particular,*

1. *Both types always repay for any m_t.*
2. *For any $b_t \in [0, \bar{b}]$ and any ρ_t^R, the country borrows $a_t(b_t, \rho_t^R|u) = \frac{R(\phi - m(1-\beta_L))}{R-1}$ for either u. This is the full-information risk-free debt limit for the bad type.*

Corollary 1 tells us about the dynamics of the infamous equilibrium. It asserts that both types borrow as much as is allowed at the risk-free rate and repay it in every period. They are constrained in this issuance by the amount specified in the corollary. Interestingly, this amount ($R(\phi - m(1 - \beta_L))/(R-1)$) is exactly equal to this risk-free limit for the *bad* type in the full-information case.

This is true regardless of investor beliefs and the reason is as follows. The good type would indeed repay more than the bad type, but there are two forces working against this taking place. First, off-equilibrium beliefs for over-issuance are met with a *worsening* of beliefs, not an improvement. Thus, if the good sovereign tried to over-issue with the hopes of distinguishing himself by paying more back, it would have the counterproductive effect of worsening his reputation today. Second, over-issuance itself at the equilibrium price schedule is sub-optimal regardless of reputational considerations. The drop in prices the good type would receive *on his entire stock of debt* is too large to justify issuing a marginal amount more into risky territory to distinguish himself.

This is why the condition on R being close to one is pivotal for existence. The closer R is to one, the greater is the capacity for *both* types to issue risk-free debt. R has no bearing on the *additional* issuance allowed in the risky region, though; this is determined entirely by m and the two discount factors. Thus, a low R implies that price reductions from risky borrowing are very expensive relative to the modest additional quantities that can be issued. This is precisely the reason for Corollary 2.

Corollary 2. *As R decreases toward unity, the range of initial beliefs (ρ_t^R) consistent with an infamous equilibrium expands.*

Corollary 2 is noteworthy because it says something about the sorts of environments in which infamous equilibria can arise. If no infamous equilibrium exists, then no equilibrium that in any way resembles it can exist at all[7]. In high interest rate environments, then, the only allowable equilibria are full-information outcomes where investors assign probability one beliefs to the correct type. As interest rates come down, though, the region of beliefs that can support infamous dynamics expands. This suggests that the

[7]This is established later in Proposition 3.

secular reduction in interest rates from the developed world that has been observed over the past several decades (Rachel and Smith [2017]) may have had the effect of inducing infamous equilibrium dynamics in more emerging markets.

Despite the fact that both types are constrained to issue debt at the risk-free price of the bad type under full-information, the pricing schedule off-equilibrium reflects the differences in default behavior across the types and accounts for investor beliefs. And there are significant differences for the good type in default behavior between the infamous equilibrium and its full-information counterpart. This is made clear in Corollary 3.

Corollary 3. *In the infamous equilibrium, the bad type defaults exactly as he would under full information but the good type defaults at lower debt levels relative to the full-information equilibrium.*

The first part of this corollary follows because the bad type receives exactly the same continuation value in the infamous equilibrium as he does under full information. Namely, he services his debt limit forever at the risk-free price. This implies that equilibrium default behavior is identical between these cases.

The second part of this corollary is more interesting. The good type receives a substantially worse pricing schedule in the infamous equilibrium relative to full-information. What this does in equilibrium is *reduce the value of repaying extant debt.* A lower value of repayment implies more frequent default. Thus, default behavior for the good type occupies significantly more of the state space in the infamous equilibrium. For instance, the good type ceases to repay with probability one at $R(\phi - m(1-\beta_H))/(R-1)$ under full-information; in the infamous equilibrium, though, this occurs at the supremum of $\mathcal{B}_2$, which is significantly less. In Figure 1 for example, the latter is nearly a quarter the size of the former.

The pooling of borrowing and default behavior implies the next result, Corollary 4, which highlights the hopelessness of the infamous equilibrium for the good type.

Corollary 4. *Beliefs never improve in the infamous equilibrium, i.e.,* $\rho_\tau = \rho_\tau^R = \rho_t$ *for any* $\tau \geq t$.

As the investors will never observe any behavior except for repayment at the prescribed equilibrium debt limit, the investors will never get an opportunity to update their beliefs. As such, their beliefs following both repayment and debt issuance will always be constant. The good type will thus never be able to 'break out' of the budget set of the bad type barring some sort of third party intervention outside the scope of the current model.

The infamous equilibrium has consequences for macroeconomic objects of interest, for instance capital inflows. This is established in Proposition 2.

Proposition 2. *In the infamous equilibrium, the good type faces a reduction in capital inflows (relative to full information) of*

$$\frac{Rm(\beta_H - \beta_L)}{R-1}$$

which is increasing as R *decreases toward unity.*

By reducing the effective debt limit, the infamous equilibrium reduces capital inflows significantly. The magnitude of this reduction increases as gross interest rates converge to unity from above. So not only are infamous equilibria more likely to emerge in low interest rate environments, but they will also be more damaging in such environments.

At a fundamental level, infamous equilibria have the property that the presence of the bad type drastically shrinks the budget set of the good type and worsens his default behavior as a consequence. It is natural to wonder

whether the reverse is possible: Does an equilibrium exist where the good type 'lifts up' the bad type, expanding his budget set and reducing his propensity to default? The next result answers this negatively.

Proposition 3. *The infamous equilibrium is the unique Markov Perfect Bayesian Equilibrium in which both types always repay and thus borrow at the risk-free rate.*

The reason why the infamous equilibrium is unique in its class is because the bad type's full-information risk-free debt limit, which is that point at which all types borrow in the infamous equilibrium, is the maximum amount that the bad type would ever repay given risk-free prices. If a greater reputation ever improved his budget set further at risk-free prices, he would simply borrow into this range and then default with strictly positive probability. But this would be inconsistent with risk-free prices and thus cannot constitute an equilibrium.

The reason why the reverse is possible is because the good type is constrained to issue at debt levels that are *less* than he would repay given risk-free prices. This implies a reduction in the value of repayment, which in turn reduces the amount he is willing to repay to retain access to credit markets.

4 Policy Consequences: Third-Party Assistance

When a country finds itself in an infamous equilibrium, there is nothing they can do on their own to escape it. All deviations from equilibrium play serve only to make a bad situation worse.

However, it is possible for an altruistic third-party, such as a multilateral development bank (MDB), to induce separation between the types. In this section we outline how this can be accomplished.

In particular, we characterize the lowest-cost way that a third-party can help a good-type debt government escape from the infamous equilibrium and separate. The environment and assumptions will all be as required for an infamous equilibrium to maintain, i.e., the assumptions underpinning Proposition 1. We know from this result that the good type cannot signal his own way out by altering his repayment, default, or issuance decision.

We assume that the third-party has linear utility, derives a strictly positive utility benefit, $S > 0$, once separation is achieved,[8] and is willing to expend (in expectation) up to some monetary amount $D > 0$. The third-party lender momentarily supplants private creditors, such that the country is never dealing with more than one creditor at a time.

We show here that this can be done by lending a to the country at sub-market interest rates. We characterize dynamic trajectories of such lending that serve to foster eventual separation. We begin by observing that there are two types of sub-market-rate loans that can be extended to the country:

1. Purchasing $\hat{b}_B \in (b_{S,L}, b_{R,L}]$ at the risk-free rate. This strategy will induce the bad type to default (but not the good type) when a default taste shock arrives. If no such shock arrives, both types repay. Repeated application of such lending would have the following effect: If the sovereign is in fact the bad type, then eventually he will reveal himself via default. If the sovereign is the good type, he will never fully reveal himself, but he will *build* his reputation (hence the 'B').

2. Purchasing $\hat{b}_S \in (b_{R,L}, b_{S,H}]$ at the risk-free rate. This strategy induces immediate separation. The bad type will default on this loan with probability 1 and the good type will repay it with probability 1 as doing so reveals his type and thus grants him the continuation value

[8] We assume this benefit is achieved even if the type is bad, but the results are robust to it being only attained if the type is revealed to be good.

associated with the (vastly better) full-information pricing schedule (hence the 'S' for *separation*).

4.1 Designing Financial Assistance Packages

We consider the following policy proposal:

- Lend a relatively small amount, $\hat{b}_B \in (b_{S,L}, b_{R,L}]$ repeatedly to build the country's reputation. To minimize losses, we will set $\hat{b}_B = b_{S,L} + \epsilon$ for some small but strictly positive ϵ.[9]
- After a finite amount of time, increase the size of the loan to $\hat{b}_S = \hat{b}_B + m$, which will by construction be $> b_{R,L}$. This will induce full separation.

This strategy reduces expected losses for a couple of reasons:

- Losses in the 'reputation-building' stage will be smaller because the quantity of debt lent is smaller *and* the risk of incurring a loss is lower (it only happens with probability p).
- By the time the third-party lends the separation-inducing larger amount, $\hat{b}_B + m$, the reputation is sufficiently established that expected losses are small.

With this in mind, we can characterize the third party's optimal strategy. First, we can say something about stopping time.

Proposition 4. *Under the gradual lending policy, a good type will separate in finite time (and a bad type will eventually default).*

If we express the expected (net present) cost associated with this policy as given a reputation, ρ_t as $EL_t(\rho_t)$, we can also say the following

[9] Because the default set of the bad type is open here, a loss minimizing strategy does not technically exist. So we assume that the third-party must lend at least some arbitrarily small ϵ more than the relevant thresholds.

Proposition 5. *$EL_t(\rho_t)$ exists and is unique, bounded, and strictly increasing in ρ_t.*

Given that EL_t is strictly increasing, it can be inverted. We define $\pi_{salvageable,t}(D) = EL_t^{-1}(D)$. This function then traces out the worst set of beliefs under which a country can be 'saved' given a finite amount of resources devoted to expected losses, $D > 0$. It is easy to see that $\pi_{salvageable,t}(D)$ is increasing, so the more resources the third party is willing to allocate to this cause, the worse can be the prior. This relationship can be observed in Figure 2, which assumes the sample parameters in the benchmark model and additionally that $S = \hat{b}_B$.

