Carry Trade Portfolios, Home Bias, and Heterogeneity*

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Abstract

In representative agent models, consumption growth risk explains currency carry trade returns only when risk aversion is implausibly high. Yet, a highly risk-averse agent would not hold a carry trade portfolio. Heterogeneity helps by allowing a risk-tolerant minority of agents to hold carry trade portfolios while a risk-intolerant majority does not. We show that with heterogeneous risk aversion, standard models of international macroeconomics can produce carry traders in low interest-rate economies and a domestic bias in aggregate portfolios, as observed in Germany, Japan, and the United States, together holding half of global debt.

JEL Codes: F31, G11, G15
Keywords: Carry trade, home bias, risk aversion, portfolio choice, heterogeneous agents

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1 Introduction

The carry trade has two parts: the return and the portfolio position. Most research on the carry trade in foreign exchange markets seeks to rationalize the return while ignoring the portfolio position. We argue that a satisfactory theory should rationalize both. In particular, a theory of the carry trade must grapple with the empirical observation that aggregate portfolio positions rarely resemble aggregate carry trades. In representative-agent economies without aggregate carry trades, the representative agent cannot be a carry trader.

Existing theories of the carry trade overlook this complication. For example, Lustig and Verdelhan (2007) argue that carry trade returns compensate a representative agent for holding high interest-rate currencies that depreciate when consumption growth is low. To generate plausible returns for the United States, they assume the representative agent is extremely risk averse. They do not consider the portfolio position of the agent. But, given the extreme risk aversion assumed, any standard extension of their model would predict domestic bias in the agent's portfolio. Domestic bias in the United States looks nothing like the carry trade, because interest rates in the United States have been low in recent decades. The same pattern holds in Germany and Japan, as we report in Table 1. Together, these three economies hold half of global debt.

Heterogeneity offers an attractive way forward. By fixing aggregate risk aversion at a high level while allowing for variation across agents, it should be possible to generate plausible carry trade returns while allowing a minority of risk-tolerant agents to hold carry trade portfolios in low interest-rate economies. Indeed, we prove sufficient conditions for exactly this result in a two-country dynamic stochastic general equilibrium model with incomplete markets and heterogeneity in risk aversion. The benefits of this approach are substantial.

First, heterogeneity bridges the gap between the finance literature on currency carry trades and the international macroeconomics literature on domestic bias in aggregate portfolio holdings. The finance literature typically employs representative-agent models to study carry trade returns. However, as discussed, the representative agent is not a carry trader in countries with low domestic interest rates and domestic bias in portfolio holdings—the empirically relevant case. Conversely, the international macroeconomics literature employs representative-agent models to study domestic bias in aggregate portfolio holdings, but often ignores relative returns across currencies and typically does not feature carry trades. Our heterogeneous-agent framework can simultaneously capture *individual carry trade positions* and *domestic bias in aggregate positions* in economies with low interest rates.

Second, the heterogeneity that drives the carry trade in our model also drives a wedge between the aggregate coefficient of risk aversion and the aggregate elasticity

of intertemporal substitution. This aggregation wedge arises even for isoelastic utility functions with constant relative risk aversion, transforming these preferences into a parsimonious and tractable alternative to the recursive preferences traditionally employed to separate risk aversion from intertemporal substitution. The aggregation wedge affects the responsiveness of macroeconomic variables to shocks, so the same heterogeneity that gives rise to individual carry traders also affects macroeconomic dynamics.

Third, the domestic bias in aggregate portfolio holdings that our model generates raises the expected level and volatility of carry trade returns. In the model, a portfolio with domestic bias amounts to a hedge against low consumption growth. If consumption growth falls, hedged portfolio values rise. When a majority of agents are hedged, this initial rise produces a positive feedback effect that raises hedged portfolio values further. The feedback effect improves the hedging properties of portfolios with domestic bias, but worsens the anti-hedging properties of portfolios with international bias, which represent carry trade portfolios in the low interest-rate economy. Thus, domestic bias in aggregate portfolio holdings amplifies the risk of the carry trade, and carry traders demand higher returns.

We acknowledge upfront one of the limitations of our approach: the aggregate risk aversion we require exceeds the already implausible risk aversion that representative-agent models require. This vice afflicts many asset pricing models. Consider Lustig and Verdelhan's model cited above, and reinterpret their pricing equation (3) for a continuum of heterogeneous agents. For an individual agent, the equation can be written as

$$\frac{1}{\gamma - 1} \mathrm{E}\Big[R_{i-j}\Big] \approx b_1' \operatorname{Cov}\Big(\Delta c(\gamma), R_{i-j}\Big) + b_2' \operatorname{Cov}\Big(\Delta d(\gamma), R_{i-j}\Big),$$

where γ denotes the agent's coefficient of risk aversion, R_{i-j} denotes the return differential $R_i - R_j$, with R_i and R_j denoting USD returns on assets denominated in currencies i and j, and $\Delta c(\gamma)$ and $\Delta d(\gamma)$ denote the agent's non-durable and durable consumption growth, respectively.[1] We define $b_1' \equiv b_1(\gamma)/(\gamma - 1)$ and $b_2' \equiv b_2(\gamma)/(\gamma - 1)$ to be independent of γ, where $b_1(\gamma)$ and $b_2(\gamma)$ are agent-specific reinterpretations of b_1 and b_2 defined in Lustig and Verdelhan's equation (4). Assuming a density function $f(\gamma)$ with support over $(1, \gamma_m]$ to describe the distribution of risk aversion across agents, we integrate to recover the aggregate pricing equation,

$$\mathrm{E}\Big[R_{i-j}\Big] \approx \frac{b_1}{\omega} \operatorname{Cov}\Big(\Delta c, R_{i-j}\Big) + \frac{b_2}{\omega} \operatorname{Cov}\Big(\Delta d, R_{i-j}\Big),$$

where b_1 and b_2 are the original coefficients in Lustig and Verdelhan (2007), but where an

[1] We omit time subscripts and simplify Lustig and Verdelhan's equation (3) by assuming that assets R_i and R_j covary equally with the return on each agent's wealth. Our arguments hold without this simplification.

Table 1 – Domestic Currency Bias and Returns on Aggregate Debt Portfolios

	Group Share of Global Holdings	Country-Level Weighted Average (in %)		
		Domestic Currency Bias	Nominal Portfolio Return	Real Portfolio Return
Germany, Japan, United States	54.0	88.6	1.2	−0.3
All Other Countries	46.0	62.4	3.2	0.7

Notes. The table reports estimated domestic currency bias and returns on debt portfolios held by Germany, Japan, and the United States versus 181 other countries. Portfolio returns and domestic currency bias are estimated from OECD and IMF interest rates, exchange rates, and CPI inflation, combined with portfolio weights estimated from IMF and BIS debt holdings and issuance data over the period 2001–2021. Shares of global holdings are calculated for each group then averaged over years; all other values are calculated for each country then averaged over countries and years, using each country's debt holdings as weights. Appendices A.2 and A.3 describe our methodology.

aggregation wedge $\omega > 1$ now appears as a consequence of Jensen's inequality.[2] With the wedge, the aggregate coefficient of risk aversion embedded in b_1 and b_2 must be larger to explain carry trade returns. Although heterogeneous-agent models increase the level of risk aversion required to explain carry trade returns, the level was already implausibly high in the representative-agent case, and the increase can be made arbitrarily small by skewing the distribution of risk aversion. Therefore, we consider the additional cost of heterogeneity to be marginal relative to the benefits of adopting our approach.

We empirically motivate our study using the International Monetary Fund's Coordinated Portfolio Investment Survey to show that carry traders are outweighed by other investors in debt markets. First, for the six countries that report them, bilateral aggregate *gross* short positions rarely exceed corresponding bilateral gross long positions. Second, across 86 countries reporting bilateral aggregate *net* positions, only 0.1% are net negative, further suggesting long positions outweigh short positions even in low interest-rate currencies. Third, our estimates of domestic-currency bias reveal that low interest-rate countries exhibit stronger domestic-currency bias than many high interest-rate countries, as documented in Table 1. Taken together, this evidence suggests that investors preferring domestic-currency assets outweigh carry traders in debt markets. While carry trades may be more prevalent in currency derivatives markets, an emerging consensus suggests that hedging, not speculation, dominates even these markets, as we review below. These patterns indicate that carry traders do not represent the average investor.

We rationalize these empirical patterns in a dynamic stochastic general equilibrium model with an infinite horizon, two countries, incomplete markets, and a continuum

[2]With $\gamma > 1$, f non-degenerate, and $\mathrm{E}_\gamma\big[g(\gamma)\big] = \int_1^{\gamma_m} g(\gamma) f(\gamma)\,\mathrm{d}\gamma$, Jensen's inequality implies $\mathrm{E}_\gamma\big[1/(\gamma-1)\big] > 1/(\mathrm{E}_\gamma[\gamma]-1)$. A wedge $\omega > 1$ equates the two sides, $\mathrm{E}_\gamma\big[1/(\gamma-1)\big] = \omega/(\mathrm{E}_\gamma[\gamma]-1)$.

of agents with heterogeneous risk aversion. In the model, individual coefficients of risk aversion equal the inverse of individual elasticities of intertemporal substitution, but this inverse relationship breaks when the individual parameters are aggregated. If aggregate risk aversion exceeds a low threshold, the aggregate portfolio displays domestic bias. While the majority of individual agents in the low interest-rate country endogenously hold portfolios with domestic bias, a minority of agents hold carry trade portfolios, consistent with the evidence.

Importantly, we present a novel method to solve for both individual and aggregate portfolio holdings, extending existing methods in international macroeconomics to a new class of heterogeneous-agent models. Our model admits approximate closed-form solutions for individual and aggregate consumption, real wealth, and real portfolio holdings, as well as for the real exchange rate and the carry trade real return. Conveniently, heterogeneity vanishes in the model's non-stochastic steady state, and the model pins down a degenerate cross-sectional distribution of zero steady-state real wealth. Around this steady state, the model generates a stationary, non-degenerate cross-sectional distribution of real wealth and consumption. Throughout the model, we prioritize tractability and simplicity in order to emphasize the mechanisms at work and highlight the key insights. For this reason, we abstract from many complexities that enrich medium scale models of international macroeconomics.

The remainder of the paper is organized as follows. In Section 2, we discuss related literature. In Section 3, we make an empirical case for heterogeneity and domestic-currency bias in portfolio positions by examining country-level debt holdings. In Section 4, we construct the two-country dynamic stochastic general equilibrium model with heterogeneity in risk aversion. Section 5 concludes.

2 Related literature

This paper bridges existing research on currency carry trades and domestic bias in portfolio holdings, using preference heterogeneity to develop a unified framework that explains both investment patterns.

In foreign exchange markets, carry trades exploit interest rate differentials across currencies. These trades violate uncovered interest parity in predictable ways, as documented by Hansen and Hodrick (1980), Fama (1984), and Engel (1984). Most explanations for these violations fall into one of three categories: financial frictions (Burnside et al., 2006; Brunnermeier et al., 2009; Gabaix and Maggiori, 2015), rare disasters (Barro, 2006; Burnside et al., 2011; Farhi and Gabaix, 2016), and risk-based factors (Lustig and Verdelhan, 2007; Lustig et al., 2011; Menkhoff et al., 2012; Lettau et al., 2014).

The consumption-based Capital Asset Pricing Model (CCAPM), adapted to carry

trades by Lustig and Verdelhan (2007), argues that profits compensate for exposure to consumption growth risk. Despite debate (Burnside, 2011; Lustig and Verdelhan, 2011), CCAPM remains influential in risk-based explanations.

Within this tradition, we are closest to Jylhä and Suominen (2011) and Filipe et al. (2023), who develop a two-country model with endogenous carry traders using two types of myopic agent—one constrained to domestic markets, the other unconstrained. Similarly, Park (2022) introduces heterogeneous beliefs in a two-country, two-period model with three agent types, showing how pessimistic beliefs raise carry trade returns. In contrast, we model a continuum of unconstrained agents with heterogeneous risk aversion who optimize over an infinite horizon. Our framework offers richer heterogeneity in a workhorse international macro model, and uniquely explains how carry traders emerge in low interest-rate economies that simultaneously exhibit domestic bias in aggregate portfolios.

The profitability of the carry trade has been extensively studied, but evidence on actual carry trade positions is more limited. Research in the mid-2000s by Galati et al. (2007) and Gagnon and Chaboud (2007) found modest evidence of carry trades primarily in derivatives markets. Curcuru et al. (2011) and Fong (2013) also documented limited evidence of carry trades, even during times of reportedly high activity. Specific carry trade channels have been observed in particular contexts—household foreign currency borrowing in Austria (Beer et al., 2010), European bank behavior during the sovereign debt crisis (Acharya and Steffen, 2015), and emerging market firms issuing USD bonds (Bruno and Shin, 2017)—but these specialized cases do not suggest widespread dominance of carry trade strategies.

In contrast, a growing literature finds that hedging, not carry trade, drives much of the currency derivatives market. Allayannis and Ofek (2001) and Allayannis et al. (2001) find that large U.S. firms use currency derivatives to reduce exchange rate exposure, enhancing firm value. Bartram et al. (2010) and Bartram (2019) find that non-financial firms globally use currency derivatives to reduce exchange rate exposure. Sialm and Zhu (2024) find that most U.S. international fixed income mutual funds hedge with currency forwards. Bräuer and Hau (2022) show that institutional investor hedging explains significant exchange rate variation. Hacioğlu-Hoke et al. (2024) show that pension and investment funds, insurers, non-financial corporations, and non-dealer banks use currency derivatives primarily for hedging, using UK micro data. Alfaro et al. (2024) survey evidence on currency derivative hedging in micro data. Liao and Zhang (2025) find that hedging demands rise in periods of financial distress, affecting exchange rates.

In bond markets, domestic bias dominates. Lane (2005) finds strong regional bias in Euro-area bonds after the Euro's introduction, anticipating a later literature on domestic-*currency* bias. Fidora et al. (2007) and Coeurdacier and Rey (2013) find that domestic bias persisted more strongly in bonds than in equities over decades of financial globalization. Coeurdacier and Gourinchas (2016) show that bonds help hedge real exchange rate

risk in portfolios with equity. Burger et al. (2018) find that investors strongly prefer domestic-currency bonds, with U.S. investors over-weighting USD bonds from foreign issuers. Recent work by Maggiori et al. (2020) and Florez-Orrego et al. (2023) using micro data shows that global investors prefer domestic-currency bonds, with USD as an exception. Faia et al. (2022) find domestic bias in Euro-area corporate bond holdings for insurers and pension funds but not mutual funds. Burietz and Ureche-Rangau (2020) find domestic bias in Euro-area syndicated lending.

These studies collectively suggest that while carry traders exist in both currency derivatives markets and specialized debt markets, they do not represent the dominant market participants. Hedging appears more important than speculation for many non-financial and financial firms. These findings reinforce our thesis that carry traders, while present, are outweighed by investors with non-speculative motives. We reconcile these findings with the carry trade literature using a heterogeneous-agent framework.

Our framework builds on influential methodological advances in solving for international portfolios in open-economy models with incomplete markets. Guu and Judd (2001), Devereux and Sutherland (2010, 2011), Tille and van Wincoop (2010), and Evans and Hnatkovska (2012) develop methods to solve open-economy models with portfolio choice under incomplete markets using perturbation methods. More recently, Saucet (2022) develops a global solution for portfolio holdings in an open-economy with heterogeneity across but not within countries. Our technical contribution extends the Devereux and Sutherland (2011) perturbation method to allow for within-country heterogeneity in risk preferences for a continuum of agents (in addition to heterogeneity across countries). We provide approximate closed-form solutions for the distinct portfolio of each individual agent, and for the aggregate economy. While we solve only for steady-state holdings, this limitation can be addressed using third-order approximations.

Our work relates to a macroeconomics literature on preference heterogeneity that until recently followed the Bewley-Ayagari-Hugget tradition, where agents differ only in their history of idiosyncratic shocks. This was true of earlier HANK models (see Kaplan et al. 2018), where heterogeneity in the marginal propensity to consume (MPC) is important for monetary policy transmission. Recently, Aguiar et al. (2020), Calvet et al. (2021b) and Gelman (2021) have shown that preference heterogeneity rather than shocks can explain much of the observed heterogeneity in MPCs and portfolio holdings. Kekre and Lenel (2022) build a closed-economy New-Keynesian model with heterogeneity in risk aversion to analyze monetary policy transmission through risk premia. Davis and Van Wincoop (2024) develop an open-economy model of within-country heterogeneity in risk aversion and risk perceptions; unlike us, they focus on capital flow retrenchment and model portfolio choice over a two-period horizon.

Our work also relates to a finance literature on preference heterogeneity. Kogan and Uppal (2001) study portfolio holdings and asset pricing in a closed economy with

heterogeneous risk aversion and incomplete markets, but unlike us they perturb parameters of the utility function around the logarithmic utility case. Guvenen (2009) studies asset prices in a two-agent closed-economy macro model with limited stock market participation and heterogeneity in the intertemporal elasticity of substitution (IES). Garleanu and Panageas (2015) study asset pricing in a closed-economy OLG model with two agents distinguished by their IES and risk aversion, and Panageas et al. (2020) surveys the literature on closed-economy asset pricing with heterogeneity and provides a unifying framework.

Our work differs from prior work on preference heterogeneity along several dimensions. First, prior work often assumes a limited number of agent types, whereas we model richer heterogeneity: a continuum of agents characterized by a distribution of risk aversion. This richness endogenizes hedgers, speculators, and hand-to-mouth consumers. Second, prior work often uses Epstein-Zin recursive preferences to separate risk aversion from the IES, whereas we show that heterogeneity naturally separates these preference parameters in aggregate, even for CRRA utility. Third, prior work has often focused on closed economies, whereas we model heterogeneity both within and across countries, solving for holdings of domestic and international assets for individual agents and the aggregate economy.

3 Empirical Motivation

We begin with the observation that if most individual investors within a country took carry trade positions in debt markets, we would observe negative aggregate positions in debt denominated in low interest-rate funding currencies. In the data, we nearly never observe this. Country-level aggregate portfolio positions, which equal the sum of individual investor positions, are positive for nearly every counterparty country and year. It follows that individual investors with long positions nearly always outweigh individual investors with short positions, even in low interest-rate assets. Furthermore, we find that investors hold domestic debt disproportionately, especially in low interest-rate countries. Thus, we argue that carry traders are outweighed by non-carry traders in global debt markets.

Our empirical analysis uses aggregate data from the International Monetary Fund's Coordinated Portfolio Investment Survey (CPIS). Section 3.1 examines data from an unbalanced panel of six countries that report aggregate gross short and aggregate net positions in the debt of 236 issuer countries over an eight-year period. We find little evidence of significant aggregate gross short positions. Section 3.2 expands this analysis to a broader set of 86 countries that report aggregate net rather than gross positions in the debt of 236 issuer countries. For this broader set of countries, we find strikingly few cases of negative aggregate net holdings of debt, and this finding does not seem to be driven by misreporting. Section 3.3 documents strong domestic-currency bias in estimated aggregate net holdings of debt for an even broader cross-section of 202 countries with 229

Table 2 – Average Aggregate Gross Short Positions: Cayman Islands and Germany

Holding Country	Average Annual Values			
	Aggregate Gross Short Positions (in USD millions)	Short Ratio (in %)	Count of Aggregate Gross Short Positions	
			Non-Zero	Non-Missing
Cayman Islands	−1 095	15	28	154
Germany	−3	3	5	235

Notes. The table summarizes IMF CPIS data on aggregate gross short positions. It shows average positions (USD millions), average short ratios defined in equation (1), and average annual counts of non-zero and non-missing positions, 2015–2019. All values are averaged over counterparties and years, and rounded to the nearest integer.

counterparty countries. The domestic-currency bias we estimate corroborates the findings of Maggiori et al. (2020), who document a similar bias in granular security-level data.

We acknowledge that CPIS aggregate debt data limits our analysis: we cannot study individual carry trade positions in this data, nor can we study carry trade positions in derivatives. But our aim is equally limited: we seek only to establish whether or not carry traders are reasonably representative investors. This limited aim justifies our use of the CPIS data. Debt markets are large, CPIS has broad coverage across countries, and the absence of dominant carry trade patterns here is persuasive.

While we find no evidence of widespread carry trade activity in debt markets, we find strong evidence of domestic bias in these markets, especially in developed economies with low interest rates. Overall, we conclude that carry traders should not be viewed as representative because a significant portion of investors pursue non-carry trade strategies, motivating the need for carry-trade models that incorporate investor heterogeneity.

3.1 Aggregate Gross Short Positions by Issuer Country

Six countries report their aggregate gross short positions in foreign debt separately from their aggregate net positions in CPIS. These annual reports cover up to 236 counterparty countries over an eight-year period. Most reported positions are zero-valued, with non-zero positions reported for only 61 counterparty countries. Among the reporting countries, only the Cayman Islands and Germany report significant positions. Aruba, Belgium, Bulgaria, and Cyprus report either zero or negligible positions. While limited in scope, this evidence may generalize to other offshore financial centers and developed economies, and suggests that aggregate gross short positions in debt are generally small and rare.

Table 2 summarizes reported aggregate gross short positions in debt for the Cayman Islands and Germany from 2015 to 2019. The positions reported by the Cayman Islands, an offshore financial center, stand out in value and frequency. Averaging across counterparty countries and over years, the Cayman Islands report an average position that is 350 times

Table 3 – Aggregate Gross Short Position Destinations: Cayman Islands and Germany

Cayman Islands' Top Destinations for Gross Short Positions		Germany's Top Destinations for Gross Short Positions	
Issuing Country	Cumulative Gross Short Position (in USD millions)	Issuing Country	Cumulative Gross Short Position (in USD millions)
United States	−253 070	France	−1 778
United Kingdom	−42 899	United States	−692
Germany	−41 506	Netherlands	−564
Italy	−31 115	Italy	−341
France	−28 423	Spain	−213

Notes. The table shows the top five destination countries for Cayman and German aggregate gross short positions, ranked by cumulative value. Cumulative gross short positions are aggregate gross short positions summed by issuing country over the period 2015–2019, reported in USD millions.

greater in value than that of Germany, and report on average six times as many non-zero positions as Germany. Even so, the reported gross short positions of the Cayman Islands are relatively small.

To measure their relative size, we define the short ratio as

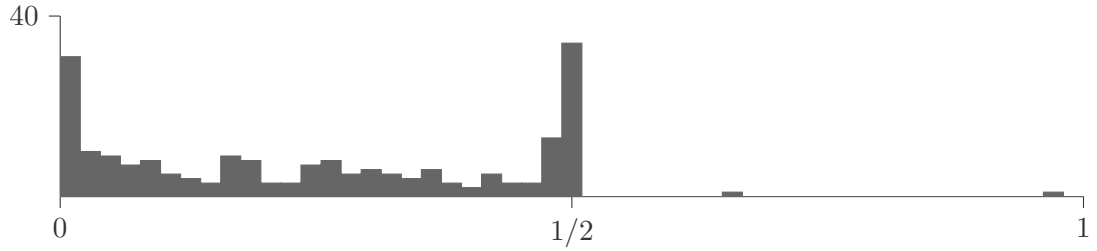
$$SR_{ij} = \frac{-B_{ij}^{k\,\text{Short}}}{B_{ij}^{k\,\text{Long}} - B_{ij}^{k\,\text{Short}}}\,, \tag{1}$$

where B_{ij}^{k}, $B_{ij}^{k\,\text{Short}}$, and $B_{ij}^{k\,\text{Long}}$ denote, respectively, the aggregate net, aggregate gross short, and aggregate gross long positions of country i in debt issued by country j and expressed in numéraire currency k. The short ratio lies between zero and one, and if a country's aggregate gross short position exceeds its aggregate gross long position with a counterparty, the short ratio exceeds one half. As Table 2 shows, the average Cayman short ratio is five times greater than the average German short ratio, but far lower than one half.

Table 3 presents the Cayman and German top five destination countries for aggregate gross short positions, ranked by cumulative value over 2015–2019. Notably, Japan is absent from these top-five lists, despite the attractiveness of Japanese debt as a low-cost funding asset for carry trades. This finding aligns with Gagnon and Chaboud (2007), who report mixed evidence of Yen-funded carry trade positions across Japanese sectors and limited evidence outside Japan. Unlike Gagnon and Chaboud, we exclude balance sheet liabilities and focus on active long and short positions booked as balance sheet assets. Nevertheless, both our findings and theirs suggest diverse investor motivations beyond carry trade returns, indicating investor heterogeneity.

Figure 1 presents a 51-bin histogram of short ratios computed from non-zero aggregate

Figure 1 – Short Ratios for Non-Zero Aggregate Gross Short Positions



Notes. The figure shows a 51-bin histogram of short ratios, as defined in equation (1), for six countries reporting gross short positions in IMF CPIS data, 2013–2020. Short ratios range from 0 to 1. Zero-valued short ratios indicate long-only positions, short ratios of 0.5 indicate equal long and short positions, and short ratios above 0.5 indicate negative net positions. Exactly zero-valued short ratios, which constitute the majority, are excluded.

gross short positions in debt for the six reporting countries in CPIS. Zero-valued positions, which constitute the majority, are excluded. The histogram shows that when short positions are taken, short ratios rarely exceed one-half, suggesting that gross short positions are usually offset by equal or greater gross long positions within the same country, indicating within-country heterogeneity in portfolio positions.

3.2 Aggregate Net Positions by Issuer Country

While CPIS data on aggregate gross short and long positions reveal within-country heterogeneity, they cover only a limited set of reporting countries. To broaden our analysis to 86 reporting countries, we now examine aggregate *net* holdings of foreign debt. Although net positions offer fewer insights into individual investor behavior than gross positions, they nonetheless reveal patterns that challenge the notion of widespread carry trading in debt markets. Notably, among this broader set of countries, aggregate net portfolio holdings in foreign debt are rarely negative. This observation suggests that individual carry traders constitute a minority of investors in most debt markets. If a majority of investors were funding carry trades by short-selling debt in key funding currencies, we would expect to observe negative aggregate net portfolio holdings with the corresponding counterparty countries, but we rarely observe this.

Table 4 presents the relative frequency and value of negative aggregate net holdings of foreign debt reported in CPIS from 2000 to 2019. The data show that such positions are rare: of the 169 732 non-missing positions reported, only 163 are negative. Negative aggregate net positions constitute less than 0.1% of non-missing and less than 1% of non-zero positions by both count and value (rows 1 and 2). Negative aggregate net positions are rare even for Japanese debt (row 3), despite the attractiveness of low-interest Japanese debt as a source of carry trade funding over this period.

Inconsistent or under-reporting of short positions might account for the infrequency of negative aggregate net positions in CPIS data, particularly in countries that rely on

Table 4 – Negative Aggregate Net Holdings of International Debt

Reported Aggregate Net Positions	Of Which, Positions With Negative Values			
	in % of Total Count		in % of Total Value	
	2001–09	2010–19	2001–09	2010–19
With Non-Missing Values	0.06	0.12	0.01	0.09
With Non-Zero Values	0.33	0.71	0.01	0.09
With Non-Zero Values JPY	0.48	0.65	0.00	0.00

Notes. The table presents the relative frequency and value of negative aggregate net positions in foreign short-term debt reported in IMF CPIS data. Row 1 shows negative position counts and values as a percentage of all non-missing position counts and values. Row 2 shows negative position counts and values as a percentage of all non-zero position counts and values. Row 3 shows negative position counts and values in Japanese short-term debt as a percentage of non-zero position counts and values in Japanese debt. Data are shown for periods before and after the 2009 IMF reporting standards change described in Appendix A.1.

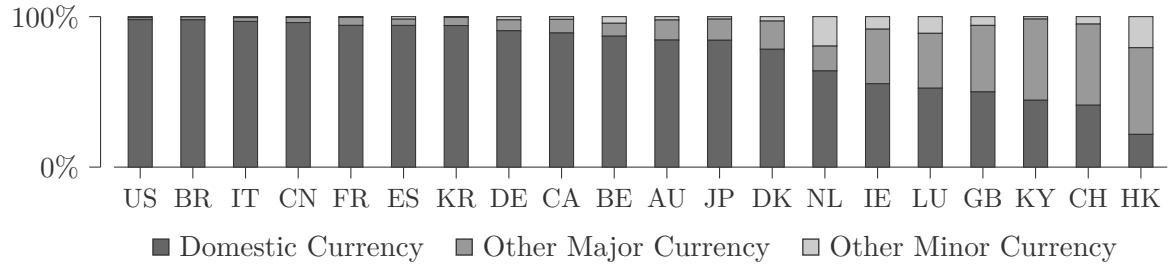
custodian bank surveys (Taub, 2008). However, new standards for short position reporting implemented by the IMF in late 2009 have mitigated these concerns. Table 4 shows an increase in both the relative frequency and value of reported negative aggregate net positions in foreign debt after the introduction of these standards (columns 3 and 5). The rise likely reflects improved reporting practices in the second half of the sample period. Nevertheless, negative aggregate net positions remain rare after the introduction of the new standards. This persistent rarity suggests a fundamental feature of debt markets rather than a reporting artifact. Even if their frequency rose one hundredfold, negative aggregate net positions would still constitute fewer than one in four reported positions. We discuss further details on the reporting of short positions and the evolution of reporting standards in Appendix A.1.

3.3 Aggregate Net Positions by Issuance Currency

While CPIS provides broad coverage of country-level debt holdings, it has limitations. It lacks information on domestic debt holdings, the currency composition of debt holdings by issuer, and the holdings of non-reporting countries. To address these gaps, we estimate the currency composition of aggregate domestic and foreign debt holdings for a broad cross-section of countries, including non-reporting countries. Our estimation procedure extends existing methods for estimating domestic debt holdings and for estimating the currency composition of foreign debt holdings.

Our approach integrates two methods. Fidora et al. (2007) develop a procedure for estimating domestic debt holdings as the difference between total issuance outstanding and rest-of-world holdings. Lane and Shambaugh (2010) develop a procedure for estimating the currency composition of debt holdings by issuer, using information on the currency

Figure 2 – Currency Composition of Net Debt Holdings for Top-Twenty Investor Countries



Notes. The figure shows the currency composition of short-term debt holdings for the top twenty investor countries, averaged over 2001–2022. Holdings are classified as domestic currency, other major currency (USD, EUR, JPY, GBP, CHF), or other minor currency, referring to any other currencies. Countries are ordered by domestic currency weight. Currency weights are estimated using the IMF's CPIS database and the BIS's DDS and IDS databases. Appendix A.2 details the methodology.

composition of debt issuances. We extend these procedures in two ways: first, we estimate domestic debt holdings for non-reporting countries using a two-stage gravity model, and second, we refine issuance-based currency weights using a biproportional fitting procedure. These contributions are novel, as far as we know, and useful because they cover both domestic and foreign debt holdings, extend to both reporting and non-reporting countries, and produce more accurate currency composition estimates. We summarize our results below, and provide an extensive description of our methodology in Appendix A.2.

Our estimates indicate a strong domestic-currency bias in aggregate net debt holdings. This finding aligns with an established literature documenting the tendency for investors to disproportionately hold domestic assets (Fidora et al., 2007; Coeurdacier and Rey, 2013) and domestic-currency assets (Maggiori et al., 2020; Burger et al., 2018). Strong domestic-currency bias in low interest-rate currencies is hard to reconcile with widespread carry trade activity. Carry trades typically involve borrowing in low interest-rate currencies and investing in high interest-rate currencies, which would result in a bias towards high interest-rate currencies, rather than the domestic-currency bias we observe in low interest-rate countries.

Figure 2 summarizes our currency composition estimates for the top twenty investor countries by total holdings. The figure shows the share of each country's debt holdings in three currency categories: domestic currency, major non-domestic currencies (USD, EUR, JPY, GBP, CHF), and minor non-domestic currencies. In most countries, domestic-currency holdings dominate portfolios, but large international financial centers like Great Britain, the Cayman Islands, Switzerland, and Hong Kong are exceptions where major non-domestic currencies play an important role. Notably, Australia (AU) and Japan (JP) each allocate a similarly large share of holdings to domestic-currency debt. This finding is hard to reconcile with widespread carry trade activity in Japan. If interest rate differentials were the primary driver of portfolio positions for Japanese investors, the

Japanese economy would hold less domestic debt, given the persistently low interest rates in Japan in recent decades.

Our empirical analysis shows that negative aggregate net positions in debt are rare, supporting our hypothesis that carry traders are outweighed by non-carry traders. Strong domestic currency bias suggests that investors favor domestic over foreign currencies, particularly in low interest-rate countries. This evidence reinforces our argument that non-carry trading strategies dominate debt markets, and sets the stage for our theoretical model, where heterogeneity in risk aversion helps to rationalize the investment patterns we document.

4 Theoretical Model

We rationalize the empirical evidence from the previous section using a two-country dynamic stochastic general equilibrium model with incomplete markets. Our model incorporates heterogeneity in risk aversion, allowing it to capture both individual carry trades and domestic portfolio bias in economies with low interest rates. This approach helps resolve an apparent tension between the finance literature on carry trades and the international macroeconomics literature on aggregate portfolio holdings, which representative-agent models cannot resolve.

In our model, agents tailor their consumption to their individual risk preferences: agents with higher risk tolerance choose higher expected consumption with higher variance, while agents with lower risk tolerance choose lower expected consumption with lower variance. Agents achieve these consumption patterns by endogenously choosing speculative, hand-to-mouth, or hedged portfolio positions that raise or lower the variance of their incomes.

The model features a determinate non-stochastic steady state and stationary dynamics for both individual agents and the aggregate economy. The common tools used to address issues of indeterminacy and non-stationarity in representative-agent models in international macroeconomics fail to address these issues at both the individual and aggregate levels in our heterogeneous-agent model. We turn to a less common tool: incorporating wealth into the utility function.

In the following paragraphs, we introduce the primitive assumptions of our model and derive a series of propositions that demonstrate how heterogeneity in risk aversion can simultaneously produce individual carry trade positions and domestic bias in aggregate portfolios in economies with low interest rates. The model allows for exact aggregation across agents in the non-stochastic steady state and approximate aggregation locally around the steady state. We use perturbation methods to derive approximate closed-form solutions for the real exchange rate, real returns, and both individual and aggregate consumption, real wealth, and portfolio holdings, underscoring the model's tractability.

4.1 Model Primitives

There are two countries, home H and foreign F. To avoid repeating home and foreign versions of equations throughout the text, we use $i, j, k \in \{H, F\}$ to index countries and currencies and write all equations in terms of these indices. Equations with multiple indices should be understood to hold for any combination of index values unless we explicitly indicate otherwise.

A continuum of agents exists in each country. We denote the set of agents $\mathcal{H}_i$ in country i and we assume a measure of one for the set of agents. Each agent is uniquely identified within country by its coefficient of relative risk aversion, denoted ρ, so we use ρ as the index for agents. Each country has an identical distribution of risk aversion across agents, so each agent has a counterpart abroad with an identical coefficient of risk aversion.

We define an inverse Pareto density function to characterize the distribution of risk aversion across agents.[3] The density function, denoted $f(\rho)$ and given by

$$f(\rho) = \frac{\kappa}{\rho}\left(\frac{\rho}{\rho_m}\right)^{\kappa}, \tag{2}$$

has positive support over $\mathcal{R} = (0, \rho_m]$, where $\kappa > 1$ and $\rho_m > (\kappa^2 + \kappa)/(\kappa^2 - 1)$ are shape and scale parameters, respectively.[4] These parameter restrictions ensure that aggregate risk aversion is well-defined and sufficiently high. We illustrate the density function in the left panel of Figure 3. The density rises from zero as ρ approaches ρ_m from below, forming a left tail of agents with decreasing coefficients of risk aversion. Agents near the tip of the tail are nearly risk neutral; these will be the carry traders in our model.

