Cyclical Returns to Scale and the Slopes of the Phillips Curves*

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Abstract

This paper shows that more procyclical returns to scale in recent decades have flattened the Phillips curve when conventional activity measures, such as the output gap, labor gap, and labor share, are used as forcing variables. In contrast, the marginal cost Phillips curve remains steep. Using a simple, intuitive model with a translog production function, we illustrate a novel channel linking input complementarity and procyclical returns to scale to the identified slopes of the Phillips curves. By utilizing the estimated production functions and quantitative models, we emphasize the importance of our mechanism for rationalizing the US inflation data.

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1 Introduction

Stable inflation dynamics and substantial business cycle fluctuations in the first two decades of the 21st century sparked a reexamination of the relationship between inflation and economic activity, i.e., the Phillips curve. A series of recent studies have documented two divergent empirical results: the Phillips curve based on marginal costs is steep and alive (Gagliardone et al., 2023), and versions based on conventional measures of economic activity, such as the output gap, labor gap, and labor shares, have flattened (Stock and Watson, 2007, 2020; Del Negro et al., 2020; Inoue, Rossi and Wang, 2024). We propose a theory that reconciles these two seemingly contradictory empirical patterns. Furthermore, our quantitative analyses predict the growing significance of inflation expectations in understanding recent US inflation data and the Phillips curve, echoing the insights from Coibion and Gorodnichenko (2015), Hazell et al. (2022), and Meeks and Monti (2023).

The commonly used Cobb–Douglas (Galí and Gertler, 1999; Sbordone, 2002; Eichenbaum and Fisher, 2007) and constant elasticity of substitution (CES, Gagnon and Khan, 2005; McAdam and Willman, 2013) production functions in the study of Phillips curves feature constant returns to scale. Switching from those tightly parameterized functional forms to a more flexible translog production function (Christensen, Jorgenson and Lau, 1973; De Loecker and Warzynski, 2012; Hyun, Kim and Lee, 2024) introduces cyclical variations around the long-term value in returns to scale.[1] This novel cyclical fluctuation in returns to scale introduces a wedge between conventional measures of economic activities and marginal costs.[2] Depending on the cyclicality of this wedge, the slopes of the Phillips curves using conventional activity measures as forcing variables could deviate from that of the marginal cost Phillips curve. We show that returns to scale have been more procyclical in recent decades, and this change has resulted in the flattening of the conventional Phillips curves. In contrast, the marginal cost Phillips curve is still steep despite decades-long highly sta-

[1] For studies assuming translog functional forms for the demand side of the economy, see, e.g., Christensen, Jorgenson and Lau (1975); Bergin and Feenstra (2000); Feenstra (2003); Bilbiie, Ghironi and Melitz (2012); Lewis and Poilly (2012); Bilbiie, Fujiwara and Ghironi (2014); Fujiwara and Matsuyama (2023).

[2] The concept of time-varying returns to scale has recently been explored in relation to the decline in information technology prices (Lashkari, Bauer and Boussard, 2024), capital misallocation (Chan et al., 2024), and the complementarity between labor and energy (Hyun, Kim and Lee, 2024); these studies employed a non-homothetic CES production function, an exogenously determined returns-to-scale process, and a translog production function, respectively. In this paper, we utilize a translog function, following Hyun, Kim and Lee (2024), to investigate the implications of endogenously cyclical returns to scale that arise from the complementarity and substitutability among capital, skilled labor, and unskilled labor.

ble inflation before the recent pandemic, exemplified by missing disinflation and reinflation episodes (Coibion and Gorodnichenko, 2015; Ball and Mazumder, 2020).

We illustrate this mechanism using analytical expressions and graphical representations based on a simple model that incorporates a translog production function in an otherwise baseline New Keynesian model (see, e.g., Woodford, 2003; Galí, 2015). We show that the translog model nests the Cobb–Douglas and CES models in the log-linearization of the equilibrium conditions. Furthermore, relaxing the restrictive Cobb–Douglas and CES functional forms introduces cyclical fluctuations in returns to scale. When factor inputs are complementary, the productivity of one input increases with the use of the other and vice versa. Thus, when inputs are used more than usual, the overall marginal productivity can rise, leading to an increase in returns to scale.

This cyclical fluctuation in returns to scale introduces a novel element in the marginal costs of production in the translog model. In contrast, in the Cobb–Douglas and CES models, the marginal costs consist only of total factor productivity (TFP) and factor prices such as wages. Because conventional measures of economic activity, such as the output gap, labor gap, and labor shares, primarily reflect factor prices and TFP, returns to scale become an omitted variable in conventional Phillips curve regressions.[3]

The Phillips curves are often estimated using instruments for aggregate demand, such as identified monetary policy shocks, to address potential endogeneity due to mismeasurement and price markup shocks (see, e.g., Mavroeidis, Plagborg-Møller and Stock, 2014; Coibion, Gorodnichenko and Kamdar, 2018; McLeay and Tenreyro, 2020; Barnichon and Mesters, 2020, 2021; Inoue, Rossi and Wang, 2024).[4] However, even when using instruments, the estimated slopes of the conventional Phillips curves could still be affected by omitted variable biases if returns to scale are cyclical conditional on monetary policy shocks. In contrast, the marginal cost Philips curve is not subject to this confounding factor because marginal costs already account for variations in returns to scale. Therefore, if the cyclicality of returns to scale and the resulting magnitude of the omitted variable bias change over time, the

[3]See, e.g., Galí and Gertler (1999); Sbordone (2002); Eichenbaum and Fisher (2007) for empirical studies employing labor shares as a forcing variable in the Phillips curve.

[4]Other intriguing approaches in the literature include introducing alternative measures of inflation and slack (Stock and Watson, 2010, 2020; Ball and Mazumder, 2011, 2019, 2020), using regional variations (Kiley, 2015; Hooper, Mishkin and Sufi, 2020; McLeay and Tenreyro, 2020; Hazell et al., 2022; Fitzgerald et al., 2024), and relying on sectoral and firm-level data (Imbs, Jondeau and Pelgrin, 2011; Gagliardone et al., 2023). In our quantitative analyses, we take a different route of structural model-based inferences following An and Schorfheide (2007); Schorfheide (2011); Del Negro et al. (2020), which is complementary to the limited information approaches mentioned above.

conventional Phillips curves could flatten without a corresponding change in the marginal cost Phillips curve.[5]

We examine the relevance of this novel mechanism to US inflation dynamics through the lens of medium-scale dynamic stochastic general equilibrium (DSGE) models. We augment the partial equilibrium, four-input, nested CES production model studied in Krusell et al. (2000) and Ohanian, Orak and Shen (2023) with a normalized translog production function proposed by Hyun, Kim and Lee (2024). Firms use structure, equipment, skilled labor, and unskilled labor for production. We combine this production block of the economy with standard elements of quantitative New Keynesian general equilibrium models known to be useful in explaining US time series data, such as sticky prices and wages, fixed costs of production, investment adjustment costs, costly capacity utilization, and consumption habits (see, e.g., Christiano, Eichenbaum and Evans, 2005; Smets and Wouters, 2007).

We consider three models based on the Cobb–Douglas, nested CES, and translog production functions. We estimate these models separately for two different time periods using an early sample (1966-99) and a late sample (2000-19). To examine structural changes, we focus on production function parameters, the cyclicality of returns to scale, and the slopes of the Phillips curves with different forcing variables. When estimating the models, we employ additional data to the standard aggregate time series in light of the two types of capital (structures and equipment) and labor (skilled and unskilled) in the models. Specifically, we use equipment stock, relative labor hours and wage rates between skilled and unskilled workers.

Our quantitative results are summarized as follows. First, the translog model better matches the US macroeconomic data than the other models. Second, when estimating the unrestricted translog model, we reject the parameter restrictions imposed by the Cobb–Douglas and nested CES models. Third, by comparing the estimated models using the pre- and post-2000 data, we find that the translog model can jointly replicate the steep marginal cost Phillips curve and the flattening of the conventional Phillips curves. Fourth, the mechanism connecting the cyclical returns to scale to marginal costs and inflation, embedded in the translog model, plays a crucial role in this result. In contrast, the Cobb–Douglas and nested CES models without cyclical returns to scale rely on unrealistically sticky prices and quite

[5]This rationalization of the flattening of the Phillips curves for conventional measures of (inverse) slack echoes the findings of Atkeson and Ohanian (2001) that the relationship between inflation and measures of slack may not be stable. This instability could be the case in our framework even when the slope of the marginal cost Phillips curve is stable, as shown in Section 2.

flat Phillips curves to match the stable inflation data after 2000. Furthermore, the translog model predicts the increased importance of inflation expectations in understanding inflation data and the Phillips curve. Finally, the historical decomposition of pandemic-era inflation data using the translog model suggests that high inflation in 2021 and 2022 was primarily driven by loose monetary policies and supply-side disturbances. This result emphasizes that monetary policies are still highly relevant to inflation despite the recent flattening of conventional Phillips curves. In contrast, the Cobb–Douglas and CES models do not predict this policy implication. With their given quite flat Phillips curves, these models primarily rely on price markup shocks and shifts in the Phillips curves to explain inflation fluctuations.

We also estimate production function parameters using multicountry industry-level panel data and instrumental variable (IV) methods as described by Hyun, Kim and Lee (2024). Our estimates using the pre- and post-2000 data align with the structural estimation results based on the translog DSGE models, providing additional empirical evidence of stronger input complementarity in recent decades than in earlier decades.

The remainder of this paper is organized as follows. Section 2 explains the mechanism behind the flattening of the Phillips curves when conventional measures of activity, such as the output gap, labor gap, and labor shares, are used as forcing variables. Using a simple tractable model with a translog production function, we illustrate the underlying intuition and introduce a novel channel linking input complementarity and procyclical returns to scale to the identified slopes of the Phillips curves. Building on this insight, Section 3 constructs a medium-scale DSGE model with a translog production function for quantitative analyses. We estimate the model parameters using Bayesian methods. Next, we investigate the slopes of the Phillips curves; the relationships among the returns to scale, marginal costs, and inflation; and the inflation dynamics under the model and during the pandemic period. Section 4 presents empirical evidence for stronger input complementarity in recent decades using multicountry industry-level panel data. Section 5 concludes the paper.

2 Illustrative Model

This section illustrates the mechanism connecting input complementarity and cyclical returns to scale with the slopes of the Phillips curves. This illustration is based on a flexible translog production function (Christensen, Jorgenson and Lau, 1973; Hyun, Kim and Lee, 2024) incorporated into an otherwise baseline New Keynesian model (see, e.g., Woodford, 2003;

Galí, 2015). This simple tractable model facilitates an intuitive exposition of the novel channel linking input complementarity to the slopes of the Phillips curves.

We describe the model below, focusing on new terms in equilibrium conditions arising from more general production functions than Cobb–Douglas functions. The derivations of standard model elements are kept brief.

Firms. Intermediate goods and their producers are indexed by $i \in [0,1]$. Firms operate in a monopolistically competitive environment. The final goods are produced by combining intermediate goods in a Dixit and Stiglitz (1977) manner. Price-setting friction of the Calvo (1983) type leads to the following New Keynesian Phillips curve in log-linearization:

$$\hat{\pi}_t = \beta \mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa \hat{mc}_t + \varepsilon^p_t, \tag{2.1}$$

where π, mc, and ε^p represent inflation, real marginal costs, and an exogenous component in price markup, respectively. Hats are used to denote log deviations from the steady-state values of the corresponding variables. The slope of the marginal cost Phillips curve, κ, is given by $\frac{(1-\beta\zeta)(1-\zeta)}{\zeta}$, where β is the discount factor and $1-\zeta$ is the probability of resetting the price of an intermediate good in each period.

Next, we discuss the production function. We drop the firm and time indices, i and t, when they are unnecessary. Suppose that intermediate goods are produced using two types of labor, l_1 and l_2, such that $y = \exp(\varepsilon^a) f(l_1, l_2)$, where ε^a represents an exogenously determined level of TFP. f is a concave and increasing function in each input locally around the steady state, $(\bar{l}_1, \bar{l}_2)$, where a bar denotes the steady-state value of a variable. We introduce two types of labor because the concept of input complementarity is not natural when a production function has only a single input. By focusing on a symmetric equilibrium, where $l_1 = l_2$, we write the equilibrium conditions in terms of the aggregate labor, $l = l_1 + l_2$, similar to the baseline New Keynesian models. Note that this modeling assumption is only for illustrative purposes. We examine the input complementarity and substitutability among equipment, skilled labor, and unskilled labor in Section 3 for the quantitative analyses.

We consider a second-order Taylor expansion of f in the logarithm. Suppose that $\alpha_j = \frac{\partial \log y}{\partial \log l_j} = 0.5$ in the steady state for $j = 1, 2$. Similarly, the second-order derivatives are denoted by $\beta_{jk} = \frac{\partial^2 \log y}{\partial \log l_j \partial \log l_k}$ in the steady state for $j, k = 1, 2$. Thus:

$$\hat{y} = \varepsilon^a + \alpha_1 \hat{l}_1 + \alpha_2 \hat{l}_2 + 0.5\beta_{11}(\hat{l}_1)^2 + \beta_{12}\hat{l}_1\hat{l}_2 + 0.5\beta_{22}(\hat{l}_2)^2. \tag{2.2}$$

Table 1: Comparison of different production functions

	Cobb–Douglas	CES	Translog
Production function	$y = l_1^{\alpha_1} l_2^{\alpha_2}$	$\frac{y}{\bar{y}} = \left[\alpha_1 \left(\frac{l_1}{\bar{l}_1}\right)^{\phi} + \alpha_2 \left(\frac{l_2}{\bar{l}_2}\right)^{\phi}\right]^{1/\phi}$	$y = f(l_1, l_2)$
Log deviation form	$\hat{y} = \alpha_1 \hat{l}_1 + \alpha_2 \hat{l}_2$	$\hat{y} = \alpha_1 \hat{l}_1 + \alpha_2 \hat{l}_2 + 0.5\alpha_1\alpha_2\phi\left[(\hat{l}_1)^2 - 2\hat{l}_1\hat{l}_2 + (\hat{l}_2)^2\right]$	$\hat{y} = \alpha_1 \hat{l}_1 + \alpha_2 \hat{l}_2 + 0.5\beta_{11}(\hat{l}_1)^2 + \beta_{12}\hat{l}_1\hat{l}_2 + 0.5\beta_{22}(\hat{l}_2)^2$
Elast. of output w.r.t. l_j	$\frac{\partial \log y}{\partial \log l_j} = \alpha_j$	$\frac{\partial \log y}{\partial \log l_j} = \alpha_j + \alpha_1\alpha_2\phi(\hat{l}_j - \hat{l}_k),\ j \neq k$	$\frac{\partial \log y}{\partial \log l_j} = \alpha_j + \beta_{jj}\hat{l}_j + \beta_{jk}\hat{l}_k,\ j \neq k$
Returns to scale	$rts = 1$	$rts = 1$	$rts = 1 + (\beta_{11} + \beta_{12})\hat{l}_1 + (\beta_{12} + \beta_{22})\hat{l}_2$

Notes: We ignore the TFP term, ε^a, in this table. The returns to scale are defined as the scale elasticity, i.e., $\frac{\partial \log f(\lambda l_1, \lambda l_2)}{\partial \log \lambda}|_{\lambda=1}$. The CES production function is written in the deviation form following Cantore and Levine (2012). For the CES and translog functions, we ignore the higher-order terms that degenerate when the equilibrium conditions are log-linearized.

Clearly, at the first order, f is proportional to a conventional Cobb–Douglas production function, $l_1^{\alpha_1} l_2^{\alpha_2}$, or l because of the symmetry ($l_1 = l_2$). However, the novelty in our theory arises from the second-order properties of the production function, captured by the translog parameters, β_{jk}.

To illustrate the workings of the translog parameters, we compare Equation (2.2) with the Cobb–Douglas and CES production functions. Table 1 shows the key differences among the three functions, ignoring the TFP and the higher-order terms that degenerate when the equilibrium conditions are log-linearized. The Cobb–Douglas function is log-linear. Thus, the elasticity of output with respect to each input, α_j, and the returns to scale, $\alpha_1 + \alpha_2$, are constant. For the CES function, one more parameter, $\phi \leq 1$, governs the coefficients on the second-order terms. At the second order, we can convert the CES function to a translog form with parameter restrictions such that $\beta_{11} = \beta_{22} = \alpha_1\alpha_2\phi$ and $\beta_{12} = -\beta_{11}$. When $\phi < 0$, the two inputs are complementary in the sense that $\beta_{12} > 0$. In this case, the elasticity of the output with respect to l_1 increases in l_2, and vice versa. However, $\frac{\partial \log y}{\partial \log l_1}$ decreases in l_1, balancing the productivity gain from the input complementarity and l_2 and making the returns to scale constant. The translog specification relaxes the restrictions on β_{jk}s. Note that without the restriction that $\beta_{11} = -\beta_{12}$, the input complementarity ($\beta_{12} > 0$) does not necessarily imply that $\frac{\partial \log y}{\partial \log l_1}$ decreases in l_1.

One of the most intriguing implications of this property is that the returns to scale can vary over time under the translog production function. When defined as the scale elasticity,

$\frac{\partial \log f(\lambda l_1, \lambda l_2)}{\partial \log \lambda}|_{\lambda=1}$, the returns to scale are given by:

$$\begin{aligned} rts_t &= 1 + (\beta_{11} + \beta_{12})\hat{l}_{1t} + (\beta_{12} + \beta_{22})\hat{l}_{2t} \\ &= 1 + (\beta_{11} + 2\beta_{12} + \beta_{22})\hat{l}_t, \end{aligned} \tag{2.3}$$

where the second line uses the symmetry between the two types of labor. For example, suppose that $\beta_{11} = \beta_{22} = 0$, as in the Cobb–Douglas case, the two types of labor are complementary ($\beta_{12} > 0$), and the aggregate labor is procyclical. In this situation, the returns to scale are also procyclical, fluctuating around the long-term value of one (see also Hyun, Kim and Lee, 2024, for empirical evidence for procyclical returns to scale). Thus, the production function (2.2) features constant returns to scale in the long run but allows for short-run variations. In contrast, for the Cobb–Douglas and CES functions, rts_t simplifies to one because the parameter restrictions imply that $\beta_{11} + \beta_{12} = \beta_{12} + \beta_{22} = 0$ in both cases.

To examine the effects of time-varying returns to scale on marginal costs, we consider the cost minimization problem of an intermediate goods producer:

$$\min_{l_1, l_2} w_1 l_1 + w_2 l_2 - mc\left[\exp(\varepsilon^a) f(l_1, l_2) - y\right],$$

where w_j is the real wage rate for type j labor for $j = 1, 2$. The first-order conditions (FOCs) imply that, in log-linearization, the aggregate marginal costs equal:

$$\hat{mc}_t = \underbrace{\hat{w}_t}_{\text{factor prices}} - \underbrace{\varepsilon^a_t}_{\text{tfp}} - \underbrace{\widehat{rts}_t}_{\text{returns to scale}}, \tag{2.4}$$

where w_t is the aggregate wage rate and $\widehat{rts}_t = (\beta_{11} + 2\beta_{12} + \beta_{22})\hat{l}_t$.[6] The first two terms in Equation (2.4), reflecting the factor prices and TFP, correspond to the marginal costs in the baseline New Keynesian models with the Cobb–Douglas or CES production functions that assume constant returns to scale. In contrast, the translog specification generates a novel element–the cyclical returns to scale–in the marginal cost. Again, suppose that aggregate labor is procyclical, $\beta_{11} = \beta_{12} = 0$, and $\beta_{12} > 0$. In this case, the marginal productivity gain from the input complementarity, along with a larger-than-usual labor input ($\hat{l} > 0$),

[6] The FOCs are given by $w_j = mc \exp(\varepsilon^a) f_j = mc \exp(\varepsilon^a) \frac{\partial \log f}{\partial \log l_j} \frac{f}{l_j} = mc \frac{y}{l_j} \frac{\partial \log f}{\partial \log l_j}$ for $j = 1, 2$. Because of the symmetry, the aggregate wage rate, w, equals $w_1 = w_2$ and, similarly, $\hat{l} = \hat{l}_1 = \hat{l}_2$. Then, Equation (2.4) follows from the fact that $\hat{y} = \varepsilon^a + \hat{l}$ in log-linearization.

decreases the marginal costs in expansions. This mechanism is captured by the procyclical variation in the returns to scale in Equation (2.4).

Households. To further simplify the expression for the marginal costs in Equation (2.4), we specify the household side of the economy. The period utility function of the type j worker is given by $\log c_j - \frac{h_j^{1+1/\sigma_l}}{1+1/\sigma_l}$, where c_j and h_j are the consumption and hours of a type j worker, respectively. The aggregate type j labor input, l_j, equals $0.5h_j$ because the population share of each type is 0.5. σ_l is the Frisch elasticity of the labor supply. Suppose that the labor market is competitive. Using the symmetry, the market clearing condition ($y = c$), and the labor supply schedule ($\text{-}\hat{c}_j + \hat{w}_j = \frac{1}{\sigma_l}\hat{h}_j$ for $j = 1, 2$), it follows that $\hat{w} - \varepsilon^a = (1 + \frac{1}{\sigma_l})\hat{l}$. Thus, Equation (2.4) implies the following:

$$\hat{mc}_t = \left(1 + \frac{1}{\sigma_l}\right)\hat{l}_t - \widehat{rts}_t = \left[1 + \frac{1}{\sigma_l} - (\beta_{11} + 2\beta_{12} + \beta_{22})\right]\hat{l}_t. \tag{2.5}$$

Phillips curves. From the marginal cost Phillips curve (2.1) and the expression for the marginal costs (2.5), we obtain the following labor Phillips curve:

$$\begin{aligned}\hat{\pi}_t &= \beta\mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa\hat{mc}_t + \varepsilon_t^p \\ &= \beta\mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa\left[1 + \frac{1}{\sigma_l} - (\beta_{11} + 2\beta_{12} + \beta_{22})\right]\hat{l}_t + \varepsilon_t^p.\end{aligned}$$

Suppose that $\beta_{11} = \beta_{22} = 0$, as in the Cobb–Douglas model. Then, according to the above equation, stronger input complementarity ($\beta_{12} > 0$) and the resulting procyclical returns to scale yield flatter labor Phillips curves. The following proposition and remark formalize this observation.

Proposition 1 (Phillips curves with cyclical returns to scale)**.** *Assume the model structure discussed in this section. For different measures of economic activity, such as the marginal cost, output gap, labor gap, and labor shares, the following hold:*

$$\begin{aligned}
&\textit{marginal cost Phillips curve:} && \hat{\pi}_t = \beta\mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa\hat{mc}_t + \varepsilon_t^p, \\
&\textit{output gap Phillips curve:} && \hat{\pi}_t = \beta\mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa(1 + 1/\sigma_l)\,\textit{output gap}_t + \varepsilon_t^p - \kappa\widehat{rts}_t, \\
&\textit{labor gap Phillips curve:} && \hat{\pi}_t = \beta\mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa(1 + 1/\sigma_l)\,\textit{labor gap}_t + \varepsilon_t^p - \kappa\widehat{rts}_t, \\
&\textit{labor share Phillips curve:} && \hat{\pi}_t = \beta\mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa\widehat{\textit{labor share}}_t + \varepsilon_t^p - \kappa\widehat{rts}_t.
\end{aligned}$$

Proof. We use $*$ to denote variables under flexible price equilibrium. Note that $\hat{mc}_t^* = 0$. Thus, Equation (2.5) implies that $\hat{l}_t^* = 0$. Furthermore, output $\text{gap}_t = \hat{y}_t - \hat{y}_t^* = (\varepsilon_t^a + \hat{l}_t) - (\varepsilon_t^a + \hat{l}_t^*) = \hat{l}_t$. Similarly, labor $\text{gap}_t = \hat{l}_t - \hat{l}_t^* = \hat{l}_t$. Then, Equation (2.5) and the marginal cost Phillips curve yield the output gap and labor gap Phillips curves above. Finally, for the labor share Phillips curve, note that $\widehat{\text{labor share}}_t = \hat{w}_t + \hat{l}_t - \hat{y}_t = \hat{w}_t - \varepsilon_t^a = \hat{mc}_t + \widehat{rts}_t$ because of Equation (2.4). □

Remark 1. In practice, when estimating a Phillips curve, IVs that are orthogonal to ε_t^p can be used to address potential endogeneity arising from supply-side disturbances (see Mavroeidis, Plagborg-Møller and Stock, 2014; Coibion, Gorodnichenko and Kamdar, 2018; Barnichon and Mesters, 2020, 2021; McLeay and Tenreyro, 2020; Inoue, Rossi and Wang, 2024). However, even in this case, if conventional measures of economic activity other than the marginal costs are used as forcing variables, the returns to scale term might behave as an omitted variable in regression equations. If the activity measure is procyclical conditional on the variations captured by the instruments, the procyclical returns to scale could induce a downward omitted variable bias in the estimated slope of the Phillips curves. Finally, if the returns to scale have become more procyclical in recent decades, the omitted variable bias in the identified slope would be more negative, leading to the flattening of the Phillips curves when the conventional activity measures are employed. Note that this prediction holds even when the "true" slope of the marginal cost Phillips curve, κ, remains large (see Gagliardone et al., 2023). Furthermore, this prediction is supported by data; we show that the returns to scale have recently become more procyclical using aggregate time series data in Section 3.

Figure 1 graphically illustrates this mechanism. Assume that $\sigma_l = \infty$ for exposition so that the Phillips curves in Proposition 1 share the same slope, κ. The Phillips curve, conditional on the expected inflation and cost-push shocks, is upward sloping on the economic activity and inflation planes. Thus, the Phillips curve serves as the aggregate supply curve, labeled AS in Figure 1. Note that estimating the slope of this curve is analogous to identifying the slope of the supply curve in the supply–demand simultaneous equation system. As emphasized by McLeay and Tenreyro (2020), the data consist of the equilibrium pairs of inflation and economic activity levels, such as point E. To properly identify κ, we need to use demand shifters, such as the monetary policy shocks employed in Barnichon and Mesters (2020, 2021), as instruments. The identification assumption is that the AD curve shifts to AD', whereas the AS curve remains the same; therefore, comparing the two equilibrium

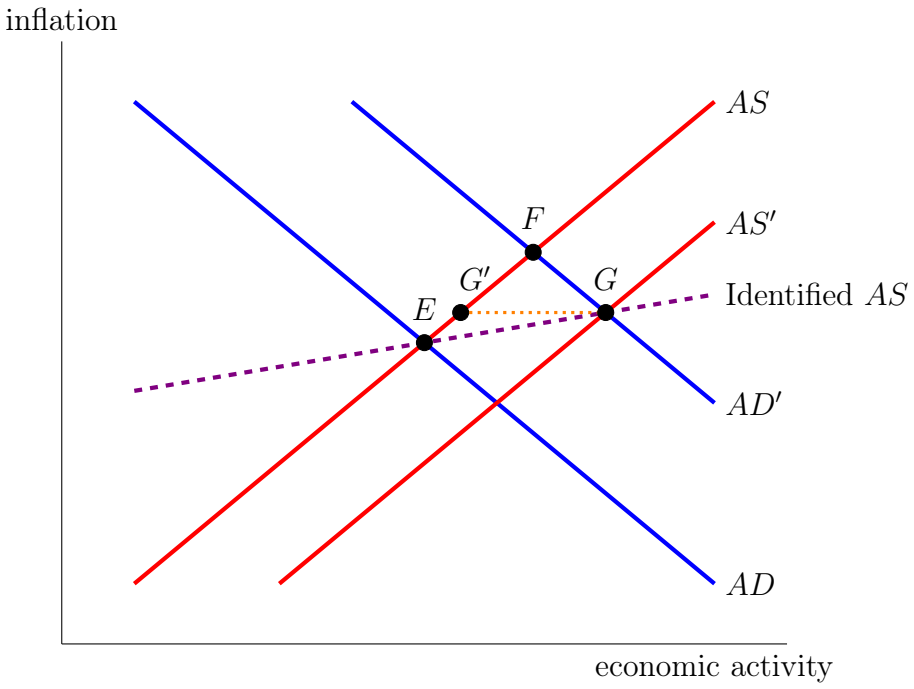


Figure 1: Graphical illustration of the Phillips curve flattening

points (E and F) allows econometricians to estimate κ. However, Proposition 1 implies that the AS curve could shift downward to AS' in response to expansionary monetary policy shocks when conventional measures of economic activity, such as the output gap, labor gap, and labor share, are employed and the returns to scale are procyclical conditional on monetary policy shocks. The increase in $\widehat{rts}_t$ works as an endogenous supply shifter, driving the economy to point G rather than F. Thus, the identified AS curve, denoted by the purple dashed line connecting points E and G in Figure 1, could be flatter than the true underlying AS curve.

Note that in the $(\hat{mc}, \hat{\pi})$ pair, the equilibrium moves from point E to G' because $\widehat{rts}$ does not appear in the marginal cost Phillips curve. However, for a conventional forcing variable, e.g., the labor gap, the new equilibrium is represented by point G. That is, the labor gap increases more than the marginal costs do, and the difference between the two, the distance between points G' and G, reflects the increase in returns to scale (see also the first equality in Equation (2.5)). If this mechanism has become more effective in recent decades, the AS curve would shift further downward. Then, econometricians can observe the flattening of the conventional Phillips curves even when the underlying slope, κ, remains largely unchanged.

Remark 2. A specific supply-side effect of demand disturbances propagating through the cyclical returns to scale is central to the above mechanism for the flattening of the conventional Phillips curves without an accompanying decrease in the slope of the marginal cost Phillips curve. Relatedly, aggregate demand might affect the supply block of the economy through other channels, such as endogenous R&D (Comin and Gertler, 2006; Anzoategui et al., 2019; Bianchi, Kung and Morales, 2019; Okada, 2022; Ma and Zimmermann, 2023; Antolin-Diaz and Surico, 2024; Cloyne et al., 2024), misallocation (Eisfeldt and Rampini, 2006; Hsieh and Klenow, 2009; Beaudry and Portier, 2014; Alam, 2020; Baqaee, Farhi and Sangani, 2024), firm dynamics (Smirnyagin, 2023; Fujiwara and Matsuyama, 2023), search effort (Qiu and Ríos-Rull, 2022; Bai, Ríos-Rull and Storesletten, 2024), and capacity utilization (Boehm and Pandalai-Nayar, 2022). If the effects of these mechanisms have recently become more procyclical, the identified Phillips curves could be flatter than previous curves for similar reasons.

A key distinction between our mechanism and other prominent supply-side channels of monetary policy is that the cyclical returns to scale do not affect TFP at the first order; the TFP in our model (ε_t^a) is exogenous and equal to the Solow residual at the first order. In contrast, several other supply-side mechanisms described above could directly affect measured aggregate TFP (see also Ma and Zimmermann, 2023; Jordà, Singh and Taylor, 2023).

Given this consideration, we empirically investigate how TFP responds to monetary policy shocks in the pre- and post-2000 samples (see Appendix C.5 for details). We document that the level of TFP responded procyclically to monetary policy shocks in the early sample. However, in the later sample, the impulse responses are close to zero and statistically insignificant. Thus, we find no significant evidence of increasingly procyclical TFP responses to monetary policy shocks in recent periods. To be clear, we do not argue that those other supply-side mechanisms are weak. Rather, our findings suggest they have not strengthened enough in the past two to three decades to drive the observed flattening of the Phillips curves. Note that our empirical results on TFP do not contradict our proposed explanation on the basis of the procyclical returns to scale because the returns to scale can endogenously vary over time without affecting ε_t^a.

Remark 3. As an alternative to Equation (2.2), we can assume a normalized translog form

proposed by Hyun, Kim and Lee (2024):

$$y_{it} = \underbrace{\exp(\varepsilon_t^a) l_{1,it}^{\alpha_1} l_{2,it}^{\alpha_2}}_{\text{Cobb–Douglas}} \times \underbrace{\exp\left(s.o.t._{.it}\right)}_{\text{Translog terms}},$$

$$\text{where} \quad s.o.t._{.it} = \beta_{11}\hat{l}_{1,it}\hat{l}_{1t} + \beta_{12}\hat{l}_{1,it}\hat{l}_{2t} + \beta_{21}\hat{l}_{2,it}\hat{l}_{1t} + \beta_{22}\hat{l}_{2,it}\hat{l}_{2t}, \tag{2.6}$$

and $\beta_{12} = \beta_{21}$. This production function has the following properties: First, the second-order term, $s.o.t._{,it}$, degenerates in the steady state because the log deviation terms are zero, $\hat{l}_{j,it} = \hat{l}_{jt} = 0$ for $j = 1, 2$. Thus, the steady state of the model is identical to that of the Cobb–Douglas model and, in particular, does not depend on the translog parameters, $\{\beta_{jk} : j, k = 1, 2\}$. This property allows us to focus on the business cycle implications of input complementarity and cyclical returns to scale without altering the long-term predictions of the model. Second, the production functions (2.2) and (2.6) yield the same aggregate equilibrium conditions in log-linearization. Furthermore, the production function in a deviation form from the steady state is given by:

$$\frac{y_{it}}{\bar{y}} = \exp(\varepsilon_t^a) \left(\frac{l_{1,it}}{\bar{l}_1}\right)^{\alpha_{1t}} \left(\frac{l_{2,it}}{\bar{l}_2}\right)^{\alpha_{2t}},$$

where $\alpha_{1t} = \alpha_1 + \beta_{11}\hat{l}_{1t} + \beta_{12}\hat{l}_{2t}$, $\alpha_{2t} = \alpha_2 + \beta_{21}\hat{l}_{1t} + \beta_{22}\hat{l}_{2t}$, and a bar denotes the steady-state values. Thus, the translog structure induces a potentially time-varying elasticity of output with respect to each input, α_{1t} and α_{2t}, leading to time-varying returns to scale, $rts_t = \alpha_{1t} + \alpha_{2t}$. Finally, the production function (2.6) does not allow each firm to choose its returns to scale, reflected by the fact that α_{1t} and α_{2t} are exogenous from firm i's perspective. This property simplifies the second-order conditions for firm i's cost minimization problem, similar to the Cobb–Douglas model (see Appendix B.1.2).

3 Quantitative Analyses

This section constructs a medium-scale DSGE model with a translog production function for quantitative analyses. We estimate the model parameters using Bayesian methods and show that the translog model fits the US macroeconomic data better than the Cobb–Douglas and nested CES models do. By comparing the estimated models using pre- and post-2000 data, we find the following: (i) The translog model successfully replicates both a steep marginal

cost Phillips curve and the flattening of the conventional Phillips curves. (ii) In doing so, our mechanism plays a crucial role by connecting the cyclical returns to scale, marginal costs, and inflation. (iii) The translog model also predicts the increased importance of the expected inflation in the US inflation dynamics in recent periods. (iv) The historical decomposition of the pandemic-era inflation data implies that high inflation in 2021 and 2022 was primarily driven by loose monetary policy as well as supply-side shocks.

3.1 Model

We augment the partial equilibrium, four-input, nested CES model of production described in Krusell et al. (2000) and Ohanian, Orak and Shen (2023) with a translog production function. We combine this novel framework for the supply block of the economy with a quantitative New Keynesian general equilibrium model. Firms use structure, equipment, skilled labor, and unskilled labor in production. Following Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2007), and Justiniano, Primiceri and Tambalotti (2010), the model features standard frictions known to be useful for explaining the US aggregate data, such as sticky prices and wages, fixed costs of production, investment adjustment costs, costly capacity utilization, and external consumption habits. The standard structure in the other parts of the model allows us to emphasize the input complementarity and substitution structure, the workings of the translog production function, the resulting cyclical returns to scale, and their contribution to the slopes of the Phillips curves. Below, we briefly introduce the model with a focus on the novel elements. See Appendix B for additional details.

3.1.1 Firms

The final goods are produced by combining intermediate goods, indexed by $i \in [0,1]$, via a Kimball (1995) aggregator. Intermediate goods producers can randomly reset their prices with a probability of $1-\zeta_p$ in each period. As a result, the model features a New Keynesian price Phillips curve, which is examined in detail in Section 3.3.

The production function for the intermediate good i is given by:

$$Y_t(i) = \exp(\varepsilon_t^a)[K_{st}(i)]^{\alpha_{ks}}[K_{et}(i)]^{\alpha_{ke}}[\gamma^t L_{st}(i)]^{\alpha_s}[\gamma^t L_{ut}(i)]^{\alpha_u} \times \exp(s.o.t._{it}) - \gamma^t \upsilon, \tag{3.1}$$

$$s.o.t._{it} = \left(\hat{k}_{i,et}, \hat{l}_{i,st}, \hat{l}_{i,ut}\right) \boldsymbol{\beta} \left(\hat{k}_{et}, \hat{l}_{st}, \hat{l}_{ut}\right)',$$

where $K_{st}(i)$, $K_{et}(i)$, $L_{st}(i)$, and $L_{ut}(i)$ are structures, capital equipment services, skilled labor hours, and unskilled labor hours, respectively. γ is the (gross) growth rate on the balanced growth path (BGP). υ represents the fixed cost in production. The profit is zero on the BGP, $(\Phi - 1)\bar{Y}_t = \gamma^t \upsilon$, where Φ and $\bar{Y}_t$ are the gross price markup and output on the BGP, respectively. The aggregate productivity, ε_t^a, follows an exogenous process. We assume that $\alpha_{ks} + \alpha_{ke} + \alpha_s + \alpha_u = 1$.

Similar to Equation (2.6) in the illustrative model, we assume a normalized translog structure for the second-order terms following Hyun, Kim and Lee (2024). We use bars and hats to denote the BGP values and log deviations from these values, respectively; $\hat{k}_{i,et} = \log\left(\frac{K_{et}(i)}{\bar{K}_{et}}\right)$, $\hat{l}_{i,st} = \log\left(\frac{L_{st}(i)}{\bar{L}_{st}}\right)$, $\hat{l}_{i,ut} = \log\left(\frac{L_{ut}(i)}{\bar{L}_{ut}}\right)$, $\hat{k}_{et} = \log\left(\frac{K_{et}}{\bar{K}_{et}}\right)$, $\hat{l}_{st} = \log\left(\frac{L_{st}}{\bar{L}_{st}}\right)$, and $\hat{l}_{ut} = \log\left(\frac{L_{ut}}{\bar{L}_{ut}}\right)$, where K_{et}, L_{st}, and L_{ut} are aggregate equipment services, skilled labor hours, and unskilled labor hours, respectively. $\boldsymbol{\beta}$ is a matrix consisting of the translog parameters:

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_{kk} & \beta_{ks} & \beta_{ku} \\ \beta_{ks} & \beta_{ss} & \beta_{su} \\ \beta_{ku} & \beta_{su} & \beta_{uu} \end{pmatrix}.$$

We assume that structures appear only in the first-order term in the production function following Krusell et al. (2000) and Ohanian, Orak and Shen (2023). However, arbitrary degrees of input complementarity and substitutability among the other three factors, represented by $\boldsymbol{\beta}$, are allowed by adopting a translog functional form.

Firms use four inputs, $K_{st}(i)$, $K_{et}(i)$, $L_{st}(i)$, and $L_{ut}(i)$, to minimize the production cost, $R_{st}K_{st}(i) + R_{et}K_{et}(i) + W_{st}L_{st}(i) + W_{ut}L_{ut}(i)$, where R_{st}, R_{et}, W_{st}, and W_{ut} are the nominal rental rate of structures, rental rate of equipment services, skilled wage rate, and unskilled wage rate, respectively. $MC_t(i)$ is the Lagrange multiplier associated with the constraint $Y_t(i) \geq Y$, representing nominal marginal costs.

The FOCs for this cost-minimization problem are as follows:

$$K_{st}(i): \quad \frac{R_{st}K_{st}(i)}{P_t\tilde{Y}_t(i)} = mc_t(i)\alpha_{ks}, \tag{3.2}$$

$$K_{et}(i): \quad \frac{R_{et}K_{et}(i)}{P_t\tilde{Y}_t(i)} = mc_t(i)\alpha_{ke,t}, \qquad \alpha_{ke,t} \equiv \left(\alpha_{ke} + \beta_{kk}\hat{k}_{et} + \beta_{ks}\hat{l}_{st} + \beta_{ku}\hat{l}_{ut}\right), \tag{3.3}$$

$$L_{st}(i): \quad \frac{W_{st}L_{st}(i)}{P_t\tilde{Y}_t(i)} = mc_t(i)\alpha_{st}, \qquad \alpha_{st} \equiv \left(\alpha_s + \beta_{ks}\hat{k}_{et} + \beta_{ss}\hat{l}_{st} + \beta_{su}\hat{l}_{ut}\right), \tag{3.4}$$

$$L_{ut}(i): \qquad \frac{W_{ut}L_{ut}(i)}{P_t\tilde{Y}_t(i)} = mc_t(i)\alpha_{ut}, \qquad \alpha_{ut} \equiv \left(\alpha_u + \beta_{ku}\hat{k}_{et} + \beta_{su}\hat{l}_{st} + \beta_{uu}\hat{l}_{ut}\right), \tag{3.5}$$

where $\tilde{Y}_t(i) = Y_t(i) + \gamma^t \upsilon$, P_t is the aggregate price level, and $mc_t(i)$ represents real marginal costs. Thus, the factor income shares, adjusted by fixed costs in production, are proportional to marginal costs times the elasticity of $\tilde{Y}_t(i)$ with respect to each input. Note that $\alpha_{ke,t}$, α_{st}, and α_{ut} are time-varying and endogenously determined. Furthermore, although the second-order term in the production function (3.1) vanishes in log linearization, the translog parameters appear in the log-linearized FOCs and influence firms' factor input decisions.

3.1.2 Households

A representative household consists of a continuum of members indexed by $j \in [0,1]$. In each period, $\chi_t \in (0,1)$ fraction of members are skilled workers, and the remaining $1-\chi_t$ fraction of workers are unskilled, where χ_t follows an exogenous process. We assume that members with $j \in [0, 1-\chi_t)$ and $j \in [1-\chi_t, 1]$ are unskilled and skilled, respectively.

The period utility function of member j is given by $\frac{A_{1j}}{1-\sigma_c}\left(C_t(j) - hC_{t-1}\right)^{1-\sigma_c} \exp\left(A_{2j}\frac{\sigma_c-1}{1+\sigma_l}H_t(j)^{1+\sigma_l}\right)$, where $C_t(j)$ is consumption, hC_{t-1} represents an external consumption habit, and $H_t(j)$ is labor hours. The preference parameters, A_{1j} and A_{2j}, vary across worker types. For skilled workers, $A_{1j} = A_{2j} = 1$. For unskilled workers, $A_{1j} = A_1$ and $A_{2j} = A_2$, where the values of A_1 and A_2 are chosen to maintain the "representativeness" of agents such that consumption and labor hours are the same across the worker types on the BGP. This property allows us to deviate minimally from the representative agent framework. However, $C_t(j)$ and $H_t(j)$ can vary across types in the short term.

A family head solves the resource allocation and portfolio choice problems by deciding the consumption ($C_t(j)$) of the family members, family-level risk-free nominal bonds (B_t), structure investment (I_{st}), equipment investment (I_{et}), structure stock ($\tilde{K}_{st}$), equipment stock ($\tilde{K}_{et}$), and equipment utilization rate (Z_t) to maximize the following utilitarian welfare function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 \left[A_{1j}\frac{1}{1-\sigma_c}\left(C_t(j) - hC_{t-1}\right)^{1-\sigma_c} \exp\left(A_{2j}\frac{\sigma_c-1}{1+\sigma_l}H_t(j)^{1+\sigma_l}\right)\right] dj$$

subject to the budget constraint as follows:

$$\int C_t(j)\,dj + I_{st} + I_{et} + \frac{B_t}{\exp(\varepsilon_t^b) R_t P_t} + T_t \tag{3.6}$$
$$\leq \frac{B_{t-1}}{P_t} + \frac{W_{st}\int_{1-\chi_t}^{1} H_t(j)\,dj}{P_t} + \frac{W_{ut}\int_0^{1-\chi_t} H_t(j)\,dj}{P_t} + \frac{R_{st}\tilde{K}_{s,t-1}}{P_t} + \frac{R_{et}Z_t\tilde{K}_{e,t-1}}{P_t} - a(Z_t)\tilde{K}_{e,t-1} + \frac{\Pi_t^w}{P_t} + \frac{\Pi_t}{P_t}$$

and the capital accumulation equations for structure and equipment: $\tilde{K}_{st} = (1-\delta_s)\tilde{K}_{s,t-1} + \left[1 - S_s\left(\frac{I_{st}}{I_{s,t-1}}\right)\right] I_{st}$ and $\tilde{K}_{et} = (1-\delta_e)\tilde{K}_{e,t-1} + \exp(\varepsilon_t^i)\left[1 - S_e\left(\frac{I_{et}}{I_{e,t-1}}\right)\right] I_{et}$. R_t is the (gross) nominal returns to bonds. ε_t^b denotes an exogenous premium in returns to bonds, reflecting a structural shock to the demand for safe and liquid assets (Fisher, 2015). T_t represents lump-sum taxes. Π_t^w and Π_t are the profits of labor unions and firms paid out as dividends. $a(\cdot)$ reflects costly capacity utilization. We assume that $a(\bar{Z}) = 0$ and $\bar{Z} = 1$, where $\bar{Z}$ is the steady-state level of utilization. Following Smets and Wouters (2007), we reparametrize $\frac{a'(\bar{Z})}{a''(\bar{Z})}$ as $\frac{1-\psi}{\psi}$. The amount of structure and equipment services households can rent to firms is given by $K_{st} = \tilde{K}_{s,t-1}$ and $K_{et} = Z_t\tilde{K}_{e,t-1}$, respectively. $S_s(\cdot)$ and $S_e(\cdot)$ are the investment adjustment costs for structure and equipment such that $S_s(\gamma) = S_e(\gamma) = 0$, $S_s'(\gamma) = S_e'(\gamma) = 0$, $S_s''(\gamma) = \varphi_s > 0$, and $S_e''(\gamma) = \varphi_e > 0$. ε_t^i is an exogenously determined (equipment) investment-specific productivity.