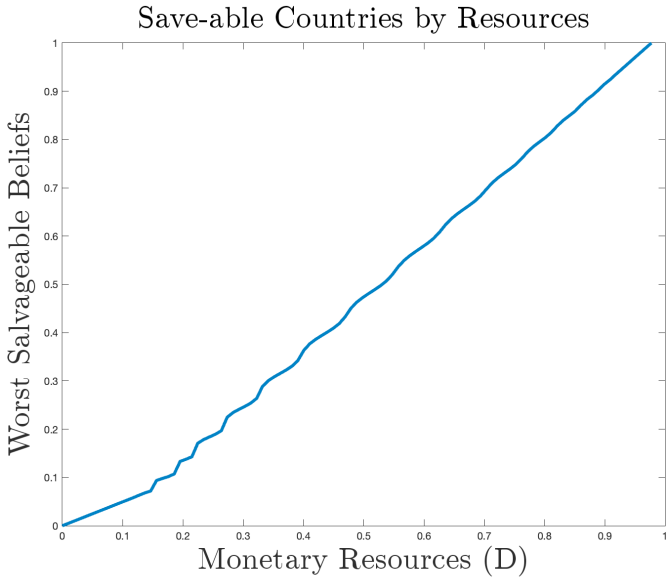
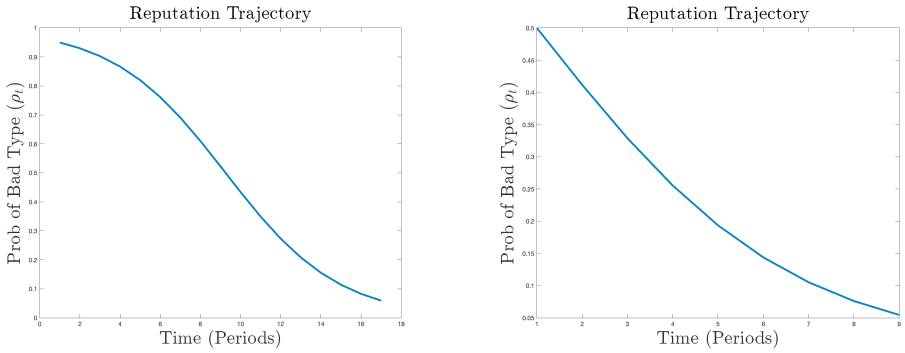


Figure 2: $\pi_{salvageable,t}(D)$

We can trace out the dynamics of this lending policy and the associated beliefs under different initial degrees of pessimism. These trajectories can be found in Figure 3, which plot the beliefs during the period of third-party intervention until the last (separating period). We observe that it takes more than twice as long to induce separation when $\pi_t = 0.95$ relative to the case when $\pi_t = 0.5$. The expected cost is also significantly higher and can

be computed from the inverse of Figure 2.



(a) High Initial Pessimism **(b)** Low Initial Pessimism

Figure 3: Sample Reputation Trajectories

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A Proofs of Analytic Findings

The proofs will not follow exactly the order laid out in the body of the paper. Rather, we will first characterize the equilibrium in the full-information case (in which investors can perfectly observe both β_u and m_t) and use it to construct the infamous equilibrium in the case of asymmetric information.

A.1 Proof of Proposition 1: Preliminaries

We will drop the u-subscript on the β for this section as all results will apply to both types so long as they satisfy Assumptions 1 and 2. The dependency on ρ_t or ρ_t^R also disappears for the relevant functions.

We first observe that the sovereign's scalar default value, in the absence of the utility shock, can be expressed in the following terms

$$V_{D,t} = \frac{y - \phi}{1 - \beta}$$

The next thing we can immediately observe as a result of lenders being risk-neutral and deep-pocketed is that the equilibrium pricing expression has the following form:

$$q_t(b_{t+1}) = \frac{\mathbb{E}_t \left[1 - d_{t+1}(b_{t+1}, \tilde{m}_{t+1})\right]}{R} \tag{8}$$

Notice that not only has the dependency on ρ_t^R disappeared, but so too has the dependency on b_t. The only reason for its inclusion in the pricing function in the first place was that its difference with b_{t+1} could be used to infer some information about the sovereign type that could be relevant for prices. In the full-information case, though, the pricing function will always be flat in b_t and so we drop this argument for brevity.

With these preliminaries, we can assert the following proposition:

Lemma 1. *$q_t(b_{t+1})$ is a decreasing step function with three non-overlapping regions.*

1. *If $b_{t+1} \in \mathcal{B}_{1,t}$, then $q_t(b_{t+1}) = \frac{1}{R}$*
2. *If $b_{t+1} \in \mathcal{B}_{2,t}$, then $q_t(b_{t+1}) = \frac{1-p}{R}$*
3. *If $b_{t+1} \in \mathcal{B}_{3,t}$, then $q_t(b_{t+1}) = 0$*

It is further the case that $\mathcal{B}_{1,t} \cup \mathcal{B}_{2,t} \cup \mathcal{B}_{3,t} = [0, \bar{b}]$.

Proof. Since $V_{D,t}$ is constant in b_t, we need only establish that $V_{R,t}$ is decreasing in b_t. This is easy to see by the principle of optimality. Let $b < b'$ and let $a_t(b')$ be the optimal policy for b_{t+1} given b'. Given that the flow payoff is decreasing in b_t, if we evaluate the payoff given an initial level of debt b using the feasible policy $a_t(b')$ (instead of optimal policy), we will find that the value is at least as large as $V_{R,t}(b')$. Further, the optimal strategy given an initial debt b must yield a higher overall value then any other feasible alternative (including $a_t(b')$). This implies that $V_{R,t}(b) \geq V_{R,t}(b')$ and thus $V_{R,t}$ is a decreasing function.

The proposition follows from the fact that the expected default frequency will be increasing in b' as a consequence. It will be stepped as the proposition asserts because of the binary nature of the shock. In $\mathcal{B}_1$, the shock $\tilde{m}$ will never sway the sovereign to default; in $\mathcal{B}_2$, the sovereign will only default if the shock is realized $\tilde{m} = m$; and in $\mathcal{B}_3$, the sovereign will default no matter what. The probabilities of these events determine the values of the prices laid out in Proposition 1. The thresholds on the regions will be determined shortly. □

It is worth noting that the step-function will feature right-hand continuities and left-hand discontinuities as a consequence of the assumption that the country repays when indifferent as spelled out in Equation 3.

We use Proposition 1 to define some new terms: Let $b_{S,t} = \sup \mathcal{B}_{1,t}$ and $b_{R,t} = \sup \mathcal{B}_{2,t}$. With these in hand we can say the following:

Lemma 2. *The optimal debt issuance policy function is constant and satisfies* $a_t(b_t) \in \{b_{S,t}, b_{R,t}\}$ *for any* $b_t \in [0, \bar{b}]$.

Proof. To see this, we will conjecture that the optimal policy lies outside of this set and show that the sovereign can always improve by moving to an object in this set.

Let us consider a candidate for optimal policy $\hat{b} \in \mathbf{int}(\mathcal{B}_{1,t})$ for some arbitrary initial b_t. If this were the case, then the price of issuance would be $1/R$, as was established in Proposition 1. In this case, the sovereign could issue some $\hat{b} + \epsilon \in \mathcal{B}_{1,t}$ for some small but positive ϵ. The utility gain in the current period would be ϵ/R and the discounted utility loss in the next period would be (at most) $\beta\epsilon$. We say 'at most,' as this would be the utility loss accrued by not changing the issuance strategy tomorrow, which would serve as a lower bound for the payoff implied by the optimal policy tomorrow. Because $1/R > \beta$, the sovereign's payoff is strictly improved by this deviation and thus $\hat{b}$ cannot be optimal. Such a deviation is always possible until the sovereign issues at $b_{S,t}$, in which case $b_{S,t}+\epsilon$ is accompanied by a discrete drop in the price for any $\epsilon > 0$. Thus, $b_{S,t}$ is a candidate for optimal issuance and is preferred over any issuance in $\mathcal{B}_{1,t}$.

Let us consider alternatively a candidate level of issuance $\hat{b} \in \mathbf{int}(\mathcal{B}_{2,t})$ for some arbitrary initial b_t. If this were the case, then the price of issuance would be $(1-p)/R$, as was established in Lemma 1. In this case, the sovereign could issue some $\hat{b} + \epsilon \in \mathcal{B}_{2,t}$ for some small but positive ϵ. The utility gain in the current period would be $\epsilon(1-p)/R$ and the discounted utility loss in the next period would be (at most) $\beta\epsilon(1-p)$, as the sovereign will only be required to honor this debt in the state in which he does not default, which happens with probability $1-p$ in this region. We say

'at most,' as this would be the utility loss accrued by not changing the issuance strategy tomorrow, which would serve as a lower bound for the payoff implied by the optimal policy tomorrow and because, if he does default, it will be because he is strictly better off by doing so. Because $1/R > \beta$, the sovereign's payoff is strictly improved by this deviation. This deviation is always possible until the sovereign issues at $b_{R,t}$, in which case $b_{R,t} + \epsilon$ is accompanied by a discrete drop in the price (to zero) for any $\epsilon > 0$. Thus, $b_{R,t}$ is a candidate for optimal issuance and is preferred over any issuance in $\mathcal{B}_{2,t}$.