Agent ρ in country i maximizes the expected present value of lifetime utility over consumption and wealth. Let $U_{it}(\rho)$ denote the agent's expected present value of lifetime utility,

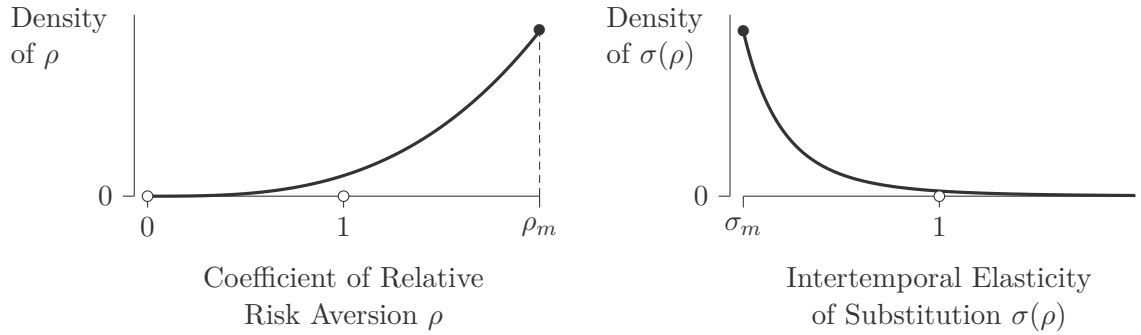
$$U_{it}(\rho) = \mathrm{E}_t\left[\sum_{s=t}^{\infty} \beta^{s-t}\Big(U_{Cis}(\rho) + U_{Wis}(\rho)\Big)\right], \tag{3}$$

where $\beta \in (0.5, 1)$ denotes the subjective discount factor, and where $U_{Cit}(\rho)$ and $U_{Wit}(\rho)$ denote the agent's period utility from consumption and real wealth, respectively. We assume constant relative risk aversion for $U_{Cit}(\rho)$ and constant absolute risk aversion for

[3] Our results do not depend on the specific distribution that we assume. For example, uniform, triangular, normal, or log-normal distributions also work. We only require the distribution to support a small measure of sufficiently risk-tolerant agents and to imply a sufficient level of aggregate risk aversion. The inverse Pareto distribution does, however, have some empirical support: Kimball et al. (2008) estimate a qualitatively similar empirical distribution from a number of large-scale surveys conducted in the United States and internationally, where respondents state their preferences over risky gambles related to employment and lifetime earnings. On the other hand, Calvet et al. (2021a) estimate a more symmetric empirical distribution of risk aversion using Swedish administrative data and assuming Epstein-Zin preferences.

[4] The agent with exactly $\rho = 1$ is measure zero, and assumed to have log utility.

Figure 3 – Distributions of Risk Aversion and Intertemporal Substitution



Notes. The figures plot the distribution of the coefficient of risk aversion ρ across individual agents (left) and the corresponding distribution of the elasticity of intertemporal substitution $\sigma(\rho)$ across individual agents (right) for an illustrative calibration of $\kappa = 4$, $\rho_m = 2$, and $\sigma_m = 1/2$. The point $\rho = \sigma(\rho) = 1$ is excluded from the support of both distributions. The two density functions plotted here represent equivalent heterogeneity across agents.

$U_{Wit}(\rho)$,

$$U_{Cit}(\rho) = \frac{1}{1-\rho}\left(\frac{C_{it}(\rho)}{C_i(\rho)}\right)^{1-\rho} \quad \text{and} \quad U_{Wit}(\rho) = -\frac{\theta}{\rho}\left(\frac{e^{W^i_{it}(\rho)}}{e^{W^i_i(\rho)}}\right)^{-\rho}, \tag{4}$$

where $C_{it}(\rho)$ denotes the agent's consumption basket, with subscript i denoting the agent's country of residence, and $W^i_{it}(\rho)$ denotes the agent's real wealth, with subscript i denoting the agent's country of residence and superscript i denoting the numéraire basket. Agents have internal habits set at the non-stochastic steady-state levels of consumption and real wealth, where the absence of time subscripts on these variables indicates their steady state levels.[5]

The parameter θ governs the importance of real wealth as a direct source of utility. We introduce wealth into the utility function for practical reasons: it ensures stationarity in the model even for arbitrarily small values of θ that have no significant impact on equilibrium outcomes. Throughout the paper, we assume $\theta \in [0, \epsilon)$, where ϵ is a small positive number. Nevertheless, wealth-in-utility can be economically justified by factors like social status and is used as a modeling device in recent studies by Michaillat and Saez (2021), He et al. (2023), Michau et al. (2023), and Zhao (2023). Unlike these papers, we show that wealth-in-utility achieves stationarity while preserving aggregability in the presence of heterogeneity.

The consumption utility function in (4) directly links the coefficient of risk aversion to the elasticity of intertemporal substitution for individual agents, meaning that heterogene-

[5]We choose these functional forms for tractability. Internal habits simplify the non-stochastic steady state and the aggregation of approximate equilibrium conditions. CARA utility over real wealth simplifies our specification of bond markets, because it admits the zero and negative values of real wealth that arise when bonds are in zero net supply. The model remains tractable if we instead specify CRRA utility over real wealth and bonds in positive net supply.

ity in the former implies heterogeneity in the latter. Letting $\sigma(\rho)$ denote the elasticity intertemporal of substitution with respect to consumption for agent ρ, the utility function in (4) implies

$$\sigma(\rho) = 1/\rho\,. \tag{5}$$

The inverse Pareto density function that we assume for ρ in (2) implies a Pareto density function for $\sigma(\rho)$ with shape parameter κ and scale parameter $\sigma_m = 1/\rho_m$. The Pareto density function falls in $\sigma(\rho)$, forming a right tail of agents with increasing elasticities of intertemporal substitution. We illustrate the density function for $\sigma(\rho)$ in the right panel of Figure 3.

Each agent's budget constraint ensures that real expenditure on consumption and bond holdings each period does not exceed the agent's real endowment income plus the gross real return on bond holdings from the previous period. We write the constraint as an equality,

$$C_{it}(\rho) + B^i_{iit}(\rho) + B^i_{ijt}(\rho) = \frac{P^i_{Cit}}{P^i_{it}} Y_{it}(\rho) + R^i_{it} B^i_{iit-1}(\rho) + R^i_{jt} B^i_{ijt-1}(\rho)\,, \quad i \neq j\,, \tag{6}$$

where $B^i_{ijt}(\rho)$ denotes the real value of bond holdings for agent ρ, with subscripts i and j denoting the agent's country of residence and the bond's country of issuance, respectively, and superscript i denoting the numéraire basket; P^i_{Cit} and P^i_{it} denote consumption good and basket prices, respectively, with subscript i denoting the country and superscript i denoting the numéraire currency; $Y_{it}(\rho)$ denotes agent ρ's real endowment, with subscript i denoting the agent's country of residence; and R^i_{jt} denotes the gross real return, with subscript j denoting the bond's country of issuance and superscript i denoting the numéraire basket. We state the equivalent nominal budget constraint in Appendix B.2.

We define agents' real wealth as the real value of their domestic and international nominal bond holdings, expressed in a common numéraire good. That is,

$$W^i_{it}(\rho) = B^i_{iit}(\rho) + B^i_{ijt}(\rho)\,, \quad i \neq j\,. \tag{7}$$

Nominal bonds are single-period assets that pay one unit of the issuer's currency at maturity with certainty. We define agents' real bond holdings and the gross real return as

$$B^i_{ijt}(\rho) = \frac{A_{ijt}(\rho) P^i_{Bjt}}{P^i_{it}} \quad \text{and} \quad R^i_{jt} = \frac{S_{ijt}/P^i_{Bjt-1}}{P^i_{it}/P^i_{it-1}}\,, \tag{8}$$

respectively, where $A_{ijt}(\rho)$ denotes the quantity of bonds held by agent ρ, with subscripts i and j denoting the bond holder's country and the bond's country of issuance, respectively; P^i_{Bjt} denotes the nominal bond price, with subscript j denoting the bond's country of issuance and superscript i denoting the numéraire currency; and S_{ijt} denotes the nominal

exchange rate in units of currency i per unit of currency j, with $S_{iit} = 1$ by definition.

All agents within a country receive the same endowment each period, and the endowment is a positive random variable that is independent and identically distributed over time. Since the measure of agents in each country equals one, each agent's endowment equals the aggregate endowment,

$$Y_{it}(\rho) = Y_{it}\,. \tag{9}$$

We normalize the steady-state endowment to one, $Y_i = 1$, where the absence of a time subscript indicates the steady state. Outside of the steady state, we assume the logarithm of endowments has finite variance.

Endowment goods trade internationally, and agents choose the quantity of each country's endowment good to consume. Agents differentiate between domestic and international endowment goods, and prefer variety in their consumption. We define each agent's consumption basket as

$$C_{it}(\rho) = \gamma C_{iit}(\rho)^{\alpha_{ii}} C_{ijt}(\rho)^{\alpha_{ij}}\,, \quad i \neq j\,, \tag{10}$$

where $C_{ijt}(\rho)$ denotes the quantity of good j that agent ρ in country i consumes, and where the parameter α_{ij} denotes country i's expenditure share on the endowment good of country j. Expenditure shares sum to one, $\alpha_{ii}+\alpha_{ij} = 1$, and we assume that the shares exogenously embed domestic bias in agents' consumption preferences, $\alpha_{ii} \in (1/2, 1)$. Agents decide the composition of their consumption baskets after first deciding on their total consumption expenditure in a two-stage budgeting procedure. Their total consumption expenditure therefore constrains their expenditure on domestic and international goods,

$$P^i_{it} C_{it}(\rho) = P^i_{Cit} C_{iit}(\rho) + P^i_{Cjt} C_{ijt}(\rho)\,, \quad i \neq j\,, \tag{11}$$

where P^j_{Cit} denotes the price of good i in numéraire currency j.

Because goods and nominal bonds are differentiated across countries, each trades in a separate market with a separate clearing condition. Market clearing requires that aggregate demand across countries equals aggregate supply,

$$C_{iit} + C_{jit} = Y_{it} \quad \text{and} \quad B^i_{iit} + B^i_{jit} = 0\,, \quad i \neq j\,, \tag{12}$$

where we assume zero net supply of each country's nominal bonds.

The law of one price holds for goods and nominal bonds, so goods prices across countries and bond prices across countries must be equalized when expressed in a common numéraire currency. That is,

$$P^i_{Cit} = S_{ijt} P^j_{Cit} \quad \text{and} \quad P^i_{Bit} = S_{ijt} P^j_{Bit}\,. \tag{13}$$

Because consumption baskets differ across countries, consumption basket prices also differ across countries and we define the real exchange rate as the ratio of consumption basket prices expressed in a common numéraire currency,

$$Q_{ijt} = S_{ijt} P^j_{jt} \Big/ P^i_{it} \,, \tag{14}$$

where Q_{ijt} denotes the real exchange rate in units of consumption basket i per unit of consumption basket j, with $Q_{iit} = 1$ by definition.

A quantity equation determines the price level in each country. We assume that each country's money supply is a strictly positive random variable that is independent and identically distributed over time. We assume symmetric money supplies across countries in the non-stochastic steady state, when expressed in a common numéraire currency,

$$M^i_{it} = Y_{it} P^i_{it} \quad \text{and} \quad M^i_i = S_{ij} M^j_j \,, \tag{15}$$

where M^j_{it} is the supply of money in country i in numéraire currency j, and where the absence of the time subscript indicates the steady state. Furthermore, we assume that the logarithms of money supplies have finite means and variances.

Because the model features four sources of uncertainty—home and foreign goods endowments and home and foreign money supplies—and only two nominal bonds, financial markets are incomplete. Goods endowments and money supplies are individually iid over time, but we allow them to covary contemporaneously within and across countries, while restricting their variance-covariance structure to ensure that portfolios with domestic bias offer lower expected real returns than portfolios with international bias in at least one country. Our restrictions allow for asymmetry in the variance-covariance structure across countries, but we do not require it. With i fixed as either H or F, we assume

$$\begin{gathered} \mathrm{E}\Big[\ln(M^i_{it}/M^j_{jt}) \ln(Y_{it}/Y_{jt})\Big] \geq 0 \,, \quad \mathrm{E}\Big[\ln(M^i_{it}/M^j_{jt}) \ln(Y_{it} Y_{jt})\Big] \geq 0 \,, \\ \text{and} \quad \mathrm{E}\Big[\ln(Y_{it})^2\Big] \geq \mathrm{E}\Big[\ln(Y_{jt})^2\Big] \,, \quad i \neq j \,. \end{gathered} \tag{16}$$

The simplest structure satisfying these restrictions is the symmetric structure with equal variances for the Home and Foreign log endowments and zero covariance between all possible pairs of log money supplies and log endowments, but the restrictions admit other, possibly asymmetric variance-covariance structures.

4.2 Model Propositions

We solve the model in two stages: first for the aggregate economy and then for individual agents, distinguishing between non-portfolio and portfolio problems at each stage. The model is tractable precisely because it separates into agent and aggregate, as well as

non-portfolio and portfolio problems.

This section is organized as follows. In Section 4.2.1 we state first-order optimality conditions. In Section 4.2.2, Propositions 1 and 2 characterize the non-stochastic steady state, where heterogeneity vanishes, contributing to the model's tractability. In Section 4.2.3, Propositions 3 and 4 introduce the concept of the aggregation wedge and demonstrate how first-order Taylor expansions of agent-specific Euler equations can be aggregated around the non-stochastic steady state. In Section 4.2.4, Proposition 5 establishes stationarity for the aggregate economy, and we solve for aggregate consumption, real wealth, and the real exchange rate. In Section 4.2.5, Propositions 6 through 9 introduce the portfolio valuation multiplier and establish sufficient conditions for domestic bias in aggregate portfolio holdings and a negative cross-country difference in real returns. In Section 4.2.6, we solve for agent-specific consumption and real wealth. In Section 4.2.7, Proposition 10, the culminating result of our model, states sufficient conditions for the existence of carry traders in low interest-rate economies with domestic bias in aggregate portfolio holdings.

These propositions provide a roadmap to our theoretical model. By incorporating heterogeneity in risk aversion, the model bridges a gap between finance and international macroeconomics, capturing interactions between individual investment decisions and macroeconomic outcomes while accommodating individual carry traders and domestic bias in aggregate portfolios within a tractable general equilibrium framework.

4.2.1 Utility Maximization.

Agents solve a two-stage utility maximization problem. In the first stage, agents maximize expected discounted lifetime utility in (3) by choosing quantities of the consumption basket to consume, and by constructing a portfolio of home and foreign nominal bonds. For agent ρ in country i, the first stage of the utility maximization problem yields the Euler equations,

$$\beta \, \mathrm{E}_t \left[\left(\frac{C_{it+1}(\rho)}{C_i(\rho)} \right)^{-\rho} \frac{1}{C_i(\rho)} R^i_{jt+1} \right] = \left(\frac{C_{it}(\rho)}{C_i(\rho)} \right)^{-\rho} \frac{1}{C_i(\rho)} - \theta \left(\frac{e^{W^i_{it}(\rho)}}{e^{W^i_i(\rho)}} \right)^{-\rho} . \tag{17}$$

If the wealth-in-utility parameter θ is positive, real wealth directly raises utility, in addition to the traditional indirect effect through consumption. A rise in present consumption that lowers real wealth will simultaneously lower utility from real wealth, and agents forming optimal consumption plans will account for this effect by subtracting the marginal utility of real wealth from the marginal utility of consumption in their Euler equations. This trade-off determines an optimal level of real wealth and ensures stationarity.

In the second stage, agents choose quantities of home and foreign goods to include

in their consumption baskets, subject to the plans they formed in the first stage of the problem. The second-stage problem yields demand equations for the home and foreign goods,

$$C_{ijt}(\rho) = \alpha_{ij} \frac{P_{it}^i}{P_{Cjt}^i} C_{it}(\rho)\,. \tag{18}$$

The demand functions in (18) can be rearranged to equate α_{ij} with the expenditure share on the good from country j for agents in country i.

Finally, the expenditure constraints in (11) and goods demands in (18) from agents' second-stage utility maximization problems together with the definition of the consumption basket in (10) imply an optimal price index. For the consumption basket in country i, the price index in numéraire currency k is

$$P_{it}^k = \left(P_{Cit}^k\right)^{\alpha_{ii}} \left(P_{Cjt}^k\right)^{\alpha_{ij}}, \quad i \neq j\,. \tag{19}$$

We derive the Euler equation in (17), the goods demands in (18), and the price index in (19) in Appendix B.3.

4.2.2 Non-Stochastic Steady State.

We now derive the non-stochastic steady-state equilibrium of the model, where all variables are constant and equal to their expected values. In the steady state, the model simplifies considerably because all heterogeneity vanishes.

Evaluating the agent-specific Euler equations in (17) at the non-stochastic steady state, we obtain

$$\theta C_i(\rho) = 1 - \beta R_j^i\,, \tag{20}$$

where the absence of time subscripts indicates the steady state. The left side of (20) is independent of j, so the steady-state real return on the home bond equals the steady-state real return on the foreign bond when returns are computed using a common numéraire currency, $R_i^i = R_j^i$.

If we set $\theta = 0$, real wealth drops out of the utility function. The agent-specific Euler equations in (20) simplify to $R_i^i = R_j^i = 1/\beta$ and impose no optimality conditions on steady-state consumption for individual agents. Without such conditions, consumption is only constrained by agents' budgets, but budget constraints alone cannot determine both consumption and real wealth, so the steady state is indeterminate. Assuming an exogenous steady-state distribution of real wealth across agents would not fully repair the model because the model is also non-stationary when we set $\theta = 0$, as we later show. For these reasons, we focus on positive values of θ in the neighborhood of zero, $\theta \in [0, \epsilon)$.

Numerous alternatives to wealth-in-utility have been proposed to induce determinacy

and stationarity in representative-agent models. Common alternatives include endogenous subjective discount factors, portfolio holding or adjustment costs, and overlapping generations, as discussed by Schmitt-Grohé and Uribe (2003) and Ghironi (2006). We find wealth in the utility function to be the simplest and most tractable device in our setting with heterogeneity in risk aversion, because other devices either hinder aggregation or introduce an incidental second dimension of heterogeneity to the model.

Because agents differ only in their aversion to risk and because risk is absent from the non-stochastic steady state, the steady-state distribution of consumption and real wealth across agents is degenerate, in the sense that all agents consume equal quantities and hold zero real wealth. This fact leads to a particularly simple steady-state solution for agent and aggregate non-portfolio variables and allows us to state the following proposition, which we prove in Appendix B.4.1.

Proposition 1 (Non-Stochastic Steady-State Non-Portfolio Equilibrium)**.** *Heterogeneity in risk aversion vanishes from the non-portfolio equilibrium conditions of the model in the non-stochastic steady state. The distribution of non-portfolio variables across agents is endogenously degenerate.*

Agent-specific and aggregate steady-state consumption baskets and goods demands are given by

$$C_i(\rho) = C_i = 1 \quad \textit{and} \quad C_{ij}(\rho) = C_{ij} = \alpha_{ij}\,, \tag{21}$$

steady-state price indices for consumption baskets, prices for consumption goods, and the real exchange rate are given by

$$P_i^j = P_{Ci}^j = M_i^j \quad \textit{and} \quad Q_{ij} = 1\,, \tag{22}$$

and agent-specific and aggregate steady-state real wealth and real returns on nominal bonds are given by

$$W_i^j(\rho) = W_i^j = 0 \quad \textit{and} \quad R_i^j = (1-\theta)/\beta\,. \tag{23}$$

Because real returns on domestic and international nominal bonds are equal in the steady state, we drop country and currency superscripts and henceforth write R.

We now turn to steady-state portfolios. In contrast to the non-portfolio steady state, the portfolio steady state does depend on agent and aggregate risk aversion. We derive steady-state portfolio solutions by extending to our heterogeneous-agent setting a method developed by Samuelson (1970) and Devereux and Sutherland (2011) for obtaining lower-order approximations of portfolio holdings from higher-order approximations of equilibrium conditions. Specifically, we derive order-zero approximate individual and aggregate portfolio holdings from second-order approximate equilibrium conditions. The latter conditions involve variance and covariance terms, which each agent evaluates differently according to their individual risk aversion. This fact produces a non-degenerate

distribution of portfolio holdings across agents, which we later characterize analytically. For now, we provide a limited characterization in the following proposition, which we prove in Appendix B.4.2.

Proposition 2 (Non-Stochastic Steady-State Portfolio Equilibrium)**.** *Agent-specific and aggregate steady-state portfolio holdings satisfy*

$$B_{ii}^{i}(\rho) + B_{ij}^{i}(\rho) = B_{ii}^{i} + B_{ij}^{i} = 0\,, \quad i \neq j\,. \tag{24}$$

Additionally, aggregate steady-state portfolio holdings satisfy

$$B_{ii}^{i} = B_{jj}^{i}\,, \tag{25}$$

while no such condition holds for agents, because bond markets must only clear in aggregate.

We later show that heterogeneity in agents' steady-state portfolios contributes to heterogeneity in consumption and real wealth dynamics through portfolio valuation effects. At the same time, the absence of heterogeneity in the non-portfolio steady state greatly simplifies aggregation, both in and around the steady state. We exploit this aggregability extensively in the following sections.

4.2.3 Model Aggregability

We aggregate variables across agents by integrating the product of agent variables and the density function $f(\rho)$,

$$\begin{gathered} Y_{it} = \int_{\mathcal{R}} Y_{it}(\rho) f(\rho)\,\mathrm{d}\rho\,, \quad C_{it} = \int_{\mathcal{R}} C_{it}(\rho) f(\rho)\,\mathrm{d}\rho\,, \quad C_{ijt} = \int_{\mathcal{R}} C_{ijt}(\rho) f(\rho)\,\mathrm{d}\rho\,, \\ W_{it}^{j} = \int_{\mathcal{R}} W_{it}^{j}(\rho) f(\rho)\,\mathrm{d}\rho\,, \quad \text{and} \quad B_{ijt}^{k} = \int_{\mathcal{R}} B_{ijt}^{k}(\rho) f(\rho)\,\mathrm{d}\rho\,, \end{gathered} \tag{26}$$

where the absence of an agent index indicates an aggregate variable. The set of agents in each country has a measure of one, so our model makes no distinction between aggregates and averages.

We apply the same procedure to agents' coefficients of risk aversion and intertemporal elasticities of substitution to obtain aggregates of these parameters. The simple inverse relationship between the agent-specific parameters in (5) breaks down in aggregate, due to a wedge that heterogeneity produces, as we emphasize in the following proposition and prove in Appendix B.6.2.

Proposition 3 (Aggregate Preference Parameters)**.** *We define the aggregate coefficient of risk aversion $\bar{\rho}$ and elasticity of intertemporal substitution $\bar{\sigma}$ as*

$$\bar{\rho} = \int_{\mathcal{R}} \rho f(\rho)\,\mathrm{d}\rho \quad \textit{and} \quad \bar{\sigma} = \int_{\mathcal{R}} \sigma(\rho) f(\rho)\,\mathrm{d}\rho\,, \tag{27}$$

respectively. Given the density function in (2)*, it follows that*

$$\bar{\sigma} = \omega/\bar{\rho}\,, \quad \textit{with} \quad \omega = \kappa^2/(\kappa^2 - 1)\,, \tag{28}$$

where ω denotes the aggregation wedge. The wedge exceeds one for finite values of the shape parameter κ, and approaches one in the limit as κ approaches infinity. As κ rises, the cross-sectional distribution of risk aversion collapses around the scale parameter ρ_m.

The aggregation wedge ω decouples aggregate risk preferences from aggregate intertemporal substitutability, offering an intermediate degree of flexibility between time-separable preferences with constant relative risk aversion and the recursive preferences of Epstein and Zin (1989).

Aggregating variables and parameters over agents is straightforward, but aggregating equilibrium conditions over agents is not. Complications arise in equilibrium conditions when an agent variable cannot be additively separated from the agent-specific coefficient of relative risk aversion. In particular, the agent Euler equations in (17) suffer from these complications and are not easily aggregable. However, in the non-stochastic steady state, heterogeneity vanishes and aggregating equilibrium conditions over agents becomes trivial. The absence of heterogeneity in the steady state makes perturbation methods particularly useful in our setting.

To approximate the model, we use Taylor expansions around the non-stochastic steady state. For positive variables like consumption baskets, goods demands, endowments, money supplies, prices, and exchange rates, we use hats to denote logarithmic deviations from the steady state. Since portfolio holdings and real wealth are not always positive, we expand these variables in levels around their steady-state values. For real wealth, hats indicate deviations relative to the steady-state endowment, $\hat{W}^j_{it}(\rho) = \big(W^j_{it}(\rho) - W^j_i(\rho)\big)/Y_j$, because steady-state real wealth is zero. For real returns, we also expand in levels, using hats to denote deviations relative to the steady-state level, $\hat{R}^j_{it} = \big(R^j_{it} - R\big)/R$.

The agent Euler equations in (17) present the only challenge to aggregation in our model, as they are the only equilibrium conditions where the agent-specific coefficient of relative risk aversion appears explicitly. However, with internal habits, this coefficient is additively separable from agent variables in first-order approximations, making the Euler equations first-order aggregable. We highlight this convenient properly in the following proposition, which we prove in Appendix B.6.3.

Proposition 4 (First-Order Aggregate Euler Equations)**.** *First-order Taylor expansions of agent Euler equations in* (17) *around the non-stochastic steady state are given by*

$$\hat{C}_{it}(\rho) = \theta \hat{W}^i_{it}(\rho) + \beta R\, \mathrm{E}_t\left[\hat{C}_{it+1}(\rho) - \frac{1}{\rho}\hat{R}^i_{jt+1}\right] + O(\epsilon^2)\,, \tag{29}$$

where the inverse of the agent-specific coefficient of relative risk aversion multiples the real return, an aggregate variable. Multiplying the agent-specific Euler equations by the density function in (2) *and integrating yields the aggregate Euler equations*

$$\hat{C}_{it} = \theta \hat{W}^i_{it} + \beta R \, \mathrm{E}_t \left[\hat{C}_{it+1} - \frac{\omega}{\bar{\rho}} \hat{R}^i_{jt+1} \right] + O(\epsilon^2) \,, \tag{30}$$

where the aggregation wedge ω defined in (28) *summarizes the first-order effect of heterogeneity on the aggregate economy.*

The agent-specific coefficient of relative risk aversion does not appear directly in the remaining agent-specific equilibrium conditions, and first-order Taylor expansions of these conditions allow for straightforward first-order aggregation. The first-order Taylor expansion of the agent-specific budget constraints in (6) and the corresponding aggregate budget constraints are given by

$$\begin{aligned} \hat{W}^i_{it}(\rho) + \hat{C}_{it}(\rho) &= R\hat{W}^i_{it-1}(\rho) + RB^i_{ii}(\rho)\left(\hat{R}^i_{it} - \hat{R}^i_{jt}\right) \\ &\quad + \hat{P}^i_{Cit} - \hat{P}^i_{it} + \hat{Y}_{it}(\rho) + O(\epsilon^2) \quad \text{and} \\ \hat{W}^i_{it} + \hat{C}_{it} &= R\hat{W}^i_{it-1} + RB^i_{ii}\left(\hat{R}^i_{it} - \hat{R}^i_{jt}\right) \\ &\quad + \hat{P}^i_{Cit} - \hat{P}^i_{it} + \hat{Y}_{it} + O(\epsilon^2) \,, \quad i \neq j \,. \end{aligned} \tag{31}$$

The first-order Taylor expansion of agent-specific goods demands in (18) and the corresponding aggregate goods demands are given by

$$\hat{C}_{ijt}(\rho) = \hat{P}^j_{it} - \hat{P}^j_{Cit} + \hat{C}_{it}(\rho) + O(\epsilon^2) \quad \text{and} \quad \hat{C}_{ijt} = \hat{P}^j_{it} - \hat{P}^j_{Cit} + \hat{C}_{it} + O(\epsilon^2) \,. \tag{32}$$

The remaining equilibrium conditions of the model depend solely on aggregate variables and require no aggregation. Their first-order Taylor expansions are standard and we provide them in Appendix B.5.

4.2.4 The Aggregate Non-Portfolio Problem

We find it convenient to solve the model first for cross-country differences. Using these solutions and market clearing conditions, we then derive country-specific solutions for aggregate variables.

The aggregate equilibrium conditions derived from first-order aggregation of agent-specific equilibrium conditions together with first-order approximations of the remaining of equilibrium conditions of the model produce a system of aggregate equations with no agent-specific variables. This aggregate system can be solved independently from the agent-specific equilibrium conditions of the model.

We solve the aggregate system first in terms of cross-country differenced aggregate consumption and real wealth. These solutions then determine the real exchange rate. To see that cross-country differences suffice, combine goods market clearing conditions in (12) with aggregate goods demand equations in (32) to obtain

$$\phi\hat{Y}_{i-jt} = \left(1-\phi^2\right)\hat{Q}_{ijt} + \phi^2\hat{C}_{i-jt} + O(\epsilon^2) \quad \text{with} \quad \phi \equiv \alpha_{ii} - \alpha_{ij}\,, \tag{33}$$

where the subscript $i-j$ denotes a cross-country difference. The parameter ϕ measures the strength of preferences for the domestic good, is symmetric across countries, and lies between zero and one, $\phi \in (0,1)$. Equation (33) states that the real exchange rate and cross-country difference in consumption adjust to clear international goods markets. The degree to which adjustment occurs through the real exchange rate or consumption depends on the strength of preferences for domestic goods and the tolerance of agents for consumption fluctuations.

The market clearing condition in (33) is one of three equations forming the cross-country differenced aggregate system. The other two are the cross-country differenced aggregate Euler equation and the cross-country differenced aggregate budget constraint. These equations depend on deflated nominal variables—specifically, aggregate real wealth and the real return on nominal bonds—and it is useful to express these in a common numéraire currency. Using definitions from (7) and (8), and noting that steady-state aggregate real wealth equals zero (Proposition 2), we obtain

$$\begin{aligned}
\hat{W}^i_{i-jt} &= \hat{W}^i_{it} - \hat{W}^i_{jt} = \hat{W}^i_{it} - \hat{W}^j_{jt} + O(\epsilon^2) \quad \text{and} \\
\hat{R}^i_{i-jt} &= \hat{R}^i_{it} - \hat{R}^i_{jt} = \hat{R}^i_{it} - \hat{R}^j_{jt} - \left(\hat{Q}_{ijt} - \hat{Q}_{ijt-1}\right) + O(\epsilon^2)\,,
\end{aligned} \tag{34}$$

where $\hat{W}^k_{i-jt}$ denotes the cross-country difference in aggregate real wealth and $\hat{R}^k_{i-jt}$ denotes the cross-country difference in real returns on nominal bonds, both in numéraire basket k.

With this notation in place, we write the cross-country differenced aggregate budget constraint and Euler equation as

$$\begin{aligned}
\hat{W}^i_{i-jt} + \hat{C}_{i-jt} &= R\hat{W}^i_{i-jt-1} - \frac{1-\phi}{\phi}\hat{Q}_{ijt} + \hat{Y}_{i-jt} + R\hat{V}^i_{i-jt} + O(\epsilon^2) \quad \text{and} \\
\hat{C}_{i-jt} - \theta\hat{W}^i_{i-jt} &= \beta R\,\mathrm{E}_t\left[\hat{C}_{i-jt+1} - \frac{\omega}{\bar{\rho}}\left(\hat{R}^i_{i-jt+1} + \hat{Q}_{ijt+1} - \hat{Q}_{ijt}\right)\right] + O(\epsilon^2)\,,
\end{aligned} \tag{35}$$

respectively, where we introduce the cross-country differenced aggregate portfolio valuation effect $\hat{V}^i_{i-jt}$, defined as

$$\hat{V}^i_{i-jt} \equiv B^i_{ii-ji}\hat{R}^i_{i-jt} + O(\epsilon^2) \quad \text{with} \quad B^i_{ii-ji} \equiv B^i_{ii} - B^i_{ji}\,. \tag{36}$$

The portfolio valuation effect is endogenous, but behaves as an independent and identically

distributed random variable, as shown by Devereux and Sutherland (2011). Consequently, we can temporarily treat the effect as if it were exogenous and derive intermediate solutions in terms of it. Later, we can use these intermediate solutions together with second-order Taylor expansions of the Euler equations in (17) to derive final solutions.

Using the market clearing condition in (33) to eliminate the real exchange from the cross-country differenced aggregate Euler equation and budget constraint in (35), we obtain a system of two first-order approximate difference equations,

$$\mathrm{E}_t\left[\hat{\boldsymbol{Z}}_{i-jt+1}\right] = \boldsymbol{\mathcal{E}}^{(-)}_{ZZ}\hat{\boldsymbol{Z}}_{i-jt} + \boldsymbol{\mathcal{E}}^{(-)}_{ZY}\hat{Y}_{i-jt} + \boldsymbol{\mathcal{E}}^{(-)}_{ZV}\hat{V}^{i}_{i-jt} + O(\epsilon^2)\,, \tag{37}$$

where $\hat{\boldsymbol{Z}}_{i-jt} = \begin{bmatrix}\hat{W}^{i}_{i-jt-1} & \hat{C}_{i-jt}\end{bmatrix}'$ denotes the (2×1) vector of cross-country differences in aggregate real wealth and aggregate consumption and where $\boldsymbol{\mathcal{E}}^{(-)}_{ZZ}$, $\boldsymbol{\mathcal{E}}^{(-)}_{ZV}$, and $\boldsymbol{\mathcal{E}}^{(-)}_{ZY}$ denote matrices of partial elasticities that depend only on the parameters of the model. The matrix $\boldsymbol{\mathcal{E}}^{(-)}_{ZZ}$, for example, is (2×2) with elements $\mathcal{E}^{(-)}_{mn}$, where $m, n \in \{W, C\}$. We provide a derivation of the cross-country differenced aggregate system in (37) in Appendix B.7, along with expressions for the full set of partial elasticities.

The cross-country differenced aggregate system in (37) has one non-predetermined variable and admits a unique non-explosive solution if the matrix of partial elasticities $\boldsymbol{\mathcal{E}}^{(-)}_{ZZ}$ has exactly one eigenvalue that lies strictly outside the unit circle, satisfying the conditions of Blanchard and Kahn (1980). If the second eigenvalue lies strictly inside the unit circle, the system is stationary.

Setting $\theta = 0$ removes wealth from the utility function and the aggregate system joins the class of non-stationary incomplete markets models discussed by Schmitt-Grohé and Uribe (2003) and Ghironi (2006). This knife-edge non-stationarity serves as a benchmark: by considering the neighborhood around $\theta = 0$, we establish a condition for stationarity, stated in the following proposition and proved in Appendix B.7.2.

Proposition 5 (Stationarity of the Cross-Country Differenced Aggregate System)**.** *Let $\theta \in [0, \epsilon)$, where ϵ is a small positive number. Then the matrix $\boldsymbol{\mathcal{E}}^{(-)}_{ZZ}$ has two distinct real-valued eigenvalues, $\lambda^{(-)}_{ZW}$ and $\lambda^{(-)}_{ZC}$. Furthermore, $\lambda^{(-)}_{ZW}\big|_{\theta=0} = 1/\beta$ and $\lambda^{(-)}_{ZC}\big|_{\theta=0} = 1$, and*

$$\partial\lambda^{(-)}_{ZW}/\partial\theta\,\big|_{\theta=0} > 0 \quad \textit{and} \quad \partial\lambda^{(-)}_{ZC}/\partial\theta\,\big|_{\theta=0} < 0\,. \tag{38}$$

Hence, there exists a $\theta > 0$ that yields a unique and stationary rational expectations solution to the cross-country differenced aggregate system in (37)*.*

The cross-country differenced aggregate system admits a unique and stationary solution when the wealth-in-utility parameter θ takes a small positive value, and we proceed under this assumption.[6]

[6]We remark that the aggregate cross-country differenced system also admits a unique and stationary solution when θ is negative and real wealth has a direct negative effect on utility.

