What drives inflation?
Lessons from disaggregated price data

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Abstract

The recent Covid pandemic impacted demand and supply in widely different ways across industries, while disrupting supply chains and labor markets. At the same time, governments implemented unprecedented demand stimulus packages. How much of the recent increase in inflation can be attributed to cross-sectional demand and supply disruptions versus aggregate stimulus policies? Building on a model with multiple industries and primary factors, I show that declines in relative productivity or increases in relative demand for specific sectors – in particular those which use inelastically supplied factors – worsen the inflation-output tradeoff, beyond the control of monetary policy. Moreover, these cross-sectional shocks induce different changes in relative prices compared to aggregate demand. Levering up on this observation, I show how to filter out the effect of cross-sectional shocks and recover the component of inflation driven by aggregate demand. Applying my decomposition to US consumer price inflation between 2020 and 2022, I find that the rapid deflation and subsequent inflation of 2020 are almost entirely driven by supply factors, while around three quarters of the increase in the CPI since 2021 is driven by aggregate demand.

1 Introduction

Several policy makers and commentators suggested that cross-sectional shocks, such as changes in relative demand or relative productivity across industries, have been an important driver of aggregate inflation during and after the Covid pandemic. Consistent with this story, during the first year of the pandemic we saw large movements in relative prices. Nonetheless, from a theoretical standpoint it is not obvious why relative price movements should translate into aggregate inflation. The traditional New Keynesian framework, on which we typically rely to analyze inflation dynamics, is ill-suited to address this question, because it features an aggregated model of production with a single representative industry. This paper enriches the New Keynesian framework with a disaggregated model of production, and it provides necessary and sufficient conditions for cross-sectional shocks to create an inflation

output tradeoff. It also shows how to identify the components of aggregate inflation coming from aggregate demand vs cross-sectional shocks based on relative price movements.

My disaggregated model of production features multiple industries and primary factors. Multiple primary factors are a novel element compared to other New Keynesian models with input-output linkages (Carvalho (2006); Nakamura and Steinsson (2010); Pasten et al. (2019); LaO and Tahbaz-Salehi (2019); Rubbo (2020)), and they are key to modeling supply-chain bottlenecks – a prominent feature of the Covid episode. After introducing the environment in Section 2, I illustrate my model of bottlenecks in Section 3. Essentially, bottlenecks are caused by a decline in relative productivity or an increase in relative demand for inelastically supplied goods. Supply may be inelastic because industries rely on inelastically-supplied primary factors, or simply because they face decreasing returns to scale in production. Importantly, inelastic industries cannot compensate for higher demand or lower productivity by using primary factors more intensively. As a result bottlenecks cause aggregate output to fall, even when aggregate productivity and aggregate demand remain constant.

After illustrating the negative consequences of supply bottlenecks for aggregate output, I turn to the implications for aggregate inflation. Before describing these, it is important to recall two fundamental concepts: the natural output and the inflation-output tradeoff. Following the New Keynesian tradition, our economy is subject to nominal rigidities. In order to tease apart the effect of exogenous shocks from the effect of nominal rigidities and monetary policy, it is useful to define a counterfactual reference economy. This economy has flexible prices, and it is subject to (a subset of) the same shocks as the actual, sticky-price economy. We define the change in natural output as the change in real GDP that would prevail in the benchmark flex-price economy. In general, the natural output will move in response to shocks. Monetary policy controls real GDP in the sticky price economy (for our analysis it won't matter through which instrument), therefore it can choose whether or not to replicate the natural output, i.e. whether or not to open an output gap.

Inflation and the output gap are positively related through the Phillips curve. Given that monetary policy can choose the output gap, we consider it responsible for the component of inflation resulting from a non-zero output gap. For example, whenever there is a supply bottleneck, the natural output declines. Monetary policy could prevent output from falling down to the natural level, and this will cause inflation. Here however we cannot attribute the increase in inflation to the bottleneck, but rather to monetary policy keeping output artificially high.

Asymmetric shocks, however – i.e. shocks that hit different industries differently, like bottlenecks – can have implications for inflation even when monetary policy lets output fall to the natural level. In fact the natural output, being a one-dimensional (scalar) object, depends on a one-dimensional combination of disaggregated industry-level shocks. In general, this one-dimensional combination is not a sufficient statistic for the full effect of shocks on inflation. In other words, we can isolate a component of shocks which determines the aggregate natural output, and a residual cross-sectional component. Industry-level as well as aggregate inflation respond to both the aggregate output gap and the cross-sectional component of shocks. Hence asymmetric shocks can result in aggregate inflation even if the central bank closes the output gap. This phenomenon is beyond the control of monetary policy, and we call it an inflation-output tradeoff.

I show that any shock which is accounted for in the natural equilibrium enters the Phillips curve only through a wedge between beginning-of-period prices and the price change induced by the shock in the benchmark flex-price economy. I analyze the response of inflation to these price wedges in Section 4, arguing that, once we control for the aggregate output gap, only relative price wedges matter. Hence price wedges are a sufficient statistic for the cross-sectional component of shocks. This allows us to decompose inflation into a part that is driven by the aggregate output gap, and a part that is driven by cross-sectional shocks through price wedges. The decomposition can be applied to both industry-level and aggregate inflation.

Specifically, cross-sectional shocks affect aggregate inflation whenever industries are asymmetric in at least one of two relevant dimensions: the degree to which they pass-through changes in their costs into final prices, and the factor supply elasticity that they face. Rubbo (2020) already pointed out the heterogeneous pass-through channel. Heterogeneous supply elasticities are novel to the environment with multiple primary factors, and, as we explained, they are essential to modeling supply bottlenecks.

I argue that bottlenecks not only reduce the natural output, but they also create a worse inflation-output tradeoff. To see why this is the case, recall that a bottleneck is an increase in relative demand or a decline in relative productivity for some inelastic industry. Nominal rigidities prevent these industries from increasing their relative price. Therefore, for any given level of aggregate output, they face inefficiently high relative demand. While relative quantity distortions cancel out on average, the resulting price distortions do not. This is because the price of inelastic factors is more sensitive to changes in employment. In other words, the fact that demand is too low in "slack" sectors does not compensate for high demand in "tight" sectors, and prices increase. This worsens the inflation-output tradeoff, beyond the control of monetary policy.

We cannot immediately take our inflation decomposition to the data, because we cannot easily observe either the aggregate output gap, or cross-sectional price wedges. To address this issue, in Section 5 I show how to implement the decomposition based on information contained in relative price movements. Intuitively, the aggregate output gap and cross-sectional price wedges must affect relative prices in different ways. Formally, the inflation response to the aggregate output gap lies outside of the space spanned by the inflation response to cross-sectional shocks. Moreover, two together span the full space. Thus we can recover the aggregate output gap by projecting realized inflation along the aggregate direction, according to the coordinates defined by the responses to aggregate and cross-sectional shocks.

The space spanned by the inflation response to cross-sectional price wedges is easy to characterize. Intuitively, industry-level inflation movements caused by cross-sectional shocks must cancel out in some appropriate weighted average. Equivalently, these movements cannot span the full space, therefore they must be orthogonal to some appropriate aggregate inflation index. I call this inflation index the divine coincidence index, and I show that it differs from traditional measures of final good inflation, such as consumer prices or the GDP deflator, in that it weights sectors according to sales shares and discounts those with flexible prices or inelastic factor supply. Our desired projection of inflation along the aggregate component is equal to the divine coincidence index divided by the slope of its Phillips curve. The slope of the divine coincidence Phillips curve is strictly positive, because the

aggregate output gap and cross-sectional shocks move sectoral inflation rates in non-collinear ways.

The aggregate component of inflation that we identify in this way is determined by the discrepancy between real GDP in the actual sticky-price economy and real GDP in a benchmark flex-price economy subject to the full set of shocks that hit the actual economy. The residual inflation is driven by cross-sectional shocks. Our decomposition cannot tell us whether the output gap is due to monetary policy pushing output away from the steady-state level (for constant natural output), or whether it is due to a change in the natural output. Moreover, the benchmark flex-price economy based on which we compute the output gap faces the same changes in the producers' desired markups as the sticky-price economy. This includes markup changes that are endogenously driven by expected inflation. Thus our decomposition is a lower bound for the impact of monetary policy on current inflation, because it does not account for its endogenous effect through inflation expectations. Properly accounting for this channel would require accurate disaggregated measures of inflation expectations, and providing these is beyond the scope of the paper.[1]

While the divine coincidence index provides us with a proxy for the aggregate output gap – which is equal to the income-share-weighted average of factor-level employment gaps – I conclude the theoretical analysis by showing how to construct inflation proxies for any weighted average of factor-specific employment gaps. This result is particularly useful in a currency union, as it allows us to proxy for regional output gaps.

In Section 6 I apply my inflation decomposition to consumer prices in the US between January 2020 and December 2022. I find that, in 2020, inflation was almost entirely driven by supply shocks. Thus traditional measures of core inflation – such as the PCE less food and energy – overestimate the role of aggregate demand. This result is largely due to the fact that, in the early phases of the pandemic, inflation has been concentrated in inelastic, capital-intensive sectors. Instead, about three quarters of the inflation that we observed from 2021 onwards can be attributed to aggregate demand, and the aggregate component that I estimate is close to the core PCE. This is consistent with the fact that in recent periods inflation has been high in most sectors. I also find that the model attributes a smaller fraction of inflation to aggregate demand when labor and capital markets are more segmented.

1.1 Related literature

Ball and Mankiw (1995) first put forward the hypothesis that relative price movements can be inflationary, by showing a positive empirical relationship between aggregate inflation and the right skewness of the distribution of sectoral price adjustments. More recently, Ruge-Murcia et al. (2022) show that a similar empirical relationship still holds in more recent years. They also argue that both the low inflation between 2012-2019 and the high inflation in the first months of the Covid pandemic were driven by a small number of sectors, while inflation has become broad based since the second half of 2021. Compared to these papers, I provide a formal theory of when and why

[1] Even when looking at inflation dynamics over time, and assuming rational expectations, we cannot separately identify changes in inflation expectations from exogenous changes in desired markups.

relative price movements matter for aggregate inflation, and clarify the distinction between the effect of shocks on the natural output versus the inflation-output tradeoff.

In closely related papers, Baqaee and Farhi (2020a) and di Giovanni et al. (2022) also provide a decomposition between supply- and demand-driven inflation, moving from a model with multiple industries and primary factors. Baqaee and Farhi (2020a) and di Giovanni et al. (2022) identify demand and supply shocks based on changes in sector-level wages and employment, assuming inelastic labor supply and downward wage rigidities. My decomposition instead is entirely based on relative prices. I consider good prices as well as wages, in an environment where labor supply is elastic, and both prices and wages are sticky (both upwards and downwards). Most importantly, Baqaee and Farhi (2020a) and di Giovanni et al. (2022) define aggregate demand shocks as changes in interest rates or in the discount factor which cause aggregate real GDP to deviate from the (pre-pandemic) steady-state. By contrast, I define aggregate demand shocks relative to an economy with flexible prices, which faces the same real shocks as the actual sticky-price economy. Accordingly, I estimate a much smaller role for aggregate demand in the early phases of the pandemic.

In other related work, Shapiro (2020) and Shapiro (2022) classify PCE categories as demand versus supply driven depending on whether quantities and prices change in the same or opposite directions, finding that demand and supply effects are equally important to explain the recent inflation episode. Ball et al. (2021) show that core inflation – as measured by the weighted median inflation rate, which is highly correlated with labor market slack – was initially subdued, while the increase in prices since 2021 was driven by inflation in non-core sectors, that only later propagated to other sectors. Relative to these empirical approaches, my decomposition is grounded in a theory of the transmission of real and monetary shocks in a multi-sector economy. As a payoff, my network model properly accounts for the fact that what looks like a demand shock in one sector – i.e. prices and quantities moving in the same direction – could actually originate from a supply shock in some customer sector (and vice versa). Moreover, I provide a theoretical rationale for aggregate inflation statistics to exclude sectors which experience disproportionately large price changes.

2 Environment

This section sets up a New Keynesian model with monopolistic competition and sticky prices, featuring multiple heterogeneous industries and primary factors. Sections 2.1 through 2.3 lay out the assumptions about preferences, production and policy instruments and derive optimality conditions for consumers and producers; section 2.4 defines the general equilibrium; and sections 2.5 and 2.6 describe the log-linearized model.

Figure 1 provides a schematic representation of the economy.

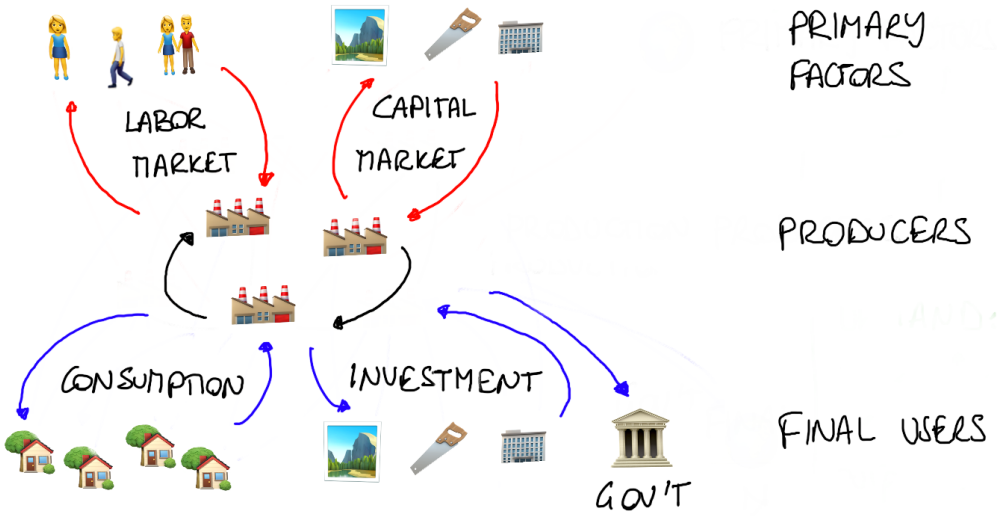


Figure 1: Schematic representation of the economy

2.1 Final users

The final users are consumers-workers, investment providers, and the government. There are N_w consumer-worker types and N_f capital asset types, each with a corresponding set of investment providers. We index final users by h or $f \in \{1, ..., N_w + N_f + 1\}$. We denote by $\mathcal{N}_w$ the set of consumer-worker types, and by $\mathcal{N}_f$ the set of capital types. We denote the price of factor h by W_h, and the quantity by L_h.