Finally, we show that the country would never issue in $\mathcal{B}_{3,t}$. If he issued some $\hat{b} \in \mathcal{B}_{3,t}$, then there would be no intertemporal transfer of resources: Debt would command a zero price today, which generates no revenue, and there is no repayment tomorrow. Further, the sovereign faces a certain future default cost of ϕ tomorrow. The sovereign could do strictly better than this by issuing some small level of debt $\epsilon \in \mathcal{B}_{1,t}$, which would engender a utility gain of ϵ/R, which is larger than the utility loss of $\beta\epsilon$. The sovereign also avoids default costs this way, so it is clearly better. And since we have already established that $b_{S,t}$ dominates all issuance levels in $\mathcal{B}_{1,t}$, $b_{S,t}$ must dominate any potential issuance in $\mathcal{B}_{3,t}$.

The constancy of the policy function follows from the linearity of the utility (and thus the constancy of the marginal utility).

□

We can now begin to sketch out exactly what the equilibrium pricing schedule and borrowing/default behavior look like in the full-information case.

Lemma 3. *The equilibrium is unique and time-invariant. Further, the budget set is increasing in β. In particular, the non-overlapping sub-intervals*

of the debt domain are given by

1. $\mathcal{B}_1 = \left[0, \frac{R(\phi - m(1-\beta))}{R-1}\right]$
2. $\mathcal{B}_2 = \left(\frac{R(\phi - m(1-\beta))}{R-1}, \frac{R(\phi - m(1-\beta))}{R-1} + m\right]$
3. $\mathcal{B}_3 = \left(\frac{R(\phi - m(1-\beta))}{R-1} + m, \bar{b}\right]$

Further, for any $b \in [0, \bar{b}]$, $b'(b) = \frac{R(\phi - m(1-\beta))}{R-1} = b_S$.

Proof. We begin by determining the thresholds of the sub-intervals. To determine the thresholds of the sub-intervals, we need to know exactly the value functions of repayment and default. Thus, we will need to know future optimal borrowing behavior. Utilizing Lemma 2, we take a guess-and-verify approach. We will suppose that country finds it optimal to borrow $b_{S,t}$ in the future and find conditions under which he finds it optimal to do so today as well.

If he is borrowing $b_{S,t}$ in the future, then he will never default in any period after tomorrow. Under this assumption, we can compute both $b_{S,t}$ and $b_{R,t}$ via the equalities $V_{R,t}(b_{S,t}) = V_{D,t} + m$ and $V_{R,t}(b_{R,t}) = V_{D,t}$. The first equality implies

$$\begin{aligned} \frac{y - b_{S,t}\left(\frac{R-1}{R}\right)}{1-\beta} &= \frac{y-\phi}{1-\beta} + m \\ b_{S,t} &= \frac{R(\phi - m(1-\beta))}{R-1} \end{aligned}$$

This expression is time-invariant, so from this point on we will refer to this threshold as b_S. The second equality is

$$y - b_{R,t} + \frac{1}{R} b_S + \frac{\beta}{1-\beta}\left(1 - b_S\left(\frac{R-1}{R}\right)\right) = \frac{y-\phi}{1-\beta}$$

Notice the assumption that b_S is optimal issuance on the left-hand side. After mild manipulation and substitution, we arrive at a similarly time-invariant expression $b_{R,t} = b_S + m$, and so we refer to this quantity as b_R.

These two are sufficient to trace out the budget set as described in Lemma 3 and establish its time-invariance.

But we still need verify that borrowing b_S is indeed optimal in all states. Since this is a limited commitment model, we can do this by ensuring that if the country finds it optimal to borrow b_S in all future periods he will also find it optimal to do so today. Thus, we need only prevent a one-shot deviation from b_S to b_R in the current period. The payoffs from following the assumed b_S strategy (always) and deviating (only once and only today) to the b_R strategy for any arbitrary b today are given respectively by

$$y - b + \frac{1}{R} b_S + \frac{\beta}{1-\beta}\left(y - b_S\left(\frac{R-1}{R}\right)\right) \tag{9}$$

$$y - b + \frac{1-p}{R} b_R + \beta \left[\begin{array}{c} (1-p)\left(y - b_R + \frac{1}{R} b_S + \frac{\beta}{1-\beta}\left(y - b_S\left(\frac{R-1}{R}\right)\right)\right) \\ +p\left(\frac{y-\phi}{1-\beta} + m\right) \end{array}\right] \tag{10}$$

Subtracting Equation 10 from Equation 9 and requiring that the difference be positive yields the condition

$$\frac{p(\phi - m(1-\beta))}{R-1} \geq \frac{(1-p)}{R} m\left(1 - \beta R\right) \tag{11}$$

The left-hand-side is the reduction in consumption resources the sovereign would receive as a result of the price drop (which pertains to the entire debt stock) were he to issue at b_R. The right-hand-side is increase in utility the sovereign receives from front-loading an additional m resources at the risky price $(1-p)/R$ were he to issue b_R. One-shot deviations will be precluded provided that the left-hand-side is greater.

Assumption 2 ensures that this is the case. Thus, existence is established and Lemma 3 holds. □

We now provide a simple calibration to illustrate the environment here.

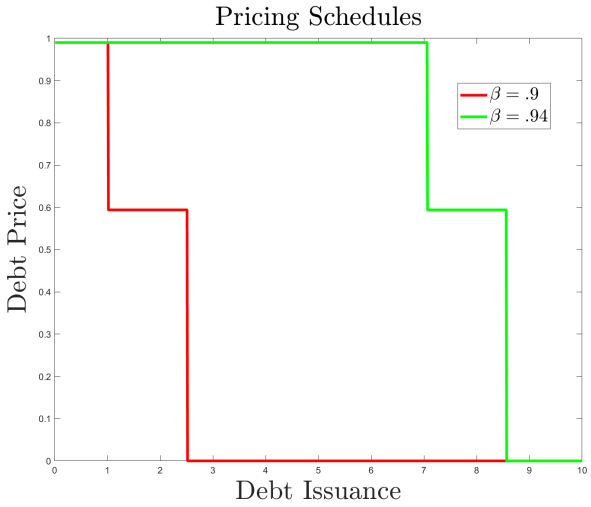


Figure 4: Full-Information Example

We set $R = 1.01$, $p = .4$, $m = 1.5$, $y = 1.0$, and $\phi = .16$. We consider two different values of β (both of which are empirically relevant) that, importantly, will satisfy one of the later assumptions required in the asymmetric information analysis. The resulting pricing functions can be observed in Figure 4.

It is worth noting the substantial difference between the two pricing schedules in Figure 4 despite their relative similarity in discount factors, e.g., the good type is allowed to borrow roughly 4x as much as the bad type. This is because a feedback loop is operative here. The good type is inclined to default less often than the bad type, which expands his budget set. But in equilibrium, this expansion of the budget set *also increases the value of repaying debt obligations relative to the value of defaulting* as credit market access becomes more lucrative. Thus the budget set expands further.

We will see this break down in the asymmetric information case: When the good type is not rewarded with better prices, he does not behave nearly as nicely as he would in the full-information case.

A.2 Proof of Proposition 1 Continued

Having characterized the full-information case in the preliminary section, we now construct the infamous equilibrium in the asymmetric information case. The first thing we observe is that the sovereign will never have an incentive to repay/default to manipulate public perception of m_t, unless (potentially) it is necessary to do so to manipulate perception of β_u. This follows from its iid and obviates many potential strategic considerations.

We employ a guess-and-verify approach in establishing the infamous equilibrium. In particular, we will build it up under the assumption that both types borrow at $b_{S,L}$, which is the largest possible safe level of borrowing for the bad type under full information. After building such an equilibrium, we will be able to verify its validity, structure, and uniqueness.

We proceed in three steps: (1) Trace out the pricing schedule and implied default behavior under the assumption that both types borrow $b_{S,L} = \frac{R(\phi - m(1-\beta_L))}{R-1}$ in equilibrium, (2) verify that over-borrowing is precluded in equilibrium, and (3) verify that under-borrowing is precluded in equilibrium.

Step 1 (Tracing out the prices and default behavior): If both types are constrained to issue $b_{S,L}$ forever, we know from the full-information case that the low-β type will be content with this arrangement by the definition of $b_{S,L}$. That is, he will choose to borrow this maximal allowed amount and repay his debt in every period just as happened in the full-information case.

The inclusion of reputation here via ρ_t^R will not matter for the calculation of this threshold because we assume $q_t(b_{S,L}, \rho_t^R)$ is the same for any ρ_t^R. Given that the country would return to $b_{S,L}$ from $b_{S,R}$ under any hypothetical deviation, the calculation of $b_{S,R}$ also does not change from the full-information case for the low-β type. Thus, we have the upper bounds

of $\mathcal{B}_1$ and $\mathcal{B}_3$, both of which are time-invariant. Since these bounds will be used to construct all other relevant objects, we declare time-invariance.

We now turn to the good type. Under the assumption that the good type is also relegated to issuing $b_{S,L}$ forever, we can construct the value of repayment, $V_{R,t}(b_t, \rho_t^R|H)$. To determine the threshold at which the good type would stop repaying with probability one, we need only equate $V_{R,t}(b_{S,L} + \epsilon_1, \rho_t^R|H) = V_{D,t}(H) + m$. Doing so yields

$$\begin{aligned} y - b_{S,L} - \epsilon_1 + \frac{1}{R} b_{S,L} + \beta_H \left(\frac{y - b_{S,L}\left(\frac{R-1}{R}\right)}{1-\beta_H} \right) &= \frac{y-\phi}{1-\beta_H} + m \\ \frac{y - b_{S,L}\left(\frac{R-1}{R}\right)}{1-\beta_H} - \epsilon_1 &= \frac{y-\phi}{1-\beta_H} + m \\ \implies \epsilon_1 &= m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right) \end{aligned}$$

where the second line follows from substituting in the known expression for $b_{S,L}$ and some simple manipulation. It is not hard to observe via simple algebra that $b_{S,L} + \epsilon_1 < b_{S,H}$ if and only if $\beta_H < 1/R$, which is assumed. This implies that the good type is made worse off by the fact that he is constrained to the (on-equilibrium) budget set of the bad type establishes Corollary 3.