To derive intermediate solutions to the aggregate system in (37), we first factorize the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(-)}_{ZZ}$ into a diagonal matrix of eigenvalues and a matrix of eigenvectors, decoupling the two difference equations in the system. We then impose the condition that the endogenous variables in the decoupled equations do not "explode too fast," as in Blanchard and Kahn (1980). This procedure yields intermediate solutions in terms of lagged aggregate real wealth, aggregate real portfolio valuation effects, and aggregate endowments in cross-country differences. Coefficients in the intermediate solution represent semi-partial elasticity—semi-partial because they fail to capture the effects of exogenous variables that act on aggregate real wealth and aggregate consumption indirectly through portfolio valuation effects.

Our intermediate solution to the cross-country differenced aggregate system in (37) is given by

$$\begin{bmatrix} \hat{W}^i_{i-jt} \\ \hat{C}_{i-jt} \end{bmatrix} = \boldsymbol{\eta}^{(-)}_{ZW} \hat{W}^i_{i-jt-1} + \boldsymbol{\eta}^{(-)}_{ZY} \hat{Y}_{i-jt} + \boldsymbol{\eta}^{(-)}_{ZV} \hat{V}^i_{i-jt} + O(\epsilon^2)\,, \tag{39}$$

where $\boldsymbol{\eta}^{(-)}_{ZW}$, $\boldsymbol{\eta}^{(-)}_{ZY}$, and $\boldsymbol{\eta}^{(-)}_{ZV}$ denote matrices of semi-partial elasticities that depend only on the parameters of the model.[7]

To obtain intermediate solutions for country-specific aggregate real wealth and consumption, we use the intermediate solutions for cross-country differences in (39) together with first-order Taylor expansions of the market clearing conditions in (12),

$$\hat{C}_{i+jt} = \hat{Y}_{i+jt} + O(\epsilon^2) \quad \text{and} \quad \hat{W}^i_{i+jt} = 0 + O(\epsilon^2)\,, \tag{40}$$

where the subscript $i+j$ denotes a cross-country sum. Intermediate solutions for country-specific aggregate consumption and real wealth are then obtained as

$$\hat{C}_{it} = \frac{1}{2}\Big(\hat{C}_{i+jt} + \hat{C}_{i-jt}\Big) + O(\epsilon^2) \quad \text{and} \quad \hat{W}^i_{it} = \frac{1}{2}\hat{W}^i_{i-jt} + O(\epsilon^2)\,, \tag{41}$$

using the market clearing conditions in (40) and the intermediate solutions for cross-country differences in (39).

We obtain an intermediate solution for the real exchange rate using the goods market clearing condition in (33) together with the intermediate solutions for cross-country differenced aggregate real wealth and aggregate consumption in (39),

$$\hat{Q}_{ijt} = \eta^{(-)}_{QW} \hat{W}^i_{i-jt-1} + \eta^{(-)}_{QY} \hat{Y}_{i-jt} + \eta^{(-)}_{QV} \hat{V}^i_{i-jt} + O(\epsilon^2)\,, \tag{42}$$

where the semi-partial elasticities $\eta^{(-)}_{QW}$, $\eta^{(-)}_{QY}$, and $\eta^{(-)}_{QV}$ depend on parameters of the model.

[7]We reiterate the distinction between partial and semi-partial elasticities using the example of $\mathcal{E}^{(-)}_{WW}$ and $\eta^{(-)}_{WW}$. The semi-partial elasticity $\eta^{(-)}_{WW}$ captures the indirect effect of past wealth on current wealth via consumption, but not via portfolio valuation effects. In contrast, the partial elasticity $\mathcal{E}^{(-)}_{WW}$ captures for neither indirect effect. Later, we will solve for general elasticities that capture both indirect effects.

We provide detailed derivations of the intermediate solutions for cross-country differenced aggregate real wealth and consumption and for the real exchange rate in Appendix B.7.3, along with expressions for the full set of semi-partial elasticities. The solutions are intermediate because they depend on the endogenous cross-country differenced portfolio valuation effect $\hat{V}^i_{i-jt}$ defined in (36). The portfolio valuation effect has two components: cross-country differenced aggregate portfolio holdings and cross-country differenced real returns on nominal bonds. In the following section, we turn to the aggregate portfolio problem and derive solutions for these two components.

4.2.5 The Aggregate Portfolio Problem

We now derive solutions for realized and expected returns, the portfolio valuation multiplier, and aggregate portfolio holdings. We first derive an intermediate solution for the cross-country differenced real return on nominal bonds. Using first-order Taylor expansions of the real returns defined in (8) and the quantity equations defined in (15) together with our intermediate solutions for cross-country differenced aggregate real wealth and consumption in (39), we obtain

$$\hat{R}^i_{i-jt} = \eta^{(-)}_{RY}\hat{Y}_{i-jt} + \eta^{(-)}_{RM}\hat{M}^{i-j}_{i-jt} + \eta^{(-)}_{RV}\hat{V}^i_{i-jt} + O(\epsilon^2)\,, \tag{43}$$

where $\hat{M}^{i-j}_{i-jt} \equiv \hat{M}^i_{it} - \hat{M}^j_{jt}$, and where the semi-partial elasticities $\eta^{(-)}_{RY}$, $\eta^{(-)}_{RM}$, and $\eta^{(-)}_{RV}$ depend only on the parameters of the model.

The valuation effect $\hat{V}^i_{i-jt}$ in (43) itself depends on the cross-country differenced real return. The expression can, therefore, be rearranged to isolate the cross-country differenced real return on the left-hand side,

$$\hat{R}^i_{i-jt} = \gamma^{(-)}_{RY}\hat{Y}_{i-jt} + \gamma^{(-)}_{RM}\hat{M}^{i-j}_{i-jt} + O(\epsilon^2)\,, \tag{44}$$

where $\gamma^{(-)}_{RY}$ and $\gamma^{(-)}_{RM}$ denote general elasticities. The elasticities in (44) are general in the sense that they account for general equilibrium effects of the exogenous domestic and international endowments and money supplies through all endogenous variables in the model, including portfolio valuation effects. These general elasticities are given by

$$\gamma^{(-)}_{RY} = \mu^{(-)}_{ii-ji}\eta^{(-)}_{RY} \quad \text{and} \quad \gamma^{(-)}_{RM} = \mu^{(-)}_{ii-ji}\eta^{(-)}_{RM} \quad \text{with} \quad \mu^{(-)}_{ii-ji} \equiv \frac{1}{1-\eta^{(-)}_{RV}B^i_{ii-ji}}\,, \tag{45}$$

where $\mu^{(-)}_{ii-ji}$ denotes the cross-country differenced portfolio valuation multiplier. The multiplier plays an important role in shaping the risk facing agents trading in bonds markets. In particular, domestic bias in aggregate portfolio holdings amplifies risk while international bias attenuates it. The following proposition, which we prove in Appendix B.8.1, emphasizes this point.

Proposition 6 (The Portfolio Valuation Multiplier)**.** *The semi-partial elasticity* $\eta^{(-)}_{RV}$ *is positive. Bounded domestic bias in aggregate steady-state portfolio holdings implies*

$$1 < \mu^{(-)}_{ii-ji} \quad \forall\, 0 < B^i_{ii-ji} < \frac{2}{\eta^{(-)}_{RV}}. \tag{46}$$

In this case, the multiplier amplifies the response of the cross-country differenced real return to fluctuations in endowments and money supplies. In contrast, international bias in aggregate steady-state portfolio holdings implies

$$0 \leq \mu^{(-)}_{ii-ji} < 1 \quad \forall\, B^i_{ii-ji} < 0. \tag{47}$$

In this case, the multiplier attenuates the response of the cross-country differenced real return to fluctuations in endowments and money supplies.

The portfolio valuation multiplier in (45) depends on cross-country differenced steady-state aggregate portfolio holdings B^i_{ii-ji}, which we have not yet determined. For this reason, (44) is not a final solution.

The first-order Taylor expansions of the aggregate equilibrium conditions used thus far leave aggregate portfolio holdings indeterminate. This indeterminacy arises because, to a first-order approximation, the bonds in our model are perfect substitutes, yielding identical expected returns. The Euler equations in (29) directly show this substitutability. Differencing across bonds, we have

$$\mathrm{E}_t\left[R^i_{it+1}\right] = \mathrm{E}_t\left[R^i_{jt+1}\right] + O(\epsilon^2)\,. \tag{48}$$

The first-order indeterminacy of portfolio holdings is a common feature of macroeconomic models with non-trivial portfolio choice, as Coeurdacier and Rey (2013) observe.

Higher-order approximation methods resolve this indeterminacy. Samuelson (1970) first developed these methods in partial equilibrium, assuming "riskiness" was "limited" and deriving order-n approximate solutions for portfolio holdings using order-$(n+2)$ approximate equilibrium conditions. Devereux and Sutherland (2011) later extended the method to general equilibrium, recognizing that Samuelson's assumption of limited risk aligns with perturbation methods used to solve dynamic stochastic general equilibrium models. We apply the Samuelson-Devereux-Sutherland method to derive aggregate portfolio holdings and later extend it to derive agent-specific portfolio holdings.

To derive order-zero approximate aggregate portfolio holdings, we require second-order approximate agent-specific Euler equations, which we then aggregate. Aggregation can be more challenging in second-order approximations because of quadratic terms, but these terms drop out when we difference Euler equations across domestic and international bonds, and the remaining terms aggregate straightforwardly. This fact allows us to state

the following proposition, which we prove in Appendix B.8.2.

Proposition 7 (Second-Order Approximate Agent and Aggregate Euler Equations)**.** *Differencing second-order Taylor expansions of the agent-specific Euler equations in* (17) *across bonds, and then differencing and summing across countries yields, respectively,*

$$\begin{aligned} 0 &= \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\hat{C}_{i-jt+1}(\rho) - \frac{1}{\rho}\hat{Q}_{ijt+1}\right)\right] + O(\epsilon^3) \quad \textit{and} \\ \frac{2}{\rho}\,\mathrm{E}_t\big[\hat{R}^i_{i-jt+1}\big] &= \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\hat{C}_{i+jt+1}(\rho) + \frac{1}{\rho}\hat{Q}_{ijt+1}\right)\right] + O(\epsilon^3)\,, \end{aligned} \tag{49}$$

where the inverse of the agent-specific coefficient of risk aversion multiples only aggregate variables. Multiplying by the density function in (2) *and integrating yields second-order approximate differenced and summed aggregate Euler equations, respectively,*

$$\begin{aligned} 0 &= \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\hat{C}_{i-jt+1} - \frac{\omega}{\bar{\rho}}\hat{Q}_{ijt+1}\right)\right] + O(\epsilon^3) \quad \textit{and} \\ \omega\frac{2}{\bar{\rho}}\,\mathrm{E}_t\big[\hat{R}^i_{i-jt+1}\big] &= \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\hat{C}_{i+jt+1} + \frac{\omega}{\bar{\rho}}\hat{Q}_{ijt+1}\right)\right] + O(\epsilon^3)\,. \end{aligned} \tag{50}$$

The second-order approximate cross-country differenced aggregate Euler equation in (50), together with our first-order approximate solutions, suffice to determine order-zero approximate aggregate portfolio holdings. The cross-country summed version of the same, together with our first-order approximate solutions, suffice to determine second-order approximate expected cross-country differenced real returns. We now solve for these two portfolio variables.

For portfolio holdings, we substitute the first-order approximate solutions (39) for $\hat{C}_{i-jt}$, (42) for $\hat{Q}_{ijt}$, and (44) for $\hat{R}^i_{i-jt+1}$ into the second-order approximate differenced Euler equation in (50). The final solution for B^i_{ii-ji} that obtains after substitution is somewhat unwieldy; we provide it in Appendix B.8.3 and focus here on an intermediate solution in terms of cross-country differenced real returns,

$$B^i_{ii-ji} = \zeta_B^{(-)}\frac{\mathrm{E}_t\big[\hat{R}^i_{i-jt+1}\hat{Y}_{i-jt+1}\big]}{\mathrm{E}_t\Big[\big(\hat{R}^i_{i-jt+1}\big)^2\Big]} + O(\epsilon)\,, \tag{51}$$

where $\zeta_B^{(-)}$ is a collection of model parameters, and where the conditional variance of the cross-country differenced real return and its conditional covariance with the cross-country differenced aggregate endowment are constant over time. The collection of parameters $\zeta_B^{(-)}$ is given by

$$\zeta_B^{(-)} = -\frac{\bar{\rho}\eta^{(-)}_{CY} - \omega\eta^{(-)}_{QY}}{\bar{\rho}\eta^{(-)}_{CV} - \omega\eta^{(-)}_{QV}}\,. \tag{52}$$

We use the bond market clearing condition in (33) together with the solution for cross-country differenced real bond holdings in (51) to obtain country-specific solutions,

$$B^i_{ii} = -B^i_{ij} = \frac{1}{2} B^i_{ii-ji} \,. \tag{53}$$

In Section 3, we showed empirically that aggregate portfolio holdings tend to favor domestic short-term debt in most countries. Our theoretical model produces this bias as an endogenous outcome when agents are sufficiently risk-averse on average. The intuition for this result is straightforward: risk averse agents want to hedge their exposure to endowment fluctuations, the cross-country differenced real return $\hat{R}^i_{i-jt+1}$ covaries negatively with the cross-country differenced aggregate endowment $\hat{Y}_{i-jt+1}$, and, therefore, a long position in the domestic asset funded by a short position in the international asset produces the desired hedge. We state the sufficient condition for domestic bias in aggregate portfolio holdings in the following proposition, which we prove in Appendix B.8.3.

Proposition 8 (Domestic Bias in Aggregate Portfolio Holdings)**.** *If $\bar{\rho} > \omega$, then steady-state aggregate portfolio holdings satisfy the order-zero approximate inequalities*

$$B^i_{ij} < 0 + O(\epsilon) < B^i_{ii} \,. \tag{54}$$

For finite values of the shape parameter κ from the distribution of risk aversion, the aggregation wedge ω exceeds one. As the shape parameter approaches infinity, heterogeneity in risk aversion vanishes and the aggregation wedge ω approaches one.

The condition that $\bar{\rho}$ exceeds ω is relatively mild when the distribution of risk aversion is sufficiently left-skewed, and is satisfied under our assumptions on ρ_m and κ. For example, the inverse Pareto distribution shown in Figure 3 with $\kappa = 4$ implies $\omega \approx 1.33$, while Lustig and Verdelhan (2007) assume a coefficient of risk aversion that exceeds 100, and Elminejad et al. (2022) estimate a median of 3.77 and a mean of 23.36 in a 92-study meta-analysis.

Having established sufficient conditions for domestic bias in aggregate portfolio holdings, we now turn to expected returns. We obtain a second-order approximation of expected cross-country differenced real returns by substituting the first-order approximate intermediate solutions (39) for $\hat{C}_{i-jt}$ and (42) for $\hat{Q}_{ijt}$ into the second-order approximate

summed Euler equation in (50). The resulting intermediate solution,

$$\begin{aligned}\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\Big] = \frac{\omega/\bar{\rho}}{2}\,\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\hat{Y}_{i+jt+1}\Big] \\ + \frac{\overset{(-)}{\eta}_{QY}}{2}\,\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\hat{Y}_{i-jt+1}\Big] \\ + \frac{\overset{(-)}{\eta}_{QV}}{2}\,\mathrm{E}_t\Big[\Big(\hat{R}^i_{i-jt+1}\Big)^2\Big] + O(\epsilon^3)\,,\end{aligned} \tag{55}$$

can be reduced to a final solution using first-order approximate realized real returns in (43) and the intermediate solution for aggregate cross-country differenced portfolio holdings in (51). In the following proposition, which we prove in Appendix B.8.4, we establish sufficient conditions for the cross-country differenced expected real return to be negative.

Proposition 9 (Negative Expected Cross-Country Differenced Real Returns)**.** *If $\bar{\rho} > \omega$, then the cross-country differenced expected real return satisfies the inequality*

$$\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\Big] < 0 + O(\epsilon^3) \tag{56}$$

and, to a second-order approximation, the domestic bond in country i offers a lower expected real return than the international bond after currency conversion.

Under Proposition 9, the real return on the domestic bond is lower than the real return on the international bond after accounting for movements in the real exchange rate. Intuitively, a relative rise in the domestic endowment lowers the domestic price level and raises the real return on the domestic nominal bond. At the same time, domestic agents with greater endowments import more, causing the real exchange rate to depreciate, which increases the real return on the international bond in terms of the domestic numéraire. The cross-country *differenced* real return will be negative when the depreciation effect dominates, which occurs when the preference for domestic goods is strong ($\phi > 0$) and aversion to consumption fluctuations is high ($\bar{\rho} > \omega$). Much of this intuition can be seen directly in the market clearing condition (33).

Under Propositions 8 and 9, country i holds an aggregate long position in the domestic bond and an aggregate short position in the international bond, leading to an expected negative real return. Thus, aggregate portfolios exhibit domestic bias despite low domestic interest rates. We now focus on individual agents to establish conditions for carry traders in such economies. We will divide the solution into non-portfolio and portfolio components, paralleling our approach for the aggregate economy.

4.2.6 The Agent Non-Portfolio Problem

Agent-specific and aggregate equilibrium conditions are structurally similar, so we solve for agent-specific variables in deviations from aggregate variables rather than from the

steady state. This approach not only provides analytical convenience but also strengthens the link between individual and aggregate solutions.

Our solution procedure follows the one used for aggregate variables, but for individual agents we must solve systems of equations in both cross-country differences and sums, because we cannot rely on market clearing conditions as we did in the aggregate problem. Once we have solutions for these differences and sums, we can derive country-specific solutions for agents' consumption, real wealth, and portfolio holdings.

Our solutions depend on two state variables: agent-specific and aggregate real wealth. Since each agent is measure zero, these variables evolve independently. The presence of aggregate real wealth as a second state variable does complicate the problem, but only slightly because agents treat it as exogenous in their decision-making.

We begin with the cross-country differenced system for individual agents. Because the steps to solve the cross-country summed system follow the same logic, we present the cross-country summed system in Appendix B.9.4 to save space.

Cross-Country Differences. We derive the cross-country differenced system for an individual agent from the first-order approximate individual and aggregate Euler equations in (29) and (30) and budget constraints in (31). We first take cross-country differences for the individual and aggregate equilibrium conditions separately, then take the difference between the individual and aggregate cross-country differences.

We obtain the first-order approximate cross-country differenced agent-specific Euler equation in deviations from aggregates,

$$\begin{aligned}&\Big(\hat{C}_{i-jt}(\rho)-\hat{C}_{i-jt}\Big)-\theta\Big(\hat{W}^i_{i-jt}(\rho)-\hat{W}^i_{i-jt}\Big)\\&\quad=\beta R\,\mathrm{E}_t\left[\Big(\hat{C}_{i-jt+1}(\rho)-\hat{C}_{i-jt+1}\Big)-\left(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\right)\Big(\hat{Q}_{ijt+1}-\hat{Q}_{ijt}\Big)\right]+O(\epsilon^2)\,,\end{aligned}\tag{57}$$

where the subscript $i-j$ on agent-specific variables denotes cross-country differences between agents with identical coefficients of relative risk aversion, and where the intermediate solution for the real exchange rate in (42) can be used to eliminate the real exchange rate. Similarly, we obtain the first-order approximate agent-specific budget constraint in deviations from aggregates,

$$\begin{aligned}\Big(\hat{W}^i_{i-jt}(\rho)-\hat{W}^i_{i-jt}\Big)&=R\Big(\hat{W}^i_{i-jt-1}(\rho)-\hat{W}^i_{i-jt-1}\Big)\\&\quad-\Big(\hat{C}_{i-jt}(\rho)-\hat{C}_{i-jt}\Big)+R\Big(\hat{V}^i_{i-jt}(\rho)-\hat{V}^i_{i-jt}\Big)+O(\epsilon^2)\,,\end{aligned}\tag{58}$$

where we introduce the cross-country differenced agent portfolio valuation effect $\hat{V}^i_{i-jt}(\rho)$, defined as

$$\hat{V}^i_{i-jt}(\rho)\equiv B^i_{ii-ji}(\rho)\hat{R}^i_{i-jt}\quad\text{with}\quad B^i_{ii-ji}(\rho)\equiv B^i_{ii}(\rho)-B^i_{ji}(\rho)\,.\tag{59}$$

In the Euler equation (57) and the budget constraint (58), the agent's consumption always appears as a deviation from aggregate consumption, which never enters separately. Therefore, we can treat the deviation of an individual's consumption from the aggregate as a single variable. Similarly, the agent's real wealth appears as a deviation from aggregate real wealth, but aggregate real wealth also affects the equations separately through the real exchange rate in (42). As a result, we must treat individual and aggregate real wealth as separate state variables and include the law of motion for aggregate real wealth (39) in the agent's cross-country differenced system.

The agent's cross-country differenced system is then given by three simultaneous difference equations: the Euler equation in (57), the budget constraint in (58), and the law of motion for aggregate real wealth in (39). After eliminating the real exchange rate using the intermediate solution in (42), we write the system in matrix form as

$$\begin{aligned} \mathrm{E}_t\Big[\hat{\boldsymbol{Z}}_{i-jt+1}(\rho)\Big] = \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}\hat{\boldsymbol{Z}}_{i-jt}(\rho) + \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Y}\hat{Y}_{i-jt} \\ + \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V(\rho)}\hat{V}^i_{i-jt}(\rho) + \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V}\hat{V}^i_{i-jt} + O(\epsilon^2)\,, \end{aligned} \tag{60}$$

where $\hat{\boldsymbol{Z}}_{i-jt}(\rho) = \Big[\hat{W}^i_{i-jt-1}(\rho) \quad \hat{C}_{i-jt}(\rho) - \hat{C}_{i-jt} \quad \hat{W}^i_{i-jt-1}\Big]'$ denotes the (3×1) vector of the agent's cross-country differenced real wealth, cross-country differenced consumption in deviation from aggregate, and cross-country differenced aggregate real wealth, and where $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$, $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Y}$, $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V(\rho)}$, and $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V}$ denote matrices of partial elasticities. For example, $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ is (3×3) with elements $\mathcal{E}^{(-)}_{mn}$, $m, n \in \{W(\rho), C(\rho), W\}$. We provide a derivation of the agent's cross-country differenced system (60) in Appendix B.9.1, along with expressions for the full set of partial elasticities.

In Proposition B1 in Appendix B.9.2, we establish uniqueness and stationarity for the agent cross-country differenced system. There, we follow the same strategy as we did with the aggregate cross-country differenced system, considering a neighborhood of small positive values for θ near the knife-edge non-stationary value of $\theta = 0$ to establish the result.

As we did for the aggregate system, we proceed under the assumption that θ takes a small positive value, and derive an intermediate solution to the agent's cross-country differenced system in (60) by factorizing the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ into a diagonal matrix of eigenvalues and a matrix of eigenvectors and by imposing the condition that the endogenous variables in the system do not "explode too fast," again as in Blanchard and Kahn (1980). Our intermediate solutions to the agent's non-portfolio problem depend on the agent's lagged real wealth in deviation from aggregate, lagged aggregate real wealth independently, endowments, and portfolio valuation effects. Later, once the agent's portfolio problem has been solved, these intermediate non-portfolio solutions will combine with portfolio solutions to yield general solutions.

We introduce the "vanishing integral property" for deviations of agent variables from

aggregate variables, denoting such deviations with tildes. Aggregating these variables across agents results in zero because the deviations average out. These variables remain specific to individual agents and retain the index ρ. For real wealth, consumption, and portfolio valuation effects, the deviations are expressed as

$$\begin{aligned} \tilde{W}^i_{i-jt}(\rho) = \hat{W}^i_{i-jt}(\rho) - \hat{W}^i_{i-jt}\,, \quad \tilde{C}_{i-jt}(\rho) = \hat{C}_{i-jt}(\rho) - \hat{C}_{i-jt}\,, \\ \text{and} \quad \tilde{V}^i_{i-jt}(\rho) = \hat{V}^i_{i-jt}(\rho) - \hat{V}^i_{i-jt}\,. \end{aligned} \tag{61}$$

Some semi-partial elasticities in our intermediate solutions below will also have this property, and we also denote these with tildes. All semi-partial elasticities without tildes are independent of agent-specific coefficients of relative risk aversion and therefore common to all agents.

With this notation in place, we write our intermediate solution for the agent's cross-country differenced system in deviations from aggregates as

$$\begin{aligned} \begin{bmatrix} \tilde{W}^i_{i-jt}(\rho) \\ \tilde{C}_{i-jt}(\rho) \end{bmatrix} &= \boldsymbol{\eta}^{(-)}_{Z(\rho)W(\rho)} \tilde{W}^i_{i-jt-1}(\rho) + \boldsymbol{\eta}^{(-)}_{Z(\rho)V(\rho)} \tilde{V}^i_{i-jt}(\rho) \\ &\quad + \tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)V} \hat{V}^i_{i-jt} + \tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)Y} \hat{Y}_{i-jt} + O(\epsilon^2)\,, \end{aligned} \tag{62}$$

where $\boldsymbol{\eta}^{(-)}_{Z(\rho)W(\rho)}$, $\boldsymbol{\eta}^{(-)}_{Z(\rho)V(\rho)}$, $\tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)V}$, and $\tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)Y}$ denote matrices of semi-partial elasticities. The tilde notation emphasizes the consistency between agent and aggregate problems, as aggregating 62 across agents results in zero. We derive this intermediate solution and provide expressions for the full set of semi-partial elasticities in Appendix B.9.3.

Cross-Country Sums. We solve the cross-country summed system using the same method as for the cross-country differenced system. The summed system is simpler because it does not involve the real exchange rate and has only one predetermined state variable, agent-specific real wealth.

For brevity, we present solution in Appendix B.9.6. There, we establish stationarity and uniqueness in Proposition B2, before solving for cross-country summed agent consumption and real wealth in deviations from aggregates. The solution depend on lagged real wealth, lagged endowments, and portfolio valuation effects, and its derivation mirrors that of the differenced system. Together, the cross-country differenced and summed solutions yield country-specific solutions for agent consumption and real wealth.

4.2.7 The Agent Portfolio Problem

In our model, agents endogenously choose a continuum of portfolio positions according to their heterogeneous risk preferences. Depending on these preferences, agents become hedgers, speculators, or hand-to-mouth consumers. Relatively risk-tolerant agents opt for

speculative carry trade positions.

We establish this result by extending the Samuelson-Devereux-Sutherland method of higher-order approximation to our heterogeneous-agent setting. This extension allows us to derive closed-form solutions for the portfolio positions of individual agents in a general equilibrium continuum economy. Intuitively, agents with higher risk aversion prefer portfolios with less volatility and lower average returns, leading to consumption that is less volatile and lower on average. In contrast, agents with lower risk aversion choose portfolios with more volatility and higher average returns, resulting in consumption that is more volatile and higher on average. This model generates rich heterogeneity in investment and consumption patterns across agents.

The portfolio problem of individual agents differs from the aggregate portfolio problem because there are no bond market clearing conditions for individual agents, unlike for the aggregate economy. Instead, we use the second-order approximate cross-country differenced and summed agent Euler equations in (49) to derive order-zero approximate cross-country differenced and summed agent portfolio holdings, respectively. We then combine these results to obtain country-specific solutions.

By differencing the second-order approximate cross-country differenced agent and aggregate Euler equations in (49) and (50), we obtain an Euler equation with cross-country differenced agent-specific consumption expressed in deviation from aggregate,

$$0 = \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\tilde{C}_{i-jt+1}(\rho) - \left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\hat{Q}_{ijt+1}\right)\right] + O(\epsilon^3)\,. \tag{63}$$

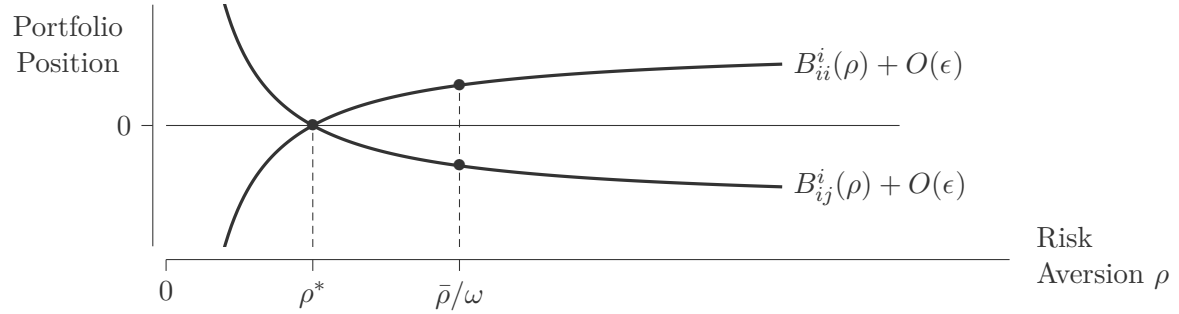
Combining (63) with our intermediate solution in (62) for agent-specific cross-country differenced consumption and the intermediate solution for the real exchange rate in (42), we obtain an intermediate order-zero approximate solution for agent-specific cross-country differenced holdings of domestic bonds in deviation from aggregate,

$$\tilde{B}^i_{ii-ji}(\rho) = \zeta^{(-)}_{B(\rho)}\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\frac{\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\hat{Y}_{i-jt+1}\right]}{\mathrm{E}_t\left[\left(\hat{R}^i_{i-jt+1}\right)^2\right]} + O(\epsilon)\,, \tag{64}$$

where $\tilde{B}^i_{ii-ji}(\rho) \equiv B^i_{ii-ji}(\rho) - B^i_{ii-ji}$, and where the variance of the cross-country differenced real return and its covariance with the cross-country differenced endowment are constant. Here, $\zeta^{(-)}_{B(\rho)}$ is a collection of model parameters independent of any individual agent's coefficient of relative risk aversion. We provide an expression for $\zeta^{(-)}_{B(\rho)}$ in Appendix B.10.

By differencing the second-order approximate cross-country summed agent and aggregate Euler equations (49) and (50), we obtain an Euler equation with cross-country

Figure 4 – Cross-Section of Agent Portfolio Holdings



Notes. The figure shows portfolio holdings of the domestic bond $B^i_{ii}(\rho)$ and international bond $B^i_{ij}(\rho)$ for agents in country i, assumed to be the low-interest rate economy. The horizontal axis shows agents' coefficients of relative risk aversion and the vertical axis shows agents' portfolio positions. Agents' domestic bond positions rise in risk aversion, while their international bond positions fall. The agent with $\rho = \rho^*$ is hand-to-mouth. Agents with $\rho < \rho^*$ are carry traders with short domestic bond positions and long international bond positions. The agent with $\rho = \bar{\rho}/\omega$ holds a portfolio with domestic bias that matches the aggregate portfolio.

summed agent-specific consumption expressed in deviation from aggregate,

$$0 = \mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\tilde{C}_{i+jt+1}(\rho) - \frac{\bar{\rho}}{\omega}\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\hat{Y}_{i+jt+1}\right)\right] + O(\epsilon^3)\,. \tag{65}$$

Combining (65) with the intermediate solution for cross-country summed agent-specific consumption in appendix equation (B114), we obtain an intermediate order-zero approximate solution for agent-specific cross-country summed bond holdings,

$$\tilde{B}^i_{ii+ji}(\rho) = \zeta^{(+)}_{B(\rho)}\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\frac{\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\hat{Y}_{i+jt+1}\right]}{\mathrm{E}_t\left[\left(\hat{R}^i_{i-jt+1}\right)^2\right]} + O(\epsilon)\,, \tag{66}$$

where $\tilde{B}^i_{ii+ji}(\rho) \equiv B^i_{ii+ji}(\rho) - B^i_{ii+ji}$, and where the variance of the cross-country differenced real return and its covariance with the cross-country summed endowment are constant. Here, $\zeta^{(+)}_{B(\rho)}$ is a collection of model parameters independent of agent-specific coefficients of relative risk aversion. We provide an expression for $\zeta^{(+)}_{B(\rho)}$ in Appendix B.10.

Combining agent-specific cross-country differenced and summed domestic bond holdings, we obtain an intermediate country-specific solution,

$$\tilde{B}^i_{ii}(\rho) = \frac{1}{2}\left(\tilde{B}^i_{ii-ji}(\rho) + \tilde{B}^i_{ii+ji}(\rho)\right) + O(\epsilon)\,, \tag{67}$$

where $\tilde{B}^i_{ii}(\rho) \equiv B^i_{ii}(\rho) - B^i_{ii}$, and where $B^i_{ii}(\rho) = -B^i_{ij}(\rho)$ from Proposition 2. A final solution for $\tilde{B}^i_{ii}(\rho)$ can be obtained directly from (67) using (43), (64), and (66).

In countries where portfolios are biased towards low interest-rate domestic bonds, aggregate portfolios do not resemble aggregate carry trades. To model carry trades in such

economies, individual carry traders must be outweighed by agents who favor domestic assets. Our heterogeneous-agent model captures this cross-sectional pattern of investment, as we establish in the following culminating proposition, which we in Appendix B.10.1.

Proposition 10 (International Bias in Agent-Specific Portfolio Holdings)**.** *If $\bar{\rho} > \omega$, then a threshold level of risk aversion $\rho^* \in (0, \bar{\rho}/\omega)$ exists such that*

$$\begin{aligned} B^i_{ij} < 0 + O(\epsilon) < B^i_{ii} \quad \textit{while} \\ B^i_{ii}(\rho) < 0 + O(\epsilon) < B^i_{ij}(\rho) \quad \forall \rho < \rho^* \,. \end{aligned} \tag{68}$$

To an order-zero approximation, the aggregate portfolio position in country i exhibits domestic bias while individual portfolio positions exhibit international bias for agents with coefficients of relative risk aversion below the threshold value ρ^*.

Figure 4 illustrates order-zero approximate holdings of domestic and international bonds for the cross-section of individual agents in country i. The illustration assumes that country i's aggregate portfolio is biased towards the low-return domestic asset. The agent with $\rho = \rho^*$ is hand-to-mouth, agents with $\bar{\rho} < \rho^*$ hold carry trade portfolios, and agents with $\bar{\rho} > \rho^*$ hold domestically-biased portfolios. The agent with $\rho = \bar{\rho}/\omega$ holds exactly the aggregate portfolio. In this way, the model endogenizes a continuum of investor types within a tractable, two-country, dynamic stochastic general equilibrium framework.

5 Conclusions

The currency carry trade literature has focused mostly on rationalizing the carry trade return while ignoring the portfolio position. But realizing a carry trade return requires an investor to hold a carry trade portfolio. Empirically, we document strong domestic bias in aggregate debt holdings by country and currency of issuance. For countries with low interest rates like Germany, Japan, and the United States, domestic bias implies the opposite of an aggregate carry trade. Individual carry traders in these countries must, therefore, be outweighed by agents who favor domestic-currency assets. Representative-agent models cannot accommodate this heterogeneity. A satisfying theory of the carry trade should rationalize both the return and the position, while confronting the empirical fact that most agents are not carry traders.

Our empirical findings highlight a tension between the finance literature on the carry trade and the international macroeconomics literature on domestic bias in aggregate portfolios. We resolve this tension using a theoretical model that features heterogeneity in risk aversion and captures a full range of individual portfolio positions. To solve the model, we extend the Samuelson-Devereux-Sutherland solution method of higher-order approximations to heterogeneous agents. The model is determinate, stationary,

and highly tractable, admitting closed-form solutions for both individual and aggregate variables in a familiar discrete-time setting, producing rich heterogeneity in investment and consumption behavior. Individual portfolio positions in the model endogenously range from speculative carry trades, to hand-to-mouth positions, to domestically-biased hedges, while the aggregate portfolio displays domestic bias. The model reconciles the existence of individual carry traders with the empirical observation of domestic bias in aggregate portfolios in low-interest rate countries.

At the core of the model is a trade-off between the hedge that portfolios with domestic bias provide and the lower returns they offer. When the Home endowment rises relative to Foreign, the price of the Home good falls and the real return on the Home nominal bond rises. Home agents consume more of both Home and Foreign goods, boosting imports and depreciating the Home real exchange rate. Consequently, the real return on the Foreign nominal bond also rises for Home agents. The *difference* in real returns—Home minus Foreign—depends on the responsiveness of the real exchange rate, which is greater when agents have higher average risk aversion and a stronger preference for the domestic good. The real exchange rate response dominates in our model, and the Foreign real return rises above Home, rendering long positions in the Home bond combined with short positions in the Foreign bond an effective hedge against consumption growth risk. Agents with higher risk aversion take this hedge, accepting a negative difference in returns, while agents with lower risk aversion take carry trades to exploit a positive difference in returns.