2.1.1 Consumers-workers

Preferences Each consumer-worker type $h \in \mathcal{N}_w$ has a representative agent, who supplies a distinct labor type and whose per-period preferences are described by the utility function

$$U_{ht} = \frac{C_{ht}^{1-\gamma_h}}{1-\gamma_h} - \frac{1}{\mathcal{E}_h} \frac{L_{ht}^{1+\varphi_h}}{1+\varphi_h} \tag{1}$$

All agents enjoy consumption (C) and dislike labor (L), with heterogeneous income effects in labor supply γ_h and Frisch elasticities $\frac{1}{\varphi_h}$. Agents also face group-specific shocks $\mathcal{E}_h$ to their relative preference for leisure. Consumption aggregators $C_{ht} \equiv \mathcal{C}_{ht}(c_{1ht}, ..., c_{Nht})$ are homothetic over the N goods produced in the economy, and can differ across groups. Consumption preferences $\mathcal{C}_{ht}$ are time-varying, to account for changes in expenditure shares across goods at constant prices.

Budget constraints Consumers-workers maximize the present discounted value of per-period utility flows, with the same discount factor ρ, subject to type-specific budget constraints

$$P_{ht}^{C} C_{ht} \leq W_{ht} L_{ht} + \sum_{f \in \mathcal{N}_f} \mathcal{Z}_{fh} \frac{\varphi_f}{1+\varphi_f} W_f L_f + \sum_{i=1}^{N} \Xi_{ih} \left(\Pi_{it} - \mathcal{T}_{it} \right) + T_{ht} \tag{2}$$

where P_{ht}^{C} is the price index implied by the consumption aggregator $\mathcal{C}_h$, W_{ht} is the nominal wage earned by workers of type h, and T_h is a lump-sum transfer received by type h.

Each consumer-worker group h owns shares in all the N industries in the economy, as described by the ownership matrix Ξ. The elements Ξ_{ih} denote the share of profits from industry i accruing to type-h agents. Profits Π_{it}, net of lump-sum taxes $\mathcal{T}_{it}$ paid by firms to the government, are allocated according to ownership shares. Agents also own shares of the N_f capital assets in the economy, as described by the matrix $\mathcal{Z}$. The elements $\mathcal{Z}_{fh}$ denote the share of asset f owned by worker type h, and the income paid out to capital asset f is equal to $\frac{\varphi_f}{1+\varphi_f} R_f K_f$ (as explained below).

Remark 1. We assume that agents cannot borrow and lend between each other. Thus the transfers $\{T_h\}_{h \in \mathcal{N}_w}$ are exogenously chosen by the government. The inflation decomposition in Section 5 would remain valid if transfers were determined endogenously by the agents' borrowing and lending decisions.

Consumption-leisure tradeoff The optimal consumption-leisure tradeoff satisfies the first order condition

$$C_{ht}^{\gamma_h} L_{ht}^{\varphi_h} = \mathcal{E}_h \frac{W_{ht}}{P_{ht}^{C}} \tag{3}$$

As explained in detail in Section 2.2 below, the flexible nominal wage W_{ht} is defined by the consumption-leisure tradeoff (3) given consumption, labor demand, and prices. Importantly, W_{ht} has no data counterpart. The model counterpart of the wages measured in the data is the sticky nominal wage paid by the firms, introduced in Section 2.2 below.

2.1.2 Investment

I adopt a stylized model of investment, with the objective to deliver capital supply curves with constant elasticity φ_f. Importantly, to simplify the analysis I abstract from the fact that demand for investment goods is more sensitive to interest rates than demand for consumption goods. In the model investment fully depreciates within one period, so that changes in interest rates have no direct effect on the relative price of consumption and investment goods.

Each capital asset f is produced by combining a fixed endowment ($\bar{K}_f$) with an investment good I_f. At each period, the supply of asset f is given by (omitting time subscripts for legibility)

$$L_f = \left[\left(1 + \varphi_f \right) I_f \right]^{\frac{1}{1+\varphi_f}} \mathcal{E}_f \bar{K}_f$$

where $\mathcal{E}_f$ is a shock to the supply of capital. For convenience we assume that the investment component I_f fully depreciates from one period to the next, while the endowment component $\bar{K}_f$ never depreciates.

In turn investment is produced with constant returns to scale, using as inputs a combination of primary factors (L_{fht}) and intermediate goods (X_{fit}), according to the production function

$$I_{ft} = G_{ft}(\{L_{fht}\}, \{X_{fit}\})$$

The production function G_{ft} is time-varying, to allow for changes in expenditure shares at constant prices.

There are N_f investment producing sectors, one for each asset type f, who sell the investment good at marginal cost P_f^I to capital retailers. Retailers purchase capital endowments from the agents, combine them with the investment good, and sell capital services to the firms at a rental rate W_f in a perfectly competitive market. Capital retailers are owned by the agents in proportion to their ownership shares $\mathcal{Z}$ in the capital endowments, and rebate their profits accordingly.

Profit maximization yields the capital supply curves

$$U_f^{\varphi_f} = \frac{W_f}{P_f} \mathcal{E}_f \bar{K}_f \tag{4}$$

where

$$U_f \equiv \left[(1+\varphi_f) I_f\right]^{\frac{1}{1+\varphi_f}}$$

can be interpreted as a measure of capital utilization. Profits are given by

$$\frac{\varphi_f}{1+\varphi_f} W_f L_f \tag{5}$$

while investment expenditures are equal to

$$\frac{1}{1+\varphi_f} W_f L_f \tag{6}$$

As we did for wages, we want to allow also for rental rates to be sticky. Thus, like the flexible wages in the consumption-leisure tradeoff (3), the flexible rental rates W_{ft} are simply defined to satisfy the capital supply equation (3) given capital demand and prices, and they have no data counterpart. The model counterpart of the rental rates measured in the data is the sticky rental rate paid by the firms, introduced in Section 2.2 below.

2.1.3 Government

The government implements transfers T_{ht} across consumers, and between itself and the consumers. We denote the total nominal transfer to the government by $G_t \geq 0$. The government's budget constraint is

$$-\sum_{h \in \mathcal{N}_w} T_{ht} = G_t \geq 0 \ \forall t$$

The government uses its revenue G to buy goods, with time-varying expenditure shares given by

$$\beta_{iGt} \equiv \frac{P_{it} G_{it}}{G_t}$$

2.2 Production

There are N good-producing industries in the economy (indexed by $i, j \in \{1, ..., N\}$). Within each industry there is a continuum of firms, producing differentiated varieties.

All firms z in industry i have the same constant returns to scale production function

$$Y_{izt} = A_{it} G_{it}(\{L_{ihzt}\}_{h \in \mathcal{N}_h \cup \mathcal{N}_f}, \{X_{ijzt}\}_{j=1}^{N})$$

where L_{ihzt} is the quantity of primary factor h hired by firm z in industry i at time t, and X_{ijzt} is the quantity of intermediate input j used by the firm. The Hicks-neutral productivity shifter A_{it} is industry-specific, and varies over time. The production function is also time-varying, to account for exogenous changes in expenditure shares on the various inputs at constant prices.[2]

Customers (consumers and other producers) buy a CES bundle of sectoral varieties, with elasticity of substitution ϵ_i. The industry output is given by

$$Y_i = \left(\int Y_{if}^{\frac{\epsilon_i - 1}{\epsilon_i}} df \right)^{\frac{\epsilon_i}{\epsilon_i - 1}}$$

and the implied sectoral price index is

$$P_i = \left(\int P_{if}^{1-\epsilon_i} df \right)^{\frac{1}{1-\epsilon_i}}$$

The government provides time-varying proportional input subsidies, fully financing them through lump-sum taxes on profits. We follow a standard practice in the literature, and assume that steady-state subsidies eliminate the markup distortions arising from monopolistic competition. Subsidies are such that at time t sector i pays a share

$$(1 - \tau_{it}) \frac{\epsilon_i - 1}{\epsilon_i}$$

[2]Relative demand shocks are isomorphic to biased productivity changes, except when production functions are Cobb-Douglass.

of its actual marginal cost. With this notation, $\tau_i = 0$ in steady state.

All producers minimize costs given input prices. With constant returns to scale marginal costs are the same for all firms within a sector i, and they all use inputs in the same proportions. The marginal cost of sector i, denoted by MC_i, is the solution of the cost minimization problem (omitting time subscripts for legibility)

$$MC_i = min_{\{X_{ij}\},\{L_{ih}\}} \sum_{h \in \mathcal{N}_h \cup \mathcal{N}_f} W_h L_{ih} + \sum_{j=1}^{N} P_j X_{ij} \quad s.t.\ A_i G_i \left(\{L_{ih}\}_{h \in \mathcal{N}_h \cup \mathcal{N}_f}, \{X_{ij}\}_{j=1}^{N} \right) = 1 \tag{7}$$

Producers are subject to nominal rigidities, modeled à la Calvo. In every sector i and at each period, only a randomly-selected fraction δ_i of the firms can update their price. They set it to maximize the present discounted value of future profits, conditional on being unable to update their price:

$$P_{it}^* = \frac{\epsilon_i}{\epsilon_i - 1} \frac{\mathbb{E} \sum_s \left[SDF_{t+s} \left(1 - \delta_i\right)^s Y_{izt+s}(P_{it}^*) \left(1 - \tau_{it+s}\right) MC_{it+s} \right]}{\mathbb{E} \sum_s \left[SDF_{t+s} \left(1 - \delta_i\right)^s Y_{izt+s}(P_{it}^*) \right]} \tag{8}$$

where $SDF_{t+s} = \rho^s \frac{U_{ct+s}}{U_{ct}} \frac{P_t^c}{P_{t+s}^c}$ is the households' stochastic discount factor, and demand functions are given by $Y_{izt+s}(P_{zt}) = Y_{it} \left(\frac{P_{zt}}{P_{it}} \right)^{-\epsilon_i}$. The firms z who cannot adjust their price instead keep it unchanged ($P_{izt} = P_{izt-1}$), and must absorb any cost changes in their markup $\mathcal{M}_{izt}$.

Sticky factor prices To model sticky factor prices we assume that primary factors (workers and capital assets) are first purchased by marketplaces, who then sell their services to producers in all the different sectors. Each marketplace deals with only one primary factor. Marketplaces are treated like any other industry. In particular there is a continuum of marketplaces for each type, with fixed unit mass, facing Calvo-style price rigidities.

Primary factor types are fixed, hence wages differ across primary factors. Nonetheless workers and investment providers can freely move across marketplaces, therefore all units of a given primary factor type earn the same flexible wage. Recall however that the flexible factor wages have no counterpart in the wages or rental rates measured in the data, nor in the actual earnings of workers and investment producers. They are simply defined to satisfy equations (3) and (4) given good prices and factor demand.

The model counterpart of factor prices measured in the data are the prices charged by factor marketplaces. Note that different marketplaces who hire the same factor type might charge different prices, due to the Calvo friction. We assume that ownership of type-h marketplaces is equally shared among all type-h factor suppliers, who earn the profit rebates. Therefore in practice all units of factor h earn the same (sticky) wage, given by the average price charged by the h-specific marketplace.

Retailers To streamline the notation below, it is convenient to augment the set of industries with one retailer for each final user. Retailers assemble consumption, investment and government spending bundles and sell them

to the relevant final user. Without loss of generality, we model relative demand shocks to consumers, investment producers, or the government, as shocks to the retailers' production function. There is a continuum of retailers for each final user type, subject to Calvo price stickiness like any other sector. Retailers are owned by the corresponding final users, to whom they rebate their profits.

Remark 2. Given our modeling of retailers and factor marketplaces, the overall number of sectors in the economy is $\bar{N} = N + 2N_w + 2N_f + 1$ (the actual production sectors, the factor marketplaces for labor and capital, and the retailers for consumption, investment and government spending).

2.2.1 Aggregation

Our goal is to relate changes in aggregate inflation, as measured by the GDP deflator, with changes in aggregate output, as measured by real GDP. These aggregate quantities are defined below. Changes in real GDP and the GDP deflator are measured relative to the initial steady-state, denoted by starred variables.

Definition 1. Nominal GDP is given by total final expenditures:

$$GDP = \sum_{h \in \mathcal{N}_w} P_h^C C_h + \sum_{f \in \mathcal{N}_f} P_f^I I_f + G_t$$

Definition 2. Infinitesimal changes in real GDP around the initial steady-state are denoted by $d \log Y_t$, and are equal to the share-weighted sum of changes in aggregate consumption, aggregate investment, and government spending:

$$d \log Y_t = \sum_{h \in \mathcal{N}_w} \frac{P_h^{*C} C_h^*}{GDP^*} d \log C_{ht} + \sum_{f \in \mathcal{N}_f} \frac{P_f^* I_f^*}{GDP^*} d \log I_{ft} + \frac{dG_t}{GDP^*}$$

Definition 3. Infinitesimal changes in the GDP deflator around the initial steady-state are denoted by $d \log P_t^Y$, and are equal to the share-weighted change in the price indices of the final users:

$$d \log P_t^Y = \sum_{h \in \mathcal{N}_w} \frac{P_h^{*C} C_h^*}{GDP^*} d \log P_{ht}^C + \sum_{f \in \mathcal{N}_f} \frac{P_f^{*C} I_f^*}{GDP^*} d \log P_{ft}^C + \frac{G}{GDP^*} d \log P_G$$

Remark 3. As a consequence of constant returns to scale, changes in real GDP equal the income-weighted sum of changes in the quantities of primary factors:

$$d \log Y = \sum_{h \in \mathcal{N}_w \cup \mathcal{N}_f} \frac{W_h^* L_h^*}{\sum_{f \in \mathcal{N}_w \cup \mathcal{N}_f} W_f^* L_f^*} d \log L_h$$

2.3 Monetary policy

At each period the economy is subject to an aggregate cash-in-advance constraint, whereby nominal GDP cannot exceed the money supply M_t chosen by the central bank:

$$P_t Y_t \leq M_t \tag{9}$$

Seignorage revenues are distributed in proportion to the agents' consumption shares, so that – to a first order – seignorage rebates are exactly equal to the amount of new money that the agents need to purchase in order to finance consumption, and the two cancel out from the budget constraint.