A symmetric argument follows for determining the threshold at which the good type 'just' repays when the shock is good, i.e., $V_{R,t}(b_{R,L} + \epsilon_2, \rho_t^R|H) = V_{D,t}(H)$. Doing so reveals that $\epsilon_2 = \epsilon_1$.

It is worth noting that $b_{S,L} + m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right)$ will be less than $b_{S,L} + m$, which is the bad-type's repayment with probability $1-p$ threshold, only under Assumption 3. We maintain this assumption such that the pricing schedule has the shape of Figure 1.

Thus, while the bad type repays with probability one up until $b_{S,L}$, the good type repays in this way up until $b_{S,L} + m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right)$. In the region

$\left(b_{S,L} + m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right), b_{S,L} + m\right]$, both types repay with probability $1 - p$. And while the bad type repays with probability $1 - p$ up until $b_{R,L}$, the good type repays in this way up until $b_{R,L} + m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right)$. Beyond this threshold, all types default with probability one. Noting this behavior and applying Equation 8 together with the prior beliefs delivers the equilibrium price in each of these regions that we find in Proposition 1. It is important to note that we are assuming (to be verified later) that investor beliefs never change over time.

It is worth noting just how different this schedule can be from the β_H type under full-information, as this is the crux of the Infamous Equilibrium. In particular, we observe that the full-information threshold is determined by a multiple of β times $\frac{R}{R-1}$. When R is close to unity, as it typically is in the quantitative literature, this multiplier becomes enormous. As such, small changes in β can result in large increases in the implied budget set (see Figure 4 for instance).

The asymmetric information budget set, however, never allows borrowing beyond $m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right)$ of the bad type. Importantly, this difference is *not* scaled by $\frac{R}{R-1}$ since it is determined only by sovereign preferences and not by the discrepancy between lender discount values $(1/R)$ and the sovereign's discount value (β). As such, the asymmetric information budget set is required to 'hug' the β_L set rather closely.

As the equilibrium always demands repayment, the only possible off-equilibrium deviation is to default. Establishing that no type would engage in off-equilibrium default is trivial. In accordance with the specified off-equilibrium beliefs, any default will instantly be met with a worse reputation, which translates to a worse budget set tomorrow. This will not matter either way, though, as it also implies permanent exclusion from credit markets, which forever nullifies the impact of reputation. Thus, no type would

ever find it worthwhile to engage in off-equilibrium default.

Step 2 (Ruling out over-issuance): We derived the budget set in the previous step from the default behavior of the two types under the assumption of borrowing $b_{S,L}$ for each type forever. Thus, default behavior is, by construction, in keeping with equilibrium conditions. What we now show is that on-equilibrium borrowing behavior is equally satisfied, first by ruling out the temptation to over-issue debt.

We rely on the restriction imposed by limited commitment here, as the country cannot deviate by selecting an alternative trajectory of debt issuance/default, but can only select debt issuance instead.

First, we note that the budget set is increasing in beliefs and that over-issuance can only worsen beliefs. This does not preclude over-issuing to eventually induce separation (a strategy we will consider), but it does imply that any trajectory that eventually induces separation will generate a non-monotonicity in beliefs. The reputation would have to get worse before it could get better.

A second cost of overborrowing is the drop in price. In equilibrium, the sovereign can issue at the risk-free price, but overborrowing will necessarily entail a costly drop in that price.

We will need to preclude all possible deviations for both types. We begin with the low type, β_L. Under Assumptions 1 and 2, we know that the β_L type would strictly prefer to borrow $b_{S,L}$ over $b_{R,L}$. We'll need to check five other deviations, though: β_L must prefer $b_{S,L}$ over anything in $\mathcal{B}_2$ or $\mathcal{B}_4$, both of which are new options in the asymmetric information case; and β_H must prefer $b_{S,L}$ to all three regions of interior price.

We do so by precluding a one-shot deviation to the supremum of $\mathcal{B}_2$, much like we did in the full-information case. To fully establish the fully general result, we will preclude these deviations given the *best* possible off-

equilibrium beliefs, which is the case where $\nu_+ = 1$ and over-issuance is met with nothing but status quo beliefs. All other allowable off-equilibrium beliefs will generate worse prices, and so if deviations are precluded in this case they are precluded in all other cases.

Thus, we'll take the difference of the following equations for any initial b.

$$y - b + \frac{1}{R} b_{S,L} + \frac{\beta_L}{1-\beta_L}\left(y - b_{S,L}\left(\frac{R-1}{R}\right)\right) \tag{12}$$

$$\begin{aligned} y - b + & \frac{1-p\rho}{R}\left(b_{S,L} + m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right)\right) + \\ & \beta_L \left[\begin{array}{c} (1-p)\left(y - b_{S,L} - m\left(\frac{\beta_H-\beta_L}{1-\beta_H}\right) + \frac{1}{R} b_{S,L} + \frac{\beta_L}{1-\beta_L}\left(y - b_{S,L}\left(\frac{R-1}{R}\right)\right)\right) \\ + p\left(\frac{y-\phi}{1-\beta_L} + m\right) \end{array}\right] \end{aligned} \tag{13}$$

Simplifying the difference and requiring it to be greater than or equal to zero, we arrive at the condition.

$$\frac{\rho p(\phi - m(1-\beta_L))}{R-1} + m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right)(1-p)\beta_L \geq m\left(\frac{\beta_H - \beta_L}{1-\beta_H}\right)(1-p)\frac{1-p\rho}{R} \tag{14}$$

This expression dictates that the costs of switching from $b_{S,L}$ to the first 'cliff' are always bigger than the benefits. The first term on the left-hand side (a cost), is the reduction in price on the outstanding stock of debt; the right-hand side (a gain), is the additional revenue/consumption gained from issuing at the equilibrium price at more than $b_{S,L}$; the second term on the left-hand side (a cost), is the cost of repaying this extra issuance, which happens with probability $1 - p$.

Notice that this condition can be satisfied by ensuring that R is larger than but very close to one, as it increases $b_{S,L}$ to infinity (the first term)

without increasing any other terms significantly.

A deviation anywhere in $\mathcal{B}_4$ is immediately precluded for the low type. Issuing into this region implies that the bad type will default with probability one tomorrow, as $b_{S,L}$ is the most the bad type can ever commit to repay even given the promise of risk-free prices int the future.

This implies that the bad type will receive the expected continuation value of default by issuing into this region, i.e., $(y-\phi)/(1-\beta_L)+pm$, which is strictly less than he attains by equilibrium play, where debt levels are chosen such that the continuation value is $(y-\phi)/(1-\beta_L)+m$. Thus, the bad type would never over-issue into $\mathcal{B}_4$.

We need to consider the deviations for β_H separately, as the necessary parameter restrictions will be slightly different.

We begin by precluding a deviation to the supremum of $\mathcal{B}_2$ for the β_H type. We'll take the difference of the following equations for any initial b.

$$y-b+\frac{1}{R}b_{S,L}+\frac{\beta_H}{1-\beta_H}\left(y-b_{S,L}\left(\frac{R-1}{R}\right)\right) \tag{15}$$

$$\begin{aligned} y-b+&\frac{1-p\rho}{R}\left(b_{S,L}+m\left(\frac{\beta_H-\beta_L}{1-\beta_H}\right)\right)+ \\ &\beta_H\left[y-b_{S,L}-m\left(\frac{\beta_H-\beta_L}{1-\beta_H}\right)+\frac{1}{R}b_{S,L}+\frac{\beta_L}{1-\beta_L}\left(y-b_{S,L}\left(\frac{R-1}{R}\right)\right)\right] \end{aligned} \tag{16}$$

Notice a couple of things here. First, in contrast to β_L, the β_H type knows he will never default in this region but instead must pay back the additional debt in all states of the world. Second, reputation first decreases (because of over-issuance) and then increases (because of repayment when the bad type might default with some probability), but neither of these effects manifests themselves in the continuation value because the sovereign must return to risk-free pricing in the subsequent period (as is demanded by limited commitment as well as the principle of one-shot deviation).

Subtracting the payoff from the deviation from that of equilibrium play and manipulating mildly, we arrive at the marginal consideration of this particular strategy

$$\underbrace{\frac{1}{R}b_{S,L} - \frac{1-p\rho}{R}\left(b_{S,L} + m\frac{\beta_H - \beta_L}{1-\beta_H}\right)}_{\text{Revenue change from price drop}} \geq \underbrace{-\beta_H m\frac{\beta_H - \beta_L}{1-\beta_H}}_{\text{Extra cost of repaying in future}} \tag{17}$$

This condition, as before, can always be satisfied as R approaches unity from above as $b_{S,L}$ grows to infinity in this case but not the rest of the terms in the inequality.

A deviation of the good to the supremum of $\mathcal{B}_3$ is precluded already by Assumptions 14 and **??**, which ensure that the marginal benefit of issuing m more is not worth the drop in the price. In this case, there is no increase in reputation for repaying debt, as both types repay debt with the same frequency. Thus, the reputation only decreases going into the next period.

Finally, we must preclude a deviation to the supremum of $\mathcal{B}_4$.