References

Acharya, V. V. and Steffen, S. (2015). The "greatest" carry trade ever? understanding eurozone bank risks. *Journal of Financial Economics*, 115(2):215–236.

Aguiar, M. A., Bils, M., and Boar, C. (2020). Who Are the Hand-to-Mouth? NBER Working Papers 26643, National Bureau of Economic Research, Inc.

Alfaro, L., Calani, M., and Varela, L. (2024). Firms, currency hedging, and financial derivatives. In *Handbook of Financial Integration*, pages 340–370. Edward Elgar Publishing.

Allayannis, G., Ihrig, J., and Weston, J. P. (2001). Exchange-rate hedging: Financial versus operational strategies. *American Economic Review*, 91(2):391–395.

Allayannis, G. and Ofek, E. (2001). Exchange rate exposure, hedging, and the use of foreign currency derivatives. *Journal of international money and finance*, 20(2):273–296.

Allen, M. C., Gautam, D., and Juvenal, L. (2023). *Currencies of external balance sheets*. International Monetary Fund.

Barro, R. J. (2006). Rare disasters and asset markets in the twentieth century. *The Quarterly Journal of Economics*, 121(3):823–866.

Bartram, S. M. (2019). Corporate hedging and speculation with derivatives. *Journal of Corporate Finance*, 57:9–34.

Bartram, S. M., Brown, G. W., and Minton, B. A. (2010). Resolving the exposure puzzle: The many facets of exchange rate exposure. *Journal of Financial Economics*, 95(2):148–173.

Beer, C., Ongena, S., and Peter, M. (2010). Borrowing in foreign currency: Austrian households as carry traders. *Journal of Banking & Finance*, 34(9):2198–2211.

Blanchard, O. J. and Kahn, C. M. (1980). The solution of linear difference models under rational expectations. *Econometrica*, 48(5):1305.

Bräuer, L. and Hau, H. (2022). Can time-varying currency risk hedging explain exchange rates? Technical report, CESifo.
Brunnermeier, M. K., Nagel, S., and Pedersen, L. H. (2009). Carry Trades and Currency Crashes. In *NBER Macroeconomics Annual 2008, Volume 23*, NBER Chapters. National Bureau of Economic Research, Inc.
Bruno, V. and Shin, H. S. (2017). Global dollar credit and carry trades: a firm-level analysis. *The Review of Financial Studies*, 30(3):703–749.
Burger, J. D. and Warnock, F. E. (2007). Foreign participation in local currency bond markets. *Review of Financial Economics*, 16(3):291–304.
Burger, J. D., Warnock, F. E., and Warnock, V. C. (2018). Currency matters: Analyzing international bond portfolios. *Journal of International Economics*, 114:376–388.
Burietz, A. and Ureche-Rangau, L. (2020). Better the devil you know: Home and sectoral biases in bank lending. *International Economics*, 164:69–85.
Burnside, C. (2011). The cross section of foreign currency risk premia and consumption growth risk: Comment. *American Economic Review*, 101(7):3456–76.
Burnside, C., Eichenbaum, M., Kleshchelski, I., and Rebelo, S. (2011). Do peso problems explain the returns to the carry trade? *The Review of Financial Studies*, 24(3):853–891.
Burnside, C., Eichenbaum, M. S., Kleshchelski, I., and Rebelo, S. (2006). The returns to currency speculation.
Calvet, L. E., Campbell, J. Y., Gomes, F., and Sodini, P. (2021a). The cross-section of household preferences. Technical report, National Bureau of Economic Research.
Calvet, L. E., Campbell, J. Y., Gomes, F., and Sodini, P. (2021b). The Cross-Section of Household Preferences. NBER Working Papers 28788, National Bureau of Economic Research, Inc.
Coeurdacier, N. and Gourinchas, P.-O. (2016). When bonds matter: Home bias in goods and assets. *Journal of Monetary Economics*, 82:119–137.
Coeurdacier, N. and Rey, H. (2013). Home bias in open economy financial macroeconomics. *Journal of Economic Literature*, 51(1):63–115.
Conte, M., P. C. and Mayer, T. (2022). The cepii gravity database. CEPII Working Paper N °2022-05.
Coppola, A., Maggiori, M., Neiman, B., and Schreger, J. (2021). Redrawing the map of global capital flows: The role of cross-border financing and tax havens. *The Quarterly Journal of Economics*, 136(3):1499–1556.
Curcuru, S. E., Vega, C., and Hoek, J. J. (2011). Measuring carry trade activity. *IFC Bulletins Chapters*, 34:436–453.
Davis, J. S. and Van Wincoop, E. (2024). A theory of capital flow retrenchment. *Journal of International Economics*, 150:103952.
Devereux, M. B. and Sutherland, A. (2010). Country portfolio dynamics. *Journal of Economic Dynamics and Control*, 34(7):1325–1342.
Devereux, M. B. and Sutherland, A. (2011). Country Portfolios In Open Economy Macro-Models. *Journal of the European Economic Association*, 9(2):337–369.
Du, W. and Schreger, J. (2016). Local currency sovereign risk. *The Journal of Finance*, 71(3):1027–1070.
Du, W. and Schreger, J. (2022). Sovereign risk, currency risk, and corporate balance sheets. *The Review of Financial Studies*, 35(10):4587–4629.
Elminejad, A., Havranek, T., and Irsova, Z. (2022). Relative risk aversion: a meta-analysis.
Engel, C. M. (1984). Testing for the absence of expected real profits from forward market speculation. *Journal of International Economics*, 17(3-4):299–308.
Epstein, L. G. and Zin, S. E. (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica*, 57(4):937–969.
European Central Bank (2014a). Implementing the new balance of payments and international investment position manual (bpm6). http://www.ecb.europa.eu/stats/external/bpm6/html/index.en.html.
European Central Bank (2014b). *The International Role of the Euro*. European Central

Bank, Frankfurt am Main, Germany.
Evans, M. D. and Hnatkovska, V. (2012). A method for solving general equilibrium models with incomplete markets and many financial assets. *Journal of Economic Dynamics and Control*, 36(12):1909–1930.
Faia, E., Salomao, J., and Ventula Veghazy, A. (2022). Granular Investors and International Bond Prices: Scarcity-Induced Safety. CEPR Discussion Papers 17454.
Fama, E. F. (1984). Forward and spot exchange rates. *Journal of Monetary Economics*, 14(3):319–338.
Farhi, E. and Gabaix, X. (2016). Rare disasters and exchange rates. *The Quarterly Journal of Economics*, 131(1):1–52.
Fidora, M., Fratzscher, M., and Thimann, C. (2007). Home bias in global bond and equity markets: the role of real exchange rate volatility. *Journal of international Money and Finance*, 26(4):631–655.
Filipe, S. F., Nissinen, J., and Suominen, M. (2023). Currency carry trades and global funding risk. *Journal of Banking & Finance*, 149:106800.
Florez-Orrego, S., Maggiori, M., Schreger, J., Sun, Z., and Tinda, S. (2023). Global capital allocation. *Annual Review of Economics*, 16.
Fong, W. M. (2013). Footprints in the market: Hedge funds and the carry trade. *Journal of International Money and Finance*, 33:41–59.
Gabaix, X. and Maggiori, M. (2015). International liquidity and exchange rate dynamics. *The Quarterly Journal of Economics*, 130(3):1369–1420.
Gagnon, J. and Chaboud, A. (2007). What can the data tell us about carry trades in japanese yen? *FRB International Finance Discussion Paper*, (899).
Galati, G., Heath, A., and McGuire, P. (2007). Evidence of carry trade activity. *BIS Quarterly Review*.
Garleanu, N. and Panageas, S. (2015). Young, old, conservative, and bold: The implications of heterogeneity and finite lives for asset pricing. *Journal of Political Economy*, 123(3):670 – 685.
Gelman, M. (2021). What drives heterogeneity in the marginal propensity to consume? temporary shocks vs persistent characteristics. *Journal of Monetary Economics*, 117:521–542.
Ghironi, F. (2006). Macroeconomic interdependence under incomplete markets. *Journal of International Economics*, 70(2):428 – 450.
Gruić, B. and Wooldridge, P. D. (2012). Enhancements to the BIS debt securities statistics. *BIS Quarterly Review December*.
Guu, S.-M. and Judd, K. L. (2001). Asymptotic methods for asset market equilibrium analysis. *Economic Theory*, 18(1):127–157.
Guvenen, F. (2009). A parsimonious macroeconomic model for asset pricing. *Econometrica*, 77(6):1711–1750.
Hacioğlu-Hoke, S., Ostry, D., Rey, H., Rousset Planat, A., Stavrakeva, V., and Tang, J. (2024). Topography of the fx derivatives market: a view from london. Technical report, Bank of England.
Hansen, L. P. and Hodrick, R. J. (1980). Forward exchange rates as optimal predictors of future spot rates: An econometric analysis. *Journal of political economy*, 88(5):829–853.
He, Q., Luo, Y., Nie, J., and Zou, H.-f. (2023). Money, growth, and welfare in a schumpeterian model with the spirit of capitalism. *Review of Economic Dynamics*, 47:346–372.
International Monetary Fund (1993). *Balance of Payments Manual*. International Monetary Fund, USA, fifth edition edition.
International Monetary Fund (1997). *Coordinated Portfolio investment Survey: Survey Guide*. International Monetary Fund, USA.
International Monetary Fund (2002). *Coordinated Portfolio Investment Survey Guide*. International Monetary Fund, USA, second edition.
International Monetary Fund (2009). *Balance of Payments Manual*. International Monetary Fund, USA, sixth edition edition.
International Monetary Fund (2010). *Handbook on Securities Statistics, Part 2: Debt*

Securities Holdings. International Monetary Fund, USA.
International Monetary Fund (2023a). *International Financial Statistics: Introductory Notes*. International Monetary Fund, USA.
International Monetary Fund (2023b). *International Financial Statistics: World Notes*. International Monetary Fund, USA.
Josyula, V. (2018). *Coordinated Portfolio Investment Survey Guide*. International Monetary Fund, USA, third edition.
Jylhä, P. and Suominen, M. (2011). Speculative capital and currency carry trades. *Journal of Financial Economics*, 99(1):60–75.
Kaplan, G., Moll, B., and Violante, G. L. (2018). Monetary policy according to hank. *American Economic Review*, 108(3):697–743.
Kekre, R. and Lenel, M. (2022). Monetary policy, redistribution, and risk premia. *Econometrica*, 90(5):2249–2282.
Kimball, M. S., Sahm, C. R., and Shapiro, M. D. (2008). Imputing risk tolerance from survey responses. *Journal of the American statistical Association*, 103(483):1028–1038.
Kogan, L. and Uppal, R. (2001). Risk aversion and optimal portfolio policies in partial and general equilibrium economies.
Lane, P. R. (2005). Global bond portfolios and emu. Technical report, ECB Working Paper.
Lane, P. R. and Shambaugh, J. C. (2010). Financial exchange rates and international currency exposures. *American Economic Review*, 100(1):518–40.
Lettau, M., Maggiori, M., and Weber, M. (2014). Conditional risk premia in currency markets and other asset classes. *Journal of Financial Economics*, 114(2):197–225.
Liao, G. Y. and Zhang, T. (2025). The hedging channel of exchange rate determination. *The Review of Financial Studies*, 38(1):1–38.
Lustig, H., Roussanov, N., and Verdelhan, A. (2011). Common risk factors in currency markets. *Review of Financial Studies*, 24(11):3731–3777.
Lustig, H. and Verdelhan, A. (2007). The cross section of foreign currency risk premia and consumption growth risk. *American Economic Review*, 97(1):89–117.
Lustig, H. and Verdelhan, A. (2011). The cross-section of foreign currency risk premia and consumption growth risk: Reply. *American Economic Review*, 101(7):3477–3500.
Maggiori, M., Neiman, B., and Schreger, J. (2020). International currencies and capital allocation. *Journal of Political Economy*, 128(6):2019–2066.
Menkhoff, L., Sarno, L., Schmeling, M., and Schrimpf, A. (2012). Carry trades and global foreign exchange volatility. *The Journal of Finance*, 67(2):681–718.
Michaillat, P. and Saez, E. (2021). Resolving new keynesian anomalies with wealth in the utility function. *Review of Economics and Statistics*, 103(2):197–215.
Michau, J.-B., Ono, Y., and Schlegl, M. (2023). Wealth preference and rational bubbles. *European Economic Review*, page 104496.
Nguyen, T. C., Castro, V., and Wood, J. (2022). A new comprehensive database of financial crises: Identification, frequency, and duration. *Economic Modelling*, 108:105770.
OECD (2025). Oecd short-term interest rates. `https://www.oecd.org/en/data/indicators/short-term-interest-rates.html` [Accessed 20250217].
Panageas, S. et al. (2020). The implications of heterogeneity and inequality for asset pricing. *Foundations and Trends® in Finance*, 12(3):199–275.
Park, S. (2022). Heterogeneous beliefs in macroeconomic growth prospects and the carry risk premium. *Journal of Banking & Finance*, 136:106393.
Samuelson, P. A. (1970). The fundamental approximation theorem of portfolio analysis in terms of means, variances and higher moments. *The Review of Economic Studies*, 37(4):537–542.
Santos Silva, J. M. and Tenreyro, S. (2022). The log of gravity at 15. *Portuguese Economic Journal*, 21(3):423–437.
Saucet, M. (2022). Asset Prices, Global Portfolios, and the International Financial System. Mimeo.
Schmitt-Grohé, S. and Uribe, M. (2003). Closing Small Open Economy Models. *Journal*

of International Economics, 61(1):163–185.
Sialm, C. and Zhu, Q. (2024). Currency management by international fixed-income mutual funds. *The Journal of Finance*, 79(6):4037–4081.
Silva, J. S. and Tenreyro, S. (2006). The log of gravity. *The Review of Economics and statistics*, 88(4):641–658.
Taub, L. (2008). Taking account of short positions in international portfolio investment statistics. *Background issue paper*, 4:142.
Tille, C. and van Wincoop, E. (2010). International capital flows. *Journal of International Economics*, 80(2):157–175.
Zhao, J. (2023). Wealth in utility, the taylor principle and determinacy. *Journal of Macroeconomics*, 76:103525.

A Empirical Appendix

Appendix A details the empirical methodology used in Section 3. It outlines the reporting standards for short positions (A.1), describes the procedures for estimating bilateral debt holdings by issuer and currency (A.2), and explains the computation of domestic currency bias and portfolio returns (A.3).

A.1 Short Position Reporting Standards

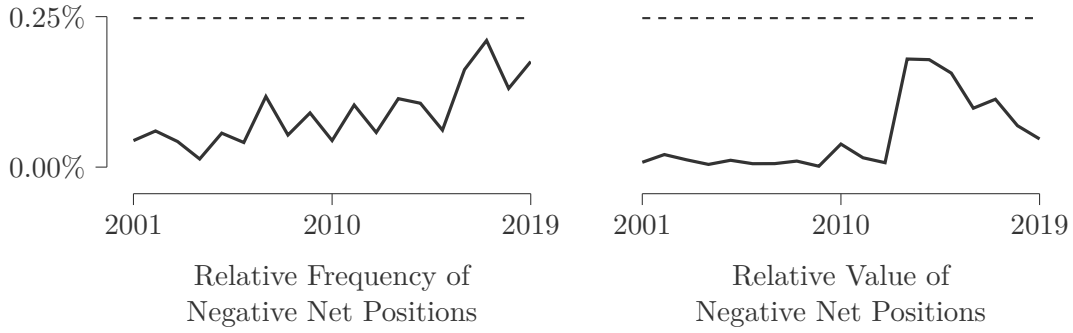
The main IMF reporting standards for international financial statistics are published in two documents: the Balance of Payments Manual (BPM) and the CPIS Survey Guide (CSG). The BPM is the more established of the two standards, it was first published in 1948, and it governs broadly the reporting of international financial statistics at the country level. The CSG is narrower in scope, and younger than the BPM. Until the turn of the century, no definitive reporting standard for short positions appeared in either publication: the fifth edition BPM makes no mention of short positions whatsoever (International Monetary Fund, 1993), and the first edition CSG describes possible methods for reporting short positions, but provides no definitive standard (International Monetary Fund, 1997, para. 93 and Box 4). After the turn of the century, the second edition CSG published clearer guidance: "If (when) the security is on-sold, the 'borrower' of the security should record a 'short' position" (International Monetary Fund, 2002, para. 3.78), and then, seven years later, the sixth edition BPM established the definitive standard that applies today:

> Short positions occur when an institutional unit sells securities for which it is not the economic owner. [...] Delivery to the purchaser is made through the use of a borrowed security. The party with the short position records a negative value for the holding of the asset. The short position is shown as a negative asset, rather than a liability. (International Monetary Fund, 2009, para. 7.28)

The most recent third edition CSG now cites the sixth edition BPM when describing the CPIS reporting standard for short positions (Josyula, 2018, para. 3.52). A third document, the Handbook on Securities Statistics (HSS), was produced jointly by the IMF, the BIS, and the ECB and released in parts over several years, beginning in 2009 (International Monetary Fund, 2010, para 4.15). The second part the HSS dealing with debt securities holdings, released in 2010, also follows the sixth edition BPM.

While 2009 marks the introduction of a definitive standard for the reporting of short positions in international financial statistics, the reporting standard was not immediately implemented by all reporting countries. Press releases and country-level reporting guides from the years following the publication of the sixth edition BPM indicate that the

Figure A1 – Negative Net Positions in International Debt Holdings



Notes. The figures show the relative count and value of reported negative net positions in short-term debt, aggregated across CPIS-reporting countries annually. Aggregate net positions equal the sum of gross long and gross short positions, with the latter being negative or zero. The left panel shows the relative frequency of negative net positions, calculated as their count divided by the count of all net positions. The right panel shows the relative value of negative net positions, computed as their value divided by the value of all net positions, multiplied by -1.

standard had been implemented by many countries by 2015.[8] For example, the ECB announced that Eurostat would disseminate statistics under the new standard beginning in October 2014 (European Central Bank, 2014a). Figure A1 shows a rise in relative counts and relative values of reported short positions in the years following the publication of the new reporting standard in 2009, though the values remain low.

A.2 Estimating Debt Holdings by Issuer and Currency

This section describes how we estimate country-level debt holdings by currency, which we use in Table 1 and Figure 2. Our approach combines the procedures in Fidora et al. (2007) for estimating investor-country holdings of domestic debt and in Lane and Shambaugh (2010) for estimating the currency composition of investor-country holdings of foreign debt.[9]

Throughout this section, *debt holdings* refer to country-level aggregate net debt holdings in nominal terms in numéraire currency USD, where aggregate net means the sum of long positions (with positive sign) and short positions (with negative sign) over all agents (households, firms, government) residing in a given country. Abusing the notation of our theoretical model, we denote these holdings B_{ijt}, although they are nominal here. Domestic debt holdings refer to debt held by residents of a given country that was issued by residents of the same country. International debt holdings refer to debt held by residents of a given country that was issued by residents of another country. Debt may be issued in

[8]The United States has not adopted the BPM6 standard for reporting short positions, instead reporting gross long positions to CPIS without netting gross short positions.

[9]Allen et al. (2023) update the work of Lane and Shambaugh (2010) using confidential IMF survey data. However, their estimates exclude domestic holdings and are available by currency only, rather than by currency and issuer, making them less suitable for our study.

domestic or international markets (as defined by the BIS), and the issuance market, as a concept, is distinct from the residence of the issuer or holder of debt.[10]

In subsection A.2.1, we use gravity models to estimate the domestic and international debt holdings of investor countries by issuer country. We first estimate a model for international debt holdings, then a second model for both domestic and international debt holdings. In subsection A.2.2, we estimate currency weights, which allow us to compute domestic and international debt holdings of investor countries, by issuer country *and* issuance currency. The initial weights are based on the reported currency composition of debt issued in international markets, but we adjust these weights to account for holdings of international debt issued in the domestic market of the issuer. Finally, we use a biproportional fitting procedure to reconcile the estimated debt holdings of investor country by issuer country and issuance currency with reported debt holdings by and issuance currency alone.

A.2.1 Estimating Debt Holdings by Issuer Country

We now estimate portfolio holdings by issuer country. We proceed in three steps. In step one, we fit a first-stage gravity model to CPIS-reported international debt holdings, and use the fitted first-stage model to obtain first-stage predictions of international debt holdings for non-reporting countries. In step two, we estimate domestic debt holdings as the residual difference between total debt issuance and rest-of-world international debt holdings for each issuer country. In step three, we fit a second-stage gravity model to domestic and international debt holdings, and use the fitted second-stage model to obtain final predictions of domestic and international debt holdings for non-reporting countries.

Step One. We fit a first-stage gravity model to CPIS-reported international debt holdings using pseudo-Poisson maximum likelihood (PPML), which performs well in the presence of heteroskedasticity (Silva and Tenreyro, 2006; Santos Silva and Tenreyro, 2022). Our unbalanced panel contains 177 981 investor-issuer-year observations spanning 2001–2021 and covering 83 investor countries and 196 issuer countries. Because we use the model to predict missing debt holdings for non-reporting countries, we limit regressors to those with good coverage across countries to minimize data loss.

The gravity variables we select, obtained from Conte and Mayer (2022), include geographic distance, a contiguity indicator, a common language indicator, GDP, GDP per capita, an EU membership indicator, and a WTO membership indicator. Additionally, we construct a common-currency indicator, a tax haven indicator based on the list in

[10]For example, a German resident might issue German debt in German debt markets under German law, that is held by a US resident. From the US perspective, this represents holdings of international debt issued in the German domestic market. Alternatively, a German resident might issue German debt in a US market under US law, that is held by a German resident. From the German perspective, this represents holdings of domestic debt issued in the German international market.

Coppola et al. (2021) (used for interaction terms), and a crisis indicator for both investor and issuer countries from the banking, debt, and currency crises identified by Nguyen et al. (2022). We also include interactions for investor and issuer EU membership, WTO membership, GDP, GDP per capita. Finally, we include investor, issuer, and year fixed effects. Our specification is given by

$$\mathrm{E}\Big[B_{ijt} \mid \boldsymbol{X}_{ijt},\, \boldsymbol{IT}_{ijt},\, \boldsymbol{FE}_{ijt}\Big] = \exp\Big(\boldsymbol{\beta}_X^\top \boldsymbol{X}_{ijt} + \boldsymbol{\beta}_{IT}^\top \boldsymbol{IT}_{ijt} + \boldsymbol{\beta}_{FE}^\top \boldsymbol{FE}_{ijt}\Big)\,, \quad i \neq j\,, \tag{A1}$$

where B_{ijt} denotes investor country i's holding of debt issued by country j in year t. The vector $\boldsymbol{X}_{ijt}$ collects a constant and the gravity and indicator variables. The vector $\boldsymbol{IT}_{ijt}$ collects interaction terms, and $\boldsymbol{FE}_{ijt}$ collects fixed effects.

After dropping 5 multicollinear fixed effects using QR decomposition, we estimate the first-stage model with 302 regressors and 171 789 observations.[11] We estimate the model both with and without L2 regularization ($\alpha = 0.0001$). Our preferred estimates, used in Table 1 and Figure 2, include regularization. First-stage results for a subset of regressors are reported in columns two and four of Table A1. The correlation between predicted and observed values is 93.0 without regularization and 92.88 with regularization.

Step Two. We compute domestic holdings of domestic debt as the residual between total reported issuance amounts outstanding and the sum of reported plus gravity-estimated rest-of-world holdings for issuer countries with available data.

Total issuance amounts outstanding are available for 38 countries in the BIS Debt Securities Statistics (DSS) with good coverage over our sample period. For 20 additional countries with poor or no DSS coverage, we approximate total issuance by summing domestic and international market issuance amounts from the BIS International Debt Statistics (IDS).[12]

To measure rest-of-world holdings of issuer-country debt, we aggregate CPIS-reported holdings for reporting countries and first-stage gravity-estimated holdings for non-reporting countries. This aggregate covers all economic sectors (including government) except for central banks. Central Bank holdings are reported separately in the IMF's Securities Held as Foreign Exchange Reserves (SEFER) survey. We add SEFER-reported holdings to the CPIS aggregate to obtain an estimate of total rest-of-world holdings.

Following Fidora et al. (2007), we define domestic holdings of domestic debt as the

[11] Our implementation of QR decomposition removes the fixed effects with the lowest-valued diagonal elements of the R matrix until the condition number equals approximately 50.

[12] As Gruić and Wooldridge (2012) note, summing domestic and international issuance may involve double-counting. However, in cases where both DSS and IDS total issuance are available, we find the totals to be very similar. DSS data represent 87.13% of total reported issuance in an average year in our sample, and IDS data represent 12.87%.

Table A1 – First- and Second-Stage PPML Gravity Model Estimation

Independent Variable	No Regularization		L2 Regularization	
	Stage One	Stage Two	Stage One	Stage Two
Constant	−17.91	−11.95	−7.07	−7.62
	(0.00)	(0.00)		
Distance	−0.39	−0.48	−0.38	−0.48
	(0.00)	(0.00)		
Common Language	0.23	0.27	0.23	0.27
	(0.00)	(0.00)		
Common Currency	0.68	0.93	0.68	0.93
	(0.00)	(0.00)		
Investor GDP	3.50	1.75	1.22	1.23
	(0.00)	(0.00)		
Issuer GDP	4.71	1.15	1.86	1.76
	(0.00)	(0.00)		
Investor GDP/Cap	−0.68	1.26	1.31	1.67
	(0.00)	(0.00)		
Issuer GDP/Cap	−1.81	1.76	0.67	1.27
	(0.00)	(0.00)		
Domestic Debt		6.19		6.14
		(0.00)		
Domestic Debt × Investor GDP		−0.49		−0.48
		(0.00)		
Domestic Debt × Investor GDP/Cap		−2.69		−2.67
		(0.00)		
Domestic Debt × Investor Tax Haven		−1.42		−1.40
		(0.00)		
Year FE	Yes	Yes	Yes	Yes
Investor and Issuer FE	Yes	Yes	Yes	Yes
Observations	171 789	172 811	171 789	172 811
Correlation	93.00	99.55	92.88	99.55

Notes. The table reports PPML estimates from the first- and second-stage gravity models described in Section A.2.1. The first stage uses CPIS-reported international debt holdings; the second stage incorporates domestic debt holdings computed as the residual between total issuance and rest-of-world holdings. P-values appear below coefficient estimates, and the observation counts and correlation between observed and predicted holdings are reported at the bottom of the table. Columns labeled "L2 Regularization" apply a penalty parameter of $\alpha = 0.0001$, and no p-values are available for those specifications. Fixed effects and less economically-significant covariates are omitted.

difference between total issuance amounts outstanding and total rest-of-world holdings:

$$B_{iit} = I_{it} - B_{-iit}\,, \quad \text{where } B_{-iit} \equiv \sum_{j \neq i} B_{jit} + B_{CBit}\,, \tag{A2}$$

where I_{it} denotes issuer-country i's total debt issuance amounts outstanding, B_{CBit} represents SEFER-reported central bank holdings of country i's debt, and B_{-iit} denotes rest-of-world holdings of debt issued by country i.

For six countries (Estonia, India, Ireland, Lebanon, Luxembourg, and Taiwan), CPIS-reported rest-of-world holdings alone, before adding gravity-estimated holdings, exceed total reported outstanding issuance. In these cases, which together account for 2.27% of total reported outstanding issuance in an average year, the residual-based estimates of domestic debt holdings become negative and we exclude them, instead relying on second-stage gravity estimates, described next.

Step Three. We fit a second-stage gravity model that incorporates both the CPIS-reported international debt holdings and the residual-based domestic debt holdings from the previous steps. We continue to use the PPML estimator and the same baseline regressors, but we now add a domestic debt indicator (equaling one if the investor and issuer countries coincide) and interact this indicator with GDP, GDP per capita, and the tax haven indicator. All continuous variables remain logged and standardized. Our specification is given by

$$\begin{aligned} &\mathrm{E}\Big[B_{ijt} \mid \boldsymbol{X}_{ijt},\, \boldsymbol{IT}_{ijt},\, \boldsymbol{FE}_{ijt},\, \boldsymbol{DOM}_{ijt}\Big] \\ &\quad = \exp\Big(\boldsymbol{\beta}_X^\top \boldsymbol{X}_{ijt} + \boldsymbol{\beta}_{IT}^\top \boldsymbol{IT}_{ijt} + \boldsymbol{\beta}_{FE}^\top \boldsymbol{FE}_{ijt} + \boldsymbol{\beta}_{DOM}^\top \boldsymbol{DOM}_{ijt}\Big)\,, \end{aligned} \tag{A3}$$

where B_{ijt} now encompasses both reported international and residual-based domestic debt holdings. The vector $\boldsymbol{DOM}_{ijt}$ contains a domestic debt indicator that equals one for $i = j$ and zero otherwise, and its interactions.

Columns three and five of Table A1 present the second-stage results. The coefficient on the domestic debt indicator is economically and statistically significant, reflecting a strong domestic bias. By contrast, the interaction terms with GDP per capita and the tax haven indicator are significantly negative, suggesting that wealthier countries and tax havens exhibit a smaller *incremental* domestic bias once we control for other factors. This does not imply that they hold less domestic debt in absolute terms or as a share of total debt; rather, it implies only that the domestic debt indicator effect is attenuated.

After fitting the second-stage model, we recalculate rest-of-world holdings by combining CPIS-reported debt holdings with second-stage gravity-estimated debt holdings for non-reporting countries, then update our residual-based estimates of domestic debt holdings accordingly. Our final data set of bilateral debt holdings thus includes both reported and

estimated holdings of international and domestic debt for all countries in our sample.

A.2.2 Estimating Weights by Issuer Country and Issuance Currency

To obtain estimates of investor-country debt holdings by issuer and currency, we proceed in five steps. First, we compute *holding weights* to capture each issuer's share in the total holdings of each investor country. Second, we compute international-market *issuance weights* to capture the currency denomination of debt issued by each investor country in international markets. Third, we compute *access weights* to capture the extent to which domestic and international investors access the issuer's domestic and international markets. Fourth, we combine these issuance and access weights to obtain initial *portfolio weights*. Fifth, we adjust the initial portfolio weights using a biproportional fitting algorithm to match the currency composition of total debt holdings reported by investor countries in CPIS. The final portfolio weights decompose each investor country's total debt holdings into holdings by issuer country and issuance currency.

Step One: Holding Weights. We compute holding weights as issuer country shares in the total debt holdings of investor countries. Holding weights for investor-country i satisfy following identity,

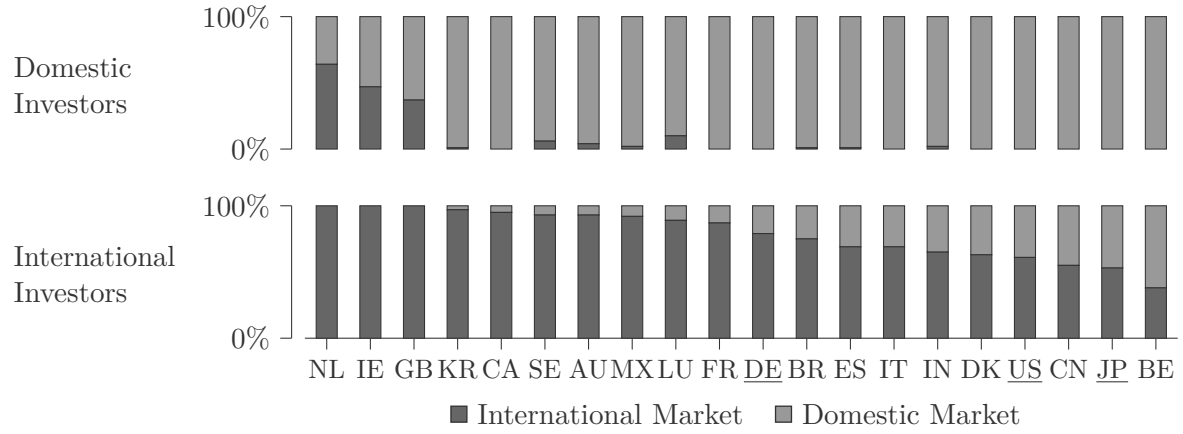
$$B_{it} = \sum_j B_{ijt} \quad \Leftrightarrow \quad 1 = \sum_j b_{ijt}\,, \tag{A4}$$

where B_{it} denotes the total debt holdings of country i and $b_{ijt} \equiv B_{ijt}/B_{it}$ denotes the share of debt issued by country j in the total debt holdings of country i.

Step Two: Access Weights. We compute access weights based on the difference between total rest-of-world holdings and total international-market debt issuance for each issuer country. If the difference is positive, with rest-of-world holdings exceeding international-market issuance, we infer that international investors hold debt from the issuer's domestic market. If the difference is negative, with international-market debt issuance exceeding rest-of-world holdings, we infer that domestic investors hold debt from the issuer's international market.[13]

[13]The BIS distinguishes between debt issued in domestic and international markets. In domestic markets, issuers predominantly issue debt in their domestic currency. Although international-market debt is increasingly also denominated in the issuer's domestic currency (see Gruić and Wooldridge 2012, Du and Schreger 2016, and Burger et al. 2018), a substantial share remains in USD or other international currencies. Because the currency composition differs by market, the market accessed by investors matters for our estimates. Gruić and Wooldridge (2012) document a divergence between international bond issues (from BIS IDS data) and cross-border bond holdings (from IMF CPIS data). This discrepancy suggests that international investors buy foreign debt in the issuer's domestic market, while domestic investors buy some 'domestic' debt in the issuer's international market. Burger and Warnock (2007) study this phenomenon in the bond markets of 40 countries.

Figure A2 – Market Access Weights for Top-Twenty Issuer Countries



Notes. The figure shows access weights for domestic and international investors in the top 20 issuer countries by total issuance, averaged over the period 2001–2021. The upper panel shows weights for domestic investors, who generally buy domestic debt from domestic markets. The lower panel shows weights for international investors, who generally buy international debt from international markets. In 5.87% of country-year observations, weights fall outside the interval $[0, 1]$ and are truncated to zero or one.

We compute separate access weights for domestic and international investors. For domestic investors residing in issuer country i, access weights satisfy the following identity,

$$\begin{aligned} B_{iit} &= \max\Big(I_{it}^{(IM)} - B_{-iit}, 0\Big) + \Big(B_{iit} - \max\Big(I_{it}^{(IM)} - B_{-iit}, 0\Big)\Big) \\ \Leftrightarrow \quad 1 &= a_{iit}^{(IM)} + a_{iit}^{(DM)}\,, \end{aligned} \tag{A5}$$

where $a_{iit}^{(IM)} \equiv \max(I_{it}^{(IM)} - B_{-iit}, 0)/B_{iit}$ denotes the domestic-investor access weight to issuer i's international market, and $a_{iit}^{(DM)} = 1 - a_{iit}^{(IM)}$ denotes the domestic-investor access weight to issuer i's international market. For international investors outside issuer country i, access weights satisfy a similar identity,

$$\begin{aligned} B_{-iit} &= \max\Big(B_{-iit} - I_{it}^{(IM)}, 0\Big) + \Big(B_{-iit} - \max\Big(B_{-iit} - I_{it}^{(IM)}, 0\Big)\Big) \\ \Leftrightarrow \quad 1 &= a_{-iit}^{(DM)} + a_{-iit}^{(IM)}\,, \end{aligned} \tag{A6}$$

where $a_{-iit}^{(DM)} \equiv \max(B_{-iit} - I_{it}^{(IM)}, 0)/B_{-iit}$ denotes the international-investor access weight to issuer i's domestic market, and $a_{-iit}^{(IM)} = 1 - a_{-iit}^{(DM)}$ denotes the international-investor access weight to issuer i's international market.

