Knowledge is (Market) Power

Jinglun Yao *
Institute for International Economic Studies, Stockholm University

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Abstract

US corporate concentration has been persistently rising over the past century and productivity growth has been concurrently declining. This paper builds a continuous-time Schumpeterian growth model and interprets higher concentration *as a result of* lower growth, rather than the reverse. A *uniform* decrease in R&D efficiency increases the *relative* growth of leading firms compared to the aggregate and endogenously thickens the Pareto tail of firms' productivity distribution. With a demand system featuring variable demand elasticities, the model explains a large part of the changes in aggregate growth, corporate concentration, markup, labor share, R&D cost, entry and exit rates, as well as job creation and destruction rates in the US since the 1980s. The model can also accommodate increasing concentration with a stable markup and labor share in the pre-1980 period by accounting for the role of economic integration.

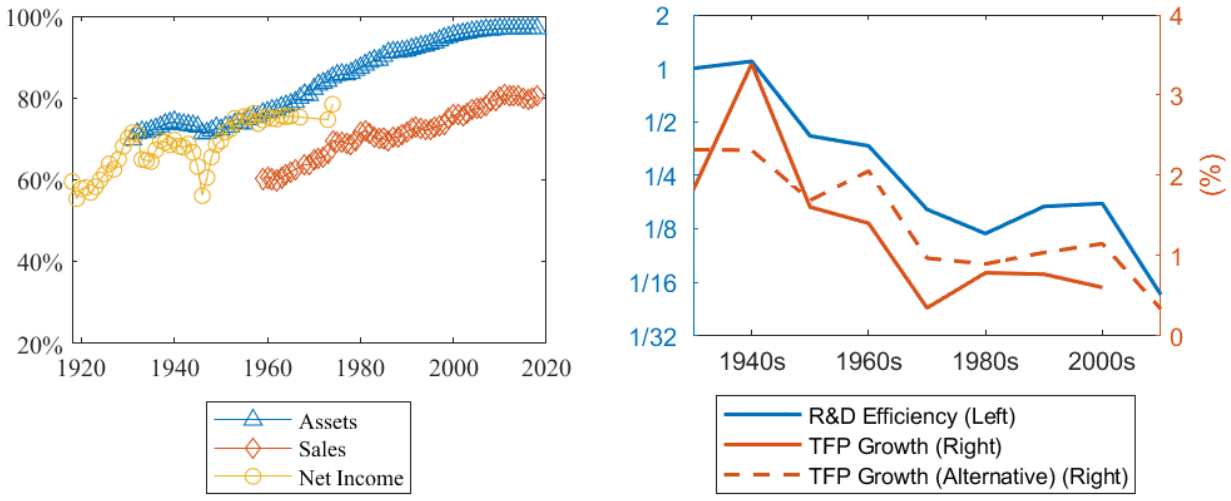
Keywords: Schumpeterian Growth, Corporate Concentration, Market Power, Business Dynamism, Heterogeneous Firms, Mean Field Game, Pareto Tail

*Institute for International Economic Studies, Stockholm University, SE-106 91 Stockholm, Sweden. Email: jinglun.yao@iies.su.se. This paper could not have taken its current form without the continuous and committed support of Hélène Rey. I thank Philippe Aghion, Nicolas Coeurdacier, Chad Jones, Pierre-Louis Lions, Joseba Martinez, Vincent Sterk, John Van Reenen, Ufuk Akcigit, Fernando Alvarez, David Argente, Antonin Bergeaud, Timo Boppart, Francisco Buera, Maarten de Ridder, Simon Gilchrist, Basile Grassi, Chenlin Gu, Stéphane Guibaud, Wei Hua, Pete Klenow, Kieran Larkin, Francesco Lippi, Isabelle Méjean, Benjamin Moll, Elias Papaioannou, Christina Patterson, Pascual Restrepo, Andrew J Scott, Petr Sedláček, Yongseok Shin, Paolo Surico and participants at various seminars for sharing their insights and valuable suggestions. I thank the UniCredit Foundation and the EEA for the Econ Job Market Best Paper Award.

1 Introduction

US corporate concentration has been rising since at least the 1930s and takes a specific form of fattening Pareto tail (Figure 1a, Kwon et al. (2022)). While many explanations have been proposed to account for this phenomenon particularly in the period after the 1980s, its strikingly long-run nature remains a puzzle. At the same time, US Total Factor Productivity (TFP) growth declined from around 2.5% in the 1930s and 1940s to near 0 today (Figure 1b, Gordon (2016) and Nordhaus (2021)). What is the driver behind increasing concentration and declining growth over such a long period?

This paper argues that decreasing Research and Development (R&D) efficiency, which has been identified as a widespread feature across scientific fields or industrial classifications,[1] can explain both long-run trends. Bloom et al. (2020) shows that the difficulty of generating TFP growth through R&D has increased by a factor of 19 since the 1930s with an annual growth rate of 3.67% (Figure 1b).[2] I interpret the evidence as higher difficulty of advancing knowledge over time, regardless of whether a firm conducts genuine research or develops new products based on existing knowledge to generate growth.



(a) Corporate Concentration

(b) TFP Growth and R&D Efficiency

Figure 1: US secular trends. Corporate concentration is the share of top 1% firms in terms of assets, receipts (sales) and net income from Kwon et al. (2022). TFP growth comes from Gordon (2016) and the alternative measure from Nordhaus (2021). R&D Efficiency is "Research Productivity" in Bloom et al. (2020) with decreasing returns to the idea production function.

To establish the link between decreasing R&D efficiency and rising concentration, I build a novel continuous-time Schumpeterian growth model which endogenizes the growth decisions

[1] See Gordon (2016), Bloom et al. (2020) and Park et al. (2023).

[2] To be consistent with my model, I assume decreasing returns to scale in the idea production function with 0.5 elasticity of growth with respect to R&D input. In the baseline setting of Bloom et al. (2020) with constant returns to scale instead, the efficiency has decreased at 5.1% per year since the 1930s.

of individual firms and generates a productivity distribution with a Pareto tail. The tail heaviness is shaped by the relative growth between the top firms and aggregate growth. These top firms face a saturated demand and thus have little incentives to improve their productivities: they grow more slowly the aggregate, which serves a mean-reverting force to generate a stationary distribution at a specific time. A uniform reduction in R&D efficiency over time implies lower aggregate growth as the mean-reverting force and thus a fatter tail. I show analytically that the model generates a one-to-one correspondence between TFP growth rate and the Pareto tail index of firm size distribution, so that the secular decline of TFP growth translates into the secular rise of corporate concentration in the data.[3] The finding is robust to whether the model includes entry/exit, or whether incumbent firms can learn from other firms in addition to innovating on their own, as long as learning by entrants or incumbent firms is imperfect. Unlike many existing papers in the literature, this does not resort to any additional assumption on the initial condition of the firms' distribution.[4]

It should be emphasized that the negative correlation between growth and concentration level should *not* be understood as higher concentration causing lower growth, which is the emphasis of the existing Schumpeterian growth literature (e.g. Aghion et al. (2005), Akcigit and Ates (2021)), but rather as lower growth generating higher concentration. Both directions between growth and concentration are present in my model and I use the Mean Field Game methodology to accommodate this two-way relationship.[5] Instead of reducing growth, higher concentration partly mitigates the impact of harder R&D and encourages growth by increasing the Schumpeterian profits of R&D. The emphasis is now shifted from how market structure determines growth to how growth determines market structure.

R&D is a fixed cost in the dynamic setting, as it reduces a firm's marginal cost in a non-rival manner (Romer (1990)). Similar to a static fixed cost, higher concentration in a closed economy or a larger market size in an open economy can increase firms' profits, thereby diluting increasing R&D difficulty. Unlike static fixed costs, however, changes in R&D costs influence concentration by shaping the equilibrium productivity distribution, meaning they can have a significant impact on concentration despite representing only a small share of GDP.[6] This paper, therefore, aligns with Kwon et al. (2022) in emphasizing the fixed-cost nature of R&D as the key in understanding rising concentration but interprets its effect through the lens of growth.

Understanding the rise of concentration is not only important for its own sake, but may also shed light on the rise of market power, the decline of labor share and the diminished business dynamism since the 1980s.[7] For this objective, I design a demand system featuring

[3] Even though growth is *negatively* correlated with the *level* of concentration, it is *positively* correlated with the *speed of concentration increase*, consistent with empirical evidence in Kwon et al. (2022). Because higher growth translates into faster decrease in R&D efficiency during a fixed period of time following the semi-endogenous growth logic.

[4] Luttmer (2012) and Perla et al. (2021) assume the initial productivity distribution to be sufficiently light-tailed in order to uniquely pin down the equilibrium Pareto tail. See Section 4.2.3 on why imperfect learning makes such an assumption unnecessary.

[5] "Mean Field Game" stands for a coupled system of an Hamilton-Jacobi-Bellman (HJB) equation with a Kolomogorov Forward (KF) equation, with possibly market clearing conditions.

[6] Total R&D expenditure as a share of GDP has never exceeded 3.5% in the past century in the U.S.

[7] See Karabarbounis and Neiman (2014), Autor et al. (2020) and Kehrig and Vincent (2021) for the labor share, De Loecker et al. (2020) for sales-weighted markups, Edmond et al. (2023) for cost-weighted markups,

variable demand elasticities and assume that firms compete monopolistically with each other. The demand system can be seen as a counterpart to nested-CES (Atkeson and Burstein (2008)) under monopolistic competition. It retains key features of nested CES regarding demand elasticities, pass-throughs and firm-size distributions, while eliminating the strategic interactions of oligopolistic competition to make the model tractable in a growth setting. With lower R&D efficiency, the fatter tail of the productivity distribution thus reallocates market share towards larger firms that charge higher markups and feature low labor share. This between-firm reallocation channel is consistent with the empirical evidence post-1980s (De Loecker et al. (2020), Autor et al. (2020)). Job creation and destruction rates decrease as lower demand elasticity of large firms translates the same idiosyncratic productivity shock into reduced job change, consistent with the evidence in Decker et al. (2020).

Such an endeavor, however, raises the doubt about concentration and market power before 1980s. While corporate concentration has been rising since the 1930s, markups were roughly stable until the 1980s.[8] How can we reconcile the fattening tail of firm size distribution with the stable market power until the 1980s? I show that economic integration across US regions can be a possible confounding factor in addition to harder R&D.[9] Economic integration reduces markups and increases labor share through the within-firm component as it brings more firms into competition with each other.[10] The between-firm reallocation component due to less efficient R&D is thus largely compensated by the within-firm component of economic integration, and aggregate markup and labor share can remain roughly stable while concentration increases more rapidly before the 1980s.[11] The model can thus deliver both the pre- and post-1980s trends. The parentheses in the title of this paper has thus two connotations: to indicate the link with the well-known phrase by Francis Bacon; to suggest that the implication on market power via concentration is nuanced.

Using sectoral-level data from the US since 1987, I find empirical evidence for the mechanism of the model which links R&D efficiency, market structure and business dynamism. R&D efficiency is significantly negatively correlated with the market concentration and profitability of the largest firms in the sector, and positively correlated with labor share and major indicators of business dynamism in the Business Dynamics Statistics (BDS). To quantify the model, I calibrate the demand system to match the heterogeneous labor shares across the establishment size distribution in the Census of Manufacturing. Other parameters are structurally estimated to match key moments around 1980. I calibrate a 2.62-fold decrease

Decker et al. (2016) and Decker et al. (2020) for business dynamism.

[8] See Edmond et al. (2023) and De Loecker et al. (2020).

[9] Other examples of rising concentration but stable markup and labor share include European countries in the last few decades. See Bighelli et al. (2023), Gutiérrez et al. (2022) and Bauer and Boussard (2020). Perhaps not coincidentally, European countries are being integrated into the European Union during the period.

[10] The productivity distribution is Pareto at the right tail but log-concave in log productivity overall. According to Autor et al. (2020), within-firm reduction in markup dominates between-firm reallocation towards higher-markups firms when the number of firms increases. The net effect coming from static integration is thus a decrease in markup. I thank Pascual Restrepo for pointing this out.

[11] When the productivity distribution is exactly Pareto, the sole force of economic integration without any change in the productivity distribution can qualitatively explain higher concentration with stable aggregate markup and labor share as in Melitz and Ottaviano (2008), but cannot explain why higher concentration takes the form of a fattening Pareto tail.

of R&D efficiency for the 1980-2020 period following Bloom et al. (2020) with aggregate US data. The model can explain most of the changes in US TFP growth, top-firm concentration, markup, labor share, entry rate, exit rate, job creation rate, job destruction rate, job reallocation rate and R&D cost since the 1980s.

The above discussion leaves unanswered the question of why R&D has become less efficient. To be sure, this has not always been the case: TFP growth went up by 1.5% from the 1900s to 1930s and peaked in the 1930s and 1940s according to Gordon (2016) and Nordhaus (2021).[12] This pattern of the rise and fall of growth is envisioned as "Schumpeterian Wave" by Aghion and Howitt (1992). The high moments of waves could be due to the arrival and *widespread* adoption of *disruptive* general-purpose technologies, such as electricity and internal combustion engine which characterized the Second Industrial Revolution. For the era under study, the lack of disruptive technologies has been extensively documented by the recent literature such as Gordon (2016). Garcia-Macia et al. (2019) demonstrates that most growth since the 1980s has resulted from incumbents' innovations on existing products. Park et al. (2023) shows a decline in the disruptiveness of patents and academic papers from the end of World War II. Nonetheless, the optimistic implication is that faster growth is possible in the future, as long as fundamental advances such as artificial intelligence can start another wave. Policies can be designed to stimulate research, particularly *fundamental* and *disruptive* ones that have a *wide scope* of applicability.