This deviation is slightly different from the others for a couple of reasons. First, this is the only deviation where true separation can occur (with some probability) on the equilibrium path. Second, because of our assumptions on the intertemporal terms of trade, issuance in this region is always reduces the net present value of consumption in the absence of reputational considerations. This means that we need to rule out the *smallest* possible level of issuance in this region, not the largest one. We can do so by precluding issuance at the infinum of $\mathcal{B}_4$ (which is $b_{S,L} + m$) at the price prescribed by $\mathcal{B}_4$. While $\mathcal{B}_4$ is open on its lower bound, by preventing this fictional deviation, we preclude any other issuance in $\mathcal{B}_4$ as well and thus achieve our objective

The two payoffs from this deviation are

$$y - b + \frac{1}{R} b_{S,L} + \frac{\beta_H}{1-\beta_H} \left(y - b_{S,L} \left(\frac{R-1}{R} \right) \right) \tag{18}$$

$$y - b + \frac{(1-p)(1-\rho)}{R} \left(b_{S,L} + m \right) + \beta_H \left[\begin{array}{c} (1-p) \left(y - b_{S,L} - m + \frac{1}{R} b_{S,H} + \frac{\beta_H}{1-\beta_H} \left(y - b_{S,H} \left(\frac{R-1}{R} \right) \right) \right) \\ + p \left(\frac{y-\phi}{1-\beta_H} + m \right) \end{array} \right] \tag{19}$$

To preclude this deviation, their difference must be weakly positive. Some simple algebraic deviations reveal that this is isomorphic to

$$\left(\frac{1}{1-p} - (1-\rho) \right) \left(\phi - m(1-\beta_L) \right) \geq \frac{\beta_H}{1-\beta_H} \times m \times (\beta_H - \beta_L) \tag{20}$$

This is the only one of the over-issuance conditions that cannot be satisfied by lowering the interest rate. This is because the benefit of separation is also increasing in the interest rate. It is easily satisfied in simple examples, nevertheless though. It can be attained as long as β_H is not too large relative to β_L for instance.

We already know that the bad type would never borrow into this range in the absence of reputational considerations. He also would never borrow into this range to increase his reputation, as he will default with probability one tomorrow regardless of his reputation at the time.

Step 3 (Ruling out under-issuance): The last step is to ensure that under-issuance is not optimal. This temptation arises for reputational considerations, as the sovereign could expand its budget set tomorrow by reducing its debt position today to increase its reputation in accordance with ν_-.

It is not difficult to preclude such deviations. We need only to ensure that beliefs do not improve *too much* for off-equilibrium under-issuance. This

can be done immediately for status quo beliefs, i.e., $\nu_- = 1$. In this case there is no reputational advantage at all to be gained in costly under-issuing and so no type engages in it.

But we can preclude such deviations under weaker restrictions on ν_-. As long as no level of under-issuance generates levels of ρ_{t+1} in the future that would violate either Equation 14, **??**, or any of the marginal conditions ruling out overissuance (which, recall, can be always be satisfied with R close enough to one) in the future, then the future payoffs of the both types will not change relative to the case of issuing equilibrium levels ($b_{S,L}$) today and thus the sovereign will not engage in it.

With this we have finally proved Proposition 1. The various relevant Corollaries were either demonstrated in the proof or will be discussed momentarily in the discussion of the implied dynamics.

Dynamics: Given that we have verified all of the equilibrium conditions. We now discuss the dynamics. We know that the equilibrium is pooling along the borrowing margin everywhere (at $b_{S,L}$ in fact). We also know that *on the equilibrium path*, the two types pool in their debt repayment (which happens with probability one). This establishes Corollary 1.

There do exist states in which the two types would separate in the default behavior, but these are always off-equilibrium. Thus, no information is ever revealed in equilibrium and provided ρ_t is large enough to ensure all equilibrium conditions are satisfied (which can be done for any $\rho_t < 1$ as long as R is sufficiently close to one). This establishes Corollary 4. The good type is thus stuck in an 'infamous equilibrium,' wherein he defaults more often as he normally would on account of receiving worse equilibrium pricing schedules. Given his inability to commit to future behavior, he is powerless to escape it.

A.3 Proof of Proposition 2

This result is much more straightforward. We need only compare the full-information level of borrowing allowed by lenders with the level allowed in the infamous equilibrium under asymmetric information. This difference is $b_{S,H} - b_{S,L}$, which is given by

$$\frac{R(\phi - m(1-\beta_H))}{R-1} - \frac{R(\phi - m(1-\beta_L))}{R-1} = \frac{Rm(\beta_H - \beta_L)}{R-1}$$

which establishes the result. It is manifest that this difference increases (even to infinity) as R decreases toward unity.

A.4 Proof of Proposition 3

To see why the infamous equilibrium is unique in its class, we consider any other alternative. That is, we consider whether any other debt level at which both types pool borrowing behavior at the risk-free rate can be sustained given equilibrium restrictions on beliefs and behavior. Let us call such a candidate threshold $\hat{b}$

We first consider the trivial case where $\hat{b} < b_{S,L}$, i.e., a debt limit less than the full-information limit for either type. Because of the linearity of sovereign utility, his would necessitate a discontinuity in the pricing function at $\hat{b}$. This could never obtain because it would be inconsistent with the market pricing condition (Equation 5). We know from the full-information case that the bad type will repay up to $b_{S,L}$ at the risk-free rate *regardless of reputational considerations*. Thus, the default probability of issuance between $\hat{b}$ and $b_{S,L}$ would have to be zero for both types. But this would violate Equation 5, as we required a kink in the pricing function in this region that could not exist. Thus such an equilibrium could not obtain. A similar argument would hold for $\hat{b} > b_{S,H}$, as the good type would not repay

in this region even if it could service that debt at the best possible price (the risk-free price). Thus, the only possible other candidates for $\hat{b}$ are in $(b_{S,L}, b_{S,H}]$.

It turns out that we cannot have $\hat{b}$ in this region either, though. The best possible price that reputation can buy the sovereign is the risk-free price. We also know that by construction $b_{S,L}$ is the greatest amount of debt that the bad type would repay at this optimal price. Thus, even with the best possible reputation, the bad type will never repay more than $b_{S,L}$ with probability one. This implies that the bad type will necessarily default with *some* probability beyond $b_{S,L}$. If $\hat{b}$ is above $b_{S,L}$ then, the bad type would borrow up to it. However, his default probability would be strictly positive in the next period, which contradicts the risk-free price he received for this issuance in accordance with Equation 5. Thus, no $\hat{b}$ in this region could ever be an equilibrium.

The reason why the good type can see his budget set contract but the bad type cannot see his budget set expand is because the bad type's value of repayment in the infamous equilibrium does not change relative to the full-information case. Thus, the threshold for repayment cannot change. However, the good type's value of repayment in the infamous equilibrium *does* change. In fact, it drops significantly enough that default behavior worsens to the point where it consistent with the monotone pricing schedule of Proposition 1.

A.5 Proof of Proposition 4

We begin by expressing the third-party's disutility recursively. We can then use this solution to characterize the expected losses associated with the policy. We define $ED_t(\rho_t)$ to be the expected disutility to the third

party given a prior ρ_t that the country is the bad type.

$$ED_t(\rho_t) = \frac{\rho_t p \hat{b}_B}{R} + \frac{(1-p)\rho_t + (1-\rho_t)}{R} \times \min \left\{ \frac{\rho_{t+1}\left[\hat{b}_B + m\right]}{R} - S, ED_{t+1}\left(\rho_{t+1}\right) \right\}$$

$$\rho_{t+1} = \frac{(1-p)\rho_t}{(1-p)\rho_t + (1-\rho_t)}$$

We establish that this expected disutility function exists and is unique.

Lemma 4. *$ED_t(\rho_t)$ exists, is unique, is bounded, is strictly increasing in ρ_t, and can be found via dynamic programming.*

We use Blackwell's sufficiency to establish that the following operator is a contraction:

$$T(ED)(\rho_t) = \frac{\rho_t p \hat{b}_B}{R} + \frac{(1-p)\rho_t + (1-\rho_t)}{R} \times \min \left\{ \frac{\rho_{t+1}\left[\hat{b}_B + m\right]}{R} - S, ED\left(\rho_{t+1}\right) \right\}$$

$$\rho_{t+1} = \frac{(1-p)\rho_t}{(1-p)\rho_t + (1-\rho_t)}$$

1. **Monotonicity:** Suppose we had two expected disutility function that satisfied $ED^1(\rho) \leq ED^2(\rho)$ for all $\rho \in [0,1]$. It is straightforward to observe that $T(ED^1)(\rho) \leq T(ED^2)(\rho)$ as well. The flow disutilities will be equal to one another, but the continuation value of $T(ED^2)$ will be greater than $T(ED^1)$.

2. **Discounting:** We set discount factor to be $\hat{\beta} = 1/R$. We now show that $T(ED + a) \leq T(ED) + \hat{\beta}a$ for any positive constant a. As flow disutilities will play no role here, we restrict attention to the continu-

ation values:

$$\begin{aligned}
&\frac{1-p\rho_t}{R}\min\left\{\frac{\rho_{t+1}\left[\hat{b}_B+m\right]}{R}-S, ED\left(\rho_{t+1}\right)+a\right\} \\
&\qquad\leq \frac{1-p\rho_t}{R}\min\left\{\frac{\rho_{t+1}\left[\hat{b}_B+m\right]}{R}-S+a, ED\left(\rho_{t+1}\right)+a\right\} \\
&\qquad= \frac{1-p\rho_t}{R}\min\left\{\frac{\rho_{t+1}\left[\hat{b}_B+m\right]}{R}-S, ED\left(\rho_{t+1}\right)\right\}+\frac{1-p\rho_t}{R}a \\
&\qquad\leq \frac{1-p\rho_t}{R}\min\left\{\frac{\rho_{t+1}\left[\hat{b}_B+m\right]}{R}-S, ED\left(\rho_{t+1}\right)\right\}+\frac{1}{R}a
\end{aligned}$$

$$\implies T(ED+a)\leq T(ED)+\hat{\beta}a$$

With these conditions established, we have a that T is a contraction, which implies existence, uniqueness, and the ability to solve via dynamic programming.

Boundedness follows immediately from a the following limiting argument. Let us conjecture that ED_t is increasing in ρ_t (this will be verified later). We can evaluate the Bellman equation in this case to derive:

1. $ED_t(0)=0$, i.e., if it is believed that the country is the good type, there are no expected losses.