Figure A2 illustrates access weights averaged over 2001–2021 for the top 20 issuer countries, separately for domestic and international investors. In the upper panel, domestic investors generally buy domestic debt in domestic markets, although in Great Britain, the Netherlands, and Ireland they buy a significant amount of domestic debt issued internationally. In the lower panel, international investors generally buy international

debt in international markets; however, for the United States, China, Japan, and Belgium, international investors often purchase debt issued in domestic markets. In 5.87% of country-year observations, weights fall outside the interval $[0, 1]$ and are truncated to zero or one.

Step Three: Issuance Weights. We compute issuance weights for 142 issuer countries that report total international-market issuance and currency-specific issuance for USD, EUR, and the issuer's domestic currency in the BIS IDS data. When values are present for at least one of these currencies, we set any remaining missing values to zero. We then define issuance in "other" currencies (neither USD, EUR, nor the issuer's domestic currency) as the remainder after subtracting these three currency totals from the overall total international-market issuance.[14]

Issuance weights for issuer-country i satisfy following identity,

$$\begin{aligned} I_{it}^{(IM)} &= I_{iit}^{(IM)} + I_{iUSDt}^{(IM)} + I_{iEURt}^{(IM)} + I_{iOTHt}^{(IM)} \\ \Leftrightarrow \quad 1 &= v_{iit}^{(IM)} + v_{iUSDt}^{(IM)} + v_{iEURt}^{(IM)} + v_{iOTHt}^{(IM)}, \end{aligned} \tag{A7}$$

where $I_{it}^{(IM)}$ denotes total international-market issuance, $I_{ijt}^{(IM)}$ denotes the portion of that issuance denominated in currency j. We define the issuance weight for issuer country i and issuance currency j as $v_{ijt}^{(IM)} \equiv I_{ijt}^{(IM)} / I_{it}^{(IM)}$. In particular, $I_{iit}^{(IM)}$ and $v_{iit}^{(IM)}$ denote the issuance amount and weight for issuer country i's international-market issuance in its domestic currency. For domestic-market issuance, we assume that all debt is issued in the issuer's domestic currency, so that $v_{iit}^{(DM)} \equiv 1$.[15]

Step Four: Portfolio Weights. We compute portfolio weights for investor-country debt holdings by combining holding weights with issuer weights and access weights. Portfolio weights allow us to decompose each investor country's total holdings into holdings by issuer country and issuance currency. These weights play an important role in the calculation of portfolio returns on investor country debt holdings in section A.3.

We compute separate portfolio weights for domestic and international debt holdings.

[14] A large share of country-year observations (72.15%) have missing values for USD, EUR, or domestic currency issuance in the IDS data. However, after setting these missing values to zero, the average share of international market issuance in the other-currency category remains low at 6.23%, compared with 8.64% for observations with no missing values. This suggests that the zero-fill assumption does not artificially inflate the other-currency category.

[15] This assumption appears reasonable for most countries. Gruić and Wooldridge (2012) observe that "bonds issued in the local market are typically issued in local currency under local law," but note international financial centers and dollarised or euroised economies as possible exceptions.

For domestic debt holdings, portfolio weights are given by

$$w_{iikt} \equiv \begin{cases} b_{iit} a_{iit}^{(IM)} v_{ikt}^{(IM)}\,, & i \neq k\,, \\ b_{iit}\left(a_{iit}^{(IM)} v_{iit}^{(IM)} + a_{iit}^{(DM)} \right), & i = k\,, \end{cases} \tag{A8}$$

where w_{iikt} denotes the portfolio weight for investor country i's holdings of domestic debt issued in currency k. For international debt holdings, portfolio weights satisfy a similar identity,

$$w_{ijkt} \equiv \begin{cases} b_{ijt} a_{-jjt}^{(IM)} v_{jkt}^{(IM)}\,, & i \neq j \neq k\,, \\ b_{ijt}\left(a_{-jjt}^{(IM)} v_{jjt}^{(IM)} + a_{-jjt}^{(DM)} \right), & i \neq j = k\,, \end{cases} \tag{A9}$$

where w_{ijkt} denotes the portfolio weight for investor country i's holdings of debt issued by country j in currency k, where $i \neq j$. For country-year observations where we lack IDS data and cannot compute access or issuance weights, we use the global average non-missing access and issuance weights to compute portfolio weights.[16]

Step Five: Biproportional Fitting. Using the portfolio weights from step four, we can derive total international debt holdings by currency for each investor country. For a subset of 66 countries holding 80.20% of global debt in an average year, we can compare this derived total with a CPIS-reported total, and thereby obtain an indirect measure of accuracy for the underlying portfolio weights. To derive total international debt holdings by currency, we simply multiply total international debt holdings by portfolio weights and aggregate over issuers.

The upper panel of Figure A3 shows the discrepancy between our derived totals and the CPIS-reported totals by currency. With accurate portfolio weights, the totals would fall on the 45-degree line. Instead, our portfolio weights overestimate international debt holdings in EUR and USD when the true values are low and underestimate when the high true values are high. The opposite is true of CHF, GBP, and JPY, but these currencies represent a much smaller fraction of international debt.

To address this issue, we adjust initial portfolio weights using a biproportional fitting algorithm. We introduce the following notation. Let B_{ijkt} denote country i's holding of debt issued by country j in currency k, computed as a portfolio weight multiplied by country i's total holdings, and let $B_{ij\bullet t}$ and $B_{i\bullet kt}$ denote total debt holdings by issuer and by currency, respectively. That is, let

$$B_{ijkt} = w_{ijkt} B_{it}\,, \quad B_{ij\bullet t} = \sum_k B_{ijkt}\,, \quad \text{and} \quad B_{i\bullet kt} = \sum_j B_{ijkt}\,. \tag{A10}$$

[16] Many of the smallest issuer countries lack IDS data. While these countries are numerous (causing 42.29% of portfolio weights to be computed with global averages), the debt they issue accounts for a negligible 0.29% of global holdings in an average year.

Note that $B_{ij\bullet t}$ has the same meaning as B_{ijt} used elsewhere; we adopt the "dot" notation here to avoid confusion with $B_{i\bullet kt}$. Finally, let $B^{\dagger}_{i\bullet kt}$ denote CPIS-reported (as opposed to derived) total holdings by currency.

For each investor–year combination, we arrange initial estimates of holdings by issuer and currency into a matrix, with issuers j in rows and currencies k in columns. We iteratively adjust rows and columns to obtain a set of holdings $B^{\dagger}_{ijkt}$ such that

$$B_{ij\bullet t} = \sum_k B^{\dagger}_{ijkt} \quad \text{and} \quad B^{\dagger}_{i\bullet kt} = \sum_j B^{\dagger}_{ijkt} \quad \forall j, k\,. \tag{A11}$$

The row target $B_{ij\bullet t}$ ensures that the issuer-level totals remain unchanged from our estimates in Section A.2.1, while the column target $B^{\dagger}_{i\bullet kt}$ forces the currency composition to align with the CPIS-reported values.

At iteration n, we first scale each row so that its sum equals the corresponding row target $B_{ij\bullet t}$. Specifically, for issuer (row) j, we update

$$B^{(n+\frac{1}{2})}_{ijkt} = B^{(n)}_{ijkt} \frac{B_{ij\bullet t}}{\sum_k B^{(n)}_{ijkt}}, \quad \forall\, k\,. \tag{A12}$$

Next, we scale each column so that its sum equals the corresponding column target $B^{\dagger}_{i\bullet kt}$. Specifically, for currency (column) k, we update

$$B^{(n+1)}_{ijkt} = B^{(n+\frac{1}{2})}_{ijkt} \frac{B^{\dagger}_{i\bullet kt}}{\sum_j B^{(n+\frac{1}{2})}_{ijkt}}, \quad \forall\, j\,. \tag{A13}$$

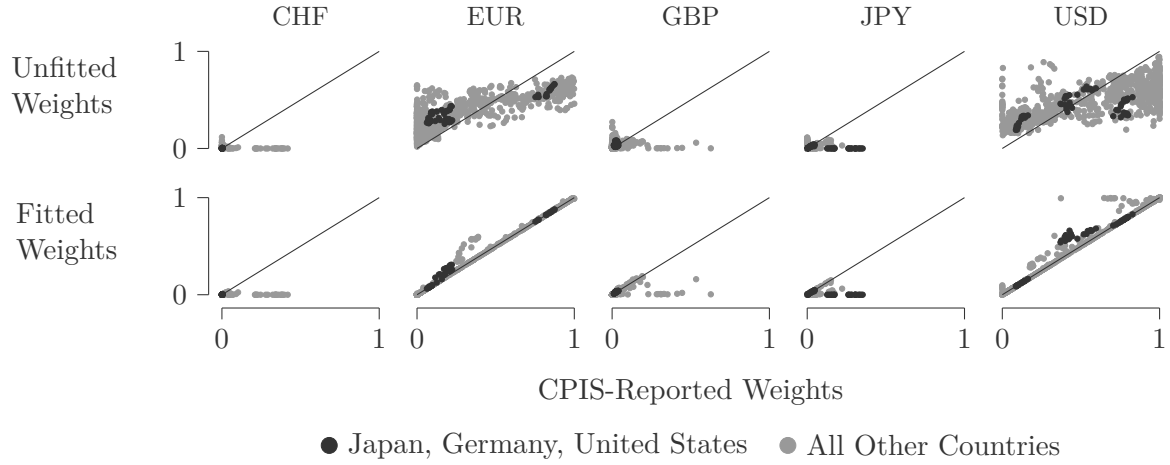
These steps are repeated until the maximum deviation between the current row and column sums and their respective targets falls below a given tolerance. If convergence is not achieved within a maximum number of iterations, we apply a final re-scaling to enforce the row constraints. The final values $B^{\dagger}_{ijkt}$ are then used to update portfolio weights w_{ijkt}.

Convergence often fails because our initial estimates B_{ijkt} are derived from IDS data that cover only USD, EUR, and domestic currency. We construct a residual "other" category that contains issuance in JPY, GBP, and CHF except when these are the issuer's domestic currency. As a result, the JPY, GBP, and CHF columns are sparsely populated, which prevents convergence to the CPIS-reported targets in these currencies. However, these currencies account for only 11.22% of total international debt holdings, and the algorithm is effective for USD and EUR. The lower panel of Figure A3 illustrates this effectiveness, with fitted values clustering around the 45-degree line for USD and EUR.

A.3 Domestic Currency Bias and Portfolio Returns

Table 1 reports average estimates of domestic currency bias and portfolio returns on debt

Figure A3 – Biproportional Fitting of Portfolio Weights to Currency Weights



Notes. The figure shows scatter plots of estimated and CPIS-reported currency weights for CHF, EUR, GBP, JPY, and USD from 2001 to 2021 before and after the biproportional fitting described in Section A.2.2, step five. Japan, Germany, and the United States are shown in black, and all other countries are shown in gray. The top row shows unfitted estimates, and the bottom row shows fitted estimates. Each plot includes a 45-degree reference line. After fitting, estimates for USD and EUR align more closely with reported values. Estimates for GBP, CHF, and JPY remain less well aligned, but these currencies account for only 11.22% of international debt holdings.

holdings for Germany, Japan, and the United States as one group, and for 181 rest-of-world countries as another group. On average, Germany, Japan, and the United States exhibit lower portfolio returns and a stronger domestic-currency bias than other countries. In this appendix section, we describe the data and methodology used to estimate domestic currency bias and portfolio returns for country-level debt holdings.

Domestic Currency Bias. We compute domestic currency bias using the debt holdings and portfolio weights estimated in Section A.2.1. Following Fidora et al. (2007), we define

$$\begin{aligned} DCB_{it} &= 100\left(w^*_{-it} - w_{-it}\right)/w^*_{-it}\,, \quad \text{with} \\ w_{-it} &= \sum_j \sum_{k\neq i} w_{ijkt} \quad \text{and} \quad w^*_{-it} = \sum_j \sum_{k\neq i} w_{ijkt} B_{it} \Big/ \sum_i B_{it}\,, \end{aligned} \tag{A14}$$

where DCB_{it} denotes domestic currency bias for country i, w_{-it} denotes the share of international-currency debt in i's debt holdings, and w^*_{-it} denotes the share of international-currency debt in global debt holdings (with "international" defined relative to country i). Domestic currency bias is bounded above by 100 and unbounded below. It is negative in only 2.38% of country-year observations, indicating that most countries overweight domestic currency in their debt holdings.

Portfolio Returns. We compute nominal portfolio returns as a weighted average of issuer-country nominal interest rates adjusted for currency returns, taken over all positions in an investor country's debt holdings. Real portfolio returns equal nominal portfolio returns adjusted for the investor country's domestic rate of CPI inflation. We obtain interest rates from the Organization for Economic Development and Co-operation (OECD) Main Economic Indicators (MEI) database and IMF International Financial Statistics (IFS) database, and exchange rates and rates of CPI inflation also from IFS.

Short-term interest rates serve as indicative interest rates for our portfolio return calculations.[17] For each issuer country, we use OECD MEI short-term interest rates where available, and IMF IFS treasury bill rates where MEI rates are unavailable.[18] These two data sources cover 2 069 country-year observations. For an additional 1 032 country-year observations, these short-term rates are unavailable but IFS deposit rates are available. In these cases, we use a fitted model to predict short-term interest rates from deposit rates,

$$R^i_{Bit} = \alpha + \beta_D\Big(R^i_{Dit}\Big) + \boldsymbol{\beta}^\top_{FE}\boldsymbol{FE}_{it} + \epsilon_t\,, \tag{A15}$$

where R^i_{Bit} and R^i_{Dit} denote gross short-term and deposit rates, respectively, with subscript i denoting the issuer country and superscript i denoting the numéraire currency, set as the domestic currency of the issuing country, and $\boldsymbol{FE}_{it}$ denotes country and year fixed effects. The regression yields an $R^2 = 0.82$ and coefficient estimates $\hat{\beta}_D = 0.01$ and $\hat{\alpha} = 98.83$, both significant at the 1% level. In total, reported or predicted rates are available for 3101 country-year observations.

End-of-year nominal exchange rates from IFS are available for 3 778 country-year observations, reported with respect to USD. With the nominal exchange rate between currency i and USD denoted $S_{i\text{USD}t}$, we define the gross nominal currency return for an arbitrary currency pair as

$$R_{Sijt} = R_{Si\text{USD}t}/R_{Sj\text{USD}t}\,, \quad \text{where} \quad R_{Si\text{USD}t} = S_{i\text{USD}t}/S_{i\text{USD}t-1} \quad \forall\, i\,. \tag{A16}$$

We define the nominal portfolio return R^i_{NPit} for country i in numéraire currency i as

$$R^i_{NPit} = \sum_j \sum_k w_{ijkt} R_{Sikt} R^j_{Bjt}\,, \tag{A17}$$

where we apply the domestic short-term interest rate R^j_{Bjt} to debt issued by j in any

[17] The debt holdings we estimate cover debt of all maturities. Ideally, we would match the term of interest rates to the maturity of debt holdings when computing portfolio returns, but coverage of short-term and long-term debt in the BIS and IMF data is poor.

[18] MEI short-term rates are "based on three-month money market rates where available. Typical standardised names are 'money market rate' and 'treasury bill rate"' (OECD, 2025). IMF treasury bill rates are rates "at which short-term government debt securities are issued or traded in the market," with details varying slightly from country to country (International Monetary Fund, 2023a,b).

Table A2 – Summary of Domestic Currency Bias and Portfolio Returns

Variable		Summary Statistics (in %)			
	Count	Mean	SD	Median	IQR
Portfolio Return, Nominal	3 778	5.6	8.8	3.9	6.5
Portfolio Return, Real	3 459	0.5	7.1	0.4	5.0
Coverage Ratio	3 778	84.7	25.6	95.6	8.8
Domestic Currency Bias	3 778	62.9	29.0	73.1	42.0
Currency Return	3 778	2.3	11.8	0.0	7.8
CPI Inflation Rate	3 459	5.3	9.4	3.3	5.1

Notes. The table presents summary statistics for country-year estimates of domestic currency bias, portfolio returns, and coverage ratios in the upper panel, and for the variables used in the calculation of portfolio returns: treasury bill rates, currency returns, and CPI rates of inflation in the lower panel. Statistics are computed for the period 2001–2021. The count column counts non-missing country-year observations. Mean, standard deviation (SD), median, and interquartile range (IQR) are presented in percentage.

currency, due to poor coverage of international-currency interest rates in the IMF and OECD data.[19] We then use domestic rates of CPI inflation for the investor country to convert nominal portfolio returns into real portfolio returns, $R_{Pit} = R^i_{NPit}/\Pi^i_{it}$.

For some debt-issuing countries and years, short-term interest rates or currency returns are unavailable. For this reason, we compute portfolio returns for each investor country on the fraction of portfolio holdings where data are available, re-scaling portfolio weights by the coverage ratio, which we define as

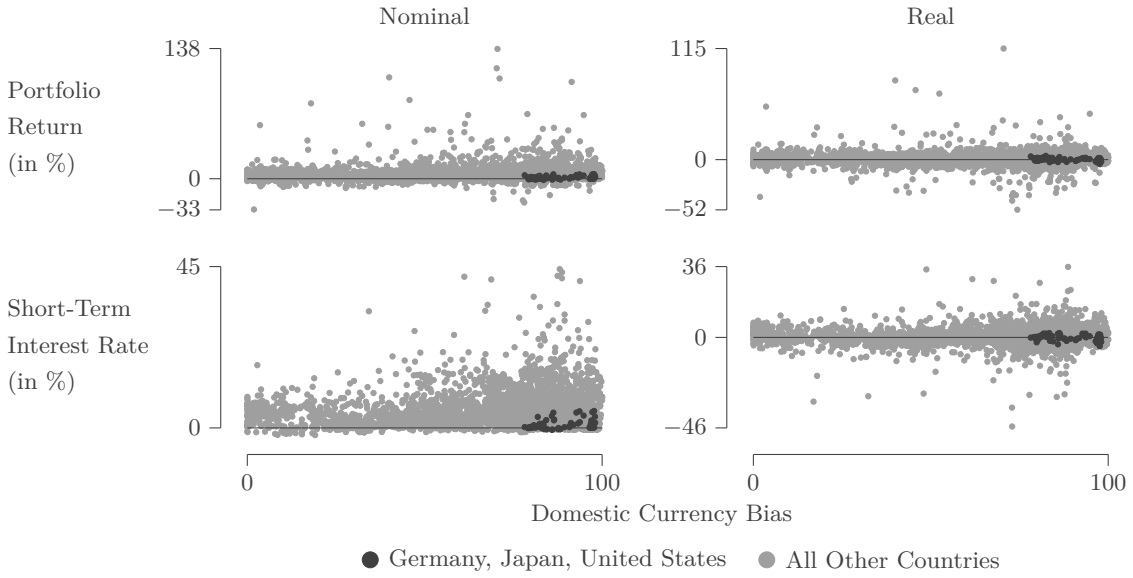
$$CR_{it} = \sum_j \sum_k w_{ijkt} \mathbb{1}_{ijk} \tag{A18}$$

for the indicator $\mathbb{1}_{ijk}$ equals zero when interest rates or exchange rates are missing for investor country i, issuer country j, or currency k. The median coverage ratio in our sample is 98.5%. Table A2 provides descriptive statistics for domestic currency bias, treasury rates, currency returns, portfolio returns, coverage ratios, and rates of inflation in our sample.

Figure A4 presents scatter plots of portfolio returns and short-term interest rates against domestic currency bias. The horizontal axes show domestic currency bias. In the upper plots, the vertical axis shows portfolio returns, nominal on the left and real on

[19]Empirically, interest rates on debt issued in domestic and international currencies do differ; for reference, the ECB reports a median international-minus-domestic currency rate spread of +0.8pp (advanced economies) and −1.6pp (emerging economies) (European Central Bank, 2014b, Section C). While these spreads are large, they would be multiplied by small portfolio weights in our analysis, and would not significantly impact the main empirical findings we report in Table 1. Du and Schreger (2016, 2022) study the spread for sovereign debt in emerging markets, emphasizing currency risk, credit risk, and corporate balance sheet mismatches as important determinants, which our simplification does not capture.

Figure A4 – Domestic Currency Bias and Portfolio Returns



Notes. The figure shows domestic currency bias and both nominal and real short-term debt portfolio returns and treasury rates for 188 countries, estimated annually. Germany, Japan, and the United States are shown in black; all other countries are shown in gray. Portfolio returns are calculated using short-term interest rates, exchange rates, and CPI inflation rates from the OECD and IMF, with portfolio weights estimated from IMF and BIS debt holdings and issuance data. Domestic currency bias is defined in equation (A14). We exclude 2.38% of estimates with negative domestic currency bias and country-year observations with interest or exchange rates more than three standard deviations above the yearly mean.

the right. In the lower plots, the vertical axis shows domestic short-term interest rates, nominal on the left and real on the right. Germany, Japan, and the United States are shown in black, and their observations cluster near zero on the vertical axis and near 100 on the horizontal axis. All other countries are shown in gray and exhibit more variation in domestic currency bias, portfolio returns, and domestic short-term interest rates.

B Theoretical Appendix

Appendix B provides derivations and proofs for the model in Section 4. It covers parameter restrictions (B.1), real and nominal budget constraint (B.2), utility maximization (B.3), the non-stochastic steady state (B.4), first-order Taylor expansions of equilibrium conditions (B.5), model aggregability (B.6), the aggregate non-portfolio problem (B.7) and portfolio problem (B.8), and the agent non-portfolio problem (B.9) and portfolio problem (B.10).

B.1 Parameter Restrictions

In this section, we collect our assumptions on parameter restrictions for easier reference throughout the appendix.

Assumption 1 (Preference Parameters)**.** *We assume that the baseline preference parameters satisfy*

$$\theta \in [0, \varepsilon)\,, \quad \beta \in (0.5, 1)\,, \quad \alpha_{ii} \in (0.5, 1)\,, \quad \textit{and} \qquad \alpha_{ii} + \alpha_{ij} = 1\,, \tag{B1}$$

so that domestic bias in consumption, measured by $\phi = \alpha_{ii} - \alpha_{ij}$*, lies in* $(0, 1)$*.*

These restrictions ensure that individual preferences yield a well-defined steady state and support the emergence of domestic portfolio bias in the model. Agents favor consumption of the domestic good when $\alpha_{ii} \in (0.5, 1)$, which strengthens the incentive to hold domestic bonds and helps to produce domestic bias in aggregate portfolio holdings.

Assumption 2 (Distribution of Risk Aversion)**.** *We assume risk aversion* ρ *lies in the interval* $(0, \rho_m]$*, and that the parameters of the cross-sectional distribution of risk aversion satisfy*

$$\kappa > 1 \quad \textit{and} \quad \rho_m > (\kappa^2 + \kappa)/(\kappa^2 - 1)\,, \tag{B2}$$

ensuring that the distribution has finite mean and variance, and that aggregate risk aversion is sufficiently high ($\bar{\rho} > \omega$*).*

Sufficiently high risk aversion is needed for Propositions 8, 9, and 10 establishing domestic portfolio bias in aggregate, negative expected cross-county differenced returns, and the existence of carry traders in low interest-rate economies.

Assumption 3 (Moments of Endowments and Money Supplies)**.** *With* i *fixed as either* H *or* F*, we assume that endowments and money supplies possess finite first and second moments and satisfy*

$$\begin{gathered} \mathrm{E}\Big[\ln(M^i_{it}/M^j_{jt})\ln(Y_{it}/Y_{jt})\Big] \geq 0\,, \quad \mathrm{E}\Big[\ln(M^i_{it}/M^j_{jt})\ln(Y_{it}Y_{jt})\Big] \geq 0\,, \\ \textit{and} \quad \mathrm{E}\Big[\ln(Y_{it})^2\Big] \geq \mathrm{E}\Big[\ln(Y_{jt})^2\Big]\,, \quad i \neq j\,. \end{gathered} \tag{B3}$$

These restrictions ensure that endowments and money supplies are well-behaved and that expected cross-country differenced returns are negative in at least one country.

B.2 Nominal and Real Budget Constraints

In this section, we relate the nominal agent budget constraint to the real agent budget constraint in (6), and the nominal bond return to the real bond return. Nominal and real budget constraints are mathematically equivalent and lead to identical first-order conditions.

Substituting the real return and real portfolio holdings into the agent's real budget constraint in (6) using the definitions in (8), we obtain

$$
\begin{aligned}
&P^i_{it}\frac{A_{iit}(\rho)P^i_{Bit}}{P^i_{it}} + P^i_{it}\frac{A_{ijt}(\rho)P^i_{Bjt}}{P^i_{it}} + P^i_{it}C_{it}(\rho) \\
&\quad = P^i_{it}\frac{S_{iit}/P^i_{it}}{P^i_{Bit-1}/P^i_{it-1}}\frac{A_{iit-1}(\rho)P^i_{Bit-1}}{P^i_{it-1}} + P^i_{it}\frac{S_{ijt}/P^i_{it}}{P^i_{Bjt-1}/P^i_{it-1}}\frac{A_{ijt-1}(\rho)P^i_{Bjt-1}}{P^i_{it-1}} + P^i_{Cit}Y_{it}(\rho)\,,
\end{aligned}
$$

which simplifies to the agent's nominal budget constraint,

$$
A_{iit}(\rho)P^i_{Bit} + A_{ijt}(\rho)P^i_{Bjt} + P^i_{it}C_{it}(\rho) = A_{iit-1}(\rho) + S_{ijt}A_{ijt-1}(\rho) + P^i_{Cit}Y_{it}(\rho)\,. \tag{B4}
$$

The agent's Euler equation in (29) would remain unchanged if we were to derive it from the nominal instead of the real budget constraint, maximizing with respect bond quantities rather than real values of bond holdings.

The Euler equations in (29) can be written in terms of nominal instead of real returns using the following expression,

$$
\begin{aligned}
R^j_{it} &= \frac{S_{jit}/P^j_{jt}}{P^j_{Bit-1}/P^j_{jt-1}} \\
&= \frac{S_{jit}/P^j_{Bit-1}}{P^j_{jt}/P^j_{jt-1}} = R^j_{Bit}/\Pi^j_{it}\,,
\end{aligned} \tag{B5}
$$

where R^j_{Bit} denotes the gross nominal return and Π^j_{it} denotes the gross rate of inflation. A version of the Fisher equation obtains as a first-order Taylor expansion of (B5),

$$
\hat{R}^j_{Bit} = \hat{R}^j_{it} + \hat{\Pi}^j_t + O(\epsilon^2)\,. \tag{B6}
$$

B.3 Utility Maximization

For the first-stage utility maximization problem, agent ρ in country i solves

$$
\begin{aligned}
\max\ \mathcal{L}^{(1)}_{it}(\rho) = &\sum_{s=t}^{\infty}\beta^{s-t}\,\mathrm{E}_t\left[\frac{1}{1-\rho}\left(\frac{C_{is}(\rho)}{C_i(\rho)}\right)^{1-\rho} + \frac{\theta}{1-\rho}\left(\frac{e^{W^i_{is}(\rho)}}{e^{W^i_i(\rho)}}\right)^{1-\rho}\right] \\
&+ \sum_{s=t}^{\infty}\beta^{s-t}\,\mathrm{E}_t\Big[\mu^{(1)}_{is}(\rho)\Big\{R^i_{is}B^i_{iis-1}(\rho) + R^i_{js}B^i_{ijs-1}(\rho) + W^i_{is}(\rho)\Big\}\Big] \\
&- \sum_{s=t}^{\infty}\beta^{s-t}\,\mathrm{E}_t\Big[\mu^{(1)}_{is}(\rho)\Big\{B^i_{iis}(\rho) + B^i_{ijs}(\rho) + C_{is}(\rho)\Big\}\Big]\,, \quad i \neq j
\end{aligned} \tag{B7}
$$

by choice of $\Big\{C_s(\rho), B^i_{iis}(\rho), B^i_{ijs}(\rho)\Big\}$ for $i \neq j$ and for $s \geq t$, where $\mathcal{L}^{(1)}_{it}(\rho)$ denotes the agent's Lagrangian for the first-stage problem and $\mu^{(1)}_{it}(\rho)$ denotes the Lagrangian multiplier on the agent's budget constraint. First-order conditions with respect to consumption and

real bond holdings are given by

$$\begin{aligned}\mu_{it}^{(1)}(\rho) &= \left(\frac{C_{it}(\rho)}{C_i(\rho)}\right)^{-\rho}\frac{1}{C_i(\rho)} \quad \text{and} \\ \mu_{it}^{(1)}(\rho) &= \theta\left(\frac{e^{W_{it}^i(\rho)}}{e^{W_i^i(\rho)}}\right)^{-\rho} + \beta\,\mathrm{E}_t\left[\mu_{it+1}^{(1)}(\rho)R_{jt+1}^i\right].\end{aligned} \tag{B8}$$

Combining the first-order conditions in (B8) yields the agent Euler equations in (17).

For the second-stage utility maximization problem, agent ρ in country i solves

$$\begin{aligned}\max\ \mathcal{L}_{it}^{(2)}(\rho) &= \gamma C_{iit}(\rho)^{\alpha_{ii}} C_{ijt}(\rho)^{\alpha_{ij}} \\ &\quad + \mu_{it}^{(2)}(\rho)\left(P_{it}^j C_{it}(\rho) - P_{Cit}^i C_{iit}(\rho) - P_{Cjt}^i C_{ijt}(\rho)\right), \quad i \neq j\end{aligned} \tag{B9}$$

by choice of $C_{iit}(\rho)$ and $C_{ijt}(\rho)$ for $i \neq j$, where $\mathcal{L}_{it}^{(2)}(\rho)$ denotes the agent's Lagrangian for the second-stage problem and $\mu_{it}^{(2)}(\rho)$ denotes the Lagrangian multiplier on the agent's expenditure constraint in (11). First-order conditions with respect to the domestic and international consumption goods are given by

$$\mu_{it}^{(2)}(\rho)P_{Cit}^i = \gamma\alpha_{ii}\left(\frac{C_{ijt}(\rho)}{C_{iit}(\rho)}\right)^{\alpha_{ij}} \quad \text{and} \quad \mu_{it}^{(2)}(\rho)P_{Cjt}^i = \gamma\alpha_{ij}\left(\frac{C_{iit}(\rho)}{C_{ijt}(\rho)}\right)^{\alpha_{ii}}, \quad i \neq j\,. \tag{B10}$$

Combining (B10) with the expenditure constraint in (11) yields the goods demands in (18). Combining (B10) with the definition of the consumption basket in (10) yields the price index in (19).

B.4 The Non-Stochastic Steady State

In this section, we prove Propositions 1 and 2 characterizing the non-stochastic steady-state equilibrium of the model.

B.4.1 Proof of Proposition 1

Proposition 1 characterizes the non-portfolio non-stochastic steady state. We begin by deriving the steady-state price indices, goods prices, and the real exchange rate. From the quantity equations and the equality of steady-state money supplies in (15),

$$M_i^i = P_{Ci}^i Y_i = S_{ij}P_{Cjt}^j Y_j = S_{ij}M_j^j = M_j^i\,. \tag{B11}$$

Using the normalization of steady-state real endowments in (9) together with the law of one price in (19), it follows that common-currency prices for the domestic and international goods are equal,

$$P_{Ci}^i = P_{Cj}^i\,, \tag{B12}$$

and using the price indices in (19) and the common-currency equality of goods prices, it follows that price indices and goods prices are equal,

$$P_i^j = P_{Ci}^j \,. \tag{B13}$$

Using the equality of prices derived above together with the definition of the real exchange rate in (14) evaluated at the steady state,

$$Q_{ij} = \frac{S_{ij} P_j^j}{P_i^i} = \frac{P_j^i}{P_i^i} = \frac{P_{Cj}^i}{P_{Ci}^i} = \frac{P_{Cj}^i}{P_{Cj}^i} = 1 \,. \tag{B14}$$

We now derive steady-state consumption. The left-hand side of the steady-state Euler equation in (20) is independent of j, so real returns on domestic and international nominal bonds are equal,

$$R_i^i = R_i^j \,, \tag{B15}$$

and the right-hand side is independent of ρ, so agent and aggregate consumption baskets are equal (given that the measure of agents equals one in each country),

$$C_i(\rho) = C_i \,. \tag{B16}$$

From the goods demands in (18), together with steady-state prices in (22),

$$C_{ij}(\rho) = C_{ij} = \alpha_{ij} C_i \,, \tag{B17}$$

and together with goods market clearing in (12), the equality of agent and aggregate endowments in (9), and the parameter restrictions in (1),

$$C_i(\rho) = C_i = Y_i = Y_i(\rho) = 1 \,. \tag{B18}$$

Since $C_i(\rho) = 1$, it follows from (20) that

$$R_i^j = \frac{1-\theta}{\beta} = R \,. \tag{B19}$$

We now derive steady-state real wealth. From the budget constraints in (6), using the definition of real wealth in (7) and the equality of steady-state real returns in (23),

$$W_i^i(\rho) + C_i(\rho) = R W_i^i(\rho) + Y_i(\rho) \,. \tag{B20}$$

Using the normalized values of steady-state consumption and endowments, and rearranging,

the budget constraint becomes

$$W_i^i(\rho)(1-R) = 0\,. \tag{B21}$$

Given that the measure of agents equals one in each country, it follows from the budget constraint that

$$W_i^i(\rho) = W_i^i = W_j^j = W_j^j(\rho)\,. \tag{B22}$$

From the bond market clearing conditions in (12), summing across bonds,

$$B_{ii}^i + B_{ji}^i + B_{jj}^i + B_{ij}^i = 0 \quad \Leftrightarrow \quad W_i^i + W_j^i = 0\,, \quad i \neq j\,, \tag{B23}$$

using the definition of real wealth in (7). From the definitions of real wealth in (7) and real bond holdings in (8), together with the steady-state real exchange rate in (22),

$$W_j^i = Q_{ij} W_j^j = W_j^j\,, \tag{B24}$$

Hence, we have

$$W_i^i = W_j^j\,, \quad W_i^i + W_j^i = 0\,, \quad \text{and} \quad W_j^i = W_j^j\,, \tag{B25}$$

and therefore

$$W_i^j(\rho) = W_i^j = 0\,. \tag{B26}$$

B.4.2 Proof of Proposition 2

Proposition 2 characterizes the portfolio non-stochastic steady state. The characterization is limited, and derives from the solution for steady-state agent and aggregate real wealth and from market clearing. From steady-state real wealth in (23), using the definition of real wealth in (7), we have

$$B_{ii}^i(\rho) = -B_{ij}^i(\rho) \quad \text{and} \quad B_{ii}^i = -B_{ij}^i\,. \tag{B27}$$

Combining these results with market clearing in (12), we have

$$B_{ii}^i = B_{jj}^i\,. \tag{B28}$$

We derive more complete characterizations of steady-state agent and aggregate portfolio holdings (to order-zero approximations) in Proposition 10.

B.5 First-Order Taylor Expansions

Several Taylor expansions already appear in the body of the paper; these include agent Euler equations in (29), agent budget constraints in (31), agent goods demands in (32),

and market clearing conditions for goods and bonds in (33). For the remaining equilibrium conditions, Taylor expansions are given below:

$$
\begin{aligned}
&\text{Agent Real Wealth:} && \hat{W}^j_{it}(\rho) = \hat{B}^j_{iit}(\rho) + \hat{B}^j_{ijt}(\rho) + O(\epsilon^2)\,, \quad i \neq j\,,\\
&\text{Real Bond Returns:} && \hat{R}^j_{it} = \left(\hat{S}_{jit} - \hat{P}^j_{Bit-1}\right) - \left(\hat{P}^j_{it} - \hat{P}^j_{it-1}\right) + O(\epsilon^2)\,,\\
&\text{Price Indices:} && \hat{P}^k_{it} = \alpha_{ii}\hat{P}^k_{Cit} + \alpha_{ij}\hat{P}^k_{Cjt} + O(\epsilon^2)\,, \quad i \neq j\,,\\
&\text{Consumption LoP:} && \hat{P}^i_{Cit} = \hat{S}_{ijt} + \hat{P}^j_{Cit} + O(\epsilon^2)\,,\\
&\text{Bond LoP:} && \hat{P}^i_{Bit} = \hat{S}_{ijt} + \hat{P}^j_{Bit} + O(\epsilon^2)\,,\\
&\text{Real Exchange Rate:} && \hat{Q}_{ijt} = \hat{S}_{ijt} + \hat{P}^j_{jt} - \hat{P}^i_{it} + O(\epsilon^2)\,, \quad \text{and}\\
&\text{Quantity Equations:} && \hat{M}^j_{it} = \hat{Y}_{it} + \hat{P}^j_{it} + O(\epsilon^2)\,.
\end{aligned}
\tag{B29}
$$

B.6 Model Aggregability

In this section, we derive expressions for aggregate risk aversion and aggregate intertemporal elasticity of substitution, and prove Proposition 3.