3.1.3 Other Components of the Model

Households relegate labor supply decisions to labor market institutions. The skilled labor supply from households is differentiated and channeled to skilled labor unions. These labor unions set nominal wages for each differentiated skilled labor service subject to Calvo-type frictions, where the resetting probability in each period is denoted by $1 - \zeta_s$. The differentiated labor services are combined into composite skilled labor services using a Kimball aggregator and sold to intermediate goods producers. This structure yields a New Keynesian skilled wage Phillips curve. We introduce a similar labor market structure for unskilled labor. With a probability of ζ_u, the unskilled labor unions cannot adjust wages in each period. The model similarly features a New Keynesian unskilled wage Phillips curve. The skilled and unskilled wage markup shocks have independent ARMA(1,1) processes, following Smets and Wouters (2007).

Given the population share of unskilled workers $(1 - \chi_t)$, in total, $(1 - \chi_t)\int_0^{1-\chi_t} H_t(j)\,dj$ hours of unskilled labor are supplied to intermediate goods producers. Thus, the market for

unskilled labor clears when $L_{ut} = \int L_{ut}(i)di = (1-\chi_t)\int_0^{1-\chi_t} H_t(j)\,dj$. Similarly, we have $L_{st} = \int L_{st}(i)di = \chi_t \int_{1-\chi_t}^{1} H_t(j)\,dj$.

The central bank's monetary policy rule is given by: $\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho}\left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{r_\pi}\left(\frac{Y_t}{Y_t^*}\right)^{r_y}\right]^{1-\rho}\left(\frac{Y_t/Y_{t-1}}{Y_t^*/Y_{t-1}^*}\right)^{r_{\Delta y}}\exp(\varepsilon_t^r)$, where $\bar{R}$ is the steady state nominal (gross) rate and Y_t^* is the natural level of output in an economy with flexible prices and wages and without markup shocks to prices and wages. ρ determines the degree of interest rate smoothing. r_π, r_y, and $r_{\Delta y}$ reflect the responsiveness of the policy rate to each forcing variable.

The aggregate resource constraint in this economy is standard: $C_t + I_t + G_t + a(Z_t)\tilde{K}_{e,t-1} = Y_t$, where aggregate consumption and investment are given by $C_t = \int C_t(j)\,dj$ and $I_t = I_{st} + I_{et}$, respectively. Government spending, G_t, is exogenously determined.

3.1.4 Cobb–Douglas and Nested CES Models

The translog model nests similar models on the basis of the Cobb–Douglas and nested CES production functions in log linearization of the corresponding equilibrium conditions. Clearly, the Cobb–Douglas model can be obtained by imposing the restriction that the translog parameters are zero, $\boldsymbol{\beta} = 0$.

Next, we consider a nested CES production function employed in Krusell et al. (2000) and Ohanian, Orak and Shen (2023). In a deviation form based on dimensionless parameters such as those in Cantore and Levine (2012); Cantore et al. (2014, 2015), we can write the nested CES production function as follows:

$$\frac{Y_t(i)+\gamma^t\upsilon}{\bar{Y}_t+\gamma^t\upsilon} = \exp(\varepsilon_t^a)\left[\frac{K_{st}(i)}{\bar{K}_{st}}\right]^{\alpha_{ks}}\left\{\tilde{\alpha}_u\left[\frac{L_{ut}(i)}{\bar{L}_{ut}}\right]^{\phi_{ux}} + \tilde{\alpha}_x\left(\tilde{\alpha}_k\left[\frac{K_{et}(i)}{\bar{K}_{et}}\right]^{\phi_{ks}} + \tilde{\alpha}_s\left[\frac{L_{st}(i)}{\bar{L}_{st}}\right]^{\phi_{ks}}\right)^{\frac{\phi_{ux}}{\phi_{ks}}}\right\}^{\frac{1-\alpha_{ks}}{\phi_{ux}}},$$

where $\tilde{\alpha}_u + \tilde{\alpha}_x = 1$, $\tilde{\alpha}_k + \tilde{\alpha}_s = 1$, and $-\infty \leq \phi_{ux}, \phi_{ks} \leq 1$. Similar to the results shown in Table 1, this nested CES function can be converted to a translog form. The steady-state income shares in the translog function are obtained as follows: $\alpha_u = (1-\alpha_{ks})\tilde{\alpha}_u$, $\alpha_{ke} = (1-\alpha_{ks})\tilde{\alpha}_x\tilde{\alpha}_k$, and $\alpha_s = (1-\alpha_{ks})\tilde{\alpha}_x\tilde{\alpha}_s$. See Appendix B.4 for similar results for the translog parameters.

Finally, all three models share the same log-linearized aggregate production functions around the BGP, $\hat{Y}_t = \Phi(\varepsilon_t^a + \alpha_{ks}\hat{K}_{st} + \alpha_{ke}\hat{K}_{et} + \alpha_s\hat{L}_{st} + \alpha_u\hat{L}_{ut})$.

3.2 Bayesian Estimation

We estimate the three models with different production functions using pre- and post-2000 US time series data and Bayesian methods. A comparison of the model fits reveals that the translog model better matches the data than the Cobb–Douglas and CES models do. Furthermore, the estimated production function parameters reflect potential structural changes in production technology between the two sample periods, which could further influence the cyclicality of returns to scale and the identified slopes of the Phillips curves.

3.2.1 Data, Priors, and Methods

Data. We use eight quarterly variables and three annual variables for estimation. The quarterly variables include skilled workers' employment shares and the seven observables employed by Smets and Wouters (2007), such as growth rates of per capita real GDP, consumption, and investment; labor hours; the growth rate of the real wage rate; price inflation; and the nominal interest rate. We replace the federal funds rate with the shadow rate of Wu and Xia (2016) from 2009:q1 to 2015:q4 because of the binding zero lower bound.

Using the Current Population Survey (CPS), we compute skilled employment shares. Following Krusell et al. (2000) and Ohanian, Orak and Shen (2023), college graduates are considered skilled workers. Because of the data limitation in the early period, this variable is only available for the first quarters until 1975. Since then, it has been available every quarter. We remove the seasonal variation using X-13 ARIMA.

In light of two types of capital (structures and equipment) and labor (skilled and unskilled), we borrow three related annual variables from Ohanian, Orak and Shen (2023) to discipline the model parameters. These variables are the growth rate of per capita equipment stock (including intellectual property products), the changes in relative labor hours, and the changes in the relative wage rates between skilled and unskilled workers. The capital stock is constructed using perpetual inventory methods. The two labor market variables are calculated using the CPS data. These annual variables are assumed to be observable in the fourth quarter of each year.

Our sample begins in 1966:q1 as in Smets and Wouters (2007) and ends in 2019:q4 before the recent pandemic. We divide the data into early (1966-99) and late (2000-19) samples and estimate each model for each sample period. In total, we have six estimated models for the three production functions and the two sample periods.

State-space system. The state equation is constructed using the Sims (2002) method for each parameter value. The measurement equation consists of available observations among the eleven variables described above each quarter.

Real GDP, consumption, investment, price inflation, and the nominal interest rate are mapped to Y_t, C_t, I_t, P_t/P_{t-1}, and R_t in the model, respectively. The aggregate labor hours are matched with $H_t \equiv \int_0^1 H_t(j)\,dj = L_{st} + L_{ut}$. The aggregate wage rate is given by the total wage bills divided by the aggregate hours, $W_t \equiv \frac{W_{st}L_{st}+W_{ut}L_{ut}}{P_t H_t}$. The skilled employment share corresponds to χ_t. We assume that $\log(\chi_t) = \log(\bar{\chi}) + \varepsilon_t^{\chi}$, where ε_t^{χ} follows a mean-zero AR(1) process. We set $\bar{\chi}$ at 0.21 for the pre-2000 sample and 0.325 for the post-2000 sample, which are the values of χ_t around the midpoint of each sample.

The three annual variables are observed (with measurement errors) in the fourth quarter. The growth rate of per capita equipment stock corresponds to $\log(\tilde{K}_{et}/\tilde{K}_{e,t-4}) + \nu_{kt}$. ν_{kt} represents a measurement error in the empirical capital stock variable computed using perpetual inventory methods. ν_{kt} has an independently and identically distributed normal density with a mean of zero and a standard deviation of $\sigma_{\nu,k}$. The relative annual labor hours, denoted by $rell_t^{ann}$, equal $\frac{L_{st}+L_{s,t-1}+L_{s,t-2}+L_{s,t-3}}{L_{ut}+L_{u,t-1}+L_{u,t-2}+L_{u,t-3}}$. The annual change in this variable, $\log(rell_t^{ann}/rell_{t-4}^{ann})$, is observed with measurement errors, $\nu_{ht} \sim N(0, \sigma_{\nu,h}^2)$, because the labor market variables in the CPS are known to be subject to nonnegligible measurement errors (see, e.g., Bound and Krueger, 1991). Similarly, the relative annual wage rate, $relw_t^{ann}$, is given by $\frac{W_{st}L_{st}+...+W_{s,t-3}L_{s,t-3}}{L_{st}+...+L_{s,t-3}} / \frac{W_{ut}L_{ut}+...+W_{u,t-3}L_{u,t-3}}{L_{ut}+...+L_{u,t-3}}$. The measurement of its annual change, $\log(relw_t^{ann}/relw_{t-4}^{ann})$, involves measurement errors denoted by $\nu_{wt} \sim N(0, \sigma_{\nu,w}^2)$.

The model includes nine structural shocks and three measurement errors. The structural shocks include shocks to TFP (ε_t^a), investment-specific productivity (ε_t^i), monetary policy (ε_t^r), risk premium (ε_t^b), government spending (ε_t^g), price markup (ε_t^p), skilled wage markup (ε_t^s), unskilled wage markup (ε_t^u), and the employment share of skilled workers (ε_t^{χ}).

Priors. We use standard priors for conventional parameters, similar to Smets and Wouters (2007) and Justiniano, Primiceri and Tambalotti (2010). For the other parameters, we set the following priors. We assume that $\alpha_{ks} \sim N(0.1, 0.005^2)$ given the results in Krusell et al. (2000) and Ohanian, Orak and Shen (2023). Combined with the prior for $\alpha_{ke} \sim N(0.25, 0.02^2)$, this assumption implies that the capital income share a priori ranges from 0.3-0.4 within the two standard deviation intervals. Given the steady increase in the skilled labor supply (Goldin and Katz, 2009), the mean of α_s is assumed to be 0.2 and 0.3 for the early and late samples, respectively.

The translog parameters, e.g., β_{kk}, have normal priors with a mean of zero and a standard deviation of 0.15. This prior is rather loose and less informative, which is illustrated by the fact that previous estimates based on nested CES functions are within a one-standard deviation interval. For example, when $\phi_{ux} = 0.401$, $\phi_{ks} = -0.495$, and $\alpha_{ks} = 0.117$, following Krusell et al. (2000) in combination with $\alpha_{ke} = 0.25$ and $\alpha_s = 0.2$, the corresponding translog parameters are less than 0.09 in absolute value (see Equation (B.19) in Appendix B.1.2).

Shocks to χ_t, ε_t^{χ}, has an AR(1) process: $\varepsilon_t^{\chi} = \rho_{\chi}\varepsilon_{t-1}^{\chi} + \eta_t^{\chi}$. Because this variable is slow-moving and steadily increases throughout the sample period, we assume that ρ_{χ} has a beta distribution with a mean of 0.9. Finally, the standard deviations of the measurement errors, $\sigma_{\nu,k}$, $\sigma_{\nu,h}$, and $\sigma_{\nu,w}$, have inverse gamma distributions with a mean of 0.15 and a standard deviation of 0.03. Thus, a priori, measurement errors are assumed not to be excessively large.

We fix δ_s and δ_e at $0.026/4$ and $0.16/4$, respectively, according to Fernald (2014, p. 18) and Ohanian, Orak and Shen (2023, fig 3.3(b)). The steady-state share of government spending (0.18), steady-state wage markups (1.5), and Kimball curvature parameters (10) are set at the same values as those in Smets and Wouters (2007).

Methods. The posterior distributions are computed using a Metropolis–Hastings algorithm with a chain length of 200,000. The likelihood of the state-space system is computed using a Kalman filter. The acceptance rates of the chain for the six cases range from 22%-30%. We summarize the posterior distributions using the mode and credible intervals. See Appendix C.3 for the complete list of the estimated parameters, priors, and posteriors for the six cases.

3.2.2 Model Comparison

This section shows that the estimated translog model better matches the data than the two alternative models do. Specifically, we test and reject the restrictions imposed on the translog parameters by the Cobb–Douglas and nested CES forms, compare the size of measurement errors across the models, and assess the log-likelihood in the posterior modes and marginal data densities (MDDs). The results favor the translog model.

The production function (3.1) implies that the aggregate returns to scale are influenced by the fixed cost and the translog structure. Let lowercase letters denote the detrended real variables around the BGP, e.g., $y_t = \frac{Y_t}{\gamma^t}$, $k_{et} = \frac{K_{et}}{\gamma^t}$, $l_{st} = L_{st}$, and $l_{ut} = L_{ut}$. In

log-linearization:

$$\widehat{rts}_t = \underbrace{-\frac{\Phi-1}{\Phi}\hat{y}_t}_{\widehat{rts}^{fc}} + \underbrace{\left(\beta_{k\cdot}\hat{k}_{et} + \beta_{s\cdot}\hat{l}_{st} + \beta_{u\cdot}\hat{l}_{ut}\right)}_{\widehat{rts}^{tl}}, \tag{3.7}$$

where $\beta_{k\cdot} = \beta_{kk}+\beta_{ks}+\beta_{ku}$, $\beta_{s\cdot} = \beta_{ks}+\beta_{ss}+\beta_{su}$, and $\beta_{u\cdot} = \beta_{ku}+\beta_{su}+\beta_{uu}$. The countercyclical variation in $\widehat{rts}_t$ due to the fixed costs in production, captured by $\widehat{rts}^{fc}$, is shared among the three models.[7] In contrast, the cyclical variation originating from the second-order properties of the production function, denoted by $\widehat{rts}^{tl}$, is unique to the translog model. For the Cobb–Douglas model, $\beta_{k\cdot} = \beta_{s\cdot} = \beta_{u\cdot} = 0$ because all six translog parameters are zero. Like $\beta_{11}+\beta_{12} = \beta_{12}+\beta_{22} = 0$ for the two-input CES model in Table 1, the translog form of the nested CES model also satisfies $\beta_{k\cdot} = \beta_{s\cdot} = \beta_{u\cdot} = 0$, implying that $\widehat{rts}^{tl}_t = 0$.[8] We test the validity of these restrictions by estimating the translog model.

Panel A in Table 2 shows the results. The table illustrates the estimated returns to scale parameters in the posterior mode and the equal-tailed 95% credible intervals. Columns (1)-(3) and (4)-(6) present the results from the three different models estimated using the early and late samples, respectively. Per the discussion above, the returns to scale parameters, $\beta_{k\cdot}$, $\beta_{s\cdot}$, and $\beta_{u\cdot}$, are zero in Columns (1), (2), (4), and (5). Note that none of the six 95% credible intervals in Panel A Columns (3) and (6) include zero, rejecting the prediction of the time-invariant $\widehat{rts}^{tl}_t$ under the Cobb–Douglas and CES models.

The results shown in Panel B imply that rationalizing the data for equipment stock, relative labor hours, and relative wage rates jointly is challenging in a general equilibrium framework. There is no case in Panel B where the magnitude of all three measurement errors is small. Given that, the translog model performs no worse than the other two models do. For example, the translog model better explains the post-2000 relative wage data ($\sigma_{\nu,w}$) than the Cobb–Douglas model does. Compared with the translog model, the nested CES model is less successful at matching the capital stock data (σ_{ν,k_e}) in the recent sample.

For the pre-2000 data, the translog model is consistently favored according to both the log-likelihood in the posterior mode and the MDD (Panel C). For the post-2000 data, the two criteria alternatively select the translog and nested CES models. Thus, given the results

[7] The fixed costs are linked to the gross price markup, Φ, through the zero-profit condition on the BGP.

[8] Choosing six translog parameters subject to these three restrictions and the additional restriction $\beta_{ku} = -\tilde{\alpha}_k\beta_{uu}$, arising from the specific nesting order of the two CES aggregators, is equivalent to determining the two CES parameters, ϕ_{ux} and ϕ_{ks}. See Appendix B.4.

Table 2: Model comparison

	(1)	(2)	(3)	(4)	(5)	(6)
	Early sample (1966-99)			Late sample (2000-19)		
	Cobb–Douglas	Nested CES	Translog	Cobb–Douglas	Nested CES	Translog
Panel A. Returns to scale parameters						
$\beta_{k\cdot}$	0	0	-0.27	0	0	-0.21
	-	-	(-0.63, -0.13)	-	-	(-0.27, -0.19)
$\beta_{s\cdot}$	0	0	-0.13	0	0	0.23
	-	-	(-0.23, -0.06)	-	-	(0.15, 0.33)
$\beta_{u\cdot}$	0	0	-0.37	0	0	-0.28
	-	-	(-0.69, -0.11)	-	-	(-0.48, -0.16)
Panel B. Measurement errors						
σ_{ν,k_e}	3.79	3.83	3.87	0.23	1.69	0.37
	(3.19, 4.58)	(3.23, 4.67)	(3.22, 4.68)	(0.19, 0.38)	(1.25, 2.34)	(0.30, 0.50)
$\sigma_{\nu,h}$	3.13	0.14	0.14	0.14	0.14	0.14
	(2.66, 3.83)	(0.10, 0.21)	(0.10, 0.21)	(0.10, 0.21)	(0.10, 0.22)	(0.12, 0.16)
$\sigma_{\nu,w}$	0.14	1.44	1.21	2.55	1.46	1.28
	(0.10, 0.22)	(1.20, 1.84)	(1.04, 1.49)	(2.11, 3.15)	(1.18, 1.86)	(1.12, 1.66)
Panel C. Log-likelihood in the posterior mode and marginal data densities						
log-likelihood	-96.10	-87.20	-70.27	71.88	85.13	114.42
log MDD	-347.75	-332.69	-311.52	-193.08	-165.21	-173.65

Notes: Panel A shows the estimated returns to scale parameters ($\beta_{k\cdot}$, $\beta_{s\cdot}$, and $\beta_{u\cdot}$) in the posterior mode and the equal-tailed 95% credible intervals. Columns (1)-(3) and (4)-(6) present the results from the three different models estimated using the early and late samples, respectively. Panel B regards the magnitude of the measurement errors. Panel C illustrates the log-likelihood in the posterior mode and the MDD. The MDD is computed using the Sims, Waggoner and Zha (2008) algorithm.

in Panels A-C, we conclude that the translog model, which predicts the time-varying returns to scale, matches the data better than the two alternative models do, at least in the posterior modes.

3.2.3 Production Function Parameters

Next, we discuss the production function parameters. Table 3 shows that the steady-state factor income shares (Panel A) and price markups (Panel C) are largely comparable across the three models in a given sample period. Between the two sample periods, the skilled labor share (α_s) increases, and the price markup (Φ) decreases. Consistent with the results in Smets and Wouters (2007, table 1A), Φ is relatively large, and capital income shares are relatively small compared with conventional calibration in all six cases.

The translog model rejects the Cobb–Douglas and nested CES specifications, which is consistent with the model comparison results in Table 2. Most of the 95% credible intervals for the translog parameters in Columns (3) and (6) in Panel B do not include zero, rejecting the Cobb–Douglas function. In Columns (2) and (5), we show the implied translog param-

Table 3: Production function parameters

	(1)	(2)	(3)	(4)	(5)	(6)
	Early sample (1966-99)			Late sample (2000-19)		
	Cobb–Douglas	Nested CES	Translog	Cobb–Douglas	Nested CES	Translog
Panel A. Factor income shares in the steady state						
α_{ks}	0.10	0.10	0.10	0.10	0.09	0.10
	(0.09, 0.11)	(0.09, 0.10)	(0.09, 0.10)	(0.09, 0.11)	(0.08, 0.10)	(0.10, 0.11)
α_{ke}	0.19	0.18	0.20	0.14	0.12	0.09
	(0.16, 0.22)	(0.15, 0.21)	(0.16, 0.22)	(0.11, 0.17)	(0.10, 0.15)	(0.08, 0.10)
α_s	0.09	0.15	0.12	0.20	0.24	0.24
	(0.06, 0.18)	(0.12, 0.19)	(0.10, 0.14)	(0.13, 0.24)	(0.23, 0.31)	(0.20, 0.26)
α_u	0.62	0.58	0.58	0.57	0.54	0.57
	(0.52, 0.66)	(0.53, 0.63)	(0.56, 0.62)	(0.53, 0.63)	(0.46, 0.56)	(0.55, 0.61)
Panel B. Translog and CES parameters						
β_{kk}	0	0.11	-0.09	0	-0.04	0.04
	-	(0.09, 0.13)	(-0.42, -0.03)	-	(-0.14, -0.03)	(0.01, 0.04)
β_{ks}	0	-0.03	-0.06	0	0.08	0.00
	-	(-0.04, -0.01)	(-0.12, -0.02)	-	(0.07, 0.19)	(-0.01, 0.04)
β_{ku}	0	-0.08	-0.12	0	-0.04	-0.25
	-	(-0.10, -0.07)	(-0.23, 0.07)	-	(-0.05, -0.03)	(-0.32, -0.23)
β_{ss}	0	0.09	0.08	0	0.00	0.13
	-	(0.07, 0.12)	(0.07, 0.10)	-	(-0.09, 0.02)	(0.04, 0.18)
β_{su}	0	-0.07	-0.16	0	-0.07	0.10
	-	(-0.09, -0.05)	(-0.20, -0.10)	-	(-0.14, -0.07)	(0.04, 0.20)
β_{uu}	0	0.15	-0.10	0	0.11	-0.13
	-	(0.12, 0.18)	(-0.33, 0.02)	-	(0.10, 0.19)	(-0.33, 0.00)
ϕ_{ux}	0	0.74	-	0	0.52	-
	-	(0.60, 0.84)	-	-	(0.47, 0.86)	-
ϕ_{ks}	0	0.78	-	0	-0.65	-
	-	(0.64, 0.92)	-	-	(-1.95, -0.57)	-
Panel C. Gross price markup in the steady state						
Φ	1.65	1.69	1.63	1.36	1.36	1.39
	(1.53, 1.77)	(1.54, 1.81)	(1.49, 1.77)	(1.23, 1.47)	(1.19, 1.50)	(1.27, 1.50)

Notes: Columns (1)-(3) and (4)-(6) illustrate the estimation results for the Cobb–Douglas, nested CES, and translog models using the early and late samples, respectively. Panel A shows the factor income shares in the steady state in the posterior mode and the equal-tailed 95% credible intervals. Panels B and C cover the translog and CES parameters and the steady-state price markups, respectively.

eters by ϕ_{ux}, ϕ_{ks} and the nested CES function. Note that the CES-based β_{kk} and β_{uu} (and β_{su} for the late sample) have different signs from the corresponding translog estimates in Columns (3) and (6). Furthermore, the magnitudes of β_{ku} and β_{ss} in Column (5) substantially differ from those in Column (6). Thus, despite being tractable, the nested CES function might not be flexible enough to capture several aspects of the input complementarity and substitution patterns in the data.

Finally, by comparing Columns (3) and (6) in Panel B, we find that β_{kk}, β_{ks}, β_{ku}, and β_{su}, in particular, changed substantially. An increase in β_{kk} probably reflects an increase

in intellectual property products capital (Koh, Santaeulàlia-Llopis and Zheng, 2020) and an innate positive externality in "ideas" (Romer, 1990). β_{ks} is closely related to the degree of the capital-skill complementarity (Griliches, 1969; Autor, Levy and Murnane, 2003; Chirinko, 2008; Chirinko and Mallick, 2017; Berlingieri et al., 2024), which also increases in the nested CES model, as shown in Columns (2) and (5). The capital deepening and concurrent automation of low- and medium-skill tasks might explain the decrease in β_{ku} (Goos and Manning, 2007; Michaels, Natraj and Van Reenen, 2014). A negative β_{su} for the pre-2000 data seems consistent with the conventional view on the substitutability between skilled and unskilled labor (Autor, Katz and Kearney, 2008). However, when estimated using post-2000 data, β_{su} becomes positive (Column (6)).[9] Furthermore, this pattern does not occur in the nested CES model; β_{su} in Columns (2) and (5) are similar to each other. In Section 4, using multicountry industry-level panel data, we present empirical results that are consistent with the translog model above.

3.3 Slopes of the Phillips Curves

This section presents the main findings from the quantitative analyses of the slopes of the Phillips curves. We show that the translog model can jointly replicate the steep marginal cost Phillips curve and the flattening of conventional Phillips curves in recent decades. In contrast, the Cobb–Douglas and nested CES models rely on unrealistically sticky prices and quite flat Phillips curves to match the inflation data after 2000.

The New Keynesian price Phillips curve in our model features real marginal costs as a forcing variable:

$$\hat{\pi}_t = \tilde{\beta}\gamma \mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa \hat{mc}_t + \varepsilon_t^p, \tag{3.8}$$

where $\tilde{\beta} = \beta\gamma^{-\sigma_c}$ is the discount factor in the detrended economy. The slope of this marginal cost Phillips curve, κ, is determined by $\frac{(1-\zeta_p\tilde{\beta}\gamma)(1-\zeta_p)}{\zeta_p}\frac{1}{1+\theta_p(\Phi-1)}$. Relative to the illustrative model in Section 2, κ includes a term representing strategic complementarity in price setting, depending on the curvature of the Kimball aggregator ($\theta_p = 10$ following Smets and Wouters 2007) and the gross price markup (Φ).

We compute the model-predicted slopes of the Phillips curves with different forcing vari-

[9] See also Havranek et al. (2024, table 5) for a wide range of estimates in the literature, including the case of gross complements.

ables as follows. Let x_t denote a measure of economic activity, such as the real marginal cost, output gap, labor gap, and labor shares. Then, the Phillips curve based on x_t is given by:

$$\hat{\pi}_t = \beta_x \mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa_x x_t + error_{x,t},$$

where κ_x represents the slope of this Phillips curve. We project $\hat{\pi}_t$ on $\mathbb{E}_t[\hat{\pi}_{t+1}]$ and x_t and obtain κ_x from the projection coefficient:

$$\begin{pmatrix} * \\ \kappa_x \end{pmatrix} = \left[\text{var}_{c,mp} \begin{pmatrix} \mathbb{E}_t[\hat{\pi}_{t+1}] \\ x_t \end{pmatrix} \right]^{-1} \text{cov}_{c,mp} \left[\begin{pmatrix} \mathbb{E}_t[\hat{\pi}_{t+1}] \\ x_t \end{pmatrix}, \hat{\pi}_t \right], \tag{3.9}$$

where subscripts c and mp indicate that we use the cyclical variation with the periodicity from 6-32 quarters driven by monetary policy shocks. We compute the (co)variances in Equation (3.9) by integrating the frequency-domain representation of the model (while only allowing monetary policy shocks) over the business cycle frequencies. Equivalently, we can obtain κ_x by applying the ideal bandpass filter to the infinite time series of $\hat{\pi}_t$, $\mathbb{E}_t[\hat{\pi}_{t+1}]$, and x_t and use monetary policy shock series as IVs.[10] Thus, our formula corresponds to the population version of empirical practices of estimating Phillips curves using detrended data by filters (Mavroeidis, Plagborg-Møller and Stock, 2014; Stock and Watson, 2020) and the identified monetary policy shocks as instruments (Barnichon and Mesters, 2020, 2021). However, our results are robust to not using filters (see Appendix C.4).

We consider the following forcing variables: marginal costs ($\hat{mc}_t$), the output gap ($\hat{y}_t - \hat{y}^*_t$), the labor gap ($\hat{h}_t - \hat{h}^*_t$), labor shares ($\widehat{w_t h_t / y_t}$) following Galí and Gertler (1999); Sbordone (2002); Eichenbaum and Fisher (2007), and fixed cost adjusted labor shares ($\widehat{w_t h_t/(y_t + \upsilon)}$). The last variable is included because it equals $\hat{mc}_t$ in the Cobb–Douglas model (see Equations (3.4) and (3.5)). Note that when $x_t = \hat{mc}_t$, κ_x coincides with the underlying slope of the New Keynesian price Phillips curve, κ, in Equation (3.8).

Panel A in Table 4 shows the Calvo price stickiness parameter, ξ_p. For the early sample, the three models feature ξ_p values of approximately two-thirds. As shown in Columns (4)

[10] When $x = mc$, this method correctly identifies κ in Equation (3.8). Let $\mathcal{B}(L)$ denote the ideal bandpass filter, where L is a lag operator. Because $\mathcal{B}(L)$ is a linear, time-invariant filter, applying it to Equation (3.8) leads to $\{\mathcal{B}(L)\hat{\pi}_t\} = \tilde{\beta}\gamma\{\mathcal{B}(L)\mathbb{E}_t[\hat{\pi}_{t+1}]\} + \kappa\{\mathcal{B}(L)\hat{mc}_t\} + \{\mathcal{B}(L)\varepsilon^p_t\}$. Thus, the slope remains to be κ. Furthermore, because monetary policy shocks $\{\varepsilon^r_t\}$ are orthogonal to the basis of the residual (i.e., price markup shocks), $\{\varepsilon^p_t\}$, the instruments' exclusion restriction is satisfied.

Table 4: Slopes of the Phillips curves

	(1)	(2)	(3)	(4)	(5)	(6)
	Early sample (1966-99)			Late sample (2000-19)		
	Cobb–Douglas	Nested CES	Translog	Cobb–Douglas	Nested CES	Translog
Panel A. Calvo price stickiness parameter						
ξ_p	0.66	0.72	0.65	0.84	0.89	0.75
	(0.53, 0.79)	(0.59, 0.85)	(0.53, 0.78)	(0.80, 0.89)	(0.89, 0.97)	(0.72, 0.77)
Panel B. Slopes of the price Phillips curves						
marginal cost	0.023	0.013	0.026	0.006	0.003	0.017
	(0.007, 0.057)	(0.004, 0.037)	(0.009, 0.059)	(0.003, 0.010)	(0.000, 0.003)	(0.014, 0.023)
output gap	0.016	0.010	0.028	0.006	0.001	0.005
	(0.004, 0.027)	(0.003, 0.021)	(0.009, 0.049)	(0.003, 0.009)	(0.000, 0.003)	(0.004, 0.007)
labor gap	0.027	0.020	0.039	0.006	0.002	0.005
	(0.007, 0.044)	(0.005, 0.037)	(0.013, 0.066)	(0.004, 0.010)	(0.000, 0.003)	(0.003, 0.006)
labor share	0.047	0.044	0.118	0.009	0.005	0.009
	(-0.140, 0.197)	(-0.084, 0.143)	(-0.481, 0.406)	(0.005, 0.015)	(0.000, 0.006)	(0.007, 0.012)
labor share,	0.023	0.016	0.047	0.006	0.003	0.006
υ adjusted	(0.007, 0.057)	(0.005, 0.041)	(0.010, 0.197)	(0.003, 0.010)	(0.000, 0.003)	(0.004, 0.009)

Notes: Columns (1)-(3) and (4)-(6) illustrate the results from the Cobb–Douglas, nested CES, and translog models estimated using the early and late samples, respectively. Panel A shows the Calvo price stickiness parameter in the posterior mode and the equal-tailed 95% credible intervals. Panel B focuses on the slopes of the price Phillips curves with different forcing variables, computed using Equation (3.9).

and (5), the estimated Cobb–Douglas and nested CES models rely on large ξ_p values for the late sample, corresponding to excessively sticky prices. For example, the average duration of prices implied by ξ_p in Column (5) is longer than nine quarters. In contrast, the translog model uses reasonable values of ξ_p to match the data. In the posterior mode, ξ_p is 0.75 in Column (6), which is consistent with the empirical evidence on the degree of price stickiness in Bils and Klenow (2004) and Nakamura and Steinsson (2008).

ξ_p under the translog model modestly increased in the posterior mode from 0.65 to 0.75 between the two sample periods. As a result, κ in Column (6), 0.017, is slightly smaller than that in Column (3), 0.026. However, this decline is statistically insignificant given that 0.017 is included in the 95% credible interval in Column (3), (0.009, 0.059).[11] Furthermore, 0.017 still corresponds to a significantly positive slope of the marginal cost Phillips curve. We obtain this result despite highly stable inflation during the late sample, exemplified by

[11] This modest decrease in κ might reflect (i) sectoral transformation toward services (Galesi and Rachedi, 2019; Cotton and Garga, 2022) and the fact that service prices are stickier than manufacturing prices (Bils and Klenow, 2004; Nakamura and Steinsson, 2008; Imbs, Jondeau and Pelgrin, 2011; Carvalho, Lee and Park, 2021), (ii) trade and globalization (Sbordone, 2007), (iii) market concentration (Wang and Werning, 2022; Fujiwara and Matsuyama, 2023; Baqaee, Farhi and Sangani, 2024), (iv) the production network and an increase in the intermediate input share (Rubbo, 2023), and (v) occupational composition in the labor market and an increase in the share of non-routine jobs (Siena and Zago, 2021).

the missing disinflation and reinflation episodes (Coibion and Gorodnichenko, 2015; Ball and Mazumder, 2020).

In short, through the lens of the translog model, which matches the data better than the other two models do, the New Keynesian price Phillips curve is not flat and is alive well. Furthermore, despite this result for the marginal cost Phillips curve, the translog model can generate substantially smaller κ_x values for the conventional activity measures in Column (6) than κ_x in Column (3), i.e., the flattening of the conventional Phillips curves. Thus, we conclude that the translog model with structural changes, captured by the differences in translog parameters between the two sample periods, can rationalize the two seemingly contradictory empirical results of a steep marginal cost Phillips curve (Gagliardone et al., 2023) and flattened conventional Phillips curves (Del Negro et al., 2020; Stock and Watson, 2020; Hazell et al., 2022; Inoue, Rossi and Wang, 2024).

The results differ for the other production functions. The Cobb–Douglas and nested CES models predict quite flat marginal cost Phillips curves for the late sample, given the large values of ξ_p. Those quite flat marginal cost Phillips curves translate into similarly flat conventional Phillips curves, as shown in Columns (4) and (5). Thus, in this case, the flattening of the conventional Phillips curves mirrors the similar flattening of the marginal cost Phillips curve, contradicting the empirical evidence in Gagliardone et al. (2023). We obtain similar results when we simulate the translog model and estimate the Cobb–Douglas and nested CES models using the simulated data (see Appendix C.4).

3.4 Marginal Costs and Cyclical Returns to Scale

Next, we discuss the mechanism involved. Specifically, we relate the non-flattening of the marginal cost Phillips curve and the flattening of the conventional Phillips curves to more procyclical returns to scale in recent decades.

Like Equation (2.4) of the illustrative model, the marginal costs consist of factor prices, TFP, and returns to scale owing to the translog structure:

$$\hat{mc}_t = \underbrace{\alpha_{ks}\hat{r}_{st} + \alpha_{ke}\hat{r}_{et} + \alpha_s\hat{w}_{st} + \alpha_u\hat{w}_{ut}}_{\text{factor prices}} - \underbrace{\varepsilon_t^a}_{\text{tfp}} - \underbrace{(\beta_k.\hat{k}_{et} + \beta_s.\hat{l}_{st} + \beta_u.\hat{l}_{ut})}_{\widehat{rts}_t^{tl}}. \tag{3.10}$$

Thus, cyclical variation in returns to scale weakens the connection between marginal costs and their conventional components, such as factor prices and TFP. Consider an expansionary

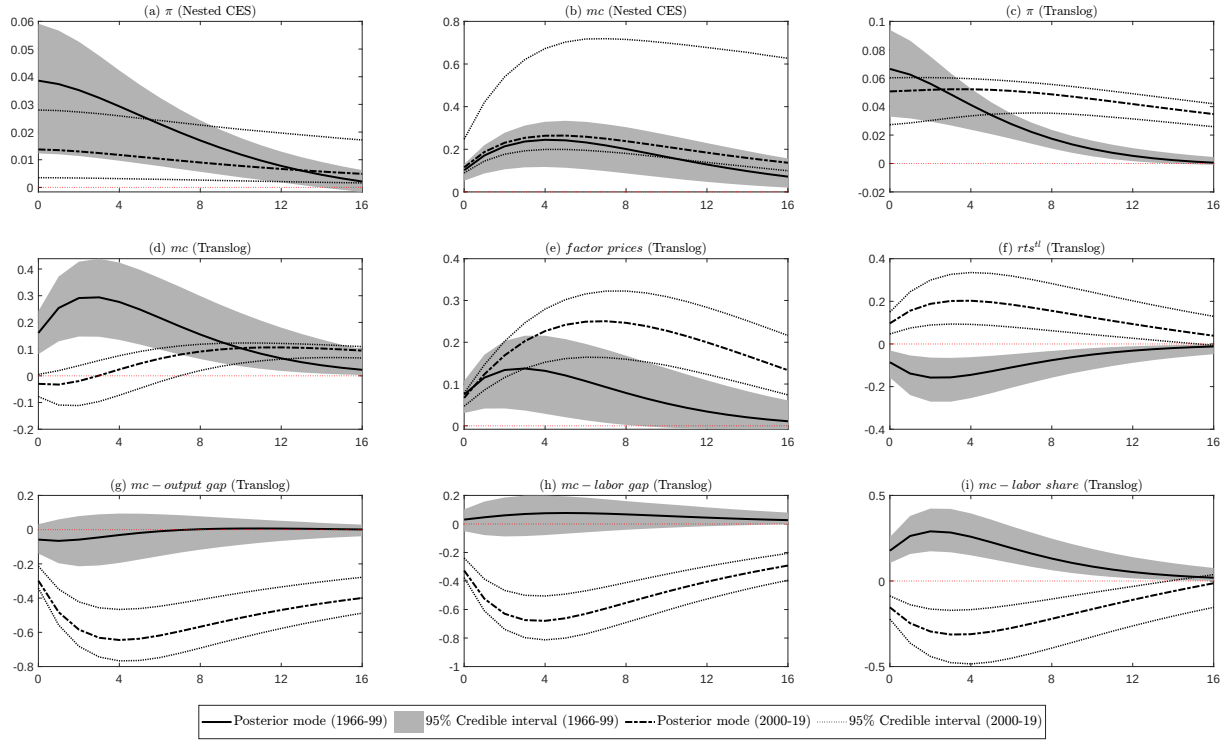


Figure 2: Impulse responses to expansionary monetary policy shocks

Notes: This figure plots the impulse responses to one-standard-deviation expansionary monetary policy shocks. We show the model-predicted impulse responses in the posterior mode with the equal-tailed 95% credible intervals for each sample period. Panels (a) and (b) are based on the nested CES model. The other panels regard the translog model.

monetary policy shock increasing the output gap. Because the wage Phillips curves are steep in the model (see the estimates of ζ_s and ζ_u in Tables C.1-C.2 in Appendix C.3) and in the data (Galí and Gambetti, 2020; Heise, Karahan and Şahin, 2022; Bernanke and Blanchard, 2023), factor prices respond procyclically to the shock, paralleling the response of the output gap. However, the response of the marginal costs could be mitigated given the procyclical response of rts_t^{tl}. In this case, inflation fluctuates less than the output gap (i.e., the output gap Phillips curve is quite flat), not because the pass-through of the marginal costs on inflation is weak (i.e., the marginal cost Phillips curve is steep) but because the marginal costs are relatively stable owing to the offsetting of procyclical factor prices and returns to scale conditional on monetary policy shocks.

To illustrate this mechanism, we plot the impulse responses of the relevant variables to one-standard-deviation expansionary monetary policy shocks in Figure 2. We first consider models without cyclical variation in $\widehat{rts}_t^{tl}$. Panel (b) shows that the marginal costs increase similarly under the nested CES model across the sample period. However, because of sub-

stantially stickier prices in the late sample, the inflation responses are much smaller for the late sample than for the early sample in Panel (a). Similarly, for the Cobb–Douglas model, the pass-through of the marginal costs on inflation has been weak in recent decades (not shown), reflecting the estimated flattening of the marginal cost Phillips curve.

The results differ in the translog model with a cyclical rts_t^{tl}. When estimated using the late sample, monetary policy shocks generate persistent inflation responses with a smaller impact response than those in the early sample (Panel (c)). These distinct inflation dynamics reflect the dissimilar responses of marginal costs between the two sample periods (Panel (d)). In contrast to the hump-shaped responses of marginal costs before 2000, the short-term responses within a year are muted after 2000. These close-to-zero responses lead to small changes in the present discounted values of marginal costs in the short term and, thus, small variations in inflation responses.[12]

These distinct dynamics of marginal costs mostly arise from more procyclical rts_t^{tl} after 2000 than before. Panels (e) and (f) decompose the response of marginal costs into that of factor prices and rts_t^{tl} considering Equation (3.10). Note that procyclical rts_t^{tl} after 2000 partially cancel out procyclical factor prices, yielding smaller responses in marginal costs.

Finally, Panels (g)-(i) show the responses of the gap between marginal cost and other measures of economic activity, constituting an omitted variable in conventional Phillips curve regressions, as illustrated in Proposition 1. More countercyclical responses in recent periods generate more negative omitted variable biases in the identified slope, κ_x. The flattening of the conventional Phillips curves follows from this result.[13]

3.5 Determinants of Inflation

The above results regarding the slopes of the Phillips curves and determinants of marginal costs imply that the inflation dynamics and its determinants could vary across the models and the sample periods. From the results of the variance decomposition analysis, we find

[12] Equation (3.8) implies that $\hat{\pi}_t = \kappa \sum_{\tau=0}^{\infty} (\tilde{\beta}\gamma)^\tau \mathbb{E}_t[\hat{mc}_{t+\tau}] + \sum_{\tau=0}^{\infty} (\tilde{\beta}\gamma)^\tau \mathbb{E}_t[\varepsilon^p_{t+\tau}]$. Thus, the responses of $\hat{\pi}_{t+h}$ to a monetary policy shock at time t for $h \geq 0$ depend on the discounted values of the expected responses of marginal costs, $\kappa \sum_{\tau=0}^{\infty} (\tilde{\beta}\gamma)^\tau \mathbb{E}_t[\hat{mc}_{t+h+\tau}]$.

[13] The conventional measures of economic activity mostly reflect factor prices in the marginal costs (Equation (3.10)). Consider the posterior mode of the translog model in recent periods. Conditional on monetary policy shocks, the cyclical correlations between factor prices ($\alpha_{ks}\hat{r}_{st} + \alpha_{ke}\hat{r}_{et} + \alpha_s\hat{w}_{st} + \alpha_u\hat{w}_{ut}$) and the output gap, labor gap, labor shares, and fixed cost adjusted labor shares are 0.95, 0.94, 0.98, and 0.97, respectively. In contrast, $\text{corr}_{c,mp}(\hat{mc}_t, \text{factor prices})$ is only 0.31, reflecting the cyclical returns to scale in the marginal costs.

Table 5: Variance decomposition of inflation

	(1)	(2)	(3)	(4)	(5)	(6)
	Early sample (1966-99)			Late sample (2000-19)		
	C–D	NCES	Translog	C–D	NCES	Translog
Panel A. Variance of inflation at the cyclical frequencies						
$\text{var}_c(\hat{\pi}_t)$	0.086	0.076	0.084	0.050	0.018	0.057
Panel B. Contribution of each structural shock, %						
Productivity (Hicks neutral)	3	2	3	3	1	1
Risk premium	1	0	1	1	6	40
Government spending	1	0	1	2	0	0
Investment-specific productivity	3	1	7	41	0	12
Monetary policy	5	4	9	9	2	8
Price markup	55	69	50	38	91	34
Wage markup, skilled	1	4	1	2	0	1
Wage markup, unskilled	32	20	27	2	1	2
Skilled worker population share	0	0	0	0	0	1

Notes: This table shows the variance of inflation at the business cycle frequencies corresponding to the periodicity of 6-32 quarters (Panel A) and its decomposition into the contribution of the nine structural shocks (Panel B) in the posterior mode of each model.

that the translog model in recent periods relies more on risk premium shocks, probably capturing the Great Recession, and less on price markup shocks than the other models do. Furthermore, inflation expectation is a more important driver of inflation in the post-2000 period than before.

Table 5 shows the variance of inflation at the business cycle frequencies (Panel A) and its decomposition into the contribution of the nine structural shocks (Panel B) in the posterior mode of each model. Given more stable inflation data in recent periods, the estimated models using the post-2000 data predict smaller variances than those using the early sample. Furthermore, all three models similarly predict that price and wage markup shocks explain the majority of the cyclical variations in inflation in Columns (1)-(3). However, for the late sample, different models attribute inflation volatility to different shocks. Given a quite flat Phillips curve, the nested CES model relies heavily on price markup shocks. The Cobb–Douglas model adds investment-specific shocks to the set of meaningful drivers of inflation, yielding a larger cyclical variance of inflation than the nested CES model does. In contrast to the results in Columns (1)-(5), the translog model captures aggregate fluctuations due to risk premium shocks and their contribution to inflation. Finally, wage markup shocks have minimal contributions to overall inflation volatility in recent periods in all three models.

