Optimal Informality*

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Abstract

What is the optimal size of the informal sector? We address this question using a mechanism design approach applied to rich administrative data from Brazil. We argue that informality presents a fundamental trade-off: while it provides a safety net for low-income workers, it also creates opportunities for tax evasion, which constrain the design of transfer policies as in Saez 2002. Our analysis reveals that the socially optimal informality rate in Brazil is 14%, exhibiting a hump-shaped pattern across the income distribution. General equilibrium reduces the optimal informality rate by raising informal efficiency wages as more individuals enter the formal sector. In contrast, alternative policies like universal basic income (UBI) increase optimal informality by reducing the necessity for low-income workers to formalize in order to receive Government transfers. These results provide novel insights into the design of tax and transfer policies in economies with large informal sectors.

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1 Introduction

Informality is a widespread phenomenon, particularly in developing countries. In Brazil, for instance, 45% of the labor force is employed in informal jobs (PNAD 2017). Several studies have highlighted benefits and costs of the informal sector (Dix-Carneiro et al. 2021). A key challenge posed by informality is its constraint on the government's ability to implement social safety nets and redistribute income effectively. For example, incorporating an informal sector participation constraint to the Saez 2002 framework restricts the set of feasible allocations that can be achieved with in-work benefits, such as the Earned Income Tax Credit (EITC). On the other hand, informality serves as a buffer, providing shelter for low-income workers who are unable to secure formal employment (Dix-Carneiro and Kovak 2019). This trade-off raises an important and understudied question: What is the optimal size of the informal sector?

In this paper, we address this question by introducing a fresh perspective that emphasizes worker heterogeneity within a mechanism design approach, applied to rich administrative data from Brazil. The key element of our model is that workers sort into informality based comparative advantages in productivity while accounting for the influence of general equilibrium forces and institutions, such as the income tax schedule and transfer policies. The optimal level of informality ultimately hinges on the distribution of worker productivity. For instance, as within-sector income inequality rises and the cross-sector correlation of productivity strengthens, reducing informality through generous in-work benefits, such as the EITC, becomes more effective. This occurs because high-income earners are sufficiently distant from lower-income individuals, making it impractical for them to manipulate their earnings to qualify for transfers. Our analysis reveals that the optimal informality rate in Brazil is 14%, a striking 70% lower than the current observed rate.

We begin by presenting empirical evidence that highlights substantial overlap in productivities across sectors. As in Maloney 2004, the data also reveals that informality in Brazil, mostly takes the form of self-employment. We incorporate these facts in a model where firms and workers are often indistinguishable, and individuals possess both formal and informal productivities, which are private information.[1] This framework is

[1] In Appendix D, we provide microfoundations for this assumption using a task-based approach. Specifically, we consider sector-specific production functions that rely on different intensities of two types of worker abilities: physical and intellectual.

represented by a two-dimensional productivity type drawn from a joint distribution. Importantly, our model allows for any degree of correlation between formal and informal productivities. Workers choose the sector that maximizes their utility, taking into account the prevailing tax/transfer scheme and their relative productivities in each sector, as in Roy 1951.

Designing optimal taxation in the presence of informality is challenging for two main reasons. First, sectoral choice is endogenous, as individuals may opt out of formal work based on the tax schedule. To address this, we solve a mechanism design problem that incorporates a participation constraint alongside the usual incentive compatibility constraint. This modified Mirrlees problem highlights the role of informality in shaping optimal taxation. Second, workers' productivity is multidimensional, complicating the implementation of a screening mechanism that accounts for both formal and informal abilities. We assume the planner observes both formal-sector income and the worker's sector choice, using the latter as an indirect signal of productivity in the other sector. This enables an additional screening tool to refine taxation strategies. Our solution shows that optimal marginal tax rates can be up to 15% lower in the presence of informality to prevent workers from taking advantage of evasion opportunities.

In the mechanism design tradition, optimal taxation aims to maximize social welfare. In our framework, this principle holds, however, informality adds a layer of complexity by influencing the design of the optimal tax schedule. The tax schedule, in turn, shapes workers' sectoral choices. The government can reduce informality by offering in-work benefits, such as the Earned Income Tax Credit (EITC), to formal workers, providing stronger incentives for low-income individuals to enter the formal sector. However, overly generous transfers can create perverse incentives for higher-skilled workers to reduce their labor supply and mimic low-skilled workers to qualify for these benefits. These distortions, known as informational rents, constrain the planner's ability to fully eliminate informality. Balancing these trade-offs is central to the planner's optimization problem, which ultimately determines the socially optimal size of the informal sector.

Computing the optimal informality rate involves a recursive process, as informality affects the optimal tax schedule, and the tax schedule in turn influences informality. We address this by taking a fixed-point approach, iteratively solving the mechanism design problem until convergence. Starting with a parametrized tax function, individuals choose sectors. Based on these choices, we re-optimize the tax system to maximize social wel-

fare. We repeat this process until the system converges. Applied to our data, this method suggests that the optimal informality rate in Brazil should be 14%. Interestingly, while observed informality decreases monotonically with productivity, the optimal informality exhibits an inverted U-shape. This reflects the planner's preference for formalizing low-skilled workers, enabling redistribution through in-work benefits, while also formalizing high-skilled workers to prevent significant revenue losses.

Our solution to the optimal size of the informal sector is shaped by tax evasion opportunities and the inability to redistribute income to individuals outside the formal system. In our framework, for the planner to transfer resources to informal workers, they must first incentivize these workers to enter the formal sector. This raises a natural question: How would welfare and optimal informality change if the Government could directly subsidize informal workers through universal basic income? UBI has recently regained prominence in policy discussions (Hoynes and Rothstein 2018; Daruich and Fernández 2020), particularly due to its straightforward implementation, which does not require observing individuals' income — an especially attractive feature for economies with large informal sectors. Our findings suggest that optimal informality rate rises in the presence of UBI.

When the income tax schedule changes, individuals re-optimize their sector choice, leading to general equilibrium effects with welfare implications. For instance, lowering income taxes attracts informal workers to the formal sector, increasing the supply of formal relative to informal labor. This reallocation raises efficiency wages in the informal sector, ultimately improving welfare for informal workers. To incorporate these general equilibrium effects into the Mirrleesian framework, we extend the approach of Rothschild and Scheuer 2013, which analyzes optimal taxation in a two-sector economy. A key distinction in our setting is that in one of the sectors —informality— the government cannot observe income at all. Our findings show that general equilibrium effects reduce the optimal informality rate and act as an indirect mechanism for transferring resources to informal workers, thereby increasing overall social welfare. This result supports the idea that formal tax policy can generate a positive productivity externality for workers who remain informal by increasing their relative productivity as the informal sector shrinks.

In the realm of informality and tax-related policies, a recurrent point in the debate is the use of enforcement as a tool to regulate the size of the informal sector (IMF 2020). While policymakers often advocate for eradicating informality as a means of improving

welfare, we have little evidence to support or refute this argument. We see the enforcement agenda as presenting a nuanced trade-off in the landscape of informality. On the one hand, eliminating the informal sector restricts workers' choices, reducing welfare for some individuals. On the other hand, it also eases constraints on the planner's problem, expanding the set of feasible policies. In the final part of the paper, we accommodate enforcement policies to our machinery, and ask the question: What are the welfare implications of shutting down the informal sector?

To answer this, we examine two extreme scenarios. In the first, informality serves as a survival mechanism, where certain economic activities exist only because they avoid the costs of formality. In this case, eliminating informality forces workers to operate with formal-sector productivity. In the second scenario, informality represents a profitable alternative, where economic activities continue under formalization but at reduced profitability. Here, eliminating informality does not affect productivity. Overall, our findings suggest that enforcing the elimination of informality is a regressive policy, mostly deteriorating the life of low-skill workers. This result underscores that informality may constrain the planner's ability to design effective social welfare policies.

Related Literature. This paper bridges the literatures on informality and optimal taxation. First, we build on the broad informality literature, which has traditionally emphasized the role of firms (e.g., La Porta and Shleifer 2008; de Paula and Scheinkman 2010; Meghir et al. 2015; Ulyssea 2018; Dix-Carneiro et al. 2021). In contrast, we examine informality from the perspective of workers. This shift in focus enables us to address a key normative question: how informality affects individual well-being? Additionally, this paper offers a novel angle to the informality literature, enhancing our understanding of the desirability of an informal sector.

Second, we explore the relationship between informality and the design of social welfare policies. Similar to Saez 2002, who examines the conditions under which in-work benefits are optimal, we build on the mechanism design framework pioneered by Mirrlees 1971b and Saez 2001 to incorporate the role of informality in shaping the design of social welfare programs. Our work also relates to Bachas et al. 2024, who provide empirical evidence that low-income households are more likely to purchase goods and services from the informal economy, with implications for the design of consumption taxes. While their study focuses on consumption, we develop a comprehensive theoretical framework that

integrates labor choices across sectors, the optimal design of income taxes, and the underlying primitives that justify the existence of a larger informal sector. In Appendix E we can take into account the type of evidence uncovered by Bachas et al. 2024, in a setting where we allow for sector-specific consumption goods.

Methodologically, our paper contributes to the literature on optimal taxation with extensive margin decisions, including migration (Lehmann et al. 2013), sectoral choice (Rothschild and Scheuer 2013; Gomes et al. 2017), and the shadow economy (Doligalski and Rojas 2023; Huang and Rios 2016). We add by leveraging the mechanism design approach not only to characterize the optimal tax structure but also to determine the optimal size of the informal sector. We differentiate ourselves from Rothschild and Scheuer 2013 by solving the bi-dimensional screening with different tax schedules across sectors, and from Gomes et al. 2017 by allowing for income effects and considering the general equilibrium impacts of policy and some other key institutional features such as universal basic income. Finally, our work contributes to the literature on UBI in redistributive policy (see Daruich and Fernández 2020 for a review). We contribute to this discussion by exploring how UBI can coexist with an optimally designed tax-and-transfer system, providing new insights into its role in shaping the optimal size of the informal sector.

The rest of the paper is organized as follows. Section 2 presents the data and some empirical motivating facts. Section 3 outlines the model. Section 4 connects model and data. Section 5 discusses optimal policies under the presence of informality, including optimal taxation, optimal size of the informal economy with universal basic income and general equilibrium. Section 6 presents counterfactual exercises to understand the social cost of informality in the context of enforcement policies. Finally, Section 7 concludes.

2 Data and Empirical Motivation

Informality, by definition, represents the less visible side of the economy, where data is inherently challenging to observe. We leverage detailed survey conducted by the Brazilian National Statistics Bureau (IBGE), *Pesquisa Mensal de Emprego (PME)*, which tracks individuals over time across six major metropolitan regions. This labor survey interviews workers for four consecutive months and then resumes interviews for another four consecutive months one year after their initial participation. This longitudinal design provides a rich dataset that captures not only informal labor income and workers' background char-

acteristics – such as education and region of residence – but also transitions into and out of informal employment. The sample ranges from 2002 to 2007, which is a relatively stable period of the Brazilian economy.

We adopt the sample restrictions outlined in Meghir et al. 2015. Specifically, we limit the sample to workers aged 23 to 65 who are either employed or unemployed, excluding inactive individuals. Workers are classified as formal if they have a signed labor card; otherwise, they are considered informal. We further exclude formal workers earning less than the minimum wage, as these cases likely represent apprentices, part-time workers, or potential data errors. These exclusions account for approximately 8% of formal workers. Given that our model accounts for worker heterogeneity, we present descriptive statistics by workers' educational levels. Table 1 categorizes workers into low, medium, or high skill levels based on the annual terciles of years of education.

In terms of earnings, Table 1 highlights two key findings. First, income increases with educational attainment, both in the formal and informal sectors. More educated workers are less likely to experience unemployment and are more likely to be formalized. Second, within any given educational group, earnings tend to be higher in the formal sector. Notice, however, that based on the standard deviations there is substantial overlap in productivities across sectors. Table 1 also reveals notable transition patterns among workers who did not remain in the same job between interview cycles. For workers with lower levels of education, informality appears to act as a safety net, helping them transition out of unemployment.

These findings are consistent with a nuanced view of informality that combines elements from the two sides of the spectrum. Magnac 1991 discusses two prevailing views of informality: a segmented labor market, which excludes less productive workers due to legal constraints, and a competitive view, where workers self-select into informality based on comparative advantages. On one hand, we find a significant overlap in worker productivity across sectors, challenging the purely segmented view. On the other hand, we observe that the lowest-skilled workers are disproportionately reliant on informal employment, suggesting some degree of segmentation at the bottom of the skill distribution.

Our theoretical framework enables us to study an economy within this nuanced view, capturing both key empirical patterns: the observed overlap in productivities and the greater prevalence of informality among low-skilled workers.

Table 1: Descriptives Statistics

	Workers' Education		
	Low	Med	High
Number of individuals			
Unemployed	23,824	26,040	6,377
Formal	99,201	128,294	55,506
Informal	117,170	75,796	34,633
Formality rate	0.41	0.56	0.58
Transitions			
Unemployed to Formal	0.08	0.11	0.10
Unemployed to Informal	0.32	0.22	0.22
Formal to Formal	0.02	0.02	0.02
Formal to Unemployed	0.02	0.02	0.01
Formal to Informal	0.01	0.00	0.00
Informal to Informal	0.06	0.04	0.03
Informal to Unemployed	0.04	0.04	0.03
Informal to Formal	0.01	0.01	0.00
Log (Wages)			
Formal Sector			
Mean	6.46	6.65	7.44
SD	0.43	0.54	0.75
Observations	83,979	113,492	49,268
Informal Sector			
Mean	6.09	6.40	7.13
SD	0.61	0.73	0.88
Observations	98,748	65,521	29,656

Note: This table presents descriptive statistics from PME (2002–2007) for workers across different educational levels. The categories *(low, med, high)* correspond to annual terciles of education, where education is measured in years. Transition rates reflect the proportion of workers moving between unemployment, formal, and informal sectors across interview cycles. Formality rates are calculated as the share of formal workers within each education group. Wage statistics are reported in logs and include the mean and standard deviation.