In the examples below we use a static version of the cash-in-advance model, with $\rho = 0$, to provide intuition. We assume that the economy enters period 0 in steady-state, so that pre-set prices from $t = -1$ are equal across producers. Only a fraction δ_i of producers in each sector i can update prices after observing the actual shocks.

2.4 Equilibrium

The equilibrium concept adapts the definition in Baqaee and Farhi (2020b) to account for the endogenous determination of markups given pricing frictions and shocks. Given sectoral markups, all markets must clear; and the evolution of markups must be consistent with Calvo pricing and the realization of monetary policy.

Definition 4. At each period t, for given sectoral probabilities of price adjustment δ_i, money supply M_t, and transfers T_{ht}, the general equilibrium is given by a vector of firm-level markups $\mathcal{M}_{ift}$, a vector of sectoral prices P_{it}, a vector of factor-specific nominal wages W_{ht}, a vector of factor supplies L_{ht}, a vector of sectoral outputs Y_{it}, a matrix of intermediate input quantities X_{ijt}, and a matrix of final demands C_{iht} such that: (i) a fraction δ_i of firms in each sector i charges the profit-maximizing price given by (8); (ii) the markup charged by adjusting firms is given by the ratio of the profit-maximizing price and marginal costs, while the markups of non-adjusting firms are such that their price remains constant; (iii) agents maximize utility subject to their budget constraint; (iv) producers in each sector i minimize costs and charge the relevant markup; and (v) markets for all goods and factors clear.

Remark 4. This equilibrium concept nests the standard one with flexible prices, which is obtained as a special case when $\delta_i = 1$ for every sector i.

2.5 Log-linearized model

The analysis is based on a log-linear approximation of the model around the initial efficient steady state. Starred variables denote the steady-state, and vectors are in bold. We assume that in steady-state sectoral productivities are equal to one ($\mathbf{A}^* = \mathbf{1}$), supply shifters are $\mathcal{E}_h^* = 1$ for all factors h, there is no government spending ($G^* = 0$), there are no transfers ($T_h^* = 0 \; \forall h$), and nominal GDP is normalized to one ($M^* = \sum_h P_h^* C_h^* + \sum_f P_f^* I_f^* = 1$).

Shocks We allow for a large set of shocks, described in Table 1.

Nominal money supply	$m_t \equiv \log M_t$
Industry-specific productivity	$\log \mathbf{A}_t \in \mathbb{R}^{\mathbb{N}}$
Factor supply	$\boldsymbol{\epsilon}_t \in \mathbb{R}^{N_w+N_f},\ \epsilon_{ht} \equiv \log \mathcal{E}_{ht}$
Desired markups	$\hat{\boldsymbol{\mu}}_t^D \in \mathbb{R}^N$
Input subsidies	$\boldsymbol{\tau}_t \in \mathbb{R}^N$
Relative demand	$\mathcal{X} \in \mathbb{R}^{\bar{N}\times\bar{N}},\ \mathcal{X}_{ij} = d\log\omega_{ij}\ \vert_{\text{const. prices}}$
Income transfers	$\mathbf{T} \in \mathbb{R}^{N_w+N_f},\ \mathbf{T} = \left(T_1, ..., T_{N_w}, \mathbf{0}_{N_f}^T\right)^T$

Table 1: Shocks

The matrix $\mathcal{X}$ denotes exogenous relative demand shocks – that is, log-changes in expenditure shares at constant prices. Given our modeling of retailers as additional production sectors, $\mathcal{X}$ also includes shocks to final expenditure shares (i.e. changes in the sectoral composition of consumption, investment, and government spending).

We do not include total government spending G in the set of shocks, because it can be expressed as a function of the transfers $\mathbf{T}$ to consumers using the government budget constraint. We assume that investment suppliers receive no lump-sum transfers.

Disaggregated variables Prior to introducing our disaggregated variables, let's begin by defining the natural equilibrium.

Definition 5. The natural equilibrium is the equilibrium that would prevail in an economy with flexible prices, subject to the same productivity shocks, labor supply shocks, relative demand shocks, production subsidies, and lump-sum transfers as the sticky-price economy.

Note that, with this definition, transfers across final users – including the transfer G to the government which finances spending – are accounted for in the natural equilibrium.

Remark 5. Real GDP in the natural equilibrium is the benchmark against which we measure aggregate demand. Therefore the component of inflation that we attribute to aggregate demand depends on which shocks are accounted for in the natural equilibrium. Our definition is motivated by what we can identify in the data. We discuss this in detail in Section 5 below.

Table 2 introduces the variables that describe the dynamics of the economy. We use lower case letters to denote log-deviations from the initial steady state, and tildes to denote log-deviations with respect to the natural equilibrium.

Employment gaps	$\tilde{\boldsymbol{\ell}}_t = \begin{pmatrix} \tilde{\ell}_{1t} & \dots & \tilde{\ell}_{N_w+N_f,t} \end{pmatrix}^T$
Factor price gaps	$\tilde{\mathbf{w}}_t = \begin{pmatrix} \tilde{w}_{1t} & \dots & \tilde{w}_{N_w+N_f,t} \end{pmatrix}^T$
Inflation rates	$\boldsymbol{\pi}_t = \begin{pmatrix} \pi_{1t} & \dots & \pi_{Nt} \end{pmatrix}^T$, $\pi_{it} \equiv p_{it} - p_{i,t-1}$

Table 2: Model variables

Remark 6. Table 2 defines employment and utilization gaps for primary factors, rather than for sectors. Knowing factor-level gaps is sufficient to characterize the evolution of inflation in all sectors.

Parameters Figure 2 provides a visual illustration of the steady-state parameters that pin down the log-linearized model. Tables 3, 4 and 5 provide formal definitions.

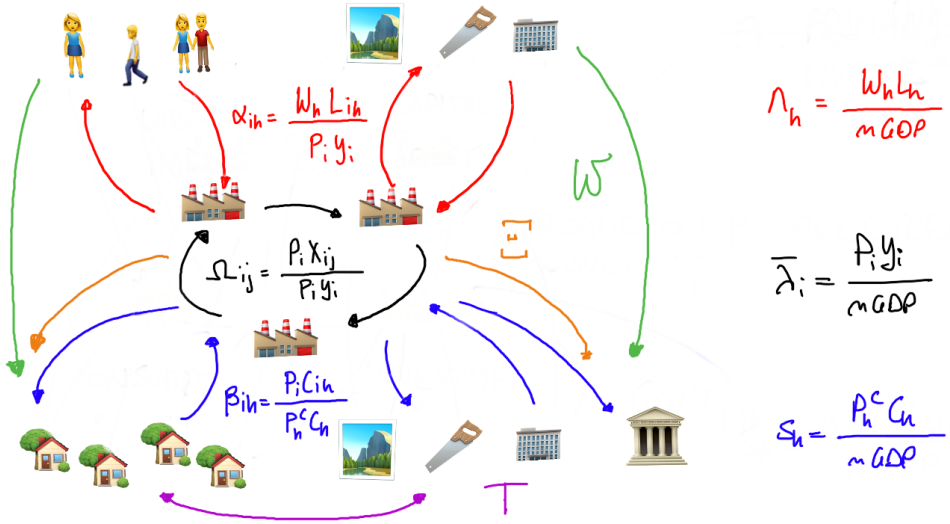


Figure 2: Log-linearized model

There are three sets of parameters: the expenditure shares of producers on primary factors (denoted by α), of producers on other producers (denoted by Ω), and of final users on producers (denoted by β); the shares of income paid by primary factors and producers to final users (denoted by $\mathcal{W}$ and Ξ); and demand and supply elasticities, plus sectoral price adjustment probabilities (defined in Table 6).

Expenditure shares and income flows correspond to the weights of the edges in figure 2. The weight of the nodes are given by the income shares of primary factors, producers, and final users in total GDP, denoted by $\boldsymbol{\Lambda}$, $\bar{\boldsymbol{\lambda}}$ and $\mathbf{s}$ respectively. We first define all of these parameters, and then show in Lemma 1 how to compute the node weights as a function of the edge weights.

Table 3 starts by introducing our notation for income flows.

Factor ownership	$\mathcal{W} \in \mathbb{R}^{N_w+N_f \times N_w+N_f}$
Sector ownership	$\Xi \in \mathbb{R}^{\bar{N},N_w+N_f}$

Table 3: Input-output definitions

The matrix $\mathcal{W} = (\mathcal{W}_{hf})$ tells us which fraction of each factor h's income accrues to each factor f. We assume that workers own their labor income. Workers also own shares of the capital assets, as described by the matrix $\mathcal{Z} \in \mathbb{R}^{N_f \times N_w}$ introduced in Section 2.1.1. As explained in Section 2.1.2, the income from each capital asset f is divided between investment expenditures (a fraction $\frac{1}{1+\varphi_f}$), and the profits of investment producers (the remaining fraction $\frac{\varphi_f}{1+\varphi_f}$), which in turn are rebated to consumers according to $\mathcal{Z}$. Denoting by $\Phi_F \equiv diag(\{\varphi_f\}_{f\in\mathcal{N}_f})$, the matrix $\mathcal{W}$ takes the following form:

$$\mathcal{W} \equiv \begin{pmatrix} I & \mathcal{Z}^T\Phi_F\left(I+\Phi_F\right)^{-1} \\ \mathbb{O} & \left(I+\Phi_F\right)^{-1} \end{pmatrix}$$

The matrix Ξ is defined in Section 2.1.1. Recall that final users own the corresponding retailers, while we assumed that the ownership shares of marketplaces are equal to the ownership shares of factor income ($\Xi_{hf} = \mathcal{W}_{hf}$ for $h, f \in \mathcal{N}_w \cup \mathcal{N}_f$). We assume that investment suppliers do not own shares in the production sectors ($\Xi_{fi} = 0$ for $i = 1, ..., N,\ f \in \mathcal{N}_f$).

Next, Table 4 defines expenditure shares for producers and final users. Note that, in our multi-factor environment, final expenditure shares β and primary factor shares α are $\bar{N} \times N_w + N_f + 1$ and $\bar{N} \times N_w + N_f$ matrices respectively.

Final use shares	$\beta \in \mathbb{R}^{N\times N_w+N_f+1},\ \beta_{ih} = \frac{P_i C_{ih}}{P_h^C C_h}$
Primary factor shares	$\alpha \in \mathbb{R}^{N\times N_w+N_f},\ \alpha_{ih} = \frac{W_h L_{ih}}{MC_i Y_i}$
Input-output matrix	$\Omega \in \mathbb{R}^N \times \mathbb{R}^N,\ \Omega_{in} = \frac{P_n X_{in}}{MC_i Y_i}$

Table 4: Input-output definitions

Table 5 defines some useful transformations of the parameters in Table 4: the Leontief inverse, the network-adjusted expenditure shares λ, the Domar weights $\bar{\boldsymbol{\lambda}}$, the factor income shares $\boldsymbol{\Lambda}$, and the income shares of final users $\mathbf{s}$.

Leontief inverse	$(I-\Omega)^{-1}$
Network-adjusted expenditure shares	$\lambda \in \mathbb{R}^{N\times(N_w+N_f+1)}$, $\lambda^T = \beta^T (I-\Omega)^{-1}$
Domar weights	$\bar{\boldsymbol{\lambda}} \in \mathbb{R}^N$, $\bar{\lambda}_i = \frac{P_i Y_i}{GDP}$
Factor income shares	$\boldsymbol{\Lambda} \in \mathbb{R}^{N_w+N_f}$, $\Lambda_h = \frac{W_h L_h}{GDP}$, $\mathcal{L} \equiv diag(\Lambda)$
Final use shares	$\mathbf{s} \in \mathbb{R}^{N_w+N_f+1}$, $s_h = \frac{P_h^C C_h}{GDP}$, $S \equiv diag(\mathbf{s})$

Table 5: Input-output definitions

While the input-output matrix captures the *direct* elasticity of each sector i's cost to every other sector j's price, the Leontief inverse captures the *total* exposure, directly and indirectly through intermediate inputs. To see this, one can expand the Leontief inverse as a geometric sum, where the n-th term is the exposure of i to j through paths of length n:

$$(I-\Omega)^{-1}_{ij} = I_{ij} + \Omega_{ij} + \Omega^2_{ij} + ...$$

In a similar way, while the matrix β captures direct final use shares, the matrix λ instead captures the direct and indirect expenditure of final users (consumers, investment producers, and the government) on each good.

Remark 7. In every sector, the expenditure shares on inputs and primary factors must sum to one: $(\Omega+\alpha)\mathbf{1} = \mathbf{1}$. This implies the following relations in steady-state:

$$(I-\Omega)^{-1}\alpha\mathbf{1} = \mathbf{1},\ \lambda^T\alpha\mathbf{1} = 1$$

To understand Remark 7 , note that $\left[(I-\Omega)^{-1}\alpha\right]_{ih}$ is the value added from primary factor h in sector i's cost, directly and through intermediate inputs. Similarly, $\left(\alpha^T\lambda\right)_{hf}$ is the share of value added from factor h embedded in the consumption of final user f. Remark 7 states that, in every sector, costs are equal to the value added from primary factors ($(I-\Omega)^{-1}\alpha\mathbf{1} = \mathbf{1}$). Hence, when producers price at marginal cost, primary factors also account for the entire value added in final consumption ($\lambda^T\alpha\mathbf{1} = \mathbf{1}$).

Assumption 1. *To ensure that all relative prices are well defined, we assume*

$$rank\left(I - \mathcal{W}\alpha^T\lambda\right) = N_w + N_f$$

Assumption 1 is equivalent to imposing that, for every pair (h,f) of a primary factor h and a final user f, there is a connected path $(h, a_1, ..., a_K, f)$ between h and f, where a_k is connected to a_{k-1} if there is an income flow from a_k to a_{k-1}.