Table 6 decomposes $\text{var}_c(\hat{\pi}_t)$ through the lens of the Phillips curve. Equation (3.8) implies that $\text{var}_c(\hat{\pi}_t) = \text{cov}_c(\hat{\pi}_t, \tilde{\beta}\gamma\mathbb{E}_t[\hat{\pi}_{t+1}]) + \text{cov}_c(\hat{\pi}_t, \kappa\hat{mc}_t) + \text{cov}_c(\hat{\pi}_t, \varepsilon_t^p)$. These three terms equal

Table 6: Variance decomposition of inflation using the Phillips curve

	(1)	(2)	(3)	(4)	(5)	(6)
	Early sample (1966-99)			Late sample (2000-19)		
	$\mathbb{E}_t[\hat{\pi}_{t+1}]$	$\hat{mc}_t$	ε_t^p	$\mathbb{E}_t[\hat{\pi}_{t+1}]$	$\hat{mc}_t$	ε_t^p
Panel A. Decomposition of $var_c(\hat{\pi}_t)$ *using the Phliips curve:* $\hat{\pi}_t = \tilde{\beta}\gamma\mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa\hat{mc}_t + \varepsilon_t^p$						
$\text{cov}_c(\hat{\pi}_t, \tilde{\beta}\gamma\mathbb{E}_t[\hat{\pi}_{t+1}]$ or $\kappa\hat{mc}_t$ or $\varepsilon_t^p)$	0.062	0.004	0.018	0.048	0.002	0.008
slope ($\tilde{\beta}\gamma$ or κ or 1)	0.997	0.026	1	0.999	0.017	1
$\text{corr}_c(\hat{\pi}_t, \mathbb{E}_t[\hat{\pi}_{t+1}]$ or $\hat{mc}_t$ or $\varepsilon_t^p)$	0.978	0.409	0.684	0.969	0.301	0.530
$\text{std}_c(\hat{\pi}_t)$	0.290	0.290	0.290	0.239	0.239	0.239
$\text{std}_c(\mathbb{E}_t[\hat{\pi}_{t+1}]$ or $\hat{mc}_t$ or $\varepsilon_t^p)$	0.219	1.428	0.089	0.206	1.391	0.061
Panel B. Contribution of each structural shock, %						
Productivity (Hicks neutral)	3	17	-	1	4	-
Risk premium	1	7	-	49	-2	-
Government spending	1	6	-	0	0	-
Investment-specific productivity	9	22	-	14	30	-
Monetary policy	11	20	-	9	-3	-
Price markup	40	-18	100	23	37	100
Wage markup, skilled	1	0	-	1	17	-
Wage markup, unskilled	34	44	-	2	15	-
Skilled worker population share	0	0	-	1	2	-

Notes: Panel A decomposes $\text{var}_c(\hat{\pi}_t)$ through the lens of the Phillips curve. Equation (3.8) implies that $\text{var}_c(\hat{\pi}_t) = \text{cov}_c(\hat{\pi}_t, \tilde{\beta}\gamma\mathbb{E}_t[\hat{\pi}_{t+1}]) + \text{cov}_c(\hat{\pi}_t, \kappa\hat{mc}_t) + \text{cov}_c(\hat{\pi}_t, \varepsilon_t^p)$. These three terms are shown in the first row based on the translog models in the posterior mode for the pre- and post-2000 samples. The remaining part of Panel A decomposes each covariance into the slope (e.g., κ), the correlation between $\hat{\pi}_t$ and, e.g., $\hat{mc}_t$, and standard deviations of these two variables. Panel B presents conditional covariances on each structural shock relative to unconditional covariances. For example, according to column (5), 4% of $\text{cov}_c(\hat{\pi}_t, \kappa\hat{mc}_t)$ is because of the covariation driven by TFP shocks.

0.062, 0.004, and 0.018 (the first three numbers in the first row in Panel A) in the posterior mode of the translog model for the pre-2000 sample. Similarly, for the translog model using the post-2000 data, $\text{var}_c(\hat{\pi}_t)$ is decomposed into 0.048 (inflation expectation), 0.002 (marginal cost), and 0.008 (price markup shock). These numbers imply that the contribution of inflation expectation to inflation volatility, $\frac{\text{cov}_c(\hat{\pi}_t, \tilde{\beta}\gamma\mathbb{E}_t[\hat{\pi}_{t+1}])}{\text{var}_c(\hat{\pi}_t)}$, increased from 74% to 84% between the two sample periods. This increased importance of inflation expectation or, relatedly, a decrease in $\text{cov}_c(\hat{\pi}_t, \kappa\hat{mc}_t)$ can be decomposed into changes in the slope (κ), the correlation between $\hat{\pi}_t$ and $\hat{mc}_t$, and the standard deviations of these two variables, as shown in Panel A. Comparing Columns (2) and (5) suggests that the decrease in $\text{cov}_c(\hat{\pi}_t, \kappa\hat{mc}_t)$ is mostly due to the moderate flattening of the marginal cost Phillips curve (κ) and a decrease in the correlation between inflation and the marginal cost.

The decrease in this correlation results from procyclical returns to scale that dampen the short-term responses of marginal costs conditional on demand shocks. Consider the impulse responses of inflation and marginal costs to monetary policy shocks (Figure 2(c)-(d)). The impact response of marginal costs is weak. In contrast, inflation responds significantly be-

cause it reflects the present discounted value of the current and future marginal costs. Given similar mechanisms for other demand shocks, the contemporaneous correlation between the marginal costs and inflation decreases after 2000 compared with before 2000.

Panel B illustrates conditional covariances on each structural shock relative to unconditional covariances. For example, according to Column (5), 4% of $\text{cov}_c(\hat{\pi}_t, \kappa\hat{mc}_t)$ is because of the comovement driven by TFP shocks. Comparing Columns (2) and (5), we find that the contribution of demand shocks (risk premium, government spending, and monetary policy) to $\text{cov}_c(\hat{\pi}_t, \kappa\hat{mc}_t)$ indeed decreased between the two periods. The contemporaneous covariance conditional on monetary policy shocks is negative in Column (5), reflecting a negative short-run response of marginal costs in Figure 2(d) for the recent period. A similar result holds for risk premium shocks (not shown) as a result of procyclical returns to scale conditional on demand shocks.

Another significant difference between Columns (2) and (5) is the correlation between inflation and the marginal costs conditional on price markup shocks. Before 2000, a positive price markup shock increased inflation and decreased the marginal cost, generating a negative correlation between them. In contrast, after 2000, price makeup shocks led to positively correlated variations between inflation and the marginal cost. This change is again caused by more procyclical returns to scale. When a price markup shock induces high inflation and low output, decreased factor inputs endogenously decrease returns to scale, pushing marginal costs upward.

3.6 Pandemic-Era Inflation through the Lens of the Model

Given the different inflation dynamics under the translog model compared with those under the Cobb–Douglas and nested CES models, this section investigates recent inflation data through the lens of all three models. Specifically, we augment the sample with the eight quarterly variables used for the Bayesian estimation in Section 3.2 from 2020:q1 to 2024:q2.[14] For each model, given the parameter values in the posterior mode in Section 3.2, we use the Kalman smoother to estimate the realized structural shocks during this extended sample period from 2020:q1-2024:q2.

Figure 3(b)-(d) shows the historical decomposition of the realized quarterly inflation into the contributions of each structural shock and the pre-2020 economic conditions for

[14]We use the skilled employment share until 2023:q4. Its values in 2024:q1-q2 are treated as missing.

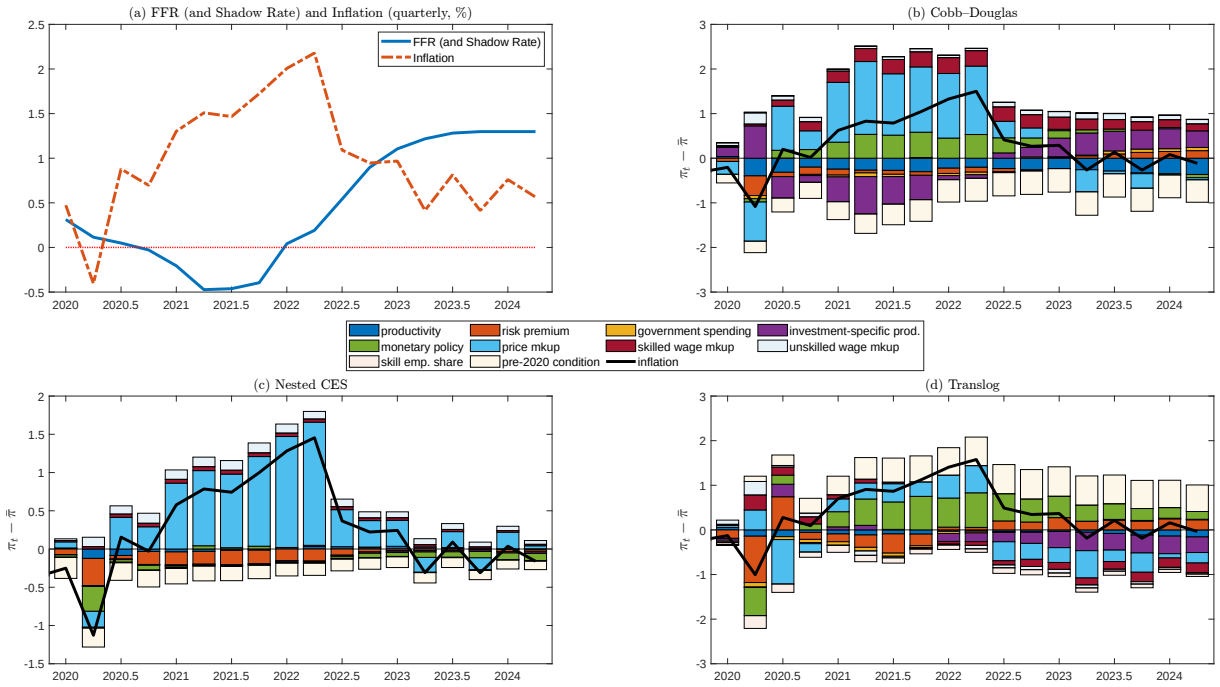


Figure 3: Historical decomposition of the pandemic-era inflation

Notes: Panel (a) presents the federal funds rate (augmented with the shadow rate of Wu and Xia (2016) when the zero lower bound was binding) and quarterly GDP deflator inflation. Panels (b)-(d) show the historical decomposition of the realized quarterly inflation into the contribution of each structural shock and the pre-2020 conditions for the Cobb–Douglas, nested CES, and translog models, respectively.

the Cobb–Douglas, nested CES, and translog models, respectively. Panel (a) presents the federal funds rate (augmented with the shadow rate of Wu and Xia (2016) when the zero lower bound was binding) and quarterly GDP deflator inflation. Clearly, US monetary policy was loose between 2021:q1 and 2022:q2 when inflation was increasing.

Panel (d) shows that the translog model emphasizes the contribution of risk premium shocks at the onset of the pandemic in 2020:q2, reflecting elevated demand for safe assets (Fisher, 2015), and loose monetary policies contributing to high inflation during 2021:q1-2022:q2 in addition to price markup shocks, probably reflecting global supply chain disruptions and the global energy crisis (Di Giovanni et al., 2022; Bernanke and Blanchard, 2023; Bai et al., 2024). Because this model features less sticky prices than the other two models do, demand shifters such as monetary policy and risk premium shocks appear to be meaningful sources of recent inflation fluctuations. Note that this result holds despite quite flat conventional Phillips curves in recent periods.

In contrast, the Cobb–Douglas (Panel (b)) and nested CES (Panel (c)) models rely heav-

ily on price markup shocks to match the realized inflation dynamics after 2020. Because the Phillips curves are quite flat in these models, demand disturbances contribute little to inflation, as shown in Table 5.[15] Instead, these models shift the Phillips curve using supply shocks to match the inflation fluctuations.

In short, the translog model provides a more comprehensive interpretation of pandemic-era inflation than the Cobb–Douglas and nested CES models do. The historical decomposition in Panel (d) reflects several aggregate developments that occurred during the period instead of relying solely on supply-side disturbances. Furthermore, this result emphasizes that monetary policies have been highly relevant to inflation in recent years, which is consistent with the results based on different methods (see Comin, Johnson and Jones, 2023; Gagliardone and Gertler, 2023; Bocola et al., 2024; Giannone and Primiceri, 2024). In contrast, the Cobb–Douglas and nested CES models do not predict this policy implication. Finally, these results are robust to two-input (labor and capital) models à la Smets and Wouters (2007) with translog production functions (see Appendix C.6).[16]

4 Empirical Analyses

This section estimates production function parameters using multicountry, industry-level data and panel data methods. We show that the estimates are consistent with the results based on the structural estimation of the translog DSGE models in Section 3.2, corroborating stronger input complementarity in recent decades than in earlier decades.

[15] See L'Huillier, Phelan and Zame (2022); Beaudry, Hou and Portier (2024*b*) for other policy implications of quite flat Phillips curves.

[16] Our historical decomposition analysis using a structural model is complementary to other explanations for recent inflation. Given the Ricardian nature of the model, fiscal shocks had limited contributions to inflation, as shown in Figure 3. For models emphasizing the role of massive fiscal stimulus packages and increases in unfunded government debt, see Chen, Leeper and Leith (2022); Bianchi, Faccini and Melosi (2023); Barro and Bianchi (2023); Di Giovanni et al. (2023). Crump et al. (2024) focuses on labor market conditions and the measurement of the natural rate of unemployment, which is absent in our model. Note that our analysis assumes full-information, linear rational expectations models. Beaudry, Hou and Portier (2024*a*) deviates from this assumption and introduces limited information and bounded rational beliefs that affect inflation through inflation expectations. Finally, several authors have questioned linearity and proposed convex price and wage Phillips curves (see Daly and Hobijn, 2014; Ball, Leigh and Mishra, 2022; Boehm and Pandalai-Nayar, 2022; Harding, Lindé and Trabandt, 2022, 2023; Benigno and Eggertsson, 2023; Schmitt-Grohé and Uribe, 2023; Blanco et al., 2024*a*,*b*). In contrast, Kocherlakota (2024) argues that the substitution effect among intermediate goods yields a concave Phillips curve. Interestingly, a nonlinear version of our model can generate convex Phillips curves even in an environment similar to that of Kocherlakota (2024) if input complementarity is sufficiently strong (see Appendix A).

4.1 Data

We utilize data from the EU KLEMS Growth and Productivity Accounts, which provide detailed industry-level information across various countries in the European Union. For our analysis, we rely on three releases of the EU KLEMS dataset: March 2008, October 2012, and February 2023. We combine these three releases to achieve the most comprehensive coverage of labor data, particularly labor usage disaggregated by educational attainment. We use the March 2008 release for the pre-2000 sample (1970-1999). The 2012 release fills a gap between the two other releases by supplying labor account data from 2002 to 2007. By combining the 2012 and 2023 releases, we create a continuous dataset for the post-2000 sample (2002-2019). Our sample period for recent decades begins in 2002 because labor data disaggregated by educational attainment are limited for 2000 and 2001.[17]

We construct a multicountry, industry-level panel dataset by integrating the EU KLEMS national, labor, and capital accounts. From the national accounts, we extract data on the volumes and prices of output, intermediate inputs, total labor hours, and compensation across industries, countries, and years. The labor accounts decompose labor inputs by education, allowing us to calculate hours worked by college graduates (skilled labor) and others (unskilled labor), as well as wages for each group. We classify college graduates as skilled workers, which is consistent with the DSGE estimation in Section 3.2.1 (see also Krusell et al., 2000; Ohanian, Orak and Shen, 2023). Finally, we obtain volumes and prices for the real capital stock from the capital accounts. Appendix D.1 reports the summary statistics of the data (see also Bontadini et al., 2023).

4.2 Empirical Framework

Estimating a translog production function is challenging because of the large number of parameters involved. To address this issue, we use the FOC for the firm's cost-minimization problem following Gandhi, Navarro and Rivers (2020), Chan et al. (2024), and Hyun, Kim and Lee (2024). This condition establishes a relationship between the marginal product of an input (e.g., skilled labor) and its price (e.g., the skilled wage rate). Using this condition, we identify the shape of the production function associated with the input considered. We

[17] Our sample covers Austria, Belgium, Germany, Denmark, Spain, Finland, France, the United Kingdom, Italy, Japan, Luxembourg, the Netherlands, Portugal, and Sweden. Labor data for 2000-2001 are only available for Finland and Italy. From 2002 (2008) onward, the data are available for Austria, Germany, Spain, France, the UK, and the Netherlands (the other countries).

utilize the time series and cross-sectional variations in (lagged) input prices as IVs to further enhance the identification, following Doraszelski and Jaumandreu (2013, 2018) and Hyun, Kim and Lee (2024).

To fix ideas, consider an economic environment similar to that in Section 3.1.1 with the following production function:

$$y = \exp(\varepsilon^a) f(\log \boldsymbol{x}) - \nu,$$

where y denotes the output, ε^a represents the level of productivity, and $\boldsymbol{x}$ is a vector of factor inputs. The fixed cost of production, ν, ensures the zero-profit condition on the BGP. The FOC for the cost-minimization problem with respect to an input x^i is given by:

$$\underbrace{\frac{P^i}{P}}_{\text{real input price}} = mc \times \underbrace{\frac{\tilde{y}}{x^i}\frac{\partial \log f}{\partial \log x^i}}_{\text{marginal product}},$$

where P^i represents the nominal price of x^i, $\tilde{y} = y + \nu$, and mc denotes the real marginal costs of production. Rearranging this equation yields:

$$s^i \equiv \frac{P^i x^i}{Py} = mc \frac{\tilde{y}}{y}\frac{\partial \log f}{\partial \log x^i}, \tag{4.1}$$

where s^i is the input expenditure share of total sales for input x^i.

We log-linearize this equation and relate the input i's expenditure share to factor inputs in period t as follows:

$$\hat{s}_t^i = \sum_k \delta_{ik} \hat{x}_t^k + \tau_t, \tag{4.2}$$

$$\text{where} \qquad \delta_{ik} = -\beta_{k\cdot} - (\Phi - 1)\alpha_k + \frac{\beta_{ik}}{\alpha_i}, \tag{4.3}$$

$\alpha_i = \frac{\partial \log f}{\partial \log x^i}$, $\beta_{ik} = \frac{\partial^2 \log f}{\partial \log x^k \partial \log x^i}$, $\beta_{k\cdot} = \sum_\ell \beta_{k\ell}$, and Φ is the gross markup along the BGP. τ_t is given by real factor prices and the level of productivity: $\tau_t = \sum_\ell \alpha_\ell \widehat{(P_t^\ell / P_t)} - \varepsilon_t^a$.

The key parameters in this section, δ_{ik}s, represent how x^k affects the input share, s^i. As shown in Equation (4.3), δ_{ik} consists of three terms. The first term, $-\beta_{k\cdot}$, emerges because x^k has effects on mc via rts^{tl} and, thus, s^i (see Equation (4.1)). When $\beta_{k\cdot} > 0$, x^k

positively contributes to rts^{tl}, and thus, mc decreases (see also Equation (3.10)). The second component, $-(\Phi-1)\alpha_k$, arises from the fixed cost in production, inducing countercyclical variations in $\frac{\tilde{y}}{y}$. Finally, $\frac{\beta_{ik}}{\alpha_i}$ captures how the output elasticity (ignoring the fixed costs), $\frac{\partial \log f}{\partial \log x^i}$, varies with x^k. Note that in a textbook New Keynesian model with a Cobb–Douglas production function ($\beta_{ik}=0$ for all i and k and $\nu=0$) (see, e.g., Galí, 2015), the input share, s^i, simplifies to $mc \times \alpha_i$. In that case, all input shares exhibit the same cyclical variations because $\hat{s}^i_t = \hat{mc}_t = \tau_t$ for all i, and δ_{ik} equals zero for all i and k.

The empirical specification for our regression analysis builds on the theoretical prediction in Equation (4.2). Specifically, we estimate the following equation using the EU KLEMS data:

$$\log\left(s^i_{cjt}\right) = \sum_k \delta_{ik} \log\left(x^k_{cjt}\right) + \alpha_c + \gamma_j + \tau_t + \varepsilon^i_{cjt}, \tag{4.4}$$

where c, j, and t denote country, industry, and year, respectively. We include country, industry, and time fixed effects (α_c, γ_j, and τ_t) to control for unobserved heterogeneity at these levels. Additionally, Equation (4.4) does not involve real output, y^i_{cjt}. The dependent variable, s^i_{cjt}, can be computed using the two nominal values, $P^i_{cjt}x^i_{cjt}$ and $P_{cjt}y_{cjt}$. This property allows us to circumvent the construction of the exact output price index (see, e.g., Hottman, Redding and Weinstein, 2016). Finally, we assume that δs do not depend on industry j. That is, we consider a representative production function to inform the aggregate production function assumed in the DSGE model in Section 3.

Equation (4.4) is based on the relationship between the real input price and the marginal product of the input. A wedge between these two quantities may arise from frictions in the economy, such as input adjustment costs (Hall, 2004), imperfect competition in output and factor markets (Rotemberg and Woodford, 1999; Berger, Herkenhoff and Mongey, 2022), and financial frictions (Jermann and Quadrini, 2012; Arellano, Bai and Kehoe, 2019; Bigio and La'O, 2020). After controlling for county, industry, and time fixed effects, the remaining idiosyncratic component of this wedge could appear in the residual, ε^i_{cjt}. This residual term may also capture measurement errors and ex post productivity shocks (Gandhi, Navarro and Rivers, 2020).

To enhance the identification of δs, following Doraszelski and Jaumandreu (2013, 2018) and Hyun, Kim and Lee (2024), we utilize lagged input prices as IVs. Our identification assumption is that after controlling for fixed effects, the idiosyncratic components of the lagged input prices are orthogonal to the idiosyncratic components of the wedge in the resid-

ual. For instance, if the residual is serially uncorrelated, the lagged input prices satisfy the exogeneity condition for the IVs. The relevance condition holds when x^i is correlated with the input price and the input price is serially correlated. In addition to this benchmark specification, we perform several robustness checks considering potential sources of persistence in the residual, such as adjustment costs and persistent country-industry-specific components in productivity. For the IV relevance, we will present the first-stage F-statistics as a measure of correlations between x^i and lagged input prices.

4.3 Estimation Results

We focus on the cases where the skilled labor share is the dependent variable in the regression equation (4.4). We show that the values of δ vary between the two sample periods, consistent with the structural estimation results in Section 3.2.

We use the skilled labor share in this section for three reasons. First, the translog model estimation results in Table 3 indicate that β_{ks} and β_{su} increased significantly between the pre- and post-2000 samples. These increases in βs suggest that δ_{ks} and δ_{su} also changed accordingly between the two sample periods. Note that this prediction is testable using the IV estimation results of Equation (4.4) when the skilled labor share is placed on the left-hand side. Second, δ_{si} includes the β_{si}/α_s term in Equation (4.3). Since α_s is relatively small (see Table 3), even a moderate change in β_{si} could result in a sizable shift in the DSGE model-implied δ_{si} for $i = k, u$. In contrast, δ_{ui} is less sensitive to changes in β_{ui} because $\alpha_s < \alpha_u$ (see Table 3). Thus, detecting a structural change in the degree of input complementarity (represented by different values of βs) from the estimates of δs in the empirical model is more viable when the skilled labor share is considered than the unskilled labor share. Finally, decomposing capital into structures and equipment, as Krusell et al. (2000) and Ohanian, Orak and Shen (2023) did for the US aggregate data, is challenging for the EU KLEMS data because of the lack of necessary information. Thus, we do not pursue empirical specifications using the equipment (or capital) expenditure shares for the comparability of the results.

To investigate potential changes in the production function over time, we split the sample into two periods: 1970-1999 and 2002-2019. Columns (1) and (2) in Table 7 present the IV estimation results for the pre- and post-2000 samples, respectively. The two-step efficient generalized method of moments (GMM) estimator is employed, with standard errors clustered at the country level. One- and two-year lagged input prices are utilized as IVs. In

Table 7: IV and structural estimation of δ

	IV Estimation		Structural Estimation	
	pre-2000	post-2000	pre-2000	post-2000
	(1)	(2)	(3)	(4)
Capital	-0.548	0.765		
	(0.091)	(0.321)		
Structure			-0.060	-0.040
			(0.008)	(0.007)
Equipment			-0.361	0.192
			(0.132)	(0.070)
Skilled Labor	0.429	-0.093	0.746	0.207
	(0.239)	(0.418)	(0.040)	(0.129)
Unskilled Labor	-0.721	0.477	-1.259	0.475
	(0.389)	(0.640)	(0.100)	(0.234)
Intermediate Goods	0.334	0.135		
	(0.322)	(0.120)		
F-stat for Capital	25.25	13.25		
F-stat for Skilled Labor	158.89	13.15		
F-stat for Unskilled Labor	61.33	23.46		
F-stat for Intermediate Goods	531.64	71.62		
Observations	2,372	2,566		

Notes: The skilled labor share is used as the dependent variable in regression equation (4.4). Columns (1) and (2) report the IV regression results for the period 1970-1999 and 2002-2019, respectively. The inputs are instrumented using lagged input prices at $t-1$ and $t-2$. The two-step efficient GMM specification is employed, with standard errors clustered at the country level, shown in parentheses. In columns (3) and (4), we calculate the values of δ in the posterior mode and the posterior standard deviation of the translog model. For this purpose, we use Equation (4.3) and the Bayesian estimation results discussed in Section 3.2.

Columns (3) and (4), we calculate the values of δ in the posterior mode and the posterior standard deviation of the translog model. For this purpose, we use Equation (4.3) and the Bayesian estimation results discussed in Section 3.2.

By comparing the results in Columns (1) and (2), we can evaluate how the production function has changed over time. First, the sign of the coefficient on capital shifts from negative to positive, indicating that capital has become more complementary to skilled labor over time. Similarly, the coefficient on unskilled labor is negative in Column (1) but becomes positive and insignificant in Column (2). This result suggests that skilled and unskilled labor were more substitutable in the earlier period but became less substitutable over time. In contrast, the coefficient on skilled labor decreases between the two sample periods. Overall, all these changes are consistent with the results from the structural estimations shown in Columns (3) and (4); δ_{ks} (equipment-skilled labor) and δ_{su} (skilled labor-unskilled labor) increase, whereas δ_{ss} decreases between the two sample periods in the posterior mode of the translog model. Despite the different methods, types of variations, and countries covered in

the data, the results from the IV and structural estimations are largely congruent with one another.

In summary, the IV analysis in this section suggests that the production function has evolved over time, consistent with the DSGE analysis in Section 3. This empirical evidence aligns well with our explanation of the flattening of the Phillips curve, which is based on changes in the degree of input complementarity and the resulting cyclicality of returns to scale. Our results also emphasize the need for a more flexible approach beyond the standard Cobb–Douglas production function.

Robustness checks. The results in this section are robust to multiple alternative specifications and data adjustments as follows. First, we consider a value-added production function and use capital services (available only in the March 2008 release) instead of capital stock. Second, we broaden the post-2000 sample by including Finnish and Italian data from 2000 instead of 2002. Third, we compute Driscoll and Kraay (1998) standard errors. Fourth, we employ two- and three-year lagged input prices as IVs to lessen the potential influence of serial correlation in the residual of regression (4.4). Fifth, we adjust the skilled labor share by average input prices (weighted by expenditure shares) in view of $\sum_k \alpha_k \hat{p}_{kt}$ in τ_t (see Equation (4.3)). Lastly, we enhance the moment conditions to account for potentially persistent, country-industry-specific components in the levels of productivity. In all these cases, the main results remain largely unchanged (see Appendix D.2).

5 Conclusion

This paper shows that more procyclical returns to scale in recent decades can reconcile the two seemingly divergent empirical findings: (i) the Phillips curve is steep and alive when the marginal costs are used as a forcing variable (Gagliardone et al., 2023), and (ii) the Phillips curves have flattened when conventional measures of economic activity, such as the output gap, labor gap, and labor shares, are employed (Del Negro et al., 2020; Stock and Watson, 2020; Inoue, Rossi and Wang, 2024). We develop a theory based on a more flexible production function than the tightly parameterized Cobb–Douglas and CES functions. Our quantitative results emphasize how changes in the cyclicality of returns to scale help explain inflation dynamics and the Phillips curve in the US. Furthermore, our model predicts the growing significance of inflation expectations, echoing the insights from Coibion and Gorodnichenko (2015), Hazell et al. (2022), and Meeks and Monti (2023).

This paper considers closed economy models and ignores the potential source of meaningful complementarity between imported intermediate goods and domestic factor inputs. The rise of globalization and its macroeconomic implications through cyclical returns to scale under the translog framework is left for future research.

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Cyclical Returns to Scale and the Slopes of the Phillips Curves

Online Appendix - Not for Publication

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Appendix A Supplementary Materials for Section 2

This section investigates the nonlinear properties of the Phillips curve induced by translog production functions. The exposition is based on the tractable illustrative model in Section 2.

We use hats to denote log deviations, e.g., $\hat{y} = \log(y/\bar{y})$, where $\bar{y}$ represents the steady state value of y. We ignore the productivity shocks to focus on the movement of the economy along the Phillips curve driven by demand shocks, as in the case of Section 2. Subscripts are dropped when they are unnecessary.

Model. The household side consists of two types of workers. The population share of each type is half. Labor hours per type i worker are denoted by h_i for $i = 1, 2$. The total supply of type i labor is denoted by $l_i = 0.5h_i$ for $i = 1, 2$. The aggregate labor input l is given by:

$$l = l_1 + l_2 = 0.5h_1 + 0.5h_2.$$

In a symmetric equilibrium, $\bar{l} = \bar{h}_1 = \bar{h}_2$, $\bar{l}_1 = \bar{l}_2 = 0.5\bar{l}$, and $\hat{l} = \hat{l}_1 = \hat{l}_2 = \hat{h}_1 = \hat{h}_2$.

Considering the first- and second-order terms, the translog production function in the log-deviation form can be expressed as:

$$\begin{aligned}\hat{y} &= \alpha_1\hat{l}_1 + \alpha_2\hat{l}_2 + 0.5\beta_{11}(\hat{l}_1)^2 + \beta_{12}\hat{l}_1\hat{l}_2 + 0.5(\hat{l}_2)^2, \\ &= \hat{l} + 0.5\beta(\hat{l})^2,\end{aligned}$$

where $\alpha_1 = \alpha_2 = 1$ and $\beta = \beta_{11} + 2\beta_{12} + \beta_{22}$. β captures the magnitude of overall input complementarity arising from the second-order terms in the translog production function.

We restrict our attention to the values of $\hat{l}$ such that output increases in labor and β is strictly positive ($\beta > 0$). Therefore:

$$\hat{l} \geq -\frac{1}{\beta} \qquad\qquad \hat{y} \geq -\frac{1}{2\beta}.$$

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Also,

$$\hat{l} = \frac{-1 + \sqrt{1 + 2\beta\hat{y}}}{\beta}.$$

Given the symmetry, $\beta_{11} = \beta_{22}$. Thus,

$$\begin{aligned}\beta_{1\cdot} &\equiv \beta_{11} + \beta_{12} = 0.5\beta, \\ \beta_{2\cdot} &\equiv \beta_{12} + \beta_{22} = 0.5\beta.\end{aligned}$$

Furthermore, the elasticity of output with respect to each input equals:

$$\begin{aligned}\frac{\partial \log y}{\partial \log l_i} = \frac{\partial \hat{y}}{\partial \hat{l_i}} &= \alpha_i + \beta_{ii}\hat{l}_i + \beta_{ij}\hat{l}_j \\ &= 0.5 + \beta_{i\cdot}\hat{l} \\ &= 0.5 + 0.5\beta\hat{l}\end{aligned}$$

for $i = 1, 2$, where $j \neq i$, $\beta_{21} = \beta_{12}$, and the second line uses the symmetry between i and j.

Let w_i be the real hourly wage rate for type i labor. The symmetry implies that $w_1 = w_2$, denoted by w. Then, an additive separable preference, $\frac{c^{1-\sigma_c}-1}{1-\sigma_c} - \frac{h^{1+1/\sigma_l}}{1+1/\sigma_l}$, and the fact that $c_1 = c_2 = y$ in equilibrium yield that:

$$w_i c_i^{-\sigma_c} = h_i^{1/\sigma_l} \quad \Rightarrow \quad \hat{w} = \sigma_c \hat{y} + \frac{1}{\sigma_l}\hat{l}.$$

Marginal costs. The cost-minimization problem of an intermediate goods producer leads to $mc = \frac{w_i}{\frac{\partial y}{\partial l_i}} = \frac{w}{\frac{\partial \hat{y}}{\partial \hat{l_i}} \frac{y}{l_i}}$. Thus,

$$\begin{aligned}\hat{mc} &= \hat{w} - \widehat{(0.5 + 0.5\beta\hat{l})} - (\hat{y} - \hat{l}) \\ &= (\sigma_c - 1)\hat{y} + (1 + \frac{1}{\sigma_l})\hat{l} - \log\left(\frac{0.5 + 0.5\beta\hat{l}}{0.5}\right) \\ &= (\sigma_c - 1)\hat{y} + (1 + \frac{1}{\sigma_l})\hat{l} - \log\left(1 + \beta\hat{l}\right).\end{aligned}$$

Note that the first two terms are standard. The translog functional form adds a new term to marginal costs, $-\log\left(1 + \beta\hat{l}\right)$, related to the returns to scale (see also Equation (B.102)).

The marginal costs are the key forcing variable in the New Keynesian Phillips curve. In contrast, the conventional forms of the Phillips curves use the output and labor gaps as forcing variables. Below, we investigate the relationship between the conventional measures of the inverse slack ($\hat{y}$ and $\hat{l}$) and the model-predicted measure of it ($\hat{mc}$). Because we only focus on demand-driven dynamics, the natural levels of output and labor do not change. Therefore, $\hat{y}$ and $\hat{l}$ coincide with the output and labor gaps.

Proposition OA.1. (Marginal costs and labor gap). If $\sigma_c \geq 1$ and $\beta > 0$, the marginal costs are a strictly increasing convex function in labor in logarithm on $\hat{l} > \frac{1}{1+1/\sigma_l} - \frac{1}{\beta}$.

Proof. Note that:

$$\begin{aligned}\frac{\partial \hat{mc}}{\partial \hat{l}} &= (\sigma_c - 1)\frac{\partial \hat{y}}{\partial \hat{l}} + (1 + \frac{1}{\sigma_l}) - \frac{\beta}{1+\beta\hat{l}} \\ &= (\sigma_c - 1)(1+\beta\hat{l}) + (1 + \frac{1}{\sigma_l}) - \frac{\beta}{1+\beta\hat{l}} \ > \ 0, \\ \frac{\partial^2 \hat{mc}}{\partial \hat{l}^2} &= (\sigma_c - 1)\beta + \frac{\beta^2}{(1+\beta\hat{l})^2} \ > \ 0. \qquad \square\end{aligned}$$

Proposition OA.2. (Marginal costs and output gap). If $\sigma_c \geq 1$ and $\beta > 0$, the marginal costs are a strictly increasing convex function in output in logarithm on $\frac{1}{1+1/\sigma_l} - \frac{1}{\beta} < \hat{l} < \frac{2}{1+1/\sigma_l} - \frac{1}{\beta}$.

Proof. Note that:

$$\begin{aligned}\frac{\partial \hat{mc}}{\partial \hat{y}} &= (\sigma_c - 1) + \left[\left(1 + \frac{1}{\sigma_l}\right) - \frac{\beta}{1+\beta\hat{l}}\right]\left(\frac{\partial \hat{y}}{\partial \hat{l}}\right)^{-1} \\ &= (\sigma_c - 1) + \left[\left(1 + \frac{1}{\sigma_l}\right) - \frac{\beta}{1+\beta\hat{l}}\right]\frac{1}{1+\beta\hat{l}} \ > \ 0, \\ \frac{\partial^2 \hat{mc}}{\partial \hat{y}^2} &= \frac{\partial^2 \hat{mc}}{\partial \hat{l}\partial \hat{y}}\left(\frac{\partial \hat{y}}{\partial \hat{l}}\right)^{-1} \\ &= \left\{\frac{\beta^2}{(1+\beta\hat{l})^3} - \left[\left(1 + \frac{1}{\sigma_l}\right) - \frac{\beta}{1+\beta\hat{l}}\right]\frac{\beta}{(1+\beta\hat{l})^2}\right\}\frac{1}{1+\beta\hat{l}} \\ &= \frac{\beta}{(1+\beta\hat{l})^4}\left\{\beta - \left(1 + \frac{1}{\sigma_l}\right)(1+\beta\hat{l}) + \beta\right\} \\ &= \frac{\beta^2}{(1+\beta\hat{l})^4}\left(1 + \frac{1}{\sigma_l}\right)\left\{\frac{2}{1+\frac{1}{\sigma_l}} - \frac{1}{\beta} - \hat{l}\right\} \ > \ 0. \qquad \square\end{aligned}$$

Thus, under certain conditions, the marginal costs convexly increase in economic activity. To examine whether this convexity in the marginal costs translates into the convexity in inflation, we investigate a static Phillips curve postulated by Kocherlakota (2024). This formulation allows us to study the curvature of the Phillips curve analytically by circumventing issues related to nonlinear dynamics in expected inflation in the conventional Calvo- and Rotemberg-type models.

Suppose that ζ-fraction of prices do not change and $1-\zeta$-fraction of firms reoptimize their prices. ϵ represents the elasticity of substitution across the intermediate goods. In this situation, we obtain the following marginal cost Phillips curve of Kocherlakota (2024, Equation (6)):

$$\Pi = \left(\frac{1 - (1-\zeta)\Phi^{1-\epsilon} mc^{1-\epsilon}}{\zeta}\right)^{-\frac{1}{1-\epsilon}},$$

where Π is the gross inflation and $\Phi = 1/\bar{mc}$ is the steady-state gross markup, implying that $\bar{\Pi} = 1$. Thus, $\Phi^{1-\epsilon} mc^{1-\epsilon} = \Phi^{1-\epsilon}\bar{mc}^{1-\epsilon}\exp((1-\epsilon)\hat{mc}) = \exp((1-\epsilon)\hat{mc})$. Furthermore:

$$\hat{\pi} \equiv \log\left(\frac{\Pi}{\bar{\Pi}}\right) = \frac{1}{\epsilon - 1}\log\left(\frac{1 - (1-\zeta)\exp((1-\epsilon)\hat{mc})}{\zeta}\right).$$

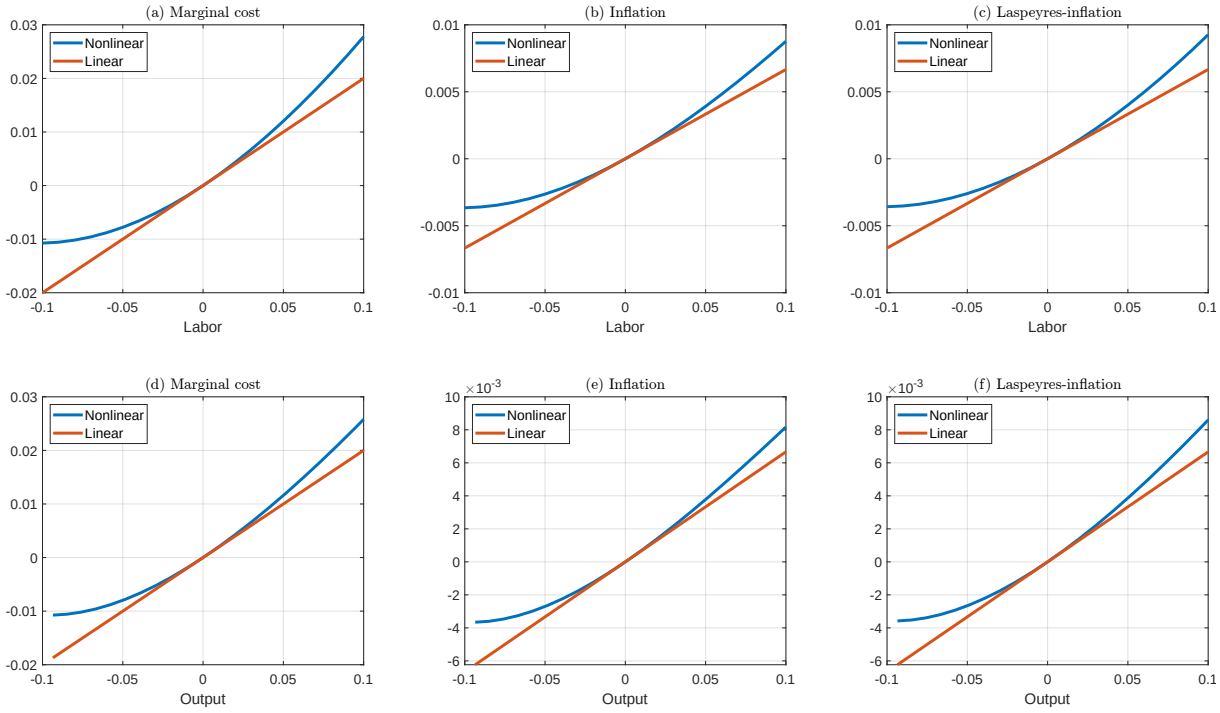


Figure A.1: Nonlinear Phillips curves under translog production functions

Note that:

$$\frac{\partial \hat{\pi}}{\partial \hat{mc}} = \frac{(1-\zeta)\exp((1-\epsilon)\hat{mc})}{1-(1-\zeta)\exp((1-\epsilon)\hat{mc})},$$
$$\frac{\partial^2 \hat{\pi}}{\partial \hat{mc}^2} = \frac{(1-\zeta)\exp((1-\epsilon)\hat{mc})}{[1-(1-\zeta)\exp((1-\epsilon)\hat{mc})]^2}(1-\epsilon).$$

We assume that $1-(1-\zeta)\exp((1-\epsilon)\hat{mc}) > 0$ or, equivalently, $\hat{mc} > \frac{1}{\epsilon-1}\log(1-\zeta)$. Also, we assume that $\epsilon > 1$. Clearly, $\frac{\partial \hat{\pi}}{\partial \hat{mc}} > 0$ and $\frac{\partial^2 \hat{\pi}}{\partial \hat{mc}^2} < 0$. Thus, the marginal cost Phillips curve is concave because of the substitution effect, represented by the fact that $1-\epsilon < 0$, as shown in Kocherlakota (2024).

How about the labor and output Phillips curves?

$$\frac{\partial \hat{\pi}}{\partial \hat{l}} = \frac{\partial \hat{\pi}}{\partial \hat{mc}}\frac{\partial \hat{mc}}{\partial \hat{l}} > 0$$
$$\frac{\partial^2 \hat{\pi}}{\partial \hat{l}^2} = \frac{\partial(\partial \hat{\pi}/\partial \hat{mc})}{\partial \hat{mc}}\left(\frac{\partial \hat{mc}}{\partial \hat{l}}\right)^2 + \frac{\partial \hat{\pi}}{\partial \hat{mc}}\frac{\partial^2 \hat{mc}}{\partial \hat{l}^2},$$
$$= \underbrace{\frac{\partial^2 \hat{\pi}}{\partial \hat{mc}^2}}_{(-)}\underbrace{\left(\frac{\partial \hat{mc}}{\partial \hat{l}}\right)^2}_{(+)} + \underbrace{\frac{\partial \hat{\pi}}{\partial \hat{mc}}}_{(+)}\underbrace{\frac{\partial^2 \hat{mc}}{\partial \hat{l}^2}}_{(+)}.$$

Thus, if the marginal costs are convex enough in labor to dominate the concavity in the marginal cost Phillips curve, the labor Phillips curve can be convex. By changing $\hat{l}$ to $\hat{y}$, we can obtain a similar result for the output Phillips curve.

Numerical example. For illustration, we plot $\hat{mc}$, $\hat{\pi}$, and $\hat{\pi}^L \equiv \log(\zeta + (1-\zeta)\exp(\hat{mc}+\hat{\pi}))$ against $\hat{l}$ and $\hat{y}$, where $\hat{\pi}^L$ is the Laspeyres price inflation in Kocherlakota (2024, Equation (9)). We set $\sigma_c = 1$, $\sigma_l = 2$, $\zeta = 0.75$, $\epsilon = 4$, and $\beta = 1.3$. ϵ is chosen according to a relatively sizeable steady-state price markup estimate in Section 3

to conventional calibration.

Consistent with the theoretical results above, a sufficiently strong input complementarity, measured by β, generates convex labor and output Phillips curves (shown in the second column in Figure A.1) through its nonlinear effects on the marginal costs (the first column). Our results hold even for the true inflation as well as the Laspeyres inflation that underestimates substitution effects (see Kocherlakota, 2024) (the third column in Figure A.1).

Appendix B Medium-scale DSGE Model

This section illustrates the model details used for quantitative analyses in Section 3. We assume a normalized translog production function following Hyun, Kim and Lee (2024). Firms use four inputs in production: structure, equipment, skilled labor, and unskilled labor, as described in Krusell et al. (2000); Ohanian, Orak and Shen (2023). Our model builds on the medium-scale DSGE models in Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007*b*), featuring sticky prices and wages, fixed costs of production, investment adjustment costs, costly capacity utilization, and external consumption habits. By deviating minimally from the Smets and Wouters (2007*b*) model, we emphasize the flexible input complementarity and substitution structure introduced by the translog production function and its contribution to the slopes of the Phillips curves.

B.1 Decision Problems and Equilibrium Conditions

B.1.1 Final good producers

The final good producers solve the following problem, as is the case in the Smets and Wouters (2007*b*) model:

$$\max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \qquad \text{s.t.} \qquad \int_0^1 G\left(\frac{Y_t(i)}{Y_t}; \lambda_t^p\right) di = 1,$$

where Y_t and $Y_t(i)$ are the final and intermediate goods, respectively, and P_t and $P_t(i)$ are the corresponding prices. G is an aggregator à la Kimball (1995), and λ_t^p represents the (net) time-varying price markup.

Given the Lagrange multiplier, μ_t, the FOCs are given by:

$$\begin{aligned} Y_t: \quad & P_t = \frac{\mu_t}{Y_t} \int_0^1 G'\left(\frac{Y_t(i)}{Y_t}; \lambda_t^p\right)\left(\frac{Y_t(i)}{Y_t}\right) di \\ Y_t(i): \quad & P_t(i) = \frac{\mu_t}{Y_t} G'\left(\frac{Y_t(i)}{Y_t}; \lambda_t^p\right). \end{aligned} \tag{B.1}$$

Combining the above equations yields the following zero-profit condition for the final good producers.

$$P_t Y_t = \int_0^1 P_t(i) Y_t(i) di. \tag{B.2}$$

Furthermore, we write the demand for each intermediate good as follows:

$$Y_t(i) = Y_t \times (G')^{-1}\left(\frac{P_t(i)}{P_t}\tau_t; \lambda_t^p\right), \tag{B.3}$$

where $\tau_t = P_t Y_t / \mu_t$.