3 The Economy

Preliminaries. The economy is inhabited by a continuum of agents homogeneous with regards to preferences defined over consumption, c, and effort, l, represented by $\mathcal{U}(c,l)$, a smooth strictly concave function increasing in c and decreasing in l. The economy is divided into two sectors, a formal and an informal one indexed by n and m, respectively. Letting N_n and H_m denote the total supply of efficiency hours in the formal and informal sectors, GDP is given by $H(N_n, N_m)$, for $H(\cdot,\cdot)$ a strictly concave constant returns to scale (CRS) function. We further assume that $\lim_{N_m \to 0} H(N_n, N_m) = AN_n$ and $\lim_{N_n \to 0} H(N_n, N_m) = AN_m$. As our prime example, consider

$$H(N_n, N_m) = A\left\{(1-a)N_m^{1-\frac{1}{\varepsilon}} + aN_n^{1-\frac{1}{\varepsilon}}\right\}^{\frac{\varepsilon}{\varepsilon-1}},$$

for $\varepsilon > 1$. Our simplified model, which we sometimes refer to as Partial Equilibrium (PE) is the limit $\varepsilon \to \infty$. Due to the CRS assumption we can write $GDP = N_n\Phi(r)$ for $r = N_m/N_n$ and $\Phi(r) = N(1,r)$. We also define $\omega(r) = \partial H(1,r)/\partial N_m$ and $\phi(r) = \partial H(1,r)/\partial N_n = \Phi(r) - \omega(r)r$. As $\varepsilon \to \infty$, the marginal productivities (wage per efficiency unit) converge to Aa in the formal sector and $(1-a)A$ in the informal sector.[2]

Productivity. The two sectors use different combinations of workers' skills. Institutional and technological differences between the sectors will usually lead to differences in the relative value of different skills.[3] We capture these differences and any other work-specific cost or benefit of participating in a given sector, by assuming that each agent's set of skills translates into bi-dimensional productivity, (n,m): the formal sector productivity, n, and the informal sector productivity, m. This reduced form set of parameters, akin to Gomes et al.'s 2017 two-sector model formulation, captures the fundamental joint distribution of skills, which is a key factor in the sector's endogenous formation. Formally, we denote the joint distribution of productivities n and m, by $F(n,m)$. With some abuse, we let $g(m)$ denote the marginal density of m, $g(m|n)$ the conditional (on n) density, $f(n)$ and $f(n|m)$ the analogous definitions for n.

[2] In our calibration, we choose the normalization of $\omega(r) = 1$, and $a = 0.5$, which implies $A = 2$, since $\omega(r) = \partial H(1,r)/\partial N_m = A\,[1-a]\,r^{-\frac{1}{\varepsilon}}\left\{\left[a + (1-a)r^{1-\frac{1}{\varepsilon}}\right]\right\}^{\frac{1}{\varepsilon-1}}$.

[3] This mapping is explained in Appendix D.

Information set. Workers productivity is private information. The Government knows the joint distribution $F(n, m)$ and can costlessly observe income earned in the formal sector. Motivated by the real-world intrinsic limitation to eliciting information from the shadow economy, we do not allow the government to observe agent's earnings in the informal sector.

Government policy. For the baseline, we consider a progressive tax/transfer system summarized by a constant progressivity (HSV) schedule that applies to formal earnings. This leads to a $c \leq \xi z^{1-\iota}$ budget set. In Section 5.1, we let the Government choose an optimal non-linear income tax schedule. In Section 5.3 we further expand the set of policies available, and allow the Government to transfer a lump sum value, B, to agents in the informal sector. We call this lump sum transfer, UBI, which is designed in combination with the optimal income tax. The fact that the transfer is only made to informal workers is without loss since the tax function includes transfers.

Worker's choice. Each agent chooses in which sector to work - extensive margin choice - and how much effort to make in the selected sector - intensive margin choice. The chosen effort level, l, by a (n, m)-type maps into $z_f = ln$ units of effective labor if the extensive choice was the formal sector and $z_i = lm$ if the agent is in the informal sector, measured in units of good, from each sector. The extensive margin decision follows Roy 1951, in which workers choose the utility-maximizing sector. We do not allow agents to work part-time in each sector; once a sector is selected, all the effort will be restricted to that sector.

Intensive margin choices If a type (n, m) agent decides to work in the formal sector, the utility attained is

$$v(n, m) = \left\{ \max_{c,z} \mathcal{U}\left(c, \frac{z}{n}\right) \quad \text{s.t.,} \quad c \leq z\phi(r) - T(z\phi(r)) \right\},$$

whereas in the informal sector,

$$\underline{v}(n, m) = \max_{c,z} \left\{ \mathcal{U}\left(c, \frac{z}{m}\right) \quad \text{s.t.,} \quad c \leq z\omega(r) + B \right\}.$$

We let $(c(n,m), z(n,m))$ denote the solution to the first and $(\underline{c}(n,m), \underline{z}(n,m))$, the solution to the second problem. Notice that the Government influences the intensive margin choice by manipulating $T(\cdot)$, which ultimately affects consumption, and labor supply. We finally define $y(v,z,n)$, the minimum income level necessary to generate utility v for an n-type who is producing z through

$$v = \mathcal{U}\left(y(v,z,n), \frac{z}{n}\right) \qquad \forall\, v,z,n.$$

Extensive margin choices. Following the Roy 1951's tradition, each person compares the utility attained by working in either sector and chooses the sector in which it is highest. Hence, a type (n,m) utility at his optimal choice is

$$\nu(n,m) = \max\{v(n,m), \underline{v}(n,m)\} \tag{1}$$

Since the tax schedule directly affects the formal sector and influences intensive margin choices, it will also have an impact on the endogenous sector decision.

The threshold function. For any n, $\underline{v}(n,m)$ is a strictly increasing function of m, and for any m, $v(n,m)$ is a strictly increasing function of n. Assume that, for a given m, there is a (n,m)-type for which $\underline{v}(n,m) = v(n,m)$, then $\underline{v}(n,m) < v(n',m)$ for all $n' > n$ and $\underline{v}(n,m) > v(n'',m)$ for all $n'' < n$. For any such m, $\tilde{n}$ defined through $\underline{v}(\tilde{n},m) = v(\tilde{n},m)$ is a participation threshold. Similarly, for any n, for which there is (n,m)-type for which $\underline{v}(n,m) = v(n,m)$, defines a participation threshold $\mathrm{m}(n)$ such that $\underline{v}(n,\mathrm{m}(n)) = v(n,\mathrm{m}(n))$ for all n. $\mathrm{m}(\cdot)$ is a threshold function mapping, assigning for each n the maximum informal sector skill m such that the type (n,m) works in the formal sector. For later reference, define the function $\alpha(\cdot)$ through $\alpha(v(n,\mathrm{m}(n))) = \mathrm{m}(n)$, $\forall\, n$. Of course, $\mathrm{m} = \alpha \circ v$.

4 Calibration

In the model, individuals make their labor decisions based not only on their relative productivity but also on their labor supply (Frisch) elasticity and the observed tax schedule. We begin by assuming the following non-linear tax function, as in Musgrave 1959, Feld-

stein 1969; Bénabou 2000, 2002:

$$T(z) = z - \zeta z^{1-\iota} \tag{2}$$

The tax parameters have been consistently estimated in several papers (e.g., Heathcote et al. 2017; Daruich and Fernández 2020). Drawing on this literature, we set $\iota = 0.18$ and $\zeta = 0.82$. To take the model to the data, we specialize the utility function by allowing it to take the standard separable form $u(c) = \ln c - \frac{l^\sigma}{\sigma}$, where the parameter σ for the disutility of labor supply can be directly mapped to the Frisch elasticity. In the numerical calibration, we adopt the Frisch elasticity estimate of 0.5 from Chetty 2012.

As presented in Section 2, we observe both formal (n) and informal (m) wages, but only for the sector in which each worker is employed. This selection process causes a truncation problem since the distribution of unobserved productivities in the non-chosen sector is censored. Consequently, only the marginal distributions could be non-parametrically identified. To tackle the problem, we normalize efficiency units so that wages are equal to one in both sectors and specify a bivariate log-normal distribution for the joint distribution of formal and informal productivities:

$$\begin{pmatrix} \log n \\ \log m \end{pmatrix} \sim \mathbb{N}\left(\begin{pmatrix} \mu_n \\ \mu_m \end{pmatrix}, \begin{pmatrix} \sigma_n^2 & \rho\sigma_n\sigma_m \\ \rho\sigma_n\sigma_m & \sigma_m^2 \end{pmatrix} \right)$$

where μ_n, μ_m, σ_n, σ_m, and ρ are parameters to be estimated. One advantage of the log-normal distribution is that it is flexible to accommodate the data in a parsimonious manner, meaning it can capture the essential features of the data with relatively few parameters: the means, variances, and the correlation ρ. For the bi-variate log-normal case, we only need five parameters to characterize the joint distribution, making it both simple and computationally efficient to estimate. This parsimony is important when we are dealing with our large dataset from Brazil, which exhibits positive skewness. In our model, productivity differences also reflect other sector-specific factors, such as limited access to capital in the informal sector and additional costs.

The correlation parameter ρ plays a fundamental role in capturing the relationship between productivity in the formal and informal sectors. Analytically, the conditional

mean and variance of informal productivity, given formal productivity, are expressed as:

$$\mathbb{E}[\log \tilde{w}_m \mid \log \tilde{w}_n] = \mu_m + \rho \frac{\sigma_m}{\sigma_n}(\log \tilde{w}_n - \mu_n)$$

$$V(\log \tilde{w}_m \mid \log \tilde{w}_n) = \sigma_m^2(1 - \rho^2)$$

These conditional moments reveal that ρ reflects the extent to which a worker's productivity in one sector is transferable to the other. The model is flexible to account for individual-specific factors, such as education, that could enhance productivity across both sectors. By the revelation principle, the distribution of productivities can be mapped to the distribution of incomes, allowing us to use income as a proxy for productivity, as in Mankiw et al. 2009.

To estimate the parameters of the joint log-normal distribution, we employ the method of moments, which minimizes the distance between model predictions and observed data across key moments. These moments include the median and average income ratios of informal to formal wages, which provide information to govern the relative levels of productivity across sectors, μ_n and μ_m. Measures of income dispersion, such as the p20/p80 income ratio within the formal sector and the cross-sector p20/p80 ratio, are used to identify the variances of formal and informal productivities, σ_n and σ_m. Finally, the formality rate is a critical moment that helps identify the correlation parameter ρ, as it reflects the degree to which high productivity in one sector translates into high productivity in the other, influencing workers' sectoral choices. Table 2 summarizes the moments used in the estimation and provides the corresponding parameter estimates, highlighting the interplay between the structural moments and the calibrated parameters.

The estimated parameters of the joint log-normal distribution indicate that individual productivities in both the formal and informal sectors are positively correlated, as evidenced by the realistic magnitude of ρ. While individuals need not have identical productivity in both sectors—given cross-sectoral structural differences—the positive correlation suggests that those who are more productive in one sector also tend to be more productive in the other. Moreover, the parameter estimates confirm that, on average, workers are more productive in the formal sector. This finding aligns with existing literature (Dix-Carneiro et al. 2021) and can be attributed to factors such as superior technology, better training opportunities, and more efficient organizational practices in the formal sector. Figure 4 illustrates the joint log-normal distribution of productivities, showing that while

Table 2: Model Fit and Estimated Parameters

Moments	Model	Data
Median income ratio	0.56	0.54
Average income ratio	0.67	0.47
Formal p20/p80 income ratio	0.38	0.36
Cross-sector p20/p80 ratio	0.90	0.45
Formality rate	0.61	0.58

Parameter	Value
μ_n	1.2463
μ_m	1.0964
σ_n	1.6182
σ_m	1.0400
ρ	0.6416

Note: The left table reports the moments used in the model and data used in the structural estimation. The right-hand side table reports the estimated parameters of the joint log-normal distribution for formal and informal productivities.

there is overlap between formal and informal productivity distributions, informal productivity is generally lower. The left tail of the distribution, in particular, exhibits a higher concentration of informal workers, suggesting that informality is more prevalent among those with the lowest productivity levels.

A moment not explicitly targeted in the calibration procedure is the share of GDP generated by the informal sector. Empirical estimates suggest that between 25% and 35% of Brazil's total output may come from the informal sector (see Schneider and Enste 2000, Table 2). Reassuringly, despite not directly targeting this moment in our calibration exercise, the estimated model produces a similar informal GDP share. From the model's perspective, we have:

$$r = \frac{\int_{\underline{n}}^{\bar{n}} \int_{\alpha(v(n))}^{\bar{m}} \underline{z}(m) g(m|n) dm f(n) dn.}{\int_{\underline{n}}^{\bar{n}} \int_{\underline{m}}^{\alpha(v(n))} z(n) g(m|n) dm f(n) dn} \approx 0.4$$

which implies an informal GDP share of, $r/(1+r)$, or approximately 28%.

When designing optimal policies for the informal sector, policymakers face the challenge of limiting available sector choices for low-income individuals without harming their overall welfare. The model and subsequent policy simulations will be carefully crafted to account for these considerations, ensuring that interventions do not exacerbate the difficulties faced by those vulnerable workers at the bottom. These observations highlight the complex challenges policymakers encounter when designing interventions for the informal sector. Restricting sectoral choices for low-income individuals can negatively affect their overall welfare, as informality often serves as a buffer for households

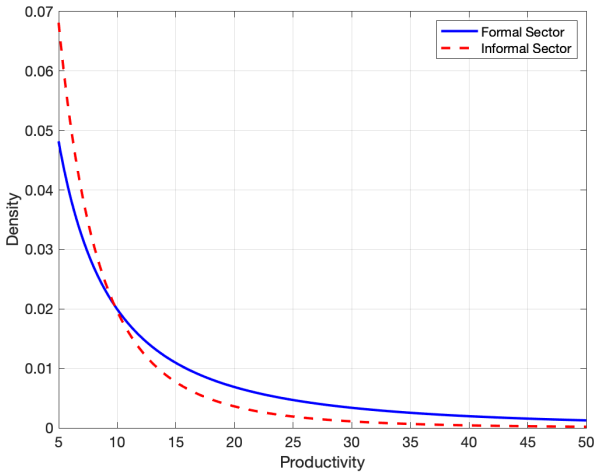


Figure 1: This figure compares the fitted log-normal probability density functions (PDFs) of productivity for formal and informal workers. The red dashed curve represents the informal sector, while the blue solid curve represents the formal sector. The informal productivity distribution is, on average, lower. The left tail of the informal productivity distribution shows a higher density, reflecting a greater proportion of workers with very low productivity in the informal sector.

with limited resources. At the same time, the presence of informality introduces additional constraints to the policymaker's optimization problem. To address this trade-off, the model and subsequent policy simulations are designed to carefully balance the role of informality to achieve optimal policy.

5 Optimal Policy

Informality limits the Government's ability to design social safety policies. In this section, we take this restriction seriously in a mechanism design framework to derive the optimal tax schedule in the presence of informality. Given that sector decisions are influenced by policy, we also seek to understand how the Government can optimally design the size of the informal sector. The answer is not obvious as there is benefits and costs associated with the existence of the informal sector.