Lemma 1 below shows how to derive steady-state income shares of final users $\mathbf{s}$, factor income shares $\boldsymbol{\Lambda}$, and Domar weights $\bar{\boldsymbol{\lambda}}$.

Lemma 1. *Under Assumption 1 , the vectors* $\mathbf{\Lambda}$, $\bar{\boldsymbol{\lambda}}$ *and* $\mathbf{s}$ *are the unique solution of the system*

$$\mathbf{s} = \mathcal{W}\mathbf{\Lambda}$$
$$\mathbf{\Lambda} = \alpha^T \bar{\boldsymbol{\lambda}}$$
$$\bar{\boldsymbol{\lambda}} = \lambda \mathbf{s}$$

such that $\mathbf{\Lambda 1} = \mathbf{s1} = 1$.

Finally, Table 6 summarizes the notation for price adjustment probabilities and for demand and supply elasticities. As derived in the Appendix, sectoral inflation rates depend on an increasing and convex function of the Calvo parameters δ_i, given by

$$\hat{\delta}_i(\rho, \delta_i) \equiv \frac{\delta_i(1 - \rho(1 - \delta_i))}{1 - \rho\delta_i(1 - \delta_i)}$$

Wealth effects	$\Gamma \in \mathbb{R}^{(N_w+N_f)\times(N_w+N_f)},\ \Gamma \equiv diag(\gamma_1, ..., \gamma_{N_w}, 0, ..., 0)$
Factor supply elasticities	$\Phi \in \mathbb{R}^{N_w+N_f},\ \Phi \equiv \Gamma\mathcal{W}\mathcal{L} + diag(\varphi_1, ..., \varphi_{N_w+N_h})$
Demand elasticities	$\theta^i_{jk} \equiv \frac{d\log \frac{X_{ij}}{X_{ik}}}{d\log \frac{P_j}{P_k}}$
Price adjustment probabilities	$\Delta \equiv diag\left(\hat{\delta}_1, ..., \hat{\delta}_N\right)$

Table 6: Input-output definitions

Aggregate variables Following definition 3, log-changes in the GDP deflator π^Y are a weighted average of sectoral inflation rates, with weights given by aggregate final expenditure shares $\bar{\boldsymbol{\beta}}$:

$$\pi^Y = \sum_{i=1}^{\bar{N}} \bar{\beta}_i \pi_i$$

In turn, aggregate final expenditure shares are a weighted average of the expenditure shares of the various final users, with weights given by income shares:

$$\bar{\beta}_i \equiv \sum_{h \in \mathcal{N}_w \cup \mathcal{N}_f} s_h \beta_{ih}$$

Definition 6. The aggregate output gap $\bar{y}$ is defined as the log difference between real GDP in the sticky-price versus the flex-price economy.

Following Definition 2 and Remark 3 , the aggregate output gap is the same as the aggregate employment gap (including employment of capital assets). Moreover, the aggregate employment gap is given by a weighted average

of factor-level employment gaps, with weights equal to factor income shares:

$$\bar{y} = \sum_{h=1}^{N_w+N_f} \Lambda_h \tilde{\ell}_h$$

Notation Following Remark 2, we include factor marketplaces and retailers among the production sectors. With this modeling choice, the matrices α, β and Ω take the specific form illustrated below. We ordered the good-producing sectors first, then the factor marketplaces, and lastly the final consumption and investment retailers.

$$\Omega = \begin{bmatrix} \Omega_P & \Omega_M & \mathbb{O}_{N,N_w+N_f} \\ \mathbb{O}_{N,N_w+N_f} & \mathbb{O}_{N_w+N_f} & \mathbb{O}_{N_w+N_f} \\ \Omega_C & \mathbb{O}_{N_w+N_f} & \mathbb{O}_{N_w+N_f} \end{bmatrix}, \ \alpha = \begin{bmatrix} \mathbb{O}_{N,N_w+N_f} \\ I \\ \mathbb{O}_{N_w+N_f} \end{bmatrix}, \ \beta = \begin{bmatrix} \mathbb{O}_{N,N_w+N_f} \\ \mathbb{O}_{N_w+N_f} \\ I_{N_w+N_f} \end{bmatrix}$$

Only the marketplaces hire primary factors directly, therefore they are the only industries with non-zero labor shares. Analogously, consumption good retailers and investment producers are the only sectors with non-zero final use shares β.

We denoted the expenditure shares of production sectors on other production sectors by $\Omega_P \in \mathbb{R}^{N\times N}$, the expenditure shares of production sectors on factor marketplaces by $\Omega_M \in \mathbb{R}^{N\times N_w+N_f}$, and the expenditure shares of retailers on production sectors by $\Omega_C \in \mathbb{R}^{N_w+N_f\times N}$. The data counterpart of Ω_M are the direct expenditure shares of producers on primary factors. A model with flexible factor prices is equivalent to a model with no labor unions and direct factor shares $\alpha_{ih} = \Omega_{i,\text{marketplace}_h}\ \forall i,h$. Similarly, final expenditure shares are the data counterpart of Ω_C. A model where retailers have flexible prices is equivalent to a model with no retailers and consumption shares $\beta_{ih} = \Omega_{\text{retailer}_h,i}\ \forall i,h$.

2.6 Log-linearized equilibrium conditions

We now derive log-linearized equilibrium conditions in gaps with respect to the natural allocation. The equilibrium of the economy is characterized by three equations, describing the supply of primary factors, the relative demand for primary factors, and the aggregate demand for primary factors. A fourth equation describes the evolution of prices. We start by introducing and commenting these four equations. We then combine the pricing equation with the other three, to express the equilibrium in terms of the Phillips curve (which takes the place of the supply equation), a modified relative demand equation, and the aggregate demand equation.

Prior to introducing the equations, we define two additional variables. The price wedges $\boldsymbol{\chi}_t$ introduced in Definition 7 summarize the effect of all the shocks that are accounted for in the natural equilibrium.

Definition 7. We denote by $\boldsymbol{\chi}_t$ the sectoral wedges between lagged prices $\mathbf{p}_{t-1}$ and prices in the natural equilibrium $\mathbf{p}_t^{nat}$:

$$\boldsymbol{\chi}_t = \mathbf{p}_{t-1} - \mathbf{p}_t^{nat}$$

As derived in the Appendix, natural prices are a function of all the shocks that are accounted for in the natural equilibrium. Importantly, these shocks will enter the equilibrium system only through $\boldsymbol{\chi}$. In particular, for a sector i will have $\chi_i < 0$ when its productivity declines, or its relative demand increases.

The variable $\boldsymbol{\xi}_t$, introduced in Definition 8, captures endogenous and exogenous changes in desired markups.

Definition 8. We denote by $\boldsymbol{\xi}_t$ the following function of desired markups and inflation expectations:

$$\boldsymbol{\xi}_t \equiv \rho \mathbb{E}\boldsymbol{\pi}_{t+1} + \Delta \left(I - \Delta\right)^{-1} \hat{\boldsymbol{\mu}}_t^D$$

From the pricing equation (8), inflation expectations are akin to changes in desired markups. In fact, for given costs producers preemptively charge a higher price today – and therefore a higher markup – if they expect high inflation in the future, in the event that they cannot update their price again. As we will see, all changes in desired markups enter the equilibrium equations in the same way, regardless of whether they are endogenous (due to inflation expectations $\mathbb{E}\boldsymbol{\pi}_{t+1}$) or exogenous (due to the shocks $\hat{\boldsymbol{\mu}}_t^D$).

Remark 8. Lagged prices ($\mathbf{p}_{t-1}$) are the only state variable in the model, and they enter the equilibrium system through the price wedges $\boldsymbol{\chi}_t$. Expected future inflation is the only forward looking variable, and it enters the equilibrium system through $\boldsymbol{\xi}_t$.

Supply equation The supply equation stacks together the log-linearized consumption-leisure tradeoff (3) and the optimal capital utilization (4):

$$\Phi \tilde{\boldsymbol{\ell}}_t = \tilde{\mathbf{w}}_t - \underline{\beta}^T \tilde{\mathbf{p}}_t \tag{10}$$

where

$$\underline{\beta}^T \equiv \beta^T - \Gamma \left(\beta^T - S^{-1} \Xi^T diag\left(\bar{\lambda}\right) \left(I - \Omega\right)\right)$$

Equation (10) tells us that factor supply is increasing in real wages, with elasticity Φ. Note that Φ is not diagonal, capturing wealth effects in labor supply from capital income. The relevant price index for real wages is weighted according to $\underline{\beta}^T$, rather than by expenditure shares β^T, to account for the discrepancy between wealth effects induced by final prices (β^T) and profit income ($S^{-1}\Xi^T diag\left(\bar{\lambda}\right)\left(I - \Omega\right)$). In a representative agent economy the two would cancel out. With heterogeneous agents, instead, the firms that each agent owns might be different from those that they buy from. Moreover some agents might not own firm shares, while they still face wealth effects from consumption prices.

Relative demand equation The relative demand equation expresses factor-level employment gaps as a function of the aggregate output gap $\bar{y}$, and sectoral price gaps:

$$\tilde{\boldsymbol{\ell}} = \mathbf{1}\bar{y} + \mathcal{D}_p \tilde{\mathbf{p}} \tag{11}$$

Factor-level gaps increase proportionately to the aggregate output gap. On top of this, relative factor demand is affected by changes in relative prices, as described by the matrix $\mathcal{D}_p$ (derived in the Appendix). Relative prices affect relative factor demand through substitution and through a reallocation of real income across final users with different consumption baskets. Note that, given our notation, changes in the sticky relative wages are included in the price vector $\tilde{\mathbf{p}}$, while changes in the underlying flex wages do not matter for relative demand.

Lemma 2 states two key properties of the matrix $\mathcal{D}_p$. First, this matrix captures changes in relative demand, which therefore cancel out in the share-weighted average. Second, proportional changes in the price of all sectors have no effect on relative factor demand.

Lemma 2. *The matrix $\mathcal{D}_p$ is such that $\mathbf{\Lambda}^T \mathcal{D}_p = \mathbf{0}^T$ and $\mathcal{D}_p \mathbf{1} = \mathbf{0}$.*

Aggregate demand equation The aggregate demand equation follows immediately from the cash-in-advance constraint. It tells us that changes in final expenditure $(\pi_t^Y + \bar{y}_t)$ must be equal to changes in money balances $(m_t - p_{t-1}^Y)$:

$$\pi_t^Y + \bar{y}_t = m_t - p_{t-1}^Y \tag{12}$$

Pricing equation The pricing equation relates good prices with factor prices $\tilde{\mathbf{w}}$, price wedges $\boldsymbol{\chi}$, and desired markups $\boldsymbol{\xi}$:

$$\tilde{\mathbf{p}}_t = \mathcal{P}\alpha\tilde{\mathbf{w}}_t + [I - \mathcal{P}(I - \Omega)](\boldsymbol{\chi}_t + \boldsymbol{\xi}_t) \tag{13}$$

In equation (13), we denoted by

$$\mathcal{P} \equiv \Delta (I - \Omega\Delta)^{-1}$$

the $\bar{N} \times \bar{N}$ matrix whose (i, j)-th element describes the propagation of a cost shock to sector j into the price of sector i.

In the definition of $\mathcal{P}$, the price adjustment probability Δ governs the response of prices to changes in marginal costs. The matrix $(I - \Omega\Delta)^{-1}$ is similar to the Leontief inverse, although it discounts supplier sectors by their price adjustment probability. Its (i, j)-th element corresponds to the propagation of a cost shock in sector j into the marginal cost of sector i, either directly or through intermediate suppliers, suppliers' suppliers, etc.

Changes in factor prices affect marginal costs proportionately to the direct factor shares α, thus their total pass-through into good prices (accounting for propagation through input-output linkages) is given by $\mathcal{P}\alpha$. Price wedges instead imply a change in costs equal to $(I - \Omega)\boldsymbol{\chi}$. If prices were fully rigid ($\mathcal{P} = \mathbb{O}$), price wedges $\boldsymbol{\chi}_t$ would translate one-for-one into price gaps $\tilde{\mathbf{p}}_t$. When prices are not fully rigid, instead, the gaps $\tilde{\mathbf{p}}_t$ are reduced by the price adjustment $\mathcal{P}(I - \Omega)\boldsymbol{\chi}_t$.

Equilibrium system Using the supply equation to make employment gaps appear in the pricing equation, and then substituting for price gaps in the relative demand equation, we obtain the following equilibrium system:[3]

$$\begin{cases} \boldsymbol{\pi}_t - \boldsymbol{\xi}_t = \kappa\Phi\tilde{\boldsymbol{\ell}}_t - \mathcal{V}\left(\boldsymbol{\chi}_t + \boldsymbol{\xi}_t\right) & \text{Phillips curves} \\ \tilde{\boldsymbol{\ell}}_t = \left(I - \hat{\mathcal{D}}_\ell\right)^{-1} \mathbf{1}\bar{y}_t + \hat{\mathcal{D}}_p\left(\boldsymbol{\chi}_t + \boldsymbol{\xi}_t\right) & \text{relative demand} \\ \pi_t^Y + \bar{y}_t = m_t - p_{t-1}^Y & \text{aggregate demand} \end{cases} \tag{14}$$

The Appendix provides explicit expressions for κ, $\hat{\mathcal{D}}_\ell$, and $\hat{\mathcal{D}}_p$ as a function of primitives. The matrix $\mathcal{V}$ is related with endogenous cost-push shocks, and we derive it in Section 4.

The first equation displays sector-by-factor Phillips curves, which relate the vector of sectoral inflation rates $\boldsymbol{\pi}_t$ with the vector of employment and utilization gaps $\tilde{\boldsymbol{\ell}}_t$, the vector of price gaps $\boldsymbol{\chi}_t$, and the vector of desired markups $\boldsymbol{\xi}_t$. The slope κ is a matrix whose elements κ_{ih} capture the response of inflation in sector i to the real wage of factor h. The inverse supply elasticity Φ maps employment into real wages. The term $-\mathcal{V}\left(\boldsymbol{\chi}_t + \boldsymbol{\xi}_t\right)$ instead describes the response of sectoral inflation to cross-sectional shocks, as explained in detail in Section 4.

