The Welfare Consequences of Countercyclical Fiscal Transfers*

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March 1, 2025

Abstract.

One-time transfers stimulate economies during recessions, but anticipation of future stimulus checks affects precautionary savings. Given these tradeoffs, this paper examines optimal stimulus check generosity and its distributional effects in a two-asset heterogeneous-agent model with aggregate shocks, adopting the global solution algorithm in Abbott and Phan (2024). Optimal transfers provide an extra $1,800 per recessionary year. Indebted households benefit most, while wealthy Hand-to-Mouth households incur welfare losses. A Krusell and Smith (1998) approach leads to overestimation of capital in prolonged recessions and underestimation of optimal transfer cyclicality and welfare gains. This highlights the need for global solutions in business cycle models with realistic Marginal Propensities to Consume.

JEL Classification: E21, E32, E63, C63
Keywords: Stimulus Check, Consumption Insurance, Business Cycle, Heterogeneous-Agent with Aggregate Shock

*I am deeply grateful to Brant Abbott for his invaluable mentorship and guidance throughout my PhD journey. I am also very grateful to Allen Head and Huw Lloyd-Ellis for their continuous guidance and support. I have greatly benefited from comments and discussions with Jason Choi, Eugene Tan, Rafael Guntin, Kyle Herkenhoff, Shuoyong Shi, Edmund Crawley, José-Víctor Ríos-Rull, and from presentations at Canadian Economics Association 2024, Midwest Macroeconomic Meetings Fall, Bank of Canada Graduate Student Paper Award, Workshop of the Australasian Macroeconomics Society, Wilfrid Laurier University, Boise State University, University of Tennessee, and numerous seminars at Queen's Economic Department. All errors are my own.

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1 Introduction

Stimulus transfers have become an important fiscal policy instrument employed by the U.S. government to stimulate the economy during recessions. However, given historical precedents, households are likely to anticipate similar government responses in future crises, which may, in turn, reduce precautionary savings and undermine the efficacy of the policy. Therefore, it is crucial to evaluate the benefits of consumption insurance against the costs of reduced long-term savings when designing the size of stimulus checks. In this paper, I develop a two-asset heterogeneous-agent model with aggregate shocks to study the welfare implications of a countercyclical fiscal transfer and adopt the method developed in Abbott and Phan (2024) to globally solve the model, overcoming the issue of inaccurate Krusell and Smith (1998) style forecasting rule in models with realistic Marginal Propensities to Consume (MPCs). A more generous transfer improves welfare by aiding households in smoothing consumption over the business cycle. However, this policy decreases precautionary savings against Total Factor Productivity (TFP) shocks, leading to a welfare loss due to a reduced long-term capital stock and implied income redistribution from capital-poor to capital-rich households.

Notably, the U.S. government allocated approximately $814 billion (equivalent to 4% of GDP) during the COVID-19 pandemic, $152 billion (1.5% of GDP) during the Global Financial Crisis, and $38 billion (0.35% of GDP) during the 2001 Dot-com recession for stimulus check programs.[1] The primary objective of these substantial stimulus payments is to promptly stimulate aggregate consumption amidst economic downturns, as noted by Ben Bernanke in his testimony before a House budget committee in January 2008: "To be useful, a fiscal stimulus package should be implemented quickly and structured so that its effects on aggregate spending are felt as much as possible within the next 12 months or so." However, the prevalence of the policy may prompt households to anticipate similar government responses in the future crises. This conjecture is reasonable, given that the stimulus during COVID-19 was supported by 9 out of 10 Democrats and 2 out of 3 Republicans.[2] Moreover, news outlets have discussed the potential for stimulus checks to be used in future recessions.[3] Additionally, Oatley (2020) coins the term "The Great Expectation"

[1] During the COVID-19 recession, three rounds of stimulus checks were delivered as part Economic Impact Payments. Respectively, for each round, tax filers receive $1200 (+$500 per child), $600 (+$600 per child) and $1400 (+$1400 per qualifying dependent adult) During the 2008-2009 Global Financial Crisis, about 130 million U.S tax-filers receive tax rebates as under the Economic Stimulus Act. Typically, single individuals received $300–$600 and couples received $600–$1,200; in addition, households received $300 per child who qualified for the child tax credit. The Dot-com recession in 2001, in spite of having a relatively short-life of 8 months, triggered the enactment of The Economic Growth and Tax Relief Reconciliation Act that sent tax rebates, typically $300 or $600 in value

[2] Historically, presidents from both the Democratic and Republican parties have used stimulus checks as an expansionary fiscal policy during recessions.

[3] For example, see https://www.cnbc.com/2022/06/22/stimulus-checks-could-be-used-in-the-next-recession-economists-say.html and https://finance.yahoo.com/news/stimulus-checks-likely-next-recession-183500580.html

to describe the growing demand that government will protect household wealth and incomes from the damage inflicted by banking crises. A natural question is what are the costs and benefits of such stimulus checks? Moreover, what factors determine how generous they should be?

The complexity of these questions is amplified by the fact that households respond heterogeneously to the stimulus checks, and a complete theory must account for this. For example, Coibion, Gorodnichenko, and Weber (2020) draw on large-scale survey data in the U.S to show that, in response to receiving the stimulus checks during the COVID-19 recession, households with low income and individuals out of the labor force are more likely to spend the checks, as are households facing liquidity constraints. High-income individuals are more likely to save, while those with mortgages and renters are much more likely to pay off debts, as are financially constrained individuals. Similarly, Baker, Farrokhnia, Meyer *et al.* (2023) analyze transaction data in the U.S and find that households with lower incomes, greater income declines, and less liquidity display stronger responses to the same stimulus checks. In particular, individuals with less than $100 in their accounts spend 50% of their stimulus payments within three months, while individuals with more than $1,000 in their accounts spend only about 15% of their payment. These empirical findings are consistent with results of Kaplan and Violante (2014), who highlight the importance of asset liquidity in studying fiscal policies.

From the perspective of macroeconomic theory, the policy aims to stabilize consumption fluctuations driven by certain households who have limited buffer stocks to rely on during declines in income. The policy can improve welfare because households are risk-averse and value a stable stream of consumption - *the uncertainty effect*. However, as households anticipate better insurance against shocks, they reduce precautionary savings, which in turn lowers the long-run capital stock, consumption, and overall welfare - *the level effect*. Additionally, a reduced capital stock decreases wages and increases interest rates. This, in turn, redistributes income from capital-poor to capital-rich households - *the inequality effect*[4]. This redistribution results in a welfare loss as resources are transferred from households with high marginal utility to those with low marginal utility[5]. The level and inequality effects represent costs of stimulus transfers that are often overlooked by policymakers.

In this paper I contribute a model that draws on the frameworks of Krusell and Smith (1998) and Kaplan, Moll, and Violante (2018) to estimate and characterize the optimal transfer that maximizes

[4] Bhandari, Evans, Golosov *et al.* (2021a) coins the terms *insurance, aggregate efficency* and *redistribution effects* to refer to these same concepts, respectively.

[5] Davila, Hong, Krusell *et al.* (2012) show that if the social planner is unable to allocate resources in a way that equates marginal utility of consumption across households, then the next best constrained efficiency is to raise the capital stock above the competitive equilibrium level. This is because capital and labor are complement so an increase in capital stock would raise the wage level in the economy which consumption poor values a lot since they do not own assets.

a Utilitarian welfare function in an economy that infrequently experiences severe recessions as per Krueger, Mitman, and Perri (2016a).[6] The model features a rich endogenous distribution of liquid and illiquid asset holdings, with a realistic fraction of indebted and hand-to-mouth (HtM) households, including both poor HtM (households with neither type of assets) or wealthy HtM (households with illiquid assets but no liquid assets). These households, characterized by high MPCs, exert a significant influence on aggregate consumption fluctuations and are among the most responsive recipients of the policy. Moreover, the proportion of these households varies countercyclically over the business cycle, a feature not captured by Two-Agent models with fixed proportions of HtM households.[7] After calibrating the model to match key observations from the Survey of Consumer Finances (SCF) 2019, it delivers an average MPC out of small windfalls of 17.73%, aligning with empirical estimates ranging from 15-25% (Parker, Souleles, Johnson *et al.* (2013); Johnson, Parker, and Souleles (2006)). The MPC is an important statistic that captures the efficacy of fiscal policy, as is well understood in the literature (Jappelli and Pistaferri (2014); Kaplan and Violante (2022)).

My second contribution is methodological. To globally solve the model, I adopt the solution method proposed in Abbott and Phan (2024), which is highly reliable and easy to adapt. The algorithm integrates neural networks into finite-difference methods, combining the best of both worlds to advance the frontier of solving heterogeneous-agent models with aggregate shocks. Importantly, the solution algorithm does not require the assumption that households form a linear forecasting rule based only on aggregate variables as in Krusell and Smith (1998) (henceforth KS forecasting rule), therefore solving the full rational expectations model. This advancement is essential because models with high MPC necessitate the inaccuracy of the KS forecasting rule, as discussed in Guvenen (2011) and Heathcote, Storesletten, and Violante (2009) among other papers. Consequently, the Approximate Aggregation algorithm, proposed by Krusell and Smith (1998), which remains a commonly used algorithm to solve this class of the model, is likely an unsuitable choice for my model.

I confirm that this is indeed the case: the KS forecasting rule fails to predict aggregate capital in periods of consecutive recessions. Specifically, examining the forecasting errors using Den Haan (2010a) statistics, I find that the maximum deviation between the forecasted and actual aggregate capital is about 7 times larger than the standard model where the Approximate Aggregate holds. Consequently, the Approximate Aggregation solution would underestimate the welfare gains pro-

[6] An alternative approach is to consider a model based on NBER recessions. However, in practice, stimulus checks are much more likely to be issued during severe recessions, as observed in the U.S. and Japan.

[7] For example, Debortoli and Galí (2017) argue that a Two-Agent New Keynesian (TANK) model with a fixed proportion of HtM households can reasonably approximate the implications of a fully-fledged HANK model in terms of the effects of aggregate shocks on output. However, when studying the welfare implications of policies over the business cycle, this framework is likely unsuitable.

vided by the transfer policy and optimal cyclicality of transfers. The true optimal transfer is about 20% larger in recessions than what the Approximate Aggregate solution suggests. These findings highlight the importance of a global solution for my model.

In my model, households are exogenously heterogeneous due to persistent idiosyncratic shocks and changes in employment status, with employment prospects worsening during recessions. They can save by accumulating liquid assets, which offer a lower rate of return, or illiquid assets, which provide a higher rate of return but involve transaction costs. Additionally, they can borrow up to a certain limit. The incomplete-markets assumption generates an endogenous joint distribution of earnings, and liquid and illiquid assets, which are influenced by policy decisions. The government implements a countercyclical transfer to all households, which they internalize, reflecting the notion of an anticipated policy. This policy is financed by issuing debt, with the government balancing the long-run budget by running deficits during recessions and surpluses during normal periods. A more generous countercyclical transfer smooths aggregate consumption because a significant portion of households are non-Ricardian. However, households anticipating greater insurance against aggregate fluctuations accumulate less precautionary savings, leading to a reduction in the long-run capital stock. The optimal transfer scheme balances the marginal benefit of stabilizing consumption over the business cycle against the marginal cost of lowering long-run consumption, reflecting a trade-off between fluctuations and level effects as discussed in Cho, Cooley, and Kim (2015).

In the baseline model, I find that the optimal stimulus transfer provides an extra $1,800 per household during each recessionary year compared to the baseline[8] (equivalent to 3.0% of GDP per capita in the U.S), resulting in welfare gains equivalent to 0.048% in Consumption Equivalent Variation (CEV). Under this countercyclical policy, the variance of aggregate consumption reduces to 0.036% compared to 0.136% under an acyclical transfer policy. However, this comes at the cost of reduced precautionary savings, leading to a 0.139% decrease in the average capital stock and a 0.043% reduction in average consumption. Performing a Benabou-Floden decomposition as in Benabou (1996) and Floden (2001), the welfare gain from the uncertainty effect is found to be 0.0893% in CEV. However, this is offset by welfare losses from the level and inequality effects , which amount to -0.0368% and -0.0009% in CEV, respectively. If households do not anticipate the stimulus checks, countercyclical policy can lead to larger welfare gains because it does not affect precautionary savings while still mitigating consumption volatility.

The welfare impact of the policy is highly heterogeneous across households. At the optimal transfer level, indebted and poor HtM households experience significant welfare gains, with CEVs of 0.1978% and 0.1747%, respectively. This outcome is intuitive, as these groups benefit the most from the insurance provided by the policy. Conversely, wealthy HtM households incur welfare

[8] Beraja and Zorzi (2023) builds an incomplete-markets model of durable spending goods and find that checks of $1,500 to $2,000 would fully close the output gap in a typical recession

losses of -0.0776% in CEV, as they own assets that help to insure them against the underlying uncertainty but suffer from the reduction in wages due to the decreased long-run capital stock.[9] Savers are neutral with respect to the policy as the benefit of consumption smoothing is offset by the long-run cost of the reduced capital stock.

I also explore alternative policy interventions and model specifications. My findings indicate that increasing unemployment benefits by the same amount allocated in the fiscal budget for uniform transfers of $1,800 per household would lead to a welfare gain of 0.0798% in CEV.[10] The higher welfare gain under this targeted approach can be attributed to the fact that unemployed households are more likely to need liquidity to smooth consumption, thus valuing the policy more. However, such a targeted policy would fail to reach indebted and poor HtM households that are employed, which represent 10% and 12% of their respective populations.

Building on this insight, I examine a mixed policy strategy wherein the government simultaneously implements a monthly unemployment insurance (UI) enhancement of $300 per household during recessions, along with a uniform transfer to all households. This is similar to the Coronavirus Aid, Relief, and Economic Security (CARES) Act in the U.S. The analysis indicates that the optimal uniform transfer amount in this mixed policy framework is $1200, yielding a CEV of 0.1055%, more than double the CEV obtained under the optimal uniform transfer in the baseline model. This policy addresses the limitations of a purely UI enhancement strategy and results in greater overall welfare improvements. Furthermore, employed households indirectly benefit from the UI enhancement, as it mitigates the negative impacts of job layoffs during recessions. As a result, the mixed policy allows for less generous uniform transfers to remain optimal.

Lastly, I examined a variation of the model that incorporates demand externalities, as outlined in Krueger, Mitman, and Perri (2016a). This specification allows for a feedback loop between aggregate consumption and aggregate output through total factor productivity (TFP), effectively introducing a New Keynesian element into the model without formally modeling nominal rigidities. In this framework, the countercyclical policy exacerbates the trade-off between consumption fluctuations and the level of consumption due to the feedback loop on the variance and average TFP. As it turns out, the negative impacts of this feedback loop on long-run consumption outweigh the benefits of stabilizing consumption fluctuations. Consequently, the overall welfare gain is lower in this scenario, amounting to 0.0393% in CEV, with an optimal transfer of $1500 per household during each recessionary year.

This paper supplements existing literature on more standard countercyclical fiscal policies, of-

[9]Similarly, Cho and Ma (2023) finds that wealth-rich households enjoy business cycles more than wealth-poor households

[10]Given the calibrated unemployment rate of 8.33% during a recession, the UI enhancement increases unemployment benefits by approximately $21500 per household annually, equivalent to $410 per month.

fering policymakers insights into the efficacy of various policy tools at their disposal. For example, in the Congressional Research Service Report, Keightley (2019) remarked that "Congress also has a range of other options it could consider when designing a stimulus package should a recession occur and automatic stabilizers are not sufficient to counteract it". One key takeaway from my model is that an anticipated stimulus check can be overly generous, and that the effect on precautionary savings motives is a critical factor that policymakers must consider when designing such policies. This consideration is particularly relevant in the context of countries such as United State, Singapore, Japan or Hong Kong, where stimulus transfers have been implemented more frequently than other developed countries.[11] Similarly, McKay and Reis (2016) examine the role of automatic stabilizer policies in the U.S business cycle. They find that most of the welfare gain from stabilizers comes from redistribution and social insurance rather than stabilizing the business cycle because the cost of latter is small to start with. They also highlight that economies without automatic stabilizer have higher average output due to increased precautionary savings as households face higher risks.

Literature Review: This paper is related to a substantial body of literature on the cost of business cycles (e.g., Lucas (1987), Otrok (2001), Lucas Jr (2003), Krusell and Smith Jr (1999), Krusell, Mukoyama, Şahin *et al.* (2009), Cho, Cooley, and Kim (2015)). My model contributes to the literature studying the aggregate and distributional effects of fiscal policy designs that mitigate the impact of business cycles (e.g., Oh and Reis (2012) Krueger, Mitman, and Perri (2016a), Krueger, Mitman, and Perri (2016b), McKay and Reis (2016), Kekre (2023), Carroll, Crawley, and Tretvoll (2023)) and characterizes an optimal fiscal policy (e.g., Chari, Christiano, and Kehoe (1994), Bassetto (2014), Werning (2007), Guerrieri, Lorenzoni, Straub *et al.* (2022)). Related to this strand of literature, this paper studies a fiscal transfer policy within a heterogeneous-agent model with a rich asset structure,[12] and examines the sources of welfare gains in an environment where any redistribution effects are channeled via capital stock (e.g., Davila, Hong, Krusell *et al.* (2012)).[13] Furthermore, this paper contributes to the growing body of research on the solution of heterogeneous-agent models under aggregate shocks (e.g., Krusell and Smith (1998), Reiter (2009), Auclert, Bardóczy, Rognlie *et al.* (2021), among many others).

The paper is organized as follows. Section 2 lays out the model. Section 3 describes the model calibration and validation. Section 4 studies the optimal countercyclical fiscal transfer. Section

[11] See https://www.rba.gov.au/publications/bulletin/2021/jun/pdf/the-global-fiscal-response-to-covid-19.pdf for a breakdown of fiscal stimulus across countries in response to the COVID-19 recession.

[12] Auclert, Rognlie, and Straub (2018) highlights the advantages of using a two-asset model in the study of fiscal policy, as it qualitatively captures the empirical intertemporal marginal propensity to consume (iMPC).

[13] In this model, the government reallocates resources from periods of economic normalcy to recessions through the issuance of government bonds while maintaining long-term fiscal balance. This setup is arguably more relevant to real-life policymaking, as opposed to models that assume government budget balancing period-to-period, which effectively transfers resources from one group to another at each point in time.

5 considers an alternative model specification and policy experiments. Section 6 discusses the solution algorithm. Section 7 concludes.

2 The Model

The model adapts that of Kaplan, Moll, and Violante (2018) by incorporating aggregate shocks, characterized by fluctuations in TFP as described in Krueger, Mitman, and Perri (2016a), but there are no nominal rigidities. A key departure lies in the inclusion of a large but finite number of heterogeneous agent households, contrasting with the conventional approach that uses a continuum of households. This assumption allows me to capture the dependence of household decisions on the distribution asset holdings in a parsimonious way, and is necessary for the computational method.

2.1 Technology

The representative firm produces output according to a standard Cobb-Douglas production function.

$$Y_t = z_t K_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

where K_t denotes capital per capita and L_t represents labor demand. TFP follows a Poisson process with two discrete states, $z_t \in \{z^N, z^R\}$, corresponding to normal and recession periods. Transitions between these states occur with probabilities determined by the Poisson process. Capital depreciates at a constant rate δ.