1.1 Related Literature

Schumpeterian Growth. This paper belongs to the Schumpeterian growth literature and owes its inspiration to the seminal work of Aghion et al. (2001). This paper innovates by generating a well-defined productivity distribution with Pareto tail.[13] Such a distribution allows us to understand why market concentration has increased via the specific form of a fattening Pareto tail. It also allows us to evaluate economic integration in the spirit of the international trade literature, and thus to study dynamic growth effects and static market structure changes in a unified framework. Importantly, the paper shifts from the traditional Schumpeterian emphasis of how market structure determines growth to how growth determines market structure. Akcigit and Ates (2021) interprets lower growth since the 1980s as a result of a change in market structure which comes from lower technological diffusion rate. In the present paper, by contrast, higher concentration has a pro-growth effect.

Mean Field Game and Pareto Tail. Consistent with the economic message that paper is conveying, I take advantage of the newly developed methodology of Mean Field Game to endogenize the productivity distribution based on firm-level growth decisions. Closest in spirit to my model are Lucas and Moll (2014) and Benhabib et al. (2021). Both papers imply a positive correlation between concentration and growth, as a fatter tail promotes social learning in the former, and aggregate growth is synonymous with frontier growth which widens the gap between leaders and laggards in the latter. Moreover, both assume firms' profits

[12] Philippon (2022) shows that productivity is best described as "additive" *per historical era*: growth rate declines during each era but can temporarily increase at some key moments.

[13] Aghion et al. (2001) generates a productivity *gap* distribution, but the productivity distribution dissipates over time as innovation and diffusion intensities do not depend on sectoral productivity *level*.

are unaffected by market structure. This paper differs by having the empirically-consistent correlation between concentration and growth. It incorporates a standard static model of firm heterogeneity in the dynamic setting, making it possible to speak to both directions between market structure and firm growth. Chen (2022) also endogenizes firm-level growth and fatter Pareto tail. While his paper focuses on the transitional dynamics when the initial distribution is assumed to be lighter-tailed than the equilibrium distribution,[14] my paper focuses on the equilibrium distribution for each historical period and the transition is understood as moving from one equilibrium to another due to changes in R&D efficiency. The paper is also related to earlier exogenous growth literature which generates Pareto tail, in particular Gabaix et al. (2016) and Luttmer (2012). Similar to Luttmer (2012), less productive firms enjoy growth advantage and are particularly hurt by lower growth, making higher concentration go hand in hand with lower growth. This paper additionally endogenizes firm growth, emphasizing the fixed-cost nature of R&D via growth.

Semi-endogenous Growth. Decreasing R&D efficiency, the driving force in this paper, draws on fundamental insights of the semi-endogenous growth literature, e.g. Jones (1995), Kortum (1997) and Segerstrom (1998).[15] Bloom et al. (2020) provides recent empirical evidence based on micro data. Existing theoretical frameworks typically assume a positive exogenous population growth which cancels out the effect of harder R&D by the increasing number of researchers, and consequently maintains constant productivity growth. Jones (2002) shows, however, that less than 20% of growth is explained by population growth in the data. Thus I assume a fixed population except when I analyze economic integration,[16] and study the effect of decreasing R&D efficiency on declining growth. Given the importance of R&D in driving firm growth and business dynamics, the sheer magnitude of the decrease in its efficiency needs to be compatible with other key macro moments.

Market Power and Business Dynamics The paper is also closely related to the large literature on US growth, market power and business dynamism since the 1980s. Notable works on the theory side include Akcigit and Ates (2021, 2023) on technological diffusion, Aghion et al. (2023) and De Ridder (2019) on intangibles, and Liu et al. (2022) on low interest rates. This paper differs by having a longer historical focus since the 1930s. Stronger intellectual property right enforcement may have decreased the diffusion rate after the 1980s, but the evidence is limited for the pre-1980 period. Moreover, lower diffusion's pro-growth effect on leaders largely compensates its anti-growth effect from higher concentration, making it difficult to speak to the secular decline in growth.[17] But my paper does concur with Akcigit and Ates (2021, 2023) in that the technological diffusion rate in the form of costly learning declines. The declining diffusion rate is now understood to be *not* due to institutional reasons, but rather to increasing difficulty of improving knowledge. De Ridder (2019) assumes ex ante differences in firms' abilities to utilize intangibles such as IT, and Aghion et al. (2023)

[14] Tail transition is fast in Chen (2022) as learning is perfect and targets only more productive firms. Consequently, when the log-productivity distribution has a tail of order Ce^{-kx}, successful learners with tail Cxe^{-kx} are injected, pushing the distribution towards a heavier tail.

[15] See Jones (2022) for a review.

[16] Even the economic integration involves not population growth but rather a reorganization of existing population.

[17] See also Aghion et al. (2001) for the non-monotonic relationship between aggregate growth and diffusion rate.

uniformly decreases the overhead cost of managing multiple product lines due to IT. Both increase the span of control of IT-capable firms, increasing concentration and depressing growth. IT was not widespread before the 1980s. Despite this, my paper agrees with higher intangibles in these papers, as measured intangibles predominantly consist of capitalized R&D expenditures[18] and the R&D share of GDP increases in my model. Liu et al. (2022) investigates the anti-growth effect of a lower interest rate when the latter is close to 0 and if there is no quick catch-up. In the present paper, a lower interest rate has a pro-growth effect and acts as a compensating force to mitigate the effects of harder R&D, regardless whether there is quick catch-up.[19] Closest in spirit to the present paper are Olmstead-Rumsey (2019) and Cavenaile et al. (2019). Olmstead-Rumsey (2019) studies the decrease in innovativeness of all firms which corresponds to decreasing R&D efficiency in this paper, and Cavenaile et al. (2019) compares different mechanisms and argues that harder R&D is likely to be the driver behind lower growth. My paper differs by generating a well-defined productivity distribution, accounting for the increasing concentration of very top firms, and accommodating different patterns of concentration and market power across historical periods. It is worth noting that the framework is not only able to match key moments of growth, market power and business dynamics, but also encompass some main drivers that are put forward to explain these moments: technological diffusion, intangibles, interest rate and innovativeness of R&D. In this sense, my work attempts to resolve the double accounting issue in the literature and hopes to avoid "explaining the labor share decline many times over" as advocated by Grossman and Oberfield (2022).

2 Empirical Evidence

I use US sectoral level data between 1987 and 2018 to show the correlations between R&D efficiency on the one hand, and TFP growth, indicators of market structure and business dynamics on the other. There are 33 sectors in total, including 19 manufacturing sectors at 3-digit NAICS level and 14 non-manufacturing sectors at 2-digit NAICS level. I follow Bloom et al. (2020) and define R&D efficiency as the ratio between TFP growth and effective number of researchers at the sectoral level, taking into account decreasing returns to scale of the idea production function.[20] The effective number of researchers is R&D expenditure deflated by scientist wage which is proxied by average annual earnings for men with four or more years of college education. R&D expenditure comes from the aggregation of firm-level R&D in Compustat at the sectoral level.[21] TFP growth comes from the KLEMS data of the

[18] See Hall et al. (2005), Bloom et al. (2013), Peters and Taylor (2017) and Haskel and Westlake (2017).

[19] Moreover, if equity cost is considered in addition to debt cost to form the Required Rate of Return on Capital (RRRC) for discounting future profits, RRRC has been above 10% since the 1980s according to Barkai (2020).

[20] The results remain robust with constant returns to scale of the idea production function (see Appendix A.2).

[21] Compustat covers public firms whose share among all firms varies across sectors and years. To alleviate the concern about the R&D data coverage in Compustat, Appendix A.2 adjusts sectoral R&D by the employment share of Compustat firms within each sector. The correlations remain robust after the adjustment.

US Bureau of Labor Statistics (BLS) and Bureau of Economic Analysis (BEA).[22] To remove the cyclical variations of TFP growth in a business cycle, I divide the 1987-2018 history into four equal periods, each with eight years. All variables are calculated as per period average.[23]. See Appendix A.1 for more details on the definition of R&D efficiency.

Dependent variables are constructed to reflect the market structure and business dynamics. (1) I construct an indicator of market concentration from Business Dynamics Statistics (BDS) which reports employment by firm size category and sector. For each sector*period, market concentration is defined as the employment share of firms with more than 5000 employees. (2) EBIT/Sales of the largest four or eight firms by sector in Compustat is used to indicate the profitability of the largest firms in the sector. (3) Sectoral labor shares come from KLEMS. (4) For business dynamics, I use entry (exit) rate in terms of the number of establishments, entry (exit) rate in terms of job creation (destruction) from birth (death), and job creation/destruction/reallocation rates, all from BDS.

For each dependent variable $y_{s,t}$, where s stands for sector and t for period, I run the following regression:

$$y_{s,t} = \beta_0 + \beta_1 \log(\text{RDeffc}_{s,t}) + \gamma_t + \epsilon_{s,t},$$

where $\text{RDeffc}_{s,t}$ is the R&D efficiency and γ_t is period fixed effect. Standard errors are clustered at sectoral level. Table 1 shows the results for growth and market structure indicators, and Table 2 for business dynamism indicators. Within-R2 reports the R^2 without fixed effects. R&D efficiency is significantly negatively correlated with market concentration and profitability of the largest firms in a sector, and positively correlated with growth, labor share and all indicators of business dynamics. The next section presents a simple model focusing on growth and concentration, and Section 4 presents a full model consistent with all the empirical findings.

Table 1: Regressions of TFP growth and market structure indicators on log R&D efficiency.

	(1) Growth	(2) Concentration	(3) EBIT/Sales 4	(4) EBIT/Sales 8	(5) Labor Share
log(RDeffc)	0.472	-4.594	-0.723	-0.817	2.664
	(0.0853)	(1.526)	(0.414)	(0.432)	(0.927)
N	132	132	132	132	132
Within R2	0.187	0.147	0.0547	0.0729	0.0877

Note: Notes: EBIT/Sales 4(8) is EBIT/Sales of the largest 4(8) firms in the sector. Intercepts and period fixed effect are included in the regressions, but omitted in the table. Standard errors are clustered at sectoral level and shown in parentheses.

[22] BEA treats R&D as intermediate input when calculating TFP and reports separately the contribution of R&D to productivity. Following Bloom et al. (2020), this contribution is added back to TFP to be consistent with the conceptual framework of endogenous growth.

[23] Such a division is also conceptually reasonable as a change in "R&D efficiency" only seems meaningful when a longer horizon is involved. Using divisions other than eight years per period, as long as each period is not too short so that cyclical variations can be removed, and not too long so that there are sufficient data points, generates similar results (See Appendix A.2)

Table 2: Regressions of business dynamism indicators on log R&D efficiency.

	(1) Entry (num)	(2) Exit (num)	(3) Entry (job)	(4) Exit (job)	(5) Job Creation	(6) Job Destruction	(7) Job Reallocation
log(RDeffc)	0.610	0.593	0.459	0.393	0.967	0.848	1.867
	(0.232)	(0.157)	(0.150)	(0.0992)	(0.279)	(0.228)	(0.493)
N	132	132	132	132	132	132	132
Within R2	0.0740	0.103	0.0991	0.110	0.0909	0.0969	0.117

Note: Notes: Entry (num) is the number of new establishments over the number of existing establishments. Entry (job) is the number of jobs created by new establishments over total jobs. Exit (num) and Exit (job) are defined analogously for exits. Job creation/destruction/reallocation are rates. Intercepts and period fixed effect are included in the regressions, but omitted in the table. Standard errors are clustered at sectoral level and shown in parentheses.

3 A Simple Model

This section presents an illustrative model in a continuous-time setting, illustrating the key mechanism that links lower R&D efficiency with increased concentration. It also paves the way for introducing additional elements in the full model. For notational conciseness, time index t is dropped whenever possible.

3.1 Simple Model and Evidence on the Mechanism

There is a continuum of firms indexed by $\theta \in [0,1]$, each of which produces one differentiated good with the production function $Y_\theta = A_\theta L_\theta$, where A_θ is the firm's productivity and L_θ the amount of labor. Firms engage in monopolistic competition with each other. A representative consumer supplies inelastically one unit of labor force and has a utility with Constant Elasticity of Substitution (CES) σ across differentiated goods. Standard calculations show that the profit of each firm is given by $\Pi_\theta = \frac{w}{\sigma-1}\tilde{A}_\theta^{\sigma-1}$, where $\tilde{A}_\theta = A_\theta/A$ is firm θ's productivity relative to the aggregate level $A = [\int_0^1 A_\theta^{\sigma-1}\,\mathrm{d}\theta]^{\frac{1}{\sigma-1}}$.

Each firm can invest in R&D to improve its productivity step-by-step. By hiring $\frac{1}{2}\alpha\tilde{A}_\theta^{\beta}\lambda^2$ number of researchers per period,[24] firm θ's innovation has Poisson rate of arrival λ, the success upon which allows its log productivity to increase from $a_\theta \triangleq \log(A_\theta)$ to $a_\theta + q$.[25] The factor $\tilde{A}_\theta^{-\beta/2}$ with $\beta > 0$ captures decreasing R&D efficiency upon more advanced technology in a cross section of firms,[26] while $1/\sqrt{\alpha}$ captures the economy-wide R&D efficiency at a specific time.[27] A larger α is used for a later historical period to capture uniformly harder

[24] The quadratic cost function is a typical assumption in Schumpeterian growth models which matches the elasticity of R&D with respect to the user costs of around -1. See Bloom et al. (2002), Acemoglu et al. (2018) and Akcigit and Kerr (2018).