B.1.2 Intermediate goods producers

We extend the problem of the intermediate goods producers in the Smets and Wouters (2007*b*) model by introducing two types of capital and labor following Krusell et al. (2000) and Ohanian, Orak and Shen (2023). We further augment a conventional Cobb–Douglas production function with second-order terms, yielding a translog production

function. Specifically, the production function for intermediate good i is given by:

$$Y_t(i) = \exp(\varepsilon_t^a)[K_{st}(i)]^{\alpha_{ks}}[K_{et}(i)]^{\alpha_{ke}}[\gamma^t L_{st}(i)]^{\alpha_s}[\gamma^t L_{ut}(i)]^{\alpha_u} \times \exp(s.o.t._{it}) - \gamma^t \upsilon, \tag{B.4}$$
$$s.o.t._{it} = \left(\hat{k}_{i,et}, \hat{l}_{i,st}, \hat{l}_{i,ut}\right) \boldsymbol{\beta} \left(\hat{k}_{et}, \hat{l}_{st}, \hat{l}_{ut}\right)',$$

where $K_{st}(i)$, $K_{et}(i)$, $L_{st}(i)$, and $L_{ut}(i)$ are structures, capital equipment services, skilled labor hours, and unskilled labor hours, respectively. γ represents the (gross) growth rate on the balanced growth path (BGP). υ captures the fixed costs in production, yielding zero profits on the BGP. Aggregate productivity, ε_t^a, follows an exogenous process. We assume that $\alpha_{ks} + \alpha_{ke} + \alpha_s + \alpha_u = 1$.

Regarding the second-order terms, we assume a normalized translog production function following Hyun, Kim and Lee (2024). We use bars and hats to denote the BGP values and log deviations from these values, respectively. Specifically, $\hat{k}_{i,et} = \log\left(\frac{K_{et}(i)}{K_{et}}\right)$, $\hat{l}_{i,st} = \log\left(\frac{L_{st}(i)}{L_{st}}\right)$, $\hat{l}_{i,ut} = \log\left(\frac{L_{ut}(i)}{L_{ut}}\right)$, $\hat{k}_{et} = \log\left(\frac{K_{et}}{K_{et}}\right)$, $\hat{l}_{st} = \log\left(\frac{L_{st}}{L_{st}}\right)$, and $\hat{l}_{ut} = \log\left(\frac{L_{ut}}{L_{ut}}\right)$, where K_{et}, L_{st}, and L_{ut} are aggregate capital services, skilled labor hours, and unskilled labor hours, respectively. $\boldsymbol{\beta}$ is a matrix consisting of the translog parameters:

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_{kk} & \beta_{ks} & \beta_{ku} \\ \beta_{ks} & \beta_{ss} & \beta_{su} \\ \beta_{ku} & \beta_{su} & \beta_{uu} \end{pmatrix}.$$

Profit of firm i in period t is given by:

$$\Pi_t(i) = P_t(i)Y_t(i) - R_{st}K_{st}(i) - R_{et}K_{et}(i) - W_{st}L_{st}(i) - W_{ut}L_{ut}(i), \tag{B.5}$$

where W_{st}, W_{ut}, R_{st}, and R_{et} are the nominal skilled wage rate, nominal unskilled wage rate, rental rate of structures, and rental rate of capital services, respectively.

Consider the following cost minimization problem:

$$\min R_{st}K_{st}(i) + R_{et}K_{et}(i) + W_{st}L_{st}(i) + W_{ut}L_{ut}(i) - MC_t(i)(Y_t(i) - Y), \tag{B.6}$$

where $MC_t(i)$ is the Lagrange multiplier associated with the constraint $Y_t(i) \geq Y$, representing the nominal marginal costs.

First-order conditions. The FOCs are as follows:

$$K_{st}(i): \quad R_{st} = MC_t(i)\frac{\tilde{Y}_t(i)}{K_{st}(i)}\alpha_{ks}, \tag{B.7}$$

$$K_{et}(i): \quad R_{et} = MC_t(i)\frac{\tilde{Y}_t(i)}{K_{et}(i)}\alpha_{ke,t}, \qquad \alpha_{ke,t} \equiv \left(\alpha_{ke} + \beta_{kk}\hat{k}_{et} + \beta_{ks}\hat{l}_{st} + \beta_{ku}\hat{l}_{ut}\right), \tag{B.8}$$

$$L_{st}(i): \quad W_{st} = MC_t(i)\frac{\tilde{Y}_t(i)}{L_{st}(i)}\alpha_{st}, \qquad \alpha_{st} \equiv \left(\alpha_s + \beta_{ks}\hat{k}_{et} + \beta_{ss}\hat{l}_{st} + \beta_{su}\hat{l}_{ut}\right), \tag{B.9}$$

$$L_{ut}(i): \quad W_{ut} = MC_t(i)\frac{\tilde{Y}_t(i)}{L_{ut}(i)}\alpha_{ut}, \qquad \alpha_{ut} \equiv \left(\alpha_u + \beta_{ku}\hat{k}_{et} + \beta_{su}\hat{l}_{st} + \beta_{uu}\hat{l}_{ut}\right), \tag{B.10}$$

where $\tilde{Y}_t(i) = Y_t(i) + \gamma^t \upsilon$.

Second-order conditions. The cost minimization problem (B.6) is equivalent to minimizing:

$$R_{st}\exp(\log(K_{st}(i))) + R_{et}\exp(\log(K_{et}(i))) + W_{st}\exp(\log(L_{st}(i))) + W_{ut}\exp(\log(L_{ut}(i)))$$

by choosing $\log(K_{st}(i))$, $\log(K_{et}(i))$, $\log(L_{st}(i))$, and $\log(L_{ut}(i))$ subject to $\log(Y_t(i) + \gamma^t \upsilon) \geq \log(Y + \gamma^t \upsilon)$. The bordered Hessian for this optimization problem is given by:

$$\begin{pmatrix} 0 & \alpha_{ks} & \alpha_{ke,t} & \alpha_{st} & \alpha_{ut} \\ \alpha_{ks} & R_{st}K_{st}(i) & 0 & 0 & 0 \\ \alpha_{ke,t} & 0 & R_{et}K_{et}(i) & 0 & 0 \\ \alpha_{st} & 0 & 0 & W_{st}L_{st}(i) & 0 \\ \alpha_{ut} & 0 & 0 & 0 & W_{ut}L_{ut}(i) \end{pmatrix}.$$

If the last three leading principal minors of the above matrix are negative, the second-order condition for the cost minimization is satisfied (see, e.g., Simon and Blume, 1994, Theorem 19.6). For simple notation, we write $a_1 = \alpha_{ks}$, $a_2 = \alpha_{ke,t}$, $a_3 = \alpha_{st}$, $a_4 = \alpha_{ut}$, $d_1 = R_{st}K_{st}(i)$, $d_2 = R_{et}K_{et}(i)$, $d_3 = W_{st}L_{st}(i)$, and $d_4 = W_{ut}L_{ut}(i)$. The last three leading principal minors, denoted by M_3, M_4, and M_5, are as follows:

$$\begin{aligned} M_3 &= -a_1^2 d_1 - a_2^2 d_1, \\ M_4 &= -a_1^2 d_2 d_3 - a_2^2 d_1 d_3 - a_3^2 d_1 d_2, \\ M_5 &= -a_1^2 d_2 d_3 d_4 - a_2^2 d_1 d_3 d_4 - a_3^2 d_1 d_2 d_4 - a_4^2 d_1 d_2 d_3. \end{aligned}$$

Clearly, when all as and ds are strictly positive, the second-order condition is satisfied. We assume all factor prices and quantities are strictly positive and $\alpha_{ks}, \alpha_{ke,t}, \alpha_{st}, \alpha_{ut} > 0$.

Marginal costs. Equation (B.7) implies that $K_{st}(i) = MC_t(i)\frac{\alpha_{ks}\tilde{Y}_t(i)}{R_{st}}$. Plugging similar equations for $K_{et}(i)$, $L_{st}(i)$, and $L_{ut}(i)$ into Equation (B.4) yields the following equation for the nominal marginal costs:

$$\begin{aligned} MC_t(i) = &\left(\frac{R_{st}}{\alpha_{ks}}\right)^{\alpha_{ks}} \left(\frac{R_{et}}{\alpha_{ke}}\right)^{\alpha_{ke}} \left(\frac{W_{st}}{\gamma^t \alpha_s}\right)^{\alpha_s} \left(\frac{W_{ut}}{\gamma^t \alpha_u}\right)^{\alpha_u} \exp(-\varepsilon_t^a) \\ &\times \left(\frac{\alpha_{ke,t}}{\alpha_{ke}}\right)^{-\alpha_{ke}} \left(\frac{\alpha_{st}}{\alpha_s}\right)^{-\alpha_s} \left(\frac{\alpha_{ut}}{\alpha_u}\right)^{-\alpha_u} \\ &\times \exp(-s.o.t._{it}). \end{aligned} \tag{B.11}$$

Similarly, we define the following aggregate variable that each firm takes as given:

$$\begin{aligned} MC_t = &\left(\frac{R_{st}}{\alpha_{ks}}\right)^{\alpha_{ks}} \left(\frac{R_{et}}{\alpha_{ke}}\right)^{\alpha_{ke}} \left(\frac{W_{st}}{\gamma^t \alpha_s}\right)^{\alpha_s} \left(\frac{W_{ut}}{\gamma^t \alpha_u}\right)^{\alpha_u} \exp(-\varepsilon_t^a) \\ &\times \left(\frac{\alpha_{ke,t}}{\alpha_{ke}}\right)^{-\alpha_{ke}} \left(\frac{\alpha_{st}}{\alpha_s}\right)^{-\alpha_s} \left(\frac{\alpha_{ut}}{\alpha_u}\right)^{-\alpha_u} \\ &\times \exp(-s.o.t._t), \end{aligned} \tag{B.12}$$

where $s.o.t._t \equiv \left(\hat{k}_{et}, \hat{l}_{st}, \hat{l}_{ut}\right) \boldsymbol{\beta} \left(\hat{k}_{et}, \hat{l}_{st}, \hat{l}_{ut}\right)'$. Note that the first line in Equation (B.12) based on factor prices and TFP resembles the nominal marginal costs in the Cobb–Douglas model. The last line disappears in log-linearization

because it is of the second order. Then, the translog structure contributes to the marginal costs through a novel first-order variation represented by the terms in the second line in Equation (B.12).

Sticky prices. Next, we investigate the price-setting problem under the Calvo pricing framework with partial indexation. Firms can reoptimize their prices, $P_t(i)$, with the probability of $1-\zeta_p$ each period. When it cannot reoptimize, $\{P_{t+s}(i)\}$ evolves as follows. For each $s \geq 0$,

$$P_{t+s}(i) = P_t(i) X_{t,s}, \qquad \text{where} \qquad X_{t,s} = \begin{cases} 1 & s = 0 \\ \left(\Pi_{t-1}^{t+s-1}\right)^{\iota_p} \left(\bar{\Pi}^s\right)^{1-\iota_p} & s \geq 1 \end{cases},$$

where $\Pi_{t_1}^{t_0} = \frac{P_{t_1}}{P_{t_0}}$ denotes the gross price inflation from period t_0 to t_1, and $\bar{\Pi}$ represents the steady-state gross price inflation. As in Smets and Wouters (2007*b*), reoptimizing firms in period t choose the optimal reset price, $\tilde{P}_t(i)$, by solving the following problem:

$$\begin{aligned} &\max_{\tilde{P}_t(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \beta^s \frac{\Xi_{t+s}/P_{t+s}}{\Xi_t/P_t} \left[\tilde{P}_t(i) X_{t,s} - MC_{t+s}\right] Y_{t+s|t} \\ &\text{s.t.} \quad Y_{t+s|t} = Y_{t+s} \times (G')^{-1} \left(\frac{\tilde{P}_t(i) X_{t,s}}{P_{t+s}} \tau_{t+s}; \lambda_{t+s}^p\right), \end{aligned}$$

where the demand function follows from Equation (B.3), and $\beta^s \frac{\Xi_{t+s}/P_{t+s}}{\Xi_t/P_t}$ is the nominal stochastic discount factor. With some algebra, we can show that:

$$\frac{\partial Y_{t+s|t}}{\partial \tilde{P}_t(i)} = Y_{t+s} \frac{\frac{X_{t,s}}{P_{t+s}} \tau_{t+s}}{G''} = \frac{Y_{t+s|t}}{(G')^{-1}} \frac{1}{\tilde{P}_t(i)} \frac{\frac{\tilde{P}_t(i) X_{t,s}}{P_{t+s}} \tau_{t+s}}{G''} = \frac{1}{\tilde{P}_t(i)} \frac{Y_{t+s|t}}{(G')^{-1}} \frac{G'}{G''},$$

where G' and G'' are evaluated at $(\frac{Y_{t+s|t}}{Y_{t+s}}; \lambda_{t+s}^p)$, and $(G')^{-1}$ is evaluated at $\left(\frac{\tilde{P}_t(i) X_{t,s}}{P_{t+s}} \tau_{t+s}; \lambda_{t+s}^p\right)$. Therefore, the FOC with respect to $\tilde{P}_t(i)$ is given by:

$$\begin{aligned} &\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \beta^s \frac{\Xi_{t+s}/P_{t+s}}{\Xi_t/P_t} \left\{ X_{t,s} Y_{t+s|t} + \left[\tilde{P}_t(i) X_{t,s} - MC_{t+s}\right] \frac{\partial Y_{t+s|t}}{\partial \tilde{P}_t(i)} \right\} = 0 \\ \Rightarrow \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \beta^s \frac{\Xi_{t+s}/P_{t+s}}{\Xi_t/P_t} Y_{t+s|t} &\left\{ \tilde{P}_t(i) X_{t,s} \left[1 + \frac{1}{(G')^{-1}} \frac{G'}{G''}\right] - MC_{t+s} \frac{1}{(G')^{-1}} \frac{G'}{G''} \right\} = 0. \end{aligned} \tag{B.13}$$

Clearly, all reoptimizing firms choose the same $\tilde{P}_t(i)$, which is denoted by $\tilde{P}_t$.

To compute the aggregate price index P_t, we use Equation (B.2). In period t, for each $s \geq 0$, there are $(1-\zeta_p)\zeta_p^s$ fractions of firms that optimized their prices in period $t-s$ for the last time. In period t, the prices of those firms' products are given by $\tilde{P}_{t-s} X_{t-s,s}$. Then,

$$\begin{aligned} P_t &= \int_0^1 P_t(i) \frac{Y_t(i)}{Y_t} di \\ &= (1-\zeta_p) \tilde{P}_t \frac{Y_{t|t}}{Y_t} + (1-\zeta_p)\zeta_p \tilde{P}_{t-1} X_{t-1,1} \frac{Y_{t|t-1}}{Y_t} + (1-\zeta_p)\zeta_p^2 \tilde{P}_{t-2} X_{t-2,2} \frac{Y_{t|t-2}}{Y_t} + \dots \\ &= (1-\zeta_p) \tilde{P}_t \times (G')^{-1} \left(\frac{\tilde{P}_t}{P_t} \tau_t; \lambda_t^p\right) + (1-\zeta_p)\zeta_p \tilde{P}_{t-1} X_{t-1,1} \times (G')^{-1} \left(\frac{\tilde{P}_{t-1} X_{t-1,1}}{P_t} \tau_t; \lambda_t^p\right) + \dots . \end{aligned} \tag{B.14}$$

Returns to scale. Let the production function (B.4) be denoted by $f_t(K_{st}(i), K_{et}(i), L_{st}(i), L_{ut}(i); \varepsilon_t^a, K_{et}, L_{st}, L_{ut})$. The returns to scale (or the scale elasticity) is defined as follows:

$$rts_{it} \equiv \frac{\partial \log \left[f_t(\lambda K_{st}(i), \lambda K_{et}(i), \lambda L_{st}(i), \lambda L_{ut}(i); \varepsilon_t^a, K_{et}, L_{st}, L_{ut}) \right]}{\partial \log \lambda} |_{\lambda=1}.$$

The output augmented with the fixed costs, $f_t + \gamma^t \upsilon$, is denoted by $\tilde{f}_t$. Then, $rts_{it} = \frac{\partial \log f_t}{\partial \log \tilde{f}_t} \frac{\partial \log \tilde{f}_t}{\partial \log \lambda} |_{\lambda=1} = \frac{\tilde{f}_t}{f_t} \frac{\partial f_t}{\partial \tilde{f}_t} \left[\alpha_{ks} + \alpha_{ke} + \alpha_s + \alpha_u + \mathbf{1}' \boldsymbol{\beta} \left(\hat{k}_{et}, \hat{l}_{st}, \hat{l}_{ut} \right)' \right] |_{\lambda=1}$, where $\mathbf{1} = (1,1,1)'$. Thus,

$$rts_{it} = \frac{Y_t(i) + \gamma^t \upsilon}{Y_t(i)} \left(1 + \beta_{k\cdot} \hat{k}_{et} + \beta_{s\cdot} \hat{l}_{st} + \beta_{u\cdot} \hat{l}_{ut} \right), \tag{B.15}$$

where $\beta_{k\cdot} = \beta_{kk} + \beta_{ks} + \beta_{ku}$, $\beta_{s\cdot} = \beta_{ks} + \beta_{ss} + \beta_{su}$, and $\beta_{u\cdot} = \beta_{ku} + \beta_{su} + \beta_{uu}$.

Remarks. (i) Structure, $K_{st}(i)$, appears only in the first-order term, as in Krusell et al. (2000) and Ohanian, Orak and Shen (2023). In contrast, an arbitrary degree of complementarity and substitutability is allowed for any pair of the remaining three factors: equipment service, skilled labor, and unskilled labor.

(ii) The second-order term degenerates on the BGP because $\hat{k}_{i,et} = \hat{l}_{i,st} = \hat{l}_{i,ut} = \hat{k}_{et} = \hat{l}_{st} = \hat{l}_{ut} = 0$ on the BGP. Thus, the translog parameters do not affect the BGP and the long-term dynamics implied by the model, which are identical to those in the Cobb–Douglas model. This property allows us to focus on the business cycle implications of input complementarity and cyclical returns to scale without affecting the long-term properties of the model, such as Kaldor's facts.

(iii) In a deviation form from the BGP, the output, augmented with the fixed costs, is given by:

$$\frac{Y_t(i) + \gamma^t \upsilon}{\bar{Y}_t + \gamma^t \upsilon} = \exp(\varepsilon_t^a) \left[\frac{K_{st}(i)}{\bar{K}_{st}} \right]^{\alpha_{ks}} \left[\frac{K_{et}(i)}{\bar{K}_{et}} \right]^{\alpha_{ke,t}} \left[\frac{L_{st}(i)}{\bar{L}_{st}} \right]^{\alpha_{st}} \left[\frac{L_{ut}(i)}{\bar{L}_{ut}} \right]^{\alpha_{ut}}.$$

This expression clearly illustrates how the translog structure affects the elasticity of output with respect to each factor and the scale elasticity (i.e., returns to scale). Ignoring the fixed costs, the elasticity of output with respect to $K_{et}(i)$, $L_{st}(i)$, and $L_{ut}(i)$, captured by $\alpha_{ke,t}$, α_{st}, and α_{ut}, are endogenously determined and time-varying around their long-term values, α_{ke}, α_s, and α_u. Furthermore, the translog structure can yield a cyclical component in the returns to scale because:

$$rts^{tl} \equiv \alpha_{ks} + \alpha_{ke,t} + \alpha_{st} + \alpha_{ut} = 1 + \beta_{k\cdot} \hat{k}_{et} + \beta_{s\cdot} \hat{l}_{st} + \beta_{u\cdot} \hat{l}_{ut}, \tag{B.16}$$

(iv) A conventional second-order Taylor expansion of a generic production function in $\hat{k}_{i,et}$, $\hat{l}_{i,st}$, and $\hat{l}_{i,ut}$ leads to

$$s.o.t. = \frac{1}{2} \left(\hat{k}_{i,et}, \hat{l}_{i,st}, \hat{l}_{i,ut} \right) \boldsymbol{\beta} \left(\hat{k}_{i,et}, \hat{l}_{i,st}, \hat{l}_{i,ut} \right)'.$$

This formulation and our specification in Equation (B.4) yield the same expressions for the production function and firms' first-order conditions for the cost minimization problems in log-linearization. However, our specification does not allow each firm to choose its own returns to scale, represented by the fact that $\alpha_{ke,t}$, α_{st}, and α_{ut} are exogenous from firm i's perspective because they are determined at the aggregate level. This property simplifies

the second-order conditions for the cost minimization problem. As shown above, under our specification (B.4), the second-order condition is identical to that in the Cobb–Douglas model.

Other production functions. Clearly, the translog production function (B.4) nests the Cobb–Douglas function with the restriction that $\boldsymbol{\beta} = 0$.

Next, we consider a nested constant elasticity of substitution (CES) production function employed in Krusell et al. (2000) and Ohanian, Orak and Shen (2023). In a deviation form with dimensionless parameters as those in Cantore and Levine (2012); Cantore et al. (2014, 2015), we can write the nested CES production function as follows:

$$\frac{Y_t(i)+\gamma^t\upsilon}{\bar{Y}_t+\gamma^t\upsilon} = \exp(\varepsilon_t^a)\left[\frac{K_{st}(i)}{\bar{K}_{st}}\right]^{\alpha_{ks}}\left\{\tilde{\alpha}_u\left[\frac{L_{ut}(i)}{\bar{L}_{ut}}\right]^{\phi_{ux}}+\tilde{\alpha}_x\left(\tilde{\alpha}_k\left[\frac{K_{et}(i)}{\bar{K}_{et}}\right]^{\phi_{ks}}+\tilde{\alpha}_s\left[\frac{L_{st}(i)}{\bar{L}_{st}}\right]^{\phi_{ks}}\right)^{\frac{\phi_{ux}}{\phi_{ks}}}\right\}^{\frac{1-\alpha_{ks}}{\phi_{ux}}}, \quad \text{(B.17)}$$

where $\tilde{\alpha}_u+\tilde{\alpha}_x=1$, $\tilde{\alpha}_k+\tilde{\alpha}_s=1$, and $-\infty \leq \phi_{ux}, \phi_{ks} \leq 1$. It is shown in Section B.4 that the equilibrium conditions based on this production function are a special case under (B.4) in log-linearization. Specifically, first, the fact that the alpha parameter gives the steady-state income share of each factor leads to the following parameter restrictions:

$$\begin{aligned}
\tilde{\alpha}_u &= \frac{\alpha_u}{1-\alpha_{ks}}, \\
\tilde{\alpha}_x &= 1-\frac{\alpha_u}{1-\alpha_{ks}}, \\
\tilde{\alpha}_k &= \frac{\alpha_{ke}}{(1-\alpha_{ks})\tilde{\alpha}_x}, \\
\tilde{\alpha}_s &= 1-\frac{\alpha_{ke}}{(1-\alpha_{ks})\tilde{\alpha}_x}.
\end{aligned} \quad \text{(B.18)}$$

Then, the translog parameters consistent with the nested CES form in (B.17) are given by:

$$\begin{aligned}
\beta_{kk} &= \alpha_{ke}(\phi_{ux}\tilde{\alpha}_k\tilde{\alpha}_u+\phi_{ks}\tilde{\alpha}_s), \\
\beta_{ks} &= \tilde{\alpha}_k\alpha_s(\phi_{ux}\tilde{\alpha}_u-\phi_{ks}), \\
\beta_{ku} &= -\phi_{ux}\tilde{\alpha}_x\tilde{\alpha}_k\alpha_u, \\
\beta_{ss} &= \phi_{ux}\alpha_s\tilde{\alpha}_s\tilde{\alpha}_u+\phi_{ks}\alpha_s\tilde{\alpha}_k, \\
\beta_{su} &= -\phi_{ux}\alpha_u\tilde{\alpha}_x\tilde{\alpha}_s, \\
\beta_{uu} &= \phi_{ux}\alpha_u\tilde{\alpha}_x.
\end{aligned} \quad \text{(B.19)}$$

B.1.3 Households

A representative household consists of a continuum of members indexed by $j \in [0,1]$. In each period, $\chi_t \in (0,1)$ fraction of members are skilled workers, and the remaining $1-\chi_t$ fraction of workers are unskilled. χ_t follows an exogenous process. For simplicity, we assume that members with $j \in [0, 1-\chi_t)$ and $j \in [1-\chi_t, 1]$ are unskilled and skilled, respectively.

The period utility function of member j is given by:

$$\frac{A_{1j}}{1-\sigma_c}\left(C_t(j)-hC_{t-1}\right)^{1-\sigma_c}\exp\left(A_{2j}\frac{\sigma_c-1}{1+\sigma_l}H_t(j)^{1+\sigma_l}\right),$$

where $C_t(j)$ is consumption, hC_{t-1} represents external consumption habits, and $H_t(j)$ is labor hours. The preference parameters, A_{1j} and A_{2j}, vary across worker types:

$$A_{1j} = \begin{cases} A_1, & j < 1-\chi_t \\ 1, & j \geq 1-\chi_t \end{cases} \qquad A_{2j} = \begin{cases} A_2, & j < 1-\chi_t \\ 1, & j \geq 1-\chi_t \end{cases}.$$

Thus, $A_{1j} = A_{2j} = 1$ for the skilled workers with $j \geq 1-\chi_t$. The unskilled workers share the same $A_1 = \exp\left(\frac{\sigma_c-1}{1+\sigma_l}(1-A_2)\bar{h}^{1+\sigma_l}\right)$ and $A_2 = \frac{\Phi_s/\Phi_u}{\alpha_s/\alpha_u}\frac{\bar{\chi}}{1-\bar{\chi}}$, where $\bar{h}$ is the value of $H_t(j)$ on the BGP for all j. The values of A_1 and A_2 are chosen to maintain the "representativeness" of agents such that consumption and labor hours are the same across worker types on the BGP. This property allows us to deviate minimally from the representative agent framework and write several equilibrium conditions using aggregate consumption, e.g., the consumption Euler equation. However, $C_t(j)$ and $H_t(j)$ can vary across types in the short term.

A family head solves the resource allocation and portfolio choice problems by deciding the consumption ($C_t(j)$) of the family members, family-level risk-free nominal bonds (B_t), structure investment (I_{st}), equipment investment (I_{et}), structure stock ($\tilde{K}_{st}$), equipment stock ($\tilde{K}_{et}$), and equipment utilization rate (Z_t) to maximize the following utilitarian welfare function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 \left[A_{1j} \frac{1}{1-\sigma_c} \left(C_t(j) - hC_{t-1}\right)^{1-\sigma_c} \exp\left(A_{2j} \frac{\sigma_c - 1}{1+\sigma_l} H_t(j)^{1+\sigma_l} \right) \right] dj$$

subject to the budget constraint as follows:

$$\begin{aligned} &\int C_t(j)\, dj + I_{st} + I_{et} + \frac{B_t}{\exp(\varepsilon_t^b) R_t P_t} + T_t \qquad \text{(B.20)} \\ &\leq \frac{B_{t-1}}{P_t} + \frac{W_{st} \int_{1-\chi_t}^1 H_t(j)\, dj}{P_t} + \frac{W_{ut} \int_0^{1-\chi_t} H_t(j)\, dj}{P_t} + \frac{R_{st} \tilde{K}_{s,t-1}}{P_t} + \frac{R_{et} Z_t \tilde{K}_{e,t-1}}{P_t} - a(Z_t)\tilde{K}_{e,t-1} + \frac{\Pi_t^w}{P_t} + \frac{\Pi_t}{P_t} \end{aligned}$$

and the capital accumulation equations for structure and equipment:

$$\tilde{K}_{st} = (1-\delta_s)\tilde{K}_{s,t-1} + \left[1 - S_s\left(\frac{I_{st}}{I_{s,t-1}}\right)\right] I_{st}, \qquad \text{(B.21)}$$

$$\tilde{K}_{et} = (1-\delta_e)\tilde{K}_{e,t-1} + \exp(\varepsilon_t^i)\left[1 - S_e\left(\frac{I_{et}}{I_{e,t-1}}\right)\right] I_{et}. \qquad \text{(B.22)}$$

R_t is the (gross) nominal return to bonds. ε_t^b denotes an exogenous premium in returns to bonds, reflecting a structural shock to the demand for safe and liquid assets (Fisher, 2015). T_t represents lump-sum taxes. Π_t^w and Π_t are the profits of labor unions and firms paid out as dividends. $a(\cdot)$ reflects costly capacity utilization. Following Smets and Wouters (2007*b*), we assume that $a(\bar{Z}) = 0$, where $\bar{Z} = 1$ is the steady state level of utilization. We reparametrize $\frac{a'(\bar{Z})}{a''(\bar{Z})}$ as $\frac{1-\psi}{\psi}$. The amount of capital services that households can rent to firms is given by:

$$K_{st} = \tilde{K}_{s,t-1}, \qquad \text{(B.23)}$$

$$K_{et} = Z_t \tilde{K}_{e,t-1}. \qquad \text{(B.24)}$$

$S_s(\cdot)$ and $S_e(\cdot)$ are the investment adjustment costs for structure and equipment such that $S_s(\gamma) = S_e(\gamma) = 0$, $S_s'(\gamma) = S_e'(\gamma) = 0$, $S_s''(\gamma) = \varphi_s > 0$, and $S_e''(\gamma) = \varphi_e > 0$. ε_t^i is an exogenously determined (equipment)

investment-specific productivity.

Let Ξ_t, Ξ_{st}, and Ξ_{et} be Lagrange multipliers associated with the budget constraint (B.20) and the capital accumulation equations (B.23)-(B.24), respectively. We obtain the following FOCs:

$$C_t(j): \quad \Xi_t = A_{1j}(C_t(j) - hC_{t-1})^{-\sigma_c} \exp\left(A_{2j}\frac{\sigma_c - 1}{1+\sigma_l}H_t(j)^{1+\sigma_l}\right), \tag{B.25}$$

$$B_t: \quad \Xi_t = \beta \exp(\varepsilon_t^b) R_t \mathbb{E}_t\left[\frac{\Xi_{t+1}}{\Pi_{t+1}}\right], \tag{B.26}$$

$$I_{st}: \quad \Xi_t = \Xi_{st}\left[1 - S_s\left(\frac{I_{st}}{I_{s,t-1}}\right) - S_s'\left(\frac{I_{st}}{I_{s,t-1}}\right) \times \left(\frac{I_{st}}{I_{s,t-1}}\right)\right] + \beta \mathbb{E}_t\left[\Xi_{s,t+1} S_s'\left(\frac{I_{s,t+1}}{I_{st}}\right) \times \left(\frac{I_{s,t+1}}{I_{st}}\right)^2\right], \tag{B.27}$$

$$I_{et}: \quad \Xi_t = \Xi_{et}\exp(\varepsilon_t^i)\left[1 - S_e\left(\frac{I_{et}}{I_{e,t-1}}\right) - S_e'\left(\frac{I_{et}}{I_{e,t-1}}\right) \times \left(\frac{I_{et}}{I_{e,t-1}}\right)\right] + \beta \mathbb{E}_t\left[\Xi_{e,t+1}\exp(\varepsilon_{t+1}^i) S_e'\left(\frac{I_{e,t+1}}{I_{et}}\right) \times \left(\frac{I_{e,t+1}}{I_{et}}\right)^2\right], \tag{B.28}$$

$$\tilde{K}_{st}: \quad \Xi_{st} = \beta \mathbb{E}_t\left[\Xi_{t+1}\left(\frac{R_{s,t+1}}{P_{t+1}}\right) + \Xi_{s,t+1}(1-\delta_s)\right], \tag{B.29}$$

$$\tilde{K}_{et}: \quad \Xi_{et} = \beta \mathbb{E}_t\left[\Xi_{t+1}\left(\frac{R_{e,t+1}Z_{t+1}}{P_{t+1}} - a(Z_{t+1})\right) + \Xi_{e,t+1}(1-\delta_e)\right], \tag{B.30}$$

$$Z_t: \quad \frac{R_{et}}{P_t} = a'(Z_t). \tag{B.31}$$

Finally, Tobin's qs for structure and equipment are given by:

$$q_{st} = \frac{\Xi_{st}}{\Xi_t},$$
$$q_{et} = \frac{\Xi_{et}}{\Xi_t}.$$

B.1.4 Labor packers, unions, and unpackers

Households relegate labor supply decisions to labor market institutions. We introduce a similar labor market structure for skilled and unskilled labor. For each type of labor, labor packers operate in a competitive environment, combine differentiated labor services using a Kimball aggregator, and sell the composite labor service to firms. Labor unions set wages for each differentiated labor service subject to Calvo-type frictions with partial indexation, similar to Smets and Wouters (2007*b*). Labor unpackers differentiate labor supply from households and channel them to the labor unions and the labor packers.

Labor unpackers. Consider the unskilled labor market and an unskilled worker $j \in [0, 1-\chi_t)$ supplying H_{ut} hours of labor. Labor unpackers in this market hire this labor, divide H_{ut} into a continuum of differentiated unskilled labor services, $\{H_{ut}(\kappa)\}$, for $\kappa \in [0,1]$, and supply them to each labor union. Specifically, the labor unpackers solve the following problem:

$$\max_{H_{ut}, H_{ut}(\kappa)} \int_0^1 W_{ut}(\kappa) H_{ut}(\kappa) d\kappa - W_{ut} H_{ut} \qquad \text{s.t.} \qquad 1 = \int_0^1 G_U\left(\frac{H_{ut}(\kappa)}{H_{ut}}; \lambda_t^u\right) d\kappa,$$

where G_U is a Kimball aggregator for unskilled labor. Then,

$$W_{ut}H_{ut} = \int W_{ut}(\kappa)H_{ut}(\kappa)d\kappa, \tag{B.32}$$

$$H_{ut}(\kappa) = H_{ut} \times (G_U')^{-1}\left(\frac{W_{ut}(\kappa)}{W_{ut}}\tau_{U,t}; \lambda_t^u\right), \tag{B.33}$$

where $\tau_{U,t} = W_{ut}H_{ut}/\mu_{U,t}$, λ_t^u is the (net) unskilled wage markup, and $\mu_{U,t}$ is the Lagrange multiplier for the above optimization problem.

Labor packers. Labor packers undo this process. They combine the differentiated labor services ($\{H_{ut}(\kappa)\}$) using the same Kimball aggregator (G_U) and supply the composite unskilled labor (H_{ut}) to firms by solving the following problem:

$$\max_{H_{ut}, H_{ut}(\kappa)} W_{ut}H_{ut} - \int_0^1 W_{ut}(\kappa)H_{ut}(\kappa)d\kappa \qquad \text{s.t.} \qquad \int_0^1 G_U\left(\frac{H_{ut}(\kappa)}{H_{ut}}; \lambda_t^u\right)d\kappa = 1.$$

The optimality conditions for the labor packers are the same as Equations (B.32) and (B.33).

Given the population share of unskilled workers $(1-\chi_t)$, in total, $(1-\chi_t)H_{ut}$ hours of unskilled labor are supplied to intermediate goods-producing firms: $L_{ut} = \int L_{ut}(i)di = (1-\chi_t)H_{ut}$.

Labor unions. The labor unions connect the labor unpackers and packers. Each labor union has market power and chooses the wage of its differentiated labor service subject to the labor demand (B.33) and the Calvo-type nominal wage rigidity governed by ζ_u. With the probability of ζ_u, the labor union cannot adjust wages in each period. For those that cannot adjust wages, $W_{ut}(\kappa)$ will be partially indexed to the past price inflation, similar to the prices of intermediate goods:

$$W_{u,t+s}(\kappa) = \tilde{W}_{ut}(\kappa)X_{t,s}^u, \qquad \text{where} \qquad X_{t,s}^u = \begin{cases} 1 & s = 0 \\ \gamma^s\left(\Pi_{t-1}^{t+s-1}\right)^{\iota_u}\left(\bar{\Pi}^s\right)^{1-\iota_u} & s \geq 1 \end{cases}.$$

To characterize the objective function of the labor unions, we compute the marginal rate of substitution between consumption and labor. We assume that labor packers demand the same hours of labor from all unskilled workers with $j \in [0, 1-\chi_t)$. The structure of the model implies that $C_t(j)$ is the same for all $j \in [0, 1-\chi_t)$. Those unskilled workers' consumption and labor are denoted by C_{ut} and H_{ut}. Similarly, the skilled workers' consumption and labor are denoted by C_{st} and H_{st}:

$$C_t(j) = \begin{cases} C_{ut}, & j < 1-\chi_t \\ C_{st}, & j \geq 1-\chi_t \end{cases}, \qquad H_t(j) = \begin{cases} H_{ut}, & j < 1-\chi_t \\ H_{st}, & j \geq 1-\chi_t \end{cases}.$$

Then, the marginal rate of substitution of unskilled workers is given by:

$$MRS_{ut} = \frac{-\frac{A_1}{1-\sigma_c}\left(C_{ut} - hC_{t-1}\right)^{1-\sigma_c}\exp\left(A_2\frac{\sigma_c-1}{1+\sigma_l}H_{ut}^{1+\sigma_l}\right)A_2(\sigma_c-1)H_{ut}^{\sigma_l}}{\Xi_t} = A_2\left(C_{ut} - hC_{t-1}\right)H_{ut}^{\sigma_l}.$$

We define a related variable based only on aggregate consumption, $\tilde{MRS}_{ut}$, as follows:

$$\tilde{MRS}_{ut} = A_2(C_t - hC_{t-1})H_{ut}^{\sigma_l}.$$

Each unskilled labor union takes $M\tilde{R}S_{ut}$ as given and solves the following problem when it can adjust its wage:

$$\max_{\tilde{W}_{ut}(\kappa)} \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_u^s \frac{\beta^s \Xi_{t+s}/P_{t+s}}{\Xi_t/P_t} \left[\tilde{W}_{ut}(\kappa) X_{t,s}^u - P_{t+s} M\tilde{R}S_{u,t+s}\right] H_{u,t+s|t}$$
$$\text{s.t.} \quad H_{u,t+s|t} = H_{u,t+s} \times (G_U')^{-1}\left(\frac{\tilde{W}_{ut}(\kappa) X_{t,s}^u}{W_{u,t+s}} \tau_{U,t+s}; \lambda_{t+s}^u\right).$$

Similar to the intermediate good producers' price-setting problem, we can show that:

$$\frac{\partial H_{u,t+s|t}}{\partial \tilde{W}_{ut}(\kappa)} = H_{u,t+s} \frac{\frac{X_{t,s}^u}{W_{u,t+s}} \tau_{U,t+s}}{G_U''} = \frac{H_{u,t+s|t}}{(G_U')^{-1}} \frac{1}{\tilde{W}_{ut}(\kappa)} \frac{\frac{\tilde{W}_{ut}(\kappa) X_{t,s}^u}{W_{u,t+s}} \tau_{U,t+s}}{G_U''} = \frac{1}{\tilde{W}_{ut}(\kappa)} \frac{H_{u,t+s|t}}{(G_U')^{-1}} \frac{G_U'}{G_U''},$$

where G_U' and G_U'' are evaluated at $\left(\frac{H_{u,t+s|t}}{H_{u,t+s}}; \lambda_{t+s}^u\right)$, and $(G_U')^{-1}$ is evaluated at $\left(\frac{\tilde{W}_{ut}(\kappa) X_{t,s}^u}{W_{u,t+s}} \tau_{U,t+s}; \lambda_{t+s}^u\right)$. Furthermore, the FOC with respect to $\tilde{W}_{ut}(\kappa)$ is given by:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_u^s \beta^s \frac{\Xi_{t+s}/P_{t+s}}{\Xi_t/P_t} \left\{ X_{t,s}^u H_{u,t+s|t} + \left[\tilde{W}_{ut}(\kappa) X_{t,s}^u - P_{t+s} M\tilde{R}S_{u,t+s}\right] \frac{\partial H_{u,t+s|t}}{\partial \tilde{W}_{ut}(\kappa)} \right\} = 0$$
$$\Rightarrow \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_u^s \beta^s \frac{\Xi_{t+s}}{\Xi_t} H_{u,t+s|t} \left\{ \frac{\tilde{W}_{ut}(\kappa) X_{t,s}^u}{P_{t+s}} \left[1 + \frac{1}{(G_U')^{-1}} \frac{G_U'}{G_U''}\right] - M\tilde{R}S_{u,t+s} \frac{1}{(G_U')^{-1}} \frac{G_U'}{G_U''} \right\} = 0. \tag{B.34}$$

Clearly, all reoptimizing unions choose the same $\tilde{W}_{ut}(\kappa)$, denoted by $\tilde{W}_{ut}$.

The above modeling choice based on $M\tilde{R}S_{ut}$ is employed to deviate minimally from the representative agent framework with a single level of consumption for all agents in, e.g., Smets and Wouters (2007*b*). Relatedly, this specification allows us to directly observe all forcing variables in the New Keynesian wage Phillips curves. Indeed, aggregate consumption data are readily available and used for Bayesian estimation in Section 3.2 together with information regarding type-specific wages and hours. Furthermore, MRS_{ut} and $M\tilde{R}S_{ut}$ coincide at least on the BGP. Finally, we can alternatively assume that the family head can allocate the same amount of consumption for all j, i.e., $C_t(j) = C_t$. Under this representative consumption decision assumption, we can obtain the same set of equilibrium conditions.

To compute the aggregate unskilled wage index, W_{ut}, we use Equations (B.32)-(B.33). In period t, for each $s \geq 0$, there are $(1-\zeta_u)\zeta_u^s$-fractions of unions that optimized their wages in $t-s$ for the last time. In period t, those unions' wages are given by $\tilde{W}_{u,t-s} X_{t-s,s}^u$. Then,

$$\begin{aligned} W_{ut} &= \int_0^1 W_{ut}(\kappa) \frac{H_{ut}(\kappa)}{H_{ut}} d\kappa \\ &= (1-\zeta_u)\tilde{W}_{ut} \frac{H_{u,t|t}}{H_{ut}} + (1-\zeta_u)\zeta_u \tilde{W}_{u,t-1} X_{t-1,1}^u \frac{H_{u,t|t-1}}{H_{ut}} + \dots \\ &= (1-\zeta_u)\tilde{W}_{ut} \times (G_U')^{-1}\left(\frac{\tilde{W}_{ut}}{W_{ut}} \tau_{U,t}; \lambda_t^u\right) \\ &\quad + (1-\zeta_u)\zeta_u \tilde{W}_{u,t-1} X_{t-1,1}^u \times (G_U')^{-1}\left(\frac{\tilde{W}_{u,t-1} X_{t-1,1}^u}{W_{ut}} \tau_{U,t}; \lambda_t^u\right) + \dots . \end{aligned} \tag{B.35}$$

Skilled labor. We similarly introduce labor unpackers, unions, and packers for skilled workers. Here, we list the notations and results for the completeness of the exposition.

The labor unpackers and packers' problem leads to the following conditions:

$$W_{st}H_{st} = \int W_{st}(\kappa)H_{st}(\kappa)d\kappa, \tag{B.36}$$

$$H_{st}(\kappa) = H_{st} \times (G_S')^{-1}\left(\frac{W_{st}(\kappa)}{W_{st}}\tau_{S,t};\lambda_t^s\right), \tag{B.37}$$

where G_S is a Kimball aggregator such that:

$$\int_0^1 G_S\left(\frac{H_{st}(\kappa)}{H_{st}};\lambda_t^s\right)d\kappa = 1,$$

$\tau_{S,t} = W_{st}H_{st}/\mu_{S,t}$, λ_t^s is the (net) skilled wage markup, and $\mu_{S,t}$ is the Lagrange multiplier associated with the Kimball aggregator equation.

Given the population share of skilled workers (χ_t), $\chi_t H_{st}$ hours of skilled labor are supplied to intermediate goods-producing firms. Thus, in equilibrium, $L_{st} = \int L_{st}(i)di = \chi_t H_{st}$.

The skilled labor unions cannot adjust their wages with the probability of ζ_s in each period. When not reoptimized, the nominal wage rates evolve according to a partial-indexation scheme with the indexation parameter, ι_s. The marginal rate of substitution of skilled workers is given by:

$$MRS_{st} = \frac{-\frac{1}{1-\sigma_c}\left(C_{st}-hC_{t-1}\right)^{1-\sigma_c}\exp\left(\frac{\sigma_c-1}{1+\sigma_l}H_{st}^{1+\sigma_l}\right)(\sigma_c-1)H_{st}^{\sigma_l}}{\Xi_t} = \left(C_{st}-hC_{t-1}\right)H_{st}^{\sigma_l}.$$

Similar to the unskilled unions, the skilled labor unions take $\tilde{MRS}_{st} = (C_t - hC_{t-1})H_{st}^{\sigma_l}$ as given and solve the following problem when it can adjust its wage:

$$\begin{aligned}\max_{\tilde{W}_{st}(\kappa)}\ & \mathbb{E}_t\sum_{s'=0}^{\infty}\zeta_s^{s'}\frac{\beta^{s'}\Xi_{t+s'}/P_{t+s'}}{\Xi_t/P_t}\left[\tilde{W}_{st}(\kappa)X_{t,s'}^s - P_{t+s'}\tilde{MRS}_{s,t+s'}\right]H_{s,t+s'|t}\\ \text{s.t.}\quad & H_{s,t+s'|t} = H_{s,t+s'}\times(G_S')^{-1}\left(\frac{\tilde{W}_{st}(\kappa)X_{t,s'}^s}{W_{s,t+s'}}\tau_{S,t+s'};\lambda_{t+s'}^s\right).\end{aligned}$$

The aggregate skilled wage index, W_{st}, satisfies the following equation:

$$\begin{aligned}W_{st} &= \int_0^1 W_{st}(\kappa)\frac{H_{st}(\kappa)}{H_{st}}d\kappa\\ &= (1-\zeta_s)\tilde{W}_{st}\times(G_S')^{-1}\left(\frac{\tilde{W}_{st}}{W_{st}}\tau_{S,t};\lambda_t^s\right)\\ &\quad + (1-\zeta_s)\zeta_s\tilde{W}_{s,t-1}X_{t-1,1}^s\times(G_S')^{-1}\left(\frac{\tilde{W}_{s,t-1}X_{t-1,1}^s}{W_{st}}\tau_{S,t};\lambda_t^s\right)+\ldots.\end{aligned} \tag{B.38}$$

B.1.5 Central bank and government policies

The central bank uses the following rule to decide the policy rate:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho} \left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{r_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{r_y}\right]^{1-\rho} \left(\frac{Y_t/Y_{t-1}}{Y_t^*/Y_{t-1}^*}\right)^{r_{\Delta y}} \exp(\varepsilon_t^r), \tag{B.39}$$

where $\bar{R}$ is the steady state nominal (gross) rate, and Y_t^* is the natural level of output. ρ determines the degree of interest rate smoothing.