B.6.1 Aggregating Preference Parameters

Given the CRRA consumption utility in (4), heterogeneity can be defined equivalently in terms of either the coefficient of relative risk aversion or the intertemporal elasticity of substitution with respect to consumption. Using the distribution in (2), the aggregate coefficient of relative risk aversion and intertemporal elasticity of substitution are given by

$$
\bar{\rho} = \int_{\mathcal{R}} \rho f(\rho)\,\mathrm{d}\rho = \frac{\kappa\rho_m}{\kappa+1} \quad \text{and} \quad \bar{\sigma} = \int_{\mathcal{R}} \sigma(\rho) f(\rho)\,\mathrm{d}\rho = \frac{\kappa\rho_m}{\kappa-1}\,,
\tag{B30}
$$

where we have used equation (5) for $\sigma(\rho)$.

B.6.2 Proof of Proposition 3

Proposition 3 introduces the aggregation wedge ω. Using the expressions for $\bar{\rho}$ and $\bar{\sigma}$ in (B30) and the definition of ω in (28), we have

$$
\bar{\sigma} = \frac{\omega}{\bar{\rho}} \quad \Leftrightarrow \quad \frac{\kappa\rho_m}{\kappa-1} = \omega\frac{\kappa+1}{\kappa\rho_m} \quad \Leftrightarrow \quad \omega = \frac{\kappa^2}{\kappa^2-1}\,.
$$

By inspection, the wedge is greater than one and decreasing in κ for $\kappa > 1$.

B.6.3 Proof of Proposition 4

Proposition 4 states the first-order approximate agent and aggregate Euler equations. We begin with the agent Euler equation. Combining fist-order conditions in (B8), we obtain

$$\left(\frac{C_{it}(\rho)}{C_i(\rho)}\right)^{-\rho}\frac{1}{C_i(\rho)} = \theta\left(\frac{e^{W^i_{it}(\rho)}}{e^{W^i_i(\rho)}}\right)^{-\rho} + \beta\,\mathrm{E}_t\left[\left(\frac{C_{it+1}(\rho)}{C_i(\rho)}\right)^{-\rho}\frac{1}{C_i(\rho)}R^i_{jt+1}\right].$$

A standard first-order Taylor expansion of this expression around the non-stochastic steady state then yields (29).

The expression in (29) aggregates straightforwardly. The coefficient of relative risk aversion is the only source heterogeneity in the expression, and it multiplies an aggregate variable. Integrating (29) over agents,

$$\begin{aligned}\int_{\mathcal{R}}\hat{C}_{it}(\rho)f(\rho)\,\mathrm{d}\rho &= \theta\int_{\mathcal{R}}\hat{W}^i_{it}(\rho)f(\rho)\,\mathrm{d}\rho \\ &+ \beta R\,\mathrm{E}_t\left[\int_{\mathcal{R}}\hat{C}_{it+1}(\rho)f(\rho)\,\mathrm{d}\rho - \int_{\mathcal{R}}\frac{1}{\rho}f(\rho)\,\mathrm{d}\rho\hat{R}^i_{jt+1}\right] + O(\epsilon^2)\,,\end{aligned} \tag{B31}$$

and noting that (26) implies

$$\hat{C}_{it} = \int_{\mathcal{R}}\hat{C}_{it}(\rho)f(\rho)\,\mathrm{d}\rho + O(\epsilon^2) \quad \text{and} \quad \hat{W}^j_{it} = \int_{\mathcal{R}}\hat{W}^j_{it}(\rho)f(\rho)\,\mathrm{d}\rho + O(\epsilon^2)\,,$$

we obtain (30) from (B31) directly. Expressions in (5) and (27) are used for the coefficient of relative risk aversion.

B.7 The Aggregate Non-Portfolio Problem

In the aggregate non-portfolio problem we solve for aggregate consumption and real wealth and for the real exchange rate in terms of lagged state variables, exogenous variables, and parameters. We first derive cross-country differenced solutions, and then use market clearing conditions to derive country-specific solutions.

B.7.1 The Aggregate Non-Portfolio System

To derive the cross-country differenced aggregate non-portfolio system in (37), we combine the market clearing condition in (33) with the aggregate budget constraints in (31) differenced across countries and aggregate Euler equations in (30) differenced across countries. Using the market clearing condition in (33) to eliminate the real exchange rate from the cross-country differenced aggregate budget constraint in (35), we obtain

$$\hat{W}^i_{i-jt} = R\hat{W}^i_{i-jt-1} - \frac{1-\phi}{1-\phi^2}\hat{C}_{i-jt} + \phi\frac{1-\phi}{1-\phi^2}\hat{Y}_{i-jt} + R\hat{V}^i_{i-jt} + O(\epsilon^2)\,. \tag{B32}$$

Similarly, using the market clearing condition in (33) to eliminate the real exchange rate from the cross-country differenced aggregate Euler equation in (35), we obtain

$$\begin{aligned}&\theta\hat{W}^i_{i-jt} + \beta R\left(1+\bar{\sigma}\frac{\phi^2}{1-\phi^2}\right)\mathrm{E}_t\left[\hat{C}_{i-jt+1}\right]\\&\quad=\left(1+\bar{\sigma}\beta R\frac{\phi^2}{1-\phi^2}\right)\hat{C}_{i-jt}+\bar{\sigma}\beta R\frac{\phi}{1-\phi^2}\hat{Y}_{i-jt}+O(\epsilon^2)\,.\end{aligned} \tag{B33}$$

These two equations, (B32) and (B33), constitute the cross-country differenced aggregate system. The system depends on three endogenous variables: cross-country differenced aggregate consumption $\hat{C}_{i-jt}$, cross-country differenced aggregate real wealth $\hat{W}^i_{i-jt}$, and the cross-country differenced aggregate portfolio valuation effect $\hat{V}^i_{i-jt}$. We solve the system initially for $\hat{C}_{i-jt}$ and $\hat{W}^i_{i-jt}$ in terms of $\hat{V}^i_{i-jt}$, describing these solutions as *intermediate solutions*. Solving for $\hat{V}^i_{i-jt}$ requires second-order Taylor expansions of Euler equations and a solution for B^i_{ii-ji}, which we derive in B.8.

We write the cross-country differenced aggregate system in (B32) and (B33) as

$$\boldsymbol{\mathcal{E}}^{(-)}_0\,\mathrm{E}_t\left[\hat{\boldsymbol{Z}}_{i-jt+1}\right]=\boldsymbol{\mathcal{E}}^{(-)}_1\hat{\boldsymbol{Z}}_{i-jt}+\boldsymbol{\mathcal{E}}^{(-)}_2\hat{Y}_{i-jt}+\boldsymbol{\mathcal{E}}^{(-)}_3\hat{V}^i_{i-jt}+O(\epsilon^2)\,, \tag{B34}$$

where the coefficient matrices are given by

$$\begin{aligned}&\boldsymbol{\mathcal{E}}^{(-)}_0=\begin{bmatrix}1 & 0\\ \theta & \beta R\left(1+\bar{\sigma}\frac{\phi^2}{1-\phi^2}\right)\end{bmatrix},\quad \boldsymbol{\mathcal{E}}^{(-)}_1=\begin{bmatrix}R & -\frac{1-\phi}{1-\phi^2}\\ 0 & 1+\bar{\sigma}\beta R\frac{\phi^2}{1-\phi^2}\end{bmatrix},\\&\boldsymbol{\mathcal{E}}^{(-)}_2=\begin{bmatrix}\phi\frac{1-\phi}{1-\phi^2}\\ -\bar{\sigma}\beta R\frac{\phi}{1-\phi^2}\end{bmatrix},\quad\text{and}\quad \boldsymbol{\mathcal{E}}^{(-)}_3=\begin{bmatrix}R\\ 0\end{bmatrix}.\end{aligned} \tag{B35}$$

The matrix $\boldsymbol{\mathcal{E}}^{(-)}_0$ is invertible if its determinant is non-zero. Therefore, we require that

$$\det\left(\boldsymbol{\mathcal{E}}^{(-)}_0\right)=\beta R\left(1+\bar{\sigma}\frac{\phi^2}{1-\phi^2}\right)\neq 0\,, \tag{B36}$$

which is always satisfied under Assumptions 1 and 2. Inverting $\boldsymbol{\mathcal{E}}^{(-)}_0$, we rewrite the aggregate system in (B34) to obtain (37), with partial elasticity matrices given by

$$\boldsymbol{\mathcal{E}}^{(-)}_{ZZ}=\begin{bmatrix}\mathcal{E}^{(-)}_{WW} & \mathcal{E}^{(-)}_{WC}\\ \mathcal{E}^{(-)}_{CW} & \mathcal{E}^{(-)}_{CC}\end{bmatrix}=\begin{bmatrix}R & -\frac{1-\phi}{1-\phi^2}\\ -\frac{\theta}{\beta}\frac{1-\phi^2}{1-(1-\bar{\sigma})\phi^2} & \frac{1}{\beta R}\frac{1+\theta(1-\phi)-(1-\bar{\sigma}\beta R)\phi^2}{1-(1-\bar{\sigma})\phi^2}\end{bmatrix},$$

$$\boldsymbol{\mathcal{E}}^{(-)}_{ZY}=\begin{bmatrix}\mathcal{E}^{(-)}_{WY}\\ \mathcal{E}^{(-)}_{CY}\end{bmatrix}=\begin{bmatrix}\phi\frac{1-\phi}{1-\phi^2}\\ -\frac{\phi}{1-(1-\bar{\sigma})\phi^2}\left(\bar{\sigma}+\frac{\theta(1-\phi)}{\beta R}\right)\end{bmatrix},\quad\text{and} \tag{B37}$$

$$\boldsymbol{\mathcal{E}}_{ZV}^{(-)} = \begin{bmatrix} \mathcal{E}_{WV}^{(-)} \\ \mathcal{E}_{CV}^{(-)} \end{bmatrix} = \begin{bmatrix} R \\ -\frac{\theta}{\beta}\frac{1-\phi^2}{1-(1-\bar{\sigma})\phi^2} \end{bmatrix}.$$

Before solving the aggregate cross-country differenced system in (37), we establish the conditions for a unique and stationary rational expectations solution following Blanchard and Kahn (1980). The eigendecomposition of $\boldsymbol{\mathcal{E}}_{ZZ}^{(-)}$ is given by

$$\boldsymbol{\mathcal{E}}_{ZZ}^{(-)} = \boldsymbol{\mathcal{V}}_{ZZ}^{(-)} \boldsymbol{\Lambda}_{ZZ}^{(-)} \boldsymbol{\mathcal{V}}_{ZZ}^{(-)^{-1}} \tag{B38}$$

where $\boldsymbol{\mathcal{V}}_{ZZ}^{(-)}$ is a matrix of eigenvectors and $\boldsymbol{\Lambda}_{ZZ}^{(-)}$ a diagonal matrix of eigenvalues,

$$\begin{aligned} \boldsymbol{\mathcal{V}}_{ZZ}^{(-)} &= \begin{bmatrix} \nu_{WW}^{(-)} & \nu_{WC}^{(-)} \\ \nu_{CW}^{(-)} & \nu_{CC}^{(-)} \end{bmatrix} = \begin{bmatrix} \mathcal{E}_{WC}^{(-)} & \mathcal{E}_{WC}^{(-)} \\ \lambda_{ZW}^{(-)} - \mathcal{E}_{WW}^{(-)} & \lambda_{ZC}^{(-)} - \mathcal{E}_{WW}^{(-)} \end{bmatrix}, \quad \text{and} \\ \boldsymbol{\Lambda}_{ZZ}^{(-)} &= \begin{bmatrix} \lambda_{ZW}^{(-)} & 0 \\ 0 & \lambda_{ZC}^{(-)} \end{bmatrix}, \end{aligned} \tag{B39}$$

where

$$\lambda_{ZW}^{(-)} = \frac{1}{2}\left(\mathcal{E}_{WW}^{(-)} + \mathcal{E}_{CC}^{(-)}\right) + \sqrt{\frac{1}{4}\left(\mathcal{E}_{WW}^{(-)} - \mathcal{E}_{CC}^{(-)}\right)^2 + \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CW}^{(-)}}, \quad \text{and} \tag{B40}$$

$$\lambda_{ZC}^{(-)} = \frac{1}{2}\left(\mathcal{E}_{WW}^{(-)} + \mathcal{E}_{CC}^{(-)}\right) - \sqrt{\frac{1}{4}\left(\mathcal{E}_{WW}^{(-)} - \mathcal{E}_{CC}^{(-)}\right)^2 + \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CW}^{(-)}}. \tag{B41}$$

B.7.2 Proof of Proposition 5

Proposition 5 states that $\lambda_{ZW}^{(-)}$ and $\lambda_{ZC}^{(-)}$ are distinct and real-valued if $\theta \geq 0$. To prove this statement, it suffices to show that the discriminant in (B40) and (B41) is positive. The discriminant Δ is given by

$$\Delta = \frac{1}{4}\left(\mathcal{E}_{WW}^{(-)} - \mathcal{E}_{CC}^{(-)}\right)^2 + \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CW}^{(-)}. \tag{B42}$$

Suppose $\theta > 0$. The squared term is always positive or zero, so we need to show that the second term is positive,

$$\mathcal{E}_{WC}^{(-)}\mathcal{E}_{CW}^{(-)} > 0 \quad \Leftrightarrow \quad \frac{\theta}{\beta}\frac{1-\phi}{1-(1-\bar{\sigma})\phi^2} > 0, \tag{B43}$$

where we have used the partial elasticities in (B37). Under Assumptions 1 and 2, the inequality in (B43) is always satisfied.

Suppose $\theta = 0$. The second term is zero, $\mathcal{E}_{WC}^{(-)}\mathcal{E}_{CW}^{(-)} = 0$, but the squared term is positive, since $\mathcal{E}_{WW}^{(-)}\Big|_{\theta=0} - \mathcal{E}_{CC}^{(-)}\Big|_{\theta=0} = 1/\beta - 1 > 0$, so the discriminant Δ is again positive. Thus, $\theta \geq 0$ is sufficient for distinct and real-valued eigenvalues.

Table B3 – Aggregate Partial Elasticities and Their Derivatives at $\theta = 0$

Partial Elasticity	Value of Partial Elasticity Evaluated at $\theta = 0$		Derivative with Respect to θ Evaluated at $\theta = 0$	
	Expression	Sign	Expression	Sign
$\mathcal{E}^{(-)}_{WW}$	$\frac{1}{\beta}$	$+$	$-\frac{1}{\beta}$	$-$
$\mathcal{E}^{(-)}_{WC}$	$-\frac{1-\phi}{1-\phi^2}$	$-$	0	$+$
$\mathcal{E}^{(-)}_{CW}$	0	$\cdot$	$-\frac{1}{\beta}\frac{1-\phi^2}{1-(1-\bar{\sigma})\phi^2}$	$-$
$\mathcal{E}^{(-)}_{CC}$	1	$+$	$\frac{2-\phi-\phi^2}{1-(1-\bar{\sigma})\phi^2}$	$+$
$\mathcal{E}^{(-)}_{WY}$	$\phi\frac{1-\phi}{1-\phi^2}$	$+$	0	$\cdot$
$\mathcal{E}^{(-)}_{CY}$	$-\frac{\phi\bar{\sigma}}{1-(1-\bar{\sigma})\phi^2}$	$-$	$-\frac{\phi(1-\phi)}{1-(1-\bar{\sigma})\phi^2}$	$-$
$\mathcal{E}^{(-)}_{WV}$	$\frac{1}{\beta}$	$+$	$-\frac{1}{\beta}$	$-$
$\mathcal{E}^{(-)}_{CV}$	0	$\cdot$	$-\frac{1}{\beta}\frac{1-\phi^2}{1-(1-\bar{\sigma})\phi^2}$	$-$

Notes. Column one lists the partial elasticities that appear in the cross-country differenced aggregate system (37). Columns two and four, respectively, give expressions for the partial elasticities and their derivatives with respect to θ, both evaluated at $\theta = 0$. Columns three and five, respectively, give the signs of the partial elasticities and their derivatives, both evaluated at $\theta = 0$. Signs are established under Assumptions 1 and 2.

The case where $\theta = 0$ serves as a helpful benchmark for proving the remaining statements in Proposition 5. Table B3 reports the partial elasticities in (37) and their derivatives with respect to θ, evaluated at $\theta = 0$.

Proposition 5 states that $\lambda^{(-)}_{ZW}$ evaluated at $\theta = 0$ is strictly greater than one and $\lambda^{(-)}_{ZC}$ evaluated at $\theta = 0$ is exactly one. Using Table B3 to evaluate (B40) at $\theta = 0$,

$$\lambda^{(-)}_{ZW}\Big|_{\theta=0} = \frac{1}{2}\left(\frac{1}{\beta}+1\right) + \frac{1}{2}\left|\frac{1}{\beta}-1\right| = \frac{1}{\beta} = \mathcal{E}^{(-)}_{WW}\Big|_{\theta=0} > 1\,, \tag{B44}$$

and (B41) at $\theta = 0$,

$$\lambda^{(-)}_{ZC}\Big|_{\theta=0} = \frac{1}{2}\left(\frac{1}{\beta}+1\right) - \frac{1}{2}\left|\frac{1}{\beta}-1\right| = \mathcal{E}^{(-)}_{CC}\Big|_{\theta=0} = 1\,. \tag{B45}$$

Proposition 5 also states that the partial derivative of $\lambda^{(-)}_{ZW}$ with respect to θ evaluated at $\theta = 0$ is positive, and the partial derivative of $\lambda^{(-)}_{ZC}$ with respect to θ evaluated at $\theta = 0$

is negative. To establish this result, we differentiate the eigenvalues with respect to θ,

$$\begin{aligned}\frac{\partial \lambda_{\pm}^{(-)}}{\partial \theta} &= \frac{1}{2}\left(\frac{\partial \mathcal{E}_{ZW}^{(-)}}{\partial \theta} + \frac{\partial \mathcal{E}_{ZC}^{(-)}}{\partial \theta}\right) \pm \frac{1}{2}\left[\frac{1}{4}\left(\mathcal{E}_{WW}^{(-)} - \mathcal{E}_{CC}^{(-)}\right)^2 + \mathcal{E}_{WC}^{(-)}\mathcal{E}_{CW}^{(-)}\right]^{-\frac{1}{2}} \\ &\times \left[\frac{1}{2}\left(\mathcal{E}_{WW}^{(-)} - \mathcal{E}_{CC}^{(-)}\right)\left(\frac{\partial \mathcal{E}_{WW}^{(-)}}{\partial \theta} - \frac{\partial \mathcal{E}_{CC}^{(-)}}{\partial \theta}\right) + \frac{\partial \mathcal{E}_{WC}^{(-)}}{\partial \theta}\mathcal{E}_{CW}^{(-)} + \mathcal{E}_{WC}^{(-)}\frac{\partial \mathcal{E}_{CW}^{(-)}}{\partial \theta}\right],\end{aligned} \tag{B46}$$

where the "+" case corresponds to eigenvalue $\lambda_{ZW}^{(-)}$ and the "−" case corresponds to eigenvalue $\lambda_{ZC}^{(-)}$. Using Table B3, we then evaluate (B46) at $\theta = 0$,

$$\begin{aligned}\left.\frac{\partial \lambda_{ZW}^{(-)}}{\partial \theta}\right|_{\theta=0} &= \left.\frac{\partial \mathcal{E}_{WW}^{(-)}}{\partial \theta}\right|_{\theta=0} + \left.\frac{\beta}{1-\beta}\mathcal{E}_{WC}^{(-)}\right|_{\theta=0}\left.\frac{\partial \mathcal{E}_{CW}^{(-)}}{\partial \theta}\right|_{\theta=0} \\ &= \frac{1}{1-\beta}\frac{1-\phi}{1-(1-\bar{\sigma})\phi^2} - \frac{1}{\beta},\end{aligned} \tag{B47}$$

and

$$\begin{aligned}\left.\frac{\partial \lambda_{ZC}^{(-)}}{\partial \theta}\right|_{\theta=0} &= \left.\frac{\partial \mathcal{E}_{CC}^{(-)}}{\partial \theta}\right|_{\theta=0} - \left.\frac{\beta}{1-\beta}\mathcal{E}_{WC}^{(-)}\right|_{\theta=0}\left.\frac{\partial \mathcal{E}_{CW}^{(-)}}{\partial \theta}\right|_{\theta=0} \\ &= \frac{1}{1-(1-\bar{\sigma})\phi^2}\left[2 - \phi - \phi^2 - \frac{1-\phi}{1-\beta}\right].\end{aligned} \tag{B48}$$

The right-hand side of (B47) is positive if

$$\beta > \frac{1-(1-\bar{\sigma})\phi^2}{2-(1-\bar{\sigma})\phi^2-\phi}, \tag{B49}$$

and the right-hand side of (B48) is negative if

$$\beta > \frac{1-\phi^2}{2-\phi^2-\phi}. \tag{B50}$$

The inequality in (B50) is implied by the inequality in (B49). Hence, (B49) provides a sufficient condition for the derivatives of $\lambda_{ZW}^{(-)}$ and $\lambda_{ZC}^{(-)}$ with respect to θ to be positive and negative, respectively, when evaluated at $\theta = 0$.

Assumptions 1 and 2 ensure that (B49) is satisfied. By continuity, a small positive value for θ then pushes the eigenvalues $\lambda_{ZC}^{(-)}$ below one and $\lambda_{ZW}^{(-)}$ further above one, ensuring a unique and stationary rational expectations solution to (37).

B.7.3 Intermediate Aggregate Non-Portfolio Solution

We now derive intermediate solutions for aggregate real wealth, aggregate consumption, and the real exchange rate in terms lagged state variables, exogenous variables and parameters, and the endogenous aggregate portfolio valuation effect. Using the eigendecomposition in

(B38), we rewrite the aggregate cross-country differenced system in (B34) as

$$\begin{aligned}&\boldsymbol{\nu}_{ZZ}^{(-)^{-1}}\,\mathrm{E}_t\Big[\hat{\boldsymbol{Z}}_{i-jt+1}\Big]\\&\quad=\boldsymbol{\Lambda}_{ZZ}^{(-)}\boldsymbol{\nu}_{ZZ}^{(-)^{-1}}\hat{\boldsymbol{Z}}_{i-jt}+\boldsymbol{\nu}_{ZZ}^{(-)^{-1}}\boldsymbol{\mathcal{E}}_{ZY}^{(-)}\hat{Y}_{i-jt}+\boldsymbol{\nu}_{ZZ}^{(-)^{-1}}\boldsymbol{\mathcal{E}}_{ZV}^{(-)}\hat{V}_{i-jt}^{i}+O(\epsilon^2)\,,\end{aligned}\tag{B51}$$

and from (B51) we extract the equation associated with the unstable eigenvalue $\lambda_{ZW}^{(-)}$,

$$\begin{aligned}\mathrm{E}_t\Big[\hat{Z}_{i-jt+1}\Big]=\lambda_{ZW}^{(-)}\hat{Z}_{i-jt}&+\Big[\Big(\lambda_{ZC}^{(-)}-\mathcal{E}_{WW}^{(-)}\Big)\mathcal{E}_{WY}^{(-)}-\mathcal{E}_{WC}^{(-)}\mathcal{E}_{CY}^{(-)}\Big]\hat{V}_{i-jt}^{i}\\&+\Big[\Big(\lambda_{ZC}^{(-)}-\mathcal{E}_{WW}^{(-)}\Big)\mathcal{E}_{WV}^{(-)}-\mathcal{E}_{WC}^{(-)}\mathcal{E}_{CV}^{(-)}\Big]\hat{Y}_{i-jt}+O(\epsilon^2)\,,\end{aligned}\tag{B52}$$

where $\hat{Z}_{i-jt+1}=\Big(\lambda_{ZC}^{(-)}-\mathcal{E}_{WW}^{(-)}\Big)\hat{W}_{i-jt}^{i}-\mathcal{E}_{WC}^{(-)}\hat{C}_{i-jt+1}$.

Because $\lambda_{ZW}^{(-)}$ is greater than one, the left-hand side of (B52) must be zero to rule out explosive paths. Setting the left-hand side to zero, we obtain

$$\hat{C}_{i-jt}=\eta_{CW}^{(-)}\hat{W}_{i-jt-1}^{i}+\eta_{CY}^{(-)}\hat{Y}_{i-jt}+\eta_{CV}^{(-)}\hat{V}_{i-jt}^{i}+O(\epsilon^2)\,,\tag{B53}$$

and using (B53) to eliminate $\hat{C}_{i-jt}$ from (B32), we obtain

$$\hat{W}_{i-jt}^{i}=\eta_{WW}^{(-)}\hat{W}_{i-jt-1}^{i}+\eta_{WY}^{(-)}\hat{Y}_{i-jt}+\eta_{WV}^{(-)}\hat{V}_{i-jt}^{i}+O(\epsilon^2)\,.\tag{B54}$$

Writing the intermediate solutions in (B53) and (B54) in more compact matrix form, we obtain (39), where the semi-partial elasticity matrices are given by

$$\begin{aligned}\boldsymbol{\eta}_{ZW}^{(-)}&=\begin{bmatrix}\eta_{WW}^{(-)}\\ \eta_{CW}^{(-)}\end{bmatrix}=\begin{bmatrix}\lambda_{ZC}^{(-)}\\ \frac{\lambda_{ZC}^{(-)}-\mathcal{E}_{WW}^{(-)}}{\mathcal{E}_{WC}^{(-)}}\end{bmatrix},\\[2ex]\boldsymbol{\eta}_{ZY}^{(-)}&=\begin{bmatrix}\eta_{WY}^{(-)}\\ \eta_{CY}^{(-)}\end{bmatrix}=\begin{bmatrix}\frac{\Big(\lambda_{ZC}^{(-)}-\mathcal{E}_{WW}^{(-)}\Big)\mathcal{E}_{WY}^{(-)}-\mathcal{E}_{WC}^{(-)}\mathcal{E}_{CY}^{(-)}}{\lambda_{ZW}^{(-)}}-\phi\mathcal{E}_{WC}^{(-)}\\ \frac{\Big(\lambda_{ZC}^{(-)}-\mathcal{E}_{WW}^{(-)}\Big)\mathcal{E}_{WY}^{(-)}-\mathcal{E}_{WC}^{(-)}\mathcal{E}_{CY}^{(-)}}{\lambda_{ZW}^{(-)}\mathcal{E}_{WC}^{(-)}}\end{bmatrix},\quad\text{and}\\[2ex]\boldsymbol{\eta}_{ZV}^{(-)}&=\begin{bmatrix}\eta_{WV}^{(-)}\\ \eta_{CV}^{(-)}\end{bmatrix}=\begin{bmatrix}\frac{\Big(\lambda_{ZC}^{(-)}-\mathcal{E}_{WW}^{(-)}\Big)\mathcal{E}_{WV}^{(-)}-\mathcal{E}_{WC}^{(-)}\mathcal{E}_{CV}^{(-)}}{\lambda_{ZW}^{(-)}}+\mathcal{E}_{WW}^{(-)}\\ \frac{\Big(\lambda_{ZC}^{(-)}-\mathcal{E}_{WW}^{(-)}\Big)\mathcal{E}_{WV}^{(-)}-\mathcal{E}_{WC}^{(-)}\mathcal{E}_{CV}^{(-)}}{\lambda_{ZW}^{(-)}\mathcal{E}_{WC}^{(-)}}\end{bmatrix}.\end{aligned}\tag{B55}$$

For the real exchange rate, we combine the market clearing condition in (33) with our intermediate solution for cross-country differenced consumption in (B53) to obtain (43),

with semi-partial elasticities

$$\begin{aligned}
\eta^{(-)}_{QW} &= -\frac{\phi^2}{1-\phi^2}\frac{\lambda^{(-)}_{ZC} - \mathcal{E}^{(-)}_{WW}}{\mathcal{E}^{(-)}_{WC}} \\
\eta^{(-)}_{QY} &= \frac{\phi}{1-\phi^2}\left(1 - \phi\frac{\left(\lambda^{(-)}_{ZC} - \mathcal{E}^{(-)}_{WW}\right)\mathcal{E}^{(-)}_{WY} - \mathcal{E}^{(-)}_{WC}\mathcal{E}^{(-)}_{CY}}{\lambda^{(-)}_{ZW}\mathcal{E}^{(-)}_{WC}}\right), \quad \text{and} \\
\eta^{(-)}_{QV} &= -\frac{\phi^2}{1-\phi^2}\frac{\left(\lambda^{(-)}_{ZC} - \mathcal{E}^{(-)}_{WW}\right)\mathcal{E}^{(-)}_{WV} - \mathcal{E}^{(-)}_{WC}\mathcal{E}^{(-)}_{CV}}{\lambda^{(-)}_{ZW}\mathcal{E}^{(-)}_{WC}}\,.
\end{aligned} \tag{B56}$$

The semi-partial elasticities in (B55) and (B56) can be written in terms of model parameters using the partial elasticities in (B37) and the eigenvalues in (B40) and (B41).

B.8 The Aggregate Portfolio Problem

In the aggregate portfolio problem we solve for cross-country differenced realized and expected real returns on nominal bonds and cross-country differenced aggregate portfolio holdings of nominal bonds.

To begin, we take the cross-country difference of realized real returns in (B29) to obtain

$$\hat{R}^i_{i-jt+1} = -\hat{S}_{jit+1} - \hat{P}^i_{Bi-jt} + O(\epsilon^2)\,, \tag{B57}$$

which we combine with the first-order expansion of expected real returns in (48) to obtain

$$\hat{R}^i_{i-jt+1} = -\Big(S_{ijt+1} - \mathrm{E}_t\Big[S_{ijt+1}\Big]\Big) + O(\epsilon^2)\,. \tag{B58}$$

Using first-order Taylor expansions of the real exchange rate, price indices, and quantity equations in 48, the nominal exchange rate can be written as

$$\hat{S}_{ijt} = \frac{1}{\phi}\hat{Q}_{ijt} + \hat{M}^{i-j}_{i-jt} - \hat{Y}_{i-jt} + O(\epsilon^2)\,, \tag{B59}$$

and combining with the intermediate solution for the real exchange rate in (43) and the expression for the cross-country differenced real return in (B58), we obtain

$$\hat{R}^i_{i-jt} = \eta^{(-)}_{RY}\hat{Y}_{i-jt} + \eta^{(-)}_{RM}\hat{M}^{i-j}_{i-jt} + \eta^{(-)}_{RV}\hat{V}^i_{i-jt} + O(\epsilon^2)\,, \tag{B60}$$

Table B4 – Aggregate Semi-Partial Elasticities at $\theta = 0$

Semi-Partial Elasticity	Evaluation at $\theta = 0$	
	Expression	Sign
$\eta^{(-)}_{CY}$	$(1-\beta)\phi + \frac{\beta\phi\bar{\sigma}}{1-(1-\bar{\sigma})\phi^2}$	$+$
$\eta^{(-)}_{WY}$	$\beta\phi\frac{(1-\phi)(1-\bar{\sigma})}{1-(1-\bar{\sigma})\phi^2}$	$+$
$\eta^{(-)}_{QY}$	$\phi\Big(1+\beta\frac{1-\bar{\sigma}}{1-(1-\bar{\sigma})\phi^2}\Big)$	$+$
$\eta^{(-)}_{RY}$	$-\beta\phi^2\frac{1-\bar{\sigma}}{1-(1-\bar{\sigma})\phi^2}$	$-$
$\eta^{(-)}_{CV}$	$\frac{1-\beta}{\beta}\frac{1-\phi^2}{1-\phi}$	$+$
$\eta^{(-)}_{WV}$	$\frac{1-\phi^2-(1-\beta)(1-\phi)}{\beta\left(1-\phi^2\right)}$	$+$
$\eta^{(-)}_{QV}$	$-\frac{1-\beta}{\beta}\frac{\phi^2}{1-\phi}$	$-$
$\eta^{(-)}_{QW}$	$-\frac{1-\beta}{\beta}\frac{\phi^2}{1-\phi}$	$-$
$\eta^{(-)}_{RV}$	$\frac{1-\beta}{\beta}\frac{1-\phi^2}{1-\phi}$	$+$
$\eta^{(-)}_{RM}$	-1	$-$

Notes. The table shows semi-partial elasticities from the cross-country differenced aggregate non-portfolio and portfolio problems, evaluated at $\theta = 0$. Column two give an expression for each elasticity, and column three gives its sign under Assumptions 1 and 2.

where the real return semi-partial elasticities are given by

$$\begin{aligned}
\eta^{(-)}_{RY} &= \frac{\phi}{1-\phi^2}\left(\frac{\left(\lambda^{(-)}_{ZC} - \mathcal{E}^{(-)}_{WW}\right)\mathcal{E}^{(-)}_{WY} - \mathcal{E}^{(-)}_{WC}\mathcal{E}^{(-)}_{CY}}{\lambda^{(-)}_{ZW}\mathcal{E}^{(-)}_{WC}} - \phi\right), \\
\eta^{(-)}_{RM} &= -1\,, \text{ and} \\
\eta^{(-)}_{RV} &= \frac{\phi}{1-\phi^2}\frac{\left(\lambda^{(-)}_{ZC} - \mathcal{E}^{(-)}_{WW}\right)\mathcal{E}^{(-)}_{WV} - \mathcal{E}^{(-)}_{WC}\mathcal{E}^{(-)}_{CV}}{\lambda^{(-)}_{ZW}\mathcal{E}^{(-)}_{WC}}\,.
\end{aligned} \tag{B61}$$

We substitute the portfolio valuation effect (36) to into (B60) and rearrange to obtain the cross-country differenced real return in terms of general elasticities in (44), where the general elasticities are given by

$$\gamma^{(-)}_{RY} = \frac{\eta^{(-)}_{RY}}{1-\eta^{(-)}_{RV}B^{i}_{ii-ji}} \quad \text{and} \quad \gamma^{(-)}_{RM} = \frac{\eta^{(-)}_{RM}}{1-\eta^{(-)}_{RV}B^{i}_{ii-ji}}\,, \tag{B62}$$

and where the semi-partial elasticities $\eta^{(-)}_{RY}$, $\eta^{(-)}_{RM}$, and $\eta^{(-)}_{RV}$ can be written in terms of model parameters using the partial elasticities in (B37), but where B^{i}_{ii-ji} remains to be solved. For use later, we report the semi-partial elasticities from the aggregate portfolio problem in Table B4, evaluate at $\theta = 0$.

B.8.1 Proof of Proposition 6

The cross-country differenced aggregate portfolio valuation multiplier $\mu^{(-)}_{ii-ji}$ in (45) depends on the semi-partial elasticity $\eta^{(-)}_{RV}$ in (B61), and exceeds one in absolute value if

$$\frac{1}{\left|1-\eta^{(-)}_{RV}B^{i}_{ii-ji}\right|}>1\,. \tag{B63}$$

From Table B4, $\eta^{(-)}_{RV}>0$ at $\theta=0$. Because $\eta^{(-)}_{RV}$ is continuous in θ in the neighborhood of $\theta=0$, its sign is preserved for $\theta\in[0,\epsilon)$.