2. $ED_t(1)=\frac{p\hat{b}_B}{R-1+p}$. This will be the case under the assumption that $\frac{p\hat{b}_B}{R-1+p}<\frac{\hat{b}_B+m}{R}$, which we maintain from now on.

Notice that if ED_t is increasing, then $ED_t(1)$ will be its upper bound. We now verify its increasing nature by taking derivatives on a case-by-case depending on which of the continuation values is smaller.

1. If the next period induces separation, we have

$$ED_t(\rho_t) = \frac{\rho_t p \hat{b}_B}{R} + \frac{1 - p\rho_t}{R} \times \left(\frac{1-p}{1-p\rho_t} \frac{\rho_t \left[\hat{b}_B + m \right]}{R} - S \right)$$

$$\implies ED'_t(\rho_t) = \frac{p\hat{b}_B}{R} + \frac{(1-p)\left[\hat{b}_B + m \right]}{R^2} - \frac{pS}{R}$$

This will be positive provided $S \leq \hat{b}_B$, which we assume from now on.

2. If the next period does not induce separation, we have

$$ED_t(\rho_t) = \frac{\rho_t p \hat{b}_B}{R} + \frac{1 - p\rho_t}{R} \times ED_{t+1}(\rho_{t+1})$$

$$ED'_t(\rho_t) = \frac{p\hat{b}_B}{R} - \frac{p}{R} ED_{t+1}(\rho_{t+1}) + \frac{1-p\rho_t}{R} ED'_{t+1}(\rho_{t+1}) \left[\frac{1-p}{1-p\rho_t} + \frac{p\rho_t(1-p)}{(1-p\rho_t)^2} \right]$$

...

$$ED'_t(\rho_t) = \frac{p\hat{b}_B}{R} - \frac{p}{R} ED_{t+1}(\rho_{t+1}) + \frac{1-p}{(1-p\rho_t)R} ED'_{t+1}(\rho_{t+1})$$

$$\implies ED'_t(\rho_t) > \frac{p\hat{b}_B}{R} - \frac{p}{R} \frac{p\hat{b}_B}{R-1+p} + \frac{1-p}{(1-p\rho_t)R} ED'_{t+1}(\rho_{t+1})$$

$$\implies ED'_t(\rho_t) > \frac{p\hat{b}_B}{R} \left[1 - \frac{p}{R-1+p} \right] + \frac{1-p}{(1-p\rho_t)R} ED'_{t+1}(\rho_{t+1})$$

This last line establishes combined with the fact that the terminal period, T, (whenever it is) will feature $ED'_T(\rho_T) > 0$ is enough to establish by induction that $ED'_t(\rho_t) > 0$ for any t and any ρ_t. This also verifies the assumption under which $ED_t(1)$ was an upper bound, which delivers boundedness as well and completes the proof.

A.6 Proof of Proposition 4

This is fairly straightforward. $ED(\rho)$ will shrink as time passes because ρ will converge to zero as time passes on. Given that $S > 0$ does not depend

on ρ, at some point the costly separation loan becomes a low enough cost strategy that it necessarily becomes optimal at some finite time T. The lower is S, the larger will T be, but $T < \infty$ will exist for any $S > 0$.

If the type is bad, then they will either default with a flow probability p before time T or with probability 1 at time T.

A.7 Proof of Proposition 5

Once we have established the existence of expected disutility, we can write out expected losses recursively like we did for expected disutility as follows.

$$
\begin{aligned}
EL_t(\rho_t) = & \frac{\rho_t p \hat{b}_B}{R} + \frac{(1-p)\rho_t + (1-\rho_t)}{R} \times \\
& \left[\mathbf{1}\left\{ \frac{\rho_{t+1}\left[\hat{b}_B + m\right]}{R} \leq ED_{t+1}\left(\rho_{t+1}\right) \right\} \frac{\rho_{t+1}\left[\hat{b}_B + m\right]}{R} + \right. \\
& \left. \mathbf{1}\left\{ \frac{\rho_{t+1}\left[\hat{b}_B + m\right]}{R} > ED_{t+1}\left(\rho_{t+1}\right) \right\} EL_{t+1}\left(\rho_{t+1}\right) \right] \\
\rho_{t+1} = & \frac{(1-p)\rho_t}{(1-p)\rho_t + (1-\rho_t)}
\end{aligned}
$$

Proposition 5 follows from the fact that Blackwell's sufficiency applies equally to the recursive operator defining $EL_t(\rho_t)$. It is thus a contraction. Strict monotonicity also follows.

B Quantitative Model Extension

As a robustness exercise, we embellish the analytic model until it resembles those that are typically used in the quantitative literature, e.g., Aguiar and Gopinath (2006) or Arellano (2008). We find that the general lessons of the analytic model go through in such an environment.

B.1 Shocks and Information Structure

We assume that publicly observable output is characterized by a Markov process following Arellano (2008). In particular, $z_t = \log(Y_t)$ and

$$z_t = \rho_z z_{t-1} + \sigma_z \epsilon_t$$

where ϵ_t is an iid standard normal. We succinctly write $s_t = [z_t, Y_t]$ as the vector of observable states.

We make two augmentations to the asymmetric information component of the model. First, we allow for some infrequent and stochastic switching across β-types, which remain unobserved by investors. Second, we allow for an additional fully transitory but unobserved shock to the sovereign's marginal utility from consumption.[10]

This dual shock structure serves a couple of purposes. First and foremost, it is empirically plausible. When lenders see a sudden surge in indebtedness or default risk, they are often left trying to tease apart how much is due to underlying institutional change in the borrowing country and how much is due to random, uncontrollable liquidity shocks that can happen even to a well-behaved government. Second, it reduces incidences of full-revelation on the equilibrium path by introducing non-degenerate belief updates in most states of the world. This increases numerical stability, even for relatively low variances.

As in the analytic model, we assume that $0 < \beta^L < \beta^H < 1/R$. In addition, we must introduce a symmetric and persistent transition probability

[10]The transitory shock is iid and can be interpreted either as an endowment shock or a preference shock to the level of subsistence consumption, which directly affects the sovereign's incentives to borrow. See Chatterjee and Eyigungor (2012) for a proof of this result.

across these types given by

$$Pr(\beta^H|\beta^L) = Pr(\beta^L|\beta^H) = p_\beta < 0.5$$

The transitory shock has any potentially type-specific distribution, $m_t \sim F_{\beta_t}$. We assume that it is continuous and iid.[11]

We will bundle these unobserved states together into a vector $u_t = [\beta_t, m_t]$. The debt stock is observed by all parties, but for expositional clarity we do not bundle it with the other observed states.

The lenders will attempt to infer the sovereign's persistent type from his borrowing and default behavior. As before, their beliefs at any time t can be summarized by the scalar

$$\rho_t = Pr(\beta_t = \beta^L|x^t)$$

Every period, the lenders update their beliefs using Bayes' rule according to x^t, which is the lenders' entire information set at time t, which consists of the history of all endowment realizations, borrowing and default decisions, and prices.

B.2 Timing

The timing will be the effectively same as the analytic model, with investors given at most two opportunities to update beliefs in a given period. A sample period t takes place as follows:

1. Period t begins with the realization of observed (s_t) and unobserved (u_t) states together with some inherited debt, B_t, and current beliefs, ρ_t

[11] Since m_t is iid, lenders will not need to carry around beliefs regarding its value. Given ρ_t, beliefs regarding it could be inferred, but they will never be used in forecasting or demand. For expositional simplicity, we omit it entirely.

2. Given $[s_t, u_t, B_t, \rho_t]$, the sovereign decides whether or not to repay B_t.

   (a) If the sovereign defaults, beliefs change to ρ_t^D via an equilibrium updating rule. The sovereign receives a default value given by $[s_t, u_t, B_t, \rho_t^D]$.[12] Following default, the sovereign will enter financial autarky so there are no other actions he can take to signal information in period t.

   (b) If the sovereign repays, beliefs change to ρ_t^R via an equilibrium updating rule. The sovereign's period t debt issuance problem thus assumes the state $[s_t, u_t, B_t, \rho_t^R]$. The choice of debt issuance further reveals information, changing ρ_t^R to ρ_t^{R+A} via an equilibrium updating rule.

3. End-of-period lender beliefs update to ρ_{t+1} from either ρ_t^D or ρ_t^{R+A} via the expected trajectory of the Markov process for β_t.

B.3 Sovereign

The sovereign borrower suffers from limited commitment. In period t, he cannot commit to either borrowing or default behavior in period $t+1$. He has a time-separable utility function, $u(\cdot)$, and is a monopolist in his own debt market. Debt is potentially long-term and matures with probability λ while issuing a fraction κ of coupons regardless of maturity.

The sovereign begins each period by making a default-repayment decision. We express the sovereign's value at period t as

$$\begin{aligned} V_t(s_t, u_t, \rho_t, B_t) &= \max\{V_{R,t}(s_t, u_t, \rho_t^R, B_t), V_{D,t}(s_t, u_t, \rho_t^D)\} \qquad (21) \\ \text{s.t.} \quad \rho_t^R &= G_{R,t}(s_t, \rho_t, B_t) \\ \rho_t^D &= G_{D,t}(s_t, \rho_t, B_t) \end{aligned}$$

[12] Debt levels will matter for the default value because the model will feature recovery.

where $G_{R,t}$ and $G_{D,t}$ are belief updating rules that will be described momentarily. Notice that beliefs can only be updated using publicly observed information, s_t and B_t. Also, and unlike the analytic model, reputation matters for default. This is because we will now allow for stochastic re-entry following default, so the beliefs engendered by the default will follow the sovereign into an eventual period of credit market access.