The economy is populated by N infinitely lived households indexed by i. The state variable x of individual i at time t is denoted as x_{it}. Let $\mathbf{A}_t = [a_{1t}, a_{2t}, ..., a_{Nt}]'$ and $\mathbf{B}_t = [b_{1t}, b_{2t}, ..., b_{Nt}]'$ respectively represent vectors of liquid and illiquid assets held by households. The supply of liquid and illiquid assets per capita is given by:

$$\bar{\mathbf{A}}_t = \frac{\sum_{i=1}^{N} a_{it}}{N};\ \bar{\mathbf{B}}_t = \frac{\sum_{i=1}^{N} b_{it}}{N} \tag{2}$$

Both types of assets are considered productive and contribute to the aggregate capital stock. Thus, the market-clearing condition for capital is given by:

$$K_t = \bar{\mathbf{A}}_t + \bar{\mathbf{B}}_t \tag{3}$$

Consequently, prices determined as functions of both the aggregate level and the distribution of asset holdings: $r_t^k(z_t, \mathbf{B}_t, \mathbf{A}_t)$ and $w(z_t, \mathbf{B}_t, \mathbf{A}_t)$.[14]

[14] Alternatively, current prices can be expressed as functions of aggregate productivity and capital. However, the

2.2 Households

Time is continuous. Households are indexed by their holdings of liquid assets b, illiquid assets a, idiosyncratic persistent productivity ϵ, and their employment status s. Households supply labors inelastically as leisure is not valued.[15] They internalize the countercyclical fiscal transfer scheme, and are subject to the evolution of the distributions of liquid and illiquid assets.

Households derive utility $u(c_t)$, which is strictly increasing and concave in consumption c_t. Formally, households choose consumption and net deposits to maximize a stream of utility, discounted by the time preference ρ.

$$V \equiv \max_{c_t, d_t} \mathbb{E} \int_0^\infty e^{-\rho t} u\big(c_t\big) dt \tag{4}$$

The expectation operator, $\mathbb{E}$, is over both aggregate shocks z_t, and individual shocks. Idiosyncratic productivity shocks are assumed to be independent of the aggregate state, while employment shocks depend on the aggregate state through the job-finding rate, $\lambda(z)$, and job separation rate, $\sigma(z)$. V is the value function associated with the optimal choices of households.

Households can save in liquid assets which yields an interest rate r^b, or borrow up to an exogenous limit $\underline{b}$ at an interest $r^b + \kappa$, where κ represents the borrowing wedge. Short positions in illiquid assets are not allowed.

Households can also deposit or withdraw from illiquid accounts a by paying a transaction cost $\chi(d_t, a_t)$. The interest rate on illiquid assets exceeds that on liquid assets by a liquidity premium ϕ. Conceptually, illiquid assets are assumed to be immediately available for rental by firms, while liquid assets require conversion to be productive. Consequently, the rental price of capital, net of depreciation, is equal to the interest rate on illiquid assets: $r^k - \delta = r^a$, highlighting the dependence of interest rates on both the aggregate state and distribution.

Households receive income y, which is labor income net of taxes when employed, and a flat unemployment benefit u_b when unemployed. They also receive the fiscal transfers $T(z)$, which can be either countercyclical or acyclical, as will be discussed in Section 2.3. Households take prices, which in this framework are functions of the distributions and aggregate state in equilibrium, as given.

forecast of prices depends on the forecast of the distributions, as we will see in the next section.

[15] Coibion, Gorodnichenko, and Weber (2020) find that 90% of employed workers who received a stimulus check reported that the transfer had no effect on their work effort.

The evolution of household holdings of liquid assets b_t and illiquid assets a_t is given by:

$$\begin{aligned}
\dot{b}_t &= r_b(z_t, \mathbf{B}_t, \mathbf{A}_t)b_t + y(s_t, \epsilon_t, z_t, \mathbf{B}_t, \mathbf{A}_t) + T(z) - c_t - d_t - \chi(d_t, a_t) \\
\dot{a}_t &= r^a(z_t, \mathbf{B}_t, \mathbf{A}_t)a_t + d_t \\
y(s_t, \epsilon_t, z_t, \mathbf{B}_t, \mathbf{A}_t) &= \begin{cases} (1-\tau)\epsilon_t w(z_t, \mathbf{B}_t, \mathbf{A}_t) & , s_t = e \\ u_b & , s_t = u \end{cases}
\end{aligned} \tag{5}$$

Savings in liquid assets, $\dot{b}_t$, are equal to capital income from liquid assets plus income and fiscal transfers, minus consumption c_t, net deposit d_t, and transaction costs $\chi(d_t, a_t)$. Savings in illiquid assets are determined by capital income from illiquid assets and deposits. Household income depends on employment status, idiosyncratic productivity, and wages.

Households are also subject to the evolution of the distributions of liquid and illiquid assets:

$$\begin{aligned}
\dot{\mathbf{B}}_t &= r^b(z_t, \mathbf{B}_t, \mathbf{A}_t)\mathbf{B}_t + y(\mathbf{S}_t, \mathbf{E}_t, z_t, \mathbf{B}_t, \mathbf{A}_t) + T(z_t) - \tilde{\mathbf{c}}_t - \tilde{\mathbf{d}}_t - \chi(\tilde{\mathbf{d}}_\mathbf{t}, \mathbf{A}_t) \\
\dot{\mathbf{A}}_t &= r^a(z_t, \mathbf{B}_t, \mathbf{A}_t)\mathbf{A}_t + \tilde{\mathbf{d}}_t \\
\dot{\mathbf{B}}_t &\geq \underline{b}, \mathbf{A}_t \geq 0
\end{aligned} \tag{6}$$

where $\dot{\mathbf{B}}_\mathbf{t}$ and $\dot{\mathbf{A}}_\mathbf{t}$ are $N \times 1$ vectors stacking up the decisions of households in the economy.[16] Households understand that the consumption and deposit decisions of others, denoted as $\tilde{c}$ and $\tilde{d}$, respectively, affect their own value functions via price effects.[17] Similarly, $\mathbf{S}_t = [s_{1t}, s_{2t}, ..., s_{Nt}]'$ and $\mathbf{E}_t = [\epsilon_{1t}, \epsilon_{2t}, ..., \epsilon_{Nt}]'$ are vectors representing the employment status and idiosyncratic productivity of N households, respectively. It is assumed that all households are subject to an independent and identically distributed stochastic processes. Lastly, all households in the economy are subject to the same constraints on liquid and illiquid asset holdings.

Prices r^b, r^a, and w are functions of the aggregate state and the distributions of liquid and illiquid assets. The equilibrium interest rates satisfy the following conditions:

$$\begin{aligned}
r_b(z_t, \mathbf{B}_t, \mathbf{A}_t) &= r^b(z_t, \mathbf{B}_t, \mathbf{A}_t) \cdot \mathcal{I}\{\mathbf{B}_t \geq 0\} + (r^b(z_t, \mathbf{B}_t, \mathbf{A}_t) + \kappa) \cdot \mathcal{I}\{\mathbf{B}_t < 0\} \\
r^b(z_t, \mathbf{B}_t, \mathbf{A}_t) &= r^a(z_t, \mathbf{B}_t, \mathbf{A}_t) - \phi
\end{aligned} \tag{8}$$

[16] Explicitly, these constraints can be rewritten as

$$\begin{aligned}
\dot{b}_{i,t} &= r^b(z_t, \mathbf{B}_t, \mathbf{A}_t)b_{it} + y(s_{it}, \epsilon_{it}, z_t, \mathbf{B}_t, \mathbf{A}_t) + T(z_t) - c(\mathbf{s}_{it}, z_t, \mathcal{S}_t) - d(\mathbf{s}_{it}, z_t, \mathcal{S}_t) - \chi(d(\mathbf{s}_{it}, z_t, \mathcal{S}_t), \mathbf{A}_t) \\
\dot{a}_{it} &= r^a(z_t, \mathbf{B}_t, \mathbf{A}_t)a_{it} + d(\mathbf{s}_{it}, z_t, \mathcal{S}_t)
\end{aligned} \tag{7}$$

for $i = 1, 2, ..., N$. Here, $\mathbf{s}_{it} = \{b_{it}, a_{it}, \epsilon_{it}, s_{it}\}$ is a tuple of individual state variables and $\mathcal{S}_t = \{\mathbf{B}_t, \mathbf{A}_t, \mathbf{S}_t, \mathbf{E}_t\}$ is a tuple of distributional state variables.

[17] In the bounded rationality models such as Krusell and Smith (1998) and Fernández-Villaverde, Hurtado, and Nuno (2023), the forecast is on the evolution of the aggregate capital, and these equations will be replaced by $\dot{K}$ which can be either linear or non-linear in K_t

where ϕ and κ will be internally calibrated.

Fluctuations in both idiosyncratic and aggregate shocks in an incomplete markets settings prompt households to engage in precautionary savings to smooth consumption. A more generous countercyclical policy embedded within $T(z_t)$, as will be discussed in the next section, diminishes the need for precautionary savings induced by aggregate fluctuations. Conversely, due to the substantial decline in both wages and interest rates during recessions, households value stimulus payments to smooth consumption. Consequently, I aim to identify the optimal transfer scheme that maximizes welfare by balancing these conflicting dynamics.

The entire state space $(\mathbf{s}, z, \mathcal{S})$ is composed of $\mathbf{s} = \{b, a, \epsilon, s\}$ representing individual state variables, including holdings of liquid and illiquid assets, idiosyncratic productivity, employment status, the aggregate risk z, and $\mathcal{S} = \{\mathbf{B}, \mathbf{A}, \mathbf{E}, \mathbf{S}\}$, representing the distributional state space, which encapsulates the distributions of these individual state variables.[18]

2.3 Government

The government transfer function takes the following form:

$$T(z_t) = \bar{T} + \mathcal{T}(z_t) \tag{9}$$

where $\bar{T}$ denotes the steady-state transfer, and $\mathcal{T}(z_t)$ is the state-dependent transfer. I assume that the state-dependent transfer does not enter the standard budget constraint and that any fluctuation in wages will be absorbed into government spending G. At each point in time, the tax revenue from labor is equal to the government spending, the state-independent transfer, and total unemployment benefits.[19]

$$\tau w(z_t, \mathbf{B_t}, \mathbf{A_t})(1 - U(z_t)) = G(z_t, \mathbf{B_t}, \mathbf{A_t}) + \bar{T} + u_b \times U(z_t) \tag{10}$$

I make a stark but simplifying assumption that the state-dependent transfer is financed separately from the standard budget constraint, and is revenue-neutral in expectation:[20]

$$\Pi^N \mathcal{T}(z^N) + \Pi^R \mathcal{T}(z^R) = 0 \tag{11}$$

[18] While the distribution of idiosyncratic shocks and employment status is ergodic given the aggregate state, it is essential to track these distributions within the state space to accurately forecast the saving decisions of each household in the economy.

[19] The average idiosyncratic productivity is normalized to 1 for clarity.

[20] Implicitly, this means that the government can borrow at a zero interest rate and roll over the debt without it exploding or defaulting. This assumption allows me to avoid including government debt as an additional state variable. Conceptually, one can imagine that the government does not know the initial conditions of the economy (whether it will be in normal times or a recession) and adjusts government spending G accordingly. When simulating the economy to study the welfare implications, I draw multiple paths of the aggregate state to eliminate the dependence of the policy on initial conditions.

where Π^N and Π^R represent the fraction of time that the economy is in normal and recessionary states, respectively. The aggregate expenditure on state-dependent transfers (e.g., stimulus checks) must be offset by a proportional increase in the lump-sum tax levied on all households. Combining Equation 9 with 10 we get:

$$\mathbb{E}(T(z)) = \bar{T} \tag{12}$$

That is, on average, the government transfers the same amount of money over the business cycle as in the acyclical transfer baseline, regardless of the size of the state-dependent transfer. A countercyclical transfer implies that $T(z^R) > T(z^N)$ (or equivalently, $\mathcal{T}(z^R) > 0$ and $\mathcal{T}(z^N) < 0$), indicating transfers increase during recessions and decrease during normal times. This policy effectively transfers resources from periods of economic stability to downturns for all households in the economy. Consequently, any redistribution across households occurs indirectly through adjustments in the capital stock, rather than through explicit policies that directly transfer resources between different groups at each point in time, such as immediate increases in tax rates to fund unemployment benefits. In contrast, an acyclical transfer would simply set $T(z^R) = T(z^N) = \bar{T}$.

2.4 Recursive Competitive Equilibrium

A recursive competitive equilibrium is a set of value function, $V(\mathbf{s}_{it}, z_t, \mathcal{S}_t)$ and decision rules $c(\mathbf{s}_{it}, z_t, \mathcal{S}_t)$ and $d(\mathbf{s}_{it}, z_t, \mathcal{S}_t)$, and pricing functions $r^b(z_t, \mathbf{B}_t, \mathbf{A}_t)$, $r^a(z_t, \mathbf{B}_t, \mathbf{A}_t)$ and $w(z_t, \mathbf{B}_t, \mathbf{A}_t)$, such that at each point in time.

1. Given the price $r^b(z_t, \mathbf{B}_t, \mathbf{A}_t)$, $r^a(z_t, \mathbf{B}_t, \mathbf{A}_t)$ and $w(z_t, \mathbf{B}_t, \mathbf{A}_t)$, households solve the utility maximization problem in Equation 4. $V(\mathbf{s}_{it}, z_t, \mathcal{S}_t)$ is the associated value function; and $c(\mathbf{s}_{it}, z_t, \mathcal{S}_t)$ and $d(\mathbf{s}_{it}, z_t, \mathcal{S}_t)$ are the associated policy function.

2. The representative firm maximizes profit given prices $r^k(z_t, \mathbf{B}_t, \mathbf{A}_t)$ and $w(z_t, \mathbf{B}_t, \mathbf{A}_t)$.

3. The government budget constraint 10 and revenue-neutral condition 11 are satisfied.

4. Prices clear the labor[21] and capital markets:

$$\begin{aligned} L_t &= \sum_{i=1}^{N} \frac{\mathcal{I}\{s_{it}(z_t) = e\}}{N} \\ K_t &= \bar{\mathbf{A}}_t + \bar{\mathbf{B}}_t \end{aligned} \tag{13}$$

[21] Notice even though households supply labor inelastically, the aggregate labor still varies with the aggregate state because of the fluctuation in employment rate.

5. The decision rule is symmetric: $\tilde{c} = c$ and $\tilde{d} = d$: households in the same state will make the same decision.[22]

3 Model Calibration and Validation

I calibrate the deterministic steady-state version of the model by assuming that $z = 1$ and the unemployment rate remains at its normal-time level. Using the calibrated parameters, I proceed to solve and simulate the full stochastic model.

3.1 Aggregate Fluctuations and Employment risk

The model calibration is based on annual data. The calibration largely follows Krusell and Smith (1998) and Krueger, Mitman, and Perri (2016a). The business cycle in consideration features infrequent but severe recessions. Let β_l and β_h denote the Poisson rates governing the expected duration of recession and normal periods, respectively. The transition matrix for the TFP process is given by:

$$\pi_z = \begin{bmatrix} -\beta_l & \beta_l \\ \beta_h & -\beta_h \end{bmatrix} \tag{14}$$

I set $\beta_l = 0.1818$ so that the average duration of recession is 5.5 years. The economy experiences recessions 16.5% of the time, requiring $\Pi^R = 0.165$ and $\Pi^N = 0.835$. To target these moments, β_h is set to 0.0359, indicating that normal periods last approximately 28 years on average. Consequently, for every \$1 distributed to households as a stimulus check during a recession, the government will finance it by reducing transfers by $\frac{0.165}{0.835} = \$0.2$ during normal periods.

I allow for correlation between aggregate and employment risks, reflected in the employment transition matrix, which depends on job separation and finding rates that vary with the aggregate state:

$$\pi_s(z) = \begin{bmatrix} -\lambda(z) & \lambda_z(z) \\ \sigma(z) & -\sigma(z) \end{bmatrix} \tag{15}$$

The job finding rates during normal and recessionary periods are set to be 2 quarters and 3.5 quarters respectively so $\lambda(z^N) = 2$ and $\lambda(z^R) = 1.14$. The unemployment rate in normal times is set at 5.33%, increasing by 3% during recessions. Calibrating the job separation rates to achieve these unemployment levels results in $\sigma(z^N) = 0.1126$ and $\sigma(z^R) = 0.1047$. Notably, the rise in unemployment during recessions primarily comes from reduced job finding rates.

[22] It is worth noticing that by construction the definition of the equilibrium in this model does not require consistency in the law-of-motion as in Krusell and Smith (1998).

TFP levels are chosen to reflect a 12% GDP decline during recessions[23]. The output ratio between normal and recessionary periods (ignoring the capital stock) is:

$$\frac{Y_N}{Y_R} = \frac{z^N}{z^R}\left(\frac{1-U(z^N)}{1-U(z^R)}\right)^{1-\alpha} \tag{16}$$

Assuming unemployment rates are fully determined by TFP, transitions between economic states trigger jumps in unemployment rates.[24] Targeting this ratio alongside z normalization implies $(z^N, z^R) = (0.9260, 1.0146)$.

3.2 Household parameters

Households have a constant relative risk aversion (CRRA) utility function for consumption $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ with $\gamma = 1.1$. The persistent idiosyncratic labor productivity, independent of aggregate fluctuations, follows a "jump-drift" process:

$$\begin{aligned} \log \epsilon_{it} &= e_{it} \\ de_{it} &= -\xi e_{it} dt + dJ_{it} \end{aligned} \tag{17}$$

where dJ_{it} captures jumps occurring at a Poisson rate λ_ϵ, with subsequent productivity levels drawn from a normal distribution with mean 0 and variance σ_ϵ. Annualizing parameters from Kaplan, Moll, and Violante (2018), I set $\xi = 0.0360$, $\lambda_\epsilon = 0.0280$ and $\sigma_\epsilon = 1.53$. On average, shocks occur every 36 years, decaying annually by 3.6% of current productivity levels.

Labor income is taxed at 30%, while steady-state transfers and unemployment benefits account for 5% and 10% of output respectively. The borrowing limit on liquid assets is set at a quarter of steady-state output, following Kaplan, Moll, and Violante (2018).

The transaction cost function is :

$$\chi(d,a) = \chi_0|d| + \frac{1}{\chi_1}\left|\frac{d}{a}\right|^{\chi_1} a \tag{18}$$

where χ_0 represents the linear component and χ_1 the convex component.

[23] Krueger, Mitman, and Perri (2016a) considers a severe recession in which GDP drops by 7%. The 12% GDP drop used in this paper reflects the inclusion of COVID-19 in the definition of a severe recession, which was naturally not accounted for in Krueger, Mitman, and Perri (2016a). Alternatively, one could consider a business cycle in which GDP drops by 7% but occurs more frequently or lasts longer than in the baseline calibration of Krueger, Mitman, and Perri (2016a).

[24] This assumption is also made in Krusell and Smith (1998). Accounting for unemployment rate transitions necessitates adding aggregate unemployment into state space, complicating household forecasts, as noted when applying the Abbott and Phan (2024) algorithm.

