Kyle in a Triangle: Asymmetric Information and Strategic Trading in Connected Markets with Redundant Assets*

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Abstract

We extend the Kyle model to analyze strategic trading in three interconnected asset markets forming a triangle relationship, where the value of one asset is determined by a combination of two others. An informed, risk-averse speculator trades both directly in the asset's market and indirectly via the triangle by simultaneously trading the other two assets. Our key insight is that in equilibrium indirect trading through the triangle leads to less strategic offsetting of noise trading, leaving more residual noise for market makers. With more residual noise, market makers struggle to extract private information from order flows and respond by making prices less sensitive to trades, lowering price impact. As a result, the speculator trades more aggressively and earns higher profits in markets where indirect trading is more prevalent. This mechanism explains a well-documented empirical pattern in FX markets: currency pairs involving the USD (the dominant vehicle currency in exchange rate triangles) exhibit higher trading volumes and lower price impact. From the lens of our model indirect trading via the USD vehicle currency introduces more residual noise, and these markets become more liquid. Our findings provide a novel microstructural explanation for cross-market liquidity dynamics and offer new insights into how strategic trading behavior influences price formation in interconnected financial markets.

JEL-Classification: D82, G14.
Keywords: Kyle, Connected Markets, FX Triangle, Asymmetric Information, Strategic Trading.

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1 Introduction

We analytically solve a Kyle model with three connected asset markets satisfying a triangle relationship, where a combination of two assets determines the value of the third (redundant) asset. One informed risk-averse speculator strategically trades independently with three uninformed risk-neutral market makers, each specializing in a market for one asset. Given private information on one asset, the speculator trades this asset directly in the corresponding market as well as indirectly via the triangle by simultaneously trading the other two assets. In equilibrium, we find that the price informativeness is smaller, and the speculator trades more aggressively and earns higher profits in markets for assets that are more intensively used for indirect trading via the triangle.

For illustration purposes we focus on the triangle relationship in foreign exchange (FX) markets. Euro (EUR) can be exchanged for Japanese Yen (JPY) directly at the EUR/JPY exchange rate. Equivalently an FX trader can use the U.S. dollar (USD) as a vehicle currency, and trade JPY/USD and USD/EUR to implement an indirect JPY/EUR trade via the FX triangle. Besides the application in FX markets a triangle relationship also exists for other markets as for instance an exchange traded fund (ETF) tracking an index and its constituent stocks, or a derivative, its underlying and the risk-free asset.

In the context of FX markets our key result implies that the price impact of trading is smaller for currency pairs that are predominantly used for indirect trading over the FX triangle, i.e., for currency pairs that involve a vehicle currencies such as the USD. Indeed, the empirical literature documents that the price impact of trading is smaller for currency pairs that include the USD (Ranaldo and Somogyi, 2021; Ranaldo and Santucci de Magistris, 2022; Lacava et al., 2023; Somogyi, 2023). It is important to understand the economic channels that determine the price impact of trading and the interconnectedness of liquidity across currency pairs. For instance, Filippou et al. (2024) show that the price impact of trading has first order implications on the profitability of FX trading strategies. While Schrimpf et al. (2024) explore an economic channel rooted in constraints of dealers to explain liquidity supply in FX markets (i.e., a supply side mechanism), our model provides a complementary mechanism based on strategic traders with superior information (i.e., a demand side mechanism).

A triangle relationship imposes a tighter structure and fundamentally differs from an imperfect correlation between multiple assets. A correlation between assets (assuming it is imperfect) does not provide an opportunity to implement the same investment in two distinct ways. In stark contrast, the triangle relationship allows for this. In equilibrium, strategic trades and market liquidity across the three markets in the triangle relationship are non-trivially interconnected. Accordingly, our setting provides novel insights which cannot be studied in a standard model of multiple correlated assets. We are the first to provide an analytical characterization of the equilibrium in a model with an informed speculator who simultaneously trades with three market makers across three markets satisfying a triangle relationship and one asset is redundant. Specifically, we prove that the equilibrium (within the class of linear solutions) is unique, and we derive closed-form solutions for the strategic demands and the price informativeness.

Our model is based on Kyle (1985) but instead of one market we assume three markets that are tightly linked through the triangle relationship. We exogenously model the fundamental values of assets in markets 1 and 2, and denote them by ν_1 and ν_2. Asset 3 is redundant and its fundamental value is determined by the triangle relationship, $\nu_3 = \nu_1 + \nu_2$. We assume that a risk-averse speculator receives a private signal that perfectly identifies the value of asset 3, $\zeta = \nu_3$. Due to the triangle relationship the perfect signal on asset 3 also provides imperfect information about the other two assets, i.e., in case of asset 1 (2) and signal $\zeta = \nu_1 + \nu_2$ we can interpret ν_2 (ν_1) as noise. The informed speculator simultaneously trades with three distinct uninformed risk-neutral dealers who make markets. The dealers specialize in one market. That is, each dealer only observe the speculator's orders in her specific market, and there is no communication across dealers.

The key economic mechanism of our model is the following. Besides trading asset 3 using her perfect signal $\zeta = \nu_3$, the speculator can (i) simultaneously trade assets 1 and 2 to implement an indirect trade of asset 3 and further exploit the perfect signal $\zeta = \nu_3$, or (ii) trade assets 1 and 2 separately using $\zeta = \nu_1 + \nu_2$ as an imperfect signal. Our first result establishes under what condition the speculator chooses either approach (i) or (ii). Specifically, when the speculator is infinitely risk averse then she only considers to trade on perfect information and she ignores any imperfect signal to avoid uncertainty. Accordingly,

she chooses approach (i) and uses assets 1 and 2 together to indirectly trade asset 3 via the triangle. Signal $\zeta = \nu_3$ is perfect for this indirect trade, and the trader does not face any uncertainty. In contrast, when the speculator is risk neutral then she does not mind the noise in the signal $\zeta = \nu_1 + \nu_2$, and she separately trades asset 1 and 2 according to (ii). In other words, the risk neutral speculator does not see any benefit to trade via the triangle to exploit the perfect signal $\zeta = \nu_3$. Finally, the speculator employs a combination of direct and indirect trading when the risk aversion coefficient is larger than zero and finite.

As in Kyle (1985) we assume exogenous random demand orders that serve as noise trading. Market makers cannot distinguish the exogenous demands from the strategic demands of the informed speculator, and thus, cannot infer the signal ζ. However, in contrast to Kyle (1985), the informed speculator observes the exogenous demands, and thus, she takes into account their price impact when deciding her strategic demand for an asset. Our model shares this feature with Rochet and Vila (1994). One interpretation (in the context of FX markets) is that the informed speculator is a broker, e.g., a bank, that trades on private information but simultaneously submits orders of her clients to market makers. The clients are uninformed and trade for other reasons not specified in the model. Therefore, we model these client orders as exogenous random demands that are known to the informed speculator.

We show that the speculator strategically offsets part of the exogenous demand. The intuition is as follows. Suppose the signal indicates that asset 1 is undervalued, and the speculator intends to submit a purchase order. If the exogenous demand also requires buying the asset, then the price impact due to the exogenous demand increases the price of the asset, reducing expected profits from the strategic purchase order. In response, the speculator optimally reduces her strategic purchase order. In contrast, when the exogenous demand calls for a sale of the asset, then this is in favor of the speculator as it reduces the price impact of the strategic purchase order and increases expected profits. In response, she optimally increases her strategic purchase order. The optimal adjustment of the strategic order in the opposite direction of the exogenous demand can be interpreted as strategically offsetting part of the exogenous demand.

A key result of our analysis is that the speculator offsets less of the exogenous demand

for assets 1 and 2 when these assets are more intensively used to indirectly trade asset 3 via the triangle (to exploit the perfect signal $\zeta = \nu_3$ on asset 3) rather than for direct trading in markets 1 and 2 (based on the imperfect signal $\zeta = \nu_1 + \nu_2$). The intuition is as follows. Suppose the speculator only trades asset 1 to exploit the imperfect signal $\zeta = \nu_1 + \nu_2$. Then, the costs due to the price impact are fully determined by the strategic trade plus the exogenous demand for asset 1. As the price impact is linear in the (Kyle) model and the speculator acts as a monopolist, it is optimal to strategically offset exactly half of the exogenous demand. In contrast, suppose that the speculator simultaneously trades assets 1 and 2 to indirectly trade asset 3 using the perfect signal $\zeta = \nu_3$. In this case the costs of the triangle trade are the sum of the costs to trade assets 1 and 2. Accordingly, as the exogenous demand for asset 1 only affects the costs to trade asset 1 but not asset 2, its impact on the total costs of the triangle trade is comparatively smaller than in the previous case where the speculator only trades asset 1. In turn, the speculator has a smaller incentive to strategically offset the exogenous demand when implementing the triangle trade. Moreover, under the assumption that markets for assets 1 and 2 are symmetric, we show that the optimal offsetting is exactly one fourth of the exogenous demand if the speculator exclusively engages in the triangle trade.

The strategic offsetting of the exogenous demand has important implications for the price informativeness. The market maker observes the total order submitted by the speculator, i.e., the strategic order plus the exogenous demand. When a large part of the exogenous demand is strategically offset, then this implies that the market maker faces less noise and can more precisely identify the speculator's signal. In turn, she sets a price that is more sensitive to the submitted order flow. As explained in the previous paragraph, when these assets are more intensively used for the triangle trade to indirectly trade asset 3, then the strategic offsetting of the exogenous demand for assets 1 and 2 is smaller. Accordingly, the price is less informative and the price impact is smaller in this case. Moreover, in response to a smaller price impact the speculator trades more aggressively and earns higher profits.

1.1 Related Literature

Our paper contributes to four strands of literature. First, we add to the theoretical literature on strategic trading models initiated by Kyle (1985) and continued with its relevant direct extensions. We are the first to analytically solve a Kyle model where an informed speculator trades across three markets connected by a triangle relationship. Kyle (1989) allows speculators to condition their orders on the equilibrium price, while in Rochet and Vila (1994) they observe the amount of noise trading. Our model shares the latter feature. Chowdhry and Nanda (1991) study price informativeness when a security trades at multiple locations simultaneously. Bhushan (1991) emphasizes that noise trading explains the interdependence between trading costs in various asset markets. Caballe and Krishnan (1994) extend to the multi-security, heterogeneous-information model in Admati (1985) the imperfectly competitive equilibrium that Kyle (1985) applied to the single-security, homogeneous-information model of Grossman and Stiglitz (1980). Pasquariello (2007) and Bernhardt and Taub (2008) find that cross-asset trading of heterogeneously informed speculators drives, respectively, financial contagion and comovement of asset prices. Boulatov et al. (2013) use a multi-period version of the multi-asset model to predict that informed institutional trading is a source of the positive auto-correlations in portfolio returns. Bouchaud et al. (2020) propose a model for multiple markets with correlated assets and use it to calibrate empirical cross-impact matrices. Finally, Huddart et al. (2001), Yang and Zhu (2020), and Cespa and Colla (2023) discuss how mixed strategies (although with a component that is purposely randomized instead of genuinely random like in our model) arise in equilibrium in response to, respectively, ex-post trade disclosure, presence of back-runners, and impaired order flow observability.

Second, we add to the few theoretical papers focused on asset market triangles. Our intuitions apply across several triangles of naturally coupled asset markets[1]. Veldkamp (2006) introduces the triangle relationship of payoffs $\nu_1 + \nu_2 = \nu_3$, but to explain the excess comovement of assets 1 and 2's prices by introducing markets for information about asset 3's payoff. More closely related to our paper, Lyons and Moore (2009) also propose a

[1]To the best of our knowledge, the only two asset market triangles modeled so far are a triplet of currencies and an exchange-traded fund (ETF) with two constituents, as we cite here. However, also others are relevant. For instance, a convertible bond with its underlying stock, a tradable portfolio of separately-tradable bonds, and a credit default swap (CDS) with its securitized debt instrument/asset-backed security.

Kyle model to analyze the FX triangle. However, their model differs from ours in several important dimensions. First, their model cannot be solved in closed form, and as such our analysis provides deeper insights to understand the economic mechanism. Second, they do not enforce the condition $\nu_1 + \nu_2 = \nu_3$, and thus, the triangle relationship does not actually hold. Third, they assume the speculator is risk-neutral. In our model it is important that the speculator is risk-averse, otherwise she does not engage in the triangle trade but speculates on the (noisy) signal in each market separately.

Voigt (2023) investigates the price informativeness in a setting with an ETF and two constituent stocks, but in his model there is no informed agent that trades across all three markets in the triangle. In the same setting, Ernst (2022) predicts that the ETF provides single stock-level price discovery. In his Glosten and Milgrom (1985)-based model multiple investors simultaneously trade the ETF and its two single stocks against a unique common market maker who posts quotes for all three assets. In contrast, our Kyle (1985)-based model is for a single investor whose private information is perfect only about the redundant asset. Crucially, simultaneous trading requires that our investor is able to contemporaneously submit orders to three separate market makers specializing in different assets and with no cross-asset learning capabilities. As such, our model better describes OTC trading and a fragmented market structure, e.g., features observed in FX markets.

Third, our paper contributes to that FX literature with a special focus on the customer-dealer segment of the market, where information is generated and prices are set. We show how information asymmetry together with the fragmented OTC market structure affect the price informativeness in the FX triangle through the speculator's optimization of her strategic and exogenous demands. Ranaldo and Somogyi (2021) are the first using CLS order flow data to document significant differences in information asymmetry across customers, time and currency pairs. Kondor and Pinter (2022) test the time variation in the number of dealers with whom a customer trades in a time period as a proxy for private information in OTC markets. Cespa et al. (2022) assess the information content of FX volume. Ranaldo and Santucci de Magistris (2022) and Lacava et al. (2023) discuss a refined Amihud (2002) illiquidity measure based on higher frequency FX data. Roussanov and Wang (2023) find that funds and non-bank financial firms, unlike FX dealers, trade with a 3 to 5 days delay on

monetary policy announcements. Bräuer and Hau (2023) show that FX hedging demands substantially impact exchange rates.

Importantly, we show that in equilibrium the price informativeness is smaller, and the speculator trades more aggressively and earns higher profits in markets 1 and 2 if these assets are more intensively used for indirect trading of asset 3 via the triangle. In FX markets, vehicle currencies and in particular the USD are widely used to trade the FX triangle[2]. Somogyi (2023) estimates that over a third of USD transactions are for indirectly exchanging two other currencies, as trading against the USD is up to an order of magnitude cheaper, as estimated in Ranaldo and Santucci de Magistris (2022). In our theory, USD dominance as a vehicle currency can be modeled by most trading occurring in markets 1 and 2, with only minimal trading in market 3 due to much lower exogenous noise trading there. While Somogyi (2023) provides a convincing micro-founded explanation based on Rostek and Yoon (2021)'s model, the latter overlooks two key features: dealers' learning and the fragmented OTC structure of FX markets. Unlike the centralized inter-dealer market that runs on platforms like Electronic Broking Services (EBS), the customer-dealer market enables strategic speculators to trade in tandem, e.g., USD/JPY with one dealer and EUR/USD with another, helping to conceal JPY/EUR information. Our model incorporates these features, offering new insights into FX markets and expanding on Somogyi (2023)'s explanation.

Finally, we contribute to the literature on market liquidity commonality and co-movement. Empirically, there is co-movement in liquidity across markets (see, e.g., Chordia et al. (2000), Chordia et al. (2001), Hasbrouck and Seppi (2001), and Chordia et al. (2005)). We establish that indirect trading via the triangle drives common liquidity across markets, a novel economic channel firmly rooted in the demand side. Karolyi et al. (2012) survey international evidence and find stronger support for demand-side explanations. However, most theories on the determinants of common market liquidity focus on the supply side. That is, market-wide shocks to capital constraints (e.g., Brunnermeier and Pedersen (2009)) or supplier-specific shocks to risk preferences (e.g., Cespa and Foucault (2014)) of market makers/dealers.

[2]The USD is the world's leading vehicle currency as it is used as a medium of exchange and unit of account in most international transactions, even when the U.S. is not a counterparty (see Krugman (1980) and Devereux and Shi (2013)). For instance, the BIS Triennial Survey in 2022 documents that almost 90% of FX spot transactions are against the USD, an average daily turnover of USD 1.8tn out of USD 2.1tn.

In FX markets, Schrimpf et al. (2024) show that reduced liquidity supply elasticity leads to a disproportionate rise in liquidity provision costs (based on the triangle relationship and triangular arbitrage trades) relative to volume when dealers face tighter intermediation constraints. The non-linearity disappears when dealers are unconstrained, in line with the evidence of FX liquidity commonality as documented by Mancini et al. (2013) and Karnaukh et al. (2015). As a complement to these papers, we argue that informed risk-averse speculators take advantage of a smaller price impact/larger liquidity for both FX pairs 1 and 2 when these are traded in tandem to implement an indirect trade of FX pair 3. As such, we justify the emergence of cross-currency dependencies in the market liquidity of the seemingly unrelated pairs 1 and 2. Moreover, since indirect FX trading is generally enabled by the USD as a vehicle currency, our model implies that there is a linkage between the use of the USD as a vehicle currency and the liquidity in markets of currency pairs that do versus do not involve USD. Finally, it is important to note that the model of Schrimpf et al. (2024) explains bid-ask spreads, while our model analyzes the price impact of trading. These two concepts of illiquidity are (in theory) fundamentally different. Empirically, they are however strongly correlated (e.g., Ranaldo and Santucci de Magistris (2022), Lacava et al. (2023), Filippou et al. (2024)). Filippou et al. (2024) further establish that the price impact of trading has more severe implications on FX trading profits than bid-ask spreads. As such it is particularly important to have models (like ours) that investigate the possible determinants of the price impact of trading.

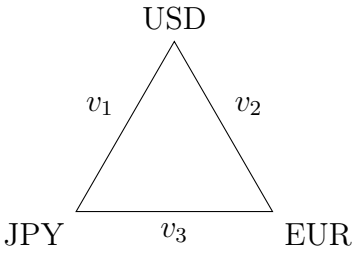
2 Model of Asset Market Triangle

We assume a static model with trade at time 0, and all payoffs realize at time 1. There are three assets that are connected by a triangle relationship. For illustrative purposes we frame our model in terms of the foreign exchange (FX) market triangle for the currencies U.S. Dollar (USD), Japanese Yen (JPY), Euro (EUR). We denote a currency pair by the index i, with $i = 1$ for JPY/USD, $i = 2$ for USD/EUR, and $i = 3$ for JPY/EUR.

At time 1 the fundamental relative valuation of currency pair i is denoted by e^{ν_i}, i.e,

$$\text{USD } 1 = \text{JPY } e^{\nu_1}, \quad \text{EUR } 1 = \text{USD } e^{\nu_2}, \quad \text{EUR } 1 = \text{JPY } e^{\nu_3}.$$

ν_1, ν_2 and ν_3 are tightly linked to each other by the triangle relationship,



$$\text{EUR } 1 = \text{USD } e^{\nu_2} = \text{JPY } e^{\nu_1} e^{\nu_2} = \text{JPY } e^{\nu_3},$$

$$\Rightarrow \nu_3 = \nu_1 + \nu_2. \tag{1}$$

This means that a position in JPY/EUR can either be implemented directly by trading JPY/EUR in market 3, or indirectly by simultaneously trading JPY/USD in market 1 and USD/EUR in market 2. That is, one asset is spanned by the other two, and thus, redundant. In the specific example, the indirect trade via the triangle uses the USD as a vehicle currency.

We model ν_1 and ν_2 exogenously, while ν_3 is endogenously determined by the triangle relationship (1). To keep the model tractable we assume the random variables ν_1 and ν_2 are independent and normally distributed with identical variances σ_ν^2,

$$\begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_1 + \nu_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \overline{\nu}_1 \\ \overline{\nu}_2 \\ \overline{\nu}_1 + \overline{\nu}_2 \end{bmatrix}, \begin{bmatrix} \sigma_\nu^2 & 0 & \sigma_\nu^2 \\ 0 & \sigma_\nu^2 & \sigma_\nu^2 \\ \sigma_\nu^2 & \sigma_\nu^2 & 2\sigma_\nu^2 \end{bmatrix} \right). \tag{2}$$

Note that the covariance matrix is singular and not invertible as the third row (column) is the sum of the first and second rows (columns) since one asset is redundant. In Section 6 we allow ν_1 and ν_2 to be either correlated or distributed with different variances.

At time 0 currencies can be exchanged at the market rates e^{q_i}, which are defined analogously to the fundamental relative valuations e^{ν_i}. However, while ν_1 and ν_2 are exogenous variables that realize at time 1 and ν_3 it is pinned down by the triangle relationship, q_i are the log-exchange rates quoted by market makers in markets $i \in \{1, 2, 3\}$ at time 0. These quoted rates are endogenous and determined in equilibrium.

We denote the speculator's trading positions in the three markets $i \in \{1,2,3\}$ by χ_i and measure them in terms of USD. That is, χ_1 denotes the USD bought in the JPY/USD market, χ_2 is the USD value of EUR bought in the USD/EUR market, and χ_3 is the USD value of EUR bought in the JPY/EUR market. At time 1 the speculator's payoff is determined by converting all her currency holdings into the USD at the fundamental relative valuations e^{ν_i}. Without loss of generality we set the risk-free interest rates to zero in the three currencies.[3] Thus, positions χ_i do not grow from time 0 to 1. Then, the trading profits π_i^* in each market i depend on the log-valuations ν_i, which determine payoffs at time 1, and the log-exchange rates q_i, which determine the cost of entering a position at time 0,

$$\begin{aligned}
\pi_1^* &= \text{JPY } \chi_1 \left(e^{\nu_1} - e^{q_1}\right) = \text{USD } \chi_1 \left(1 - e^{-(\nu_1 - q_1)}\right), \\
\pi_2^* &= \text{USD } \chi_2 e^{-q_2} \left(e^{\nu_2} - e^{q_2}\right) = \text{USD } \chi_2 \left(e^{\nu_2 - q_2} - 1\right), \\
\pi_3^* &= \text{JPY } \chi_3 e^{-q_2} \left(e^{\nu_3} - e^{q_3}\right) = \text{USD } \chi_3 e^{-q_2} \left(e^{\nu_3} - e^{q_3}\right) e^{-\nu_1} \\
&= \text{USD } \chi_3 \left(e^{\nu_2 - q_2} - e^{\nu_2 - q_2} e^{-(\nu_3 - q_3)}\right).
\end{aligned}$$

Assuming $\nu_i - q_i$ is small enough we can use the log-linearization $e^{\nu_i - q_i} \approx 1 + \nu_i - q_i$ and $(\nu_2 - q_2)(\nu_3 - q_3) \approx 0$ to obtain the approximate total profit Π denominated in USD,

$$\Pi = \chi_1 (\nu_1 - q_1) + \chi_2 (\nu_2 - q_2) + \chi_3 (\nu_3 - q_3) \approx \pi_1^* + \pi_2^* + \pi_3^*. \tag{3}$$

Note that this profit function is a straightforward extension of Kyle (1985) by summing profits across the three markets.

Before trade takes place at time 0 the speculator receives the private signal $\zeta = \nu_3$. That is, she has perfect private information when trading JPY/EUR. Due to the triangle relationship the perfect signal on JPY/EUR (which is a combination of JPY/USD and USD/EUR) also provides imperfect information (i.e., a noisy signal) when trading JPY/USD in market 1 or USD/EUR in market 2. For instance, when trading JPY/USD in market 1 the speculator would like to know ν_1, but the signal $\zeta = \nu_3 = \nu_1 + \nu_2$ (JPY/EUR) does not

[3] If interest rates were not zero, we would need a simple adjustment of the payoffs using the interest rate differentials across currencies. Such an adjustment has no material effect as interest rates are exogenous.

allow to disentangle ν_1 (JPY/USD) from ν_2 (USD/EUR). Accordingly, ν_2 can be understood as the noise of the imperfect signal.

Based on the signal ζ the speculator strategically submits trade orders χ_i for the three currency pairs $i \in \{1, 2, 3\}$ with the aim to maximize mean-variance preferences with risk aversion γ defined over trading profits,

$$\max_{\{\chi_1,\chi_2,\chi_3\}} \left\{ \mathrm{E}\left[\Pi|\,\zeta\right] - \frac{\gamma}{2}\mathrm{Var}\left[\Pi|\,\zeta\right] \right\}. \tag{4}$$

The key economic mechanism of our model is the following. Besides trading JPY/EUR in market 3 using her perfect signal $\zeta = \nu_3$, the speculator has two options. First, she can simultaneously trade JPY/USD in market 1 and USD/EUR in market 2 to implement an indirect trade of JPY/EUR using the USD as a vehicle currency and further exploit the perfect signal $\zeta = \nu_3$. Second, she can trade JPY/USD in market 1 and independently USD/EUR in market 2 using $\zeta = \nu_1 + \nu_2$ as an imperfect signal for each trade separately. We show later that the risk aversion γ is a key determinant of how intensively the speculator uses the first approach and trades JPY/EUR indirectly using the USD as a vehicle currency. Intuitively, when the speculator is infinitely risk averse then she only considers to trade on perfect information and she ignores any imperfect signal. Accordingly, she chooses the first approach and only trades in markets 1 and 2 to indirectly trade JPY/EUR. In contrast, when the speculator is risk neutral then she does not mind the noise in the imperfect signal $\zeta = \nu_1 + \nu_2$, and she separately trades JPY/USD in market 1 and USD/EUR in market 2 according to the second approach. In other words, the risk neutral speculator does not see any benefit to trade via the triangle to exploit the perfect signal $\zeta = \nu_3$. Finally, the speculator employs a combination of direct and indirect trading when the risk aversion coefficient is larger than zero and finite.

Besides the information-driven orders χ_i, the informed speculator also submits exogenous and price-inelastic demands δ_i in each market $i \in \{1, 2, 3\}$ with

$$\delta_i \sim \mathcal{N}\left(0, \sigma_i^2\right).$$

These demands are independent across markets, and they are independent of the signal or the fundamental relative valuations. We require that the variances of δ_1 and δ_2 are identical to obtain closed form solutions, $\sigma_\delta^2 = \sigma_{\delta 1}^2 = \sigma_{\delta 2}^2$. However, we can allow for $\sigma_{\delta 3}^2 \neq \sigma_\delta^2$. The exogenous demands serve the purpose of noise trading as in Kyle (1985). Market makers do not observe δ_i, and thus, cannot infer the signal ζ from the observable total trade order $\chi_i + \delta_i$. In contrast to Kyle (1985), the informed speculator observes the exogenous demands, and thus, her strategic trade orders χ_i are a function of δ_i. Our model shares this feature with Rochet and Vila (1994). One interpretation is that the informed speculator is a bank that trades on private information but also submits orders of her clients (which are uninformed and trade for other reasons not specified in the model) to the market maker.

To obtain closed form solutions we require that the informed trader perfectly observes the exogenous demands. The reason is that the informed speculator is risk averse and uncertainty due to noise trading (if not observed by her) complicates her optimization problem and leads to non-trivial interactions between the strategic demands across the three markets. In extensions that rely on numerical solutions we show that our main mechanism is also at work when there is additional noise trading that is not observed by either the market maker or the informed speculator. In the extended version of the model we further show that the additional noise trading generates a supplementary interesting interaction in the strategic trading behavior of the informed speculator and the price impact functions across the three markets.

Each market $i \in \{1, 2, 3\}$ has a separate market maker, and market makers across markets do not communicate. This allows us to consider each market maker problem in isolation for each market. The market maker in market i observes the total order flow $\chi_i + \delta_i$ but she does not observe what happens in the other two markets. Accordingly, she sets a competitive log-exchange rate quote q_i that is a function of only the order flow in market i,

$$q_i = \mathrm{E}\left[\nu_i \middle| \chi_i + \delta_i\right]. \tag{5}$$

3 Equilibrium Derivation

The equilibrium is defined as a straightforward extension of Kyle (1985). The informed speculator strategically takes into account the price impact of her trade orders, and market makers set competitive quotes.

Definition 1 *An equilibrium is a set of trade orders* $\chi_i(\zeta, \delta_1, \delta_2, \delta_3)$ *and price functions* $q_i(\chi_i + \delta_i)$ *for all* $i \in \{1, 2, 3\}$ *such that*

1. $\chi_i(\zeta, \delta_1, \delta_2, \delta_3)$ *are the optimal controls that solve the speculator's utility maximization problem,*

$$\max_{\{\chi_1, \chi_2, \chi_3\}} \left\{ \mathrm{E}[\Pi|\,\zeta] - \frac{\gamma}{2}\mathrm{Var}[\Pi|\,\zeta] \right\},$$
$$\text{with} \quad \Pi = \chi_1(\nu_1 - q_1(\chi_1 + \delta_1)) + \chi_2(\nu_2 - q_2(\chi_2 + \delta_2)) + \chi_3(\nu_3 - q_3(\chi_3 + \delta_3)),$$

2. $q_i(\chi_i + \delta_i) = \mathrm{E}[\nu_i|\,\chi_i + \delta_i]$, $\forall i \in \{1, 2, 3\}$.

Theorem 1 *There exists a unique linear equilibrium with*

$$\begin{aligned}
\chi_1(\zeta, \delta_1, \delta_2, \delta_3) &= \beta(\zeta - \overline{\nu}_1 - \overline{\nu}_2) + \alpha\delta_1 + \alpha^*\delta_2, \\
\chi_2(\zeta, \delta_1, \delta_2, \delta_3) &= \beta(\zeta - \overline{\nu}_1 - \overline{\nu}_2) + \alpha\delta_2 + \alpha^*\delta_1, \\
\chi_3(\zeta, \delta_1, \delta_2, \delta_3) &= \beta_3(\zeta - \overline{\nu}_1 - \overline{\nu}_2) + \alpha_3\delta_3, \\
q_1(\chi_1 + \delta_1) &= \overline{\nu}_1 + \lambda(\chi_1 + \delta_1), \\
q_2(\chi_2 + \delta_2) &= \overline{\nu}_2 + \lambda(\chi_2 + \delta_2), \\
q_3(\chi_3 + \delta_3) &= \overline{\nu}_1 + \overline{\nu}_2 + \lambda_3(\chi_3 + \delta_3),
\end{aligned} \tag{6}$$

for constants β, β_3, α, α_3, α^*, λ *and* λ_3.

Following the extant literature our focus is on linear equilibria. We provide closed-form solutions for the constants β, β_3, α, α_3, α^*, λ and λ_3 in (13), (16), (20), and (28). There are two important observations. First, markets 1 and 2 are symmetric, and therefore, the

optimal trade orders and equilibrium prices in these markets share the same constants β, α, α^*, and λ.

Second, trading in market 3 is independent of the other two markets. The reason is that the speculator has a perfect signal for market 3, and there is no benefit to construct a trading strategy that interacts market 3 with either a simultaneous trade in market 1 or 2. Thus, the equilibrium price q_3 is independent of the other two markets.

The constants have clear interpretations. As is standard in Kyle models β and β_3 capture the intensity of strategic trading due to the signal ζ. Intuitively, the speculator submits buy (sell) orders when $\zeta > 0$ ($\zeta < 0$) to exploit her informational advantage over the market makers. Therefore, we expect $\beta > 0$ and $\beta_3 > 0$, and these coefficients are large (small) when the price impact of trading is low (high).

The constants α and α_3 capture the intensity of strategic trading in market i in response to the speculator's exogenous demand δ_i for currency pair i. The intuition is as follows. Suppose the speculator intends to submit a purchase order of asset i due to a positive signal $\zeta > 0$. If the exogenous demand also requires buying the asset ($\delta_i > 0$), then the price impact due to the exogenous demand increases the price of the asset, reducing expected profits from the strategic purchase order. In response, the speculator optimally reduces her strategic purchase order. In contrast, when the exogenous demand calls for a sale of the asset ($\delta_i < 0$), then this is in favor of the speculator as it reduces the price impact of the strategic purchase order and increases expected profits. In response, she optimally increases her strategic purchase order. The optimal adjustment of the strategic order is in the opposite direction of the exogenous demand and we expect $\alpha < 0$ and $\alpha_3 < 0$. We interpret this effect as strategically offsetting part of the exogenous demand.

The constant α^* captures the intensity of strategic trading in market 1 (respectively market 2) in response to the exogenous demand in market 2 (respectively market 1). Intuitively, if the speculator simultaneously trades JPY/USD in markets 1 and USD/EUR in market 2 to indirectly trade JPY/EUR, then a strategic order for JPY/USD in market 1 (respectively USD/EUR in market 2) accounts for the price impact and exogenous demands in both markets 1 and 2. Accordingly, α^* describes the indirect strategic offsetting of the exogenous

demand in the market which constitutes the second leg of the indirect trade via the triangle. We expect $\alpha^* < 0$ for the same reason as $\alpha < 0$. Moreover, we expect the magnitude of α^* is larger if the speculator engages in more indirect trading via the triangle.

Finally, the constants λ and λ_3 describe the price impact or price informativeness in markets 1, 2 and 3. Generally, the price impact is smaller if it is more challenging for market makers to identify the signal ζ from the submitted order $\chi_i + \delta_i$. We show below that the amount of strategic offsetting of the exogenous demand (α, α^*, α_3) is crucial to determine the price impact. Intuitively, the exogenous demand is noise from the perspective of the market maker. Therefore, more strategic offsetting of the exogenous demand implies less noise to the market maker, and thus, makes it easier for the market maker to learn about ζ from the submitted order $\chi_i + \delta_i$. As a result, the price impact is smaller (and hence the market is more liquid) when there is less strategic offsetting of the exogenous demand.

In the following we provide details to Theorem 1 in two stages. First, we solve the problem of the informed speculator for the parameters β, β_3, α, α^*, α_3. Then, we plug the speculator's strategic demands into the price impact functions and solve for λ, λ_3 to characterize market liquidity in terms of the exogenous model parameters.

3.1 Informed Speculator Problem

The speculator is an information monopolist and her strategic trades account for the price impact as specified in the conjectured quoting rules q_i in Theorem 1. Plugging these pricing functions into the mean-variance optimization problem (4) yields,

$$\max_{\{\chi_1,\chi_2,\chi_3\}} \left\{ \begin{array}{c} \chi_1 \left(\mathrm{E}\left[\nu_1 \middle| \zeta\right] - \overline{\nu}_1 - \lambda \left(\chi_1 + \delta_1 \right) \right) + \chi_2 \left(\mathrm{E}\left[\nu_2 \middle| \zeta\right] - \overline{\nu}_2 - \lambda \left(\chi_2 + \delta_2 \right) \right) \\ + \chi_3 \left(\nu_1 + \nu_2 - \overline{\nu}_1 - \overline{\nu}_2 - \lambda_3 \left(\chi_3 + \delta_3 \right) \right) \\ - \frac{\gamma}{2} \left(\chi_1^2 \mathrm{Var}\left[\nu_1 \middle| \zeta\right] + \chi_2^2 \mathrm{Var}\left[\nu_2 \middle| \zeta\right] + 2\chi_1\chi_2 \mathrm{Cov}\left[\nu_1, \nu_2 \middle| \zeta\right] \right) \end{array} \right\}, \tag{7}$$

with:

$$\mathrm{E}\left[\nu_i \middle| \zeta\right] = \overline{\nu}_i + \frac{1}{2}\left(\zeta - \overline{\nu}_1 - \overline{\nu}_2\right), \;\; \mathrm{Var}\left[\nu_i \middle| \zeta\right] = \frac{\sigma_\nu^2}{2}, \;\; \mathrm{Cov}\left[\nu_1, \nu_2 \middle| \zeta\right] = -\frac{\sigma_\nu^2}{2}, \;\; \forall i \in \{1, 2\}.$$

The conditional expectations and covariances are solved using the linear projection theorem, the specific distribution assumption for ν_1, ν_2, ν_3 in (2), the fact that the signal is perfect $\zeta = \nu_3$, and the triangle relationship $\nu_3 = \nu_1 + \nu_2$ in (1).

There are two important observations. First, the signal $\zeta = \nu_1 + \nu_2$ provides perfect information about the valuation of JPY/EUR, which means that the informed speculator faces no uncertainty when placing trading order χ_3 in market 3. Mathematically, this means that $(\nu_1 + \nu_2)|\,\zeta$ is degenerate and has no variation, and specifically does not co-vary with either $\nu_1|\,\zeta$ or $\nu_2|\,\zeta$. It follows that trading order χ_3 is decoupled from χ_1 and χ_2 in the optimization problem, and the strategic demand χ_3 is independently determined from the strategic demands in the other two markets.

Second, the signal ζ provides only partial information for JPY/USD and USD/EUR in markets 1 and 2. Mathematically, $\nu_1|\,\zeta$ and $\nu_2|\,\zeta$ are random variables. Interestingly, the signal introduces a non-zero co-variation between two random variables $\nu_1|\,\zeta$ and $\nu_2|\,\zeta$ even if ν_1 and ν_2 are uncorrelated. Risk aversion $\gamma > 0$ together with $\mathrm{Cov}\,[\nu_1, \nu_2|\,\zeta] \neq 0$ are the key elements to introduce a non-trivial interaction between the strategic demands χ_1 and χ_2 in markets 1 and 2. Intuitively, when the speculator is risk averse then she is more hesitant to trade JPY/USD in market 1 (respectively USD/EUR in market 2) as this trade is based on a noisy signal. However, she can alternatively trade JPY/USD in market 1 and simultaneously USD/EUR in market 2 to indirectly trade JPY/EUR. For this trade the signal ζ is perfect, and leaves no uncertainty to the speculator. Indeed, it is the case in optimization (7) that χ_1 and χ_2 are equal in the limit when $\gamma \to \infty$, i.e., the speculator only trades JPY/USD and USD/EUR in markets 1 and 2 to indirectly trade JPY/EUR. On the other hand, if the speculator is risk-neutral, then she does not care about any noise in the signal, and the strategic demands χ_1 and χ_2 are determined independently of each other. Mathematically, when $\gamma = 0$, then all interaction terms between χ_1 and χ_2 disappear in the optimization problem (7). Generally, the larger the risk aversion γ the more intensively the speculator uses the USD as a vehicle currency to indirectly trade JPY/EUR via markets 1 and 2.

The first order conditions of the mean-variance problem (7) with respect to the strategic

trading orders χ_1, χ_2, χ_3 are as follows,

$$\begin{aligned}
&\lambda\left(-2\chi_1-\delta_1\right)+\frac{1}{2}\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)+\frac{1}{2}\gamma\sigma_\nu^2\left(\chi_2-\chi_1\right)=0,\\
&\lambda\left(-2\chi_2-\delta_2\right)+\frac{1}{2}\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)+\frac{1}{2}\gamma\sigma_\nu^2\left(\chi_1-\chi_2\right)=0,\\
&\lambda_3\left(-2\chi_3-\delta_3\right)+\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)=0.
\end{aligned} \tag{8}$$

We solve them and verify the linear conjectures in Theorem 1 for χ_1, χ_2, χ_3. We find the trading intensity parameters β, β_3 (driven by information) and α, α_3, α^* (driven by the exogenous demand) as,

$$\begin{aligned}
&\beta=\frac{1}{4\lambda}, \qquad \alpha=-\frac{4\lambda+\gamma\sigma_\nu^2}{4(2\lambda+\gamma\sigma_\nu^2)}, \qquad \alpha^*=-\frac{\gamma\sigma_\nu^2}{4(2\lambda+\gamma\sigma_\nu^2)},\\
&\beta_3=\frac{1}{2\lambda_3}, \qquad \alpha_3=-\frac{1}{2}.
\end{aligned} \tag{9}$$

3.2 Market Maker Problem

Once we obtain the equilibrium trading rules in the three markets, we apply the linear projection theorem to compute the log-quotes set by the market makers,

$$q_i=\mathrm{E}\left[\nu_i\middle|\,\chi_i+\delta_i\right]=\overline{\nu}_i+\frac{\mathrm{Cov}\left[\nu_i,\chi_i+\delta_i\right]}{\mathrm{Var}\left[\chi_i+\delta_i\right]}\left(\chi_i+\delta_i-\mathrm{E}\left[\chi_i+\delta_i\right]\right), \quad \forall i\in\{1,2,3\}. \tag{10}$$

Since $\mathrm{E}\left[\chi_i+\delta_i\right]=0$ we can easily map the terms in (10) to the parameters λ and λ_3 in the linear conjectures of $q_i\left(\chi_i+\delta_i\right)$ in Theorem 1. The price impacts λ and λ_3 are

$$\begin{aligned}
\lambda&=\frac{\mathrm{Cov}\left[\nu_1,\chi_1+\delta_1\right]}{\mathrm{Var}\left[\chi_1+\delta_1\right]}=\frac{\mathrm{Cov}\left[\nu_2,\chi_2+\delta_2\right]}{\mathrm{Var}\left[\chi_2+\delta_2\right]}=\frac{\beta\sigma_\nu^2}{2\beta^2\sigma_\nu^2+(\alpha+1)^2\sigma_\delta^2+(\alpha^*)^2\sigma_\delta^2},\\
\lambda_3&=\frac{\mathrm{Cov}\left[\nu_1+\nu_2,\chi_3+\delta_3\right]}{\mathrm{Var}\left[\chi_3+\delta_3\right]}=\frac{2\beta_3\sigma_\nu^2}{2\beta_3^2\sigma_\nu^2+(\alpha_3+1)^2\sigma_{\delta 3}^2},
\end{aligned} \tag{11}$$

where we use the linear trading rules specified in Theorem 1, and the assumption that the exogenous demands δ_i are independent of the fundamental values ν_j $\forall i\in\{1,2,3\}$ and $j\in\{1,2,3\}$.

The price impact λ is the same in markets 1 and 2 because of the symmetry in the distri-

bution of the fundamental values specified in (2), and the fact that the exogenous demands in markets 1 and 2 have the same variance, $\sigma_{\delta 1}^2 = \sigma_{\delta 2}^2$. Crucially, the learning capability of market makers depends on the amount of informed trading relative to uninformed trading. The former is characterized by $\beta_i^2 \sigma_v^2$. The latter is $(\alpha - 1)^2 \sigma_\delta^2 + (\alpha^*)^2 \sigma_\delta^2$ in case of market 1 or 2 respectively $(\alpha_3 - 1)^2 \sigma_{\delta 3}^2$ in case of market 3, and describes the variance of the residual exogenous demands after accounting for the strategic off-setting by the speculator.

To obtain solutions for λ and λ_3 and in turn $\beta, \beta_3, \alpha, \alpha^*, \alpha_3$ in terms of only the exogenous model parameters, we plug the trading rule functions (9) into the price impacts functions (11), and obtain two decoupled, non-linear equations in λ and λ_3,

$$\begin{aligned} &\lambda - \sigma_\nu^2 \left(4\lambda \left(\frac{\sigma_\nu^2}{8\lambda^2} + \left(-\frac{\gamma\sigma_\nu^2 + 4\lambda}{4\gamma\sigma_\nu^2 + 8\lambda} + 1 \right)^2 \sigma_\delta^2 + \frac{\gamma^2 \sigma_\nu^4 \sigma_\delta^2}{(4\gamma\sigma_\nu^2 + 8\lambda)^2} \right) \right)^{-1} = 0, \\ &\lambda_3 - \sigma_\nu^2 \left(\lambda_3 \left(\frac{\sigma_\nu^2}{2\lambda_3^2} + \frac{\sigma_{\delta 3}^2}{4} \right) \right)^{-1} = 0. \end{aligned} \tag{12}$$

In general, solving these is a messy non-linear fixed point problem, potentially with many possible or no solutions. However, we show later that we are able to obtain a unique closed-form solution. Given the closed-form solutions for λ and λ_3 we further obtain closed-form solutions for β, β_3, α, α_3, and α^* using (9). This completes the derivation of Theorem 1.

4 Equilibrium Characterization

As we anticipated, the equilibrium for market 3 is easy to characterize because it is independent of the speculator's risk aversion γ. For market 3, the trading position and quoting rule in equilibrium Theorem 1 specify to,

$$\chi_3 = \underbrace{\frac{\sqrt{2}\sigma_{\delta_3}}{4\sigma_\nu}}_{=\beta_3 = \frac{1}{2\lambda_3}} (\zeta - \overline{\nu}_1 - \overline{\nu}_2) \underbrace{-\frac{1}{2}}_{=\alpha_3} \delta_3, \tag{13}$$

$$q_3 = \overline{\nu}_1 + \overline{\nu}_2 + \underbrace{\frac{\sqrt{2}\sigma_\nu}{\sigma_{\delta_3}}}_{=\lambda_3} (\chi_3 + \delta_3) = \overline{\nu}_1 + \overline{\nu}_2 + \frac{\zeta - \overline{\nu}_1 - \overline{\nu}_2}{2} + \frac{\sqrt{2}\sigma_\nu}{2\sigma_{\delta_3}} \delta_3. \tag{14}$$

The costs due to the price impact are fully determined by the strategic trade in market 3 plus the exogenous demand δ_3 for asset 3. As the price impact is linear in the (Kyle) model and the speculator acts as a monopolist, it is optimal to strategically offset exactly half of the exogenous demand, i.e., $\alpha_3 = -\frac{1}{2}$.

Importantly, the price impact λ_3 is a proxy for the learning capability of the market maker in market 3. Specifically, we compute the variance of the order flow in market 3 as,

$$\text{Var}\left[\chi_3 + \delta_3\right] = \beta_3^2 2\sigma_\nu^2 + \frac{1}{4}\sigma_{\delta_3}^2. \tag{15}$$

Our interpretation of it is the following. Fixing β_3, a smaller σ_ν or a larger $\sigma_{\delta 3}$ makes it more difficult for the JPY/EUR dealer to learn about the signal ζ. In turn, as the dealer learns less, she sets a quote that is less sensitive to the order flow she observes and the resulting price impact λ_3 is smaller. Reversely, a larger σ_ν or a smaller $\sigma_{\delta 3}$ leads to a larger λ_3.

With respect to the equilibrium for market 3, the equilibrium for markets 1 and 2 is much more challenging to characterize because of the non-trivial interaction between the risk aversion $\gamma > 0$ and the co-variation $\text{Cov}\left[\nu_1, \nu_2 | \zeta\right] \neq 0$ that we discussed earlier. Therefore, before analyzing the general equilibrium solution for an informed speculator with limited risk aversion ($0 < \gamma < \infty$) we first discuss two important limit cases. Specifically, we compare the case in which the speculator is risk-neutral ($\gamma = 0$) with the case in which she is infinitely risk-averse ($\gamma \to \infty$).

4.1 Limit Case: Risk Neutrality ($\gamma = 0$)

For markets 1 and 2, when $\gamma = 0$ the equilibrium Theorem 1 specifies to,

$$\chi_i = \underbrace{\frac{\sqrt{2}\sigma_\delta}{4\sigma_\nu}}_{=\beta=\frac{1}{4\lambda}} \left(\zeta - \overline{\nu}_1 - \overline{\nu}_2\right) \underbrace{-\frac{1}{2}}_{=\alpha} \delta_i, \tag{16}$$

$$q_i = \overline{\nu}_i + \underbrace{\frac{\sigma_\nu}{\sqrt{2}\sigma_\delta}}_{=\lambda} \left(\chi_i + \delta_i\right) = \overline{\nu}_i + \frac{\zeta - \overline{\nu}_1 - \overline{\nu}_2}{4} + \frac{\sqrt{2}\sigma_\nu}{4\sigma_\delta}\delta_i, \tag{17}$$

$\forall i \in \{1,2\}$. The costs due to the price impact are fully determined by the strategic trade in market i plus the exogenous demand δ_i for asset i. As the price impact is linear in the (Kyle) model and the speculator acts as a monopolist, it is optimal to strategically offset exactly half of the exogenous demand, i.e., $\alpha = -\frac{1}{2}$ (same as α_3 for market 3).

We find that the risk-neutral equilibrium (16-17) and the equilibrium for market 3 (13-14) are equivalent to the equilibrium for our single-market model, i.e., the model only allowing for simple direct trading. We derive the single-market model in Appendix A. Such model is consistent with Rochet and Vila (1994) and Kyle (1989). Our intuition is the following. When the speculator is neutral to risk she is indifferent to the noise in the signal. As such, she has no preference for trading on the perfect signal (in market 3 or in markets 1 and 2 combined) rather than on the imperfect signal (in markets 1 and 2 separate). In other words, her risk neutrality implies that there is no benefit in trading asset 3 over the triangle. As a consequence, she chooses to trade only directly in all three markets, which are independent of each other.

The only difference across the three markets is that the trade of asset 3 is based on a perfect signal, while the trade of asset 1 and the trade of asset 2 are both based on an imperfect signal. Then, once we adjust for the mapping of variables, we see that the equilibrium parameters in (13-14) and in (16-17) correspond exactly to the single-market equilibrium parameters (70) without noise ($\sigma_\epsilon = 0$) and with noise ($\sigma_\epsilon \neq 0$), respectively. In particular, σ_δ and σ_ν in (70) with $\sigma_\epsilon = 0$ maps to $\sigma_{\delta 3}$ and $\sqrt{2}\sigma_\nu$ in (13-14), while σ_ϵ in (70) maps to σ_ν in (16-17). This is because the triangle relationship (1) implies that the complete information in market 3 (i.e., $2\sigma_\nu^2$) is exactly given by the partial information in market 1 (i.e., σ_ν^2) plus the partial information in market 2 (i.e., σ_ν^2), as per (2). Then, the "missing information" (i.e., the noise σ_ϵ^2) in market 1 must equal the partial information in market 2 and the "missing information" in market 2 must equal the partial information in market 1. Naturally, the equivalence between the risk-neutral specifications of the market-triangle model and the single-market model extends to the equilibrium pricing rules set by the market makers. That is, once we adjust for the same mapping of variables, we see that the quoting rules (14) and (17) correspond exactly to the single-market log-quote (71) without noise ($\sigma_\epsilon = 0$) and with noise ($\sigma_\epsilon \neq 0$), respectively.

Interestingly, in our equilibrium there is a violation of the triangular no-arbitrage condition. That is, we find that $q_3 \neq q_1 + q_2$ given (14) and (17). This is not a concern. The empirical literature provides evidence of frequent and short-lived triangular arbitrage opportunities (e.g., Chaboud and Vega (2014) and Foucault et al. (2017)). Note that our setting relies on the assumptions of (i) no non-negligible jumps in exchange rates (to justify the log-linearization of the speculator's profits) and (ii) imperishable information asymmetry. In reality, only sufficiently short time horizons accommodate these features. As such, our static (one-shot trading) Kyle model is most naturally interpreted as a stylized snapshot of the microstructure that is typical of the higher frequencies[4].

One might wonder why our speculator would not engage in the triangular arbitrage trade since such opportunity arises. In Appendix B we first show that for a simple model of two markets with one identical asset a speculator trading as a monopolist would not trade the arbitrage. Then we prove that the same argument holds for our market triangle, where the triangular arbitrage opportunity enters the utility as $\theta(q_1 + q_2 - q_3)$, i.e., the speculator sells θ in markets 1 and 2 and buys θ in market 3. Specifically, we show that solving the new first order conditions for χ_1, χ_2, χ_3 and θ yields,

$$\begin{aligned}
\chi_i &= \frac{\zeta - \overline{\nu}_1 - \overline{\nu}_2}{4\lambda} - \frac{4\lambda + \gamma\sigma_\nu^2}{4(2\lambda + \gamma\sigma_\nu^2)}\delta_i - \frac{\gamma\sigma_\nu^2}{4(2\lambda + \gamma\sigma_\nu^2)}\delta_j + \theta, \quad \forall\{i,j\} \in \{\{1,2\},\{2,1\}\} \\
\chi_3 &= \frac{\zeta - \overline{\nu}_1 - \overline{\nu}_2}{2\lambda_3} - \frac{\delta_3}{2} - \theta, \\
\theta &= \theta.
\end{aligned} \tag{18}$$

Note that (i) $\theta \in \mathbb{R}$ is free/not determined and (ii) χ_1, χ_2, χ_3 are exactly the same as in the original equilibrium (Theorem 1's χ_1, χ_2, χ_3 with parameters (9)) but with the additional $+\theta$ term for χ_1, χ_2 and $-\theta$ term for χ_3. That is, the strategic demands χ_1 and χ_2 each should be offsetting the selling of any θ in their corresponding market. Although in a model allowing for transaction costs (in the form of bid-ask spreads), Schrimpf et al. (2024) document empirically that violations of the law of one price need not translate into profitable

[4]In contrast, in a dynamic model the asymmetry in information should dissipate almost immediately as a specialized dealer can adjust her own quote based on others' quotes or order flows, as well as share the adverse selection risk with other dealers by trading in the inter-dealer market. Our model instead does not allow for cross-market order flow/quote observability and inter-dealer communication or trading.

triangular arbitrage opportunities, in line with our theoretical explanation.

Finally we calculate the utility U_0, i.e., the conditional profits for $\gamma = 0$. We plug the specified equilibrium trading positions (13) and (16) into the expectation terms of the objective function (with $\zeta = \zeta$ realized) for the mean-variance optimization problem. Note that the mean-variance optimization problem (7) simplifies to a mean-optimization problem since the variance terms vanish for $\gamma = 0$. Conditional on knowing ζ (i.e., after learning the realization of ζ at the beginning of the period), the risk-neutral informed speculator expects profits,

$$U_0 = \mathrm{E}\left[\Pi|\zeta=\zeta\right] = \mathrm{E}\left[\pi_1|\zeta=\zeta\right] + \mathrm{E}\left[\pi_2|\zeta=\zeta\right] + \mathrm{E}\left[\pi_3|\zeta=\zeta\right], \tag{19}$$

with:

$$\mathrm{E}\left[\pi_i|\zeta=\zeta\right] = \frac{\sqrt{2}\sigma_\delta}{16\sigma_\nu}\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)^2 - \frac{\delta_i}{4}\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right) + \frac{\sqrt{2}\sigma_\nu}{8\sigma_\delta}\delta_i^2, \quad \forall i \in \{1,2\},$$

$$\mathrm{E}\left[\pi_3|\zeta=\zeta\right] = \frac{\sqrt{2}\sigma_{\delta 3}}{8\sigma_\nu}\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)^2 - \frac{\delta_3}{2}\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right) + \frac{\sqrt{2}\sigma_\nu}{4\sigma_{\delta 3}}\delta_3^2.$$

We confirm that the equivalence between the risk-neutral specifications of the market-triangle model and of the single-market model carries over to the conditional profits. After mapping our variables we observe that the profits in each market of the triangle (19) follow the profits in the single-market model (72).

To summarize, a risk-neutral speculator only engages in direct trading as predicted in our single-market model in Appendix A, in line with Rochet and Vila (1994) and Kyle (1989). That is, she trades separately JPY/USD in market 1, USD/EUR in market 2, and JPY/EUR in market 3. However, a risk-neutral speculator never trades JPY/USD in market 1 and simultaneously USD/EUR in market 2 to indirectly trade JPY/EUR. There is no value in trading indirectly via the triangle because she does not care about any noise in the signal. Mathematically, when $\gamma = 0$ all interaction terms between the strategic demands χ_1 and χ_2 disappear in the optimization problem (7). Then, χ_1, χ_2, χ_3 are determined independently of each other.

4.2 Limit Case: Infinite Risk Aversion ($\gamma \to \infty$)

For market 1 and 2, in the limit for $\gamma \to \infty$ the equilibrium Theorem 1 specifies to,

$$\chi_i = \underbrace{\frac{\sqrt{5}\sigma_\delta}{4\sigma_\nu}}_{=\beta=\frac{1}{4\lambda}} (\zeta - \overline{\nu}_1 - \overline{\nu}_2) \underbrace{-\frac{1}{4}}_{=\alpha} \delta_i \underbrace{-\frac{1}{4}}_{=\alpha^*} \delta_j, \tag{20}$$

$$q_i = \overline{\nu}_i + \underbrace{\frac{\sigma_\nu}{\sqrt{5}\sigma_\delta}}_{=\lambda} (\chi_i + \delta_i) = \overline{\nu}_i + \frac{\zeta - \overline{\nu}_1 - \overline{\nu}_2}{4} + \frac{\sqrt{5}\sigma_\nu}{20\sigma_\delta} (3\delta_i - \delta_j), \tag{21}$$

$\forall \{i, j\} \in \{\{1, 2\}, \{2, 1\}\}$. If the speculator is infinitely averse to risk she is not willing to take any risk at all. She gets infinitely punished for any variation in the optimization problem (7), i.e., whenever the factor multiplying $-\frac{\gamma}{2}$ differs from zero. To avoid any risk, an infinitely risk-averse speculator has two options. First, she does not trade asset 1 or asset 2 at all (for which she has an imperfect signal), hence accepting to give up any profit in these markets. Second, she trades an optimal combination of quantities for asset 1 and asset 2 to synthetically get rid of any risk.

In particular, we show that she trades the same amount of asset 1 and asset 2 (simultaneously) to implement a triangle trade of asset 3, for which she has a perfect signal. In other words, she trades asset 3 twice, once directly and once indirectly, but with different dealers. Both trades are riskless, since the payoff for the direct trade is $\chi_3\nu_3$ and for the indirect trade is,

$$\chi_1\nu_1 + \chi_2\nu_2 = \chi_1 (\nu_1 + \nu_2) = \chi_1\nu_3, \tag{22}$$

for equal order size $\chi_1 = \chi_2$ and triangle relationship (1).

We find that the infinitely risk-averse speculator offsets less of her exogenous demand in asset i when she trades this asset as part of the indirect triangle trade rather than standalone. The reason is that the price impact of the exogenous demand only affects half of the indirect trade. Note that the cost of the indirect trade is the sum of the costs to trade assets 1 and 2. Because of the triangle relationship (1), this sum must be the same as the cost of trading

asset 3 directly,

$$\lambda(\chi_1 + \chi_2) = \lambda_3 \chi_3. \tag{23}$$

Recall that α captures the intensity of strategic trading in market 1 (2) in response to the speculator's exogenous demand δ_1 (δ_2) for asset 1 (2), while α^* the intensity of strategic trading in market 1 (2) in response to the exogenous demand in market 2 (1). In the limit case of infinitely risk-averse speculator, the constant $\alpha = -\frac{1}{4}$ (20) is half the magnitude of $\alpha = -\frac{1}{2}$ as in market 3 (13) and as in the limit case of risk-neutral speculator (16). We explain this halving as follows.

Suppose the infinitely risk-averse speculator wants to buy asset 3 indirectly and hence asset 1 and asset 2 in tandem. Let us focus on the strategic demand χ_1 for asset 1. If the exogenous demand δ_1 for asset 1 is a positive realization (another buy), then it increases the price impact in market 1 but it has no impact on the price in market 2. In other words, the price impact of δ_1 only affects the costs for one (χ_1) of the two legs of the triangle trade and hence half of the latter's total costs (23). In turn, the speculator offsets only half of the amount of exogenous demand δ_3 that she offsets for asset 3 (or only half of the amount of exogenous demand δ_1 that she offsets for asset 1 if she were risk-neutral). Moreover, if also the exogenous demand δ_2 for asset 2 is a positive realization, then it increases the price impact in market 2 and hence affects the strategic demand χ_2 for asset 2. However, since $\chi_1 = \chi_2$, the price impact of δ_2 must also affect the strategic demand χ_1 for asset 1 in exactly the same way, and the speculator has to offset δ_2 as well. This is why χ_1 includes a quartering of not only δ_1 but also δ_2. A specular argument can be made for the strategic demand χ_2 for asset 2.

Our main finding is that the price informativeness/price impact is smaller (and hence there is more liquidity) in markets for assets that are more intensively used for indirect trading via the triangle. More precisely, the price impact λ (21) in the limit case of infinitely risk-averse speculator is smaller by a factor of $\sqrt{\frac{5}{2}}$ compared to the λ (17) in the limit case of risk-neutral speculator. This result is driven by the residual amount of noise trading that the market maker is left to face after the speculator offsets a portion of it for price impact reasons. Such residual amount differs for the two limit cases.

Implicitly, we write the variance of the order flow observed by the market maker for market i as,

$$\begin{aligned}\text{Var}\left[\chi_i + \delta_i\right] &= \text{Var}\left[\beta\left(\zeta - \overline{\nu}_1 - \overline{\nu}_2\right) + \alpha\delta_i + \alpha^*\delta_j + \delta_i\right] \\ &= \beta^2 2\sigma_\nu^2 + (\alpha+1)^2\sigma_\delta^2 + (\alpha^*)^2\sigma_\delta^2, \end{aligned} \tag{24}$$

$\forall i \in \{1,2\}$. From the perspective of the market maker, the first term is the informative component of the order flow while the second and third terms constitute the confounding component. The two components are driven by two different sources of variation. The informative component is driven by σ_ν, while the confounding component is driven by σ_δ. Note that if $\delta_i = 0$ and $\delta_j = 0$ the variance of the order flow would be fully informative. That is, the market maker would know the β and perfectly infer the signal, leading to no trading at all as shown in Milgrom and Stokey (1982). This is the reason why all Kyle-style models require noise trading for the informed speculator to join at the onset. However, our model departs in a major way from Kyle (1985), as well as from Rochet and Vila (1994) and thus Kyle (1989).

In Kyle (1985), the overall noise affecting the market maker's pricing is simply the exogenous noise trading σ_u^2. The reason is that the informed speculator does not observe the noise trading u and therefore cannot strategically offset any fraction of it. This is equivalent to setting $\alpha = 0$ (and of course $\alpha^* = 0$) and $\sigma_\delta = \sigma_u$ in (24). Instead, the model of Rochet and Vila (1994) allows the speculator to observe and hence condition on the noise trades. Accordingly, they show that $\alpha = -\frac{1}{2}$ as in our market-triangle model (16) and in our single-market model (70) for the risk-neutral limit case. The speculator strategically offsets exactly half of the noise because she is an information monopolist, a standard result. This means that in our models specified with risk neutrality and in Rochet and Vila (1994) the residual noise is not anymore σ_δ^2, or σ_u^2 if we use the same notation of Kyle (1985), but just a fourth of that,

$$\text{Var}\left[\chi_i + \delta_i\right] = \beta^2 2\sigma_\nu^2 + \left(1 - \frac{1}{2}\right)^2 \sigma_\delta^2 = \beta^2 2\sigma_\nu^2 + \underbrace{\frac{1}{4}\sigma_\delta^2}_{<\sigma_\delta^2}, \tag{25}$$

$\forall i \in \{1,2\}$. This is equivalent to setting $\alpha = -\frac{1}{2}$ (and of course $\alpha^* = 0$) in (24). In our setting noise trading is modeled as exogenous random demand orders. Our informed speculator offsets part of this exogenous demand which she observes because she wants to minimize her price impact of trading. We motivate the speculator's ability to observe noise trading by interpreting her as a bank that trades FX on private information but also submits orders of her clients (who are uninformed and trade for other exogenous reasons not specified in the model) to the market maker.

However, in reality, a speculating bank is risk-averse and submits proprietary and client orders for more than one currency pair at once (and to more than one dealer at once). As such, the bank must strategically take into account the triangle relationship (1) because it drives the interconnectedness of valuations for multiple currency pairs. For instance, suppose the bank has perfect private information about JPY/EUR. When the bank chooses her strategic demand for JPY/USD, she considers (and thus partially mitigates) the price impact not only of her client order for JPY/USD but also of her client order for USD/EUR. In a specular way, when the bank chooses her strategic demand for USD/EUR, she considers the price impact not only of her client order for USD/EUR but also of her client order for JPY/USD. Importantly, such concern for cross-market price impacts only arises if the speculator is (i) risk-averse and (ii) allowed to trade in multiple markets. These two realistic and important features are ignored in Rochet and Vila (1994), as well as in our single-market model and risk-neutral specification of the market-triangle model.

In the limit case of infinitely risk-averse speculator, we specify the variance of the order flow observed by the market maker for market i as,

$$\text{Var}\left[\chi_i + \delta_i\right] = \beta^2 2\sigma_\nu^2 + \left(\left(1-\frac{1}{4}\right)^2 + \left(-\frac{1}{4}\right)^2\right)\sigma_\delta^2 = \beta^2 2\sigma_\nu^2 + \underbrace{\frac{5}{8}\sigma_\delta^2}_{=\frac{5}{2}\left(\frac{\sigma_\delta^2}{4}\right)}, \tag{26}$$

$\forall i \in \{1,2\}$. This is equivalent to setting $\alpha = \alpha^* = -\frac{1}{4}$ in (24), with α contributing 9 times more than α^* to the residual noise. Compared to the limit case of risk-neutral speculator (25), in the limit case of infinitely risk-averse speculator there is $\frac{5}{2}$ more residual noise from the market maker's perspective. As we explained before, the reason is that

the infinitely risk-averse speculator offsets less of the exogenous demand δ_i in asset i (and offsets some of the exogenous demand δ_j in market i) when she trades this asset as part of the indirect triangle trade. Moreover, because her risk aversion is infinite the speculator engages exclusively in the indirect triangle trade and never in the standalone direct trade. In other words, the market makers for asset 1 and 2 face 2.5 times more residual noise trading when these assets are traded simultaneously in equal amount (to trade asset 3 over the triangle) rather than separately. Accordingly, they learn less from indirect triangle trading (in the combined markets) than from direct trading (in the standalone markets) and set prices that comparatively are less sensitive to the absorbed order flow. In turn, price informativeness/price impact is smaller (liquidity is higher) and the speculator trades more aggressively in markets for assets that are more intensively used for indirect trading via the triangle. Mathematically, the factor 2 in λ and β for the equilibrium with risk neutrality (16-17) is multiplied by 2.5 and thus becomes 5 in λ and β for the equilibrium with infinite risk aversion (20-21).

Note that we observe the same scaling if we compute the utility U_∞, i.e., the conditional profits in the limit for $\gamma \to \infty$. We plug the specified equilibrium trading positions (13) and (20) into the objective function (with $\zeta = \zeta$ realized) for the mean-variance optimization problem. Conditional on knowing ζ (i.e., after learning the realization of ζ at the beginning of the period), the infinitely risk-averse informed speculator expects variance-adjusted profits,

$$\begin{aligned} U_\infty &= \mathrm{E}\Big[\Pi|\zeta=\zeta\Big] - \frac{\gamma}{2}\mathrm{Var}\Big[\Pi|\zeta=\zeta\Big] \\ &= \frac{\sqrt{5}\sigma_\delta + \sqrt{2}\sigma_{\delta 3}}{8\sigma_\nu}\big(\zeta - \overline{\nu}_1 - \overline{\nu}_2\big)^2 + \big(-\frac{\delta_1}{4} - \frac{\delta_2}{4} - \frac{\delta_3}{2}\big)\big(\zeta - \overline{\nu}_1 - \overline{\nu}_2\big) \\ &\quad + \frac{\sqrt{5}\sigma_\nu}{40\sigma_\delta}(\delta_1^2 + \delta_2^2) + \frac{\sqrt{5}\sigma_\nu}{20\sigma_\delta}\delta_1\delta_2 + \frac{\sqrt{2}\sigma_\nu}{4\sigma_{\delta 3}}\delta_3^2 > U_0. \end{aligned} \tag{27}$$

Consistently with β (20) $>$ β (16), the infinitely risk-averse speculator expects larger conditional (variance-adjusted) profits than those of the risk-neutral speculator (19). The boost in profits stems from the indirect triangle trade.