[25] The equilibrium distribution is virtually the same under an alternative model in which innovation is modelled as a continuous process (i.e. a dt term) rather than as a Poisson process.

[26] Bloom et al. (2020) shows decreasing R&D efficiency as a firm grows larger, which translates into $\beta > 0$ in a cross section as firms are synonymous in the model.

[27] To see where the 1/2 comes from, denote $\tilde{R} = \frac{1}{2}\alpha\tilde{A}_\theta^{\beta}\lambda^2$, then $\lambda = \sqrt{2/\alpha}\tilde{A}_\theta^{-\beta/2}\tilde{R}^{1/2}$ which is the "idea

R&D over time. In a fully dynamic setting, this would be justified by the semi-endogenous assumption $\alpha = A^{\beta_a}$ where $\beta_a > 0$.[28] For simplicity, however, I will focus on solving balanced growth path (BGP) with a fixed α, then comparing different BGPs with different α.[29] Adjusted by the researchers' wage w, the R&D cost is $R = \frac{1}{2}\alpha\tilde{A}_\theta^\beta\lambda^2 w$. In this simple model, each firm is myopic and considers only marginal profit when taking the R&D decision, i.e. it does not solve an infinite-horizon optimization problem with discounted future profits.

The R&D decision of each firm is thus:

$$\max_\lambda \left\{\lambda q \frac{\mathrm{d}\Pi_\theta}{\mathrm{d}a_\theta} - R\right\}$$

whose solution is $\lambda^* = \frac{q}{\alpha}\tilde{A}_\theta^{\sigma-1-\beta}$, which decreases with $\tilde{A}_\theta$ if $\beta > \sigma - 1$, which is assumed in this section. With a given σ, marginal profit of R&D increases with the firm's productivity with the elasticity of $\sigma-1$: ideas are non-rival so that a larger firm can better take advantage of a technological improvement. A lower σ implies a more saturated demand faced by a firm: it is more difficult for the firm to increase its sales and profits by cutting its price. If the demand is saturated enough ($\sigma - 1 < \beta$), large firms have little incentive to overcome the low R&D efficiency so that their growth rates are close to 0.

In addition to innovation, each firm is subjected to idiosyncratic log-productivity shocks modelled as a Brownian motion with a standard deviation of ν.[30] Along the BGP, the log-productivity Probability Density Function (PDF) ϕ maintains its shape and travels at a constant speed g, which is also the aggregate growth rate as A is a homothetic aggregator of $\{A_\theta\}_{\theta\in[0,1]}$. The distribution $\tilde{\phi}$ of normalized log productivities $\tilde{a}_\theta \triangleq \log(\tilde{A}_\theta)$ is stationary. Taken together, each firm's normalized log productivity follows the stochastic process:

$$\mathrm{d}\tilde{a}_{\theta,t} = (-\frac{1}{2}\nu^2 - g)\,\mathrm{d}t + \nu\,\mathrm{d}B_{\theta,t} + \mathrm{d}J_{\theta,t} \tag{1}$$

where $B_{\theta,t}$ is Brownian motion independent across firms, $-\frac{1}{2}\nu^2\,\mathrm{d}t$ is a normalization term for the Brownian motion so that the expected productivity change due to Brownian motion is null, and $J_{\theta,t}$ is the jump process due to innovation. A Kolmogorov Forward (KF) equation describes the dynamics of distribution $\tilde{\phi}$ whose equilibrium solution is:

$$\tilde{\phi}(\tilde{a}) = C_1 \exp(-C_2\tilde{a} - C_3 e^{-(\beta-\sigma+1)\tilde{a}}), \quad \tilde{a} \in \mathbb{R} \tag{2}$$

where $C_1 > 0$, $C_2 = 1 + 2g/\nu^2$ and $C_3 > 0$. See Appendix B.1 for the derivations. g is pinned down by the normalization condition $A(\tilde{\phi}) = 1$. Asymptotically, $\tilde{\phi}(\tilde{a})$ has an exponential right tail with tail index C_2, or equivalently, the productivity distribution has a Pareto right tail with tail index C_2. C_2 decreases with aggregate growth so that a fatter tail is associated with lower growth. Moreover, $\log(\tilde{\phi})$ is concave in $\tilde{a}$, the importance of which will be seen with economic integration in Section 8.

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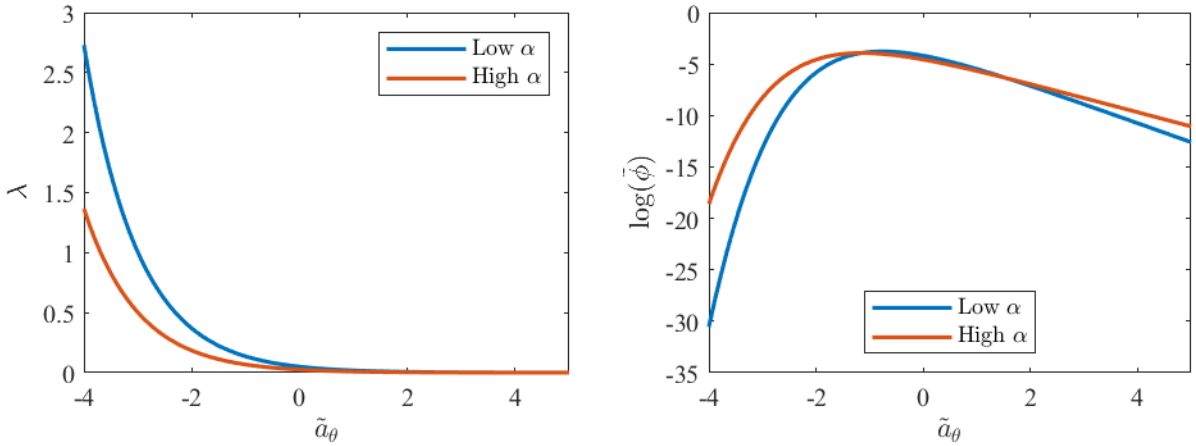
production function" in Bloom et al. (2020), and $\sqrt{2/\alpha}\tilde{A}_\theta^{-\beta/2}$ is firm-level R&D efficiency.

[28] If $\beta_a = \beta$, then $\alpha\tilde{A}_\theta^\beta = A_\theta^\beta$. A priori they do not have to be the same, as evidenced by Bloom et al. (2020).

[29] It is well-understood that baseline random growth models generate slow transitions of Pareto tail. See Luttmer (2011) and Gabaix et al. (2016).

[30] The probability of Brownian motion and innovation happening together is of order of magnitude $o(\mathrm{d}t)$ so that it can be ignored. Thus when making the R&D decision, each firm ignores the Brownian motion.

Like in random growth models, I have assumed idiosyncratic shocks to generate the Pareto tail. The tail heaviness depends on the joint force of random growth and endogenous growth. As largest firms have little growth, aggregate growth allows the rest of the economy to catch with the them. A *uniform* increase in α for all the firms decreases aggregate growth and thus increases tail fatness, as the *relative* growth rate of large firms increases. The *absolute* growth of large firms, however, still decreases. With CES demand, a change in productivity distribution does not affect individual firm profit and thus growth incentives given the firm's relative productivity. Thus higher concentration is understood *as a result of* lower growth instead of the other way around.



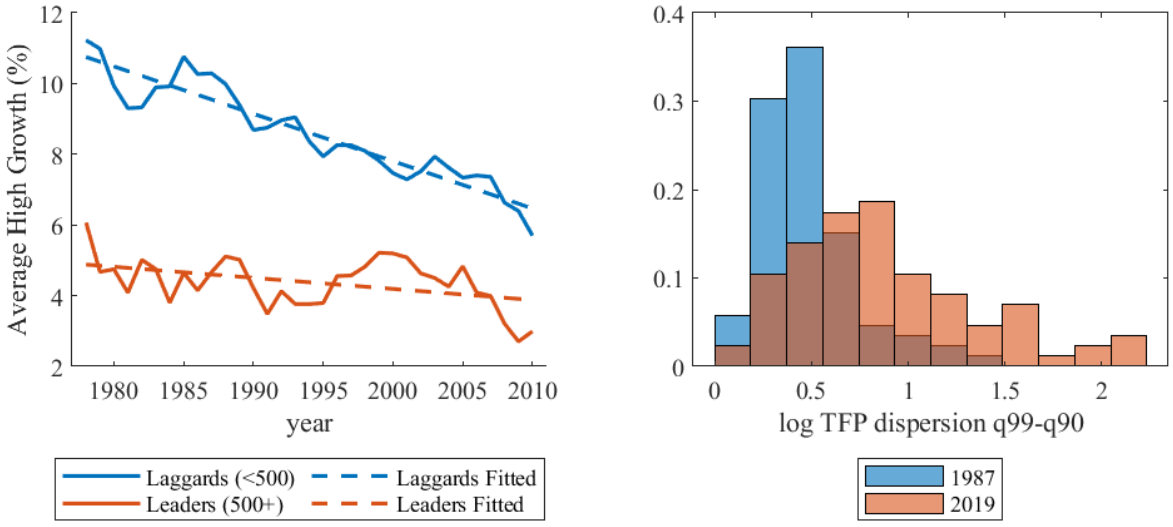
(a) Innovation intensity

(b) Log productivity distribution in log scale

Figure 2: Illustrative model solutions for illustrative purposes. $\sigma = 2$, $\beta = 2$, $q = 0.2$, $\nu = 0.15$, $\alpha = 4$ for low α case and $\alpha = 8$ for high α case.

Innovation intensity decreases with firm size, which is a departure from Gibrat's law. This is consistent with the evidence in Akcigit and Kerr (2018) that growth rate decreases with firm size among innovating firms. Moreover, even though there is only one type of firms in the model, in reality there can be multiple types: the pattern of decreasing absolute growth but increasing relative growth of large firms only need to hold for the high-growth type to make the mechanism work. Following Decker et al. (2016), high growth firms are defined as those with more than 30% annual growth in employment. Figure 3a shows that the high growth firms among the laggards grow more quickly than those of leaders in any given year. Both leaders and laggards witness a decline in growth, but the decline is more pronounced for laggards. All of these empirical patterns are consistent with the model in Figure 2a. Moreover, Figure 3b shows the histograms of log TFP dispersion between the 99th quantile and the 90th quantile, where each observation is a 4-digit NAICS manufacturing sector. The productivity dispersion at the tail has in general increased,[31] which is consistent with the model's prediction that the productivity Pareto tail fattens.

[31] If I track each 4-digit sector and calculate its change in log TFP dispersion between 1987 and 2019, 84% of sectors see an increase in the dispersion.



(a) High growth among leaders and laggards

(b) Histogram of TFP dispersion at tail

Figure 3: Checking the model's mechanism with data. Panel (a): Laggards are firms with less than 500 employees and leaders more than 500. For each year, the plot shows the employment-weighted average growth of high growth firms among leaders and laggards respectively. Data source: Decker et al. (2016). Panel (b): Data source: US Census Bureau, Dispersion Statistics on Productivity (DiSP).

3.2 Necessity of a Richer Model

Apart from illustrating the key mechanism at work, the simple model also clarifies the necessity of a richer model:

1. Feedback from market structure to growth decisions
   Firms evaluate the effectiveness of their individual growth against aggregate growth, which affects the growth decisions if they are forward-looking. Growth decisions thus depend on the distribution via aggregate growth which includes distribution as weights. Distribution thus affects grow decisions even if the demand is CES where profits do not depend on distribution. This is even more the case with a more general demand system: firms' profit typically depend on the productivity distribution. Consequently, we need to complement the simple model by describing how market structure (i.e. distribution) shapes the growth decisions of individual firms. In addition to the emphasis of how firm-level growth shapes market structure, this gives rise to the Mean Field Game (MFG) system in the full model.

2. Heterogeneous demand elasticities
   Introducing heterogeneous demand elasticities not only allows me to analyze how market structure affects growth via profits as discussed above, but also to think about market power when concentration increases. Following the macroeconomic literature on market power, I assume demand elasticity decreases with firm size in the full model. Moreover, this allows me to relax the assumption of $\beta > \sigma - 1$ in the simple model to $\beta > \underline{\sigma} - 1$ where $\underline{\sigma}$ is the lowest possible demand elasticity faced by largest firms, which is easily satisfied by the data.[32]

3. Entry and Exit
   If demand elasticity σ declines with firm size such that $\sigma - 1 - \beta$ is first positive and then negative, the innovation intensity λ then features an inverted U that is often found in Schumpeterian growth models. The low growth of the least productive firm implies that a dynamic equilibrium may not exist, and additional stabilizing forces such as entry and exit are needed. [33] Adding entry and exit also gives a more realistic description of business dynamics.

4 Model

Like in the last section, a representative consumer consumes differentiated goods per period; each good is produced by one firm with constant returns to scale to labor; firms with different productivities compete monopolistically with each other and can improve their productivities via R&D. In the full model, incumbent firms can die due to large idiosyncratic shocks to their productivities or due to being too unproductive, and entrants keep joining the pool of incumbent firms by learning from the latter. Figure 4 illustrates the structure of the baseline

[32] Largest firm only needs to stretch out relative to moderately large firms. Figure A2 re-plots Figure 3a with finer size bin, and shows this is the case in the data.

[33] Appendix F introduces incumbent learning as an additional stabilizing force, which also explains higher growth of laggards in the data.

full model. For notational simplicity, I shall drop the time index t whenever possible. A tilde denotes normalization by the aggregate productivity.