Externally specified	Syntax	Value	Source
1. Technology			
Depreciation rate	δ	6%	Assumption
Capital share of GDP	α	1/3	Assumption
Capital net rate of return	$r_k - \delta$	4.31%	Target K-to-Y: 3.2
2. Aggregate Fluctuation			
Aggregate transition: $R \to N$	β_l	0.182	Recession Length: 5.5 years
Aggregate transition : $N \to R$	β_n	0.036	Normal time length: 28 years
TFP in recession	z^R	0.9260	GDP drops of 12%
TFP in normal time	z^N	1.0146	Normalization
Job finding rate - R	$\lambda(z^R)$	1.140	Unemp length in R: 10 months
Job separation rate - R	$\sigma(z^R)$	0.105	Unemp. rate in R: 8.33%
Job finding rate - N	$\lambda(z^N)$	2	Unemp length in N: 6 months
Job seperation rate - N	$\sigma(z^R)$	0.113	Unemp. rate in N: 5.33%
3. Household and Government			
CRRA coefficient	γ	1.1	Assumption
Borrowing limit	$\underline{a}$	\$16000	0.25 $\times$ GDP
Labor tax rate	τ	0.30	Assumption
Mean reversion rate	ξ	0.036	Kaplan, Moll, and Violante (2018)
Arrival rate	λ_ϵ	0.028	Kaplan, Moll, and Violante (2018)
Standard deviation of innovations	σ_ϵ	1.53	Kaplan, Moll, and Violante (2018)
Unemployment benefit	u_b	\$6500	0.10 $\times$ GDP
Steady-State Transfer	$\bar{T}$	\$3250	0.05 $\times$ GDP

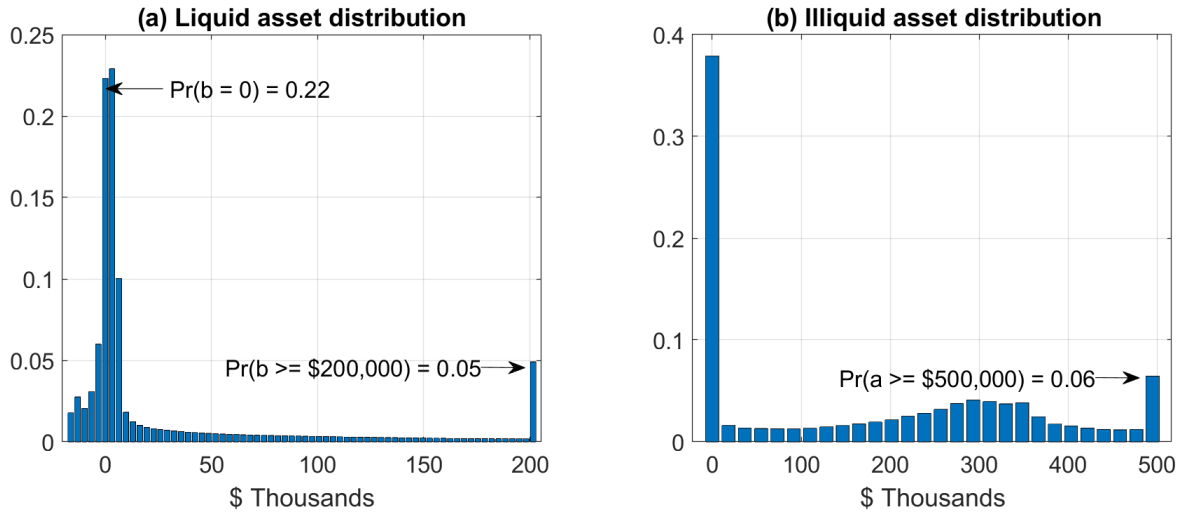
Table 1: Externally calibrated parameters. Values are reported in annual frequency.

3.3 Calibration result

I set the capital share $\alpha = 1/3$ and the depreciation rate $\delta = 0.06$. Targeting a capital-to-output ratio of 3.2 implies a capital rental rate net of depreciation rate $r^k - \delta = 4.31\%$, which also serves as the interest rate paid on illiquid assets r^a. The externally-calibrated parameters are reported in Table 1.

Internally Calibrated Parameters: Five parameters $(\rho, \kappa, \phi, \chi_0, \chi_1)$ are internally calibrated to match five key moments characterizing the distribution of household wealth: the mean of liquid and illiquid assets, the fractions of poor and wealthy HtM, and the fraction of indebted households (i.e households with negative liquid assets). Data is from the SCF 2019 with the top 5% by net worth removed.[25] The SCF classifies net worth into various categories, and I adopt the definitions of liquid and illiquid assets as outlined by Kaplan and Violante (2022). In particular, Liquid

[25]In the full sample, the mean of illiquid asset is about 3 times as much as the mean of liquid assets. It is very difficult to match this moment while targeting the fraction of poor hand-to-mouth of 7%. Meanwhile, this ratio is about six when removing the 5% by networth out of sample, implying that the top 5% hold a large fraction of liquid assets. Arguably the top 5% will not benefit from a stimulus check as they have enough liquid assets to smooth out consumption.



Internally Calibrated	Syntax	Value	Moment	Data	Model
Mean discount rate	ρ	0.043	Mean illiquid assets	2.75	2.76
Borrowing wedge	κ	3.13%	Mean liquid assets	0.46	0.45
Liquidity premium	ϕ	2.00%	Frac. of $b = 0$, $a > 0$	0.15	0.18
Linear cost parameter	χ_0	0.4	Frac. of $b = 0$, $a = 0$	0.07	0.04
Convex cost parameter	χ_1	1.2	Frac. of $b < 0$	0.15	0.15

(1) Means are expressed in multiple of annual output
(2) HtM cutoffs in the data are $1000
(3) Parameters are in annual frequency

Figure 1: Calibration Result at the Deterministic Steady-State

assets comprise Total financial assets (FIN) net of Certificates of deposits (CDS), Savings bonds (SAVBND), Cash value of whole life insurance (CASHLI), Other managed assets (OTHMA), and Quasi-liquid retirement accounts (RETQLIQ). Liquid debt includes Other lines of credit (OTHLOC), Credit card balances after last payment (CCBAL), and Other debt (ODEBT). Net Liquid assets are the difference between Liquid assets and Liquid debt. Illiquid assets are the difference between Net worth and net liquid assets. The calibration results and distributions of liquid and illiquid are reported in Figure 1.

The model replicates the mean of liquid and illiquid asset holdings, the overall fraction of HtM households, and the fraction of indebted households very closely.[26] Nevertheless, there are somewhat fewer poor HtM and more wealthy HtM in the model than in the data. In the calibration, the rate of return on liquid assets is 2% lower than on illiquid assets, and the borrowing rate is 3.13% higher than the lending rate. In the steady-state, 54.31% of households make deposits or

[26]In the model, the poor HtM is households with $b = 0$ and $a = 0$. The wealthy HtM is households with $b = 0$ and $a > 0$. Indebt households are ones with $b < 0$

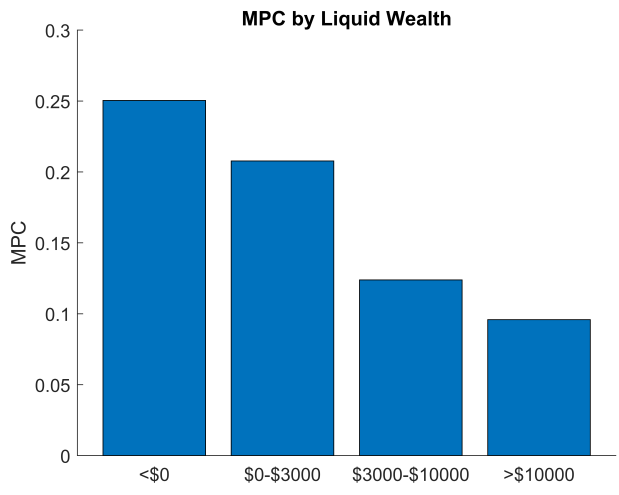


Figure 2: Distribution of MPC by Liquid Wealth

	Data	Model
Top 1% share: Liquid	23.28%	10.64%
Top 5% share: Liquid	58.61%	40.83%
Top 10% share: Liquid	76.52%	67.68%
Top 25% share: Liquid	97.19%	97.85%
Top 50% share: Liquid	104.28%	104.18%
Top 1% share: Illiquid	8.82%	6.49%
Top 5% share: Illiquid	31.74%	18.73%
Top 10% share: Illiquid	50.01%	28.03%
Top 25% share: Illiquid	78.47%	64.81%
Top 50% share: Illiquid	97.13%	94.82%

Table 2: Untargeted moments of the asset distribution in the deterministic steady state

withdrawals, with the total transaction cost amounting to 4.69% of the steady-state GDP.

3.4 Model Validation

To validate the model, I compare several moments of the model to untargeted moments in the data. Table 2 shows that the model closely matches the top 25% and top 50% of both the liquid and illiquid asset distributions. It does relatively poorer at the very top of the distribution, though this deviation is within a reasonable margin given that these moments are untargeted. It is argued that policymakers often prioritize the bottom and middle of the distribution when designing the policy. More importantly, the model delivers an average MPC of 17.73% which aligns well with many empirical estimates. Figure 2 displays the distribution of MPC over Liquid Wealth. The size of MPC is largely driven by households with low liquid wealth, which is consistent with the data (e.g Coibion, Gorodnichenko, and Weber (2020), Baker, Farrokhnia, Meyer *et al.* (2023)).

Additionally, I compare the model's business cycle properties with empirical data. Table 3 reports the business cycle statistics for HP-filtered Output, Consumption and Capital of the data, and compares to those from model.[27] The model matches qualitative patterns in the consumption and capital with output observed in the data. While the model's magnitudes differ somewhat, it successfully captures several key properties: both model and data exhibit higher output variance compared to consumption, and capital shows the highest autocorrelation, followed by consumption

[27]I divide all aggregate variables in the data by the labor force, then take logs and apply HP filter to extract the cyclical components. The first and last three observations in the filtered series are dropped before calculating the business cycle statistics. Data are at annual frequency from 1950 to 2019. All data are taken from FRED: Output (GDPC1), Consumption (PCECCA), Capital stock (RKNANPUSA666NRUG), Labor Force (CLF16OV)

	Data			Model		
	$\sigma(\mathbf{x_t})$	$\rho(\mathbf{x_t}, \mathbf{x_{t-1}})$	$\rho(\mathbf{x_t}, \mathbf{Y_t})$	$\sigma(\mathbf{x_t})$	$\rho(\mathbf{x_t}, \mathbf{x_{t-1}})$	$\rho(\mathbf{x_t}, \mathbf{Y_t})$
$\mathbf{Y_t}$	0.0197	0.5406		0.0482	0.8770	
$\mathbf{C_t}$	0.0168	0.6220	0.8804	0.0302	0.9485	0.9190
$\mathbf{K_t}$	0.0058	0.7632	0.5534	0.0320	0.9802	0.6713

Table 3: Standard Deviation, Auto-correlation and Correlation with Output for Output, Consumption and Capital: Data versus Model

and then output. A caveat is the model's higher standard deviation of capital relative to the data.[28] Despite the simplified TFP process, the model reasonably replicates business cycle characteristics observed in the data.

4 Policy Analysis

4.1 Concept of welfare

I consider a Utilitarian and "behind the veil of ignorance" welfare function. The Utilitarian aspect means that a social planner assigns the same weight to all households and weights normal versus recessionary states equally. The "veil of ignoranc" aspect implies that households are unaware of their idiosyncratic initial states and the history of aggregate states. Formally, welfare under a given policy is defined by the function $\mathcal{W}(\mathcal{T})$ as follows:

$$\mathcal{W}(\mathcal{T}) = \int_{z\times\mathcal{S}} \int_{\mathbf{s}} V(\mathbf{s}, z, \mathcal{S}, \mathcal{T}) d\mu(\mathbf{s}|z, \mathcal{S}) dg(z, \mathcal{S}) \tag{19}$$

where $dg(z, \mathcal{S})$ is the probability density function of potential histories of aggregate shocks, joint with distributions of idiosyncratic states that can arise, and the integration $\int_{z\times\mathcal{S}}$ reflects the notion that welfare reflects the indirect utility of a household that is ignorant of this history. $\mu(\mathbf{s}|z, \mathcal{S})$ is the conditional probability density function of the idiosyncratic states that can arise upon reaching a particulate state of the economy. The integration $\int_{\mathbf{s}}$ then implies that welfare captures the utility of a household that is also uninformed about their idiosyncratic initial conditions.[29]

[28] Incorporating adjustment cost into capital accumulation can improve the fit of the variance of capital (e.g Boldrin, Christiano, and Fisher (2001) and Christiano, Eichenbaum, and Evans (2005) among others).

[29] Krusell and Smith (1998) show computationally that the probability density functions $\mu(\mathbf{s}|z, \mathcal{S})$ and $g(z, \mathcal{S})$ are well-defined within this model framework, and coins the term "stationary stochastic equilibrium". Other authors define stationary stochastic equilibrium as a state of the economy characterized by a prolonged period without aggregate shocks, despite households being aware of the potential for such shocks.

The value function, by definition, is:

$$V(\mathbf{s}, z, \mathcal{S}, \mathcal{T}) = \mathbb{E} \int_0^\infty e^{-\rho t} u(c_t(\mathbf{s}, z, \mathcal{S}, \mathcal{T}))dt \tag{20}$$

Here, households value both the certainty and level of consumption. As the transfer becomes more cyclical, individual and thus aggregate consumption will be more certain, thereby improving welfare. On the other hand, the increase in transfer size reduces precautionary savings, in turns reduces the long-run capital stock and thus level of long-run consumption, posing a welfare loss. I solve the model for various sizes of countercyclical transfers to seek an optimal policy. This involves finding $\tau(z^R)$, which can be thought of as size of government stimulus that maximizes the welfare function defined in Equation 19, subject to the revenue-neutrality condition 12.[30]

4.2 Simulation

To illustrate the trade-off between level and fluctuation, I conduct simulations under two scenarios: one with an acyclical policy, and another with a countercyclical policy involving a government transfer of $1,800 per household during each recessionary year. I draw a path of the TFP shocks and N paths of idiosyncratic labor productivity and employment shocks (one path for each of the N households). Idiosyncratic shock draws are constrained such that the cross-sectional distribution remains at its ergodic distribution at all points in time. Further details can be found in Appendix A.

After simulating the model, I calculate and plot aggregate statistics and the fractions of HtM and indebted households in both scenarios. Let $X^{avg} = \frac{1}{T}\frac{1}{N}\sum_{t=1}^{T}\sum_{i=1}^{N} x_{it}$ denote the average of variable x_{it} over both time and individuals. X^{avg} captures the notion of "long-run" as there are only normal and recession states in the economy. I also discard the first two decades of simulated data to eliminate the economy's dependence on initial conditions.

Figure 3 displays the aggregate consumption, capital, liquid assets and illiquid assets over time. Panel 3a illustrates the trade-off between fluctuations and the level of consumption. With an acyclical transfer, consumption initially drops by approximately 4% upon entering a recession, continuing to decline as households deplete their savings. The first simulated recession lasts approximately 7 years, with consumption decreasing by about 6% by its end. Conversely, with a

[30] In practice, the probability density functions used to calculate welfare in Equation 19 are found numerically by simulating the model for a long period of time. In all scenarios, I simulate 10000 households over 1000 years. The integrations are then replaced here by averaging over N households across time T:

$$\mathcal{W}(\mathcal{T}) = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} V(b_{it}, a_{it}, \epsilon_{it}, s_{it}, z_t, \mathbf{B}_t, \mathbf{A}_t, \mathbf{E}_t, \mathbf{S}_t, \mathcal{T})}{N \times T} \tag{21}$$

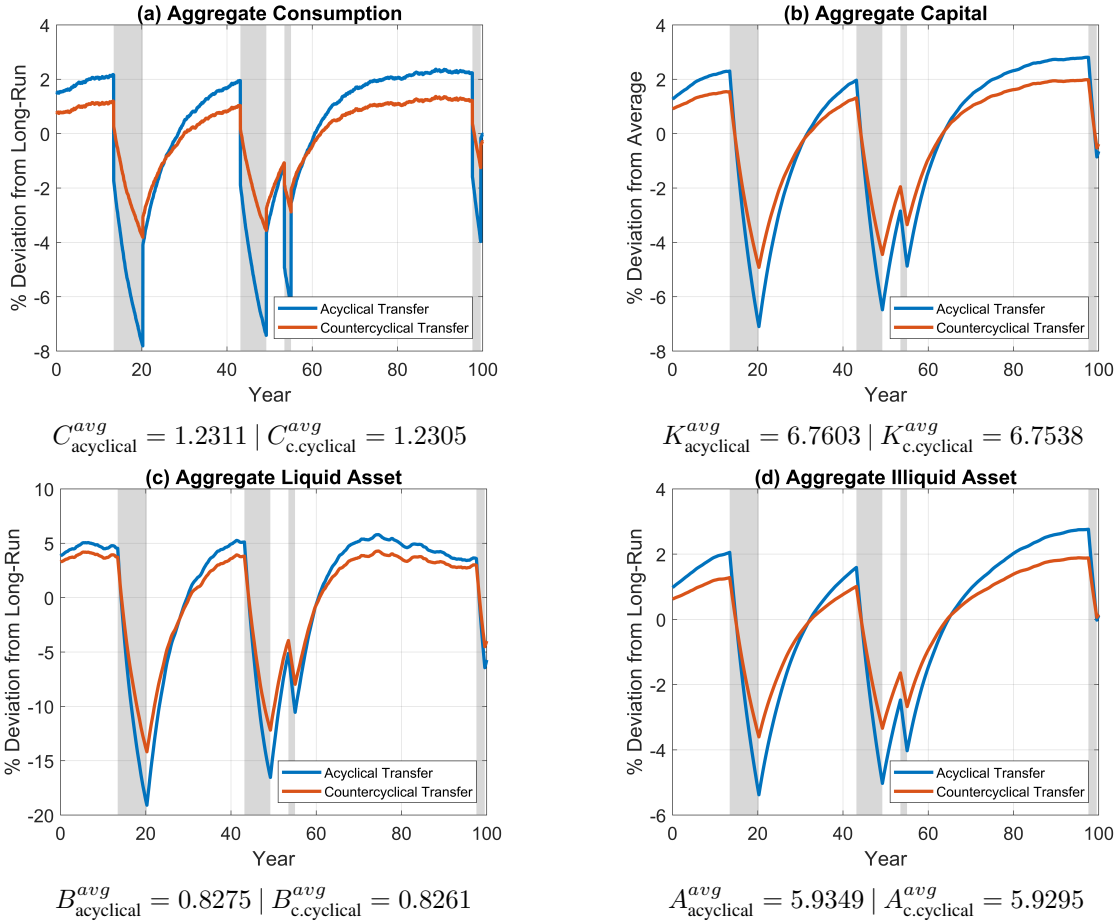


Figure 3: Aggregate Statistics of the Simulated model

countercyclical policy, consumption exhibits smoother dynamics over the business cycle, with an initial drop of about 1.5% and a subsequent decline of 4% during the first recession. Risk-averse households value stable consumption streams, and the reduced fluctuation enhances their welfare. However, anticipated policy transfers prompt households to reduce precautionary savings, leading to a decrease in aggregate capital. Panel 3b indicates that aggregate capital declines from 6.7603 units to 6.7538 units, a reduction of 0.0065 units (approximately \$190 per household). This reduction in long-run capital translates to a decrease in long-run consumption, from 1.2311 units to 1.2305 units. Moreover, Panel 3b shows that aggregate capital exhibits greater stability under the countercyclical policy, which implies a more stable path for interest rates and wages over the business cycles.

Panels 3c and 3d respectively depict the trajectories of aggregate liquid and illiquid assets, whose sum equals aggregate capital. Upon entering recessions, households initially draw down liquid assets, with those lacking liquid assets tapping into illiquid assets to smooth consumption. Consequently, the percentage deviation drop in liquid assets is greater than that in illiquid assets.

For instance, with the acyclical policy, during the first recession, liquid assets decrease by approximately 25%, compared to a 7% decline in illiquid assets.