To summarize, an infinitely risk-averse speculator engages in direct trading exclusively of JPY/EUR in market 3. As an alternative, she trades JPY/USD in market 1 and simultane-

ously USD/EUR in market 2 to indirectly trade JPY/EUR. Indeed, for both trading routes her signal is perfect. However, an infinitely risk-averse speculator never trades JPY/USD in market 1 (respectively USD/EUR in market 2) alone, as this standalone trade is based on a noisy signal. Mathematically, in the limit for $\gamma \to \infty$ the strategic demands χ_1 and χ_2 become equal, i.e., she only trades JPY/USD and USD/EUR in markets 1 and 2 to indirectly trade JPY/EUR. Moreover, the speculator offsets less of the exogenous demand for JPY/USD (USD/EUR) —and offsets some of the exogenous demand for USD/EUR (JPY/USD) in market 1 (market 2)— when she trades JPY/USD (USD/EUR) as part of the indirect triangle trade (rather than as a standalone trade). In turn, the JPY/USD (USD/EUR) dealer is left to face more residual noise trading and so she learns less from the order flow in market 1 (market 2). Hence, the price impact is smaller and the speculator trades more aggressively and earns higher profits in markets 1 and 2 if JPY/USD and USD/EUR are more intensively used for indirect trading of JPY/EUR via the triangle (i.e., if the USD is more intensively used as a vehicle currency).

4.3 General Case: Finite Risk Aversion ($0 < \gamma < \infty$)

For markets 1 and 2, the general equilibrium solution for an informed speculator with limited risk aversion ($0 < \gamma < \infty$) becomes significantly more complicated. Such informed speculator employs a combination of direct and indirect trading when the risk aversion coefficient is larger than zero and finite. The basic intuition is that the speculator faces a trade-off between trading JPY/USD in market 1 and USD/EUR in market 2 on the imprecise signals versus trading indirectly JPY/EUR via the triangle on the perfect signal. Generally, the larger the risk aversion γ the more intensively the speculator uses the USD as a vehicle currency to indirectly trade JPY/EUR via markets 1 and 2.

To simplify our notation, we define a new combined parameter $\Gamma \coloneqq \gamma\sigma_\nu\sigma_\delta$. Then, for $0 < \gamma < \infty$, the equilibrium Theorem 1 specifies to,

$$\chi_i = \underbrace{\frac{\sigma_\delta}{4\Lambda(\Gamma)\sigma_\nu}}_{=\beta=\frac{1}{4\lambda}}(\zeta - \overline{\nu}_1 - \overline{\nu}_2) \underbrace{- \frac{4\Lambda(\Gamma)+\Gamma}{4\left(2\Lambda(\Gamma)+\Gamma\right)}}_{=\alpha}\delta_i \underbrace{- \frac{\Gamma}{4\left(2\Lambda(\Gamma)+\Gamma\right)}}_{=\alpha^*}\delta_j, \tag{28}$$

$$
\begin{aligned}
q_i &= \overline{\nu}_i + \underbrace{\Lambda(\Gamma)\frac{\sigma_\nu}{\sigma_\delta}}_{=\lambda}(\chi_i + \delta_i) \\
&= \overline{\nu}_i + \frac{\zeta - \overline{\nu}_1 - \overline{\nu}_2}{4} + \frac{\Lambda(\Gamma)\sigma_\nu\left(4\Lambda(\Gamma)+3\Gamma\right)}{4\sigma_\delta\left(2\Lambda(\Gamma)+\Gamma\right)}\delta_i - \frac{\Lambda(\Gamma)\gamma\sigma_\nu^2}{4\left(2\Lambda(\Gamma)+\Gamma\right)}\delta_j,
\end{aligned}
\tag{29}
$$

$\forall\{i,j\} \in \{\{1,2\},\{2,1\}\}$. Note that $\Lambda(\Gamma)$ is a known function of Γ. Importantly, the price impact λ is the typical Kyle-style ratio but adjusted for $\Lambda(\Gamma)$. Therefore, it is not immediately obvious that λ is increasing in the fundamental risk and decreasing in the amount of noise trading as in the standard framework of Kyle (1985). Naturally, such $\Lambda(\Gamma)$ factor also enters the trading intensity β and the partial offsetting α and α^* of the exogenous demands. We find that $\Lambda(\Gamma)$ is the unique positive root of the quartic equation,

$$
8\Lambda(\Gamma)^4 + 12\Gamma\Lambda(\Gamma)^3 + (5\Gamma^2 - 4)\Lambda(\Gamma)^2 - 4\Gamma\Lambda(\Gamma) - \Gamma^2 = 0. \tag{30}
$$

In Appendix C we provide a rigorous proof of the uniqueness of our model equilibrium, a key contribution of our paper. Importantly, we show that the uniqueness extends to a more general version of the model that allows assets to be correlated. In Appendix D we solve such generalized model and compare it to the baseline model without asset correlation. The analytical expression for $\Lambda(\Gamma)$ is long and convoluted but we can compactly write it as,

$$
\begin{aligned}
&\Lambda(\Gamma) = -\frac{3\Gamma}{8} + \frac{\sqrt{3}\sqrt{\frac{2\eta(\Gamma)}{\omega(\Gamma)} + 21\Gamma^2 + 48}}{24} + \frac{\sqrt{-\frac{6\eta(\Gamma)\sqrt{\frac{2\eta(\Gamma)}{\omega(\Gamma)}+21\Gamma^2+48} - 18\Gamma\omega(\Gamma)\sqrt{3}(3\Gamma^2+8)}{\omega(\Gamma)\sqrt{\frac{2\eta(\Gamma)}{\omega(\Gamma)}+21\Gamma^2+48}}}}{24} > 0, \\
&\text{with } \eta(\Gamma) = \omega(\Gamma)^2 + (-7\Gamma^2 - 16)\omega(\Gamma) + 25\Gamma^4 + 8\Gamma^2 + 16, \\
&\text{where } \omega(\Gamma) = (125\Gamma^6 + 276\Gamma^4 - 48\Gamma^2 + 12\sqrt{3}\sqrt{125\Gamma^{10} + 68\Gamma^8 - 144\Gamma^6 - 128\Gamma^4} - 64)^{\frac{1}{3}}.
\end{aligned}
\tag{31}
$$

Note that $\Lambda(\Gamma)$ depend only on Γ. In Fig. 1a we plot $\Lambda(\Gamma)$ as the unique positive root (colored in black) of the quartic equation (30). For $\Gamma = 0$ the function $\Lambda(\Gamma)$ converges to $\frac{1}{\sqrt{2}}$, while for $\Gamma \to \infty$ it converges to $\frac{1}{\sqrt{5}}$. Given these limits and the smoothness of the curve, we

find a very simple and reasonably accurate (error $< 2\%$) approximation $\widehat{\Lambda(\Gamma)}$ for $\Lambda(\Gamma)$,

$$\Lambda(\Gamma) \approx \widehat{\Lambda(\Gamma)} = \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{5}}\right) \frac{\mathrm{e}^{-\frac{\Gamma}{8}}}{1+\frac{5\Gamma}{6}} + \frac{1}{\sqrt{5}}. \tag{32}$$

The limits $\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{5}}$ are of course consistent with the price impact λ limits in (17) and (21), respectively. Since $\Lambda(\Gamma)$ depend only on Γ, also α and α^* depend only on Γ. In Fig. 1b we plot α and α^*, including their limits for $\Gamma = 0$ and $\Gamma \to \infty$. These limits are the special cases α in (16) and α, α^* in (20), respectively.

Next we calculate the utility U, i.e., the conditional profits in the general case for $0 < \gamma < \infty$. To do so, we proceed as we did for the limit case $\gamma \to \infty$. That is, we plug the specified equilibrium trading positions (13) and (28) into the objective function (with $\zeta = \zeta$ realized) for the mean-variance optimization problem. Conditional on knowing ζ (i.e., after learning the realization of ζ at the beginning of the period), the risk-averse informed speculator expects variance-adjusted profits,

$$\begin{aligned} U &= \mathrm{E}\Big[\Pi|\zeta=\zeta\Big] - \frac{\gamma}{2}\mathrm{Var}\Big[\Pi|\zeta=\zeta\Big] = \\ &= \frac{\sqrt{2}\Lambda(\Gamma)\sigma_{\delta 3}+\sigma_\delta}{8\Lambda(\Gamma)\sigma_\nu}\big(\zeta-\overline{\nu}_1-\overline{\nu}_2\big)^2 + (-\frac{\delta_1}{4}-\frac{\delta_2}{4}-\frac{\delta_3}{2})\big(\zeta-\overline{\nu}_1-\overline{\nu}_2\big)+ \\ &\quad + \frac{\Lambda(\Gamma)\sigma_\nu(4\Lambda(\Gamma)+\Gamma)}{8\sigma_\delta(2\Lambda(\Gamma)+\Gamma)}(\delta_1^2+\delta_2^2) + \frac{\Lambda(\Gamma)\gamma\sigma_\nu^2}{4(2\Lambda(\Gamma)+\Gamma)}\delta_1\delta_2 + \frac{\sqrt{2}\sigma_\nu}{4\sigma_{\delta 3}}\delta_3^2. \end{aligned} \tag{33}$$

We can compare these profits with those for the limit cases, i.e., (19) and (27).

5 Sensitivity Analysis

Our sensitivity analysis proceeds as follows. First we compute first order and second order derivatives of $\Lambda(\Gamma)$ and then we analyze the sensitivity of the equilibrium (28-29) with respect to changes in the underlying distribution parameters γ, σ_ν, and σ_δ. Finally we propose an example of a preliminary comparative analysis to elucidate how one sensitivity relative to another depends on the values taken by two parametric combinations, namely, $\Gamma = \gamma\sigma_\nu\sigma_\delta$ and $\mathrm{K} = \gamma\sigma_\delta^2$.

As the $\Lambda(\Gamma)$ function is already complicated, its derivatives are even more complicated. However, instead of relying on the simpler approximation $\widehat{\Lambda(\Gamma)}$ (32), we found a way to synthetically represent these derivatives in terms of the $\Lambda(\Gamma)$ function itself. Doing so ensures that we are always working with exact quantities. We rewrite the first and second order derivatives of $\Lambda(\Gamma)$ with respect to Γ as,

$$\frac{\mathrm{d}\Lambda(\Gamma)}{\mathrm{d}\Gamma} = -\frac{6\Lambda(\Gamma)^3 + 5\Gamma\Lambda(\Gamma)^2 - 2\Lambda(\Gamma) - \Gamma}{16\Lambda(\Gamma)^3 + 18\Gamma\Lambda(\Gamma)^2 + (5\Gamma^2 - 4)\Lambda(\Gamma) - 2\Gamma} < 0,$$
$$\frac{\mathrm{d}^2\Lambda(\Gamma)}{\mathrm{d}\Gamma^2} = \frac{p^{(8)}(\Lambda(\Gamma))}{(16\Lambda(\Gamma)^3 + 18\Gamma\Lambda(\Gamma)^2 + (5\Gamma^2 - 4)\Lambda(\Gamma) - 2\Gamma)^3} > 0,$$

with:

$$\begin{aligned} p^{(8)}(\Lambda(\Gamma)) = {} & 448\Lambda(\Gamma)^8 + 1632\Gamma\Lambda(\Gamma)^7 + 8(265\Gamma^2 - 26)\Lambda(\Gamma)^6 + 80(15\Gamma^3 - 8\Gamma)\Lambda(\Gamma)^5 \\ & + 2(125\Gamma^4 - 4(79\Gamma^2 - 2))\Lambda(\Gamma)^4 + 16(-15\Gamma^3 + 2\Gamma)\Lambda(\Gamma)^3 - 25\Gamma^4\Lambda(\Gamma)^2 \\ & - 16\Gamma^3\Lambda(\Gamma) - 5\Gamma^4. \end{aligned} \tag{34}$$

The polynomial $p^{(8)}(\Lambda(\Gamma))$ in $\Lambda(\Gamma)$ is of degree eight. In Fig. 2a and 2b we plot the two derivatives. We show that their signs are consistent with $\Lambda(\Gamma)$ being a positive and monotonically decreasing function of Γ, as in Fig. 1a.

As we know the signs of $\Lambda(\Gamma)$ and its derivatives, we can now infer the signs of the partial derivatives of the equilibrium parameters in (28-29) for markets 1 and 2 with respect to γ, σ_ν, and σ_δ. We apply the chain rule and derive the partial derivatives but we keep them in implicit form, to facilitate their interpretation. The economic intuition is best emphasized if we start by explaining the partial derivative for α, then λ, and finally β.

First, we want to know how the price impact λ changes with increasing risk aversion γ. Recall that α captures the intensity of strategic trading in market 1 (market 2) in response to the speculator's exogenous demand δ_1 (δ_2) for asset 1 (asset 2), while α^* the intensity of strategic trading in market 1 (market 2) in response to the exogenous demand in market 2 (market 1). Such strategic offsetting α is increasing in the speculator's risk aversion γ,

$$\frac{\partial\alpha}{\partial\gamma} = -\frac{\partial\alpha^*}{\partial\gamma} = -\frac{\sigma_\nu\sigma_\delta\left(\frac{\mathrm{d}\Lambda(\Gamma)}{\mathrm{d}\Gamma}\Gamma - \Lambda(\Gamma)\right)}{2\left(\Gamma + 2\Lambda(\Gamma)\right)^2} > 0. \tag{35}$$

Since α is negative, an increasing α means that it gets closer to zero. Indeed, as discussed before and according to Fig. 1b, α transitions from $-\frac{1}{2}$ at $\gamma = 0$ to $-\frac{1}{4}$ in the limit for $\gamma \to \infty$. Our intuition is as follows. As the speculator becomes more risk-averse, she is more hesitant to trade on her noisy signal in either market 1 or market 2. Instead, the speculator substitutes more of the standalone direct trades in markets 1 and 2 with the combined indirect trade via markets 1 and 2 simultaneously. Then, as she engages more in indirect trading (of asset 3) she offsets less of the exogenous demand, and hence leaves more noise in the order flow for assets 1 and 2 submitted to the respective market makers. These market makers learn less and set less informative quotes, which leads to a smaller price impact in markets 1 and 2,

$$\frac{\partial \lambda}{\partial \gamma} = \frac{\mathrm{d}\Lambda(\Gamma)}{\mathrm{d}\Gamma}\sigma_\nu^2 < 0. \tag{36}$$

In turn, a smaller price impact incentivizes the risk-averse speculator to trade more aggressively,

$$\frac{\partial \beta}{\partial \gamma} = -\frac{\mathrm{d}\Lambda(\Gamma)}{\mathrm{d}\Gamma}\frac{\sigma_\delta^2}{4\Lambda(\Gamma)^2} > 0. \tag{37}$$

Second, we want to know how the price impact λ changes with increasing σ_ν. Crucially, from the speculator's perspective σ_ν represents both the fundamental risk and the noise in the signal. On one hand, a larger σ_ν is more beneficial for trading asset 3 because it provides the speculator with a greater information advantage (larger information asymmetry) on the perfect signal $\zeta = \nu_3$. On the other hand, a larger σ_ν is less beneficial for the standalone trades of assets 1 and 2 because σ_ν constitutes the noise in the imperfect signal $\zeta = \nu_1 + \nu_2$. The signal ζ is the same but because of the triangle relationship (1) it is perfect for trading one asset while imperfect for trading the other two assets in the triangle.

With a larger σ_ν the risk-averse speculator is less prone to trade on the more noisy information and thus exits some of the standalone trades for assets 1 and 2 in order to engage more in indirect trading of asset 3 via the triangle (i.e., via markets 1 and 2 simultaneously). As more triangle trading stimulates less strategic offsetting of the exogenous demand, the

offsetting parameter α (which is negative) grows closer to zero,

$$\frac{\partial \alpha}{\partial \sigma_\nu} = -\frac{\partial \alpha^*}{\partial \sigma_\nu} = -\frac{\Gamma\left(\frac{\mathrm{d}\Lambda(\Gamma)}{\mathrm{d}\Gamma}\Gamma - \Lambda(\Gamma)\right)}{2\sigma_\nu\left(\Gamma + 2\Lambda(\Gamma)\right)^2} > 0. \tag{38}$$

At this point, less offsetting should translate into more residual noise left to handle to the market maker and hence smaller price impact, as predicted in (36). However, this is not what we observe. Instead, we find that an increasing σ_ν drives a larger (rather than smaller) price impact λ,

$$\frac{\partial \lambda}{\partial \sigma_\nu} = \frac{\frac{\mathrm{d}\Lambda(\Gamma)}{\mathrm{d}\Gamma}\Gamma + \Lambda(\Gamma)}{\sigma_\delta} > 0, \tag{39}$$

which the speculator has to mitigate by trading with less intensity,

$$\frac{\partial \beta}{\partial \sigma_\nu} = -\frac{\sigma_\delta\left(\frac{\mathrm{d}\Lambda(\Gamma)}{\mathrm{d}\Gamma}\Gamma + \Lambda(\Gamma)\right)}{4\sigma_\nu^2\Lambda(\Gamma)^2} < 0. \tag{40}$$

Our intuition for the puzzling behavior is as follows. Let us focus on the market maker's learning problem, as captured by the variance of the order flow (24). Overall, the variation in the order flow is driven by two different sources, namely, σ_ν and σ_δ. Depending on which one of the two dominates the other, the order flow is more or less informative to the market maker, respectively. Given that we find (39), we conclude that the exogenous increase in σ_ν must dominate the endogenous increase in σ_δ that is caused by the speculator's deeper involvement with indirect trading. That is, a larger σ_ν means that more of the order flow variance is due to informed trading and thus the market maker learns more, leading to larger price impact.

Besides the economics, note that also mathematically the derivatives (39) and (40) have ambiguous signs at a first sight. Specifically, the numerator of both (39) and (40) is the sum of a negative term and a positive term. We nondimensionalize both partial derivatives to infer their signs. We multiply (39) by the positive σ_δ and (40) by the positive $\sigma_\nu^2/\sigma_\delta$. Note that such scaling exercise does not alter the signs and by doing so we obtain two expressions only dependent on $\Gamma = \gamma\sigma_\nu\sigma_\delta$. Then we can directly plot them against Γ, as we show in Fig.

3a and in Fig. 3b, respectively. We confirm that $\partial\lambda/\partial\sigma_\nu$ (39) is positive and $\partial\beta/\partial\sigma_\nu$ (40) is negative.

Third, we want to know how the price impact λ changes with increasing noise trading variation σ_δ. Since σ_δ is the amount of noise trading in both market 1 and market 2, noise trading increases in tandem in the two markets as σ_δ gets larger. This presents a unique opportunity for the risk-averse speculator. The larger σ_δ in general promotes more direct trading in assets 1 or 2 separately (a standard Kyle effect) even if the speculator is risk-averse. However, exactly because the speculator is risk-averse such standalone trades (which are still based on a noisy signal) are not as palatable as the indirect triangle trade (which synthetically eradicates any risk). Therefore, a larger σ_δ leads to more direct trading but even more indirect trading as the amount of noise trading grows one-to-one in markets 1 and 2. As always, more indirect trading calls for less strategic offsetting of the exogenous demand and α grows closer to zero,

$$\frac{\partial\alpha}{\partial\sigma_\delta} = -\frac{\partial\alpha^*}{\partial\sigma_\delta} = -\frac{\Gamma\left(\frac{\mathrm{d}\Lambda(\Gamma)}{\mathrm{d}\Gamma}\Gamma - \Lambda(\Gamma)\right)}{2\sigma_\delta\left(\Gamma + 2\Lambda(\Gamma)\right)^2} > 0. \tag{41}$$

Less offsetting of the exogenous demand translates into more residual noise trading, further amplifying the amount of noise (which was already increasing through the exogenous rise in σ_δ). In turn, the learning capability of the market maker is hampered even more and the price impact is smaller,

$$\frac{\partial\lambda}{\partial\sigma_\delta} = \frac{\sigma_\nu\left(\frac{\mathrm{d}\Lambda(\Gamma)}{\mathrm{d}\Gamma}\Gamma - \Lambda(\Gamma)\right)}{\sigma_\delta^2} < 0, \tag{42}$$

allowing the speculator to trade more aggressively,

$$\frac{\partial\beta}{\partial\sigma_\delta} = \frac{-\frac{\mathrm{d}\Lambda(\Gamma)}{\mathrm{d}\Gamma}\Gamma + \Lambda(\Gamma)}{4\sigma_\nu\Lambda(\Gamma)^2} > 0. \tag{43}$$

Next, we turn to the second order partial derivatives. We note that the sign of the second order partial derivatives is difficult to infer by just looking at the analytical expressions. Therefore, we rely again on directly plotting the non-dimensionalized versions of these ex-

pressions, which preserve the signs. We include $\partial^2\lambda/\partial\gamma\partial\sigma_\nu$ and $\partial^2\beta/\partial\gamma\partial\sigma_\nu$ in Fig. 4a and Fig. 4b, $\partial^2\lambda/\partial\gamma\partial\sigma_\delta$ and $\partial^2\beta/\partial\gamma\partial\sigma_\delta$ in Fig. 5a and Fig. 5b. We include $\partial^2\alpha/\partial\gamma\partial\sigma_\nu$, $\partial^2\alpha/\partial\gamma\partial\sigma_\delta$, $\partial^2\alpha^*/\partial\gamma\partial\sigma_\nu$, and $\partial^2\alpha^*/\partial\gamma\partial\sigma_\delta$ in Fig. 6a and Fig. 6b. We use the sign of the second order partial derivatives to develop further intuition about the first order sensitivities.

For example, we now know that $\partial\lambda/\partial\sigma_\nu > 0$ (39) but its magnitude is driven by two opposite effects, as we discussed before. First, there is the standard positive effect also captured by Kyle (1985). That is, an increase in the fundamental risk leads to more learning for the market maker (because the order flow becomes more informative) and hence a larger price impact. Second, there is the negative effect provoked by the change in α which in turn also affects λ. The first effect is due to direct trading, while the second effect is due to indirect trading. Overall, $\partial\lambda/\partial\sigma_\nu$ (39) should be smaller than in the limit case with risk neutrality (equivalent to the single-market model with only direct trading) but larger than in the limit case with infinite risk aversion (where the speculator trades exclusively over the triangle),

$$\left.\frac{\partial\lambda}{\partial\sigma_\nu}\right|_{\gamma\to\infty} < \frac{\partial\lambda}{\partial\sigma_\nu} < \left.\frac{\partial\lambda}{\partial\sigma_\nu}\right|_{\gamma=0}. \tag{44}$$

That is, the sensitivity of the price impact λ to changes in σ_ν is bounded between the derivative in the limit for $\gamma\to\infty$ and the derivative for $\gamma = 0$,

$$\left.\frac{\partial\lambda}{\partial\sigma_\nu}\right|_{\gamma=0} = \frac{1}{\sqrt{2}\sigma_\delta}, \qquad \left.\frac{\partial\lambda}{\partial\sigma_\nu}\right|_{\gamma\to\infty} = \frac{1}{\sqrt{5}\sigma_\delta}. \tag{45}$$

We can easily derive these from the price impact λ in (17) and in (21), respectively. Mathematically, such sensitivity decreases monotonically from $\gamma = 0$ to $\gamma\to\infty$ since,

$$\frac{\partial^2\lambda}{\partial\gamma\partial\sigma_\nu} = \frac{4\sigma_\nu(-4p_1(\Gamma)\Lambda(\Gamma)^3 - 2p_2(\Gamma)\Lambda(\Gamma)^2 - p_3(\Gamma)\Lambda(\Gamma) + p_4(\Gamma))}{\Gamma(125\Gamma^6 + 68\Gamma^4 - 144\Gamma^2 - 128)^2} < 0,$$

with:

$$p_1(\Gamma) = 3125\Gamma^{10} + 550\Gamma^8 - 9784\Gamma^6 - 1536\Gamma^4 + 19712\Gamma^2 + 12288,$$

$$p_2(\Gamma) = 9375\Gamma^{11} + 3900\Gamma^9 - 27360\Gamma^7 - 12512\Gamma^5 + 36480\Gamma^3 + 22528\Gamma,$$

$$p_3(\Gamma) = -7875\Gamma^{10} - 3936\Gamma^8 + 25200\Gamma^6 + 10112\Gamma^4 - 39936\Gamma^2 - 24576,$$

$$p_4(\Gamma) = 3750\Gamma^{11} + 720\Gamma^9 - 11512\Gamma^7 - 2368\Gamma^5 + 15872\Gamma^3 + 8192\Gamma. \tag{46}$$

By just looking at this closed-form expression its sign is not obvious, so we plot it in Fig. 4a as a function of Γ.

A similar argument holds for $\partial\beta/\partial\sigma_\nu < 0$ (40). Overall, this derivative should be smaller than in the limit case with risk neutrality but larger than in the limit case with infinite risk aversion,

$$\left.\frac{\partial\beta}{\partial\sigma_\nu}\right|_{\gamma\to\infty} < \frac{\partial\beta}{\partial\sigma_\nu} < \left.\frac{\partial\beta}{\partial\sigma_\nu}\right|_{\gamma=0}. \tag{47}$$

That is, the sensitivity of the trading aggressiveness β to changes in σ_ν is bounded between the derivative in the limit for $\gamma \to \infty$ and the derivative for $\gamma = 0$,

$$\left.\frac{\partial\beta}{\partial\sigma_\nu}\right|_{\gamma=0} = -\frac{\sqrt{2}\sigma_\delta}{4\sigma_\nu^2}, \qquad \left.\frac{\partial\beta}{\partial\sigma_\nu}\right|_{\gamma\to\infty} = -\frac{\sqrt{5}\sigma_\delta}{4\sigma_\nu^2}. \tag{48}$$

We can easily derive these from the β in (16) and in (20), respectively. Mathematically, such sensitivity decreases monotonically from $\gamma = 0$ to $\gamma \to \infty$ since,

$$\frac{\partial^2\beta}{\partial\gamma\partial\sigma_\nu} = \frac{\sigma_\delta^2(-8p_1(\Gamma)\Lambda(\Gamma)^3 - 4p_2(\Gamma)\Lambda(\Gamma)^2 - p_3(\Gamma)\Lambda(\Gamma) + p_4(\Gamma))}{\Gamma^2\sigma_\nu(125\Gamma^6 + 68\Gamma^4 - 144\Gamma^2 - 128)^2} < 0,$$

with:

$$\begin{aligned}
p_1(\Gamma) &= -1375\Gamma^9 - 12816\Gamma^7 - 3024\Gamma^5 + 8704\Gamma^3 + 3072\Gamma,\\
p_2(\Gamma) &= -9125\Gamma^{10} - 20808\Gamma^8 + 18816\Gamma^6 + 20736\Gamma^4 - 10240\Gamma^2 - 8192,\\
p_3(\Gamma) &= -24375\Gamma^{11} + 660\Gamma^9 + 102256\Gamma^7 + 27200\Gamma^5 - 63488\Gamma^3 - 28672\Gamma,\\
p_4(\Gamma) &= -6250\Gamma^{12} - 22200\Gamma^{10} - 2024\Gamma^8 + 49888\Gamma^6 + 16128\Gamma^4 - 30720\Gamma^2 - 16384.
\end{aligned} \tag{49}$$

By just looking at this closed-form expression its sign is not obvious, so we plot it in Fig. 4b as a function of Γ.

We follow a specular line of reasoning to gain a deeper understanding of the derivatives $\partial\lambda/\partial\sigma_\delta < 0$ (42) and $\partial\beta/\partial\sigma_\delta > 0$ (43). The derivative $\partial\lambda/\partial\sigma_\delta < 0$ (42) should be larger than in the limit case with risk neutrality (equivalent to the single-market model with only direct

trading) but smaller than in the limit case with infinite risk aversion (where the speculator trades exclusively over the triangle),

$$\left.\frac{\partial\lambda}{\partial\sigma_\delta}\right|_{\gamma=0} < \frac{\partial\lambda}{\partial\sigma_\delta} < \left.\frac{\partial\lambda_\infty}{\partial\sigma_\delta}\right|_{\gamma\to\infty}. \tag{50}$$

That is, the sensitivity of the price impact λ to changes in σ_δ is bounded between the derivative for $\gamma = 0$ and the derivative in the limit for $\gamma \to \infty$,

$$\left.\frac{\partial\lambda}{\partial\sigma_\delta}\right|_{\gamma=0} = -\frac{\sqrt{2}\sigma_\nu}{2\sigma_\delta^2}, \qquad \left.\frac{\partial\lambda}{\partial\sigma_\delta}\right|_{\gamma\to\infty} = -\frac{\sqrt{5}\sigma_\nu}{5\sigma_\delta^2}. \tag{51}$$

We can easily derive these from the λ in (17) and in (21), respectively. Mathematically, such sensitivity increases monotonically from $\gamma = 0$ to $\gamma \to \infty$ since,

$$\frac{\partial^2\lambda}{\partial\gamma\partial\sigma_\delta} = \frac{4\sigma_\nu^2(8p_1(\Gamma)\Lambda(\Gamma)^3 + 64p_2(\Gamma)\Lambda(\Gamma)^2 + p_3(\Gamma)\Lambda(\Gamma) + p_4(\Gamma))}{\Gamma\sigma_\delta(125\Gamma^6 + 68\Gamma^4 - 144\Gamma^2 - 128)^2} > 0,$$

with:

$$\begin{aligned}
p_1(\Gamma) &= -2175\Gamma^8 - 2404\Gamma^6 + 160\Gamma^4 - 2432\Gamma^2 - 2048,\\
p_2(\Gamma) &= -275\Gamma^9 - 90\Gamma^7 + 213\Gamma^5 - 316\Gamma^3 - 256\Gamma,\\
p_3(\Gamma) &= -1875\Gamma^{10} + 10632\Gamma^8 + 8560\Gamma^6 - 5248\Gamma^4 + 9216\Gamma^2 + 8192,\\
p_4(\Gamma) &= 1250\Gamma^{11} + 3860\Gamma^9 - 2184\Gamma^7 - 2816\Gamma^5 + 6656\Gamma^3 + 4096\Gamma.
\end{aligned} \tag{52}$$

By just looking at this closed-form expression its sign is not obvious, so we plot it in Fig. 5a as a function of Γ.

Finally we analyze $\partial\beta/\partial\sigma_\delta > 0$ (43). The derivative $\partial\beta/\partial\sigma_\delta > 0$ (43) should be larger than in the limit case with risk neutrality but smaller than in the limit case with infinite risk aversion,

$$\left.\frac{\partial\beta}{\partial\sigma_\delta}\right|_{\gamma=0} < \frac{\partial\beta}{\partial\sigma_\delta} < \left.\frac{\partial\beta}{\partial\sigma_\delta}\right|_{\gamma\to\infty}. \tag{53}$$

That is, the sensitivity of the trading aggressiveness β to changes in σ_δ is bounded between

the derivative for $\gamma = 0$ and the derivative in the limit for $\gamma \to \infty$,

$$\left.\frac{\partial\beta}{\partial\sigma_\delta}\right|_{\gamma=0} = \frac{\sqrt{2}}{4\sigma_\nu}, \qquad \left.\frac{\partial\beta}{\partial\sigma_\delta}\right|_{\gamma\to\infty} = \frac{\sqrt{5}}{4\sigma_\nu}. \tag{54}$$

We can easily derive these from the β in (16) and in (20), respectively. Mathematically, such sensitivity increases monotonically from $\gamma = 0$ to $\gamma \to \infty$ since,

$$\frac{\partial^2\beta}{\partial\gamma\partial\sigma_\delta} = \frac{\sigma_\delta(8p_1(\Gamma)\Lambda(\Gamma)^3 + 4p_2(\Gamma)\Lambda(\Gamma)^2 + p_3(\Gamma)\Lambda(\Gamma) + p_4(\Gamma))}{\Gamma(125\Gamma^6 + 68\Gamma^4 - 144\Gamma^2 - 128)^2} > 0$$

with:

$$\begin{aligned}
p_1(\Gamma) &= 1625\Gamma^8 + 6952\Gamma^6 - 528\Gamma^4 - 2048\Gamma^2 + 3072 \\
p_2(\Gamma) &= 4875\Gamma^9 + 9496\Gamma^7 - 10816\Gamma^5 - 1664\Gamma^3 + 10240\Gamma \\
p_3(\Gamma) &= 8125\Gamma^{10} - 8500\Gamma^8 - 42992\Gamma^6 + 10048\Gamma^4 + 16384\Gamma^2 - 12288 \\
p_4(\Gamma) &= -7300\Gamma^9 - 4968\Gamma^7 + 13152\Gamma^5 - 2048\Gamma^3 - 10240\Gamma
\end{aligned} \tag{55}$$

By just looking at this closed-form expression its sign is not obvious, so we plot it in Fig. 5b as a function of Γ.

We will include a more detailed discussion of the second order sensitivities in an updated draft of this paper. For now we conclude by summarizing our observation of critical values for Γ. The non-dimensionalized derivative $\partial^2\lambda/\partial\gamma\partial\sigma_\delta$ has a maximum,

$$\left.\frac{\partial^2\lambda}{\partial\gamma\partial\sigma_\delta}\right|^{(\mathrm{ND})}_{\mathrm{max}} = 0.092, \qquad \text{for} \quad \Gamma_{\lambda_{\mathrm{max}}} = 0.681.$$

The non-dimensionalized derivative $\partial^2\beta/\partial\gamma\partial\sigma_\nu$ has a minimum,

$$\left.\frac{\partial^2\beta}{\partial\gamma\partial\sigma_\nu}\right|^{(\mathrm{ND})}_{\mathrm{min}} = -0.051, \qquad \text{for} \quad \Gamma_{\beta_{\mathrm{min}}} = 0.987.$$

The non-dimensionalized derivative $\partial^2\alpha/\partial\gamma\partial\sigma_\delta = \partial^2\alpha/\partial\gamma\partial\sigma_\nu$ has a minimum and a zero,

$$\left.\frac{\partial^2\alpha}{\partial\gamma\partial\sigma_\delta}\right|^{(\mathrm{ND})}_{\mathrm{min}} = \left.\frac{\partial^2\alpha}{\partial\gamma\partial\sigma_\nu}\right|^{(\mathrm{ND})}_{\mathrm{min}} = -0.012, \qquad \text{for} \quad \Gamma_{\alpha_{\mathrm{min}}} = 2.014.$$

$$\frac{\partial^2 \alpha}{\partial \gamma \partial \sigma_\delta}^{(\mathrm{ND})} = \frac{\partial^2 \alpha}{\partial \gamma \partial \sigma_\nu}^{(\mathrm{ND})} = 0, \qquad \text{for} \quad \Gamma_{\alpha_{\mathrm{zero}}} = 1.095.$$

The non-dimensionalized derivative $\partial^2 \alpha^* / \partial \gamma \partial \sigma_\delta = \partial^2 \alpha^* / \partial \gamma \partial \sigma_\nu$ has a maximum and a zero,

$$\frac{\partial^2 \alpha^*}{\partial \gamma \partial \sigma_\delta}\bigg|_{\mathrm{max}}^{(\mathrm{ND})} = \frac{\partial^2 \alpha^*}{\partial \gamma \partial \sigma_\nu}\bigg|_{\mathrm{max}}^{(\mathrm{ND})} = 0.012, \qquad \text{for} \quad \Gamma_{\alpha^*_{\mathrm{max}}} = 2.014.$$

$$\frac{\partial^2 \alpha^*}{\partial \gamma \partial \sigma_\delta}^{(\mathrm{ND})} = \frac{\partial^2 \alpha^*}{\partial \gamma \partial \sigma_\nu}^{(\mathrm{ND})} = 0, \qquad \text{for} \quad \Gamma_{\alpha^*_{\mathrm{zero}}} = 1.095.$$

Since we have analytical expressions for the second order derivatives, we can cross-compare them to discuss the relative second order sensitivity of one derivative with respect to another. We can neatly distinguish for which pair of values taken by the two parameter combinations $\Gamma = \gamma \sigma_\nu \sigma_\delta$ and $\mathrm{K} = \gamma \sigma_\delta^2$ the magnitude of one second order derivative is larger, equal, or smaller than the magnitude of another derivative. In Fig. 7a and Fig. 7b we show just two of the many possible comparisons we can analyze. The plot in Fig. 7a shows the two parametric regions for which $\partial^2 \lambda / \partial \gamma \partial \sigma_\delta$ is larger or smaller than $|\partial^2 \lambda / \partial \gamma \partial \sigma_\nu|$. The blue curve is the equality curve, which identifies for which γ, σ_ν, and σ_δ values the two derivatives coincide (in magnitude). Note that the intersection between the blue curve and the grey vertical line ($\Gamma_{\lambda_{\mathrm{max}}} = 0.681$) indicates the exact coordinates where such equal derivatives are maximal. A similar line of reasoning holds when discussing more complicated comparisons, like $|\partial^2 \alpha / \partial \gamma \partial \sigma_\nu|$ versus $|\partial^2 \beta / \partial \gamma \partial \sigma_\nu|$ shown in the plot of Fig. 7b. The plot illustrates how, given the sensitivities of the offsetting intensity (of some of the exogenous demand) and trading aggressiveness to changes in risk aversion, the rate of change of the offsetting intensity with respect to changes in information asymmetry (or noise for the imperfect signal) relates to the respective rate of change for the trading aggressiveness. The plot shows two (local) maximal equal derivatives, depending on the sign of $\partial^2 \alpha / \partial \gamma \partial \sigma_\nu$ that changes at $\Gamma_{\alpha_{\mathrm{zero}}} = 1.095$ (the red vertical line).

6 Model Extensions

In Appendix D we provide analytical solutions for a model with a more general information structure of the assets' payoffs. Specifically, we generalize the covariance matrix (2) to allow for the correlation of assets 1 and 2 ($\rho \neq 0$),

$$\begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_1 + \nu_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \overline{\nu}_1 \\ \overline{\nu}_2 \\ \overline{\nu}_1 + \overline{\nu}_2 \end{bmatrix}, \begin{bmatrix} \sigma_\nu^2 & \rho\sigma_\nu^2 & \sigma_\nu^2 + \rho\sigma_\nu^2 \\ \rho\sigma_\nu^2 & \sigma_\nu^2 & \sigma_\nu^2 + \rho\sigma_\nu^2 \\ \sigma_\nu^2 + \rho\sigma_\nu^2 & \sigma_\nu^2 + \rho\sigma_\nu^2 & 2\sigma_\nu^2 + 2\rho\sigma_\nu^2 \end{bmatrix} \right). \tag{56}$$

The case of correlated assets is interesting because on ρ depends the symmetry of the assets' triangle. For example, for $\rho = -1/2$ the three assets have identical variance for their payoffs: $\text{Var}[\nu_3] = 2\sigma_\nu^2(1+\rho) = \sigma_\nu^2 = \text{Var}[\nu_1] = \text{Var}[\nu_2]$. In other words, for $\rho = -1/2$ the assets' triangle is fully symmetric in terms of payoff's risk. In general, when assets 1 and 2 are correlated, the strategic behaviour of the speculator is as follows,

$$\frac{\partial \alpha}{\partial \rho} = \frac{\partial(-\alpha^*)}{\partial \rho} < 0, \qquad \frac{\partial \lambda}{\partial \rho} > 0, \qquad \frac{\partial \beta}{\partial \rho} < 0. \tag{57}$$

In the extreme case of $\rho = 1$, the speculator does not need to engage in triangle trading at all because trading asset 1 is exactly the same as trading asset 2. Moreover, the variance for asset 3 becomes $4\sigma_\nu^2$, that is, double the variance for the same asset in the baseline model with no correlation. On top of that, the signal becomes more precise for trading only asset 1 or asset 2. The intuition is the following. For trading only asset 1 (asset 2) the signal $\zeta = \nu_1 + \nu_2$ provides noisy information, where the noise component is ν_2 (ν_1). Then, as assets 1 and 2 become more (positively) correlated, the signal becomes less noisy and more valuable for trading either asset separately. As such, the speculator chooses more direct trading (as if she were less risk-averse) and hence strategically offsets more of the exogenous demand (α becomes even more negative). More direct trading in just one asset also follows from a pure correlation effect, since the larger the correlation with the other asset the more the two assets are substitute of each other.

In contrast, as assets 1 and 2 become less correlated or more negatively correlated, the

variance for asset 3 becomes smaller and so does the informational advantage of the speculator (in the extreme case of $\rho = -1$ the signal becomes useless). However, at the same time, the noise effect described above is also triggered. That is, the signal becomes less noisy and hence more valuable for direct trading also when the assets become more negatively correlated. To compare the magnitude of the two effects we compute the ratio given by the variance of the noise $\text{Var}[\nu_3|\nu_1]$ divided by the total variation in the signal $\text{Var}[\nu_3]$. We find that such ratio increases as the correlation becomes more negative, which translates into more indirect trading as the speculator strategically offsets less of the exogenous demand (α becomes less negative). More indirect trading also follows from the pure correlation effect, since the more negative the correlation between assets 1 and 2 the less incentive has the speculator to trade these assets separately.

The implication on the price impact naturally follows, as more strategic offsetting (more direct trading) leads to a larger price impact compared to less strategic offsetting (more indirect trading). Then, the larger the price impact the smaller the orders submitted by the speculator.

In Appendix E we provide analytical solutions for another model with a more general information structure of the assets' payoffs. Specifically, we generalize the covariance matrix (2) to allow for asymmetric risk in the payoff of assets 1 and 2 ($\sigma_{\nu 1} \neq \sigma_{\nu 2}$),

$$\begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_1 + \nu_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \overline{\nu}_1 \\ \overline{\nu}_2 \\ \overline{\nu}_1 + \overline{\nu}_2 \end{bmatrix}, \begin{bmatrix} \sigma_{\nu 1}^2 & 0 & \sigma_{\nu 1}^2 \\ 0 & \sigma_{\nu 2}^2 & \sigma_{\nu 2}^2 \\ \sigma_{\nu 1}^2 & \sigma_{\nu 2}^2 & \sigma_{\nu 1}^2 + \sigma_{\nu 2}^2 \end{bmatrix} \right). \tag{58}$$

The case of asymmetric assets is interesting because the wedge $\sigma_{\nu 1} \neq \sigma_{\nu 2}$ in assets' fundamental risk drives a wedge $\lambda_1 \neq \lambda_2$ in price impacts. Accordingly, the equilibrium conjectures of Theorem 1 rewrite as,

$$\begin{aligned} \chi_i &= \beta_i \left(\zeta - \overline{\nu}_i - \overline{\nu}_j\right) + \alpha_i \delta_i + \alpha_j^* \delta_j, & \chi_3 &= \beta_3 \left(\zeta - \overline{\nu}_1 - \overline{\nu}_2\right) + \alpha_3 \delta_3, \\ q_i &= \overline{\nu}_i + \lambda_i \left(\chi_i + \delta_i\right), & q_3 &= \overline{\nu}_1 + \overline{\nu}_2 + \lambda_3 \left(\chi_3 + \delta_3\right), \\ & \forall \{i, j\} \in \{\{1,2\},\{2,1\}\}. \end{aligned} \tag{59}$$

When the volatility of asset 1's payoff is different from the volatility of asset 2's payoff, the strategic behaviour of the speculator depends as follows on her increasing risk aversion,

$$\frac{\partial \alpha_i}{\partial \gamma} = \frac{\partial(-\alpha_j^*)}{\partial \gamma} > 0, \qquad \forall \{i,j\} \in \{\{1,2\},\{2,1\}\},$$

$$\frac{\partial \lambda_i}{\partial \gamma} < 0, \qquad \forall i \in \{1,2\},$$

$$\frac{\partial \beta_1}{\partial \gamma} \begin{cases} > 0 & \text{if } \Gamma_2 < 0.97 + 1.32\Gamma_1 \\ \leq 0 & \text{otherwise} \end{cases}, \qquad \frac{\partial \beta_2}{\partial \gamma} \begin{cases} > 0 & \text{if } \Gamma_2 > -0.73 + 0.76\Gamma_1 \\ \leq 0 & \text{otherwise} \end{cases}. \tag{60}$$

The effect of risk aversion on the strategic offsetting of the exogenous demand is standard. That is, a more risk averse speculator engages more in indirect trading of asset 3 in markets 1 and 2. As the speculator strategically offsets less of the exogenous demand δ_i in market i (and offsets some of the exogenous demand δ_j in market i), the dealer faces more residual variation of noise trading and the price impact is smaller in market i. However, this does not necessarily incentivize the speculator to trade more aggressively in that market. Specifically, we find that the trading aggressiveness of the speculator depends on the magnitude of $\Gamma_1 = \gamma\sigma_\delta\sigma_{\nu 1}$ vs. $\Gamma_2 = \gamma\sigma_\delta\sigma_{\nu 2}$.

Let us focus on β_1, the trading aggressiveness for asset 1. As $\sigma_{\nu 1}$ is sufficiently larger than $\sigma_{\nu 2}$ (i.e., $\Gamma_2 < 0.97 + 1.32\Gamma_1$), the private signal $\zeta = \nu_1 + \nu_2$ becomes more precise to trade asset 1 (rather than asset 2) directly. On the other hand, if $\sigma_{\nu 2}$ is sufficiently larger than $\sigma_{\nu 1}$ (i.e., $\Gamma_2 > 0.97 + 1.32\Gamma_1$), the inferred information about asset 1 becomes more imprecise. Then, as the speculator is more risk-averse, she is concerned about the extra noise in the signal and trades even less. A specular argument holds for β_2, the trading aggressiveness for asset 2.

Next, we explain how the speculator's behaviour changes as $\sigma_{\nu i}$ increases,

$$\frac{\partial \alpha_j}{\partial \sigma_{\nu i}} = \frac{\partial(-\alpha_i^*)}{\partial \sigma_{\nu i}} > 0, \qquad \frac{\partial \alpha_i}{\partial \sigma_{\nu i}} = \frac{\partial(-\alpha_j^*)}{\partial \sigma_{\nu i}} < 0,$$

$$\frac{\partial \lambda_j}{\partial \sigma_{\nu i}} < 0, \qquad \frac{\partial \beta_j}{\partial \sigma_{\nu i}} < 0, \qquad \frac{\partial \lambda_i}{\partial \sigma_{\nu i}} > 0, \qquad \frac{\partial \beta_i}{\partial \sigma_{\nu i}} < 0. \tag{61}$$

$\forall\{i,j\} \in \{\{1,2\},\{2,1\}\}$. Recall that $\sigma_{\nu i}$ is the noise for the imprecise signal about asset j. Suppose $i = 2, j = 1$. Then $\sigma_{\nu 2}$ is the noise in the signal for trading asset 1. Let us look at the left-hand side. As the noise increases the speculator uses asset 1 less for direct trading and more for indirect trading, with less strategic offsetting of the exogenous demand (α becomes less negative). Now suppose $i = 1, j = 2$. Let us look at the right-hand side. As $\sigma_{\nu 1}$ increases, the signal-to-noise ratio for asset 1 goes up. A more precise signal encourages the speculator to trade more on it (more direct trading), and hence to strategically offset more of the exogenous demand (α becomes even more negative). The effect on price impact naturally follows from the strategic offsetting. However, similarly to before, the speculator's trading aggressiveness is affected by two dynamics. Suppose $i = 2, j = 1$. The increased noise drives more indirect trading, which lowers the price impact, but not enough to compensate for the worry of a risk-averse speculator about an increasingly noisy signal. Overall, the speculator trades less aggressively.

7 Conclusion

We characterize the closed-form solution of a Kyle model involving three interlinked asset markets that form a triangular relationship, where the value of one asset is derived from the combination of the other two assets. An informed, risk-averse speculator engages in strategic trading with three uninformed, risk-neutral market makers, each specializing in one of the asset markets. After receiving perfect private information about one asset, the speculator trades it directly in its market and indirectly through the triangle by simultaneously trading the other two assets. In our model, the informed trader observes and strategically offsets some of the exogenous noise trading. These offsetting trades are smaller in markets where the speculator trades indirectly via the triangle, hence leading to a larger residual amount of noise trading in these markets. More residual noise trading means the market maker learns less about the private information. In equilibrium, this results in lower price informativeness and price impact. Then, in markets where indirect trading via the triangle is more common the speculator is willing to trade more aggressively in order to earn higher profits.

In the context of FX markets our key result implies that the price impact of trading

is smaller for currency pairs that are pre-dominantly used for indirect trading over the FX triangle. Indeed, the empirical literature documents that the price impact of trading is smaller for currency pairs that include the USD, i.e., the dominant vehicle currency (Ranaldo and Somogyi, 2021; Ranaldo and Santucci de Magistris, 2022; Lacava et al., 2023; Somogyi, 2023). It is important to understand the determinants of the price impact of trading and the interconnectedness across currency pairs. For instance, Filippou et al. (2024) show that the price impact of trading has first order implications on the profitability of FX trading strategies.

References

Admati, Anat, 1985, A Noisy Rational Expectations Equilibrium for Multi-Asset Securities Markets, *Econometrica* 53, 629–658.

Amihud, Yakov, 2002, Illiquidity and stock returns: cross-section and time-series effects, *Journal of financial markets* 5, 31–56.

Bernhardt, Dan, and Bart Taub, 2008, Cross-Asset Speculation in Stock Markets, *The Journal of Finance* 63, 2385–2427.

Bhushan, Ravi, 1991, Trading Costs, Liquidity, and Asset Holdings, *The Review of Financial Studies* 4, 343–360.

Bouchaud, Jean-Philippe, Luis Carlos Garcia del Molino, Iacopo Mastromatteo, and Michael Benzaquen, 2020, The multivariate kyle model: More is different, *SIAM Journal on Financial Mathematics* 11.

Boulatov, Alex, Terrence Hendershott, and Dmitry Livdan, 2013, Informed tradingand portfolio returns, *Review of Economic Studies* 80, 35–72.

Bräuer, Leonie, and Harald Hau, 2023, Can time varying currency risk hedging explain exchange rates?, Working paper.

Brunnermeier, Markus, and Lasse Pedersen, 2009, Market liquidity and funding liquidity, *The Review of Financial Studies* 22, 2201–2238.

Caballe, Jordi, and Murugappa Krishnan, 1994, Imperfect Competition in a Multi-Security Market with Risk Neutrality, *Econometrica* 62, 695–704.

Cespa, Giovanni, and Paolo Colla, 2023, Transparency and Fragmentation with Strategic Trading, Available at ssrn: Working paper no. 2879273.

Cespa, Giovanni, and Thierry Foucault, 2014, Illiquidity Contagion and Liquidity Crashes, *The Review of Financial Studies* 27, 1615–1660.

Cespa, Giovanni, Antonio Gargano, Steven J Riddiough, and Lucio Sarno, 2022, Foreign exchange volume, *The Review of Financial Studies* 35, 2386–2427.

Chaboud, Alain, and Clara Vega, 2014, Rise of the machines: Algorithmic trading in the foreign exchange market, *The Journal of Finance* 69, 2045–2084.

Chordia, Tarun, Richard Roll, and Avanidhar Subrahmanyam, 2000, Commonality in liquidity, *Journal of Financial Economics* 56, 3–28.

Chordia, Tarun, Richard Roll, and Avanidhar Subrahmanyam, 2001, Market liquidity and trading activity, *The Journal of Finance* 56, 501–530.

Chordia, Tarun, Asani Sarkar, and Avanidhar Subrahmanyam, 2005, An empirical analysis of stock and bond market liquidity, *The Review of Financial Studies* 18, 85–129.

Chowdhry, Bhagwan, and Vikram Nanda, 1991, Multimarket Trading and Market Liquidity, *The Review of Financial Studies* 4, 483–511.

Collins, George, 1975, Quantifier Elimination for Real Closed Fields by Cylindrical Algebraic Decompostion, in *Lecture Notes in Computer Science*, volume 33, 134–183 (Springer).

Devereux, Michael, and Shouyong Shi, 2013, Vehicle currency, *International Economic Review* 54, 97–133.

Ernst, Thomas, 2022, Stock-specific price discovery from etfs, Working paper.

Filippou, Ilias, Thomas Maurer, Luca Pezzo, and Mark P. Taylor, 2024, Importance of transaction costs for asset allocation in foreign exchange markets, *Journal of Financial Economics* 103886.

Foucault, Thierry, Roman Kozhan, and Wing Wah Tham, 2017, Toxic arbitrage, *The Review of Financial Studies* 30, 1053–1094.

Glosten, Lawrence, and Paul Milgrom, 1985, Bid, ask and transaction prices in a specialist market with heterogeneously informed traders, *Journal of Financial Economics* 14, 71–100.

Grossman, Sanford, and Joseph Stiglitz, 1980, On the impossibility of informationally efficient markets, *The American Economic Review* 70, 393–408.

Hasbrouck, Joel, and Duane Seppi, 2001, Common factors in prices, order flows, and liquidity, *Journal of Financial Economics* 59, 383–411.

Hubert, Evelyne, and George Labahn, 2012, Rational invariants of scalings from hermite normal forms, *Proceedings of International Symposium of Symbolic and Algebraic Computation, ISSAC'12* 219–226.

Hubert, Evelyne, and George Labahn, 2013, Scaling invariants and symmetry reduction of dynamical systems, *Foundations of Computational Mathematics* 13, 479–516.

Huddart, Steven, John S. Hughes, and Carolyn B. Levine, 2001, Public Disclosure and Dissimulation of Insider Trades, *Econometrica* 69, 665–681.

Karnaukh, Nina, Angelo Ranaldo, and Paul Soederlind, 2015, Understanding FX Liquidity, *The Review of Financial Studies* 28, 3073–3108.

Karolyi, Andrew, Kuan-Hui Lee, and Mathijs van Dijk, 2012, Understanding commonality in liquidity around the world, *The Journal of Financial Economics* 105, 82–112.

Kondor, Peter, and Gabor Pinter, 2022, Clients' Connections: Measuring the Role of Private Information in Decentralized Markets, *The Journal of Finance* 77, 505–544.

Krugman, Paul, 1980, Vehicle currencies and the structure of international exchange, *Journal of Money, Credit and Banking* 12, 513–526.

Kyle, Albert S., 1985, Continuous auctions and insider trading, *Econometrica* 53.

Kyle, Albert S., 1989, Informed Speculation with Imperfect Competition, *The Review of Economic Studies* 56, 317–355.

Lacava, Demetrio, Angelo Ranaldo, and Paolo Santucci de Magistris, 2023, Realized Illiquidity, Working paper.

Lazard, Daniel, and Fabrice Rouillier, 2007, Solving parametric polynomial systems, *Journal of Symbolic Computation* 42, 636–667.

Liang, Songxin, Jurgen Gerhard, DJ Jeffrey, and Guillaume Moroz, 2009, A Package for Solving Parametric Polynomial Systems, Communications in computer algebra, University of Western Ontario.

Lyons, Richard, and Michael Moore, 2009, An information approach to international currencies, *Journal of International Economics* 79, 211–221.

Mancini, Loriano, Angelo Ranaldo, and Jan Wrampelmeyer, 2013, Liquidity in the foreign exchange market: Measurement, commonality, and risk premiums, *The Journal of Finance* 68, 1805–1841.

Maza, Marc Moreno, 2005, On triangular decompositions of algebraic varieties, *Elsevier Science* .

Milgrom, Paul, and Nancy Stokey, 1982, Information, trade and common knowledge, *Journal of Economic Theory* 26, 17–27.

Pasquariello, Paolo, 2007, Imperfect Competition, Information Heterogeneity, and Financial Contagion, *The Review of Financial Studies* 20, 391–426.

Porcu, Emilio, Moreno Bevilacqua, Robert Schaback, and Chris Oates, 2024, The matern model: A journey through statistics, numerical analysis and machine learning, *Statistical Science* 39, 469–492.

Ranaldo, Angelo, and Paolo Santucci de Magistris, 2022, Liquidity in the global currency market, *Journal of Financial Economics* 146, 859–883.

Ranaldo, Angelo, and Fabricius Somogyi, 2021, Asymmetric information risk in fx markets, *Journal of Financial Economics* 140, 391–411.

Rochet, Jean-Charles, and Jean-Luc Vila, 1994, Insider trading without normality, *The Review of Economic Studies* 61, 131–152.

Rostek, Marzena, and Ji Hee Yoon, 2021, Exchange design and efficiency, *Econometrica* 89, 2887–2928.

Roussanov, Nikolai, and Xiaoliang Wang, 2023, Following the fed: Limits of arbitrage and the dollar, Working paper.

Schrimpf, Andreas, Wenqian Huang, Angelo Ranaldo, and Fabricius Somogyi, 2024, Constrained liquidity provision in currency markets, *BIS Working Papers* .

Somogyi, Fabricius, 2023, Dollar Dominance in FX Trading, Working paper.

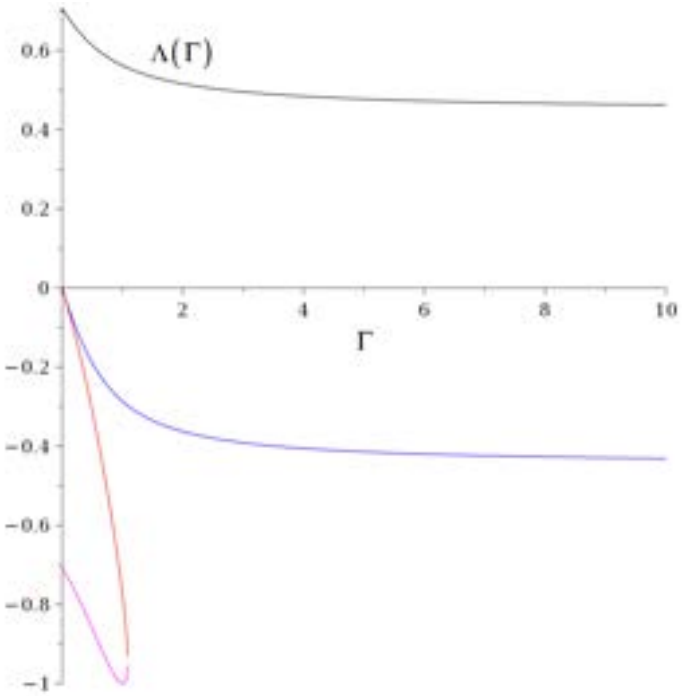
Tuo, Rui, and Wenjia Wang, 2020, Kriging prediction with isotropic matern correlations: Robustness and experimental designs, *Journal of Machine Learning Research* 21, 1–38.

Veldkamp, Laura, 2006, Information Markets and the Comovement of Asset Prices, *The Review of Economic Studies* 73, 823–845.

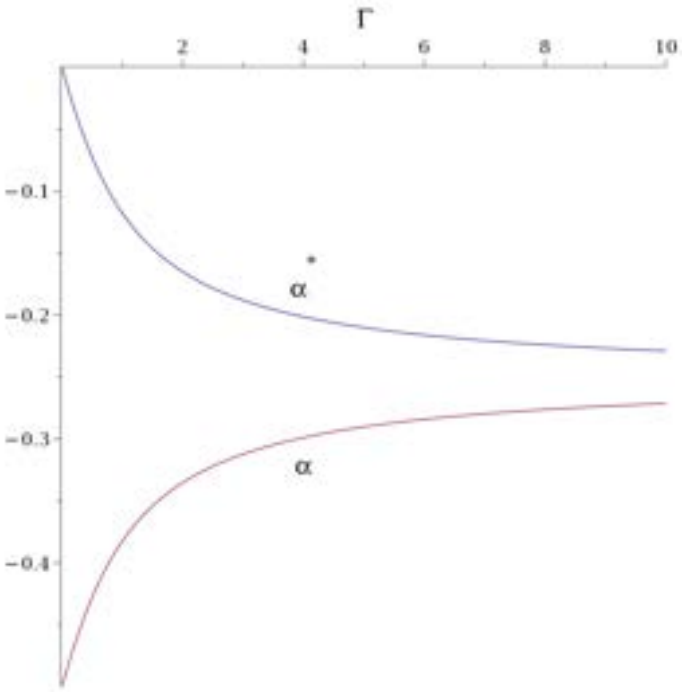
Voigt, Maximilian, 2023, Learning and strategic trading in etf markets, Working paper.

Yang, Liyan, and Haoxiang Zhu, 2020, Back-running: Seeking and hiding fundamental information in order flows, *The Review of Financial Studies* 33, 1484–1533.

Figures

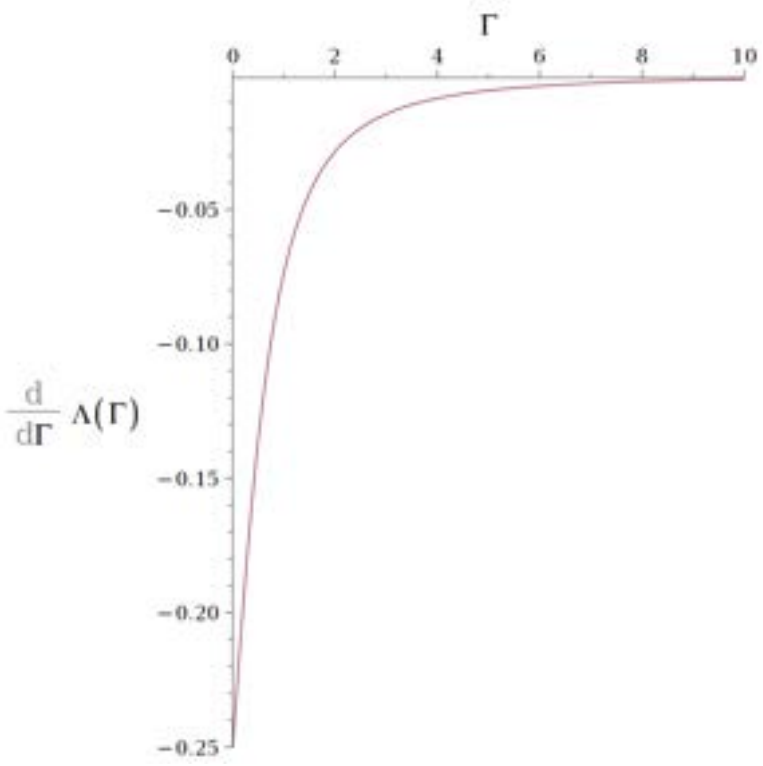


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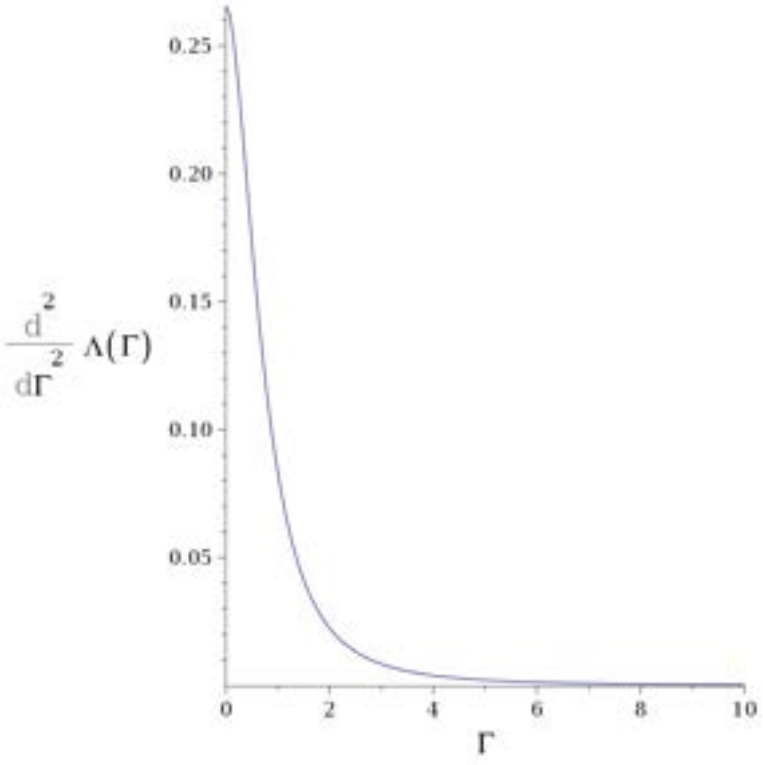


(b)

Figure 1: (a) The function $\Lambda(\Gamma)$ (depicted in black) is the unique positive root of the quartic equation (30). It converges to $1/\sqrt{2}$ for $\Gamma = 0$ and to $1/\sqrt{5}$ for $\Gamma \to \infty$. These limits are consistent with the price impact limits λ in (17) and in (21), respectively. (b) The equilibrium parameters α and α^* are functions of Γ and they are both negative. Their limits for $\Gamma = 0$ is the special case $\alpha = -1/2$ in (16), and their limits for $\Gamma \to \infty$ are the special cases $\alpha = -1/4$ and $\alpha^* = -1/4$ in (20).

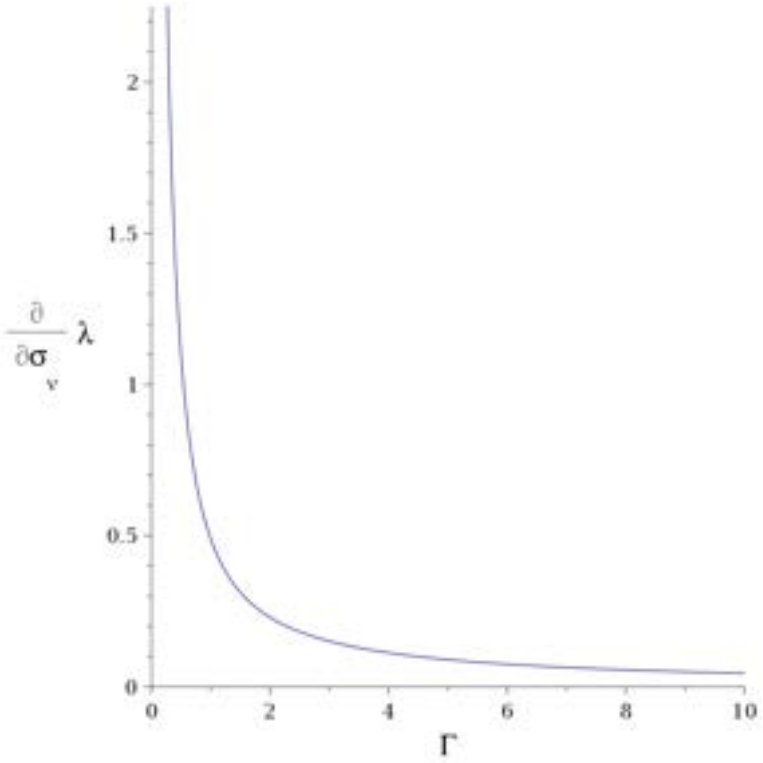


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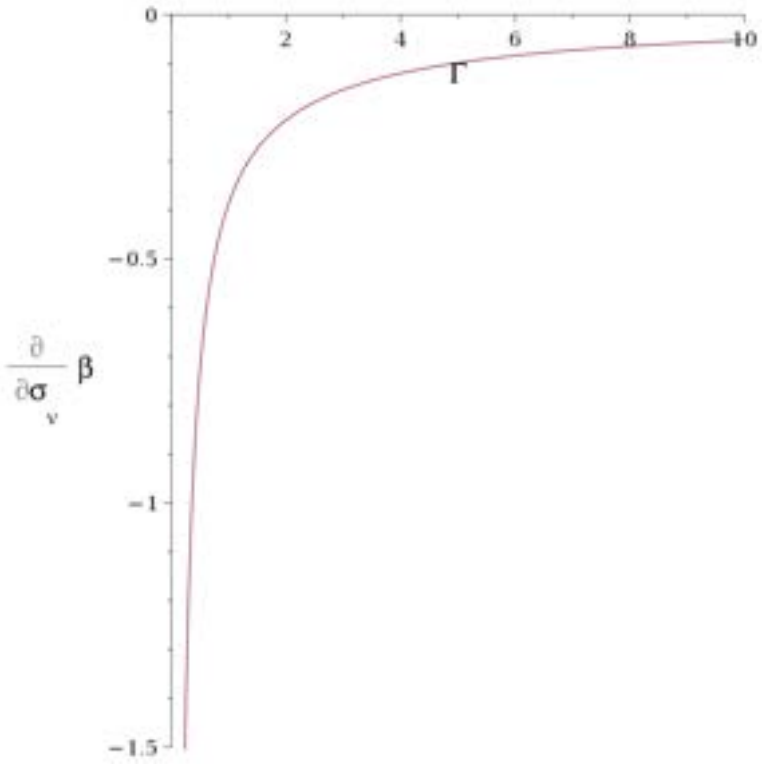


(b)

Figure 2: (a) The first derivative of $\Lambda(\Gamma)$ is negative. (b) The second derivative is positive. The signs (34) are consistent with $\Lambda(\Gamma)$ being a positive and monotonically decreasing function of Γ.