Case one: $1-\eta^{(-)}_{RV}B^{i}_{ii-ji}>0$. In this case, $1>1-\eta^{(-)}_{RV}B^{i}_{ii-ji}$ is equivalent to $0<B^{i}_{ii-ji}$. Hence, $0<B^{i}_{ii-ji}<1/\eta^{(-)}_{RV}$ implies $\left|\mu^{(-)}_{ii-ji}\right|>1$. Case two: $1-\eta^{(-)}_{RV}B^{i}_{ii-ji}<0$. In this case, $1>\eta^{(-)}_{RV}B^{i}_{ii-ji}-1$ is equivalent to $B^{i}_{ii-ji}<2/\eta^{(-)}_{RV}$. Hence, $\frac{1}{\eta^{(-)}_{RV}}<B^{i}_{ii-ji}<\frac{2}{\eta^{(-)}_{RV}}$ implies $\left|\mu^{(-)}_{ii-ji}\right|>1$. Combining cases one and two,

$$0<B^{i}_{ii-ji}<\frac{2}{\eta^{(-)}_{RV}}\quad\Rightarrow\quad\left|\mu^{(-)}_{ii-ji}\right|>1\,. \tag{B64}$$

As an illustration, suppose $\theta=0$, $\phi=0.5$, and $\beta=0.95$. In this case, $2/\eta^{(-)}_{RV}=38$; domestic bias in aggregate portfolio holdings amplifies the response of the cross-country differenced real return to endowment and money supply shocks, provided that the cross-country differenced aggregate holdings of country i's bond are less than 38 times the size of the steady-state economy, which is normalized to one.

In contrast, $\mu^{(-)}_{ii-ji}$ lies between zero and one if

$$0\leq\frac{1}{1-\eta^{(-)}_{RV}B^{i}_{ii-ji}}<1\,. \tag{B65}$$

Given that $\eta^{(-)}_{RV}$ is positive, these inequalities are satisfied for any $B^{i}_{ii-ji}<0$. In other words, any amount of international bias in aggregate portfolio holdings attenuate rather than amplify the response of the cross-country differenced real return to endowment and money supply shocks.

B.8.2 Proof of Proposition 7

Second-order Taylor expansions of the agent Euler equations in (17) are given by

$$\begin{aligned}&\hat{C}_{it}(\rho)-\frac{\rho}{2}\hat{C}_{it}(\rho)^{2}-\theta\hat{W}^{i}_{it}(\rho)+\frac{\theta\rho}{2}\hat{W}^{i}_{it}(\rho)\\&\quad=\beta R\,\mathrm{E}_t\left[\hat{C}_{it+1}(\rho)-\frac{\rho}{2}\hat{C}_{it+1}(\rho)^{2}-\frac{1}{\rho}\hat{R}^{i}_{jt+1}+\hat{C}_{it+1}(\rho)\hat{R}^{i}_{jt+1}\right]+O(\epsilon^{3})\,.\end{aligned} \tag{B66}$$

Differencing (B66) across bonds, i minus j, for agent ρ in country i, we obtain

$$\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\Big] = \rho\,\mathrm{E}_t\Big[\hat{C}_{it+1}(\rho)\hat{R}^i_{i-jt+1}\Big] + O(\epsilon^3)\,. \tag{B67}$$

We use this second-order approximate agent Euler equation to obtain the cross-country differenced and cross-country summed Euler equations that appear in Proposition 7.

We begin by deriving the second-order approximate cross-country differenced agent Euler equations in (49). Differencing (B67) across countries for agents with identical coefficients of relative risk aversion, we obtain

$$\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1} - \hat{R}^j_{i-jt+1}\Big] = \rho\,\mathrm{E}_t\Big[\hat{C}_{it+1}(\rho)\hat{R}^i_{i-jt+1} - \hat{C}_{jt+1}(\rho)\hat{R}^j_{i-jt+1}\Big] + O(\epsilon^3). \tag{B68}$$

The cross-country differenced real return in numéraire currency j, denoted $\hat{R}^j_{i-jt+1}$, now appears on both sides of the cross-country differenced Euler equations in (B68), which we convert to numéraire currency i before proceeding. To preserve the order of approximation in (B68), we need a second-order approximate conversion for the left-hand side of (B68) and a first-order approximate conversion for the right-hand side. Using the definition of the real return in (8) and the definition of the real exchange rate in (14), we obtain

$$\begin{aligned} &\hat{R}^j_{i-jt+1} = \hat{R}^i_{i-jt+1} + O(\epsilon^2) \quad \text{and} \\ &\hat{R}^j_{i-jt+1} = \hat{R}^i_{i-jt+1} - \hat{R}^i_{i-jt+1}\Big(\hat{Q}_{ijt+1} - \hat{Q}_{ijt}\Big) + O(\epsilon^3)\,. \end{aligned} \tag{B69}$$

Using (B69) to convert $\hat{R}^j_{i-jt+1}$ to numéraire currency i in (B68), we obtain the second-order approximate cross-country differenced agent Euler equation in equation (49).

We next derive the second-order approximate cross-country summed agent Euler equations in (49). Summing (B67) across countries for agents with identical coefficients of relative risk aversion ρ, we obtain

$$\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1} + \hat{R}^j_{i-jt+1}\Big] = \rho\,\mathrm{E}_t\Big[\hat{C}_{it+1}(\rho)\hat{R}^i_{i-jt+1} + \hat{C}_{jt+1}(\rho)\hat{R}^j_{i-jt+1}\Big] + O(\epsilon^3)\,. \tag{B70}$$

Again using (B69) to convert $\hat{R}^j_{i-jt+1}$ to numéraire currency i, we obtain the second-order approximate cross-country summed agent Euler equation in (49).

To obtain aggregate second-order approximate cross-country differenced and summed agent Euler equations, we exploit the fact that ρ multiplies no agent variables in (49) and integrate with respect to ρ, leading straightforwardly to the expressions in (50).

B.8.3 Proof of Proposition 8

To prove Proposition 8 on domestic bias in aggregate portfolio holdings, we first derive a final solution for cross-country differenced portfolio holdings, which we omitted from Section 4.2.5.

We derive the intermediate solution in (51) by combining the aggregate cross-country differenced portfolio valuation effect in (36), cross-country differenced aggregate consumption in (39) and the real exchange rate in (42) with the cross-country differenced aggregate Euler equations (50). This solution is intermediate because it depends on $\hat{R}^i_{i-jt+1}$.

Using the first-order approximate solution for $\hat{R}^i_{i-jt+1}$ in (43), and rearranging to isolate B^i_{ii-ji} on the left-hand side, the final solution is given by,

$$B^i_{ii-ji} = \frac{\mathrm{E}_{t-1}\left[\zeta^{(-)}_{B_1}\left(\hat{Y}_{i-jt}\right)^2 + \zeta^{(-)}_{B_2}\left(\hat{Y}_{i-jt}\hat{M}^{i-j}_{i-jt}\right)\right]}{\mathrm{E}_{t-1}\left[\zeta^{(-)}_{B_3}\left(\hat{Y}_{i-jt}\right)^2 + \zeta^{(-)}_{B_4}\left(\hat{Y}_{i-jt}\hat{M}^{i-j}_{i-jt}\right) + \zeta^{(-)}_{B_5}\left(\hat{M}^{i-j}_{i-jt}\right)^2\right]} + O(\epsilon)\,, \tag{B71}$$

where the coefficients given by

$$\begin{aligned}
\zeta^{(-)}_{B_1} &= \eta^{(-)}_{RY}\left(\eta^{(-)}_{CY} - \bar{\sigma}\eta^{(-)}_{QY}\right),\\
\zeta^{(-)}_{B_2} &= \eta^{(-)}_{RM}\left(\eta^{(-)}_{CY} - \bar{\sigma}\eta^{(-)}_{QY}\right),\\
\zeta^{(-)}_{B_3} &= \eta^{(-)}_{RY}\eta^{(-)}_{RV}\left(\eta^{(-)}_{CY} - \bar{\sigma}\eta^{(-)}_{QY}\right) - \left(\eta^{(-)}_{RY}\right)^2\left(\eta^{(-)}_{CV} - \bar{\sigma}\eta^{(-)}_{QV}\right),\\
\zeta^{(-)}_{B_4} &= \eta^{(-)}_{RM}\eta^{(-)}_{RV}\left(\eta^{(-)}_{CY} - \bar{\sigma}\eta^{(-)}_{QY}\right) - 2\eta^{(-)}_{RM}\eta^{(-)}_{RY}\left(\eta^{(-)}_{CV} - \bar{\sigma}\eta^{(-)}_{QV}\right), \text{ and}\\
\zeta^{(-)}_{B_5} &= -\left(\eta^{(-)}_{RM}\right)^2\left(\eta^{(-)}_{CV} - \bar{\sigma}\eta^{(-)}_{RV}\right),
\end{aligned} \tag{B72}$$

and where the semi-partial elasticities that appear in the coefficients are defined in equations (B55), (B56), and (B61). The coefficients in (B72) are continuous in θ in the neighborhood of $\theta = 0$, so it will suffice for the proof to establish signs for the coefficients at $\theta = 0$, because these signs are preserved for θ sufficiently close to zero.

To establish signs for the coefficients in (B72), we must first establish signs for the semi-partial elasticities that appear in the coefficients. The semi-partial elasticities are defined in (B55), (B56), and (B61), and their values evaluated at $\theta = 0$ are given in Table B4. Using the table, we can further establish signs of two compound terms that appear repeatedly in the coefficients in (B72),

$$\left(\eta^{(-)}_{CY} - \bar{\sigma}\eta^{(-)}_{QY}\right)\Big|_{\theta=0} > 0 \quad \text{and} \quad \left(\eta^{(-)}_{CV} - \bar{\sigma}\eta^{(-)}_{RV}\right)\Big|_{\theta=0} > 0\,, \tag{B73}$$

under Assumptions 1 and 2.

Using these terms, Table B4, and (B73), we find that the coefficients in (B72) are negative when evaluated at $\theta = 0$:

$$\begin{gathered}
\zeta^{(-)}_{B_1}\Big|_{\theta=0} < 0\,, \quad \zeta^{(-)}_{B_2}\Big|_{\theta=0} < 0\,, \quad \zeta^{(-)}_{B_3}\Big|_{\theta=0} < 0\\
\zeta^{(-)}_{B_4}\Big|_{\theta=0} < 0\,, \quad \text{and} \quad \zeta^{(-)}_{B_5}\Big|_{\theta=0} < 0\,.
\end{gathered} \tag{B74}$$

Assumption 3, together with the results in (B71) and (B74), are sufficient to conclude

that $B^i_{ii-ji} > 0$. From the definition of cross-country differenced steady-state bond holdings in (36) and the non-stochastic steady-state portfolio equilibrium in Proposition 2, it follows that $B^i_{ij} < 0 + O(\epsilon) < B^i_{ii}$ as stated in Proposition 8.

B.8.4 Proof of Proposition 9

Proposition 9 states that the cross-country differenced real return is negative. To establish this result, we begin from the intermediate solution for expected cross-country differenced real returns in (55). We need to establish the signs for the three terms under expectation operators on the right-hand side of this expression, as well as for the coefficients on these terms.

We begin with the coefficients. The coefficient $\omega\bar{\rho}/2$ is positive by inspection, under Assumption 2. The signs of the remaining two coefficients are determined by the signs of the semi-partial elasticities $\eta^{(-)}_{QY}$ and $\eta^{(-)}_{QV}$. From Table B4, $\eta^{(-)}_{QY}$ and $\eta^{(-)}_{QV}$ are positive and negative at $\theta = 0$, respectively, under Assumptions 1 and 2. By continuity, these signs are preserved for small positive values of θ.

It remains to establish the signs of the three terms under expectation operators on the right-hand side of (55). The first term is $\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\hat{Y}_{i+jt+1}\Big]$. We use the cross-country differenced realized return in (44) to obtain

$$\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\hat{Y}_{i+jt+1}\Big] = \gamma^{(-)}_{RY}\,\mathrm{E}_t\Big[\hat{Y}_{i-jt+1}\hat{Y}_{i+jt+1}\Big] + \gamma^{(-)}_{RM}\,\mathrm{E}_t\Big[\hat{M}^{i-j}_{i-jt+1}\hat{Y}_{i+jt+1}\Big] + O(\epsilon^3)\,.$$

Under Assumption 3, the first term is negative if the general elasticities $\gamma^{(-)}_{RY}$ and $\gamma^{(-)}_{RM}$ are negative. These general elasticities are given in (45). Their numerators $\eta^{(-)}_{RY}$ and $\eta^{(-)}_{RM}$ are negative at $\theta = 0$, as shown in Table B4. Their denominator $1 - \eta^{(-)}_{RV}B^i_{ii-ji}$ is positive if

$$\eta^{(-)}_{RV}B^i_{ii-ji} < 1\,,$$

or, using the solution for B^i_{ii-ji} in (B71), if

$$\begin{aligned}&\Big(\eta^{(-)}_{CV} - \bar{\sigma}\eta^{(-)}_{QV}\Big)\Big(\eta^{(-)}_{RY}\Big)^2\,\mathrm{E}_t\Big[\Big(\hat{Y}_{i-jt+1}\Big)^2\Big]\\ &\qquad + 2\Big(\eta^{(-)}_{CV} - \bar{\sigma}\eta^{(-)}_{QV}\Big)\eta^{(-)}_{RY}\eta^{(-)}_{RM}\,\mathrm{E}_t\Big[\hat{M}^{i-j}_{i-jt+1}\hat{Y}_{i-jt+1}\Big] > \zeta^{(-)}_{B_5}\,\mathrm{E}_t\Big[\Big(\hat{M}^{i-j}_{i-jt+1}\Big)^2\Big]\,.\end{aligned} \tag{B75}$$

Evaluating this inequality at $\theta = 0$ using the semi-partial elasticity signs from Table B4, and using the signs for the cross-country differenced bond holding coefficients in (B74), the left-hand side of (B75) is positive while the right-hand side is negative under Assumptions 1–3. By continuity, the inequality is preserved for values of θ in the neighborhood of zero. Thus, $\gamma^{(-)}_{RY} < 0$ and $\gamma^{(-)}_{RM} < 0$, and therefore $\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\hat{Y}_{i+jt+1}\Big] \leq 0$.

The second term is $\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\hat{Y}_{i-jt+1}\Big]$. We again use the cross-country differenced real

returns in (44) to obtain

$$\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\hat{Y}_{i-jt+1}\Big] = \gamma^{(-)}_{RY}\,\mathrm{E}_t\Big[\Big(\hat{Y}_{i-jt+1}\Big)^2\Big] + \gamma^{(-)}_{RM}\,\mathrm{E}_t\Big[\hat{M}^{i-j}_{i-jt+1}\hat{Y}_{i-jt+1}\Big] + O(\epsilon^3)\,,$$

and using $\gamma^{(-)}_{RY} < 0$ and $\gamma^{(-)}_{RM} < 0$ from above, we immediately obtain $\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\hat{Y}_{i-jt+1}\Big] < 0$ under Assumptions 1–3. The third term, $\mathrm{E}_t\Big[\Big(\hat{R}^i_{i-jt+1}\Big)^2\Big]$, is squared and therefore non-negative.

The signs of the three coefficients and three terms under expectation operators on the right-hand side of (55) under Assumptions 1–3 imply that $\mathrm{E}_t\Big[\hat{R}^i_{i-jt+1}\Big] < 0 + O(\epsilon^3)$, as stated in Proposition 9. Importantly, these assumptions were also sufficient for domestic bias in aggregate portfolio holdings, as shown in Proposition 8. Propositions 8 and 9 together capture an empirical pattern that we observe in Germany, Japan, and the United States and document in Table 1 and Figure A4: strong domestic bias in aggregate portfolio holdings despite relatively low domestic returns.

B.9 The Agent Non-Portfolio Problem

In the agent non-portfolio problem we solve for agent consumption and real wealth in terms of exogenous variables and parameters. Unlike the aggregate system, where we solved for cross-country differences and used market clearing conditions to derive country-specific solutions, the agent system requires us to solve for both cross-country differences and cross-country sums. We derive these solutions in the following two subsections.

B.9.1 The Agent Non-Portfolio System (−)

The agent Euler equation and budget constraint were first-order approximated, cross-country differenced, and differenced with aggregate Euler and budget equations in (57) and (58). We begin from these equations.

Eliminating the real exchange rate from the cross-country differenced agent Euler equation (57) using the intermediate solution for the real exchange rate in (42), we obtain

$$\begin{aligned}&\theta\hat{W}^i_{i-jt}(\rho) + \beta R\,\mathrm{E}_t\Big[\tilde{C}_{i-jt+1}(\rho)\Big] - \left(\theta + \beta R\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\eta^{(-)}_{QV}\right)\hat{W}^i_{i-jt} \\ &\quad = \tilde{C}_{i-jt}(\rho) - \beta R\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\Big(\eta^{(-)}_{QW}\hat{W}^i_{i-jt-1} + \eta^{(-)}_{QV}\hat{Y}_{i-jt} + \eta^{(-)}_{QV}\hat{V}^i_{i-jt}\Big) \quad + O(\epsilon^2)\,,\end{aligned} \tag{B76}$$

where $\tilde{C}_{i-jt}(\rho) = \hat{C}_{i-jt}(\rho) - \hat{C}_{i-jt}$ denotes the difference between cross-country differenced agent and aggregate consumption, using the tilde notation to denote differences between agent and aggregate variables or parameters that integrate to zero.

Cross-country differenced aggregate real wealth $\hat{W}^i_{i-jt}$ now appears in (B76). A third

equation, the law of motion for cross-country differenced aggregate real wealth in (39), must therefore join the system. We write the system in matrix form as

$$\begin{aligned}\boldsymbol{\mathcal{E}}^{(-)}_{0(\rho)}\,\mathrm{E}_t\Big[\hat{\boldsymbol{Z}}_{i-jt+1}(\rho)\Big] = \boldsymbol{\mathcal{E}}^{(-)}_{1(\rho)}\hat{\boldsymbol{Z}}_{i-jt}(\rho) + \boldsymbol{\mathcal{E}}^{(-)}_{2(\rho)}\hat{Y}_{i-jt} \\ + \boldsymbol{\mathcal{E}}^{(-)}_{3(\rho)}\hat{V}^{i}_{i-jt}(\rho) + \boldsymbol{\mathcal{E}}^{(-)}_{4(\rho)}\hat{V}^{i}_{i-jt} + O(\epsilon^2)\,,\end{aligned} \tag{B77}$$

where $\hat{\boldsymbol{Z}}_{i-jt}(\rho) = \begin{bmatrix}\hat{W}^{i}_{i-jt-1}(\rho) & \tilde{C}_{i-jt}(\rho) & \hat{W}^{i}_{i-jt-1}\end{bmatrix}'$ denotes the (3×1) vector of cross-country differenced agent real wealth, cross-country differenced agent consumption in deviations from aggregate, and cross-country differenced aggregate real wealth, and where

$$\boldsymbol{\mathcal{E}}^{(-)}_{0(\rho)} = \begin{bmatrix}1 & 0 & -1 \\ \theta & \beta R & -\Big(\theta + \beta R\big(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\big)\eta^{(-)}_{QW}\Big) \\ 0 & 0 & 1\end{bmatrix},$$

$$\boldsymbol{\mathcal{E}}^{(-)}_{1(\rho)} = \begin{bmatrix}R & -1 & -R \\ 0 & 1 & -\beta R\big(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\big)\eta^{(-)}_{QW} \\ 0 & 0 & \eta^{(-)}_{WW}\end{bmatrix}, \quad \boldsymbol{\mathcal{E}}^{(-)}_{2(\rho)} = \begin{bmatrix}0 \\ -\beta R\big(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\big)\eta^{(-)}_{QY} \\ \eta^{(-)}_{WY}\end{bmatrix},$$

$$\boldsymbol{\mathcal{E}}^{(-)}_{3(\rho)} = \begin{bmatrix}R \\ 0 \\ 0\end{bmatrix}, \quad \text{and} \quad \boldsymbol{\mathcal{E}}^{(-)}_{4(\rho)} = \begin{bmatrix}0 \\ -\beta R\big(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\big)\eta^{(-)}_{QV} \\ \eta^{(-)}_{WV}\end{bmatrix}.$$

The matrix $\boldsymbol{\mathcal{E}}^{(-)}_{0(\rho)}$ is invertible if its determinant is non-zero. Therefore, we require that $\det\big(\boldsymbol{\mathcal{E}}^{(-)}_{0(\rho)}\big) = \beta R \neq 0$, which is always satisfied under Assumption 1.

Inverting $\boldsymbol{\mathcal{E}}^{(-)}_{0(\rho)}$, we can rewrite the system in (B77) to obtain the system as it appears in (60), where matrices of partial elasticities are given by

$$\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)} = \begin{bmatrix}\mathcal{E}^{(-)}_{W(\rho)W(\rho)} & \mathcal{E}^{(-)}_{W(\rho)C(\rho)} & \mathcal{E}^{(-)}_{W(\rho)W} \\ \mathcal{E}^{(-)}_{C(\rho)W(\rho)} & \mathcal{E}^{(-)}_{C(\rho)C(\rho)} & \tilde{\mathcal{E}}^{(-)}_{C(\rho)W} \\ \mathcal{E}^{(-)}_{WW(\rho)} & \mathcal{E}^{(-)}_{WC(\rho)} & \mathcal{E}^{(-)}_{WW}\end{bmatrix} = \begin{bmatrix}R & -1 & \eta^{(-)}_{WW}-R \\ -\frac{\theta}{\beta} & \frac{1+\theta}{\beta R} & \frac{\theta}{\beta}-\eta^{(-)}_{QW}\Big(1-\eta^{(-)}_{WW}\Big)\Big(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\Big) \\ 0 & 0 & \eta^{(-)}_{WW}\end{bmatrix},$$

$$\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Y} = \begin{bmatrix}\mathcal{E}^{(-)}_{W(\rho)Y} \\ \tilde{\mathcal{E}}^{(-)}_{C(\rho)Y} \\ \mathcal{E}^{(-)}_{WY}\end{bmatrix} = \begin{bmatrix}\eta^{(-)}_{WY} \\ \Big(\frac{1}{\rho}-\frac{\omega}{\bar{\rho}}\Big)\Big(\eta^{(-)}_{QW}\eta^{(-)}_{WY}-\eta^{(-)}_{QY}\Big) \\ \eta^{(-)}_{WY}\end{bmatrix}, \tag{B78}$$

$$\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V(\rho)} = \begin{bmatrix}\mathcal{E}^{(-)}_{W(\rho)V(\rho)} \\ \mathcal{E}^{(-)}_{C(\rho)V(\rho)} \\ \mathcal{E}^{(-)}_{WV(\rho)}\end{bmatrix} = \begin{bmatrix}R \\ -\frac{\theta}{\beta} \\ 0\end{bmatrix}, \quad \text{and}$$

$$\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V} = \begin{bmatrix} \mathcal{E}^{(-)}_{W(\rho)V} \\ \tilde{\mathcal{E}}^{(-)}_{C(\rho)V} \\ \mathcal{E}^{(-)}_{WV} \end{bmatrix} = \begin{bmatrix} \eta^{(-)}_{WV} \\ \left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\left(\eta^{(-)}_{QW}\eta^{(-)}_{WV} - \eta^{(-)}_{QV}\right) \\ \eta^{(-)}_{WV} \end{bmatrix}.$$

Before solving the cross-country differenced agent system, we decompose the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ into eigenvalues and eigenvectors and establish the conditions for a unique and stationary rational expectations solution following Blanchard and Kahn (1980). The eigendecomposition of $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ is given by

$$\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)} = \boldsymbol{\mathcal{V}}^{(-)}_{Z(\rho)Z(\rho)} \boldsymbol{\Lambda}^{(-)}_{Z(\rho)Z(\rho)} \boldsymbol{\mathcal{V}}^{(-)^{-1}}_{Z(\rho)Z(\rho)} \tag{B79}$$

where $\boldsymbol{\mathcal{V}}^{(-)}_{Z(\rho)Z(\rho)}$ is a matrix of eigenvectors of $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ and where $\boldsymbol{\Lambda}^{(-)}_{Z(\rho)Z(\rho)}$ is a diagonal matrix of eigenvalues of $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$. We write the matrix of eigenvectors as

$$\begin{aligned} \boldsymbol{\mathcal{V}}^{(-)}_{Z(\rho)Z(\rho)} &= \begin{bmatrix} \nu^{(-)}_{W(\rho)W(\rho)} & \nu^{(-)}_{W(\rho)C(\rho)} & \nu^{(-)}_{W(\rho)W} \\ \nu^{(-)}_{C(\rho)W(\rho)} & \nu^{(-)}_{C(\rho)C(\rho)} & \nu^{(-)}_{C(\rho)W} \\ \nu^{(-)}_{WW(\rho)} & \nu^{(-)}_{WC(\rho)} & \nu^{(-)}_{WW} \end{bmatrix} \\ &= \begin{bmatrix} \mathcal{E}^{(-)}_{W(\rho)C(\rho)} & \mathcal{E}^{(-)}_{W(\rho)C(\rho)} & \mathcal{E}^{(-)}_{W(\rho)C(\rho)} \\ \lambda^{(-)}_{Z(\rho)W(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)} & \lambda^{(-)}_{ZC(\rho)} - \mathcal{E}^{(-)}_{WW} & \lambda^{(-)}_{ZW} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W} \\ 0 & 0 & \mathcal{E}^{(-)}_{W(\rho)C(\rho)} \end{bmatrix}, \end{aligned} \tag{B80}$$

and the diagonal matrix of eigenvalues as

$$\boldsymbol{\Lambda}^{(-)}_{Z(\rho)Z(\rho)} = \begin{bmatrix} \lambda^{(-)}_{Z(\rho)W(\rho)} & 0 & 0 \\ 0 & \lambda^{(-)}_{Z(\rho)C(\rho)} & 0 \\ 0 & 0 & \lambda^{(-)}_{Z(\rho)W} \end{bmatrix}, \tag{B81}$$

where $\lambda^{(-)}_{Z(\rho)W} = \eta^{(-)}_{WW} = \lambda^{(-)}_{ZW}$, with $\lambda^{(-)}_{ZW}$ given in equation (B40), and where

$$\begin{aligned} \lambda^{(-)}_{Z(\rho)W(\rho)} = \frac{1}{2}\left(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} + \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\right) \\ + \sqrt{\frac{1}{4}\left(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\right)^2 + \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)}} \end{aligned} \tag{B82}$$

and

$$\begin{aligned} \lambda^{(-)}_{Z(\rho)C(\rho)} = \frac{1}{2}\left(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} + \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\right) \\ - \sqrt{\frac{1}{4}\left(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\right)^2 + \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)}}. \end{aligned} \tag{B83}$$

B.9.2 Stationarity of the Agent System (−)

We now establish uniqueness and stationarity for the cross-country differenced agent system. Following the same strategy as we did with the cross-country differenced aggregate system, we consider the neighborhood of values for θ around the knife-edge non-stationary case of $\theta = 0$ to establish the result. We state the following proposition.

Proposition B1 (Stationarity of the Cross-Country Differenced Agent System)**.** *The matrix $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ has two distinct real-valued eigenvalues, $\lambda^{(-)}_{Z(\rho)W(\rho)}$ and $\lambda^{(-)}_{Z(\rho)C(\rho)}$. Furthermore, $\lambda^{(-)}_{Z(\rho)W(\rho)}\Big|_{\theta=0} = 1/\beta$ and $\lambda^{(-)}_{Z(\rho)C(\rho)}\Big|_{\theta=0} = 1$, and*

$$\partial\lambda^{(-)}_{Z(\rho)W(\rho)}/\partial\theta\,\Big|_{\theta=0} > 0 \quad \textit{and} \quad \partial\lambda^{(-)}_{ZC(\rho)}/\partial\theta\,\Big|_{\theta=0} < 0\,. \tag{B84}$$

The third eigenvalue $\lambda^{(-)}_{Z(\rho)C}$ equals the aggregate eigenvalue $\lambda^{(-)}_{ZC}$, which lies between zero and one if θ takes a small positive value (Proposition 5). Hence, there exists a $\theta > 0$ that yields a unique and stationary rational expectations solution to the cross-country differenced agent system in (60)*. All three eigenvalues are independent of ρ.*

To prove this statement, it suffices to show that the discriminant in (B82) and (B83) is positive. The discriminant, denoted $\Delta^{(-)}_{\rho}$, is given by

$$\Delta^{(-)}_{\rho} = \frac{1}{4}\big(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\big)^2 + \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)}\,. \tag{B85}$$

Suppose $\theta > 0$. The squared term is always positive or zero, so it suffices to show that the second term is positive. We have

$$\mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)} = \frac{\theta}{\beta} > 0\,, \tag{B86}$$

where we have used the partial elasticities in (B78) under Assumptions 1 and 2. Suppose $\theta = 0$. The second term is zero, $\mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)} = 0$, but the squared term is positive, since $\mathcal{E}^{(-)}_{W(\rho)W(\rho)}\Big|_{\theta=0} - \mathcal{E}^{(-)}_{C(\rho)C(\rho)}\Big|_{\theta=0} = 1/\beta - 1 > 0$ under Assumptions 1, so the discriminant $\Delta^{(-)}_{\rho}$ is again positive. Thus, $\theta \geq 0$ is sufficient to show that (B82) and (B83) are distinct and real-valued eigenvalues.

As in the cross-country differenced aggregate problem, the case where $\theta = 0$ serves as a helpful benchmark here for proving the further statements in Proposition B1. In Table B5, we evaluate at $\theta = 0$ the partial elasticities in (B78) and their derivatives with respect to θ.

Proposition B1 states that $\lambda^{(-)}_{Z(\rho)W(\rho)}$ evaluated at $\theta = 0$ exceeds one and $\lambda^{(-)}_{Z(\rho)C(\rho)}$ evaluated at $\theta = 0$ equals one. Using Table B5 to evaluate (B82) and (B83) at $\theta = 0$,

$$\lambda^{(-)}_{Z(\rho)W(\rho)}\Big|_{\theta=0} = \mathcal{E}^{(-)}_{WW}\Big|_{\theta=0} > 1 \quad \text{and} \quad \lambda^{(-)}_{Z(\rho)C(\rho)}\Big|_{\theta=0} = \mathcal{E}^{(-)}_{CC}\Big|_{\theta=0} = 1\,. \tag{B87}$$

Table B5 – Agent Cross-Country Differenced Partial Elasticities at $\theta = 0$

Partial Elasticity	Value of Partial Elasticity Evaluated at $\theta = 0$		Derivative with Respect to θ Evaluated at $\theta = 0$	
	Expression	Sign	Expression	Sign
$\mathcal{E}^{(-)}_{W(\rho)W(\rho)}$	$\frac{1}{\beta}$	$+$	$-\frac{1}{\beta}$	$-$
$\mathcal{E}^{(-)}_{W(\rho)C(\rho)}$	-1	$-$	0	$\cdot$
$\mathcal{E}^{(-)}_{C(\rho)W(\rho)}$	0	$\cdot$	$-\frac{1}{\beta}$	$-$
$\mathcal{E}^{(-)}_{C(\rho)C(\rho)}$	1	$+$	$1+\frac{1}{\beta}$	$+$

Notes. Column one lists the partial elasticities that appear in the cross-country differenced aggregate system (37). Columns two and four, respectively, give expressions for the partial elasticities and their derivatives with respect to θ, both evaluated at $\theta = 0$. Columns three and five, respectively, give the signs of the partial elasticities and their derivatives, both evaluated at $\theta = 0$. Signs are established under Assumptions 1 and 2.

Proposition B1 further states that the partial derivative of $\lambda^{(-)}_{Z(\rho)W(\rho)}$ with respect to θ evaluated at $\theta = 0$ is positive, and the partial derivative of $\lambda^{(-)}_{Z(\rho)C(\rho)}$ with respect to θ evaluated at $\theta = 0$ is negative. To establish this result, we differentiate the eigenvalues with respect to θ,

$$\begin{aligned}\frac{\partial \lambda^{(-)}_{\pm}}{\partial \theta} &= \frac{1}{2}\left(\frac{\partial \mathcal{E}^{(-)}_{C(\rho)C(\rho)}}{\partial \theta} + \frac{\partial \mathcal{E}^{(-)}_{W(\rho)W(\rho)}}{\partial \theta}\right) \\ &\pm \frac{1}{2}\left[\frac{1}{4}\left(\mathcal{E}^{(-)}_{C(\rho)C(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\right)^2 + \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)W(\rho)}\right]^{-\frac{1}{2}} \\ &\times \left[\frac{1}{2}\left(\mathcal{E}^{(-)}_{C(\rho)C(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\right)\left(\frac{\partial \mathcal{E}^{(-)}_{C(\rho)C(\rho)}}{\partial \theta} - \frac{\partial \mathcal{E}^{(-)}_{W(\rho)W(\rho)}}{\partial \theta}\right)\right. \\ &\qquad \left. + \frac{\partial \mathcal{E}^{(-)}_{W(\rho)C(\rho)}}{\partial \theta}\mathcal{E}^{(-)}_{C(\rho)W(\rho)} + \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\frac{\partial \mathcal{E}^{(-)}_{C(\rho)W(\rho)}}{\partial \theta}\right],\end{aligned} \tag{B88}$$

where the "+" case corresponds to $\lambda^{(-)}_{Z(\rho)W(\rho)}$ and the "−" case corresponds to $\lambda^{(-)}_{Z(\rho)C(\rho)}$. Using Table B5, we evaluate (B88) at $\theta = 0$ to obtain

$$\left.\frac{\partial \lambda^{(-)}_{Z(\rho)W(\rho)}}{\partial \theta}\right|_{\theta=0} = \frac{1}{1-\beta} - \frac{1}{\beta} \quad \text{and} \quad \left.\frac{\partial \lambda^{(-)}_{Z(\rho)C(\rho)}}{\partial \theta}\right|_{\theta=0} = \frac{1}{\beta} - \frac{1}{1-\beta}\,. \tag{B89}$$

The derivatives with respect to $\lambda^{(-)}_{Z(\rho)W(\rho)}$ and $\lambda^{(-)}_{Z(\rho)C(\rho)}$ are positive and negative, respectively, if $1/2 < \beta < 1$, which is satisfied under Assumption 1.

Finally, Proposition B1 states that the three eigenvalues of the matrix $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ are independent of ρ. This result follows straightforwardly from the definitions of the eigenvalues in (B41), (B82), and (B83), together with the definitions of the partial

elasticities in (B37) and (B78).

In view of the eigenvalues at $\theta = 0$ in (B45) and (B87), and the derivatives at $\theta = 0$ in (B50), (B89) and (B89) under Assumptions 1 and 2, a small positive value for θ pushes $\lambda^{(-)}_{Z(\rho)C(\rho)}$ and $\lambda^{(-)}_{Z(\rho)C}$ below one while pushing $\lambda^{(-)}_{Z(\rho)W(\rho)}$ further above one, thus satisfying the conditions in Blanchard and Kahn (1980) for a unique and stationary rational expectations solution to the cross-country differenced agent system in (60).