Behind these aggregate statistics lies a joint distribution of assets, idiosyncratic productivity, and employment status, which varies over the business cycle and is influenced by policy decisions. Figure 4 presents the fractions of HtM and indebted households over the business cycle. These households, characterized by high MPC, exert significant influence on aggregate consumption fluctuations, and are among the most responsive recipients of the policy.

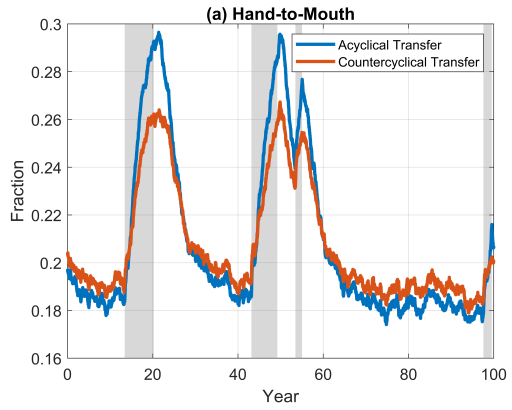
Panel 4a shows that the fraction of HtM households increases during recessions as households deplete liquid assets to smooth consumption.[31] However, this happens to a lesser extent with the countercyclical policy. During the first recession, with the acyclical policy, this results in an increase of approximately 12%, whereas with the countercyclical policy, it increases by only 7%. An immediate consequence of the transfer policy is the need for government surplus during normal times to balance the budget. This leaves households with fewer resources and potentially increases the prevalence of HtM and indebted households (as depicted by the higher orange line relative to the blue line in Panels 4a and 4b during normal times). An unintended consequence of the policy is that households, anticipating better insurance through the policy, reduce precautionary savings, increasing the likelihood of vulnerability. This effect leads to a decrease in the long-run fractions of HtM and indebted households under the policy of approximately 0.08% and 0.04%, respectively. Models assuming a fixed proportion of HtM to generate high average MPC likely fail to capture this dynamic.[32]

Similarly, Panel 4b demonstrates that households without liquid assets may borrow and become indebted during recessions, although the countercyclical policy almost entirely eliminates that need. A comparison between Panels 4a and 4b highlights that countercyclical transfers primarily stabilize the economy by reducing the fraction of households becoming indebted during recessions rather than reducing the fraction of HtM. This observation results from many HtM being wealthy, relying on their illiquid assets rather than borrowing against liquid assets.

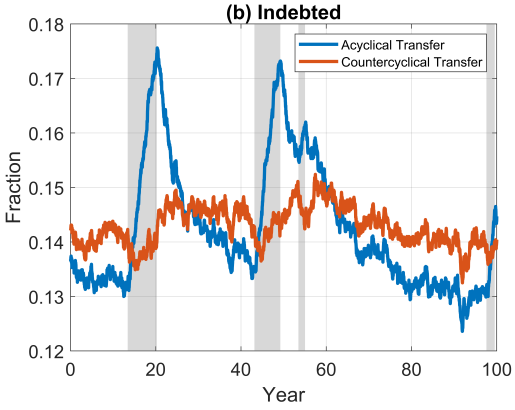
Panels 4c and 4d further decompose the dynamics of HtM into poor HtM and wealthy HtM. Households depleting both types of assets transition into poor HtM, including those with no illiquid assets exhausting liquid assets, those lacking liquid assets but having illiquid assets withdrawn entirely, and those depleting both asset types. Panel 4c indicates that the fraction of poor HtM households increases by approximately 2.5% during recessions with the acyclical policy, compared to only about 1.5% with the countercyclical policy in place. Finally, households with both liquid

[31] Appendix E shows the data counterpart of Figure 4, and illustrates that the model predictions for fraction of HtM and Indebted households are qualitatively in line with data.

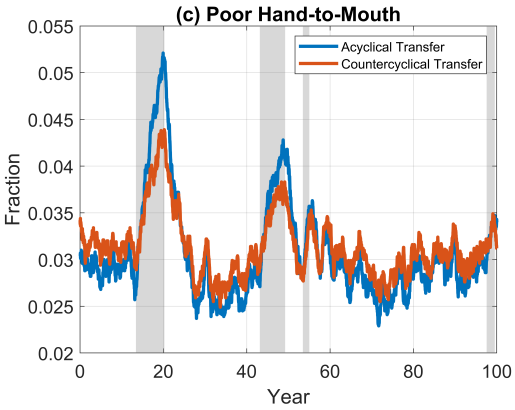
[32] Gross, Notowidigdo, and Wang (2020) shows that empirically the average MPC is countercyclical, which is consistent with our model due to the increase in the fraction of HtM and indebted households during the recession.



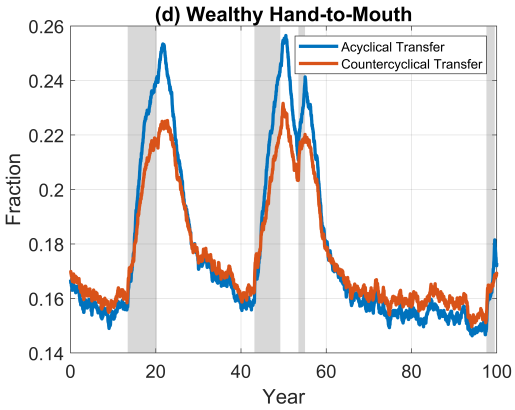
$HtM^{avg}_{\text{acyclical}} = 0.2110 \mid HtM^{avg}_{\text{c.cyclical}} = 0.2118$



$Indebt^{avg}_{\text{acyclical}} = 0.1425 \mid Indebt^{avg}_{\text{c.cyclical}} = 0.1429$



$pHtM^{avg}_{\text{acyclical}} = 0.0272 \mid pHtM^{avg}_{\text{c.cyclical}} = 0.0274$



$wHtM^{avg}_{\text{acyclical}} = 0.1838 \mid wHtM^{avg}_{\text{c.cyclical}} = 0.1844$

Figure 4: Fraction of HtM and Indebted households of the Simulated model

$\mathcal{T}(\mathbf{z^R})$	**\$300**	**\$600**	**\$900**	**\$1200**	**\$1500**	**\$1800**	**\$2100**	**\$2400**
C^{avg}	$\downarrow$ 0.0068%	$\downarrow$ 0.0138%	$\downarrow$ 0.0209%	$\downarrow$ 0.0280%	$\downarrow$ 0.0353%	$\downarrow$ 0.0428%	$\downarrow$ 0.0503%	$\downarrow$ 0.0579%
K^{avg}	$\downarrow$ 0.0240%	$\downarrow$ 0.0477%	$\downarrow$ 0.0711%	$\downarrow$ 0.0940%	$\downarrow$ 0.01167%	$\downarrow$ 0.1389%	$\downarrow$ 0.1606%	$\downarrow$ 0.1817%
Var(C_t)	0.1144%	0.0945%	0.0767%	0.0610%	0.0472%	0.0355%	0.0258%	0.0182%
CEV (in %)	0.0155	0.0282	0.0378	0.0443	0.0478	0.0483	0.0456	0.0398

(1) The first two rows are reported as relative to numbers from the acyclical policy.
(2) With the acyclical policy, the Variance of C_t is 0.1363%

Table 4: Welfare calculation and model statistics for various transfer sizes in the baseline model.

and illiquid assets may draw down liquid assets during recessions, becoming wealthy HtM. The large buffer stock of illiquid assets held by households ensures that most of the increase in HtM households during recessions comes from wealthy HtM. Notably, this group does not represent a vulnerable segment of the economy, and countercyclical policies enhance welfare by supporting households at risk of becoming poor HtM or indebted.

4.3 Optimal Transfer Size

The Consumption Equivalent Variation (CEV) is computed using the following welfare metric:

$$\omega = \left(\frac{\mathcal{W}(\mathcal{T}^{new})}{\mathcal{W}(\mathcal{T}^{old})} \right)^{\frac{1}{1-\gamma}} - 1 \tag{22}$$

where $\mathcal{W}(\mathcal{T}^{new})$ and $\mathcal{W}(\mathcal{T}^{old})$ represent welfare under the countercyclical and acyclical transfer policies, respectively. The CEV measures the percentage of consumption that a household under an acyclical policy would require to achieve the same level of welfare as in an economy with a countercyclical policy.

Table 4 quantifies the trade-off between consumption volatility and levels, with the model evaluated for $\mathcal{T}(z^R)$ values ranging from \$0 to \$2400 in \$300 increments. For each \$300 increment in the recessionary transfer, long-run consumption and capital decrease by approximately 0.007% and 0.0240%, respectively. The reduction in the variance of aggregate consumption diminishes with larger transfers, resulting in the CEV exhibiting an inverse U-shaped relationship with transfer size, peaking at \$1800 with a value of 0.0483%. At the optimal transfer size, the long-run consumption decreases by 0.0428%, and the variance of aggregate consumption is 0.0355%, which is about 1/4 of that from the acyclical policy.

4.4 Benabou-Floden Welfare Decomposition

I employ the Benabou-Floden decomposition method, as outlined in Benabou (1996) and Floden (2001), to analyze the sources of welfare gains. Detailed calculations are provided in Appendix D.

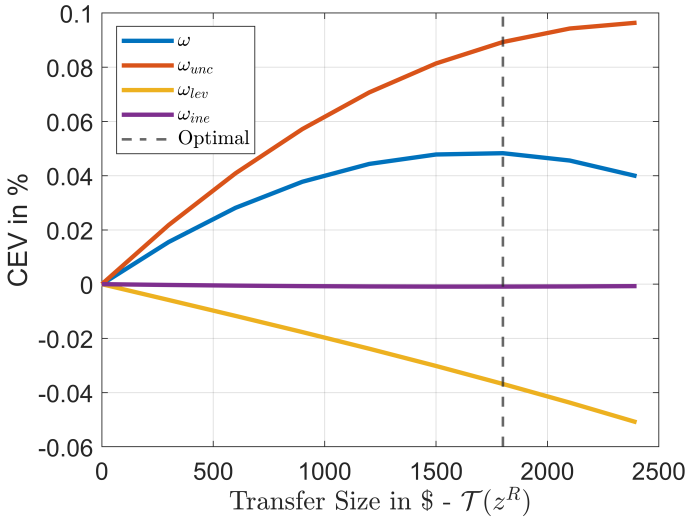


Figure 5: Benabou-Floden decomposition for various transfer size

The Benabou-Floden method decomposes the total welfare gain, ω, into three distinct components:

- ω^{unc}: the part due to reduction in uncertainty about future consumption - the *uncertainty effect*
- ω^{lev}: the part due to the change in average consumption - the *level effect*
- ω^{ine}: the part due to the change in current inequality - the *inequality effect*.

Figure 5 illustrates the decomposition for various transfer sizes. The only source of welfare gain is from the reduced uncertainty about future consumption, aligning with the intended benefits of countercyclical fiscal policies. Notably, ω_{unc} exhibits diminishing returns as transfer size increases, indicating decreasing marginal benefits of consumption stabilization.

The majority of welfare loss associated with the policy comes from the level effect, driven by a reduction in average consumption due to lower precautionary savings and decreased average capital. Subsequently, this reduction in average capital leads to income redistribution from capital-poor to capital-rich households. In particular, the diminishing marginal product of capital raises interest rates, disproportionately benefiting wealthier households. Additionally, because capital and labor are complementary, the decline in average capital lowers average wages, adversely affecting income for capital-poor households reliant on labor income. Although the welfare contribution of the inequality effect is negative, it is quantitatively trivial compared to the level effect. At the optimal transfer size of \$1800, the values are $\omega^{unc} = 0.0893\%$, $\omega^{lev} = -0.0368\%$, and $\omega^{ine} = -0.0009\%$.

4.5 Welfare Implications of an Unanticipated Policy

The anticipation of stimulus checks reduces precautionary savings, thereby partially counteracting the policy's intended benefits. A natural question to ask is what would happen if the policy was unanticipated? To address this question, I consider an experiment where households believe that the transfers will always be at the normal-time level, and unexpectedly receive transfers during recessions.

Figure 6a zooms in on a recessionary period to illustrate the consumption responses between an anticipated versus unanticipated countercyclical transfer of the same size, and compares these responses to an acyclical policy. We can see that when a recession occurs, the drop in consumption on impact is larger with unanticipated transfers compared to the anticipated policy. The MPCs from anticipated and unanticipated transfers turn out to be 0.60 and 0.25, respectively.

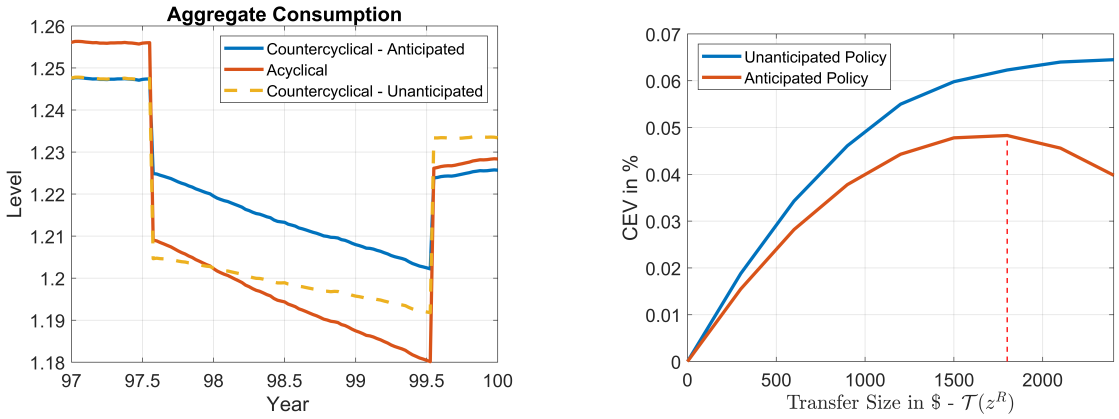
This is reasonable because if households do not anticipate continual transfers, they will not be confident spending as much upon receiving them in recessions, compared to the case where they anticipate future transfers. The first takeaway is that an anticipated policy more effectively alleviates the impact of a recession on aggregate consumption than an unanticipated transfer.

Figure 6b illustratesthe difference in welfare gains between anticipated and unanticipated policies. First, looking at Figure 6a, we can see that the variance of consumption under the unanticipated policy is smaller compared to the acyclical policy, which implies that the unanticipated policy helps reduce consumption fluctuations, though to a lesser degree than anticipated policy. On the other hand, unanticipated policies do not alter precautionary savings. Consequently, the welfare gain is strictly increasing in the size of transfers if they are unanticipated. Thus, a second key takeaway is that if the countercyclical policy is truly unanticipated, then the welfare gain would be greater than what is observed in the more realistic case that households anticipate them.

4.6 Welfare Gain by Distribution

As discussed in Section 4.4, the benefit of the policy comes from a reduction in consumption uncertainty. This implies that households with a large buffer-stock may not value the policy as highly. Conversely, some households bear costs from a lower capital stock in the economy and income redistribution more than others. Consequently, the welfare gains from the policy are heterogeneous across the distribution of liquid and illiquid assets. Figure 7 depicts the CEV for indebted, poor HtM, wealthy HtM, and Savers (defined as households with positive liquid assets) for different transfer sizes.

Several points are noteworthy. First, the CEV for indebted households increases monotonically, indicating that this group always prefers more generous transfers. This arises because they



(a) Consumption Responses

(b) Welfare in CEV

Figure 6: Comparing Consumption and Welfare between anticipated and unanticipated policies

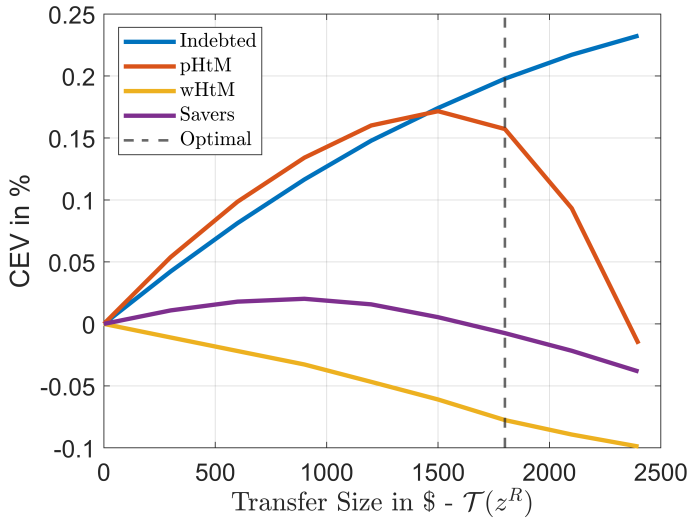
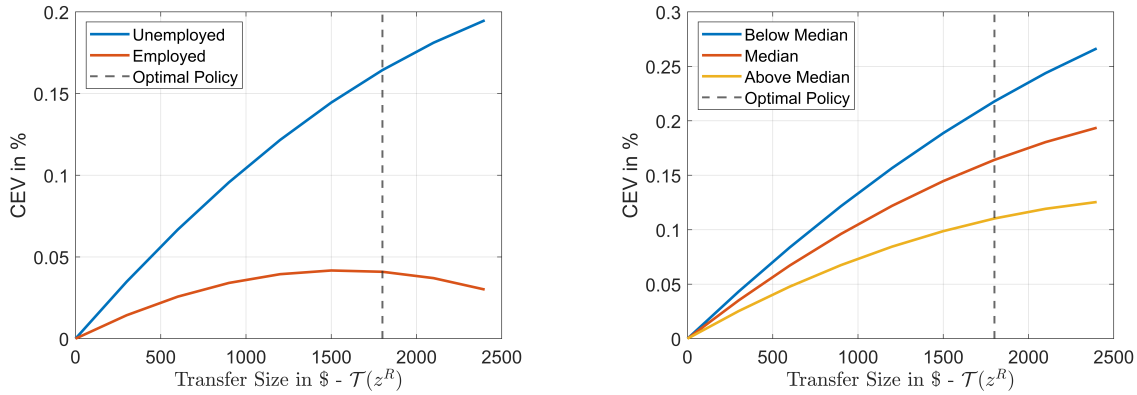


Figure 7: Welfare across different groups of asset distribution



(a) Unemployed versus Employed

(b) Unemployed w/ different Productivity

Figure 8: Welfare Gain across Employment Status

are already borrowers and can only borrow up to the credit limit; thus, they value transfers during recessions as a form of insurance. Second, the CEV for poor HtM exhibits an inverted U-shape, suggesting there exists an transfer level that maximizes welfare for this group, beyond which welfare decreases. This group values the transfer because it helps avoid expensive credit, though it remains an option if necessary. However, they also bear the policy cost due to lower wages and consumption resulting from the decreased capital stock. This group would prefer a transfer size of about $1,500. Third, the CEV for wealthy HtM decreases with the transfer size, indicating they consistently incur losses from the policy. This group can access their stock of illiquid assets to smooth consumption over the business cycle, thus they only benefit marginally from the policy. Moreover, since they do not own liquid assets, they rely on wages as their primary source of liquid income and suffer from wage decreases. At the optimal policy of $1,800, which maximizes the welfare of the economy, Indebted and poor HtM households benefit from the policy with CEVs of 0.1978% and 0.1747%, respectively, while wealthy HtM households experience a loss with a CEV of -0.0776%. Savers are neutral with respect to the policy.

Additionally, we can examine the welfare gains across employment status, as illustrated in Figure 8. Panel 8a contrasts the welfare gains between unemployed and employed households. Since unemployed households are more likely to face financial constraints, they strictly prefer a more generous policy. In contrast, the welfare gains for employed households follows an inverted U-shape and peaked at $1,500, which suggests that there is a redistribution of welfare from employed to unemployed households at the optimal policy of $1,800. Notably, the welfare gain for unemployed households is approximately four times larger than that for employed households under the optimal policy, suggesting that targeted policies, such as enhanced unemployment benefits, could be more effective in improving overall welfare. Panel 8b illustrates the welfare gains for unemployed households across different levels of idiosyncratic productivity. Households with lower

productivity levels, who are more likely to be either indebted or poor HtM, experience higher welfare gains as productivity decreases.