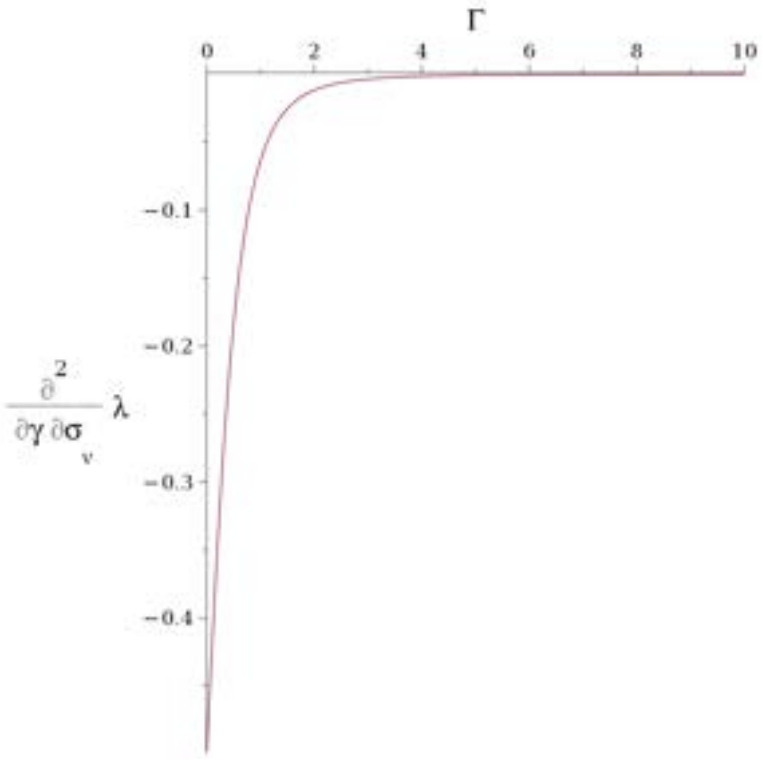


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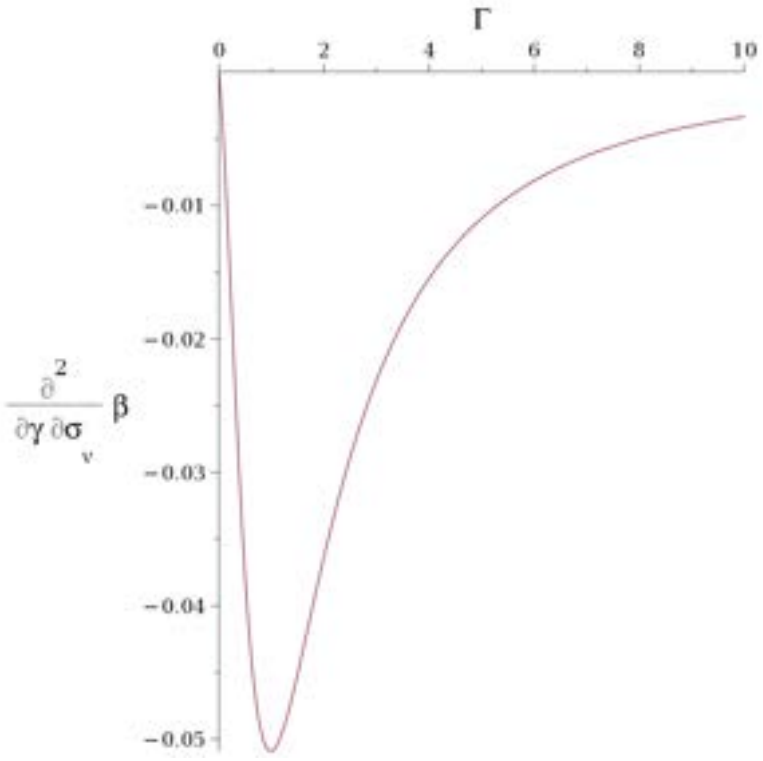


(b)

Figure 3: (a) The partial derivative $\partial\lambda/\partial\sigma_\nu$ (39) is positive. (b) The partial derivative $\partial\beta/\partial\sigma_\nu$ (40) is negative.

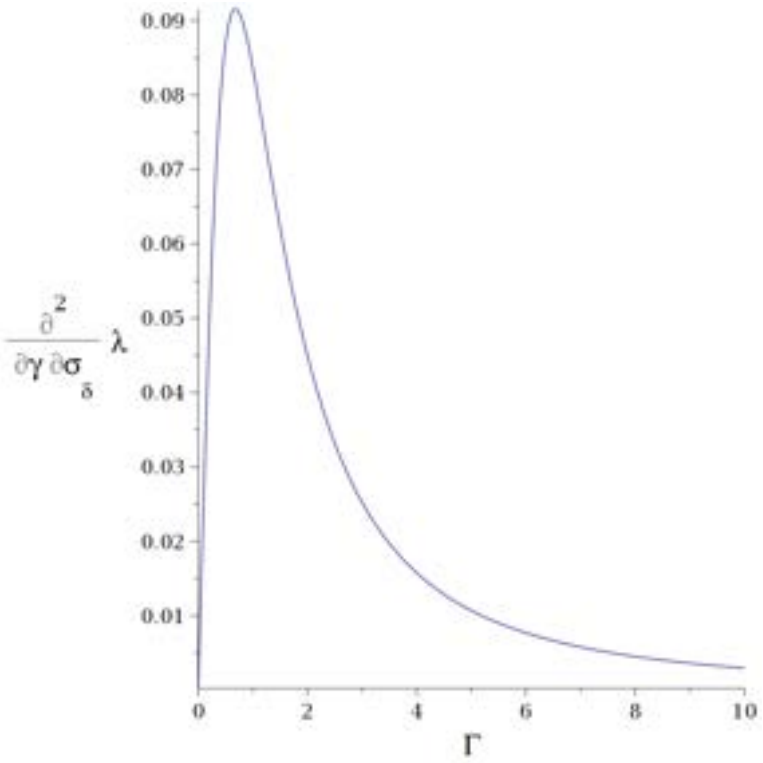


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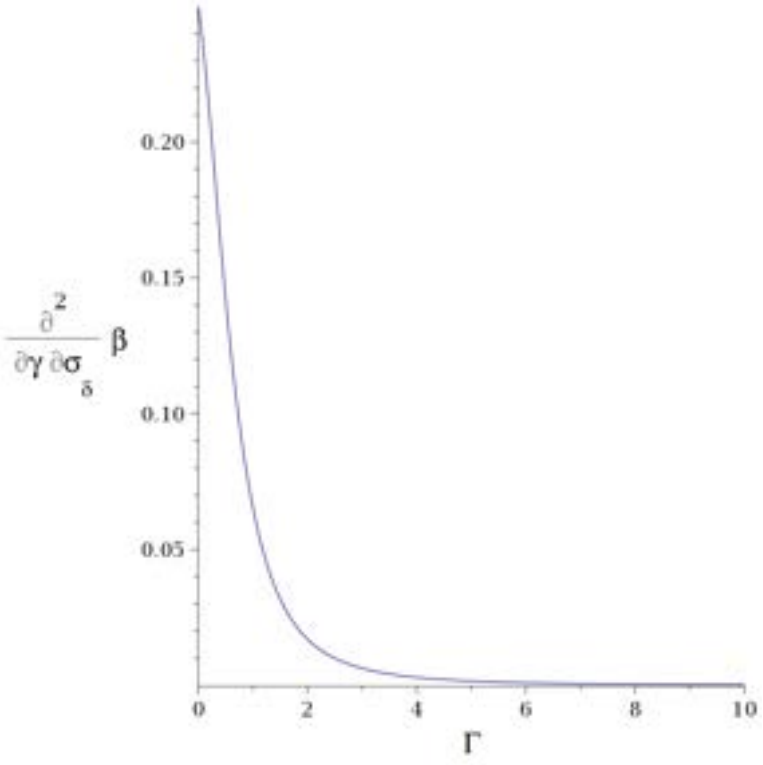


(b)

Figure 4: (a) The partial derivative $\partial^2\lambda/\partial\gamma\partial\sigma_\nu$ (46) is negative. (b) The partial derivative $\partial^2\beta/\partial\gamma\partial\sigma_\nu$ (49) is negative.

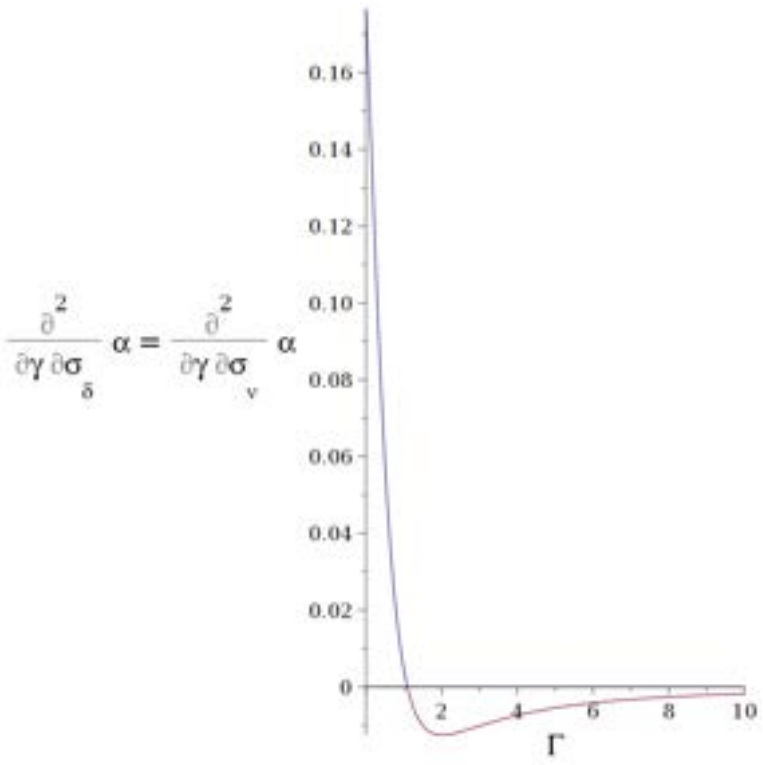


(a)

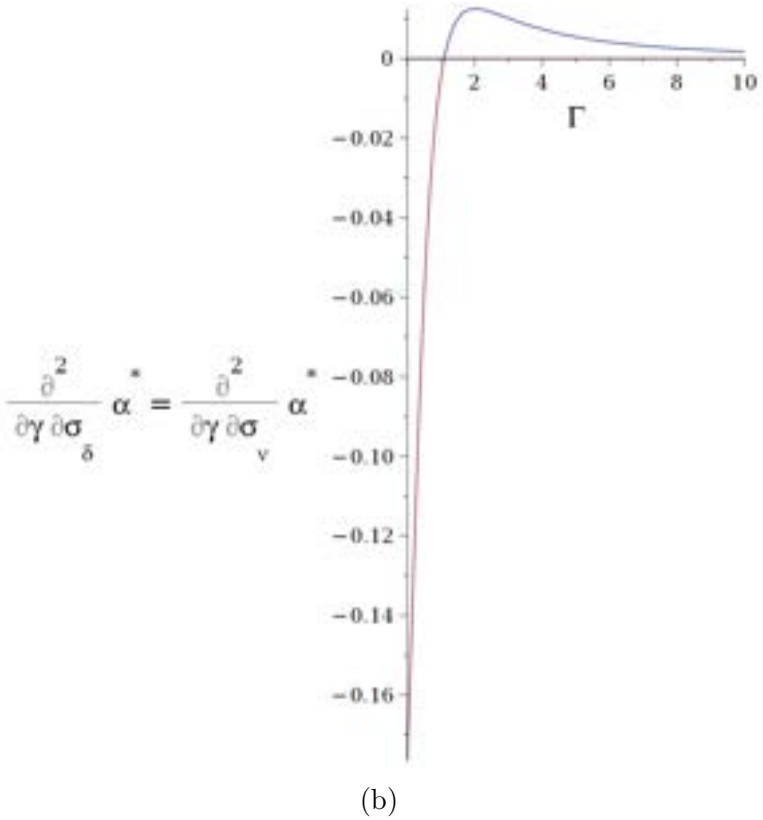


(b)

Figure 5: (a) The partial derivative $\partial^2\lambda/\partial\gamma\partial\sigma_\delta$ (52) is positive. (b) The partial derivative $\partial^2\beta/\partial\gamma\partial\sigma_\delta$ (55) is positive.

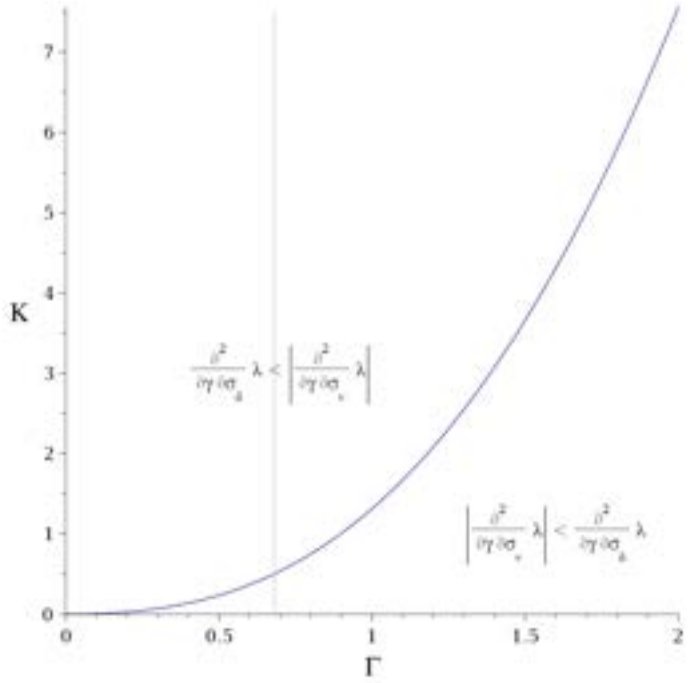


(a)

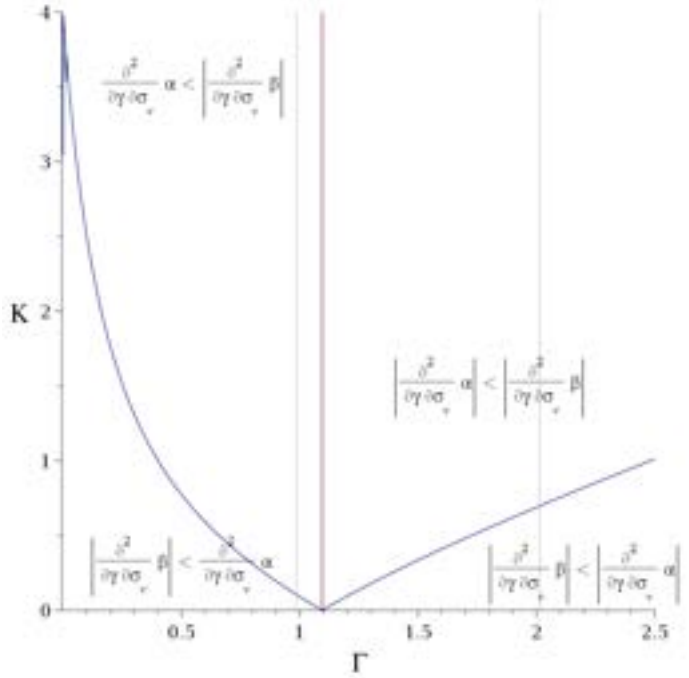


(b)

Figure 6: (a) The partial derivatives $\partial^2\alpha/\partial\gamma\partial\sigma_\nu$ and $\partial^2\alpha/\partial\gamma\partial\sigma_\delta$ are equivalent after scaling. They both change sign. (b) The partial derivatives $\partial^2\alpha^*/\partial\gamma\partial\sigma_\nu$ and $\partial^2\alpha^*/\partial\gamma\partial\sigma_\delta$ are equivalent after scaling. They both change sign and they are exactly the opposite of $\partial^2\alpha/\partial\gamma\partial\sigma_\nu$ and $\partial^2\alpha/\partial\gamma\partial\sigma_\delta$.

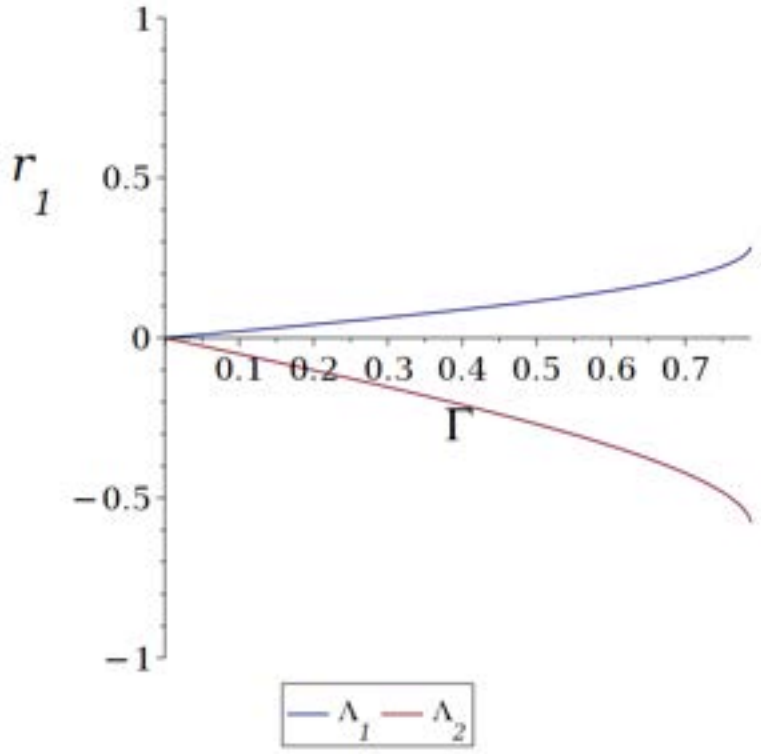


(a)

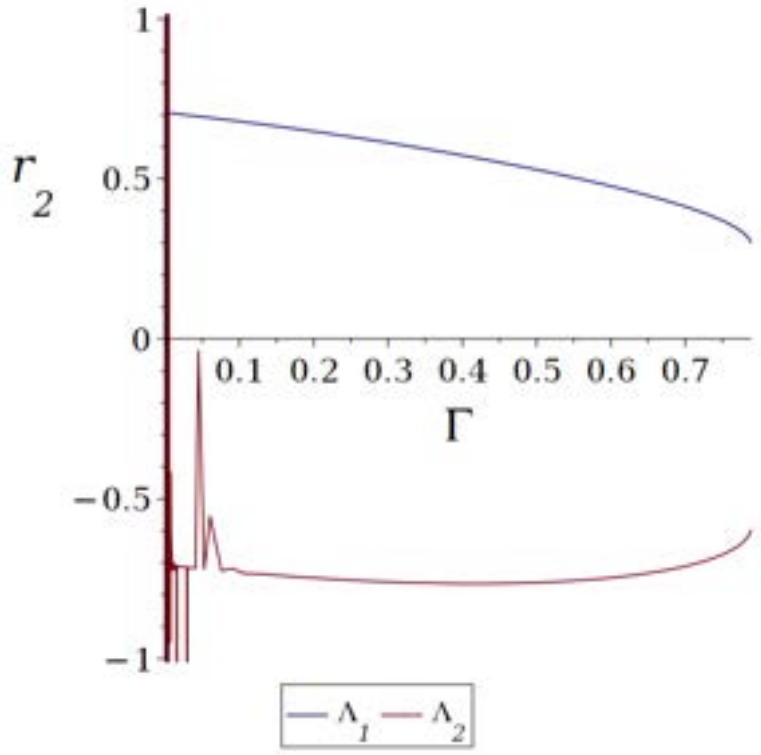


(b)

Figure 7: (a) The two parametric regions for which $\partial^2\lambda/\partial\gamma\partial\sigma_\delta$ (52) is larger or smaller than $|\partial^2\lambda/\partial\gamma\partial\sigma_\nu|$ (absolute value of (46)). The blue curve is the equality curve. The intersection between the blue curve and the grey vertical line ($\Gamma_{\lambda_{\max}} = 0.681$) indicates the exact coordinates where such equal derivatives are maximal. (b) An example of a more complicated comparison of second order sensitivity. The plot shows two (local) maximal equal derivatives, depending on the sign of $\partial^2\alpha/\partial\gamma\partial\sigma_\nu$ that changes at $\Gamma_{\alpha_{\text{zero}}} = 1.095$ (the red vertical line).

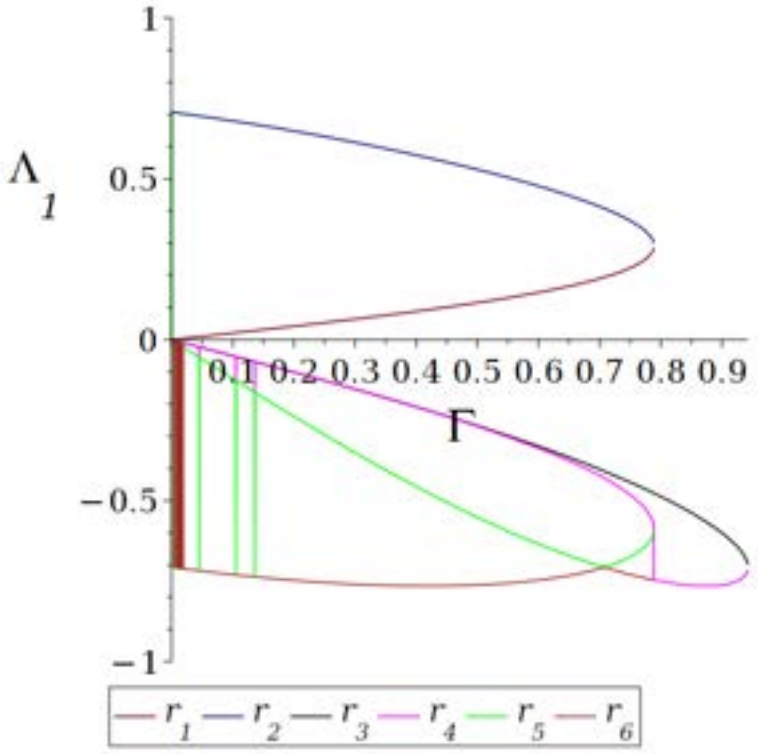


(a)

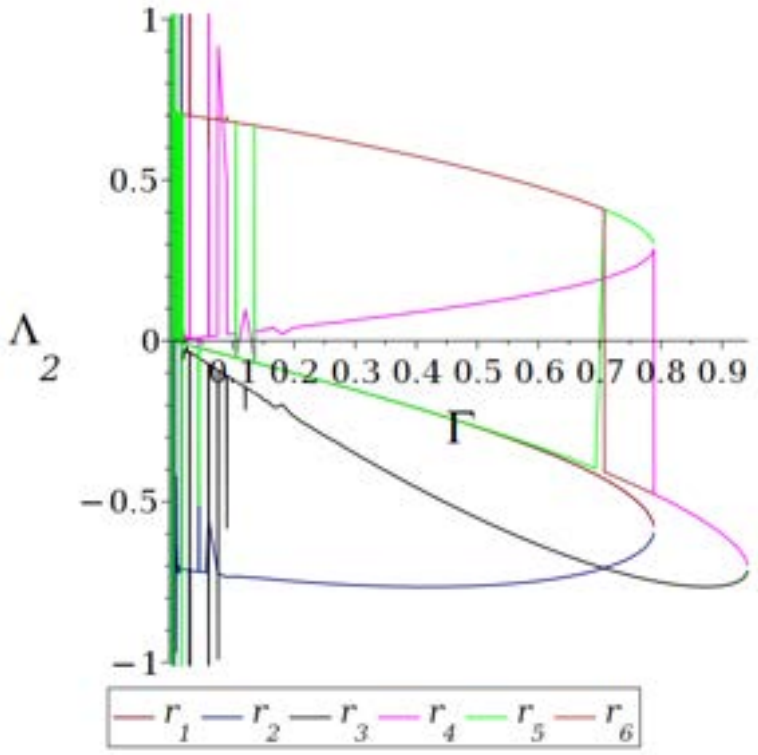


(b)

Figure 8: (a) The first root (r_1) of the degree-6 polynomial is such that Λ_1 and Λ_2 cannot be both positive for any $\Gamma > 0$. (b) Also the second root (r_2) of the degree-6 polynomial is such that Λ_1 and Λ_2 cannot be both positive for any $\Gamma > 0$.

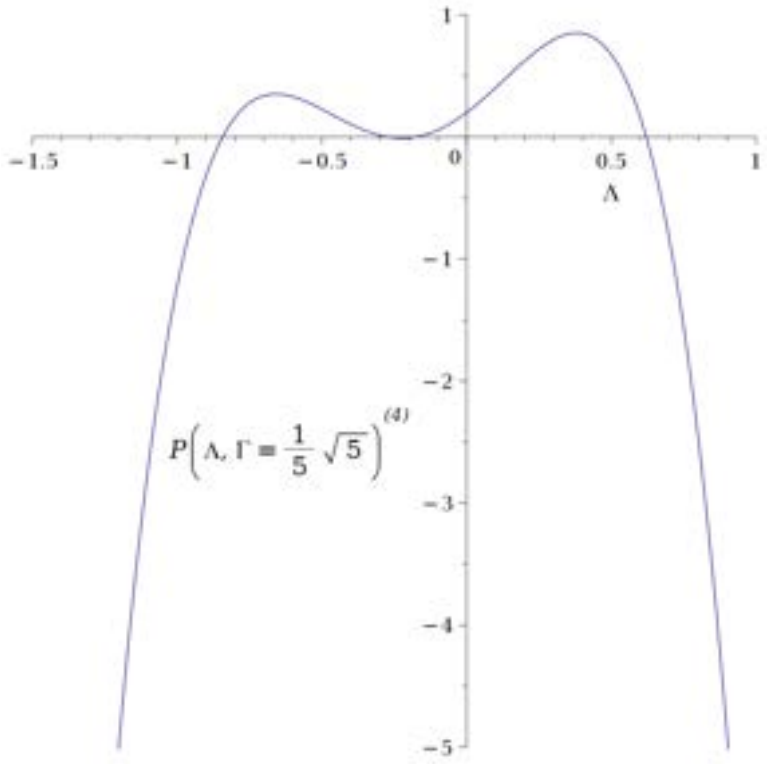


(a)

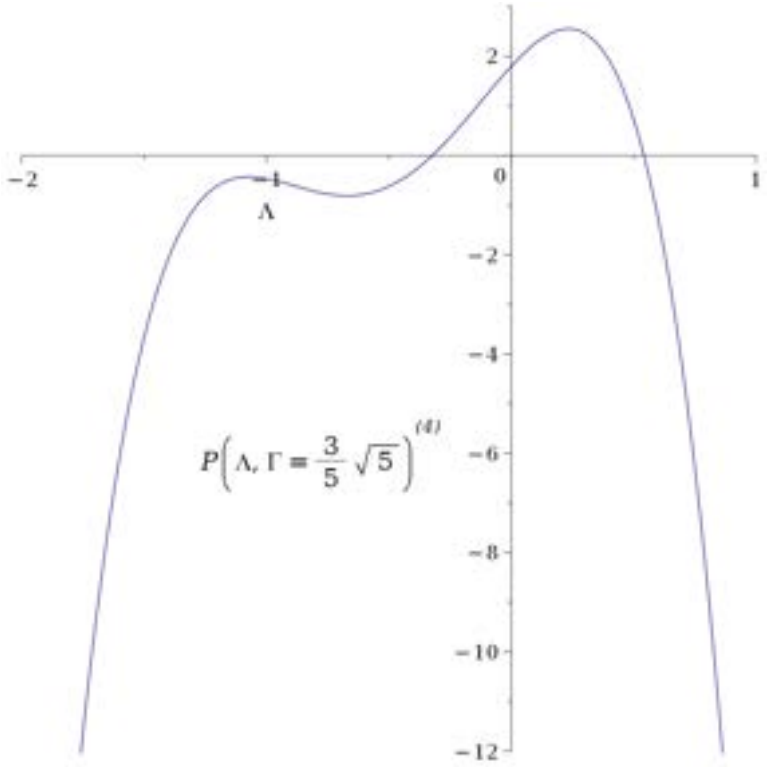


(b)

Figure 9: (a) The first root (r_1) of the degree-6 polynomial is such that Λ_1 and Λ_2 cannot be both positive for any $\Gamma > 0$. (b) Also the second root (r_2) of the degree-6 polynomial is such that Λ_1 and Λ_2 cannot be both positive for any $\Gamma > 0$.



(a)



(b)

Figure 10: (a) The quartic (109) has one positive root and three negative roots for $\Gamma = \sqrt{5}/5$. (b) The quartic (109) has one positive root and one negative root for $\Gamma = 3\sqrt{5}/5$.

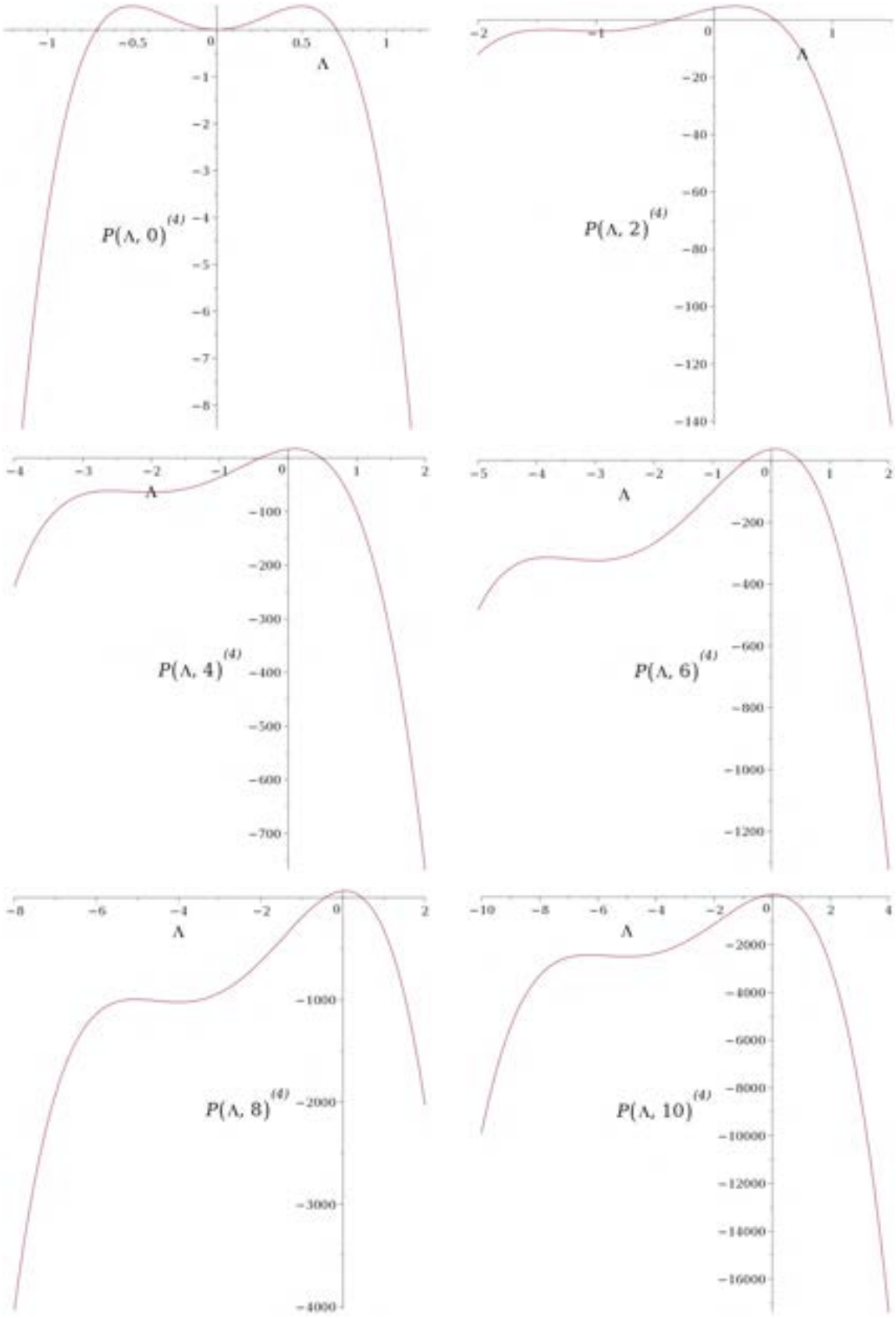
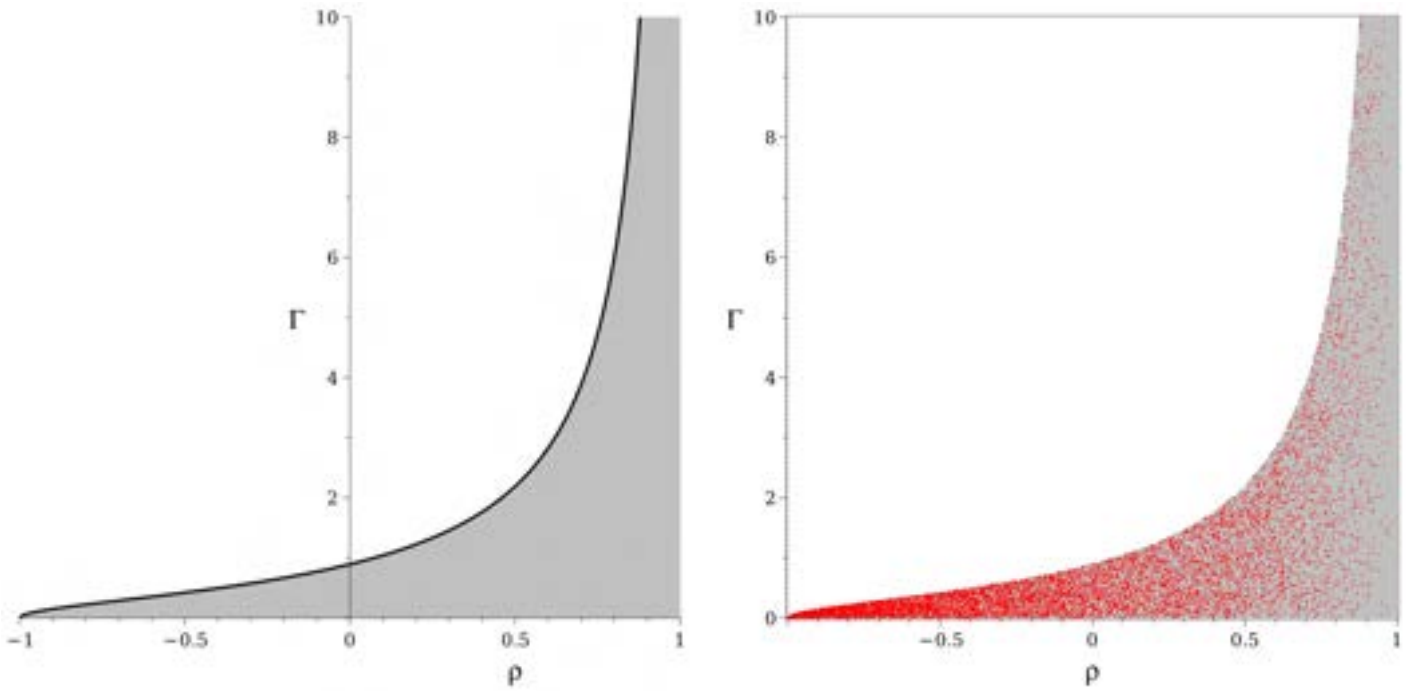
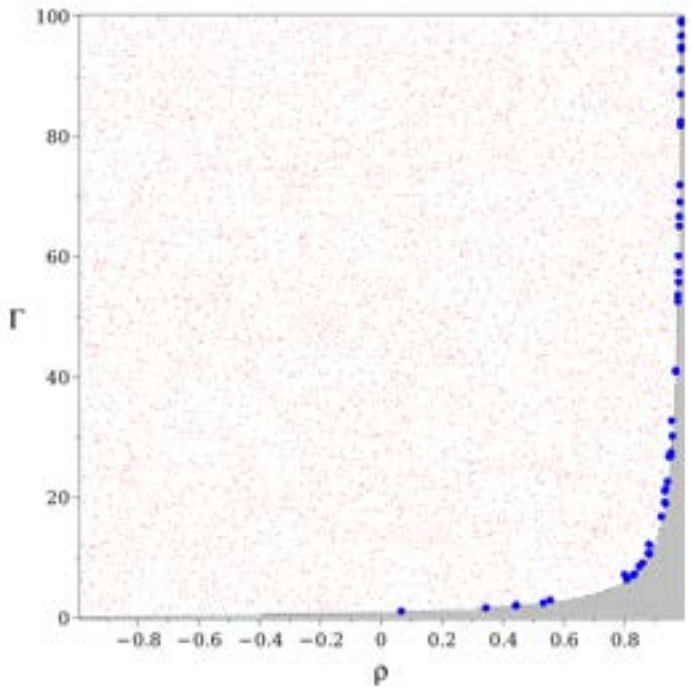


Figure 11: The quartic (109) always has a unique positive root, e.g., for $\Gamma = \{0, 2, 4, 6, 8, 10\}$.



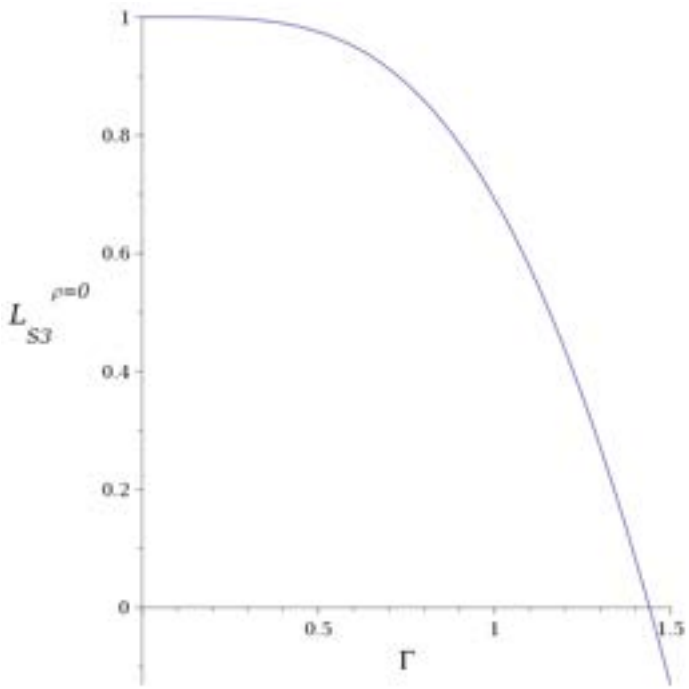
(a)

(b)

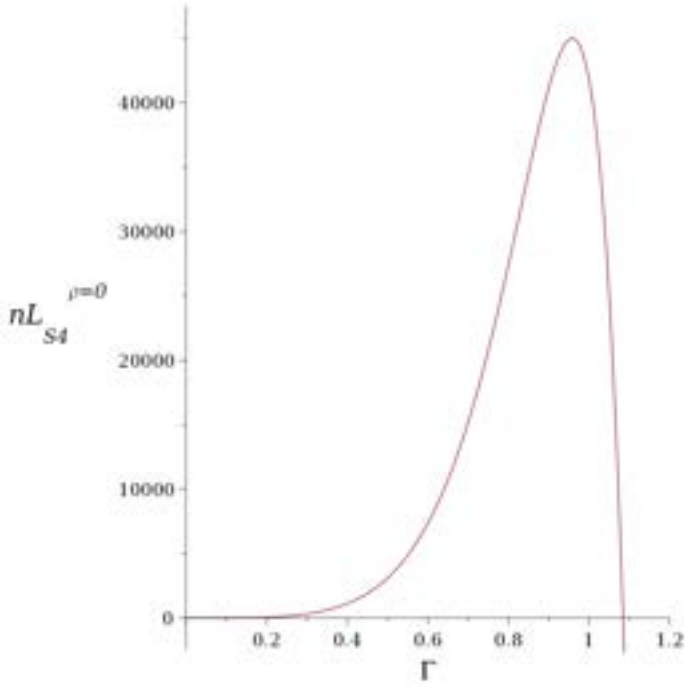


(c)

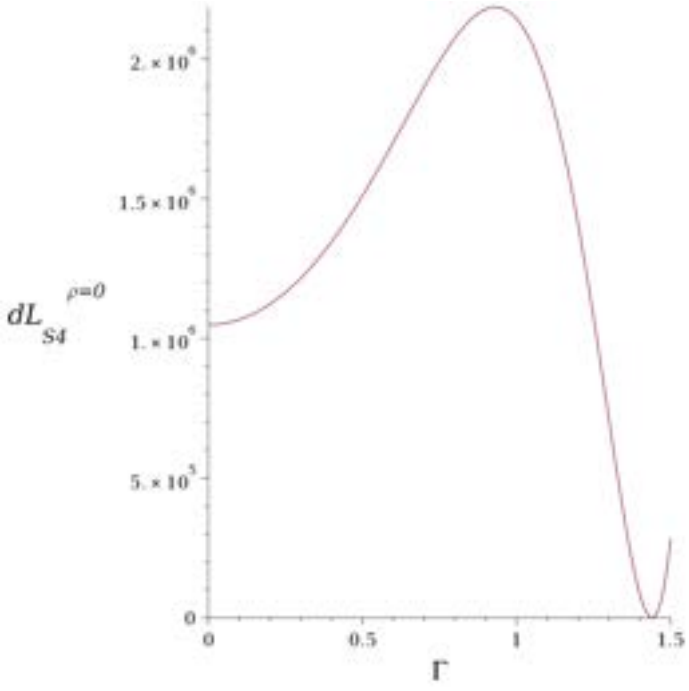
Figure 12: (a) The quadratic term (113) is positive when $\Gamma \in (0, \Gamma^*(\rho))$ (gray region), where $\Gamma^*(\rho)$ (114) is the boundary curve (black curve). (b) Our first numerical simulation shows that the quartic (111) always has four real $\Lambda(\Gamma, \rho)$ roots when $\Gamma \in (0, \Gamma^*(\rho))$. We reproduce the solving domain (gray region) together with the scatter plot (opaque red dots) of the 10000 $\left(\widehat{\Gamma}, \rho^*\right)$ coordinates. (c) Our second numerical simulation shows that the probability that (111) has four real roots when $\Gamma \notin (0, \Gamma^*(\rho))$ is only 0.48% or, in other words, there are 99.52% chances that the quartic has only two real roots when $\Gamma > \Gamma^*(\rho)$. We reproduce the solving domain (gray region) together with the scatter plots of the (Γ, ρ) coordinates where the quartic has, respectively, two real roots (9952 dots in transparent red) and four real roots (48 dots in blue).



(a)

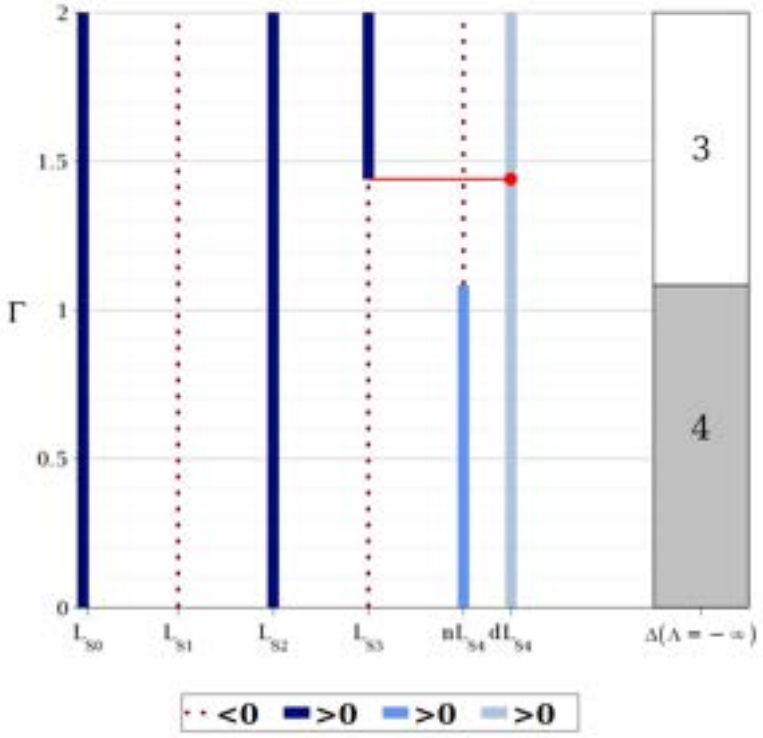


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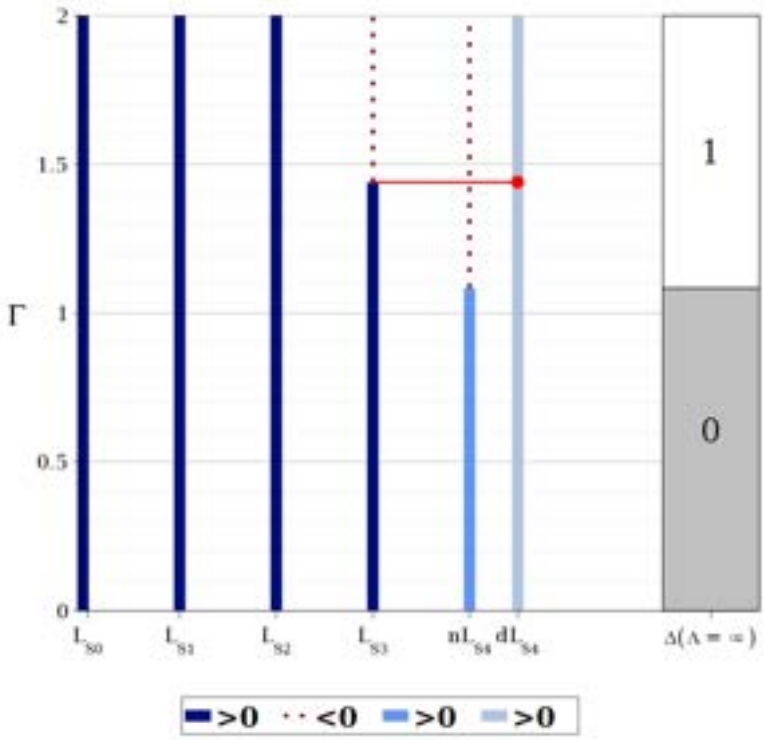


(c)

Figure 13: (a) Sturm's fourth leading coefficient L_{S3} (126) for $\rho = 0$, with root $\Gamma_{LS3}^{\rho=0} = 1.4397$. (b) Numerator of Sturm's fifth leading coefficient nL_{S4} (133) for $\rho = 0$, with root $\Gamma_{nLS4}^{\rho=0} = 1.0844$. (c) Denominator of Sturm's fifth leading coefficient dL_{S4} (134) for $\rho = 0$, with pole $\Gamma_{dLS4}^{\rho=0} = \Gamma_{LS3}^{\rho=0} = 1.4397$.

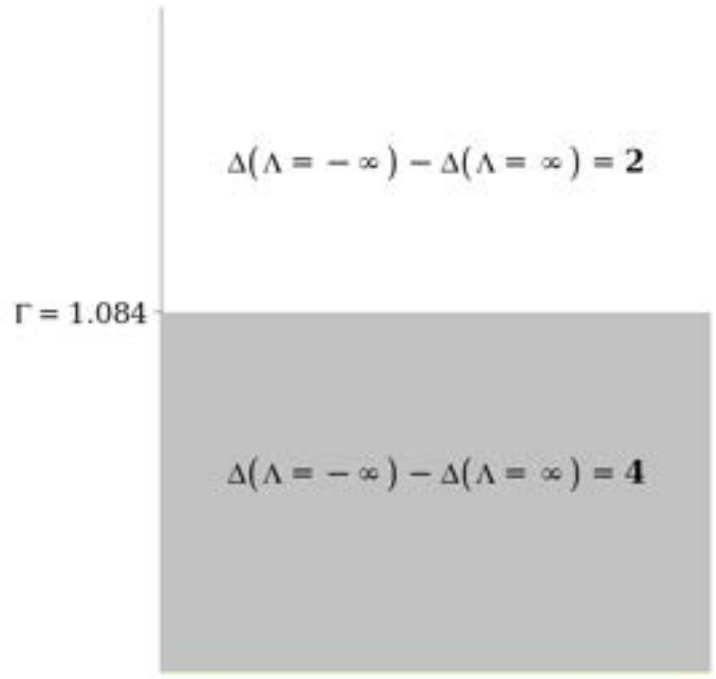


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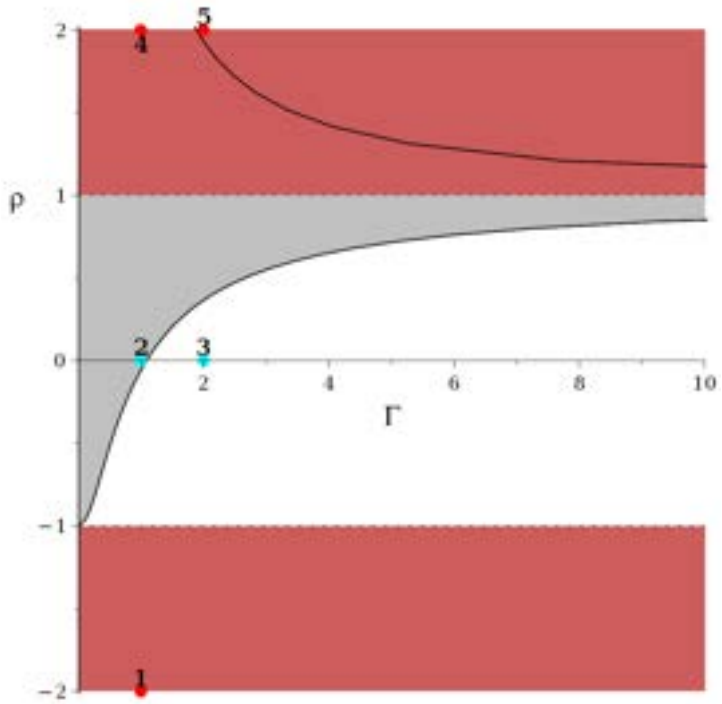


(b)

Figure 14: (a) Signs of Sturm's limits for $\Lambda(\Gamma)$ or z to $-\infty$. (b) Signs of Sturm's limits for $\Lambda(\Gamma)$ or z to $+\infty$. For each figure we also count the number of times a sign changes across the corresponding series of limits, $\Delta(\Lambda = -\infty)$ or $\Delta(\Lambda = +\infty)$, from the first leading coefficient (L_{S0}) to the last one (L_{S4}).

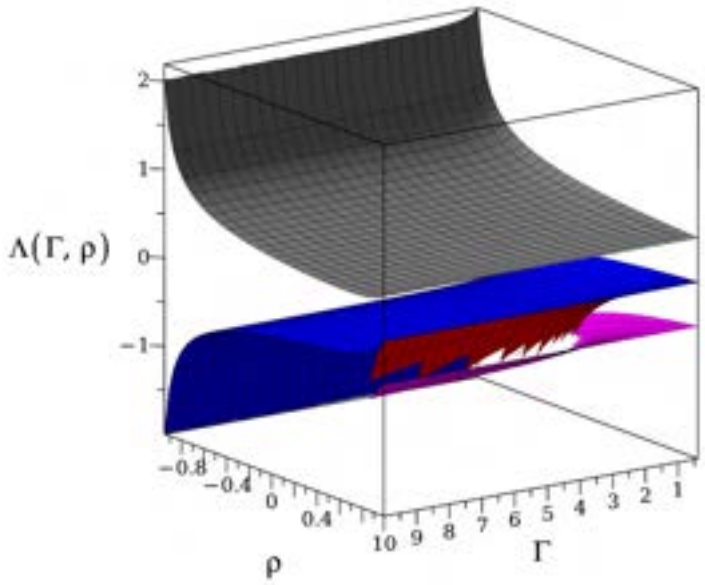


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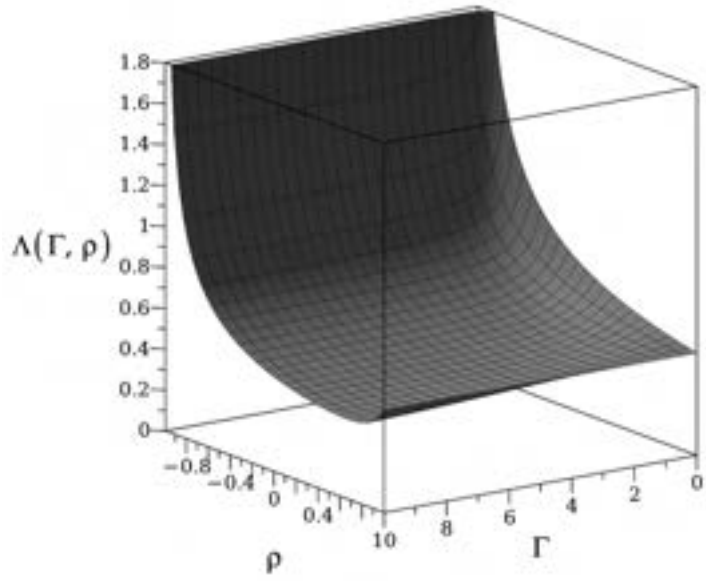


(b)

Figure 15: (a) Sturm's method for counting the number of distinct real roots (143) of the baseline ($\rho = 0$) quartic (30). We subtract the counters (in the "Δ bars") in Fig. 14b from the counters in Fig. 14a. (b) Cylindrical algebraic decomposition (CAD) of our parametric polynomial system (144). Cell 2 (in gray, with sample point $(\Gamma = 1, \rho = 0)$) and cell 3 (in white, with sample point $(\Gamma = 2, \rho = 0)$) contain four and two distinct real roots, respectively. With $\Lambda(\Gamma, \rho) > 0$ there exists only one real root in cells 2 and 3. Both Sturm and CAD's methods formally prove Fig. 1a.

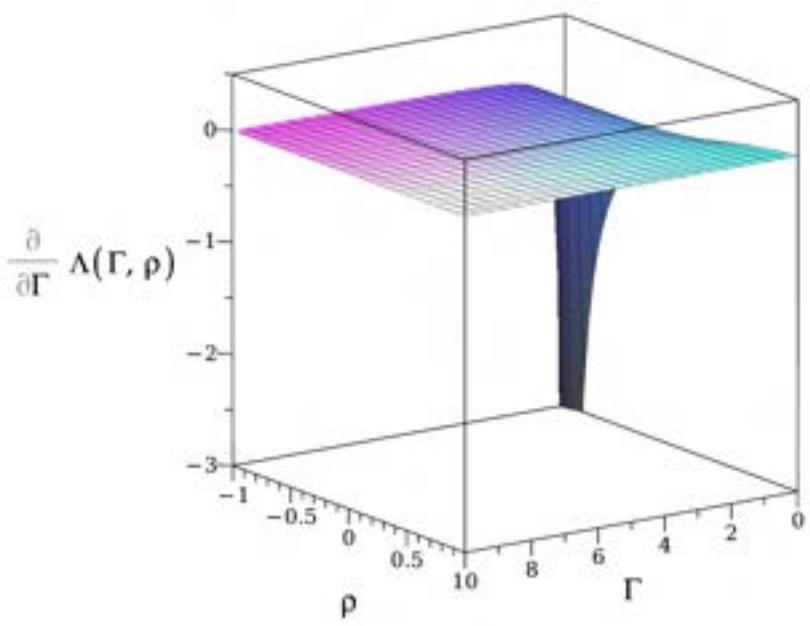


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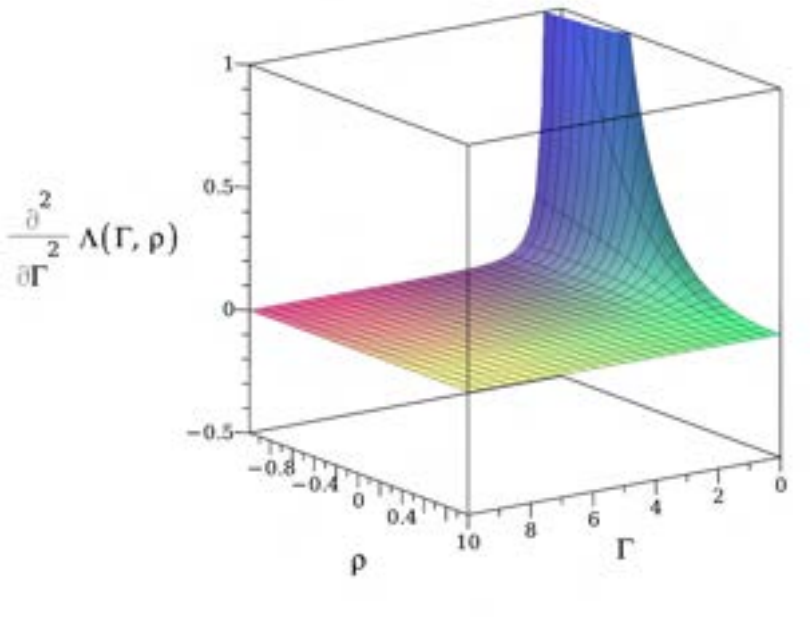


(b)

Figure 16: (a) $\Lambda(\Gamma, \rho)$ as the unique positive root (gray surface) of the quartic equation (158). The surface is a generalization for $\rho \neq 0$ of Fig. 1a. (b) $\Lambda(\Gamma, \rho)$ (159) is a generalization for $\rho \neq 0$ of $\Lambda(\Gamma)$ (31) in Fig. 1a. In particular, the solution for the baseline model with uncorrelated assets is just a specification of the solution for the extended model with correlated assets.

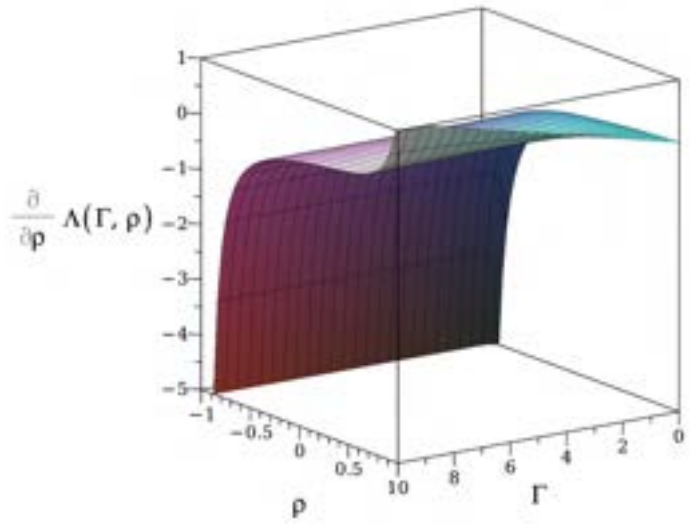


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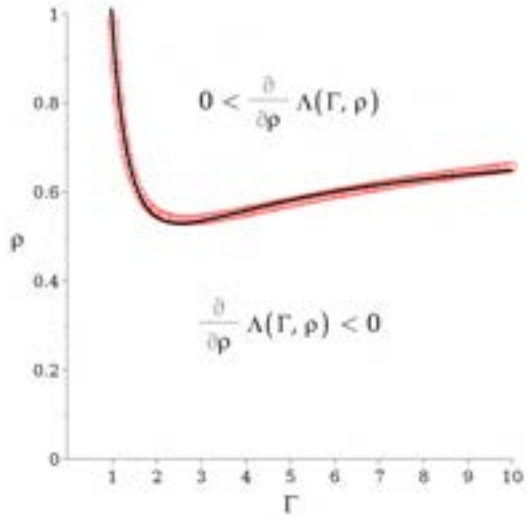


(b)

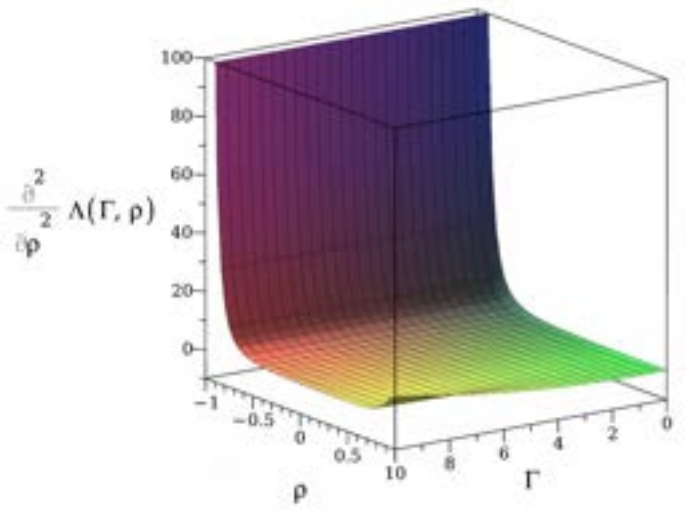
Figure 17: First and second order derivatives of $\Lambda(\Gamma, \rho)$ with respect to Γ. The first derivative (a) is negative and the second derivative (b) is positive (164).



(a)

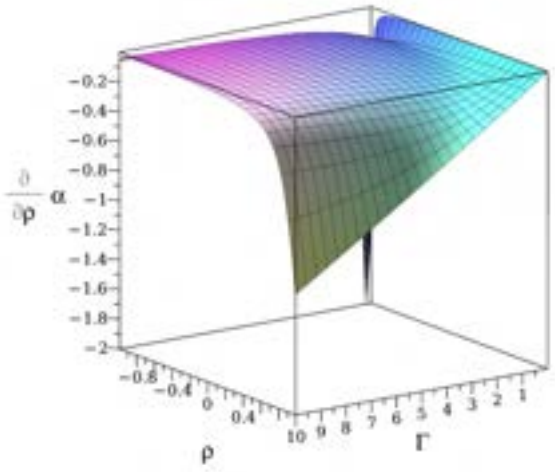


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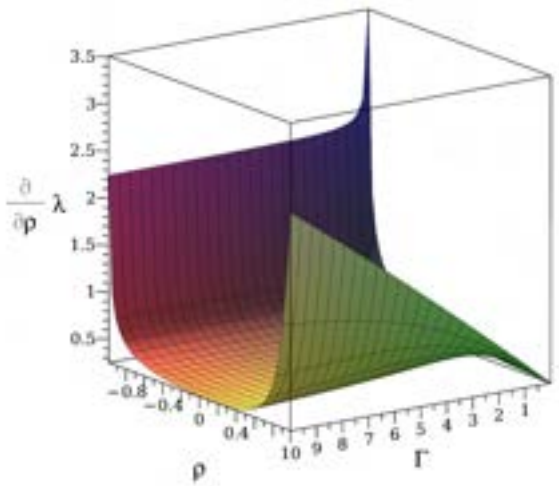


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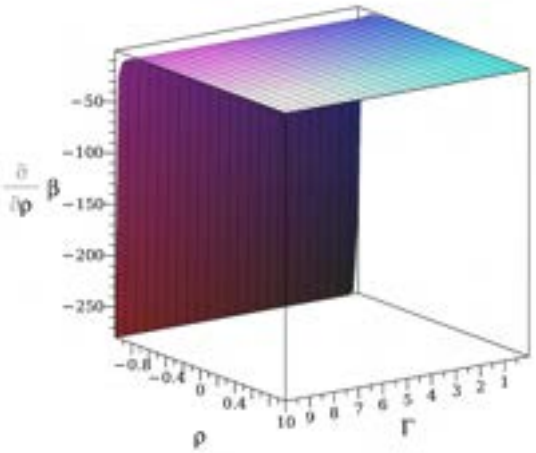
Figure 18: (a) The first derivative of $\Lambda(\Gamma, \rho)$ with respect to ρ changes sign. (b) Such derivative is positive for $\rho > p_{LJ}$ and negative otherwise, where p_{LJ} is approximated by a Lennard-Jones potential-like function (166) (depicted in black). (c) The second derivative of $\Lambda(\Gamma, \rho)$ with respect to ρ is positive.



(a)

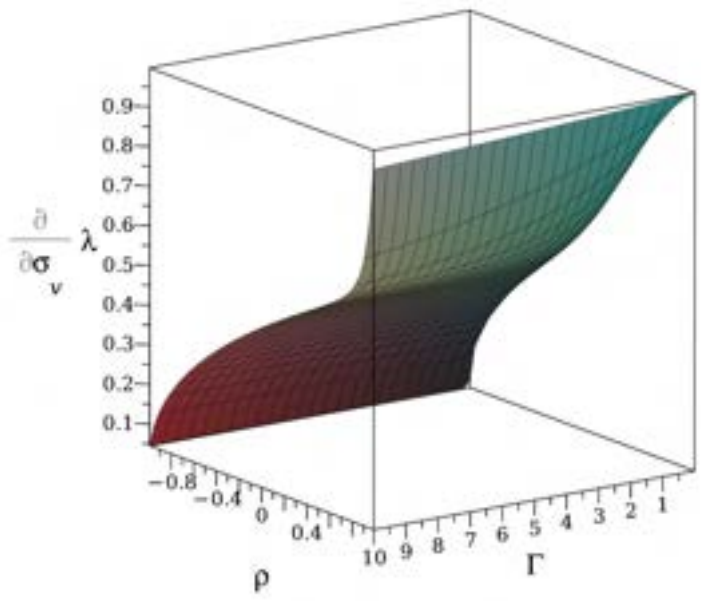


(b)

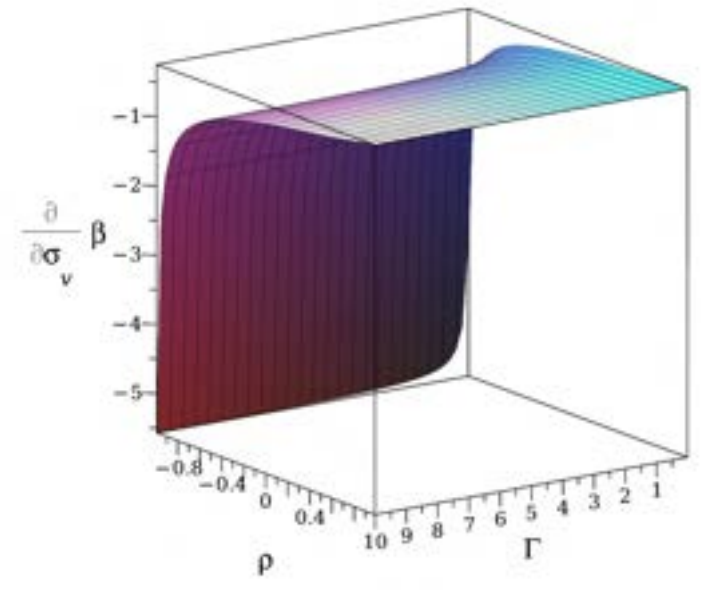


(c)

Figure 19: (a) $\partial\alpha/\partial\rho$ (167) is negative. (b) $\partial\lambda/\partial\rho$ (168) is positive. (c) $\partial\beta/\partial\rho$ (169) is negative.