The government budget constraint is as follows:

$$P_t G_t + B_{t-1} = P_t T_t + \frac{B_t}{\exp(\varepsilon_t^b) R_t}, \tag{B.40}$$

where T_t are nominal lump-sum taxes (or transfers).

Government spending relative to GDP on the BGP is denoted by $\exp(g_t) = \frac{G_t}{Y_t}$, which is assumed to be exogenously determined.

B.1.6 Resource constraints

From the household budget constraint and the government budget constraint, we obtain:

$$\begin{aligned}
&(1-\chi_t)C_{ut} + \chi_t C_{st} + I_{st} + I_{et} + G_t \\
&= \frac{W_{st}\chi_t H_{st}}{P_t} + \frac{W_{ut}(1-\chi_t)H_{ut}}{P_t} + \frac{R_{st}\tilde{K}_{s,t-1}}{P_t} + \frac{R_{et}Z_t\tilde{K}_{e,t-1}}{P_t} - a(Z_t)\tilde{K}_{e,t-1} + \frac{\Pi_t^w}{P_t} + \frac{\Pi_t}{P_t}.
\end{aligned}$$

Also, note that:

$$\begin{aligned}
\Pi_t &= \int P_t(i)Y_t(i) - R_{st}K_{st}(i) - R_{et}K_{et}(i) - W_{st}L_{st}(i) - W_{ut}L_{ut}(i)di \\
&= P_tY_t - R_{st}K_{st} - R_{et}K_{et} - W_{st}L_{st} - W_{ut}L_{ut}, \\
\Pi_t^w &= 0. \qquad \because \text{Equations (B.32) and (B.36)}
\end{aligned}$$

Therefore:

$$C_t + I_t + G_t + a(Z_t)\tilde{K}_{e,t-1} = Y_t, \tag{B.41}$$

where aggregate consumption and investment are given by $C_t = (1-\chi_t)C_{ut} + \chi_t C_{st}$ and $I_t = I_{st} + I_{et}$.

B.1.7 Equilibrium conditions

We define the following detrended and real variables: $y_t = \frac{Y_t}{\gamma^t}$, $\tilde{y}_t = \frac{\tilde{Y}_t}{\gamma^t} = \frac{Y_t + \gamma^t \upsilon}{\gamma^t}$, $c_t = \frac{C_t}{\gamma^t}$, $c_{ut} = \frac{C_{ut}}{\gamma^t}$, $c_{st} = \frac{C_{st}}{\gamma^t}$, $i_t = \frac{I_t}{\gamma^t}$, $i_{st} = \frac{I_{st}}{\gamma^t}$, $i_{et} = \frac{I_{et}}{\gamma^t}$, $\tilde{k}_{st} = \frac{\tilde{K}_{st}}{\gamma^t}$, $k_{st} = \frac{K_{st}}{\gamma^t} = \frac{1}{\gamma}\tilde{k}_{s,t-1}$, $\tilde{k}_{et} = \frac{\tilde{K}_{et}}{\gamma^t}$, $k_{et} = \frac{K_{et}}{\gamma^t} = \frac{1}{\gamma}Z_t\tilde{k}_{e,t-1}$, $r_{st} = \frac{R_{st}}{P_t}$, $r_{et} = \frac{R_{et}}{P_t}$, $z_t = Z_t$, $q_{st} = \Xi_{st}/\Xi_t$, $q_{et} = \Xi_{et}/\Xi_t$, $mc_t = \frac{MC_t}{P_t}$, $\tilde{p}_t = \frac{\tilde{P}_t}{P_t}$, $w_{ut} = \frac{W_{ut}}{\gamma^t P_t}$, $w_{st} = \frac{W_{st}}{\gamma^t P_t}$, $\tilde{w}_{ut} = \frac{\tilde{W}_{ut}}{\gamma^t P_t}$, $\tilde{w}_{st} = \frac{\tilde{W}_{st}}{\gamma^t P_t}$, $h_{ut} = H_{ut}$, $h_{st} = H_{st}$, $l_{ut} = L_{ut}$, $l_{st} = L_{st}$, $mrs_{ut} = \frac{MRS_{ut}}{\gamma_t}$, $mrs_{st} = \frac{MRS_{st}}{\gamma_t}$, $\tilde{mrs}_{ut} = \frac{\tilde{MRS}_{ut}}{\gamma_t}$, $\tilde{mrs}_{st} = \frac{\tilde{MRS}_{st}}{\gamma_t}$, $\xi_t = \Xi_t \gamma^{\sigma_c t}$, $\tilde{\beta} = \beta\gamma^{-\sigma_c}$, and $\exp(g_t) = \frac{G_t}{\gamma^t \bar{y}}$, where $\bar{y}$ is the steady state value of y_t. We further define $x_{t,s} = \frac{X_{t,s}}{\Pi_t^{t+s}}$, $x_{t,s}^u = \frac{X_{t,s}^u}{\gamma^s \Pi_t^{t+s}}$, and $x_{t,s}^s = \frac{X_{t,s}^s}{\gamma^s \Pi_t^{t+s}}$.

In terms of the detrended variables, we have the following equilibrium conditions.

$$(B.12) \quad \Rightarrow \quad mc_t = \left(\frac{r_{st}}{\alpha_{ks}}\right)^{\alpha_{ks}} \left(\frac{r_{et}}{\alpha_{ke}}\right)^{\alpha_{ke}} \left(\frac{w_{st}}{\alpha_s}\right)^{\alpha_s} \left(\frac{w_{ut}}{\alpha_u}\right)^{\alpha_u} \exp(-\varepsilon_t^a) \times \left(\frac{\alpha_{ke,t}}{\alpha_{ke}}\right)^{-\alpha_{ke}} \left(\frac{\alpha_{st}}{\alpha_s}\right)^{-\alpha_s} \left(\frac{\alpha_{ut}}{\alpha_u}\right)^{-\alpha_u} \times \exp(-s.o.t._t). \tag{B.42}$$

$$(B.13) \quad \Rightarrow \quad \mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_p \tilde{\beta}\gamma)^s \xi_{t+s} y_{t+s|t} \left\{ \tilde{p}_t x_{t,s} \left[1 + \frac{1}{(G')^{-1}} \frac{G'}{G''}\right] - mc_{t+s} \frac{1}{(G')^{-1}} \frac{G'}{G''} \right\} = 0, \tag{B.43}$$

where G' and G'' are evaluated at $\left(\frac{y_{t+s|t}}{y_{t+s}}; \lambda_{t+s}^p\right)$, and $(G')^{-1}$ is evaluated at $\left(\tilde{p}_t x_{t,s} \tau_{t+s}; \lambda_{t+s}^p\right)$.

$$(B.14) \quad \Rightarrow \quad 1 = (1-\zeta_p)\tilde{p}_t \times (G')^{-1}\left(\tilde{p}_t \tau_t; \lambda_t^p\right) + (1-\zeta_p)\zeta_p \tilde{p}_{t-1} x_{t-1,1} \times (G')^{-1}\left(\tilde{p}_{t-1} x_{t-1,1} \tau_t; \lambda_t^p\right) + \ldots. \tag{B.44}$$

Because the cross-sectional dispersion across firms is of the second order, firms' input choices, captured by Equations (B.7)-(B.10), are given by the following equations at the first order:

$$(B.7) \quad \Rightarrow \quad r_{st} = mc_t \frac{\tilde{y}_t}{k_{st}} \alpha_{ks}, \tag{B.45}$$

$$(B.8) \quad \Rightarrow \quad r_{et} = mc_t \frac{\tilde{y}_t}{k_{et}} \alpha_{ke,t}, \tag{B.46}$$

$$(B.9) \quad \Rightarrow \quad w_{st} = mc_t \frac{\tilde{y}_t}{l_{st}} \alpha_{st}, \tag{B.47}$$

$$(B.10) \quad \Rightarrow \quad w_{ut} = mc_t \frac{\tilde{y}_t}{l_{ut}} \alpha_{ut}. \tag{B.48}$$

Similarly, in the first order,

$$(B.4) \quad \Rightarrow \quad y_t = \exp(\varepsilon_t^a)[k_{st}]^{\alpha_{ks}}[k_{et}]^{\alpha_{ke}}[l_{st}]^{\alpha_s}[l_{ut}]^{\alpha_u} - \upsilon. \tag{B.49}$$

The household's optimal decisions, represented by Equations (B.25)-(B.31), yield the following conditions:

$$(B.25) \quad \Rightarrow \quad \xi_t = (c_{st} - \frac{h}{\gamma} c_{t-1})^{-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} h_{st}^{1+\sigma_l}\right) = A_1 (c_{ut} - \frac{h}{\gamma} c_{t-1})^{-\sigma_c} \exp\left(A_2 \frac{\sigma_c - 1}{1+\sigma_l} h_{ut}^{1+\sigma_l}\right), \tag{B.50}$$

$$(B.26) \quad \Rightarrow \quad \xi_t = \tilde{\beta} \exp(\varepsilon_t^b) R_t \mathbb{E}_t \left[\frac{\xi_{t+1}}{\Pi_{t+1}}\right], \tag{B.51}$$

$$(B.27) \quad \Rightarrow \quad 1 = q_{st}\left[1 - S_s\left(\frac{\gamma i_{st}}{i_{s,t-1}}\right) - S_s'\left(\frac{\gamma i_{st}}{i_{s,t-1}}\right) \times \left(\frac{\gamma i_{st}}{i_{s,t-1}}\right)\right] + \tilde{\beta}\mathbb{E}_t\left[\frac{\xi_{t+1}}{\xi_t} q_{s,t+1} S_s'\left(\frac{\gamma i_{s,t+1}}{i_{st}}\right) \times \left(\frac{\gamma i_{s,t+1}}{i_{st}}\right)^2\right], \tag{B.52}$$

$$(B.28) \quad \Rightarrow \quad 1 = q_{et} \exp(\varepsilon_t^i)\left[1 - S_e\left(\frac{\gamma i_{et}}{i_{e,t-1}}\right) - S_e'\left(\frac{\gamma i_{et}}{i_{e,t-1}}\right) \times \left(\frac{\gamma i_{et}}{i_{e,t-1}}\right)\right]$$

$$+\tilde{\beta}\mathbb{E}_t\left[\frac{\xi_{t+1}}{\xi_t}q_{e,t+1}\exp(\varepsilon^i_{t+1})S'_e\left(\frac{\gamma i_{e,t+1}}{i_{et}}\right)\times\left(\frac{\gamma i_{e,t+1}}{i_{et}}\right)^2\right], \tag{B.53}$$

$$(B.29) \quad \Rightarrow \quad q_{st}=\tilde{\beta}\mathbb{E}_t\left[\frac{\xi_{t+1}}{\xi_t}\left\{r_{s,t+1}+q_{s,t+1}(1-\delta_s)\right\}\right], \tag{B.54}$$

$$(B.30) \quad \Rightarrow \quad q_{et}=\tilde{\beta}\mathbb{E}_t\left[\frac{\xi_{t+1}}{\xi_t}\left\{r_{e,t+1}z_{t+1}-a(z_{t+1})+q_{e,t+1}(1-\delta_e)\right\}\right], \tag{B.55}$$

$$(B.31) \quad \Rightarrow \quad r_{et}=a'(z_t). \tag{B.56}$$

The capital accumulation equations (B.21) and (B.22) and the corresponding amount of capital services (B.23) and (B.24) imply that:

$$(B.21) \quad \Rightarrow \quad \tilde{k}_{st}=\frac{1-\delta_s}{\gamma}\tilde{k}_{s,t-1}+\left[1-S_s\left(\frac{\gamma i_{st}}{i_{s,t-1}}\right)\right]i_{st}, \tag{B.57}$$

$$(B.22) \quad \Rightarrow \quad \tilde{k}_{et}=\frac{1-\delta_e}{\gamma}\tilde{k}_{e,t-1}+\exp(\varepsilon^i_t)\left[1-S_e\left(\frac{\gamma i_{et}}{i_{e,t-1}}\right)\right]i_{et}, \tag{B.58}$$

$$(B.23) \quad \Rightarrow \quad k_{st}=\frac{1}{\gamma}\tilde{k}_{s,t-1}. \tag{B.59}$$

$$(B.24) \quad \Rightarrow \quad k_{et}=\frac{1}{\gamma}z_t\tilde{k}_{e,t-1}. \tag{B.60}$$

The labor market institutions for the unskilled labor lead to the following equilibrium conditions:

$$(B.34) \quad \Rightarrow \quad \mathbb{E}_t\sum_{s=0}^{\infty}(\zeta_u\tilde{\beta}\gamma)^s\xi_{t+s}h_{u,t+s|t}\left\{\tilde{w}_{ut}x^u_{t,s}\left[1+\frac{1}{(G'_U)^{-1}}\frac{G'_U}{G''_U}\right]-m\tilde{r}s_{u,t+s}\frac{1}{(G'_U)^{-1}}\frac{G'_U}{G''_U}\right\}=0. \tag{B.61}$$

$$(B.35) \quad \Rightarrow \quad w_{ut}=(1-\zeta_u)\tilde{w}_{ut}\times(G'_U)^{-1}\left(\frac{\tilde{w}_{ut}}{w_{ut}}\tau_{U,t};\lambda^u_t\right)$$
$$+(1-\zeta_u)\zeta_u\tilde{w}_{u,t-1}x^u_{t-1,1}\times(G'_U)^{-1}\left(\frac{\tilde{w}_{u,t-1}x^u_{t-1,1}}{w_{ut}}\tau_{U,t};\lambda^u_t\right)+\ldots, \tag{B.62}$$

where $m\tilde{r}s_{ut}=A_2(c_t-h/\gamma c_{t-1})h^{\sigma_l}_{ut}$. Furthermore, G'_U and G''_U are evaluated at $\left(\frac{h_{u,t+s|t}}{h_{u,t+s}};\lambda^u_{t+s}\right)$, and $(G'_U)^{-1}$ is evaluated at $\left(\frac{\tilde{w}_{st}x^u_{t,s}}{w_{u,t+s}}\tau_{U,t+s};\lambda^u_{t+s}\right)$.

The corresponding equations for the skilled labor are as follows:

$$\mathbb{E}_t\sum_{s'=0}^{\infty}(\zeta_s\tilde{\beta}\gamma)^{s'}\xi_{t+s'}h_{s,t+s'|t}\left\{\tilde{w}_{st}x^s_{t,s'}\left[1+\frac{1}{(G'_S)^{-1}}\frac{G'_S}{G''_S}\right]-m\tilde{r}s_{s,t+s'}\frac{1}{(G'_S)^{-1}}\frac{G'_S}{G''_S}\right\}=0. \tag{B.63}$$

$$(B.38) \quad \Rightarrow \quad w_{st}=(1-\zeta_s)\tilde{w}_{st}\times(G'_S)^{-1}\left(\frac{\tilde{w}_{st}}{w_{st}}\tau_{S,t};\lambda^s_t\right)$$
$$+(1-\zeta_s)\zeta_s\tilde{w}_{s,t-1}x^s_{t-1,1}\times(G'_S)^{-1}\left(\frac{\tilde{w}_{s,t-1}x^s_{t-1,1}}{w_{st}}\tau_{S,t};\lambda^s_t\right)+\ldots, \tag{B.64}$$

where $m\tilde{r}s_{st}=(c_t-h/\gamma c_{t-1})h^{\sigma_l}_{st}$.

The labor market clearing conditions, $L_{ut} = (1-\chi_t)H_{ut}$ and $L_{st} = \chi_t H_{st}$, become:

$$l_{ut} = (1-\chi_t)h_{ut}, \tag{B.65}$$

$$l_{st} = \chi_t h_{st}. \tag{B.66}$$

The aggregate resource constraint and the monetary policy rule are as follows:

$$\text{(B.41)} \quad \Rightarrow \quad c_t + i_{st} + i_{et} + \exp(g_t)\bar{y} + a(z_t)\frac{\tilde{k}_{e,t-1}}{\gamma} = y_t, \tag{B.67}$$

$$\text{(B.39)} \quad \Rightarrow \quad \frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho}\left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{r_\pi}\left(\frac{y_t}{y_t^*}\right)^{r_y}\right]^{1-\rho}\left(\frac{y_t/y_{t-1}}{y_t^*/y_{t-1}^*}\right)^{r_{\Delta y}}\exp(\varepsilon_t^r), \tag{B.68}$$

where $i_t = i_{st} + i_{et}$ and y_t^* is the detrended natural level of output under the flexible prices and wages equilibrium without price and wage markup shocks.

B.2 Steady State

We assume the following steady-state values:

$$\bar{\varepsilon}^a = \bar{\varepsilon}^i = \bar{\varepsilon}^b = \bar{\varepsilon}^r = 0.$$

Note that:

$$\text{(B.43)} \quad \Rightarrow \quad \Phi \equiv 1 + \bar{\lambda}^p = \frac{1}{\bar{m}c} = \frac{\frac{1}{(G')^{-1}}\frac{G'}{G''}}{1 + \frac{1}{(G')^{-1}}\frac{G'}{G''}}, \tag{B.69}$$

where G' and G'' are evaluated at $(1;\bar{\lambda}^p)$, and $(G')^{-1}$ is evaluated at $\left(\bar{\tau};\bar{\lambda}^p\right)$. Because the steady-state profit of the intermediate goods producers is assumed to be zero to rule out new firms' entry, it follows that $\bar{\lambda}^p\bar{y} - \upsilon = 0$. Furthermore:

$$\text{(B.44)} \quad \Rightarrow \quad \tilde{p}_t = 1, \qquad G'(1;\bar{\lambda}^p) = \bar{\tau}. \tag{B.70}$$

Similarly:

$$\text{(B.61)} \quad \Rightarrow \quad \Phi_u \equiv 1 + \bar{\lambda}^u = \frac{\bar{w}_u}{\bar{m}\bar{r}s_u} = \frac{\frac{1}{(G'_U)^{-1}}\frac{G'_U}{G''_U}}{1 + \frac{1}{(G'_U)^{-1}}\frac{G'_U}{G''_U}} \quad \Rightarrow \quad \bar{w}_u = \Phi_u \bar{m}\bar{r}s_u, \tag{B.71}$$

$$\text{(B.62)} \quad \Rightarrow \quad \bar{w}_u = \bar{\tilde{w}}_u, \qquad G'_U(1;\bar{\lambda}^u) = \bar{\tau}_U, \tag{B.72}$$

$$\text{(B.63)} \quad \Rightarrow \quad \Phi_s \equiv 1 + \bar{\lambda}^s = \frac{\bar{w}_s}{\bar{m}\bar{r}s_s} = \frac{\frac{1}{(G'_S)^{-1}}\frac{G'_S}{G''_S}}{1 + \frac{1}{(G'_S)^{-1}}\frac{G'_S}{G''_S}} \quad \Rightarrow \quad \bar{w}_s = \Phi_s \bar{m}\bar{r}s_s, \tag{B.73}$$

$$\text{(B.64)} \quad \Rightarrow \quad \bar{w}_s = \bar{\tilde{w}}_s, \qquad G'_S(1;\bar{\lambda}^s) = \bar{\tau}_S, \tag{B.74}$$

where $\bar{m}\bar{r}s_u = A_2(\bar{c} - h/\gamma\bar{c})\bar{h}_u^{\sigma_l}$, $\bar{m}\bar{r}s_s = (\bar{c} - h/\gamma\bar{c})\bar{h}_s^{\sigma_l}$. In the above equations, G'_U and G''_U are evaluated at $(1;\bar{\lambda}^u)$, and $(G'_U)^{-1}$ is evaluated at $\left(\bar{\tau}_U;\bar{\lambda}^u\right)$. Similarly, G'_S and G''_S are evaluated at $(1;\bar{\lambda}^s)$, and $(G'_S)^{-1}$ is evaluated at $\left(\bar{\tau}_S;\bar{\lambda}^s\right)$.

Next, we discuss the firm side of the economy. Because $\upsilon = \bar{\lambda}^p \bar{y}$, $\bar{\tilde{y}} = \bar{y} + \upsilon = (1 + \bar{\lambda}^p)\bar{y} = \Phi \bar{y} = \frac{1}{\bar{m}\bar{c}} \bar{y}$,

$$
\begin{aligned}
\text{(B.45)} \quad &\Rightarrow \quad \bar{r}_s \bar{k}_s = \bar{m}\bar{c}\bar{\tilde{y}}\alpha_{ks} = \alpha_{ks}\bar{y}, && \text{(B.75)} \\
\text{(B.46)} \quad &\Rightarrow \quad \bar{r}_e \bar{k}_e = \bar{m}\bar{c}\bar{\tilde{y}}\alpha_{\bar{ke},t} = \alpha_{ke}\bar{y}, && \text{(B.76)} \\
\text{(B.47)} \quad &\Rightarrow \quad \bar{w}_s \bar{l}_s = \bar{m}\bar{c}\bar{\tilde{y}}\alpha_{\bar{s}t} = \alpha_s \bar{y}, && \text{(B.77)} \\
\text{(B.48)} \quad &\Rightarrow \quad \bar{w}_u \bar{l}_u = \bar{m}\bar{c}\bar{\tilde{y}}\alpha_{\bar{u}t} = \alpha_u \bar{y}. && \text{(B.78)}
\end{aligned}
$$

Thus, αs capture the steady-state income share of each factor. The production function in the steady state is given by:

$$
\text{(B.4) and (B.49)} \quad \Rightarrow \quad \bar{\tilde{y}} = \bar{y} + \upsilon = \Phi \bar{y} = [\bar{k}_s]^{\alpha_{ks}} [\bar{k}_e]^{\alpha_{ke}} [\bar{l}_s]^{\alpha_s} [\bar{l}_u]^{\alpha_u}. \tag{B.79}
$$

The household-side conditions, evaluated at the steady state, yield the following equations:

$$
\begin{aligned}
\text{(B.50)} \quad &\Rightarrow \quad \bar{\xi}_s = (\bar{c}_s - \frac{h}{\gamma}\bar{c})^{-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1 + \sigma_l} \bar{h}_s^{1+\sigma_l}\right), \\
&\qquad\quad \bar{\xi}_u = A_1 (\bar{c}_u - \frac{h}{\gamma}\bar{c})^{-\sigma_c} \exp\left(A_2 \frac{\sigma_c - 1}{1 + \sigma_l} \bar{h}_u^{1+\sigma_l}\right), \\
&\qquad\quad \bar{\xi} = \bar{\xi}_s = \bar{\xi}_u, && \text{(B.80)} \\
\text{(B.51)} \quad &\Rightarrow \quad \bar{\xi} = \tilde{\beta}\bar{R}\frac{\bar{\xi}}{\bar{\Pi}} \quad \Rightarrow \quad \bar{\Pi} = \tilde{\beta}\bar{R}, && \text{(B.81)} \\
\text{(B.52)} \quad &\Rightarrow \quad 1 = \bar{q}_s, && \text{(B.82)} \\
\text{(B.53)} \quad &\Rightarrow \quad 1 = \bar{q}_e, && \text{(B.83)} \\
\text{(B.54)} \quad &\Rightarrow \quad \bar{r}_s = \frac{1}{\tilde{\beta}} - (1 - \delta_s), && \text{(B.84)} \\
\text{(B.55)} \quad &\Rightarrow \quad \bar{r}_e = \frac{1}{\tilde{\beta}} - (1 - \delta_e), && \text{(B.85)} \\
\text{(B.56)} \quad &\Rightarrow \quad \bar{r}_e = a'(\bar{z}) = a'(1), && \text{(B.86)} \\
\text{(B.57)} \quad &\Rightarrow \quad \frac{\bar{i}_s}{\bar{k}_s} = 1 - \frac{1 - \delta_s}{\gamma}, && \text{(B.87)} \\
\text{(B.58)} \quad &\Rightarrow \quad \frac{\bar{i}_e}{\bar{k}_e} = 1 - \frac{1 - \delta_e}{\gamma}, && \text{(B.88)} \\
\text{(B.59)} \quad &\Rightarrow \quad \bar{k}_s = \frac{1}{\gamma}\bar{\tilde{k}}_s, && \text{(B.89)} \\
\text{(B.60)} \quad &\Rightarrow \quad \bar{k}_e = \frac{1}{\gamma}\bar{\tilde{k}}_e. && \text{(B.90)}
\end{aligned}
$$

Also:

$$
\begin{aligned}
\text{(B.65)} \quad &\Rightarrow \quad \bar{l}_u = (1 - \bar{\chi})\bar{h}_u, && \text{(B.91)} \\
\text{(B.66)} \quad &\Rightarrow \quad \bar{l}_s = \bar{\chi}\bar{h}_s, && \text{(B.92)} \\
\text{(B.67)} \quad &\Rightarrow \quad \bar{c} + \bar{i}_s + \bar{i}_e + \exp(\bar{g})\bar{y} = \bar{y}. && \text{(B.93)}
\end{aligned}
$$

The following ratios are used in the log-linearized system:

$$\frac{\bar{i}_s}{\bar{y}} = \frac{\bar{i}_s}{\bar{\tilde{k}}_s}\frac{\bar{\tilde{k}}_s}{\bar{k}_s}\frac{\bar{k}_s\bar{r}_s}{\bar{y}}\frac{1}{\bar{r}_s} = \left(1 - \frac{1-\delta_s}{\gamma}\right)\gamma\frac{\alpha_{ks}}{\frac{1}{\bar{\beta}} - (1-\delta_s)}, \tag{B.94}$$

$$\frac{\bar{i}_e}{\bar{y}} = \frac{\bar{i}_e}{\bar{\tilde{k}}_e}\frac{\bar{\tilde{k}}_e}{\bar{k}_e}\frac{\bar{k}_e\bar{r}_e}{\bar{y}}\frac{1}{\bar{r}_e} = \left(1 - \frac{1-\delta_e}{\gamma}\right)\gamma\frac{\alpha_{ke}}{\frac{1}{\bar{\beta}} - (1-\delta_e)}, \tag{B.95}$$

$$\frac{\bar{c}}{\bar{y}} = 1 - \frac{\bar{i}_s}{\bar{y}} - \frac{\bar{i}_e}{\bar{y}} - \exp(\bar{g}), \tag{B.96}$$

$$\frac{m\bar{r}s_u \times \bar{h}_u}{\bar{c}_u} = \frac{1}{\Phi_u}\frac{\bar{w}_u\bar{h}_u(1-\bar{\chi})}{\bar{y}}\frac{1}{1-\bar{\chi}}\frac{\bar{y}}{\bar{c}_u} = \frac{1}{\Phi_u}\frac{\alpha_u}{1-\bar{\chi}}\left[\frac{\bar{c}_u}{\bar{y}}\right]^{-1}, \tag{B.97}$$

$$\frac{m\bar{r}s_s \times \bar{h}_s}{\bar{c}_s} = \frac{1}{\Phi_s}\frac{\bar{w}_s\bar{h}_s\bar{\chi}}{\bar{y}}\frac{1}{\bar{\chi}}\frac{\bar{y}}{\bar{c}_s} = \frac{1}{\Phi_s}\frac{\alpha_s}{\bar{\chi}}\left[\frac{\bar{c}_s}{\bar{y}}\right]^{-1}. \tag{B.98}$$

Finally, we find the steady state such that $\bar{c} = \bar{c}_u = \bar{c}_s$ and $\bar{h} = \bar{h}_u = \bar{h}_s$ in the following manner. By combining Equations (B.73), (B.77), and (B.92), we have:

$$\begin{aligned}\alpha_s\bar{y} = \bar{w}_s\bar{l}_s = \bar{w}_s\bar{\chi}\bar{h}_s = \Phi_s m\bar{r}s_s\bar{\chi}\bar{h}_s = \Phi_s(\bar{c}_s - h/\gamma\bar{c})\bar{\chi}\bar{h}_s^{1+\sigma_l}\\ \Rightarrow \quad \bar{h}_s = \left[\frac{\alpha_s}{\Phi_s(1-h/\gamma)\bar{\chi}}\left(\frac{\bar{c}}{\bar{y}}\right)^{-1}\right]^{\frac{1}{1+\sigma_l}},\end{aligned} \tag{B.99}$$

where $\bar{c}/\bar{y}$ follows from Equation (B.96). In the last line, we used $\bar{c}_s = \bar{c}$. Next, we turn to $\bar{h}_u$. Similarly, by combining Equations (B.71), (B.78), and (B.91), we have:

$$\begin{aligned}\alpha_u\bar{y} = \bar{w}_u\bar{l}_u = \bar{w}_u(1-\bar{\chi})\bar{h}_u = \Phi_u m\bar{r}s_u(1-\bar{\chi})\bar{h}_u = \Phi_u A_2(\bar{c}_u - h/\gamma\bar{c})(1-\bar{\chi})\bar{h}_u^{1+\sigma_l}\\ \Rightarrow \quad \bar{h}_u = \left[\frac{\alpha_u}{\Phi_u A_2(1-h/\gamma)(1-\bar{\chi})}\left(\frac{\bar{c}}{\bar{y}}\right)^{-1}\right]^{\frac{1}{1+\sigma_l}} = \left[\frac{\alpha_s}{\Phi_s(1-h/\gamma)\bar{\chi}}\left(\frac{\bar{c}}{\bar{y}}\right)^{-1}\right]^{\frac{1}{1+\sigma_l}} = \bar{h}_s,\end{aligned} \tag{B.100}$$

where we used $\bar{c}_u = \bar{c}$ and $A_2 = \frac{\Phi_s/\Phi_u}{\alpha_s/\alpha_u}\frac{\bar{\chi}}{1-\bar{\chi}}$ in the second line. The above result implies that $\bar{h}_s = \bar{h}_u = \bar{h}$ and their values are determined by structural parameters. We further need to show that $\bar{c}_s = \bar{c}_u = \bar{c}$ and the above value of $\bar{h}$ satisfy Equation (B.80). Note that when $\bar{c}_s = \bar{c}_u = \bar{c}$ and $\bar{h}_s = \bar{h}_u = \bar{h}$, Equation (B.80), specifically, $\bar{\xi}_s = \bar{\xi}_u = \bar{\xi}$, follows because:

$$A_1 = \exp\left(\frac{\sigma_c - 1}{1+\sigma_l}(1-A_2)\bar{h}^{1+\sigma_l}\right).$$

Next, Equations (B.87), (B.88), (B.94), and (B.95) determine $\bar{k}_s/\bar{y}$ and $\bar{k}_e/\bar{y}$:

$$\begin{aligned}\frac{\bar{k}_s}{\bar{y}} &= \frac{\gamma\alpha_{ks}}{\frac{1}{\bar{\beta}} - (1-\delta_s)},\\ \frac{\bar{k}_e}{\bar{y}} &= \frac{\gamma\alpha_{ke}}{\frac{1}{\bar{\beta}} - (1-\delta_e)}.\end{aligned}$$

Then, from Equation (B.79), it follows that:

$$\bar{y} = \left[\frac{1}{\Phi}\left(\frac{\bar{k}_s}{\bar{y}}\right)^{\alpha_{ks}}\left(\frac{\bar{k}_e}{\bar{y}}\right)^{\alpha_{ke}}\bar{\chi}^{\alpha_s}(1-\bar{\chi})^{\alpha_u}\bar{h}^{\alpha_s+\alpha_u}\right]^{\frac{1}{\alpha_s+\alpha_u}}$$

because $\alpha_{ks}+\alpha_{ke}+\alpha_s+\alpha_u=1$. Thus, $\bar{y}$ is determined by structural parameters. It is straightforward to pin down the steady-state values of the remaining variables.

B.3 Log-linearization

B.3.1 Economy with sticky prices and wages

This section illustrates the log-linearized equilibrium conditions. Hatted variables represent log deviations from their steady-state values.

In view of Equation (B.15), we define the aggregate returns to scale as follows:

$$rts_t = \underbrace{\frac{y_t+\upsilon}{y_t}}_{rts^{fc}}\underbrace{(1+\beta_k.\hat{k}_{et}+\beta_s.\hat{l}_{st}+\beta_u.\hat{l}_{ut})}_{rts^{tl}},$$

where rts^{fc} and rts^{tl} represent the contribution of fixed costs and the translog structure in the production function to the returns to scale, respectively. Then:

$$\begin{aligned} \hat{rts}_t &= \hat{rts}_t^{fc} + \hat{rts}_t^{tl}, \\ \hat{rts}_t^{fc} &= -\left(\frac{\Phi-1}{\Phi}\right)\hat{y}_t \\ \hat{rts}_t^{tl} &= \beta_k.\hat{k}_{et}+\beta_s.\hat{l}_{st}+\beta_u.\hat{l}_{ut}, \end{aligned} \tag{B.101}$$

where Φ is the gross price markup in the steady state.

Next, we discuss the equilibrium conditions from the firm side of the economy.

$$\text{(B.42)} \quad \Rightarrow \quad \hat{mc}_t = \alpha_{ks}\hat{r}_{st}+\alpha_{ke}\hat{r}_{et}+\alpha_s\hat{w}_{st}+\alpha_u\hat{w}_{ut}-\varepsilon_t^a-(\beta_k.\hat{k}_{et}+\beta_s.\hat{l}_{st}+\beta_u.\hat{l}_{ut}). \tag{B.102}$$

Note that the real marginal costs consist of factor prices ($\alpha_{ks}\hat{r}_{st}+\alpha_{ke}\hat{r}_{et}+\alpha_s\hat{w}_{st}+\alpha_u\hat{w}_{ut}$), TFP ($\varepsilon_t^a$), and $\hat{rts}_t^{tl}$ ($\beta_k.\hat{k}_{et}+\beta_s.\hat{l}_{st}+\beta_u.\hat{l}_{ut}$).

The sticky price equations, (B.43) and (B.44), yield a New Keynesian price Phillips curve with a suitable normalization for $\hat{\lambda}_t^p$:

$$\hat{\pi}_t = \frac{\iota_p}{1+\tilde{\beta}\gamma\iota_p}\hat{\pi}_{t-1}+\frac{\tilde{\beta}\gamma}{1+\tilde{\beta}\gamma\iota_p}\mathbb{E}_t[\hat{\pi}_{t+1}]+\frac{(1-\zeta_p\tilde{\beta}\gamma)(1-\zeta_p)}{\zeta_p(1+\tilde{\beta}\gamma\iota_p)}\frac{1}{1+\theta_p(\Phi-1)}\hat{mc}_t+\hat{\lambda}_t^p, \tag{B.103}$$

where θ_p is the curvature of the Kimball aggregator G (see, e.g., Hyun, Kim and Lee, 2024, Online Appendix C.3.1).

Given the real marginal cost equation (B.102), we need only three additional conditions among Equations

(B.45)-(B.48). Here, without loss of generality, we drop the condition for equipment services.

$$(B.45) \quad \Rightarrow \quad \hat{r}_{st} + \hat{k}_{st} = \hat{mc}_t + \hat{\tilde{y}}_t, \tag{B.104}$$

$$(B.47) \quad \Rightarrow \quad \hat{w}_{st} + \hat{l}_{st} = \hat{mc}_t + \hat{\tilde{y}}_t + \hat{\alpha}_{st}, \tag{B.105}$$

$$(B.48) \quad \Rightarrow \quad \hat{w}_{ut} + \hat{l}_{ut} = \hat{mc}_t + \hat{\tilde{y}}_t + \hat{\alpha}_{ut}, \tag{B.106}$$

where $\hat{\tilde{y}}_t = \frac{1}{\Phi}\hat{y}_t$, $\hat{\alpha}_{st} = \frac{1}{\alpha_s}(\beta_{ks}\hat{k}_{et} + \beta_{ss}\hat{l}_{st} + \beta_{su}\hat{l}_{ut})$, and $\hat{\alpha}_{ut} = \frac{1}{\alpha_u}(\beta_{ku}\hat{k}_{et} + \beta_{su}\hat{l}_{st} + \beta_{uu}\hat{l}_{ut})$.

The production function resembles a Cobb–Douglas function at the first order:

$$(B.49) \quad \Rightarrow \quad \hat{y}_t = \Phi(\alpha_{ks}\hat{k}_{st} + \alpha_{ke}\hat{k}_{et} + \alpha_s\hat{l}_{st} + \alpha_u\hat{l}_{ut} + \varepsilon_t^a). \tag{B.107}$$

Regarding the household-side conditions, we begin with the Euler equation. Let ξ_{st} and ξ_{ut} denote the detrended marginal consumption utilities of the skilled and unskilled workers, respectively:

$$\xi_{st} = (c_{st} - \frac{h}{\gamma}c_{t-1})^{-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} h_{st}^{1+\sigma_l}\right),$$
$$\xi_{ut} = A_1(c_{ut} - \frac{h}{\gamma}c_{t-1})^{-\sigma_c} \exp\left(A_2\frac{\sigma_c - 1}{1+\sigma_l} h_{ut}^{1+\sigma_l}\right).$$

Then:

$$\hat{\xi}_{st} = -\sigma_c\left(\frac{1}{1-h/\gamma}\hat{c}_{st} - \frac{h/\gamma}{1-h/\gamma}\hat{c}_{t-1}\right) + \frac{\sigma_c - 1}{1-h/\gamma}\frac{m\bar{r}s_s \times \bar{h}_s}{\bar{c}_s}\hat{h}_{st},$$

because $\bar{c} = \bar{c}_s = \bar{c}_u$ and $m\bar{r}s_s = (1-h/\gamma)\bar{c}_s\bar{h}_s^{\sigma_l}$, where the value of $\frac{m\bar{r}s_s \times \bar{h}_s}{\bar{c}_s}$ is determined by Equation (B.98). Similarly:

$$\hat{\xi}_{ut} = -\sigma_c\left(\frac{1}{1-h/\gamma}\hat{c}_{ut} - \frac{h/\gamma}{1-h/\gamma}\hat{c}_{t-1}\right) + \frac{\sigma_c - 1}{1-h/\gamma}\frac{m\bar{r}s_u \times \bar{h}_u}{\bar{c}_u}\hat{h}_{ut},$$

because $m\bar{r}s_u = A_2(1-h/\gamma)\bar{c}_u\bar{h}_u^{\sigma_l}$. Also, $\frac{m\bar{r}s_s \times \bar{h}_s}{\bar{c}_s}$ follows from Equation (B.97).

Next, note that:

$$\begin{aligned} & c_t = (1-\chi_t)c_{ut} + \chi_t c_{st} \\ \Rightarrow \quad & dc_t = -d\chi_t\bar{c}_u + (1-\bar{\chi})dc_{ut} + d\chi_t\bar{c}_s + \bar{\chi}dc_{st} \\ \Rightarrow \quad & \hat{c}_t = \frac{dc_t}{\bar{c}} = (1-\bar{\chi})\hat{c}_{ut} + \bar{\chi}\hat{c}_{st} \end{aligned}$$

because $\bar{c} = \bar{c}_u = \bar{c}_s$. Also, Equation (B.50) implies that $\xi_t = \xi_{st} = \xi_{ut}$, and, specifically, $\hat{\xi}_t = (1-\bar{\chi})\hat{\xi}_{ut} + \bar{\chi}\hat{\xi}_{st}$. Thus:

$$\hat{\xi}_t = -\sigma_c\left(\frac{1}{1-h/\gamma}\hat{c}_t - \frac{h/\gamma}{1-h/\gamma}\hat{c}_{t-1}\right) + \frac{\sigma_c - 1}{1-h/\gamma}\left[(1-\bar{\chi})\frac{m\bar{r}s_u \times \bar{h}_u}{\bar{c}_u}\hat{h}_{ut} + \bar{\chi}\frac{m\bar{r}s_s \times \bar{h}_s}{\bar{c}_s}\hat{h}_{st}\right].$$

By log-linearizing Equation (B.51), we obtain:

$$\hat{\xi}_t = \varepsilon_t^b + \hat{r}_t + \mathbb{E}_t\left[\hat{\xi}_{t+1} - \hat{\pi}_{t+1}\right],$$

where $\hat{r}_t \equiv \log(R_t/\bar{R})$ and $\hat{\pi}_t = \log(\Pi_t/\bar{\Pi})$. By combining the two equations above, we have the following Euler equation:

$$\begin{aligned}\hat{c}_t = &\frac{1}{1+h/\gamma}\mathbb{E}_t[\hat{c}_{t+1}] + \frac{h/\gamma}{1+h/\gamma}\hat{c}_{t-1} - \frac{1-h/\gamma}{\sigma_c(1+h/\gamma)}\left(\hat{r}_t - \mathbb{E}_t[\hat{\pi}_{t+1}]\right) \\ &- \frac{\sigma_c - 1}{\sigma_c(1+h/\gamma)}\left\{(1-\bar{\chi})\frac{m\bar{r}s_u \times \bar{h}_u}{\bar{c}_u}(\mathbb{E}_t[\hat{h}_{u,t+1}] - \hat{h}_{ut}) + \bar{\chi}\frac{m\bar{r}s_s \times \bar{h}_s}{\bar{c}_s}(\mathbb{E}_t[\hat{h}_{s,t+1}] - \hat{h}_{st})\right\} + \hat{\varepsilon}^b_t, \end{aligned} \tag{B.108}$$

where we define $\hat{\varepsilon}^b_t$ as $-\frac{1-h/\gamma}{\sigma_c(1+h/\gamma)}\varepsilon^b_t$ following Smets and Wouters (2007*b*).

We also have nine equations for capital stock, investment, utilization, and Tobin's Qs.

$$\text{(B.52)} \quad \Rightarrow \quad \hat{i}_{st} = \frac{1}{1+\tilde{\beta}\gamma}\hat{i}_{s,t-1} + \frac{\tilde{\beta}\gamma}{1+\tilde{\beta}\gamma}\mathbb{E}_t[\hat{i}_{s,t+1}] + \frac{1}{(1+\tilde{\beta}\gamma)\varphi_s\gamma^2}\hat{q}_{st}, \tag{B.109}$$

$$\text{(B.53)} \quad \Rightarrow \quad \hat{i}_{et} = \frac{1}{1+\tilde{\beta}\gamma}\hat{i}_{e,t-1} + \frac{\tilde{\beta}\gamma}{1+\tilde{\beta}\gamma}\mathbb{E}_t[\hat{i}_{e,t+1}] + \frac{1}{(1+\tilde{\beta}\gamma)\varphi_e\gamma^2}\hat{q}_{et} + \hat{\varepsilon}^i_t, \tag{B.110}$$

$$\text{(B.54)} \quad \Rightarrow \quad \hat{q}_{st} = -(\hat{r}_t - \mathbb{E}_t[\hat{\pi}_{t+1}]) + \tilde{\beta}\bar{r}_s\mathbb{E}_t[\hat{r}_{s,t+1}] + \tilde{\beta}(1-\delta_s)\mathbb{E}_t[\hat{q}_{s,t+1}] + \frac{\sigma_c(1+h/\gamma)}{1-h/\gamma}\hat{\varepsilon}^b_t, \tag{B.111}$$

$$\text{(B.55)} \quad \Rightarrow \quad \hat{q}_{et} = -(\hat{r}_t - \mathbb{E}_t[\hat{\pi}_{t+1}]) + \tilde{\beta}\bar{r}_e\mathbb{E}_t[\hat{r}_{e,t+1}] + \tilde{\beta}(1-\delta_e)\mathbb{E}_t[\hat{q}_{e,t+1}] + \frac{\sigma_c(1+h/\gamma)}{1-h/\gamma}\hat{\varepsilon}^b_t, \tag{B.112}$$

$$\text{(B.56)} \quad \Rightarrow \quad \hat{z}_t = \frac{a'(1)}{a''(1)}\hat{r}_{et} = \frac{1-\psi}{\psi}\hat{r}_{et}, \tag{B.113}$$

$$\text{(B.57)} \quad \Rightarrow \quad \hat{\tilde{k}}_{st} = \frac{1-\delta_s}{\gamma}\hat{\tilde{k}}_{s,t-1} + \left(1 - \frac{1-\delta_s}{\gamma}\right)\hat{i}_{st}, \tag{B.114}$$

$$\text{(B.58)} \quad \Rightarrow \quad \hat{\tilde{k}}_{et} = \frac{1-\delta_e}{\gamma}\hat{\tilde{k}}_{e,t-1} + \left(1 - \frac{1-\delta_e}{\gamma}\right)\hat{i}_{et} + \left(1 - \frac{1-\delta_e}{\gamma}\right)(1+\tilde{\beta}\gamma)\varphi_e\gamma^2\hat{\varepsilon}^i_t, \tag{B.115}$$

$$\text{(B.59)} \quad \Rightarrow \quad \hat{k}_{st} = \hat{\tilde{k}}_{s,t-1}, \tag{B.116}$$

$$\text{(B.60)} \quad \Rightarrow \quad \hat{k}_{et} = \hat{z}_t + \hat{\tilde{k}}_{e,t-1}. \tag{B.117}$$

We used φ_s and φ_e to denote $S''_s(\gamma)$ and $S''_e(\gamma)$, respectively. Like $\hat{\varepsilon}^b_t$ in Equations (B.108), (B.111), and (B.112), we define $\hat{\varepsilon}^i_t$ as $\frac{1}{(1+\tilde{\beta}\gamma)\varphi_e\gamma^2}\varepsilon^i_t$ and use $\hat{\varepsilon}^i_t$ in Equations (B.110) and (B.115). In Equation (B.113), we define $\psi = (1 + a'(1)/a''(1))^{-1}$ following Smets and Wouters (2007*b*).