5.1 Optimal Tax Formula with Informality

Following the tradition established by Mirrlees 1971a, we assume that while the government can observe formal-sector income, it cannot observe productivity directly. Motivated by the natural challenges to elicit information from the informal sector, we assume that for informal workers even income is unobserved, which restricts the set of feasible policies. This creates two primary challenges: (i) we have a bi-dimensional screening problem; (ii) we can't use informal income to screen informal productivity. The planner's

program then becomes,

$$\max \int_{\underline{n}}^{\bar{n}} \int_{\underline{m}}^{\alpha(v(n))} \varphi\left(v(n)\right) g(m|n) dm f(n) dn + \int_{\underline{n}}^{\bar{n}} \int_{\alpha(v(n))}^{\bar{m}} \varphi(\underline{v}(m)) g(m|n) dm f(n) dn$$

subject to the incentive compatibility constraint,

$$\dot{v}(n) = \frac{\partial \mathcal{U}\big(c(v(n), z(n), n), z(n)/n\big)}{\partial n} \frac{z(n)}{n^2},$$

$$z(n) \text{ increasing in } n,$$

the resource constraint,

$$\int_{\underline{n}}^{\bar{n}} \int_{\underline{m}}^{\alpha(v(n))} c(v(n), z(n), n) g(m|n) dm f(n) dn + \omega(r) \int_{\underline{n}}^{\bar{n}} \int_{\alpha(v(n))}^{\bar{m}} \underline{z}(m) g(m|n) dm f(n) dn + G \leq \Phi(r) \int_{\underline{n}}^{\bar{n}} \int_{\underline{m}}^{\alpha(v(n))} z(n) g(m|n) dm f(n) dn,$$

and the consistency constraint,

$$r \int_{\underline{n}}^{\bar{n}} \int_{\underline{m}}^{\alpha(v(n))} z(n) g(m|n) dm f(n) dn = \int_{\underline{n}}^{\bar{n}} \int_{\alpha(v(n))}^{\bar{m}} \underline{z}(m) g(m|n) dm f(n) dn.$$

where $\Phi(r) = F(1, r)$, $\omega(r) = \frac{\partial F(N_i, N_f)}{\partial N_i} = \frac{\partial F(1, r)}{\partial N_i}$, and r is an endogenous variable that balances the consistency constraint. A few things to note. When a tax perturbation changes the composition of the informal sector, general equilibrium forces will change the value of informality, which is captured by $\omega(r)$, affecting the participation constraint. The planner must take the impact of taxes on r when designing the optimal system.

To save on notation, we write $v(n) = v(n, m)$, and $z(n) = z(n, m)$ for all (n, m) that opts to work in the formal sector, and $\underline{v}(m) = \underline{v}(n, m)$ and $\underline{z}(m) = \underline{z}(n, m)$ for all (n, m) that opts to work in the formal sector. In what follows we specialize to the standard

utilitarian case with the specification for preferences,

$$\mathcal{U}\big(c, z/n\big) = \ln(c) - \frac{1}{\sigma}\left(\frac{z}{n}\right)^{\sigma}$$

Proposition 1. *The optimal non-linear tax formula that accounts for informality, endogenous sector choice and general equilibrium effects takes the form of a modified Saez ABC formula,*

$$\frac{\tau(n)}{1-\tau(n)} = \mathcal{A}(n)\mathcal{B}(n)\mathcal{C}(n), \tag{3}$$

where,

$$\mathcal{A}(n) = \sigma,$$

$$\mathcal{B}(n) = \frac{\sigma}{\mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) n},$$

$$\mathcal{C}(n) = \frac{1}{\Gamma(\bar{n})}\Bigg\{\Bigg[\Gamma(n)\int_{n}^{\bar{n}} \frac{c(\tilde{n})}{c(n)}\mathcal{G}\left(\alpha\left(v(\tilde{n})\right)\right) f(\tilde{n}) d\tilde{n} - \left[\Gamma(\bar{n}) - \Gamma(n)\right]\int_{\underline{n}}^{n} \frac{c(\tilde{n})}{c(n)}\mathcal{G}\left(\alpha\left(v(\tilde{n})\right)\right) f(\tilde{n}) d\tilde{n}\Bigg] + \frac{\Gamma(\bar{n})\rho(n) - \Gamma(n)\rho(\bar{n})}{c(n)}\Bigg\},$$

$$\mathcal{G}\left(\alpha\left(v(n)\right)\right) = \int_{\underline{m}}^{\alpha(v(n))} g(m|n) dm,$$

$$\Gamma(n) = \int_{\underline{n}}^{n} \mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) dn,$$

$$\rho(n) = \int_{\underline{n}}^{n} \left[\left(\Phi(r) - \omega(r) r\right) z(\tilde{n}) - c(\tilde{n}) + B\right] \nu_g(n) \mathcal{G}\left(\alpha\left(v(\tilde{n})\right)\right) f(\tilde{n}) d\tilde{n},$$

and

$$\nu_g(n) = \frac{\mathcal{G}'\left(\alpha\left(v(n)\right)\right)\alpha'\left(v(n)\right)}{\mathcal{G}\left(\alpha\left(v(n)\right)\right)} = \frac{g\left(\alpha(v(n))|n\right)\alpha'\left(v(n)\right)}{\mathcal{G}\left(\alpha\left(v(n)\right)\right)},$$

is the semi-elasticity of participation with respect to $v(n)$.

Proof. See Appendix A. □

An increase in $v(n)$ induces higher formal sector participation thus increasing (reducing) government net revenues whenever $(\Phi(r) - \omega(r)r)\, z(n) - c(n) + B > 0$ (≤ 0). $\nu_g(n)$ quantifies the impact at each formal sector productivity level, n.[4]

Important to note a few differences between Equation 3 and the standard optimal tax formula. In the presence of informality, an increase in the optimal marginal tax rate (left-hand side) creates an incentive for marginal workers to become informal. This is captured in the last term of the $\mathbb{C}$ term, which also captures the fiscal consequence of such evasion. When this happens, there is an impact on the level of tax collected as the marginal workers move all their earnings (not only the marginal earnings) to the informal economy. Important to notice that the optimal tax formula is weighted by $\mathcal{G}(\cdot)$ because in the context of informality, an increase in marginal income tax rates affects tax collection only among the pool of formal workers.

To use our optimal tax formula to concretely inform policy design, we take our model to the Brazilian data, and simulate the optimal tax following the fixed-point algorithm described in Appendix B. The algorithm goes as follows, the government sets taxes according to Equation 3, individuals respond by optimizing their choices along both the intensive and extensive margins. Taking individual decisions into account, the government iteratively adjusts the tax schedule until convergence is achieved. This implementation allows us to examine how tax policy interacts with key features of the informal sector, including enforcement, general equilibrium effects, and transfer-based interventions such as universal basic income. Before turning to these applications, we first leverage the mechanism design approach to study the fundamental forces that shape the size of the informal sector.

5.2 Optimal Informality

At the heart of this paper lies the question: what is the optimal size of the informal sector? Informality impacts social welfare through two opposing channels. On one hand, it constrains the set of feasible policies, thereby reducing welfare. On the other hand, it

[4]Indeed, it is often convenient to define, $T(n) = (\Phi(r) - \omega(r)r)\, z(n) - c(n) + B$ as the net marginal revenues that result from increasing formal participation at the skill level n. Then, $\rho(n)$ becomes

$$\rho(n) = \int_{\underline{n}}^{n} T(n)\nu_g(n)\mathcal{G}\left(\alpha\left(v(\tilde{n})\right)\right) f(\tilde{n})d\tilde{n}.$$

offers workers additional choices, which can enhance welfare by serving as a buffer for those unable to secure formal employment. The tension between these forces shapes the planner's perspective on the desirable size of the informal sector. Governments can reduce informality by offering generous tax transfers to low-income workers who participate in the formal sector. However, excessively generous transfer systems risk creating perverse incentives, where higher-skilled workers mimic low-skilled workers by reducing their labor supply to qualify for benefits. These distortions, referred to as informational rents, constrain the planner's ability to further reduce informality. Within a mechanism design framework, the planner must balance these trade-offs to determine the socially optimal size of the informal sector.

Intuitive example. To be a bit more formal (and precise) about the desirability of the informal sector consider an intuitive example for an economy with only two types of agents, (n, m) and (n', m') with $n > n'$. Let $(c(n), z(n))_n$ denote the solution to the unconstrained (by the existence of an informal sector) Mirrlees' program such that,

$$\mathcal{U}\left(c(n), \frac{z(n)}{n}\right) = \mathcal{U}\left(c(n'), \frac{z(n')}{n}\right)$$

Let $\hat{B}$ be the level of lump sum transfers to the informal sector that leaves the least productive agent indifferent between formality and informality,[5]

$$\underline{v}(\hat{B}, m') = \mathcal{U}\left(c(n'), \frac{z(n')}{n'}\right).$$

If $\mathcal{U}\left(c(n), z(n)/n\right) > \underline{v}(\hat{B}, m)$, then the existence of an informal sector allows one to increase utilitarian welfare. At the Mirrlees' solution, $\hat{B} < -T(n')$ since the bundle is distorted. If the (n', m') agent moves to the informal sector, the resources saved can be reallocated to improve the utility of the (n, m) agent. Importantly, it is not necessary for m' to be greater than n'. What makes the informal sector attractive is that the (n, m)-agent can attain a level of utility in the informal sector with a lump sum transfer without making (n', m') worse-off.

A simple and intuitive case is when $m = m' = n'$, meaning the low-productivity agent

[5] Allowing the planner to make lump sum transfers, which we explore with detail in Section 5.3, greatly increases the scope of informality in permitting redistribution to workers with relatively low formal skills.

uses the same skills in both the formal and informal sectors, while the (n, m) type possesses skills that are only valuable in the formal sector. In practice, some agents are more productive in the informal sector, though for most, transitioning to informality results in a productivity loss. This trade-off must be weighed against the screening benefits that arise due to the relative reduction in productivity. More broadly, this example illustrates the key consideration in assessing the optimal size of the informal sector: as long as productivity differences are not too large, a sizable informal sector is desirable when high-productivity agents have more to lose. Specifically, the lower the correlation between productivity in the formal and informal sectors, the larger the optimal informal sector.

Procedure. Computing the optimal informality involves solving a recursive problem, as informality influences the optimal tax schedule, which in turn affects the level of informality. We address this challenge using a fixed-point approach, iteratively solving the welfare-maximizing mechanism design problem until convergence. The process begins with our parameterized tax function, where taxes are defined as $T(z) = z - \xi z^{1-\iota}$. For a given tax system, the informal productivity m that makes individuals indifferent across the two sectors, $v_f = v_i$, is:

$$\log m = \log \xi + (1-\iota)\log n + \sigma \log \chi + \frac{1}{\sigma}\left[\log(1-\iota) + 1 - \log \chi\right]$$

$$\alpha = \exp\left(\log \xi + (1-\iota)\log n + \sigma \log \chi + \frac{1}{\sigma}\left[\log(1-\iota) + 1 - \log \chi\right]\right)$$

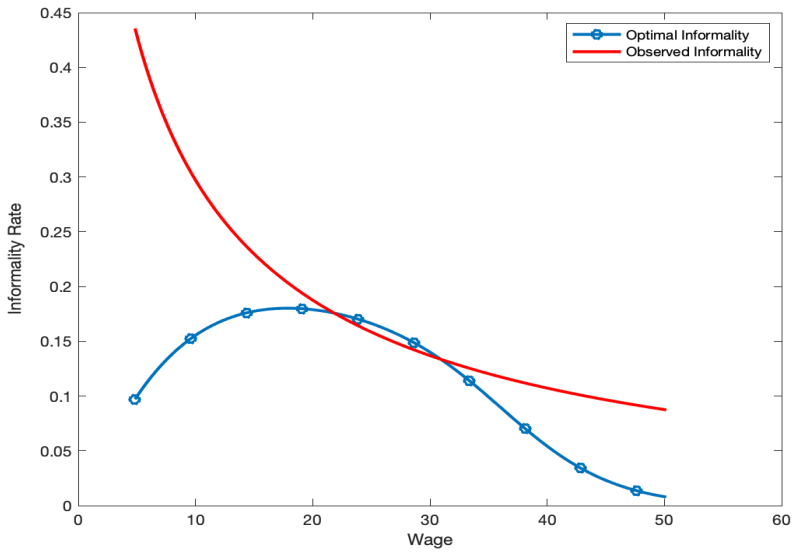
The threshold function α represents the highest level of informal productivity at which a worker still opts to participate in the formal sector. This threshold depends not only on relative productivities but also on the tax schedule, which is a policy choice and directly influences workers' decisions. Once individuals make a sector choice informed by α, we leverage the mechanism design to define the optimal tax formula that incorporates endogenous sectoral choice, as shown in Equation 3. The tax system updates the formal sector indirect utility, which in turn updates α according to the following relationship:

$$\alpha = \exp(v_f + ((1./\sigma)(\log(\chi) + 1)));$$

There is a direct mapping between α and the formality rate captured by the condi-

tional cumulative distribution $G(\alpha(v(s))|s)$, which represents the probability of choosing formality given productivity level s. With the formality rate updated, the government re-optimizes the tax schedule to maximize social welfare. This iterative process continues until convergence is achieved.

Figure 2: Optimal vs Observed Informality Rate



Note: This figure plots the optimal informality calibrated for the Brazilian economy in blue. In red we plot the observed informality in Brazil.

Optimal informality in Brazil. When applied to Brazilian data, this approach suggests that the optimal informality rate is 14%, which is 70% lower than the observed rate. Interestingly, Figure 2 shows that while observed informality decreases monotonically with productivity, the optimal informality exhibits an inverted U-shape. This reflects the planner's preference for making a more effective social safety net at the bottom of the income distribution, while also formalizing high-skilled workers to prevent significant revenue losses. That is, at the very bottom the planner increases redistribution thus bringing more low-skill agents to the formal sector. However, to disincentivize very productive agents to reduce their earnings by mimicking mid-range productivity types the planner adopts a policy that discourages participation in the formal sector at intermediate skill levels.

The effects described above align with the insights of Gomes et al. 2017 regarding optimal deviations from production efficiency in a sectoral choice model. In their frame-

work, as in ours, production efficiency corresponds to an allocation in which any (n, m)-agent with $n \geq m$ works in the sector that utilizes their skill n, while any (n, m)-agent with $n < m$ works in the sector that utilizes their skill m. In Gomes et al. 2017, both sectors can be taxed independently, whereas in our setting, only sector n is subject to taxation. Consequently, production efficiency is feasible in their model, requiring justification for any deviation. In contrast, we preclude taxes in the informal sector, any positive taxes for formal workers induces some form of production inefficiency. Even though efficiency is not attainable, it is still a useful reference in highlighting that the informal sector is for some workers the one that best uses his or her set of skills.

As discussed above, a key feature of our model is that, due to limited information on the informal sector, taxation is only feasible in the formal sector. To relax this restriction, next, we consider the use of a lump sum transfer, or Universal Basic Income – UBI –, and discuss its implications for the optimal level of informality.