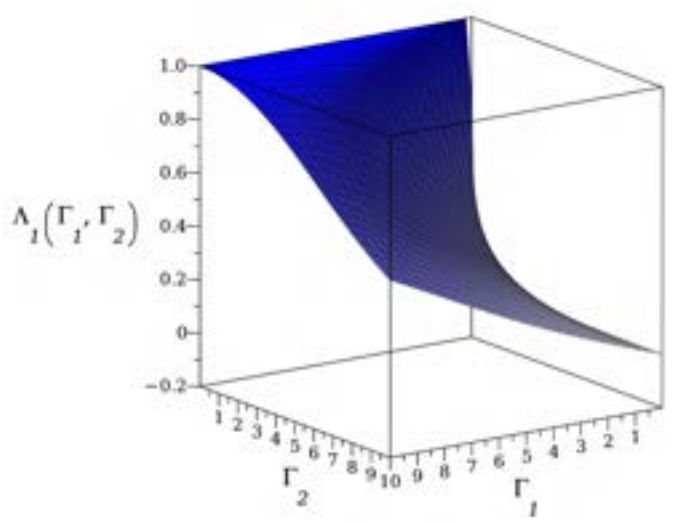


(a)

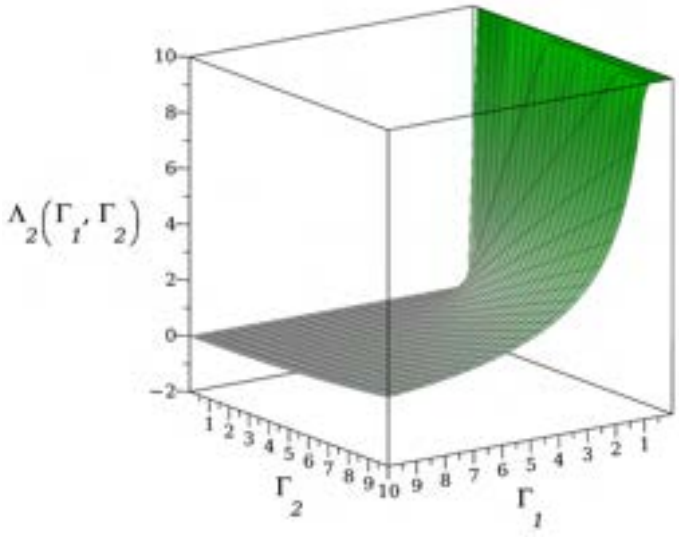


(b)

Figure 20: (a) $\partial\lambda/\partial\sigma_\nu$ (174) is positive. (b) $\partial\beta/\partial\sigma_\nu$ (175) is negative.

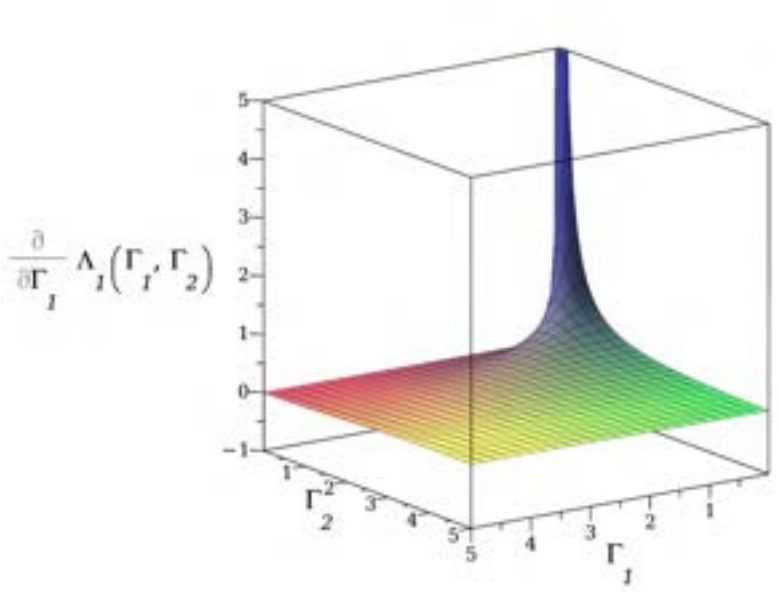


(a)

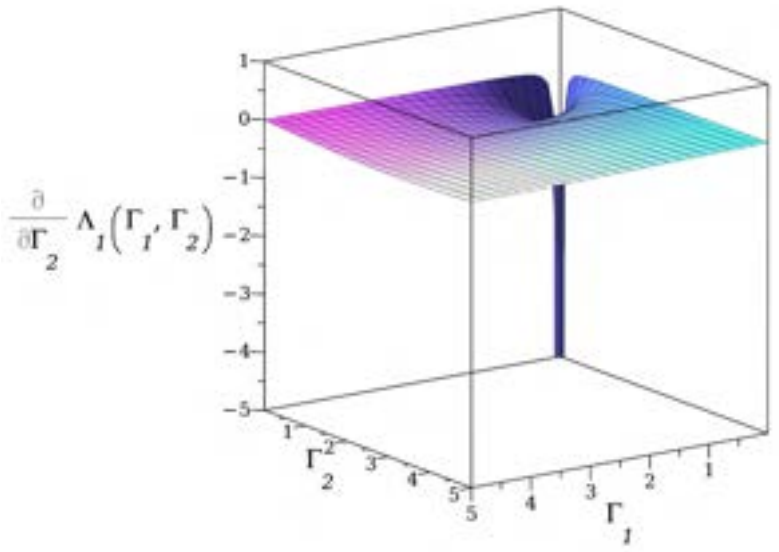


(b)

Figure 21: (a) $\Lambda_1(\Gamma_1, \Gamma_2) > 0$ is the greatest (most positive) real root of a degree-10 polynomial (194). (b) $\Lambda_2(\Gamma_1, \Gamma_2) > 0$ is a rational function of the $\Lambda_1(\Gamma_1, \Gamma_2) > 0$ just mentioned.

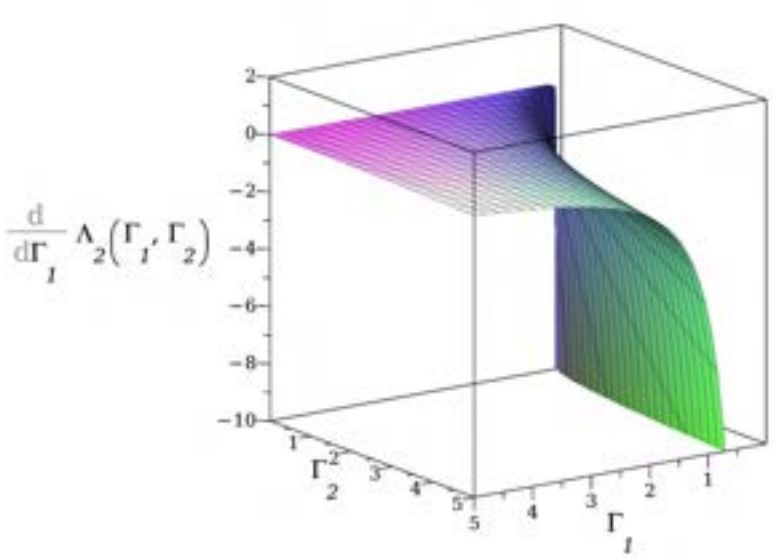


(a)

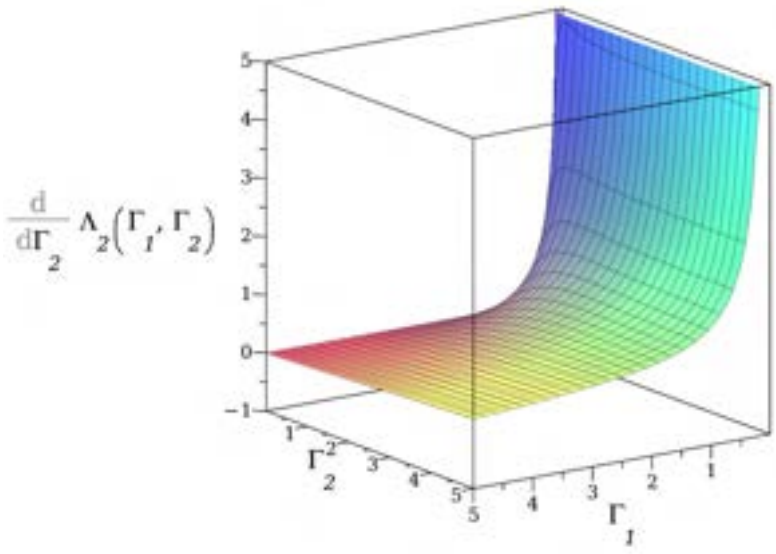


(b)

Figure 22: (a) $\partial\Lambda_1(\Gamma_1, \Gamma_2)/\partial\Gamma_1$ (205) is positive. (b) $\partial\Lambda_1(\Gamma_1, \Gamma_2)/\partial\Gamma_2$ (205) is negative.



(a)



(b)

Figure 23: (a) $d\Lambda_2(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)/d\Gamma_1$ (206) is negative. (b) $d\Lambda_2(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)/d\Gamma_2$ (206) is positive.

Appendix

A Single-Market Model

Our single-market model is in the spirit of Kyle (1989), who models strategic trading in a noisy rational expectation equilibrium framework. His model features $n = n_I + n_U$ risk-averse investors (n_I informed and n_U uninformed) in a setup with noise traders, where the investors condition their orders on the equilibrium price[5]. In contrast, our model introduces just one risk-averse speculator and we allow her to condition the order on her exogenous demand, that is the noise trading in the market order game of Rochet and Vila (1994)[6].

We assume a model with a single currency pair/market. We denote the corresponding log-exchange rate by $\nu \sim \mathcal{N}(\overline{\nu}, \sigma_\nu^2)$. The risk-averse speculator receives a noisy private signal $\zeta = \nu + \widetilde{\epsilon}$, where the noise $\widetilde{\epsilon} \sim \mathcal{N}(0, \sigma_\epsilon^2)$ is independent of the log-exchange rate. In essence, the model is a one-dimensional (single-market) version of the model we introduced in this paper. As such, all the equations in this section are simpler specifications of the equations we already described earlier. Specifically, the informed speculator problem (4) and the market maker problem (5) become, respectively,

$$\max_{\chi} \left\{ \mathrm{E}\Big[\pi|\zeta\Big] - \frac{\gamma}{2}\mathrm{Var}\Big[\pi|\zeta\Big] \right\} \text{ with } \pi = \chi\big(\nu - q\big) \text{ and } q(\widetilde{\chi} + \delta) = \mathrm{E}\Big[\nu|\widetilde{\chi} + \delta\Big]. \tag{62}$$

The conjectured equilibrium trading position and quoting rule Theorem 1 simplifies to,

$$\chi = \beta_s\big(\zeta - \overline{\nu}\big) + \alpha_s\delta, \qquad\qquad q = \overline{\nu} + \lambda_s\big(\chi + \delta\big). \tag{63}$$

[5]Note that price-contingent orders are limit orders and conditioning on price is equivalent to conditioning on the realized exogenous exogenous demands (or noise trades) if and only if the model setup features just one speculator.

[6]See their Appendix A. They show that when they specify Kyle (1989) model of informed speculation with a unique insider and a large number of uninformed traders, they obtain Kyle (1985) model of insider trading where the insider observes the amount of noise trading.

Then we can re-write the mean-variance optimization problem (7) as,

$$\max_{\chi} \left\{ \chi(-\lambda_s \chi - \lambda_s \delta - \overline{\nu}) + \chi \mathrm{E}\Big[\nu|\zeta\Big] - \frac{\gamma}{2}\chi^2 \mathrm{Var}\Big[\nu|\zeta\Big] \right\},$$

with:

$$\mathrm{E}\Big[\nu|\zeta\Big] = \mathrm{E}\big[\nu\big] + \mathrm{Cov}\Big[\nu,\zeta\Big]\mathrm{Var}^{-1}\Big[\zeta\Big]\Big(\zeta - \mathrm{E}\Big[\zeta\Big]\Big) = \overline{\nu} + \frac{\sigma_\nu^2}{\sigma_\nu^2 + \sigma_\epsilon^2}\Big(\zeta - \overline{\nu}\Big),$$

$$\mathrm{Var}\Big[\nu|\zeta\Big] = \mathrm{Var}\big[\nu\big] - \mathrm{Cov}\Big[\nu,\zeta\Big]\mathrm{Var}^{-1}\Big[\zeta\Big]\mathrm{Cov}\Big[\zeta,\nu\Big] = \sigma_\nu^2 - \frac{\sigma_\nu^4}{\sigma_\nu^2 + \sigma_\epsilon^2}. \tag{64}$$

Given the first order condition,

$$-2\chi\lambda_s - \delta\lambda_s + \frac{\sigma_\nu^2}{\sigma_\nu^2 + \sigma_\epsilon^2}\Big(\zeta - \overline{\nu}\Big) - \gamma\chi\left(\sigma_\nu^2 - \frac{\sigma_\nu^4}{\sigma_\nu^2 + \sigma_\epsilon^2}\right) = 0, \tag{65}$$

we find the trading and offsetting intensity equilibrium parameters satisfying the linear conjecture,

$$\beta_s = \frac{\sigma_\nu^2}{2\lambda(\sigma_\nu^2 + \sigma_\epsilon^2) + \gamma\sigma_\nu^2\sigma_\epsilon^2}, \quad \alpha_s = -\frac{\lambda(\sigma_\nu^2 + \sigma_\epsilon^2)}{2\lambda(\sigma_\nu^2 + \sigma_\epsilon^2) + \gamma\sigma_\nu^2\sigma_\epsilon^2}. \tag{66}$$

Since there is only one currency pair in this model, the market maker problem (10) collapses to,

$$q = \mathrm{E}\Big[\nu|\widetilde{\chi} + \delta\Big] = \mathrm{E}\big[\nu\big] + \mathrm{Cov}\Big[\nu,\widetilde{\chi} + \delta\Big]\mathrm{Var}^{-1}\Big[\widetilde{\chi} + \delta\Big]\Big(\widetilde{\chi} + \delta\Big), \tag{67}$$

and we compute the price impact in the market as,

$$\lambda_s = \frac{\mathrm{Cov}\Big[\nu,\widetilde{\chi} + \delta\Big]}{\mathrm{Var}\Big[\widetilde{\chi} + \delta\Big]} = \frac{\beta_s\sigma_\nu^2}{\beta_s^2(\sigma_\nu^2 + \sigma_\epsilon^2) + (\alpha_s + 1)^2\sigma_\delta^2}, \tag{68}$$

where we used $\mathrm{Cov}\Big[\nu,\delta\Big] = 0$ and the linear trading position. We plug (66) into (68) and obtain the equation in λ_s,

$$\lambda_s = \frac{\sigma_\nu^4(2\lambda_s(\sigma_\nu^2 + \sigma_\epsilon^2) + \gamma\sigma_\nu^2\sigma_\epsilon^2)}{\sigma_\nu^6 + \sigma_\nu^4(\sigma_\delta^2(\gamma\sigma_\epsilon^2 + \lambda_s)^2 + \sigma_\epsilon^2) + 2\lambda_s\sigma_\delta^2\sigma_\epsilon^2(\lambda_s(\sigma_\epsilon^2/2 + \sigma_\nu^2) + \gamma\sigma_\nu^2\sigma_\epsilon^2)}. \tag{69}$$

Once we solve for λ_s explicitly, we can pin down β_s and α_s (66). We have now characterized our equilibrium conjectures (63). Next we compare the extreme case in which the speculator is risk-neutral ($\gamma \to 0$) with the extreme case in which she is infinitely risk-averse ($\gamma \to \infty$). After that we discuss the general case with limited risk aversion ($0 < \gamma < \infty$).

A.1 Limit Case: Risk Neutrality ($\gamma = 0$)

For $\gamma = 0$ the equilibrium (63) specifies to,

$$\lambda_{s_0} = \frac{\sigma_\nu^2}{\sigma_\delta \sqrt{\sigma_\nu^2 + \sigma_\epsilon^2}}, \qquad \beta_{s_0} = \frac{\sigma_\delta}{2\sqrt{\sigma_\nu^2 + \sigma_\epsilon^2}}, \qquad \alpha_{s_0} = -\frac{1}{2}, \tag{70}$$

$$q_0 = \overline{\nu} + \frac{\sigma_\nu^2}{2(\sigma_\nu^2 + \sigma_\epsilon^2)}\big(\zeta - \overline{\nu}\big) + \frac{\sigma_\nu^2}{2\sigma_\delta \sqrt{\sigma_\nu^2 + \sigma_\epsilon^2}}\delta. \tag{71}$$

Next we calculate the utility u_0, i.e., the conditional profits in the limit for $\gamma \to 0$. We plug the specified equilibrium trading position (i.e., the χ in (63) with parameters (70)) into the expectation terms of the objective function (with $\zeta = \zeta$) for the mean-variance optimization problem (64). Conditional on knowing ζ (i.e., after learning the realization of ζ at the beginning of the period), the risk-neutral informed speculator expects the profit,

$$u_0 = \mathrm{E}\Big[\pi | \zeta = \zeta\Big] = \frac{\sigma_\nu^2 \sigma_\delta}{4(\sigma_\nu^2 + \sigma_\epsilon^2)^{3/2}}\big(\zeta - \overline{\nu}\big)^2 - \frac{\sigma_\nu^2}{2(\sigma_\nu^2 + \sigma_\epsilon^2)}\delta\big(\zeta - \overline{\nu}\big) + \frac{\sigma_\nu^2}{4\sigma_\delta \sqrt{\sigma_\nu^2 + \sigma_\epsilon^2}}\delta^2. \tag{72}$$

We also compute the partial derivatives of the parameters (70) with respect to σ_ν and σ_δ

$$\frac{\partial \lambda_{s_0}}{\partial \sigma_\nu} = \frac{\sigma_\nu(\sigma_\nu^2 + 2\sigma_\epsilon^2)}{\sigma_\delta(\sigma_\nu^2 + \sigma_\epsilon^2)^{3/2}}, \qquad \frac{\partial \beta_{s_0}}{\partial \sigma_\nu} = -\frac{\sigma_\nu \sigma_\delta}{2(\sigma_\nu^2 + \sigma_\epsilon^2)^{3/2}}, \tag{73}$$

$$\frac{\partial \lambda_{s_0}}{\partial \sigma_\delta} = -\frac{\sigma_\nu^2}{\sigma_\delta^2 \sqrt{\sigma_\nu^2 + \sigma_\epsilon^2}}, \qquad \frac{\partial \beta_{s_0}}{\partial \sigma_\delta} = \frac{1}{2\sqrt{\sigma_\nu^2 + \sigma_\epsilon^2}}. \tag{74}$$

A.2 Limit Case: Infinite Risk Aversion ($\gamma \to \infty$)

In the limit for $\gamma \to \infty$ the equilibrium (63) specifies to,

$$\lambda_{s_\infty} = 0, \qquad \beta_{s_\infty} = 0, \qquad \alpha_{s_\infty} = 0, \tag{75}$$

$$q_\infty = \overline{\nu}. \tag{76}$$

Next we calculate the utility u_∞, i.e., the conditional profits in the limit for $\gamma \to \infty$. We plug the specified equilibrium trading position (i.e., the χ in (63) with parameters (75)) into the objective function (with $\zeta = \zeta$) for the mean-variance optimization problem (64). Conditional on knowing ζ (i.e., after learning the realization of ζ at the beginning of the period), the infinitely risk-averse informed speculator expects a variance-adjusted profit,

$$u_\infty = \mathrm{E}\Big[\pi|\zeta = \zeta\Big] - \frac{\gamma}{2}\mathrm{Var}\Big[\pi|\zeta = \zeta\Big] = 0. \tag{77}$$

A.3 General Case: Finite Risk Aversion ($0 < \gamma < \infty$)

We solve the price impact equation (69) and pin down the positive solution for λ_s. For $0 < \gamma < \infty$, the equilibrium (63) generalizes to,

$$\lambda_s = \frac{\sigma_\nu^2(\sqrt{4(\sigma_\nu^2+\sigma_\epsilon^2)+\gamma^2\sigma_\delta^2\sigma_\epsilon^4} - \gamma\sigma_\delta\sigma_\epsilon^2)}{2\sigma_\delta(\sigma_\nu^2+\sigma_\epsilon^2)},$$
$$\beta_s = \frac{\sigma_\delta}{\sqrt{4(\sigma_\nu^2+\sigma_\epsilon^2)+\gamma^2\sigma_\delta^2\sigma_\epsilon^4}}, \qquad \alpha_s = \frac{\gamma\sigma_\delta\sigma_\epsilon^2 - \sqrt{4(\sigma_\nu^2+\sigma_\epsilon^2)+\gamma^2\sigma_\delta^2\sigma_\epsilon^4}}{2\sqrt{4(\sigma_\nu^2+\sigma_\epsilon^2)+\gamma^2\sigma_\delta^2\sigma_\epsilon^4}}, \tag{78}$$

$$q = \overline{\nu} + \frac{\sigma_\nu^2(\sqrt{4(\sigma_\nu^2+\sigma_\epsilon^2)+\gamma^2\sigma_\delta^2\sigma_\epsilon^4} - \gamma\sigma_\delta\sigma_\epsilon^2)}{2(\sigma_\nu^2+\sigma_\epsilon^2)\sqrt{4(\sigma_\nu^2+\sigma_\epsilon^2)+\gamma^2\sigma_\delta^2\sigma_\epsilon^4}}\big(\zeta - \overline{\nu}\big) + \frac{\sigma_\nu^2}{\sigma_\delta\sqrt{4(\sigma_\nu^2+\sigma_\epsilon^2)+\gamma^2\sigma_\delta^2\sigma_\epsilon^4}}\delta. \tag{79}$$

Next we calculate the utility u, i.e., the conditional profits when the speculator is finitely risk averse. We plug the specified equilibrium trading position (i.e., the χ in (63) with parameters (78)) into the objective function (with $\zeta = \zeta$) for the mean-variance optimization problem (64). Conditional on knowing ζ (i.e., after learning the realization of ζ at the beginning of the period), the finitely risk-averse informed speculator expects a variance-adjusted profit,

$$u = \mathrm{E}\Big[\pi|\zeta = \zeta\Big] - \frac{\gamma}{2}\mathrm{Var}\Big[\pi|\zeta = \zeta\Big] = c_1\big(\zeta - \overline{\nu}\big)^2 + c_2\delta\big(\zeta - \overline{\nu}\big) + c_3\delta^2,$$

with:

$$c_1 = \frac{\sigma_\nu^2\sigma_\delta}{2(\sigma_\nu^2+\sigma_\epsilon^2)\sqrt{4(\sigma_\nu^2+\sigma_\epsilon^2)+\gamma^2\sigma_\delta^2\sigma_\epsilon^4}},$$

$$c_2 = \frac{\sigma_\nu^2(\gamma\sigma_\delta\sigma_\epsilon^2 - \sqrt{4(\sigma_\nu^2+\sigma_\epsilon^2)+\gamma^2\sigma_\delta^2\sigma_\epsilon^4})}{2(\sigma_\nu^2+\sigma_\epsilon^2)\sqrt{4(\sigma_\nu^2+\sigma_\epsilon^2)+\gamma^2\sigma_\delta^2\sigma_\epsilon^4}},$$
$$c_3 = \frac{\sigma_\nu^2(\gamma^2\sigma_\delta^2\sigma_\epsilon^4 + 2(\sigma_\nu^2+\sigma_\epsilon^2) - \gamma\sigma_\delta\sigma_\epsilon^2\sqrt{4(\sigma_\nu^2+\sigma_\epsilon^2)+\gamma^2\sigma_\delta^2\sigma_\epsilon^4})}{4\sigma_\delta(\sigma_\nu^2+\sigma_\epsilon^2)\sqrt{4(\sigma_\nu^2+\sigma_\epsilon^2)+\gamma^2\sigma_\delta^2\sigma_\epsilon^4}}. \tag{80}$$

B Triangular Arbitrage Trade

The triangle relationship (1) is the basic underlying assumption in our model. However, the same restriction needs not to apply to the the log-quotes in the three markets. In other words, our equilibrium derivation shows a violation of the triangular no-arbitrage condition or, equivalently, we find that $q_3 \neq q_1 + q_2$. That is, (14) is not the sum of the log-quotes in (17), or (21), or (29). Note that our theoretical finding is consistent with the empirical evidence documented in the extant literature. For example, Chaboud and Vega (2014) and Foucault et al. (2017) find frequent triangular arbitrage opportunities between the limit order books operated by EBS and Reuters for different currency pairs. They also find that these opportunities last no more than fractions of a second and are merely caused by technological constraints. Both findings squares well with the fact that the time horizon in our model is assumed sufficiently short to capture high-frequency dynamics, where our static (one-shot trading) Kyle setting accommodates for short-lived asymmetric information across agents and when there are no large jumps in exchange rates[7].

One might wonder why a speculator would not engage in triangular arbitrage trades since arbitrage opportunities seem to arise. Schrimpf et al. (2024) provide empirical evidence in support of the argument that violations of the law of one price do not translate into profitable triangular arbitrage opportunities if transaction costs in the form of bid-ask spreads are sufficiently large. Accordingly, their model abstracts away from any cross-market arbitrageurs. However, our theoretical model is based on an order-driven trading mechanism à-la Kyle (1985) (rather than a quote-driven mechanism à-la Glosten and Milgrom (1985)) and hence does not accommodate for transaction costs or bid-ask spreads. As such, we cannot argue that our speculator does not trade an arbitrage because it would be too expensive to do so. In this section we provide an alternative justification for the violation of the triangular

[7]Recall that we justified the log-linearization of the speculator's profits with the same assumption.

no-arbitrage condition that we derive at the equilibrium. To illustrate the basic intuition, we first show that for a simple model of two markets with one identical asset a speculator trading as a monopolist would not trade the arbitrage. Then we prove that the same argument holds for our market triangle.

B.1 Arbitrage in a Simple Model of Monopoly

We build a simple model of a speculator who is an information monopolist. The speculator can trade in two separate markets A and B but only the same asset. The asset traded has a fundamental value ν. We denote as a and b the two strategic trades, respectively, and as θ the cross-market arbitrage trade, i.e., buy θ in market A and sell θ in market B. The pricing functions for the (same) asset in the two markets are,

$$\begin{aligned} p_a(a,\theta) &= p_A + \lambda_A(a+\theta), \\ p_b(b,\theta) &= p_B + \lambda_B(b-\theta). \end{aligned} \tag{81}$$

The speculator chooses the optimal a, b, and θ maximizing her utility,

$$\begin{aligned} &\max_{a,b,\theta}\{a(\nu - p_a(a,\theta)) + b(\nu - p_b(b,\theta)) + \theta(p_b(b,\theta) - p_a(a,\theta))\} = \\ &\quad = \max_{a,b,\theta}\{a(\nu - p_A - \lambda_A(a+\theta)) + b(\nu - p_B - \lambda_B(b-\theta)) + \\ &\quad + \theta(p_B + \lambda_B(b-\theta) - p_A - \lambda_A(a+\theta))\}. \end{aligned} \tag{82}$$

where the first order conditions of (82) for the three choice variables are,

$$\begin{aligned} &\nu - p_A - \lambda_A(a+\theta) - a\lambda_A - \theta\lambda_A = 0, \\ &\nu - p_B - \lambda_B(b-\theta) - b\lambda_B + \theta\lambda_B = 0, \\ &-a\lambda_A + b\lambda_B + p_B + \lambda_B(b-\theta) - p_A - \lambda_A(a+\theta) + \theta(-\lambda_B - \lambda_A) = 0. \end{aligned} \tag{83}$$

We solve the first order conditions (83) for a, b, and θ,

$$\begin{aligned} a &= \frac{\nu - p_A}{2\lambda_A} - \theta, \\ b &= \frac{\nu - p_B}{2\lambda_B} + \theta, \\ \theta &= \theta. \end{aligned} \tag{84}$$

The solution $\theta = \theta$ implies that if we substitute the optimal a and b (first two equations in (84)) into the first order condition for θ (last equation in (83)) the latter would result in the identity $0 = 0$. In other words, $\theta \in \mathbb{R}$ is free/not determined. Note that a and b are the standard demands of a monopolist trading separately in each market and we just showed that they perfectly offset *any* arbitrage demand θ.

Then we can rewrite the prices in the two markets (81) as,

$$\begin{aligned} p_a &= p_A + \lambda_A \left(\frac{\nu - p_A}{2\lambda_A} - \theta + \theta \right) = \frac{p_A + \nu}{2}, \\ p_b &= p_B + \lambda_B \left(\frac{\nu - p_B}{2\lambda_B} + \theta - \theta \right) = \frac{p_B + \nu}{2}, \end{aligned} \tag{85}$$

and the profits Π (i.e., the objective function in (82)) as,

$$\begin{aligned} \Pi &= \left(\frac{\nu - p_A}{2\lambda_A} - \theta \right) \left(\nu - \frac{p_A + \nu}{2} \right) + \left(\frac{\nu - p_B}{2\lambda_B} + \theta \right) \left(\nu - \frac{p_B + \nu}{2} \right) + \\ &+ \theta \left(\frac{p_B + \nu}{2} - \frac{p_A + \nu}{2} \right) = \\ &= \frac{(\nu - p_A)^2}{4\lambda_A} + \frac{(\nu - p_B)^2}{4\lambda_B}. \end{aligned} \tag{86}$$

In other words, as θ has no effect on the total orders $a + \theta$ and $b - \theta$, it does not affect prices or profits. In particular, prices (85) are as if the monopolist would trade in only one market and profits (86) are the sum of the standard monopoly profits from trading separately in the two markets.

B.2 Arbitrage in Our Model of Asset Market Triangle

Now we show how the same insights from the simple model of monopoly applies to our model of asset market triangle. In our model of asset market triangle, the direct trade in market 3 maps to the trade in market A in the simple model of monopoly. Similarly, the (combined and contemporaneous) indirect trade in market 1 and 2 corresponds to the trade in market B in the simple model of monopoly. Accordingly, we map the fundamental value as $\nu = \nu_3 = \nu_1 + \nu_2$ (1) and the two strategic trades as $a = \chi_3$ and $b = \chi_1 + \chi_2$. We denote as θ the triangular arbitrage trade, i.e., buy θ in market 3 and sell θ in markets 1 and 2. Therefore, if we explicitly take into account the triangular arbitrage trade, the conjectured quoting rules Theorem 1 set by the dealers become,

$$\begin{aligned} q_1 &= \overline{\nu}_1 + \lambda\left(\chi_1 + \delta_1 - \theta\right), \\ q_2 &= \overline{\nu}_2 + \lambda\left(\chi_2 + \delta_2 - \theta\right), \\ q_3 &= \overline{\nu}_1 + \overline{\nu}_2 + \lambda_3\left(\chi_3 + \delta_3 + \theta\right). \end{aligned} \tag{87}$$

Accordingly, we rewrite the speculator's mean-variance optimization problem (7) to include the profits earned from trading the triangular arbitrage,

$$\begin{aligned} \max_{\chi_1,\chi_2,\chi_3} \Big\{ & \chi_1(-\overline{\nu}_1 - \lambda(\chi_1 + \delta_1 - \theta)) + \theta(\overline{\nu}_1 + \lambda\left(\chi_1 + \delta_1 - \theta\right)) + \\ & + \chi_2(-\overline{\nu}_2 - \lambda(\chi_2 + \delta_2 - \theta)) + \theta(\overline{\nu}_2 + \lambda\left(\chi_2 + \delta_2 - \theta\right)) + \\ & + \chi_3(\nu_3 - \overline{\nu}_1 - \overline{\nu}_2 - \lambda_3(\chi_3 + \delta_3 + \theta)) - \theta(\overline{\nu}_1 + \overline{\nu}_2 + \lambda_3\left(\chi_3 + \delta_3 + \theta\right)) + \\ & + \chi_1\overline{\nu}_1 + \chi_2\overline{\nu}_2 + (\chi_1 + \chi_2)\left(\frac{\nu_3 - \overline{\nu}_1 - \overline{\nu}_2}{2}\right) - \gamma\frac{\sigma_\nu^2}{2}\left(\frac{\chi_1^2}{2} + \frac{\chi_2^2}{2} - \chi_1\chi_2\right)\Big\}. \end{aligned} \tag{88}$$

As such, with respect to the original first order conditions (8), the new first order conditions for χ_1, χ_2, and χ_3 embed a θ-term,

$$\begin{aligned} & 2\lambda\theta - 2\lambda\chi_1 - \lambda\delta_1 + \frac{1}{2}\left(\nu_3 - \overline{\nu}_1 - \overline{\nu}_2\right) - \frac{1}{2}\gamma\sigma_\nu^2\chi_1 + \frac{1}{2}\gamma\sigma_\nu^2\chi_2 = 0, \\ & 2\lambda\theta - 2\lambda\chi_2 - \lambda\delta_2 + \frac{1}{2}\left(\nu_3 - \overline{\nu}_1 - \overline{\nu}_2\right) - \frac{1}{2}\gamma\sigma_\nu^2\chi_2 + \frac{1}{2}\gamma\sigma_\nu^2\chi_1 = 0, \\ & -2\lambda_3\theta - 2\lambda_3\chi_3 - \lambda_3\delta_3 + \nu_3 - \overline{\nu}_1 - \overline{\nu}_2 = 0, \end{aligned} \tag{89}$$

and there is an additional first order condition for θ,

$$\lambda(2\chi_1 + 2\chi_2 - 4\theta + \delta_1 + \delta_2) - \lambda_3(2\chi_3 + 2\theta + \delta_3) = 0. \tag{90}$$

We solve the first order conditions (89) and (90) for χ_1, χ_2, χ_3, and θ,

$$\begin{aligned}
\chi_1 &= \frac{\nu_3 - \overline{\nu}_1 - \overline{\nu}_2}{4\lambda} - \frac{4\lambda + \gamma\sigma_\nu^2}{4(2\lambda + \gamma\sigma_\nu^2)}\delta_1 - \frac{\gamma\sigma_\nu^2}{4(2\lambda + \gamma\sigma_\nu^2)}\delta_2 + \theta, \\
\chi_2 &= \frac{\nu_3 - \overline{\nu}_1 - \overline{\nu}_2}{4\lambda} - \frac{4\lambda + \gamma\sigma_\nu^2}{4(2\lambda + \gamma\sigma_\nu^2)}\delta_2 - \frac{\gamma\sigma_\nu^2}{4(2\lambda + \gamma\sigma_\nu^2)}\delta_1 + \theta, \\
\chi_3 &= \frac{\nu_3 - \overline{\nu}_1 - \overline{\nu}_2}{2\lambda_3} - \frac{\delta_3}{2} - \theta, \\
\theta &= \theta.
\end{aligned} \tag{91}$$

Note that (i) $\theta \in \mathbb{R}$ is free/not determined and (ii) χ_1, χ_2, and χ_3 are exactly the same as in the original equilibrium but with the additional $+\theta$ term for χ_1 and χ_2 and $-\theta$ term for χ_3[8]. The economic intuition for the irrelevance of the arbitrage position is equivalent to what we already described for the simple model of monopoly. In the simple model, the cross-market arbitrage trade enters the utility as $\theta(p_b - p_a)$. Similarly, in our model of asset market triangle the triangular arbitrage opportunity is captured by $\theta(q_1 + q_2 - q_3)$, i.e., the speculator sells θ in markets 1 and 2 and buys θ in market 3. Thus, χ_1 and χ_2 each should be offsetting the selling of θ in their corresponding market.

C Equilibrium Uniqueness

If we assume that $\sigma_{\nu 1} = \sigma_{\nu 2} = \sigma_\nu$ (2) then markets 1 and 2 share the same parameters β, α, α^*, and hence λ. Based on this key assumption we verified the equilibrium Theorem 1 with parameters (9). In this section we generalize our conjectures Theorem 1 to allow for heterogeneous parameters across markets 1 and 2 and formally prove that the only equilibrium, given (2), is still Theorem 1. We do so by showing that the only real and positive solution for the system of nonlinear price impact equations derived from (10) is one

[8]That is, if it was not for θ, (91) would coincide with Theorem 1 where (9) pins down the equilibrium parameters.

where $\lambda_1 = \lambda_2 = \lambda$. We formally prove that such unique λ is proportional to the only real and positive root of the quartic equation (30).

C.1 Set of Solutions

First, we generalize the conjectures of Theorem 1 to accommodate for the possibility that markets 1 and 2 are endogenously asymmetric despite a symmetric variance-covariance matrix (2) for their fundamentals,

$$\begin{aligned}
\chi_1(\zeta, \delta_1, \delta_2, \delta_3) &= \beta_1(\zeta - \overline{\nu}_1 - \overline{\nu}_2) + \alpha_1\delta_1 + \alpha_2^*\delta_2, \\
\chi_2(\zeta, \delta_1, \delta_2, \delta_3) &= \beta_2(\zeta - \overline{\nu}_1 - \overline{\nu}_2) + \alpha_2\delta_2 + \alpha_1^*\delta_1, \\
\chi_3(\zeta, \delta_1, \delta_2, \delta_3) &= \beta_3(\zeta - \overline{\nu}_1 - \overline{\nu}_2) + \alpha_3\delta_3, \\
q_1(\chi_1 + \delta_1) &= \overline{\nu}_1 + \lambda_1(\chi_1 + \delta_1), \\
q_2(\chi_2 + \delta_2) &= \overline{\nu}_2 + \lambda_2(\chi_2 + \delta_2), \\
q_3(\chi_3 + \delta_3) &= \overline{\nu}_1 + \overline{\nu}_2 + \lambda_3(\chi_3 + \delta_3).
\end{aligned} \tag{92}$$

We solve the corresponding mean-variance optimization problem of the speculator and the equilibrium parameters (9) generalize as,

$$\begin{aligned}
\beta_1 &= \frac{2\lambda_2 + \gamma\sigma_\nu^2}{2(4\lambda_1\lambda_2 + \gamma\sigma_\nu^2(\lambda_1 + \lambda_2))}, \\
\alpha_1 = -\frac{\lambda_1(4\lambda_2 + \gamma\sigma_\nu^2)}{2(4\lambda_1\lambda_2 + \gamma\sigma_\nu^2(\lambda_1 + \lambda_2))}, &\qquad \alpha_2^* = -\frac{\lambda_2\gamma\sigma_\nu^2}{2(4\lambda_1\lambda_2 + \gamma\sigma_\nu^2(\lambda_1 + \lambda_2))}, \\
\beta_2 &= \frac{2\lambda_1 + \gamma\sigma_\nu^2}{2(4\lambda_1\lambda_2 + \gamma\sigma_\nu^2(\lambda_1 + \lambda_2))}, \\
\alpha_2 = -\frac{\lambda_2(4\lambda_1 + \gamma\sigma_\nu^2)}{2(4\lambda_1\lambda_2 + \gamma\sigma_\nu^2(\lambda_1 + \lambda_2))}, &\qquad \alpha_1^* = -\frac{\lambda_1\gamma\sigma_\nu^2}{2(4\lambda_1\lambda_2 + \gamma\sigma_\nu^2(\lambda_1 + \lambda_2))}, \\
\beta_3 = \frac{1}{2\lambda_3}, &\qquad \alpha_3 = -\frac{1}{2}.
\end{aligned} \tag{93}$$

We proceed with solving the problem faced by the market makers. As before, each market maker observes the pooled order flow in her own market and sets the corresponding log-quote competitively. However, as the new conjectures (92) accommodate three λ instead of just two, the price impact equations (11) become three now,

$$\begin{aligned}
\lambda_1 &= \frac{\beta_1 \sigma_\nu^2}{2\beta_1^2\sigma_\nu^2 + (\alpha_1+1)^2\sigma_\delta^2 + (\alpha_2^*)^2\sigma_\delta^2}, \\
\lambda_2 &= \frac{\beta_2 \sigma_\nu^2}{2\beta_2^2\sigma_\nu^2 + (\alpha_2+1)^2\sigma_\delta^2 + (\alpha_1^*)^2\sigma_\delta^2}, \\
\lambda_3 &= \frac{2\beta_3 \sigma_\nu^2}{2\beta_3^2\sigma_\nu^2 + (\alpha_3+1)^2\sigma_{\delta 3}^2}.
\end{aligned} \tag{94}$$

We plug the parameters (93) into the system of three equations (94). The resulting equations are highly nonlinear and significantly convoluted, so it is not easy to solve them explicitly for the three λ. Note that the right hand sides of the first two equations in (94) depend on both λ_1 and λ_2, while the third equation only depends on λ_3 and hence we can solve it directly to obtain,

$$\lambda_3 = \sqrt{2}\frac{\sigma_\nu}{\sigma_{\delta 3}}, \tag{95}$$

as in (13). However, the solutions for λ_1 and λ_2 are lengthy and unrevealing nonlinear expressions which require finding the roots of a degree-4 polynomial, $P^{(4)}$, and a degree-6 polynomial, $P^{(6)}$, whose implicit variable we can denote as Z,

$$\begin{aligned}
P^{(4)} &= 8\sigma_\delta^2 Z^4 + 12\gamma\sigma_\delta^2\sigma_\nu Z^3 + (5\gamma^2\sigma_\delta^2\sigma_\nu^2 - 4)Z^2 - 4\gamma\sigma_\nu Z - \gamma^2\sigma_\nu^2 = 0, \\
P^{(6)} &= (8\gamma^2\sigma_\delta^4\sigma_\nu^2 + 32\sigma_\delta^2)Z^6 + (8\gamma^3\sigma_\delta^4\sigma_\nu^3 + 80\gamma\sigma_\delta^2\sigma_\nu)Z^5 + (4\gamma^4\sigma_\delta^4\sigma_\nu^4 + 72\gamma^2\sigma_\delta^2\sigma_\nu^2 - 16)Z^4 + \\
&\quad + (36\gamma^3\sigma_\delta^2\sigma_\nu^3 - 32\gamma\sigma_\nu)Z^3 + (8\gamma^4\sigma_\delta^2\sigma_\nu^4 - 16\gamma^2\sigma_\nu^2)Z^2 + \gamma^4\sigma_\nu^4 = 0.
\end{aligned} \tag{96}$$

Both polynomials depend on multiple parameters (γ, σ_ν, and σ_δ) and hence it is difficult to infer information about their roots. In order to reduce the complexity, we manipulate our system with a "scaling exercise". Specifically, we proceed with the non-dimensionalization of the price impact equations. To illustrate the idea, we first non-dimensionalize the equation

for λ_3 by hand. From (95), we define a new non-dimensional variable $\Lambda_3 \coloneqq \lambda_3\sigma_{\delta 3}/\sigma_\nu$. Note that if we isolate λ_3 from Λ_3 and plug it into the third equation in (94) scaled by the factor $\sigma_{\delta 3}/\sigma_\nu$, then we end up with no parameters at all in the equation,

$$-\frac{\Lambda_3(\Lambda_3^2-2)}{\Lambda_3^2+2}=0 \Rightarrow \Lambda_3=\{0,\sqrt{2},-\sqrt{2}\}.$$

In general, we can non-dimensionalize the first two equations in (94) by a systematic process, but let's guess for simplicity that Λ_1 and Λ_2 are defined similarly to Λ_3. That is, $\Lambda_1 \coloneqq \lambda_1\sigma_\delta/\sigma_\nu$ and $\Lambda_2 \coloneqq \lambda_2\sigma_\delta/\sigma_\nu$. We proceed in three steps. First, we isolate λ_1 from Λ_1 and λ_2 from Λ_2[9]. We plug λ_1 and λ_2 as functions of Λ_1 and Λ_2, respectively, into the first two price impact equations in (94). Second, we normalize the resulting equations (we rewrite each of the two as a single numerator divided by a single denominator) and only look at their numerators,

$$\begin{aligned}
&-\sigma_\nu(\gamma^2\Lambda_1^3\sigma_\delta^2\sigma_\nu^2+4\gamma^2\Lambda_1^2\Lambda_2\sigma_\delta^2\sigma_\nu^2+5\gamma^2\Lambda_1\Lambda_2^2\sigma_\delta^2\sigma_\nu^2-2\gamma^2\Lambda_2\sigma_\delta^2\sigma_\nu^2+8\gamma\Lambda_1^3\Lambda_2\sigma_\delta\sigma_\nu+\\
&\quad+16\gamma\Lambda_1^2\Lambda_2^2\sigma_\delta\sigma_\nu-4\gamma\Lambda_1\Lambda_2\sigma_\delta\sigma_\nu-4\gamma\Lambda_2^2\sigma_\delta\sigma_\nu+16\Lambda_1^3\Lambda_2^2-8\Lambda_1\Lambda_2^2)=0,\\
&-\sigma_\nu(5\gamma^2\Lambda_1^2\Lambda_2\sigma_\delta^2\sigma_\nu^2+4\gamma^2\Lambda_1\Lambda_2^2\sigma_\delta^2\sigma_\nu^2+\gamma^2\Lambda_2^3\sigma_\delta^2\sigma_\nu^2-2\gamma^2\Lambda_1\sigma_\delta^2\sigma_\nu^2+8\gamma\Lambda_1\Lambda_2^3\sigma_\delta\sigma_\nu+\\
&\quad+16\gamma\Lambda_1^2\Lambda_2^2\sigma_\delta\sigma_\nu-4\gamma\Lambda_1\Lambda_2\sigma_\delta\sigma_\nu-4\gamma\Lambda_1^2\sigma_\delta\sigma_\nu+16\Lambda_1^2\Lambda_2^3-8\Lambda_1^2\Lambda_2)=0.
\end{aligned} \tag{97}$$

We divide through by σ_ν, which is in every term, and see that γ, σ_ν, and σ_δ always multiply each other. Thus, we define the new parameter $\Gamma \coloneqq \gamma\sigma_\nu\sigma_\delta$. We isolate γ from Γ and substitute it into (97) to obtain, respectively,

$$\begin{aligned}
&(-5\Gamma^2\Lambda_1-16\Gamma\Lambda_1^2-16\Lambda_1^3+4\Gamma+8\Lambda_1)\Lambda_2^2-4\Gamma(\Gamma+2\Lambda_1)(\Lambda_1^2-1/2)\Lambda_2-\Gamma^2\Lambda_1^3=0,\\
&(-5\Gamma^2\Lambda_2-16\Gamma\Lambda_2^2-16\Lambda_2^3+4\Gamma+8\Lambda_2)\Lambda_1^2-4\Gamma(\Gamma+2\Lambda_2)(\Lambda_2^2-1/2)\Lambda_1-\Gamma^2\Lambda_2^3=0.
\end{aligned} \tag{98}$$

The non-dimensional price impact equations (98) look much simpler than the original price impact equations for λ_1 and λ_2 in (94) with parameters (93). We proceed with solving (98)

[9]Note how the non-dimensional Λ_1, Λ_2, and Λ_3 are like multipliers to the original Kyle (1985)'s λ forms.

for Λ_1 and Λ_2 using a powerful solver that is available on Maple software[10]. Based on (96), we now define some aliases (or placeholders) as the respective roots of polynomial equations in the variable Z,

$$\begin{aligned}
\eta \coloneqq&\ \text{the } Z \text{ that solves } (\Gamma^2+4)Z^2+4\Gamma Z+\Gamma^2=0,\\
\mu \coloneqq&\ \text{the } Z \text{ that solves } 8Z^4+12\Gamma Z^3+(5\Gamma^2-4)Z^2-4\Gamma Z-\Gamma^2=0,\\
\omega \coloneqq&\ \text{the } Z \text{ that solves } (8\Gamma^2+32)Z^6+(8\Gamma^3+80\Gamma)Z^5+(4\Gamma^4+72\Gamma^2-16)Z^4+\\
&+(36\Gamma^3-32\Gamma)Z^3+(8\Gamma^4-16\Gamma^2)Z^2+\Gamma^4=0.
\end{aligned} \tag{99}$$

Then we can rewrite in a compact form the complete solution set, which includes four solutions[11],

$$\begin{aligned}
&\text{Solution 1: } \{\Lambda_1=0,\ \Lambda_2=0\};\\
&\text{Solution 2: } \left\{\Lambda_1=\eta,\ \Lambda_2=-\frac{\Gamma^2\eta+4\Gamma+4\eta}{\Gamma^2+4}\right\};\\
&\text{Solution 3: } \left\{\Lambda_1=\omega,\ \Lambda_2=-\frac{\Omega_1+\Omega_2}{\Gamma^6(2\Gamma^4+7\Gamma^2-4)}\right\},\\
&\text{with:}\\
&\Omega_1=(4\omega^3+8\omega)\Gamma^{10}+(8\omega^4+42\omega^2+9)\Gamma^9+(8\omega^5+48\omega^3-25\omega)\Gamma^8+\\
&\quad+(12\omega^4-292\omega^2-38)\Gamma^7+(-48\omega^5-528\omega^3+108\omega)\Gamma^6+\\
&\quad+(-592\omega^4+920\omega^2+80)\Gamma^5,\\
&\Omega_2=(-32\omega^5+1664\omega^3-176\omega)\Gamma^4+(2304\omega^4-1152\omega^2-32)\Gamma^3+\\
&\quad+(1024\omega^5-1216\omega^3+64\omega)\Gamma^2+(-1024\omega^4+384\omega^2)\Gamma-\\
&\quad-512\omega^5+256\omega^3;\\
&\text{Solution 4: } \{\Lambda_1=\mu,\ \Lambda_2=\mu\}.
\end{aligned} \tag{100}$$

[10]The solver implements the algorithm developed in Maza (2005) to compute the triangular decomposition of a polynomial system.

[11]We solve in both orders, i.e., for Λ_1 first and then for Λ_2 as well as vice versa, and verify that the solutions are the same for both orders but Λ_1 and Λ_2 swap with each other, which is consistent with (98) being specular to each other. Because of this symmetry (and in the interest of brevity) here we only report the solutions we obtain when we solve for Λ_2 first and then for Λ_1.

We analyze the four solutions (100), one by one. Solution 1 is the trivial solution $\lambda_1 = \lambda_2 = \lambda_3 = 0$. We can disregard it because it has no economic meaning and it is mathematically generated from the conversion of the price impact equations into their normal forms. Next we analyze Solution 2, where Λ_1 looks simpler than Λ_2. However, recall that η is the root of a quadratic polynomial equation in Z and after we express it in explicit form we find that the complexity of Λ_1 and Λ_2 is actually the same,

$$\begin{aligned}
&\left\{\Lambda_1 = \frac{(\iota\Gamma - 2)\Gamma}{\Gamma^2 + 4},\ \Lambda_2 = -\frac{(\iota\Gamma + 2)\Gamma}{\Gamma^2 + 4}\right\},\\
&\left\{\Lambda_1 = -\frac{(\iota\Gamma + 2)\Gamma}{\Gamma^2 + 4},\ \Lambda_2 = \frac{(\iota\Gamma - 2)\Gamma}{\Gamma^2 + 4}\right\}.
\end{aligned} \tag{101}$$

Note that these Λ_1 and Λ_2 are not real, as they are complex conjugates of each other. Thus we can disregard Solution 2 and move to the next solution. Solution 3 has $\Lambda_1 = \omega$, that is the root of a degree-6 polynomial equation (96). As such, there is no guarantee that a closed form expression exists but we can still try to simplify ω, convert it to radical, or split it in order to isolate the real from the imaginary parts, if any. Sometimes a closed form can be found for polynomial of degree higher than four. However, in this case the software Maple cannot find any closed form expression for ω. Moreover, Descartes' rule of signs does not help to find more information about the roots for the degree-6 polynomial,

$$\begin{aligned}
&(8\Gamma^2 + 32)Z^6 + (8\Gamma^3 + 80\Gamma)Z^5 + (4\Gamma^4 + 72\Gamma^2 - 16)Z^4 + (36\Gamma^3 - 32\Gamma)Z^3 +\\
&+ (8\Gamma^4 - 16\Gamma^2)Z^2 + \Gamma^4 = 0,\ \text{where } \Gamma > 0.
\end{aligned} \tag{102}$$

For positive roots, the signs are $+, +, ?, +, ?, +$ since the sign of the coefficients on Z^4 and Z^2 depends on the value of Γ, so it is not clear how many times the sign changes. For negative roots (i.e., for $Z = -Z$), the signs are $+, -, ?, -, ?, +$, so we either have 2 sign changes (2 or 0 negative roots) or 4 sign changes (4, 2, or 0 negative roots). All we can do is to analyze the six roots r_i, $i \in \{1, .., 6\}$, numerically (i.e., by plotting them). First of all, we confirm that all roots nullify both the non-dimensional price impact equations (98), thus verifying that ω in (99) is indeed a mathematically valid solution. Next we focus on the first two roots, r_1 and r_2, and we plot them in Fig. 8a and Fig. 8b, respectively. We show that for $\Gamma > 0$,

$\Lambda_1 > 0$ when $\Lambda_2 < 0$. Therefore we can disregard both r_1 and r_2. Next we color each of the six roots according to its index and we plot the corresponding Λ_1 in Fig. 9a and Λ_2 in Fig. 9b. In each plot the different pieces are in different roots. Fig. 9a shows that only r_1 and r_2 generate $\Lambda_1 > 0$ for any $\Gamma > 0$. Fig. 9b shows that only r_4, r_5, and r_6 generate $\Lambda_2 > 0$ for some (but not any) $\Gamma > 0$. By comparing the two plots we see that none of the six roots are such that Λ_1 and Λ_2 are both positive for any $\Gamma > 0$. Thus we can disregard also Solution 3 and move to the next solution, i.e., Solution 4.

To sum up the complete solution set (100), Solution 1 is trivial and meaningless ($\Lambda_1 = \Lambda_2 = 0$). Solution 2 is such that Λ_1 and Λ are complex conjugates. Solution 3 is such that Λ_1 and Λ_2 are never both positive for any $\Gamma > 0$. Therefore, none of the three solutions generate real and positive Λ_1 and Λ_2. As such, we disregard them and further analyze the only meaningful solution that survives, i.e., Solution 4: $\Lambda_1 = \Lambda_2 = \mu$.

C.2 Admissible Polynomial Solution

The only admissible solution is $\Lambda_1 = \Lambda_2 = \mu$, where μ is defined in (99). Since $\Lambda_1 \coloneqq \lambda_1\sigma_\delta/\sigma_\nu$ and $\Lambda_2 \coloneqq \lambda_2\sigma_\delta/\sigma_\nu$, $\Lambda_1 = \Lambda_2$ implies $\lambda_1 = \lambda_2 = \lambda$ and, in turn, the equilibrium parameters (93) specify to (9). In other words, markets 1 and 2 are perfectly symmetric. As such, (94) specify to (11) or, in other words, the second equation in (94) becomes redundant and we just need (11) to fully characterize the unique equilibrium.

Formally, we take the first equation in (94) with parameters (93) and evaluate it for $\lambda_1 = \lambda$ and $\lambda_2 = \lambda$, imposing the identity ex-ante, and thus obtain,

$$-\frac{\lambda(5\gamma^2\lambda^2\sigma_\nu^4\sigma_\delta^2 - \gamma^2\sigma_\nu^6 + 12\gamma\lambda^3\sigma_\nu^2\sigma_\delta^2 - 4\gamma\lambda\sigma_\nu^4 + 8\lambda^4\sigma_\delta^2 - 4\lambda^2\sigma_\nu^2)}{5\gamma^2\lambda^2\sigma_\nu^4\sigma_\delta^2 + \gamma^2\sigma_\nu^6 + 12\gamma\lambda^3\sigma_\nu^2\sigma_\delta^2 + 4\gamma\lambda\sigma_\nu^4 + 8\lambda^4\sigma_\delta^2 + 4\lambda^2\sigma_\nu^2} = 0. \tag{103}$$

We divide out the $\lambda = 0$ solution and take the numerator,

$$-5\gamma^2\lambda^2\sigma_\nu^4\sigma_\delta^2 + \gamma^2\sigma_\nu^6 - 12\gamma\lambda^3\sigma_\nu^2\sigma_\delta^2 + 4\gamma\lambda\sigma_\nu^4 - 8\lambda^4\sigma_\delta^2 + 4\lambda^2\sigma_\nu^2 = 0. \tag{104}$$

Note that from just looking at the explicit solution of (104) we cannot tell which is the positive one since it will depend on the values of the parameters. Non-dimensionalization

allows us to work on λ-polynomials with just one parameter: the product $\gamma\sigma_\nu\sigma_\delta$, which we denote as $\Gamma > 0$. We can find the non-dimensionalization by a systematic process, but let's guess that the non-dimensional variable Λ is defined as the Λ_1, Λ_2, and Λ_3 in the previous subsection. That is,

$$\Lambda := \lambda\frac{\sigma_\delta}{\sigma_\nu}. \tag{105}$$

If we isolate λ from (105) and plug it into (104) we obtain,

$$-5\gamma^2\Lambda^2\sigma_\nu^6 + \gamma^2\sigma_\nu^6 - \frac{12\gamma\Lambda^3\sigma_\nu^5}{\sigma_\delta} + \frac{4\gamma\Lambda\sigma_\nu^5}{\sigma_\delta} - \frac{8\Lambda^4\sigma_\nu^4}{\sigma_\delta^2} + \frac{4\Lambda^2\sigma_\nu^4}{\sigma_\delta^2} = 0. \tag{106}$$

We divide through by σ_ν^4, which is in every term, and multiply by σ_δ^2 to get rid of the denominators,

$$-5\sigma_\delta^2\sigma_\nu^2\gamma^2\Lambda^2 - 12\sigma_\delta\sigma_\nu\gamma\Lambda^3 + \sigma_\delta^2\sigma_\nu^2\gamma^2 - 8\Lambda^4 + 4\sigma_\delta\sigma_\nu\gamma\Lambda + 4\Lambda^2 = 0. \tag{107}$$

We see that γ always appears multiplied by $\sigma_\nu\sigma_\delta$, so we define Γ as such product,

$$\Gamma := \gamma\sigma_\nu\sigma_\delta. \tag{108}$$

Finally, if we isolate γ from (108) and plug it into (107) we obtain,

$$-8\Lambda^4 - 12\Gamma\Lambda^3 + (4 - 5\Gamma^2)\Lambda^2 + 4\Gamma\Lambda + \Gamma^2 = 0. \tag{109}$$

We can make three observations. First, the quartic (109) is what we would obtain if we took the first equation in (98), evaluate it for $\Lambda_1 = \Lambda_2 = \Lambda$, and divide it by 2Λ. Second, it shares the same roots with the degree-4 polynomial $P^{(4)}$ in (96) and, as such, its roots solve the price impact equation(s). In particular, it is exactly the quartic (30) introduced in the main body of the paper for the case where the speculator is partially risk averse. Third, the quartic (109) is a one-parameter (i.e., parameter $\Gamma > 0$) equation for which we can easily apply Descartes' rule of signs to learn more about its roots. According to such rule, it seems that (109) has two types of behavior, depending on whether $\Gamma < 2\sqrt{5}/5$ or $\Gamma > 2\sqrt{5}/5$.

However, we show in Fig. 10a that the quartic has one positive root for $\Gamma = \sqrt{5}/5$, an example of $\Gamma < 2\sqrt{5}/5$, and in Fig. 10b that it still has a positive root for $\Gamma = 3\sqrt{5}/5$, an example of $\Gamma > 2\sqrt{5}/5$. Moreover, in Fig. 11 we plot the quartic for six different values of positive Γ, i.e., $\Gamma = \{0, 2, 4, 6, 8, 10\}$. The curve jumps at three places and seems to always have one positive root. We also computes the two-dimensional plot of the implicitly defined curve (109) and find that there is always a positive root for $\Gamma > 0$. In order to identify the positive one among the four, we tried to solve explicitly (109) for Λ in parametric form, which should have given us the different solutions for different ranges of Γ. However, it only gives us values at special values of Γ, perhaps because this case is too hard to solve. Then, we could simply plot the four roots for Γ from 0 to 10 and color them by index. We find that the positive root is consistently the first one. As we wanted to prove, such root exactly overlaps with the curve colored in black in Fig. 1a, where we plot $\Lambda(\Gamma)$ (31) against Γ as the unique positive root of the quartic equation (30).

Finally, note that in Appendix D.2 we prove that the solution for the baseline model with uncorrelated assets is just a specification of the solution for the extended model with correlated assets. There we note how the quartic equation (158) collapses exactly to the quartic equation for the baseline model (30) when ρ is set to 0. Then, we repeat all the steps described so far for the baseline model (in C.1 and C.2) to the extended model with correlated assets[12]. We confirm that also for the extended model the only admissible solution is the root of a quartic, specifically, the $\Lambda(\Gamma, \rho)$ (159) which solves the quartic equation (158). As such, we proceed with analyzing the quartic (158) and treat the quartic (30) as its key specification.

C.3 Roots of Polynomial Solution

In this section we formally characterize the quartic equation (158) (and hence the quartic (30)). We do so both numerically and analytically. For $\Gamma = \gamma\sigma_\nu\sigma_\delta > 0$ and $-1 < \rho < 1$, we conclude that (i) the quartic equation has exactly four real roots and (ii) only one of these

[12] All the corresponding equations and figures become significantly more complicated when we introduce correlation, but reporting them here would be redundant as exactly the same logic applies for each step.

roots is positive, thus confirming the uniqueness of our model equilibrium[13].

C.3.1 Preliminary Analysis

We rearrange (158) as,

$$\begin{aligned}&-8(\rho+1)^4\Lambda(\Gamma,\rho)^4+12(\rho+1)^3\Gamma(\rho-1)\Lambda(\Gamma,\rho)^3-5(\rho+1)^2\big(-4/5+\Gamma^2\rho^2\\&+2\left(-2/5-\Gamma^2\right)\rho+\Gamma^2\big)\Lambda(\Gamma,\rho)^2-4(\rho+1)\Gamma(\rho^2-1)\Lambda(\Gamma,\rho)\\&+\Gamma^2(\rho+1)(\rho-1)^2,\end{aligned}\tag{110}$$

and then we simplify it dividing each term by $\rho+1$,

$$\begin{aligned}&-8(\rho+1)^3\Lambda(\Gamma,\rho)^4+12(\rho+1)^2\Gamma(\rho-1)\Lambda(\Gamma,\rho)^3-5(\rho+1)\big(-4/5+\Gamma^2\rho^2\\&+2\left(-2/5-\Gamma^2\right)\rho+\Gamma^2\big)\Lambda(\Gamma,\rho)^2-4\Gamma(\rho^2-1)\Lambda(\Gamma,\rho)+\Gamma^2(\rho-1)^2.\end{aligned}\tag{111}$$

We only allow $\rho\in(-1,+1)$ and $\Gamma\in(0,+\infty)$, with bounds excluded, and look at the signs of the coefficients of (111),

$$\begin{bmatrix}1 & \Gamma^2(\rho-1)^2\\ \Lambda(\Gamma,\rho) & -4\Gamma(\rho^2-1)\\ \Lambda(\Gamma,\rho)^2 & -5\left(\rho+1\right)\left(-4/5+\Gamma^2\rho^2+2\left(-2/5-\Gamma^2\right)\rho+\Gamma^2\right)\\ \Lambda(\Gamma,\rho)^3 & 12(\rho+1)^2\Gamma(\rho-1)\\ \Lambda(\Gamma,\rho)^4 & -8(\rho+1)^3\end{bmatrix}.\tag{112}$$

We can see that the constant term is positive, as well as the coefficient on $\Lambda(\Gamma,\rho)$. The coefficient on $\Lambda(\Gamma,\rho)^3$ is negative, as well as the coefficient on $\Lambda(\Gamma,\rho)^4$. Therefore, only the coefficient on $\Lambda(\Gamma,\rho)^2$ has an ambiguous sign so we focus on such quadratic term,

$$-5\left(\rho+1\right)\left(-4/5+\Gamma^2\rho^2+2\left(-2/5-\Gamma^2\right)\rho+\Gamma^2\right).\tag{113}$$

[13]Note that the Fundamental Theorem of Algebra only states that the field of complex numbers is algebraically closed, implying that any polynomial of degree 4 with complex coefficients has 4 complex roots (counting multiplicity). However, we want to count the number of distinct real roots and prove that the positive one is unique.

We equate (113) to 0 and solve the resulting equation for Γ to find $\Gamma^*(\rho)$,

$$\Gamma^*(\rho) = \frac{2\sqrt{5}\sqrt{(\rho-1)^2(\rho+1)}}{5(\rho-1)^2}. \tag{114}$$

We factor the coefficient of Γ^2 in (113) and find that it is negative for all real ρ,

$$-5(\rho-1)^2(\rho+1) < 0, \tag{115}$$

concluding that the quadratic term (113) is positive when $\Gamma \in (0, \Gamma^*(\rho))$. We plot this region in gray color in Fig. 12a, where the curve in black at the intersection of the white and gray regions is $\Gamma^*(\rho)$ (114).

To learn more, we reduce the quartic (111) into another one which does not contain $\Lambda(\rho,\Gamma)^3$. To do so, we first divide each term of the quartic (111) by its coefficient on $\Lambda(\Gamma,\rho)^4$. Then we take the resulting expression and substitute every $\Lambda(\Gamma,\rho)$ in it with a new variable, i.e., z minus a fourth of the coefficient on $\Lambda(\Gamma,\rho)^3$ of such resulting expression. After the change of variable we factor out z to obtain the expanded form of the reduced quartic, which in turn we can concisely rewrite as,

$$\begin{aligned}
&z^4 - \frac{1}{32}a_1 z^2 + \frac{1}{64}a_2 z + \frac{1}{4096}a_3, \\
&\text{with:} \\
&a_1 = \frac{7\Gamma^2\rho^2 - 14\Gamma^2\rho + 7\Gamma^2 + 16\rho + 16}{(\rho+1)^2}, \\
&a_2 = \frac{\Gamma\left(3\Gamma^2\rho^3 - 9\Gamma^2\rho^2 + 9\Gamma^2\rho - 3\Gamma^2 + 8\rho^2 - 8\right)}{(\rho+1)^3}, \\
&a_3 = \frac{\Gamma^2(\rho-1)(117\Gamma^2\rho^3 - 351\Gamma^2\rho^2 + 351\Gamma^2\rho - 117\Gamma^2 - 32\rho^2 + 32)}{(\rho+1)^4}.
\end{aligned} \tag{116}$$

Note that if $a_2 = 0$ the reduced form quartic would be of the form $z^4 + pz^2 + r$. So we take the numerator of a_2 (the denominator is always nonzero), rewrite it by factoring out the ρ, and set it equal to 0,

$$3\Gamma^3\rho^3 + \Gamma(-9\Gamma^2 + 8)\rho^2 + 9\Gamma^3\rho + \Gamma(-3\Gamma^2 - 8) = 0. \tag{117}$$

We solve (117) for ρ while assuming $\Gamma \in (0, \Gamma^*(\rho))$ and obtain,

$$1, \frac{3\Gamma^2 + 4\sqrt{-3\Gamma^2+1} - 4}{3\Gamma^2}, \frac{3\Gamma^2 - 4\sqrt{-3\Gamma^2+1} - 4}{3\Gamma^2}. \tag{118}$$

This proves that a_2 in (116) is never equal to 0 in the open interval $\rho \in (-1, +1)$, provided $\Gamma \in (0, \Gamma^*(\rho))$.