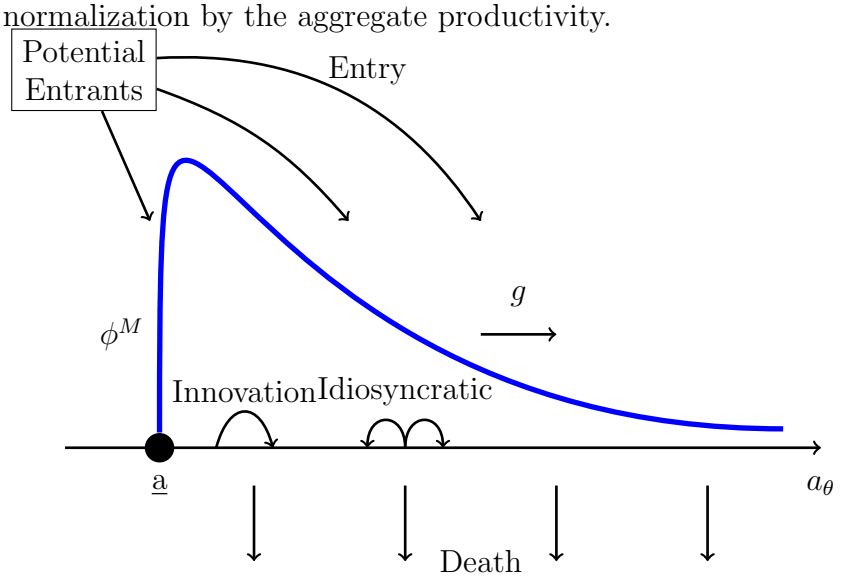


Figure 4: Illustration of the baseline full model. a_θ is an individual firm's log-productivity, and the bold blue line represents its distribution.

4.1 Market Structure

4.1.1 Preference

The representative consumer's utility Y from goods consumption is defined implicitly by a homothetic Kimball aggregator:

$$M \int_0^1 \gamma(\frac{Y_\theta}{Y}) \mathrm{d}\theta = 1 \tag{3}$$

where M denotes the total measure of varieties, Y_θ is the consumption of variety $\theta \in [0,1]$ and γ is an increasing and concave function with $\gamma(0) = 0$ and $\gamma(1) = 1$.

Each variety is produced by one firm. Note that a higher θ does not necessarily corresponds to a higher productivity: θ indexes a firm whose productivity evolves in a dynamic setting. In any static moment, nevertheless, one can re-index firms such that $\hat{\theta} \in [0,1]$ indexes log-productivities in an orderly way. With an abuse of notation, I shall use θ as $\hat{\theta}$ for denoting the productivity quantile. Then $\theta = \Phi(a_\theta)$ is the cumulative probability of the log-productivity distribution, where a_θ is the log productivity of firm θ and Φ is the Cumulative Distribution Function (CDF) of log productivities.[34]

It should be emphasized that equation 3 implicitly incorporates the distribution of productivities: change variable from θ to a_θ to obtain $M \int_{\mathbb{R}} \gamma(\frac{Y(a_\theta)}{Y}) \phi(a_\theta) \mathrm{d}a_\theta = 1$,[35] where ϕ is

[34] To see why cumulative probability is used for indexing firms, consider the discrete counterpart of equation 3: $M \frac{1}{I} \sum_{i=1}^{I} \gamma(\frac{Y_{i/I}}{Y}) = 1$. In this case, $\theta(a_{i/I}) = \frac{i}{I}$ for the i-th firm.

[35] The variable to be integrated can of course be written as any variable such as x instead of a_θ, but I shall often use a_θ to emphasize the nature of individual firms.

the PDF of log productivities. Denote $\phi^M = M\phi$ to be the generalized PDF which has a total measure of M. The distribution and its change will be the key concern in the paper.

The representative consumer maximizes his/her utility Y subject to the budget constraint $M\int_0^1 P_\theta Y_\theta \,\mathrm{d}\theta = W$, where P_θ is the price of variety θ and W the income of the consumer. Solving the consumer's maximization problem gives the inverse-demand curve for each variety θ:

$$\frac{P_\theta}{P} = \gamma'(\frac{Y_\theta}{Y}) \tag{4}$$

where the price aggregator P satisfies $\frac{P}{\zeta}Y = W$ and ζ is defined as $\zeta = \left[M\int_0^1 \gamma'(\frac{Y_\theta}{Y})\frac{Y_\theta}{Y}\,\mathrm{d}\theta\right]^{-1}$. Note that there are two price indices when the preference is Kimball: P and ζ.[36] The ideal price index, which is used to deflate nominal income to evaluate welfare, is P/ζ and *not* P.

For notational simplicity, I shall denote $Z_\theta = \frac{Y_\theta}{Y}$ as the relative consumption and $z_\theta = \log(Z_\theta)$. The price elasticity of demand for variety θ is:

$$\sigma_\theta = \frac{\gamma'(Z_\theta)}{-Z_\theta\gamma''(Z_\theta)} \tag{5}$$

The CES preference is a special case of the Kimball preference with $\gamma(Z) = Z^{\frac{\sigma-1}{\sigma}}$, where $\sigma > 1$ is the elasticity of substitution between varieties of goods. By introducing a different functional form of γ, we can generate heterogeneous demand elasticities and hence heterogeneous markups for heterogeneous firms. I use a new parametric specification in the spirit of Atkeson and Burstein (2008) which is standard in the literature on market power.[37] In particular, define γ' for the inverse demand curve 4:

$$\gamma'(Z_\theta) = C\left[\frac{1}{\underline{\sigma}}Z_\theta^{\frac{k\underline{\sigma}}{\overline{\sigma}-\underline{\sigma}}} + \frac{1}{\overline{\sigma}}Z_\theta^{\frac{k\overline{\sigma}}{\overline{\sigma}-\underline{\sigma}}}\right]^{-\frac{\overline{\sigma}-\underline{\sigma}}{k\overline{\sigma}\underline{\sigma}}} \tag{6}$$

where $1 < \underline{\sigma} \leq \overline{\sigma}$, $k > 0$ and $C > 0$. $\overline{\sigma}$ is the upper bound of demand elasticities and $\underline{\sigma}$ is the lower bound.[38] k governs the transition from the highest demand elasticity to the lowest when relative consumption increases. $C > 0$ is a constant pinned down by normalizing conditions. The special case of $\underline{\sigma} = \overline{\sigma}$ corresponds to CES. When $\underline{\sigma} < \overline{\sigma}$, larger firms face lower demand elasticities and charge higher markups. To see the transition of demand elasticity more clearly, use equation 5 for calculating the demand elasticity:

$$\sigma(z_\theta) = \overline{\sigma} + \frac{\underline{\sigma} - \overline{\sigma}}{1 + \exp(-kz_\theta)} \tag{7}$$

i.e. σ is a logistic function of z_θ, which decreases monotonically with z_θ and has limits $\lim_{z_\theta\to+\infty}\sigma(z_\theta) = \underline{\sigma}$ and $\lim_{z_\theta\to-\infty}\sigma(z_\theta) = \overline{\sigma}$. The specification resembles the nested CES of Atkeson and Burstein (2008) as it combines two power functions, each corresponding to one limiting demand elasticity. Section 4.1.3 discusses the properties of the demand system and shows that the resemblance is not only in appearance but also in essence.

[36] This is a general feature of Homothetic Direct Implicit Additivity (HDIA) preferences of which Kimball is a special example, see Matsuyama (2023).

[37] The current literature has often adopted the Klenow and Willis (2016) specification of Kimball. However, as Baqaee et al. (2023) remarks, the Klenow and Willis (2016) specification implies that the demand elasticity converges to 0 too quickly as the consumption of this variety increases, which implies excessively low sales and high markups for these varieties.

[38] See Appendix D.1 for the numerical calculation of γ.

4.1.2 Firms' Production Decision

Optimal pricing under monopolistic competition implies the markup:

$$\mu_\theta \triangleq \frac{P_\theta/P}{MC_\theta/P} = \frac{\sigma(z_\theta)}{\sigma(z_\theta)-1} \tag{8}$$

where $MC_\theta = w/A_\theta$ is the marginal cost of production and w is the wage rate. Equation 8 is similar to the markup formula under CES, with the sole exception that σ now depends on z_θ. With given aggregate variables w and P, the intersection of the inverse demand curve 4 and the pricing equation 8 solves a firm's static production problem at each time. Both price and quantity are functions of the firm's productivity.

Production incurs a fixed cost of fA per period, where A reflects higher wages as the economy grows. Firms with operational profits less than fA per period die immediately. Denote the cutoff log productivity to be $\underline{a}$ which is determined by the break-even condition $\Pi(\underline{a}; \phi^M) = fA$.

4.1.3 Properties of the Demand System

Aggregate productivity is defined by $A = Y/L$ where $L = M\int_0^1 L_\theta \, \mathrm{d}\theta$ is the total amount of labor inelastically supplied by the consumer. It is easy to check that

$$A = \left[M \int_0^1 Z_\theta A_\theta^{-1} \, \mathrm{d}\theta\right]^{-1} \tag{9}$$

Given the measure of firms M, all variables on the right hand side are determined by the productivity distribution so that the equation can be represented as $A = \mathcal{A}(\{A_\theta\}_{\theta\in[0,1]})$.[39]

Proposition 1 (Homotheticity of the Productivity Aggregator)**.** *$\mathcal{A}$ is a homothetic aggregator of $\{A_\theta\}_{\theta\in[0,1]}$, i.e. $\mathcal{A}(\{sA_\theta\}_{\theta\in[0,1]}) = s\mathcal{A}(\{A_\theta\}_{\theta\in[0,1]})$, $\forall s > 0$.*

See Appendix C.1.2 for the proof. The key in the proof is that: a constant adjustment of all productivities adjusts wage-price ratio by the same percentage but keeps relative quantities $\{Z_\theta\}_{\theta\in[0,1]}$ unchanged. Homotheticity connects micro-level speed of the log-productivity distribution with macro-level growth. Proposition 1 is true for a general Kimball demand, while the following proposition is valid under the specific form in this paper.

Proposition 2 (Demand Elasticities, Passthroughs and Pareto Tail)**.** *If $1 < \underline{\sigma} < \overline{\sigma}$, then*

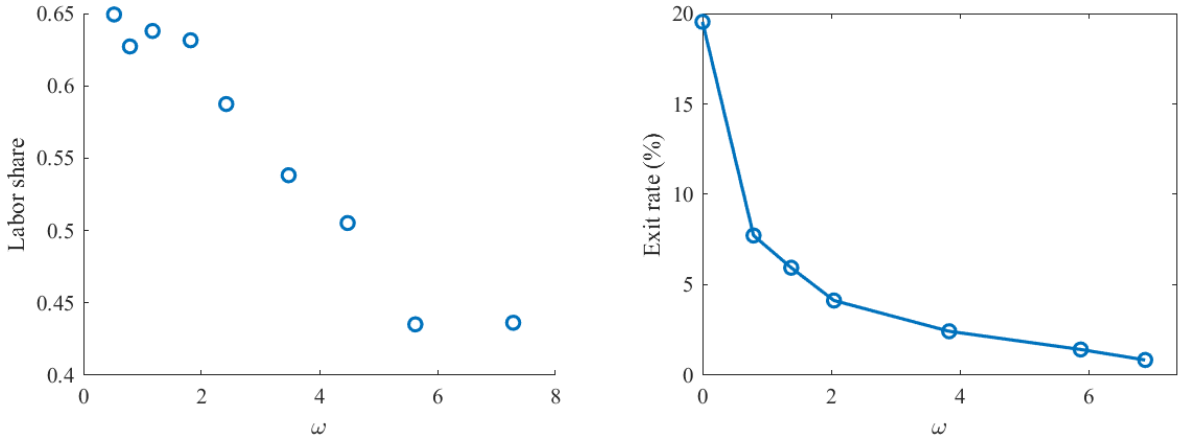
1. *$\frac{\partial\sigma}{\partial a_\theta} < 0$, i.e. demand elasticity decreases with a firm's productivity, so that more productive firms charge higher markups.*
2. *There exists a threshold $\hat{a}$, such that passthrough $\frac{\partial \log(P_\theta)}{\partial \log(MC_\theta)}$ as a function of a_θ is decreasing on $(-\infty, \hat{a}]$ and increasing on $[\hat{a}, +\infty)$, i.e. the passthrough of cost shocks to price decreases with a firm's productivity when the latter does not exceed a certain threshold $\hat{a}$.*

[39] In the special case of CES, $A = [M\int_0^1 A_\theta^{\sigma-1} \, \mathrm{d}\theta]^{\frac{1}{\sigma-1}}$.

3. *$P_\theta Y_\theta \sim C_1 A_\theta^{\sigma-1}$ and $L_\theta \sim C_2 A_\theta^{\sigma-1}$ as $a_\theta \to +\infty$, where C_1 and C_2 are positive constants. Moreover, the distributions of these variables are Pareto-tailed if the productivity distribution is Pareto-tailed.*

See Appendix C.1.3 for the proof. The notation ∂ is used to emphasize that only the direct effect of a_θ on the respective variable is considered: indirect effects through aggregate variables are ignored as each firm is infinitesimal by assumption.

As my specification of Kimball is in the same spirit as nested-CES, it is of no surprise that it satisfies the properties of markup, passthrough and large firm asymptotics like Atkeson and Burstein (2008). These properties are consistent with the empirical evidence, as surveyed by Arkolakis and Morlacco (2017),[40] and is crucial for Schumpeterian incentives of growth.[41] Figure 5a shows labor shares across US manufacturing establishments, where each point represents a bin of employment size range. With the abuse of notation, θ is the share of bottom firms and $\omega = -\log(1-\theta)$ is used as index instead to take into account the Pareto tail,[42] with a larger ω meaning a larger firm. Labor share decreases with establishment size with possible bounds on the two extremities. Equation 7 is a natural specification to capture the feature.