5 Alternative Model Specification and Policies

5.1 Aggregate Demand Externality

Despite featuring rich heterogeneity, the baseline model, is, effectively, a Real Business Cycle model characterized by minimal feedback from aggregate demand to aggregate supply. Incorporating nominal rigidities could enable such a feedback loop; however, this would necessitate adding inflation as an extra state variable, complicating the model. Instead, following Krueger, Mitman, and Perri (2016a), I consider a production function where TFP directly depends on aggregate consumption:

$$\begin{aligned} Y_t &= z_t^* K_t^{\alpha} L_t^{1-\alpha} \\ z_t^* &= z_t C_t^{\psi} \end{aligned} \tag{23}$$

Here, ψ denotes the degree of demand externality. When $\psi > 0$, a decline in aggregate consumption leads to a decrease in productivity z_t, which further contracts wages and interest rates, reducing aggregate consumption in a feedback loop. Consequently, fiscal transfers become more effective in stabilizing consumption during economic downturns. Conversely, a reduction in government transfers in normal times to finance recessionary transfers negatively impacts aggregate consumption and productivity. In this context, a similarly-sized transfer has a more pronounced impact on long-run capital because a decrease in precautionary savings leads to a reduction in capital and consumption, which lowers average TFP and further reduces the long-run capital stock. Additionally, households recognize that countercyclical transfers offer more insurance compared to the baseline model, leading to a further reduction in precautionary savings. As a result, while the dynamics of the model are similar to the baseline, the quantitative trade-off between fluctuations and levels differs.

I study the implication of same policy $\psi = 0.1$, while keeping the processes for TFP and employment risks unchanged. Table 5 reports the quantitative findings. In this specification, the reduction in long-run consumption is significantly steeper: for every \$300 increase in transfer size, the reductions are approximately 0.0105% in consumption, compared to 0.007% in the baseline model. However, this policy also results in a larger reduction in consumption fluctuations. For a \$1,500 transfer, the variance of consumption declines by about 0.1047 p.p. (from 0.1656% to 0.0609%), compared to a 0.0891 p.p. decrease (from 0.1363% to 0.0472%) in the baseline model. It turns out that the increase in the cost of policy outweighs the increase in benefits from

Baseline Model ($\psi = 0$)

$\mathcal{T}(\mathbf{z}^{\mathbf{R}})$	**\$300**	**\$600**	**\$900**	**\$1200**	**\$1500**	**\$1800**	**\$2100**	**\$2400**
C^{avg}	$\downarrow$ 0.0068%	$\downarrow$ 0.0138%	$\downarrow$ 0.0209%	$\downarrow$ 0.0280%	$\downarrow$ 0.0353%	$\downarrow$ 0.0428%	$\downarrow$ 0.0503%	$\downarrow$ 0.0579%
Var(C_t)	0.1144%	0.0945%	0.0767%	0.0610%	0.0472%	0.0355%	0.0258%	0.0182%
CEV (in %)	0.0155	0.0282	0.0378	0.0443	0.0478	0.0483	0.0456	0.0398

(1) With the Acyclical Transfer, the Variance of C_t is 0.1363%
(2) Numbers in the first row are relative to the acyclical transfer for $\psi = 0$

Model with Demand Externality ($\psi = 0.1$)

$\mathcal{T}(\mathbf{z}^{\mathbf{R}})$	**\$300**	**\$600**	**\$900**	**\$1200**	**\$1500**	**\$1800**	**\$2100**	**\$2400**
C^{avg}	$\downarrow$ 0.0105%	$\downarrow$ 0.0212%	$\downarrow$ 0.0320%	$\downarrow$ 0.0430%	$\downarrow$ 0.0542%	$\downarrow$ 0.0657%	$\downarrow$ 0.0774%	$\downarrow$0.0894%
Var(C_t)	0.1401%	0.1168%	0.0959%	0.0772%	0.0609%	0.0467%	0.0348%	0.0252%
CEV (in %)	0.0135	0.0242	0.0321	0.0372	0.0393	0.0383	0.0344	0.0273

(1) With the Acyclical Transfer, the Variance of C_t is 0.1656%.
(2) Numbers in the first row are relative to the acyclical transfer for $\psi = 0.1$

Table 5: Welfare calculation and model statistics for various transfer sizes in the model with Demand Externality, compared to the baseline model.

the uncertainty effect. Consequently, the welfare gain from the policy is smaller in this model, peaking at 0.0393% in CEV with the optimal transfer size being \$1,500.

5.2 Alternative Policy Instruments

5.2.1 Enhancing Unemployment Benefit

The Coronavirus Aid, Relief, and Economic Security (CARES) Act, enacted on March 27, 2020, in the United States, introduced three federally funded temporary unemployment insurance (UI) programs. These programs (1) expanded eligibility for UI benefits to self-employed workers, independent contractors, and part-time workers; (2) extended benefits by 13 weeks for individuals who exhausted their regular UI benefits; and (3) provided a \$600-per-week supplement to UI benefits through July 31, 2020. Subsequently, on August 8, 2020, an additional \$300-per-week supplement for up to six weeks of unemployment was implemented.[33]

To analyze the impact of a similar policy in the model, I first consider a UI enhancement policy that increases unemployment benefits by a fixed amount during a recession, offset by reduced transfers during normal economic conditions for all households. This approach leads to a cross-sectional redistribution of resources, with non-recipients of the policy (i.e., employed households that do not experience layoffs during recessions) sharing the cost with the recipients.

[33] Similar policies were adopted in other countries, such as Canada's Canada Emergency Response Benefit (CERB), which provided \$2,000 CAD for workers who ceased working due to COVID-19-related reasons.

	Enhancing Unemployment Benefits	$1800 Transfer to All Households	Acyclical Policy Baseline
C^{avg}	$\downarrow$ 0.0302%	$\downarrow$ 0.0428%	$36350 per HH
Var(C_t)	0.0388%	0.0355%	0.1363%
CEV (in %)	0.0798	0.0483	

Table 6: Welfare calculations and model statistics for a policy that enhances unemployment benefits at a cost to the government equivalent to $1800 transfers to all households.

Specifically, I set $\mathcal{T}(z^R) = 0$, modifying the labor income function as follows:

$$y(s_t, \epsilon_t, z_t, \mathbf{B}_t, \mathbf{A}_t) = \begin{cases} (1-\tau)\epsilon_t w(z_t, \mathbf{B}_t, \mathbf{A}_t) & , s_t = e \\ u_b + T_u \times \mathcal{I}(z_t = z^R) & , s_t = u \end{cases} \tag{24}$$

To finance the enhanced benefits, the transfer during normal periods is adjusted to $\mathcal{T}(z^N) = -T_u \times U(R) \times \frac{\Pi^R}{\Pi^N}$. This implies that for every $1 increase in unemployment benefits during a recession, the government reduces transfers to all households by approximately $0.016.

As discussed in Section 4.3, the optimal uniform transfer is determined to be $1,800. Instead of allocating the budget to a uniform transfer policy, I explore the implications of reallocating it towards enhancing unemployment benefits. Given a calibrated unemployment rate of 8.33% during recessions, this policy results in a top-up of approximately $21,500 per unemployed household during a recessionary year, equivalent to around $410 per week. Setting $T_u = \$21500$ and $\mathcal{T}(z^R) = 0$, I resolve the model and compare the outcomes with a countercyclical transfer scenario where $T_u = 0$ and $\mathcal{T}(z^R) = \$1800$.

Table 6 presents the results of this experiment, contrasting them with the optimal uniform transfer of $1,800. The findings indicate that both long-run consumption and the variance of consumption decrease to a lesser extent under the unemployment benefit enhancement policy compared to the $1800 transfer policy. Additionally, the welfare gain from the enhancement policy is significantly higher, amounting to 0.0798% in CEV. [34]This outcome is driven by the higher welfare gains among unemployed households, as illustrated in 8a. However, the targeted policy also fails to reach many employed households, particularly those who are HtM and indebted, who highly value the insurance provided by the policy. Specifically, approximately 10% of indebted and 12% of poor HtM households are employed in the simulations. These findings suggest that a mixed policy combining enhanced UI benefits with uniform transfers may yield greater welfare improvements.

[34]Similarly, Carroll, Crawley, and Tretvoll (2023) find that the welfare gain from a UI extension is higher than that of the stimulus check policy. Ganong, Greig, Noel *et al.* (2024) uses micro data to show that the increase in unemployment benefits in the US during COVID-19 recession had large spending impacts, and argued that temporary benefit supplements are a promising countercyclical tool.

Mixed Policy: Enhancing UI by $300 per month + Varying Uniform Transfer

$\mathcal{T}(\mathbf{z^R})$	**$300**	**$600**	**$900**	**$1200**	**$1500**	**$1800**	**$2100**
C^{avg}	$\downarrow$ 0.0497%	$\downarrow$ 0.0583%	$\downarrow$ 0.0668%	$\downarrow$ 0.0758%	$\downarrow$ 0.0847%	$\downarrow$ 0.0941%	$\downarrow$ 0.1037%
Var(C_t)	0.0458%	0.0353%	0.0267%	0.0201%	0.0154%	0.0127%	0.0102%
CEV (in %)	0.0818	0.0992	0.1048	0.1055	0.1027	0.0970	0.0922

(1) With the UI enhancement only, the Variance of C_t is 0.0575% and the CEV is 0.0670%
(2) The reduction in C^{avg} is relative to the acyclical transfer + no UI enhancement policy

Baseline: No UI enhancement - Varying Uniform Transfer

$\mathcal{T}(\mathbf{z^R})$	**$300**	**$600**	**$900**	**$1200**	**$1500**	**$1800**	**$2100**
C^{avg}	$\downarrow$ 0.0068%	$\downarrow$ 0.0138%	$\downarrow$ 0.0209%	$\downarrow$ 0.0280%	$\downarrow$ 0.0353%	$\downarrow$ 0.0428%	$\downarrow$ 0.0503%
Var(C_t)	0.1144%	0.0945%	0.0767%	0.0610%	0.0472%	0.0355%	0.0258%
CEV (in %)	0.0155	0.0282	0.0378	0.0443	0.0478	0.0483	0.0456

(1) With the acyclical transfer, the Variance of C_t is 0.1348%
(2) The reduction in C^{avg} is relative to the acyclical transfer + no UI enhancement policy

Table 7: Welcare calculation and model statistics for the Mixed Policy under various transfer size, compared to the uniform transfer policy as in the baseline model

5.3 Mixed Policy: Enhancing Unemployment Benefits and Implementing Uniform Transfers

I consider a policy that simultaneously tops up the UI by $300 per household for each recessionary month (equivalent to $15600 for each recessionary year), and implements a uniform transfer. This policy is financed by a reduction in transfers to households during non-recession periods. Specifically, the policy sets $T_u = \$15600$ and $\mathcal{T}(z^R) = \$X$. To maintain budget neutrality, the government reduces transfers by $\mathcal{T}(z^N) = -(T_u \times U(R) + \mathcal{T}(z^R)) \times \frac{\Pi^R}{\Pi^N}$. The model is solved for varying transfer sizes to identify the optimal policy mix, with results presented in Table 7, comparing the mixed policy with the baseline with uniform transfers only.

There are two key observations to note. First, the UI enhancement attenuates the impact of uniform transfers on reducing the variance of aggregate consumption. For example, when the uniform transfer increases from $300 to $600, the reduction in consumption variance under the mixed policy is 0.011 p.p, approximately half the reduction observed under the baseline policy, where this figure is 0.019 p.p. Second, the optimal uniform transfer in the mixed policy scenario is $1200, resulting in a welfare gain of 0.1055% in CEV, which is more than double the welfare gain under the optimal uniform transfer policy. This greater welfare improvement is attributed to the UI enhancement benefiting a significant portion of the economy that highly values insurance, while uniform transfers provide liquidity to the remaining employed households during recessions. Moreover, employed households indirectly benefit from the UI enhancement by reducing the adverse effects of layoffs during recessions, thereby necessitating less generous uniform transfers in the mixed policy compared to the baseline policy.

A key takeaway is that the distribution of welfare gains is an important consideration alongside the aggregate trade-off between level and fluctuation when evaluating fiscal policies. Indeed, the model does not account for real-life complications and indirect consequences of enhancing unemployment benefits, such as potential discouragement of labor market participation or operational and time costs.[35]

6 Computation

6.1 Solution Algorithm

Approximate Aggregation, initially proposed by Krusell and Smith (1998), is a widely adopted algorithm for solving heterogeneous-agent models with aggregate shocks. This algorithm assumes that households are boundedly-rational and form a linear forecast based on the aggregate capital. In this model, adopting a linear forecasting approach effectively replaces the laws of motion for distributions, $\dot{\mathbf{A}}$ and $\dot{\mathbf{B}}$, which are $N \times 1$ vectors, with a single equation governing the law of motion for aggregate capital:

$$\dot{K}_t = \alpha_0 + \alpha_1 \log(K_t) + \alpha_2 \mathcal{I}(z_t = z^N) + \alpha_3 (\mathcal{I}(z_t = z^N) \times \log(K_t)) \tag{25}$$

Krusell and Smith (1998) then solve the household problem using this linear forecasting rule for some initial guesses for $(\alpha_0, \alpha_1, \alpha_2, \alpha_3)$, and iteratively updates these guesses by running the regression on a simulated series of capital and aggregate shocks until convergence. They verify the accuracy of the forecasting rule by examining the R^2, which is usually very high in many model setups. Its implementability and efficiency make it a compelling choice of solution algorithm for macroeconomists over the past two decades.[36] A main criticism of the algorithm, perhaps philosophical, is that it solves a self-justifying equilibrium, for which we never know whether it is close to the rational expectations equilibrium, even if R^2 is very high.

Although Approximate Aggregation is a powerful algorithm, it is unlikely a suitable choice for this model. The main reason is in models with high MPCs, the saving decision of households is non-linear in wealth, and the mass of households in the non-linear region is substantial. However, this non-linearity in saving decisions contradicts the forecasting rule, which assumes that the saving decision of households is linear in wealth. Furthermore, in my model, about 22% of the households are at the "kink" (both zero kink and the liquidity constraint), where saving decisions are highly

[35] Ganong, Greig, Noel *et al.* (2024) found that the unemployment benefits in the U.S during the pandemic had small effects on job finding rates

[36] Examples include Krueger, Mitman, and Perri (2016a), Bloom, Floetotto, Jaimovich *et al.* (2018), Kaplan, Mitman, and Violante (2020) and Gilchrist, Sim, and Zakrajšek (2014) among many others

non-linear. This implies that if one applies a KS style forecasting rule in this model, the welfare implications may be very different than the rational-expectation solution.

To overcome this issue, I adopt the technique developed in Abbott and Phan (2024). Heuristically, we can write the Hamiltonian-Jacobian-Bellman (HJB) equation that solves the household problem in 4 as follows:[37]

$$\begin{aligned}\rho V(\mathbf{s}, z, \mathcal{S}) = \max_{c,d} u(c) + V_b \dot{b} + V_a \dot{a} + V_\epsilon \dot{\epsilon} + V_s \dot{s} + V_z \dot{z} \\ + V_{\mathbf{B}} \cdot \dot{\mathbf{B}} + V_{\mathbf{A}} \cdot \dot{\mathbf{A}}\end{aligned} \tag{26}$$

where $V_{\mathbf{B}}$ and $V_{\mathbf{A}}$ are a $N \times 1$ vector stacking up the derivative of value functions with respect to liquid and illiquid asset holding of each household i.[38] Importantly, with the assumption of a N finite-households households in a continuous-time environment, the only additional complication is to find these derivatives. If we know these derivatives, we can adopt a finite-difference (FD) method as in Achdou, Han, Lasry *et al.* (2022) to solve this HJB equation.

Indeed, a standard discretization method cannot be applied to the distributional state space due to the curse of dimensionality. To this end, Abbott and Phan (2024) proposes to solve the HJB problem of type 26 by (1) training a neural network to learn the value function and its derivatives, and (2) solving the HJB equation with a FD method to update the value function, given these derivatives. This procedure is repeated until the value function that the neural network learns is also the one that solves the HJB equation. The key insight for the successful implementation of the algorithm in Abbott and Phan (2024) is the symmetric nature of this dynamic program: the position of each household in the economy can be shuffled without affecting the household problem. In this context, Kahou, Fernández-Villaverde, Perla *et al.* (2021) demonstrates that there exists a nested neural network that reduces the dimensionality of the distribution, facilitating the learning of this high-dimensional value function. Appendix C provides a detailed description of the solution algorithm.

[37] Here, it is assumed that the distribution of idiosyncratic shocks and employment status is always at its ergodic state, thus $\dot{\mathbf{E}} = 0$ and $\dot{\mathbf{S}} = 0$, meaning that there is no direct effect of these transitions on the HJB equation.

[38] Formally, these derivatives also depend on state variables so we have:

$$V_{\mathbf{B}} = \begin{bmatrix} \frac{\partial V(\mathbf{s},z,\mathcal{S})}{\partial b_{1t}} \\ \frac{\partial V(\mathbf{s},z,\mathcal{S})}{\partial b_{2t}} \\ \vdots \\ \frac{\partial V(\mathbf{s},z,\mathcal{S})}{\partial b_{Nt}} \end{bmatrix}; V_{\mathbf{A}} = \begin{bmatrix} \frac{\partial V(\mathbf{s},z,\mathcal{S})}{\partial a_{1t}} \\ \frac{\partial V(\mathbf{s},z,\mathcal{S})}{\partial a_{2t}} \\ \vdots \\ \frac{\partial V(\mathbf{s},z,\mathcal{S})}{\partial a_{Nt}} \end{bmatrix} \tag{27}$$

6.2 Comparison to the Approximate Aggregation

To examine the implications of KS style forecasting rule in this context, I compare the welfare implications of the Global solution derived from the algorithm in Abbott and Phan (2024) with the same model solved using the Approximate Aggregation algorithm from Krusell and Smith (1998). Specifically, the household's utility maximization problem is given by:

$$
\begin{aligned}
&\max_{c_t, d_t} \mathbb{E} \int_0^\infty e^{-\rho t} u\big(c_t\big) dt \\
&\dot{b_t} = r_b(z_t, K_t) b_t + y(s_t, \epsilon_t, z_t, K_t) + T(z) - c_t - d_t - \chi(d_t, a_t) \\
&\dot{a}_t = r^a(z_t, K_t) a_t + d_t \\
&y(s_t, \epsilon_t, z_t, K_t) = \begin{cases} (1-\tau)\epsilon_t w(z_t, K_t) & , s_t = e \\ u_b & , s_t = u \end{cases} \\
&\dot{K}_t = \alpha_0 + \alpha_1 \log(K_t) + \alpha_2 \mathcal{I}(z_t = z^N) + \alpha_3 (\mathcal{I}(z_t = z^N) \times \log(K_t))
\end{aligned}
\tag{28}
$$

This problem can be solved by a FD method with discretized grid on the K dimension. In this bounded-rationality framework, households forecast the evolution of aggregate capital based only on the current aggregate capital and TFP.[39]

I solve the model in Equation 28 for similar cyclical transfer policies as in 4.3, and compare the CEV between the Global and Approximate Aggregation solutions. In theory, the welfare implications between the two can differ due to the difference in both decisions rules and the value functions.[40] To decompose the source of welfare discrepancy, I calculated the CEV from the value function of the global solution but evaluated it using the state variables generated by the Approximate Aggregation algorithm.