Next we look at a_3 in (116). If $a_3 = 0$ then the quartic would have the obvious root $\Lambda(\Gamma, \rho) = 0$. So we take a_3 and set it equal to 0,

$$\frac{\Gamma^2(\rho-1)(117\Gamma^2\rho^3 - 351\Gamma^2\rho^2 + 351\Gamma^2\rho - 117\Gamma^2 - 32\rho^2 + 32)}{(\rho+1)^4} = 0. \tag{119}$$

We solve (119) for both Γ and ρ and obtain,

$$\left\{[\Gamma = \Gamma, \rho = 1], \left[\Gamma = -\Gamma^0(\rho), \rho = \rho\right], \left[\Gamma = \Gamma^0(\rho), \rho = \rho\right], [\Gamma = 0, \rho = \rho]\right\},$$

with:

$$\Gamma^0(\rho) = -\frac{4\sqrt{26(\rho+1)}}{39(\rho-1)}. \tag{120}$$

We exclude the solution with $\rho = 1$, the solution with $\Gamma = 0$, and also the solution with $\Gamma = -\Gamma^0(\rho)$ since $-\Gamma^0(\rho) < 0$ for $\rho \in (-1, +1)$. Then $a_3 = 0$ only in one situation (regardless of the value of ρ). That is, the quartic has root $\Lambda(\Gamma, \rho) = 0$ if and only if $\Gamma = \Gamma^0(\rho)$, where $0 < \Gamma^0(\rho) < \Gamma^*(\rho)$.

C.3.2 Numerical Simulations

First of all, we run a simulation to show that the quartic (111) always has four real $\Lambda(\Gamma, \rho)$ roots when $\Gamma \in (0, \Gamma^*(\rho))$, i.e., when Γ is chosen inside the gray domain of Fig. 12a. Specifically, we implement a procedure to pick a random $\rho^* \in (-1, +1)$, to compute its corresponding $\Gamma^*(\rho^*)$ and to randomly choose a $\widehat{\Gamma} \in (0, \Gamma^*(\rho^*))$ whenever it is called in the computation. We call such procedure 10000 times and, for each run, we solve (111) with parameters $\Gamma = \widehat{\Gamma}$ and $\rho = \rho^*$ for $\Lambda(\Gamma, \rho)$, while keeping track of the number of real roots. The number of real roots is always four, i.e., for 10000 times. In Fig. 12b we reproduce the

solving domain $0 < \Gamma < \Gamma^*(\rho)$ (colored in gray) together with the scatter plot (opaque red dots) of the 10000 $\left(\widehat{\Gamma}, \rho^*\right)$ coordinates.

Next, we run a simulation to compute the probability that (111) has four real $\Lambda(\Gamma, \rho)$ roots when $\Gamma \notin (0, \Gamma^*(\rho))$, i.e., when Γ is chosen outside the gray domain of Fig. 12a. Our procedure replicates the one we described above but applies to $\widehat{\Gamma} > \Gamma^*(\rho)$. We call such procedure 10000 times and, for each run, we solve (111) and keep track of the number of real $\Lambda(\Gamma, \rho)$ roots. Moreover, for each run, if the roots counter measures four we store the corresponding (Γ, ρ) coordinates. We find that the number of real roots is two for 9952 times and four for 48 times. That is, the probability that (111) has four real roots when $\Gamma \notin (0, \Gamma^*(\rho))$ is only 0.48% or, in other words, there are 99.52% chances that the quartic has only two real roots when $\Gamma > \Gamma^*(\rho)$. In Fig. 12c we reproduce the solving domain $\Gamma > \Gamma^*(\rho)$ (in white color) together with the scatter plots of the (Γ, ρ) coordinates where the quartic has, respectively, two real roots (9952 dots in transparent red) and four real roots (48 dots in blue). As the plot suggests, the four real roots situation comes when Γ is slightly above $\Gamma^*(\rho)$, the boundary curve depicted in black in Fig. 12a.

Finally, we use Sturm's theorem to demonstrate that (111) has four real $\Lambda(\Gamma, \rho)$ roots when $\Gamma = \Gamma^*(\rho)$, i.e., when Γ lies on the boundary curve. First, we evaluate the quartic for $\Gamma = \Gamma^*(\rho)$ to obtain an expression in ρ only,

$$\begin{aligned} &-8(\rho+1)^3\Lambda(\Gamma,\rho)^4+\frac{24(\rho+1)^2\sqrt{5}\sqrt{(\rho-1)^2(\rho+1)}\Lambda(\Gamma,\rho)^3}{5\rho-5}+\frac{4}{5}\\ &-\frac{8\sqrt{5}\sqrt{(\rho-1)^2(\rho+1)}(\rho+1)\Lambda(\Gamma,\rho)}{5\rho-5}+\frac{4\rho}{5}. \end{aligned} \tag{121}$$

We form the corresponding Sturm's sequence of polynomials S_0^*, S_1^*, S_2^*, S_3^*, and S_4^*. We form S_0^* by dividing (121) for its coefficient on $\Lambda(\Gamma, \rho)^4$, expanding it (distributing products over sums) and keeping the $\Lambda(\Gamma, \rho)$ factored out. We form S_1^* by differentiating S_0^* with respect to $\Lambda(\Gamma, \rho)$. We compute the remainder r_0^* of polynomial S_0^* divided by polynomial S_1^* (both in $\Lambda(\Gamma, \rho)$) where $S_0^* = S_1^* q + r_0^*$ and q is the quotient. Then we form S_2^* as the negative of such r_0^*. Next, we compute the remainder r_1^* of polynomial S_1^* divided by polynomial S_2^* and form S_3^* as the negative of r_1^*. Finally, we compute the remainder r_2^* of polynomial S_2^*

divided by polynomial S_3^* and form S_4^* as the negative of r_2^*. All in all, we write the Sturm's polynomial chain as,

$$\begin{aligned}
S_0^* &= \Lambda(\Gamma,\rho)^4 - \frac{3\sqrt{5}\sqrt{(\rho+1)(\rho-1)^2}}{5\rho^2-5}\Lambda(\Gamma,\rho)^3 + \frac{\sqrt{5}\sqrt{(\rho+1)(\rho-1)^2}}{5(\rho-1)(\rho+1)^2}\Lambda(\Gamma,\rho) \\
&\quad - \frac{1}{10(\rho+1)^2}, \\
S_1^* &= 4\Lambda(\Gamma,\rho)^3 - \frac{9\sqrt{5}\sqrt{(\rho+1)(\rho-1)^2}}{5\rho^2-5}\Lambda(\Gamma,\rho)^2 + \frac{\sqrt{5}\sqrt{(\rho+1)(\rho-1)^2}}{5(\rho-1)(\rho+1)^2}, \\
S_2^* &= \frac{27}{80(\rho+1)}\Lambda(\Gamma,\rho)^2 - \frac{3\sqrt{5}\sqrt{(\rho+1)(\rho-1)^2}}{20(\rho-1)(\rho+1)^2}\Lambda(\Gamma,\rho) + \frac{1}{16(\rho+1)^2}, \\
S_3^* &= \frac{64}{81(\rho+1)}\Lambda(\Gamma,\rho) - \frac{248\sqrt{5}\sqrt{(\rho+1)(\rho-1)^2}}{1215(\rho-1)(\rho+1)^2}, \\
S_4^* &= \frac{477}{25600(\rho+1)^2}.
\end{aligned} \tag{122}$$

After forming the Sturm's chain, we compute the limits of the five polynomials (122) for $\Lambda(\Gamma,\rho)$ to $-\infty$ and to $+\infty$, and then collect them in two separate lists. For each one of the two lists, we evaluate the signs of these limits (taking into account that $-1<\rho<1$) and count the number of changes of sign,

$$\begin{aligned}
&\text{signum}\left(S_{-\infty}^*\right) = [1,-1,1,-1,1]\,, \quad \Delta^*(\Lambda=-\infty) = 4, \\
&\text{signum}\left(S_{+\infty}^*\right) = [1,1,1,1,1]\,, \quad \Delta^*(\Lambda=+\infty) = 0.
\end{aligned} \tag{123}$$

Then, according to Sturm's method, the number of real roots of the quartic at the boundary $\Gamma=\Gamma^*(\rho)$ (121) is given by,

$$\Delta^*(\Lambda=-\infty) - \Delta^*(\Lambda=+\infty) = 4. \tag{124}$$

That is, the quartic (111) has four real $\Lambda(\Gamma,\rho)$ roots when $\Gamma=\Gamma^*(\rho)$, which is not surprising given what we observed in Fig. 12c.

C.3.3 Analytical Investigation

We develop a study based on Sturm's theorem to understand our findings from the numerical simulations. We start by forming the Sturm's sequence of polynomials S_0, S_1, S_2, S_3, and S_4 following the procedure we described for (122). However, we start the chain with the polynomial S_0 as the reduced form quartic (116). We form S_1 by differentiating S_0 with respect to z. We compute the remainder r_0 of polynomial S_0 divided by polynomial S_1 (both in z) where $S_0 = S_1 q + r_0$ and q is the quotient. Then we form S_2 as the negative of such r_0. Next, we compute the remainder r_1 of polynomial S_1 divided by polynomial S_2 and form S_3 as the negative of r_1. Finally, we compute the remainder r_2 of polynomial S_2 divided by polynomial S_3 and form S_4 as the negative of r_2. All in all, after due simplifications, we write the Sturm's polynomial chain as,

$$
\begin{aligned}
S_0 &= z^4 + \left(\frac{-7(\rho-1)^2\Gamma^2 - 16\rho - 16}{32(\rho+1)^2}\right)z^2 + \left(\frac{3(\rho-1)\Gamma\left((\rho-1)^2\Gamma^2 + (8/3)\rho + 8/3\right)}{64(\rho+1)^3}\right)z \\
&\quad + \frac{117(\rho-1)^2\Gamma^2((\rho-1)^2\Gamma^2 - (32/117)\rho - 32/117)}{4096(\rho+1)^4}, \\
S_1 &= 4z^3 + \left(\frac{-7(\rho-1)^2\Gamma^2 - 16\rho - 16}{16(\rho+1)^2}\right)z + \frac{3(\rho-1)\Gamma\left((\rho-1)^2\Gamma^2 + (8/3)\rho + 8/3\right)}{64(\rho+1)^3}, \\
S_2 &= \left(\frac{7(\rho-1)^2\Gamma^2 + 16\rho + 16}{64(\rho+1)^2}\right)z^2 - \left(\frac{9(\rho-1)((\rho-1)^2\Gamma^2 + (8/3)\rho + 8/3)\Gamma}{256(\rho+1)^3}\right)z \\
&\quad - \frac{117\left((\rho-1)^2\Gamma^2 - (32/117)\rho - 32/117\right)(\rho-1)^2\Gamma^2}{4096(\rho+1)^4}, \\
S_3 &= \left(\frac{-50(\rho-1)^6\Gamma^6 - 64(\rho+1)(\rho-1)^4\Gamma^4 + 224(\rho^2-1)^2\Gamma^2 + 256(\rho+1)^3}{\left(7(\rho-1)^2\Gamma^2 + 16\rho + 16\right)^2(\rho+1)^2}\right)z \\
&\quad - \frac{75\Gamma(\rho-1)\left((\rho-1)^2\Gamma^2 + (8/3)\rho + 8/3\right)\left((\rho-1)^4\Gamma^4 + \frac{8(\rho+1)(\rho-1)^2\Gamma^2}{25} + \frac{16(\rho+1)^2}{25}\right)}{4\left(7(\rho-1)^2\Gamma^2 + 16\rho + 16\right)^2(\rho+1)^3}, \\
S_4 &= \frac{n_{S4}(\Gamma,\rho)}{d_{S4}(\Gamma,\rho)},
\end{aligned}
$$

with:

$$
\begin{aligned}
n_{S4}(\Gamma,\rho) = -\Bigg(49\Big((\rho-1)^2\Gamma^2 + (16/7)\rho + 16/7\Big)^2\Gamma^4(\rho-1)^4\Big((\rho-1)^6\Gamma^6 - (128/125) \\
(\rho+1)^3 + (68/125)(\rho+1)(\rho-1)^4\Gamma^4 - (144/125)(\rho-1)^2(\rho+1)^2\Gamma^2\Big)\Bigg),
\end{aligned}
$$

$$d_{S4}(\Gamma,\rho) = 320(\rho+1)^3\Big((\rho-1)^6\Gamma^6 + (32/25)(\rho+1)(\rho-1)^4\Gamma^4 - (128/25)(\rho+1)^3 - (112/25)(\rho-1)^2(\rho+1)^2\Gamma^2\Big)^2. \tag{125}$$

Then we consider the coefficients of the leading monomials, i.e., the coefficient on z^4 in S_0, the coefficient on z^3 in S_1, the coefficient on z^2 in S_2, the coefficient on z in S_3, and the whole S_4 as the last coefficient. We define these, respectively, as the five leading coefficients of the Sturm's polynomial chain (125) and we study their signs.

The first three leading coefficients are clearly positive, given $\rho > -1$. So we study the sign of the fourth leading coefficient, which we denote as L_{S3} and representing the coefficient on z in S_3,

$$L_{S3} = \frac{-50(\rho-1)^6\Gamma^6 - 64(\rho+1)(\rho-1)^4\Gamma^4 + 224(\rho^2-1)^2\Gamma^2 + 256(\rho+1)^3}{\left(7(\rho-1)^2\Gamma^2 + 16\rho + 16\right)^2(\rho+1)^2}. \tag{126}$$

As a numerical example, it is natural to consider the meaningful no correlation case. Recall that such case characterizes our baseline model's equilibrium with the quartic polynomial (30) as the solution. Thus we evaluate L_{S3} (126) for $\rho = 0$, set the expression to 0 and solve numerically to find its root, $\Gamma_{LS3}^{\rho=0} = 1.4397$. We plot it in Fig. 13a.

To confirm it analytically, we focus on the numerator of L_{S3} (126), since its denominator is strictly positive. We rewrite the numerator by implementing the substitution $\Gamma^2 = Z$,

$$-50(\rho-1)^6Z^3 - 64(\rho+1)(\rho-1)^4Z^2 + 224(\rho^2-1)^2Z + 256(\rho+1)^3. \tag{127}$$

We compute the discriminant of (127) which is obviously positive,

$$1601175552(\rho+1)^6(\rho-1)^{12} > 0, \tag{128}$$

meaning that (127) has three real roots[14]. To find them, we proceed in three steps. First, we divide all terms in (127) by its own coefficient on Z^3, $-50(\rho-1)^6$, and rewrite it by defining

[14]Moreover, note that no root can be equal to 0 because of the polynomial's constant/trailing term $256(\rho+1)^3$ being non null, i.e., (127) cannot be written as $Z(Z^2 + aZ + b)$.

the new variable $\phi \coloneqq (\rho+1)/(\rho-1)^2$,

$$Z^3 + \frac{32}{25}\phi Z^2 - \frac{112}{25}\phi^2 Z - \frac{128}{25}\phi^3. \tag{129}$$

Second, we set (129) to 0, solve the expression analytically, and convert the exponentials into trigonometric functions to obtain the three roots in real form,

$$\left\{ \frac{8\phi(-4+\cos(\arctan\left(\frac{450\sqrt{509}}{10063}\right)/3)\sqrt{589})}{75}, \right.$$
$$-\frac{4\phi(\sqrt{3}\sin(\arctan\left(\frac{450\sqrt{509}}{10063}\right)/3)\sqrt{589}+\cos(\arctan\left(\frac{450\sqrt{509}}{10063}\right)/3)\sqrt{589}+8)}{75},$$
$$\left. \frac{4\phi(\sqrt{3}\sin(\arctan(\frac{450\sqrt{509}}{10063})/3)\sqrt{589}-\cos(\arctan(\frac{450\sqrt{509}}{10063})/3)\sqrt{589}-8)}{75} \right\}. \tag{130}$$

Third, we select the only positive real Z-root, the first one, so that we can take its square root to get back to a Γ-root (recall the change of variable $\Gamma^2 = Z$),

$$\Gamma_{LS3}(\rho) = \frac{2\sqrt{6}}{15}\sqrt{\frac{(\rho+1)\left(-4+\cos\left(\arctan\left(\frac{450\sqrt{509}}{10063}\right)/3\right)\sqrt{589}\right)}{(\rho-1)^2}}, \tag{131}$$

where we only considered the positive one as admissible (since we know that $\Gamma = \gamma\sigma_\nu\sigma_\delta$ must be positive to be valid) and also substituted back ϕ with $(\rho+1)/(\rho-1)^2$. Finally, we numerically evaluate $\Gamma_{LS3}(\rho)$ (131) for $\rho = 0$ and confirm the root $\Gamma_{LS3}^{\rho=0} = 1.4397$ that we previously found by solving (126) numerically. In summary, as shown in Fig. 13a,

$$L_{S3}^{\rho=0} \begin{cases} > 0 & \text{if } 0 < \Gamma < \Gamma_{LS3}^{\rho=0} \\ = 0 & \text{if } \Gamma = \Gamma_{LS3}^{\rho=0} \\ < 0 & \text{if } \Gamma > \Gamma_{LS3}^{\rho=0} \end{cases} \tag{132}$$

Next, we study the sign of the fifth leading coefficient L_{S4}, which is S_4 in (125). For

convenience, we denote its numerator as,

$$nL_{S4} = -\Big(49\Big((\rho-1)^2\Gamma^2 + (16/7)\rho + 16/7\Big)^2\Gamma^4(\rho-1)^4\Big((\rho-1)^6\Gamma^6 - (128/125)$$
$$(\rho+1)^3 + (68/125)(\rho+1)(\rho-1)^4\Gamma^4 - (144/125)(\rho-1)^2(\rho+1)^2\Gamma^2\Big)\Big), \quad (133)$$

and its denominator as,

$$dL_{S4} = 320(\rho+1)^3\Big((\rho-1)^6\Gamma^6 + (32/25)(\rho+1)(\rho-1)^4\Gamma^4 - (128/25)(\rho+1)^3$$
$$-(112/25)(\rho-1)^2(\rho+1)^2\Gamma^2\Big)^2. \quad (134)$$

We start from its denominator dL_{S4} (134). Note that after the change of variable $\Gamma^2 = Z$, we can rewrite the squared factor as,

$$(\rho-1)^6Z^3 + (32/25)(\rho+1)(\rho-1)^4Z^2 - (112/25)(\rho-1)^2(\rho+1)^2Z$$
$$-(128/25)(\rho+1)^3, \quad (135)$$

and if we change its sign and multiply it by 50 we obtain exactly the numerator of L_{S3} (127). In other words, the only valid Γ-root of dL_{S4} (134),

$$\Gamma_{dLS4}(\rho) = \frac{2\sqrt{6}}{15}\sqrt{\frac{(\rho+1)\left(-4+\cos\left(\arctan\left(\frac{450\sqrt{509}}{10063}\right)/3\right)\sqrt{589}\right)}{(\rho-1)^2}}, \quad (136)$$

coincides with the only valid root of L_{S3} (131) and, in particular, $\Gamma^{\rho=0}_{dLS4} = \Gamma^{\rho=0}_{LS3} = 1.4397$. We now move on to the numerator nL_{S4} (133), where we only look at the factor that is not surely positive. We apply the usual change of variable $\Gamma^2 = Z$ to it and divide its terms by the coefficient on Z^3, $(\rho-1)^6$, to obtain,

$$Z^3 + \frac{68(\rho+1)}{125(\rho-1)^2}Z^2 - \frac{144(\rho+1)^2}{125(\rho-1)^4}Z - \frac{128(\rho+1)^3}{125(\rho-1)^6}. \quad (137)$$

We compute the discriminant of (137) and find that it is negative,

$$-\frac{2342039552(\rho+1)^6}{244140625(\rho-1)^{12}}<0, \tag{138}$$

meaning that (137) has only one real root. To find it, we apply the same steps as before to obtain the only valid Γ-root of nL_{S4} (133) as,

$$\Gamma_{nLS4}(\rho)=\frac{\sqrt{30}}{75}\sqrt{\frac{(\rho+1)\left((2k)^{2/3}+2^{1/3}7328-34k^{1/3}\right)}{(\rho-1)^2k^{1/3}}},$$

with:

$$k=661799+31125\sqrt{249}. \tag{139}$$

We numerically evaluate $\Gamma_{nLS4}(\rho)$ (139) for $\rho=0$ and find the root $\Gamma_{nLS4}^{\rho=0}=1.0844$. In summary, as shown in Fig. 13b,

$$nL_{S4}^{\rho=0}\begin{cases}>0 & \text{if } 0<\Gamma<\Gamma_{nLS4}^{\rho=0}\\ =0 & \text{if } \Gamma=\Gamma_{nLS4}^{\rho=0}\\ <0 & \text{if } \Gamma>\Gamma_{nLS4}^{\rho=0}\end{cases} \tag{140}$$

and as shown in Fig. 13c,

$$dL_{S4}^{\rho=0}\begin{cases}>0 & \text{if } \Gamma>0\ \wedge\ \Gamma\neq\Gamma_{dLS4}^{\rho=0}\\ =0 & \text{if } \Gamma=\Gamma_{dLS4}^{\rho=0}\end{cases} \tag{141}$$

Note that $\Gamma_{LS3}(\rho)$ (131), thus $\Gamma_{dLS4}(\rho)$ (136), and $\Gamma_{nLS4}(\rho)$ (139) are both functions of assets' correlation ρ,

$$\begin{aligned}\Gamma_{LS3}(\rho)=\Gamma_{dLS4}(\rho)&\coloneqq\rho\mapsto 1.4397\sqrt{\frac{\rho+1}{(\rho-1)^2}},\\ \Gamma_{nLS4}(\rho)&\coloneqq\rho\mapsto 1.0844\sqrt{\frac{\rho+1}{(\rho-1)^2}}.\end{aligned} \tag{142}$$

We have found the signs of all the leading coefficients L_{Si}, $i = \{0, 1, 2, 3, 4\}$, in the Sturm's sequence of polynomials. Thus, we can finally apply the Sturm's theorem to formally count the number of distinct real $\Lambda(\Gamma, \rho)$ roots in the unbounded interval $(-\infty, +\infty)$. We evaluate the signs of the five Sturm's polynomials (125) for z to $-\infty$ and to $+\infty$. Note that the sign at $+\infty$ of a polynomial is the sign of its leading coefficient, while at $-\infty$ the sign of a polynomial is the sign of its leading coefficient for a polynomial of even degree, and the opposite sign for a polynomial of odd degree. We focus on the baseline ($\rho = 0$) case, with no correlation among assets.

In Fig. 14a we plot the signs of Sturm's limits for $\Lambda(\Gamma)$ or z to $-\infty$, while in Fig. 14b we plot the signs of Sturm's limits for $\Lambda(\Gamma)$ or z to $+\infty$. For each one of the two figures we also count the number of times a sign changes across the corresponding series of limits, $\Delta(\Lambda = -\infty)$ or $\Delta(\Lambda = +\infty)$, from the first leading coefficient (L_{S0}) to the last one (L_{S4}). As we can see, such count in general depends on the location of the root $\Gamma^{\rho=0}_{LS3} = 1.4397$ (or pole $\Gamma^{\rho=0}_{dLS4} = 1.4397$ depicted in red) and of the root $\Gamma^{\rho=0}_{nLS4} = 1.0844$. Then, according to Sturm's method, the number of distinct real roots of the baseline ($\rho = 0$) quartic (30) is given by,

$$\Delta(\Lambda = -\infty) - \Delta(\Lambda = +\infty) = \begin{cases} 4 - 0 = 4 & \text{if } 0 < \Gamma < \Gamma^{\rho=0}_{nLS4} \\ 3 - 1 = 2 & \text{if } \Gamma^{\rho=0}_{nLS4} < \Gamma < \infty \end{cases}$$

with:

$$\Gamma^{\rho=0}_{nLS4} = 1.0844, \tag{143}$$

as we show in Fig. 15a, which is built by subtracting the counters (in the "Δ bars") in Fig. 14b from the counters in Fig. 14a. Note how the result (143) is exactly what we anticipated informally by directly plotting the roots of the baseline ($\rho = 0$) quartic (30) in Fig. 1a.

As we previously mentioned, the baseline ($\rho = 0$) quartic (30) is nothing more than a specification of the quartic with assets' correlation $\rho \neq 0$ (158). As such, we would like to know for which values of the parameters Γ and ρ the generic ($\rho \neq 0$) quartic has real $\Lambda(\Gamma, \rho)$ roots and how many. To learn this, we decompose the parameter space of our parametric

polynomial system made of the quartic and the three constraints,

$$\begin{cases} (8\rho^3+24\rho^2+24\rho+8)\Lambda(\Gamma,\rho)^4+(-12\Gamma\rho^3-12\Gamma\rho^2+12\Gamma\rho+12\Gamma)\Lambda(\Gamma,\rho)^3 \\ +(5\Gamma^2\rho^3-5\Gamma^2\rho^2-5\Gamma^2\rho+5\Gamma^2-4\rho^2-8\rho-4)\Lambda(\Gamma,\rho)^2 \\ +(4\Gamma\rho^2-4\Gamma)\Lambda(\Gamma,\rho)-\Gamma^2\rho^2+2\Gamma^2\rho-\Gamma^2=0, \\ \Gamma>0, \\ \rho>-1, \\ \rho<1, \end{cases} \tag{144}$$

into cells in which the quartic has a constant number of solutions. We use the implementation included in a subpackage of Maple software for performing such type of structural analysis[15].

In Fig. 15b we plot the cells for which our quartic with constrained parameters has a given number of roots. There are five parametric cells in total. For each cell the procedure returns the number of real roots found in it, as well as a sample point strictly in the interior of the cell. We can then find all real roots of the non-parametric quartic resulting from substituting the parameter values identified by the given sample point in each cell. The cells are labeled from 1 to 5. Cells 1, 4, and 5, which are depicted in opaque red, have no real roots at all. This squares well with the fact that the asset correlation parameter ρ is only meaningful if its values are bounded in $(-1,+1)$. However, cells 2 and 3 do contain real roots. Cell 2 is colored in gray and contains four distinct real roots which are, for sample point $(\Gamma=1,\rho=0)$,

$$\{\Lambda_1(1,0)=0.5623,\ \Lambda_2(1,0)=-0.2866,\ \Lambda_3(1,0)=-0.7757,\ \Lambda_4(1,0)=-1\}. \tag{145}$$

Cell 3 is colored in white and contains two distinct real roots which are, for sample point

[15]The underlying routine of the implementation is based on the concept of discriminant variety and leverages numerical algorithms for the cylindrical algebraic decomposition of the parameter space through the Groebner basis method. The concept of discriminant variety was proposed in Lazard and Rouillier (2007), while the cylindrical algebraic decomposition dates back to Collins (1975) and it is now widely used for problems in computational real algebraic geometry. Liang et al. (2009) provide a comprehensive description of the Maple's package we use.

$(\Gamma = 2, \rho = 0)$,

$$\{\Lambda_1(2,0) = 0.5161,\ \Lambda_2(2,0) = -0.3627\}. \tag{146}$$

Note that $\Gamma = 1 < 1.0844$ but $\Gamma = 2 > 1.0844$, so (145) and (146) are consistent with the finding (143) from our Sturm-based analytical investigation of the baseline ($\rho = 0$) quartic (30). Moreover, we colored cell 2 in gray and cell 3 in white to emphasize how such exercise of parameter space's decomposition formally proves what we showed with the numerical simulations in Fig. 12b and Fig. 12c for the generic ($\rho \neq 0$) quartic (158).

Last but not least, (145) and (146) seem to show that there is always a unique positive root in cells 2 and 3, i.e., anywhere in between the cells in opaque red of Fig. 15b. We confirm it by repeating the same cylindrical algebraic decomposition of our parametric polynomial system (144) but with the additional constraint,

$$\Lambda(\Gamma, \rho) > 0. \tag{147}$$

The plot of the decomposition remains the same, but with the further constraint there exists only one real root in cells 2 and 3. The root's uniqueness is consistent with what we showed in Fig. 16a and Fig. 16b, i.e., for the region $\Lambda(\Gamma, \rho) > 0$ there is everywhere only a single root (the top sheet in dim gray) for the quartic (158). Such root is expressed in closed-form as (159). For sample point $(\Gamma = 1, \rho = 0)$ in cell 2 and for sample point $(\Gamma = 2, \rho = 0)$ in cell 3, this root specifies to

$$\Lambda(1,0) = 0.5623 \quad \text{and} \quad \Lambda(2,0) = 0.5161, \tag{148}$$

which is consistent with $\Lambda(\Gamma)$ (31) being a positive and monotonically decreasing function of Γ, as in Fig. 1a.

D Analytical Model Extension: Correlated Assets

We provide analytical solutions for a model with a more general information structure of the assets' payoffs. Specifically, we generalize the covariance matrix (2) to allow for the correlation of assets 1 and 2 ($\rho \neq 0$),

$$\begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_1 + \nu_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \overline{\nu}_1 \\ \overline{\nu}_2 \\ \overline{\nu}_1 + \overline{\nu}_2 \end{bmatrix}, \begin{bmatrix} \sigma_\nu^2 & \rho\sigma_\nu^2 & \sigma_\nu^2 + \rho\sigma_\nu^2 \\ \rho\sigma_\nu^2 & \sigma_\nu^2 & \sigma_\nu^2 + \rho\sigma_\nu^2 \\ \sigma_\nu^2 + \rho\sigma_\nu^2 & \sigma_\nu^2 + \rho\sigma_\nu^2 & 2\sigma_\nu^2 + 2\rho\sigma_\nu^2 \end{bmatrix} \right). \tag{149}$$

The case of correlated assets is interesting because on ρ depends the symmetry of the assets' triangle. For example, for $\rho = -1/2$ the three assets have identical variance for their payoffs: $\text{Var}[\nu_3] = 2\sigma_\nu^2(1+\rho) = \sigma_\nu^2 = \text{Var}[\nu_1] = \text{Var}[\nu_2]$. In other words, for $\rho = -1/2$ the assets' triangle is fully symmetric in terms of payoff's risk.

D.1 Equilibrium Derivation

We re-write the mean-variance optimization problem (4) as,

$$\begin{aligned} \max_{\chi_1,\chi_2,\chi_3} \Big\{ & \chi_1 \left(-\lambda\chi_1 - \lambda\delta_1 - \overline{\nu}_1\right) + \chi_2 \left(-\lambda\chi_2 - \lambda\delta_2 - \overline{\nu}_2\right) + \chi_3 \left(-\lambda_3\chi_3 - \lambda_3\delta_3 - \overline{\nu}_1 - \overline{\nu}_2\right) \\ & + \left(\chi_1 + \chi_3\right)\left(\overline{\nu}_1 + \frac{\sigma_\nu^2(\rho+1)}{2\sigma_\nu^2(\rho+1)}\left(\zeta - \overline{\nu}_1 - \overline{\nu}_2\right)\right) + \left(\chi_2 + \chi_3\right)\left(\overline{\nu}_2 + \frac{\sigma_\nu^2(\rho+1)}{2\sigma_\nu^2(\rho+1)}\left(\zeta - \overline{\nu}_1 - \overline{\nu}_2\right)\right) \\ & - \frac{\gamma}{2}\left(\chi_1 + \chi_3\right)^2\left(\sigma_\nu^2 - \frac{\left(\sigma_\nu^2(\rho+1)\right)^2}{2\sigma_\nu^2(\rho+1)}\right) - \frac{\gamma}{2}\left(\chi_2 + \chi_3\right)^2\left(\sigma_\nu^2 - \frac{\left(\sigma_\nu^2(\rho+1)\right)^2}{2\sigma_\nu^2(\rho+1)}\right) \\ & - \gamma\left(\chi_1 + \chi_3\right)\left(\chi_2 + \chi_3\right)\left(\rho\sigma_\nu^2 - \frac{\left(\sigma_\nu^2(\rho+1)\right)^2}{2\sigma_\nu^2(\rho+1)}\right) \Big\}. \end{aligned} \tag{150}$$

With due simplifications the first order conditions (8) become,

$$\begin{aligned} & \lambda\left(-2\chi_1 - \delta_1\right) + \frac{1}{2}\left(\zeta - \overline{\nu}_1 - \overline{\nu}_2\right) + \frac{1}{2}\gamma\sigma_\nu^2\left(\rho - 1\right)\left(\chi_1 - \chi_2\right) = 0, \\ & \lambda\left(-2\chi_2 - \delta_2\right) + \frac{1}{2}\left(\zeta - \overline{\nu}_1 - \overline{\nu}_2\right) + \frac{1}{2}\gamma\sigma_\nu^2\left(\rho - 1\right)\left(\chi_2 - \chi_1\right) = 0, \\ & \lambda_3\left(-2\chi_3 - \delta_3\right) + \left(\zeta - \overline{\nu}_1 - \overline{\nu}_2\right) = 0. \end{aligned} \tag{151}$$

The trading and offsetting intensity parameters (9) in the equilibrium Theorem 1 now include the correlation coefficient,

$$\begin{aligned}\beta &= \frac{1}{4\lambda}, \qquad \alpha = \frac{4\lambda - \gamma\sigma_\nu^2(\rho-1)}{4\left(-2\lambda + \gamma\sigma_\nu^2(\rho-1)\right)}, \qquad \alpha^* = -\frac{\gamma\sigma_\nu^2(\rho-1)}{4(-2\lambda + \gamma\sigma_\nu^2(\rho-1))},\\ \beta_3 &= \frac{1}{2\lambda_3}, \qquad \alpha_3 = -\frac{1}{2}.\end{aligned} \tag{152}$$

We solve the corresponding market maker problem for the model with correlated assets and the price impacts (11) now include the correlation coefficient,

$$\begin{aligned}\lambda &= \frac{\beta\sigma_\nu^2(\rho+1)}{2\beta^2\sigma_\nu^2(\rho+1) + (\alpha+1)^2\sigma_\delta^2 + (\alpha^*)^2\sigma_\delta^2},\\ \lambda_3 &= \frac{2\beta_3\sigma_\nu^2(\rho+1)}{2\beta_3^2\sigma_\nu^2(\rho+1) + (\alpha_3+1)^2\sigma_{\delta 3}^2}.\end{aligned} \tag{153}$$

We plug the trading and offsetting intensity parameters (152) into the price impacts (153) and obtain two nonlinear equations in λ and in λ_3, respectively. Once we solve for λ and λ_3 explicitly, we can pin down β, β_3, α, α_3, and α^* (152). We have now pinned down our equilibrium conjectures Theorem 1.

D.2 Equilibrium Characterization

As in the baseline model without correlation, the equilibrium for market 3 is independent of the speculator's risk aversion. Accordingly, irrespective of γ, for market 3 the equilibrium parameters (13) and quoting rule (14) become, respectively,

$$\lambda_3 = \sqrt{2(\rho+1)}\frac{\sigma_\nu}{\sigma_{\delta 3}}, \qquad \beta_3 = \frac{\sigma_{\delta 3}}{2\sqrt{2(\rho+1)}\sigma_\nu}, \qquad \alpha_3 = -\frac{1}{2}, \tag{154}$$

$$q_3 = \overline{\nu}_1 + \overline{\nu}_2 + \frac{(\zeta - \overline{\nu}_1 - \overline{\nu}_2)}{2} + \frac{\sqrt{2(\rho+1)}\sigma_\nu}{2\sigma_{\delta 3}}\delta_3. \tag{155}$$

For market 1 and 2, we do not discuss the limit cases of $\gamma = 0$ and $\gamma \to \infty$ for the model with correlated assets as the same underlying mechanism of the baseline model carries over to this model extension. In the following, we allow the informed speculator to have a positive

but finite risk aversion coefficient.

As in the baseline model, we define the new combined parameter $\Gamma = \gamma\sigma_\nu\sigma_\delta$ in order to facilitate the nondimensionalization/scaling of all the equations which follow. Then, for $0 < \gamma < \infty$, the equilibrium parameters (28) and quotes (29) generalize to,

$$\lambda = (\rho+1)\Lambda(\Gamma,\rho)\frac{\sigma_\nu}{\sigma_\delta}, \qquad \beta = \frac{\sigma_\delta}{4\sigma_\nu(\rho+1)\Lambda(\Gamma,\rho)},$$
$$\alpha = \frac{\Gamma(\rho-1)-4(\rho+1)\Lambda(\Gamma,\rho)}{4(2(\rho+1)\Lambda(\Gamma,\rho)-\Gamma(\rho-1))}, \qquad \alpha^* = \frac{\Gamma(\rho-1)}{4(2(\rho+1)\Lambda(\Gamma,\rho)-\Gamma(\rho-1))}, \tag{156}$$

$$\begin{aligned} q_1 &= \overline{\nu}_1 + \frac{\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)}{4} + \frac{(\rho+1)\Lambda(\Gamma,\rho)\sigma_\nu(-4(\rho+1)\Lambda(\Gamma,\rho)+3\Gamma(\rho-1))}{4\sigma_\delta(-2(\rho+1)\Lambda(\Gamma,\rho)+\Gamma(\rho-1))}\delta_1 \\ &\quad + \frac{(\rho+1)(\rho-1)\Lambda(\Gamma,\rho)\gamma\sigma_\nu^2}{4(2(\rho+1)\Lambda(\Gamma,\rho)-\Gamma(\rho-1))}\delta_2, \\ q_2 &= \overline{\nu}_2 + \frac{\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)}{4} + \frac{(\rho+1)\Lambda(\Gamma,\rho)\sigma_\nu(-4(\rho+1)\Lambda(\Gamma,\rho)+3\Gamma(\rho-1))}{4\sigma_\delta(-2(\rho+1)\Lambda(\Gamma,\rho)+\Gamma(\rho-1))}\delta_2 \\ &\quad + \frac{(\rho+1)(\rho-1)\Lambda(\Gamma,\rho)\gamma\sigma_\nu^2}{4(2(\rho+1)\Lambda(\Gamma,\rho)-\Gamma(\rho-1))}\delta_1, \end{aligned} \tag{157}$$

where $\Lambda(\Gamma,\rho)$ is a (known) convoluted function of Γ and ρ. Specifically, $\Lambda(\Gamma,\rho)$ is the unique positive root of the quartic equation,

$$\begin{aligned} &(8\rho^3+24\rho^2+24\rho+8)\Lambda(\Gamma,\rho)^4 + (-12\Gamma\rho^3-12\Gamma\rho^2+12\Gamma\rho+12\Gamma)\Lambda(\Gamma,\rho)^3 \\ &+ (5\Gamma^2\rho^3-5\Gamma^2\rho^2-5\Gamma^2\rho+5\Gamma^2-4\rho^2-8\rho-4)\Lambda(\Gamma,\rho)^2 \\ &+ (4\Gamma\rho^2-4\Gamma)\Lambda(\Gamma,\rho)-\Gamma^2\rho^2+2\Gamma^2\rho-\Gamma^2 = 0. \end{aligned} \tag{158}$$

We can explicitly write $\Lambda(\Gamma,\rho)$ as,

$$\Lambda(\Gamma,\rho) = \frac{\sqrt{15}\sqrt{\frac{\omega(\Gamma,\rho)}{\eta(\Gamma,\rho)^{1/3}(\rho+1)^6}}}{120} + \frac{\sqrt{150}\sqrt{\frac{\phi(\Gamma,\rho)-5\theta(\Gamma,\rho)\sqrt{\frac{\omega(\Gamma,\rho)}{\eta(\Gamma,\rho)^{1/3}(\rho+1)^6}}}{\eta(\Gamma,\rho)^{1/3}(\rho+1)^6\sqrt{\frac{\omega(\Gamma,\rho)}{\eta(\Gamma,\rho)^{1/3}(\rho+1)^6}}}}}{3000} + \frac{3\Gamma(\rho-1)}{8(\rho+1)} > 0,$$

with:

$$\tau(\Gamma,\rho) = 125(\rho-1)^6\Gamma^6 + 68(\rho+1)(\rho-1)^4\Gamma^4 - 128(\rho+1)^3 - 144(\rho-1)^2(\rho+1)^2\Gamma^2,$$

$$
\begin{aligned}
\eta(\Gamma,\rho) &= (\rho+1)^{12}\left(\frac{12\Gamma^2\sqrt{3}(\rho+1)^5\sqrt{\frac{(\rho-1)^4\tau(\Gamma,\rho)}{(\rho+1)^9}}}{125}+(\rho-1)^6\Gamma^6+\frac{276(\rho+1)(\rho-1)^4\Gamma^4}{125}\right.\\
&\quad\left.-\frac{48(\rho-1)^2(\rho+1)^2\Gamma^2}{125}-\frac{64(\rho+1)^3}{125}\right),\\
\omega(\Gamma,\rho) &= 50\eta(\Gamma,\rho)^{\frac{2}{3}}+(35\Gamma^2\rho^6+(70\Gamma^2+80)\rho^5+(-35\Gamma^2+400)\rho^4+(-140\Gamma^2+800)\rho^3\\
&\quad+(-35\Gamma^2+800)\rho^2+(70\Gamma^2+400)\rho+35\Gamma^2+80)\eta(\Gamma,\rho)^{\frac{1}{3}}+50\Gamma^4\rho^{12}\\
&\quad+(200\Gamma^4+16\Gamma^2)\rho^{11}+(100\Gamma^4+112\Gamma^2+32)\rho^{10}+(-600\Gamma^4+304\Gamma^2+320)\rho^9\\
&\quad+(-850\Gamma^4+336\Gamma^2+1440)\rho^8+(400\Gamma^4-96\Gamma^2+3840)\rho^7\\
&\quad+(1400\Gamma^4-672\Gamma^2+6720)\rho^6+(400\Gamma^4-672\Gamma^2+8064)\rho^5\\
&\quad+(-850\Gamma^4-96\Gamma^2+6720)\rho^4+(-600\Gamma^4+336\Gamma^2+3840)\rho^3\\
&\quad+(100\Gamma^4+304\Gamma^2+1440)\rho^2+(200\Gamma^4+112\Gamma^2+320)\rho+50\Gamma^4+16\Gamma^2+32,\\
\theta(\Gamma,\rho) &= 625\eta(\Gamma,\rho)^{\frac{2}{3}}+(-875\Gamma^2\rho^6+(-1750\Gamma^2-2000)\rho^5+(875\Gamma^2-10000)\rho^4\\
&\quad+(3500\Gamma^2-20000)\rho^3+(875\Gamma^2-20000)\rho^2+(-1750\Gamma^2-10000)\rho\\
&\quad-875\Gamma^2-2000)\eta(\Gamma,\rho)^{\frac{1}{3}}+625\Gamma^4\rho^{12}+(2500\Gamma^4+200\Gamma^2)\rho^{11}\\
&\quad+(1250\Gamma^4+1400\Gamma^2+400)\rho^{10}+(-7500\Gamma^4+3800\Gamma^2+4000)\rho^9\\
&\quad+(-10625\Gamma^4+4200\Gamma^2+18000)\rho^8+(5000\Gamma^4-1200\Gamma^2+48000)\rho^7\\
&\quad+(17500\Gamma^4-8400\Gamma^2+84000)\rho^6+(5000\Gamma^4-8400\Gamma^2+100800)\rho^5\\
&\quad+(-10625\Gamma^4-1200\Gamma^2+84000)\rho^4+(-7500\Gamma^4+4200\Gamma^2+48000)\rho^3\\
&\quad+(1250\Gamma^4+3800\Gamma^2+18000)\rho^2+(2500\Gamma^4+1400\Gamma^2+4000)\rho\\
&\quad+625\Gamma^4+200\Gamma^2+400,\\
\phi(\Gamma,\rho) &= -1875(\rho-1)(\rho+1)^3\eta(\Gamma,\rho)^{\frac{1}{3}}\Gamma\sqrt{15}(3\Gamma^2\rho^2-6\Gamma^2\rho+3\Gamma^2+8\rho+8). \qquad (159)
\end{aligned}
$$

Note that $\Lambda(\Gamma,\rho)$, and therefore also α and α^* in (156), depend only on Γ and on ρ. In Fig. 16a we plot $\Lambda(\Gamma,\rho)$ as the unique positive root (colored in gray) of the quartic equation (158). Note how the three-dimensional plot is exactly a generalization for $\rho \neq 0$ of the two-dimensional plot in Fig. 1a. In particular, the $\Lambda(\Gamma,\rho)$ (159) we plot in Fig. 16b is just a generalization for $\rho \neq 0$ of the $\Lambda(\Gamma)$ (31) depicted in gray in Fig. 1a. This is not surprising, since the solution for the baseline model with uncorrelated assets is just a specification of

the solution for the extended model with correlated assets. Note how the quartic equation (158) collapses exactly to the quartic equation for the baseline model (30) when ρ is set to 0.

D.3 Sensitivity Analysis

Our sensitivity analysis mirrors that of Section 5 but also includes the dependency on the correlation coefficient ρ. First we compute first order and second order derivatives of $\Lambda(\Gamma, \rho)$ and then we analyze the sensitivity of the equilibrium parameters (156) with respect to changes in ρ. For completeness, we also report the partial derivatives with respect to the other underlying distribution parameters σ_δ, σ_ν, and γ but we will not discuss them since the signs are the same as for the baseline model without asset correlation.

We can synthetically represent the derivatives in terms of the $\Lambda(\Gamma, \rho)$ function itself. The first and second order derivatives of $\Lambda(\Gamma, \rho)$ with respect to Γ are, respectively,

$$\frac{\partial \Lambda(\Gamma,\rho)}{\partial \Gamma} = \frac{n^{(\Gamma)}_{\partial\Lambda}(\Lambda(\Gamma,\rho),\Gamma,\rho)}{d^{(\Gamma)}_{\partial\Lambda}(\Lambda(\Gamma,\rho),\Gamma,\rho)},$$

where:

$$n^{(\Gamma)}_{\partial\Lambda}(\Lambda(\Gamma,\rho),\Gamma,\rho) = -\varepsilon_1(\rho)\Lambda(\Gamma,\rho)^3 - \varepsilon_2(\Gamma,\rho)\Lambda(\Gamma,\rho)^2 - \varepsilon_3(\rho)\Lambda(\Gamma,\rho) + \Gamma\rho^2 - 2\Gamma\rho + \Gamma,$$

$$d^{(\Gamma)}_{\partial\Lambda}(\Lambda(\Gamma,\rho),\Gamma,\rho) = \varepsilon_4(\rho)\Lambda(\Gamma,\rho)^3 + \varepsilon_5(\Gamma,\rho)\Lambda(\Gamma,\rho)^2 + \varepsilon_6(\Gamma,\rho)\Lambda(\Gamma,\rho) + 2\Gamma\rho^2 - 2\Gamma,$$

with:

$$\varepsilon_1(\rho) = -6\rho^3 - 6\rho^2 + 6\rho + 6,$$

$$\varepsilon_2(\Gamma,\rho) = 5\Gamma\rho^3 - 5\Gamma\rho^2 - 5\Gamma\rho + 5\Gamma,$$

$$\varepsilon_3(\rho) = 2\rho^2 - 2,$$

$$\varepsilon_4(\rho) = 16\rho^3 + 48\rho^2 + 48\rho + 16,$$

$$\varepsilon_5(\Gamma,\rho) = -18\Gamma\rho^3 - 18\Gamma\rho^2 + 18\Gamma\rho + 18\Gamma,$$

$$\varepsilon_6(\Gamma,\rho) = 5\Gamma^2\rho^3 - 5\Gamma^2\rho^2 - 5\rho\Gamma^2 + 5\Gamma^2 - 4\rho^2 - 8\rho - 4. \tag{160}$$

and

$$\frac{\partial^2\Lambda(\Gamma,\rho)}{\partial\Gamma^2}=\frac{n^{(\Gamma)}_{\partial^2\Lambda}(\Lambda(\Gamma,\rho),\Gamma,\rho)}{d^{(\Gamma)}_{\partial^2\Lambda}(\Lambda(\Gamma,\rho),\Gamma,\rho)},$$

where:

$$\begin{aligned}
n^{(\Gamma)}_{\partial^2\Lambda}(\Lambda(\Gamma,\rho),\Gamma,\rho) &= \varepsilon_1(\rho)\Lambda(\Gamma,\rho)^8+\varepsilon_2(\Gamma,\rho)\Lambda(\Gamma,\rho)^7+\varepsilon_3(\Gamma,\rho)\Lambda(\Gamma,\rho)^6+\varepsilon_4(\Gamma,\rho)\Lambda(\Gamma,\rho)^5\\
&\quad+\varepsilon_5(\Gamma,\rho)\Lambda(\Gamma,\rho)^4+\varepsilon_6(\Gamma,\rho)\Lambda(\Gamma,\rho)^3+\varepsilon_7(\Gamma,\rho)\Lambda(\Gamma,\rho)^2\\
&\quad+\varepsilon_8(\Gamma,\rho)\Lambda(\Gamma,\rho)+\varepsilon_9(\Gamma,\rho),\\
d^{(\Gamma)}_{\partial^2\Lambda}(\Lambda(\Gamma,\rho),\Gamma,\rho) &= (\varepsilon_{10}(\rho)\Lambda(\Gamma,\rho)^3+\varepsilon_{11}(\Gamma,\rho)\Lambda(\Gamma,\rho)^2+\varepsilon_{12}(\Gamma,\rho)\Lambda(\Gamma,\rho)+\varepsilon_{13}(\Gamma,\rho))\\
&\quad(\varepsilon_{14}(\rho)\Lambda(\Gamma,\rho)^3+\varepsilon_{15}(\Gamma,\rho)\Lambda(\Gamma,\rho)^2+\varepsilon_{16}(\Gamma,\rho)\Lambda(\Gamma,\rho)+\varepsilon_{17}(\Gamma,\rho))^2,
\end{aligned}$$

with:

$$\begin{aligned}
\varepsilon_1(\rho) &= 448\rho^8+1792\rho^7+1792\rho^6-1792\rho^5-4480\rho^4-1792\rho^3+1792\rho^2+1792\rho+448,\\
\varepsilon_2(\Gamma,\rho) &= -1632\Gamma\rho^8-3264\Gamma\rho^7+3264\Gamma\rho^6+9792\Gamma\rho^5-9792\Gamma\rho^3-3264\Gamma\rho^2+3264\Gamma\rho\\
&\quad+1632\Gamma,\\
\varepsilon_3(\Gamma,\rho) &= 2120\Gamma^2\rho^8-8480\Gamma^2\rho^6-208\rho^7+12720\Gamma^2\rho^4-624\rho^6-208\rho^5-8480\Gamma^2\rho^2+1040\rho^4\\
&\quad+1040\rho^3+2120\Gamma^2-208\rho^2-624\rho-208,\\
\varepsilon_4(\Gamma,\rho) &= -1200\Gamma^3\rho^8+2400\Gamma^3\rho^7+2400\Gamma^3\rho^6-7200\Gamma^3\rho^5+640\Gamma\rho^7+640\Gamma\rho^6+7200\Gamma^3\rho^3\\
&\quad-1920\Gamma\rho^5-2400\Gamma^3\rho^2-1920\Gamma\rho^4-2400\Gamma^3\rho+1920\Gamma\rho^3+1200\Gamma^3+1920\Gamma\rho^2\\
&\quad-640\Gamma\rho-640\Gamma,\\
\varepsilon_5(\Gamma,\rho) &= 250\Gamma^4\rho^8-1000\Gamma^4\rho^7+1000\Gamma^4\rho^6+1000\Gamma^4\rho^5-632\Gamma^2\rho^7-2500\Gamma^4\rho^4+632\Gamma^2\rho^6\\
&\quad+1000\Gamma^4\rho^3+1896\Gamma^2\rho^5+1000\Gamma^4\rho^2-1896\Gamma^2\rho^4+16\rho^6-1000\Gamma^4\rho-1896\Gamma^2\rho^3\\
&\quad+32\rho^5+250\Gamma^4+1896\Gamma^2\rho^2-16\rho^4+632\Gamma^2\rho-64\rho^3-632\Gamma^2-16\rho^2+32\rho+16,\\
\varepsilon_6(\Gamma,\rho) &= 240\Gamma^3\rho^7-720\Gamma^3\rho^6+240\Gamma^3\rho^5+1200\Gamma^3\rho^4-32\Gamma\rho^6-1200\Gamma^3\rho^3-240\Gamma^3\rho^2+96\Gamma\rho^4\\
&\quad+720\Gamma^3\rho-240\Gamma^3-96\Gamma\rho^2+32\Gamma,\\
\varepsilon_7(\Gamma,\rho) &= -25\Gamma^4\rho^7+125\Gamma^4\rho^6-225\Gamma^4\rho^5+125\Gamma^4\rho^4+125\Gamma^4\rho^3-225\Gamma^4\rho^2+125\Gamma^4\rho-25\Gamma^4,\\
\varepsilon_8(\Gamma,\rho) &= 16\Gamma^3\rho^6-64\Gamma^3\rho^5+80\Gamma^3\rho^4-80\Gamma^3\rho^2+64\Gamma^3\rho-16\Gamma^3,\\
\varepsilon_9(\Gamma,\rho) &= -5\Gamma^4\rho^6+30\Gamma^4\rho^5-75\Gamma^4\rho^4+100\Gamma^4\rho^3-75\Gamma^4\rho^2+30\Gamma^4\rho-5\Gamma^4,
\end{aligned}$$

$$\varepsilon_{10}(\rho) = 16\rho^2 + 32\rho + 16,$$

$$\varepsilon_{11}(\Gamma, \rho) = -18\Gamma\rho^2 + 18\Gamma,$$

$$\varepsilon_{12}(\Gamma, \rho) = 5\Gamma^2\rho^2 - 10\Gamma^2\rho + 5\Gamma^2 - 4\rho - 4,$$

$$\varepsilon_{13}(\Gamma, \rho) = 2\Gamma\rho - 2\Gamma,$$

$$\varepsilon_{14}(\rho) = 16\rho^3 + 48\rho^2 + 48\rho + 16,$$

$$\varepsilon_{15}(\Gamma, \rho) = -18\Gamma\rho^3 - 18\Gamma\rho^2 + 18\Gamma\rho + 18\Gamma,$$

$$\varepsilon_{16}(\Gamma, \rho) = 5\Gamma^2\rho^3 - 5\Gamma^2\rho^2 - 5\Gamma^2\rho + 5\Gamma^2 - 4\rho^2 - 8\rho - 4,$$

$$\varepsilon_{17}(\Gamma, \rho) = 2\Gamma\rho^2 - 2\Gamma. \tag{161}$$

The first and second order derivatives of $\Lambda(\Gamma, \rho)$ with respect to ρ are, respectively,

$$\frac{\partial\Lambda(\Gamma, \rho)}{\partial\rho} = \frac{n^{(\rho)}_{\partial\Lambda}(\Lambda(\Gamma, \rho), \Gamma, \rho)}{d^{(\rho)}_{\partial\Lambda}(\Lambda(\Gamma, \rho), \Gamma, \rho)},$$

where:

$$n^{(\rho)}_{\partial\Lambda}(\Lambda(\Gamma, \rho), \Gamma, \rho) = -o_1(\rho)\Lambda(\Gamma, \rho)^4 - o_2(\Gamma, \rho)\Lambda(\Gamma, \rho)^3 - o_3(\Gamma, \rho)\Lambda(\Gamma, \rho)^2 - o_4(\Gamma, \rho)\Lambda(\Gamma, \rho) + 2\rho\Gamma^2 - 2\Gamma^2,$$

$$d^{(\rho)}_{\partial\Lambda}(\Lambda(\Gamma, \rho), \Gamma, \rho) = 2(o_5(\rho)\Lambda(\Gamma, \rho)^3 + o_6(\Gamma, \rho)\Lambda(\Gamma, \rho)^2 + o_7(\Gamma, \rho)\Lambda(\Gamma, \rho) + 2\Gamma\rho^2 - 2\Gamma),$$

with:

$$o_1(\rho) = 24\rho^2 + 48\rho + 24,$$

$$o_2(\Gamma, \rho) = -36\Gamma\rho^2 - 24\Gamma\rho + 12\Gamma,$$

$$o_3(\Gamma, \rho) = 15\Gamma^2\rho^2 - 10\Gamma^2\rho - 5\Gamma^2 - 8\rho - 8,$$

$$o_4(\Gamma, \rho) = 8\Gamma\rho,$$

$$o_5(\rho) = 16\rho^3 + 48\rho^2 + 48\rho + 16,$$

$$o_6(\Gamma, \rho) = -18\Gamma\rho^3 - 18\Gamma\rho^2 + 18\Gamma\rho + 18\Gamma,$$

$$o_7(\Gamma, \rho) = 5\Gamma^2\rho^3 - 5\Gamma^2\rho^2 - 5\Gamma^2\rho + 5\Gamma^2 - 4\rho^2 - 8\rho - 4. \tag{162}$$

and

$$\frac{\partial^2\Lambda(\Gamma,\rho)}{\partial\rho^2} = \frac{n^{(\rho)}_{\partial^2\Lambda}(\Lambda(\Gamma,\rho),\Gamma,\rho)}{d^{(\rho)}_{\partial^2\Lambda}(\Lambda(\Gamma,\rho),\Gamma,\rho)},$$

where:

$$\begin{aligned}
n^{(\rho)}_{\partial^2\Lambda}(\Lambda(\Gamma,\rho),\Gamma,\rho) = {} & o_1(\rho)\Lambda(\Gamma,\rho)^{10} + o_2(\Gamma,\rho)\Lambda(\Gamma,\rho)^9 + o_3(\Gamma,\rho)\Lambda(\Gamma,\rho)^8 + o_4(\Gamma,\rho)\Lambda(\Gamma,\rho)^7 \\
& + o_5(\Gamma,\rho)\Lambda(\Gamma,\rho)^6 + o_6(\Gamma,\rho)\Lambda(\Gamma,\rho)^5 + o_7(\Gamma,\rho)\Lambda(\Gamma,\rho)^4 + o_8(\Gamma,\rho) \\
& \Lambda(\Gamma,\rho)^3 + o_9(\Gamma,\rho)\Lambda(\Gamma,\rho)^2 + o_{10}(\Gamma,\rho)\Lambda(\Gamma,\rho) + o_{11}(\Gamma,\rho), \\
d^{(\rho)}_{\partial^2\Lambda}(\Lambda(\Gamma,\rho),\Gamma,\rho) = {} & 4(o_{12}(\rho)\Lambda(\Gamma,\rho)^3 + o_{13}(\Gamma,\rho)\Lambda(\Gamma,\rho)^2 + o_{14}(\Gamma,\rho)\Lambda(\Gamma,\rho) + o_{15}(\Gamma,\rho)) \\
& (o_{16}(\rho)\Lambda(\Gamma,\rho)^3 + o_{17}(\Gamma,\rho)\Lambda(\Gamma,\rho)^2 + o_{18}(\Gamma,\rho)\Lambda(\Gamma,\rho) + o_{19}(\Gamma,\rho))^2,
\end{aligned}$$

with:

$$\begin{aligned}
o_1(\rho) = {} & 21504\rho^6 + 129024\rho^5 + 322560\rho^4 + 430080\rho^3 + 322560\rho^2 + 129024\rho + 21504, \\
o_2(\Gamma,\rho) = {} & -80640\Gamma\rho^6 - 347136\Gamma\rho^5 - 526080\Gamma\rho^4 - 245760\Gamma\rho^3 + 157440\Gamma\rho^2 + 199680\Gamma\rho \\
& + 56064\Gamma, \\
o_3(\Gamma,\rho) = {} & 127680\Gamma^2\rho^6 + 336512\Gamma^2\rho^5 + 122944\Gamma^2\rho^4 - 319744\Gamma^2\rho^3 - 18176\rho^5 - 246976\Gamma^2\rho^2 \\
& - 90880\rho^4 + 40576\Gamma^2\rho - 181760\rho^3 + 53696\Gamma^2 - 181760\rho^2 - 90880\rho - 18176, \\
o_4(\Gamma,\rho) = {} & -110016\Gamma^3\rho^6 - 112128\Gamma^3\rho^5 + 191808\Gamma^3\rho^4 + 172032\Gamma^3\rho^3 + 57344\Gamma\rho^5 \\
& - 105792\Gamma^3\rho^2 + 191488\Gamma\rho^4 - 59904\Gamma^3\rho + 192512\Gamma\rho^3 + 24000\Gamma^3 \\
& + 2048\Gamma\rho^2 - 94208\Gamma\rho - 37888\Gamma, \\
o_5(\Gamma,\rho) = {} & 54960\Gamma^4\rho^6 - 28480\Gamma^4\rho^5 - 102480\Gamma^4\rho^4 + 56960\Gamma^4\rho^3 - 74176\Gamma^2\rho^5 + 40080\Gamma^4\rho^2 \\
& - 126144\Gamma^2\rho^4 - 28480\Gamma^4\rho + 41088\Gamma^2\rho^3 + 7440\Gamma^4 + 138368\Gamma^2\rho^2 + 5120\rho^4 \\
& + 19776\Gamma^2\rho + 20480\rho^3 - 25536\Gamma^2 + 30720\rho^2 + 20480\rho + 5120, \\
o_6(\Gamma,\rho) = {} & -15300\Gamma^5\rho^6 + 30000\Gamma^5\rho^5 - 2100\Gamma^5\rho^4 - 24000\Gamma^5\rho^3 + 50048\Gamma^3\rho^5 + 14100\Gamma^5\rho^2 \\
& + 5632\Gamma^3\rho^4 - 6000\Gamma^5\rho - 89856\Gamma^3\rho^3 + 3300\Gamma^5 - 1024\Gamma^3\rho^2 - 13056\Gamma\rho^4 \\
& + 39808\Gamma^3\rho - 31232\Gamma\rho^3 - 4608\Gamma^3 - 15360\Gamma\rho^2 + 10752\Gamma\rho + 7936\Gamma, \\
o_7(\Gamma,\rho) = {} & 1875\Gamma^6\rho^6 - 6250\Gamma^6\rho^5 + 7125\Gamma^6\rho^4 - 3500\Gamma^6\rho^3 - 18316\Gamma^4\rho^5 + 2125\Gamma^6\rho^2 \\
& + 25540\Gamma^4\rho^4 - 2250\Gamma^6\rho + 12072\Gamma^4\rho^3 + 875\Gamma^6 - 26520\Gamma^4\rho^2 + 12992\Gamma^2\rho^4
\end{aligned}$$

$$
\begin{aligned}
&+ 6244\Gamma^4\rho + 10240\Gamma^2\rho^3 + 980\Gamma^4 - 15488\Gamma^2\rho^2 - 9728\Gamma^2\rho \\
&- 512\rho^3 + 3008\Gamma^2 - 1536\rho^2 - 1536\rho - 512, \\
o_8(\Gamma,\rho) = {} & 3320\Gamma^5\rho^5 - 9240\Gamma^5\rho^4 + 7600\Gamma^5\rho^3 - 560\Gamma^5\rho^2 - 6144\Gamma^3\rho^4 - 1320\Gamma^5\rho + 4992\Gamma^3\rho^3 \\
&+ 200\Gamma^5 + 6784\Gamma^3\rho^2 - 4992\Gamma^3\rho + 1024\Gamma\rho^3 - 640\Gamma^3 + 1536\Gamma\rho^2 - 512\Gamma, \\
o_9(\Gamma,\rho) = {} & -200\Gamma^6\rho^5 + 800\Gamma^6\rho^4 - 1200\Gamma^6\rho^3 + 800\Gamma^6\rho^2 + 1248\Gamma^4\rho^4 - 200\Gamma^6\rho - 3136\Gamma^4\rho^3 \\
&+ 1920\Gamma^4\rho^2 + 576\Gamma^4\rho - 768\Gamma^2\rho^3 - 608\Gamma^4 + 768\Gamma^2\rho, \\
o_{10}(\Gamma,\rho) = {} & -16\Gamma^5\rho^4 + 192\Gamma^5\rho^3 - 480\Gamma^5\rho^2 + 448\Gamma^5\rho + 256\Gamma^3\rho^3 - 144\Gamma^5 - 384\Gamma^3\rho^2 + 128\Gamma^3, \\
o_{11}(\Gamma,\rho) = {} & -20\Gamma^6\rho^4 + 80\Gamma^6\rho^3 - 120\Gamma^6\rho^2 + 80\Gamma^6\rho - 32\Gamma^4\rho^3 - 20\Gamma^6 + 96\Gamma^4\rho^2 - 96\Gamma^4\rho \\
&+ 32\Gamma^4, \\
o_{12}(\rho) = {} & 16\rho^2 + 32\rho + 16, \\
o_{13}(\Gamma,\rho) = {} & -18\Gamma\rho^2 + 18\Gamma, \\
o_{14}(\Gamma,\rho) = {} & 5\Gamma^2\rho^2 - 10\Gamma^2\rho + 5\Gamma^2 - 4\rho - 4, \\
o_{15}(\Gamma,\rho) = {} & 2\Gamma\rho - 2\Gamma, \\
o_{16}(\rho) = {} & 16\rho^3 + 48\rho^2 + 48\rho + 16, \\
o_{17}(\Gamma,\rho) = {} & -18\Gamma\rho^3 - 18\Gamma\rho^2 + 18\Gamma\rho + 18\Gamma, \\
o_{18}(\Gamma,\rho) = {} & 5\Gamma^2\rho^3 - 5\Gamma^2\rho^2 - 5\Gamma^2\rho + 5\Gamma^2 - 4\rho^2 - 8\rho - 4, \\
o_{19}(\Gamma,\rho) = {} & 2\Gamma\rho^2 - 2\Gamma.
\end{aligned}
\tag{163}
$$

The four derivatives are extremely convoluted and the only way to assess their signs is through numeric differentiation. However, these derivatives are expressed in synthetic forms, that is, they are written as functions of the "differentiand" $\Lambda(\Gamma,\rho)$ which, in turn, is not necessarily the meaningful root (159) but a generic unspecified root of the quartic equation (158). As such, there could be issues with numeric differentiation[16]. We overcome these

[16]First of all, a generic unspecified RootOf (for a polynomial) does not necessarily follow a single continuous curve, and even the indexed RootOf's do not do that. The indexed RootOf's are sorted, and the generic RootOf is taken from the end of those values, so it does not follow any single one of them throughout. Moreover, the numeric differentiation across the $\Gamma = 0$ and $\rho = 1$ boundaries will produce bad results since the generic RootOf formula switches curve at the boundary on account of the appearance of multiple roots (which affects the principal root via sorting). Note that this "switching" affects not just parametrized indexed RootOf's. It also occurs with roots in explicit parametrized radical form. To sidestep these challenges, perhaps we could have used slightly narrower regions but this is not very robust, as it is not known a priori

issues by noting that the greatest real root in the multiple root cases (of $\Gamma < 0$, Γ near 0, or $\rho > 1$, ρ near 1) happens to "match" the surface induced by the single real root within $\Gamma > 0, -1 < \rho < 1$. Then, we can compute all four roots (via indexed RootOf's), select the real ones, take the max of those, and get a smooth function across the boundaries $\Gamma = 0$, $\rho = 1$ so that numeric differentiation behaves there. At $\rho = -1$ we just went with a slightly narrower range in order to avoid artifacts while ensuring that no feature is lost.

In Fig. 17a and 17b we show that the signs of the derivatives with respect to Γ, (160) and (161), are consistent with $\Lambda(\Gamma, \rho)$ being a positive and monotonically decreasing function of Γ (as evident from Fig. 16b),

$$\frac{\partial \Lambda(\Gamma, \rho)}{\partial \Gamma} < 0, \qquad \frac{\partial^2 \Lambda(\Gamma, \rho)}{\partial \Gamma^2} > 0. \tag{164}$$

Note that the signs (164) are the same as in the baseline model (34).