The relative demand equation in (14) is obtained by combining sectoral Phillips curves with equation (11), and it relates factor-level gaps with the aggregate gap, price wedges, and desired markups.

Remark 9. Once we account for the response of relative prices through the Phillips curve, changes in the aggregate output gap have an effect on relative factor demand ($\left(I - \hat{\mathcal{D}}_\ell\right)^{-1} \mathbf{1} \neq \mathbf{1}$). Moreover we can think of the output gap as a measure of aggregate demand, which is controlled by monetary policy. The coefficient $\left(I - \hat{\mathcal{D}}_\ell\right)^{-1} \mathbf{1}$ then corresponds to the direction along which monetary policy is constrained to move relative factor demand.

With homothetic preferences and production, shocks to the output gap still have a proportional impulse effect on the demand for all factors, as captured by the vector $\mathbf{1}$. This uniform impulse effect however propagates asymmetrically into prices and incomes. Specifically, relative prices decline for sectors that use elastic factors or where nominal rigidities are less severe. This feeds back into demand – as captured by the matrix $\hat{\mathcal{D}}_\ell$ – which then feeds back again into prices, and so on. The fixed point is given by the multiplier $\left(I - \hat{\mathcal{D}}_\ell\right)^{-1}$.[4]

Like in the old relative demand equation (11), the matrices $\hat{\mathcal{D}}_\ell$ and $\hat{\mathcal{D}}_p$ describe changes in relative demand which cancel out in the aggregate. Moreover, proportional changes in price gaps and desired markups across all sectors have no effect on relative factor demand. This is stated in Lemma 3, which is the counterpart of Lemma 2.

Lemma 3. *It holds that* $\boldsymbol{\Lambda}^T\hat{\mathcal{D}}_\ell = \boldsymbol{\Lambda}^T\hat{\mathcal{D}}_p = \mathbf{0}^T$ *and* $\hat{\mathcal{D}}_p\mathbf{1} = \mathbf{0}$.

Proof. See Appendix. □

[3] See the Appendix for the full derivation.
[4] See Rubbo (2023) for an in-depth discussion.

3 A simple model of supply bottlenecks

We model supply bottlenecks as declines in the relative productivity or increases in the relative demand for inelastically supplied goods. In our model, the supply of goods can be inelastic because it is constrained by semi-fixed factors (such as machines or land), or simply because of decreasing returns. Note that our model can accommodate decreasing returns to scale by adding industry-specific capital assets f with high inverse supply elasticity ($\varphi_f \to \infty$).

Inelastic sectors generate bottlenecks because they cannot compensate for higher demand or lower productivity by utilizing their primary factors more intensively. In this section we formalize this intuition, using a simplified model with two industries and two primary factors. Specifically, we show that there are two ways in which shocks to inelastic sectors create bottlenecks. First, as we will illustrate in this section, increases in the relative demand, declines in the relative productivity (paired with complementarities) or declines in the relative factor supply for inelastic sectors cause the natural output to fall. Second, as we will illustrate in the next sections, with nominal rigidities these shocks create an inflation-output tradeoff.

Specifically, we will see that with nominal rigidities the price of inelastic sectors does not increase enough in response to increases in relative demand, declines in relative productivity, or declines in relative factor supply. This in turn raises the relative demand for inelastic factors compared to the flex-price economy. If monetary policy closes the output gap (i.e. it sets real GDP in the sticky-price economy equal to the natural one), demand wedges cancel out on average. However the price wedges that they induce do not. This is because the price of inelastic factors increases by more than the price of elastic factors falls, thereby raising average factor prices and final consumption prices. This creates an inflation-output tradeoff, that is, inflation increases even if monetary policy closes the output gap. Examples 2 and 4 build on the simplified environment introduced here to illustrate this mechanism.

Consider an economy with two sectors (sector 1 and sector 2) and two primary factors (labor and capital). Primary factors have different supply elasticities $\varphi_L < \varphi_K$. Sector 1 uses only labor in production, while sector 2 uses only capital, with constant returns to scale. Consumers and investment producers have equal expenditure shares on the two goods, with substitution elasticity θ.

Sectors face productivity shocks $\log A_1$, $\log A_2$, and relative demand shocks $d\log b_1 = -d\log b_2$ (where we denote by $d\log b_1$ the change in the expenditure share on good 1 at constant prices). Primary factors face supply shocks ϵ_L and ϵ_K. All good and factor prices are flexible.

We now derive equilibrium equations (supply, demand, and pricing) in this simplified model, expressing them in log-changes with respect to the steady-state. Note that these equations are different from (10)-(13) and (14) above, which are expressed in gaps with respect to the flex-price economy considered here. Equations in levels for the general model are reported in the appendix.

The supply equation is:

$$\begin{pmatrix} \ell_L \\ \ell_K \end{pmatrix} = \begin{pmatrix} \frac{1}{\varphi_L} & 0 \\ 0 & \frac{1}{\varphi_K} \end{pmatrix} \left[\begin{pmatrix} w_L \\ w_K \end{pmatrix} - \mathbf{1}\bar{p} + \begin{pmatrix} \epsilon_L \\ \epsilon_K \end{pmatrix} \right]$$

where, for any given vector $\mathbf{x}$, we denoted by $\bar{x}$ the equally weighted average of its components: $\bar{x} \equiv \frac{1}{2}(x_1 + x_2)$.

The demand equation is:

$$l_L - l_K = -\theta(p_1 - p_2) - (\log A_1 - \log A_2) + (d\log b_1 - d\log b_2)$$

The pricing equation is:

$$\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} w_L \\ w_K \end{pmatrix} - \begin{pmatrix} \log A_1 \\ \log A_2 \end{pmatrix}$$

Combining the supply, demand and pricing equations allows us to relate changes in natural aggregate real GDP with changes in productivity, relative demand, and labor supply:

$$y^{nat} = \left[\mathbb{E}_\Lambda\left(\frac{1}{\varphi}\right) - \frac{Var_\Lambda\left(\frac{1}{\varphi}\right)}{\theta + \mathbb{E}_\Lambda\left(\frac{1}{\varphi}\right)}\right]\log \bar{A} - \frac{1-\theta}{\theta + \mathbb{E}_\Lambda\left(\frac{1}{\varphi}\right)} Cov_\Lambda\left(\frac{1}{\varphi}, \log A\right) + \frac{Cov_\Lambda\left(\frac{1}{\varphi}, d\log b\right)}{\theta + \mathbb{E}_\Lambda\left(\frac{1}{\varphi}\right)} + \mathbb{E}_\Lambda\left(\frac{\epsilon}{\varphi}\right) - \frac{Cov_\Lambda\left(\frac{1}{\varphi}, \frac{\epsilon}{\varphi}\right)}{\theta + \mathbb{E}_\Lambda\left(\frac{1}{\varphi}\right)} \tag{15}$$

In equation (15), the natural level of output is lower when relative demand increases for sectors which use inelastic factors ($\frac{Cov_\Lambda\left(\frac{1}{\varphi}, d\log b\right)}{\theta + \mathbb{E}_\Lambda\left(\frac{1}{\varphi}\right)} < 0$) or when the supply of inelastic factors exogenously declines ($Cov_\Lambda\left(\frac{1}{\varphi}, \frac{\epsilon}{\varphi}\right) > 0$). Moreover, the natural level of output is also lower when productivity declines in inelastic sectors and goods are complementary ($Cov_\Lambda\left(\frac{1}{\varphi}, \log A\right)$ and $\theta < 1$), and vice versa when goods are substitutes.

Thus equation (15) formalizes the notion that inelastic sectors create bottlenecks, because they cannot compensate for higher demand or lower productivity by utilizing their primary factors more intensively. On top of this, in the sticky price economy inelastic sectors further create bottlenecks due to incomplete relative price adjustment. Examples 2 and 4 below build on the simple model introduced here to illustrate this in detail. To set the stage for the analysis to come, let's solve for relative price changes in the flex-price economy:

$$p_1^{nat} - p_2^{nat} = -\frac{1 + \mathbb{E}_\Lambda\left(\frac{1}{\varphi}\right)}{\theta + \mathbb{E}_\Lambda\left(\frac{1}{\varphi}\right)}(\log A_1 - \log A_2) + \frac{d\log b_1 - d\log b_2}{\theta + \mathbb{E}_\Lambda\left(\frac{1}{\varphi}\right)} - \frac{1}{\theta}\frac{\frac{\epsilon_L}{\varphi_L} - \frac{\epsilon_K}{\varphi_K}}{\theta + \mathbb{E}_\Lambda\left(\frac{1}{\varphi}\right)} - \frac{1}{\theta}\frac{\left(\frac{1}{\varphi_L} - \frac{1}{K}\right)}{\theta + \mathbb{E}_\Lambda\left(\frac{1}{\varphi}\right)}\log \bar{A} \tag{16}$$

As expected relative prices increase when relative productivity or relative factor supply fall, or when relative demand increases. In the sticky-price economy, instead, relative prices are prevented from fully adjusting, which will result in an inflation-output tradeoff. Also note that natural relative prices respond to aggregate productivity, and not just to relative productivity. This is because higher productivity translates in higher supply for the elastic factor (labor), and higher price for the inelastic one (capital). In turn, these changes in the cost of primary factors are passed-through into good prices.

Remark 10. In our model (both the simplified in this section and the general version) factor supply elasticities are exogenous and constant. In practice, factor supply may become more inelastic as employment increases. If this is the case, and supply elasticities are identical ex-ante, then *any* large cross-sectional (productivity, demand, factor

supply) shock would result in a bottleneck, through a non-linear effect. That is, any large cross-sectional shock would reduce the natural output y^{nat}.

4 Aggregate and cross-sectional drivers of inflation

In Section 4.1 we move from the equilibrium equations (14) to isolate a component of inflation coming from aggregate demand, and a component coming from cross-sectional shocks. As it is natural, cross-sectional shocks affect relative prices. We illustrate this in Section 4.2. However – and this is less obvious – cross-sectional shocks also affect aggregate prices, as we illustrate in Section 4.3.

Cross-sectional shocks have aggregate effects whenever industries are asymmetric in two relevant dimensions. First, the pass-through of cost or markup shocks into aggregate prices is heterogeneous across industries. Second, industries face heterogeneous factor supply elasticities. The implications of heterogeneous cost pass-through are the same as in Rubbo (2020), which studies them in the context of a multi-sector model with only one primary factor. Heterogeneity in supply elasticity is novel to the multi-factor environment, and – as we explained in Section 3 – it is key to model supply chain bottlenecks.

4.1 Inflation decomposition

Combining the relative demand equation with sector-by-factor Phillips curves, we can express sector-level inflation rates as a function of the aggregate output gap, price wedges, and desired markups:

$$\boldsymbol{\pi}_t - \boldsymbol{\xi}_t = \hat{\boldsymbol{\kappa}}\bar{y}_t - \hat{\mathcal{V}}\left(\boldsymbol{\chi}_t + \boldsymbol{\xi}_t\right) \tag{17}$$

where

$$\hat{\boldsymbol{\kappa}} \equiv \kappa\Phi\left(I - \hat{\mathcal{D}}_\ell\right)^{-1}\mathbf{1}$$
$$\hat{\mathcal{V}} \equiv \mathcal{V} - \kappa\Phi\hat{\mathcal{D}}_p$$

The right hand side of Equation (17) distinguishes a component of inflation driven by the aggregate output gap $\bar{y}$, from a component driven by price gaps $\boldsymbol{\chi}$ and desired markups $\boldsymbol{\xi}$. In Section 4.2 we show that the matrix $\hat{\mathcal{V}}$ filters out the aggregate component of $\boldsymbol{\chi}$ and $\boldsymbol{\xi}$, so that inflation only depends on relative price wedges and desired markups.

In equation (17), the aggregate output gap moves sectoral inflation rates along a fixed direction $\hat{\boldsymbol{\kappa}}$ (consistent with Remark 9).[5] Moreover, given that the matrix $\hat{\mathcal{V}}$ filters out aggregate shocks, it cannot span all possible relative

[5] See Rubbo (2023) for an in-depth discussion of what determines $\hat{\boldsymbol{\kappa}}$.

price changes. In particular, in Section 5 we show that $\hat{\boldsymbol{\kappa}}$ does not belong to the image of $\hat{\mathcal{V}}$. That is, aggregate demand and cross-sectional shocks have distinct effects on relative prices. This insight will be key to identifying the component of inflation coming from a non-zero aggregate output gap.

We can associate the component $\hat{\boldsymbol{\kappa}}\bar{y}_t$ most closely with monetary policy. In fact the aggregate output gap is the only variable that monetary policy can influence directly, through the cash-in-advance constraint. Monetary policy also indirectly influences $\boldsymbol{\xi}_t$, because inflation expectations depend on the future path of output gaps.[6] To illustrate how monetary policy determines the aggregate output gap, we can combine the Phillips curves (17) with the cash-in-advance constraint. This gives us a mapping between the aggregate output gap and real money balances, conditional on price gaps and desired markups:

$$\bar{y}_t = \frac{m_t - p_{t-1}^Y + \bar{\beta}^T \hat{\mathcal{V}} \left(\boldsymbol{\chi}_t + \boldsymbol{\xi}_t \right)}{1 + \hat{\kappa}^Y} \tag{18}$$

4.1.1 Comparison with alternative decompositions

4.2 Cross-sectional shocks and sector-level inflation

Lemma 4 below provides an explicit expression for $\hat{\mathcal{V}}$ in terms of the model primitives. The Lemma also shows that – once we control for the effect of shocks to $\boldsymbol{\chi}$ on the natural output (i.e. once we set $\bar{y} = 0$ in equation (17)) and for the one-to-one effect of desired markups $\boldsymbol{\xi}$ on inflation rates (on the left-hand-side of equation (17)) – the inflation response $-\hat{\mathcal{V}} \left(\boldsymbol{\chi} + \boldsymbol{\xi} \right)$ only depends on the cross-sectional component of $\boldsymbol{\chi}$ and $\boldsymbol{\xi}$. Formally, Lemma 4 states that the matrix $\hat{\mathcal{V}}$ has zero-sum rows. This means that $\hat{\mathcal{V}}$ filters out aggregate shocks that hit all sectors proportionately (i.e. shocks parallel to the vector $\mathbf{1}$).