Like the price Phillips curve, the equilibrium conditions for sticky wages, (B.61)-(B.64), lead to New Keynesian wage Phillips curves for each type of labor:

$$\begin{aligned}\hat{w}_{ut} - \frac{1}{1+\tilde{\beta}\gamma}\hat{w}_{u,t-1} - \frac{\tilde{\beta}\gamma}{1+\tilde{\beta}\gamma}\mathbb{E}_t[\hat{w}_{u,t+1}] = &\frac{(1-\zeta_u\tilde{\beta}\gamma)(1-\zeta_u)}{\zeta_u(1+\tilde{\beta}\gamma)}\frac{1}{1+\theta_u(\Phi_u-1)}(\hat{m\tilde{r}s}_{ut} - \hat{w}_{ut}) \\ &- \frac{1+\tilde{\beta}\gamma\iota_u}{1+\tilde{\beta}\gamma}\hat{\pi}_t + \frac{\iota_u}{1+\tilde{\beta}\gamma}\hat{\pi}_{t-1} + \frac{\tilde{\beta}\gamma}{1+\tilde{\beta}\gamma}\mathbb{E}_t[\hat{\pi}_{t+1}] + \hat{\lambda}^u_t, \end{aligned} \tag{B.118}$$

$$\begin{aligned}\hat{w}_{st} - \frac{1}{1+\tilde{\beta}\gamma}\hat{w}_{s,t-1} - \frac{\tilde{\beta}\gamma}{1+\tilde{\beta}\gamma}\mathbb{E}_t[\hat{w}_{s,t+1}] = &\frac{(1-\zeta_s\tilde{\beta}\gamma)(1-\zeta_s)}{\zeta_s(1+\tilde{\beta}\gamma)}\frac{1}{1+\theta_s(\Phi_s-1)}(\hat{m\tilde{r}s}_{st} - \hat{w}_{st}) \\ &- \frac{1+\tilde{\beta}\gamma\iota_s}{1+\tilde{\beta}\gamma}\hat{\pi}_t + \frac{\iota_s}{1+\tilde{\beta}\gamma}\hat{\pi}_{t-1} + \frac{\tilde{\beta}\gamma}{1+\tilde{\beta}\gamma}\mathbb{E}_t[\hat{\pi}_{t+1}] + \hat{\lambda}^s_t, \end{aligned} \tag{B.119}$$

where $\hat{m\tilde{r}s}_{ut} = \frac{1}{1-h/\gamma}\hat{c}_t - \frac{h/\gamma}{1-h/\gamma}\hat{c}_{t-1} + \sigma_l\hat{h}_{ut}$ and $\hat{m\tilde{r}s}_{st} = \frac{1}{1-h/\gamma}\hat{c}_t - \frac{h/\gamma}{1-h/\gamma}\hat{c}_{t-1} + \sigma_l\hat{h}_{st}$ (see, e.g., Hyun, Kim and Lee, 2024, Online Appendic C.3.2). Note that we properly normalized $\hat{\lambda}^u_t$ and $\hat{\lambda}^s_t$. θ_u and θ_s are the curvature of

the Kimball aggregators for G_U and G_S, respectively. Φ_u and Φ_s are the steady-state unskilled and skilled wage markups, respectively.

Alternatively, we can express the wage Phillips curves in terms of the nominal wage inflation: $\pi_t^u \equiv \log\left(\frac{P_t w_{ut}}{P_{t-1} w_{u,t-1}}\right) = \pi_t + \hat{w}_{ut} - \hat{w}_{u,t-1}$ and $\pi_t^s \equiv \log\left(\frac{P_t w_{st}}{P_{t-1} w_{s,t-1}}\right) = \pi_t + \hat{w}_{st} - \hat{w}_{s,t-1}$.

$$\pi_t^u = \tilde{\beta}\gamma \mathbb{E}_t[\pi_{t+1}^u] + \frac{(1-\zeta_u\tilde{\beta}\gamma)(1-\zeta_u)}{\zeta_u}\frac{1}{1+\theta_u(\Phi_u-1)}(\hat{mrs}_{ut} - \hat{w}_{ut}) - \iota_u(\tilde{\beta}\gamma\hat{\pi}_t - \hat{\pi}_{t-1}) + (1+\tilde{\beta}\gamma)\hat{\lambda}_t^u.$$
$$\pi_t^s = \tilde{\beta}\gamma \mathbb{E}_t[\pi_{t+1}^s] + \frac{(1-\zeta_s\tilde{\beta}\gamma)(1-\zeta_s)}{\zeta_s}\frac{1}{1+\theta_s(\Phi_s-1)}(\hat{mrs}_{st} - \hat{w}_{st}) - \iota_s(\tilde{\beta}\gamma\hat{\pi}_t - \hat{\pi}_{t-1}) + (1+\tilde{\beta}\gamma)\hat{\lambda}_t^s.$$

The labor market clearing conditions in log-linearization are as follows:

$$\text{(B.65)} \quad \Rightarrow \quad \hat{l}_{ut} = -\frac{\bar{\chi}}{1-\bar{\chi}}\varepsilon_t^\chi + \hat{h}_{ut}, \tag{B.120}$$
$$\text{(B.66)} \quad \Rightarrow \quad \hat{l}_{st} = \varepsilon_t^\chi + \hat{h}_{st}, \tag{B.121}$$

where $\chi_t = \bar{\chi}\exp(\varepsilon_t^\chi)$ and ε_t^χ follows an exogenous process.

The monetary policy rule and the aggregate resource constraint in log-linearization are given by:

$$\text{(B.68)} \quad \Rightarrow \quad \hat{r}_t = \rho\hat{r}_{t-1} + (1-\rho)(r_\pi\hat{\pi}_t + r_y\hat{x}_t) + r_{\Delta y}(\hat{x}_t - \hat{x}_{t-1}) + \varepsilon_t^r, \tag{B.122}$$
$$\text{(B.67)} \quad \Rightarrow \quad \frac{\bar{c}}{\bar{y}}\hat{c}_t + \frac{\bar{i}_s}{\bar{y}}\hat{i}_{st} + \frac{\bar{i}_e}{\bar{y}}\hat{i}_{et} + \hat{g}_t + \frac{\bar{r}_e\bar{k}_e}{\bar{y}}\hat{z}_t = \hat{y}_t, \tag{B.123}$$

where the output gap $\hat{x}_t = \log(y_t/\bar{y}) - \log(y_t^*/\bar{y})$, y_t^* is the detrended natural level of output, and $\hat{g}_t \equiv \exp(\bar{g})(g_t - \bar{g})$.

B.3.2 Economy with flexible prices and wages and without markup shocks

This section describes the model economy with flexible prices, flexible wages, and no price and wage markup shocks. We use it to determine the natural level of output, y_t^*. We use $*$ to denote the variables in this economy.

When prices are flexible ($\zeta_p = 0$), and there are no price markup shocks, real marginal costs (and, therefore, price markup) are constant (see Equation (B.103)). Thus:

$$0 = \hat{mc}_t^* = \alpha_{ks}\hat{r}_{st}^* + \alpha_{ke}\hat{r}_{et}^* + \alpha_s\hat{w}_{st}^* + \alpha_u\hat{w}_{ut}^* - \varepsilon_t^a - (\beta_{k\cdot}\hat{k}_{et}^* + \beta_{s\cdot}\hat{l}_{st}^* + \beta_{u\cdot}\hat{l}_{ut}^*). \tag{B.124}$$

Similarly, when wages are flexible ($\zeta_u = \zeta_s = 0$) and there are no wage markup shocks, wage markups ($w_u/\widetilde{mrs}_u$ and $w_s/\widetilde{mrs}_s$) are constants (Φ_u and Φ_s). Thus:

$$\hat{w}_{ut}^* = \hat{mrs}_{ut}^* = \frac{1}{1-h/\gamma}\hat{c}_t^* - \frac{h/\gamma}{1-h/\gamma}\hat{c}_{t-1}^* + \sigma_l\hat{h}_{ut}^*, \tag{B.125}$$
$$\hat{w}_{st}^* = \hat{mrs}_{st}^* = \frac{1}{1-h/\gamma}\hat{c}_t^* - \frac{h/\gamma}{1-h/\gamma}\hat{c}_{t-1}^* + \sigma_l\hat{h}_{st}^*, \tag{B.126}$$

In this economy, the monetary policy rule (B.122) is not necessary. Furthermore, the real interest rate $\hat{r}_t^* = \hat{r}_t - \mathbb{E}_t[\hat{\pi}_{t+1}]$ is directly determined. The other equilibrium conditions are as follows:

$$\text{(B.104)} \quad \Rightarrow \quad \hat{r}_{st}^* + \hat{k}_{st}^* = \hat{\hat{y}}_t^*, \tag{B.127}$$

(B.104) $\Rightarrow$ $\hat{w}^*_{st} + \hat{l}^*_{st} = \hat{\tilde{y}}^*_t + \hat{\alpha}^*_{st}$, (B.128)

(B.104) $\Rightarrow$ $\hat{w}^*_{ut} + \hat{l}^*_{ut} = \hat{\tilde{y}}^*_t + \hat{\alpha}^*_{ut}$, (B.129)

(B.107) $\Rightarrow$ $\hat{y}^*_t = \Phi(\alpha_{ks}\hat{k}^*_{st} + \alpha_{ke}\hat{k}^*_{et} + \alpha_s\hat{l}^*_{st} + \alpha_u\hat{l}^*_{ut} + \varepsilon^a_t)$. (B.130)

(B.108) $\Rightarrow$ $\hat{c}^*_t = \frac{1}{1+h/\gamma}\mathbb{E}_t[\hat{c}^*_{t+1}] + \frac{h/\gamma}{1+h/\gamma}\hat{c}^*_{t-1} - \frac{1-h/\gamma}{\sigma_c(1+h/\gamma)}\hat{r}^*_t$

$- \frac{\sigma_c - 1}{\sigma_c(1+h/\gamma)}\left\{(1-\bar{\chi})\frac{m\bar{r}s_u \times \bar{h}_u}{\bar{c}_u}(\mathbb{E}_t[\hat{h}^*_{u,t+1}] - \hat{h}^*_{ut}) + \bar{\chi}\frac{m\bar{r}s_s \times \bar{h}_s}{\bar{c}_s}(\mathbb{E}_t[\hat{h}^*_{s,t+1}] - \hat{h}^*_{st})\right\} + \hat{\varepsilon}^b_t$, (B.131)

(B.109) $\Rightarrow$ $\hat{i}^*_{st} = \frac{1}{1+\tilde{\beta}\gamma}\hat{i}^*_{s,t-1} + \frac{\tilde{\beta}\gamma}{1+\tilde{\beta}\gamma}\mathbb{E}_t[\hat{i}^*_{s,t+1}] + \frac{1}{(1+\tilde{\beta}\gamma)\varphi_s\gamma^2}\hat{q}^*_{st}$, (B.132)

(B.110) $\Rightarrow$ $\hat{i}^*_{et} = \frac{1}{1+\tilde{\beta}\gamma}\hat{i}^*_{e,t-1} + \frac{\tilde{\beta}\gamma}{1+\tilde{\beta}\gamma}\mathbb{E}_t[\hat{i}^*_{e,t+1}] + \frac{1}{(1+\tilde{\beta}\gamma)\varphi_e\gamma^2}\hat{q}^*_{et} + \hat{\varepsilon}^i_t$, (B.133)

(B.111) $\Rightarrow$ $\hat{q}^*_{st} = -\hat{r}^*_t + \tilde{\beta}\bar{r}_s\mathbb{E}_t[\hat{r}^*_{s,t+1}] + \tilde{\beta}(1-\delta_s)\mathbb{E}_t[\hat{q}^*_{s,t+1}] + \frac{\sigma_c(1+h/\gamma)}{1-h/\gamma}\hat{\varepsilon}^b_t$, (B.134)

(B.112) $\Rightarrow$ $\hat{q}^*_{et} = -\hat{r}^*_t + \tilde{\beta}\bar{r}_e\mathbb{E}_t[\hat{r}^*_{e,t+1}] + \tilde{\beta}(1-\delta_e)\mathbb{E}_t[\hat{q}^*_{e,t+1}] + \frac{\sigma_c(1+h/\gamma)}{1-h/\gamma}\hat{\varepsilon}^b_t$, (B.135)

(B.113) $\Rightarrow$ $\hat{z}^*_t = \frac{a'(1)}{a''(1)}\hat{r}^*_{et} = \frac{1-\psi}{\psi}\hat{r}^*_{et}$, (B.136)

(B.114) $\Rightarrow$ $\hat{\tilde{k}}^*_{st} = \frac{1-\delta_s}{\gamma}\hat{\tilde{k}}^*_{s,t-1} + \left(1 - \frac{1-\delta_s}{\gamma}\right)\hat{i}^*_{st}$, (B.137)

(B.115) $\Rightarrow$ $\hat{\tilde{k}}^*_{et} = \frac{1-\delta_e}{\gamma}\hat{\tilde{k}}^*_{e,t-1} + \left(1 - \frac{1-\delta_e}{\gamma}\right)\hat{i}^*_{et} + \left(1 - \frac{1-\delta_e}{\gamma}\right)(1+\tilde{\beta}\gamma)\varphi_e\gamma^2\hat{\varepsilon}^i_t$, (B.138)

(B.116) $\Rightarrow$ $\hat{k}^*_{st} = \hat{\tilde{k}}^*_{s,t-1}$, (B.139)

(B.117) $\Rightarrow$ $\hat{k}^*_{et} = \hat{z}^*_t + \hat{\tilde{k}}^*_{e,t-1}$. (B.140)

(B.120) $\Rightarrow$ $\hat{l}^*_{ut} = -\frac{\bar{\chi}}{1-\bar{\chi}}\varepsilon^\chi_t + \hat{h}^*_{ut}$, (B.141)

(B.121) $\Rightarrow$ $\hat{l}^*_{st} = \varepsilon^\chi_t + \hat{h}^*_{st}$, (B.142)

(B.123) $\Rightarrow$ $\frac{\bar{c}}{\bar{y}}\hat{c}^*_t + \frac{\bar{i}_s}{\bar{y}}\hat{i}^*_{st} + \frac{\bar{i}_e}{\bar{y}}\hat{i}^*_{et} + \hat{g}_t + \frac{\bar{r}_e\bar{k}_e}{\bar{y}}\hat{z}^*_t = \hat{y}^*_t$. (B.143)

B.3.3 Exogenous processes

There are nine exogenous processes in this model. ε^b, ε^i, g, λ^p, λ^u, and λ^s shocks are normalized as discussed above.

Technology:

$$\varepsilon^a_t = \rho_a\varepsilon^a_{t-1} + \eta^a_t,\ \eta^a_t \sim N(0, \sigma^2_a). \tag{B.144}$$

Intertemporal preference shifter (financial risk-premium process):

$$\hat{\varepsilon}^b_t = \rho_b\hat{\varepsilon}^b_{t-1} + \eta^b_t,\ \eta^b_t \sim N(0, \sigma^2_b). \tag{B.145}$$

Government spending:

$$\hat{g}_t = \rho_g\hat{g}_{t-1} + \eta^g_t + \mu_{ga}\eta^a_t,\ \eta^g_t \sim N(0, \sigma^2_g). \tag{B.146}$$

Investment-specific productivity:

$$\hat{\varepsilon}_t^i = \rho_i \hat{\varepsilon}_{t-1}^i + \eta_t^i,\ \eta_t^i \sim N(0, \sigma_i^2). \tag{B.147}$$

Monetary policy:

$$\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r,\ \eta_t^r \sim N(0, \sigma_r^2). \tag{B.148}$$

Price mark-up process:

$$\hat{\lambda}_t^p = \rho_p \hat{\lambda}_{t-1}^p - \mu_p \eta_{t-1}^p + \eta_t^p,\ \eta_t^p \sim N(0, \sigma_p^2). \tag{B.149}$$

Unskilled wage mark-up process:

$$\hat{\lambda}_t^u = \rho_u \hat{\lambda}_{t-1}^u - \mu_u \eta_{t-1}^u + \eta_t^u,\ \eta_t^u \sim N(0, \sigma_u^2). \tag{B.150}$$

Skilled wage mark-up process:

$$\hat{\lambda}_t^s = \rho_s \hat{\lambda}_{t-1}^s - \mu_s \eta_{t-1}^s + \eta_t^s,\ \eta_t^s \sim N(0, \sigma_s^2). \tag{B.151}$$

Employment share of skilled workers process:

$$\varepsilon_t^\chi = \rho_\chi \varepsilon_{t-1}^\chi + \eta_t^\chi,\ \eta_t^\chi \sim N(0, \sigma_\chi^2). \tag{B.152}$$

With a slight abuse of notation, we denote those exogenous variables using ε with corresponding superscripts in the main text.

In the measurement equation of the state-space representation of the model, we introduce three independent measurement errors for (i) the growth rate of per capita capital equipment stock (ν_{kt}), (ii) changes in relative labor supply (ν_{ht}), and (iii) changes in the relative wage rates between skilled and unskilled workers (ν_{wt}):

$$\begin{aligned}
\nu_{kt} &\sim N(0, \sigma_{\nu,k}^2), \\
\nu_{ht} &\sim N(0, \sigma_{\nu,h}^2), \\
\nu_{wt} &\sim N(0, \sigma_{\nu,w}^2).
\end{aligned}$$

B.3.4 Comparison with the Smets and Wouters model

The basic structure of our model is adopted from Smets and Wouters (2007*b*) to emphasize the role of more flexible production functions than the Cobb–Douglas function. Although we deviate minimally from the Smets and Wouters (2007*b*) model, there are a few differences:

- Firms use four inputs in production, including structure, equipment, skilled labor, and unskilled labor, similar to the models discussed in Krusell et al. (2000); Ohanian, Orak and Shen (2023). In contrast, the Smets and Wouters (2007*b*) model features two inputs (capital and labor).
- We assume a normalized translog production function following Hyun, Kim and Lee (2024) rather than the Cobb–Douglas function.

- Because we have the skilled and unskilled labor, the model features two wage Phillips curves and two wage markup shocks.
- Number of the skilled workers is a new exogenous variable in our model.
- We rule out price and wage indexation: $\iota_p = \iota_u = \iota_s = 0$ (see Woodford, 2007; Cogley and Sbordone, 2008; Phaneuf, Sims and Victor, 2018).
- We assume that government spending does not depend on productivity shocks: $\rho_{ga} = 0$ (see Equation (B.146)).

B.4 Translog Approximation of Nested CES Production Functions

Equations (B.17)-(B.19) in Section B.1.2 show the mapping from a nested CES production function employed in Krusell et al. (2000) and Ohanian, Orak and Shen (2023) to our translog form. In log-linearization, the equilibrium conditions in the nested CES model are equivalent to those in the translog model subject to relevant restrictions imposed on βs. This section presents these results. For simple notation and because the cost minimization is a static problem, we drop time subscript t and firm index (i) below.

First, we derive a log-linear approximation of a generic two-factor CES function.

Lemma. Let y be $\left(a_1 x_1^{\phi} + a_2 x_2^{\phi}\right)^{1/\phi}$ for $\phi \leq 1$. Log deviations of y, x_1, and x_2 from $\bar{y}$, $\bar{x}_1$, and $\bar{x}_2$ are denoted by $\hat{y}$, $\hat{x}_1$, and $\hat{x}_2$, where $\bar{y} = \left(a_1 \bar{x}_1^{\phi} + a_2 \bar{x}_2^{\phi}\right)^{1/\phi}$. Then, at the first order,

$$\begin{aligned}
\hat{y} &= \alpha_1 \hat{x}_1 + \alpha_2 \hat{x}_2, \\
\alpha_1 &= \frac{a_1 \bar{x}_1^{\phi}}{a_1 \bar{x}_1^{\phi} + a_2 \bar{x}_2^{\phi}}, \\
\alpha_2 = 1 - \alpha_1 &= \frac{a_2 \bar{x}_2^{\phi}}{a_1 \bar{x}_1^{\phi} + a_2 \bar{x}_2^{\phi}}.
\end{aligned}$$

Proof. Note that $\frac{\partial \log y}{\partial \log x_i} = \frac{\partial \log y}{\partial x_i} \frac{\partial x_i}{\partial \log x_i} = \frac{1}{\phi} \frac{a_i \phi x_i^{\phi-1}}{a_1 x_1^{\phi} + a_2 x_2^{\phi}} x_i = \frac{a_i x_i^{\phi}}{a_1 x_1^{\phi} + a_2 x_2^{\phi}}$ for $i = 1, 2$. Then, a Taylor expansion of $\log y$ around $\log \bar{y}$ yields the result. □

Consider the following nested CES production function:

$$\begin{aligned}
Y &= \exp(\varepsilon^a) K_s^{\alpha_{ks}} Y_{CES}^{1-\alpha_{ks}} - \gamma^t \upsilon, \\
Y_{CES} &= \left(a_1 (\gamma^t L_u)^{\phi_{ux}} + a_2 X^{\phi_{ux}}\right)^{1/\phi_{ux}}, \\
X &= \left(b_1 K_e^{\phi_{ks}} + b_2 (\gamma^t L_s)^{\phi_{ks}}\right)^{1/\phi_{ks}},
\end{aligned} \tag{B.153}$$

where a_1, a_2, b_1, and b_2 are positive constants and ϕ_{ux}, $\phi_{ks} \in [-\infty, 1]$.

FOCs. The FOCs for the cost minimization problem (B.6) are as follows:

$$K_s: \qquad R_s = MC \frac{\tilde{Y}}{K_s} \alpha_{ks},$$

$$
\begin{aligned}
K_e: \quad & R_e = MC\frac{\tilde{Y}}{K_e}(1-\alpha_{ks})\left[a_2\left(\frac{X}{Y_{CES}}\right)^{\phi_{ux}}\right]\left[b_1\left(\frac{K_e}{X}\right)^{\phi_{ks}}\right], \\
L_s: \quad & W_s = MC\frac{\tilde{Y}}{L_s}(1-\alpha_{ks})\left[a_2\left(\frac{X}{Y_{CES}}\right)^{\phi_{ux}}\right]\left[b_2\left(\frac{\gamma^t L_s}{X}\right)^{\phi_{ks}}\right], \\
L_u: \quad & W_u = MC\frac{\tilde{Y}}{L_u}(1-\alpha_{ks})\left[a_1\left(\frac{\gamma^t L_u}{Y_{CES}}\right)^{\phi_{ux}}\right],
\end{aligned}
$$

where $\tilde{Y} = Y + \gamma^t \upsilon$. By detrending the relevant variables and changing nominal costs into real variables, we have:

$$r_s = mc\frac{\tilde{y}}{k_s}\alpha_{ks}, \tag{B.154}$$

$$r_e = mc\frac{\tilde{y}}{k_e}(1-\alpha_{ks})\left[a_2\left(\frac{x}{y_{CES}}\right)^{\phi_{ux}}\right]\left[b_1\left(\frac{k_e}{x}\right)^{\phi_{ks}}\right], \tag{B.155}$$

$$w_s = mc\frac{\tilde{y}}{l_s}(1-\alpha_{ks})\left[a_2\left(\frac{x}{y_{CES}}\right)^{\phi_{ux}}\right]\left[b_2\left(\frac{l_s}{x}\right)^{\phi_{ks}}\right], \tag{B.156}$$

$$w_u = mc\frac{\tilde{y}}{l_u}(1-\alpha_{ks})\left[a_1\left(\frac{l_u}{y_{CES}}\right)^{\phi_{ux}}\right], \tag{B.157}$$

Steady-state income shares. Given the lemma, we define the following parameters using the steady-state values of the related variables:

$$
\begin{aligned}
\tilde{\alpha}_u &= a_1\left(\frac{\bar{l}_u}{\bar{y}_{CES}}\right)^{\phi_{ux}}, \\
\tilde{\alpha}_x &= 1-\tilde{\alpha}_u = a_2\left(\frac{\bar{x}}{\bar{y}_{CES}}\right)^{\phi_{ux}}, \\
\tilde{\alpha}_k &= b_1\left(\frac{\bar{k}_e}{\bar{x}}\right)^{\phi_{ks}}, \\
\tilde{\alpha}_s &= 1-\tilde{\alpha}_k = b_2\left(\frac{\bar{l}_s}{\bar{x}}\right)^{\phi_{ks}},
\end{aligned}
$$

The steady-state income shares are denoted by α_{ks}, α_{ke}, α_s, and α_u as before. Because $\bar{mc} = 1/\Phi$ and $\bar{\tilde{y}} = \Phi\bar{y}$:

$$(\text{B.157}) \quad \Rightarrow \quad \alpha_u = \frac{\bar{w}_u\bar{l}_u}{\bar{y}} = (1-\alpha_{ks})\tilde{\alpha}_u, \tag{B.158}$$

$$(\text{B.155}) \quad \Rightarrow \quad \alpha_{ke} = \frac{\bar{r}_e\bar{k}_e}{\bar{y}} = (1-\alpha_{ks})\tilde{\alpha}_x\tilde{\alpha}_k, \tag{B.159}$$

$$(\text{B.156}) \quad \Rightarrow \quad \alpha_s = \frac{\bar{w}_s\bar{l}_s}{\bar{y}} = (1-\alpha_{ks})\tilde{\alpha}_x\tilde{\alpha}_s. \tag{B.160}$$

Thus, specifying $(\alpha_{ks}, \tilde{\alpha}_u, \tilde{\alpha}_x, \tilde{\alpha}_k, \tilde{\alpha}_s)$ subject to $\tilde{\alpha}_u + \tilde{\alpha}_x = 1$ and $\tilde{\alpha}_k + \tilde{\alpha}_s = 1$ is equivalent to determining $(\alpha_{ks}, \alpha_{ke}, \alpha_s, \alpha_u)$ subject to $\alpha_{ks} + \alpha_{ke} + \alpha_s + \alpha_u = 1$.

Log-linearization. Next, we discuss the log-linearization of the production function. The lemma implies that:

$$\hat{x} = \tilde{\alpha}_k\hat{k}_e + \tilde{\alpha}_s\hat{l}_s = \frac{1}{(1-\alpha_{ks})\tilde{\alpha}_x}(\alpha_{ke}\hat{k}_e + \alpha_s\hat{l}_s), \tag{B.161}$$

$$\hat{y}_{CES} = \tilde{\alpha}_u \hat{l}_u + \tilde{\alpha}_x \hat{x} = \tilde{\alpha}_u \hat{l}_u + \tilde{\alpha}_x(\tilde{\alpha}_k \hat{k}_e + \tilde{\alpha}_s \hat{l}_s) = \frac{1}{1-\alpha_{ks}}(\alpha_{ke}\hat{k}_e + \alpha_s \hat{l}_s + \alpha_u \hat{l}_u). \tag{B.162}$$

Thus:

$$\begin{aligned}
\hat{y} &= \Phi(\varepsilon^a + \alpha_{ks}\hat{k}_s + (1-\alpha_{ks})\hat{y}_{CES}) \\
&= \Phi[\varepsilon^a + \alpha_{ks}\hat{k}_s + (1-\alpha_{ks})(\tilde{\alpha}_u \hat{l}_u + \tilde{\alpha}_x \hat{x})] \\
&= \Phi\{\varepsilon^a + \alpha_{ks}\hat{k}_s + (1-\alpha_{ks})[\tilde{\alpha}_u \hat{l}_u + \tilde{\alpha}_x(\tilde{\alpha}_k \hat{k}_e + \tilde{\alpha}_s \hat{l}_s)]\} \\
&= \Phi(\alpha_{ks}\hat{k}_s + \alpha_{ke}\hat{k}_e + \alpha_s \hat{l}_s + \alpha_u \hat{l}_u + \varepsilon^a).
\end{aligned}$$

We use Equations (B.158)-(B.160) in the derivation. The log-linearized production function is the same as Equation (B.107) in the translog production model.

Similarly, we log-linearize the FOCs:

$$(\text{B.154}) \quad \Rightarrow \quad \hat{r}_s + \hat{k}_s = \hat{mc} + \hat{\tilde{y}}, \tag{B.163}$$

$$(\text{B.155}) \quad \Rightarrow \quad \hat{r}_e + \hat{k}_e = \hat{mc} + \hat{\tilde{y}} + \phi_{ux}(\hat{x} - \hat{y}_{CES}) + \phi_{ks}(\hat{k}_e - \hat{x}), \tag{B.164}$$

$$(\text{B.156}) \quad \Rightarrow \quad \hat{w}_s + \hat{l}_s = \hat{mc} + \hat{\tilde{y}} + \phi_{ux}(\hat{x} - \hat{y}_{CES}) + \phi_{ks}(\hat{l}_s - \hat{x}), \tag{B.165}$$

$$(\text{B.157}) \quad \Rightarrow \quad \hat{w}_u + \hat{l}_u = \hat{mc} + \hat{\tilde{y}} + \phi_{ux}(\hat{l}_u - \hat{y}_{CES}). \tag{B.166}$$

Equations (B.161) and (B.162) imply that the above equilibrium conditions are the same in the translog model when:

$$\hat{\alpha}_{ket} = \frac{1}{\alpha_{ke}}(\beta_{kk}\hat{k}_e + \beta_{ks}\hat{l}_s + \beta_{ku}\hat{l}_u) \qquad = \phi_{ux}(\hat{x} - \hat{y}_{CES}) + \phi_{ks}(\hat{k}_e - \hat{x}), \tag{B.167}$$

$$\hat{\alpha}_{st} = \frac{1}{\alpha_s}(\beta_{ks}\hat{k}_e + \beta_{ss}\hat{l}_s + \beta_{su}\hat{l}_u) \qquad = \phi_{ux}(\hat{x} - \hat{y}_{CES}) + \phi_{ks}(\hat{l}_s - \hat{x}), \tag{B.168}$$

$$\hat{\alpha}_{ut} = \frac{1}{\alpha_u}(\beta_{ku}\hat{k}_e + \beta_{su}\hat{l}_s + \beta_{uu}\hat{l}_u) \qquad = \phi_{ux}(\hat{l}_u - \hat{y}_{CES}). \tag{B.169}$$

By comparing the coefficients on $\hat{k}_e$, $\hat{l}_s$, and $\hat{l}_u$ on both sides of the above three equations, we can derive the results in Equation (B.19):

$$\begin{aligned}
\beta_{kk} &= \alpha_{ke}(\phi_{ux}\tilde{\alpha}_k\tilde{\alpha}_u + \phi_{ks}\tilde{\alpha}_s), \\
\beta_{ks} &= \alpha_{ke}\tilde{\alpha}_s(\phi_{ux}\tilde{\alpha}_u - \phi_{ks}) = \tilde{\alpha}_k\alpha_s(\phi_{ux}\tilde{\alpha}_u - \phi_{ks}), \\
\beta_{ku} &= -\alpha_{ke}\phi_{ux}\tilde{\alpha}_u = -\phi_{ux}\tilde{\alpha}_x\tilde{\alpha}_k\alpha_u, \\
\beta_{ss} &= \alpha_s(\phi_{ux}\tilde{\alpha}_s\tilde{\alpha}_u + \phi_{ks}\tilde{\alpha}_k), \\
\beta_{su} &= -\alpha_s\phi_{ux}\tilde{\alpha}_u = -\phi_{ux}\alpha_u\tilde{\alpha}_x\tilde{\alpha}_s, \\
\beta_{uu} &= \alpha_u\phi_{ux}\tilde{\alpha}_x.
\end{aligned}$$

Finally, note that $\beta_{k\cdot} = \beta_{s\cdot} = \beta_{u\cdot} = 0$ in this case. Thus:

$$\hat{mc} = \alpha_{ks}\hat{r}_s + \alpha_{ke}\hat{r}_e + \alpha_s\hat{l}_s + \alpha_u\hat{l}_u - \varepsilon^a.$$

Furthermore:

$$-\tilde{\alpha}_k\beta_{uu} = -\tilde{\alpha}_k\alpha_u\phi_{ux}\tilde{\alpha}_x = -\phi_{ux}\frac{\alpha_{ke}}{1-\alpha_{ks}}\alpha_u = -\alpha_{ke}\phi_{ux}\tilde{\alpha}_u = \beta_{ku}. \tag{B.170}$$

Therefore, given the steady-state income shares (α_{ks}, α_{ke}, α_s, and α_u), two degrees of freedom that the nested CES function has for the second-order terms (ϕ_{ux} and ϕ_{ks}) lead to six translog parameters (β_{kk}, β_{ks}, β_{ku}, β_{ss}, β_{su}, and β_{uu}) subject to four restrictions $\beta_{k\cdot} = \beta_{s\cdot} = \beta_{u\cdot} = 0$ and $\beta_{ku} = -\tilde{\alpha}_k\beta_{uu}$.

Appendix C Supplementary Materials for Section 3

C.1 Data Used in Section 3

Our observables include the seven quarterly variables employed in Smets and Wouters (2007*b*). These variables include growth rates of per capita real GDP, consumption, and investment; labor hours; the growth rate of the real wage rate; price inflation; and the nominal interest rate. We extend the data constructed in Smets and Wouters (2007*b*) to later periods. We replace the federal funds rate with the shadow rate of Wu and Xia (2016) from 2009:q1 to 2015:q4 because of the binding zero lower bound.

In view of two types of capital (structure and equipment) and labor (skilled and unskilled), we further borrow three annual variables from Ohanian, Orak and Shen (2023) to discipline the model parameters.[1] These variables are the growth rate of per capita capital equipment stock (including intellectual property products), relative labor supply changes, and the relative wage rate changes between skilled and unskilled workers. The capital stock is constructed using perpetual inventory methods. The two other labor market variables are calculated using the Current Population Survey (CPS). Those three annual variables are observable in the fourth quarter of each year.

Finally, we compute the skilled employment shares using the CPS data. Following Krusell et al. (2000) and Ohanian, Orak and Shen (2023), college graduates are considered skilled workers. In the early period, the required variables to construct the series were available only in the March surveys. Thus, this series is available only in the first quarter until 1975. Since then, it has been available every quarter. We remove the seasonal variation using X-13 ARIMA.

Our sample begins in 1966:q1 following as in Smets and Wouters (2007*b*) and ends in 2019:q4 before the recent pandemic. We divide the data into the early (1966-99) and late (2000-19) samples and estimate each model for each sample period. In total, we have six estimated models for the three production functions and the two sample periods.

C.2 Observables

We first define (detrended) aggregate investment (i_t), labor hours (h_t), and the real wage rate (w_t):

$$\begin{aligned} i_t &= i_{st} + i_{et}, \\ h_t &= (1-\chi_t)h_{ut} + \chi_t h_{st}, \\ w_t &= \frac{w_{ut}l_{ut} + w_{st}l_{st}}{h_t} = \frac{w_{ut}(1-\chi_t)h_{ut} + w_{st}\chi_t h_{st}}{h_t}. \end{aligned}$$

Like the skilled and unskilled wage rates, the aggregate nominal wage rate W_t is given by $\gamma^t P_t w_t$. In log-linearization, we have:

$$\begin{aligned} \hat{i}_t &= \frac{\bar{i}_s/\bar{y}}{\bar{i}_s/\bar{y} + \bar{i}_e/\bar{y}}\hat{i}_{st} + \frac{\bar{i}_e/\bar{y}}{\bar{i}_s/\bar{y} + \bar{i}_e/\bar{y}}\hat{i}_{et}, \\ \hat{h}_t &= (1-\bar{\chi})\hat{h}_{ut} + \bar{\chi}\hat{h}_{st}, \\ \hat{w}_t &= \frac{\bar{w}_u\bar{l}_u}{\bar{w}_u\bar{l}_u + \bar{w}_s\bar{l}_s}(\hat{w}_{ut} - \frac{\bar{\chi}}{1-\bar{\chi}}\varepsilon_t^{\chi} + \hat{h}_{ut}) + \frac{\bar{w}_s\bar{l}_s}{\bar{w}_u\bar{l}_u + \bar{w}_s\bar{l}_s}(\hat{w}_{st} + \varepsilon_t^{\chi} + \hat{h}_{st}) - \hat{h}_t \end{aligned}$$

[1] Musa Orak kindly shared the dataset constructed for Ohanian, Orak and Shen (2023) with us, which is greatly appreciated.

$$= \frac{\alpha_u}{\alpha_u + \alpha_s}(\hat{w}_{ut} - \frac{\bar{\chi}}{1-\bar{\chi}}\varepsilon_t^{\chi} + \hat{h}_{ut}) + \frac{\alpha_s}{\alpha_u + \alpha_s}(\hat{w}_{st} + \varepsilon_t^{\chi} + \hat{h}_{st}) - \hat{h}_t,$$

where $\frac{\bar{i}_s}{\bar{y}}$ and $\frac{\bar{i}_e}{\bar{y}}$ are obtained from Equations (B.94) and (B.95). In deriving the result for $\hat{h}_t$, the fact that $\bar{h}_s = \bar{h}_u$ is used. Finally, Equations (B.77) and (B.78) are used for $\hat{w}_t$.

Growth rates of real GDP per capita in data are mapped to $\log\gamma + \Delta\hat{y}_t$ in the model. Consumption, investment, and the real wage growth rates are similar. Following Smets and Wouters (2007*b*), log labor hours are demeaned because an index variable is involved in computing this empirical measure. This measure is compared with $\bar{l} + \hat{h}_t$, where $\bar{l}$ is a parameter to be estimated. Inflation is matched with $\bar{\pi} + \hat{\pi}_t$, where $\bar{\pi}$ is similarly estimated. The nominal interest rate corresponds to $\bar{r} + \hat{r}_t$, where $\bar{r} = \log(\bar{R}) = \bar{\pi} - \log(\beta\gamma^{-\sigma_c})$ because of Equation (B.80). The log employment share of skilled workers in the data follows an exogenous process in the model such that $\log(\chi_t) = \log(\bar{\chi}) + \varepsilon_t^{\chi}$. We set $\bar{\chi}$ at 0.21 for the pre-2000 sample and 0.325 for the post-2000 sample. Around the midpoint of each sample, i.e., in 1983 and 2010, χ_t was approximately 0.21 and 0.325, respectively.

Next, we address annual variables. The growth rate of per capita capital equipment stock corresponds to $\log(\tilde{K}_{et}/\tilde{K}_{e,t-4}) + \nu_{kt} = 4\log(\gamma) + \log(\tilde{k}_{et}/\tilde{k}_{e,t-4}) + \nu_{kt} = 4\log(\gamma) + \hat{\tilde{k}}_{et} - \hat{\tilde{k}}_{e,t-4} + \nu_{kt}$. This variable is observable only in the fourth quarter. ν_{kt} represents a measurement error in the empirical capital stock variable computed using perpetual inventory methods. ν_{kt} has an independently and identically distributed (iid) normal density with a mean of zero and a standard deviation of $\sigma_{\nu,k}$.

For the relative labor supply, we first consider annual labor supply of each type:

$$\begin{aligned}
l_{ut}^{ann} &= l_{ut} + l_{u,t-1} + l_{u,t-2} + l_{u,t-3} \\
&= (1-\chi_t)h_{ut} + (1-\chi_{t-1})h_{u,t-1} + (1-\chi_{t-2})h_{u,t-2} + (1-\chi_{t-3})h_{u,t-3}, \\
l_{st}^{ann} &= l_{st} + l_{s,t-1} + l_{s,t-2} + l_{s,t-3} \\
&= \chi_t h_{st} + \chi_{t-1}h_{s,t-1} + \chi_{t-2}h_{s,t-2} + \chi_{t-3}h_{s,t-3}.
\end{aligned}$$

Then, $\bar{l}_u^{ann} = 4\bar{l}_u = 4(1-\bar{\chi})\bar{h}_u$ and $\bar{l}_s^{ann} = 4\bar{l}_s = 4\bar{\chi}\bar{h}_s$. Furthermore:

$$\begin{aligned}
\hat{l}_{ut}^{ann} &= \frac{1}{4}(\hat{l}_{ut} + \hat{l}_{u,t-1} + \hat{l}_{u,t-2} + \hat{l}_{u,t-3}), \\
\hat{l}_{st}^{ann} &= \frac{1}{4}(\hat{l}_{st} + \hat{l}_{s,t-1} + \hat{l}_{s,t-2} + \hat{l}_{s,t-3}).
\end{aligned}$$

Thus, the relative quarterly and annual labor supply are given by:

$$\begin{aligned}
rell_t &\equiv \frac{l_{st}}{l_{ut}}, & \widehat{rell}_t &= \hat{l}_{st} - \hat{l}_{ut}, \\
rell_t^{ann} &\equiv \frac{l_{st}^{ann}}{l_{ut}^{ann}}, & \widehat{rell}_t^{ann} &= \hat{l}_{st}^{ann} - \hat{l}_{ut}^{ann} = \frac{1}{4}(\widehat{rell}_t + \widehat{rell}_{t-1} + \widehat{rell}_{t-2} + \widehat{rell}_{t-3}).
\end{aligned}$$

Finally, the change in (log) relative annual labor supply in the data corresponds to $\widehat{rell}_t^{ann} - \widehat{rell}_{t-4}^{ann} + \nu_{ht}$, observable in the fourth quarters only. The labor market variables in the CPS are known to be subject to nonnegligible measurement errors (see, e.g. Bound and Krueger, 1991); ν_{ht} reflects measurement errors in the survey data. We assume that ν_{ht} has an iid normal distribution with a mean of zero and a standard deviation of $\sigma_{\nu,h}$.

The nominal annual wage rate for each type is defined as the total annual wage bill divided by the total annual

hours:

$$W_{st}^{ann} \equiv \frac{W_{st}L_{st} + \ldots + W_{s,t-3}L_{s,t-3}}{L_{st}^{ann}},$$
$$W_{ut}^{ann} \equiv \frac{W_{ut}L_{ut} + \ldots + W_{u,t-3}L_{u,t-3}}{L_{ut}^{ann}},$$

where $L_{st}^{ann} = l_{st}^{ann}$ and $L_{ut}^{ann} = l_{ut}^{ann}$. Thus, the relative real wage rate is given by:

$$relw_t^{ann} \equiv \frac{W_{st}^{ann}/(\gamma^t P_t)}{W_{ut}^{ann}/(\gamma^t P_t)} = \frac{wbr_t^{ann}}{rell_t^{ann}},$$
$$\text{where} \quad wbr_t^{ann} \equiv \frac{(W_{st}L_{st} + \ldots + W_{s,t-3}L_{s,t-3})/(\gamma^t P_t)}{(W_{ut}L_{ut} + \ldots + W_{u,t-3}L_{u,t-3})/(\gamma^t P_t)} = \frac{W_{st}^{ann}L_{st}^{ann}/(\gamma^t P_t)}{W_{ut}^{ann}L_{ut}^{ann}/(\gamma^t P_t)}.$$

The following hold for the total annual real wage bills for each type of worker:

$$\begin{aligned}
\hat{w}_{st}^{ann} + \hat{l}_{st}^{ann} &= \left\{ w_{st}l_{st} + \ldots + \widehat{\frac{1}{\gamma^3 \Pi_{t-3}^t} w_{s,t-3}l_{s,t-3}} \right\} \\
&= \frac{1}{\mathcal{C}}\Big[\hat{w}_{st} + \hat{l}_{st} + \frac{1}{\gamma\bar{\Pi}}(\hat{w}_{s,t-1} + \hat{l}_{s,t-1} - \hat{\pi}_t) + \frac{1}{\gamma^2\bar{\Pi}^2}(\hat{w}_{s,t-2} + \hat{l}_{s,t-2} - \hat{\pi}_t - \hat{\pi}_{t-1}) \\
&+ \frac{1}{\gamma^3\bar{\Pi}^3}(\hat{w}_{s,t-3} + \hat{l}_{s,t-3} - \hat{\pi}_t - \hat{\pi}_{t-1} - \hat{\pi}_{t-2})\Big], \\
\hat{w}_{ut}^{ann} + \hat{l}_{ut}^{ann} &= \left\{ w_{ut}l_{ut} + \ldots + \widehat{\frac{1}{\gamma^3 \Pi_{t-3}^t} w_{u,t-3}l_{u,t-3}} \right\} \\
&= \frac{1}{\mathcal{C}}\Big[\hat{w}_{ut} + \hat{l}_{ut} + \frac{1}{\gamma\bar{\Pi}}(\hat{w}_{u,t-1} + \hat{l}_{u,t-1} - \hat{\pi}_t) + \frac{1}{\gamma^2\bar{\Pi}^2}(\hat{w}_{u,t-2} + \hat{l}_{u,t-2} - \hat{\pi}_t - \hat{\pi}_{t-1}) \\
&+ \frac{1}{\gamma^3\bar{\Pi}^3}(\hat{w}_{u,t-3} + \hat{l}_{u,t-3} - \hat{\pi}_t - \hat{\pi}_{t-1} - \hat{\pi}_{t-2})\Big], \\
\text{where} \quad \mathcal{C} &\equiv 1 + \frac{1}{\gamma\bar{\Pi}} + \frac{1}{\gamma^2\bar{\Pi}^2} + \frac{1}{\gamma^3\bar{\Pi}^3}.
\end{aligned}$$

We further define the quarterly wage bill ratio as:

$$\begin{aligned}
wbr_t &\equiv \frac{(W_{st}L_{st})/(\gamma^t P_t)}{(W_{ut}L_{ut})/(\gamma^t P_t)} \\
\Rightarrow \quad \widehat{wbr}_t &= (\hat{w}_{st} + \hat{l}_{st}) - (\hat{w}_{ut} + \hat{l}_{ut}).
\end{aligned}$$

Note that:

$$\begin{aligned}
\widehat{wbr}_t^{ann} &= (\hat{w}_{st}^{ann} + \hat{l}_{st}^{ann}) - (\hat{w}_{ut}^{ann} + \hat{l}_{ut}^{ann}) \\
&= \frac{1}{\mathcal{C}}\Big[\widehat{wbr}_t + \frac{1}{\gamma\bar{\Pi}}\widehat{wbr}_{t-1} + \frac{1}{\gamma^2\bar{\Pi}^2}\widehat{wbr}_{t-2} + \frac{1}{\gamma^3\bar{\Pi}^3}\widehat{wbr}_{t-3}\Big].
\end{aligned}$$

Finally,

$$\widehat{relw}_t^{ann} = \widehat{wbr}_t^{ann} - \widehat{rell}_t^{ann},$$

and its annual change augmented with measurement errors, $\widehat{relw}_t^{ann} - \widehat{relw}_{t-4}^{ann} + \nu_{wt}$, is associated with the change in the relative wage rates between skilled and unskilled workers in the data. This variable is assumed to be observable

only in the fourth quarter. ν_{wt} captures measurement errors and has an iid normal distribution with a mean of zero and a standard deviation of $\sigma_{\nu,w}$.