In Fig. 18a and 18c we show the sign of the derivatives with respect to ρ, (162) and (163). The second derivative is positive but the first derivative changes sign[17],

$$\frac{\partial \Lambda(\Gamma, \rho)}{\partial \rho} \begin{cases} > 0 & \text{if } \rho > p_{LJ}(\Gamma) \\ \leq 0 & \text{otherwise} \end{cases}, \qquad \frac{\partial^2 \Lambda(\Gamma, \rho)}{\partial \rho^2} > 0, \tag{165}$$

where $p_{LJ}(\Gamma)$ is named after its striking similarity to the Lennard-Jones potential in molecular physics. The curve $p_{LJ}(\Gamma)$ is the level curve $h(\Gamma, \rho) = 0$ with $h(\Gamma, \rho) = \partial \Lambda(\Gamma, \rho)/\partial \rho$. To get some intuition about its shape and related analytical form, we extract some points $(x, y) \mapsto (\Gamma, \rho)$ (removing duplicate abscissas) and build an ad hoc approximation of the curve passing through these points. The resulting curve is an approximation of the function $z : \Gamma \to z(\Gamma)$ such that $h(\Gamma, z(\Gamma)) = 0$,

$$p_{LJ}(\Gamma) \approx 0.74 + 1.18 \left(\left(\frac{1.18}{\Gamma} \right)^{1.81} - \left(\frac{1.15}{\Gamma} \right)^{1.06} \right). \tag{166}$$

how much narrower and whether features will surely not get missed.

[17] Note that the change of sign of the first derivative was anticipated in Fig. 16b, as it is most evident for $\Gamma = 10$ and ρ around 0.8.

We plot $p_{LJ}(\Gamma)$'s fitting points and the approximation form in Fig. 18b. We fit the points by solving a nonlinear least squares problem for a model of Lennard-Jones functional form to find the optimal values of the six parameters. As such, the approximation accuracy is measured as the sum of squares of the residual and amounts to 0.006. For a more accurate result there exist very versatile interpolation models that could be used instead of the more classical nonlinear least squares but we are more interested in the interpretability of the closed-form expression rather than in the accuracy of the approximation[18].

As we know the signs of $\Lambda(\Gamma, \rho)$ and its derivatives, we can now infer the signs of the partial derivatives of the equilibrium parameters (156) for markets 1 and 2 with respect to ρ, γ, σ_ν, and σ_δ. We apply the chain rule and derive the partial derivatives but we keep them in implicit form, to facilitate their interpretation. According to our plots, the signs of all these partial derivatives are consistent with our economic intuitions about the strategic behaviour of a risk-averse speculator trading in a triangle where $\rho \neq 0$. We report our economic intuitions in Section 6, where we only discuss those that are unique to this model extension and not already encountered in the baseline model. In the following we only report the analytical expressions and their signs.

When asset 1 and asset 2 are correlated, the strategic behaviour of the speculator depends as follows on the increasing ρ,

$$\frac{\partial \alpha}{\partial \rho} = -\frac{\partial \alpha^*}{\partial \rho} = \frac{\Gamma\left(\frac{\partial \Lambda(\Gamma,\rho)}{\partial \rho}(\rho-1)(\rho+1) - 2\Lambda(\Gamma,\rho)\right)}{2\left(-2(\rho+1)\Lambda(\Gamma,\rho) + \Gamma(\rho-1)\right)^2} < 0, \tag{167}$$

[18]For instance, we used an elementary Kriging (Gaussian) emulator to find the "approximation function" that is an exact interpolator which passes through all points. Its approximation error out of these points is less than $1e^{-7}$. Note that the interpolator depends on a single parameter (ψ) whose meaning is a correlation length, that is how far from a point x_i the information it contains (y_i) "propagates" to affect the value y_k at any other point x_k. The fact that the smoothness of the Kriging emulator is almost the only parameter to fit makes Kriging one of the most powerful tool to build high dimensional metamodels (in dimension d ψ contains only d parameters, while the fit a polynomial of total degree q in dimension d depends on $(q+d)!/q!/d!$ parameters). However, note that a Kriging emulator is always used as a black-box function where one provides the value of the inputs (Γ) and it returns the corresponding value of the output (ρ): despite the fact the interpolator has a closed-form expression this latter can be that complex that it is never displayed. For more details on Kriging and Matér correlations the interested reader can refer to Porcu et al. (2024) and Tuo and Wang (2020).

$$\frac{\partial \lambda}{\partial \rho} = \frac{\sigma_\nu \left(\frac{\partial \Lambda(\Gamma,\rho)}{\partial \rho}(\rho+1) + \Lambda(\Gamma,\rho) \right)}{\sigma_\delta} > 0, \tag{168}$$

$$\frac{\partial \beta}{\partial \rho} = -\frac{\sigma_\delta \left(\frac{\partial \Lambda(\Gamma,\rho)}{\partial \rho}(\rho+1) + \Lambda(\Gamma,\rho) \right)}{4\sigma_\nu \Lambda(\Gamma,\rho)^2(\rho+1)^2} < 0. \tag{169}$$

Note that because of the change of sign of $\partial\Lambda(\Gamma,\rho)/\partial\rho$ (165) the sign of the partial derivatives (167), (168), and (169) is not obvious from the analytical expressions so we infer the signs by directly plotting their nondimensionalized forms in Fig. 19a, Fig. 19b, and Fig. 19c, respectively.

When asset 1 and asset 2 are correlated, the strategic behaviour of the speculator depends as follows on her increasing γ,

$$\frac{\partial \alpha}{\partial \gamma} = -\frac{\partial \alpha^*}{\partial \gamma} = \frac{\sigma_\delta \sigma_\nu (\rho-1)(\rho+1) \left(\frac{\partial \Lambda(\Gamma,\rho)}{\partial \Gamma}\Gamma - \Lambda(\Gamma,\rho) \right)}{2(-2(\rho+1)\Lambda(\Gamma,\rho) + \Gamma(\rho-1))^2} > 0, \tag{170}$$

$$\frac{\partial \lambda}{\partial \gamma} = \frac{\partial \Lambda(\Gamma,\rho)}{\partial \Gamma}\sigma_\nu^2(\rho+1) < 0, \tag{171}$$

$$\frac{\partial \beta}{\partial \gamma} = -\frac{\frac{\partial \Lambda(\Gamma,\rho)}{\partial \Gamma}\sigma_\delta^2}{4\Lambda(\Gamma,\rho)^2(\rho+1)} > 0. \tag{172}$$

When asset 1 and asset 2 are correlated, the strategic behaviour of the speculator changes as follows as σ_ν increases,

$$\frac{\partial \alpha}{\partial \sigma_\nu} = -\frac{\partial \alpha^*}{\partial \sigma_\nu} = \frac{\Gamma(\rho-1)(\rho+1) \left(\frac{\partial \Lambda(\Gamma,\rho)}{\partial \Gamma}\Gamma - \Lambda(\Gamma,\rho) \right)}{2\sigma_\nu(-2(\rho+1)\Lambda(\Gamma,\rho) + \Gamma(\rho-1))^2} > 0, \tag{173}$$

$$\frac{\partial \lambda}{\partial \sigma_\nu} = \frac{(\rho+1)\left(\frac{\partial \Lambda(\Gamma,\rho)}{\partial \Gamma}\Gamma + \Lambda(\Gamma,\rho)\right)}{\sigma_\delta} > 0, \tag{174}$$

$$\frac{\partial \beta}{\partial \sigma_\nu} = -\frac{\sigma_\delta\left(\frac{\partial \Lambda(\Gamma,\rho)}{\partial \Gamma}\Gamma + \Lambda(\Gamma,\rho)\right)}{4\sigma_\nu^2(\rho+1)\Lambda(\Gamma,\rho)^2} < 0. \tag{175}$$

Note that at a first sight the derivatives (174) and (175) have ambiguous signs so we infer the signs by directly plotting their nondimensionalized forms in Fig. 20a and Fig. 20b, respectively. We find that $\partial\lambda/\partial\sigma_\nu$ (174) is positive but $\partial\beta/\partial\sigma_\nu$ (175) is negative.

Finally, when asset 1 and asset 2 are correlated, the strategic behaviour of the speculator responds as follows as σ_δ increases,

$$\frac{\partial \alpha}{\partial \sigma_\delta} = -\frac{\partial \alpha^*}{\partial \sigma_\delta} = \frac{\Gamma(\rho-1)(\rho+1)\left(\frac{\partial \Lambda(\Gamma,\rho)}{\partial \Gamma}\Gamma - \Lambda(\Gamma,\rho)\right)}{2\sigma_\delta(-2(\rho+1)\Lambda(\Gamma,\rho)+\Gamma(\rho-1))^2} > 0, \tag{176}$$

$$\frac{\partial \lambda}{\partial \sigma_\delta} = \frac{\sigma_\nu(\rho+1)\left(\frac{\partial \Lambda(\Gamma,\rho)}{\partial \Gamma}\Gamma - \Lambda(\Gamma,\rho)\right)}{\sigma_\delta^2} < 0, \tag{177}$$

$$\frac{\partial \beta}{\partial \sigma_\delta} = \frac{-\frac{\partial \Lambda(\Gamma,\rho)}{\partial \Gamma}\Gamma + \Lambda(\Gamma,\rho))}{4\sigma_\nu(\rho+1)\Lambda(\Gamma,\rho)^2} > 0. \tag{178}$$

As mentioned, we discuss in Section 6 our economic intuitions for all these sensitivities.

E Analytical Model Extension: Asymmetric Assets

We provide analytical solutions for another model with a more general information structure of the assets' payoffs. Specifically, we generalize the covariance matrix (2) to allow for

asymmetric risk in the payoff of assets 1 and 2 ($\sigma_{\nu 1} \neq \sigma_{\nu 2}$),

$$\begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_1 + \nu_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \overline{\nu}_1 \\ \overline{\nu}_2 \\ \overline{\nu}_1 + \overline{\nu}_2 \end{bmatrix}, \begin{bmatrix} \sigma_{\nu 1}^2 & 0 & \sigma_{\nu 1}^2 \\ 0 & \sigma_{\nu 2}^2 & \sigma_{\nu 2}^2 \\ \sigma_{\nu 1}^2 & \sigma_{\nu 2}^2 & \sigma_{\nu 1}^2 + \sigma_{\nu 2}^2 \end{bmatrix} \right). \tag{179}$$

The case of asymmetric assets is interesting because the wedge in assets' risk ($\sigma_{\nu 1} \neq \sigma_{\nu 2}$) drives a wedge in price impacts ($\lambda_1 \neq \lambda_2$). Accordingly, the equilibrium conjectures of Theorem 1 rewrite as in (92), i.e.,

$$\begin{aligned} \chi_i &= \beta_i \left(\zeta - \overline{\nu}_i - \overline{\nu}_j\right) + \alpha_i \delta_i + \alpha_j^* \delta_j, & \chi_3 &= \beta_3 \left(\zeta - \overline{\nu}_1 - \overline{\nu}_2\right) + \alpha_3 \delta_3, \\ q_i &= \overline{\nu}_i + \lambda_i \left(\chi_i + \delta_i\right), & q_3 &= \overline{\nu}_1 + \overline{\nu}_2 + \lambda_3 \left(\chi_3 + \delta_3\right), \\ & \forall \{i,j\} \in \{\{1,2\},\{2,1\}\}. \end{aligned} \tag{180}$$

E.1 Equilibrium Derivation

We re-write the mean-variance optimization problem (4) as,

$$\begin{aligned} \max_{\chi_1,\chi_2,\chi_3} \Big\{ & \chi_1 \left(-\lambda \chi_1 - \lambda \delta_1 - \overline{\nu}_1\right) + \chi_2 \left(-\lambda \chi_2 - \lambda \delta_2 - \overline{\nu}_2\right) + \chi_3 \left(-\lambda_3 \chi_3 - \lambda_3 \delta_3 - \overline{\nu}_1 - \overline{\nu}_2\right) \\ & + \left(\chi_1 + \chi_3\right) \left(\overline{\nu}_1 + \frac{\sigma_{\nu 1}^2}{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2} \left(\zeta - \overline{\nu}_1 - \overline{\nu}_2\right)\right) + \left(\chi_2 + \chi_3\right) \left(\overline{\nu}_2 + \frac{\sigma_{\nu 2}^2}{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2} \left(\zeta - \overline{\nu}_1 - \overline{\nu}_2\right)\right) \\ & - \frac{\gamma}{2} \left(\chi_1 + \chi_3\right)^2 \left(\sigma_{\nu 1}^2 - \frac{\sigma_{\nu 1}^4}{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2}\right) - \frac{\gamma}{2} \left(\chi_2 + \chi_3\right)^2 \left(\sigma_{\nu 2}^2 - \frac{\sigma_{\nu 2}^4}{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2}\right) \\ & + \gamma \left(\chi_1 + \chi_3\right) \left(\chi_2 + \chi_3\right) \left(\frac{\sigma_{\nu 1}^2 \sigma_{\nu 2}^2}{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2}\right) \Big\}. \end{aligned} \tag{181}$$

With due simplifications, the first order conditions (8) become,

$$\begin{aligned} & \lambda_1 \left(-2\chi_1 - \delta_1\right) + \frac{\sigma_{\nu 1}^2}{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2} \left(\zeta - \overline{\nu}_1 - \overline{\nu}_2\right) + \frac{\sigma_{\nu 1}^2 \sigma_{\nu 2}^2}{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2} \gamma \left(\chi_2 - \chi_1\right) = 0, \\ & \lambda_2 \left(-2\chi_2 - \delta_2\right) + \frac{\sigma_{\nu 2}^2}{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2} \left(\zeta - \overline{\nu}_1 - \overline{\nu}_2\right) + \frac{\sigma_{\nu 1}^2 \sigma_{\nu 2}^2}{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2} \gamma \left(\chi_1 - \chi_2\right) = 0, \\ & \lambda_3 \left(-2\chi_3 - \delta_3\right) + \left(\zeta - \overline{\nu}_1 - \overline{\nu}_2\right) = 0, \end{aligned} \tag{182}$$

and the trading and offsetting intensity parameters (9) for the new equilibrium strategies in (180) become eight,

$$\beta_1 = \frac{\sigma_{\nu 1}^2\left(\gamma\sigma_{\nu 2}^2 + 2\lambda_2\right)}{2\left((\lambda_1+\lambda_2)\gamma\sigma_{\nu 1}^2\sigma_{\nu 2}^2 + 2\lambda_1\lambda_2(\sigma_{\nu 1}^2+\sigma_{\nu 2}^2)\right)},$$

$$\alpha_1 = -\frac{\lambda_1((\gamma\sigma_{\nu 2}^2+2\lambda_2)\sigma_{\nu 1}^2 + 2\lambda_2\sigma_{\nu 2}^2)}{2\left((\lambda_1+\lambda_2)\gamma\sigma_{\nu 1}^2\sigma_{\nu 2}^2 + 2\lambda_1\lambda_2(\sigma_{\nu 1}^2+\sigma_{\nu 2}^2)\right)},\ \alpha_2^* = -\frac{\lambda_2\gamma\sigma_{\nu 1}^2\sigma_{\nu 2}^2}{2\left((\lambda_1+\lambda_2)\gamma\sigma_{\nu 1}^2\sigma_{\nu 2}^2 + 2\lambda_1\lambda_2(\sigma_{\nu 1}^2+\sigma_{\nu 2}^2)\right)},$$

$$\beta_2 = \frac{\sigma_{\nu 2}^2(\gamma\sigma_{\nu 1}^2 + 2\lambda_1)}{2\left((\lambda_1+\lambda_2)\gamma\sigma_{\nu 1}^2\sigma_{\nu 2}^2 + 2\lambda_1\lambda_2(\sigma_{\nu 1}^2+\sigma_{\nu 2}^2)\right)},$$

$$\alpha_2 = -\frac{\lambda_2((\gamma\sigma_{\nu 2}^2+2\lambda_1)\sigma_{\nu 1}^2 + 2\lambda_1\sigma_{\nu 2}^2)}{2\left((\lambda_1+\lambda_2)\gamma\sigma_{\nu 1}^2\sigma_{\nu 2}^2 + 2\lambda_1\lambda_2(\sigma_{\nu 1}^2+\sigma_{\nu 2}^2)\right)},\ \alpha_1^* = -\frac{\lambda_1\gamma\sigma_{\nu 1}^2\sigma_{\nu 2}^2}{2\left((\lambda_1+\lambda_2)\gamma\sigma_{\nu 1}^2\sigma_{\nu 2}^2 + 2\lambda_1\lambda_2(\sigma_{\nu 1}^2+\sigma_{\nu 2}^2)\right)},$$

$$\beta_3 = \frac{1}{2\lambda_3}, \qquad \alpha_3 = -\frac{1}{2}. \tag{183}$$

We solve the corresponding market maker problem for the model with asymmetric assets and the price impacts (11) split into three,

$$\begin{aligned}
\lambda_1 &= \frac{\beta_1\sigma_{\nu 1}^2}{\beta_1^2(\sigma_{\nu 1}^2+\sigma_{\nu 2}^2) + (\alpha_1+1)^2\sigma_\delta^2 + (\alpha_2^*)^2\sigma_\delta^2},\\
\lambda_2 &= \frac{\beta_2\sigma_{\nu 2}^2}{\beta_2^2(\sigma_{\nu 1}^2+\sigma_{\nu 2}^2) + (\alpha_2+1)^2\sigma_\delta^2 + (\alpha_1^*)^2\sigma_\delta^2},\\
\lambda_3 &= \frac{\beta_3(\sigma_{\nu 1}^2+\sigma_{\nu 2}^2)}{\beta_3^2(\sigma_{\nu 1}^2+\sigma_{\nu 2}^2) + (\alpha_3+1)^2\sigma_{\delta 3}^2}.
\end{aligned} \tag{184}$$

We plug the trading and offsetting intensity parameters (183) into the price impacts (184) and obtain, respectively, two nonlinear equations in both λ_1 and λ_2, and one in λ_3 only. Once we solve for λ_1, λ_2 and λ_3 explicitly, we can pin down β_1, β_2, β_3, α_1, α_2, α_3, α_2^*, and α_1^* (183). We have now pinned down our new equilibrium (180).

E.2 Equilibrium Characterization

Also for the extended model with asymmetric assets the equilibrium for market 3 is straightforward to characterize. Note that the third equation in (184) can be directly solved for λ_3.

Moreover, such equilibrium is independent of the speculator's risk aversion. Accordingly, irrespective of γ, for market 3 the equilibrium parameters (13) and quoting rule (14) become, respectively,

$$\lambda_3 = \frac{\sqrt{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2}}{\sigma_{\delta 3}}, \qquad \beta_3 = \frac{\sigma_{\delta 3}}{2\sqrt{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2}}, \qquad \alpha_3 = -\frac{1}{2}, \tag{185}$$

$$q_3 = \overline{\nu}_1 + \overline{\nu}_2 + \frac{(\zeta - \overline{\nu}_1 - \overline{\nu}_2)}{2} + \frac{\sqrt{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2}}{2\sigma_{\delta 3}}\delta_3. \tag{186}$$

For market 1 and 2, we do not discuss the limit cases of $\gamma = 0$ and $\gamma \to \infty$ for the model with asymmetric assets. Therefore, in the following, we allow the informed speculator to have a positive but finite risk aversion coefficient γ.

In contrast to the equilibrium for market 3, the equilibrium for market 1 and the equilibrium for market 2 are challenging to characterize. Once we plug (183) into the first two equations in (184), these become even more convoluted than for the two models previously described. The further complexity stems from the price impact wedge $\lambda_1 \neq \lambda_2$, which follows from the asymmetry $\sigma_{\nu 1} \neq \sigma_{\nu 2}$. We overcome the complexity by reworking the original nonlinear system in the primitive parameters $\gamma, \sigma_\delta, \sigma_{\nu 1}, \sigma_{\nu 2}$ into a scaled version only dependent on products of them. The logic of this non-dimensionalization procedure is the same as before but, rather than by hand, we derive the non-dimensionalization substitutions by implementing an automated procedure based on Hubert and Labahn (2013)[19]. The non-dimensionalization substitutions are as follows,

$$\Gamma_1 \coloneqq \sigma_\delta \sigma_{\nu 1} \gamma, \ \Gamma_2 \coloneqq \sigma_\delta \sigma_{\nu 2} \gamma, \ \Lambda_1 \coloneqq \frac{\sigma_\delta \lambda_1}{\sigma_{\nu 1}}, \ \Lambda_2 \coloneqq \frac{\sigma_\delta \lambda_2}{\sigma_{\nu 1}}, \ \Lambda_3 \coloneqq \frac{\sigma_{\delta 3} \lambda_3}{\sigma_{\nu 1}}. \tag{187}$$

We solve (187) for $\lambda_1, \lambda_2, \lambda_3, \gamma, \sigma_{\nu 1}$ to rewrite the latter as,

$$\gamma = \frac{\Gamma_2}{\sigma_\delta \sigma_{\nu 2}}, \ \lambda_1 = \frac{\Lambda_1 \Gamma_1 \sigma_{\nu 2}}{\Gamma_2 \sigma_\delta}, \ \lambda_2 = \frac{\Lambda_2 \Gamma_1 \sigma_{\nu 2}}{\Gamma_2 \sigma_\delta}, \ \lambda_3 = \frac{\Lambda_3 \Gamma_1 \sigma_{\nu 2}}{\Gamma_2 \sigma_{\delta 3}}, \ \sigma_{\nu 1} = \frac{\Gamma_1 \sigma_{\nu 2}}{\Gamma_2}. \tag{188}$$

[19]The authors propose an algorithmic scheme for the symmetry reduction of dynamical systems. They also demonstrate how it can be applied to reduce the number of parameters in models of mathematical biology. The algorithm leverages tools from linear algebra, specifically the Hermite normal form of the matrix of exponents along with its normalized unimodular multiplier. In Hubert and Labahn (2012) they introduced a scaling symmetry reduction scheme for polynomial system.

In two steps we derive the non-dimensionalized counterpart for each one of the three original price impact equations obtained by evaluating (184) for (183). First, we evaluate the respective original equation for (188) and convert it to factored normal form numerator/denominator, where numerator and denominator are expanded polynomials. Second, we take its numerator, factorize it, and divide by $\sigma_{\nu 2}$ and Γ_1. The three non-dimensionalized price impact equations we just obtained only depend on the two new parameters Γ_1 and Γ_2. The first and second equations are,

$$\begin{aligned}
&-4\Gamma_1^4\Lambda_1^3\Lambda_2^2 - 4\Gamma_1^3\Gamma_2^2\Lambda_1^3\Lambda_2 - 8\Gamma_1^3\Gamma_2^2\Lambda_1^2\Lambda_2^2 - \Gamma_1^2\Gamma_2^4\Lambda_1^3 - 4\Gamma_1^2\Gamma_2^4\Lambda_1^2\Lambda_2 - 5\Gamma_1^2\Gamma_2^4\Lambda_1\Lambda_2^2 - 8\Gamma_1^2\Gamma_2^2\Lambda_1^3\Lambda_2^2 \\
&\quad - 4\Gamma_1\Gamma_2^4\Lambda_1^3\Lambda_2 - 8\Gamma_1\Gamma_2^4\Lambda_1^2\Lambda_2^2 - 4\Gamma_2^4\Lambda_1^3\Lambda_2^2 + 4\Gamma_1^4\Lambda_1\Lambda_2^2 + 4\Gamma_1^3\Gamma_2^2\Lambda_1\Lambda_2 + 4\Gamma_1^3\Gamma_2^2\Lambda_2^2 + \Gamma_1^2\Gamma_2^4\Lambda_1 \\
&\quad + 2\Gamma_1^2\Gamma_2^4\Lambda_2 + 4\Gamma_1^2\Gamma_2^2\Lambda_1\Lambda_2^2 - \Gamma_2^6\Lambda_1 = 0, \qquad (189)
\end{aligned}$$

$$\begin{aligned}
&-4\Gamma_1^6\Lambda_1^2\Lambda_2^3 - 8\Gamma_1^5\Gamma_2^2\Lambda_1^2\Lambda_2^2 - 4\Gamma_1^5\Gamma_2^2\Lambda_1\Lambda_2^3 - 5\Gamma_1^4\Gamma_2^4\Lambda_1^2\Lambda_2 - 4\Gamma_1^4\Gamma_2^4\Lambda_1\Lambda_2^2 - \Gamma_1^4\Gamma_2^4\Lambda_2^3 - 8\Gamma_1^4\Gamma_2^2\Lambda_1^2\Lambda_2^3 \\
&\quad - 8\Gamma_1^3\Gamma_2^4\Lambda_1^2\Lambda_2^2 - 4\Gamma_1^3\Gamma_2^4\Lambda_1\Lambda_2^3 - 4\Gamma_1^2\Gamma_2^4\Lambda_1^2\Lambda_2^3 - \Gamma_1^4\Gamma_2^4\Lambda_2 + 2\Gamma_1^2\Gamma_2^6\Lambda_1 + \Gamma_1^2\Gamma_2^6\Lambda_2 + 4\Gamma_1^2\Gamma_2^4\Lambda_1^2\Lambda_2 \\
&\quad + 4\Gamma_1\Gamma_2^6\Lambda_1^2 + 4\Gamma_1\Gamma_2^6\Lambda_1\Lambda_2 + 4\Gamma_2^6\Lambda_1^2\Lambda_2 = 0. \qquad (190)
\end{aligned}$$

The third equation is,

$$-\Gamma_1^2\Lambda_3^2 + \Gamma_1^2 + \Gamma_2^2 = 0, \qquad (191)$$

where we divided it by Λ_3 to get rid of the zero solution $\Lambda_3 = 0$. First, we solve (191) and find one positive and one negative solution, but only the positive solution is admissible of course,

$$\Lambda_3(\Gamma_1, \Gamma_2) = \frac{\sqrt{\Gamma_1^2 + \Gamma_2^2}}{\Gamma_1}. \qquad (192)$$

Note how plugging (187) into (192) and re-arranging allows us to obtain λ_3 exactly as in (185). Next, we solve (189) and (190) as a system of two equations in the two variables $\Lambda_1(\Gamma_1, \Gamma_2)$ and $\Lambda_2(\Gamma_1, \Gamma_2)$. We obtain three solutions. The first one is the trivial solution

$\{\Lambda_1 = 0, \Lambda_2 = 0\}$, while the second one is such that Λ_1 is complex,

$$\left\{\frac{\Gamma_1(-2\Gamma_1^2 - 2\Gamma_2^2 + \iota\sqrt{2}\Gamma_1^2\sqrt{\Gamma_1^2+\Gamma_2^2})}{2\Gamma_1^4 + 4\Gamma_1^2 + 4\Gamma_2^2},\ -\frac{\Gamma_1(2\Gamma_1^2 + 2\Gamma_2^2 + \iota\sqrt{2}\Gamma_1^2\sqrt{\Gamma_1^2+\Gamma_2^2})}{2\Gamma_1^4 + 4\Gamma_1^2 + 4\Gamma_2^2}\right\}. \tag{193}$$

Only the third solution is meaningful. In this solution, $\Lambda_1(\Gamma_1, \Gamma_2) > 0$ is the greatest (most positive) real root of a degree-10 polynomial,

$$\begin{aligned}
&8\tau_1(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^{10} + 8\tau_2(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^9 - 4\tau_3(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^8 - 8\tau_4(\Gamma_1,\Gamma_2)\\
&\quad\Lambda_1(\Gamma_1,\Gamma_2)^7 + 4\tau_5(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^6 + 4\tau_6(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^5 + 4\tau_7(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^4\\
&\quad + 4\tau_8(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^3 - \tau_9(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^2 - 4\tau_{10}(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)\\
&\quad - \tau_{11}(\Gamma_1,\Gamma_2),
\end{aligned}$$

with:

$$\begin{aligned}
\tau_1(\Gamma_1,\Gamma_2) &= \Gamma_1^2(\Gamma_1^4 + 2\Gamma_1^2 + 2\Gamma_2^2)(\Gamma_1^2+\Gamma_2^2)^4,\\
\tau_2(\Gamma_1,\Gamma_2) &= \Gamma_1^3\left(5\Gamma_1^4\Gamma_2^2 + 2\Gamma_1^4 + 16\Gamma_1^2\Gamma_2^2 + 14\Gamma_2^4\right)\left(\Gamma_1^2+\Gamma_2^2\right)^3,\\
\tau_3(\Gamma_1,\Gamma_2) &= (-21\Gamma_1^8\Gamma_2^4 + 5\Gamma_1^{10} - 17\Gamma_1^8\Gamma_2^2 - 109\Gamma_1^6\Gamma_2^4 - 87\Gamma_1^4\Gamma_2^6 + 12\Gamma_1^8 + 24\Gamma_1^6\Gamma_2^2 + 8\Gamma_1^4\Gamma_2^4\\
&\quad - 8\Gamma_1^2\Gamma_2^6 - 4\Gamma_2^8)(\Gamma_1^2+\Gamma_2^2)^2,\\
\tau_4(\Gamma_1,\Gamma_2) &= -11\Gamma_1^{11}\Gamma_2^6 - 11\Gamma_1^9\Gamma_2^8 + 10\Gamma_1^{13}\Gamma_2^2 - 7\Gamma_1^{11}\Gamma_2^4 - 120\Gamma_1^9\Gamma_2^6 - 179\Gamma_1^7\Gamma_2^8 - 76\Gamma_1^5\Gamma_2^{10}\\
&\quad + 6\Gamma_1^{13} + 52\Gamma_1^{11}\Gamma_2^2 + 118\Gamma_1^9\Gamma_2^4 + 92\Gamma_1^7\Gamma_2^6 - 2\Gamma_1^5\Gamma_2^8 - 32\Gamma_1^3\Gamma_2^{10} - 10\Gamma_1\Gamma_2^{12},\\
\tau_5(\Gamma_1,\Gamma_2) &= \Gamma_1^2\big(10\Gamma_1^8\Gamma_2^8 - 32\Gamma_1^{10}\Gamma_2^4 + 42\Gamma_1^8\Gamma_2^6 + 232\Gamma_1^6\Gamma_2^8 + 158\Gamma_1^4\Gamma_2^{10} + 3\Gamma_1^{12} - 48\Gamma_1^{10}\Gamma_2^2\\
&\quad - 271\Gamma_1^8\Gamma_2^4 - 368\Gamma_1^6\Gamma_2^6 - 85\Gamma_1^4\Gamma_2^8 + 108\Gamma_1^2\Gamma_2^{10} + 45\Gamma_2^{12} + 12\Gamma_1^{10} + 36\Gamma_1^8\Gamma_2^2\\
&\quad + 28\Gamma_1^6\Gamma_2^4 - 12\Gamma_1^4\Gamma_2^6 - 24\Gamma_1^2\Gamma_2^8 - 8\Gamma_2^{10}\big),\\
\tau_6(\Gamma_1,\Gamma_2) &= 8\Gamma_1^3\big(7\Gamma_2^{12} + ((23/2)\Gamma_1^4 + (47/4)\Gamma_1^2 - 4)\Gamma_2^{10} + ((57/8)\Gamma_1^6 - 22\Gamma_1^4 - 9\Gamma_1^2)\Gamma_2^8\\
&\quad + (-(23/8)\Gamma_1^8 - (159/4)\Gamma_1^6 + (1/2)\Gamma_1^4)\Gamma_2^6 + (-12\Gamma_1^8 + (27/2)\Gamma_1^6)\Gamma_2^4\\
&\quad + (\Gamma_1^{10} + (19/2)\Gamma_1^8)\Gamma_2^2 + (3/2)\Gamma_1^{10}\big),\\
\tau_7(\Gamma_1,\Gamma_2) &= \Gamma_1^4\big(\Gamma_1^{10} + \left(6\Gamma_2^4 + 43\Gamma_2^2 - 4\right)\Gamma_1^8 + \left(-6\Gamma_2^8 - 103\Gamma_2^6 + 133\Gamma_2^4 - 8\Gamma_2^2\right)\Gamma_1^6\\
&\quad + \left(24\Gamma_2^{10} - 144\Gamma_2^8 + 63\Gamma_2^6\right)\Gamma_1^4 + \left(41\Gamma_2^{10} - 82\Gamma_2^8 + 8\Gamma_2^6\right)\Gamma_1^2\\
&\quad + 40\Gamma_2^{12} - 54\Gamma_2^{10} + 4\Gamma_2^8\big),
\end{aligned}$$

$$\tau_8(\Gamma_1,\Gamma_2) = 15\Gamma_1^5\Gamma_2^{12} + (7\Gamma_1^7 - 46\Gamma_1^5)\Gamma_2^{10} + 6(-8\Gamma_1^9 - 7\Gamma_1^7 + 2\Gamma_1^5)\Gamma_2^8 - 2\Gamma_1^7(\Gamma_1^4 - 37\Gamma_1^2 - 8)\Gamma_2^6$$
$$+ 58(\Gamma_1^2 - (4/29))\Gamma_1^9\Gamma_2^4 + 4(\Gamma_1^{13} - 4\Gamma_1^{11})\Gamma_2^2 - 4\Gamma_1^{13},$$
$$\tau_9(\Gamma_1,\Gamma_2) = \Gamma_1^6(15\Gamma_1^4\Gamma_2^8 - 9\Gamma_2^{12} - 24\Gamma_1^6\Gamma_2^4 - 116\Gamma_1^4\Gamma_2^6 + 36\Gamma_1^2\Gamma_2^8 + 80\Gamma_2^{10} + 4\Gamma_1^8 + 40\Gamma_1^6\Gamma_2^2$$
$$+ 48\Gamma_1^4\Gamma_2^4 - 40\Gamma_1^2\Gamma_2^6 - 52\Gamma_2^8),$$
$$\tau_{10}(\Gamma_1,\Gamma_2) = 2\big(2\Gamma_2^6 + (-(3/2)\Gamma_1^2 - 3)\Gamma_2^4 + 2\Gamma_1^2\Gamma_2^2 + \Gamma_1^4)\Gamma_1^7\Gamma_2^2\left(\Gamma_1^2 + \Gamma_2^2\right),$$
$$\tau_{11}(\Gamma_1,\Gamma_2) = \Gamma_1^8\Gamma_2^4(\Gamma_1^2\Gamma_2^4 + \Gamma_2^6 + 4\Gamma_1^4 - 4\Gamma_2^4). \tag{194}$$

In the same solution, $\Lambda_2(\Gamma_1,\Gamma_2)$ is a rational function of the $\Lambda_1(\Gamma_1,\Gamma_2) > 0$ just introduced (i.e., the greatest $\Lambda_1(\Gamma_1,\Gamma_2)$ solving (194)),

$$\Lambda_2(\Gamma_1,\Gamma_2) = \frac{n_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)}{d_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)} > 0,$$

where:

$$n_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2) = -\Gamma_1\kappa_1(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)\big(8\kappa_2(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^7 + 8\kappa_3(\Gamma_1,\Gamma_2)$$
$$\Lambda_1(\Gamma_1,\Gamma_2)^6 - 2\kappa_4(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^5 - 4\kappa_5(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^4$$
$$+ 4\kappa_6(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^3 + \kappa_7(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^2$$
$$+ 2\kappa_8(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2) - \kappa_9(\Gamma_1,\Gamma_2)\big),$$
$$d_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2) = 32\kappa_{10}(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^8 - 8\kappa_{11}(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^7 - 8\kappa_{12}(\Gamma_1,\Gamma_2)$$
$$\Lambda_1(\Gamma_1,\Gamma_2)^6 - 2\kappa_{13}(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^5 + 2\kappa_{14}(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^4$$
$$+ 4\kappa_{15}(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^3 - \kappa_{16}(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^2$$
$$- 2\kappa_{17}(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2) - 2\kappa_{18}(\Gamma_1,\Gamma_2),$$

with:

$$\kappa_1(\Gamma_2) = \Gamma_2^2,$$
$$\kappa_2(\Gamma_1,\Gamma_2) = (\Gamma_1^4 + 4\Gamma_1^2 + 4\Gamma_2^2)(\Gamma_1^2 + \Gamma_2^2)^3,$$
$$\kappa_3(\Gamma_1,\Gamma_2) = \Gamma_1(\Gamma_1^2 + \Gamma_2^2)^2(3\Gamma_1^4\Gamma_2^2 + 2\Gamma_1^4 + 20\Gamma_1^2\Gamma_2^2 + 18\Gamma_2^4),$$
$$\kappa_4(\Gamma_1,\Gamma_2) = 8(\Gamma_1^2 + \Gamma_2^2)\Gamma_1^2(\Gamma_1^6 + (-13/8\Gamma_2^4 - 3\Gamma_2^2 + 4)\Gamma_1^4 + (-43/2\Gamma_2^4 + 8\Gamma_2^2)\Gamma_1^2$$
$$- (35\Gamma_2^6)/2 + 4\Gamma_2^4),$$
$$\kappa_5(\Gamma_1,\Gamma_2) = 9\big(-8\Gamma_2^8 + \left(-(2/9)\Gamma_1^4 - 11\Gamma_1^2 + (56/9)\right)\Gamma_2^6 + \left(-2\Gamma_1^4 + (40/3)\Gamma_1^2\right)\Gamma_2^4$$

$$
\begin{aligned}
&+\left(\Gamma_1^6+8\Gamma_1^4\right)\Gamma_2^2+(8/9)\Gamma_1^6\Big)\Gamma_1^3,\\
\kappa_6(\Gamma_1,\Gamma_2) &= 2(\Gamma_1^6+\big(-(7/2)\Gamma_2^4-11\Gamma_2^2+4\big)\Gamma_1^4+\big((21/2)\Gamma_2^6-51\Gamma_2^4+8\Gamma_2^2\big)\Gamma_1^2+\big((39/2)\Gamma_2^8\\
&\quad -39\Gamma_2^6+4\Gamma_2^4\big)\Gamma_1^4),\\
\kappa_7(\Gamma_1,\Gamma_2) &= (12\Gamma_2^2+16)\Gamma_1^9+(-7\Gamma_2^6-108\Gamma_2^4+96\Gamma_2^2)\Gamma_1^7+(33\Gamma_2^8-196\Gamma_2^6+80\Gamma_2^4)\Gamma_1^5+4\Gamma_1^3\Gamma_2^8,\\
\kappa_8(\Gamma_1,\Gamma_2) &= \Gamma_1^8\Gamma_2^4-23\Gamma_1^6\Gamma_2^6+2\Gamma_1^4\Gamma_2^8+16\Gamma_1^8\Gamma_2^2+32\Gamma_1^6\Gamma_2^4,\\
\kappa_9(\Gamma_1,\Gamma_2) &= \Gamma_2^4\Gamma_1^5\left((\Gamma_2^2-16)\Gamma_1^2-\Gamma_2^4\right),\\
\kappa_{10}(\Gamma_1,\Gamma_2) &= (\Gamma_1^2+\Gamma_2^2)^5,\\
\kappa_{11}(\Gamma_1,\Gamma_2) &= \Gamma_1\Gamma_2^2(\Gamma_1^4-20\Gamma_1^2-20\Gamma_2^2)(\Gamma_1^2+\Gamma_2^2)^3,\\
\kappa_{12}(\Gamma_1,\Gamma_2) &= (\Gamma_1^6+(4\Gamma_2^4+\Gamma_2^2+8)\Gamma_1^4+(-44\Gamma_2^4+16\Gamma_2^2)\Gamma_1^2-44\Gamma_2^6+8\Gamma_2^4)(\Gamma_1^2+\Gamma_2^2)^2\Gamma_1^2,\\
\kappa_{13}(\Gamma_1,\Gamma_2) &= \Gamma_1^3\Gamma_2^2(\Gamma_1^2+\Gamma_2^2)\big(25\Gamma_1^4\Gamma_2^4+12\Gamma_1^6+12\Gamma_1^4\Gamma_2^2-212\Gamma_1^2\Gamma_2^4-212\Gamma_2^6+128\Gamma_1^4\\
&\quad +256\Gamma_1^2\Gamma_2^2+128\Gamma_2^4\big),\\
\kappa_{14}(\Gamma_1,\Gamma_2) &= 8\Gamma_1^4\big((145/8)\Gamma_2^{10}+\left(-(7/4)\Gamma_1^4+(137/8)\Gamma_1^2-(53/2)\right)\Gamma_2^8+\big(-(27/8)\Gamma_1^4\\
&\quad -53\Gamma_1^2+2\big)\Gamma_2^6+\big(-(19/8)\Gamma_1^6-(51/2)\Gamma_1^4+6\Gamma_1^2\big)\Gamma_2^4+\left(2\Gamma_1^6+6\Gamma_1^4\right)\Gamma_2^2\\
&\quad +\Gamma_1^8+2\Gamma_1^6\big),\\
\kappa_{15}(\Gamma_1,\Gamma_2) &= 26\Gamma_1^5\Gamma_2^{10}+(-8\Gamma_1^7-88\Gamma_1^5)\Gamma_2^8+(-11\Gamma_1^9-88\Gamma_1^7+24\Gamma_1^5)\Gamma_2^6+12\Gamma_1^7(\Gamma_1^2+4)\Gamma_2^4\\
&\quad +12\Gamma_1^9(\Gamma_1^2+2)\Gamma_2^2,\\
\kappa_{16}(\Gamma_1,\Gamma_2) &= \Gamma_1^6(27\Gamma_1^2\Gamma_2^8-15\Gamma_2^{10}-54\Gamma_1^4\Gamma_2^4+150\Gamma_2^8+8\Gamma_1^6+8\Gamma_1^4\Gamma_2^2-104\Gamma_1^2\Gamma_2^4-104\Gamma_2^6),\\
\kappa_{17}(\Gamma_1,\Gamma_2) &= \Gamma_1^7\Gamma_2^2(-11\Gamma_1^2\Gamma_2^4+15\Gamma_2^6+8\Gamma_1^4-24\Gamma_2^4),\\
\kappa_{18}(\Gamma_1,\Gamma_2) &= \Gamma_1^8\Gamma_2^4(\Gamma_2^4+4\Gamma_1^2-4\Gamma_2^2).
\end{aligned}
\tag{195}
$$

Note that the greatest $\Lambda_1(\Gamma_1,\Gamma_2)$ solving (194), as the root of a degree-10 polynomial, is unsurprisingly not representable explicitly, i.e., in closed-form. Moreover, the reason why we pick the greatest $\Lambda_1(\Gamma_1,\Gamma_2)$ is that the greatest root is the only root for which the corresponding $\Lambda_2(\Gamma_1,\Gamma_2)$ is also positive. To prove uniqueness in a rigorous way (e.g., as we did for the other two models) is difficult, but just testing numerically over a grid shows that the solution exists everywhere and is unique.

Our methodology consists of six steps. First, we numerically solve (194) in $\Lambda_1(\Gamma_1,\Gamma_2)$

for Γ_1 and Γ_2 ranging from almost zero to five with a solver which reliably searches for all solutions, including complex solutions. Second, we do a floating-point normalization and call a sequence which replaces complex numerics with their corresponding real parts and non-real objects with their corresponding imaginary parts. Third, we filter out the complex and negative solutions so that only real and positive solutions survive. Fourth, we make a function that returns the numeric value of the greatest solution among all the positive ones and change the "root of" implicit form just to such numeric value (similar to if we had solved without backsubstitution). Fifth, we use the numeric value of the greatest $\Lambda_1(\Gamma_1,\Gamma_2)$ to find the corresponding $\Lambda_2(\Gamma_1,\Gamma_2)$. Finally, we implement a procedure to count positive pairs (Λ_1,Λ_2) for given Γ_1 and Γ_2, i.e., we check all ten $\Lambda_1(\Gamma_1,\Gamma_2)$ values with their corresponding $\Lambda_2(\Gamma_1,\Gamma_2)$ values for cases when both are positive. We find no cases of two or more valid solutions, thus proving that the only positive pair (Λ_1,Λ_2) is the one where $\Lambda_1(\Gamma_1,\Gamma_2)$ is the greatest root of the polynomial (194). We plot the positive pair (Λ_1,Λ_2) in Fig. 21a and Fig. 21b.

All in all, given (188) and assuming $0<\gamma<\infty$, the parameters in equilibrium (180) for markets 1 and 2 are,

$$\lambda_i = \frac{\Lambda_i(\Gamma_1,\Gamma_2)\Gamma_1\sigma_{\nu 2}}{\Gamma_2\sigma_\delta} \quad \forall i\in\{1,2\},$$

$$\beta_i = \begin{cases} \dfrac{\sigma_\delta\left(2\Gamma_1\Gamma_2\Lambda_j(\Gamma_1,\Gamma_2)+\Gamma_2^3\right)}{4\sigma_{\nu 2}\left(\left(\left(\Gamma_1^2+\Gamma_2^2\right)\Lambda_i(\Gamma_1,\Gamma_2)+\frac{\Gamma_1\Gamma_2^2}{2}\right)\Lambda_j(\Gamma_1,\Gamma_2)+\Lambda_i(\Gamma_1,\Gamma_2)\frac{\Gamma_1\Gamma_2^2}{2}\right)} & \text{if } \{i,j\}=\{1,2\}, \\ \dfrac{\Gamma_2^3\sigma_\delta(2\Lambda_j(\Gamma_1,\Gamma_2)+\Gamma_1)}{4\Gamma_1\sigma_{\nu 2}\left(\left((\Gamma_1^2+\Gamma_2^2)\Lambda_i(\Gamma_1,\Gamma_2)+\frac{\Gamma_1\Gamma_2^2}{2}\right)\Lambda_j(\Gamma_1,\Gamma_2)+\Lambda_i(\Gamma_1,\Gamma_2)\frac{\Gamma_1\Gamma_2^2}{2}\right)} & \text{if } \{i,j\}=\{2,1\}, \end{cases}$$

$$\alpha_i = \frac{-(2(\Gamma_1^2+\Gamma_2^2)\Lambda_j(\Gamma_1,\Gamma_2)+\Gamma_1\Gamma_2^2)\Lambda_i(\Gamma_1,\Gamma_2)}{((4\Gamma_1^2+4\Gamma_2^2)\Lambda_j(\Gamma_1,\Gamma_2)+2\Gamma_1\Gamma_2^2)\,\Lambda_i(\Gamma_1,\Gamma_2)+2\Lambda_j(\Gamma_1,\Gamma_2)\Gamma_1\Gamma_2^2} \quad \forall\{i,j\}\in\{\{1,2\},\{2,1\}\},$$

$$\alpha_j^* = \frac{-\Lambda_j(\Gamma_1,\Gamma_2)\Gamma_1\Gamma_2^2}{((4\Gamma_1^2+4\Gamma_2^2)\Lambda_j(\Gamma_1,\Gamma_2)+2\Gamma_1\Gamma_2^2)\,\Lambda_i(\Gamma_1,\Gamma_2)+2\Lambda_j(\Gamma_1,\Gamma_2)\Gamma_1\Gamma_2^2} \quad \forall\{i,j\}\in\{\{1,2\},\{2,1\}\}, \tag{196}$$

and the quotes in equilibrium (180) for markets 1 and 2 can be rearranged as,

$$
\begin{aligned}
q_i &= \overline{\nu}_i + s_i\left(\zeta - \overline{\nu}_i - \overline{\nu}_j\right) + d_i\delta_i + d_j^*\delta_j \\
&\quad \forall\{i,j\} \in \{\{1,2\},\{2,1\}\}\,, \\
&\text{with:} \\
s_i &= \begin{cases} \dfrac{\Lambda_i(\Gamma_1,\Gamma_2)\Gamma_1(2\Lambda_j(\Gamma_1,\Gamma_2)\Gamma_1+\Gamma_2^2)}{((4\Gamma_1^2+4\Gamma_2^2)\Lambda_j(\Gamma_1,\Gamma_2)+2\Gamma_1\Gamma_2^2)\Lambda_i(\Gamma_1,\Gamma_2)+2\Lambda_j(\Gamma_1,\Gamma_2)\Gamma_1\Gamma_2^2}, & \text{if } \{i,j\}=\{1,2\} \\ \dfrac{\Lambda_i(\Gamma_1,\Gamma_2)\Gamma_2^2(2\Lambda_j(\Gamma_1,\Gamma_2)+\Gamma_1)}{((4\Gamma_1^2+4\Gamma_2^2)\Lambda_j(\Gamma_1,\Gamma_2)+2\Gamma_1\Gamma_2^2)\Lambda_i(\Gamma_1,\Gamma_2)+2\Lambda_j(\Gamma_1,\Gamma_2)\Gamma_1\Gamma_2^2}, & \text{if } \{i,j\}=\{2,1\} \end{cases} \\
d_i &= \frac{\Lambda_i(\Gamma_1,\Gamma_2)\left(\left((\Gamma_1^2+\Gamma_2^2)\Lambda_j(\Gamma_1,\Gamma_2)+\frac{\Gamma_1\Gamma_2^2}{2}\right)\Lambda_i(\Gamma_1,\Gamma_2)+\Lambda_j(\Gamma_1,\Gamma_2)\Gamma_1\Gamma_2^2\right)\sigma_{\nu 2}\Gamma_1}{2\left(\left((\Gamma_1^2+\Gamma_2^2)\Lambda_j(\Gamma_1,\Gamma_2)+\frac{\Gamma_1\Gamma_2^2}{2}\right)\Lambda_i(\Gamma_1,\Gamma_2)+\Lambda_j(\Gamma_1,\Gamma_2)\frac{\Gamma_1\Gamma_2^2}{2}\right)\sigma_\delta\Gamma_2} \\
&\quad \forall\{i,j\} \in \{\{1,2\},\{2,1\}\} \\
d_j^* &= \frac{-\Lambda_i(\Gamma_1,\Gamma_2)\Gamma_1^2\sigma_{\nu 2}\Gamma_2\Lambda_j(\Gamma_1,\Gamma_2)}{4\left(\left((\Gamma_1^2+\Gamma_2^2)\Lambda_j(\Gamma_1,\Gamma_2)+\frac{\Gamma_1\Gamma_2^2}{2}\right)\Lambda_i(\Gamma_1,\Gamma_2)+\Lambda_j(\Gamma_1,\Gamma_2)\frac{\Gamma_1\Gamma_2^2}{2}\right)\sigma_\delta}, \\
&\quad \forall\{i,j\} \in \{\{1,2\},\{2,1\}\}\,. \qquad (197)
\end{aligned}
$$

We use the notation with i and j to emphasize symmetries. For example, note how in (196) λ_2 is λ_1 with $\Lambda_1(\Gamma_1,\Gamma_2)$ replaced by $\Lambda_2(\Gamma_1,\Gamma_2)$. Similarly, if we were to interchange $\Lambda_1(\Gamma_1,\Gamma_2)$ and $\Lambda_2(\Gamma_1,\Gamma_2)$ in α_1 we would obtain exactly α_2. The same symmetry holds for α_j^*, as well as for d_i and d_j^* in q_i (197). However, the symmetry breaks for β_i in (196) and for s_i in (197). That is, even if we were to replace $\Lambda_1(\Gamma_1,\Gamma_2)$ with $\Lambda_2(\Gamma_1,\Gamma_2)$ and vice versa in β_1, we would still not obtain β_2. The same holds for s_i. However, the symmetry is recovered for β_i and s_i if $\Gamma_1=\Gamma_2$, i.e., if $\sigma_{\nu 1}=\sigma_{\nu 2}$.

E.3 Sensitivity Analysis

We can synthetically represent the derivatives of $\Lambda_1(\Gamma_1, \Gamma_2)$ in terms of the $\Lambda_1(\Gamma_1, \Gamma_2)$ function itself. The first order derivative of $\Lambda_1(\Gamma_1, \Gamma_2)$ with respect to Γ_1 is,

$$\frac{\partial \Lambda_1(\Gamma_1, \Gamma_2)}{\partial \Gamma_1} = \frac{n^{(\Gamma 1)}_{\partial \Lambda 1}(\Lambda_1(\Gamma_1, \Gamma_2), \Gamma_1, \Gamma_2)}{d^{(\Gamma 1)}_{\partial \Lambda 1}(\Lambda_1(\Gamma_1, \Gamma_2), \Gamma_1, \Gamma_2)},$$

where:

$$\begin{aligned}
n^{(\Gamma 1)}_{\partial \Lambda 1}(\Lambda_1(\Gamma_1, \Gamma_2), \Gamma_1, \Gamma_2) = &-8\varepsilon_1(\Gamma_1, \Gamma_2)\Lambda_1(\Gamma_1, \Gamma_2)^{10} - 4\varepsilon_2(\Gamma_1, \Gamma_2)\Lambda_1(\Gamma_1, \Gamma_2)^9 + 4\varepsilon_3(\Gamma_1, \Gamma_2) \\
&\Lambda_1(\Gamma_1, \Gamma_2)^8 + 4\varepsilon_4(\Gamma_1, \Gamma_2)\Lambda_1(\Gamma_1, \Gamma_2)^7 - 4\varepsilon_5(\Gamma_1, \Gamma_2)\Lambda_1(\Gamma_1, \Gamma_2)^6 \\
&- 2\varepsilon_6(\Gamma_1, \Gamma_2)\Lambda_1(\Gamma_1, \Gamma_2)^5 - 4\varepsilon_7(\Gamma_1, \Gamma_2)\Lambda_1(\Gamma_1, \Gamma_2)^4 - 2\varepsilon_8(\Gamma_1, \Gamma_2) \\
&\Lambda_1(\Gamma_1, \Gamma_2)^3 + \varepsilon_9(\Gamma_1, \Gamma_2)\Lambda_1(\Gamma_1, \Gamma_2)^2 + 2\varepsilon_{10}(\Gamma_1, \Gamma_2)\Lambda_1(\Gamma_1, \Gamma_2) \\
&+ \varepsilon_{11}(\Gamma_1, \Gamma_2), \\
d^{(\Gamma 1)}_{\partial \Lambda 1}(\Lambda_1(\Gamma_1, \Gamma_2), \Gamma_1, \Gamma_2) = &\, 40\varepsilon_{12}(\Gamma_1, \Gamma_2)\Lambda_1(\Gamma_1, \Gamma_2)^9 + 36\varepsilon_{13}(\Gamma_1, \Gamma_2)\Lambda_1(\Gamma_1, \Gamma_2)^8 - 16\varepsilon_{14}(\Gamma_1, \Gamma_2) \\
&\Lambda_1(\Gamma_1, \Gamma_2)^7 - 28\varepsilon_{15}(\Gamma_1, \Gamma_2)\Lambda_1(\Gamma_1, \Gamma_2)^6 + 12\varepsilon_{16}(\Gamma_1, \Gamma_2)\Lambda_1(\Gamma_1, \Gamma_2)^5 \\
&+ 10\varepsilon_{17}(\Gamma_1, \Gamma_2)\Lambda_1(\Gamma_1, \Gamma_2)^4 + 8\varepsilon_{18}(\Gamma_1, \Gamma_2)\Lambda_1(\Gamma_1, \Gamma_2)^3 + 6\varepsilon_{19}(\Gamma_1, \Gamma_2) \\
&\Lambda_1(\Gamma_1, \Gamma_2)^2 - \varepsilon_{20}(\Gamma_1, \Gamma_2)\Lambda_1(\Gamma_1, \Gamma_2) - 2\varepsilon_{21}(\Gamma_1, \Gamma_2),
\end{aligned}$$

with:

$$\begin{aligned}
\varepsilon_1(\Gamma_1, \Gamma_2) &= \Gamma_1(7\Gamma_1^6 + 3\Gamma_1^4\Gamma_2^2 + 12\Gamma_1^4 + 14\Gamma_1^2\Gamma_2^2 + 2\Gamma_2^4)(\Gamma_1^2 + \Gamma_2^2)^3, \\
\varepsilon_2(\Gamma_1, \Gamma_2) &= \Gamma_1^2(\Gamma_1^2 + \Gamma_2^2)^2(65\Gamma_1^6\Gamma_2^2 + 35\Gamma_1^4\Gamma_2^4 + 26\Gamma_1^6 + 190\Gamma_1^4\Gamma_2^2 + 206\Gamma_1^2\Gamma_2^4 + 42\Gamma_2^6), \\
\varepsilon_3(\Gamma_1, \Gamma_2) &= (-126\Gamma_1^8\Gamma_2^4 - 84\Gamma_1^6\Gamma_2^6 + 35\Gamma_1^{10} - 77\Gamma_1^8\Gamma_2^2 - 613\Gamma_1^6\Gamma_2^4 - 675\Gamma_1^4\Gamma_2^6 - 174\Gamma_1^2\Gamma_2^8 + 72\Gamma_1^8 \\
&\quad + 168\Gamma_1^6\Gamma_2^2 + 104\Gamma_1^4\Gamma_2^4 - 8\Gamma_1^2\Gamma_2^6 - 16\Gamma_2^8)\Gamma_1(\Gamma_1^2 + \Gamma_2^2), \\
\varepsilon_4(\Gamma_1, \Gamma_2) &= (130\Gamma_2^2 + 78)\Gamma_1^{12} + (-121\Gamma_2^6 - 77\Gamma_2^4 + 572\Gamma_2^2)\Gamma_1^{10} + (-99\Gamma_2^8 - 1080\Gamma_2^6 + 1062\Gamma_2^4) \\
&\quad \Gamma_1^8 + (-1253\Gamma_2^8 + 644\Gamma_2^6)\Gamma_1^6 + (-380\Gamma_2^{10} - 10\Gamma_2^8)\Gamma_1^4 - 96\Gamma_1^2\Gamma_2^{10} - 10\Gamma_2^{12}, \\
\varepsilon_5(\Gamma_1, \Gamma_2) &= 21\Gamma_1^{13} + (-192\Gamma_2^4 - 288\Gamma_2^2 + 72)\Gamma_1^{11} + (50\Gamma_2^8 + 210\Gamma_2^6 - 1355\Gamma_2^4 + 180\Gamma_2^2)\Gamma_1^9 \\
&\quad + (928\Gamma_2^8 - 1472\Gamma_2^6 + 112\Gamma_2^4)\Gamma_1^7 + (474\Gamma_2^{10} - 255\Gamma_2^8 - 36\Gamma_2^6)\Gamma_1^5 + (216\Gamma_2^{10} - 48\Gamma_2^8) \\
&\quad \Gamma_1^3 + (45\Gamma_2^{12} - 8\Gamma_2^{10})\Gamma_1, \\
\varepsilon_6(\Gamma_1, \Gamma_2) &= (104\Gamma_2^2 + 156)\Gamma_1^{12} + (-253\Gamma_2^6 - 1056\Gamma_2^4 + 836\Gamma_2^2)\Gamma_1^{10} + (513\Gamma_2^8 - 2862\Gamma_2^6 + 972\Gamma_2^4)
\end{aligned}$$

$$\Gamma_1^8 + (644\Gamma_2^{10} - 1232\Gamma_2^8 + 28\Gamma_2^6)\Gamma_1^6 + (470\Gamma_2^{10} - 360\Gamma_2^8)\Gamma_1^4 + (168\Gamma_2^{12} - 96\Gamma_2^{10})\Gamma_1^2,$$

$$\varepsilon_7(\Gamma_1,\Gamma_2) = 7\Gamma_1^{13} + (36\Gamma_2^4 + 258\Gamma_2^2 - 24)\Gamma_1^{11} + (-30\Gamma_2^8 - 515\Gamma_2^6 + 665\Gamma_2^4 - 40\Gamma_2^2)\Gamma_1^9 + (96\Gamma_2^{10}$$
$$- 576\Gamma_2^8 + 252\Gamma_2^6)\Gamma_1^7 + (123\Gamma_2^{10} - 246\Gamma_2^8 + 24\Gamma_2^6)\Gamma_1^5 + (80\Gamma_2^{12} - 108\Gamma_2^{10} + 8\Gamma_2^8)\Gamma_1^3,$$

$$\varepsilon_8(\Gamma_1,\Gamma_2) = \Gamma_1^4(-22\Gamma_1^6\Gamma_2^6 - 432\Gamma_1^4\Gamma_2^8 + 49\Gamma_1^2\Gamma_2^{10} + 75\Gamma_2^{12} + 52\Gamma_1^8\Gamma_2^2 + 638\Gamma_1^6\Gamma_2^4 + 666\Gamma_1^4\Gamma_2^6$$
$$- 294\Gamma_1^2\Gamma_2^8 - 230\Gamma_2^{10} - 52\Gamma_1^8 - 176\Gamma_1^6\Gamma_2^2 - 72\Gamma_1^4\Gamma_2^4 + 112\Gamma_1^2\Gamma_2^6 + 60\Gamma_2^8),$$

$$\varepsilon_9(\Gamma_1,\Gamma_2) = -27\Gamma_1^5\Gamma_2^{12} + 240\Gamma_1^5\Gamma_2^{10} + (75\Gamma_1^9 + 144\Gamma_1^7 - 156\Gamma_1^5)\Gamma_2^8 - 580\Gamma_1^7(\Gamma_1^2 + \frac{8}{29})\Gamma_2^6 - 144$$
$$\Gamma_1^9(\Gamma_1^2 - \frac{5}{3})\Gamma_2^4 + 240\Gamma_1^{11}\Gamma_2^2 + 28\Gamma_1^{13},$$

$$\varepsilon_{10}(\Gamma_1,\Gamma_2) = \Gamma_1^6\Gamma_2^2(-33\Gamma_1^4\Gamma_2^4 + 9\Gamma_1^2\Gamma_2^6 + 28\Gamma_2^8 + 26\Gamma_1^6 + 66\Gamma_1^4\Gamma_2^2 - 18\Gamma_1^2\Gamma_2^4 - 42\Gamma_2^6),$$

$$\varepsilon_{11}(\Gamma_1,\Gamma_2) = \Gamma_1^7\Gamma_2^4(5\Gamma_1^2\Gamma_2^4 + 4\Gamma_2^6 + 24\Gamma_1^4 - 16\Gamma_2^4),$$

$$\varepsilon_{12}(\Gamma_1,\Gamma_2) = \Gamma_1^2(\Gamma_1^4 + 2\Gamma_1^2 + 2\Gamma_2^2)(\Gamma_1^2 + \Gamma_2^2)^4,$$

$$\varepsilon_{13}(\Gamma_1,\Gamma_2) = \Gamma_1^3(\Gamma_1^2 + \Gamma_2^2)^3(5\Gamma_1^4\Gamma_2^2 + 2\Gamma_1^4 + 16\Gamma_1^2\Gamma_2^2 + 14\Gamma_2^4),$$

$$\varepsilon_{14}(\Gamma_1,\Gamma_2) = (\Gamma_1^2 + \Gamma_2^2)^2(-21\Gamma_1^8\Gamma_2^4 + 5\Gamma_1^{10} - 17\Gamma_1^8\Gamma_2^2 - 109\Gamma_1^6\Gamma_2^4 - 87\Gamma_1^4\Gamma_2^6 + 12\Gamma_1^8 + 24\Gamma_1^6\Gamma_2^2$$
$$+ 8\Gamma_1^4\Gamma_2^4 - 8\Gamma_1^2\Gamma_2^6 - 4\Gamma_2^8),$$

$$\varepsilon_{15}(\Gamma_1,\Gamma_2) = -11\Gamma_1^{11}\Gamma_2^6 - 11\Gamma_1^9\Gamma_2^8 + 10\Gamma_1^{13}\Gamma_2^2 - 7\Gamma_1^{11}\Gamma_2^4 - 120\Gamma_1^9\Gamma_2^6 - 179\Gamma_1^7\Gamma_2^8 - 76\Gamma_1^5\Gamma_2^{10} + 6\Gamma_1^{13}$$
$$+ 52\Gamma_1^{11}\Gamma_2^2 + 118\Gamma_1^9\Gamma_2^4 + 92\Gamma_1^7\Gamma_2^6 - 2\Gamma_1^5\Gamma_2^8 - 32\Gamma_1^3\Gamma_2^{10} - 10\Gamma_1\Gamma_2^{12},$$

$$\varepsilon_{16}(\Gamma_1,\Gamma_2) = \Gamma_1^2\big(10\Gamma_1^8\Gamma_2^8 - 32\Gamma_1^{10}\Gamma_2^4 + 42\Gamma_1^8\Gamma_2^6 + 232\Gamma_1^6\Gamma_2^8 + 158\Gamma_1^4\Gamma_2^{10} + 3\Gamma_1^{12} - 48\Gamma_1^{10}\Gamma_2^2 - 271$$
$$\Gamma_1^8\Gamma_2^4 - 368\Gamma_1^6\Gamma_2^6 - 85\Gamma_1^4\Gamma_2^8 + 108\Gamma_1^2\Gamma_2^{10} + 45\Gamma_2^{12} + 12\Gamma_1^{10} + 36\Gamma_1^8\Gamma_2^2 + 28\Gamma_1^6\Gamma_2^4$$
$$- 12\Gamma_1^4\Gamma_2^6 - 24\Gamma_1^2\Gamma_2^8 - 8\Gamma_2^{10}\big),$$

$$\varepsilon_{17}(\Gamma_1,\Gamma_2) = 8\big(7\Gamma_2^{12} + \big((23/2)\Gamma_1^4 + (47/4)\Gamma_1^2 - 4\big)\Gamma_2^{10} + \big((57/8)\Gamma_1^6 - 22\Gamma_1^4 - 9\Gamma_1^2\big)\Gamma_2^8$$
$$+ \big(- (23/8)\Gamma_1^8 - (159/4)\Gamma_1^6 + (1/2)\Gamma_1^4\big)\Gamma_2^6 + \big(- 12\Gamma_1^8 + (27/2)\Gamma_1^6\big)\Gamma_2^4 + \big(\Gamma_1^{10}$$
$$+ (19/2)\Gamma_1^8\big)\Gamma_2^2 + (3/2)\Gamma_1^{10}\big)\Gamma_1^3,$$

$$\varepsilon_{18}(\Gamma_1,\Gamma_2) = \Gamma_1^{10} + (6\Gamma_2^4 + 43\Gamma_2^2 - 4)\Gamma_1^8 + (-6\Gamma_2^8 - 103\Gamma_2^6 + 133\Gamma_2^4 - 8\Gamma_2^2)\Gamma_1^6 + (24\Gamma_2^{10}$$
$$- 144\Gamma_2^8 + 63\Gamma_2^6)\Gamma_1^4 + (41\Gamma_2^{10} - 82\Gamma_2^8 + 8\Gamma_2^6)\Gamma_1^2 + 40\Gamma_2^{12} - 54\Gamma_2^{10} + 4\Gamma_2^8)\Gamma_1^4,$$

$$\varepsilon_{19}(\Gamma_1,\Gamma_2) = 15\Gamma_1^5\Gamma_2^{12} + (7\Gamma_1^7 - 46\Gamma_1^5)\Gamma_2^{10} + 6(-8\Gamma_1^9 - 7\Gamma_1^7 + 2\Gamma_1^5)\Gamma_2^8 - 2\Gamma_1^7(\Gamma_1^4 - 37\Gamma_1^2 - 8)\Gamma_2^6$$
$$+ 58\Gamma_1^9(\Gamma_1^2 - \frac{4}{29})\Gamma_2^4 + 4(\Gamma_1^{13} - 4\Gamma_1^{11})\Gamma_2^2 - 4\Gamma_1^{13},$$

$$\varepsilon_{20}(\Gamma_1,\Gamma_2) = \Gamma_1^6(15\Gamma_1^4\Gamma_2^8 - 9\Gamma_2^{12} - 24\Gamma_1^6\Gamma_2^4 - 116\Gamma_1^4\Gamma_2^6 + 36\Gamma_1^2\Gamma_2^8 + 80\Gamma_2^{10} + 4\Gamma_1^8 + 40\Gamma_1^6\Gamma_2^2$$

$$+ 48\Gamma_1^4\Gamma_2^4 - 40\Gamma_1^2\Gamma_2^6 - 52\Gamma_2^8),$$

$$\varepsilon_{21}(\Gamma_1,\Gamma_2) = 2\Gamma_1^7\big(2\Gamma_2^6 + \big(-(3/2)\Gamma_1^2 - 3\big)\Gamma_2^4 + 2\Gamma_1^2\Gamma_2^2 + \Gamma_1^4\big)(\Gamma_1^2+\Gamma_2^2)\Gamma_2^2. \tag{198}$$