(a) Labor share (b) Exit rate

Figure 5: Implications of variable demand elasticities. Left panel: US Census of Manufacturing 2002 corrected by KLEMS and the Annual Survey of Manufacturing. Each dot is the average labor share of firms within the employment distribution bin $\omega \in [\omega_i, \omega_{i+1}]$. Right panel: average of US Business Dynamic Statistics 1978-1982, all sectors at the establishment level.

Why not using the nested-CES of Atkeson and Burstein (2008) directly? The oligopolistic competition setting of nested-CES implies strategic interactions between firms which makes a dynamic model generally intractable. By contrast, monopolistic competition allows

[40] The only concern is increasing passthrough when productivity is excessively large. This feature also exists in nested-CES: asymptotically, the firm is competing with other *sectors* with CES aggregation.

[41] In fact, the demand system in the Schumpeterian growth models of Aghion et al. (2001) and Aghion et al. (2005) is a simplified version of nested-CES.

[42] See Appendix E.1 for detailed discussions.

individual firms to interact with the whole distribution but not with specific firms: this matches the requirement of the Mean Field Games methodology. My demand system is thus a counterpart of Atkeson and Burstein (2008) under monopolistic competition.[43]

4.2 Exit and Entry

4.2.1 Exit

Incumbents face idiosyncratic shocks of log productivity modelled as Brownian motion with a standard deviation of ν, regardless of their current productivity.[44] Sales are more volatile for smaller firms facing higher demand elasticities. A firm dies if the decrease in sales per period exceeds κ log-points. Formally, death rate $\lambda_{d,\theta}$ is defined as:

$$\lambda_{d,\theta} = \lambda_d(a_\theta; \phi^M) = \mathbb{P}\Big(\frac{\partial \log(P_\theta Y_\theta)}{\partial a_\theta} \cdot \nu B_1 < -\kappa\Big) \tag{10}$$

where $B_1 \sim \mathcal{N}(0,1)$.[45] $\lambda_{d,\theta}$ thus decreases with firm size, consistent with the empirical evidence in Figure 5b. Heterogeneous demand elasticities generate heterogeneous death rates.

A second source of exit concerns only firms close to the left boundary $\underline{a}$. A negative productivity shock can bring these firms to the left of $\underline{a}$ and thus force them to exit. The total number of firms dying from this second channel is $M\frac{\nu^2}{2}\frac{\mathrm{d}\phi}{\mathrm{d}a}(\underline{a})$ per period.[46]

4.2.2 Entry

At any time, there are M_e potential entrants or outsiders. Each of them invests in learning and enters if the learning is successful. If not successful, the opportunity to enter lapses and a new set of M_e outsiders prepares for entry. As any log productivity below $\underline{a}$ is not profitable to exploit, outsiders have free access to $\underline{a}$ as its initial log productivity. The learning cost takes the form $R_e = \frac{1}{2}\alpha_e\lambda_e^2 A$, where λ_e is the success rate. If the entry is successful, the outsider becomes an incumbent by randomly matching with incumbent firms: An incumbent with log productivity a_j is randomly drawn to be matched the entrant, which then enters at position $a_\psi \in (\underline{a}, a_j]$ with probability

$$\psi(a_\psi; \underline{a}, a_j) = k_\psi e^{-k_\psi(a_\psi - \underline{a})} + e^{-k_\psi(a_j - \underline{a})}\delta_{a_j}(a_\psi) \tag{11}$$

where δ_{a_j} is the Dirac mass function at point a_j and $k_\psi > 0$ suggests imperfect learning as explained in Section 4.2.3. $\underline{a}$ is included in the notation $\psi(a_\psi; \underline{a}, a_j)$ to emphasize the initial

[43] Commonly used demand systems with heterogeneous elasticities under monopolistic competition do not satisfy these properties very well. Translog preference (Feenstra (2003)) and Generalized CES (Arkolakis et al. (2019)) imply counterfactually *higher* passthroughs for more productive firms. Even though the quadratic preference (Melitz and Ottaviano (2008)) and the Klenow and Willis (2016) specification of Kimball satisfy the first two properties, both have the issue discussed in Footnote 37.

[44] Put in discrete terms, during each period a productivity shock of $\mathcal{N}(0, \nu^2)$ is realized at the log level.

[45] To incentivize the definition, I have stayed slightly outside the continuous-time setting and considered its discrete counterpart. Nonetheless, equation 10 only depends on variables at time t and is equally suitable for the continuous-time setting.

[46] See Online Appendix G.2 for the proof.

position $\underline{a}$ before entry. Equation 11 nests the special case of Lucas and Moll (2014) with $k_\psi = 0$. Figure 6 illustrates two matchings, each with a different incumbent.

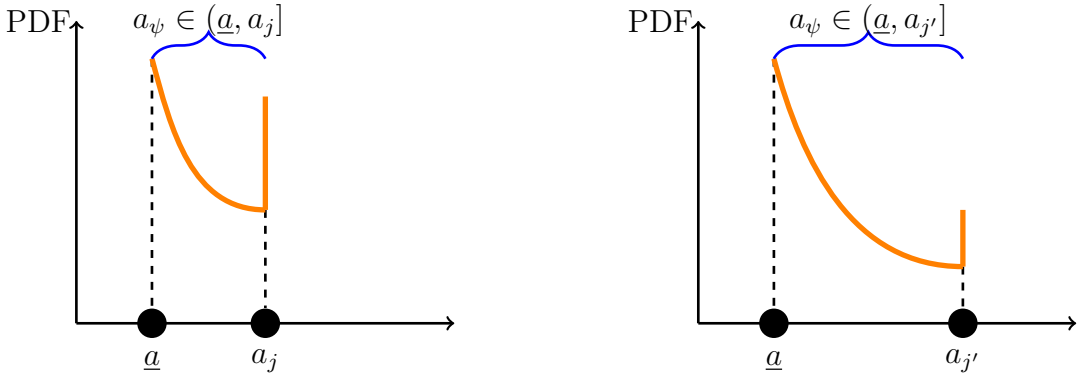


Figure 6: Illustration of an entrant being matched with an incumbent a_j or a'_j. The vertical line at a_j or a'_j illustrates the Dirac mass.

Considering all possible matchings, entry occurs at position a_ψ with PDF:

$$\psi_e(a_\psi; \phi) = \int_{\underline{a}}^{+\infty} \psi(a_\psi; \underline{a}, a_j)\phi(a_j)\, \mathrm{d}a_j = [k_\psi \overline{\Phi}(a_\psi) + \phi(a_\psi)]e^{-k_\psi(a_\psi - \underline{a})} \tag{12}$$

where $\overline{\Phi}(a_\psi) = 1 - \Phi(a_\psi) = \mathbb{P}(a_\theta > a_\psi)$ is the survival function of the log productivity distribution. Moreover, the survival function corresponding to ψ_e is $\overline{\Psi}_e(a_\psi) = \overline{\Phi}(a_\psi)e^{-k_\psi(a_\psi - \underline{a})}$ such that Φ first-order stochastically dominates Ψ_e. We will shortly compare the right tail of the two distributions.

4.2.3 Properties of the Entry Specification

Equation 11 is a succinct description of step-by-step learning *with commitment*, and can be understood intuitively by resorting to its discrete counterpart illustrated in Figure 7.[47] Discretize the segment between $\underline{a}$ and a_j into equally-spaced N steps. An entrant starts from $\underline{a}$ and goes up step by step, each step with success probability $p = e^{-\frac{1}{N}k_\psi(a_j - \underline{a})}$, with a_j setting the upper bound of movement. The probability of entering at position $n < N$ is thus $p^n(1-p)$. Using the fact that at position n, $a_\psi = \underline{a} + \frac{n}{N}(a_j - \underline{a})$, we have

$$p^n(1-p) = e^{-k_\psi(a_\psi - \underline{a})}(1 - e^{-\frac{1}{N}k_\psi(a_j - \underline{a})}) \approx k_\psi e^{-k_\psi(a_\psi - \underline{a})} \cdot \frac{1}{N}(a_j - \underline{a})$$

which is just the continuous part of equation 11 times the step size. The Dirac mass of equation 11 is the probability of entering at a_j after going through all N steps, i.e. p^N.

$k_\psi > 0$ (or equivalently $p < 1$ in the discrete case) is named "imperfect learning" as the entrant may not be able to learn all the knowledge from a_j. Equations 11 and 12 look more complicated with $k_\psi > 0$ than with $k_\psi = 0$, but are intrinsically simpler with $k_\psi > 0$ as it renders the solution to the KF equation unique. As an illustrative example,

[47] See also König et al. (2016) for the discrete specification.

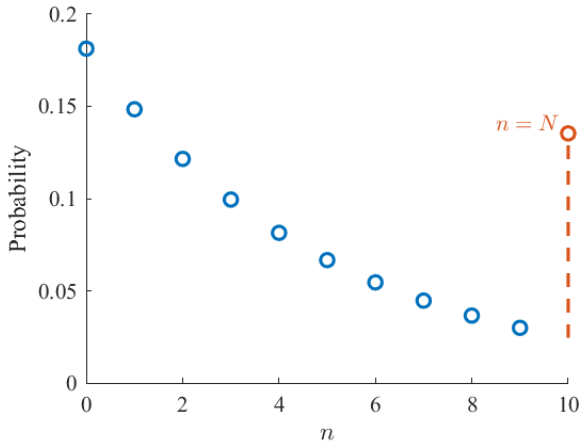


Figure 7: Discrete counterpart of an entrant learning from a_j. $n = 0$ corresponds to $a_\psi = \underline{a}$, and $n = N$ to $a_\psi = a_j$. For illustration purposes with $\underline{a} = -1$, $k_\psi = 1$, $a_j = 1$ and $N = 10$.

if the log productivity distribution is exponential, i.e. $\phi(a) = k_\phi e^{-k_\phi(a-\underline{a})}\ \forall a \geq \underline{a}$, then $\psi_e(a_\psi;\phi) = (k_\phi + k_\psi)e^{-(k_\phi+k_\psi)(a_\psi-\underline{a})}$, which has the same tail as ϕ if $k_\psi = 0$ and a thinner tail if $k_\psi > 0$. Thus if $k_\psi = 0$ and the model starts from an initial ϕ with a sufficiently heavy tail, the injected ψ_e keeps interfering with ϕ, preventing the latter from evolving to a lighter tail. The model then features multiple equilibria with the equilibrium ϕ depending on the initial ϕ like in Lucas and Moll (2014). Adding the Brownian motion as in Luttmer (2012) and Perla et al. (2021) does not essentially make the equilibrium distribution unique, as both papers require the initial condition to be thin-tailed enough to avoid the interference.[48] Such an assumption in this paper would be equivalent to assuming higher concentration over time which I seek to explain.[49] On the other hand, if $k_\psi > 0$, ψ_e always has a lighter tail than ϕ under very weak assumptions, as proved in the following proposition. It serves the basis for proving the unique equilibrium tail index of the KF equation and is a precondition for the numerical algorithm to start from *any* initial condition of ϕ.[50]

Proposition 3 (Non-interference Condition of Entry)**.** *If ϕ is ultimately monotone (i.e. ϕ is monotone on $[\hat{a}, +\infty)$ for some $\hat{a}$), then $\forall k_s \in (-\infty, k_\psi)$, $\lim_{a_\psi \to +\infty} \frac{\psi_e(a_\psi;\phi)}{\phi(a_\psi)} e^{k_s a_\psi} = 0$. In particular, if $k_\psi > 0$, then $\lim_{a_\psi \to +\infty} \frac{\psi_e(a_\psi;\phi)}{\phi(a_\psi)} = 0$ by setting $k_s = 0$.*

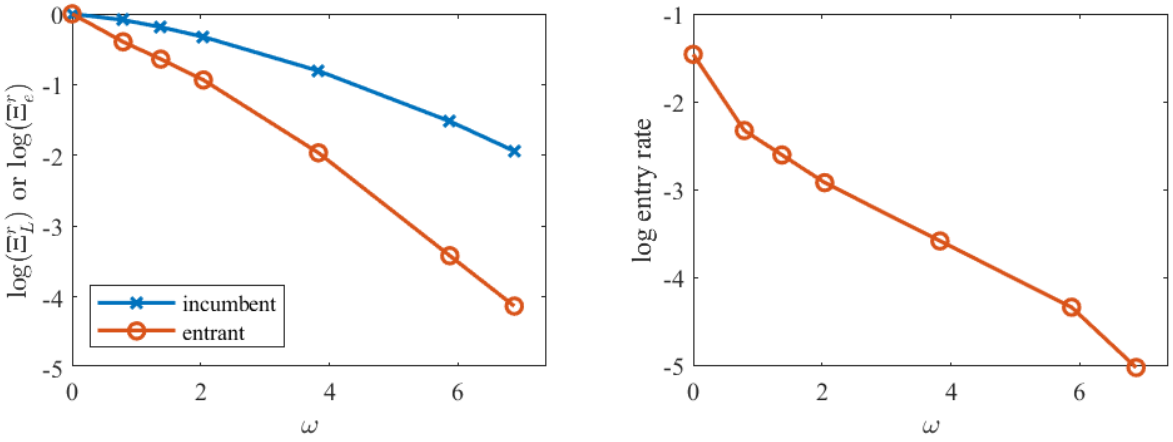
See Appendix C.2 for the sketch of proof and Online Appendix H for the detailed proof. The assumption of imperfect learning is not only theoretically convenient, but also empirically plausible. US BDS data reports the number of establishments, incumbent employment and job creation from entrants by the size bin of establishments. $k_\psi > 0$ can be validated by a lighter tail of entrants than incumbents. To this end, for each ω I calculate the complementary cumulated share of employment respectively for incumbents and entrants, $\log(\xi_L^r)$

[48] The property of the Fisher-KPP type of KF equation in Luttmer (2012) is well understood; see McKean (1975). In particular, if the initial condition is sufficiently heavy-tailed, multiple equilibria emerge.