Figure 9 illustrates this exercise. It reveals that the measured welfare gains under the Approximate Aggregation solution are significantly smaller in magnitude compared to the Global solution. Notably, the welfare gain peaks at \$1,500 under Approximate Aggregation, whereas the global solution yields a peak at \$1,800. This highlights the importance of employing a global solution for the model - not only does it provide a more precise measure of welfare, but the policy implications derived from it also differ substantially from those measured under the approximate solution. One might argue that the welfare loss from a suboptimal policy appears small, given the relatively flat

[39] We can also assume that households could forecast aggregate liquid and illiquid savings separately. However, given that individual savings decisions are non-linear with respect to both types of assets, Approximate Aggregation would still fail to hold.

[40] The value function depends on state variables $V(\mathbf{s}, z, \mathcal{S})$, meaning welfare gains are influenced by both the state variables $\mathbf{s}$ and the value function V, both of which depend on the solution algorithm. The Krusell and Smith (1998) algorithm assumes that the value function depends only on current aggregate capital, K, while my method allows the value function to depend on the entire distribution $\mathcal{S}$.

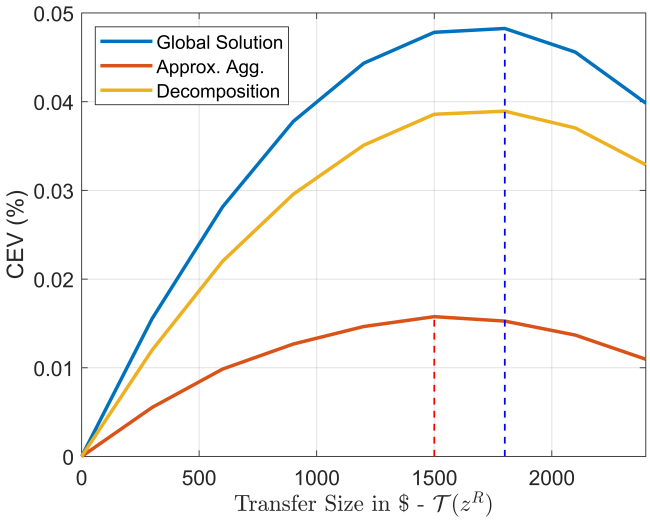


Figure 9: Comparison of Welfare gain between the Global solution and Approximate Aggregation solution.

optimal policy area. However, in a model where business cycles entail greater welfare costs, the impact of suboptimal policies could be amplified significantly. Lastly, the decomposition reveals that about 25% of the welfare difference is attributable to the differences in decision rules (the difference between blue and yellow lines), with the remaining discrepancy arising from differences in value functions.

Accuracy of the KS forecasting rule: To evaluate the forecasting rule, I adopt the approach from Den Haan (2010b). The linear forecasting rule after solving the model with Krusell and Smith (1998) algorithm is:

$$\frac{K_{t+1}-K_t}{dt} = 1.239 - 0.701\log(K_t) + 0.104\mathcal{I}(z_t = z^N) + 0.178\big(\mathcal{I}(z_t = z^N)\times\log(K_t)\big), R^2 = 0.99 \tag{29}$$

The R^2 from this regression is quite high, despite theoretical expectation that the linear forecasting rule should be inaccurate for this model. As noted by Den Haan (2010a), a high R^2 can be misleading because it only captures the one-period-ahead forecast error.[41] To address this, I conduct the Den Haan (2010b) procedure, outlined below:

1. Simulate a path of aggregate shocks different from the one used to solve the model.

2. Simulate individual decisions using this path of aggregate shocks and an initial capital stock K_0, and aggregate the individual asset holdings to calculate the aggregate capital stock over time. Call this K_t^*.

[41] This issue is exacerbated in continuous-time frameworks, where the time lapse between periods is short.

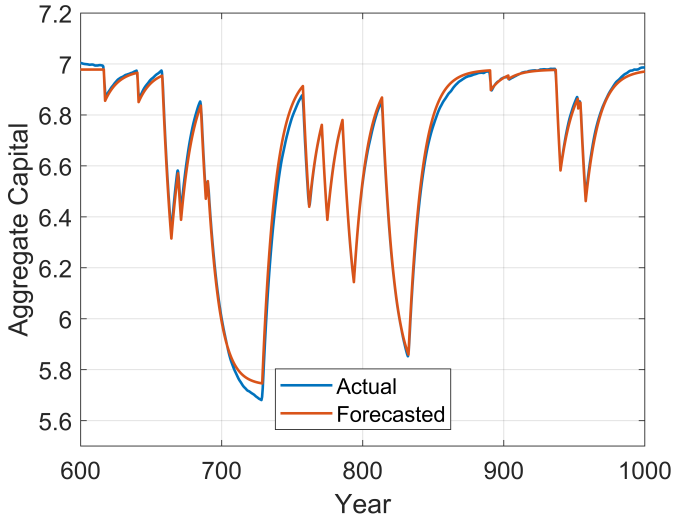


Figure 10: Den Haan (2010b) simulation for this model on the Approximate Aggregation solution of the model.

3. Sequentially simulate a path of aggregate capital using the KS forecasting rule.[42] This is done by plugging in values of K_0 and realization of shocks at time $t = 0$ into Equation 29 to calculate K_1, and replug this K_1 with realization of shock at $t = 1$ to calculate K_2 and so on. Call this series of capital K_t^{lin}.

4. Compute the log difference between the two capital series: $\epsilon_{DH} = |log(K_t^*) - log(K_t^{lin})|$

This approach assesses the out-of-sample accuracy of the forecasting rule, and allows the forecast error to accumulate over time. I compare these errors with those from the standard Krusell-Smith model, where Approximate Aggregation holds.[43] The results are reported in Table 8, and Figure 10 displays these over a time sample with periods of severe recession. In this model, where the linear forecasting rule is less likely to hold, the average and maximum forecasting error are approximately five times larger (0.22% versus 0.04%) than in the Krusell-Smith model. This highlights the lower accuracy of the linear forecasting rule in models with realistic MPC. The discrepancy is especially pronounced when examining the maximum forecasting error (1.25% compared to 0.18%), as linear rules tend to mispredict aggregate capital during periods of consecutive severe recessions, as observed around the year 700 in Figure 10 . Because the value function from the Approximate Aggregation solution fails to internalize the insurance provided by the policy during severe

[42]This principle can be applied to any forecasting rule. For example, Ahn, Kaplan, Moll *et al.* (2018) assesses the accuracy of their pertubation method with the same method.

[43]The Krusell-Smith model used here is the baseline version without a stochastic discount rate, so the fraction of households at the borrowing constraint is marginal. Even in the version with stochastic β (Krusell and Smith, 1998), the model's calibration to match wealth quantiles does not generate realistic marginal propensities to consume (MPC) because the fraction of constrained households remains small.

Model	Avg ϵ_{DH}	Max ϵ_{DH}
This Model	0.22%	1.25%
Krusell-Smith (1998)	0.04%	0.18%

Table 8: Assessing the Den Haan (2010b) statistics between my model and the Krusell and Smith (1998) model. The code used to solve the Krusell and Smith (1998) model is borrowed from Fernandez-Villaverde, Nuno, Sorg-Langhans *et al.* (2020) .

downtimes, it would underestimate the welfare gain and therefore misidentify the optimal policy. Indeed, the algorithm in Abbott and Phan (2024) finds the global solution, eliminating forecasting errors by fully internalizing the decisions of all other households in the economy. Therefore, the value of a global solution is significantly higher in my model.

7 Conclusion

Stimulus checks have been widely utilized by the US government and other developed countries to mitigate the impact of recessions on the economy. However, due to the recurrent nature of this policy, households may come to expect similar interventions in future economic downturns. This anticipation could lead to a reduction in precautionary savings, thereby undermining some of the policy's intended objectives. Examining the benefits and costs of stimulus policies enables policymakers to make more informed decisions during future economic crises.

This paper studies the welfare implications of countercyclical transfers through the lens of a two-asset heterogeneous agent model with aggregate shocks. The model is calibrated to match the distribution of liquid and illiquid assets observed in the data, yielding an average MPC that aligns well with empirical estimates. The economy experiences infrequent but severe recessions, capturing the nature of stimulus checks deployed during significant downturns, such as the Global Financial Crisis of 2008 and the COVID-19 pandemic. Validation exercises demonstrate that the model accurately reflects untargeted moments of asset distribution and business cycle statistics.

The optimal fiscal transfer policy is found to be countercyclical. In the baseline model, the optimal size amounts to $1,800 per household during each recessionary year, resulting in a CEV of 0.0483%. The policy balances the trade-off between consumption fluctuations and levels. The Benabou-Floden decomposition shows that the welfare gain comes from the uncertainty effect, while the costs are largely due to welfare losses associated with the level effect. The inequality effect, however, contributes only marginally to welfare losses. If the policy was truly modelled as unanticipated, the countercyclical policy will lead to a larger welfare gain than are realistic because precautionary savings are not affected, but consumption volatility is still mitigated.

The welfare gain is highly heterogenous across the distribution of asset holdings. At the op-

timal policy, indebted and poor HtM households benefit, while wealthy HtM households incur welfare losses. Savers remain neutral to the policy. A mixed policy, which combines enhanced unemployment insurance (UI) benefits with uniform transfers, outperforms either policy in isolation, delivering a welfare gain that is more than double that of either policy alone.

On the computational front, I show that a conventional Mean-based forecasting rule would fail to predict the aggregate capital in periods of consecutive recessions. Therefore, adapting the Approximate Aggregation solution to my model would lead to underestimation of the welfare gains and miscalculation of the optimal policy. These findings highlight the value of adapting the Abbott and Phan (2024) method for a global solution. Indeed, a lesson from this exercise is that it is essential to have a global solution to study the welfare implications of cyclical policies, where the interaction between the MPCs and aggregate fluctuations is key.

Further research could extend the model by introducing nominal rigidities to incorporate the propagation of aggregate shocks through aggregate demand (Kaplan, Moll, and Violante (2018)). Stabilization policies, such as transfers, could be more welfare-improving in this environment. The model can also be adapted to study optimal monetary policy with rich heterogeneity (Acharya, Challe, and Dogra (2023), Bhandari, Evans, Golosov *et al.* (2021b), McKay and Wolf (2022)). Additionally, there is a vast literature on Central Bank credibility and forward guidance highlighting the importance of monetary policy being anticipated and trusted by households (Galí (2015), McKay, Nakamura, and Steinsson (2016), Dávila and Schaab (2023)). Techniques developed in Abbott and Phan (2024) can be extended to tackle the computational challenges in this frontier of macroeconomic research.

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APPENDIX NOT FOR PUBLICATION

A Simulation of the model

I simulate the decisions of N households over time by discretizing the time space into steps of size Δ. Each household potentially faces different draws of exogenous idiosyncratic shocks. To simulate exogenous shocks for TFP, employment and idiosyncratic risks, I convert a continuous-time Poisson transition matrix to a discrete-time transition probability matrix. For example, the continuous-time transition matrix for the TFP process is:

$$\pi_z = \begin{bmatrix} -\beta_l & \beta_l \\ \beta_h & -\beta_h \end{bmatrix} \tag{30}$$

In a time step Δ, the probability that the economy stays in a recession is $\exp(-\beta_l)\Delta$, and the probability that it transitions to normal times is $1 - \exp(-\beta_l)\Delta$. The discrete-time transition probability matrix for the TFP process is then:

$$P_z = \begin{bmatrix} \exp(-\beta_l)\Delta & 1 - \exp(-\beta_l)\Delta \\ 1 - \exp(-\beta_h)\Delta & \exp(-\beta_h)\Delta \end{bmatrix} \tag{31}$$

The TFP process can then be simulated as a Markov chain. The transition matrix for employment risk is also two-state and can be constructed in a similar way.

Converting a multiple Poisson process, such as those involving idiosyncratic shocks, requires appropriate weighting of the arrival probabilities. In particular, let N_ϵ be the number of points in the discretized grid for the idiosyncratic shock. The "jump" component implies that the Poisson transition matrix will not be sparse because once the jump occurs, a household gets a random draw to transition to any productivity level. The generic continuous-time transition matrix for the jump-drift process takes the following form:

$$\pi_\epsilon = \begin{bmatrix} -\varphi_{1,1} & \varphi_{1,2} & \cdots & \varphi_{1,N_z} \\ \varphi_{21} & -\varphi_{22} & \cdots & \varphi_{2,N_z} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_{N_z,1} & -\varphi_{N_z,2} & \cdots & \varphi_{N_z,N_z} \end{bmatrix} \tag{32}$$

with the restriction that each row sums to 0. The off-diagonals φ_{ij} represent the rate at which a person transitions from state i to j, while the absolute values of the diagonal elements represent the rate at which a person remains in the same state. Within a time step Δ, the probability that a person with productivity state i stays there is $\exp(-\xi_{ii})\Delta$. The probability that he leaves state i is $1 - \exp(-\xi_{ii})\Delta$. The probability that an individual arrives at state j is then $\left(1 - \exp(-\xi_{ij})\right)\frac{\xi_{ij}}{\sum_{j \neq i} \xi_{ij}}$.

With the discrete-time probability transition matrices, I simulate these shocks to track how idiosyncratic shock and employment status of each individual evolves over time. An important assumption is that the distribution of idiosyncratic shocks and employment status is always at its ergodic state at any given point

in time. This is not guaranteed in the model with many-finite agent, as the law of large number does not hold here. To this end, the simulation of household transitions is implemented by sampling the proportions of 'movers' and 'stayers' according to the probabilities in the transition matrix. For example, if the ergodic distribution of employment has 9500 employed and 500 unemployed, and the job finding probability is 5%, then I will sample 25 unemployed individuals to transition to employed status. If the law of larger number holds, this is equivalent to giving each unemployed person a 5% chance of transitioning to employment.

Lastly, the simulation of employment risks assumes that once the aggregate state switches, there will be a jump in the number of employed/unemployed individuals to the ergodic distribution associated with the new aggregate state. For example, if the normal and recession states have 9500 and 9000 employed individuals, respectively, then when the economy transitions from the normal state to recession, 500 employed individuals will be moved to the unemployment pool within the time step Δ.

The endogenous state variables of individual i at time t are $b_{i,t}$ and the illiquid asset $a_{i,t}$, which evolve within the time step Δ according to the following equations:

$$\begin{aligned} b_{i,t+1} &= \Big(r_b(z_t, \mathbf{B}_t, \mathbf{A}_t) b_{i,t} + y(s_{i,t}, \epsilon_{i,t}, z_t, \mathbf{B}_t, \mathbf{A}_t) + T(z^t) - c_{i,t} - d_{i,t} - \chi(d_{i,t}, a_{i,t}) \Big) \Delta + b_{i,t} \\ a_{i,t+1} &= \Big(r^a(z_t, \mathbf{B}_t, \mathbf{A}_t) a_{i,t} + d_{i,t} \Big) \Delta + a_{i,t} \end{aligned} \tag{33}$$

Once the decision rules c and d are solved, these calculations become straightforward. Stacking up the saving decisions gives us the evolution of the distribution of liquid and illiquid assets over time:

$$\mathbf{B}_{t+1} = \begin{bmatrix} b_{1,t+1} \\ b_{2,t+1} \\ \dots \\ b_{N,t+1} \end{bmatrix} ; \ \mathbf{A}_{t+1} = \begin{bmatrix} a_{1,t+1} \\ a_{2,t+1} \\ \dots \\ a_{N,t+1} \end{bmatrix} \tag{34}$$

Finally, because state-dependent transfers are revenue-neutral in expectation, the timing of TFP realization plays a crucial role in policy evaluation.[44] To eliminate the dependence of welfare study on the initial conditions,, instead of simulating a 1000 years of the one series of TFP, I simulate 10 paths of TFP, each with 100 years of length, and then take averages of model statistics.

[44] As an extreme example, if by chance the economy frontloads all recessions at the beginning of the simulation, the countercyclical transfer policy would provide more net benefit compared to an acyclical policy. This is because, technically, the government could borrow at a zero interest rate to finance the transfers.

B The HJB equations

The HJB equation for employed households in the normal state is:

$$
\begin{aligned}
&\rho V(b,a,\epsilon,e,z^N,\mathcal{S}) = \max_{c,d} u(c) \\
&+ V_b(\mathbf{s},z,\mathcal{S})\big(r_b(z^N,\mathbf{B},\mathbf{A})b + (1-\tau)\epsilon w(z^N,\mathbf{B},\mathbf{A}) + T(z^N) - c - d - \chi(d,a)\big) \\
&+ V_a(\mathbf{s},z,\mathcal{S})\big(r^a(z^N,\mathbf{B},\mathbf{A})a + d\big) \\
&+ V_\epsilon(\mathbf{s},z,\mathcal{S})(-\xi\epsilon) + \lambda_\epsilon \int_{-\infty}^{\infty} \big(V(b,a,x,e,z^N,\mathcal{S}) - V(b,a,\epsilon,e,z^N,\mathcal{S})\phi(x)dx\big) \\
&+ \sigma(z^N)\big(V(b,a,\epsilon,u,z^N,\mathcal{S}) - V(b,a,\epsilon,e,z^N,\mathcal{S})\big) \\
&+ \beta_h\big(Q_{eu}V(b,a,\epsilon,u,z^R,\mathcal{S}) + (1-Q_{eu})V(b,a,\epsilon,e,z^R,\mathcal{S}) - V(b,a,\epsilon,u,z^N,\mathcal{S})\big) \\
&+ V_{\mathbf{B}}(\mathbf{s},z,\mathcal{S})\cdot\big(r^b(z^N,\mathbf{B},\mathbf{A})\mathbf{B} + y(\mathbf{S},\mathbf{E},z^N,\mathbf{B},\mathbf{A}) + T(z^N) - \tilde{\mathbf{c}} - \tilde{\mathbf{d}} - \chi(\tilde{\mathbf{d}},\mathbf{A})\big) \\
&+ V_{\mathbf{A}}(\mathbf{s},z,\mathcal{S})\cdot\big(r^a(z^N,\mathbf{B},\mathbf{A})\mathbf{A} + \tilde{\mathbf{d}}\big)
\end{aligned}
\tag{35}
$$

Lines 1 to 5 of Equation 35 are standard: the flow of the value function on the LHS is equal to the RHS, consisting of the instantaneous utility from consumption and the changes in individual state variables, multiplied by the derivatives of the value function with respect to those state variables. This logic extends to Line 6, with an added complexity that, when the economy transitions from a normal state to a recession, a proportion Q_{eu} of employed households will move to unemployment, so the associated distribution of employment status will reach its ergodic state upon the transition.[45] Employed households account for this additional layer of risk when making decisions.

Lines 7 and 8 reflect the value function's dependence on other households' saving decisions, where $V_{\mathbf{B}}$ and $V_{\mathbf{A}}$ are $N \times 1$ vectors of derivatives of the household's value function with respect to liquid and illiquid asset holdings of N households in the economy.[46] The HJB equation for employed households in recession and unemployed households in either state can be derived similarly.

[45] Krusell and Smith (1998) also implements this principle when simulating the model. This assumption eliminates the dependence of the value function on the distribution of employment status.