Lemma 4. *The matrix $\hat{\mathcal{V}}$ is given by*

$$\hat{\mathcal{V}} = \mathcal{V} - \kappa \Phi \hat{\mathcal{D}}_p \tag{19}$$

where

$$\mathcal{V} = \mathcal{P} \left(I - \Omega \right) - \kappa \underline{\beta}^T \left[I - \mathcal{P} \left(I - \Omega \right) \right] \tag{20}$$

It holds that $-1 \leq \hat{\mathcal{V}}_{ij} \leq 0$ for $i \neq j$, $0 \leq \hat{\mathcal{V}}_{ii} \leq 1$ and $\sum_j \hat{\mathcal{V}}_{ij} = 0$, and analogously for $\mathcal{V}$.

Proof. See Appendix. □

As we can see from equation (14), the matrix $\mathcal{V}$ describes the response of inflation to $\boldsymbol{\chi}$ and $\boldsymbol{\xi}$ when all the employment gaps are closed. The expression for $\mathcal{V}$ in equation (20) starts with the direct effect of $\boldsymbol{\chi}$ and $\boldsymbol{\xi}$ on sectoral inflation – which is given by $\mathcal{P} \left(I - \Omega \right) \left(\boldsymbol{\chi} + \boldsymbol{\xi} \right)$, from the pricing equation (13) – and it subtracts the price response to the change in natural employment of all the various factors. The latter is given by $\kappa \underline{\beta}^T \left[I - \mathcal{P} \left(I - \Omega \right) \right]$. To understand

[6] If preferences were not homothetic, monetary policy would also endogenously affect the relative demand shock $\mathcal{X}$. Through $\mathcal{X}$, monetary policy would then also impact the price gaps $\boldsymbol{\chi}$.

this expression recall that, to close employment gaps, real wages must change so as to replicate the natural ones, as implied by the supply equation (10). Hence nominal wage gaps must equal weighted-average price gaps, with weights given by $\underline{\beta}^T$. The pricing equation (13) implies that price gaps are equal to $[I - \mathcal{P}(I - \Omega)](\boldsymbol{\chi} + \boldsymbol{\xi})$, so wage gaps must move by $\underline{\beta}^T [I - \mathcal{P}(I - \Omega)](\boldsymbol{\chi} + \boldsymbol{\xi})$. Finally, the Phillips curve slope κ describes the response of prices to wages.

The matrix $\hat{\mathcal{V}}$ instead describes the response of inflation to $\boldsymbol{\chi}$ and $\boldsymbol{\xi}$ when the aggregate output gap is closed, as we can read from equation (17). Note that, in general, closing the aggregate output gap is not enough to close all factor-level gaps. Therefore, on top of the matrix $\mathcal{V}$, we need to add the inflation response to the relative employment wedges opened by $\boldsymbol{\chi}$ and $\boldsymbol{\xi}$. This is given by $\kappa\Phi\hat{\mathcal{D}}_p$. From the relative demand equation (11), relative demand gaps are given by $\hat{\mathcal{D}}_p(\boldsymbol{\chi} + \boldsymbol{\xi})$. The inverse supply elasticity Φ maps factor demand into factor prices, and the slope κ gives us the response of good prices to factor prices. Prices increase for factors h whose relative demand is too high compared to the efficient equilibrium ($\left[\hat{\mathcal{D}}_p(\boldsymbol{\chi} + \boldsymbol{\xi})\right]_h > 0$), and more so when these factors are inelastic ($\Phi_{:,h}$ is large).

4.3 Cross-sectional shocks and aggregate inflation

Having characterized the effect of cross-sectional shocks on sectoral inflation rates in Section 4.2, we now discuss their effect on aggregate inflation. We first derive general results for any aggregate inflation index, and then focus specifically on the GDP deflator. We show that cross-sectional shocks worsen the inflation-output tradeoff when productivity declines, or relative demand increases ($\boldsymbol{\chi} > 0$) in sectors with more flexible prices, or which rely on inelastic factors.

Definition 9. Given a vector of weights $\boldsymbol{\phi} = (\phi_1, ..., \phi_{\bar{N}})^T$, such that $\sum_{i=1}^{\bar{N}} \phi_i = 1$, the aggregate inflation index

$$\pi^{\phi} \equiv \sum_i \phi_i \pi_i$$

is the weighted average of sectoral inflation rates according to $\boldsymbol{\phi}$.

Following Definition 9 one can construct infinitely many inflation indices, each characterized by the weights that it assigns to the various sectors. Corollary 1 below provides a general expression for the response of any aggregate inflation index to cross-sectional shocks. It then specializes it to the GDP deflator.

Corollary 1. *The Phillips curve for the aggregate inflation index $\pi^{\phi} \equiv \sum_i \phi_i \pi_i$ is*

$$\pi^{\phi} - \boldsymbol{\phi}^T \boldsymbol{\xi} = \kappa^{\phi} \bar{y}_t + u_t^{\phi}$$

where

$$\kappa^{\phi} \equiv \boldsymbol{\phi}^T \hat{\boldsymbol{\kappa}}$$

is the slope and

$$u_t^{\phi} = \boldsymbol{\phi}^T \hat{\mathcal{V}} \left(\boldsymbol{\chi}_t + \boldsymbol{\xi}_t \right)$$

is the response of π^{ϕ} to cross-sectional shocks. Denoting by

$$\kappa^Y \equiv \bar{\boldsymbol{\beta}}^T \hat{\boldsymbol{\kappa}}$$

the Phillips curve for the GDP deflator is

$$\pi^Y - \bar{\boldsymbol{\beta}}^T \boldsymbol{\xi} = \kappa^Y \bar{y}_t + u_t^Y \tag{21}$$

When there are no wealth effects in labor supply ($\Gamma = \mathbb{O}$) and all final users consume goods in the same proportions, the effect of cross-sectional shocks on the GDP deflator is given by

$$u_j^Y = \frac{\bar{\boldsymbol{\beta}}^T \mathcal{P} \bar{\boldsymbol{\alpha}}}{1 - \bar{\boldsymbol{\beta}}^T \mathcal{P} \bar{\boldsymbol{\alpha}}} \left[Cov_{\bar{\boldsymbol{\beta}}} \left(\frac{\mathbf{PT}}{\bar{\boldsymbol{\beta}}^T \mathcal{P} \bar{\boldsymbol{\alpha}}}, -\left(\boldsymbol{\chi}_t + \boldsymbol{\xi}_t \right) \right) + Cov_{\frac{\bar{\boldsymbol{\beta}}^T \mathcal{P} \alpha}{\bar{\boldsymbol{\beta}}^T \mathcal{P} \bar{\boldsymbol{\alpha}}}} \left(\boldsymbol{\varphi}, \hat{\mathcal{D}}_p \left(\boldsymbol{\chi}_t + \boldsymbol{\xi}_t \right) \right) + \mathbb{E}_{\frac{\bar{\boldsymbol{\beta}}^T \mathcal{P} \alpha}{\bar{\boldsymbol{\beta}}^T \mathcal{P} \bar{\boldsymbol{\alpha}}}} \left(\boldsymbol{\varphi} \right) Cov_{\boldsymbol{\Lambda}} \left(\bar{\boldsymbol{\beta}}^T \mathcal{P} \alpha, \hat{\mathcal{D}}_p \left(\boldsymbol{\chi}_t + \boldsymbol{\xi}_t \right) \right) \right] \tag{22}$$

where we denoted by $\bar{\boldsymbol{\alpha}} \equiv \alpha \mathbf{1}$ the $\bar{N} \times 1$ vector of total expenditure shares on primary factors, and by

$$PT_i \equiv \frac{\left[\bar{\boldsymbol{\beta}}^T \mathcal{P} \left(I - \Omega \right) \right]_i}{\bar{\beta}_i}$$

the pass-through of a cost shock to sector i into the GDP deflator, relative to the final expenditure share on sector i.

The covariances on the right-hand-side of equation (22) capture the two relevant dimensions of heterogeneity through which cross-sectional shocks affect aggregate inflation: heterogeneous pass-through of cost and markup shocks into prices, and heterogeneous factor supply elasticities. The scaling factor $\frac{\bar{\boldsymbol{\beta}}^T \mathcal{P} \bar{\boldsymbol{\alpha}}}{1 - \bar{\boldsymbol{\beta}}^T \mathcal{P} \bar{\boldsymbol{\alpha}}}$ is simply the network-adjusted counterpart of the traditional Phillips curve slope $\frac{\hat{\delta}}{1 - \hat{\delta}}$. Here, $\bar{\boldsymbol{\beta}}^T \mathcal{P} \bar{\boldsymbol{\alpha}}$ is the pass-through of a proportional 1% increase in all nominal factor prices into the GDP deflator.[7]

The covariance

$$Cov_{\bar{\boldsymbol{\beta}}} \left(\frac{\mathbf{PT}}{\bar{\boldsymbol{\beta}}^T \mathcal{P} \bar{\boldsymbol{\alpha}}}, -\left(\boldsymbol{\chi}_t + \boldsymbol{\xi}_t \right) \right)$$

means that there is a worse inflation-output tradeoff if $\chi_i < 0$ (productivity declines, or relative demand increases), in sectors i with larger cost pass-through PT_i. In turn, these are the sectors with more flexible prices, or whose suppliers have more flexible prices, etc., as captured by the pass-through matrix $\mathcal{P}$. This effect is present in any network model, even with just one primary factor (see Rubbo (2020)).

The second and third terms instead are novel to the multi-factor setup. The covariance

$$Cov_{\frac{\bar{\boldsymbol{\beta}}^T \mathcal{P} \alpha}{\bar{\boldsymbol{\beta}}^T \mathcal{P} \bar{\boldsymbol{\alpha}}}} \left(\boldsymbol{\varphi}, \hat{\mathcal{D}}_p \left(\boldsymbol{\chi}_t + \boldsymbol{\xi}_t \right) \right)$$

[7] Also note that, by Remark 7, $\bar{\boldsymbol{\beta}}^T \mathcal{P} \bar{\boldsymbol{\alpha}} = \bar{\boldsymbol{\beta}}^T \mathcal{P} \left(I - \Omega \right) \mathbf{1}$.

weights each primary factor h according to its wage pass-through in the GDP deflator ($\left[\bar{\boldsymbol{\beta}}^T \mathcal{P} \alpha\right]_h$) relative to the average pass-through ($\bar{\boldsymbol{\beta}}^T \mathcal{P} \bar{\boldsymbol{\alpha}}$). This covariance tells us that there is a worse inflation-output tradeoff when the relative demand for inelastic factors is higher than in the natural equilibrium ($\left[\hat{\mathcal{D}}_p \left(\boldsymbol{\chi}_t + \boldsymbol{\xi}_t\right)\right]_h > 0$ for factors h with larger φ_h). Intuitively this is the case when productivity declines, or relative demand increases, for the sectors which use these factors,[8] because their prices are sticky and do not increase fast enough in response to the shock. Importantly, for zero aggregate gap ($\bar{y} = 0$) relative factor demand wedges cancel out in the aggregate ($\boldsymbol{\Lambda}^T \hat{\mathcal{D}}_p = \mathbf{0}^T$). However relative factor price gaps do not cancel out ($\boldsymbol{\Lambda}^T \Phi \hat{\mathcal{D}}_p = \mathbf{0}^T$), because the prices of inelastic factors respond more. This is why relative price gaps, through their effect on factor demand wedges, can result in an aggregate inflation-output tradeoff.

Finally, the covariance

$$Cov_{\boldsymbol{\Lambda}} \left(\bar{\boldsymbol{\beta}}^T \mathcal{P} \alpha, \hat{\mathcal{D}}_p \left(\boldsymbol{\chi}_t + \boldsymbol{\xi}_t\right) \right)$$

tells us that there is a worse inflation-output tradeoff also when demand is inefficiently high for factors with higher wage pass-through into the GDP deflator. Again, this is the case when productivity declines, or relative demand increases, in the sectors which use these factors.

Remark 11. As we already pointed out in Remark 10, factor supply elasticities are exogenous and fixed in our model. Realistically, supply elasticities are likely increasing in employment. If this is the case, and supply elasticities are identical ex-ante, *any* large cross-sectional shock will cause bottlenecks through a non-linear effect. Specifically, any large cross-sectional shock will worsen the inflation-output tradeoff.

In a simple economy with only two goods, Examples 1 and 2 illustrate the response of aggregate inflation to asymmetric productivity shocks, combined with heterogeneous cost pass-through or heterogeneous factor supply elasticities.

Example 1. Heterogeneous cost pass-through

Consider a horizontal economy with two sectors (sector 1 and sector 2) and one primary factor (labor). Sectors face different productivity shocks $d \log A_1$ and $d \log A_2$, and have different price adjustment probabilities δ_1 and δ_2. Consumers have equal expenditure shares on the two goods.

The Phillips curve for the GDP deflator is

$$\pi^Y = \kappa^Y \bar{y} - \frac{Cov_\beta \left(\delta, d \log A\right)}{1 - \bar{\delta}}$$

while relative prices evolve according to

$$\pi_1 - \pi_2 = \frac{\delta_1 - \delta_2}{\bar{\delta}} \kappa^Y \bar{y} - \left[\bar{\delta} + \frac{1}{4} \frac{\left(\delta_1 - \delta_2\right)^2}{1 - \bar{\delta}} \right] \left(\log A_1 - \log A_2 \right)$$

[8] The exact propagation of $\boldsymbol{\chi}_t$ and $\boldsymbol{\xi}_t$ into relative demand is described by the matrix $\hat{\mathcal{D}}_p$. Appendix **??** derives this matrix as a function of primitives.