The first order derivative of $\Lambda_1(\Gamma_1,\Gamma_2)$ with respect to Γ_2 is,

$$\frac{\partial\Lambda_1(\Gamma_1,\Gamma_2)}{\partial\Gamma_2} = \frac{n_{\partial\Lambda 1}^{(\Gamma 2)}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)}{d_{\partial\Lambda 1}^{(\Gamma 2)}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)},$$

where:

$$\begin{aligned} n_{\partial\Lambda 1}^{(\Gamma 2)}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2) = -\Gamma_2(&16o_1(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^{10} + 8o_2(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^9 + 4o_3(\Gamma_1,\Gamma_2)\\ &\Lambda_1(\Gamma_1,\Gamma_2)^8 - 8o_4(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^7 - 8o_5(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^6\\ &+ 4o_6(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^5 + 4o_7(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^4 + 4o_8(\Gamma_1,\Gamma_2)\\ &\Lambda_1(\Gamma_1,\Gamma_2)^3 + 2o_9(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)^2 - 4o_{10}(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)\\ &- o_{11}(\Gamma_1,\Gamma_2)), \end{aligned}$$

$$d_{\partial\Lambda 1}^{(\Gamma 2)}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2) = d_{\partial\Lambda 1}^{(\Gamma 1)}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2),$$

with:

$$o_1(\Gamma_1,\Gamma_2) = (2\Gamma_1^4 + 5\Gamma_1^2 + 5\Gamma_2^2)\Gamma_1^2(\Gamma_1^2+\Gamma_2^2)^3,$$

$$o_2(\Gamma_1,\Gamma_2) = \Gamma_1^3(5\Gamma_1^6 + 20\Gamma_1^4\Gamma_2^2 + 22\Gamma_1^4 + 92\Gamma_1^2\Gamma_2^2 + 70\Gamma_2^4)(\Gamma_1^2+\Gamma_2^2)^2,$$

$$\begin{aligned} o_3(\Gamma_1,\Gamma_2) = 42\big(&(\Gamma_2^2 + 1/6)\Gamma_1^{10} + (2\Gamma_2^4 + (269/42)\Gamma_2^2 - 8/7)\Gamma_1^8 + ((697/42)\Gamma_2^4 - (44/21)\Gamma_2^2)\Gamma_1^6\\ &+ ((145/14)\Gamma_2^6 - (4/21)\Gamma_2^4)\Gamma_1^4 + (4/3)\Gamma_1^2\Gamma_2^6 + (4/7)\Gamma_2^8\big)(\Gamma_1^2+\Gamma_2^2), \end{aligned}$$

$$\begin{aligned} o_4(\Gamma_1,\Gamma_2) = &10\Gamma_1^{13} + (-33\Gamma_2^4 - 14\Gamma_2^2 + 52)\Gamma_1^{11} + (-44\Gamma_2^6 - 360\Gamma_2^4 + 236\Gamma_2^2)\Gamma_1^9 + (-716\Gamma_2^6\\ &+ 276\Gamma_2^4)\Gamma_1^7 + (-380\Gamma_2^8 - 8\Gamma_2^6)\Gamma_1^5 - 160\Gamma_1^3\Gamma_2^8 - 60\Gamma_1\Gamma_2^{10}, \end{aligned}$$

$$\begin{aligned} o_5(\Gamma_1,\Gamma_2) = &(32\Gamma_2^2 + 24)\Gamma_1^{12} + (-20\Gamma_2^6 - 63\Gamma_2^4 + 271\Gamma_2^2 - 18)\Gamma_1^{10} + (-464\Gamma_2^6 + 552\Gamma_2^4 - 28\Gamma_2^2)\\ &\Gamma_1^8 + (-395\Gamma_2^8 + 170\Gamma_2^6 + 18\Gamma_2^4)\Gamma_1^6 + (-270\Gamma_2^8 + 48\Gamma_2^6)\Gamma_1^4 + (-135\Gamma_2^{10} + 20\Gamma_2^8)\Gamma_1^2, \end{aligned}$$

$$\begin{aligned} o_6(\Gamma_1,\Gamma_2) = \Gamma_1^3\big(&-69\Gamma_1^8\Gamma_2^4 + 228\Gamma_1^6\Gamma_2^6 + 460\Gamma_1^4\Gamma_2^8 + 8\Gamma_1^{10} - 192\Gamma_1^8\Gamma_2^2 - 954\Gamma_1^6\Gamma_2^4 - 704\Gamma_1^4\Gamma_2^6\\ &+ 470\Gamma_1^2\Gamma_2^8 + 336\Gamma_2^{10} + 76\Gamma_1^8 + 216\Gamma_1^6\Gamma_2^2 + 12\Gamma_1^4\Gamma_2^4 - 288\Gamma_1^2\Gamma_2^6 - 160\Gamma_2^8\big), \end{aligned}$$

$$\begin{aligned} o_7(\Gamma_1,\Gamma_2) = 12\Gamma_1^4\big(&20\Gamma_2^{10} + \big(10\Gamma_1^4 + (205/12)\Gamma_1^2 - 45/2\big)\Gamma_2^8 + \big(-2\Gamma_1^6 - 48\Gamma_1^4 - (82/3)\Gamma_1^2\\ &+ 4/3\big)\Gamma_2^6 + \big(-(103/4)\Gamma_1^6 + (63/4)\Gamma_1^4 + 2\Gamma_1^2\big)\Gamma_2^4 + \big(\Gamma_1^8 + (133/6)\Gamma_1^6\big)\Gamma_2^2 \end{aligned}$$

$$
\begin{aligned}
&+ (43/12)\Gamma_1^8 - (2/3)\Gamma_1^6\big),\\
o_8(\Gamma_1,\Gamma_2) &= 4\Gamma_1^5\big((45/2)\Gamma_2^{10} + \big((35/4)\Gamma_1^2 - 115/2\big)\Gamma_2^8 + \big(-48\Gamma_1^4 - 42\Gamma_1^2 + 12\big)\Gamma_2^6 - (3/2)\Gamma_1^2\\
&\quad (\Gamma_1^4 - 37\Gamma_1^2 - 8)\Gamma_2^4 + \big(29\Gamma_1^6 - 4\Gamma_1^4\big)\Gamma_2^2 + \Gamma_1^8 - 4\Gamma_1^6\big),\\
o_9(\Gamma_1,\Gamma_2) &= 27\Gamma_1^6\Gamma_2^{10} - 200\Gamma_1^6\Gamma_2^8 + (-30\Gamma_1^{10} - 72\Gamma_1^8 + 104\Gamma_1^6)\Gamma_2^6 + 174\Gamma_1^8(\Gamma_1^2 + 10/29)\Gamma_2^4\\
&\quad + 24\Gamma_1^{10}(\Gamma_1^2 - 2)\Gamma_2^2 - 20\Gamma_1^{12},\\
o_{10}(\Gamma_1,\Gamma_2) &= \Gamma_1^7\left(-9\Gamma_1^4\Gamma_2^4 + 4\Gamma_1^2\Gamma_2^6 + 20\Gamma_2^8 + 2\Gamma_1^6 + 12\Gamma_1^4\Gamma_2^2 - 6\Gamma_1^2\Gamma_2^4 - 24\Gamma_2^6\right),\\
o_{11}(\Gamma_1,\Gamma_2) &= 5\Gamma_1^8\Gamma_2^8 + 4(\Gamma_1^{10} - 4\Gamma_1^8)\Gamma_2^6 + 8\Gamma_1^{12}\Gamma_2^2.
\end{aligned}
\tag{199}
$$

The derivatives for $\Lambda_2(\Gamma_1,\Gamma_2)$ are more complicated because the latter is also a function of $\Lambda_1(\Gamma_1,\Gamma_2)$, that is, $\Lambda_2(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)$ (195). As such, we apply the chain rule to derive the total derivatives. The total derivative of $\Lambda_2(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)$ with respect to Γ_1 is,

$$
\frac{\mathrm{d}\Lambda_2(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)}{\mathrm{d}\Gamma_1} = \left(\frac{\partial\Lambda_2(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)}{\partial\Lambda_1(\Gamma_1,\Gamma_2)}\right)\left(\frac{\partial\Lambda_1(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}\right) + \frac{\partial\Lambda_2(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)}{\partial\Gamma_1},
\tag{200}
$$

where $\partial\Lambda_1/\partial\Gamma_1$ is (198). The total derivative of $\Lambda_2(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)$ with respect to Γ_2 is,

$$
\frac{\mathrm{d}\Lambda_2(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)}{\mathrm{d}\Gamma_2} = \left(\frac{\partial\Lambda_2(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)}{\partial\Lambda_1(\Gamma_1,\Gamma_2)}\right)\left(\frac{\partial\Lambda_1(\Gamma_1,\Gamma_2)}{\partial\Gamma_2}\right) + \frac{\partial\Lambda_2(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)}{\partial\Gamma_2},
\tag{201}
$$

where $\partial\Lambda_1/\partial\Gamma_2$ is (199). Note that the two total derivatives share the same $\partial\Lambda_2/\partial\Lambda_1$, which we write as,

$$
\begin{aligned}
\frac{\partial\Lambda_2(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)}{\partial\Lambda_1(\Gamma_1,\Gamma_2)} &= \frac{-\Gamma_1\kappa_1(\Gamma_1,\Gamma_2)}{d_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)}\big(n^*_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2) + 2\Lambda_1(\Gamma_1,\Gamma_2)\\
&\quad \big(28\Lambda_1(\Gamma_1,\Gamma_2)^6\kappa_2(\Gamma_1,\Gamma_2) + 24\Lambda_1(\Gamma_1,\Gamma_2)^5\kappa_3(\Gamma_1,\Gamma_2) - 5\Lambda_1(\Gamma_1,\Gamma_2)^4\\
&\quad \kappa_4(\Gamma_1,\Gamma_2) - 8\Lambda_1(\Gamma_1,\Gamma_2)^3\kappa_5(\Gamma_1,\Gamma_2) + 6\Lambda_1(\Gamma_1,\Gamma_2)^2\kappa_6(\Gamma_1,\Gamma_2)\\
&\quad + \Lambda_1(\Gamma_1,\Gamma_2)\kappa_7(\Gamma_1,\Gamma_2) + \kappa_8(\Gamma_1,\Gamma_2)\big)\big)\\
&\quad + \frac{\Gamma_1\kappa_1(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)}{d_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)^2}\big(n^*_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)\big(256\Lambda_1(\Gamma_1,\Gamma_2)^7
\end{aligned}
$$

$$\kappa_{10}(\Gamma_1,\Gamma_2) - 56\Lambda_1(\Gamma_1,\Gamma_2)^6\kappa_{11}(\Gamma_1,\Gamma_2) - 48\Lambda_1(\Gamma_1,\Gamma_2)^5\kappa_{12}(\Gamma_1,\Gamma_2)$$
$$- 10\Lambda_1(\Gamma_1,\Gamma_2)^4\kappa_{13}(\Gamma_1,\Gamma_2) + 8\Lambda_1(\Gamma_1,\Gamma_2)^3\kappa_{14}(\Gamma_1,\Gamma_2) + 12\Lambda_1(\Gamma_1,\Gamma_2)^2$$
$$\kappa_{15}(\Gamma_1,\Gamma_2) - 2\Lambda_1(\Gamma_1,\Gamma_2)\kappa_{16}(\Gamma_1,\Gamma_2) - 2\kappa_{17}(\Gamma_1,\Gamma_2)\Big)\Big). \tag{202}$$

The only term left in (200) is $\partial\Lambda_2/\partial\Gamma_1$, which we write as,

$$\frac{\partial\Lambda_2(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)}{\partial\Gamma_1} = \frac{-\Lambda_1(\Gamma_1,\Gamma_2)}{d_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)}\Bigg(\kappa_1(\Gamma_1,\Gamma_2)n^*_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)$$
$$+ \Gamma_1\frac{\partial\kappa_1(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}n^*_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2) + \Gamma_1\kappa_1(\Gamma_1,\Gamma_2)\Bigg(8\Lambda_1(\Gamma_1,\Gamma_2)^7$$
$$\frac{\partial\kappa_2(\Gamma_1,\Gamma_2)}{\partial\Gamma_1} + 8\Lambda_1(\Gamma_1,\Gamma_2)^6\frac{\partial\kappa_3(\Gamma_1,\Gamma_2)}{\partial\Gamma_1} - 2\Lambda_1(\Gamma_1,\Gamma_2)^5\frac{\partial\kappa_4(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}$$
$$- 4\Lambda_1(\Gamma_1,\Gamma_2)^4\frac{\partial\kappa_5(\Gamma_1,\Gamma_2)}{\partial\Gamma_1} + 4\Lambda_1(\Gamma_1,\Gamma_2)^3\frac{\partial\kappa_6(\Gamma_1,\Gamma_2)}{\partial\Gamma_1} + \Lambda_1(\Gamma_1,\Gamma_2)^2$$
$$\frac{\partial\kappa_7(\Gamma_1,\Gamma_2)}{\partial\Gamma_1} + 2\Lambda_1(\Gamma_1,\Gamma_2)\frac{\partial\kappa_8(\Gamma_1,\Gamma_2)}{\partial\Gamma_1} - \frac{\partial\kappa_9(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}\Bigg)\Bigg)$$
$$+ \frac{\Gamma_1\kappa_1(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)}{d_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)^2}\Bigg(n^*_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)\Bigg(32\Lambda_1(\Gamma_1,\Gamma_2)^8$$
$$\frac{\partial\kappa_{10}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1} - 8\Lambda_1(\Gamma_1,\Gamma_2)^7\frac{\partial\kappa_{11}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1} - 8\Lambda_1(\Gamma_1,\Gamma_2)^6\frac{\partial\kappa_{12}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}$$
$$- 2\Lambda_1(\Gamma_1,\Gamma_2)^5\frac{\partial\kappa_{13}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1} + 2\Lambda_1(\Gamma_1,\Gamma_2)^4\frac{\partial\kappa_{14}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}$$
$$+ 4\Lambda_1(\Gamma_1,\Gamma_2)^3\frac{\partial\kappa_{15}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1} - \Lambda_1(\Gamma_1,\Gamma_2)^2\frac{\partial\kappa_{16}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}$$
$$- 2\Lambda_1(\Gamma_1,\Gamma_2)\frac{\partial\kappa_{17}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1} - 2\frac{\partial\kappa_{18}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}\Bigg)\Bigg). \tag{203}$$

The only term left in (201) is $\partial\Lambda_2/\partial\Gamma_2$, which we write as,

$$\frac{\partial\Lambda_2(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)}{\partial\Gamma_2} = \frac{-\Lambda_1(\Gamma_1,\Gamma_2)}{d_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)}\Bigg(\Gamma_1\frac{\partial\kappa_1(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}n^*_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)$$
$$+ \Gamma_1\kappa_1(\Gamma_1,\Gamma_2)\Bigg(8\Lambda_1(\Gamma_1,\Gamma_2)^7\frac{\partial\kappa_2(\Gamma_1,\Gamma_2)}{\partial\Gamma_1} + 8\Lambda_1(\Gamma_1,\Gamma_2)^6\frac{\partial\kappa_3(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}$$
$$- 2\Lambda_1(\Gamma_1,\Gamma_2)^5\frac{\partial\kappa_4(\Gamma_1,\Gamma_2)}{\partial\Gamma_1} - 4\Lambda_1(\Gamma_1,\Gamma_2)^4\frac{\partial\kappa_5(\Gamma_1,\Gamma_2)}{\partial\Gamma_1} + 4\Lambda_1(\Gamma_1,\Gamma_2)^3$$
$$\frac{\partial\kappa_6(\Gamma_1,\Gamma_2)}{\partial\Gamma_1} + \Lambda_1(\Gamma_1,\Gamma_2)^2\frac{\partial\kappa_7(\Gamma_1,\Gamma_2)}{\partial\Gamma_1} + 2\Lambda_1(\Gamma_1,\Gamma_2)\frac{\partial\kappa_8(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}$$
$$- \frac{\partial\kappa_9(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}\Bigg)\Bigg) + \frac{\Gamma_1\kappa_1(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)}{d_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)^2}\Bigg(n^*_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)$$

$$\Big(32\Lambda_1(\Gamma_1,\Gamma_2)^8\frac{\partial\kappa_{10}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}-8\Lambda_1(\Gamma_1,\Gamma_2)^7\frac{\partial\kappa_{11}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}$$
$$-8\Lambda_1(\Gamma_1,\Gamma_2)^6\frac{\partial\kappa_{12}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}-2\Lambda_1(\Gamma_1,\Gamma_2)^5\frac{\partial\kappa_{13}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}$$
$$+2\Lambda_1(\Gamma_1,\Gamma_2)^4\frac{\partial\kappa_{14}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}+4\Lambda_1(\Gamma_1,\Gamma_2)^3\frac{\partial\kappa_{15}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}$$
$$-\Lambda_1(\Gamma_1,\Gamma_2)^2\frac{\partial\kappa_{16}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}-2\Lambda_1(\Gamma_1,\Gamma_2)\frac{\partial\kappa_{17}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}$$
$$-2\frac{\partial\kappa_{18}(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}\Big)\Big). \tag{204}$$

In (202), (203), and (204) the terms $d_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)$ and $\kappa_i(\Gamma_1,\Gamma_2)$, $i\in\{1...18\}$ are defined exactly as in $\Lambda_2(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)$ (195). The term $n^*_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)$ is equal to $n_{\Lambda 2}(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)$ in (195) but divided by $-\Gamma_1\kappa_1(\Gamma_1,\Gamma_2)\Lambda_1(\Gamma_1,\Gamma_2)$.

We determine the signs of the four convoluted derivatives (198), (199), (200), and (201) by numerical differentiation, i.e., by computing a numerical approximation for the derivatives evaluated for Γ_1 and Γ_2 ranging from 0 to 5,

$$\frac{\partial\Lambda_1(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}>0,\qquad \frac{\partial\Lambda_1(\Gamma_1,\Gamma_2)}{\partial\Gamma_2}<0, \tag{205}$$

$$\frac{\mathrm{d}\Lambda_2(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)}{\mathrm{d}\Gamma_1}<0,\qquad \frac{\mathrm{d}\Lambda_2(\Lambda_1(\Gamma_1,\Gamma_2),\Gamma_1,\Gamma_2)}{\mathrm{d}\Gamma_2}>0. \tag{206}$$

We plot (205) in Fig. 22a and Fig. 22b, respectively, and (206) in Fig. 23a and Fig. 23b. From now onwards, we redefine $\mathrm{d}\Lambda_2(\cdot)/\mathrm{d}\Gamma_1$ (200) and $\mathrm{d}\Lambda_2(\cdot)/\mathrm{d}\Gamma_2$ (201) as, respectively, $\partial\Lambda_2(\cdot)/\partial\Gamma_1$ and $\partial\Lambda_2(\cdot)/\partial\Gamma_2$. We do this so that we can use a more flexible indexed notation with i and j. However, whenever we call the derivatives of $\Lambda_2(\cdot)$ we always mean their total derivatives rather than their actual partial derivatives.

As we know the signs of $\Lambda_1(\Gamma_1,\Gamma_2)$, $\Lambda_2(\Gamma_1,\Gamma_2)$ and their derivatives, we can now infer the signs of the partial derivatives of the equilibrium parameters (196) for markets 1 and 2 with respect to γ, $\sigma_{\nu 1}$, $\sigma_{\nu 2}$ and σ_δ. We apply the chain rule and derive the partial derivatives but we keep them in implicit form, to facilitate their interpretation. We implement an efficient method for the evaluation of derivatives of these original (dimensioned) variables. To infer their signs, the idea is to plot (numerically evaluate) the scaled dimensionless versions of

these vs. Γ_1 and Γ_2, keeping all calculations in terms of dimensionless variables as much as possible. We create a procedure that takes Γ_1 and Γ_2 and produces the scaled derivative in four steps. First, we evaluate $\Lambda_1(\Gamma_1, \Gamma_2)$ numerically only once, extracting the most positive root. Second, we use this to evaluate $\Lambda_2(\Gamma_1, \Gamma_2)$ numerically only once. Third, after substituting the pieces we will do numerically with functions, we evaluate the derivative through numeric differentiation. Last, we we combine the previous three steps to obtain the required derivative. According to our plots (not reported), the signs of all these partial derivatives are consistent with our economic intuitions about the strategic behaviour of a risk-averse speculator trading in a triangle where $\sigma_{\nu 1} \neq \sigma_{\nu 2}$. We report our economic intuitions in Section 6, where we only discuss those that are unique to this model extension and not already encountered in the baseline model. In the following we only report the analytical expressions and their signs.

When the volatility of asset 1's payoff is different from the volatility of asset 2's payoff, the strategic behaviour of the speculator depends as follows on her increasing γ,

$$\begin{aligned}
&\frac{\partial \alpha_i}{\partial \gamma} = -\frac{\partial \alpha_j^*}{\partial \gamma} = \\
&= \left(\frac{-1}{4\gamma(\Lambda_i(\Gamma_1,\Gamma_2)\Lambda_j(\Gamma_1,\Gamma_2)\Gamma_1^2 + \Gamma_2^2(\Lambda_i(\Gamma_1,\Gamma_2) + \Lambda_j(\Gamma_1,\Gamma_2))\Gamma_1/2 + \Lambda_i(\Gamma_1,\Gamma_2)\Lambda_j(\Gamma_1,\Gamma_2)\Gamma_2^2)^2} \right) \\
&\Gamma_1\Gamma_2^2 \Bigg(\Gamma_1^3 \Lambda_j(\Gamma_1,\Gamma_2)^2 \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_1} + \Gamma_1^2 \Bigg((\Gamma_2^2/2) \left(-\Lambda_i(\Gamma_1,\Gamma_2) \frac{\partial \Lambda_j(\Gamma_1,\Gamma_2)}{\partial \Gamma_1} + \Lambda_j(\Gamma_1,\Gamma_2) \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_1} \right) \\
&+ \Gamma_2 \Lambda_j(\Gamma_1,\Gamma_2)^2 \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_2} - \Lambda_i(\Gamma_1,\Gamma_2)\Lambda_j(\Gamma_1,\Gamma_2)^2 \Bigg) \\
&- (\Gamma_2^2\Gamma_1/2) \Bigg(\Gamma_2 \left(\Lambda_i(\Gamma_1,\Gamma_2) \frac{\partial \Lambda_j(\Gamma_1,\Gamma_2)}{\partial \Gamma_2} - \Lambda_j(\Gamma_1,\Gamma_2) \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_2} \right) - 2\Lambda_j(\Gamma_1,\Gamma_2)^2 \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_1} \Bigg) \\
&+ \Gamma_2^2 \Lambda_j(\Gamma_1,\Gamma_2)^2 \left(\frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_2} \Gamma_2 - \Lambda_i(\Gamma_1,\Gamma_2) \right) \Bigg) > 0, \quad \forall \{i,j\} \in \{\{1,2\},\{2,1\}\}\,. \qquad (207)
\end{aligned}$$

$$\frac{\partial \lambda_i}{\partial \gamma} = (\Gamma_1/\gamma^2\sigma_d^2) \left(\frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_1}\Gamma_1 + \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_2}\Gamma_2 \right) < 0, \quad \forall i \in \{1,2\}\,. \qquad (208)$$

$$\frac{\partial \beta_1}{\partial \gamma} =$$
$$= \Big(\frac{-1}{2\left(((\Gamma_1/2)(\Lambda_i(\Gamma_1,\Gamma_2)+\Lambda_j(\Gamma_1,\Gamma_2))+\Lambda_j(\Gamma_1,\Gamma_2)\Lambda_i(\Gamma_1,\Gamma_2)\right)\Gamma_2^2+\Gamma_1^2\Lambda_j(\Gamma_1,\Gamma_2)\Lambda_i(\Gamma_1,\Gamma_2)\right)^2} \Big)$$
$$\sigma_\delta^2 \Big(\frac{\Gamma_2^5}{2} \Big(\Big(\frac{\partial \Lambda_j(\Gamma_1,\Gamma_2)}{\partial \Gamma_2} + \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_2} \Big) \frac{\Gamma_1}{2} + \Lambda_i(\Gamma_1,\Gamma_2) \frac{\partial \Lambda_j(\Gamma_1,\Gamma_2)}{\partial \Gamma_2} + \Lambda_j(\Gamma_1,\Gamma_2) \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_2} \Big)$$
$$+ \frac{\Gamma_2^4}{2} \Big(\frac{\Gamma_1^2}{2} \Big(\frac{\partial \Lambda_j(\Gamma_1,\Gamma_2)}{\partial \Gamma_1} + \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_1} \Big) + \Gamma_1 \Big(\Lambda_j(\Gamma_1,\Gamma_2) \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_1} + \Lambda_i(\Gamma_1,\Gamma_2) \frac{\partial \Lambda_j(\Gamma_1,\Gamma_2)}{\partial \Gamma_1} \Big)$$
$$- \Lambda_j(\Gamma_1,\Gamma_2)\Lambda_i(\Gamma_1,\Gamma_2) \Big) + \Gamma_1 \Lambda_j(\Gamma_1,\Gamma_2) \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_2} (\Gamma_1 + \Lambda_j(\Gamma_1,\Gamma_2))\Gamma_2^3 + \Gamma_1^2\Gamma_2^2\Lambda_j(\Gamma_1,\Gamma_2)$$
$$\Big(\frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_1}\Gamma_1 + \Lambda_j(\Gamma_1,\Gamma_2) \Big(\frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_1} + 1/2 \Big) \Big) + \Gamma_1^3\Gamma_2\Lambda_j(\Gamma_1,\Gamma_2)^2 \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_2}$$
$$+ \Gamma_1^4 \Lambda_j(\Gamma_1,\Gamma_2)^2 \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_1} \Big), \quad \text{if } \{i,j\} = \{1,2\}. \tag{209}$$

$$\frac{\partial \beta_2}{\partial \gamma} =$$
$$= \Big(\frac{-1}{4\Gamma_1((\Gamma_1\Gamma_2^2/2)(\Lambda_i(\Gamma_1,\Gamma_2)+\Lambda_j(\Gamma_1,\Gamma_2))+\Gamma_2^2\Lambda_j(\Gamma_1,\Gamma_2)\Lambda_i(\Gamma_1,\Gamma_2)+\Gamma_1^2\Lambda_j(\Gamma_1,\Gamma_2)\Lambda_i(\Gamma_1,\Gamma_2))^2} \Big)$$
$$\sigma_\delta^2\Gamma_2^2 \Big(\Gamma_1^4 \Big(\Lambda_i(\Gamma_1,\Gamma_2) \frac{\partial \Lambda_j(\Gamma_1,\Gamma_2)}{\partial \Gamma_1} + \Lambda_j(\Gamma_1,\Gamma_2) \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_1} \Big) + \Gamma_1^3 \Big((\Gamma_2^2/2) \Big(\frac{\partial \Lambda_j(\Gamma_1,\Gamma_2)}{\partial \Gamma_1}$$
$$+ \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_1} \Big) + \Gamma_2 \Big(\Lambda_i(\Gamma_1,\Gamma_2) \frac{\partial \Lambda_j(\Gamma_1,\Gamma_2)}{\partial \Gamma_2} + \Lambda_j(\Gamma_1,\Gamma_2) \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_2} \Big) + 2\Lambda_j(\Gamma_1,\Gamma_2)^2$$
$$\frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_1} - \Lambda_j(\Gamma_1,\Gamma_2)\Lambda_i(\Gamma_1,\Gamma_2) \Big) + (\Gamma_2\Gamma_1^2/2) \Big(\Gamma_2^2 \Big(\frac{\partial \Lambda_j(\Gamma_1,\Gamma_2)}{\partial \Gamma_2} + \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_2} \Big)$$
$$+ 4\Gamma_2\Lambda_j(\Gamma_1,\Gamma_2) \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_1} + 4\Lambda_j(\Gamma_1,\Gamma_2)^2 \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_2} \Big) + 2\Lambda_j(\Gamma_1,\Gamma_2)\Gamma_2^2\Gamma_1 \Big(\Gamma_2 \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_2}$$
$$+ \Lambda_j(\Gamma_1,\Gamma_2) \Big(\frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_1} + 1/2 \Big) \Big) + 2\Gamma_2^3\Lambda_j(\Gamma_1,\Gamma_2)^2 \frac{\partial \Lambda_i(\Gamma_1,\Gamma_2)}{\partial \Gamma_2} \Big), \quad \text{if } \{i,j\} = \{2,1\}. \tag{210}$$

Their signs vary as,

$$\frac{\partial \beta_1}{\partial \gamma} \begin{cases} > 0 & \text{if } \Gamma_2 < p_1(\Gamma_1) \\ \leq 0 & \text{otherwise} \end{cases}, \qquad \frac{\partial \beta_2}{\partial \gamma} \begin{cases} > 0 & \text{if } \Gamma_2 > p_2(\Gamma_1) \\ \leq 0 & \text{otherwise} \end{cases},$$

with:

$$p_1(\Gamma_1) \approx 0.97 + 1.32\Gamma_1, \qquad p_2(\Gamma_1) \approx -0.73 + 0.76\Gamma_1. \tag{211}$$

We can plot $p_1(\Gamma_1)$'s fitting points and the approximation form, and $p_2(\Gamma_1)$ fitting points and the approximation form . We fit the points by solving a linear least squares problem.

When the volatility of asset 1's payoff is different from the volatility of asset 2's payoff, the strategic behaviour of the speculator changes as follows as $\sigma_{\nu i}$ increases,

$$\frac{\partial \alpha_i}{\partial \sigma_{\nu i}} = -\frac{\partial \alpha_j^*}{\partial \sigma_{\nu i}} =$$
$$\left(\frac{-1}{4(\Lambda_2(\Gamma_i,\Gamma_j)\Gamma_i^2\Lambda_1(\Gamma_i,\Gamma_j) + \Gamma_j^2(\Lambda_1(\Gamma_i,\Gamma_j) + \Lambda_2(\Gamma_i,\Gamma_j))\Gamma_i/2 + \Gamma_j^2\Lambda_1(\Gamma_i,\Gamma_j)\Lambda_2(\Gamma_i,\Gamma_j))^2}\right)$$
$$\gamma\sigma_\delta\Gamma_j^2\Big(\Gamma_i^3\Lambda_2(\Gamma_i,\Gamma_j)^2\frac{\partial\Lambda_1(\Gamma_i,\Gamma_j)}{\partial\Gamma_i} + \Gamma_i^2\Big((\Gamma_j^2/2)\Big(\Lambda_2(\Gamma_i,\Gamma_j)\frac{\partial\Lambda_1(\Gamma_i,\Gamma_j)}{\partial\Gamma_i} - \Lambda_1(\Gamma_i,\Gamma_j)\frac{\partial\Lambda_2(\Gamma_i,\Gamma_j)}{\partial\Gamma_i}\Big)$$
$$+ \Lambda_1(\Gamma_i,\Gamma_j)\Lambda_2(\Gamma_i,\Gamma_j)^2\Big) + \Gamma_i\Gamma_j^2\Lambda_2(\Gamma_i,\Gamma_j)^2\frac{\partial\Lambda_1(\Gamma_i,\Gamma_j)}{\partial\Gamma_i} - \Gamma_j^2\Lambda_1(\Gamma_i,\Gamma_j)\Lambda_2(\Gamma_i,\Gamma_j)^2\Big) < 0,$$
$$\forall\{i,j\} \in \{\{1,2\},\{2,1\}\}. \tag{212}$$

$$\frac{\partial \alpha_j}{\partial \sigma_{\nu i}} = -\frac{\partial \alpha_i^*}{\partial \sigma_{\nu i}} =$$
$$\left(\frac{-1}{4(\Lambda_2(\Gamma_i,\Gamma_j)\Gamma_i^2\Lambda_1(\Gamma_i,\Gamma_j) + \Gamma_j^2(\Lambda_1(\Gamma_i,\Gamma_j) + \Lambda_2(\Gamma_i,\Gamma_j))\Gamma_i/2 + \Gamma_j^2\Lambda_1(\Gamma_i,\Gamma_j)\Lambda_2(\Gamma_i,\Gamma_j))^2}\right)$$
$$\gamma\sigma_\delta\Gamma_j^2\Big(\Gamma_i^3\Lambda_1(\Gamma_i,\Gamma_j)^2\frac{\partial\Lambda_2(\Gamma_i,\Gamma_j)}{\partial\Gamma_i} + \Gamma_i^2\Big((\Gamma_j^2/2)\Big(\Lambda_1(\Gamma_i,\Gamma_j)\frac{\partial\Lambda_2(\Gamma_i,\Gamma_j)}{\partial\Gamma_i} - \Lambda_2(\Gamma_i,\Gamma_j)\frac{\partial\Lambda_1(\Gamma_i,\Gamma_j)}{\partial\Gamma_i}\Big)$$
$$+ \Lambda_2(\Gamma_i,\Gamma_j)\Lambda_1(\Gamma_i,\Gamma_j)^2\Big) + \Gamma_i\Gamma_j^2\Lambda_1(\Gamma_i,\Gamma_j)^2\frac{\partial\Lambda_2(\Gamma_i,\Gamma_j)}{\partial\Gamma_i} - \Gamma_j^2\Lambda_2(\Gamma_i,\Gamma_j)\Lambda_1(\Gamma_i,\Gamma_j)^2\Big) > 0,$$
$$\forall\{i,j\} \in \{\{1,2\},\{2,1\}\}. \tag{213}$$

$$\frac{\partial \lambda_i}{\partial \sigma_{\nu i}} = \frac{1}{\sigma_\delta}\Big(\Gamma_i \frac{\partial \Lambda_1(\Gamma_i,\Gamma_j)}{\partial \Gamma_i} + \Lambda_1(\Gamma_i,\Gamma_j)\Big) > 0, \quad \forall\{i,j\} \in \{\{1,2\},\{2,1\}\}\,. \tag{214}$$

$$\frac{\partial \lambda_j}{\partial \sigma_{\nu i}} = \frac{1}{\sigma_\delta}\Big(\Gamma_i \frac{\partial \Lambda_2(\Gamma_i,\Gamma_j)}{\partial \Gamma_i} + \Lambda_2(\Gamma_i,\Gamma_j)\Big) < 0, \quad \forall\{i,j\} \in \{\{1,2\},\{2,1\}\}\,. \tag{215}$$

$$\begin{aligned}
&\frac{\partial \beta_i}{\partial \sigma_{\nu i}} = \\
&= \Big(\frac{-1}{2\left(\Gamma_j^2((\Gamma_i/2)\Lambda_1(\Gamma_i,\Gamma_j) + \Lambda_2(\Gamma_i,\Gamma_j)(\Lambda_1(\Gamma_i,\Gamma_j) + \Gamma_i/2)) + \Lambda_2(\Gamma_i,\Gamma_j)\Gamma_i^2\Lambda_1(\Gamma_i,\Gamma_j)\right)^2}\Big) \\
&\sigma_\delta^3\gamma^2\Big(\frac{\Gamma_j^4}{2}\Big(\Big(\frac{\partial \Lambda_1(\Gamma_i,\Gamma_j)}{\partial \Gamma_i} + \frac{\partial \Lambda_2(\Gamma_i,\Gamma_j)}{\partial \Gamma_i}\Big)\frac{\Gamma_i}{2} + \Lambda_2(\Gamma_i,\Gamma_j)\Big(\frac{\partial \Lambda_1(\Gamma_i,\Gamma_j)}{\partial \Gamma_i} + 1/2\Big) + \Lambda_1(\Gamma_i,\Gamma_j) \\
&\Big(\frac{\partial \Lambda_2(\Gamma_i,\Gamma_j)}{\partial \Gamma_i} + 1/2\Big)\Big) + \Lambda_2(\Gamma_i,\Gamma_j)\Gamma_j^2\Big(\Lambda_2(\Gamma_i,\Gamma_j)\Big(\frac{\partial \Lambda_1(\Gamma_i,\Gamma_j)}{\partial \Gamma_i}\Gamma_i - \Lambda_1(\Gamma_i,\Gamma_j)\Big) \\
&+ \Gamma_i\Big(\frac{\partial \Lambda_1(\Gamma_i,\Gamma_j)}{\partial \Gamma_i}\Gamma_i + \Lambda_1(\Gamma_i,\Gamma_j)\Big)\Big) + \Gamma_i^2\Lambda_2(\Gamma_i,\Gamma_j)^2\Big(\frac{\partial \Lambda_1(\Gamma_i,\Gamma_j)}{\partial \Gamma_i}\Gamma_i + \Lambda_1(\Gamma_i,\Gamma_j)\Big)\Big) < 0, \\
&\forall\{i,j\} \in \{\{1,2\},\{2,1\}\}\,.
\end{aligned} \tag{216}$$

$$\begin{aligned}
&\frac{\partial \beta_j}{\partial \sigma_{\nu i}} = \\
&= \Big(\frac{-1}{4\Gamma_i^2(\Lambda_2(\Gamma_i,\Gamma_j)\Gamma_i^2\Lambda_1(\Gamma_i,\Gamma_j) + \Gamma_j^2(\Lambda_1(\Gamma_i,\Gamma_j) + \Lambda_2(\Gamma_i,\Gamma_j))\Gamma_i/2 + \Gamma_j^2\Lambda_1(\Gamma_i,\Gamma_j)\Lambda_2(\Gamma_i,\Gamma_j))^2}\Big) \\
&\sigma_\delta^3\gamma^2\Gamma_j^2\Big(\Gamma_i^4\Big(\Lambda_1(\Gamma_i,\Gamma_j)\frac{\partial \Lambda_2(\Gamma_i,\Gamma_j)}{\partial \Gamma_i} + \Lambda_2(\Gamma_i,\Gamma_j)\frac{\partial \Lambda_1(\Gamma_i,\Gamma_j)}{\partial \Gamma_i}\Big) + \Gamma_i^3\Big((\Gamma_j^2/2)\Big(\frac{\partial \Lambda_1(\Gamma_i,\Gamma_j)}{\partial \Gamma_i} \\
&+ \frac{\partial \Lambda_2(\Gamma_i,\Gamma_j)}{\partial \Gamma_i}\Big) + 2\Lambda_1(\Gamma_i,\Gamma_j)^2\frac{\partial \Lambda_2(\Gamma_i,\Gamma_j)}{\partial \Gamma_i} + 2\Lambda_1(\Gamma_i,\Gamma_j)\Lambda_2(\Gamma_i,\Gamma_j)\Big) + \Gamma_i^2\Big(\Gamma_j^2\Big(\Lambda_1(\Gamma_i,\Gamma_j) \\
&\Big(2\frac{\partial \Lambda_2(\Gamma_i,\Gamma_j)}{\partial \Gamma_i} + 1/2\Big) + \Lambda_2(\Gamma_i,\Gamma_j)/2\Big) + 6\Lambda_1(\Gamma_i,\Gamma_j)^2\Lambda_2(\Gamma_i,\Gamma_j)\Big) + 2\Gamma_i\Gamma_j^2\Lambda_1(\Gamma_i,\Gamma_j) \\
&\Big(\Lambda_1(\Gamma_i,\Gamma_j)\Big(1 + \frac{\partial \Lambda_2(\Gamma_i,\Gamma_j)}{\partial \Gamma_i}\Big) + \Lambda_2(\Gamma_i,\Gamma_j)\Big) + 2\Gamma_j^2\Lambda_1(\Gamma_i,\Gamma_j)^2\Lambda_2(\Gamma_i,\Gamma_j)\Big) < 0,
\end{aligned}$$

$$\forall\{i,j\} \in \{\{1,2\},\{2,1\}\}\,. \tag{217}$$

Finally, when the volatility of asset 1's payoff is different from the volatility of asset 2's payoff, the strategic behaviour of the speculator responds as follows as σ_δ increases,

$$\frac{\partial \alpha_i}{\partial \sigma_\delta} = -\frac{\partial \alpha_j^*}{\partial \sigma_\delta} =$$
$$\Bigg(\frac{-1}{4\sigma_\delta(\Lambda_i(\Gamma_1,\Gamma_2)\Gamma_1^2\Lambda_j(\Gamma_1,\Gamma_2)+\Gamma_2^2(\Lambda_i(\Gamma_1,\Gamma_2)+\Lambda_j(\Gamma_1,\Gamma_2))\Gamma_1/2+\Gamma_2^2\Lambda_i(\Gamma_1,\Gamma_2)\Lambda_j(\Gamma_1,\Gamma_2))^2}\Bigg)$$
$$\Gamma_2^2\Gamma_1\Bigg(\Gamma_1^3\Lambda_j(\Gamma_1,\Gamma_2)^2\frac{\partial\Lambda_i(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}+\Gamma_1^2\Bigg((\Gamma_2^2/2)\Bigg(\Lambda_j(\Gamma_1,\Gamma_2)\frac{\partial\Lambda_i(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}-\Lambda_i(\Gamma_1,\Gamma_2)\frac{\partial\Lambda_j(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}\Bigg)$$
$$+\Gamma_2\Lambda_j(\Gamma_1,\Gamma_2)^2\frac{\partial\Lambda_i(\Gamma_1,\Gamma_2)}{\partial\Gamma_2}-\Lambda_i(\Gamma_1,\Gamma_2)\Lambda_j(\Gamma_1,\Gamma_2)^2\Bigg)-(\Gamma_2^2\Gamma_1/2)\Bigg(\Gamma_2\Bigg(\Lambda_i(\Gamma_1,\Gamma_2)\frac{\partial\Lambda_j(\Gamma_1,\Gamma_2)}{\partial\Gamma_2}$$
$$-\Lambda_j(\Gamma_1,\Gamma_2)\frac{\partial\Lambda_i(\Gamma_1,\Gamma_2)}{\partial\Gamma_2}\Bigg)-2\Lambda_j(\Gamma_1,\Gamma_2)^2\frac{\partial\Lambda_i(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}\Bigg)+\Gamma_2^2\Lambda_j(\Gamma_1,\Gamma_2)^2\Bigg(\frac{\partial\Lambda_i(\Gamma_1,\Gamma_2)}{\partial\Gamma_2}\Gamma_2$$
$$-\Lambda_i(\Gamma_1,\Gamma_2)\Bigg)\Bigg)>0,\quad \forall\{i,j\}\in\{\{1,2\},\{2,1\}\}\,. \tag{218}$$

$$\frac{\partial\lambda_i}{\partial\sigma_\delta}=(\Gamma_1/\sigma_\delta^3\gamma)\Bigg(\Gamma_1\frac{\partial\Lambda_i(\Gamma_1,\Gamma_2)}{\partial\Gamma_1}+\Gamma_2\frac{\partial\Lambda_i(\Gamma_1,\Gamma_2)}{\partial\Gamma_2}-\Lambda_i(\Gamma_1,\Gamma_2)\Bigg)<0,\quad \forall i\in\{1,2\}\,. \tag{219}$$

$$\frac{\partial\beta_i}{\partial\sigma_\delta}=$$
$$\Bigg(\frac{-1}{8(\Gamma_j^2((\Gamma_i/2)(\Lambda_1(\Gamma_i,\Gamma_j)+\Lambda_2(\Gamma_i,\Gamma_j))+\Lambda_2(\Gamma_i,\Gamma_j)\Lambda_1(\Gamma_i,\Gamma_j))\Gamma_j^2+\Lambda_2(\Gamma_i,\Gamma_j)\Gamma_i^2\Lambda_1(\Gamma_i,\Gamma_j))^2}\Bigg)$$
$$\gamma\sigma_\delta\Bigg(\Gamma_j^5\Bigg(\Gamma_i\Bigg(\frac{\partial\Lambda_2(\Gamma_i,\Gamma_j)}{\partial\Gamma_j}+\frac{\partial\Lambda_1(\Gamma_i,\Gamma_j)}{\partial\Gamma_j}\Bigg)+2\Lambda_1(\Gamma_i,\Gamma_j)\frac{\partial\Lambda_2(\Gamma_i,\Gamma_j)}{\partial\Gamma_j}+2\Lambda_2(\Gamma_i,\Gamma_j)\frac{\partial\Lambda_1(\Gamma_i,\Gamma_j)}{\partial\Gamma_j}\Bigg)$$
$$+\Gamma_j^4\Bigg(\Gamma_i^2\Bigg(\frac{\partial\Lambda_1(\Gamma_i,\Gamma_j)}{\partial\Gamma_i}+\frac{\partial\Lambda_2(\Gamma_i,\Gamma_j)}{\partial\Gamma_i}\Bigg)+\Gamma_i\Bigg(\Lambda_2(\Gamma_i,\Gamma_j)\Bigg(2\frac{\partial\Lambda_1(\Gamma_i,\Gamma_j)}{\partial\Gamma_i}-1\Bigg)+\Lambda_1(\Gamma_i,\Gamma_j)$$
$$\Bigg(2\frac{\partial\Lambda_2(\Gamma_i,\Gamma_j)}{\partial\Gamma_i}-1\Bigg)\Bigg)-4\Lambda_2(\Gamma_i,\Gamma_j)\Lambda_1(\Gamma_i,\Gamma_j)\Bigg)+4\Gamma_i\Lambda_2(\Gamma_i,\Gamma_j)\frac{\partial\Lambda_1(\Gamma_i,\Gamma_j)}{\partial\Gamma_j}\Gamma_j^3(\Gamma_i+\Lambda_2(\Gamma_i,\Gamma_j))$$
$$+4\Gamma_i\Gamma_j^2\Lambda_2(\Gamma_i,\Gamma_j)(\Gamma_i+\Lambda_2(\Gamma_i,\Gamma_j))\Bigg(\Gamma_i\frac{\partial\Lambda_1(\Gamma_i,\Gamma_j)}{\partial\Gamma_i}-\Lambda_1(\Gamma_i,\Gamma_j)\Bigg)+4\Gamma_i^3\Gamma_j\Lambda_2(\Gamma_i,\Gamma_j)^2$$

$$\frac{\partial \Lambda_1(\Gamma_i,\Gamma_j)}{\partial \Gamma_j}+4\Gamma_i^3\Lambda_2(\Gamma_i,\Gamma_j)^2\left(\Gamma_i\frac{\partial \Lambda_1(\Gamma_i,\Gamma_j)}{\partial \Gamma_i}-\Lambda_1(\Gamma_i,\Gamma_j)\right)>0,\quad \forall\{i,j\}\in\{\{1,2\},\{2,1\}\}\,. \tag{220}$$

As mentioned, we discuss in Section 6 our economic intuitions for all these sensitivities.

F Numerical Model Extensions

We provide numerical solutions for two more model extensions, i.e., one where the private signal is noisy and one where noise traders enter the markets. We compare these numerical extensions to a baseline model where the signal is perfect and there are no noise traders, like in the original model we solved analytically. However, in contrast to the original model, the baseline model in this section is solved numerically because we allow for both asset correlation and asymmetric assets (and not just for either of the two at once). Moreover, we allow the variances of δ_1 and δ_2 to be different from each other, i.e., $\sigma_{\delta 1}\neq\sigma_{\delta 2}\neq\sigma_{\delta 3}$.

For both numerical model extensions, the covariance matrix (2) generalizes to allow for asset correlation and asymmetric assets at the same time,

$$\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_1+\nu_2\end{bmatrix}\sim\mathcal{N}\left(\begin{bmatrix}\overline{\nu}_1\\ \overline{\nu}_2\\ \overline{\nu}_1+\overline{\nu}_2\end{bmatrix},\begin{bmatrix}\sigma_{\nu1}^2 & \rho\sigma_{\nu1}\sigma_{\nu2} & \sigma_{\nu1}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\\ \rho\sigma_{\nu1}\sigma_{\nu2} & \sigma_{\nu2}^2 & \sigma_{\nu2}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\\ \sigma_{\nu1}^2+\rho\sigma_{\nu1}\sigma_{\nu2} & \sigma_{\nu2}^2+\rho\sigma_{\nu1}\sigma_{\nu2} & \sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}\end{bmatrix}\right). \tag{221}$$

This section only includes the derivations and a description of our parameter calibrations. We leave the sensitivity analysis for Section 6.

F.1 Noisy Signal Derivation

We embed noise in the signal,

$$\zeta=\nu_3+\epsilon,\quad \epsilon\sim\mathcal{N}\left(0,\sigma_\epsilon^2\right). \tag{222}$$

Since $\epsilon \perp \nu_3$ we write the variance of the signal as,

$$\text{Var}[\zeta] = \sigma_{\nu 1}^2 + \sigma_{\nu 2}^2 + \sigma_\epsilon^2 + 2\rho\sigma_{\nu 1}\sigma_{\nu 2}. \tag{223}$$

This model extension is interesting because for $\sigma_\epsilon \neq 0$ the trade orders χ_1, χ_2, χ_3 become all entangled. Accordingly, the equilibrium conjectures of Theorem 1 rewrite as,

$$\begin{aligned}
\chi_1(\zeta, \delta_1, \delta_2, \delta_3) &= \beta_1(\zeta - \overline{\nu}_1 - \overline{\nu}_2) + \alpha_{11}\delta_1 + \alpha_{12}\delta_2 + \alpha_{13}\delta_3, \\
\chi_2(\zeta, \delta_1, \delta_2, \delta_3) &= \beta_2(\zeta - \overline{\nu}_1 - \overline{\nu}_2) + \alpha_{21}\delta_1 + \alpha_{22}\delta_2 + \alpha_{23}\delta_3, \\
\chi_3(\zeta, \delta_1, \delta_2, \delta_3) &= \beta_3(\zeta - \overline{\nu}_1 - \overline{\nu}_2) + \alpha_{31}\delta_1 + \alpha_{32}\delta_2 + \alpha_{33}\delta_3, \\
q_1(\chi_1 + \delta_1) &= \overline{\nu}_1 + \lambda_1(\chi_1 + \delta_1), \\
q_2(\chi_2 + \delta_2) &= \overline{\nu}_2 + \lambda_2(\chi_2 + \delta_2), \\
q_3(\chi_3 + \delta_3) &= \overline{\nu}_1 + \overline{\nu}_2 + \lambda_3(\chi_3 + \delta_3).
\end{aligned} \tag{224}$$

We re-write the mean-variance optimization problem (4) as,

$$\begin{aligned}
&\max_{\chi_1,\chi_2,\chi_3} \Bigg\{ \chi_1(-\lambda_1\chi_1 - \lambda_1\delta_1 - \overline{\nu}_1) + \chi_2(-\lambda_2\chi_2 - \lambda_2\delta_2 - \overline{\nu}_2) + \chi_3(-\lambda_3\chi_3 - \lambda_3\delta_3 - \overline{\nu}_1 - \overline{\nu}_2) \\
&+ (\chi_1 + \chi_3)\left(\overline{\nu}_1 + \frac{\sigma_{\nu 1}^2 + \rho\sigma_{\nu 1}\sigma_{\nu 2}}{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2 + \sigma_\epsilon^2 + 2\rho\sigma_{\nu 1}\sigma_{\nu 2}}(\zeta - \overline{\nu}_1 - \overline{\nu}_2)\right) \\
&+ (\chi_2 + \chi_3)\left(\overline{\nu}_2 + \frac{\sigma_{\nu 2}^2 + \rho\sigma_{\nu 1}\sigma_{\nu 2}}{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2 + \sigma_\epsilon^2 + 2\rho\sigma_{\nu 1}\sigma_{\nu 2}}(\zeta - \overline{\nu}_1 - \overline{\nu}_2)\right) \\
&- \frac{\gamma}{2}(\chi_1 + \chi_3)^2\left(\sigma_{\nu 1}^2 - \frac{(\sigma_{\nu 1}^2 + \rho\sigma_{\nu 1}\sigma_{\nu 2})^2}{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2 + \sigma_\epsilon^2 + 2\rho\sigma_{\nu 1}\sigma_{\nu 2}}\right) \\
&- \frac{\gamma}{2}(\chi_2 + \chi_3)^2\left(\sigma_{\nu 2}^2 - \frac{(\sigma_{\nu 2}^2 + \rho\sigma_{\nu 1}\sigma_{\nu 2})^2}{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2 + \sigma_\epsilon^2 + 2\rho\sigma_{\nu 1}\sigma_{\nu 2}}\right) \\
&- \gamma(\chi_1 + \chi_3)(\chi_2 + \chi_3)\left(\rho\sigma_{\nu 1}\sigma_{\nu 2} - \frac{(\sigma_{\nu 1}^2 + \rho\sigma_{\nu 1}\sigma_{\nu 2})(\sigma_{\nu 2}^2 + \rho\sigma_{\nu 1}\sigma_{\nu 2})}{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2 + \sigma_\epsilon^2 + 2\rho\sigma_{\nu 1}\sigma_{\nu 2}}\right) \Bigg\}.
\end{aligned} \tag{225}$$

The first order conditions (8) become,

$$\lambda_1(-2\chi_1 - \delta_1) - \gamma(\chi_2 + \chi_3)\left(\rho\sigma_{\nu 1}\sigma_{\nu 2} - \frac{(\sigma_{\nu 1}^2 + \rho\sigma_{\nu 1}\sigma_{\nu 2})(\sigma_{\nu 2}^2 + \rho\sigma_{\nu 1}\sigma_{\nu 2})}{\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2 + \sigma_\epsilon^2 + 2\rho\sigma_{\nu 1}\sigma_{\nu 2}}\right)$$

$$
\begin{aligned}
&-\gamma\left(\chi_1+\chi_3\right)\left(\sigma_{\nu 1}^2-\frac{\left(\sigma_{\nu 1}^2+\rho\sigma_{\nu 1}\sigma_{\nu 2}\right)^2}{\sigma_{\nu 1}^2+\sigma_{\nu 2}^2+\sigma_\epsilon^2+2\rho\sigma_{\nu 1}\sigma_{\nu 2}}\right)\\
&+\frac{\sigma_{\nu 1}^2+\rho\sigma_{\nu 1}\sigma_{\nu 2}}{\sigma_{\nu 1}^2+\sigma_{\nu 2}^2+\sigma_\epsilon^2+2\rho\sigma_{\nu 1}\sigma_{\nu 2}}\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)=0,\\
&\lambda_2\left(-2\chi_2-\delta_2\right)-\gamma\left(\chi_1+\chi_3\right)\left(\rho\sigma_{\nu 1}\sigma_{\nu 2}-\frac{\left(\sigma_{\nu 1}^2+\rho\sigma_{\nu 1}\sigma_{\nu 2}\right)\left(\sigma_{\nu 2}^2+\rho\sigma_{\nu 1}\sigma_{\nu 2}\right)}{\sigma_{\nu 1}^2+\sigma_{\nu 2}^2+\sigma_\epsilon^2+2\rho\sigma_{\nu 1}\sigma_{\nu 2}}\right)\\
&-\gamma\left(\chi_2+\chi_3\right)\left(\sigma_{\nu 2}^2-\frac{\left(\sigma_{\nu 2}^2+\rho\sigma_{\nu 1}\sigma_{\nu 2}\right)^2}{\sigma_{\nu 1}^2+\sigma_{\nu 2}^2+\sigma_\epsilon^2+2\rho\sigma_{\nu 1}\sigma_{\nu 2}}\right)\\
&+\frac{\sigma_{\nu 2}^2+\rho\sigma_{\nu 1}\sigma_{\nu 2}}{\sigma_{\nu 1}^2+\sigma_{\nu 2}^2+\sigma_\epsilon^2+2\rho\sigma_{\nu 1}\sigma_{\nu 2}}\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)=0,\\
&\lambda_3\left(-2\chi_3-\delta_3\right)-\gamma\left(\chi_2+\chi_3\right)\left(\rho\sigma_{\nu 1}\sigma_{\nu 2}-\frac{\left(\sigma_{\nu 1}^2+\rho\sigma_{\nu 1}\sigma_{\nu 2}\right)\left(\sigma_{\nu 2}^2+\rho\sigma_{\nu 1}\sigma_{\nu 2}\right)}{\sigma_{\nu 1}^2+\sigma_{\nu 2}^2+\sigma_\epsilon^2+2\rho\sigma_{\nu 1}\sigma_{\nu 2}}\right)\\
&-\gamma\left(\chi_1+\chi_3\right)\left(\rho\sigma_{\nu 1}\sigma_{\nu 2}-\frac{\left(\sigma_{\nu 1}^2+\rho\sigma_{\nu 1}\sigma_{\nu 2}\right)\left(\sigma_{\nu 2}^2+\rho\sigma_{\nu 1}\sigma_{\nu 2}\right)}{\sigma_{\nu 1}^2+\sigma_{\nu 2}^2+\sigma_\epsilon^2+2\rho\sigma_{\nu 1}\sigma_{\nu 2}}\right)\\
&-\gamma\left(\chi_2+\chi_3\right)\left(\sigma_{\nu 2}^2-\frac{\left(\sigma_{\nu 2}^2+\rho\sigma_{\nu 1}\sigma_{\nu 2}\right)^2}{\sigma_{\nu 1}^2+\sigma_{\nu 2}^2+\sigma_\epsilon^2+2\rho\sigma_{\nu 1}\sigma_{\nu 2}}\right)\\
&-\gamma\left(\chi_1+\chi_3\right)\left(\sigma_{\nu 1}^2-\frac{\left(\sigma_{\nu 1}^2+\rho\sigma_{\nu 1}\sigma_{\nu 2}\right)^2}{\sigma_{\nu 1}^2+\sigma_{\nu 2}^2+\sigma_\epsilon^2+2\rho\sigma_{\nu 1}\sigma_{\nu 2}}\right)\\
&+\frac{\sigma_{\nu 2}^2+\rho\sigma_{\nu 1}\sigma_{\nu 2}}{\sigma_{\nu 1}^2+\sigma_{\nu 2}^2+\sigma_\epsilon^2+2\rho\sigma_{\nu 1}\sigma_{\nu 2}}\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)\\
&+\frac{\sigma_{\nu 1}^2+\rho\sigma_{\nu 1}\sigma_{\nu 2}}{\sigma_{\nu 1}^2+\sigma_{\nu 2}^2+\sigma_\epsilon^2+2\rho\sigma_{\nu 1}\sigma_{\nu 2}}\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)=0.
\end{aligned}
\tag{226}
$$

The trading and offsetting intensity parameters (9) for the new equilibrium strategies in (224) become twelve,

$$
\begin{aligned}
\beta_1=&\Big(((\gamma(\rho^2-1)\sigma_{\nu 2}^2-2\lambda_2)\sigma_{\nu 1}-2\rho\lambda_2\sigma_{\nu 2})\lambda_3\sigma_{\nu 1}\Big)\Big((\gamma(\rho+1)(\sigma_\epsilon^2(\lambda_1+\lambda_2+\lambda_3)\gamma+2(\lambda_1\\
&+\lambda_2)\lambda_3)(\rho-1)\sigma_{\nu 2}^2-2\lambda_2(\sigma_\epsilon^2(\lambda_1+\lambda_3)\gamma+2\lambda_1\lambda_3))\sigma_{\nu 1}^2-4\rho\lambda_1\lambda_2\sigma_{\nu 1}(\gamma\sigma_\epsilon^2+2\lambda_3)\sigma_{\nu 2}\\
&-2\lambda_1((\sigma_\epsilon^2(\lambda_2+\lambda_3)\gamma+2\lambda_2\lambda_3)\sigma_{\nu 2}^2+2\lambda_2\lambda_3\sigma_\epsilon^2)\Big)^{-1},\\
\alpha_{11}=&-\Big(\lambda_1((\gamma(\rho-1)(\rho+1)(\gamma\sigma_\epsilon^2+2\lambda_3)\sigma_{\nu 1}^2-2\sigma_\epsilon^2(\lambda_2+\lambda_3)\gamma-4\lambda_2\lambda_3)\sigma_{\nu 2}^2-4\rho\lambda_2\sigma_{\nu 1}(\gamma\sigma_\epsilon^2\\
&+2\lambda_3)\sigma_{\nu 2}-2\lambda_2((\gamma\sigma_\epsilon^2+2\lambda_3)\sigma_{\nu 1}^2+2\lambda_3\sigma_\epsilon^2))\Big)\Big((2\gamma(\rho+1)(\sigma_\epsilon^2(\lambda_1+\lambda_2+\lambda_3)\gamma+2(\lambda_1\\
&+\lambda_2)\lambda_3)(\rho-1)\sigma_{\nu 1}^2-4\lambda_1(\sigma_\epsilon^2(\lambda_2+\lambda_3)\gamma+2\lambda_2\lambda_3))\sigma_{\nu 2}^2-8\rho\lambda_1\lambda_2\sigma_{\nu 1}(\gamma\sigma_\epsilon^2+2\lambda_3)\sigma_{\nu 2}
\end{aligned}
$$

$$- 4\lambda_2((\sigma_\epsilon^2(\lambda_1+\lambda_3)\gamma + 2\lambda_1\lambda_3)\sigma_{\nu 1}^2 + 2\lambda_1\lambda_3\sigma_\epsilon^2)\Big)^{-1},$$

$$\alpha_{12} = -\Big(\lambda_2\gamma(\sigma_{\nu 2}(\rho-1)(\rho+1)(\gamma\sigma_\epsilon^2+2\lambda_3)\sigma_{\nu 1} + 2\rho\lambda_3\sigma_\epsilon^2)\sigma_{\nu 2}\sigma_{\nu 1}\Big)\Big((2\gamma(\rho+1)(\sigma_\epsilon^2(\lambda_1+\lambda_2 + \lambda_3)\gamma + 2(\lambda_1+\lambda_2)\lambda_3)(\rho-1)\sigma_{\nu 2}^2 - 4\lambda_2(\sigma_\epsilon^2(\lambda_1+\lambda_3)\gamma + 2\lambda_1\lambda_3)\sigma_{\nu 1}^2 - 8\rho\lambda_1\lambda_2\sigma_{\nu 1}(\gamma\sigma_\epsilon^2 + 2\lambda_3)\sigma_{\nu 2} - 4((\sigma_\epsilon^2(\lambda_2+\lambda_3)\gamma + 2\lambda_2\lambda_3)\sigma_{\nu 2}^2 + 2\lambda_2\lambda_3\sigma_\epsilon^2)\lambda_1\Big)^{-1},$$

$$\alpha_{13} = \Big(\gamma((\gamma(\rho^2-1)\sigma_{\nu 2}^2 - 2\lambda_2)\sigma_{\nu 1} - 2\rho\lambda_2\sigma_{\nu 2})\lambda_3\sigma_\epsilon^2\sigma_{\nu 1}\Big)\Big((2\gamma(\rho+1)(\sigma_\epsilon^2(\lambda_1+\lambda_2+\lambda_3)\gamma + 2(\lambda_1+\lambda_2)\lambda_3)(\rho-1)\sigma_{\nu 2}^2 - 4\lambda_2(\sigma_\epsilon^2(\lambda_1+\lambda_3)\gamma + 2\lambda_1\lambda_3))\sigma_{\nu 1}^2 - 8\rho\lambda_1\lambda_2\sigma_{\nu 1}^2(\gamma\sigma_\epsilon^2 + 2\lambda_3)\sigma_{\nu 2} - 4((\sigma_\epsilon^2(\lambda_2+\lambda_3)\gamma + 2\lambda_2\lambda_3)\sigma_{\nu 2}^2 + 2\lambda_2\lambda_3\sigma_\epsilon^2)\lambda_1\Big)^{-1},$$

$$\beta_2 = \Big(\lambda_3((\gamma(\rho^2-1)\sigma_{\nu 1}^2 - 2\lambda_1)\sigma_{\nu 2} - 2\rho\lambda_1\sigma_{\nu 1})\sigma_{\nu 2}\Big)\Big((\gamma(\rho+1)(\sigma_\epsilon^2(\lambda_1+\lambda_2+\lambda_3)\gamma + 2(\lambda_1+\lambda_2)\lambda_3)(\rho-1)\sigma_{\nu 1}^2 - 2\lambda_1(\sigma_\epsilon^2(\lambda_2+\lambda_3)\gamma + 2\lambda_2\lambda_3))\sigma_{\nu 2}^2 - 4\rho\lambda_1\lambda_2\sigma_{\nu 1}(\gamma\sigma_\epsilon^2 + 2\lambda_3)\sigma_{\nu 2} - 2\lambda_2((\sigma_\epsilon^2(\lambda_1+\lambda_3)\gamma + 2\lambda_1\lambda_3)\sigma_{\nu 1}^2 + 2\lambda_1\lambda_3\sigma_\epsilon^2)\Big)^{-1},$$

$$\alpha_{21} = -\Big(\gamma(\sigma_{\nu 2}(\rho-1)(\rho+1)(\gamma\sigma_\epsilon^2+2\lambda_3)\sigma_{\nu 1} + 2\rho\lambda_3\sigma_\epsilon^2)\lambda_1\sigma_{\nu 2}\sigma_{\nu 1}\Big)\Big((2\gamma(\rho+1)(\sigma_\epsilon^2(\lambda_1+\lambda_2 + \lambda_3)\gamma + 2(\lambda_1+\lambda_2)\lambda_3)(\rho-1)\sigma_{\nu 2}^2 - 4\lambda_2(\sigma_\epsilon^2(\lambda_1+\lambda_3)\gamma + 2\lambda_1\lambda_3)\sigma_{\nu 1}^2 - 8\rho\lambda_1\lambda_2\sigma_{\nu 1}(\gamma\sigma_\epsilon^2 + 2\lambda_3)\sigma_{\nu 2} - 4((\sigma_\epsilon^2(\lambda_2+\lambda_3)\gamma + 2\lambda_2\lambda_3)\sigma_{\nu 2}^2 + 2\lambda_2\lambda_3\sigma_\epsilon^2)\lambda_1\Big)^{-1},$$

$$\alpha_{22} = -\Big(\lambda_2\big((\gamma(\rho-1)(\rho+1)(\gamma\sigma_\epsilon^2+2\lambda_3)\sigma_{\nu 2}^2 - 2\sigma_\epsilon^2(\lambda_1+\lambda_3)\gamma - 4\lambda_1\lambda_3)\sigma_{\nu 1}^2 - 4\rho\lambda_1\sigma_{\nu 2}(\gamma\sigma_\epsilon^2 + 2\lambda_3)\sigma_{\nu 1} - 2((\gamma\sigma_\epsilon^2+2\lambda_3)\sigma_{\nu 2}^2 + 2\lambda_3\sigma_\epsilon^2)\lambda_1)\big)\Big)\Big((2\gamma(\rho+1)(\sigma_\epsilon^2(\lambda_1+\lambda_2+\lambda_3)\gamma + 2(\lambda_1+\lambda_2)\lambda_3)(\rho-1)\sigma_{\nu 2}^2 - 4\lambda_2(\sigma_\epsilon^2(\lambda_1+\lambda_3)\gamma + 2\lambda_1\lambda_3))\sigma_{\nu 1}^2 - 8\rho\lambda_1\lambda_2\sigma_{\nu 1}(\gamma\sigma_\epsilon^2 + 2\lambda_3)\sigma_{\nu 2} - 4((\sigma_\epsilon^2(\lambda_2+\lambda_3)\gamma + 2\lambda_2\lambda_3)\sigma_{\nu 2}^2 + 2\lambda_2\lambda_3\sigma_\epsilon^2)\lambda_1\Big)^{-1},$$

$$\alpha_{23} = \Big(\gamma\lambda_3\sigma_\epsilon^2((\gamma(\rho^2-1)\sigma_{\nu 1}^2 - 2\lambda_1)\sigma_{\nu 2} - 2\rho\lambda_1\sigma_{\nu 1})\sigma_{\nu 2}\Big)\Big((2\gamma(\rho+1)(\sigma_\epsilon^2(\lambda_1+\lambda_2+\lambda_3)\gamma + 2(\lambda_1+\lambda_2)\lambda_3)(\rho-1)\sigma_{\nu 1}^2 - 4\lambda_1(\sigma_\epsilon^2(\lambda_2+\lambda_3)\gamma + 2\lambda_2\lambda_3))\sigma_{\nu 2}^2 - 8\rho\lambda_1\lambda_2\sigma_{\nu 1}(\gamma\sigma_\epsilon^2 + 2\lambda_3)\sigma_{\nu 2}$$

$$- 4\lambda_2((\sigma_\epsilon^2(\lambda_1+\lambda_3)\gamma + 2\lambda_1\lambda_3)\sigma_{\nu 1}^2 + 2\lambda_1\lambda_3\sigma_\epsilon^2)\Big)^{-1},$$

$$\beta_3 = \Big((\gamma(\rho-1)(\rho+1)(\lambda_1+\lambda_2)\sigma_{\nu 2}^2 - 2\lambda_1\lambda_2)\sigma_{\nu 1}^2 - 4\rho\lambda_1\lambda_2\sigma_{\nu 1}\sigma_{\nu 2} - 2\lambda_1\lambda_2\sigma_{\nu 2}^2\Big)\Big((\gamma(\rho+1)(\sigma_\epsilon^2$$
$$(\lambda_1+\lambda_2+\lambda_3)\gamma + 2(\lambda_1+\lambda_2)\lambda_3)(\rho-1)\sigma_{\nu 2}^2 - 2\lambda_2(\sigma_\epsilon^2(\lambda_1+\lambda_3)\gamma + 2\lambda_1\lambda_3))\sigma_{\nu 1}^2$$
$$- 4\rho\lambda_1\lambda_2\sigma_{\nu 1}(\gamma\sigma_\epsilon^2 + 2\lambda_3)\sigma_{\nu 2} - 2((\sigma_\epsilon^2(\lambda_2+\lambda_3)\gamma + 2\lambda_2\lambda_3)\sigma_{\nu 2}^2 + 2\lambda_2\lambda_3\sigma_\epsilon^2)\lambda_1\Big)^{-1},$$

$$\alpha_{31} = \Big(\gamma((\gamma(\rho^2-1)\sigma_{\nu 2}^2 - 2\lambda_2)\sigma_{\nu 1} - 2\rho\lambda_2\sigma_{\nu 2})\sigma_\epsilon^2\lambda_1\sigma_{\nu 1}\Big)\Big((2\gamma(\rho+1)(\sigma_\epsilon^2(\lambda_1+\lambda_2+\lambda_3)\gamma + 2(\lambda_1$$
$$+ \lambda_2)\lambda_3)(\rho-1)\sigma_{\nu 2}^2 - 4\lambda_2(\sigma_\epsilon^2(\lambda_1+\lambda_3)\gamma + 2\lambda_1\lambda_3))\sigma_{\nu 1}^2 - 8\rho\lambda_1\lambda_2\sigma_{\nu 1}(\gamma\sigma_\epsilon^2 + 2\lambda_3)\sigma_{\nu 2}$$
$$- 4((\sigma_\epsilon^2(\lambda_2+\lambda_3)\gamma + 2\lambda_2\lambda_3)\sigma_{\nu 2}^2 + 2\lambda_2\lambda_3\sigma_\epsilon^2)\lambda_1\Big)^{-1},$$

$$\alpha_{32} = \Big(\lambda_2\gamma\sigma_\epsilon^2((\gamma(\rho^2-1)\sigma_{\nu 1}^2 - 2\lambda_1)\sigma_{\nu 2} - 2\rho\lambda_1\sigma_{\nu 1})\sigma_{\nu 2}\Big)\Big((2\gamma(\rho+1)(\sigma_\epsilon^2(\lambda_1+\lambda_2+\lambda_3)\gamma + 2(\lambda_1$$
$$+ \lambda_2)\lambda_3)(\rho-1)\sigma_{\nu 1}^2 - 4\lambda_1(\sigma_\epsilon^2(\lambda_2+\lambda_3)\gamma + 2\lambda_2\lambda_3))\sigma_{\nu 2}^2 - 8\rho\lambda_1\lambda_2\sigma_{\nu 1}(\gamma\sigma_\epsilon^2 + 2\lambda_3)\sigma_{\nu 2}$$
$$- 4\lambda_2((\sigma_\epsilon^2(\lambda_1+\lambda_3)\gamma + 2\lambda_1\lambda_3)\sigma_{\nu 1}^2 + 2\lambda_1\lambda_3\sigma_\epsilon^2)\Big)^{-1},$$

$$\alpha_{33} = -\Big(\lambda_3((\gamma(\rho-1)(\rho+1)(\gamma\sigma_\epsilon^2 + 2\lambda_1 + 2\lambda_2)\sigma_{\nu 2}^2 - 2\lambda_2(\gamma\sigma_\epsilon^2 + 2\lambda_1))\sigma_{\nu 1}^2 - 8\rho\lambda_1\lambda_2\sigma_{\nu 1}\sigma_{\nu 2}$$
$$- 2((\gamma\sigma_\epsilon^2 + 2\lambda_2)\sigma_{\nu 2}^2 + 2\lambda_2\sigma_\epsilon^2)\lambda_1)\Big)\Big(\big(2\gamma(\rho+1)(\sigma_\epsilon^2(\lambda_1+\lambda_2+\lambda_3)\gamma + 2(\lambda_1+\lambda_2)\lambda_3)$$
$$(\rho-1)\sigma_{\nu 2}^2 - 4\lambda_2(\sigma_\epsilon^2(\lambda_1+\lambda_3)\gamma + 2\lambda_1\lambda_3)\big)\sigma_{\nu 1}^2 - 8\rho\lambda_1\lambda_2\sigma_{\nu 1}(\gamma\sigma_\epsilon^2 + 2\lambda_3)\sigma_{\nu 2}$$
$$- 4((\sigma_\epsilon^2(\lambda_2+\lambda_3)\gamma + 2\lambda_2\lambda_3)\sigma_{\nu 2}^2 + 2\lambda_2\lambda_3\sigma_\epsilon^2)\lambda_1\Big)^{-1}. \tag{227}$$

We solve the corresponding market maker problem for the model with noisy signal and the price impacts (11) are as follows,

$$\lambda_1 = \frac{\beta_1\left(\sigma_{\nu 1}^2 + \rho\sigma_{\nu 1}\sigma_{\nu 2}\right)}{\beta_1^2\left(\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2 + \sigma_\epsilon^2 + 2\rho\sigma_{\nu 1}\sigma_{\nu 2}\right) + \left(\alpha_{11}+1\right)^2\sigma_{\delta 1}^2 + \alpha_{12}^2\sigma_{\delta 2}^2 + \alpha_{13}^2\sigma_{\delta 3}^2},$$
$$\lambda_2 = \frac{\beta_2\left(\sigma_{\nu 2}^2 + \rho\sigma_{\nu 1}\sigma_{\nu 2}\right)}{\beta_2^2\left(\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2 + \sigma_\epsilon^2 + 2\rho\sigma_{\nu 1}\sigma_{\nu 2}\right) + \alpha_{21}^2\sigma_{\delta 1}^2 + \left(\alpha_{22}+1\right)^2\sigma_{\delta 2}^2 + \alpha_{23}^2\sigma_{\delta 3}^2},$$
$$\lambda_3 = \frac{\beta_3\left(\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2 + 2\rho\sigma_{\nu 1}\sigma_{\nu 2}\right)}{\beta_3^2\left(\sigma_{\nu 1}^2 + \sigma_{\nu 2}^2 + \sigma_\epsilon^2 + 2\rho\sigma_{\nu 1}\sigma_{\nu 2}\right) + \alpha_{31}^2\sigma_{\delta 1}^2 + \alpha_{32}^2\sigma_{\delta 2}^2 + \left(\alpha_{33}+1\right)^2\sigma_{\delta 3}^2}. \tag{228}$$

We plug the trading and offsetting intensity parameters (227) into the price impacts (228) and obtain three nonlinear equations each in $\lambda_1, \lambda_2, \lambda_3$. Once we solve numerically for λ_1, λ_2 and λ_3, we can pin down $\beta_1, \beta_2, \beta_3, \alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{31}, \alpha_{32}, \alpha_{33}$ (227). We have now pinned down our new equilibrium (224). We compute the corresponding equations for the baseline model without noise in the signal by simply setting $\sigma_\epsilon = 0$ in (225), (226), (227), and (228). Moreover, such baseline model has no noise traders since we have not introduced them yet. As such, the same baseline model acts as a benchmark of comparison for both the noisy signal extension and the noise traders extension.