[49] The assumption is not important for their main results though.

[50] While multiple equilibria in the case of $k_\psi = 0$ is not a problem per se, the above argument shows that it is the result of a cutting-edge assumption unlikely to hold empirically.

and $\log(\xi_e^r)$, where r stands for "right" to emphasize the accumulation from the right tail. Linearity of $\log(\xi_L^r)$ ($\log(\xi_e^r)$) in large values of ω implies Pareto tail of incumbent (entrant) employment distribution. A flatter line suggests a heavier tail. Figure 8a shows that entrants indeed exhibit a lighter tail than incumbents. Another way to see this is job creation rate from birth, i.e. entry rate, by the size bin of establishments. If $k_\psi > 0$, then the log entry rate should be a linear and decreasing function of ω at the right tail. Figure 8b confirms it.



(a) (b)

Figure 8: Check $k_\psi > 0$. Left panel: complementary cumulated share of employment for incumbents and entrants, as a function of ω, where the same ω is used for incumbents and entrants of the same bin. Right panel: log entry rate as a function of ω. Data source: Average of US Business Dynamics Statistics 1978-1982, all sectors.

4.3 Evolution of Incumbents

R&D takes the same form as in the simple model of Section 3: spending $R_{i,\theta} = R(\lambda_{i,\theta}; \tilde{a}_\theta) = \frac{1}{2}\alpha \tilde{A}_\theta^\beta \lambda_{i,\theta}^2 A$ allows an incumbent firm to realize an innovation with Poisson rate of $\lambda_{i,\theta}$ whose success increases the log productivity by q.[51] $\tilde{a}_\theta = a_\theta - a$ is the relative log TFP of firm θ versus the aggregate. Like in the simple model, $\tilde{A}_\theta^{-\beta/2}$ captures R&D efficiency in a cross-section and $1/\sqrt{\alpha}$ captures general R&D efficiency for all firms at a time. Again, I will focus on solving an equilibrium distribution with a constant α.[52] The stochastic process of the normalized log productivity follows Equation 1, except that $J_{\theta,t}$ now consists of two parts: death and innovation. If death is realized with Poisson intensity $\lambda_{d,\theta,t}$, then $a_{\theta,t}$ jumps directly to $-\infty$ and firm θ exits. If innovation is realized with intensity $\lambda_{i,\theta,t}$, then the log productivity jumps up by q.

[51] The expected per-period log-productivity improvement is $\lambda_{i,\theta} q$: choosing the success rate $\lambda_{i,\theta}$ or the step size q are equivalent formulations, and the paper uses the former.

[52] To be more self-contained, we can introduce a government that keeps α constant by investing in fundamental researches and taxing the representative consumer accordingly. Since the final good is a homothetic aggregator, this does not change the result of the model.

4.4 Innovation Decisions and Evolution of Productivity Distribution

Given the building blocks in previous sections, each incumbent solves its optimal innovation decision by a Hamilton-Jacobi-Bellman (HJB) equation, and each potential entrant solves its optimal entry. The productivity distribution evolves based on these optimal decisions via a Kolmogorov forward (KF) equation. I will focus on the balanced growth path in which the log-productivity distribution keeps its shape and travels at a constant speed, i.e. a travelling-wave solution. Online Appendix G derives the equations in an accessible way.

Each incumbent's optimisation problem can be formulated as an HJB equation:

$$(r-g)\tilde{V}(\tilde{a}_\theta) = \max_{\lambda_i}\left\{\Pi(\tilde{a}_\theta;\tilde{\phi}^M) - f - \tilde{R}(\lambda_i;\tilde{a}_\theta) + \tilde{\mathcal{F}}\tilde{V}\right\} \tag{13}$$

with boundary condition $\tilde{V}(\underline{\tilde{a}}) = 0$ for the left absorbing barrier[53], where

$$\tilde{\mathcal{F}}\tilde{V} = \Big[-\frac{1}{2}\nu^2 - g\Big]\tilde{V}' + \frac{1}{2}\nu^2\tilde{V}'' + \lambda_i\big[\tilde{V}(\tilde{a}_\theta+q) - \tilde{V}(\tilde{a}_\theta)\big] - \lambda_d(\tilde{a}_\theta;\tilde{\phi}^M)\tilde{V}, \tag{14}$$

r is the interest rate, Π is per-period production profit, and $\tilde{R}(\lambda_i;\tilde{a}_\theta) = \frac{1}{2}\alpha e^{\beta\tilde{a}_\theta}\lambda_i^2$ is R&D cost.

Equation 13 has an intuitive interpretation: the per period flow value should be equal to the sum of current-period profit net of fixed cost and innovation cost, plus a change of value due to firm dynamics, i.e. $\tilde{\mathcal{F}}\tilde{V}$. The first two terms of $\tilde{\mathcal{F}}\tilde{V}$ correspond to the drift and Brownian motion, and the last two terms to innovation and death. The optimal innovation intensity is thus determined by the first order condition $\lambda_i^*(\tilde{a}_\theta) = 1/(\alpha e^{\beta\tilde{a}_\theta})\cdot\big[\tilde{V}(\tilde{a}_\theta+q) - \tilde{V}(\tilde{a}_\theta)\big]$.

Each potential entrant solves the optimal entry decision:

$$\max_{\lambda_e}\Big\{-\tilde{R}_e(\lambda_e) + \lambda_e\int_{\mathbb{R}}\tilde{V}(\tilde{a}_\psi)\psi_e(\tilde{a}_\psi;\tilde{\phi})\,\mathrm{d}\tilde{a}_\psi\Big\} \tag{15}$$

where $\tilde{R}_e(\lambda_e) = \frac{1}{2}\alpha_e\lambda_e^2$. The maximisation problem involves only the instant period as the opportunity to enter lapses if not taken advantage of. The optimal entry intensity is thus $\lambda_e^* = 1/\alpha_e\cdot\int_{\mathbb{R}}\tilde{V}(\tilde{a}_\psi)\psi_e(\tilde{a}_\psi;\tilde{\phi})\,\mathrm{d}\tilde{a}_\psi$.

Given the choice of λ_i and λ_e, the Kolmogorov Forward (KF) equation is:

$$0 = \tilde{\mathcal{K}}\tilde{\phi}(\tilde{a}_\theta) + \frac{\lambda_e M_e}{M}\psi_e(\tilde{a}_\theta;\tilde{\phi}) \tag{16}$$

with boundary condition $\tilde{\phi}(\underline{\tilde{a}}) = 0$,[54] where

$$\tilde{\mathcal{K}}\tilde{\phi} = (\frac{1}{2}\nu^2+g)\tilde{\phi}' + \frac{1}{2}\nu^2\tilde{\phi}'' + \lambda_i(\tilde{a}_\theta-q)\tilde{\phi}(\tilde{a}_\theta-q) - \lambda_i(\tilde{a}_\theta)\tilde{\phi}(\tilde{a}_\theta) - \lambda_d(\tilde{a}_\theta;\tilde{\phi}^M)\tilde{\phi}. \tag{17}$$

[53] At the algorithm level, another boundary condition is needed on the right as a computer can only handle finite segments. A natural choice is a "reflecting barrier": $\tilde{V}'(\bar{\tilde{a}}) = 0$. Note that the reflecting barrier is *not* used for stabilizing the productivity distribution. As shall be seen, the most productive firms have growth rates lower than average, which already stabilizes the distribution.

[54] At the algorithm level, the boundary condition for the right reflecting barrier is $(\frac{1}{2}\nu^2+g)\tilde{\phi}(\bar{\tilde{a}}) + \frac{1}{2}\nu^2\tilde{\phi}'(\bar{\tilde{a}}) = 0$.

Online Appendix G.5 shows that $\tilde{\mathcal{K}}$ is the adjoint operator of $\tilde{\mathcal{F}}$ so that we can denote $\tilde{\mathcal{K}} = \tilde{\mathcal{F}}^\intercal$. In numerical calculations where $\tilde{\mathcal{F}}$ is discretized into a matrix, the discrete counterpart of $\tilde{\mathcal{F}}^\intercal$ is then the transpose of this matrix.

Equation 16 says that the dynamics of incumbents ($\tilde{\mathcal{K}}\tilde{\phi}$) and the dynamics of entry, taken together, imply a time-invariant distribution after normalization. Within $\tilde{\mathcal{K}}\tilde{\phi}$, $\lambda_i(\tilde{a}_\theta - q)\tilde{\phi}(\tilde{a}_\theta - q)$ captures the inflow into $\tilde{a}_\theta$ from $\tilde{a}_\theta - q$ due to successful innovation from the latter, $-\lambda_i(\tilde{a}_\theta)\tilde{\phi}(\tilde{a}_\theta)$ captures the outflow from $\tilde{a}_\theta$, and $-\lambda_d(\tilde{a}_\theta; \tilde{\phi}^M)\tilde{\phi}$ captures the first type of death.[55]

Entry and exit are balanced along a BGP:

$$\lambda_e M_e = M\Big[\int_{\mathbb{R}} \lambda_d(\tilde{a}_\theta; \tilde{\phi}^M)\tilde{\phi}(\tilde{a}_\theta)\,\mathrm{d}\tilde{a}_\theta + \frac{\nu^2}{2}\tilde{\phi}'(\underline{\tilde{a}})\Big] \tag{18}$$

The following definition summarizes the equations constituting a travelling wave equilibrium.

Definition 1 (Travelling Wave Equilibrium). A travelling wave equilibrium (i.e. balanced growth path), is defined by equations 13-18, together with zero-profit condition at the lower boundary $\Pi(\underline{\tilde{a}}; \tilde{\phi}^M) = f$ and the normalization condition $\mathcal{A}(\tilde{\phi}^M) = 1$. The last two conditions pin down respectively $\underline{\tilde{a}}$ and g.

The following proposition is the most important analytical result of this paper: it establishes the fundamental link between lower aggregate growth and fatter Pareto tail seen in Figure 1, regardless of market power.

Proposition 4 (Unique Tail of Equilibrium Distribution)**.** *If $\beta > \underline{\sigma} - 1$ and $k_\psi > 0$, then*

1. *Any solution $\tilde{\phi}$ to KF equation 16 must be ultimately monotone.*
2. *$\tilde{\phi}$ has an exponential tail with unique tail index $-\xi$, where ξ is the unique negative root of $-\frac{1}{2}\nu^2\xi^2 - (\frac{1}{2}\nu^2 + g)\xi + \lambda_d(\infty) = 0$ and where $\lambda_d(\infty) = \Phi_{\mathcal{N}}(-\frac{\kappa}{\nu(\underline{\sigma}-1)})$ and $\Phi_{\mathcal{N}}$ is the CDF of standard normal distribution $\mathcal{N}(0,1)$. In other words, the equilibrium productivity distribution has a unique Pareto tail index of $-\xi$. Moreover, equilibrium sales and employment distributions have a unique Pareto tail index of $-\xi/(\underline{\sigma} - 1)$.*
3. *$-\xi$ is strictly increasing in g, i.e. lower growth is associated with fatter Pareto tails.*

See Appendix C.2 for the sketch of proof and Online Appendix I for the detailed proof. The non-interference condition under $k_\psi > 0$ (Propositions 3) is key for ensuring the uniqueness.[56] The insight remains valid in an extended model with incumbent firms learning from others, as long as learning is similarly imperfect (See Appendix F). Moreover, as in the simple model, demand needs to be sufficiently saturated compared to R&D efficiency for the largest firms, i.e. $\beta > \underline{\sigma} - 1$. This implies little growth for the largest firms so that aggregate growth summarizes the ability of other firms to catch up.

[55] The second type of death near the left boundary is incorporated into the boundary condition.

[56] Proposition 4 only claims uniqueness of KF equation but not of the HJB-KF system, even though the properties of HJB are used for proving it. The uniqueness of the system remains an open question, even though numerical solutions have been reassuringly unique.

The next proposition illustrates the fixed cost nature of innovation and learning in a dynamic setting. A larger market size L can dilute innovation and learning costs, as well as static fixed cost.

Proposition 5 (Indistinguishable Economies)**.** *An economy with labor L, fixed cost f, innovation cost parameter α and entry cost parameter α_e is indistinguishable from another economy with parameters NL, Nf, $N\alpha$ and $N\alpha_e$ ($N > 0$), keeping all the other parameters the same, except that all the firms and their value functions in the second economy are N times large.*

The indistinguishability is in both the static and the dynamic senses; see Appendix C.3 for a proof by verification. The pro-growth effect of a larger market size should be distinguished from semi-endogenous growth models in which population growth cancels the harder research to maintain constant growth. In the present paper, by contrast, it is by higher profits incentivising Schumpeterian growth. Economic integration involves a one-time increase in market size: it is a reorganization of population without population growth.[57]

5 Calibration

Comparative statics in Section 6 will cover the period from 1980 to 2020, while those in Section 8 will cover the period from 1960 to 1980. The choice of 1980 as the watershed year is motivated by most papers in the literature, which examine declining growth, increasing concentration and market power, declining labor share, and diminishing business dynamism in the post-1980 period.[58] Most parameters remain unchanged from 1980 to 2020, except research cost parameter α, entry learning cost parameter α_e which is intimately tied to α, and the interest rate r. r affects how a firm discounts future profits and thus its growth incentives: it needs to be changed in accordance with the data to evaluate the performance of the model.[59] Table 3 summarizes all the externally calibrated parameters for 1980 and 2020.