5.3 Universal Basic Income

Up to this point, the Government's redistribution efforts have been limited to adjustments in the income tax schedule. In this context, informality poses a significant challenge to welfare distributive policies. If the Government tries to attract low-skilled workers to the formal sector by offering generous in-work transfer policies, it risks creating socially undesirable informational rents for inframarginal formal workers. To overcome this limitation, we extend our analysis to consider an additional policy instrument: universal basic income (UBI), which has recently gained prominence in policy discussions (Hoynes and Rothstein 2018; Daruich and Fernández 2020). UBI has been discussed as an alternative to providing an unconditional minimum income to every adult citizen. A notable advantage of UBI is that its implementation does not require observing individuals' income, making it especially attractive for economies with large informal sectors.

Introducing UBI improves the welfare of informal workers (Figure 3, Panel C), but requires a comprehensive re-optimization of the tax system to fund the program. Since UBI makes informal employment relatively more attractive, the participation constraint becomes harder to satisfy, limiting the government's ability to keep workers in the formal sector when taxes increase. On the positive side, UBI enables direct transfers to low-skill informal workers without requiring the government to pay informational rents to high-

skill formal workers that would arise in a system where generous benefits are used to incentivize formalization. As a result, the overall welfare impact is not immediately clear when UBI is introduced. Solving the modified Mirrlees problem with both informality and UBI, we find that optimal informality increases, while social welfare declines.

For this analysis, we introduce a UBI transfer, denoted by B, which affects both the government's budget constraint and the indirect utility of informal workers.[6] Including B complicates the utility maximization problem of informal workers, which now takes the following form:

$$v_m(B, m) = \max_l \left[\log(ml + B) - \frac{l^\sigma}{\sigma} \right] \tag{4}$$

With the introduction of B, the informal sector's problem no longer has a closed-form solution. Instead, we solve Equation 4 numerically for each m and B, considering a range of B values from 0 to 90% of median income. By influencing informal indirect utility, UBI also affects the threshold function $\alpha(v(m, B)|n)$, which informs about the productivity level of the marginal worker who is indifferent between the formal and informal sectors. Once we compute $\alpha(v(m, B)|n)$, we have all the necessary components to determine the optimal marginal tax rate. Using the iterative procedure described in Section 5.2, we then calculate the optimal level of informality under UBI.

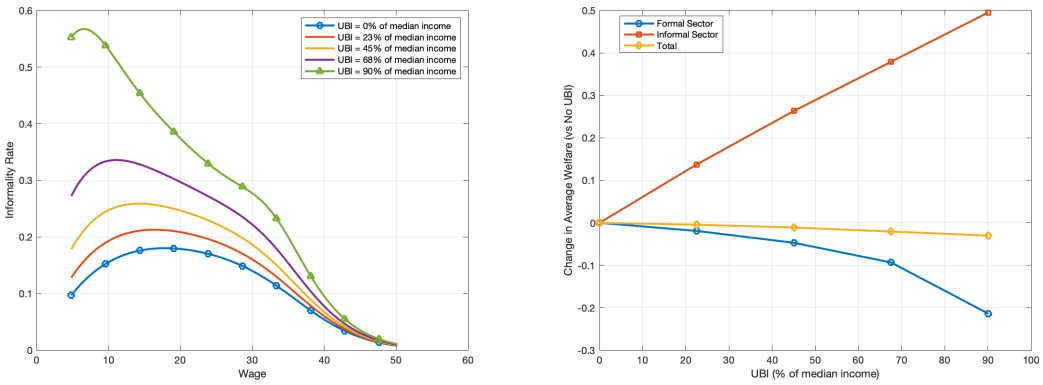
Figure 3 summarizes the results. Compared to the baseline where $B = 0$, we find that optimal informality increases with UBI. This occurs because, at the margin, the planner faces a lower welfare cost when low-income workers transition to informality, as they continue to receive UBI. One might argue that introducing UBI mechanically increases welfare simply by adding another policy instrument. However, this is not necessarily the case, as we impose a constraint that UBI must be strictly positive.

As illustrated in the intuitive example on page 18, UBI enables the planner to implement a transfer system that incentivizes low-skill workers to leave the formal sector. This shift results in an efficiency loss, as workers allocate their skills to the informal sector instead. However, UBI also generates redistributive gains by relaxing incentive compatibility constraints. Figure 3 presents the welfare change (in consumption terms) relative to the baseline of zero UBI, highlighting as expected, a redistribution of welfare from formal to informal workers. Since UBI changes the composition of the formal workforce, we

[6] We do not need to adjust B in the formal sector's indirect utility, as the income tax schedule can directly account for any lump-sum benefit or cost added to individual consumption.

also report the aggregate welfare effect in both sectors to account for these composition changes. Our results indicate a slight overall decline in welfare. These findings suggest that, given the broader set of worker types and the parameterization used to approximate the Brazilian economy, the efficiency costs of UBI outweigh its redistributive benefits.

Figure 3: Optimal Tax with Informality and UBI



(a) Optimal Informality

(b) Welfare Change with Optimal Tax and UBI

Note: This figure summarizes the consequences of increasing the levels of UBI on the shape of theoptimal informality (Panel A), and social welfare change relative to the case without UBI (Panel B). Each line in the plot to a different level of UBI, ranging from zero to 90% of the median income.

5.4 General Equilibrium

When individuals transition from the informal to the formal sector, the relative labor supply across sectors changes, potentially generating general equilibrium responses in wages.[7] These effects are governed by the elasticity of substitution between formal and informal labor, ε, within the economy's production function:

$$H(N_n, N_m) = A\left\{(1-a)N_m^{1-\frac{1}{\varepsilon}} + aN_n^{1-\frac{1}{\varepsilon}}\right\}^{\frac{\varepsilon}{\varepsilon-1}}$$

This function is versatile and can accommodate various scenarios. When $\varepsilon = 1$, the model corresponds to the Cobb-Douglas case. When $\varepsilon \to \infty$, formal and informal labor

[7] Ordover and Phelps 1979, Stiglitz 1982a are early investigations of general equilibrium effects in a Mirrlees's environment. Rothschild and Scheuer 2013 add by taking it into account in a context of sectoral choice closer to what we present here.

are perfect substitutes. Finally, when $\varepsilon = 0$, the model corresponds to the Leontief case. Up to this point, we have designed optimal policies in a partial equilibrium framework, which is a specific case where $\varepsilon \to \infty$.

Introducing general equilibrium considerations is of particular interest from a policy perspective because it allows the Government to indirectly improve the welfare of informal workers. By decreasing marginal tax rates, individuals are incentivized to move from informal to formal markets. This shift enhances the welfare of inframarginal workers who remain in the informal sector because it reduces the labor supply of informal workers, therefore, increasing the efficiency wages for those that stay informal. Under the CES assumption, we have

$$\omega(r) = A\left[1-a\right] r^{-\frac{1}{\varepsilon}} \left\{ \left[a + (1-a) r^{1-\frac{1}{\varepsilon}} \right] \right\}^{\frac{1}{\varepsilon-1}},$$

and

$$\phi(r) = Aa \left\{ \left[a + (1-a) r^{1-\frac{1}{\varepsilon}} \right] \right\}^{\frac{1}{\varepsilon-1}}$$

For any ε, imposing $\omega(r) = \phi(r)$ in the baseline value for r pins down a through

$$\left[\frac{1-a}{a} \right]^{\varepsilon} = r.$$

Then, we choose A to guarantee that our normalization $\omega(r) = \phi(r) = 1$ is satisfied. All the calculations above are for a given value for ε. In this section, we consider the case where $\varepsilon = 1$, thus spanning welfare and marginal tax rates (MTR) between the two extreme cases of general equilibrium. To compute the optimal policy under general equilibrium, it is necessary to correctly adjust for the fiscal externality in the Government budget constraint:

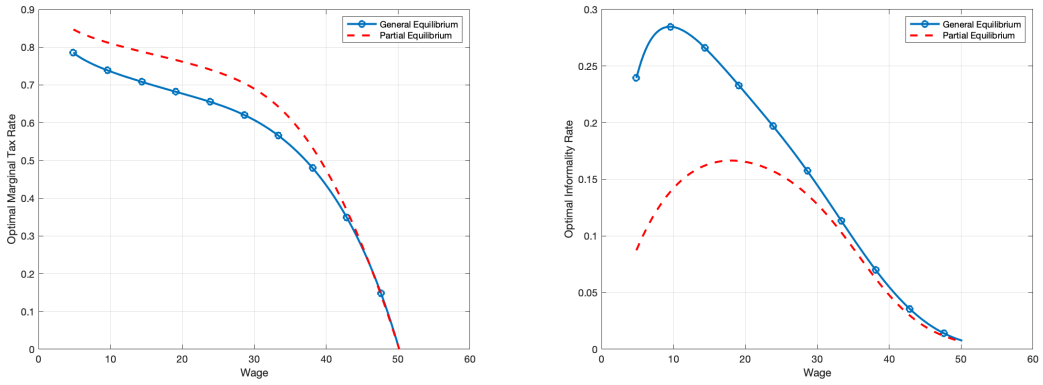
$$T(n) = \left(\Phi(r) - \omega(r) r\right) z(n) - c(n) + B,$$

representing the net marginal revenues from increasing formal participation at skill level n. In this context, $\phi(r) = (\Phi(r) - \omega(r) r) = \partial H(1, r)/\partial N_n$. In the edge case where $\varepsilon = 1$, $\phi(r) = a r^{1-a}$, and $r = N_m/N_n$. As general equilibrium forces change the relative attractiveness of the two sectors, it will alter the threshold function that defines the marginal worker:

$$\alpha = \exp\left(v_n - \log(\omega(r)) + \frac{1}{\sigma} \right).$$

In Figure 4 we compare outcomes for the two polar cases, $\varepsilon = \infty$ (partial equilibrium) and $\varepsilon = 1$ (general equilibrium).[8] Marginal tax rates are lower in the presence of general equilibrium effects, which is reminiscent of Rothschild and Scheuer's 2013 trickle-down effect. In fact, albeit in a world without selection, Stiglitz 1982b had already considered this income transfer through general equilibrium effects. Here, by inducing agents to work harder in the formal sector through lower marginal tax rates the planner increases informal sector wages through general equilibrium effects. Since informal workers have a higher average social marginal value of income, this helps advance the planner's objective.

Figure 4: Optimal Policy with General Equilibrium



(a) Optimal Marginal Tax Rates

(b) Optimal Informality

Note: This figure illustrates the impact of general equilibrium for the design of optimal taxes (Panel A), and for the optimal size of the informal sector (Panel B).

Participation in the formal sector is, however, lower with general equilibrium effects. This suggests that these transfers through general equilibrium are accomplished by making participation in the formal sector less attractive for low-productivity agents, but effort conditional on participating more attractive. While we have described the differences in tax policies under the two scenarios by highlighting the new opportunities available to the planner, we can also view them from the opposite perspective—as constraints on the extent to which the planner can implement redistributive policies. If labor effort is discouraged, informal workers are adversely affected by general equilibrium effects, which

[8] Our assumption in page 8 is that the two sectors are that the two sectors can survive in the limit in which the other disappears. This requires $\varepsilon > 1$ in the case of CES. Our general equilibrium exercise is to be understood as taking the limit from above, $\varepsilon \to 1^+$.

in turn limits the planner's ability to impose higher taxes on formal workers at the margin.

6 The Cost of Informality

While informality is often blamed for many of the challenges faced by developing countries,[9] it is widely acknowledged that informality may not be the root cause but rather a symptom of poorly designed institutions (Dix-Carneiro and Kovak 2019). Despite this nuance, policymakers often advocate for the elimination of informality as a welfare-improving policy, ignoring its critical role as the sole income source for billions of people, as recognized by multilateral institutions (IMF 2020).

In this section, we assess the welfare implications of a policy that eradicates the informal sector. To tackle this relevant counterfactual policy question, we begin by considering what would happen to agents currently in the informal sector if they were forced to participate in the formal sector. We answer this question with two edge cases that reflect different views of informality. One is the view of informality as a survival strategy, where certain economic activities exist only because they evade the costs of formality. Second, is the view of informality as a profitable opportunity, where activities would exist in a less profitable format under formalization. Eliminating informality has different welfare and distributional consequences, under these two scenarios, as we illuminate below.

6.1 Informality as a Survival Strategy

To capture the idea that some economic activities would not be feasible if they had to operate in the formal sector, we assume that shutting down the informal sector forces each (n, m)-agent to use only their formal-sector skill, n. We benchmark the optimal tax policy discussed in Subsection 5.1 against the standard Saez 2001 framework, where informality does not exist.

To simulate the optimal tax system in the one-sector economy, we let the Government optimize taxes based on the distribution of formal sector productivities in a standard Mirrlees one-sector economy. This exercise captures the counterfactual of what would happen if regulatory enforcement shuts down the informal sector. For consistency, we

[9] "The high incidence and persistence of informal labor, particularly in emerging market and developing economies, is increasingly recognized as an obstacle to sustainable development.", IMF 2020

stick with the assumption of a bi-variate log-normal distribution for productivity in the formal and informal sectors, allowing us to draw the the associated one dimensional distribution of formal sector productivities based on our the calibrated parameters.

Using this distribution within the Saez's framework and employing the fixed-point algorithm detailed in Appendix B, we simulate the optimal tax formula. From the optimal policy in the counterfactual one-sector economy, we compute the associated welfare and compare it to our baseline. Eliminating the informal sector results in a 8% welfare decrease. However, this loss is not uniform across the income distribution. Figure 6.1 shows that the welfare change displays a U-shaped pattern, and high skill individuals are not affected by the closure of the informal sector. Eliminating informality brings us back to the Saez 2002 world, and despite of relaxing constraints in the planner's problem, it eliminates an advantageous market for many workers.