F.2 Noise Traders Derivation

So far we have modeled noise trading as exogenous random demand orders submitted to the dealers by the same speculator who submits strategic demand orders. That is, the noise trades are perfectly known to the speculator. Now we allow for actual noise traders to participate in the market, as it is standard in Kyle (1985) and most of its derivative literature. In this model extension the overall noise trading is in part unknown to the speculator, since it includes a component that she cannot observe, i.e., the orders submitted to the dealers by the noise traders. We model the additional unobservable noise trading as in Kyle (1985) but we also allow for correlation in noise trading across markets (although with the same correlation coefficient),

$$\begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{\mu 1}^2 & \rho_\mu \sigma_{\mu 1} \sigma_{\mu 2} & \rho_\mu \sigma_{\mu 1} \sigma_{\mu 3} \\ \rho_\mu \sigma_{\mu 1} \sigma_{\mu 2} & \sigma_{\mu 2}^2 & \rho_\mu \sigma_{\mu 2} \sigma_{\mu 3} \\ \rho_\mu \sigma_{\mu 1} \sigma_{\mu 3} & \rho_\mu \sigma_{\mu 2} \sigma_{\mu 3} & \sigma_{\mu 3}^2 \end{bmatrix} \right). \tag{229}$$

This model extension is interesting because with the introduction of noise traders the order flow observed by the dealers becomes more uninformative. Accordingly, the equilibrium conjectures of Theorem 1 rewrite as,

$$\chi_1(\zeta, \delta_1, \delta_2, \delta_3) = \beta_1(\zeta - \overline{\nu}_1 - \overline{\nu}_2) + \alpha_{11}\delta_1 + \alpha_{12}\delta_2 + \alpha_{13}\delta_3,$$
$$\chi_2(\zeta, \delta_1, \delta_2, \delta_3) = \beta_2(\zeta - \overline{\nu}_1 - \overline{\nu}_2) + \alpha_{21}\delta_1 + \alpha_{22}\delta_2 + \alpha_{23}\delta_3,$$

$$\chi_3\left(\zeta,\delta_1,\delta_2,\delta_3\right)=\beta_3\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)+\alpha_{31}\delta_1+\alpha_{32}\delta_2+\alpha_{33}\delta_3,$$

$$\begin{aligned}
q_1\left(\chi_1+\delta_1+\mu_1\right)&=\overline{\nu}_1+\lambda_1\left(\chi_1+\delta_1+\mu_1\right),\\
q_2\left(\chi_2+\delta_2+\mu_2\right)&=\overline{\nu}_2+\lambda_2\left(\chi_2+\delta_2+\mu_2\right),\\
q_3\left(\chi_3+\delta_3+\mu_3\right)&=\overline{\nu}_1+\overline{\nu}_2+\lambda_3\left(\chi_3+\delta_3+\mu_3\right).
\end{aligned}\tag{230}$$

Note that $\beta_1,\beta_2,\beta_3,\alpha_{11},\alpha_{12},\alpha_{13},\alpha_{21},\alpha_{22},\alpha_{23},\alpha_{31},\alpha_{32},\alpha_{33}$ in (230) are not the same as in (224). The speculator's trading strategies in the model extension with noise traders are significantly more convoluted because the speculator cannot observe the trades submitted by the noise traders. Mathematically, the presence of noise traders is encapsulated by the $\sigma_{\mu1},\sigma_{\mu2},\sigma_{\mu3},\rho_\mu$ dependencies showing up in the conditional variance term of the speculator's objective function. Specifically, we re-write the mean-variance optimization problem (4) as,

$$\begin{aligned}
&\max_{\chi_1,\chi_2,\chi_3}\Bigg\{\chi_1\left(-\lambda_1\chi_1-\lambda_1\delta_1-\overline{\nu}_1\right)+\chi_2\left(-\lambda_2\chi_2-\lambda_2\delta_2-\overline{\nu}_2\right)+\chi_3\left(-\lambda_3\chi_3-\lambda_3\delta_3-\overline{\nu}_1-\overline{\nu}_2\right)\\
&+\left(\chi_1+\chi_3\right)\left(\overline{\nu}_1+\frac{\sigma_{\nu1}^2+\rho\sigma_{\nu1}\sigma_{\nu2}}{\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}}\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)\right)\\
&+\left(\chi_2+\chi_3\right)\left(\overline{\nu}_2+\frac{\sigma_{\nu2}^2+\rho\sigma_{\nu1}\sigma_{\nu2}}{\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}}\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)\right)\\
&-\frac{\gamma}{2}\left(\chi_1+\chi_3\right)^2\left(\sigma_{\nu1}^2-\frac{\left(\sigma_{\nu1}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)^2}{\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}}\right)\\
&-\frac{\gamma}{2}\left(\chi_2+\chi_3\right)^2\left(\sigma_{\nu2}^2-\frac{\left(\sigma_{\nu2}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)^2}{\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}}\right)\\
&-\gamma\left(\chi_1+\chi_3\right)\left(\chi_2+\chi_3\right)\left(\rho\sigma_{\nu1}\sigma_{\nu2}-\frac{\left(\sigma_{\nu1}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)\left(\sigma_{\nu2}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)}{\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}}\right)\\
&-\frac{\gamma}{2}\left(\chi_1^2\lambda_1^2\sigma_{\mu1}^2+\chi_2^2\lambda_2^2\sigma_{\mu2}^2+\chi_3^2\lambda_3^2\sigma_{\mu3}^2\right)\\
&-\gamma\left(\chi_1\chi_2\rho_\mu\lambda_1\lambda_2\sigma_{\mu1}\sigma_{\mu2}+\chi_1\chi_3\rho_\mu\lambda_1\lambda_3\sigma_{\mu1}\sigma_{\mu3}+\chi_2\chi_3\rho_\mu\lambda_2\lambda_3\sigma_{\mu2}\sigma_{\mu3}\right)\Bigg\}.
\end{aligned}\tag{231}$$

The first order conditions (8) become,

$$-\gamma\left(\chi_1+\chi_3\right)\left(\sigma_{\nu1}^2-\frac{\left(\sigma_{\nu1}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)^2}{\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}}\right)+\frac{\sigma_{\nu1}^2+\rho\sigma_{\nu1}\sigma_{\nu2}}{\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}}\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)$$

$$
\begin{aligned}
&+\lambda_1\left(-2\chi_1-\delta_1\right)-\gamma\left(\chi_2+\chi_3\right)\left(\rho\sigma_{\nu1}\sigma_{\nu2}-\frac{\left(\sigma_{\nu1}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)\left(\sigma_{\nu2}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)}{\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}}\right)\\
&-\gamma\chi_1\lambda_1^2\sigma_{\mu1}^2-\gamma(\chi_2\rho_\mu\sigma_{\mu1}\sigma_{\mu2}\lambda_1\lambda_2+\chi_3\rho_\mu\sigma_{\mu1}\sigma_{\mu3}\lambda_1\lambda_3)=0,\\
&-\gamma\left(\chi_2+\chi_3\right)\left(\sigma_{\nu2}^2-\frac{\left(\sigma_{\nu2}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)^2}{\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}}\right)+\frac{\sigma_{\nu2}^2+\rho\sigma_{\nu1}\sigma_{\nu2}}{\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}}\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)\\
&+\lambda_2\left(-2\chi_2-\delta_2\right)-\gamma\left(\chi_1+\chi_3\right)\left(\rho\sigma_{\nu1}\sigma_{\nu2}-\frac{\left(\sigma_{\nu1}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)\left(\sigma_{\nu2}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)}{\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}}\right)\\
&-\gamma\chi_2\lambda_2^2\sigma_{\mu2}^2-\gamma(\chi_1\rho_\mu\sigma_{\mu1}\sigma_{\mu2}\lambda_1\lambda_2+\chi_3\rho_\mu\sigma_{\mu2}\sigma_{\mu3}\lambda_2\lambda_3)=0,\\
&-\gamma\left(\chi_2+\chi_3\right)\left(\sigma_{\nu2}^2-\frac{\left(\sigma_{\nu2}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)^2}{\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}}\right)-\gamma\left(\chi_1+\chi_3\right)\left(\sigma_{\nu1}^2-\frac{\left(\sigma_{\nu1}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)^2}{\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}}\right)\\
&+\lambda_3\left(-2\chi_3-\delta_3\right)-\gamma\left(\chi_2+\chi_3\right)\left(\rho\sigma_{\nu1}\sigma_{\nu2}-\frac{\left(\sigma_{\nu1}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)\left(\sigma_{\nu2}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)}{\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}}\right)\\
&+\frac{\sigma_{\nu2}^2+\rho\sigma_{\nu1}\sigma_{\nu2}}{\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}}\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)+\frac{\sigma_{\nu1}^2+\rho\sigma_{\nu1}\sigma_{\nu2}}{\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}}\left(\zeta-\overline{\nu}_1-\overline{\nu}_2\right)\\
&-\gamma\left(\chi_1+\chi_3\right)\left(\rho\sigma_{\nu1}\sigma_{\nu2}-\frac{\left(\sigma_{\nu1}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)\left(\sigma_{\nu2}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)}{\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}}\right)\\
&-\gamma\chi_3\lambda_3^2\sigma_{\mu3}^2-\gamma(\chi_1\rho_\mu\sigma_{\mu1}\sigma_{\mu3}\lambda_1\lambda_3+\chi_2\rho_\mu\sigma_{\mu2}\sigma_{\mu3}\lambda_2\lambda_3)=0.
\end{aligned}
\tag{232}
$$

The trading and offsetting intensity parameters (9) for the new equilibrium strategies in (230) become twelve,

$$
\begin{aligned}
\beta_1=&\Big(\sigma_{\mu3}(2\sigma_{\mu2}^2(((\rho_\mu\lambda_1\sigma_{\mu1}-\lambda_3\sigma_{\mu3}(\rho_\mu+1))\sigma_{\nu1}^2)/2+(\rho_\mu\lambda_1\sigma_{\mu1}-\lambda_3\sigma_{\mu3}(\rho_\mu+1)/2)\sigma_{\nu2}\rho\sigma_{\nu1}\\
&+\sigma_{\nu2}^2\rho_\mu\lambda_1\sigma_{\mu1}/2)(\rho_\mu-1)\lambda_2^2+(\sigma_{\nu2}(\rho^2-1)\sigma_{\nu1}^2+\rho\sigma_{\nu1}\lambda_1\lambda_3\sigma_{\mu1}\sigma_{\mu3}(\rho_\mu-1)+\sigma_{\nu2}\lambda_1\lambda_3\sigma_{\mu1}\sigma_{\mu3}\\
&(\rho_\mu-1))\sigma_{\mu2}\sigma_{\nu2}\rho_\mu\lambda_2+\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1)(\rho_\mu\sigma_{\mu1}\lambda_1-\sigma_{\mu3}\lambda_3))\gamma^2+2(\sigma_{\nu1}\sigma_{\mu2}^2(\rho\sigma_{\nu2}\\
&+\sigma_{\nu1})\lambda_2^2+((-\rho_\mu\sigma_{\mu1}\sigma_{\mu3}\lambda_1+\sigma_{\mu3}^2\lambda_3)\sigma_{\nu1}^2-(\rho_\mu\sigma_{\mu1}(\sigma_{\mu2}+2\sigma_{\mu3})\lambda_1-\lambda_3\sigma_{\mu3}^2)\sigma_{\nu2}\rho\sigma_{\nu1}\\
&-\sigma_{\nu2}^2\rho_\mu\lambda_1\sigma_{\mu1}(\sigma_{\mu2}+\sigma_{\mu3}))\lambda_2-\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1))\gamma+4\sigma_{\nu1}\lambda_2(\rho\sigma_{\nu2}+\sigma_{\nu1})\Big)\\
&\Big(\sigma_{\mu3}^2((((\rho^2\rho_\mu+\rho^2-\rho_\mu-1)\sigma_{\nu2}^2+2\lambda_1^2\sigma_{\mu1}^2(\rho_\mu-1)(\rho_\mu+1/2))\sigma_{\nu1}^2+4\lambda_1^2\sigma_{\mu1}^2(\rho_\mu-1)\sigma_{\nu2}\rho(\rho_\mu\\
&+1/2)\sigma_{\nu1}+2\lambda_1^2\sigma_{\mu1}^2(\rho_\mu-1)\sigma_{\nu2}^2(\rho_\mu+1/2))\sigma_{\mu2}^2\lambda_2^2+2\sigma_{\nu1}^2\sigma_{\nu2}^2\rho_\mu\lambda_1\sigma_{\mu1}\sigma_{\mu2}(\rho-1)(\rho+1)\lambda_2\\
&+\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1^2\sigma_{\mu1}^2(\rho-1)(\rho+1)(\rho_\mu+1))(\rho_\mu-1)\lambda_3\gamma^3+2(-((\rho^2-1)\sigma_{\nu2}^2+\lambda_1(\rho_\mu-1)(\rho_\mu
\end{aligned}
$$

$$
\begin{aligned}
&+1)(\sigma_{\mu1}^2\lambda_1+\sigma_{\mu3}^2\lambda_3)\sigma_{\nu1}^2+2\rho\sigma_{\nu2}\lambda_1(\rho_\mu-1)(\rho_\mu+1)(\sigma_{\mu1}^2\lambda_1+\sigma_{\mu3}^2\lambda_3)\sigma_{\nu1}+\sigma_{\nu2}^2\lambda_1(\rho_\mu\\
&-1)(\rho_\mu+1)(\sigma_{\mu1}^2\lambda_1+\sigma_{\mu3}^2\lambda_3)\sigma_{\mu2}^2\lambda_2^2+((-2(\rho+1)(\rho_\mu\lambda_1\sigma_{\mu1}\sigma_{\mu2}+\lambda_3\sigma_{\mu3}^2/2)(\rho-1)\sigma_{\nu2}^2\\
&-\lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu-1)(\rho_\mu+1))\sigma_{\nu1}^2-2\rho\sigma_{\nu2}\lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu-1)(\rho_\mu+1)\sigma_{\nu1}\\
&-\sigma_{\nu2}^2\lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu-1)(\rho_\mu+1))\lambda_2-\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1(\rho-1)(\rho+1)(\sigma_{\mu1}^2\lambda_1+\sigma_{\mu3}^2\lambda_3))\gamma^2\\
&+4(2(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\lambda_1\sigma_{\mu2}^2\lambda_2^2+(((-\rho^2+1)\sigma_{\nu2}^2+\lambda_1^2\sigma_{\mu1}^2+\lambda_1\lambda_3\sigma_{\mu3}^2)\sigma_{\nu1}^2\\
&+2\rho\sigma_{\nu2}\lambda_1(\sigma_{\mu1}^2\lambda_1+\sigma_{\mu3}^2\lambda_3)\sigma_{\nu1}+\sigma_{\nu2}^2\lambda_1(\sigma_{\mu1}^2\lambda_1+\sigma_{\mu3}^2\lambda_3))\lambda_2-\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1(\rho-1)(\rho+1))\gamma\\
&+16(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\lambda_1\lambda_2\Big)^{-1},
\end{aligned}
$$

$$
\begin{aligned}
\alpha_{11}=&\Big(\lambda_1\big(\sigma_{\mu3}^2\lambda_3\big(2(\rho_\mu+1)(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\sigma_{\mu2}^2(\rho_\mu-1)\lambda_2^2+\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1)\big)\gamma^2\\
&+\big(-4(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\sigma_{\mu2}^2\lambda_2^2-4(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\sigma_{\mu3}^2\lambda_3\lambda_2\\
&+2\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1)\big)\gamma-8(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\lambda_2\big)\Big)\\
&\Big(\sigma_{\mu3}^2((((\rho^2\rho_\mu+\rho^2-\rho_\mu-1)\sigma_{\nu2}^2+2\lambda_1^2\sigma_{\mu1}^2(\rho_\mu-1)(\rho_\mu+1/2))\sigma_{\nu1}^2+4\lambda_1^2\sigma_{\mu1}^2(\rho_\mu-1)\sigma_{\nu2}\rho(\rho_\mu\\
&+1/2)\sigma_{\nu1}+2\lambda_1^2\sigma_{\mu1}^2(\rho_\mu-1)\sigma_{\nu2}^2(\rho_\mu+1/2))\sigma_{\mu2}^2\lambda_2^2+2\sigma_{\nu1}^2\sigma_{\nu2}^2\rho_\mu\lambda_1\sigma_{\mu1}\sigma_{\mu2}(\rho-1)(\rho+1)\lambda_2\\
&+\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1^2\sigma_{\mu1}^2(\rho-1)(\rho+1)(\rho_\mu+1))(\rho_\mu-1)\lambda_3\gamma^3+2(-((\rho^2-1)\sigma_{\nu2}^2+\lambda_1(\rho_\mu-1)(\rho_\mu\\
&+1)(\sigma_{\mu1}^2\lambda_1+\sigma_{\mu3}^2\lambda_3)\sigma_{\nu1}^2+2\rho\sigma_{\nu2}\lambda_1(\rho_\mu-1)(\rho_\mu+1)(\sigma_{\mu1}^2\lambda_1+\sigma_{\mu3}^2\lambda_3)\sigma_{\nu1}+\sigma_{\nu2}^2\lambda_1(\rho_\mu\\
&-1)(\rho_\mu+1)(\sigma_{\mu1}^2\lambda_1+\sigma_{\mu3}^2\lambda_3)\sigma_{\mu2}^2\lambda_2^2+((-2(\rho+1)(\rho_\mu\lambda_1\sigma_{\mu1}\sigma_{\mu2}+\lambda_3\sigma_{\mu3}^2/2)(\rho-1)\sigma_{\nu2}^2\\
&-\lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu-1)(\rho_\mu+1))\sigma_{\nu1}^2-2\rho\sigma_{\nu2}\lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu-1)(\rho_\mu+1)\sigma_{\nu1}\\
&-\sigma_{\nu2}^2\lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu-1)(\rho_\mu+1))\lambda_2-\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1(\rho-1)(\rho+1)(\sigma_{\mu1}^2\lambda_1+\sigma_{\mu3}^2\lambda_3))\gamma^2\\
&+4(2(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\lambda_1\sigma_{\mu2}^2\lambda_2^2+(((-\rho^2+1)\sigma_{\nu2}^2+\lambda_1^2\sigma_{\mu1}^2+\lambda_1\lambda_3\sigma_{\mu3}^2)\sigma_{\nu1}^2\\
&+2\rho\sigma_{\nu2}\lambda_1(\sigma_{\mu1}^2\lambda_1+\sigma_{\mu3}^2\lambda_3)\sigma_{\nu1}+\sigma_{\nu2}^2\lambda_1(\sigma_{\mu1}^2\lambda_1+\sigma_{\mu3}^2\lambda_3))\lambda_2-\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1(\rho-1)(\rho+1))\gamma\\
&+16(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\lambda_1\lambda_2\Big)^{-1},
\end{aligned}
$$

$$
\begin{aligned}
\alpha_{12}=&\Big(\gamma\big(\sigma_{\mu3}^2\lambda_3\big(-2\lambda_1\big(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2\big)\sigma_{\mu1}\sigma_{\mu2}(\rho_\mu-1)\rho_\mu\lambda_2+\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho\\
&+1)\big)\gamma+4\lambda_1\big(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2\big)\sigma_{\mu1}\sigma_{\mu2}\rho_\mu\lambda_2+2\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1)\big)\lambda_2\Big)\\
&\Big(\sigma_{\mu3}^2((((\rho^2\rho_\mu+\rho^2-\rho_\mu-1)\sigma_{\nu2}^2+2\lambda_1^2\sigma_{\mu1}^2(\rho_\mu-1)(\rho_\mu+1/2))\sigma_{\nu1}^2+4\lambda_1^2\sigma_{\mu1}^2(\rho_\mu-1)\sigma_{\nu2}\rho(\rho_\mu
\end{aligned}
$$

$$
\begin{aligned}
&+ 1/2)\sigma_{\nu 1} + 2\lambda_1^2\sigma_{\mu 1}^2(\rho_\mu - 1)\sigma_{\nu 2}^2(\rho_\mu + 1/2))\sigma_{\mu 2}^2\lambda_2^2 + 2\sigma_{\nu 1}^2\sigma_{\nu 2}^2\rho_\mu\lambda_1\sigma_{\mu 1}\sigma_{\mu 2}(\rho - 1)(\rho + 1)\lambda_2 \\
&+ \sigma_{\nu 1}^2\sigma_{\nu 2}^2\lambda_1^2\sigma_{\mu 1}^2(\rho - 1)(\rho + 1)(\rho_\mu + 1))(\rho_\mu - 1)\lambda_3\gamma^3 + 2(-((\rho^2 - 1)\sigma_{\nu 2}^2 + \lambda_1(\rho_\mu - 1)(\rho_\mu \\
&+ 1)(\sigma_{\mu 1}^2\lambda_1 + \sigma_{\mu 3}^2\lambda_3)\sigma_{\nu 1}^2 + 2\rho\sigma_{\nu 2}\lambda_1(\rho_\mu - 1)(\rho_\mu + 1)(\sigma_{\mu 1}^2\lambda_1 + \sigma_{\mu 3}^2\lambda_3)\sigma_{\nu 1} + \sigma_{\nu 2}^2\lambda_1(\rho_\mu \\
&- 1)(\rho_\mu + 1)(\sigma_{\mu 1}^2\lambda_1 + \sigma_{\mu 3}^2\lambda_3)\sigma_{\mu 2}^2\lambda_2^2 + ((-2(\rho + 1)(\rho_\mu\lambda_1\sigma_{\mu 1}\sigma_{\mu 2} + \lambda_3\sigma_{\mu 3}^2/2)(\rho - 1)\sigma_{\nu 2}^2 \\
&- \lambda_1^2\lambda_3\sigma_{\mu 1}^2\sigma_{\mu 3}^2(\rho_\mu - 1)(\rho_\mu + 1))\sigma_{\nu 1}^2 - 2\rho\sigma_{\nu 2}\lambda_1^2\lambda_3\sigma_{\mu 1}^2\sigma_{\mu 3}^2(\rho_\mu - 1)(\rho_\mu + 1)\sigma_{\nu 1} \\
&- \sigma_{\nu 2}^2\lambda_1^2\lambda_3\sigma_{\mu 1}^2\sigma_{\mu 3}^2(\rho_\mu - 1)(\rho_\mu + 1))\lambda_2 - \sigma_{\nu 1}^2\sigma_{\nu 2}^2\lambda_1(\rho - 1)(\rho + 1)(\sigma_{\mu 1}^2\lambda_1 + \sigma_{\mu 3}^2\lambda_3))\gamma^2 \\
&+ 4(2(\rho\sigma_{\nu 1}\sigma_{\nu 2} + 1/2\sigma_{\nu 1}^2 + 1/2\sigma_{\nu 2}^2)\lambda_1\sigma_{\mu 2}^2\lambda_2^2 + (((-\rho^2 + 1)\sigma_{\nu 2}^2 + \lambda_1^2\sigma_{\mu 1}^2 + \lambda_1\lambda_3\sigma_{\mu 3}^2)\sigma_{\nu 1}^2 \\
&+ 2\rho\sigma_{\nu 2}\lambda_1(\sigma_{\mu 1}^2\lambda_1 + \sigma_{\mu 3}^2\lambda_3)\sigma_{\nu 1} + \sigma_{\nu 2}^2\lambda_1(\sigma_{\mu 1}^2\lambda_1 + \sigma_{\mu 3}^2\lambda_3))\lambda_2 - \sigma_{\nu 1}^2\sigma_{\nu 2}^2\lambda_1(\rho - 1)(\rho + 1))\gamma \\
&+ 16(\rho\sigma_{\nu 1}\sigma_{\nu 2} + 1/2\sigma_{\nu 1}^2 + 1/2\sigma_{\nu 2}^2)\lambda_1\lambda_2\Big)^{-1},
\end{aligned}
$$

$$
\begin{aligned}
\alpha_{13} = -\Big(&\big(\big(\big(2(\rho\sigma_{\nu 1}\sigma_{\nu 2} + 1/2\sigma_{\nu 1}^2 + 1/2\sigma_{\nu 2}^2)\sigma_{\mu 2}^2(\rho_\mu - 1)\lambda_2^2 + \sigma_{\nu 1}^2\sigma_{\nu 2}^2(\rho - 1)(\rho + 1)\big)\sigma_{\mu 1}\lambda_1 \\
&+ \sigma_{\nu 1}^2\sigma_{\nu 2}^2(\rho - 1)(\rho + 1)\lambda_2\sigma_{\mu 2})\gamma - 4\lambda_1(\rho\sigma_{\nu 1}\sigma_{\nu 2} + 1/2\sigma_{\nu 1}^2 + 1/2\sigma_{\nu 2}^2)\sigma_{\mu 1}\lambda_2)\gamma\sigma_{\mu 3}\lambda_3\rho_\mu\Big) \\
&\Big(\sigma_{\mu 3}^2((((\rho^2\rho_\mu + \rho^2 - \rho_\mu - 1)\sigma_{\nu 2}^2 + 2\lambda_1^2\sigma_{\mu 1}^2(\rho_\mu - 1)(\rho_\mu + 1/2))\sigma_{\nu 1}^2 + 4\lambda_1^2\sigma_{\mu 1}^2(\rho_\mu - 1)\sigma_{\nu 2}\rho(\rho_\mu \\
&+ 1/2)\sigma_{\nu 1} + 2\lambda_1^2\sigma_{\mu 1}^2(\rho_\mu - 1)\sigma_{\nu 2}^2(\rho_\mu + 1/2))\sigma_{\mu 2}^2\lambda_2^2 + 2\sigma_{\nu 1}^2\sigma_{\nu 2}^2\rho_\mu\lambda_1\sigma_{\mu 1}\sigma_{\mu 2}(\rho - 1)(\rho + 1)\lambda_2 \\
&+ \sigma_{\nu 1}^2\sigma_{\nu 2}^2\lambda_1^2\sigma_{\mu 1}^2(\rho - 1)(\rho + 1)(\rho_\mu + 1))(\rho_\mu - 1)\lambda_3\gamma^3 + 2(-((\rho^2 - 1)\sigma_{\nu 2}^2 + \lambda_1(\rho_\mu - 1)(\rho_\mu \\
&+ 1)(\sigma_{\mu 1}^2\lambda_1 + \sigma_{\mu 3}^2\lambda_3)\sigma_{\nu 1}^2 + 2\rho\sigma_{\nu 2}\lambda_1(\rho_\mu - 1)(\rho_\mu + 1)(\sigma_{\mu 1}^2\lambda_1 + \sigma_{\mu 3}^2\lambda_3)\sigma_{\nu 1} + \sigma_{\nu 2}^2\lambda_1(\rho_\mu \\
&- 1)(\rho_\mu + 1)(\sigma_{\mu 1}^2\lambda_1 + \sigma_{\mu 3}^2\lambda_3)\sigma_{\mu 2}^2\lambda_2^2 + ((-2(\rho + 1)(\rho_\mu\lambda_1\sigma_{\mu 1}\sigma_{\mu 2} + \lambda_3\sigma_{\mu 3}^2/2)(\rho - 1)\sigma_{\nu 2}^2 \\
&- \lambda_1^2\lambda_3\sigma_{\mu 1}^2\sigma_{\mu 3}^2(\rho_\mu - 1)(\rho_\mu + 1))\sigma_{\nu 1}^2 - 2\rho\sigma_{\nu 2}\lambda_1^2\lambda_3\sigma_{\mu 1}^2\sigma_{\mu 3}^2(\rho_\mu - 1)(\rho_\mu + 1)\sigma_{\nu 1} \\
&- \sigma_{\nu 2}^2\lambda_1^2\lambda_3\sigma_{\mu 1}^2\sigma_{\mu 3}^2(\rho_\mu - 1)(\rho_\mu + 1))\lambda_2 - \sigma_{\nu 1}^2\sigma_{\nu 2}^2\lambda_1(\rho - 1)(\rho + 1)(\sigma_{\mu 1}^2\lambda_1 + \sigma_{\mu 3}^2\lambda_3))\gamma^2 \\
&+ 4(2(\rho\sigma_{\nu 1}\sigma_{\nu 2} + 1/2\sigma_{\nu 1}^2 + 1/2\sigma_{\nu 2}^2)\lambda_1\sigma_{\mu 2}^2\lambda_2^2 + (((-\rho^2 + 1)\sigma_{\nu 2}^2 + \lambda_1^2\sigma_{\mu 1}^2 + \lambda_1\lambda_3\sigma_{\mu 3}^2)\sigma_{\nu 1}^2 \\
&+ 2\rho\sigma_{\nu 2}\lambda_1(\sigma_{\mu 1}^2\lambda_1 + \sigma_{\mu 3}^2\lambda_3)\sigma_{\nu 1} + \sigma_{\nu 2}^2\lambda_1(\sigma_{\mu 1}^2\lambda_1 + \sigma_{\mu 3}^2\lambda_3))\lambda_2 - \sigma_{\nu 1}^2\sigma_{\nu 2}^2\lambda_1(\rho - 1)(\rho + 1))\gamma \\
&+ 16(\rho\sigma_{\nu 1}\sigma_{\nu 2} + 1/2\sigma_{\nu 1}^2 + 1/2\sigma_{\nu 2}^2)\lambda_1\lambda_2\Big)^{-1},
\end{aligned}
$$

$$
\begin{aligned}
\beta_2 = \Big(&\sigma_{\mu 3}(2\sigma_{\mu 1}^2(\rho_\mu - 1)(((\rho_\mu\lambda_2\sigma_{\mu 2} - \lambda_3\sigma_{\mu 3}(\rho_\mu + 1))\sigma_{\nu 2}^2)/2 + \sigma_{\nu 1}\rho(\rho_\mu\lambda_2\sigma_{\mu 2} - \lambda_3\sigma_{\mu 3}(\rho_\mu + 1)/2) \\
&\sigma_{\nu 2} + \sigma_{\nu 1}^2\rho_\mu\lambda_2\sigma_{\mu 2}/2)\lambda_1^2 + \sigma_{\nu 1}(\sigma_{\nu 1}(\rho^2 - 1)\sigma_{\nu 2}^2 + \rho\sigma_{\nu 2}\lambda_2\lambda_3\sigma_{\mu 2}\sigma_{\mu 3}(\rho_\mu - 1) + \sigma_{\nu 1}\lambda_2\lambda_3\sigma_{\mu 2}\sigma_{\mu 3}
\end{aligned}
$$

$$
\begin{aligned}
&(\rho_\mu - 1))\sigma_{\mu1}\rho_\mu\lambda_1 + \sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1)(\rho_\mu\sigma_{\mu2}\lambda_2 - \sigma_{\mu3}\lambda_3)\gamma^2 + 2(\sigma_{\nu2}\sigma_{\mu1}^2(\rho\sigma_{\nu1}+\sigma_{\nu2})\lambda_1^2 \\
&+ ((-\rho_\mu\sigma_{\mu2}\sigma_{\mu3}\lambda_2 + \sigma_{\mu3}^2\lambda_3)\sigma_{\nu2}^2 - \rho\sigma_{\nu1}(\rho_\mu\sigma_{\mu2}(\sigma_{\mu1}+2\sigma_{\mu3})\lambda_2 - \lambda_3\sigma_{\mu3}^2)\sigma_{\nu2} - \sigma_{\nu1}^2\rho_\mu\lambda_2\sigma_{\mu2}(\sigma_{\mu1} \\
&+ \sigma_{\mu3})\lambda_1 - \sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1))\gamma + 4\sigma_{\nu2}\lambda_1(\rho\sigma_{\nu1}+\sigma_{\nu2})\Big) \\
&\Big(\sigma_{\mu3}^2((((\rho^2\rho_\mu + \rho^2 - \rho_\mu - 1)\sigma_{\nu2}^2 + 2\lambda_1^2\sigma_{\mu1}^2(\rho_\mu-1)(\rho_\mu+1/2))\sigma_{\nu1}^2 + 4\lambda_1^2\sigma_{\mu1}^2(\rho_\mu-1)\sigma_{\nu2}\rho(\rho_\mu \\
&+ 1/2)\sigma_{\nu1} + 2\lambda_1^2\sigma_{\mu1}^2(\rho_\mu-1)\sigma_{\nu2}^2(\rho_\mu+1/2))\sigma_{\mu2}^2\lambda_2^2 + 2\sigma_{\nu1}^2\sigma_{\nu2}^2\rho_\mu\lambda_1\sigma_{\mu1}\sigma_{\mu2}(\rho-1)(\rho+1)\lambda_2 \\
&+ \sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1^2\sigma_{\mu1}^2(\rho-1)(\rho+1)(\rho_\mu+1))(\rho_\mu-1)\lambda_3\gamma^3 + 2(-((\rho^2-1)\sigma_{\nu2}^2 + \lambda_1(\rho_\mu-1)(\rho_\mu \\
&+ 1)(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3)\sigma_{\nu1}^2 + 2\rho\sigma_{\nu2}\lambda_1(\rho_\mu-1)(\rho_\mu+1)(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3)\sigma_{\nu1} + \sigma_{\nu2}^2\lambda_1(\rho_\mu \\
&- 1)(\rho_\mu+1)(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3)\sigma_{\mu2}^2\lambda_2^2 + ((-2(\rho+1)(\rho_\mu\lambda_1\sigma_{\mu1}\sigma_{\mu2} + \lambda_3\sigma_{\mu3}^2/2)(\rho-1)\sigma_{\nu2}^2 \\
&- \lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu-1)(\rho_\mu+1))\sigma_{\nu1}^2 - 2\rho\sigma_{\nu2}\lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu-1)(\rho_\mu+1)\sigma_{\nu1} \\
&- \sigma_{\nu2}^2\lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu-1)(\rho_\mu+1))\lambda_2 - \sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1(\rho-1)(\rho+1)(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3))\gamma^2 \\
&+ 4(2(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\lambda_1\sigma_{\mu2}^2\lambda_2^2 + (((-\rho^2+1)\sigma_{\nu2}^2 + \lambda_1^2\sigma_{\mu1}^2 + \lambda_1\lambda_3\sigma_{\mu3}^2)\sigma_{\nu1}^2 \\
&+ 2\rho\sigma_{\nu2}\lambda_1(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3)\sigma_{\nu1} + \sigma_{\nu2}^2\lambda_1(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3))\lambda_2 - \sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1(\rho-1)(\rho+1))\gamma \\
&+ 16(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\lambda_1\lambda_2\Big)^{-1},
\end{aligned}
$$

$$
\begin{aligned}
\alpha_{21} = &\Big(\lambda_1\gamma(\sigma_{\mu3}^2\lambda_3(-2\lambda_1(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\sigma_{\mu1}\sigma_{\mu2}(\rho_\mu-1)\rho_\mu\lambda_2 + \sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho \\
&+ 1))\gamma + 4\lambda_1(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\sigma_{\mu1}\sigma_{\mu2}\rho_\mu\lambda_2 + 2\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1)\Big) \\
&\Big(\sigma_{\mu3}^2((((\rho^2\rho_\mu + \rho^2 - \rho_\mu - 1)\sigma_{\nu2}^2 + 2\lambda_1^2\sigma_{\mu1}^2(\rho_\mu-1)(\rho_\mu+1/2))\sigma_{\nu1}^2 + 4\lambda_1^2\sigma_{\mu1}^2(\rho_\mu-1)\sigma_{\nu2}\rho(\rho_\mu \\
&+ 1/2)\sigma_{\nu1} + 2\lambda_1^2\sigma_{\mu1}^2(\rho_\mu-1)\sigma_{\nu2}^2(\rho_\mu+1/2))\sigma_{\mu2}^2\lambda_2^2 + 2\sigma_{\nu1}^2\sigma_{\nu2}^2\rho_\mu\lambda_1\sigma_{\mu1}\sigma_{\mu2}(\rho-1)(\rho+1)\lambda_2 \\
&+ \sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1^2\sigma_{\mu1}^2(\rho-1)(\rho+1)(\rho_\mu+1))(\rho_\mu-1)\lambda_3\gamma^3 + 2(-((\rho^2-1)\sigma_{\nu2}^2 + \lambda_1(\rho_\mu-1)(\rho_\mu \\
&+ 1)(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3)\sigma_{\nu1}^2 + 2\rho\sigma_{\nu2}\lambda_1(\rho_\mu-1)(\rho_\mu+1)(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3)\sigma_{\nu1} + \sigma_{\nu2}^2\lambda_1(\rho_\mu \\
&- 1)(\rho_\mu+1)(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3)\sigma_{\mu2}^2\lambda_2^2 + ((-2(\rho+1)(\rho_\mu\lambda_1\sigma_{\mu1}\sigma_{\mu2} + \lambda_3\sigma_{\mu3}^2/2)(\rho-1)\sigma_{\nu2}^2 \\
&- \lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu-1)(\rho_\mu+1))\sigma_{\nu1}^2 - 2\rho\sigma_{\nu2}\lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu-1)(\rho_\mu+1)\sigma_{\nu1} \\
&- \sigma_{\nu2}^2\lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu-1)(\rho_\mu+1))\lambda_2 - \sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1(\rho-1)(\rho+1)(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3))\gamma^2 \\
&+ 4(2(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\lambda_1\sigma_{\mu2}^2\lambda_2^2 + (((-\rho^2+1)\sigma_{\nu2}^2 + \lambda_1^2\sigma_{\mu1}^2 + \lambda_1\lambda_3\sigma_{\mu3}^2)\sigma_{\nu1}^2 \\
&+ 2\rho\sigma_{\nu2}\lambda_1(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3)\sigma_{\nu1} + \sigma_{\nu2}^2\lambda_1(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3))\lambda_2 - \sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1(\rho-1)(\rho+1))\gamma
\end{aligned}
$$

$$+ 16(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\lambda_1\lambda_2\Big)^{-1},$$

$$\alpha_{22} = \Big(\lambda_2\big(\big(2\big(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2\big)\sigma_{\mu1}^2(\rho_\mu + 1)(\rho_\mu - 1)\lambda_1^2 + \sigma_{\nu1}^2\sigma_{\nu2}^2(\rho - 1)(\rho + 1)\big)$$
$$\sigma_{\mu3}^2\lambda_3\gamma^2 + \big(-4\big(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2\big)\,\sigma_{\mu1}^2\lambda_1^2 - 4\sigma_{\mu3}^2(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)$$
$$\lambda_3\lambda_1 + 2\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho - 1)(\rho + 1)\big)\gamma - 8\lambda_1(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2))\Big)$$
$$\Big(\sigma_{\mu3}^2((((\rho^2\rho_\mu + \rho^2 - \rho_\mu - 1)\sigma_{\nu2}^2 + 2\lambda_1^2\sigma_{\mu1}^2(\rho_\mu - 1)(\rho_\mu + 1/2))\sigma_{\nu1}^2 + 4\lambda_1^2\sigma_{\mu1}^2(\rho_\mu - 1)\sigma_{\nu2}\rho(\rho_\mu$$
$$+ 1/2)\sigma_{\nu1} + 2\lambda_1^2\sigma_{\mu1}^2(\rho_\mu - 1)\sigma_{\nu2}^2(\rho_\mu + 1/2))\sigma_{\mu2}^2\lambda_2^2 + 2\sigma_{\nu1}^2\sigma_{\nu2}^2\rho_\mu\lambda_1\sigma_{\mu1}\sigma_{\mu2}(\rho - 1)(\rho + 1)\lambda_2$$
$$+ \sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1^2\sigma_{\mu1}^2(\rho - 1)(\rho + 1)(\rho_\mu + 1))(\rho_\mu - 1)\lambda_3\gamma^3 + 2(-((\rho^2 - 1)\sigma_{\nu2}^2 + \lambda_1(\rho_\mu - 1)(\rho_\mu$$
$$+ 1)(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3)\sigma_{\nu1}^2 + 2\rho\sigma_{\nu2}\lambda_1(\rho_\mu - 1)(\rho_\mu + 1)(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3)\sigma_{\nu1} + \sigma_{\nu2}^2\lambda_1(\rho_\mu$$
$$- 1)(\rho_\mu + 1)(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3)\sigma_{\mu2}^2\lambda_2^2 + ((-2(\rho + 1)(\rho_\mu\lambda_1\sigma_{\mu1}\sigma_{\mu2} + \lambda_3\sigma_{\mu3}^2/2)(\rho - 1)\sigma_{\nu2}^2$$
$$- \lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu - 1)(\rho_\mu + 1))\sigma_{\nu1}^2 - 2\rho\sigma_{\nu2}\lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu - 1)(\rho_\mu + 1)\sigma_{\nu1}$$
$$- \sigma_{\nu2}^2\lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu - 1)(\rho_\mu + 1))\lambda_2 - \sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1(\rho - 1)(\rho + 1)(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3))\gamma^2$$
$$+ 4(2(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\lambda_1\sigma_{\mu2}^2\lambda_2^2 + (((-\rho^2 + 1)\sigma_{\nu2}^2 + \lambda_1^2\sigma_{\mu1}^2 + \lambda_1\lambda_3\sigma_{\mu3}^2)\sigma_{\nu1}^2$$
$$+ 2\rho\sigma_{\nu2}\lambda_1(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3)\sigma_{\nu1} + \sigma_{\nu2}^2\lambda_1(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3))\lambda_2 - \sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1(\rho - 1)(\rho + 1))\gamma$$
$$+ 16(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\lambda_1\lambda_2\Big)^{-1},$$

$$\alpha_{23} = -\Big(\gamma\sigma_{\mu3}\lambda_3\rho_\mu\big(\big(2\big(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2\big)\sigma_{\mu1}^2\sigma_{\mu2}\lambda_2(\rho_\mu - 1)\lambda_1^2 + \sigma_{\nu1}^2\sigma_{\nu2}^2\sigma_{\mu1}(\rho - 1)(\rho$$
$$+ 1)\lambda_1 + (\rho - 1)(\rho + 1)\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2\sigma_{\mu2}\big)\gamma - 4\lambda_1(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\sigma_{\mu2}\lambda_2\big)\Big)$$
$$\Big(\sigma_{\mu3}^2((((\rho^2\rho_\mu + \rho^2 - \rho_\mu - 1)\sigma_{\nu2}^2 + 2\lambda_1^2\sigma_{\mu1}^2(\rho_\mu - 1)(\rho_\mu + 1/2))\sigma_{\nu1}^2 + 4\lambda_1^2\sigma_{\mu1}^2(\rho_\mu - 1)\sigma_{\nu2}\rho(\rho_\mu$$
$$+ 1/2)\sigma_{\nu1} + 2\lambda_1^2\sigma_{\mu1}^2(\rho_\mu - 1)\sigma_{\nu2}^2(\rho_\mu + 1/2))\sigma_{\mu2}^2\lambda_2^2 + 2\sigma_{\nu1}^2\sigma_{\nu2}^2\rho_\mu\lambda_1\sigma_{\mu1}\sigma_{\mu2}(\rho - 1)(\rho + 1)\lambda_2$$
$$+ \sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1^2\sigma_{\mu1}^2(\rho - 1)(\rho + 1)(\rho_\mu + 1))(\rho_\mu - 1)\lambda_3\gamma^3 + 2(-((\rho^2 - 1)\sigma_{\nu2}^2 + \lambda_1(\rho_\mu - 1)(\rho_\mu$$
$$+ 1)(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3)\sigma_{\nu1}^2 + 2\rho\sigma_{\nu2}\lambda_1(\rho_\mu - 1)(\rho_\mu + 1)(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3)\sigma_{\nu1} + \sigma_{\nu2}^2\lambda_1(\rho_\mu$$
$$- 1)(\rho_\mu + 1)(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3)\sigma_{\mu2}^2\lambda_2^2 + ((-2(\rho + 1)(\rho_\mu\lambda_1\sigma_{\mu1}\sigma_{\mu2} + \lambda_3\sigma_{\mu3}^2/2)(\rho - 1)\sigma_{\nu2}^2$$
$$- \lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu - 1)(\rho_\mu + 1))\sigma_{\nu1}^2 - 2\rho\sigma_{\nu2}\lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu - 1)(\rho_\mu + 1)\sigma_{\nu1}$$
$$- \sigma_{\nu2}^2\lambda_1^2\lambda_3\sigma_{\mu1}^2\sigma_{\mu3}^2(\rho_\mu - 1)(\rho_\mu + 1))\lambda_2 - \sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1(\rho - 1)(\rho + 1)(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3))\gamma^2$$

$$+ 4(2(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\lambda_1\sigma_{\mu2}^2\lambda_2^2 + (((-\rho^2+1)\sigma_{\nu2}^2 + \lambda_1^2\sigma_{\mu1}^2 + \lambda_1\lambda_3\sigma_{\mu3}^2)\sigma_{\nu1}^2$$
$$+ 2\rho\sigma_{\nu2}\lambda_1(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3)\sigma_{\nu1} + \sigma_{\nu2}^2\lambda_1(\sigma_{\mu1}^2\lambda_1 + \sigma_{\mu3}^2\lambda_3))\lambda_2 - \sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_1(\rho-1)(\rho+1))\gamma$$
$$+ 16(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\lambda_1\lambda_2\Big)^{-1},$$

$$\beta_3 = \Big(\big(- (2(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)(\rho_\mu+1)\sigma_{\mu2}^2(\rho_\mu-1)\lambda_2^2 - \sigma_{\nu2}\rho_\mu\lambda_3\sigma_{\mu2}\sigma_{\mu3}(\rho_\mu-1)(\rho\sigma_{\nu1}$$
$$+ \sigma_{\nu2})\lambda_2 + \sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1))\sigma_{\mu1}^2\lambda_1^2 - 2\sigma_{\nu1}\sigma_{\mu1}(-\lambda_3\sigma_{\mu2}^2\sigma_{\mu3}(\rho_\mu-1)(\rho\sigma_{\nu2}+\sigma_{\nu1})\lambda_2^2/2$$
$$+ \sigma_{\nu1}\sigma_{\nu2}^2\sigma_{\mu2}(\rho-1)(\rho+1)\lambda_2 - \sigma_{\nu1}\sigma_{\nu2}^2\lambda_3\sigma_{\mu3}(\rho-1)(\rho+1)/2)\rho_\mu\lambda_1 + \sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2\sigma_{\mu2}(\rho-1)(\rho$$
$$+ 1)(\rho_\mu\sigma_{\mu3}\lambda_3 - \sigma_{\mu2}\lambda_2))\gamma^2 + (4(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\sigma_{\mu1}^2\lambda_2\lambda_1^2 + (4(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2$$
$$+ 1/2\sigma_{\nu2}^2)\sigma_{\mu2}^2\lambda_2^2 - 2\sigma_{\mu3}(\sigma_{\nu1}^2\sigma_{\mu1} + \rho\sigma_{\nu2}(\sigma_{\mu1}+\sigma_{\mu2})\sigma_{\nu1} + \sigma_{\nu2}^2\sigma_{\mu2})\lambda_3\rho_\mu\lambda_2 - 2\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho$$
$$+ 1))\lambda_1 - 2\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2(\rho-1)(\rho+1))\gamma + 8\lambda_1(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\lambda_2\Big)$$
$$\Big(\big(\sigma_{\mu3}^2(\sigma_{\mu1}^2(4(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)(\rho_\mu+1/2)\sigma_{\mu2}^2(\rho_\mu-1)\lambda_2^2 + \sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho$$
$$+ 1)(\rho_\mu+1))\lambda_1^2 + 2\sigma_{\nu1}^2\sigma_{\nu2}^2\rho_\mu\lambda_1\sigma_{\mu1}\sigma_{\mu2}(\rho-1)(\rho+1)\lambda_2 + \sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2^2\sigma_{\mu2}^2(\rho-1)(\rho+1)(\rho_\mu$$
$$+ 1))\lambda_3(\rho_\mu-1)\gamma^3 + (-2\sigma_{\mu1}^2(2(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)(\rho_\mu+1)\sigma_{\mu2}^2(\rho_\mu-1)\lambda_2^2$$
$$+ 2\sigma_{\mu3}^2(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)(\rho_\mu+1)\lambda_3(\rho_\mu-1)\lambda_2 + \sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1))\lambda_1^2$$
$$+ (-4\sigma_{\mu3}^2(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)(\rho_\mu+1)\sigma_{\mu2}^2\lambda_3(\rho_\mu-1)\lambda_2^2 - 4\sigma_{\nu1}^2\sigma_{\nu2}^2\rho_\mu\lambda_2\sigma_{\mu1}\sigma_{\mu2}(\rho$$
$$- 1)(\rho+1) - 2\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_3\sigma_{\mu3}^2(\rho-1)(\rho+1))\lambda_1 - 2\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2(\rho-1)(\rho+1)(\sigma_{\mu2}^2\lambda_2$$
$$+ \sigma_{\mu3}^2\lambda_3))\gamma^2 + \big(8(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\sigma_{\mu1}^2\lambda_2\lambda_1^2 + \big(8(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2$$
$$+ 1/2\sigma_{\nu2}^2)\sigma_{\mu2}^2\lambda_2^2 + 8\sigma_{\mu3}^2(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\lambda_3\lambda_2 - 4\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1)\big)\lambda_1$$
$$- 4\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2(\rho-1)(\rho+1)\big)\gamma + 16\lambda_1(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\lambda_2\big)\lambda_3\Big)^{-1},$$

$$\alpha_{31} = -\Big(\big(((2(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\sigma_{\mu2}^2(\rho_\mu-1)\lambda_2^2 + \sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1))\sigma_{\mu1}\lambda_1$$
$$+ \sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2(\rho-1)(\rho+1)\sigma_{\mu2})\gamma - 4\lambda_1(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)\sigma_{\mu1}\lambda_2)\lambda_1\gamma\sigma_{\mu3}\rho_\mu\Big)$$
$$\Big(\sigma_{\mu3}^2(\sigma_{\mu1}^2(4(\rho\sigma_{\nu1}\sigma_{\nu2} + 1/2\sigma_{\nu1}^2 + 1/2\sigma_{\nu2}^2)(\rho_\mu+1/2)\sigma_{\mu2}^2(\rho_\mu-1)\lambda_2^2 + \sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho$$
$$+ 1)(\rho_\mu+1))\lambda_1^2 + 2\sigma_{\nu1}^2\sigma_{\nu2}^2\rho_\mu\lambda_1\sigma_{\mu1}\sigma_{\mu2}(\rho-1)(\rho+1)\lambda_2 + \sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2^2\sigma_{\mu2}^2(\rho-1)(\rho+1)(\rho_\mu$$

$$
\begin{aligned}
&+1))\lambda_3(\rho_\mu-1)\gamma^3+(-2\sigma_{\mu1}^2(2(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)(\rho_\mu+1)\sigma_{\mu2}^2(\rho_\mu-1)\lambda_2^2\\
&+2\sigma_{\mu3}^2(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)(\rho_\mu+1)\lambda_3(\rho_\mu-1)\lambda_2+\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1))\lambda_1^2\\
&+(-4\sigma_{\mu3}^2(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)(\rho_\mu+1)\sigma_{\mu2}^2\lambda_3(\rho_\mu-1)\lambda_2^2-4\sigma_{\nu1}^2\sigma_{\nu2}^2\rho_\mu\lambda_2\sigma_{\mu1}\sigma_{\mu2}(\rho\\
&-1)(\rho+1)-2\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_3\sigma_{\mu3}^2(\rho-1)(\rho+1))\lambda_1-2\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2(\rho-1)(\rho+1)(\sigma_{\mu2}^2\lambda_2\\
&+\sigma_{\mu3}^2\lambda_3))\gamma^2+(8(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\sigma_{\mu1}^2\lambda_2\lambda_1^2+(8(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2\\
&+1/2\sigma_{\nu2}^2)\sigma_{\mu2}^2\lambda_2^2+8\sigma_{\mu3}^2(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\lambda_3\lambda_2-4\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1))\lambda_1\\
&-4\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2(\rho-1)(\rho+1))\gamma+16\lambda_1(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\lambda_2\Big)^{-1},
\end{aligned}
$$

$$
\begin{aligned}
\alpha_{32}=&-\Big(\gamma\sigma_{\mu3}\lambda_2\rho_\mu\big(((2(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\sigma_{\mu1}^2(\rho_\mu-1)\lambda_1^2+\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1))\sigma_{\mu2}\lambda_2\\
&+\sigma_{\nu1}^2\sigma_{\nu2}^2\sigma_{\mu1}(\rho-1)(\rho+1)\lambda_1\big)\gamma-4\lambda_1(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\sigma_{\mu2}\lambda_2\big)\Big)\\
&\Big(\sigma_{\mu3}^2(\sigma_{\mu1}^2(4(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)(\rho_\mu+1/2)\sigma_{\mu2}^2(\rho_\mu-1)\lambda_2^2+\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho\\
&+1)(\rho_\mu+1))\lambda_1^2+2\sigma_{\nu1}^2\sigma_{\nu2}^2\rho_\mu\lambda_1\sigma_{\mu1}\sigma_{\mu2}(\rho-1)(\rho+1)\lambda_2+\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2^2\sigma_{\mu2}^2(\rho-1)(\rho+1)(\rho_\mu\\
&+1))\lambda_3(\rho_\mu-1)\gamma^3+(-2\sigma_{\mu1}^2(2(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)(\rho_\mu+1)\sigma_{\mu2}^2(\rho_\mu-1)\lambda_2^2\\
&+2\sigma_{\mu3}^2(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)(\rho_\mu+1)\lambda_3(\rho_\mu-1)\lambda_2+\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1))\lambda_1^2\\
&+(-4\sigma_{\mu3}^2(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)(\rho_\mu+1)\sigma_{\mu2}^2\lambda_3(\rho_\mu-1)\lambda_2^2-4\sigma_{\nu1}^2\sigma_{\nu2}^2\rho_\mu\lambda_2\sigma_{\mu1}\sigma_{\mu2}(\rho\\
&-1)(\rho+1)-2\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_3\sigma_{\mu3}^2(\rho-1)(\rho+1))\lambda_1-2\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2(\rho-1)(\rho+1)(\sigma_{\mu2}^2\lambda_2\\
&+\sigma_{\mu3}^2\lambda_3))\gamma^2+(8(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\sigma_{\mu1}^2\lambda_2\lambda_1^2+(8(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2\\
&+1/2\sigma_{\nu2}^2)\sigma_{\mu2}^2\lambda_2^2+8\sigma_{\mu3}^2(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\lambda_3\lambda_2-4\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1))\lambda_1\\
&-4\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2(\rho-1)(\rho+1))\gamma+16\lambda_1(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\lambda_2\Big)^{-1},
\end{aligned}
$$

$$
\begin{aligned}
\alpha_{33}=&\Big(\big(\sigma_{\mu1}^2(2(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)(\rho_\mu+1)\sigma_{\mu2}^2(\rho_\mu-1)\lambda_2^2+\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1))\lambda_1^2\\
&+2\sigma_{\nu1}^2\sigma_{\nu2}^2\rho_\mu\lambda_1\sigma_{\mu1}\sigma_{\mu2}(\rho-1)(\rho+1)\lambda_2+\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2^2\sigma_{\mu2}^2(\rho-1)(\rho+1)\big)\gamma^2+(-4(\rho\sigma_{\nu1}\sigma_{\nu2}\\
&+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\sigma_{\mu1}^2\lambda_2\lambda_1^2+(-4(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\sigma_{\mu2}^2\lambda_2^2+2\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho\\
&-1)(\rho+1))\lambda_1+2\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2(\rho-1)(\rho+1))\gamma-8\lambda_1(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\lambda_2\Big)\\
&\Big(\sigma_{\mu3}^2(\sigma_{\mu1}^2(4(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)(\rho_\mu+1/2)\sigma_{\mu2}^2(\rho_\mu-1)\lambda_2^2+\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho
\end{aligned}
$$

$$+1)(\rho_\mu+1))\lambda_1^2+2\sigma_{\nu1}^2\sigma_{\nu2}^2\rho_\mu\lambda_1\sigma_{\mu1}\sigma_{\mu2}(\rho-1)(\rho+1)\lambda_2+\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2^2\sigma_{\mu2}^2(\rho-1)(\rho+1)(\rho_\mu$$
$$+1))\lambda_3(\rho_\mu-1)\gamma^3+(-2\sigma_{\mu1}^2(2(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)(\rho_\mu+1)\sigma_{\mu2}^2(\rho_\mu-1)\lambda_2^2$$
$$+2\sigma_{\mu3}^2(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)(\rho_\mu+1)\lambda_3(\rho_\mu-1)\lambda_2+\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1))\lambda_1^2$$
$$+(-4\sigma_{\mu3}^2(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)(\rho_\mu+1)\sigma_{\mu2}^2\lambda_3(\rho_\mu-1)\lambda_2^2-4\sigma_{\nu1}^2\sigma_{\nu2}^2\rho_\mu\lambda_2\sigma_{\mu1}\sigma_{\mu2}(\rho$$
$$-1)(\rho+1)-2\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_3\sigma_{\mu3}^2(\rho-1)(\rho+1))\lambda_1-2\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2(\rho-1)(\rho+1)(\sigma_{\mu2}^2\lambda_2$$
$$+\sigma_{\mu3}^2\lambda_3))\gamma^2+(8(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\sigma_{\mu1}^2\lambda_2\lambda_1^2+(8(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2$$
$$+1/2\sigma_{\nu2}^2)\sigma_{\mu2}^2\lambda_2^2+8\sigma_{\mu3}^2(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\lambda_3\lambda_2-4\sigma_{\nu1}^2\sigma_{\nu2}^2(\rho-1)(\rho+1))\lambda_1$$
$$-4\sigma_{\nu1}^2\sigma_{\nu2}^2\lambda_2(\rho-1)(\rho+1))\gamma+16\lambda_1(\rho\sigma_{\nu1}\sigma_{\nu2}+1/2\sigma_{\nu1}^2+1/2\sigma_{\nu2}^2)\lambda_2\Big)^{-1}. \tag{233}$$

We solve the corresponding market maker problem for the model with noise traders and the price impacts (11) are as follows,

$$\lambda_1=\frac{\beta_1\left(\sigma_{\nu1}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)}{\beta_1^2\left(\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}\right)+\left(\alpha_{11}+1\right)^2\sigma_{\delta1}^2+\alpha_{12}^2\sigma_{\delta2}^2+\alpha_{13}^2\sigma_{\delta3}^2+\sigma_{\mu1}^2},$$
$$\lambda_2=\frac{\beta_2\left(\sigma_{\nu2}^2+\rho\sigma_{\nu1}\sigma_{\nu2}\right)}{\beta_2^2\left(\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}\right)+\alpha_{21}^2\sigma_{\delta1}^2+\left(\alpha_{22}+1\right)^2\sigma_{\delta2}^2+\alpha_{23}^2\sigma_{\delta3}^2+\sigma_{\mu2}^2},$$
$$\lambda_3=\frac{\beta_3\left(\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}\right)}{\beta_3^2\left(\sigma_{\nu1}^2+\sigma_{\nu2}^2+2\rho\sigma_{\nu1}\sigma_{\nu2}\right)+\alpha_{31}^2\sigma_{\delta1}^2+\alpha_{32}^2\sigma_{\delta2}^2+\left(\alpha_{33}+1\right)^2\sigma_{\delta3}^2+\sigma_{\mu3}^2}. \tag{234}$$

We plug the trading and offsetting intensity parameters (233) into the price impacts (234) and obtain three nonlinear equations each in $\lambda_1,\lambda_2,\lambda_3$. Once we solve numerically for λ_1, λ_2 and λ_3, we can pin down $\beta_1,\beta_2,\beta_3,\alpha_{11},\alpha_{12},\alpha_{13},\alpha_{21},\alpha_{22},\alpha_{23},\alpha_{31},\alpha_{32},\alpha_{33}$ (233). We have now pinned down our new equilibrium (230).