Demand parameters $\underline{\sigma}$, $\overline{\sigma}$ and k are estimated by labor shares in a cross section of firms. For a few vintages, the US Census Bureau publishes data on payrolls, material costs, sales, value added and number of establishments for each employment size range of manufacturing establishments. Appendix E.2 and E.3 discusses the calculation and adjustments of labor share based on the Census data to be consistent with the model. $\underline{\sigma}$ is estimated as 1.93 which corresponds to the lowest labor share of the largest establishments in the data, consistent with the estimates of other papers with nested-CES under oligopolistic competition (see Atkeson and Burstein (2008), De Loecker et al. (2021), Gaubert and Itskhoki (2021) and Burstein et al. (2020)). The adjusted labor share of the smallest firms, $\overline{\chi}$, is close to 1. As

[57] The pro-growth effect of a larger market size is reminiscent of the argument in Chandler (1990) that firms expand their geographical reach as a growth strategy.

[58] Such a focus in the literature is partly driven by data availability, as US Business Dynamics Statistics (BDS) has only been available since 1978, and partly by data patterns, as the declining labor share and increasing markup have been most apparent since the 1980s.

[59] Appendix E.5 shows that changing r *understates* the impact of increasing α, as lower r encourages growth and thus mitigates the effects of higher α.

Table 3: Summary of external parameters for 1980 and 2020.

Parameter	Description	Value	Source
$\overline{\sigma}$	Upper bound of demand elasticity	6.5	External
$\underline{\sigma}$	Lower bound of demand elasticity	1.93	Estimated from US Census
k	Transition speed of demand elasticity	0.32	Estimated from US Census
$\alpha_{t=2020}/\alpha_{t=1980}$	Change in R&D cost	6.84	Estimated from US aggregate
β	Harder research for higher TFP	1.6	Estimated from Compustat
$r_{t=1980}$	Interest rate	0.0469	External
$r_{t=2020}$	Interest rate	0.0109	External
L	Labor force	1	Normalization
$\alpha_{e,t}$	R&D cost of entrants	$\alpha_t \underline{\tilde{A}_t}^{\beta}$	Model assumption

Note: Parameters that change between 1980 and 2020 are indexed by t.

Table 4: Estimated parameters for 1980 by moment matching.

Parameter	Description	Value	Main Identifying Moment	Moment Data	Moment Model
$\alpha_{t=1980}$	R&D cost parameter in 1980	557	TFP growth rate	0.95%	0.98%
q	Innovation step	0.201	R&D share of GDP	2.4%	2.3%
ν	Std of idiosyncratic shocks	0.118	Job reallocation rate	29%	28%
κ/ν	Relative death threshold	2.99	Exit rate	5.6%	5.6%
M_e	Mass of potential entrants	0.71	Entry rate	5.6%	5.6%
f	Fixed cost	0.104	Bottom 55% firms' exit rate	20%	20%
k_ψ	Learning parameter	1.37	Growth due to net entry	0%	-0.08%

$\overline{\sigma} = \frac{1}{1-\overline{\chi}}$, $\overline{\sigma}$ is sensitive to a small change in $\overline{\chi}$. Thus I have calibrated $\overline{\sigma}$ as 6.5, which is in the middle of the range used in these nested-CES papers. k matches the decrease of labor share as firm size increases.

The R&D cost function implies that $\lambda_i = (\frac{2}{\alpha})^{\frac{1}{2}} e^{-\frac{\beta}{2}\tilde{a}_\theta} \tilde{R}^{\frac{1}{2}}$, which is the idea production function in Bloom et al. (2020) with decreasing returns to scale of degree $\frac{1}{2}$. Following their paper, the effective number of researchers $\tilde{R}$ is measured as R&D expenditures deflated by the average male wage with four years or more of college education, and λ_i is TFP growth (up to a constant multiplier). With aggregate data, the decrease in R&D efficiency $(\frac{2}{\alpha})^{\frac{1}{2}} e^{-\frac{\beta}{2}\tilde{a}_\theta}$ over time reflects a general increase in α across all firms. α is estimated to increase by 6.84-fold from 1980 to 2020 corresponding to a $\sqrt{6.84} = 2.62$-fold decrease in R&D efficiency. β governs R&D efficiency in a cross section of firms and is estimated from micro-level evidence with Compustat.[60] I set α_e to be $\alpha e^{\beta \underline{\tilde{a}}}$ so that entrants' learning cost function also reflects the industry-average R&D efficiency α and cross-sectional differences $e^{\beta \underline{\tilde{a}}}$.[61] The adjustment factor $e^{\beta \underline{\tilde{a}}}$ reflects the more difficult knowledge at higher levels of technology, independently from how likely the entrant can be matched with a productive firm via ψ_e.[62]

I follow Liu et al. (2022) in using the US AA corporate bond rate net of current inflation for calibrating interest rates in 1980 and 2020, as it is more relevant than the 10-year treasury rate as firms' discount rate. They are 0.0469 for 1980 and 0.0109 for 2020.[63]

According to Proposition 5, we can normalize the labor force in 1980 as 1 so that the other parameters can be identified by targeting data moments in 1980. Some moments are especially useful for identifying specific parameters: The structurally estimated parameters and their main identifying moments are summarized in Table 4. $\alpha_{t=1980}$ targets the average TFP growth of 0.95% in the 1970s and 1980s according to BLS-KLEMS data, and q targets the 2.4% share of R&D expenditure over GDP in the National Income and Product Accounts (NIPA) around 1980. ν is identified by the job reallocation rate of 29% around 1980 according to the BDS.[64] The BGP features balanced entry/exit, so I take the average of the entry rate and exit rate around 1980 to be the targets for both moments, which is 5.6% according to the BDS. By the definition of the first kind of death in equation 10, κ/ν rather than κ drives the death rate and is used to target it. The number of potential entrants M_e targets the entry rate of 5.6%. Per-period fixed cost f determines the lower-bound productivity through the zero-profit condition, and thus shapes the demand elasticities and exit rates of the least productive firms. I use it to match the 20% exit rate of the smallest size bin of the bottom 55% firms in the BDS. Finally, this paper takes a productivity view of firm size distribution. As entrants' size distribution closely matches the dying firms' size distribution in the BDS, the contribution of net entry to growth is tiny from this point of view. I estimate k_ψ to match a 0 contribution.[65]

[60] The cross-sectional elasticity of R&D efficiency with respect to technology level is $-\beta/2 = -0.8$.

[61] Despite the same cost function of entry and incumbent innovation, k_ψ and q can adjust the relative effectiveness of one with respect to the other.

[62] Under the interpretation of step-by-step learning with commitment, entrant's learning cost should sum over the costs on the interval $[\underline{\tilde{a}}, +\infty)$, whence the factor $e^{\beta \underline{\tilde{a}}}$.

[63] In standard consumption-based asset pricing models, such a decrease in r can be endogenized by a decrease in g. I remain agnostic about the source of interest rate decline and calibrate an exogenous change.

[64] See Appendix D.6 for the computation of the job creation rate and job destruction rate in the model.

[65] See Appendix D.7 for the computation of the net entry contribution to growth.

The parameters are estimated by the Generalized Method of Moments (GMM). Weights are the inverse of the square moments to represent errors in percentage terms, except for the growth contribution of net entry whose weight is the same as that of TFP growth.

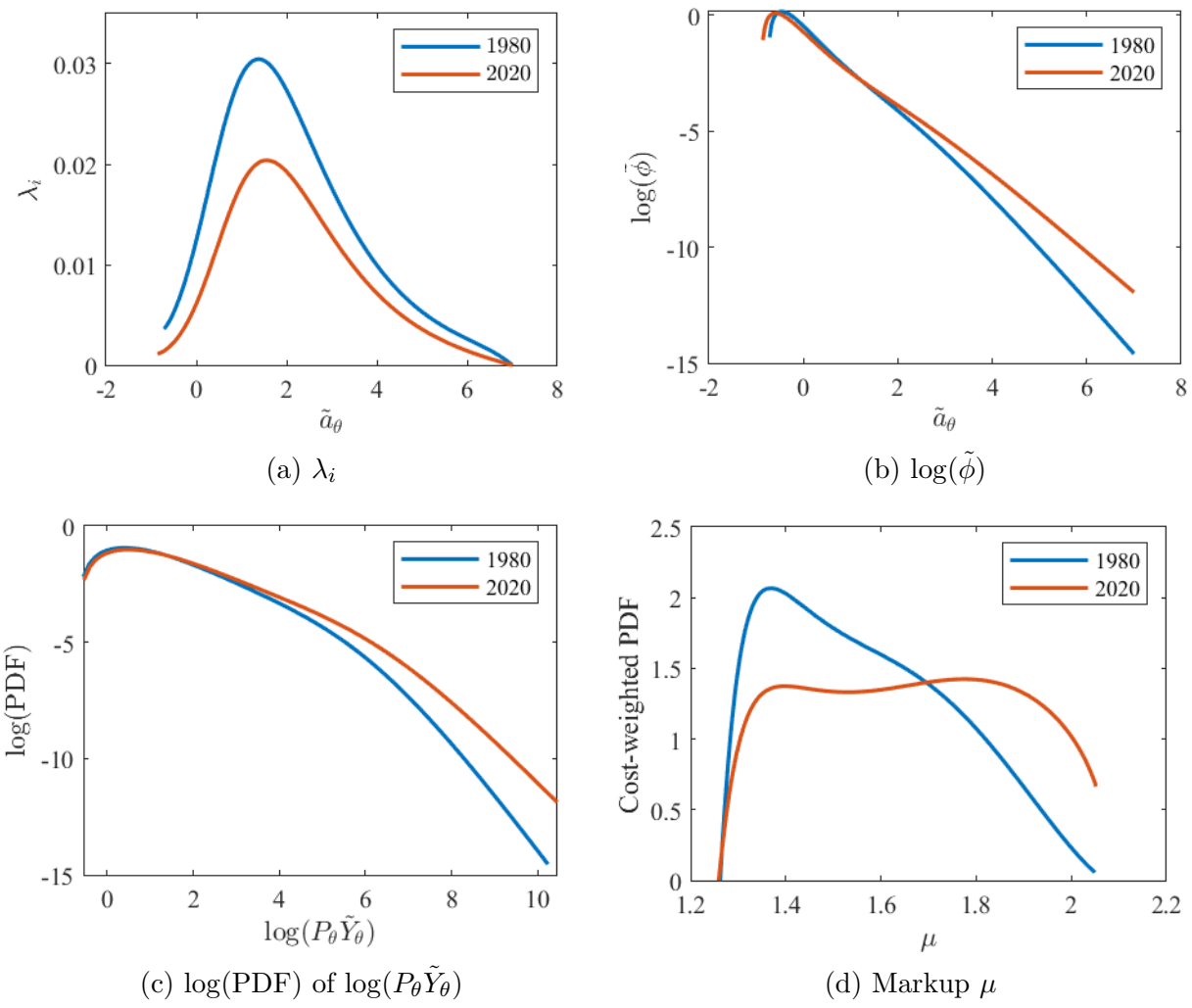
6 Comparative Statics When R&D Becomes Less Efficient

Figure 9a compares the innovation intensities in 1980 and 2020 when R&D becomes less efficient. Each of the curves features an inverted-U between growth and productivity, but the reason is different from existing Schumpeterian growth papers. In the present paper, both the marginal cost and the marginal benefit of innovation increase with a firm's productivity: the former due to less efficient R&D, the latter due to R&D's economies of scale. Marginal benefit however increases slowly for the largest firms due to saturated demand, in particular more slowly than the increase in marginal cost so that growth decreases. The opposite can be said for the smallest firms.

Given the *uniform* decrease in R&D efficiency across all firms, the intensity curve shifts down so that aggregate growth declines. Even though the growth rate of the most productive firms declines in absolute terms, it increases *in relative terms* compared to that of medium-sized firms and the economy in general, which allows leaders to stretch out further to the right tail *relative to* others. Productivity distribution and firm size distribution thus have a fatter Pareto tail in Figures 9b and 9c. This translates into higher markup, as shown in the cost-weighted markup distributions in Figure 9d.

Table 5 summarizes the changes in key macroeconomic moments from 1980 to 2020 in the model and in the data. The model explains a majority of the data changes in all cases despite all these moments being non-targeted. (1) The sales share of the top 1% firms increases as the Pareto tail of the sales distribution fattens. (2) TFP growth declines as harder R&D dominates all other mitigating forces including the lower interest rate. (3) Aggregate markup increases with the tail fatness of the productivity distribution *other things being equal*. (4) The decrease in labor share mirrors the increase in markup as labor is the only factor of production. (5) Table 6 conducts a Melitz and Polanec (2015) decomposition of markup and labor share changes in the model into a within-firm component, a reallocation component between firms, and a net entry component. Both the increase in markup and the decrease in labor share predominantly come from reallocation from low markup (i.e. high labor share) to high markup (i.e. low labor share) firms, which is consistent with the micro-level empirical evidence. [66] (6) The lower demand elasticity that accompanies higher markup also means that the same idiosyncratic productivity shock translates into less job creation or destruction, consistent with the empirical evidence in Decker et al. (2020). So does their death rate. (7) The entry rate declines as learning becomes more difficult for potential entrants. (8) Combining (6) and (7) together, the job reallocation rate decreases. (9) If r remains constant from 1980 to 2020, the above changes remain qualitatively the same but quantitatively larger (Appendix E.5), as a lower r encourages growth and mitigates the harder R&D effect. If r does not change, R&D cost over GDP decreases with α (Table A4).