In the symmetric economy where all sectors have the same price adjustment probability (so that $\delta_1 - \delta_2 = 0$ and $Cov_\beta(\delta, d\log A) = 0$), the GDP deflator only depends on the aggregate output gap, while relative prices only depend on cross-sectional productivity shocks. In the asymmetric economy with heterogeneous price stickiness, instead, the GDP deflator depends on both the aggregate output gap and cross-sectional productivity. Specifically, there is a "bad" cost-push shock whenever the relative productivity of the flex-price sector declines. This is because the flex-price sector has higher cost pass-through into the GDP deflator, while labor costs decline by less than productivity in the flexible sector, and by more than productivity in the sticky sector.

Likewise, in the asymmetric economy relative prices depend on the aggregate output gap, because wages are increasing in the output gap, and the more flexible sector responds more strongly to changes in wages.

Example 2. Heterogeneous supply elasticity

Consider the simplified environment introduced in Section 3.

The Phillips curve for the GDP deflator is

$$\pi^Y = \kappa^Y \bar{y} - \frac{\theta\delta\bar{\varphi}}{1+\theta\delta\bar{\varphi}} \frac{Cov_\beta(\varphi, d\log A)}{\bar{\varphi}}$$

while relative prices evolve according to

$$\pi_1 - \pi_2 = \frac{\delta}{1-\delta} \frac{\varphi_L - \varphi_K}{1+\theta\delta\bar{\varphi}} \bar{y} - \delta \frac{1+\theta\bar{\varphi}}{1+\theta\delta\bar{\varphi}} (\log A_1 - \log A_2)$$

Again, in the symmetric economy where all factors have the same supply elasticity (so that $\varphi_L - \varphi_K = 0$ and $Cov_\beta(\varphi, d\log A) = 0$), the GDP deflator only depends on the aggregate output gap, while relative prices only depend on cross-sectional productivity shocks. In the asymmetric economy with heterogeneous supply elasticities, instead, the GDP deflator depends on both the aggregate output gap and cross-sectional productivity.

Specifically, there is a "bad" cost-push shock whenever the relative productivity of the inelastic sector declines. With sticky prices the relative price of this sector remains too low, so that final users substitute away too little from the unproductive sector. This in turn raises the relative demand for the factor used more intensively by this sector. If the central bank closes the output gap we must have $\tilde{\ell}_K = -\tilde{\ell}_L > 0$, which however implies $\tilde{w}_K > -\tilde{w}_L$, because capital is less elastic than labor, therefore its price responds more to a given employment gap. As a result factor prices increase on average, and therefore final prices also increase, creating an inflation-output tradeoff. The relative employment gap comes from missing substitution, therefore it is proportional to the demand elasticity θ:

$$\tilde{\ell}_1 - \tilde{\ell}_2 = \frac{\theta(1-\delta)\bar{\varphi}}{1+\theta\delta\bar{\varphi}} (\log A_1 - \log A_2)$$

Hence the cost-push shock is also scaled by θ.

Note that, like in the previous example, in the asymmetric economy relative prices depend on the aggregate output gap. This is because the price of capital (the inelastic factor) responds more to the output gap, therefore the relative price of the capital intensive good increases.

5 Identification

Equation (21) isolates two drivers of the GDP deflator: the aggregate output gap, and cross-sectional shocks (captured by u_t^Y). In practice, however, it is hard to tease apart these two components in the data, because both the output gap $\bar{y}$ and the shocks that determine $\boldsymbol{\chi}$ and $\boldsymbol{\xi}$ are difficult to measure, especially in real time. To address this issue, in Section 5.1 I show how to construct an inflation proxy for the aggregate output gap, which allows us to recover the component of inflation driven by aggregate demand. I then apply my decomposition in Section 5.2, to estimate whether the inflation that we observed during and after the Covid pandemic is driven by aggregate demand versus cross-sectional shocks outside the control of monetary policy.

Our inflation proxy for the aggregate output gap is an inflation index – which I call the divine coincidence index – that aggregates sectoral inflation rates so as to filter out the effect of cross-sectional shocks. In a way, it is not surprising that such an index exists: after all, cross-sectional shocks must cancel out on average. Formally, Lemma 4 tells us that the elasticity $\hat{\mathcal{V}}$ of prices with respect to cross-sectional shocks is such that $\hat{\mathcal{V}}\mathbf{1} = \mathbf{0}$. Using a result from linear algebra, there must exist some vector $\boldsymbol{\phi}$ such that $\boldsymbol{\phi}^T\mathcal{V} = \mathbf{0}^T$. This vector is our desired weighting scheme. Proposition 1 derives an explicit expression, showing that the divine coincidence index discounts sectors with flexible prices or inelastic factor supply. Importantly, the divine coincidence weights can be very different from final expenditure shares (as in the GDP deflator) or consumption shares (as in the consumer price index or in the PCE price index). Therefore cross-sectional shocks can potentially have a large effect on other aggregate inflation indices, different from the divine coincidence one.

Section 5.3 further extends the divine coincidence result, showing how to construct inflation proxies for factor-specific gaps. Notably, this allows us to construct inflation proxies for regional output gaps in a currency union.

5.1 Aggregate divine coincidence index

Proposition 1 characterizes the divine coincidence index as the unique inflation index which only depends on the aggregate output gap.

Proposition 1. *There exists a unique vector $\boldsymbol{\phi}$ such that $\boldsymbol{\phi}^T\hat{\mathcal{V}} = \mathbf{0}^T$. The inflation index $\pi_t^{DC} = \sum_i \phi_i \pi_{it}$ does not depend on cross-sectional shocks, and its Phillips curve is given by*

$$\pi_t^{DC} - \xi_t^{DC} = \kappa^{DC}\bar{y} \tag{23}$$

where

$$\kappa^{DC} \equiv \boldsymbol{\phi}^T\hat{\boldsymbol{\kappa}},\ \xi_t^{DC} \equiv \boldsymbol{\phi}^T\boldsymbol{\xi}_t$$

When there are no wealth effects in labor supply and all final users have the same expenditure shares, the divine coincidence weights are

$$\boldsymbol{\phi}^T \propto \left[\bar{\lambda}^T - Cov_{\boldsymbol{\Lambda}}\left(\boldsymbol{\varphi}, \mathcal{D}_p\right)\right](I - \Delta)\,\Delta^{-1} \tag{24}$$

The Appendix derives $\boldsymbol{\phi}$ as a function of primitives in the general case.

The divine coincidence weights account for the two reasons why cross-sectional shocks can have aggregate effects: heterogeneous pass-through of cost shocks into prices, and heterogeneous factor supply elasticities. First, sectoral inflation rates are discounted by a decreasing function $(I - \Delta)\,\Delta^{-1}$ of price adjustment probabilities, to account for heterogeneous price stickiness. Moreover, sectors are weighed according to their total sales shares to account for input-output linkages. This weighting scheme is the same as in an economy with one primary factor only (see Rubbo (2020)), and it eliminates aggregate effects coming from heterogeneous cost pass-through.

Second, the term $Cov_{\boldsymbol{\Lambda}}\left(\boldsymbol{\varphi}, \mathcal{D}_p\right)$ eliminates aggregate effects coming from heterogeneous factor supply elasticity. To do so, it discounts sectors that use inelastic factors. Recall that positive inflation means that prices are below the producers' desired ones. Thus, if a sector experiences higher inflation than the average, this means that its relative price is too low compared to the efficient equilibrium, and therefore demand for the primary factors that it uses is too high. Hence the covariance $Cov_{\boldsymbol{\Lambda}}\left(\boldsymbol{\varphi}, (\mathcal{D}_p)_{:,i}\right)$ is positive for sectors i which use inelastic factors more intensively than the average, so that these sectors get discounted. Note that, from Lemma 3, proportional changes in sectoral price gaps have no effect on relative demand ($\mathcal{D}_p\mathbf{1} = \mathbf{0}$). Therefore these covariances sum to zero across sectors ($Cov_{\boldsymbol{\Lambda}}\left(\boldsymbol{\varphi}, \mathcal{D}_p\right)\mathbf{1} = 0$).

In a simple economy with only two goods, Examples 3 and 4 illustrate the weighting scheme of the divine coincidence index when sectors have heterogeneous cost pass-through or heterogeneous factor supply elasticities.

Example 3. Heterogeneous price stickiness

Recall the economy in Example 1. In this economy inflation in the GDP deflator depends on the covariance between productivity shocks and price adjustment probabilities. Inflation is positive when productivity declines in sectors with more flexible prices. Accordingly, the divine coincidence index reweighs sectors to account for the covariance between sectoral inflation rates and price adjustment probabilities:

$$\pi^{DC} \propto \pi^{Y} - \frac{Cov_{\beta}\left(\delta, \pi\right)}{\left(1-\bar{\delta}\right)\left[\bar{\delta} + \frac{1}{4}\frac{(\delta_1-\delta_2)^2}{1-\bar{\delta}}\right]}$$

Note that this amounts to assigning a smaller weight to sectors with more flexible prices.

Example 4. Heterogeneous supply elasticity

Recall the economy in Section 3 and Example 2. In this economy, inflation in the GDP deflator depends on the covariance between productivity shocks and factor supply elasticities. Inflation is positive when productivity declines in sectors that use less elastic factors. Accordingly, the divine coincidence index corrects CPI inflation to account for the covariance between sectoral inflation rates and factor supply elasticities:

$$\pi^{DC} \propto \pi^{Y} - \frac{\theta\bar{\varphi}}{1+\theta\bar{\varphi}}\frac{Cov_{\beta}\left(\varphi, \pi\right)}{\bar{\varphi}}$$

Note that this amounts to assigning a smaller weight to sectors that use inelastically supplied factors.

5.2 Inflation decomposition using price data

Using equations (21) and (23) we can express the Phillips curve for the GDP deflator as a function of the divine coincidence index and cross-sectional shocks:

$$\pi_t^Y - \xi_t^Y = \kappa^Y \frac{\pi_t^{DC} - \xi_t^{DC}}{\kappa^{DC}} + u_t^Y \tag{25}$$

where we denoted by $\xi_t^Y \equiv \bar{\boldsymbol{\beta}}^T \boldsymbol{\xi}_t$ and $\xi_t^{DC} \equiv \boldsymbol{\phi}^{DC} \boldsymbol{\xi}_t$. The divine coincidence index, net of desired markups $\boldsymbol{\xi}$, is a proxy for the aggregate output gap. Therefore the component of the GDP deflator, also net of desired markups, driven by the aggregate output gap is

$$\kappa^Y \frac{\pi_t^{DC} - \xi_t^{DC}}{\kappa^{DC}}$$

If we could control for desired markups and expected inflation $\boldsymbol{\xi}$, we could directly implement equation (25) in the data and identify this aggregate component. In practice, however, we do not observe changes in desired markups and inflation expectations accurately. We can still implement our decomposition in the data without controlling for $\boldsymbol{\xi}$, however this changes the interpretation of the aggregate component.

Specifically, if we do not control for $\boldsymbol{\xi}$, the aggregate component is driven by deviations of aggregate output with respect to a flex-price economy subject to the same shocks as in the natural benchmark, plus a change in desired markups equal to $(I - \Delta)\,\Delta^{-1}\boldsymbol{\xi}_t$. This is equivalent to redefining the natural output to include these (endogenous and exogenous) shocks to desired markups. In the Appendix we show that the output gap with respect to this new benchmark is

$$\hat{y}_t \equiv \bar{y}_t + \xi_t^{DC}$$

Likewise, price wedges with respect to the new reference equilibrium are given by

$$\hat{\boldsymbol{\chi}}_t = \mathbf{p}_{t-1} - \hat{\mathbf{p}}_t$$

where $\hat{\mathbf{p}}_t$ are prices in the flex-price economy that faces all the shocks including those to desired markups. Finally, we have $\hat{\boldsymbol{\xi}} \equiv \mathbf{0}$ by construction.

With this redefinition, the Phillips curves for the GDP deflator and the divine coincidence index are given by

$$\begin{aligned} \pi_t^Y &= \kappa^Y \frac{\pi_t^{DC}}{\kappa^{DC}} + \hat{u}_t^Y \\ \pi_t^{DC} &= \kappa^{DC} \hat{y}_t \end{aligned} \tag{26}$$

where

$$\hat{u}_t^Y \equiv -\boldsymbol{\phi}_{DC}^T \hat{\mathcal{V}} \hat{\boldsymbol{\chi}}_t$$

and κ^Y, κ^{DC}, and $\hat{\mathcal{V}}$ are the same functions of primitives as before.

Without data on $\boldsymbol{\xi}$, the aggregate component $\kappa^Y \frac{\pi_t^{DC}}{\kappa^{DC}}$ in equation (26) is only an imperfect proxy for the effect of monetary policy on inflation, because monetary policy endogenously affects desired markups through inflation expectations. There is no way to tell, using only price data, whether the aggregate output gap is moving because output in the benchmark flex-price equilibrium is moving (including due to endogenous desired markups) or because monetary policy is actively managing aggregate demand.

Remark 12. The ratio

$$\frac{\pi_t^{DC}}{\kappa^{DC}} = \frac{\boldsymbol{\phi}_{DC}^T \boldsymbol{\pi}_t}{\boldsymbol{\phi}_{DC}^T \hat{\boldsymbol{\kappa}}}$$

in equation (26) is the coefficient on $\hat{\boldsymbol{\kappa}}$ when we write the inflation vector $\boldsymbol{\pi}$ in components with respect to a basis of $\mathbb{R}^{\bar{N}}$ formed by $\hat{\boldsymbol{\kappa}}$ and any basis of the orthogonal space to $\boldsymbol{\phi}_{DC}$.

In other words $\frac{\pi_t^{DC}}{\kappa^{DC}}$ is the projection of sectoral inflation rates on $\hat{\boldsymbol{\kappa}}$, according to a specific basis. This basis contains the vector $\hat{\boldsymbol{\kappa}}$, along which the aggregate output gap moves prices, and a basis of the space $\boldsymbol{\phi}_{DC}^{\perp}$, where the inflation response to cross-sectional shocks must lie. Importantly, this basis is not orthogonal. Therefore our decomposition is different from the traditional principal component analysis, which imposes that the principal components must be orthogonal.