C.3 Prior and Posterior Distributions

This section illustrates the prior and posterior distributions of the model parameters. We consider the results for the early sample (1966-1999) and the late sample (2000-2019), and the three models with different intermediate goods production functions: (i) Cobb–Douglas (B.4) subject to $\boldsymbol{\beta} = 0$, (ii) nested CES (B.153), and (iii) translog (B.4).

We use standard priors for conventional parameters, similar to Smets and Wouters (2007*b*) and Justiniano, Primiceri and Tambalotti (2010). For novel parameters, we have the following priors. We assume that $\alpha_{ks} \sim N(0.1, 0.005^2)$ given the results in Krusell et al. (2000) and Ohanian, Orak and Shen (2023). Combined with the prior for $\alpha_{ke} \sim N(0.25, 0.02^2)$, this assumption implies that the capital income share a priori ranges from 0.3-0.4 within the two standard deviation intervals. The skilled labor share, α_s, has a normal distribution with a standard deviation of 0.05 and a mean of 0.2 and 0.3 for the early and late samples, respectively. We increase the mean of the skilled labor income share for the post-2000 sample given the steady increase in supply of skilled labor (Goldin and Katz, 2009).

The translog parameters, e.g., β_{kk}, have normal priors with a mean of zero and a standard deviation of 0.15. This prior is rather loose and less informative in the sense that the posterior modes lie within the two-standard deviation interval, (-0.3, 0.3). Furthermore, when $\phi_{ux} = 0.401$, $\phi_{ks} = -0.495$, and $\alpha_{ks} = 0.117$, following Krusell et al. (2000) and in combination with $\alpha_{ke} = 0.25$ and $\alpha_s = 0.2$, the corresponding translog parameters obtained from Equation (B.19) are less than 0.09 in absolute value. Thus, a priori standard deviation of 0.15 is large enough to cover previous estimates based on the nested CES functions as reasonable cases. For the nested CES model, we assume that $1 - \phi_{ux}$ and $1 - \phi_{ks}$ have Gamma distributions with a mean of 1 and a standard deviation of 0.5. Thus, ϕ_{ux} and ϕ_{ks} are zero on average, corresponding to a Cobb–Douglas function.

The employment share of skilled labor, χ_t, evolves according to an AR(1) process: $\log(\chi_t) - \log(\bar{\chi}) = \varepsilon_t^{\chi} = \rho_{\chi}\varepsilon_{t-1}^{\chi} + \eta_t^{\chi}$. Because this series steadily increases throughout the sample period, we assume that $\rho_{\chi} \sim B(0.9, 0.05^2)$ to capture the persistence process in the data, where B denotes a beta distribution. In contrast, the other AR(1) coefficients follow $B(0.5, 0.2^2)$ a priori. Finally, measurement errors are assumed not to be excessively large. The standard deviations of the measurement errors, $\sigma_{\nu,k}$, $\sigma_{\nu,h}$, and $\sigma_{\nu,w}$, have inverse gamma distributions with a mean of 0.15 and a standard deviation of 0.03.

We fix the following parameters:

$$\delta_s = 0.026/4, \qquad \delta_e = 0.16/4, \qquad \exp(\bar{g}) = \frac{\bar{G}_t}{\bar{Y}_t} = 0.18,$$

$$\theta_p = 10, \qquad \theta_s = 10, \qquad \theta_u = 10,$$

$$\Phi_s = 1 + \bar{\lambda}^s = 1.5, \qquad \Phi_u = 1 + \bar{\lambda}^u = 1.5, \qquad \bar{\chi} = \begin{cases} 0.21, & \text{pre-2000 sample} \\ 0.325, & \text{post-2000 sample} \end{cases}.$$

The depreciation rates for each type of capital are chosen according to Fernald (2014*a*, p. 18) and Ohanian, Orak and Shen (2023, fig 3.3(b)). The steady state share of government spending, wage markups, and Kimball curvature parameters are set at the same values as those in Smets and Wouters (2007*b*).

Next, we show the complete list of estimated parameters and their prior and posterior distributions. We summarize the posterior distributions using the mode and the equal-tailed 95% credible intervals. We present the results for the early sample (1966-1999) and the late sample (2000-2019) in separate tables. Each table covers the results based on the three production functions: Cobb–Douglas, nested CES, and translog. For the nested CES model, we show the implied translog parameters using Equation (B.19). We employ a standard Markov Chain Monte Carlo (MCMC) technique to obtain the posterior distribution. Specifically, we use a Metropolis–Hastings algorithm with a chain length of 200,000. The chain starts at the posterior mode, computed using interior-point methods. We use either the inverse of numerical hessian at the mode or the variance of the posterior distribution obtained from a preliminary MCMC algorithm with a chain length of 10,000 as a variance of the jump distribution in our algorithm. The step sizes are adjusted to obtain reasonable acceptance rates. The acceptance rates for the three models and the two sample periods range from 22%-30%.

Table C.1: Estimation results for the early sample (1966-1999)

Parameter	Priors			Cobb–Douglas		Nested CES		Translog	
	Mean	Std.	Family	Mode	(2.5%, 97.5%)	Mode	(2.5%, 97.5%)	Mode	(2.5%, 97.5%)
$-100 \log \beta$	0.25	0.1	Gamma	0.20	(0.07, 0.35)	0.26	(0.11, 0.40)	0.21	(0.08, 0.34)
σ_c	1.5	0.25	Normal	1.37	(1.13, 1.69)	1.20	(1.06, 1.47)	1.21	(1.06, 1.51)
h	0.7	0.1	Beta	0.68	(0.58, 0.78)	0.72	(0.61, 0.81)	0.70	(0.61, 0.79)
σ_l	1.5	0.25	Normal	1.16	(0.68, 1.81)	1.62	(1.24, 2.11)	1.72	(1.29, 2.18)
φ_s	4	1.5	Normal	5.04	(3.17, 7.88)	5.16	(3.40, 7.99)	4.65	(3.01, 7.59)
φ_e	4	1.5	Normal	4.19	(2.53, 6.36)	4.99	(3.30, 7.22)	3.71	(2.36, 6.14)
ψ	0.5	0.15	Beta	0.28	(0.08, 0.44)	0.35	(0.24, 0.56)	0.31	(0.14, 0.52)
ξ_p	0.5	0.1	Beta	0.66	(0.53, 0.79)	0.72	(0.59, 0.85)	0.65	(0.53, 0.78)
ξ_s	0.5	0.1	Beta	0.62	(0.53, 0.90)	0.63	(0.47, 0.74)	0.45	(0.35, 0.61)
ξ_u	0.5	0.1	Beta	0.61	(0.52, 0.82)	0.65	(0.58, 0.81)	0.69	(0.59, 0.83)
α_{ks}	0.1	0.005	Normal	0.10	(0.09, 0.11)	0.10	(0.09, 0.10)	0.10	(0.09, 0.10)
α_{ke}	0.25	0.02	Normal	0.19	(0.16, 0.22)	0.18	(0.15, 0.21)	0.20	(0.16, 0.22)
α_s	0.2	0.05	Normal	0.09	(0.06, 0.18)	0.15	(0.12, 0.19)	0.12	(0.10, 0.14)
β_{kk}	0	0.15	Normal	0	-	0.11	(0.09, 0.13)	-0.09	(-0.42, -0.03)
β_{ks}	0	0.15	Normal	0	-	-0.03	(-0.04, -0.01)	-0.06	(-0.12, -0.02)
β_{ku}	0	0.15	Normal	0	-	-0.08	(-0.10, -0.07)	-0.12	(-0.23, 0.07)
β_{ss}	0	0.15	Normal	0	-	0.09	(0.07, 0.12)	0.08	(0.07, 0.10)
β_{su}	0	0.15	Normal	0	-	-0.07	(-0.09, -0.05)	-0.16	(-0.20, -0.10)
β_{uu}	0	0.15	Normal	0	-	0.15	(0.12, 0.18)	-0.10	(-0.33, 0.02)
$1-\phi_{ux}$	1	0.5	Gamma	1	-	0.26	(0.16, 0.40)	-	-
$1-\phi_{ks}$	1	0.5	Gamma	1	-	0.22	(0.08, 0.36)	-	-
Φ	1.25	0.1	Normal	1.65	(1.53, 1.77)	1.69	(1.54, 1.81)	1.63	(1.49, 1.77)
ρ	0.75	0.1	Beta	0.80	(0.76, 0.87)	0.83	(0.79, 0.89)	0.82	(0.79, 0.87)
r_π	1.5	0.25	Normal	2.08	(1.76, 2.44)	1.95	(1.62, 2.32)	2.06	(1.78, 2.43)
r_y	0.125	0.05	Normal	0.08	(0.06, 0.15)	0.10	(0.07, 0.18)	0.12	(0.09, 0.19)
$r_{\Delta y}$	0.125	0.05	Normal	0.21	(0.15, 0.27)	0.22	(0.17, 0.29)	0.21	(0.16, 0.27)
$\bar{\pi}$	0.625	0.1	Gamma	0.78	(0.52, 0.96)	0.72	(0.49, 0.92)	0.68	(0.47, 0.85)
$\bar{\gamma}$	0.4	0.1	Normal	0.41	(0.38, 0.44)	0.40	(0.34, 0.44)	0.44	(0.41, 0.47)
$\bar{l}$	0	2	Normal	0.62	(-0.59, 3.65)	0.82	(-2.26, 4.12)	-0.93	(-3.58, 1.20)
ρ_a	0.5	0.2	Beta	0.94	(0.88, 0.99)	0.91	(0.86, 0.98)	0.87	(0.82, 0.98)
ρ_b	0.5	0.2	Beta	0.21	(0.03, 0.42)	0.20	(0.03, 0.47)	0.21	(0.05, 0.37)
ρ_g	0.5	0.2	Beta	0.95	(0.92, 0.97)	0.96	(0.94, 0.99)	0.96	(0.93, 0.98)
ρ_I	0.5	0.2	Beta	0.68	(0.55, 0.82)	0.64	(0.54, 0.80)	0.69	(0.57, 0.82)
ρ_r	0.5	0.2	Beta	0.18	(0.02, 0.27)	0.16	(0.02, 0.25)	0.15	(0.02, 0.25)
ρ_p	0.5	0.2	Beta	0.86	(0.77, 0.98)	0.80	(0.68, 0.91)	0.88	(0.75, 0.98)
ρ_s	0.5	0.2	Beta	0.98	(0.93, 0.99)	1.00	(0.99, 1.00)	0.99	(0.99, 1.00)
ρ_u	0.5	0.2	Beta	0.98	(0.98, 0.99)	0.99	(0.98, 1.00)	0.99	(0.97, 1.00)
ρ_χ	0.9	0.05	Beta	0.98	(0.98, 0.99)	0.98	(0.98, 0.99)	0.98	(0.98, 0.99)
μ_p	0.5	0.2	Beta	0.47	(0.16, 0.87)	0.46	(0.14, 0.71)	0.54	(0.06, 0.79)
μ_s	0.5	0.2	Beta	0.94	(0.03, 0.99)	0.28	(0.02, 0.74)	0.47	(0.28, 0.85)
μ_u	0.5	0.2	Beta	0.41	(0.19, 0.96)	0.88	(0.78, 0.97)	0.82	(0.29, 0.96)
σ_a	0.1	2	Inv.Gamma	0.39	(0.36, 0.46)	0.38	(0.34, 0.45)	0.32	(0.28, 0.40)
σ_b	0.1	2	Inv.Gamma	0.25	(0.21, 0.33)	0.26	(0.20, 0.33)	0.26	(0.22, 0.33)
σ_g	0.1	2	Inv.Gamma	0.60	(0.53, 0.68)	0.60	(0.53, 0.68)	0.58	(0.51, 0.66)
σ_I	0.1	2	Inv.Gamma	0.64	(0.52, 0.81)	0.65	(0.49, 0.81)	0.66	(0.55, 0.83)
σ_r	0.1	2	Inv.Gamma	0.24	(0.21, 0.27)	0.24	(0.21, 0.27)	0.23	(0.20, 0.27)
σ_p	0.1	2	Inv.Gamma	0.11	(0.07, 0.17)	0.13	(0.08, 0.17)	0.13	(0.07, 0.17)
σ_s	0.1	2	Inv.Gamma	0.90	(0.06, 1.00)	0.11	(0.05, 0.30)	0.41	(0.23, 0.75)
σ_u	0.1	2	Inv.Gamma	0.10	(0.06, 0.31)	0.27	(0.21, 0.34)	0.19	(0.09, 0.27)
σ_χ	0.1	2	Inv.Gamma	1.10	(0.97, 1.27)	1.10	(0.97, 1.27)	1.10	(0.97, 1.27)
$\sigma_{\nu,k}$	0.15	0.03	Inv.Gamma	3.79	(3.19, 4.58)	3.83	(3.23, 4.67)	3.87	(3.22, 4.68)
$\sigma_{\nu,h}$	0.15	0.03	Inv.Gamma	3.13	(2.66, 3.83)	0.14	(0.10, 0.21)	0.14	(0.10, 0.21)
$\sigma_{\nu,w}$	0.15	0.03	Inv.Gamma	0.14	(0.10, 0.22)	1.44	(1.20, 1.84)	1.21	(1.04, 1.49)

Table C.2: Estimation results for the late sample (2000-2019)

Parameter	Priors			Cobb–Douglas		Nested CES		Translog	
	Mean	Std.	Family	Mode	(2.5%, 97.5%)	Mode	(2.5%, 97.5%)	Mode	(2.5%, 97.5%)
$-100\log\beta$	0.25	0.1	Gamma	0.13	(0.06, 0.30)	0.55	(0.34, 0.97)	0.08	(0.07, 0.10)
σ_c	1.5	0.25	Normal	0.93	(0.89, 0.97)	0.86	(0.70, 1.01)	0.87	(0.85, 0.97)
h	0.7	0.1	Beta	0.89	(0.86, 0.92)	0.58	(0.42, 0.67)	0.75	(0.75, 0.76)
σ_l	1.5	0.25	Normal	1.50	(1.16, 1.98)	1.38	(0.73, 1.72)	0.96	(0.61, 1.14)
φ_s	4	1.5	Normal	6.27	(4.98, 8.99)	6.22	(3.51, 8.42)	4.67	(3.81, 5.34)
φ_e	4	1.5	Normal	7.81	(6.28, 9.87)	2.31	(1.39, 3.54)	7.49	(5.52, 8.34)
ψ	0.5	0.15	Beta	0.92	(0.85, 0.97)	0.94	(0.72, 0.95)	0.38	(0.36, 0.43)
ξ_p	0.5	0.1	Beta	0.84	(0.80, 0.89)	0.89	(0.89, 0.97)	0.75	(0.72, 0.77)
ξ_s	0.5	0.1	Beta	0.70	(0.57, 0.78)	0.75	(0.69, 0.89)	0.72	(0.69, 0.76)
ξ_u	0.5	0.1	Beta	0.65	(0.52, 0.74)	0.86	(0.60, 0.90)	0.75	(0.73, 0.76)
α_{ks}	0.1	0.005	Normal	0.10	(0.09, 0.11)	0.09	(0.08, 0.10)	0.10	(0.10, 0.11)
α_{ke}	0.25	0.02	Normal	0.14	(0.11, 0.17)	0.12	(0.10, 0.15)	0.09	(0.08, 0.10)
α_s	0.3	0.05	Normal	0.20	(0.13, 0.24)	0.24	(0.23, 0.31)	0.24	(0.20, 0.26)
β_{kk}	0	0.15	Normal	0	-	-0.04	(-0.14, -0.03)	0.04	(0.01, 0.04)
β_{ks}	0	0.15	Normal	0	-	0.08	(0.07, 0.19)	0.00	(-0.01, 0.04)
β_{ku}	0	0.15	Normal	0	-	-0.04	(-0.05, -0.03)	-0.25	(-0.32, -0.23)
β_{ss}	0	0.15	Normal	0	-	0.00	(-0.09, 0.02)	0.13	(0.04, 0.18)
β_{su}	0	0.15	Normal	0	-	-0.07	(-0.14, -0.07)	0.10	(0.04, 0.20)
β_{uu}	0	0.15	Normal	0	-	0.11	(0.10, 0.19)	-0.13	(-0.33, 0.00)
$1-\phi_{ux}$	1	0.5	Gamma	1	-	0.48	(0.14, 0.53)	-	-
$1-\phi_{ks}$	1	0.5	Gamma	1	-	1.65	(1.57, 2.95)	-	-
Φ	1.25	0.1	Normal	1.36	(1.23, 1.47)	1.36	(1.19, 1.50)	1.39	(1.27, 1.50)
ρ	0.75	0.1	Beta	0.81	(0.77, 0.84)	0.91	(0.87, 0.96)	0.86	(0.86, 0.86)
r_π	1.5	0.25	Normal	1.06	(1.05, 1.13)	1.43	(0.95, 1.90)	1.88	(1.79, 2.18)
r_y	0.125	0.05	Normal	-0.01	(-0.02, -0.01)	0.17	(0.09, 0.25)	-0.05	(-0.07, -0.04)
$r_{\Delta y}$	0.125	0.05	Normal	0.03	(0.00, 0.06)	0.17	(0.15, 0.28)	0.15	(0.13, 0.20)
$\bar{\pi}$	0.625	0.1	Gamma	0.68	(0.49, 0.84)	0.72	(0.59, 0.90)	0.60	(0.48, 0.83)
$\bar{\gamma}$	0.4	0.1	Normal	0.10	(0.01, 0.18)	0.09	(0.03, 0.31)	0.21	(0.19, 0.29)
$\bar{l}$	0	2	Normal	4.94	(1.79, 7.12)	3.85	(1.54, 7.22)	4.07	(1.92, 5.34)
ρ_a	0.5	0.2	Beta	0.87	(0.82, 0.91)	0.93	(0.90, 0.99)	0.85	(0.85, 0.85)
ρ_b	0.5	0.2	Beta	0.20	(0.09, 0.34)	0.95	(0.94, 0.99)	0.98	(0.98, 0.98)
ρ_g	0.5	0.2	Beta	0.98	(0.96, 0.99)	0.94	(0.91, 0.99)	1.00	(1.00, 1.00)
ρ_I	0.5	0.2	Beta	1.00	(1.00, 1.00)	1.00	(1.00, 1.00)	0.94	(0.94, 0.94)
ρ_r	0.5	0.2	Beta	0.56	(0.49, 0.67)	0.54	(0.36, 0.68)	0.52	(0.39, 0.55)
ρ_p	0.5	0.2	Beta	0.99	(0.99, 1.00)	0.55	(0.20, 0.82)	1.00	(1.00, 1.00)
ρ_s	0.5	0.2	Beta	0.98	(0.98, 0.99)	0.74	(0.06, 0.80)	0.18	(0.17, 0.19)
ρ_u	0.5	0.2	Beta	0.97	(0.96, 0.98)	0.17	(0.04, 0.99)	0.98	(0.98, 0.98)
ρ_χ	0.9	0.05	Beta	1.00	(0.99, 1.00)	0.99	(0.98, 1.00)	0.98	(0.98, 0.98)
μ_p	0.5	0.2	Beta	0.97	(0.96, 0.98)	0.39	(0.09, 0.76)	0.72	(0.67, 0.74)
μ_s	0.5	0.2	Beta	0.30	(0.04, 0.64)	0.30	(0.21, 0.79)	0.42	(0.38, 0.49)
μ_u	0.5	0.2	Beta	0.89	(0.79, 0.93)	0.45	(0.30, 0.97)	0.92	(0.92, 0.92)
σ_a	0.1	2	Inv.Gamma	0.40	(0.36, 0.48)	0.42	(0.33, 0.48)	0.31	(0.27, 0.37)
σ_b	0.1	2	Inv.Gamma	0.25	(0.20, 0.32)	0.07	(0.04, 0.08)	0.02	(0.02, 0.02)
σ_g	0.1	2	Inv.Gamma	0.40	(0.34, 0.46)	0.36	(0.29, 0.41)	0.26	(0.21, 0.29)
σ_I	0.1	2	Inv.Gamma	0.42	(0.30, 0.53)	1.36	(0.72, 1.78)	0.13	(0.11, 0.13)
σ_r	0.1	2	Inv.Gamma	0.11	(0.09, 0.13)	0.11	(0.09, 0.14)	0.11	(0.10, 0.12)
σ_p	0.1	2	Inv.Gamma	0.21	(0.18, 0.25)	0.15	(0.11, 0.20)	0.11	(0.10, 0.12)
σ_s	0.1	2	Inv.Gamma	0.08	(0.05, 0.19)	0.12	(0.12, 1.95)	1.51	(1.25, 1.93)
σ_u	0.1	2	Inv.Gamma	0.73	(0.59, 0.93)	0.84	(0.13, 0.85)	0.30	(0.25, 0.38)
σ_χ	0.1	2	Inv.Gamma	0.74	(0.65, 0.89)	0.75	(0.64, 0.88)	0.81	(0.69, 0.86)
$\sigma_{\nu,k}$	0.15	0.03	Inv.Gamma	0.23	(0.19, 0.38)	1.69	(1.25, 2.34)	0.37	(0.30, 0.50)
$\sigma_{\nu,h}$	0.15	0.03	Inv.Gamma	0.14	(0.10, 0.21)	0.14	(0.10, 0.22)	0.14	(0.12, 0.16)
$\sigma_{\nu,w}$	0.15	0.03	Inv.Gamma	2.55	(2.11, 3.15)	1.46	(1.18, 1.86)	1.28	(1.12, 1.66)

C.4 Robustness Check: Slopes of the Phillips Curves

This section evaluates the robustness of the results for the slopes of the Phillips curves presented in Table 4. First, we estimate the Phillips curves without applying bandpass filters. In this case, we use monetary policy shocks at lags 0 to 20 as IVs, following Barnichon and Mesters (2020). Second, we employ the translog model as the data generating process (DGP) and estimate both the Cobb–Douglas and nested CES models using simulated data. We show that our findings in the main text are robust to these two exercises.

In the main text, we considered the Phillips curve with a forcing variable, x_t:

$$\hat{\pi}_t = \beta_x \mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa_x x_t + error_{x,t},$$

where κ_x represents the slope of this Phillips curve. We computed the model-predicted slope, κ_x, by projecting $\hat{\pi}_t$ on $\mathbb{E}_t[\hat{\pi}_{t+1}]$ and x_t using the cyclical variation with the periodicity from 6-32 quarters driven by monetary policy shocks. Note that calculating κ_x in this manner is equivalent to applying the ideal bandpass filter to the infinite time series of $\hat{\pi}_t$, $\mathbb{E}_t[\hat{\pi}_{t+1}]$, and x_t and utilizing monetary policy shock series as IVs.

In this section, we employ an alternative specification that does not rely on filters. Specifically, we examine the population version of the two-stage least squares estimation of the above equation utilizing monetary policy shocks at lags 0 to 20, $Z_t \equiv (\varepsilon^r_t, \ldots, \varepsilon^r_{t-20})'$, as IVs. Let $P_Z(\cdot)$ be the projection operator on the space spanned by Z_t:

$$P_Z(\cdot) = Z_t'[\mathrm{var}(Z_t)^{-1}\mathrm{cov}(Z_t, \cdot)].$$

Because Z_t consists of monetary policy shocks, the first-stage projections of $\hat{\pi}_t$, $\mathbb{E}_t[\hat{\pi}_{t+1}]$, and x_t yield the following results:

$$\begin{aligned}
P_Z(\hat{\pi}_t) &= \sum_{i=0}^{20} \phi^r_{\pi,i}\varepsilon^r_{t-i}, \\
P_Z(\mathbb{E}_t[\hat{\pi}_{t+1}]) &= \sum_{i=0}^{20} \phi^r_{\pi,i+1}\varepsilon^r_{t-i}, \\
P_Z(x_t) &= \sum_{i=0}^{20} \phi^r_{x,i}\varepsilon^r_{t-i},
\end{aligned}$$

where $\phi^r_{\pi,i}$ and $\phi^r_{x,i}$ represent the impulse response coefficients of π_{t+i} and x_{t+i} to the monetary policy shock, ε^r_t, respectively. Then, the two-stage least squares estimate of $(\beta_x, \kappa_x)'$ is given by:

$$\begin{aligned}
\begin{pmatrix}\beta_x \\ \kappa_x\end{pmatrix} &= \left[\mathrm{var}\begin{pmatrix}P_Z(\mathbb{E}_t[\hat{\pi}_{t+1}]) \\ P_Z(x_t)\end{pmatrix}\right]^{-1} \mathrm{cov}\left[\begin{pmatrix}P_Z(\mathbb{E}_t[\hat{\pi}_{t+1}]) \\ P_Z(x_t)\end{pmatrix}, P_Z(\hat{\pi}_t)\right] \\
&= \left[\mathrm{var}\begin{pmatrix}\sum_{i=0}^{20}\phi^r_{\pi,i+1}\varepsilon^r_{t-i} \\ \sum_{i=0}^{20}\phi^r_{x,i}\varepsilon^r_{t-i}\end{pmatrix}\right]^{-1} \mathrm{cov}\left[\begin{pmatrix}\sum_{i=0}^{20}\phi^r_{\pi,i+1}\varepsilon^r_{t-i} \\ \sum_{i=0}^{20}\phi^r_{x,i}\varepsilon^r_{t-i}\end{pmatrix}, \sum_{i=0}^{20}\phi^r_{\pi,i}\varepsilon^r_{t-i}\right] \\
&= \begin{bmatrix}\sum_{i=0}^{20}(\phi^r_{\pi,i+1})^2 & \sum_{i=0}^{20}\phi^r_{\pi,i+1}\phi^r_{x,i} \\ \sum_{i=0}^{20}\phi^r_{\pi,i+1}\phi^r_{x,i} & \sum_{i=0}^{20}(\phi^r_{\pi,x})^2\end{bmatrix}^{-1} \begin{bmatrix}\sum_{i=0}^{20}\phi^r_{\pi,i+1}\phi^r_{\pi,i} \\ \sum_{i=0}^{20}\phi^r_{x,i}\phi^r_{\pi,i}\end{bmatrix}.
\end{aligned}$$

We use this formula to compute κ_x for different forcing variables.

Table C.3: Slopes of the Phillips curves (without filters)

	(1)	(2)	(3)	(4)	(5)	(6)
	Early sample (1966-99)			Late sample (2000-19)		
	Cobb–Douglas	Nested CES	Translog	Cobb–Douglas	Nested CES	Translog
Panel A. Calvo price stickiness parameter						
ξ_p	0.66	0.72	0.65	0.84	0.89	0.75
	(0.53, 0.79)	(0.59, 0.85)	(0.53, 0.78)	(0.80, 0.89)	(0.89, 0.97)	(0.72, 0.77)
Panel B. Slopes of the price Phillips curves						
marginal costs	0.023	0.013	0.026	0.006	0.003	0.017
	(0.007, 0.057)	(0.004, 0.037)	(0.009, 0.059)	(0.003, 0.010)	(0.000, 0.003)	(0.014, 0.023)
output gap	0.016	0.009	0.028	0.005	0.002	0.007
	(0.005, 0.027)	(0.002, 0.019)	(0.010, 0.048)	(0.003, 0.008)	(0.000, 0.003)	(0.005, 0.011)
labor gap	0.028	0.023	0.041	0.007	0.001	0.004
	(0.008, 0.046)	(0.007, 0.040)	(0.015, 0.069)	(0.004, 0.011)	(0.000, 0.002)	(-0.007, 0.007)
labor share	0.045	0.003	0.118	0.009	0.004	-0.001
	(-0.117, 0.176)	(-0.038, 0.042)	(-0.366, 0.324)	(0.005, 0.017)	(0.000, 0.004)	(-0.006, 0.008)
labor share,	0.023	0.020	0.047	0.006	0.002	0.003
υ adjusted	(0.007, 0.057)	(0.006, 0.045)	(0.005, 0.182)	(0.003, 0.010)	(0.000, 0.003)	(-0.003, 0.008)

Table C.3 shows the results in the posterior mode and the equal-tailed 95% credible intervals. Note that when real marginal costs are used, the results are the same as those in Table 4. In this case, the regression equation reduces to the price Phillips curve in the model, where $error_{x,t} = \varepsilon_t^p$. Then, monetary policy shocks satisfy the relevance and exclusion restrictions of IVs, resulting in $\kappa_x = \kappa = \frac{(1-\zeta_p\tilde{\beta}\gamma)(1-\zeta_p)}{\zeta_p}\frac{1}{1+\theta_p(\Phi-1)}$, similar to the specification employed in the main text.[2] For the conventional Phillips curves, the computed slopes slightly differ from those in Table 4. However, the results remain largely robust in the sense that the translog model can jointly replicate the steep marginal cost Phillips curve and the flattening of conventional Phillips curves in recent decades. In contrast, the Cobb–Douglas and nested CES models feature unrealistically sticky prices and quite flat Phillips curves to explain the post-2000 data.

Our second robustness exercise employs the translog model in the posterior mode as the DGP to simulate the data. Using the simulated data and the same Bayesian estimation method in Section 3.2, we estimate the Cobb–Douglas and nested CES models. The results align with the analysis in the main text (Table 4) based on the US aggregate time series data.

Using the Kalman smoother, the translog model in the posterior mode, and the US time series data, we first estimate the model's state variable in the first period (1966:Q1 and 2000:Q1) for each sample. Next, we simulate the model starting from this initial state for 136 periods in the case of the early sample to match the sample size in the data. For the late sample, we simulate the model for 80 periods. For each set of simulated data, we estimate the Cobb–Douglas and nested CES models and compute the corresponding posterior modes. We repeat this process 20 times.

For the early sample, the averages of the estimated Calvo price stickiness parameter, ζ_p, for the Cobb–Douglas and nested CES models are approximately 0.6, similar to the value under the DGP, 0.65. As a result, the price Phillips curves are also steep, similar to the results in Table C.3 based on the US data. In contrast, for the late sample, the averages of the estimated ζ_p are greater than 0.9. Thus, to match the data simulated by the translog

[2] $\tilde{\beta} = \beta\gamma^{-\sigma_c}$ is the discount factor in the detrended economy, where γ is the gross growth rate on the BGP and σ_c is the consumption utility parameter. θ_p represents the curvature of the Kimball aggregator, while Φ denotes the gross price markup.

model in the late sample, the other two models rely on excessively sticky prices and quite flat price Phillips curves. This result is also consistent with the results in Table C.3 and the discussion in the main text, emphasizing more procyclical returns to scale in recent decades than in earlier decades.

C.5 Empirical Responses of TFP and Capital Utilization to Monetary Policy Shocks

A specific supply-side effect of demand disturbances propagating through the cyclical returns to scale is central to our mechanism for the flattening of the Phillips curve. Relatedly, aggregate demand might affect the supply block of the economy through other channels, such as endogenous R&D (Comin and Gertler, 2006; Anzoategui et al., 2019; Bianchi, Kung and Morales, 2019; Okada, 2022; Ma and Zimmermann, 2023; Antolin-Diaz and Surico, 2024; Cloyne et al., 2024), misallocation (Eisfeldt and Rampini, 2006; Hsieh and Klenow, 2009; Alam, 2020; Baqaee, Farhi and Sangani, 2024), firm dynamics (Smirnyagin, 2023; Fujiwara and Matsuyama, 2023), and capacity utilization (Boehm and Pandalai-Nayar, 2022). If the effects of these mechanisms have become more procyclical in recent decades, the identified slopes of Phillips curves could be smaller than previous slopes for similar reasons to our explanation.

An important feature distinguishing our mechanism from other prominent supply-side channels of monetary policy is that the cyclical returns to scale do not affect TFP at the first order; the TFP in the model (ε_t^a) is exogenous and equal to the Solow residual at the first order. In contrast, the influence of several supply-side mechanisms described above could appear in the responses of the measured aggregate TFP (see also Ma and Zimmermann, 2023; Jordà, Singh and Taylor, 2023).

Given this consideration, we empirically investigate how TFP responds to monetary policy shocks in the pre-2000 and post-2000 samples. We document that the level of the measured TFP (i.e., the Solow residual) procyclically responded to monetary policy shocks in the early sample. However, in the later sample, the impulse responses are close to zero and statistically insignificant. Thus, we cannot find significant evidence for more procyclical TFP responses to monetary policy shocks in recent decades. To be clear, we do not argue that those other supply-side mechanisms are weak. Rather, we observe that they did not become sufficiently strong in the last two to three decades to decrease the identified slopes of the Phillips curves. Note that this empirical result for TFP does not contradict our proposed explanation based on the procyclical returns to scale because the returns to scale can endogenously vary over time without affecting ε_t^a.

To estimate the impulse response function of TFP to monetary policy shocks, we use the following local projections à la Jordà (2005):

$$TFP_{t+h} - TFP_{t-1} = \psi_h \eta_t + \Gamma_h' control_t + error, \tag{C.1}$$

where $\{\psi_0, \psi_1, \ldots\}$ constitutes an impulse response function of TFP to a monetary policy shock η_t. We include an intercept and four lagged values of $\Delta TFP_t \equiv TFP_t - TFP_{t-1}$ and η_t as controls. We use heteroskedasticity-autocorrelation-robust standard errors for inferences. Following Christiano, Eichenbaum and Evans (1996), Coibion (2012), Gorodnichenko and Lee (2020), and many others, the impact responses to monetary policy shocks are assumed to be zero. Impulse responses of other variables are estimated similarly.

We use the TFP measure computed by Fernald (2014*a*,*b*). For η_t, we rely on high-frequency monetary policy surprises around the FOMC meetings. Specifically, we use the monetary policy surprise series orthogonal to predictable variations in the instrument presented by Bauer and Swanson (2023). The sample period for this

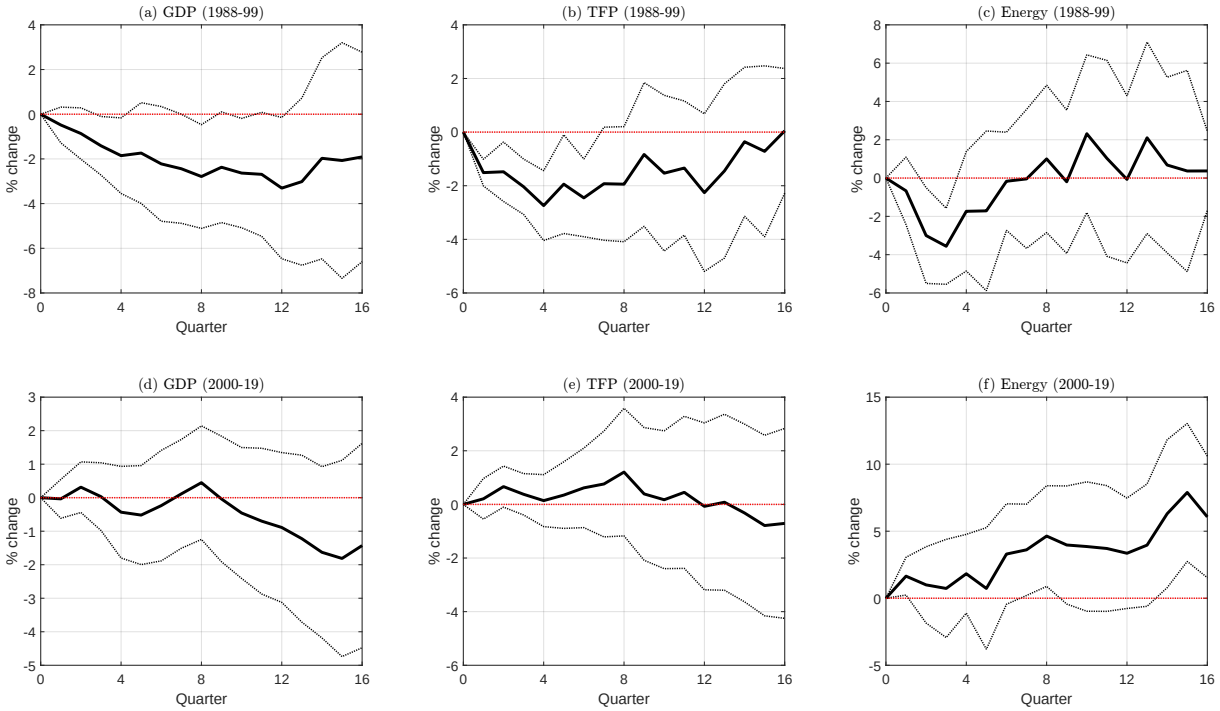


Figure C.1: Empirical impulse responses of GDP, TFP, and energy to contractionary monetary policy shocks

exercise begins in 1988:q1 and ends in 2019:q4. We divide the sample around 2000 as is the case in Section 3.

Figure C.1 shows the annualized responses to a one-standard-deviation contractionary monetary policy shock. The first and second rows illustrate the results based on the early and late samples, respectively. The confidence intervals are at the 90% level. In the early sample, TFP was conditionally procyclical on monetary policy shocks; GDP and TFP decreased simultaneously. In the late sample, GDP responds in a similarly hump-shaped manner. However, TFP does not respond much. The point estimates are close to zero and statistically insignificant at all horizons. Therefore, it is less likely that the TFP-affecting supply-side mechanisms of monetary policy have become sufficiently strong in recent periods to flatten the Phillips curves.

In the third column, we plot the responses of the US industrial energy use to the same monetary policy shock during the same sample period. This variable is interpreted as a proxy for the capacity utilization rate, z_t, in the model. We obtain the monthly data from U.S. Energy Information Administration (2021). We remove the seasonal variation using X-13 ARIMA and aggregate the series to the quarterly frequency.

In the translog model, the procyclical rts_t^{tl} arises from the changes in the translog parameters (Table 3) and related factor input decisions (Equations (3.2)-(3.5)). The response of $\hat{k}_{et}$ is of particular interest in this regard. In the early sample, expansionary monetary policy shocks lead to an increase in the utilization rate, z_t, and equipment services, k_{et}. In contrast, in recent periods, z_t and k_{et} have decreased in the short term (not shown). A smaller k_{et} than usual, combined with a negative $\beta_{k.}$, contributes to procyclical returns to scale ($\widehat{rts}_t^{tl} = \beta_{k.}\hat{k}_{et} + \beta_{s.}\hat{l}_{st} + \beta_{u.}\hat{l}_{ut}$) in the estimated model in the late sample.

The results in Figure C.1 imply that US industrial use of energy, a proxy for the utilization rate, significantly decreased in response to expansionary monetary policy shocks in the post-2000 sample, whereas the signs are opposite before 2000. Thus, the data and the model exhibit the same signs of energy (utilization) responses in each sample period, further supporting the mechanism in our model.

Therefore, we conclude that the empirical evidence in this section is broadly consistent with the role of the

procyclical returns to scale in recent decades in flattening the Phillips curves. Furthermore, the signs of energy responses in the data and the model in each sample period are the same and change between the two sample periods. Because energy usage is not directly used for the Bayesian estimation of the DSGE models, this result constitutes suggestive evidence for the mechanism in our model.

C.6 Robustness Check: Two-Input Models

Although we adopt the basic structure of a medium-scale DSGE model in Smets and Wouters (2007*b*), there are a few differences between our model and the Smets–Wouters model. The major differences include (i) production functions, (ii) the number of factor inputs, and (iii) additional variables used for Bayesian estimation (see also Online Appendix B.3.4).

This section considers simpler models with two inputs (capital and labor) than the model in Section 3 with four inputs (structure, equipment, skilled labor, and unskilled labor) for a robustness check. Furthermore, we use the same seven quarterly variables to estimate the model as those in Smets and Wouters (2007*b*). Like Section 3, we investigate the three models with different production functions: (i) Cobb–Douglas, (ii) CES, and (iii) translog, and the two sample periods (1966-1999 and 2000-2019).

The main results are robust to those changes. The translog production function yields stable slopes of the New Keynesian price Phillips curve to some extent and a decrease in the slopes of the conventional Phillips curves based on, e.g., the output and labor gaps. Regarding the pandemic-era inflation, the translog model emphasizes the contribution of risk premium shocks at the onset of the pandemic in 2020:q2 and loose monetary policies for the increase in inflation during 2021:q1-2022:q2. In contrast, the Cobb–Douglas and CES models almost entirely rely on price markup shocks to explain the realized inflation dynamics after 2020.

C.6.1 Model

The intermediate goods producers employ labor and use capital services in production. We consider the following three production functions:

$$
\begin{aligned}
&\text{Cobb–Douglas:} && Y_t(i) = \exp(\varepsilon_t^a)[K_t(i)]^{\alpha}[\gamma^t L_t(i)]^{1-\alpha} - \gamma^t \upsilon, \\
&\text{CES:} && Y_t(i) = \exp(\varepsilon_t^a)\left(a_1[K_t(i)]^{\phi} + a_2[\gamma^t L_t(i)]^{\phi}\right)^{1/\phi} - \gamma^t \upsilon, \\
&\text{Translog:} && Y_t(i) = \exp(\varepsilon_t^a)[K_t(i)]^{\alpha}[\gamma^t L_t(i)]^{1-\alpha} \\
& && \quad \times \exp\left(\beta_{kk}\hat{k}_{it}\hat{k}_t + \beta_{lk}\hat{k}_{it}\hat{l}_t + \beta_{lk}\hat{l}_{it}\hat{k}_t + \beta_{ll}\hat{l}_{it}\hat{l}_t\right) - \gamma^t \upsilon,
\end{aligned}
$$

where α is the steady-state capital income share. The CES parameter ϕ implies that the elasticity of substitution between labor and capital is $\frac{1}{1-\phi}$. βs are translog parameters. Other variables are similar to Online Appendix B.

The above CES production function can be converted to a deviation form using the steady-state income shares as follows:

$$
\text{CES (deviation form):} \qquad \frac{Y_t(i) + \gamma^t \upsilon}{\bar{Y}_t + \gamma^t \upsilon} = \exp(\varepsilon_t^a)\left(\alpha\left[\frac{K_t(i)}{\bar{K}_t}\right]^{\phi} + (1-\alpha)\left[\frac{\gamma^t L_t(i)}{\gamma^t \bar{L}_t}\right]^{\phi}\right)^{1/\phi}.
$$

Furthermore, like Online Appendix B.4, it can be shown that the equilibrium conditions in the CES model are

equivalent to those in the translog model in log linearization when:

$$\begin{aligned}\beta_{kk} &= \alpha(1-\alpha)\phi,\\ \beta_{lk} &= -\alpha(1-\alpha)\phi,\\ \beta_{ll} &= \alpha(1-\alpha)\phi.\end{aligned} \tag{C.2}$$

Other elements of the models are the same as those in the Smets and Wouters (2007*b*) model. The model features Calvo-type sticky prices and wages with Kimball (1995) aggregators, investment-adjustment costs, costly capacity utilization, and external consumption habits. Like Section 3, we rule out price and wage indexation: $\iota_p = \iota_w = 0$ (see Woodford, 2007; Cogley and Sbordone, 2008; Phaneuf, Sims and Victor, 2018). Furthermore, we assume that government spending does not depend on productivity shocks: $\rho_{ga} = 0$. We fix the following parameters: the capital depreciation rate, δ, is 0.025; G_t/Y_t is 0.18 in the steady state; the gross wage markup in the steady state, Φ_w, is 1.5; the Kimball curvature parameters for goods and labor aggregator, θ_p and θ_w, are both 10. For the remaining details, see Smets and Wouters (2007*a*,*b*).

C.6.2 Data, Priors, and Posteriors

We use seven quarterly variables for estimation. These variables include growth rates of per capita real GDP, consumption, and investment; labor hours; the growth rate of the real wage rate; price inflation; and the nominal interest rate (replaced by the shadow rate of Wu and Xia (2016) when the zero lower bound was binding). Following Smets and Wouters (2007*b*), the sample begins in 1966:q1. We divide the data into two periods, 1966-1999 and 2000-2019.

We summarize the posterior distributions using the mode and credible intervals. We present the results for the early sample (1966-1999) and the late sample (2000-2019) in separate tables. Each table covers the results based on the three production functions: Cobb–Douglas, CES, and translog. For the CES model, we show the implied translog parameters using Equation (C.2). We employ a standard MCMC technique to obtain the posterior distribution. Specifically, we use a Metropolis–Hastings algorithm with a chain length of 200,000. The chain starts at the posterior mode, computed using interior-point methods. We use the inverse of numerical hessian at the mode as a variance of the jump distribution in our algorithm. The step sizes are adjusted to obtain reasonable acceptance rates. The acceptance rates for the three models and the two sample periods range from 23%-32%.

Tables C.4 and C.5 show the results of Bayesian estimation for each subsample, respectively. For each parameter, its prior mean, standard deviation, distributional family, posterior mode, and the equal-tailed 95% credible interval are shown.