[66] see De Loecker et al. (2020) for markup, Autor et al. (2020) and Kehrig and Vincent (2021) for labor share.



(a) λ_i (b) $\log(\tilde{\phi})$

(c) log(PDF) of $\log(P_\theta \tilde{Y}_\theta)$ (d) Markup μ

Figure 9: Comparison between 1980 and 2020. Panel (a): innovation intensity; Panel (b): log PDF of log productivity; Panel (c): log PDF of log sales; Panel (d): cost-weighted markup distribution.

The joint increase of R&D difficulty and R&D cost share in the data is reconciled by lower interest rate.

Table 5: Moment changes between 1980 and 2020. All the changes are non-targeted.

Moment	Data Δ	Model Δ
Sales share of top 1% firms	10.07%	10.31%
TFP growth	-0.41%	−0.44%
Cost-weighted markup	11.40%	8.86%
Labor share	-5.39%	−3.41%
Job creation by birth (entry)	-2.70%	−1.93%
Job destruction by death (exit)	-1.67%	−1.67%
Job creation rate	-5.13%	−2.82%
Job destruction rate	-4.06%	−2.88%
Job reallocation rate	-8.12%	−5.69%
R&D over value added	64.50%	75.83%

Table 6: Melitz-Polanec decomposition of markup and labor share changes between 1980 and 2020.

	Within	Between	Net entry	Total
ΔLabor share	−0.22%	−3.15%	−0.03%	−3.41%
ΔMarkup	0.59%	8.31%	−0.04%	8.86%

7 Market Structure and Firm Growth

In the existing Schumpeterian growth literature à la Aghion et al. (2001), growth often declines as a result of higher concentration. Firms with close level of productivity have high incentives to innovate in order to escape competition. Unevenness in the productivity of firms thus reduces the incentives of individual firm growth.

The mechanism in this paper is different. Higher concentration actually encourages firm growth by increasing firm profits. To separate this direction from the opposite direction (i.e. from growth to concentration), I will fix the productivity distribution and consider how different distribution affects growth choice. The key is to understand how concentration affects individual firm profits given its productivity.

In the special case of CES, firms profit does not depend on the distribution. This independence does not hold in the more general setting of Kimball preferences. Firm profit is $\Pi_\theta = \frac{wL}{\sigma(z_\theta)-1} Z_\theta \tilde{A}_\theta^{-1}$, where z_θ is determined by w/A_θ and P. Thus the distribution enters the profit function by the price aggregator P. To see how a change in distribution affects individual firm growth, we proceed in two steps: how a change in distribution affects P; how a change in P affects marginal profit. For the first step, we need to specify the exact meaning of "higher concentration" to derive analytical results. Due to the focus on Pareto tail, I assume that the log-productivity distribution is overall exponential, i.e. $\phi(a_\theta; \xi) = \xi e^{-\xi(a_\theta - \underline{a})}$

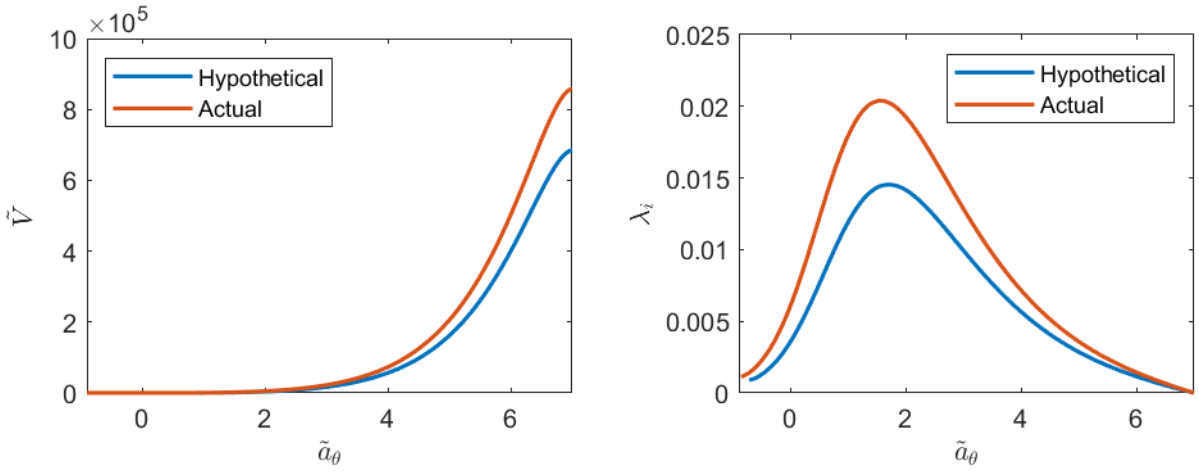
$\forall a_\theta \geq \underline{a}$, where $\underline{a}$ is pinned down by the normalization condition of $A = 1$. I set $M = 1$ for simplicity. Higher concentration is understood as lower ξ.

Proposition 6 (Concentration and Profits). *1. Under the specification of Kimball preference in equation 6, if* $k < 2(1 - 1/\underline{\sigma})$,[67] *then* $\frac{\mathrm{d}\log(P)}{\mathrm{d}\xi} < 0$.

2. $\frac{\partial^2 \Pi_\theta}{\partial \tilde{a}_\theta \partial \log(P)} > 0$ *and* $\frac{\partial \Pi_\theta}{\partial \log(P)} > 0$, *i.e. a higher price index* P *increases both the marginal profit and the profit of every firm.*

See Appendix C.1.4 for the proof. The calibrated parameters fall safely into the range in the proposition, thus higher concentration increases P and profits, consequently encouraging individual firm growth.

The intuition still holds for the solved distributions in Section 6. Consider a hypothetical case in which market structure $\tilde{\phi}^M$ retains its 1980 value in 2020, while all the other parameters have changed to their 2020 value. In the hypothetical case with the 1980 $\tilde{\phi}^M$ and 2020 parameters, the value function increases less quickly with productivity than in the actual case because of lower profits (Figure 10a). Consequently, growth is lower in the hypothetical case (Figure 10b). Therefore, higher concentration on its own raises growth.



(a) Value function

(b) Innovation intensity λ_i

Figure 10: Comparison between the hypothetical case and actual case in 2020. The red line in Panel (b) is the same as the red line in Figure 9a.

8 Introducing Economic Integration

While the previous section accounts well for the key macroeconomic moments since 1980s, a major puzzle remains: before the 1980s, how could concentration increase with a fattening Pareto tail without a trend in markup or labor share?

[67] This sufficient condition is *much stronger* than needed to get $\frac{\mathrm{d}\log(P)}{\mathrm{d}\xi} < 0$, as can be seen in the proof. Even if the parameters fall outside the range, numerical calculations always show a negative sign.

This section argues that introducing regional integration in addition to lower R&D efficiency can account for the pattern, acting as another possible first-order economic force during the earlier periods. Historical evidence motivates such an investigation. The 1920s-1970s saw the heyday of US government spending on highways, interrupted only by World War II (Gordon (2016)). The inflation-adjusted domestic airfare per mile decreased by -4.55% per year between 1940 and 1980, and only by -0.1% per year after 1980. These could have significantly reduced non-tariff trade costs across US regions. The effect of regional integration is potentially large as US regions are close to each other, each of which can be thought of as a small economy. As is well-known, smaller economies are more open to trade, and geographical distance is key.[68] Perhaps not coincidentally, in the last few decades some European countries also witnessed rising concentration with stable market power during the European integration. Such a similar experience also motivates us to consider economic integration as a confounding economic force.

I study the 1960-1980 period in the US to show the effect of economic integration *in conjunction with* less efficient R&D. 1960 is chosen as the starting point for the earlier period, as sales concentration measured by Kwon et al. (2022) is only available from that date. Consider a thought experiment in which there are N identical economies in autarky in 1960, each of which is on the same BGP described by the model. From 1960 to 1980, I calibrate a 7.93-fold increase in α, i.e. $\sqrt{7.93} = 2.82$-fold decrease in R&D efficiency, again using the methodology of Bloom et al. (2020). I also change the interest rate to match the decline in natural interest rate estimated by Holston et al. (2017).[69] To capture integration, I allow these N economies to merge into one: labor force L and the potential number of entrants M_e are multiplied by N.[70] Because of the pro-growth effect of integration, I estimate N by targeting the TFP growth rate of 1.6% around 1960. Concentration, markup and labor share remain non-targeted. Table 7 summarizes the related parameters for 1960.

Table 7: Summary of parameters for 1960.

Parameter	Description	Value	Source
$\alpha_{t=1980}/\alpha_{t=1960}$	Change in R&D cost	7.93	Estimated from US aggregate
$r_{t=1960}$	Interest rate	0.0669	External
N	Magnitude of economic integration	3.96	Structural estimation

I compare the new travelling wave equilibrium of the integrated economy in 1980 with the old one of segregated economies in 1960. The top 1% firm sales concentration increases by 14.43%, which is in the ballpark of the estimate in Kwon et al. (2022) and slightly greater than the change after 1980. Changes in markup and labor share, however, are significantly attenuated. Table 8 shows that while market share is reallocated towards high

[68] Globalization during the post-1980 period should in principle also have an effect through the logic of this section. However, international trade openness measured as the trade-to-GDP ratio increased at least as rapidly in the 1940-1980 period as in 1980-2020. Moreover, the US economy as a whole is large and thus more closed to trade. Thus the current section emphasizes the role of regional integration within the US.

[69] Real corporate rate is not used for the pre-1980 period, as the US experienced high inflation before Volcker's disinflation policies around 1980. High inflation implies that the measured real rate was lower before 1980 than afterwards, even though the natural interest rate was higher before 1980.

[70] Aggregating N representative consumers into one is feasible, as the Kimball preference is homothetic.

markup and low labor share firms as in the 1980-2020 period, this reallocation component is compensated by the within-firm component due to economic integration. As integration increases the number of firms competing with each other, in a purely static setting it reduces within-firm markups more than it reallocates markups towards higher-markup firms because the equilibrium log-productivity distribution $\tilde{\phi}$ is overall log-concave.[71]

To see why we have the concavity of $\log(\tilde{\phi})$, it is instrumental to consider an illustrative example with exogenous growth. It shows that decreasing growth in a cross-section of firms, which was the key for generating higher concentration when aggregate R&D efficiency declines, is also the key for generating concavity of $\log(\tilde{\phi})$.

Example 1. Consider a KF equation

$$0 = -\frac{\mathrm{d}[(\mu - g)\tilde{\phi}]}{\mathrm{d}\tilde{a}} + \frac{1}{2}\nu^2 \frac{\mathrm{d}^2\tilde{\phi}}{\mathrm{d}\tilde{a}^2} \tag{19}$$

with firm-level growth $\mu(\tilde{a})$, and possibly a left reflecting barrier as stabilizing force.

Proposition 7. *If* $\tilde{\phi}(\tilde{a})$ *is a solution to 19, then* $\frac{\mathrm{d}^2 \log(\tilde{\phi})}{\mathrm{d}\tilde{a}^2} \gtrless 0$ *if and only if* $\mu'(\tilde{a}) \gtrless 0$.

The combined effect of integration and harder R&D means strongly attenuated markup and labor share changes, despite a slightly higher increase in concentration. The stable labor share in this context is not understood as an economic regularity as in Kaldor (1961), but rather as a historical coincidence. Such an analysis is only possible with a framework in which a well-defined Pareto-tailed productivity distribution is endogenously generated by firm-specific growth. The model provides a way to study the question in a unified setting.

Table 8: Melitz-Polanec decomposition of markup and labor share changes between 1960 and 1980.

	Within	Between	Net entry	Total
ΔMarkup	−6.18%	9.00%	−0.41%	2.41%
ΔLabor share	3.24%	−4.39%	0.16%	−0.99%

9 Conclusion

The present study constructs a continuous-time Schumpeterian growth model which contributes to the existing literature by emphasizing how growth shapes market structure. As R&D becomes less efficient, growth declines *and consequently* corporate concentration concentrates. It provides a unified explanation linking key macroeconomic moments, and suggests decreasing R&D efficiency as the "fundamental cause" in the words of Grossman and Oberfield (2022).

Changing the perspective from how market structure determines growth to how growth determines market structure implies dramatically different policy recommendations. In particular, anti-trust policies are generally *not* key for promoting growth, contrary to what the

[71] See Footnote 10 on this point.

literature has previously concluded. Undoubtedly, anti-trust policies can still be pro-growth in specific cases where the traditional Schumpeterian logic of higher concentration prohibiting growth is present, or should be applied for ethical and social considerations when they are justified. However, this analysis suggests growth policies should center around promoting technological opportunities for every firm, large or small. This approach is reminiscent of the East Asian Miracles which involved strong development policies designed by the government.[72] In the case of the US, Antolin-Diaz and Surico (2022) shows that military spending increases productivity in the long run through public R&D. Gruber and Johnson (2019) interprets the post-war experience of the US in a similar fashion, suggesting policy implications centering around public R&D. A full analysis of these policies is beyond the scope of this paper and left for future research.

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[72] See for instance Evans (1995), Lane (2022) and Liu (2019). This model however abstracts from capital accumulation and learning from foreign countries which are usually involved in the East Asian case.

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