6.2 Informality as an Evasion Strategy

In contrast to the survival strategy view one may consider that all the economic activities could be profitably exerted formally but that some activities are costlessly hidden from the authorities. In this case, individuals may choose the informal sector either because they are more productive there, or because the tax evasion opportunities outweigh their productivity loss. In our baseline analysis of eliminating the informal sector, both forces play a crucial role in determining the welfare consequences. To isolate these effects, we now consider an alternative policy in which the Government can influence sectoral productivity such that workers always operate at their maximum productivity, regardless of their chosen sector. Under this framework, we adopt the standard Mirrlees model, where the skill of an (n, m) agent is redefined as $\tilde{n} = \max\{n, m\}$. The joint log-normal distribution of productivities allows us to compute the distribution of the maximum productivity, as detailed in Appendix C. We derive a closed-form solution for the probability density function (pdf) of the maximum:

$$f_{\max}(z) = \frac{1}{z}\left[\frac{1}{\sigma_n}\phi\left(\frac{\log(z)-\mu_n}{\sigma_n}\right)\Phi\left(\frac{\frac{\log(z)-\mu_m}{\sigma_m}-\rho\frac{\log(z)-\mu_n}{\sigma_n}}{\sqrt{1-\rho^2}}\right)+\frac{1}{\sigma_m}\phi\left(\frac{\log(z)-\mu_m}{\sigma_m}\right)\Phi\left(\frac{\frac{\log(z)-\mu_n}{\sigma_n}-\rho\frac{\log(z)-\mu_m}{\sigma_m}}{\sqrt{1-\rho^2}}\right)\right].$$

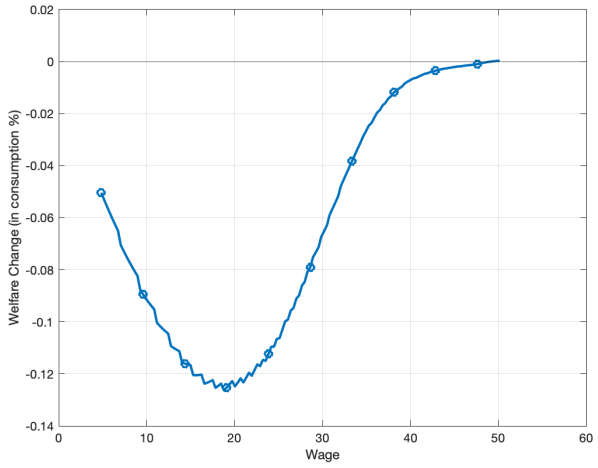


Figure 5: Welfare variation when the informal sector is eliminated: This figure shows welfare changes from eliminating the informal sector. Percentage changes are based on welfare under optimal policy with and without informality.

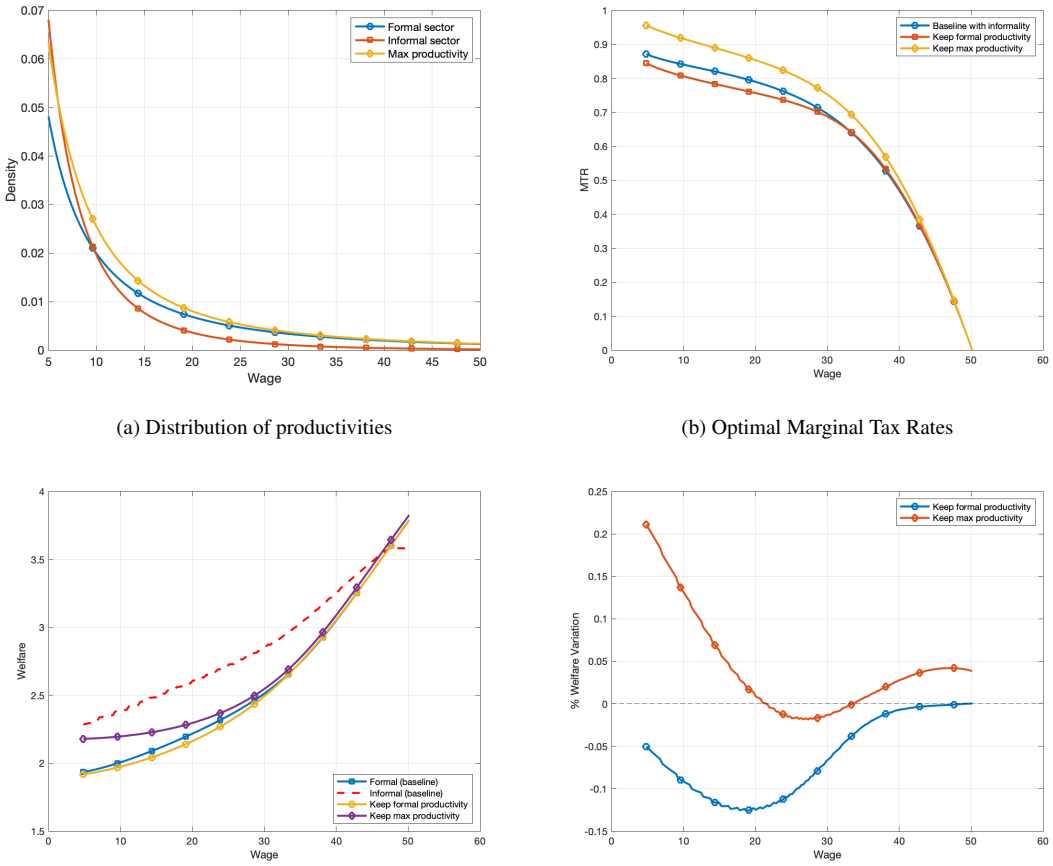
Using this distribution, we compute the optimal taxation in a one-sector economy, following the methodology outlined in Section 6.1. Panel A of Figure 6 plots the density of distributions, highlighting that formal and informal productivities have some overlap. Panel B shows that when the dominating productivity is considered, the optimal marginal tax rates are higher at the lower end of the productivity spectrum. This occurs because allowing for the maximum productivity leads to a greater skill upgrade for low-skill formal workers, who are typically more productive in the informal sector.

The results show that informality systematically reduces optimal marginal tax rates. This reduction arises because informality limits the Government's ability to effectively tax formal workers, particularly those with low productivity who are more likely to opt for informality. The gap in marginal tax rates between the two extreme scenarios can reach up to 15%. The iterative nature of our algorithm captures the endogenous response of informality to tax policy, recalibrating sectoral participation thresholds at each step. These findings highlight the importance of incorporating informality into the design of efficient and equitable tax systems.

Not surprisingly, Panel C demonstrates that welfare increases with productivity, and it is the highest in the counterfactual world where there is only the formal sector and individuals are able to keep their maximum productivity. One might wonder why average welfare is highest among those who chose informality. Since the x-axis represents formal sector productivity n, individuals opt for informality when their informal productivity m exceeds the threshold $\alpha(v(n))$. Finally, Panel D highlights the welfare variation from the baseline two-sector economy compared to two counterfactual scenarios: one in which

individuals retain their formal-sector productivity and another where they operate at their maximum productivity. The welfare variation is expressed in consumption units, and our analysis reveals that eliminating informality and allowing individuals to achieve their maximum productivity leads to a 6.5% welfare increase.

Figure 6: Eliminate Informality and keep Maximum Productivity



(a) Distribution of productivities

(b) Optimal Marginal Tax Rates

(c) Welfare

(d) Welfare Variation

Note: This figure depicts the consequences of eliminating informality and keeping the maximum productivity. Panel A compares the formal, informal, and maximum distribution of productivities. Panel B compares the optimal MTR in a two-sector economy vs one sector, where individuals keep either the formal or the maximum productivity. Panel C plots the welfare across cases, and Panel D plots the welfare variation in consumption units between the baseline economy with two sectors and the alternative scenarios where informality is eliminated.

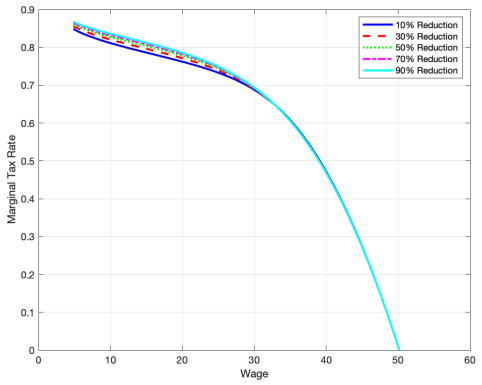
6.3 Enforcement Policies

A recent agenda in Public Finance have highlighted the trade-off between production efficiency and tax evasion within an empirical (Best et al. 2015), theoretical (Keen and Slemrod 2017), and experimental (Bergeron et al. 2024) approaches. We build on these insights by providing a theoretical framework to study the interaction of informality and the design of optimal tax policy when enforcement policies are available. Concretely, in this subsection we allow the Government to implement enforcement policies that penalize informal work while still permitting the informal sector to exist. We show that by increasing enforcement, the planner relaxes the participation constraint, enabling optimal taxes to rise.

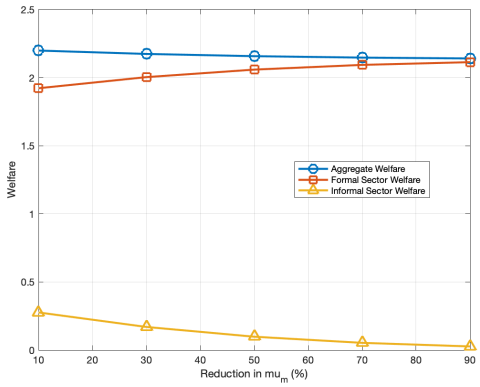
To analyze this mechanism in our framework, we introduce shocks to the parameter μ_m, which governs the distribution of informal productivity. Specifically, we examine reductions in mean informal productivity ranging from 10% to 90%. The underlying assumption is that enforcement works by reducing the return to work in the informal sector. Consistent with the findings of Bergeron et al. 2024, we show that stronger enforcement in the informal labor market leads to higher optimal income tax rates, as illustrated in Figure 7, Panel A.

The welfare effects of enforcement policy are ambiguous ex-ante. On the one hand, stricter enforcement alleviates the participation constraint, which should enhance social welfare. On the other hand, it reduces informal productivity, which negatively impacts welfare. Panel B shows that, overall, welfare declines slightly as a result of the enforcement policy. However, there are winners and losers: formal workers benefit because enforcement alleviates participation constraints leading to a more efficient tax system, while informal workers lose as they bear the burden of enforcement. Finally, Panel C demonstrates that optimal informality decreases when informality is further penalized. This result suggests that when informal work is subject to additional penalties, the design of the optimal welfare system will direct fewer individuals into the informal sector.

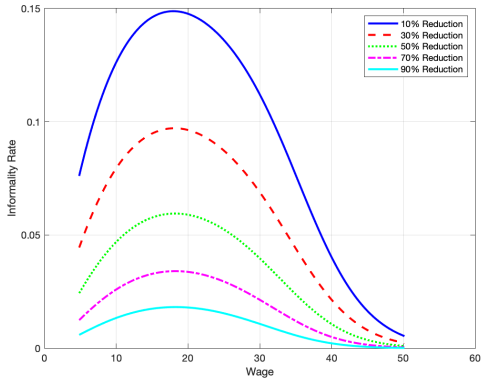
Figure 7: Enforcement Policies



(a) Optimal Marginal Tax Rates



(b) Welfare



(c) Optimal Informality

Note: This figure depicts the consequence of several enforcement policies that punish informal work. We capture these punishments based on different degrees of informal productivity loss, varying from 10% to 90% reduction in informal mean productivity. Panel A illustrates the consequences of enforcement on optimal tax rates. Panel B shows the welfare effects. Whereas Panel C computes the optimal informality under each of the counterfactual enforcement policies.

7 Conclusion

We carefully examine the trade-offs involved in manipulating the size of the informal sector. While informality provides a safety net for some individuals, it also creates opportunities for tax evasion, which constrains the design of social welfare transfer policies, as discussed in Saez 2002. This trade-off implies that the optimal size of the informal sector is not in a corner solution but lies somewhere in between. Using a calibrated model

with rich microdata from Brazil, we estimate that the optimal size of the informal sector is 14%, strikingly 70% lower than the currently observed levels in the Brazilian labor market. Interestingly, while observed informality decreases monotonically with productivity, our results show that the optimal informality rate follows an inverted U-shape. This reflects the planner's preference to formalize low-skilled workers, enhancing redistribution through in-work benefits, and to formalize high-skilled workers to minimize revenue losses.

Our analysis incorporates a comprehensive welfare framework that accounts for key aspects of the policy debate, including the interplay between optimal taxes, informality, and universal basic income. We also evaluate the general equilibrium effects and enforcement policies aimed at regulating the size of the informal sector. Overall, we find that allowing some degree of informality yields important social benefits while also having significant distributional implications. Our results are specific to Brazil, given the underlying data, but the mechanism design approach can be applied to estimate the optimal size of the informal sector in other countries.

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A Solving the Planner's Program

Recall the planner's program,

$$\max \int_{\underline{n}}^{\bar{n}} \int_{\underline{m}}^{\alpha(v(n))} \varphi\left(v(n)\right) g(m|n) dm f(n) dn + \int_{\underline{n}}^{\bar{n}} \int_{\alpha(v(n))}^{\bar{m}} \varphi(\underline{v}(m)) g(m|n) dm f(n) dn$$

subject to the incentive compatibility constraint,

$$\dot{v}(n) = \frac{\partial \mathcal{U}\big(c(v(n), z(n), n), z(n)/n\big)}{\partial n} \frac{z(n)}{n^2},$$

and

$$z(n) \text{ increasing in } n,$$

the resource constraint,

$$\int_{\underline{n}}^{\bar{n}} \int_{\underline{m}}^{\alpha(v(n))} c(v(n), z(n), n) g(m|n) dm f(n) dn + \omega(r) \int_{\underline{n}}^{\bar{n}} \int_{\alpha(v(n))}^{\bar{m}} \underline{z}(m) g(m|n) dm f(n) dn + G \leq \Phi(r) \int_{\underline{n}}^{\bar{n}} \int_{\underline{m}}^{\alpha(v(n))} z(n) g(m|n) dm f(n) dn,$$

where $\Phi(r) = F(1, r)$, and the consistency constraint,

$$\int_{\underline{n}}^{\bar{n}} \int_{\underline{m}}^{\alpha(v(n))} z(n) g(m|n) dm f(n) dn = r \int_{\underline{n}}^{\bar{n}} \int_{\alpha(v(n))}^{\bar{m}} \underline{z}(m) g(m|n) dm f(n) dn.$$

Ignoring the monotonicity constraint, We can write the Lagrangian

$$\begin{aligned}\mathcal{L} = &\int_{\underline{n}}^{\bar{n}} \varphi\left(v(n)\right) \mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) dn + \int_{\underline{n}}^{\bar{n}} \int_{\alpha(v(n))}^{\bar{m}} \varphi(\underline{v}(m)) g(m|n) dm f(n) dn \\ &- \int_{\underline{n}}^{\bar{n}} \left[\dot{\mu}(n) v(n) + \mu(n) \frac{\partial \mathcal{U}\big(c(v(n), z(n), n), z(n)/n\big)}{\partial n} \frac{z(n)}{n^2} \right] dn \\ &+ \lambda \left[\Phi(r) \int_{\underline{n}}^{\bar{n}} z(n) \mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) dn - \int_{\underline{n}}^{\bar{n}} c(v(n), z(n), n) \mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) dn \right. \\ &\left. - \omega(r) \int_{\underline{n}}^{\bar{n}} \underline{\mathcal{Z}}\left(\alpha\left(v(n)\right)\right) f(n) dn - B \int_{\underline{n}}^{\bar{n}} \left[1 - \mathcal{G}\left(\alpha\left(v(n)\right)\right)\right] f(n) dn - G \right] + \\ &\gamma \left[r \int_{\underline{n}}^{\bar{n}} z(n) \mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) dn - \int_{\underline{n}}^{\bar{n}} \underline{\mathcal{Z}}\left(\alpha\left(v(n)\right)\right) f(n) dn \right]\end{aligned}$$