Debt is short-term and pays a coupon, κ in the next period, implying that the Bellman equation conditional on repayment of the current debt stock, B_t is given by

$$\begin{aligned} V_{R,t}(s_t, u_t, \rho_t^R, B_t) = \max_{B_{t+1} \in \mathcal{B}} & \, u\left(C_t - m_t\right) + \beta_t E_t \left[V_{t+1}(\tilde{s}_{t+1}, \tilde{u}_{t+1}, \rho_{t+1}, B_{t+1})\right] \\ \text{s.t.} \;\; C_t = & \, Y_t - (1+\kappa)B_t + q_t(B_{t+1}|s_t, \rho_t^R, B_t)B_{t+1} \\ \rho_{t+1} = & \, p_\beta + (1-2p_\beta)G_{A,t}(s_t, \rho_t^R, B_t, B_{t+1}) \end{aligned} \tag{22}$$

$G_{A,t}$ is a belief updating rule that takes into account information revealed at issuance. Notice that belief updating also accounts for the expected trajectory of the Markov process between periods t and $t+1$.

If the sovereign defaults he is excluded from credit markets temporarily, receiving a value $V_{D,t}$, which entails no liquidity shocks (m_t) but a state-contingent, weakly positive default cost $\psi(Y_t)$. There will be exogenous and stochastic re-entry with recovery following Arellano (2008). In particular, during the period of exclusion the sovereign faces a Poisson probability of market re-access. Upon re-entry, debt levels are set to zero.

The Bellman for default is thus given by

$$\begin{aligned} V_{D,t}(s_t, u_t, \rho_t^D) = & \, u(Y_t - \psi(Y_t)) + \\ \beta_t E_t \left[(1-\phi)V_{D,t+1}(\tilde{s}_{t+1}, \tilde{u}_{t+1}, \rho_{t+1}) \right. & \left. + \phi V_{t+1}(\tilde{s}_{t+1}, \tilde{u}_{t+1}, \rho_{t+1}, 0)\right] \\ \rho_{t+1} = & \, p + (1-2p)\rho_t^D \end{aligned} \tag{23}$$

We follow Chatterjee and Eyigungor (2012) and assume that default costs must be always weakly positive and that $\psi(Y_t) = \max\{0, \psi_1 Y_t + \psi_2 Y_t^2\}$. Notice that, once in default, the type and belief process evolve exogenously with no chance for information to influence beliefs. This is because the sovereign takes no actions in default.

We finally assume that the sovereign defaults if it is not possible for him to raise enough revenue to satisfy liquidity needs.

B.4 Foreign Lenders

As in the analytic model, we assume that foreign lenders are competitive and risk-neutral. They arrive in overlapping generations with a wealth w_t and have access to a gross risk-free return of R against which they price default risk.

Upon arriving in an observable state, (s_t, B_t, ρ_t^R), they observe aggregate issuance, B_{t+1}, and then solve the following problem.

$$\max_{b_{t+1}} \; E_t\left[\tilde{c}_{L,t+1}\right] \tag{24}$$

$$\text{where} \quad c_{L,t+1} = (w_t - q_t b_{t+1})R + b_{t+1}\left[\rho_{t+1} R^L(s_{t+1}, m_{t+1}, \rho_{t+1}, B_{t+1}) + (1-\rho_{t+1}) R^H(s_{t+1}, m_{t+1}, \rho_{t+1}, B_{t+1})\right]$$

$$\text{and} \quad R^i(s_{t+1}, m_{t+1}, \rho_{t+1}, B_{t+1}) = \left(1 - D_{t+1}(s_{t+1}, (\beta^i, m_{t+1}), \rho_{t+1}, B_{t+1})\right) \times [1+\kappa]$$

$$\rho_{t+1} = p + (1-2p) G_{A,t}(s_t, \rho_t^R, B_t, B_{t+1})$$

Notice we define R^i as the gross return function given a hidden type $i \in \{L, H\}$.

B.5 Belief Updating

In equilibrium, lenders update their beliefs regarding β every period in response to sovereign borrowing and default behavior. They do so optimally, i.e., according to Bayes' rule.

To define this, we start with the default behavior. Denote the default

policy by $D_t(s_t, u_t, \rho_t, B_t)$. We define a subset of the domain of the m-shock by $M_D^i(s_t, \rho_t, B_t) = \{m | D_t(s_t, (\beta_i, m), \rho_t, B_t) = 1\}$. $G_{D,t}(s_t, \rho_t, B_t)$ describes the immediate window in which investors update their beliefs following a default: The first argument is the current beliefs were the sovereign to repay and the second is the current level of debt. Bayes' rule implies

$$G_{D,t}(s_t, \rho_t, B_t) = \tag{25}$$
$$\begin{cases} 1, & M_D^i(s_t, \rho_t, B_t) \text{ empty for } i \in \{L, H\} \\ \frac{\rho_t \int_{m \in M_D^L(s_t, \rho_t, B_t)} f_{\beta_L}(m) dm}{\rho_t \int_{m \in M_D^L(s_t, \rho_t, B_t)} f_{\beta_L}(m) dm + (1-\rho_t) \int_{m \in M_D^H(s_t, \rho_t, B_t)} f_{\beta_H}(m) dm}, & o/w \end{cases}$$

For the benchmark quantitative model we select the strongest response of beliefs to off-equilibrium behavior from the analytic model. That is, if the sovereign ever defaults when neither the good nor the bad type are prescribed to default, lenders will automatically assume him to be the bad type.

We can define a symmetric function for the case when the sovereign repays, noting that the repayment set over m will be the complement of the default set, i.e., $M_D^{i,C}(s_t, \rho_t, B_t)$. The only difference here will be off-equilibrium beliefs, which we assume to be maximally responsive but in the other direction. If the sovereign ever repays when both types prescribe default for all possible unobserved states, then lenders instantly assume him to be the good type.

$$G_{R,t}(s_t, \rho_t, B_t) = \tag{26}$$
$$\begin{cases} 0, & M_D^{i,C}(s_t, \rho_t, B_t) \text{ empty for } i \in \{L, H\} \\ \frac{\rho_t \int_{m \in M_D^{L,C}(s_t, \rho_t, B_t)} f_{\beta_L}(m) dm}{\rho_t \int_{m \in M_D^{L,C}(s_t, \rho_t, B_t)} f_{\beta_L}(m) dm + (1-\rho_t) \int_{m \in M_D^{H,C}(s_t, \rho_t, B_t)} f_{\beta_H}(m) dm}, & o/w \end{cases}$$

Now for the Bayesian response to borrowing. To define this, let $A_t(s_t, u_t, \rho_t^R, B_t)$

be the sovereign's borrowing policy function in equilibrium. For a given (s_t, ρ_t^R, B_t), define
$M^i(s_t, \rho_t^R, B_t, B_{t+1}) = \{m | A_t(s_t, (\beta_i, m), \rho_t^R, B_t) = B_{t+1}\}$. In states of repayment, we can define the following function:

$$G_{A,t}(s_t, \rho_t^R, B_t, B_{t+1}) = \tag{27}$$
$$\begin{cases} 1, & B_{t+1} > A_t(s_t, (\beta_i, m), \rho_t^R, B_t) \ \forall m, \ i \in \{L, H\} \\ 0, & B_{t+1} < A_t(s_t, (\beta_i, m), \rho_t^R, B_t) \ \forall m, \ i \in \{L, H\} \\ \frac{\rho_t^R \int_{m \in M^L(s_t, \rho_t^R, B_t, B_{t+1})} f_{\beta_L}(m) dm}{\rho_t^R \int_{m \in M^L(s_t, \rho_t^R, B_t, B_{t+1})} f_{\beta_L}(m) dm + (1-\rho_t^R) \int_{m \in M^H(s_t, \rho_t^R, B_t, B_{t+1})} f_{\beta_H}(m) dm}, & o/w \end{cases}$$

Again, maximal belief responsiveness to off-equilibrium behavior is maintained: Off-equilibrium overborrowing is instantly perceived as the bad type while off-equilibrium underborrowing is instantly perceived as the good type.

B.6 Market Clearing

The debt market is the only active market here, so our market clearing condition during periods of repayment is simply that

$$\underbrace{B_{t+1}}_{Sovereign\ Issuance} = \underbrace{B_{t+1}^D(s_t, \rho_t, B_t, B_{t+1})}_{New\ Lender\ Demand} \tag{28}$$

The price of bonds, q_t, adjusts to ensure this equation holds.

B.7 Equilibrium Definition

Definition 1. *A **Markov Perfect Bayesian Equilibrium (MPBE)** is a set of value, policy, belief updating, and price functions such that Equations (21)-(28) are satisfied.*

When solving for an equilibrium, we always compute the MPBE that is

the limit of the finite-horizon game as in Hatchondo and Martinez (2009). While multiple MPBEs may exist, this mechanism will allow us to select consistently from among them.

B.8 Numerical Solution Technique

We solve the model using machine learning techniques. In particular, we employ the Gaussian Process Dynamic Programming algorithm described in Gu and Stangebye (2023), which builds on work by Scheidigger and Bilionis (2019).[13] We iterate on the finite-horizon game until stationarity in the value functions is achieved via convergence on the order of $1.0e-7$.

The goal is not to match a particular empirical regularity but rather to demonstrate the generalizability of the benchmark analytic results. As such, our calibration will be guided by proximity to the extant literature rather than determined by a moment-matching exercise. The model is calibrated at a quarterly frequency with the parameters found in Table 1.