B.9.3 Intermediate Agent Non-Portfolio Solution (−)

We now derive intermediate solutions for agent real wealth and consumption in terms of lagged agent and aggregate state variables, exogenous variables and parameters, and the endogenous agent and aggregate portfolio valuation effects. Using the eigendecomposition in (60), we re-write the agent cross-country differenced system in (B77) as

$$\begin{aligned}\boldsymbol{\mathcal{V}}^{(-)^{-1}}_{Z(\rho)Z(\rho)}\,\mathrm{E}_t\Big[\hat{\boldsymbol{Z}}_{i-jt+1}(\rho)\Big] &= \boldsymbol{\Lambda}^{(-)}_{Z(\rho)Z(\rho)}\boldsymbol{\mathcal{V}}^{(-)^{-1}}_{Z(\rho)Z(\rho)}\hat{\boldsymbol{Z}}_{i-jt}(\rho)\\ &+ \boldsymbol{\mathcal{V}}^{(-)^{-1}}_{Z(\rho)Z(\rho)}\Big(\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Y}\hat{Y}_{i-jt} + \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V(\rho)}\hat{V}^i_{i-jt}(\rho) + \boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)V}\hat{V}^i_{i-jt}\Big) + O(\epsilon^2)\,,\end{aligned} \tag{B90}$$

and from (B90) we extract the equation associated with the unstable eigenvalue $\lambda^{(-)}_{Z(\rho)W(\rho)}$,

$$\begin{aligned}&\mathrm{E}_t\Big[\hat{Z}_{i-jt+1}(\rho)\Big] = \lambda^{(-)}_{Z(\rho)W(\rho)}\hat{Z}_{i-jt}(\rho)\\ &+ \Big[\Big(\lambda^{(-)}_{Z(\rho)W} - \mathcal{E}^{(-)}_{W(\rho)W}\Big)\eta^{(-)}_{WY} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\mathcal{E}^{(-)}_{W(\rho)Y} - \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\tilde{\mathcal{E}}^{(-)}_{C(\rho)Y}\Big]\hat{Y}_{i-jt}\\ &+ \Big[\Big(\lambda^{(-)}_{Z(\rho)W} - \mathcal{E}^{(-)}_{W(\rho)W}\Big)\eta^{(-)}_{WV} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\mathcal{E}^{(-)}_{W(\rho)V} - \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\tilde{\mathcal{E}}^{(-)}_{C(\rho)V}\Big]\hat{V}^i_{i-jt}\\ &+ \Big[\Big(\lambda^{(-)}_{Z(\rho)C(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\Big)\mathcal{E}^{(-)}_{W(\rho)V(\rho)} - \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\mathcal{E}^{(-)}_{C(\rho)V(\rho)}\Big]\tilde{V}^i_{i-jt}(\rho) + O(\epsilon^2)\,,\end{aligned} \tag{B91}$$

where $\hat{Z}_{i-jt+1}(\rho) = \Big(\lambda^{(-)}_{Z(\rho)C(\rho)} - \mathcal{E}^{(-)}_{W(\rho)W(\rho)}\Big)\hat{W}^i_{i-jt}(\rho) - \mathcal{E}^{(-)}_{W(\rho)C(\rho)}\tilde{C}_{i-jt+1}(\rho)$.

Because $\lambda^{(-)}_{Z(\rho)W(\rho)}$ is greater than one, the left-hand side of (B52) must be zero to rule out explosive paths. Setting the left-hand side to zero, we obtain

$$\begin{aligned}\tilde{C}_{i-jt}(\rho) &= \eta^{(-)}_{C(\rho)W(\rho)}\tilde{W}^i_{i-jt-1}(\rho) + \eta^{(-)}_{C(\rho)C(\rho)}\tilde{V}^i_{i-jt}(\rho)\\ &+ \tilde{\eta}^{(-)}_{C(\rho)V}\hat{V}^i_{i-jt} + \tilde{\eta}^{(-)}_{C(\rho)Y}\hat{Y}_{i-jt} + O(\epsilon^2)\,,\end{aligned} \tag{B92}$$

and using (B92) to eliminate $\tilde{C}_{i-jt}(\rho)$ from the cross-country differenced agent budget constraint in (58), we obtain

$$\begin{aligned}\tilde{W}^i_{i-jt}(\rho) &= \eta^{(-)}_{W(\rho)W(\rho)}\tilde{W}^i_{i-jt-1}(\rho) + \eta^{(-)}_{W(\rho)V(\rho)}\tilde{V}^i_{i-jt}(\rho)\\ &+ \tilde{\eta}^{(-)}_{W(\rho)V}\hat{V}^i_{i-jt} + \tilde{\eta}^{(-)}_{W(\rho)Y}\hat{Y}_{i-jt} + O(\epsilon^2)\,.\end{aligned} \tag{B93}$$

Writing the intermediate solutions (B92) and (B93) in more compact matrix form, we

obtain (62), where the semi-partial elasticity matrices are given by

$$\boldsymbol{\eta}^{(-)}_{Z(\rho)W(\rho)} = \begin{bmatrix} \eta^{(-)}_{W(\rho)W(\rho)} \\ \eta^{(-)}_{C(\rho)W(\rho)} \end{bmatrix} = \begin{bmatrix} \lambda^{(-)}_{Z(\rho)C(\rho)} \\ \mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \lambda^{(-)}_{Z(\rho)C(\rho)} \end{bmatrix},$$

$$\boldsymbol{\eta}^{(-)}_{Z(\rho)V(\rho)} = \begin{bmatrix} \eta^{(-)}_{W(\rho)V(\rho)} \\ \eta^{(-)}_{C(\rho)V(\rho)} \end{bmatrix} = \begin{bmatrix} \frac{1-\theta}{\beta} - \frac{\left(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \lambda^{(-)}_{Z(\rho)C(\rho)}\right)\mathcal{E}^{(-)}_{W(\rho)V(\rho)} - \mathcal{E}^{(-)}_{C(\rho)V(\rho)}}{\lambda^{(-)}_{Z(\rho)W(\rho)}} \\ \frac{\left(\mathcal{E}^{(-)}_{W(\rho)W(\rho)} - \lambda^{(-)}_{Z(\rho)C(\rho)}\right)\mathcal{E}^{(-)}_{W(\rho)V(\rho)} - \mathcal{E}^{(-)}_{C(\rho)V(\rho)}}{\lambda^{(-)}_{Z(\rho)W(\rho)}} \end{bmatrix},$$

$$\tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)V} = \begin{bmatrix} \tilde{\eta}^{(-)}_{W(\rho)V} \\ \tilde{\eta}^{(-)}_{C(\rho)V} \end{bmatrix} = \begin{bmatrix} (\bar{\sigma} - \sigma(\rho))\frac{\eta^{(-)}_{QV} - \eta^{(-)}_{QW}\eta^{(-)}_{WV}}{\lambda^{(-)}_{Z(\rho)W(\rho)}} \\ (\sigma(\rho) - \bar{\sigma})\frac{\eta^{(-)}_{QV} - \eta^{(-)}_{QW}\eta^{(-)}_{WV}}{\lambda^{(-)}_{Z(\rho)W(\rho)}} \end{bmatrix}, \quad \text{and} \tag{B94}$$

$$\tilde{\boldsymbol{\eta}}^{(-)}_{Z(\rho)Y} = \begin{bmatrix} \tilde{\eta}^{(-)}_{W(\rho)Y} \\ \tilde{\eta}^{(-)}_{C(\rho)Y} \end{bmatrix} = \begin{bmatrix} (\bar{\sigma} - \sigma(\rho))\frac{\eta^{(-)}_{QY} - \eta^{(-)}_{QW}\eta^{(-)}_{WY}}{\lambda^{(-)}_{Z(\rho)W(\rho)}} \\ (\sigma(\rho) - \bar{\sigma})\frac{\eta^{(-)}_{QY} - \eta^{(-)}_{QW}\eta^{(-)}_{WY}}{\lambda^{(-)}_{Z(\rho)W(\rho)}} \end{bmatrix}.$$

Next, we follow a similar procedure to derive an intermediate solution to the agent's cross-country summed non-portfolio system.

B.9.4 The Agent Non-Portfolio System (+)

We use the first-order approximate agent and aggregate Euler equations in (29) and (30) and the first-order approximate agent and aggregate budget constraints in (31) to derive the cross-country summed agent system in deviations from aggregates. This system is simpler than the cross-country differenced agent system, first because the real exchange rate drops out of the system and second because the system has only one predetermined state variable, agent real wealth. Aggregate real wealth does not enter because its cross-country sum is zero with assets in zero net supply.

Using first-order approximate agent and aggregate Euler equations, we obtain the first-order approximate cross-country summed agent Euler equation in deviations from aggregates,

$$\begin{aligned} \theta\left(\hat{W}^{i}_{i+jt}(\rho) - \hat{W}^{i}_{i+jt}\right) + \beta R\, \mathrm{E}_t\left[\left(\hat{C}_{i+jt+1}(\rho) - \hat{C}_{i+jt+1}\right)\right] \\ = \left(\hat{C}_{i+jt}(\rho) - \hat{C}_{i+jt}\right) - \frac{\bar{\rho}}{\omega}\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\hat{Y}_{i+jt} + O(\epsilon^2)\,, \end{aligned} \tag{B95}$$

where the subscript $i+j$ on agent variables denotes cross-country sums between agents with identical coefficients of relative risk aversion. Similarly, using first-order approximate agent and aggregate budget constraints, we obtain the first-order approximate cross-country

summed agent budget constraint in deviations from aggregates,

$$\begin{aligned}\left(\hat{W}^i_{i+jt}(\rho) - \hat{W}^i_{i+jt}\right) &= R\left(\hat{W}^i_{i+jt-1}(\rho) - \hat{W}^i_{i+jt-1}\right) \\ &\quad - \left(\hat{C}_{i+jt}(\rho) - \hat{C}_{i+jt}\right) + R\left(\hat{V}^i_{i+jt}(\rho) - \hat{V}^i_{i+jt}\right) + O(\epsilon^2)\,,\end{aligned} \tag{B96}$$

where we have introduced a new variable $\hat{V}^i_{i+jt}(\rho)$ in the budget constraint, which we call the cross-country summed agent portfolio valuation effect and define as

$$\hat{V}^i_{i+jt}(\rho) = B^i_{ii+ji}(\rho)\hat{R}^i_{i-jt}\,, \quad \text{where} \quad B^i_{ii+ji}(\rho) = B^i_{ii}(\rho) + B^i_{ji}(\rho)\,. \tag{B97}$$

Unlike in the cross-country differenced agent problem, neither the cross-country summed Euler equation in (B95) nor the cross-country summed budget constraint in (B96) depends on the real exchange rate.

The absence of the real exchange rate allows use to write the cross-country summed agent Euler equation and budget constraint directly in deviations from aggregates. Continuing to use tildes to indicate the vanishing integral property, we have

$$\begin{gathered}\tilde{W}^i_{i+jt}(\rho) = \hat{W}^i_{i+jt}(\rho) - \hat{W}^i_{i+jt}\,, \quad \tilde{C}_{i+jt}(\rho) = \hat{C}_{i+jt}(\rho) - \hat{C}_{i+jt}\,, \\ \text{and} \quad \tilde{V}^i_{i+jt}(\rho) = \hat{V}^i_{i+jt}(\rho) - \hat{V}^i_{i+jt}\,,\end{gathered} \tag{B98}$$

where, due to our assumption that bonds are in zero net supply, cross-country summed aggregate real wealth and portfolio valuation effects equal zero, $\hat{W}^i_{i+jt} = \hat{V}^i_{i+jt} = 0$. To maintain consistency in our notation, however, we still use the tilde to denote cross-country summed agent real wealth and portfolio valuation effects in deviations from aggregates.

The two equations (B95) and (B96) form the cross-country summed agent system, which we write in matrix form as

$$\boldsymbol{\mathcal{E}}^{(+)}_{0(\rho)}\,\mathrm{E}_t\left[\tilde{\boldsymbol{Z}}_{i+jt+1}(\rho)\right] = \boldsymbol{\mathcal{E}}^{(+)}_{1(\rho)}\tilde{\boldsymbol{Z}}_{i+jt}(\rho) + \tilde{\boldsymbol{\mathcal{E}}}^{(+)}_{2(\rho)}\hat{Y}_{i+jt} + \boldsymbol{\mathcal{E}}^{(+)}_{3(\rho)}\tilde{V}^i_{i+jt}(\rho) + O(\epsilon^2)\,, \tag{B99}$$

where $\tilde{\boldsymbol{Z}}_{i+jt}(\rho) = \begin{bmatrix}\tilde{W}^i_{i+jt-1}(\rho) & \tilde{C}_{i+jt}(\rho)\end{bmatrix}'$ denotes the (2×1) vector of cross-country summed agent real wealth and consumption in deviation from aggregate, where

$$\begin{aligned}\boldsymbol{\mathcal{E}}^{(+)}_{0(\rho)} &= \begin{bmatrix}1 & 0 \\ \frac{\theta}{\beta R} & 1\end{bmatrix}, & \boldsymbol{\mathcal{E}}^{(+)}_{1(\rho)} &= \begin{bmatrix}R & -1 \\ 0 & \frac{1}{\beta R}\end{bmatrix}, \\ \tilde{\boldsymbol{\mathcal{E}}}^{(+)}_{2(\rho)} &= \begin{bmatrix}0 \\ -\frac{1}{\beta R}\frac{\bar{\rho}}{\omega}\left(\frac{1}{\rho} - \frac{\omega}{\rho}\right)\end{bmatrix}, \quad \text{and} & \boldsymbol{\mathcal{E}}^{(+)}_{3(\rho)} &= \begin{bmatrix}R \\ 0\end{bmatrix}.\end{aligned} \tag{B100}$$

The matrix $\mathcal{E}^{(+)}_{0(\rho)}$ is invertible if its determinant is non-zero. We have

$$\det\left(\boldsymbol{\mathcal{E}}^{(+)}_{0(\rho)}\right) = 1 \neq 0\,, \tag{B101}$$

so $\boldsymbol{\mathcal{E}}^{(+)}_{0(\rho)}$ is invertible, and we rewrite the cross-country summed agent system as

$$\mathrm{E}_t\Big[\tilde{\boldsymbol{Z}}_{i+jt+1}(\rho)\Big] = \boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}\tilde{\boldsymbol{Z}}_{i+jt}(\rho) + \tilde{\boldsymbol{\mathcal{E}}}^{(+)}_{Z(\rho)Y}\hat{Y}_{i+jt} + \boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)V(\rho)}\tilde{V}^{i}_{i+jt}(\rho) + O(\epsilon^2)\,, \tag{B102}$$

where

$$\begin{aligned}
\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)} &= \begin{bmatrix} \mathcal{E}^{(+)}_{W(\rho)W(\rho)} & \mathcal{E}^{(+)}_{W(\rho)C(\rho)} \\ \mathcal{E}^{(+)}_{C(\rho)W(\rho)} & \mathcal{E}^{(+)}_{C(\rho)C(\rho)} \end{bmatrix} = \begin{bmatrix} R & -1 \\ -\frac{\theta}{\beta} & \frac{1+\theta}{\beta R} \end{bmatrix}, \\
\tilde{\boldsymbol{\mathcal{E}}}^{(+)}_{Z(\rho)Y} &= \begin{bmatrix} \mathcal{E}^{(+)}_{W(\rho)Y} \\ \tilde{\mathcal{E}}^{(+)}_{C(\rho)Y} \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{1}{\beta R}\frac{\bar{\rho}}{\omega}\left(\frac{1}{\rho} - \frac{\omega}{\rho}\right) \end{bmatrix}, \quad \text{and} \\
\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)V(\rho)} &= \begin{bmatrix} \mathcal{E}^{(+)}_{W(\rho)V(\rho)} \\ \mathcal{E}^{(+)}_{C(\rho)V(\rho)} \end{bmatrix} = \begin{bmatrix} R \\ -\frac{\theta}{\beta} \end{bmatrix}.
\end{aligned} \tag{B103}$$

Before solving the cross-country summed agent system, we decompose the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ into eigenvalues and eigenvectors and establish the conditions for a unique and stationary rational expectations solution following Blanchard and Kahn (1980). The eigendecomposition of $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ is given by

$$\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)} = \boldsymbol{\mathcal{V}}^{(+)}_{Z(\rho)Z(\rho)}\boldsymbol{\Lambda}^{(+)}_{Z(\rho)Z(\rho)}\boldsymbol{\mathcal{V}}^{(+)^{-1}}_{Z(\rho)Z(\rho)} \tag{B104}$$

where $\boldsymbol{\mathcal{V}}^{(+)}_{Z(\rho)Z(\rho)}$ is a matrix of eigenvectors of $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ and where $\boldsymbol{\Lambda}^{(+)}_{Z(\rho)Z(\rho)}$ is a diagonal matrix of eigenvalues of $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$,

$$\begin{aligned}
\boldsymbol{\mathcal{V}}^{(+)}_{Z(\rho)Z(\rho)} &= \begin{bmatrix} \nu^{(+)}_{W(\rho)W(\rho)} & \nu^{(+)}_{W(\rho)C(\rho)} \\ \nu^{(+)}_{C(\rho)W(\rho)} & \nu^{(+)}_{C(\rho)C(\rho)} \end{bmatrix} \\
&= \begin{bmatrix} \mathcal{E}^{(+)}_{W(\rho)C(\rho)} & \mathcal{E}^{(+)}_{W(\rho)C(\rho)} \\ \lambda^{(+)}_{Z(\rho)W(\rho)} - \mathcal{E}^{(+)}_{W(\rho)W(\rho)} & \lambda^{(+)}_{Z(\rho)C(\rho)} - \mathcal{E}^{(+)}_{W(\rho)W(\rho)} \end{bmatrix}, \quad \text{and} \\
\boldsymbol{\Lambda}^{(+)}_{Z(\rho)Z(\rho)} &= \begin{bmatrix} \lambda^{(+)}_{Z(\rho)W(\rho)} & 0 \\ 0 & \lambda^{(+)}_{Z(\rho)C(\rho)} \end{bmatrix},
\end{aligned} \tag{B105}$$

where

$$\begin{aligned}
\lambda^{(+)}_{Z(\rho)W(\rho)} = \frac{1}{2}\Big(\mathcal{E}^{(+)}_{W(\rho)W(\rho)} + \mathcal{E}^{(+)}_{C(\rho)C(\rho)}\Big) \\
+ \sqrt{\frac{1}{4}\Big(\mathcal{E}^{(+)}_{W(\rho)W(\rho)} - \mathcal{E}^{(+)}_{C(\rho)C(\rho)}\Big)^2 + \mathcal{E}^{(+)}_{W(\rho)C(\rho)}\mathcal{E}^{(+)}_{C(\rho)W(\rho)}}
\end{aligned} \tag{B106}$$

and

$$\lambda^{(+)}_{Z(\rho)C(\rho)} = \frac{1}{2}\Big(\mathcal{E}^{(+)}_{W(\rho)W(\rho)} + \mathcal{E}^{(+)}_{C(\rho)C(\rho)}\Big) - \sqrt{\frac{1}{4}\Big(\mathcal{E}^{(+)}_{W(\rho)W(\rho)} - \mathcal{E}^{(+)}_{C(\rho)C(\rho)}\Big)^2 + \mathcal{E}^{(+)}_{W(\rho)C(\rho)}\mathcal{E}^{(+)}_{C(\rho)W(\rho)}}\,. \tag{B107}$$

B.9.5 Stationarity of the Agent System (+)

We now establish uniqueness and stationarity for the cross-country differenced agent system. Stationarity of the system follows straightforwardly from a comparison of the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ in (B103) with the partial elasticity matrix $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ in (B78). We state the following proposition.

Proposition B2 (Stationarity of the Cross-Country Summed Agent System)**.** *The eigenvalues of* $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ *are given by*

$$\lambda^{(+)}_{Z(\rho)W(\rho)} = \lambda^{(-)}_{Z(\rho)W(\rho)} \quad \textit{and} \quad \lambda^{(+)}_{Z(\rho)C(\rho)} = \lambda^{(-)}_{Z(\rho)C(\rho)}\,. \tag{B108}$$

From Proposition B1, there exists a $\theta > 0$ *that yields a unique and stationary rational expectations solution to the cross-country summed agent system in* (B102)*.*

Let $\Big[\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}\Big]_{33}$ denote the (2×2) submatrix of $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$ formed by deleting the third row and third column from $\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}$. Then, by inspection, $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)} = \Big[\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}\Big]_{33}$, and the eigenvalues of $\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Z(\rho)}$ equal the eigenvalues of the submatrix $\Big[\boldsymbol{\mathcal{E}}^{(-)}_{Z(\rho)Z(\rho)}\Big]_{33}$. That is, $\lambda^{(+)}_{Z(\rho)W(\rho)} = \lambda^{(-)}_{Z(\rho)W(\rho)}$ and $\lambda^{(+)}_{Z(\rho)C(\rho)} = \lambda^{(-)}_{Z(\rho)C(\rho)}$. Thus, the stationarity and uniqueness conditions for the cross-country summed and differenced agent systems are the same, and satisfied under Assumptions 1 and 2 as shown in Proposition B1.

B.9.6 Intermediate Agent Non-Portfolio Solution (+)

We now derive intermediate solutions for cross-country summed agent real wealth and consumption in deviation from aggregate, written in terms of lagged agent state variables, exogenous variables and parameters, and the endogenous agent portfolio valuation effect. Using the eigendecomposition in (B104), we re-write the system in (B77) as

$$\begin{aligned}\boldsymbol{\mathcal{V}}^{(+)^{-1}}_{Z(\rho)Z(\rho)}\,\mathrm{E}_t\Big[\tilde{\boldsymbol{Z}}_{i+jt+1}(\rho)\Big] &= \boldsymbol{\Lambda}^{(+)}_{Z(\rho)Z(\rho)}\boldsymbol{\mathcal{V}}^{(+)^{-1}}_{Z(\rho)Z(\rho)}\tilde{\boldsymbol{Z}}_{i+jt}(\rho) \\ &+ \boldsymbol{\mathcal{V}}^{(+)^{-1}}_{Z(\rho)Z(\rho)}\Big(\boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)Y}\hat{Y}_{i+jt} + \boldsymbol{\mathcal{E}}^{(+)}_{Z(\rho)V(\rho)}\hat{V}^{i}_{i+jt}(\rho)\Big) + O(\epsilon^2)\,,\end{aligned} \tag{B109}$$

and from (B109) we extract the equation associated with the unstable eigenvalue $\lambda^{(+)}_{Z(\rho)W(\rho)}$,

$$\begin{aligned}\mathrm{E}_t\Big[\tilde{Z}_{i+jt+1}(\rho)\Big] &= \lambda^{(+)}_{Z(\rho)W(\rho)}\tilde{Z}_{i+jt}(\rho) - \mathcal{E}^{(+)}_{W(\rho)C(\rho)}\mathcal{E}^{(+)}_{C(\rho)Y}\hat{Y}_{i+jt} \\ &+ \Big[\Big(\lambda^{(+)}_{Z(\rho)C(\rho)} - \mathcal{E}^{(+)}_{W(\rho)W(\rho)}\Big)\mathcal{E}^{(+)}_{W(\rho)W(\rho)} - \mathcal{E}^{(+)}_{W(\rho)C(\rho)}\mathcal{E}^{(+)}_{C(\rho)V(\rho)}\Big]\tilde{V}^{i}_{i+jt}(\rho) + O(\epsilon^2)\,,\end{aligned} \tag{B110}$$

where $\tilde{Z}_{i+jt+1}(\rho) = \left(\lambda^{(+)}_{Z(\rho)W(\rho)} - \mathcal{E}^{(+)}_{W(\rho)W(\rho)}\right)\tilde{W}^i_{i+jt}(\rho) - \mathcal{E}^{(+)}_{W(\rho)W(\rho)}\tilde{C}_{i+jt+1}(\rho)$.

Because $\lambda^{(+)}_{Z(\rho)W(\rho)}$ is greater than one, the left-hand side of (B110) must be zero to rule out explosive paths. Setting the left-hand side to zero, we obtain

$$\tilde{C}_{i+jt}(\rho) = \eta^{(+)}_{C(\rho)W(\rho)}\tilde{W}^i_{i+jt-1}(\rho) + \tilde{\eta}^{(+)}_{C(\rho)Y}\hat{Y}_{i+jt} + \eta^{(+)}_{C(\rho)V(\rho)}\tilde{V}^i_{i+jt}(\rho) + O(\epsilon^2)\,, \tag{B111}$$

and using (B111) to eliminate $\tilde{C}_{i+jt}(\rho)$ from the cross-country summed agent budget constraint in (B96), we obtain

$$\tilde{W}^i_{i+jt}(\rho) = \eta^{(+)}_{W(\rho)W(\rho)}\tilde{W}^i_{i+jt-1}(\rho) + \tilde{\eta}^{(+)}_{C(\rho)Y}\hat{Y}_{i+jt} + \eta^{(+)}_{W(\rho)V(\rho)}\tilde{V}^i_{i+jt}(\rho) + O(\epsilon^2)\,, \tag{B112}$$

where the cross-country summed agent wealth and consumption semi-partial elasticity matrices are given by

$$\begin{aligned}
\boldsymbol{\eta}^{(+)}_{Z(\rho)W(\rho)} &= \begin{bmatrix}\eta^{(+)}_{W(\rho)W(\rho)}\\ \eta^{(+)}_{C(\rho)W(\rho)}\end{bmatrix} = \begin{bmatrix}\lambda^{(+)}_{Z(\rho)C(\rho)}\\ \mathcal{E}^{(+)}_{W(\rho)W(\rho)} - \lambda^{(+)}_{Z(\rho)C(\rho)}\end{bmatrix},\\
\tilde{\boldsymbol{\eta}}^{(+)}_{Z(\rho)Y} &= \begin{bmatrix}\tilde{\eta}^{(+)}_{W(\rho)Y}\\ \tilde{\eta}^{(+)}_{C(\rho)Y}\end{bmatrix} = \begin{bmatrix}\frac{\tilde{\mathcal{E}}^{(+)}_{C(\rho)Y}}{\lambda^{(+)}_{Z(\rho)W(\rho)}}\\ -\frac{\tilde{\mathcal{E}}^{(+)}_{C(\rho)Y}}{\lambda^{(+)}_{Z(\rho)W(\rho)}}\end{bmatrix}, \quad \text{and}\\
\boldsymbol{\eta}^{(+)}_{Z(\rho)V(\rho)} &= \begin{bmatrix}\eta^{(+)}_{W(\rho)V(\rho)}\\ \eta^{(+)}_{C(\rho)V(\rho)}\end{bmatrix} = \begin{bmatrix}\frac{1-\theta}{\beta} - \frac{\left(\mathcal{E}^{(+)}_{W(\rho)W(\rho)} - \lambda^{(+)}_{Z(\rho)C(\rho)}\right)\mathcal{E}^{(-)}_{W(\rho)V(\rho)} - \mathcal{E}^{(-)}_{C(\rho)V(\rho)}}{\lambda^{(+)}_{Z(\rho)W(\rho)}}\\ \frac{\left(\mathcal{E}^{(+)}_{W(\rho)W(\rho)} - \lambda^{(+)}_{Z(\rho)C(\rho)}\right)\mathcal{E}^{(-)}_{W(\rho)V(\rho)} - \mathcal{E}^{(-)}_{C(\rho)V(\rho)}}{\lambda^{(+)}_{Z(\rho)W(\rho)}}\end{bmatrix}.
\end{aligned} \tag{B113}$$

Writing the intermediate solutions in (B111) and (B112) in matrix form, we have

$$\begin{bmatrix}\tilde{W}^i_{i+jt}(\rho)\\ \tilde{C}_{i+jt}(\rho)\end{bmatrix} = \boldsymbol{\eta}^{(+)}_{Z(\rho)W(\rho)}\tilde{W}^i_{i+jt-1}(\rho) + \tilde{\boldsymbol{\eta}}^{(+)}_{Z(\rho)Y}\hat{Y}_{i+jt} + \boldsymbol{\eta}^{(+)}_{Z(\rho)V(\rho)}\tilde{V}^i_{i+jt}(\rho) + O(\epsilon^2)\,. \tag{B114}$$

As with the cross-country differenced solution in (62), the cross-country summed solution in (B114) is intermediate in that it depends on the endogenous portfolio valuation effect $\tilde{V}^i_{i+jt}(\rho)$. This effect in turn depend on the endogenous steady-state cross-country sum of agent portfolio holdings, for which we solve next.

B.10 The Agent Portfolio Problem

We now solve for zero-order approximate cross-country differenced and summed agent portfolio holdings. These solutions then combine to yield country-specific solutions for the agent's holdings of the domestic and international bond.

Table B6 – Agent Semi-Partial Elasticities at $\theta = 0$.

Semi-Partial Elasticity	Evaluation at $\theta = 0$	
	Expression	Sign
$\eta^{(-)}_{C(\rho)V(\rho)}$	$\frac{1-\beta}{\beta}$	$+$
$\eta^{(+)}_{C(\rho)V(\rho)}$	$\frac{1-\beta}{\beta}$	$+$

Notes. The table shows semi-partial elasticities from the cross-country differenced and summed agent non-portfolio problems, evaluated at $\theta = 0$. Column two give an expression for each elasticity, and column three gives its sign under Assumptions 1 and 2.

To derive cross-country differenced agent portfolio holdings, we begin from equation (63). Using the real exchange rate in (42), the definition of the cross-country differenced agent portfolio valuation effect in (59), and the first-order approximate intermediate solutions for cross-country differenced agent consumption in (62), we obtain the intermediate solution in (64). The collection of parameters $\zeta^{(-)}_{B(\rho)}$ in (64) is given by

$$\begin{aligned}\zeta^{(-)}_{B(\rho)} = \frac{1}{\eta^{(-)}_{C(\rho)V(\rho)}}\left[\left(1-\frac{1}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\right)\eta^{(-)}_{QY}+\frac{\eta^{(-)}_{QW}\eta^{(-)}_{WY}}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\right] \\ -\frac{1}{\eta^{(-)}_{C(\rho)V(\rho)}}\frac{\eta^{(-)}_{CY}-\eta^{(-)}_{QY}\frac{\omega}{\bar{\rho}}}{\eta^{(-)}_{CV}-\eta^{(-)}_{QV}\frac{\omega}{\bar{\rho}}}\left[\left(1-\frac{1}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\right)\eta^{(-)}_{QV}+\frac{\eta^{(-)}_{QW}\eta^{(-)}_{WV}}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\right].\end{aligned} \tag{B115}$$

To derive cross-country summed agent portfolio holdings, we begin from equation (65). Using the real exchange rate in (42), the definition of the cross-country summed agent portfolio valuation effect in (B97), and the first-order approximate intermediate solutions for cross-country summed agent consumption in (B114), we obtain the intermediate solution in (66). The collection of parameters $\zeta^{(+)}_{B(\rho)}$ in (66) is given by

$$\zeta^{(+)}_{B(\rho)} = \frac{\bar{\rho}}{\omega}\frac{1}{\eta^{(+)}_{C(\rho)V(\rho)}}\left(1-\frac{1}{(1-\theta)\lambda^{(-)}_{Z(\rho)W(\rho)}}\right). \tag{B116}$$

The solutions in equations (64) and (66) are intermediate because the cross-country differenced real return appears on the right-hand side of each. Final solutions obtain straightforwardly by eliminating the cross-country differenced real return using the first-order approximate general solution in (44). For brevity, we omit this derivation here, noting that (64) and (66) are sufficient to establish our main result for agent portfolio holdings in Proposition (10). For use later, we evaluate two semi-partial elasticities that appear in these solutions at $\theta = 0$ and report them in Table B6.

B.10.1 Proof of Proposition 10

Proposition 10 establishes sufficient conditions for international portfolio bias in agent portfolio holdings for a subset of risk-tolerant agents. These conditions simultaneously satisfy the conditions for domestic bias in aggregate portfolio holdings in Proposition 8 and for negative expected cross-country differenced real returns in Proposition 9.

We begin with equation (67) for agent holdings of the domestic bond in country i. Using the intermediate solutions for cross-country differenced and summed agent holdings of this bond in (64) and (66), respectively, we obtain

$$\tilde{B}^i_{ii}(\rho) = \frac{1}{2}\left(\frac{1}{\rho} - \frac{\omega}{\bar{\rho}}\right)\frac{\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\left(\zeta^{(-)}_{B(\rho)}\hat{Y}_{i-jt+1} + \zeta^{(+)}_{B(\rho)}\hat{Y}_{i+jt+1}\right)\right]}{\mathrm{E}_t\left[\left(\hat{R}^i_{i-jt+1}\right)^2\right]} + O(\epsilon)\,, \tag{B117}$$

from which a final solution in terms of model parameters can be obtained directly using first-order approximate realized real returns in (43).

In the proof of Proposition (9) we established $\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\hat{Y}_{i-jt+1}\right] \leq 0$ and $\mathrm{E}_t\left[\hat{R}^i_{i-jt+1}\hat{Y}_{i+jt+1}\right] < 0$ under Assumptions 1 and 2. To establish the sign of $\tilde{B}^i_{ii}(\rho)$, it remains to establish the signs of $\zeta^{(-)}_{B(\rho)}$ and $\zeta^{(+)}_{B(\rho)}$.

We first establish the sign of $\zeta^{(-)}_{B(\rho)}$ assuming $\theta = 0$. Splitting the expression in (B115) into four terms, we have

$$\zeta^{(-)}_{B(\rho)} = \zeta^{(-)(T_1)}_{B(\rho)}\left(\zeta^{(-)(T_2)}_{B(\rho)} + \zeta^{(-)(T_3)}_{B(\rho)}\zeta^{(-)(T_4)}_{B(\rho)}\right), \tag{B118}$$

where

$$\begin{aligned}
\zeta^{(-)(T_1)}_{B(\rho)} &= \frac{1}{\eta^{(-)}_{C(\rho)V(\rho)}}\,,\\
\zeta^{(-)(T_2)}_{B(\rho)} &= \left(1 - \frac{1}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\right)\eta^{(-)}_{QY} + \frac{1}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\eta^{(-)}_{QW}\eta^{(-)}_{WY}\,,\\
\zeta^{(-)(T_3)}_{B(\rho)} &= -\frac{\eta^{(-)}_{C}oo - \bar{\sigma}\eta^{(-)}_{QY}}{\eta^{(-)}_{CV} - \bar{\sigma}\eta^{(-)}_{QV}}\,, \quad \text{and}\\
\zeta^{(-)(T_4)}_{B(\rho)} &= \left(1 - \frac{1}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\right)\eta^{(-)}_{QV} + \frac{1}{\lambda^{(-)}_{Z(\rho)W(\rho)}}\eta^{(-)}_{QW}\eta^{(-)}_{WV}\,.
\end{aligned} \tag{B119}$$

Using Table B4, Table B6, and the expression for $\lambda^{(-)}_{Z(\rho)W}\Big|_{W=0}$ in Proposition B1, we obtain

$$\begin{aligned}
\zeta^{(-)(T_1)}_{B(\rho)}\Big|_{\theta=0} &= \frac{\beta}{1-\beta}\Big|_{\theta=0} > 0\,, \\
\zeta^{(-)(T_2)}_{B(\rho)}\Big|_{\theta=0} &= (1-\beta)\left(1+\beta\frac{\left(1-\phi^2\right)(1-\bar{\sigma})}{1-(1-\bar{\sigma})\phi^2}\right) > 0\,, \\
\zeta^{(-)(T_3)}_{B(\rho)}\Big|_{\theta=0} &= -\phi(1-\bar{\sigma})\frac{(1-\beta)^2}{\beta}\frac{1-(1-\bar{\sigma})\phi^2}{1-\phi} < 0\,, \quad \text{and} \\
\zeta^{(-)(T_4)}_{B(\rho)}\Big|_{\theta=0} &= -\frac{(1-\beta)^2}{\beta}\frac{\phi^2}{1-\phi}\left(1+\frac{1-\phi^2-(1-\beta)(1-\phi)}{1-\phi^2}\right) < 0\,.
\end{aligned} \tag{B120}$$

Next, we establish the sign of $\zeta^{(+)}_{B(\rho)}$ assuming $\theta = 0$. Using Table B4 and the expression for the eigenvalue $\lambda^{(-)}_{Z(\rho)W}$ in Proposition B1, we obtain $\zeta^{(+)}_{B(\rho)}\Big|_{\theta=0} = \beta\bar{\rho}/\omega > 0$ under Assumptions 1 and 2. By continuity, the signs of $\zeta^{(-)}_{B(\rho)}$ and $\zeta^{(+)}_{B(\rho)}$ are preserved for small positive values of θ.

Returning to equation (B117), we conclude that $\tilde{B}^i_{ii}(\rho)$ is negative for any $\rho < \bar{\rho}/\omega$ under Assumptions 1–3, and that $\tilde{B}^i_{ii}(\rho)$ approaches negative infinity as ρ approaches zero from above,

$$\lim_{\rho \downarrow 0} \tilde{B}^i_{ii}(\rho) = -\infty\,. \tag{B121}$$

Thus, recalling that $\tilde{B}^i_{ii}(\rho) \equiv B^i_{ii}(\rho) - B^i_{ii}$, there exists a $\rho^* \in (0, \bar{\rho}/\omega)$ such that $B^i_{ii}(\rho) < 0$ for any $\rho < \rho^*$ and any finite value of B^i_{ii}.