Noting that the consistency constraint will hold as an equality we can incorporate it in the resource constraint to simplify the Lagrangian,

$$\begin{aligned}\mathcal{L} = &\int_{\underline{n}}^{\bar{n}} \varphi\left(v(n)\right) \mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) dn + \int_{\underline{n}}^{\bar{n}} \int_{\alpha(v(n))}^{\bar{m}} \varphi(\underline{v}(m)) g(m|n) dm f(n) dn \\ &- \int_{\underline{n}}^{\bar{n}} \left[\dot{\mu}(n) v(n) + \mu(n) \frac{\partial \mathcal{U}\big(c(v(n), z(n), n), z(n)/n\big)}{\partial n} \frac{z(n)}{n^2} \right] dn \\ &+ \lambda \left[\left[\Phi(r) - \omega(r) r\right] \int_{\underline{n}}^{\bar{n}} z(n) \mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) dn \right. \\ &\left. - \int_{\underline{n}}^{\bar{n}} c(v(n), z(n), n) \mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) dn - B \int_{\underline{n}}^{\bar{n}} \left[1 - \mathcal{G}\left(\alpha\left(v(n)\right)\right)\right] f(n) dn - G \right]\end{aligned}$$

Now, recalling the specification for preferences,

$$\mathcal{U}\big(c, z/n\big) = \ln(c) - \frac{1}{\sigma} \left(\frac{z}{n}\right)^{\sigma},$$

the Lagrangian becomes

$$\begin{aligned}\mathcal{L} = &\int_{\underline{n}}^{\bar{n}} v(n)\mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n)dn + \int_{\underline{n}}^{\bar{n}}\int_{\alpha(v(n))}^{\bar{m}} \underline{v}(m)g(m|n)dm f(n)dn \\ &- \int_{\underline{n}}^{\bar{n}} \left[\dot{\mu}(n)v(n) + \mu(n)\frac{z(n)^{\sigma}}{n^{1+\sigma}}\right] dn \\ &+ \lambda\left[\left[\Phi(r) - \omega(r)r\right]\int_{\underline{n}}^{\bar{n}} z(n)\mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n)dn\right. \\ &- \int_{\underline{n}}^{\bar{n}} \exp\left\{v(n) + \frac{1}{\sigma}\left(\frac{z(n)}{n}\right)^{\sigma}\right\}\mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n)dn \\ &\left.-B\int_{\underline{n}}^{\bar{n}}\left[1 - \mathcal{G}\left(\alpha\left(v(n)\right)\right)\right] f(n)dn - G\right]\end{aligned}$$

The first order condition with respect to $z(n)$ is, therefore,

$$-\sigma\mu(n)\frac{z(n)^{\sigma-1}}{n^{1+\sigma}} + \lambda\left[\left[\Phi(r) - \omega(r)r\right] - c(n)\frac{z(n)^{\sigma-1}}{n^{\sigma}}\right]\mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) = 0,$$

which we reorganize,

$$\underbrace{\left[\Phi(r) - \omega(r)r\right] - c(n)\frac{z(n)^{\sigma-1}}{n^{\sigma}}}_{\tau(n)[\Phi(r)-\omega(r)r]} = \frac{\mu(n)}{\lambda\mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n)n}\left(\sigma\right)\frac{z(n)^{\sigma-1}}{n^{\sigma}},$$

to obtain.

$$\frac{\tau(n)}{1-\tau(n)} = \frac{\mu(n)}{\lambda\mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n)n}\frac{\sigma}{c(n)}$$

To get an expression for $\mu(n)$ we differentiate the Lagrangian with respect to $v(n)$ and obtain

$$\begin{aligned}\dot{\mu}(n) = \mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) + \lambda\Big\{\Big[&\left(\Phi(r) - \omega(r)r\right) z(n) - \\ &c(n) + B\Big]\mathcal{G}'\left(\alpha\left(v(n)\right)\right)\alpha'\left(v(n)\right) - c(n)\mathcal{G}\left(\alpha\left(v(n)\right)\right)\Big\} f(n) \quad (5)\end{aligned}$$

Then,

$$\frac{\mu(n)}{\lambda} = \int_{\underline{n}}^{n} \left[\left(\Phi(r) - \omega(r)r\right) z(\tilde{n}) - c(\tilde{n}) + B\right] \mathcal{G}'\left(\alpha\left(v(\tilde{n})\right)\right) \alpha'\left(v(\tilde{n})\right) f(\tilde{n}) d\tilde{n} + \frac{1}{\lambda} \int_{\underline{n}}^{n} \mathcal{G}\left(\alpha\left(v(\tilde{n})\right)\right) f(\tilde{n}) d\tilde{n} - \int_{\underline{n}}^{n} c(\tilde{n}) \mathcal{G}\left(\alpha\left(v(\tilde{n})\right)\right) f(\tilde{n}) d\tilde{n}.$$

Therefore,

$$\frac{\tau(n)}{1-\tau(n)} = \frac{\sigma}{\mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) n} \frac{1}{c(n)} \left\{ \frac{1}{\lambda} \int_{\underline{n}}^{n} \mathcal{G}\left(\alpha\left(v(\tilde{n})\right)\right) f(\tilde{n}) d\tilde{n} - \int_{\underline{n}}^{n} c(\tilde{n}) \mathcal{G}\left(\alpha\left(v(\tilde{n})\right)\right) f(\tilde{n}) d\tilde{n} + \underbrace{\int_{\underline{n}}^{n} \left[\left(\Phi(r) - \omega(r)r\right) z(\tilde{n}) - c(\tilde{n}) + B\right] \mathcal{G}'\left(\alpha\left(v(\tilde{n})\right)\right) \alpha'\left(v(\tilde{n})\right) f(\tilde{n}) d\tilde{n}}_{\rho(n)} \right\}$$

Let also

$$\Gamma(n) = \int_{\underline{n}}^{n} \mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) dn,$$

then

$$\frac{1}{\lambda} \int_{\underline{n}}^{\bar{n}} \mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) dn = \int_{\underline{n}}^{\bar{n}} c(n) \mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) dn - \int_{\underline{n}}^{\bar{n}} \left[\left(\Phi(r) - \omega(r)r\right) z(n) - c(n) + B\right] \mathcal{G}'\left(\alpha\left(v(n)\right)\right) \alpha'\left(v(n)\right) f(\tilde{n}) d\tilde{n}$$

Let $\Gamma = \Gamma(\bar{n})$ be the mass of formal workers, then,

$$\frac{1}{\lambda} = \frac{1}{\Gamma} \int_{\underline{n}}^{\bar{n}} c(n) \mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) dn - \underbrace{\frac{1}{\Gamma} \int_{\underline{n}}^{\bar{n}} \left[\left(\Phi(r) - \omega(r)r\right) z(n) - c(n) + B\right] \mathcal{G}'\left(\alpha\left(v(n)\right)\right) \alpha'\left(v(n)\right) f(\tilde{n}) d\tilde{n}}_{\rho = \rho(\bar{n})},$$

i.e.,

$$\frac{1}{\lambda} = \frac{1}{\Gamma} \left[\int_{\underline{n}}^{\bar{n}} c(n) \mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) dn - \rho(\bar{n}) \right].$$

We can finally rearrange terms in such a way that

$$\frac{\tau(n)}{1-\tau(n)} = \frac{\sigma}{\mathcal{G}\left(\alpha\left(v(n)\right)\right) f(n) n} \frac{1}{\Gamma} \Bigg\{ \Bigg[\Gamma(n) \int_{n}^{\bar{n}} \frac{c(\tilde{n})}{c(n)} \mathcal{G}\left(\alpha\left(v(\tilde{n})\right)\right) f(\tilde{n}) d\tilde{n} - \left[\Gamma - \Gamma(n)\right] \int_{\underline{n}}^{n} \frac{c(\tilde{n})}{c(n)} \mathcal{G}\left(\alpha\left(v(\tilde{n})\right)\right) f(\tilde{n}) d\tilde{n} \Bigg] + \frac{\Gamma\rho(n) - \Gamma(n)\rho}{c(n)} \Bigg\}. \quad (6)$$

B Quantitative Procedure for Optimal Taxation

The optimal tax formula (6) used in the simulation exercise can be equivalently written:

$$\frac{T'}{1-T'} = \mathcal{A}(n)\mathcal{B}(n)\mathcal{C}(n), \quad (7)$$

where,

$$\mathcal{A}(n) = \sigma,$$

$$\mathcal{B}(n) = \frac{1-F(n)}{\mathcal{G}(\alpha(v(n))|n) f(n) n},$$

and

$$\mathcal{C}(n) = \frac{1}{[1-F(n)]c(n)} \int_{n}^{\bar{n}} \Bigg\{ \frac{-\mathcal{G}(\alpha(v(s))|s)}{\lambda} + \Bigg[\mathcal{G}(\alpha(v(s))|s)(c(s)) - T(s) g(\alpha(v(s))|s) \alpha'(v(s)) \Bigg] \Bigg\} f(s) ds, \quad (8)$$

where

$$T(s) = \left(\Phi(r) - \omega(r) r\right) z(n) - c(n) + \left[1 - \mathcal{G}(\alpha(v(s))|s)\right] B$$

To implement Equation (8), we adapt the algorithm proposed by Mankiw et al. (2009) to incorporate modifications in the optimal tax schedule that arise due to informality. The algorithm iteratively updates both the tax schedules and the sector choices until convergence is achieved. Next, we provide a detailed explanation of this algorithm. Figure **??** illustrates the simulated optimal marginal tax rate for the calibrated economy using

Brazilian data. As expected, there is no distortion at the top of the distribution. For the most productive individuals, no distortion is necessary since there is no risk of information rents being extracted by higher types.

The methodology adapts the discrete-grid approach proposed by Mankiw et al. 2009 while addressing the complications that arise due to informality, as detailed in the main text.

Given the complexity of continuous distributions, we discretize the wage distribution over a grid of types $\{n_i\}_{i=1}^N$ to approximate the integral terms. The algorithm iteratively adjusts tax schedules until convergence to the optimal solution. The algorithm proceeds as follows:

Step 1: Discretize the Productivity Distribution. We divide the range of productivity $[\underline{n}, \overline{n}]$ into N bins of equal width Δ. The midpoints of the bins form the grid $\{n_i\}_{i=1}^N$. The probability mass function $\pi(n_i)$ for each grid point is computed as:

$$\pi(n_i) = \Psi(n_i + \Delta/2) - \Psi(n_i - \Delta/2), \tag{9}$$

where $\Psi(\cdot)$ is the cumulative distribution function of n.

Step 2: Initialize the Tax Schedule. Start with an initial guess for the marginal tax rate, e.g., a flat tax $T_0'(n_i) = 0.35$ for all n_i, and a small lump-sum transfer $t_0 = 0.0001$.

Step 3: Extrapolate the Tax Schedule. Define the tax schedule $T_{k-1}(n)$ at points outside the grid by extrapolating the marginal tax rates $T_{k-1}'(n_i)$:

$$T_{k-1}(n) = \begin{cases} T_{k-1}'(n_1)z(n) - t_{k-1}, & \text{for } n \leq n_1, \\ \sum_{j=1}^{i-1} T_{k-1}'(n_j)[z(n_j) - z(n_{j-1})] & \\ \qquad + T_{k-1}'(n_i)[z(n) - z(n_{i-1})] - t_{k-1}, & \text{for } n \in [n_{i-1}, n_i]. \end{cases} \tag{10}$$

Step 4: Solve for Optimal Labor Supply. For each type n_i, solve the individual labor supply problem:

$$\max_z \ln(z - T_{k-1}(z)) - n_i \frac{z^\sigma}{\sigma} \tag{11}$$

The optimal labor supply $z^*(n)$ satisfies the first-order condition:

$$1 - T'_{k-1}(z) = nz^{\sigma-1}\left(z - T_{k-1}(z)\right). \tag{12}$$

Step 5: Update Consumption. Compute consumption for each type as:

$$c(n) = z^*(n) - T_{k-1}(z^*(n)). \tag{13}$$

Step 6: Update the Marginal Tax Rate. Use the discrete approximation of the optimal tax formula (Equation **??**):

$$\frac{T'(z(n))}{1 - T'(z(n))} = \frac{1}{nf(n)}\frac{\sigma}{c(n)}\left[\sum_{n_i \geq n} \pi(n_i)\left\{c(n_i) - \omega(n_i)\sum_{n_j} c(n_j)\pi(n_j)\right\}\right]. \tag{14}$$

Step 7: Ensure Government Budget Balance. Adjust the lump-sum transfer t_k to satisfy the government budget constraint:

$$t_k = \sum_{j=1}^{N} T_k(n_j)\pi(n_j). \tag{15}$$

Step 8: Check Convergence. Repeat Steps 3–7 until the marginal tax rates converge, satisfying:

$$\max_i \left|T'_k(n_i) - T'_{k-1}(n_i)\right| < 10^{-3}. \tag{16}$$

Step 9: Verify Monotonicity. After getting a fixed point of this operator, the only thing to be checked is the monotonicity to guarantee that the found tax schedule is implementable. In this step, we check the monotonicity of $z^*(n)$.