[46] One can write Lines 7 and 8 compactly as $V_{\mathbf{B}}(\mathbf{s},z,\mathcal{S})\cdot\dot{\mathbf{B}} + V_{\mathbf{A}}(\mathbf{s},z,\mathcal{S})\cdot\dot{\mathbf{A}}$. In Krusell and Smith (1998) or Fernandez-Villaverde, Nuno, Sorg-Langhans *et al.* (2020) these would be replaced by $V_K\dot{K}$ where $\dot{K}$ is the forecasting rule on mean of capital, which is linear in the former paper and non-linear in the latter

Similarly, the HJB equation for employed households in recession state is:

$$
\begin{aligned}
&\rho V(b,a,\epsilon,e,z^R,\mathcal{S}) = \max_{c,d} u(c) \\
&+ V_b(\mathbf{s},z,\mathcal{S})\big(r_b(z^R,\mathbf{B},\mathbf{A})b + (1-\tau)\epsilon w(z^R,\mathbf{B},\mathbf{A}) + T(z^R) - c - d - \chi(d,a)\big) \\
&+ V_a(\mathbf{s},z,\mathcal{S})\big(r^a(z^R,\mathbf{B},\mathbf{A})a + d\big) \\
&+ V_\epsilon(\mathbf{s},z,\mathcal{S})(-\xi\epsilon) + \lambda_\epsilon \int_{-\infty}^{\infty} \big(V(b,a,x,e,z^R,\mathcal{S}) - V(b,a,\epsilon,e,z^R,\mathcal{S})\phi(x)dx\big) \\
&+ \sigma(z^R)\big(V(b,a,\epsilon,u,z^R,\mathcal{S}) - V(b,a,\epsilon,e,z^R,\mathcal{S})\big) \\
&+ \beta_l\big(V(b,a,\epsilon,u,z^N,\mathcal{S}) - V(b,a,\epsilon,u,z^R,\mathcal{S})\big) \\
&+ V_{\mathbf{B}}(\mathbf{s},z,\mathcal{S}) \cdot \big(r^b(z^R,\mathbf{B},\mathbf{A})B + y(\mathbf{S},\mathbf{E},z^R,\mathbf{B},\mathbf{A}) + T(z^R) - \tilde{\mathbf{c}} - \tilde{\mathbf{d}} - \chi(\tilde{\mathbf{d}},\mathbf{A})\big) \\
&+ V_{\mathbf{A}}(\mathbf{s},z,\mathcal{S}) \cdot \big(r^a(z^R,\mathbf{B},\mathbf{A}) + \tilde{\mathbf{d}}\big)
\end{aligned}
\tag{36}
$$

The HJB equation for unemployed households in normal state is:

$$
\begin{aligned}
&\rho V(b,a,\epsilon,u,z^N,\mathcal{S}) = \max_{c,d} u(c) \\
&+ V_b(\mathbf{s},z,\mathcal{S})\big(r_b(z^N,\mathbf{B},\mathbf{A})b + u_b + T(z^N) - c - d - \chi(d,a)\big) \\
&+ V_a(\mathbf{s},z,\mathcal{S})\big(r^a(z^N,\mathbf{B},\mathbf{A})a + d\big) \\
&+ V_\epsilon(\mathbf{s},z,\mathcal{S})(-\xi\epsilon) + \lambda_\epsilon \int_{-\infty}^{\infty} \big(V(b,a,x,u,z^N,\mathcal{S}) - V(b,a,\epsilon,u,z^N,\mathcal{S})\phi(x)dx\big) \\
&+ \lambda(z^N)\big(V(b,a,\epsilon,e,z^N,\mathcal{S}) - V(b,a,\epsilon,u,z^N,\mathcal{S})\big) \\
&+ \beta_h\big(V(b,a,\epsilon,u,z^N,\mathcal{S}) - V(b,a,\epsilon,u,z^N,\mathcal{S})\big) \\
&+ V_{\mathbf{B}}(\mathbf{s},z,\mathcal{S}) \cdot \big(r^b(z^N,\mathbf{B},\mathbf{A})\mathbf{B} + y(\mathbf{S},\mathbf{E},z^N,\mathbf{B},\mathbf{A}) + T(z^N) - \tilde{\mathbf{c}} - \tilde{\mathbf{d}} - \chi(\tilde{d},\mathbf{A})\big) \\
&+ V_{\mathbf{A}}(\mathbf{s},z,\mathcal{S}) \cdot \big(r^a(z^N,\mathbf{B},\mathbf{A})\mathbf{A} + \tilde{\mathbf{d}}\big)
\end{aligned}
\tag{37}
$$

The HJB equation for unemployed households in recession state is:

$$
\begin{aligned}
&\rho V(b,a,\epsilon,u,z^R,\mathcal{S}) = \max_{c,d} u(c) \\
&+ V_b(\mathbf{s},z,\mathcal{S})\big(r_b(z,\mathbf{B},\mathbf{A})b + u_b + \bar{T} + \mathcal{T}(z^R) - c - d - \chi(d,a)\big) \\
&+ V_a(\mathbf{s},z,\mathcal{S})\big(r^a(z,\mathbf{B},\mathbf{A})a + d\big) \\
&+ V_\epsilon(\mathbf{s},z,\mathcal{S})(-\xi\epsilon) + \lambda_\epsilon \int_{-\infty}^{\infty} \big(V(b,a,x,e,z^R,\mathcal{S}) - V(b,a,\epsilon,e,z^R,\mathcal{S})\phi(x)dx\big) \\
&+ \lambda(z^R)\big(V(b,a,\epsilon,e,z^R,\mathcal{S}) - V(b,a,\epsilon,u,z^R,\mathcal{S})\big) \\
&+ \beta_l\big(Q_{ue}V(b,a,\epsilon,e,z^N,\mathcal{S}) + (1-Q_{ue})V(b,a,\epsilon,u,z^N,\mathcal{S}) - V(b,a,\epsilon,u,z^R,\mathcal{S})\big) \\
&+ V_{\mathbf{B}}(\mathbf{s},z,\mathcal{S}) \cdot \big(r^b(z^R,\mathbf{B},\mathbf{A})B + y(\mathbf{S},\mathbf{E},z^R,\mathbf{B},\mathbf{A}) + T(z^R) - \tilde{\mathbf{c}} - \tilde{\mathbf{d}} - \chi(\tilde{d},\mathbf{A})\big) \\
&+ V_{\mathbf{A}}(\mathbf{s},z,\mathcal{S}) \cdot \big(r^a(z^R,\mathbf{B},\mathbf{A}) + \tilde{\mathbf{d}}\big)
\end{aligned}
\tag{38}
$$

C The Solution Algorithm

The HJB equations in Section 36 can be written compactly as:

$$
\begin{aligned}
&\rho V(b,a,\epsilon,s,z,\mathcal{S}) = \max_{c,d} u(c) \\
&+ V_b\big(r_b(z,\mathbf{B},\mathbf{A})b + y(s,\epsilon,z) + \bar{T} + \mathcal{T}(z) - c - d - \chi(d,a)\big) \\
&+ V_a\big(r^a(z,\mathbf{B},\mathbf{A})a + d\big) + V_\epsilon(b,a,\epsilon,s,z,\mathcal{S})(-\xi\epsilon) \\
&+ \lambda_\epsilon \int_{-\infty}^{\infty} \big(V(b,a,x,e,z,\mathcal{S}) - V(b,a,\epsilon,e,z,\mathcal{S})\phi(x)dx\big) \\
&+ \Pi_s(z)\big(Q(z)V(b,a,\epsilon,s',z',\mathcal{S}) + (1-Q(z))V(b,a,\epsilon,s,z',\mathcal{S}) - V(b,a,\epsilon,s',z,\mathcal{S})\big) \\
&+ \Pi_z\big(V(b,a,\epsilon,s,z',\mathcal{S}) - V(b,a,\epsilon,s,z,\mathcal{S})\big) \\
&+ V_{\mathbf{B}} \cdot \big(r^b(z,\mathbf{B},\mathbf{A})\mathbf{B} + y(\mathbf{S},\mathbf{E},z) + \bar{T} + \mathcal{T}(z_t) - \tilde{c}(\mathbf{B},\mathbf{A},\mathbf{E},\mathbf{S},z,\mathcal{S}) - \tilde{d}(\mathbf{B},\mathbf{A},\mathbf{E},\mathbf{S},z,\mathcal{S}) - \chi(\tilde{d},\mathbf{A})\big) \\
&+ V_{\mathbf{A}} \cdot \big(r^a(z,\mathbf{B},\mathbf{A})\mathbf{A} + \tilde{d}\big)
\end{aligned}
\tag{39}
$$

where s' and z' denote the switching states of s and z, and $Q(z)$ compactly summarizes the jump in employment status upon switches in aggregate states.[47]

To understand the logic behind Abbott and Phan (2024), suppose we know $V_{\mathbf{B}}$ and $V_{\mathbf{A}}$. The First Order Conditions (FOCs) characterizing optimal decisions is standard as in the model without aggregate shocks because households do not internalize that their decisions affect prices via changes in distributions, therefore treating $\tilde{c}$ and $\tilde{d}$ as given:

$$
\begin{aligned}
u'(c) &= V_b(\mathbf{s},z,\mathcal{S}) \\
V_b(\mathbf{s},z,\mathcal{S}) + \chi_d(d,a) &= V_a(\mathbf{s},z,\mathcal{S})
\end{aligned}
\tag{40}
$$

These FOCs pin down the optimal consumption and deposit decisions, $c(\mathbf{s},z,\mathcal{S})$ and $d(\mathbf{s},z,\mathcal{S})$. Plugging in the these FOCs, we can rewrite the HJB Equation 26 as following:

$$
\begin{aligned}
&\rho V(b,a,\epsilon,s,z,\mathcal{S}) = u(c(\mathbf{s},z,\mathcal{S})) + \mathbf{g}_{\mathbf{B}}(\mathbf{s},z,\mathcal{S}) + \mathbf{g}_{\mathbf{A}}(\mathbf{s},z,\mathcal{S}) \\
&+ V_b\big(r_b(z,\mathbf{B},\mathbf{A})b + y(s,\epsilon,z) + \bar{T} + \mathcal{T}(z) - c(\mathbf{s},z,\mathcal{S}) - d(\mathbf{s},z,\mathcal{S}) - \chi(d,a)\big) \\
&+ V_a\big(r^a(z,\mathbf{B},\mathbf{A})a + d\big) + V_\epsilon(b,a,\epsilon,s,z,\mathcal{S})(-\xi\epsilon) \\
&+ \lambda_\epsilon \int_{-\infty}^{\infty} \big(V(b,a,x,e,z,\mathcal{S}) - V(b,a,\epsilon,e,z,\mathcal{S})\phi(x)dx\big) \\
&+ \Pi_s(z)\big(Q(z)V(b,a,\epsilon,s',z',\mathcal{S}) + (1-Q(z))V(b,a,\epsilon,s,z',\mathcal{S}) - V(b,a,\epsilon,s',z,\mathcal{S})\big) \\
&+ \Pi_z\big(V(b,a,\epsilon,s,z',\mathcal{S}) - V(b,a,\epsilon,s,z,\mathcal{S})\big)
\end{aligned}
\tag{41}
$$

where $\mathbf{g}_{\mathbf{B}}(\mathbf{s},z,\mathcal{S}) \equiv V_{\mathbf{B}}(\mathbf{s},z,\mathcal{S}) \cdot \big(r^b(z,\mathbf{B},\mathbf{A})B + y(\mathbf{S},\mathbf{E},z,\mathbf{B},\mathbf{A}) + T(z) - \tilde{c}(\mathbf{B},\mathbf{A},\mathbf{E},\mathbf{S},z,\mathcal{S}) - \tilde{d}(\mathbf{B},\mathbf{A},\mathbf{E},\mathbf{S},z,\mathcal{S}) - \chi(\tilde{d},\mathbf{A})\big)$ and $\mathbf{g}_{\mathbf{A}}(\mathbf{s},z,\mathcal{S}) \equiv V_{\mathbf{A}}(\mathbf{s},z,\mathcal{S}) \cdot \big(r^a(z,\mathbf{B},\mathbf{A}) + \tilde{d}(\mathcal{S})\big)$. Here, $\tilde{c}(\mathbf{B},\mathbf{A},\mathbf{E},\mathbf{S},z,\mathcal{S})$ and $\tilde{d}(\mathbf{B},\mathbf{A},\mathbf{E},\mathbf{S},z,\mathcal{S})$ are $N \times 1$ vectors of consumption and deposit decisions of N households in the

[47] For example if $s = u$ and $z = z^R$ then $s' = e$ and $z = z^N$, and vice versa.

economy, where the first four inputs are individual state variables, and the last two inputs are respectively the aggregate state and distributional inputs.[48] Equation 42 can be solved efficiently with an upwind scheme as in Achdou, Han, Lasry *et al.* (2022) with two additional additive terms into the utility function. The only unknown in this equation is $\tilde{c}$ and $\tilde{d}$, but in a symmetric equilibrium these decisions also satisfy the FOCs in 40. Consequently, we can solve the following nested problem:

1. Start with a guess of $\tilde{c}(\mathbf{B}, \mathbf{A}, \mathbf{E}, \mathbf{S}, z, \mathcal{S})$ and $\tilde{d}(\mathbf{B}, \mathbf{A}, \mathbf{E}, \mathbf{S}, z, \mathcal{S})$
2. Solve equation 42 using finite-difference method with the FOCs from 42. Obtain $c(\mathbf{s}, z, \mathcal{S})$ and $d(\mathbf{s}, z, \mathcal{S})$
3. Update $\tilde{c}(\mathbf{B}, \mathbf{A}, \mathbf{E}, \mathbf{S}, z, \mathcal{S}) = c(\mathbf{s}, \mathcal{S})$ and $\tilde{d}(\mathbf{B}, \mathbf{A}, \mathbf{E}, \mathbf{S}, z, \mathcal{S}) = d(\mathbf{s}, \mathcal{S})$
4. Repeat (1)-(3) until convergence in decision rules.

The remaining challenge is to find $V_{\mathbf{A}}$ and $V_{\mathbf{B}}$ that satisfies Equation 39. Abbott and Phan (2024) addresses this by employing a neural network to learn $V(\mathbf{s}, z, \mathcal{S})$ and extract these derivatives. The efficiency of derivative calculations is greatly enhanced by advancements in automatic differentiation. The idea of algorithm is to "discretize" the distributional state space $\mathcal{S}$ by drawing H batches of training data, each contains a $N \times 4$ matrix reflecting holdings of liquid asset b_i, illiquid asset holding a_i, idiosyncratic productivity z_i and employment status s_i for $i = 1, 2, ..., N$ households. The neural network learns the value function over these training data batches, and the variation in distribution across these batches aids the neural network in learning the value function's dependence on the distributions.

The solution algorithm consists of three main steps. The first step is to solve the model without aggregate shock and draw the training data from the deterministic steady-state. In the second step, I solve the "myopic" model with aggregate shocks assuming the derivatives of the value function with respect to distributions are zero.[49] These two steps are served as intermediate steps. In step 3, I learn these distributional derivatives and solve the full model, and repeat until convergence. I also resample the training data from simulating the full model to ensure that the correlations between idiosyncratic variables over the business cycle are preserved.

1. **Initialization: Solve the deterministic-steady state**: The deterministic steady-state model is solved with a standard finite-difference method as in Kaplan, Moll, and Violante (2018) and Ahn, Kaplan, Moll *et al.* (2018). Absent of the aggregate shock, households take price as given and solve the

[48] Explicitly these vectors can be rewritten as:

$$\tilde{c}(\mathbf{B}, \mathbf{A}, \mathbf{E}, \mathbf{S}, z, \mathcal{S}) = \begin{bmatrix} \tilde{c}(b_1, a_1, \epsilon_1, s_1, z, \mathcal{S}) \\ \tilde{c}(b_2, a_2, \epsilon_2, s_2, z, \mathcal{S}) \\ \cdots \\ \tilde{c}(b_N, a_N, \epsilon_N, s_N, z, \mathcal{S}) \end{bmatrix}; \ \tilde{d}(\mathbf{B}, \mathbf{A}, \mathbf{E}, \mathbf{S}, z, \mathcal{S}) = \begin{bmatrix} \tilde{d}(b_1, a_1, \epsilon_1, s_1, z, \mathcal{S}) \\ \tilde{d}(b_2, a_2, \epsilon_2, s_2, z, \mathcal{S}) \\ \cdots \\ \tilde{d}(b_N, a_N, \epsilon_N, s_N, z, \mathcal{S}) \end{bmatrix} \tag{42}$$

[49] This step is analogous to assuming that the coefficients on the law-of-motion for aggregate capital are zeros in the Krusell and Smith (1998) method

following problem:

$$\begin{aligned}
&\max_{c_t,d_t} \mathbb{E}\int_0^\infty e^{-\rho t}u\big(c_t\big)dt \\
\text{s.t: } &\dot{b_t} = r_b b_t + y(s_t,\epsilon_t) + \bar{T} - c_t - d_t - \chi(d_t,a_t) \\
&\dot{a_t} = r^a a_t + d_t \\
&y(s_t,\epsilon_t,z_t) = \begin{cases} (1-\tau)\epsilon_t w & ,s_t = e \\ u_b & ,s_t = u \end{cases}
\end{aligned} \tag{43}$$

Let $g(b,a,\epsilon,s)$ be the steady-state distribution over the state variables. The market clearing condition for capital is:

$$K = \int_b\int_a\int_\epsilon \sum_{s=\{e,u\}} (b+a)g(b,a,\epsilon,s)dbdad\epsilon \tag{44}$$

The deterministic steady-state is solved when the prices that households take when solving 43 are also market-clearing.

I discretize the grid for liquid and illiquid assets and idiosyncratic shocks into $I = 86$, $J = 40$, and $M = 5$, respectively. With the upwind scheme the system of HJB equations can be written compactly in the matrix form as:

$$\rho\mathbf{v} = \mathbf{u}(\mathbf{v}) + \big(\mathbb{B}(\mathbf{v}) + \mathbb{A}(\mathbf{v}) + \mathbb{E} + \mathbb{S}\big)\mathbf{v} \tag{45}$$

where each matrix in the second term of the RHS equation is a discrete Poisson process summarizing how associated state variables evolves. Let π_s be the transition matrix for employment status:

$$\pi_s = \begin{bmatrix} -\lambda & \lambda \\ \sigma & -\sigma \end{bmatrix} \tag{46}$$

For the ordering of the state variable (b,a,ϵ,s), the transition matrix in the HJB is then:

$$\mathbb{S} = \Big[\pi_s \bigotimes \mathbf{I}_{I\cdot J\cdot M}\Big] = \begin{bmatrix} -\lambda\mathbf{I}_{I\cdot J\cdot M} & \lambda\mathbf{I}_{I\cdot J\cdot M} \\ \sigma\mathbf{I}_{I\cdot J\cdot M} & -\sigma\mathbf{I}_{I\cdot J\cdot M} \end{bmatrix} \tag{47}$$

where $\mathbf{I}$ is a $(I\cdot J\cdot M)\times(I\cdot J\cdot M)$ identity matrix and $\bigotimes$ is the Kronecker product. Likewise:

$$\mathbb{E} = \Big[\mathbf{I}_2 \bigotimes \Big(\pi_\epsilon \bigotimes \mathbf{I}_{I\cdot J}\Big)\Big] = \begin{bmatrix} \pi_\epsilon \bigotimes \mathbf{I}_{I\cdot J} & \mathbf{O}_{I\cdot J\cdot M} \\ \mathbf{O}_{I\cdot J\cdot M} & \pi_\epsilon \bigotimes \mathbf{I}_{I\cdot J} \end{bmatrix} \tag{48}$$

where π_ϵ is the Poisson intensity matrix in Equation 32 reflecting the jump-drift process, and $\mathbf{O}_{I\cdot J\cdot M}$ is a zero matrix of size $(I\cdot J\cdot M)\times(I\cdot J\cdot M)$. Lastly, the "split the drift" scheme is used to construct the drifts for liquid asset $\mathbb{B}(\mathbf{v})$ and illiquid asset $\mathbb{A}(\mathbf{v})$ as in the Fortran code of Kaplan, Moll, and Violante (2018).