5.3 Local divine coincidence indices

Proposition 1 constructs an inflation proxy for the aggregate output gap. In a similar way, Proposition 2 shows that it is possible to construct inflation proxies for any linear combination of factor-specific employment gaps. This result is particularly useful in a currency union, where it allows us to construct proxies for regional output gaps.

Proposition 2. *For any weighted average employment gap,*

$$\bar{y}_\zeta \equiv \sum_h \zeta_h \tilde{\ell}_h$$

with $\zeta_h \in [0,1]$, $\sum_h \zeta_h = 1$, *there exists a unique inflation index*

$$\pi_\zeta^{DC} \equiv \boldsymbol{\phi}_\zeta^T \boldsymbol{\pi}$$

such that the Phillips curve

$$\pi_{\zeta,t}^{DC} - \xi_{\zeta,t}^{DC} = \kappa_\zeta^{DC} \bar{y}_{\zeta,t}$$

does not depend on cross-sectional shocks. When there are no wealth effects in labor supply ($\Gamma = \mathbb{O}$) *and final expenditure shares are the same across all final users, the divine coincidence weights* $\boldsymbol{\phi}_\zeta$ *are given by*

$$\boldsymbol{\phi}_\zeta^T = \left[\bar{\lambda}^T + \boldsymbol{\Lambda}^T \Phi \mathcal{D}_{\zeta,p}\right] (I - \Delta)\, \Delta^{-1}$$

where

$$\mathcal{D}_{\zeta,p} \equiv \left(I - \mathbf{1}\boldsymbol{\zeta}^T\right)\mathcal{D}_p$$

The key result behind Proposition 2 is that the relative demand equation (11) can be expressed as a function of any weighted average employment gap $\bar{y}_\zeta$, not just the share-weighted aggregate gap $\bar{y}$. By combining the relative demand equation expressed in terms of $\bar{y}_\zeta$ with the sector-by-factor Phillips curves in (14), we can express also sectoral and aggregate Phillips curves as a function of $\bar{y}_\zeta$. Regardless of the weights $\boldsymbol{\zeta}$, it still holds that inflation only depends on the cross-sectional component of $\boldsymbol{\chi}$ and $\boldsymbol{\xi}$ (i.e. $\hat{\mathcal{V}}_\zeta \mathbf{1} = \mathbf{0}$), like in equation 17, once we control for the gap $\bar{y}_\zeta$ and for the one-for-one effect of desired markups $\boldsymbol{\xi}$ on inflation. The same argument as in Section 5 then guarantees that there exists a weighted average inflation rate which depends only on the aggregate gap $\bar{y}_\zeta$.

In particular $\bar{y}_\zeta$ could be a measure of regional gaps in a currency union, if it assigns weights to the factors proportionately to the ownership shares of agents living in that region. Like the aggregate divine coincidence index, the local divine coincidence indices weight sectors according to sales shares and discount those with more flexible prices. The difference between local and aggregate indices is that local indices assign a higher weight to the sectors which use local factors more intensively. To see this, note that local indices rescale the relative demand coefficient $\mathcal{D}_p$ to express factor-specific gaps relative to the local gap $\bar{y}_\zeta$. Therefore, compared to the aggregate divine coincidence index, the term $\boldsymbol{\Lambda}^T \Phi \mathcal{D}_{\zeta,p}$ assigns a higher weight to sectors which hire factors with large ζ_h relative to Λ_h.

Examples 5 through 7 below illustrate local divine coincidence indices in simple economies.

Example 5. Symmetric two-region economy

Consider an economy with two goods and two primary factors, H and F. Each good uses a different primary factor. Both factors have the same supply elasticity φ, and both goods have the same price adjustment probability δ. All consumers have equal expenditure shares on the two goods, with elasticity of substitution θ. The local divine coincidence index for factor H is given by

$$\pi_H^{DC} = \frac{1-\delta}{\delta}\left[\pi^Y + \frac{1}{2}\frac{\theta\varphi}{1+\theta\varphi}\left(\pi_H - \pi_F\right)\right]$$

Example 6. Sticky-price and flex-price region

Consider the same economy as in Example 1, but now assume that the two goods have different price adjustment probabilities δ_H and δ_F. The divine coincidence index for factor H is

$$\pi_H^{DC} = \bar{\pi}^{DC} + \frac{1}{2}\frac{\theta\varphi}{1+\theta\varphi}\left(\frac{1-\delta_H}{\delta_H}\pi_H - \frac{1-\delta_F}{\delta_F}\pi_F\right)$$

Example 7. Elastic and inelastic region

Consider the same economy as in Example 2, but now assume that the two factors have different supply φ_H and

φ_F. The divine coincidence index for factor H is

$$\pi_H^{DC} = \frac{1-\delta}{\delta}\left[\pi^Y + \frac{1}{2}\frac{\theta\varphi_F}{1+\theta\varphi_F}\left(\pi_H - \pi_F\right)\right]$$

6 Drivers of inflation during the Covid pandemic

In this section, I calibrate the model using data from the US input-output tables and construct a time series of the aggregate component of inflation (i.e. the component driven by the aggregate output gap) by combining sectoral inflation series provided by the Producer Price Index (PPI) database by the Bureau of Labor Statistics (BLS). Section 6.1 describes the data, and Section 6.2 presents the results.

6.1 Data

In the calibration I consider: 22 worker types, corresponding to the 22 major occupations in the Standard Occupation Classification (SOC) by the BLS; 31 capital asset types, corresponding to the equipment, structures, and intellectual property categories in the NIPA tables; and 71 production industries, as classified in the BEA input-output tables.

The parameters describing workers, capital assets, industries and their interactions are reported below.

6.1.1 Expenditure shares

Input-output network I calibrate the input-output matrix Ω based on input-output tables reported by the Bureau of Economic Analysis (BEA), for the year 2012. I compute input shares as the ratio between industry-level expenditures on each intermediate input and industry-level costs. Industry-level costs are the sum of expenditures on intermediate inputs, labor, and capital assets. I describe the methodology to measure expenditures on capital assets next.

Factor input shares I compute the 71×31 component of the matrix α of industry-level expenditures on primary factors ($W_f L_f$, $f \in \mathcal{N}_f$ in our notation) by combining industry-by-asset stock values ($P_f L_{if}$) from the NIPA tables with the values of asset-level rental rates relative to prices ($\frac{W_f}{P_f}$) computed by vom Lehn and Winberry (2021). For both, I use the 2012 values.

The 71×22 component of the matrix α corresponding to labor contains the expenditure shares of each industry on each occupation. I use data from the American Community Survey (ACS) for the year 2012, to compute the share of each industry's wage payroll that goes to each occupation, and I calibrate the overall labor share in total costs at the industry level using the BEA input-output tables.

Final expenditure shares I calibrate the 71×31 component of the matrix β corresponding to investment producers based on the investment network constructed by vom Lehn and Winberry (2021), who provide bridge tables containing the expenditure shares on each good in the production of each capital assets.

I calibrate the 71×22 component of the matrix β corresponding to final consumption by combining data from the input-output tables and the Consumer Expenditure Survey (CEX) for the year 2012. From the CEX, I compute the share of expenditures on each good coming from each occupation, and from the BEA input-output tables I measure the total expenditure by final consumers on each industry.

Price adjustment probabilities I use the price adjustment probabilities computed by Pasten et al. (2019) using micro-data underlying the Producer Price Index published by the Bureau of Labor Statistics (BLS).

6.1.2 Other parameters

All other parameters are calibrated to consensus values in the literature, reported in Table 6.1.2.

Discount factor	$\rho = 0.9975$
Wealth effects in labor supply	$\gamma = 1$
Frish elasticity (labor)	$\varphi_L = 2$
Frish elasticity (structures)	$\varphi_S = 20$
Frish elasticity (heavy equipment)	$\varphi_{EH} = 10$
Frish elasticity (R&D)	$\varphi_{RD} = 5$
Frish elasticity (light equipment software, artistic originals)	$\varphi_{EL} = 2$
Substitution between labor types	$\theta^i_{k,h} = 0.1 \ \forall k, h \in \mathcal{N}_w, i \in \{1, ..., N\}$
Substitution between capital types	$\theta^i_{k,h} = 0.1 \ \forall k, h \in \mathcal{N}_f, i \in \{1, ..., N\}$
Substitution between labor and capital	$\theta^i_{k,h} = 0.1 \ k \in \mathcal{N}_w, h \in \mathcal{N}_f, i \in \{1, ..., N\}$
Substitution between factors and intermediates	$\theta^i_{h,j} = 0.1 \ h \in \mathcal{N}_w \cup \mathcal{N}_f, i, j \in \{1, ..., N\}$
Substitution between intermediates	$\theta^i_{j,n} = 0.5 \ i, j, n \in \{1, ..., N\}$
Substitution between final goods	$\theta^h_{i,j} = 2 \ h \in \mathcal{N}_w \cup \mathcal{N}_f, i, j \in \{1, ..., N\}$
Ownership shares of firms and assets (Ξ, $\mathcal{Z}$)	Survey of consumer finances

Table 7: Model calibration

6.1.3 Time series for prices and wages

Data for the Consumer Price Index (CPI) and the Personal Consumption Expenditure price index less food and energy (core PCE) is from the St Louis Fed.

To construct the divine coincidence index, we use industry-level price series from the BLS PPI dataset, and time series for occupation-level wages from the CPS. We assume that the rental rates for capital are flexible, and hence they have a zero weight in the index.

6.2 Quantitative inflation decomposition

Figure 6.2 applies our decomposition to the CPI between January 2019 and July 2022. The blue line is the official CPI, and the red line is the core PCE. The yellow line plots the component of the CPI due to the aggregate output gap, which is equal to the ratio between the divine coincidence index and the calibrated slope of its Phillips curve. The difference between the blue and the yellow line is the component of CPI inflation driven by cross-sectional shocks.

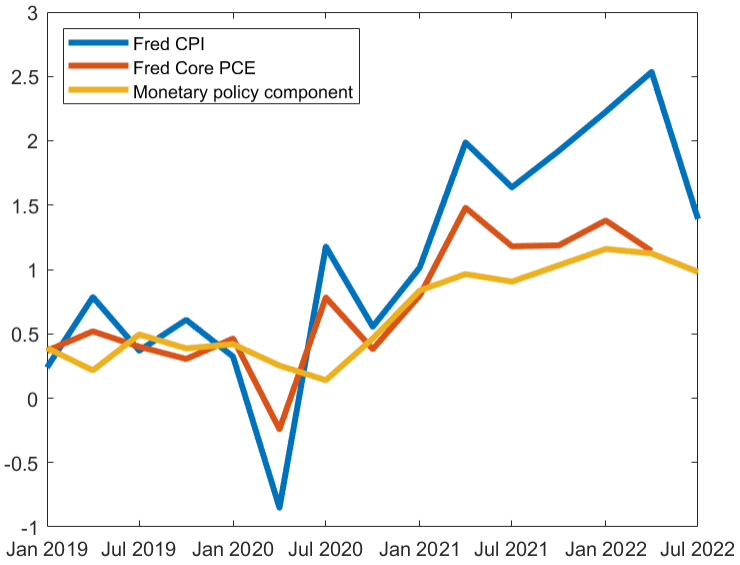


Figure 3: Inflation decomposition, baseline calibration

We see from Figure 6.2 that consumer price inflation was entirely driven by cross-sectional shocks in the early phase of the Covid pandemic (from February to October 2020). Compared to our decomposition, core inflation overstates the role of aggregate demand. The subsequent increase in inflation until February 2021 is almost entirely driven by a positive aggregate output gap. Aggregate demand still explains a large fraction of CPI inflation after February 2021, but cross-sectional shocks again have an important role, likely due to higher energy prices. By construction the core PCE filters out movements in inflation caused by energy prices, therefore it has been a better proxy for the aggregate demand component in recent periods.

Figure 6.2 replicates the inflation decomposition in an alternative calibration, where capital assets are not traded across industries. That is, each industry has its own specific capital asset, with supply elasticity equal to the share-weighted average of the relevant elasticities from Table 6.1.2. The Figure shows that the model with more segmented capital markets attributes a smaller share of inflation to aggregate demand.

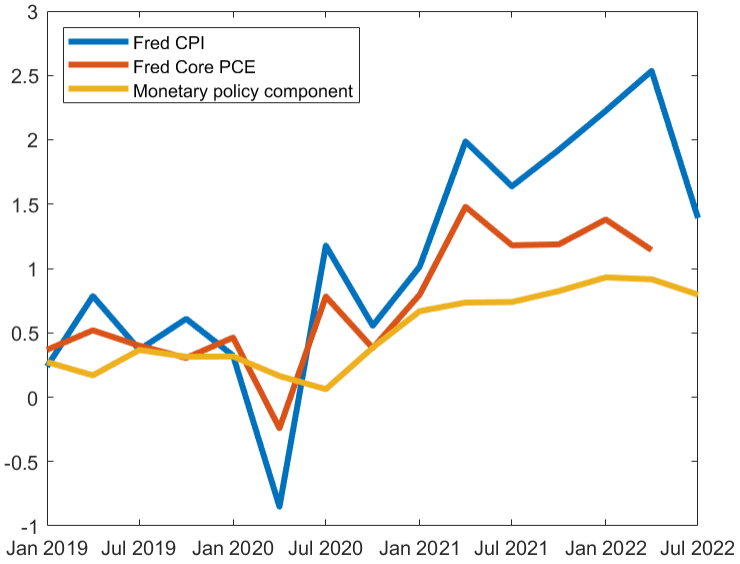


Figure 4: Inflation decomposition, fully segmented capital markets

7 Conclusion

I augment the New Keynesian framework with a disaggregated model of production, featuring multiple industries and primary factors. I argue that cross-sectional shocks create an inflation output tradeoff, when they increase the relative demand for or reduce the relative productivity of industries that have more flexible prices or use less elastic primary factors. In this case consumer prices increase even when monetary policy closes the output gap. I then show how to identify the components of aggregate inflation driven by the aggregate output gap versus cross-sectional shocks, based on relative price movements. Applying my decomposition to the US during and after the Covid pandemic, I find that inflation in the early phases of the pandemic (until around October 2020) is entirely driven by cross-sectional shocks, whereas most of the subsequent increase in consumer prices is determined by aggregate demand.

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