C Distribution of the Maximum Productivity

Let θ and ε be two positive random variables such that

$$\begin{pmatrix} \log(\theta) \\ \log(\varepsilon) \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_n \\ \mu_m \end{pmatrix}, \begin{pmatrix} \sigma_n^2 & \rho\,\sigma_n\sigma_m \\ \rho\,\sigma_n\sigma_m & \sigma_m^2 \end{pmatrix} \right).$$

Define

$$X \;=\; \max(\theta, \varepsilon).$$

We wish to find (i) the CDF $F_{\max}(z) \;=\; P(X \;\leq\; z)$ and (ii) the corresponding PDF $f_{\max}(z)$. By definition of the maximum,

$$F_{\max}(z) \;=\; P\big(X \leq z\big) \;=\; P\big(\max(\theta, \varepsilon) \leq z\big) \;=\; P\big(\theta \leq z,\; \varepsilon \leq z\big).$$

Taking logs, we see:

$$\theta \leq z \quad \Longleftrightarrow \quad \log(\theta) \leq \log(z),$$

and likewise for ε. Denote

$$x_1 \;=\; \frac{\log(z) - \mu_n}{\sigma_n}, \quad x_2 \;=\; \frac{\log(z) - \mu_m}{\sigma_m}.$$

Since $(\log(\theta), \log(\varepsilon))$ is bivariate normal with correlation ρ, we have

$$F_{\max}(z) \;=\; P\big(\log(\theta) \leq \log(z),\; \log(\varepsilon) \leq \log(z)\big) \;=\; \Phi_2\big(x_1, x_2\,; \rho\big),$$

where $\Phi_2(\cdot, \cdot; \rho)$ denotes the CDF of a standard bivariate normal distribution with correlation ρ. To find the PDF, we differentiate $F_{\max}(z)$ with respect to z:

$$f_{\max}(z) \;=\; \frac{d}{dz}\, F_{\max}(z) \;=\; \frac{d}{dz}\, \Phi_2\big(x_1, x_2; \rho\big).$$

Using the chain rule,

$$\frac{d}{dz}\, \Phi_2\big(x_1, x_2; \rho\big) \;=\; \frac{\partial \Phi_2}{\partial x_1} \cdot \frac{dx_1}{dz} \;+\; \frac{\partial \Phi_2}{\partial x_2} \cdot \frac{dx_2}{dz}.$$

Note that

$$x_1 \;=\; \frac{\log(z)-\mu_n}{\sigma_n}, \quad x_2 \;=\; \frac{\log(z)-\mu_m}{\sigma_m},$$

so

$$\frac{dx_1}{dz} \;=\; \frac{1}{\sigma_n}\frac{1}{z}, \qquad \frac{dx_2}{dz} \;=\; \frac{1}{\sigma_m}\frac{1}{z}.$$

It is a standard result that if $\Phi_2(x_1, x_2; \rho)$ is the bivariate standard normal CDF with correlation ρ, then

$$\frac{\partial}{\partial x_1}\,\Phi_2(x_1, x_2; \rho) \;=\; \phi(x_1)\,\Phi\left(\frac{x_2 - \rho\, x_1}{\sqrt{1-\rho^2}}\right),$$

$$\frac{\partial}{\partial x_2}\,\Phi_2(x_1, x_2; \rho) \;=\; \phi(x_2)\,\Phi\left(\frac{x_1 - \rho\, x_2}{\sqrt{1-\rho^2}}\right),$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the univariate standard normal PDF and CDF, respectively. Putting these together, we get

$$\frac{d}{dz}\,\Phi_2\big(x_1, x_2; \rho\big) \;=\; \phi(x_1)\,\Phi\Big(\tfrac{x_2-\rho\,x_1}{\sqrt{1-\rho^2}}\Big)\cdot\frac{dx_1}{dz} \;+\; \phi(x_2)\,\Phi\Big(\tfrac{x_1-\rho\,x_2}{\sqrt{1-\rho^2}}\Big)\cdot\frac{dx_2}{dz}.$$

Since $\frac{dx_1}{dz} = \frac{1}{\sigma_n z}$ and $\frac{dx_2}{dz} = \frac{1}{\sigma_m z}$, the result is:

$$f_{\max}(z) \;=\; \frac{1}{z}\left[\frac{\phi(x_1)}{\sigma_n}\,\Phi\Big(\tfrac{x_2-\rho\,x_1}{\sqrt{1-\rho^2}}\Big) \;+\; \frac{\phi(x_2)}{\sigma_m}\,\Phi\Big(\tfrac{x_1-\rho\,x_2}{\sqrt{1-\rho^2}}\Big)\right],$$

where

$$x_1 = \frac{\log(z)-\mu_n}{\sigma_n}, \quad x_2 = \frac{\log(z)-\mu_m}{\sigma_m}.$$

CDF:

$$F_{\max}(z) \;=\; \Phi_2\left(\frac{\log(z)-\mu_n}{\sigma_n}, \frac{\log(z)-\mu_m}{\sigma_m};\, \rho\right).$$

PDF:

$$f_{\max}(z) = \frac{1}{z}\left[\frac{\phi\left(\frac{\log(z)-\mu_n}{\sigma_n}\right)}{\sigma_n}\,\Phi\left(\frac{\frac{\log(z)-\mu_m}{\sigma_m} - \rho\,\frac{\log(z)-\mu_n}{\sigma_n}}{\sqrt{1-\rho^2}}\right) + \frac{\phi\left(\frac{\log(z)-\mu_m}{\sigma_m}\right)}{\sigma_m}\,\Phi\left(\frac{\frac{\log(z)-\mu_n}{\sigma_n} - \rho\,\frac{\log(z)-\mu_m}{\sigma_m}}{\sqrt{1-\rho^2}}\right)\right].$$

Supplemental Appendix

D A more fundamental distribution of skills

Our characterization of an agent's type is through a bi-dimensional vector representing his productivity in the formal and informal sectors. Of course, how productive an agent is in either sector is not the sole consequence of an innate capacity to work under the rules that prevail in the formal market or under the more informal set of conventions that guide the workings of informal markets. It is rather a combination of each person's abilities and the set of skills that are required to produce the tasks associated with the technologies that are used in each sector. In this section, we show how a model which incorporates these elements is mapped into the simple bi-dimensional model we study here. The advantage of making this connection explicit is that we provide a framework for understanding how changes in the underlying technologies translate into changes in the distribution of types. This is important if we want to account for the heterogeneity in the relative importance of informality across countries and time.

An agents' type is a vector of skills $\boldsymbol{\varpi}$ such that to produce a vector $\boldsymbol{a} = (a_1, ..., a_n)$ of activities, $i = 1, ..., n.$, the agent needs to spend an effort

$$l = \mathcal{m}\left(\frac{a_1}{\varpi_1}, ..., \frac{a_n}{\varpi_n}\right).$$

$\mathcal{m}$ is an aggregator aiming at capturing the fact that some activities are more complementary than others from the perspective of whoever is supplying the effort necessary to undertake it. We assume that$\mathcal{m}$ is homothetic.[10] Let $\boldsymbol{w}_k = (w_1^k, ..., w_n^k)$ be the vector of prices for activities 1 to n in sector k. That is, let w_i^k denote the market price of activity i in sector k. In this case, an agent who supplies $\boldsymbol{a} = (a_1, ..., a_n)$ in sector k earns exactly $z = \sum_i a_i w_i^k$. The effort cost for earning z in sector k is, for an agent with a skill vector $\boldsymbol{\varpi} = (\varpi_1, ..., \varpi_n)$,

$$C(z, \boldsymbol{w}_k|\boldsymbol{\varpi}) := \min_{\{a_i\}_i} M\left(\frac{a_1}{\varpi_1}, ..., \frac{a_n}{\varpi_n}\right)$$

[10] Without loss we let $\mathcal{m}$ be homogeneous of degree one.

subject to

$$\sum_i a_i w_i^k = z.$$

Hence, we may define the agent's productivity in sector k as

$$W_k(z) = \frac{z}{C(z, \boldsymbol{w}_k|\boldsymbol{\varpi})},$$

which, under our homotheticity assumption, leads to

$$n = \frac{1}{C(1, \boldsymbol{w}_F|\boldsymbol{\varpi})} = \frac{1}{\mathcal{C}(\boldsymbol{w}_F|\boldsymbol{\varpi})}, \qquad m = \frac{1}{C(1, \boldsymbol{w}_I|\boldsymbol{\varpi})} = \frac{1}{\mathcal{C}(\boldsymbol{w}_I|\boldsymbol{\varpi})}$$

as the formal and informal sector abilities of such an agent, respectively.

Implementation We first assume the underlying technology to be linear, $Y^k = \sum_{i=1}^n w_i^k A_i^k$ where A_i^k is the aggregate supply of skill i to sector k for $k = F, I$. Note that this assumption is necessary for m and n to remain exogenous.

Assume that there are only to activities, $\boldsymbol{a} = (a_1, a_2)$, and let

$$m\left(\frac{a_1}{\varpi_1}, \frac{a_2}{\varpi_2}\right) := \max\left\{\frac{a_1}{\varpi_1}, \frac{a_2}{\varpi_2}\right\}$$

In this case,

$$n = \varpi_1 w_1^F + \varpi_2 w_2^F \quad \text{and} \quad m = \varpi_1 w_1^I + \varpi_2 w_2^I.$$

In this case, we easily recover the fundamental distribution of skills, through

$$\begin{bmatrix} E[\varpi_1] \\ E[\varpi_2] \end{bmatrix} = \Omega \begin{pmatrix} \mu_n \\ \mu_m \end{pmatrix},$$

and

$$\begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix} = \Omega \begin{bmatrix} \sigma_n^2 & \sigma_{mn} \\ \sigma_{mn} & \sigma_m^2 \end{bmatrix} \Omega^T$$

where

$$\Omega := \begin{bmatrix} w_1^F & w_2^F \\ w_1^I & w_2^I \end{bmatrix}^{-1}.$$

E Extensions

E.1 Consumption Taxes

We have assumed that the planner cannot tax informal agents. While income earned in the informal sector cannot be observed consumption of formal workers can.[11] In this case, we have

Intensive margin choices If a type (n, m) agent decides to work in the formal sector, the utility attained is

$$v(n,m) = \left\{ \max_{c,z} \mathcal{U}\left(c, \frac{z}{n}\right) \quad \text{s.t.,} \quad c[1+\tau_c] \leq z\phi(r) - T(z\phi(r)) \right\},$$

whereas in the informal sector,

$$\underline{v}(n,m) = \max_{c,z} \left\{ \mathcal{U}\left(c, \frac{z}{m}\right) \quad \text{s.t.,} \quad c[1+\tau_c] \leq z\omega(r) + B \right\}.$$

We let $(c_\tau(n,m), z_\tau(n,m))$ denote the solution to the first and $(\underline{c}_\tau(n,m), \underline{z}_\tau(n,m))$, the solution to the second problem.

The Government influences the intensive margin choice of formal workers by manipulating $T(\cdot)$ and τ_c, which ultimately affects consumption and labor supply. However, the introduction of consumption taxes is irrelevant to the implementation of formal allocations *if we hold participation fixed*. Any allocation $(c_\tau(n,m), z_\tau(n,m)$ that can be implemented with $\tau \neq 0$ can be implemented with $\tau = 0$ with a suitable re-normalization of the income tax schedule.

For the informal sector, the combination of a consumption tax and a UBI is equivalent to the introduction of an affine schedule. The Government influences the intensive margin choice of formal workers by manipulating B and τ_c, which ultimately affects consumption and labor supply.

Extensive margin choices Because we have restricted the UBI, B, to be positive, we have given the planner no instruments to directly disincentivize informality. By adding

[11] Consumption of goods produced in the informal sector, in contrast, often may not. We consider this in Appendix E.2.

τ_c to the planner's toolbox, this is made feasible at the cost of introducing inefficiency in the intensive margin of informal workers.

While the analysis with consumption taxes is a straightforward extension of our main assessments, there is an important assumption in our formulation that we want to relax. We are assuming that all goods can be taxed independently of where they were produced. A more interesting possibility arises when goods produced in the formal sector, that can be taxed are not perfect substitutes for goods produced in the formal sector, that cannot be taxed.

E.2 Imperfect Substitute Goods

We have assumed that there is a single composite commodity that affects households' utility. In this section, we retain the assumption that firms are competitive and run a straightforward technology, z_f produces $c_f = z_f$ consumption units of formal goods and z_i produces $c_i = z_i$ units of the informal goods. But, instead of assuming that formal and informal goods are perfect substitutes, we allow for a finite elasticity of substitution.

We normalize the producer price of the informal good to 1 and let p denote the producer price of the formal good. Hence, the earnings of a worker supplying e_f in the formal sector are $z = pe_f$ and the earnings of a worker supplying e_i in the informal sector are $\underline{z} = e_i$.

Let

$$\hat{\phi}(c, q) := \min_{c_i, c_f} c_i + qc_f \quad \text{s.t.,} \quad \phi(c_f, c_i) \geq c,$$

where $q = p + \tau_c$ is the cum taxes price of formal goods, and p, the marginal cost of formal goods. We use $(\hat{c}_f(c), \hat{c}_f(c))$ to denote the solution to the problem above, omitting the dependence on q for convenience. We also define

$$\bar{c}(I, q) := \max_{c_i, c_f} \phi(c_f, c_i) \quad \text{s.t.,} \quad c_i + qc_f \leq I,$$

and the solution $(\bar{c}_f(I, q), \bar{c}_f(I, q))$.

Naturally, $c = \bar{c}(\hat{\phi}(c, q), q)$.

Intensive Margin Choices If a type (n, m) agent decides to work in the formal sector, the utility attained is

$$v(\tilde{n}) = \left\{ \max_{c,z} \mathcal{U}\left(c, \frac{z}{\tilde{n}}\right) \quad \text{s.t.,} \quad \hat{\phi}(c, q_f) \leq z - T(z) \right\},$$

for $\tilde{n} = pn$, whereas in the informal sector,

$$\underline{v}(m) = \max_{c,z} \left\{ \mathcal{U}\left(c, \frac{z}{m}\right) \quad \text{s.t.,} \quad \hat{\phi}(c, q_f) \leq z \right\}.$$

We let $(c(\tilde{n}), z(\tilde{n}))$ denote the solution to the first and $(\underline{c}(m), \underline{z}(m))$, the solution to the second problem. We finally define $\mathtt{y}(q_f, v, z, n)$, the minimum income level necessary to generate utility v for an n-type who is producing z through

$$v = \max_{c} \mathcal{U}\left(c, \frac{z}{\tilde{n}}\right) \quad \text{s.t.,} \quad \hat{\phi}\left(c, q_f\right) = \mathtt{y}(q, v, z, \tilde{n}).$$

Note that we have refrained from explicitly including the dependence on q of v, $\underline{v}$, z, and $\underline{z}$. Whenever this leads to no ambiguity, we keep this omission, similarly use $y(v, z, \tilde{n})$ instead of $\mathtt{y}(q, v, z, \tilde{n})$, and define $\mathtt{c}(q, v, z, \tilde{n}) = \hat{\mathtt{c}}(\mathtt{y}(q, v, z, \tilde{n}), q)$, along with the simplified counterpart $c(v, z, \tilde{n})$.[12]

Extensive Margin Choices As in our baseline model, to work in the formal sector the agent must also incur a utility cost κ. Hence, a type (n, m) utility at his optimal choice is

$$\nu(n, m) = \max\{v(\tilde{n}), \underline{v}(m)\} \tag{17}$$

The threshold function For any n, $\underline{v}(m)$ is a strictly increasing function of m, and that for any m, $v(\tilde{n})$ is a strictly increasing function of n. Assume that, for a given m, there is a (n, m)-type for which $\underline{v}(m) = v(\tilde{n})$, then $\underline{v}(m) < v(\tilde{n}')$ for all $n' > n$ and $\underline{v}(m) > v(\tilde{n}'')$ for all $n'' < n$. For any such m, $\tilde{n}$ defined through $\underline{v}(m) = v(\tilde{n})$ is a participation threshold. Similarly, for any n, for which there is (n, m)-type for which $\underline{v}(m) = v(\tilde{n})$, defines a participation threshold. Note also that since $v(\tilde{n}) = v(pn)$, the

[12]The counterparts, $(\mathtt{v}(\tilde{n}, q), \mathtt{z}(\tilde{n}, q))$ and $(\underline{\mathtt{v}}(m, q), \underline{\mathtt{z}}(m, q)$, are also useful when optimization with respect to q is considered. Otherwise, it is the simplified notation that is used.

threshold for all types will depend on p.[13]

Let the inverse of $\underline{v}(m)$ be $\underline{v}^{-1}(\cdot)$. We then define $\alpha(v(\tilde{n})) := \underline{v}^{-1}(v(\tilde{n}))$, and $\alpha \circ v$ is a threshold function mapping, assigning for each n the maximum informal sector skill m such that the type (n, m) works in the formal sector.