Draw training data: once we obtain the steady-state distribution $g^{dss}(b,a,\epsilon,s)$, we sample H

batches the training data from this distribution. Each batch, indexed by S^h is a matrix

$$\begin{bmatrix} b_1 & a_1 & z_1 & s_1 \\ b_2 & a_2 & z_2 & s_2 \\ \vdots & \vdots & \vdots & \vdots \\ b_N & a_N & z_N & s_N \end{bmatrix} \tag{49}$$

of the joint distribution of idiosyncratic state variables.

2. **Initialization: Solve the "myopic" model with aggregate shock**: the household problem in the model with aggregate shock assuming that households do not internalize the saving decisions of other households is:

$$\begin{aligned} &\max_{c_t, d_t} \mathbb{E} \int_0^\infty e^{-\rho t} u\big(c_t\big) dt \\ \text{s.t: } &\dot{b_t} = r_b(z_t, \mathbf{B}_t, \mathbf{A}_t) b_t + y(s_t, \epsilon_t, z_t) + \bar{T} + \mathcal{T}(z_t) - c_t - d_t - \chi(d_t, a_t) \\ &\dot{a}_t = r^a(z_t, \mathbf{B}_t, \mathbf{A}_t) a_t + d_t \\ &y(s_t, \epsilon_t, z_t) = \begin{cases} (1-\tau)\epsilon_t w(z_t, \mathbf{B}_t, \mathbf{A}_t) & , s_t = e \\ u_b & , s_t = u \end{cases} \\ &r_b(z_t, \mathbf{B}_t, \mathbf{A}_t) = r^b(z_t, \mathbf{B}_t, \mathbf{A}_t) \cdot \mathcal{I}\{\mathbf{B}_t \geq 0\} + (r^b(z_t, \mathbf{B}_t, \mathbf{A}_t) + \kappa) \cdot \mathcal{I}\{\mathbf{B}_t < 0\} \\ &r^b(z_t, \mathbf{B}_t, \mathbf{A}_t) = r^a(z_t, \mathbf{B}_t, \mathbf{A}_t) - \phi \end{aligned} \tag{50}$$

Similar to 4, now both aggregate shock and distributions become state variables. The former is a Two-state Poisson process therefore can be discretized in the same manner as the process for employment risk. The latter, as argued above, will be "discretized" by H batches of training data. Let $\mathbf{v}_h$ index the value function over the individual state space (b, a, ϵ, s, z) for each sample of the training data index by $h = 1, 2, ..., H$. The HJB equation in matrix form is:

$$\rho \mathbf{v}_h = \mathbf{u}(\mathbf{v}_h) + \big(\mathbb{B}(\mathbf{v}_h) + \mathbb{A}(\mathbf{v}_h) + \mathbb{E} + \mathbb{S} + \mathbb{Z}\big)\mathbf{v}_h \tag{51}$$

The exogenous transition matrices now account for an extra dimension:

$$\mathbb{E} = \left[\mathbf{I}_4 \bigotimes \left(\pi_\epsilon \bigotimes \mathbf{I}_{I \cdot J}\right)\right] \tag{52}$$

The transition matrix for the aggregate state is:

$$\mathbb{Z} = \left[\pi_z \bigotimes \mathbf{I}_{I \cdot J \cdot M \cdot 2}\right] \tag{53}$$

Finally, the transition matrix for employment status now also accounts for the fact that when there is a switch in the aggregate state, there will be a jump in employment/unemployment, as discussed in

Section A:

$$\mathbb{S} = \begin{bmatrix} -\lambda & \lambda & \beta_l(1-Q_{ue}) & \beta_l Q_{ue} \\ \sigma & -\sigma & 0 & 0 \\ 0 & 0 & -\lambda & \lambda \\ \beta_h Q_{eu} & (1-\beta_h)Q_{eu} & \sigma & -\sigma \end{bmatrix} \bigotimes \mathbf{I}_{I\cdot J\cdot M} \tag{54}$$

Households internalize this extra risk in their value function and decision making, therefore value insurance more.

For each h, Equation 51 can easily be solved with a finite-difference scheme. Parallelization over h can be implemented to speed up the computation for this step.

3. **Solve the full model**: the discretized HJB equation for the full model, as in Equation 39, can be written as:

$$\rho \mathbf{v}_h = \left(\mathbf{u}(\mathbf{v}_h) + \mathbf{v}_{\mathbf{B}} \cdot \dot{\mathbf{B}}_h + \mathbf{v}_{\mathbf{A}} \cdot \dot{\mathbf{A}}_h\right) + \left(\mathbb{B}(\mathbf{v}_h) + \mathbb{A}(\mathbf{v}_h) + \mathbb{E} + \mathbb{S} + \mathbb{Z}\right)\mathbf{v}_h \tag{55}$$

The main challenge is to find the value function such that its distributional derivatives satisfy Equation 55. This step requires iteration. Let k index iterations for the "implicit method". We have:

$$\frac{\mathbf{v}_h^{k+1} - \mathbf{v}_h^k}{\Delta} + \rho \mathbf{v}_h^{k+1} = \left(\mathbf{u}(\mathbf{v}_h^k) + \mathbf{v}_{\mathbf{B}}^k \cdot \dot{\mathbf{B}}_h + \mathbf{v}_{\mathbf{A}}^k \cdot \dot{\mathbf{A}}_h\right) + \left(\mathbb{B}(\mathbf{v}_h^k) + \mathbb{A}(\mathbf{v}_h^k) + \mathbb{E} + \mathbb{S} + \mathbb{Z}\right)\mathbf{v}_h^{k+1} \tag{56}$$

with the understanding that $\mathbf{v}_{\mathbf{A}}^k$ and $\mathbf{v}_{\mathbf{B}}^k$ are distributional derivatives of the value function indexed $\mathbf{v}^k$. The initial value function is taken from solution in Step 2.

Learn $\mathbf{v}_{\mathbf{A}}$ and $\mathbf{v}_{\mathbf{B}}$: I train the neural network Θ to learn $\mathbf{v}$ from the variation of the training data. Because the dynamic programming is symmetric, Kahou, Fernández-Villaverde, Perla *et al.* (2021) shows that the value function can be represented by:

$$V(b, a, \epsilon, s, z, \mathbf{B}, \mathbf{A}, \mathbf{E}, \mathbf{S}) = \rho\left(b, a, \epsilon, s, z, \frac{1}{N}\sum_{i=1}^{N} \phi(\mathbf{B}_i, \mathbf{A}_i, \mathbf{E}_i, \mathbf{S}_i)\right) \tag{57}$$

where $\rho : \mathbb{R}^{L+5} \to \mathbb{R}$ with $L \leq N$, and $\phi : \mathbb{R}^4 \to \mathbb{R}^L$. Here, ρ and ϕ are both neural networks, with potentially different number of neurons and layers. Intuitively, the ϕ layer compacts the relevant information of the distributions into L moments that households need to know in making decisions. The variation in the training data helps to learn ϕ. The neural network Θ is then the composite of ϕ and ρ networks.

Let Θ^k index the neural network that have learned $\mathbf{v}^k$. The associated distributional derivatives indexed $\mathbf{v}_{\mathbf{A}}^k$ and $\mathbf{v}_{\mathbf{B}}^k$ can be easily extracted from the neural network. I can then solve 56 to obtain $\mathbf{v}^{k+1}$. As before, parallel over h can be utilized to speed up this step. I repeat this step until Θ^k well approximates $\mathbf{v}^{k+1}$.

I set $N = 10000$ and draw 150 batches of training data. I set $L = 5$. The ϕ network has 3 layers and

64 neurons. The ρ network has 3 layers and 128 neurons. Both networks use $tanh$ as activation functions. Adaptive moment estimation (ADAM) is used as optimization algorithm. The value function is standardized before training. To ensure that I solve the model that are close to the data generated by model, I resample the training data from simulated data two times.

The full algorithm is as follows:

- **Initialization**:
  1. Discretize the individual state variable (b, a, ϵ, s) into a grid with $(I \times J \times M \times 2)$
  2. Solve the deterministic steady-state 43 with the finite-difference method. Obtain the deterministic steady-state distribution g^{dss}.
  3. Draw H batches of training data from g^{dss}.
  4. For each $h \in \{1, 2, ..., H\}$, solve the HJB Equation 51 on the grid of $(I \times J \times M \times 2 \times 2)$ points corresponding to the state space (b, a, ϵ, s, z). Obtain the initial value function $\mathbf{v}_h^0$.
- **Iteration**: for $k = 0, 1, 2...$
  5. Construct a neural network Θ^k with two layers ρ and ϕ.
  6. Train the neural network over H batches of training data to learn $\mathbf{v}_h^k$.
  7. Extract derivatives $\mathbf{v}_{\mathbf{A}}^k$ and $\mathbf{v}_{\mathbf{B}}^k$.
  8. For each $h = \{1, 2, ..., H\}$: solve Equation 56 with the finite-difference method to get $\mathbf{v}_h^{k+1}$.
  9. Repeat (5)-(8) until Θ^k well approximates $\mathbf{v}_h^{k+1}$.
- **Resampling**: I resample the training data by simulating model to ensure that we train the value function on the regions where households "live".

Upon solving 56, I obtain the decision rules $c(b, a, \epsilon, s, z, \mathbf{B}, \mathbf{A}, \mathbf{E}, \mathbf{S})$ and $d(b, a, \epsilon, s, z, \mathbf{B}, \mathbf{A}, \mathbf{E}, \mathbf{S})$. I then employ another neural network to learn these decision rules over the same training data used to solve the value function:

$$
\begin{aligned}
c(b, a, \epsilon, s, z, \mathbf{B}, \mathbf{A}, \mathbf{E}, \mathbf{S}) &= \rho^C \left(b, a, \epsilon, s, z, \frac{1}{N} \sum_{i=1}^{N} \phi^{CD}(\mathbf{B}_i, \mathbf{A}_i, \mathbf{E}_i, \mathbf{S}_i) \right) \\
d(b, a, \epsilon, s, z, \mathbf{B}, \mathbf{A}, \mathbf{E}, \mathbf{S}) &= \rho^D \left(b, a, \epsilon, s, z, \frac{1}{N} \sum_{i=1}^{N} \phi^{CD}(\mathbf{B}_i, \mathbf{A}_i, \mathbf{E}_i, \mathbf{S}_i) \right)
\end{aligned}
\tag{58}
$$

The composite neural network consists of three network layers. $\phi^{CD} : \mathbb{R}^4 \rightarrow$ that maps distributions into relevant information for decision makings for both consumption and deposit. $\phi^C : \mathbb{R}^{L+5} \rightarrow \mathbb{R}$ and $\phi^D : \mathbb{R}^{L+5} \rightarrow \mathbb{R}$ learns the consumption and deposit decisions over the individual state variables and L moments of the distribution. These decision rules are then used to simulate saving decisions in Equation 33.

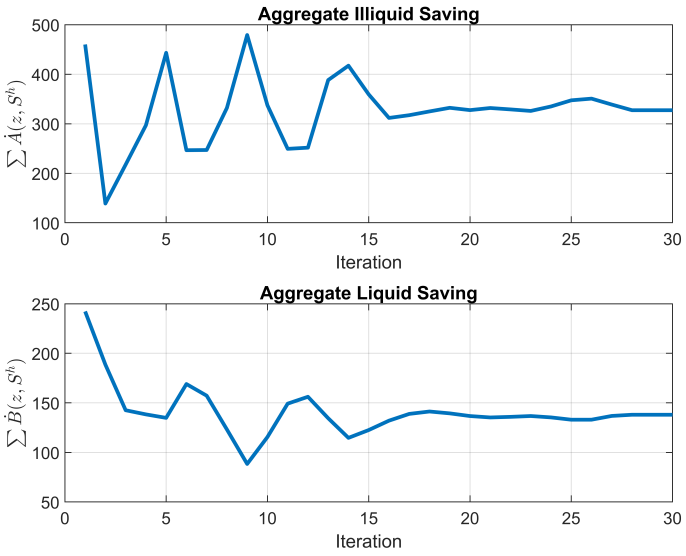


Figure 11: Aggregate Savings over iterations at a particular batch of training data.

Finally, calculating CEV requires evaluating the value functions on the distributions, which is done on the neural network for value function Θ.

Convergence of the value function: the value function in this model is a high-dimensional object, featuring non-linearity in state variables. Furthermore, two continuous choices (the consumption and deposit decisions) add an additional layer of complexity compared to the standard Krusell and Smith (1998) model. Nevertheless, the algorithm achieves convergence in only 30 iterations.

An intuitive way to examine the convergence in the value function is to look at the convergence in the aggregate saving decisions for a given aggregate state and distribution of assets. First, define:

$$\begin{aligned} \sum \dot{B}(z, S^h) &= \sum_{i=1}^{N} \dot{b}(b_i, a_i, \epsilon_i, s_i, z, S^h) \\ \sum \dot{A}(z, S^h) &= \sum_{i=1}^{N} \dot{a}(b_i, a_i, \epsilon_i, s_i, z, S^h) \end{aligned} \tag{59}$$

These are the aggregate liquid and illiquid savings evaluated at the aggregate state z and batch h of training data.[50] Figure 11 plots these at the normal state z^N and for a distribution of liquid and illiquid assets that correpond to the state of low capital stock. For example, the economy has just experienced a series of recessions in which households draw down assets. As the economy recovers, households will now reaccumulate stocks of assets, therefore savings in both type of assets are positive. As discussed above, we intialize the first iteration by assuming that one make decisions based on myopic expectation, meaning they do not internalize that other households will also stock up their savings. In turn, they *overpredict* the future interest

[50]Notice that these saving decisions can be recovered from the consumption and deposit decisions from Equation 33

rates, and save too much. Starting from the second iteration onwards, one will not save as much as they internalize the decisions of other households in the economy and understand that interest rates will not be as high. These predictions are updated over iterations until there is convergence in decision rules, in turns implying convergence in value function.

D Benabou-Floden decomposition

The Benabou-Floden decomposition requires to calculate the individual certainty equivalent consumption $\bar{c}(\mathbf{s}_{it}, \mathcal{S})$ for each individual at each point in time:

$$\begin{aligned} \frac{\bar{c}(\mathbf{s}_{it}, z_t, \mathcal{S}_t)^{1-\gamma}}{1-\gamma} &= \rho V(\mathbf{s}_{it}, z_t, \mathcal{S}_t) \\ \iff \bar{c}(\mathbf{s}_{it}, z_t, \mathcal{S}_t) &= \Big(\rho V(\mathbf{s}_{it}, z_t, \mathcal{S}_t)(1-\gamma)\Big)^{\frac{1}{1-\gamma}} \end{aligned} \tag{60}$$

$\bar{c}(\mathbf{s}_{it}, z_t, \mathcal{S}_t)$ can be interpreted as level of consumption that makes a household with individual state $\mathbf{s}_{it}$ at the distribution $\mathcal{S}_t$ as well off as facing both individual and aggregate uncertainty about his value function in the stochastic economy. The average certainty-equivalence consumption is then:

$$\bar{\mathcal{C}}^{avg} = \frac{1}{T}\frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\bar{c}(\mathbf{s}_{it}, z_t, \mathcal{S}_t) \tag{61}$$

The price of uncertainty is:

$$P^{unc} = 1 - \frac{\bar{\mathcal{C}}^{avg}}{C^{avg}} \tag{62}$$

This is how much average consumption this economy is willing to forego to get certainty. We can calculate this price for the economy with countercyclical and acyclical plicies then calculate the uncertainty effect:

$$\omega^{unc} = \frac{1 - P^{unc}_{\text{c.cyclical}}}{1 - P^{unc}_{\text{acyclical}}} - 1 \tag{63}$$

The level effect is the ratio of average consumption between the economy with and without policy:

$$\omega^{lev} = \frac{C^{avg}_{\text{c.cyclical}}}{C^{avg}_{\text{acyclical}}} - 1 \tag{64}$$

Finally, the inequality effect is:

$$(1 + \omega_{ine}) = \frac{(1+\omega)}{(1+\omega_{unc})(1+\omega_{lev})} \tag{65}$$

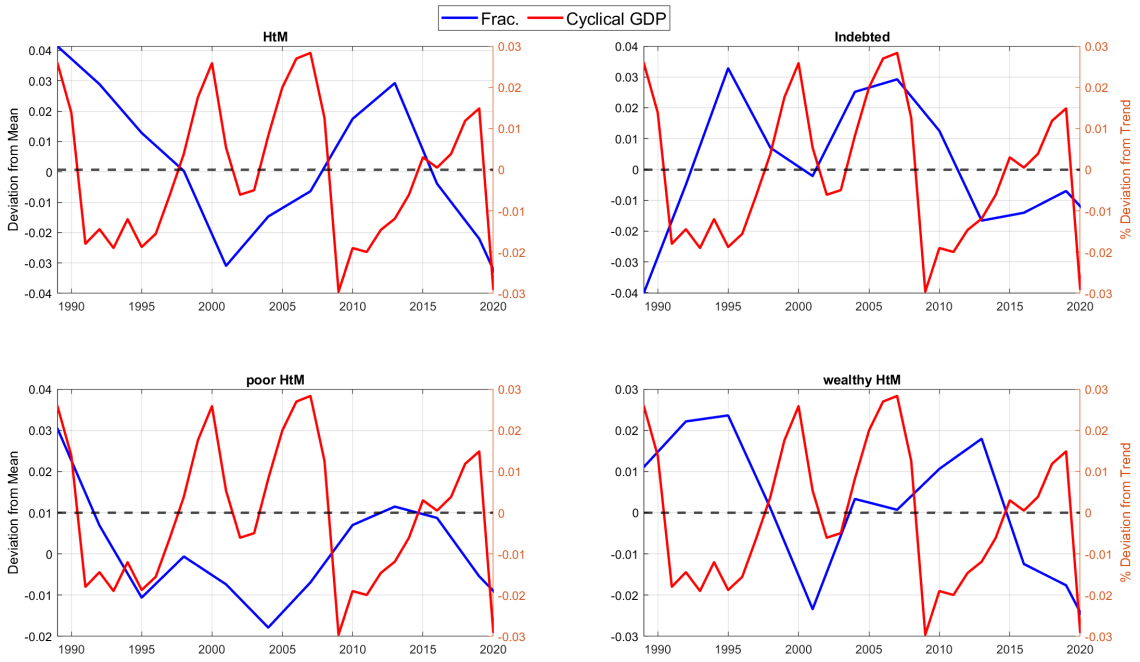


Figure 12: Fraction of HtM and Indebted households over the Business Cycle

E HtM and Indebted over the Business Cycle in data

I calculate the fraction of demeaned HtM (including poor HtM and wealthy HtM), and Indebted households from the SCF 1989 to 2022, and plot it again the cyclical component of HP-filtered GDP series in the U.S. The definition of liquid and illiquid assets similarly follow Kaplan and Violante (2022). The SCF data is deflated with 2016 being the base year. Figure 12 illustrates that the fraction of HtM and wealthy HtM are strongly countercyclical in the data. The fraction of indebted households is weaky countercyclical. The correlations between fractions of HtM, Indebted, poor HtM and wealthy HtM to the cyclical component of GDP are -0.30, -0.49, 0.06 and -0.20, respectively.