C.6.3 Results

Model comparison. Table C.6 shows the returns to scale parameters, $\beta_{k\cdot} = \beta_{kk} + \beta_{lk}$ and $\beta_{l\cdot} = \beta_{ll} + \beta_{lk}$, log-likelihood at the mode, and the marginal data density (MDD). The Cobb–Douglas and CES models feature $\beta_{k\cdot} = \beta_{l\cdot} = 0$ by assumption and, therefore, $\hat{rts}_t^{tl} = \beta_{k\cdot}\hat{k}_t + \beta_{l\cdot}\hat{l}_t = 0$. In contrast, the translog model does not impose such restrictions so that the returns to scale $\hat{rts}_t^{tl}$ can be time-varying.

In log-linearization, the translog model nests the other two models. Therefore, we can test the validity of the restrictions imposed by the Cobb–Douglas and CES functions by testing whether zero is included in the credible

Table C.4: Estimation results for the early sample (1966-1999) and the two-input models

Parameter	Priors			Cobb–Douglas		CES		Translog	
	Mean	Std.	Family	Mode	(2.5%, 97.5%)	Mode	(2.5%, 97.5%)	Mode	(2.5%, 97.5%)
φ	4	1.5	Normal	5.65	(3.93, 7.93)	5.83	(3.89, 8.09)	5.31	(3.50, 7.74)
σ_c	1.5	0.25	Normal	1.48	(1.24, 1.80)	1.48	(1.21, 1.82)	1.40	(1.14, 1.72)
h	0.7	0.1	Beta	0.68	(0.60, 0.78)	0.69	(0.61, 0.79)	0.70	(0.62, 0.81)
ξ_w	0.5	0.1	Beta	0.71	(0.62, 0.86)	0.70	(0.61, 0.84)	0.68	(0.59, 0.82)
σ_l	1.5	0.25	Normal	1.50	(1.07, 2.02)	1.51	(1.08, 2.02)	1.43	(0.99, 1.97)
ξ_p	0.5	0.1	Beta	0.67	(0.56, 0.80)	0.69	(0.54, 0.81)	0.65	(0.53, 0.76)
ψ	0.5	0.15	Beta	0.46	(0.25, 0.72)	0.43	(0.24, 0.75)	0.45	(0.22, 0.81)
Φ	1.25	0.1	Normal	1.59	(1.46, 1.71)	1.60	(1.46, 1.73)	1.60	(1.45, 1.73)
r_π	1.5	0.25	Normal	2.03	(1.71, 2.41)	2.02	(1.73, 2.42)	2.04	(1.75, 2.42)
ρ	0.75	0.1	Beta	0.81	(0.77, 0.86)	0.81	(0.76, 0.86)	0.80	(0.76, 0.85)
r_y	0.125	0.05	Normal	0.09	(0.06, 0.15)	0.09	(0.05, 0.15)	0.11	(0.07, 0.17)
$r_{\Delta y}$	0.125	0.05	Normal	0.22	(0.16, 0.28)	0.22	(0.16, 0.28)	0.22	(0.16, 0.29)
$\bar{\pi}$	0.625	0.1	Gamma	0.83	(0.59, 1.02)	0.82	(0.59, 1.01)	0.81	(0.57, 1.01)
$-100 \log \beta$	0.25	0.1	Gamma	0.20	(0.07, 0.34)	0.20	(0.08, 0.35)	0.21	(0.08, 0.36)
$\bar{l}$	0	2	Normal	2.06	(0.04, 4.74)	2.10	(0.13, 4.88)	1.89	(0.06, 4.53)
$\bar{\gamma}$	0.4	0.1	Normal	0.42	(0.39, 0.45)	0.42	(0.39, 0.45)	0.42	(0.39, 0.46)
α	0.3	0.05	Normal	0.20	(0.16, 0.24)	0.20	(0.15, 0.24)	0.21	(0.16, 0.25)
σ_a	0.1	2	Inv. Gamma	0.44	(0.39, 0.50)	0.43	(0.38, 0.50)	0.41	(0.37, 0.49)
σ_b	0.1	2	Inv. Gamma	0.25	(0.20, 0.31)	0.25	(0.20, 0.31)	0.26	(0.21, 0.33)
σ_g	0.1	2	Inv. Gamma	0.57	(0.51, 0.65)	0.57	(0.51, 0.65)	0.57	(0.51, 0.65)
σ_I	0.1	2	Inv. Gamma	0.46	(0.36, 0.57)	0.46	(0.35, 0.56)	0.49	(0.38, 0.60)
σ_r	0.1	2	Inv. Gamma	0.25	(0.22, 0.29)	0.25	(0.22, 0.29)	0.25	(0.22, 0.29)
σ_p	0.1	2	Inv. Gamma	0.13	(0.09, 0.18)	0.13	(0.08, 0.18)	0.13	(0.07, 0.17)
σ_w	0.1	2	Inv. Gamma	0.20	(0.16, 0.25)	0.20	(0.16, 0.25)	0.20	(0.16, 0.25)
ρ_a	0.5	0.2	Beta	0.95	(0.92, 0.99)	0.95	(0.92, 0.99)	0.95	(0.92, 0.99)
ρ_b	0.5	0.2	Beta	0.18	(0.04, 0.44)	0.18	(0.05, 0.41)	0.18	(0.02, 0.37)
ρ_g	0.5	0.2	Beta	0.98	(0.96, 0.99)	0.98	(0.96, 1.00)	0.97	(0.96, 1.00)
ρ_I	0.5	0.2	Beta	0.65	(0.54, 0.81)	0.64	(0.53, 0.84)	0.63	(0.52, 0.80)
ρ_r	0.5	0.2	Beta	0.14	(0.02, 0.23)	0.14	(0.02, 0.24)	0.14	(0.02, 0.24)
ρ_p	0.5	0.2	Beta	0.85	(0.74, 0.96)	0.84	(0.73, 0.98)	0.88	(0.78, 0.99)
ρ_w	0.5	0.2	Beta	0.97	(0.93, 0.99)	0.97	(0.94, 0.99)	0.97	(0.95, 0.99)
μ_p	0.5	0.2	Beta	0.52	(0.18, 0.79)	0.51	(0.14, 0.80)	0.54	(0.07, 0.80)
μ_w	0.5	0.2	Beta	0.84	(0.74, 0.95)	0.84	(0.72, 0.95)	0.83	(0.70, 0.95)
β_{lk}	0	0.15	Normal	0.00	-	-0.05	(-0.10, 0.15)	-0.16	(-0.29, 0.05)
β_{ll}	0	0.15	Normal	0.00	-	0.05	(-0.15, 0.10)	-0.21	(-0.46, -0.02)
β_{kk}	0	0.15	Normal	0.00	-	0.05	(-0.15, 0.10)	-0.04	(-0.37, 0.04)
$1-\phi$	1	0.5	Gamma	1.00	-	0.72	(0.38, 1.99)	-	-

interval for the returns to scale parameters in columns (3) and (6) for the early and late samples, respectively. Except for the 95% credible interval for $\beta_{l\cdot}$ in Column (6), all the other credible intervals in Panel A do not include zero. Thus, the data reject the predictions of the restrictive Cobb–Douglas and CES models at the 5% level.

The data consistently favor the translog model when the three models are compared using the likelihood in the mode and the MDD. We compute the MDD using the algorithm proposed by Sims, Waggoner and Zha (2008). For both sample periods, the translog model has the largest log-likelihood in the mode and the MDD among the three models. Thus, we conclude that the translog model matches the data better than the two alternative models.

It is also worth noting that the returns to scale parameters increase statistically significantly between Columns (3) and (6), hinting at potential structural changes in the long term that might have affected the comovement between inflation and economic activity.

Production function parameters. Next, we discuss the individual parameters characterizing the production function. As shown in Panel A, the steady-state capital income share, α, is estimated to be small. The estimates in Columns (4) and (5) are particularly small, less than 10%. The steady state markup is estimated to be approximately 60% for the pre-2000 sample. Those results are similar to the results in Smets and Wouters (2007*b*, Table

Table C.5: Estimation results for the late sample (2000-2019) and the two-input models

Parameter	Priors			Cobb–Douglas		CES		Translog	
	Mean	Std.	Family	Mode	(2.5%, 97.5%)	Mode	(2.5%, 97.5%)	Mode	(2.5%, 97.5%)
φ	4	1.5	Normal	5.90	(4.10, 8.53)	6.25	(4.36, 8.76)	5.74	(3.87, 8.32)
σ_c	1.5	0.25	Normal	1.04	(0.92, 1.61)	1.13	(0.96, 1.61)	1.16	(0.87, 1.52)
h	0.7	0.1	Beta	0.62	(0.46, 0.69)	0.58	(0.45, 0.68)	0.61	(0.49, 0.73)
ξ_w	0.5	0.1	Beta	0.84	(0.75, 0.88)	0.84	(0.76, 0.89)	0.86	(0.80, 0.91)
σ_l	1.5	0.25	Normal	1.35	(0.87, 1.88)	1.38	(0.92, 1.91)	1.41	(0.96, 1.93)
ξ_p	0.5	0.1	Beta	0.87	(0.71, 0.92)	0.86	(0.72, 0.92)	0.72	(0.61, 0.87)
ψ	0.5	0.15	Beta	0.82	(0.70, 0.98)	0.81	(0.73, 0.97)	0.20	(0.03, 0.31)
Φ	1.25	0.1	Normal	1.35	(1.19, 1.49)	1.35	(1.21, 1.51)	1.39	(1.26, 1.57)
r_π	1.5	0.25	Normal	1.38	(1.06, 2.07)	1.39	(1.03, 2.03)	1.69	(1.35, 2.18)
ρ	0.75	0.1	Beta	0.89	(0.86, 0.94)	0.90	(0.86, 0.94)	0.90	(0.86, 0.94)
r_y	0.125	0.05	Normal	0.17	(0.10, 0.26)	0.17	(0.10, 0.25)	0.17	(0.10, 0.26)
$r_{\Delta y}$	0.125	0.05	Normal	0.16	(0.12, 0.22)	0.16	(0.12, 0.23)	0.16	(0.11, 0.23)
$\bar{\pi}$	0.625	0.1	Gamma	0.58	(0.44, 0.73)	0.56	(0.43, 0.73)	0.62	(0.47, 0.84)
$-100 \log \beta$	0.25	0.1	Gamma	0.15	(0.05, 0.31)	0.15	(0.06, 0.32)	0.12	(0.04, 0.26)
$\bar{l}$	0	2	Normal	0.96	(-0.96, 2.57)	0.35	(-1.48, 2.23)	-0.01	(-3.09, 1.77)
$\bar{\gamma}$	0.4	0.1	Normal	0.27	(0.24, 0.37)	0.25	(0.23, 0.35)	0.26	(0.21, 0.30)
α	0.3	0.05	Normal	0.09	(0.08, 0.17)	0.09	(0.08, 0.17)	0.14	(0.11, 0.18)
σ_a	0.1	2	Inv. Gamma	0.44	(0.37, 0.52)	0.44	(0.37, 0.52)	0.35	(0.30, 0.43)
σ_b	0.1	2	Inv. Gamma	0.06	(0.04, 0.07)	0.06	(0.04, 0.07)	0.05	(0.04, 0.07)
σ_g	0.1	2	Inv. Gamma	0.31	(0.28, 0.40)	0.30	(0.27, 0.39)	0.33	(0.29, 0.40)
σ_I	0.1	2	Inv. Gamma	0.25	(0.19, 0.36)	0.20	(0.11, 0.26)	0.19	(0.12, 0.27)
σ_r	0.1	2	Inv. Gamma	0.11	(0.09, 0.14)	0.11	(0.09, 0.14)	0.10	(0.09, 0.12)
σ_p	0.1	2	Inv. Gamma	0.14	(0.10, 0.19)	0.14	(0.10, 0.19)	0.14	(0.09, 0.18)
σ_w	0.1	2	Inv. Gamma	0.60	(0.47, 0.72)	0.60	(0.48, 0.71)	0.59	(0.50, 0.74)
ρ_a	0.5	0.2	Beta	0.92	(0.88, 0.98)	0.92	(0.89, 0.97)	0.90	(0.86, 0.99)
ρ_b	0.5	0.2	Beta	0.93	(0.87, 0.97)	0.93	(0.88, 0.97)	0.91	(0.86, 0.96)
ρ_g	0.5	0.2	Beta	0.68	(0.61, 0.97)	0.61	(0.52, 0.97)	0.95	(0.93, 1.00)
ρ_I	0.5	0.2	Beta	0.77	(0.74, 0.98)	0.82	(0.78, 0.97)	0.88	(0.71, 0.97)
ρ_r	0.5	0.2	Beta	0.56	(0.37, 0.69)	0.56	(0.39, 0.70)	0.53	(0.35, 0.66)
ρ_p	0.5	0.2	Beta	0.67	(0.30, 0.95)	0.66	(0.31, 0.95)	0.94	(0.87, 1.00)
ρ_w	0.5	0.2	Beta	0.36	(0.02, 0.38)	0.36	(0.05, 0.46)	0.27	(0.03, 0.36)
μ_p	0.5	0.2	Beta	0.48	(0.05, 0.91)	0.46	(0.06, 0.91)	0.79	(0.59, 0.95)
μ_w	0.5	0.2	Beta	0.62	(0.27, 0.91)	0.62	(0.26, 0.67)	0.53	(0.30, 0.74)
β_{lk}	0	0.15	Normal	0.00	-	-0.06	(-0.11, 0.00)	0.09	(-0.04, 0.25)
β_{ll}	0	0.15	Normal	0.00	-	0.06	(0.00, 0.11)	0.05	(-0.24, 0.34)
β_{kk}	0	0.15	Normal	0.00	-	0.06	(0.00, 0.11)	0.40	(0.28, 0.58)
$1-\phi$	1	0.5	Gamma	1.00	-	0.35	(0.10, 0.98)	-	-

Table C.6: Model comparison

	(1)	(2)	(3)	(4)	(5)	(6)
	Early sample (1966-99)			Late sample (2000-19)		
	Cobb–Douglas	CES	Translog	Cobb–Douglas	CES	Translog
Panel A. Returns to scale parameters						
$\beta_{k\cdot}$	0	0	-0.19	0	0	0.49
	-	-	(-0.54, -0.05)	-	-	(0.30, 0.77)
$\beta_{l\cdot}$	0	0	-0.36	0	0	0.14
	-	-	(-0.66, -0.08)	-	-	(-0.21, 0.50)
Panel B. Log-likelihoods and marginal data densities						
log-likelihood	153.47	153.83	157.67	224.61	226.89	237.72
log MDD	55.23	54.80	56.49	129.92	132.48	136.64

1A). Φ estimates are smaller for the post-2000 sample, ranging from 1.35 to 1.39.

When the CES production function is assumed, capital and labor are estimated to be more substitutable in the mode for the recent sample than in the mode for the early sample. This result is consistent with Cantore,

Table C.7: Production function parameters

	(1)	(2)	(3)	(4)	(5)	(6)
	Early sample (1966-99)			Late sample (2000-19)		
	Cobb–Douglas	CES	Translog	Cobb–Douglas	CES	Translog
Panel A. Factor income shares in the steady state						
α	0.20	0.20	0.21	0.09	0.09	0.14
	(0.16, 0.24)	(0.15, 0.24)	(0.16, 0.25)	(0.08, 0.17)	(0.08, 0.17)	(0.11, 0.18)
$1-\alpha$	0.80	0.80	0.79	0.91	0.91	0.86
	(0.76, 0.84)	(0.76, 0.85)	(0.75, 0.84)	(0.83, 0.92)	(0.83, 0.92)	(0.82, 0.89)
Panel B. Translog and CES parameters						
β_{kk}	0	0.05	-0.04	0	0.06	0.40
	-	(-0.15, 0.10)	(-0.37, 0.04)	-	(0.00, 0.11)	(0.28, 0.58)
β_{kl}	0	-0.05	-0.16	0	-0.06	0.09
	-	(-0.10, 0.15)	(-0.29, 0.05)	-	(-0.11, 0.00)	(-0.04, 0.25)
β_{ll}	0	0.05	-0.21	0	0.06	0.05
	-	(-0.15, 0.10)	(-0.46, -0.02)	-	(0.00, 0.11)	(-0.24, 0.34)
ϕ	0	0.28	-	0	0.65	-
	-	(-0.99, 0.62)	-	-	(0.02, 0.90)	-
Panel C. Gross markup in the steady state						
Φ	1.59	1.60	1.60	1.35	1.35	1.39
	(1.46, 1.71)	(1.46, 1.73)	(1.45, 1.73)	(1.19, 1.49)	(1.21, 1.51)	(1.26, 1.57)

Ferroni and Leon-Ledesma (2017), illustrating that the increase in the supply of skilled labor relative to the supply of unskilled labor and their aggregation to a single labor factor can be a source of an increase in the elasticity of substitution between capital and labor, $1/(1-\phi)$ (see also Papageorgiou and Saam, 2008). Furthermore, ϕ is not precisely estimated. The 95% credible interval for $1/(1-\phi)$ varies from 0.5-2.6 for the early sample and from 1-10 for the late sample.

The implied translog parameters by the CES model are shown in Columns (2) and (5). Those values are substantially different from the directly estimated translog parameter values in Columns (3) and (6). In four cases among the six (three parameters and two sample periods), the CES β in the mode is not included in the 95% credible interval for the corresponding estimates in Columns (3) and (6). Thus, similar to the model comparison results, we conclude that the restrictions imposed by the Cobb–Douglas and CES production functions might be too restrictive. The data consistently favor the translog model.

Finally, by comparing Columns (3) and (6) in Panel B, we find that all three translog parameters increased substantially between the two sample periods.

Slopes of the Phillips curves. We compute the model-predicted slopes of the Phillips curves as follows. Let a measure of economic activity, such as the real marginal cost, output gap, labor gap, and labor share (Galí and Gertler, 1999; Sbordone, 2002; Eichenbaum and Fisher, 2007), be denoted by x_t. Then, the Phillips curve based on x_t is given by:

$$\hat{\pi}_t = \beta_x \mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa_x x_t + error_{x,t},$$

Table C.8: Slopes of the Phillips curves

	(1)	(2)	(3)	(4)	(5)	(6)
	Early sample (1966-99)			Late sample (2000-19)		
	Cobb–Douglas	CES	Translog	Cobb–Douglas	CES	Translog
Panel A. Calvo price stickiness parameter						
ξ_p	0.67	0.69	0.65	0.87	0.86	0.72
	(0.56, 0.80)	(0.54, 0.81)	(0.53, 0.76)	(0.71, 0.92)	(0.72, 0.92)	(0.61, 0.87)
Panel B. Slope of the price Phillips curves						
marginal costs	0.023	0.019	0.027	0.005	0.005	0.023
	(0.008, 0.051)	(0.007, 0.057)	(0.011, 0.062)	(0.002, 0.028)	(0.002, 0.026)	(0.004, 0.047)
output gap	0.013	0.012	0.022	0.002	0.002	0.009
	(0.003, 0.026)	(0.004, 0.026)	(0.008, 0.039)	(0.001, 0.009)	(0.001, 0.007)	(0.003, 0.015)
labor gap	0.022	0.020	0.034	0.002	0.003	0.009
	(0.005, 0.041)	(0.006, 0.041)	(0.012, 0.056)	(0.001, 0.011)	(0.001, 0.009)	(0.003, 0.014)
labor share	0.045	0.041	0.111	0.007	0.009	0.202
	(-0.073, 0.123)	(-0.082, 0.128)	(-0.368, 0.336)	(-0.046, 0.077)	(-0.031, 0.050)	(-0.160, 0.216)
labor share,	0.023	0.020	0.042	0.005	0.005	0.028
υ adjusted	(0.008, 0.051)	(0.007, 0.059)	(0.014, 0.175)	(0.002, 0.028)	(0.002, 0.023)	(0.005, 0.062)

where κ_x represents the slope of the Phillips curve. When $x_t = \hat{mc}_t$, it follows from a New Keynesian Phillips curve (see, e.g., Equation (B.103)) that $\beta_x = \tilde{\beta}\gamma$, $\kappa_x = \frac{(1-\zeta_p\tilde{\beta}\gamma)(1-\zeta_p)}{\zeta_p(1+\tilde{\beta}\gamma\iota_p)}\frac{1}{1+\theta_p(\Phi-1)}$ and $error_{x,t} = \hat{\lambda}^p_t$. We project $\hat{\pi}_t$ on $\mathbb{E}_t[\hat{\pi}_{t+1}]$ and x_t and obtain κ_x from the projection coefficient:

$$\begin{pmatrix} * \\ \kappa_x \end{pmatrix} = \left[\text{var}_{c,mp}\begin{pmatrix} \mathbb{E}_t[\hat{\pi}_{t+1}] \\ x_t \end{pmatrix}\right]^{-1} \text{cov}_{c,mp}\left[\begin{pmatrix} \mathbb{E}_t[\hat{\pi}_{t+1}] \\ x_t \end{pmatrix}, \hat{\pi}_t\right],$$

where subscripts c and mp indicate that we use the cyclical variation with the periodicity from 6-32 quarters driven by monetary policy shocks. Equivalently, we can obtain κ_x by applying the ideal bandpass filter to the infinite time series of $\hat{\pi}_t$, $\mathbb{E}_t[\hat{\pi}_{t+1}]$, and x_t and use monetary policy shock series as instrumental variables. Thus, our formula corresponds to the population version of empirical practices of estimating the Phillips curves using the detrended data by filters (Mavroeidis, Plagborg-Møller and Stock, 2014; Stock and Watson, 2020) and identified monetary policy shocks as instruments (Barnichon and Mesters, 2020, 2021).

Table C.8 shows the results for the Calvo price stickiness parameter (Panel A), ξ_p, and the slope of the Phillips curves (Panel B), κ_x, in the posterior mode with the 95% credible intervals. For the early sample, all three models feature ξ_p of approximately two-thirds. In contrast, the Cobb–Douglas and CES models yield large ξ_p and excessively sticky prices for the late sample. The corresponding average durations of a price are greater than seven quarters. The results based on the translog model differ remarkably. ξ_p is 0.72, consistent with the empirically reasonable degree of price stickiness reported in Bils and Klenow (2004) and Nakamura and Steinsson (2008).

The slopes of the marginal cost Phillips curve in Panel B echo the results for ξ_p. The slopes are not small and similar across Columns (1)-(3) and (6). Thus, through the lens of the translog model, which matches the data better as shown above, the New Keynesian Phillips curve did not flatten and is alive well (see also Gagliardone et al., 2023). In contrast, the Cobb–Douglas and CES models predict quite flat marginal cost Phillips curves for the late sample, given the large values of ξ_p.

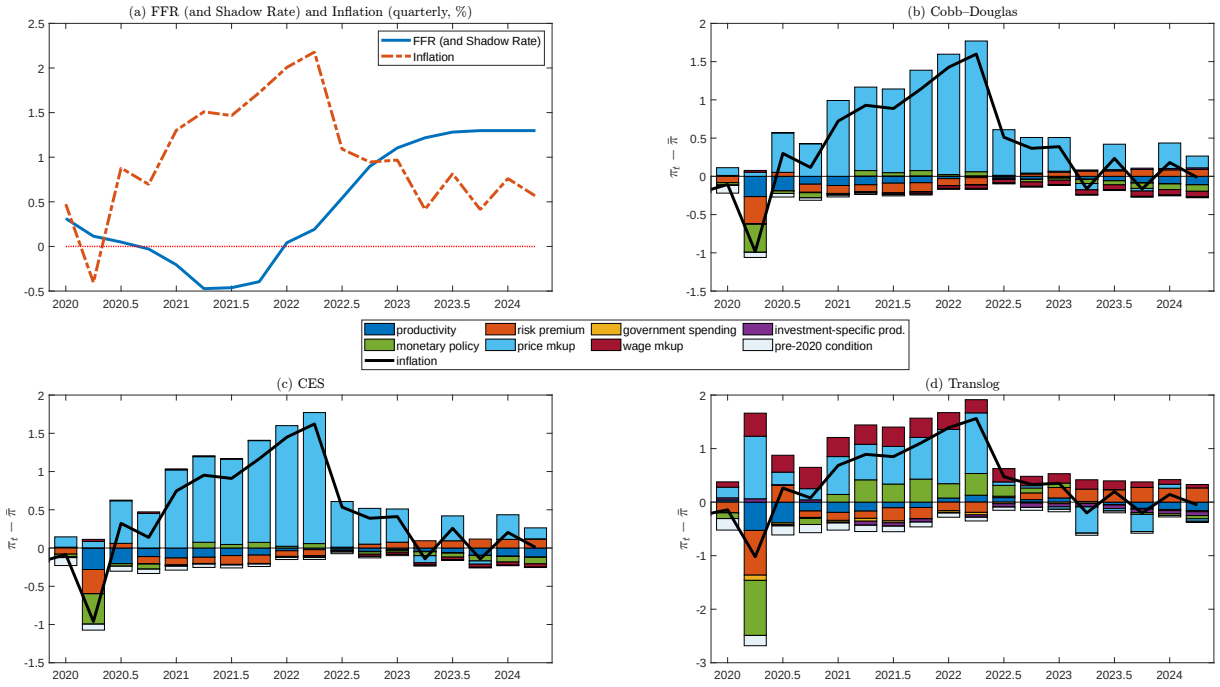


Figure C.2: Historical decomposition of the pandemic-era inflation

The quite flat marginal cost Phillips curves in Columns (4) and (5) translate into similarly flat conventional Phillips curves for the output gap, labor gap, and labor shares. We also consider the labor share adjusted by fixed costs in production, $\frac{w_t l_t}{y_t + v}$, which equals the real marginal costs in the Cobb–Douglas model. Other than the case of labor share, the translog model generates the flattening of these conventional Phillips curves, although the marginal cost Phillips curve did not flatten (Columns (3) and (6)).

Thus, the translog model with structural changes, captured by the different β coefficient values between the two sample periods, can explain the two seemingly contradictory empirical results that the marginal cost Phillips curve is steep and alive (Gagliardone et al., 2023) and the conventional Phillips curve flattened (Barnichon and Mesters, 2020; Del Negro et al., 2020; Hazell et al., 2022; Inoue, Rossi and Wang, 2024).

Pandemic-Era Inflation. Next, we present the historical decomposition of the pandemic-era inflation. We augment the sample with the same seven quarterly variables from 2020:q1 to 2024:q2. For each model, given the parameter values in the posterior mode in Table C.5, we use the Kalman smoother to estimate the realized structural shocks during this extended sample period from 2020:q1-2024:q2.

Figure C.2(b)-(d) shows the decomposition of the realized quarterly inflation into the contribution of each structural shock and the pre-2020 economic condition for the Cobb–Douglas, CES, and translog models, respectively. Panel (a) shows the federal funds rate (augmented with the shadow rate of Wu and Xia (2016) when the zero lower bound was binding) and quarterly GDP deflator inflation. Clearly, the US monetary policy was loose between 2021:q1-2022:q2 when the inflation was increasing.

Panel (d) shows that the translog model emphasizes the contribution of risk premium shocks at the onset of the pandemic in 2020:q2, reflecting elevated demand for safe assets (Fisher, 2015), and loose monetary policies for high inflation during 2021:q1-2022:q2. In contrast, the Cobb–Douglas and CES models almost entirely rely on price markup shocks to match the realized inflation dynamics after 2020. Because the Phillips curves are quite flat in the Cobb–Douglas and CES models, these models need to shift the Phillips curves using price markup shocks to

match the variation in inflation. In contrast, the translog model with less sticky prices and a steeper Phillips curve incorporates demand shifters like monetary policy shocks into the quantitative analysis as a meaningful source of inflation dynamics.

Appendix D Supplementary Materials for Section 4

D.1 Summary Statistics

Tables D.1 and D.2 report the basic summary statistics for the EU KLEMS database used in our baseline analysis. We construct a multi-country, industry-level panel database by integrating the national, labor, and capital accounts from the EU KLEMS dataset. For the pre-2000 sample (1970–1999), we use the March 2008 release. For the post-2000 sample (2002–2019), we combine the 2012 and 2023 releases to create a continuous dataset. Our baseline analysis includes data for Austria, Belgium, Denmark, Finland, France, Germany, Italy, Japan, Luxembourg, the Netherlands, Portugal, Spain, Sweden, and the United Kingdom—countries for which we were able to consistently obtain the necessary information across the two datasets we constructed.

We extract data from the national accounts on the volumes and prices of output, intermediate inputs, total labor hours, and total labor compensation across industries, countries, and years. The labor accounts break down labor inputs by education level, enabling us to calculate hours worked by college graduates (skilled labor) and others (unskilled labor), along with the wages for each group. Additionally, we obtain the volumes and prices of real capital stock from the capital accounts.

We drop observations with negative values for key variables such as output, real capital stock, intermediate inputs, and labor hours. Our key dependent variable, the cost share of skilled labor, is measured by dividing the labor compensation for skilled labor by total nominal output. We construct the price indices for capital and intermediate inputs by dividing the nominal values by the real values. For the pre-2000 analysis, nominal values for capital stock are unavailable, so we construct the price index for capital services by dividing nominal capital services by their real counterpart.[3] Lastly, skilled and unskilled wages are derived by dividing the total compensation for each labor group by the total hours worked by that group in a given year.

Table D.1: Summary Statistics (Pre-2000 Analysis)

	Mean	Std. Dev.	P25	P50	P75	N
Gross output (Nominal)	7375391.21	31393663.17	10409.64	32264.23	131930.00	2372
Gross value-added (Nominal)	3745028.74	13423626.63	5823.17	17672.12	70553.98	2372
Intermediate Inputs (Nominal)	3630362.45	18979350.35	3960.08	13400.88	60570.64	2372
Capital Services (Nominal)	1539879.22	6336040.40	1361.14	4954.79	18735.00	2359
Labor Compensation by Skilled Labor (Nominal)	499056.63	1826917.45	202.93	1042.46	5269.12	2372
Labor Compensation by Unskilled Labor (Nominal)	1370416.67	5102660.32	2122.06	8310.41	35063.90	2372
Hours worked by skilled labor	311317.98	693935.23	16617.13	61459.31	225367.02	2372
Hours worked by unskilled labor	2304998.98	3978051.17	253718.02	675825.12	2392444.88	2372
Capital Stock (Real)	26183669.34	118401167.25	36547.59	109569.70	488850.61	2372
Gross Output (Real)	7653806.99	30794245.26	14743.83	49637.35	148653.30	2372
Gross value-added (Real)	4001638.41	13487293.69	8779.23	26947.33	84943.36	2372
Intermediate Inputs (Real)	3601328.80	18203988.44	5738.12	20361.74	67829.65	2372
Capital Services (Real)	1425721.59	5880110.02	1984.89	6053.09	23789.57	2372

Notes: This table presents summary statistics for the data used in our pre-2000 analysis (1970–1999). Our baseline analysis includes data for Austria, Belgium, Denmark, Finland, France, Germany, Italy, Japan, Luxembourg, the Netherlands, Portugal, Spain, Sweden, and the United Kingdom. All variables are expressed in millions, except for hours worked, which are expressed in thousands, with nominal values reported in their respective national currencies. Capital stock variables are available only in real terms in the March 2008 release. *Sources*: The March 2008 release of EU KLEMS.

[3] Capital services are measured in terms of the user cost. The March 2008 release includes information on the volume index of capital services, which is used to construct the volume of capital services. Specifically, we calculate the volume of capital services by multiplying the volume indices by the capital services in the base year.

Table D.2: Summary Statistics (Post-2000 Analysis)

	Mean	Std. Dev.	P25	P50	P75	N
Gross output (Nominal)	4029025.53	24897641.41	25911.00	90266.00	232755.80	2566
Gross value-added (Nominal)	2127887.31	12179543.84	12782.20	47341.00	115549.00	2566
Intermiedate Inputs (Nominal)	1901138.20	13925085.56	11313.90	40720.00	117038.90	2566
Capital Stock (Nominal)	773413.26	3325113.71	32939.04	78577.39	261008.00	2566
Labor Compensation by Skilled Labor (Nominal)	22364.69	40089.09	1698.62	7402.52	26938.32	2566
Labor Compensation by Unskilled Labor (Nominal)	27227.34	40550.98	3090.86	12972.64	32222.29	2566
Hours worked by skilled labor (in millions)	417804.89	598070.37	45371.35	165152.95	510874.38	2566
Hours worked by unskilled labor (in millions)	1020254.83	1489607.29	97743.02	373289.12	1320759.50	2566
Capital Stock (Real)	755820.78	3127297.99	35049.26	81939.43	277916.00	2566
Gross Output (Real)	4068907.01	25124917.36	26763.75	92752.51	236993.19	2566
Gross value-added (Real)	2152839.65	12312930.73	13476.80	51356.48	120256.81	2566
Intermediate Inputs (Real)	1917842.42	13952868.79	11417.89	42883.11	118729.10	2566

Notes: This table presents summary statistics for the data used in our post-2000 analysis (2002-2019). Our baseline analysis includes data for Austria, Belgium, Denmark, Finland, France, Germany, Italy, Japan, Luxembourg, the Netherlands, Portugal, Spain, Sweden, and the United Kingdom. All variables are expressed in millions, except for hours worked, which are expressed in thousands, with nominal values reported in their respective national currencies. *Sources*: October 2012 and February 2023 Releases of EU KLEMS

D.2 Empirical Results: Robustness

We check the robustness of our results in Section 4.3 of the main text. Specifically, we show that our findings remain robust when we (i) use two- and three-year lagged input prices as IVs to address potential serial correlation in the baseline regression's residual, (ii) employ the adjusted skilled labor share by input prices as the dependent variable, (iii) adopt a value-added production function, (iv) include different sets of countries, (v) substitute capital services (available only in the March 2008 release) for capital stock, (vi) apply the Driscoll and Kraay (1998) standard errors, and (vii) account for persistent components in the residual, such as country-industry-specific productivity levels.

D.2.1 Using Two- and Three-year Lagged Input Prices

This section presents the estimation results when using two- and three-year lagged input prices as instrumental variables. As shown in Table D.3, our findings are robust to the use of different sets of instrumental variables. Specifically, the coefficient on capital shifts from negative to positive, suggesting that capital has become more complementary to skilled labor over time. Similarly, the coefficient on unskilled labor is negative in Column (1) but turns positive in Column (2).

Table D.3: Estimation Results, Using Two- and Three-year Lagged Instruments

	1970-1999	2002-2019
$s_{cjt}^{\texttt{skilled labor}}$	(1)	(2)
Capital	-0.486	0.619
	(0.086)	(0.266)
Skilled Labor	0.340	-0.564
	(0.151)	(0.345)
Unskilled Labor	-0.664	0.220
	(0.447)	(0.542)
Intermediate Goods	0.175	-0.067
	(0.386)	(0.138)
Observations	2240	2371

Notes: Column (1) reports the IV regression results for the period 1970–1999, and Column (2) for the period 2002–2019. All four inputs–skilled labor, unskilled labor, capital, and intermediate goods–are logged. The inputs are instrumented using lagged input prices at $t-2$ and $t-3$. The analysis includes data from Austria, Belgium, Germany, Denmark, Spain, Finland, France, the United Kingdom, Italy, Japan, Luxembourg, the Netherlands, Portugal, and Sweden. The two-step efficient GMM specification is employed, with standard errors clustered at the country level, shown in parentheses. *Sources*: The March 2008 release, the October 2012 release, and the February 2023 release of EU KLEMS.

D.2.2 Using Alternative Dependent Variables

This section presents the estimation results when using different dependent variables. First, we use the expenditure shares out of the value-added, instead of the gross output. Columns (1) and (2) of Table D.4 show the results. Second, we use the skilled labor share adjusted by $\sum_k \alpha_{jkt}(\widehat{P_{jkt}/P_{jt}})$ in view of the fact that $\tau_t = \sum_k \alpha_k \hat{p}_{kt} + \varepsilon_t^a$. As shown in Table D.4, our findings are robust to the use of alternative dependent variables.

Table D.4: Estimation Results, Using Alternative Dependent Variables

	$s_{cjt}^{\texttt{skilled labor}}$ out of value-added		$s_{cjt}^{\texttt{skilled labor}} - \sum_k \alpha_k \hat{p}_{kt}$	
	1970-1999	2002-2019	1970-1999	2002-2019
	(1)	(2)	(3)	(4)
Capital	-0.775	0.752	-0.800	0.865
	(0.082)	(0.584)	(0.139)	(0.362)
Skilled Labor	0.047	0.892	0.889	-0.415
	(0.204)	(0.686)	(0.281)	(0.749)
Unskilled Labor	-0.097	0.615	-0.597	0.511
	(0.138)	(0.810)	(0.519)	(0.776)
Intermediate goods			-0.913	0.473
			(0.361)	(0.118)
Observations	2391	2622	2303	2566

Notes: Columns (1) and (2) report the IV regression results using the expenditure share of value-added as the dependent variable, while Columns (3) and (4) show the results using the expenditure share adjusted by $\sum_k \alpha_{jkt}(\widehat{P_{jkt}/P_{jt}})$ as the dependent variable. Columns (1) and (3) present the results for the period 1970–1999, and Columns (2) and (4) for the period 2002–2019. All four inputs—skilled labor, unskilled labor, capital, and intermediate goods—are logged. The inputs are instrumented using lagged input prices at $t-1$ and $t-2$. The analysis includes data from Austria, Belgium, Germany, Denmark, Spain, Finland, France, the United Kingdom, Italy, Japan, Luxembourg, the Netherlands, Portugal, and Sweden. The two-step efficient GMM specification is employed, with standard errors clustered at the country level, shown in parentheses. *Sources*: The March 2008 release, the October 2012 release, and the February 2023 release of EU KLEMS.

D.2.3 Using Different Samples

In our baseline specification, for consistency, we use a subset of countries available in both the March 2008 release and the February 2023 release. In this section, we repeat the analysis by including additional countries in our sample. First, we include data for Finland and Italy for 2000–2001 in the post-2000 sample. Second, we repeat the analysis using all available data from each release. Table D.5 presents the results, which show that our findings in the main text are not dependent on the set of countries.

Table D.5: Estimation Results, Using Different Samples

	1970-1999	2000-2019	2002-2019
	(1)	(2)	(3)
Capital	-0.595	0.885	1.420
	(0.088)	(0.279)	(0.331)
Skilled Labor	0.521	-0.497	-0.192
	(0.245)	(0. 319)	(0.478)
Unskilled Labor	-0.690	0.623	0.468
	(0.421)	(0.526)	(0.426)
Intermediate goods	0.321	0.076	0.004
	(0.301)	(0.117)	(0.098)
Observations	2666	2622	3679

Notes: This table reports the IV regression results using different samples. Column (1) shows the results for the period 1970–1999, including all available countries from the March 2008 release of EU KLEMS. Column (2) shows the results for the period 2000–2019. Labor data for 2000–2001 is available only for Finland and Italy, and they are utilized for the results shown in Column (2). Column (3) shows the results for the period 2002–2019, using all available countries from our post-2000 dataset. All four inputs—skilled labor, unskilled labor, capital, and intermediate goods—are logged. The inputs are instrumented using lagged input prices at $t-1$ and $t-2$. The two-step efficient GMM specification is employed, with standard errors clustered at the country level, shown in parentheses. *Sources*: The March 2008 release, the October 2012 release, and the February 2023 release of EU KLEMS.

D.2.4 Using Driscoll and Kraay (1998) Standard Errors

Our baseline specification employs standard errors clustered at the country level. To ensure the robustness of our findings, we also repeat the analysis using Driscoll and Kraay (1998) standard errors, given the relatively long time series in our data compared to the number of countries included. Table D.6 shows that our results are robust to the use of Driscoll and Kraay (1998) standard errors instead.

Table D.6: Estimation Results, Using Driscoll and Kraay (1998) Standard Errors

	1970-1999	2002-2019
$s_{cjt}^{\texttt{skilled labor}}$	(1)	(2)
Capital	-0.466	0.933
	(0.086)	(0.145)
Skilled Labor	0.080	-0.130
	(0.033)	(0.569)
Unskilled Labor	-0.708	0.561
	(0.059)	(0.105)
Intermediate Goods	0.310	0.077
	(0.076)	(0.193)
Observations	2372	2566

Notes: Column (1) reports the IV regression results for the period 1970–1999, and Column (2) for the period 2002–2019. All four inputs–skilled labor, unskilled labor, capital, and intermediate goods–are logged. The inputs are instrumented using lagged input prices at $t-1$ and $t-2$. The analysis includes data from Austria, Belgium, Germany, Denmark, Spain, Finland, France, the United Kingdom, Italy, Japan, Luxembourg, the Netherlands, Portugal, and Sweden. The two-step efficient GMM specification is employed, with Driscoll and Kraay (1998) standard errors, shown in parentheses. *Sources*: The March 2008 release, the October 2012 release, and the February 2023 release of EU KLEMS.

D.2.5 Using Capital Services

The March 2008 release includes capital services in addition to the values of real capital stock. To check the robustness of our results, we repeat our analysis for the pre-2000 sample using capital services instead of capital stock. Table D.7 shows that our results are robust to the use of an alternative measure of capital.

Table D.7: Estimation Results, Using Capital Services (Only Available in March 2008 Release)

	Out of Output	Out of Value-Added
$s_{cjt}^{\texttt{skilled labor}}$	(1)	(2)
Capital Services	-0.223	-0.428
	(0.036)	(0.052)
Skilled Labor	0.339	0.480
	(0.101)	(0.130)
Unskilled Labor	-0.615	-0.601
	(0.370)	(0.135)
Intermediate Goods	-0.156	
	(0.323)	
Observations	2311	2311

Notes: Column (1) reports the IV regression results using the expenditure share of skilled labor out of gross output, and Column (2) reports the results for the expenditure share out of value-added. All four inputs—skilled labor, unskilled labor, capital services, and intermediate goods—are logged. The inputs are instrumented using lagged input prices at $t-1$ and $t-2$. The analysis includes data from Austria, Belgium, Germany, Denmark, Spain, Finland, France, the United Kingdom, Italy, Japan, Luxembourg, the Netherlands, Portugal, and Sweden. The two-step efficient GMM specification is employed, with standard errors clustered at the country level, shown in parentheses. *Sources*: The March 2008 release of EU KLEMS.

D.2.6 Country-industry-specific Productivity in the Residuals

In the main text, we estimated the following equation:

$$\log\left(s^i_{cjt}\right) = \sum_k \delta_{ik} \log\left(x^k_{cjt}\right) + \alpha_c + \gamma_j + \tau_t + \varepsilon^i_{cjt}.$$

In this section, we interpret ε_{cjt} as the level of productivity, We assume that ε_{cjt} has a persistent process represented by an AR(1) process:

$$\varepsilon_{cjt} = \rho_j \varepsilon_{cj,t-1} + \eta_{cjt}.$$

We allow the degree of persistence, captured by the AR(1) coefficient, ρ_j, to vary across industries. With the remaining idiosyncratic components in the residual such as measurement errors, denoted by ω^i_{cjt}, we have the following equation:

$$\log\left(s^i_{cjt}\right) = \sum_k \delta_{ik} \log\left(x^k_{cjt}\right) + \alpha_c + \gamma_j + \tau_t + \varepsilon_{cjt} + \omega^i_{cjt}.$$

We assume that ω^i_{cjt} is serially uncorrelated.

For a simple notation, we define a function, φ, as follows:

$$\varphi^i_{cjt}(\delta, \boldsymbol{\rho}) \equiv \log\left(s^i_{cjt}\right) - \left[\sum_k \delta_{ik} \log\left(x^k_{cjt}\right) + \alpha_c + \gamma_j + \tau_t\right] = \varepsilon_{cjt} + \omega^i_{cjt},$$

where $\boldsymbol{\rho}$ is a vector of industry-specific AR coefficients, ρ_j.

We utilize the orthogonality between lagged input prices and productivity shocks (η_{cjt}) for identification. Note that:

$$\begin{aligned}\varphi^i_{cjt}(\delta, \boldsymbol{\rho}) - \rho_j \varphi^i_{cj,t-1}(\delta, \boldsymbol{\rho}) &= (\varepsilon_{cjt} + \omega^i_{cjt}) - \rho_j(\varepsilon_{cj,t-1} + \omega^i_{cj,t-1}) \\ &= \eta_{cjt} + \omega^i_{cjt} - \rho_j \omega^i_{cj,t-1}.\end{aligned}$$

Thus, for two- and three-year lagged input prices, $Z_{cj,t-2}$, we have the following moment conditions:

$$\mathbb{E}\left\{Z_{cj,t-2}\left[\varphi^i_{cjt}(\delta, \boldsymbol{\rho}) - \rho_j \varphi^i_{cj,t-1}(\delta, \boldsymbol{\rho})\right]\right\} = 0.$$

We jointly estimate δ and $\boldsymbol{\rho}$ using the above moment conditions and GMM methods. As in the main text, we focus on the skilled labor share. We also divide the data into the pre- and post-2000 samples.

Table D.8 shows the estimated δ parameters with standard errors. Similar to the baseline results shown in Table 7, the coefficients on capital and unskilled labor increase over time, while the coefficient on skilled labor decreases. Thus, we conclude that our results are robust to explicitly controlling for persistence in the level of productivity driven by country-industry-specific productivity shocks.

Table D.8: Estimation Results, Assuming AR(1)
Country-industry-specific Productivity Shocks

	1970-1999	2002-2019
	(1)	(2)
Capital	0.037	0.330
	(0.047)	(0.141)
Skilled Labor	0.662	0.361
	(0.097)	(0.217)
Unskilled Labor	-0.117	0.026
	(0.281)	(0.174)
Intermediate Goods	-0.171	-0.623
	(0.162)	(0.079)
Observations	2166	2371

Notes: Column (1) reports the GMM estimation results for the period 1970–1999, and Column (2) for the period 2002–2019. All four inputs–skilled labor, unskilled labor, capital, and intermediate goods–are logged. We use lagged input prices at $t-2$ and $t-3$ as instruments. The analysis includes data from Austria, Belgium, Germany, Denmark, Spain, Finland, France, the United Kingdom, Italy, Japan, Luxembourg, the Netherlands, Portugal, and Sweden. *Sources*: The March 2008 release, the October 2012 release, and the February 2023 release of EU KLEMS.

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