Equivalent Tax Systems The government has two tax instruments here, a consumption tax, τ_c, and a non-linear income tax schedule, $T(\cdot)$. Now, for every q, let $\bar{\tau}_q(p)$ be defined by $q - p = \bar{\tau}_q(p)$, the tax required to hold q fixed as we vary p.

Now assume for the moment that $p = 1$ and let $T(\cdot)$ be the prevailing labor income tax schedule. Then, for every p, define $T_p(\cdot)$ through $T_p(pz) = T(z) - z(1-p)$ for all z. Now consider an agent who is solving

$$\max_z \mathcal{U}\left(\bar{c}\big(z - T(z), q\big), \frac{z}{n}\right),$$

and let the producer price of formal goods change to p, and $T(\cdot)$ be replaced by $T_p(\cdot)$, and τ_c changed to keep q fixed. Then, if we define $\tilde{n} := pn$, the agent's problem becomes

$$\max_z \mathcal{U}\left(\bar{c}\big(z - T_p(z), q\big), \frac{z}{\tilde{n}}\right) = \max_{\tilde{z}} \mathcal{U}\left(\bar{c}\big(p\tilde{z} - T_p(\tilde{z}), q\big), \frac{\tilde{z}}{n}\right)$$
$$= \max_{\tilde{z}} \mathcal{U}\left(\bar{c}\big(\tilde{z} - T(\tilde{z}), q\big), \frac{\tilde{z}}{n}\right).$$

That is, by adjusting τ, the planner can keep q fixed as p varies. Then, if p changes from $p = 1$ to $p = p'$ the same labor supply and consumption choices can be induced if q is held fixed and $T(\cdot)$ is replaced by $T_p(\cdot)$ such that $T_p(z) = T(z) + (p-1)z$ for all z.

E.2.1 Specializing the Model

In all that follows we focus on the case of preferences of the form

$$\mathcal{U}(c, l) = u(c) - \psi(l).$$

[13]The informal utility, in contrast does not depend directly on p, it only depends on the government-controled q.

We may in this case define the planner's program as

$$\max \int_{\underline{n}}^{\bar{n}} \int_{\underline{m}}^{\alpha(v(n))} \varphi\left(v(n)-\kappa\right) g(m|n)dmf(n)dn + \int_{\underline{n}}^{\bar{n}} \int_{\alpha(v(n))}^{\bar{m}} \varphi(\underline{v}(m)) g(m|n)dmf(n)dn$$

s.t.,

$$\dot{v}(n) = \psi'\left(\frac{z(n)}{n}\right)\frac{z(n)}{n^2}, \tag{18}$$

$$z(n) \text{ increasing in } n, \tag{19}$$

$$\int_{\underline{n}}^{\bar{n}} \int_{\underline{m}}^{\alpha(v(n))} \left[z(n) - \hat{c}_f\left(c(v(n), z(n), n)\right)\right] g(m|n)dmf(n)dn \geq \int_{\underline{n}}^{\bar{n}} \int_{\alpha(v(n))}^{\bar{m}} \hat{c}_f\left(\underline{c}(m)\right) g(m|n)dmf(n)dn + R, \tag{20}$$

and

$$\int_{\underline{n}}^{\bar{n}} \int_{\alpha(v(n))}^{\bar{m}} \left[\underline{z}(m) - \hat{c}_i\left(\underline{c}(m)\right)\right] g(m|n)dmf(n)dn \geq \int_{\underline{n}}^{\bar{n}} \int_{\underline{m}}^{\alpha(v(n))} \hat{c}_i\left(c(v(n), z(n), n)\right) g(m|n)dmf(n)dn \tag{21}$$

where, $c(v(n), z(n), n)$ is defined by

$$c(v(n), z(n), n) = u^{-1}\left(v(n) + \psi\left(\frac{z(n)}{n}\right)\right) \quad \forall n.$$

The first order conditions for the planner's program with respect to $v(n)$ and to $z(n)$

are, respectively,

$$0 = G(\alpha(v(n))|n)\varphi'\left(v(n)-\kappa\right)f(n) - \dot{\mu}(n) + \alpha'(v(n))\left\{\lambda_f\left[z(n) - \hat{c}_f\left(c(v(n),z(n),n)\right) + \hat{c}_f\left(\underline{c}(\alpha(v(n)))\right)\right] - \lambda_i\left[\underline{z}(\alpha(v(n))) - \hat{c}_i\left(\underline{c}(\alpha(v(n)))\right) + \hat{c}_i\left(c(v(n),z(n),n)\right)\right]\right\} g(\alpha(v(n))|n)f(n) - \left[\lambda_f\hat{c}_f'\left(c(v(n),z(n),n)\right) + \lambda_i\hat{c}_i'\left(c(v(n),z(n),n)\right)\right] c_v(v(n),z(n),n)G(\alpha(v(n))|n)f(n), \tag{22}$$

and

$$0 = \mu(n)\psi''\left(\frac{z(n)}{n}\right)\frac{z(n)}{n^3} + \Big\{\lambda_f - \Big[\lambda_f\hat{c}_f'\left(c(v(n),z(n),n)\right) + \lambda_i\hat{c}_i'\left(c(v(n),z(n),n)\right)\Big]c_z(v(n),z(n),n)\Big\}G(\alpha(v(n))|n)f(n) \tag{23}$$

with complementary slackness

$$\gamma(n)[z(n) - z_{\min}] = 0. \tag{24}$$

In what follows, to facilitate the algebra we define

$$\Pi(n) := \left[z(n) - \hat{c}_f\left(c(v(n),z(n),n)\right) + \hat{c}_f\left(\underline{c}(\alpha(v(n)))\right)\right] - \frac{\lambda_i}{\lambda_f}\left[\underline{z}(\alpha(v(n))) - \hat{c}_i\left(\underline{c}(\alpha(v(n)))\right) + \hat{c}_i\left(c(v(n),z(n),n)\right)\right]$$

and

$$\Psi(n) := \hat{c}_f'\left(c(v(n),z(n),n)\right) + \frac{\lambda_i}{\lambda_f}\hat{c}_i'\left(c(v(n),z(n),n)\right),$$

which allows us to simplify the first order conditions to

$$\gamma(n) = G(\alpha(v(n))|n)\varphi'\left(v(n)-\kappa\right)f(n) - \dot{\mu}(n) + \lambda_f\left[\alpha'(v(n))\Pi(n)g(\alpha(v(n))|n) - \Psi(n)c_v(v(n),z(n),n)G(\alpha(v(n))|n)\right]f(n), \tag{25}$$

and

$$0 = \mu(n)\psi''\left(\frac{z(n)}{n}\right)\frac{z(n)}{n^3} + \lambda_f\left\{1 - \Psi(n)c_z(v(n),z(n),n)\right\}G(\alpha(v(n))|n)f(n). \tag{26}$$

E.3 Minimum Wage

In our baseline economy, we consider that workers in the formal sector cannot earn less than a value stipulated in the form of a minimum wage. To assess the optimal policy we dropped this constraint while allowing a minimum wage to arise at the solution *if it turns out to be optimal*.

Here we impose a minimum wage as an additional constraint, possibly due to political restrictions. That is we add a constraint,

$$z(n) \geq z_{\min}, \tag{27}$$

to the planner's program from the section E.2.1.

From the monotonicity condition, there is a value $n_{\min}$ such that $z(n) - z_{\min} > 0$ for all $n > n_{\min}$. We first characterize the labor wedge for $n \in (n_{\min}, \bar{n}]$, then we characterize the labor wedge for those who earn the minimum wage.

Note that since we have consumption taxes, the labor wedge is not necessarily the same as the marginal tax rate. Indeed, using the definition for the labor wedge as the difference between the marginal rate of transformation and the marginal rate of substitution between labor and consumption, we have $t(n) = 1 - c_z(v(n), z(n), n)\hat{\phi}_c(c(n), 1)$, whereas with $\tau_c \neq 0$ labor wedges are not, in general, equal to marginal tax rates, since

$$1 - T'(z(n)) = c_z(v(n), z(n), n)\hat{\phi}_c(c(n), q) = [1 - t(n)]\frac{\hat{\phi}_c(c(n), q)}{\hat{\phi}_c(c(n), 1)}. \tag{28}$$

Both $\hat{\phi}_c(\cdot, 1)$ and $\hat{\phi}_c(\cdot, 1)$ are known. Therefore, we focus on the determination of $c_z(v(n), z(n), n)$. For $n \in (n_{\min}, \bar{n}]$, $\gamma(n) = 0$ and we have

$$\frac{\mu(n)}{\lambda_f}\psi'\left(\frac{z(n)}{n}\right)\frac{\varepsilon(z(n), n)}{n^2} = -\left\{1 - \Psi(n)c_z(v(n), z(n), n)\right\} G(\alpha(v(n))|n)f(n), \tag{29}$$

or

$$\frac{\mu(n)}{\lambda_f}c_z(v(n), z(n), n) = -\frac{\left\{1 - \Psi(n)c_z(v(n), z(n), n)\right\}}{\varepsilon(z(n), n)u'(c(n))}G(\alpha(v(n))|n)f(n)n. \tag{30}$$

Using (28)) we find that for $n \in [n_{\min}, \bar{n}]$,

$$\frac{T'(z(n))}{1-T'(z(n))} = \frac{\mu(n)\varepsilon(z(n),n)u'(c(n))}{\lambda_f \hat{\phi}_c(c(n),q_f)G(\alpha(v(n))|n)f(n)n} + \Psi(n) - \hat{\phi}_c(c(n),q_f). \tag{31}$$

For $n \in [\underline{n}, n_{\min}]$, $T'(z(n)) = T'(z_{\min})$, however, the labor wedge defined in (28)) varies with n.

Hence, it all boils down to finding an expression for $\mu(n)$. For any $n \in (z_{\min}, \bar{z})$, we can integrate (25) from n to $\bar{n}$ to obtain an expression for $\mu(n)$,

$$\mu(n) = -\int_n^{\bar{n}} G(\alpha(v(t))|t)\varphi'\left(v(t)-\kappa\right) f(t)dt + \lambda_f \int_n^{\bar{n}} \Big\{\alpha'(v(t))\Pi(t)g(\alpha(v(t))|t) - \Psi(t)c_v(v(t),z(t),t)G(\alpha(v(t))|t)\Big\} f(t)dt. \tag{32}$$

This naturally defines $\mu(n_{\min})$ as well. For $v_{\min} = \lim_{n\to n_{\min}} v(n)$, we define $c_{\min}$ through $v_{\min} = \mathcal{U}(c_{\min}, z_{\min}/n_{\min})$, and use this to define $v(n) = \mathcal{U}(c_{\min}, z_{\min}/n)$ for $n \in [\underline{n}, n_{\min}]$. With this understanding, we expand μ to this region through

$$\begin{aligned}\mu(n) = \mu(n_{\min}) &- \int_n^{n_{\min}} G(\alpha(v(t))|t)\varphi'\left(v(t)-\kappa\right) f(t)dt \\ &+ \lambda_f \int_n^{n_{\min}} \Big\{\alpha'(v(t))\Pi(t)g(\alpha(v(t))|t) - \Psi(t)c_v(v(t),z_{\min},t)G(\alpha(v(t))|t)\Big\} f(t)dt,\end{aligned} \tag{33}$$

noting that, with separable preferences, $c_v(v(t), z_{\min}, t) = 1/u'(c_{\min})$ for all $t \in [\underline{n}, n_{\min}]$.

In possession of $\mu(n)$, we recover $\gamma(n)$ using

$$\mu(n)\psi''\left(\frac{z_{\min}}{n}\right)\frac{z_{\min}}{n^3} + \lambda_f \left\{1 - \Psi(n)c_z(v(n), z_{\min}, n)\right\} G(\alpha(v(n))|n)f(n) = \gamma(n) \tag{34}$$

Integrating (25) with respect to n and using the boundary conditions $\mu(\underline{n}) = \mu(\bar{n}) = 0$, gives an expression for λ_f,

$$\lambda_f = \frac{\int_{\underline{n}}^{\bar{n}} G(\alpha(v(n))|n)\varphi'\left(v(n)-\kappa\right) f(n)dn}{\int_{\underline{n}}^{\bar{n}} \Big\{\alpha'(v(n))\Pi(n)g(\alpha(v(n))|n) - \Psi(n)c_v(v(n),z(n),n)G(\alpha(v(n))|n)\Big\} f(n)dn.} \tag{35}$$

Finally, note that

$$\frac{\lambda_i}{\lambda_f} = \frac{\int_{\underline{n}}^{\bar{n}} \left[\hat{c}'_f(c(v(n), z(n), n)) - 1\right] c_z(c(v(n), z(n), n)) G(\alpha(v(n))|n) f(n) dn}{\int_{\underline{n}}^{\bar{n}} \hat{c}'_i(c(v(n), z(n), n)) c_z(c(v(n), z(n), n)) G(\alpha(v(n))|n) f(n) dn}.$$

Using (28)) we find that for $n \in [n_{\min}, \bar{n}]$,

$$\begin{aligned}\frac{T'(z(n))}{1 - T'(z(n))} = \frac{-\varepsilon(z(n), n) u'(c(n))}{\hat{\phi}_c(c(n), q_f) G(\alpha(v(n))|n) f(n) n} \Bigg\{ & \int_n^{\bar{n}} G(\alpha(v(t))|t) \varphi' \left(v(t) - \kappa\right) f(t) dt \\ & - \lambda_f \int_n^{\bar{n}} \Bigg[\alpha'(v(t)) \Pi(t) g(\alpha(v(t))|t) - \\ & \frac{\Psi(t)}{u'(c(t))} G(\alpha(v(t))|t) \Bigg] f(t) dt \Bigg\} - \Psi(n) + \hat{\phi}_c(c(n), q). \quad (36)\end{aligned}$$