Table 1: Parameter Calibration

Parameter	Value	Source
Sovereign $u(\cdot)$	$u(C) = C^{1-\gamma}/(1-\gamma)$ with $\gamma = 2$	Arellano (2008)
R	1.017	Arellano (2008)
(ψ_1, ψ_2)	$(-.956, .998)$	Arellano (2008)
ϕ	.083	Mendoza and Yue (2012)
(β_L, β_H)	$(.93, .975)$	Proximity to Arellano (2008)
(ρ_z, σ_z)	$(.9212, .0226)$	Gu and Stangebye (2023)
σ_m	.005	Unobservable shocks $\approx 1/4$ size of observable shocks
p	.025	Types change ≈ 10 years

Only a few parameters are worth some discussion. Arellano (2008) employs a default cost function wherein defaulting with output below a certain threshold is costless and defaulting above this threshold reduces output to

[13] The only relevant deviation we invoke is the following: Gu and Stangebye (2023) assume a constant lengthscale across all states, while we allow for the reputation state (ρ_t) to have a potentially different lengthscale than the other states. The algorithm could thus find reputation to be less, more, or as important as other states. Endogenously, we find as a result that it is less important.

the level of the threshold. The chosen parameters (ψ_1, ψ_2) engender very nearly the same dynamic in the smoother cost function of Chatterjee and Eyigungor (2012). In particular, they imply that default is costless when $Y_t \leq 0.96$ and that output is nearly flat at .96 for all $Y_t > 0.96$.[14] We also select the discount factors to be modestly below and above that of Arellano (2008), in which the constant discount factor is $\beta = 0.953$.

The output process is taken from Gu and Stangebye (2023), who employ a bandpass filter on Russian output data at business cycle frequencies. The resulting process very closely resembles that used by Arellano (2008) or Chatterjee and Eyigungor (2012), who calibrate to Argentine output data from the 1990s.

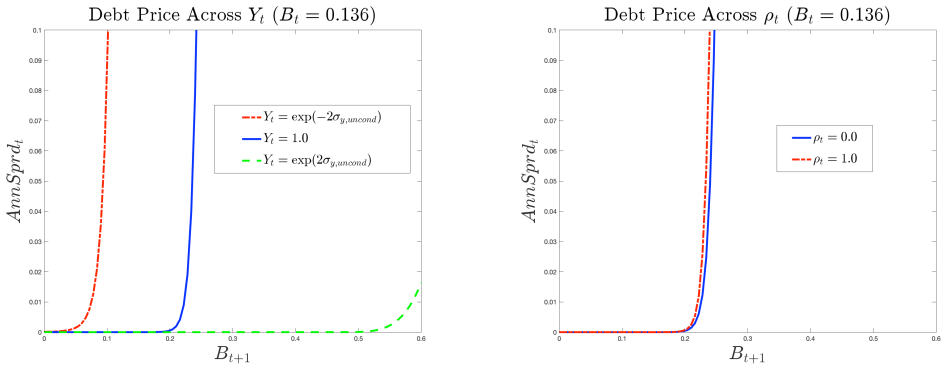
The introduction of the two new parameters in the model, i.e., unobservable liquidity shock volatility and the probability of switching types, are disciplined as follows. σ_m is set such that it is non-trivial in size but also significantly smaller than observable output shocks (σ_z). This jointly ensures that the investor's inference problem is non-trivial and that the model's aggregate dynamics closely resemble its predecessors in the literature. p is set such that types change once every ten years on average.

B.9 Quantitative Findings: Behavior

We begin by exploring policy and pricing functions. We will find that many of the infamous dynamics translate over to the more general quantitative environment. We first observe that, as in the analytic model, reputation has very little impact on aggregate dynamics. This can be observed in Figure 5, which depicts the equilibrium spreads at which debt can be issued at the

[14] In particular, it ensures that output three unconditional standard deviations above the steady state, which is roughly 1.1903, is 0.23 lower in default relative to repayment.

ergodic mean of the current debt stock[15] Figure 5a shows that, as expected,



(a) **(b)**

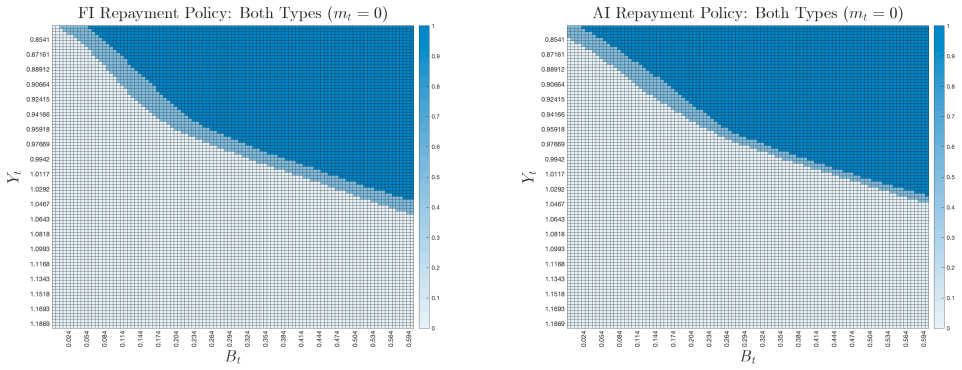
Figure 5: Equilibrium Spread Schedules

output shocks have a major impact on borrowing costs. A country in a substantial boom can borrow up to five times as much debt at the risk-free rate as it can borrow in a severe recession. In contrast, but in keeping with the intuition of the benchmark model, the impact of reputation is much smaller. This can be observed in Figure 5b, which reveals how steady state prices change from the best possible beliefs ($\rho_t = 0.0$) to the worst possible beliefs ($\rho_t = 1.0$). The difference in debt demand from investors not zero, but it is miniscule compared to output shocks, which are freely observable.

The reason is that, as in the benchmark model, asymmetric information introduces an attenuation in the default behavior across the two types. This can be seen in Figure 6, which reveals the default/repayment behavior for both types in the case of full information (6a) and asymmetric information (6b). This particular heatmap is set to zero at each point in the state space if the country where the country defaults but adds 0.5 for each type that repays. Thus, in the dark blue regions both types are repaying, in the light blue regions only the good type repays, and in the white regions both types

[15]This is relevant because the difference between B_{t+1} and B_t can reveal information that can influence prices.

are defaulting. We observe from Figure 6 that asymmetric information



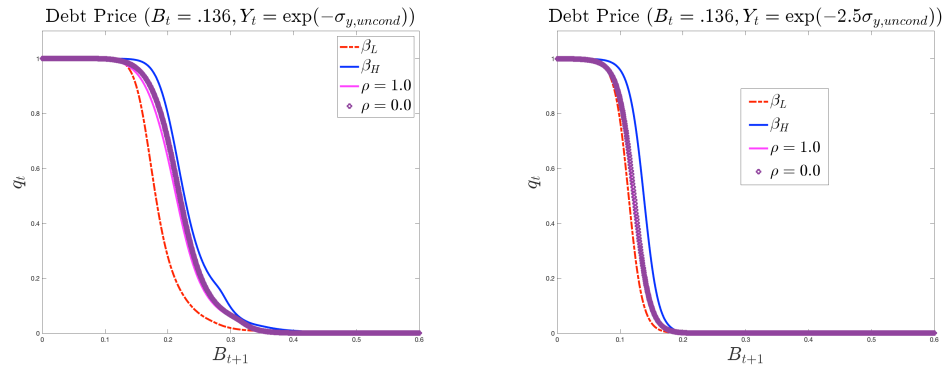
(a) **(b)**

Figure 6: Equilibrium Repayment Sets

leads to an attenuation in default behavior across the types, i.e., the region in which only the bad type defaults shrinks relative to the full-information case. This is because the good type is hurt relatively more than the bad type by the asymmetric information. This is consistent with the 'infamous dynamics' described in the analytic model.

The analytic model, however, did not have the larger state space present in the quantitative model. It turns out that this infamous effect is somewhat state-contingent and worse in deep recessions, as can be observed in Figure 7, which gives the pricing schedules across fundamental output states for the full-information and asymmetric information cases. We see in these figures both the significant attenuation in behavior across types, which can be observed in the closeness of the schedules in the asymmetric information case relative to full information, and the fact that both types in the AI case tend to attenuate tend to attenuate to the FI bad type's as the recession deepens.

The final corroboration of the quantitative model with the infamous equilibrium is the fact that asymmetric information has a disproportionate

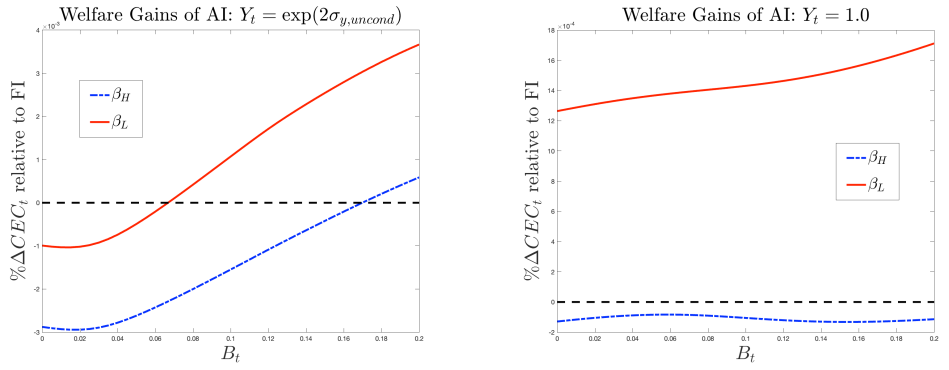


(a) **(b)**

Figure 7: Equilibrium Budget Sets Across States

negative impact on the good type relative to the bad type. This can be observed in the certainty-equivalent-consumption figures in Figure 8. We see in this figure that, during a boom when default risk is very low (as in the analytic model), the bad type always loses less (or gains more) than the good type. For low debt levels where default risk is negligible, the bad type is effectively unaffected by asymmetric information in welfare terms, as in the analytic model. The good type, however, is significantly harmed.

vspace.1in



(a) **(b)**

Figure 8: Welfare Impact of Asymmetric Information

This general dynamic holds across the state space as well, as we observe in Figure 8b. When default risk is a bit higher, the bad type benefits

significantly from asymmetric information as his budget set expands a bit relative to full-information. The good type, however, continues to suffer in steady state.