# Reverse Mortgages, Housing, and Consumption: An Equilibrium Approach*

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**Abstract**

Reverse Mortgages (RMs) enable eligible homeowners 62 years and older access to the liquidity of their home without them moving out or repaying before loan termination when the borrowers die or move to long-term care facilities. We incorporate RMs into a quantitative equilibrium life-cycle model to assess their impacts on household decisions, the mortgage, and the housing market. We show that the volatility of consumption growth decreases for RM borrowers. Additionally, introducing RMs enhances the house's perceived value to households, making homeownership a more financially attractive option and stimulating housing demand. These effects increase overall household welfare in our model, highlighting the positive impact of RMs.

JEL Classification: G21, E21, J14

Key Words: Reverse Mortgage, Mortgage, Housing, Retirement, Welfare

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# 1 Introduction

Financial instability in later life has become a global challenge over the past decades due to factors such as rising life expectancy, growing socioeconomic inequality, and inadequate public resources for social security and the pension system, which puts pressure on private savings for retirement.[1] At the same time, most household wealth accumulates in the form of illiquid housing assets.[2] Moreover, some households make suboptimal decisions regarding their pension schemes. All of these lead to the phenomenon of "equity-rich and cash-poor" households (see Caplin (2000)).

How shall we tackle this challenge? Reverse Mortgages (RMs hereafter) could be a potential solution. RMs have emerged as a popular financial tool for older households to finance their retirement living expenses which have gained popularity globally in developed economies and several emerging economies.[3,4] They allow homeowners to extract the value of their homes to supplement their retirement income without having to move out or downsize. Moreover, borrowers are not required to make any payments on the loan until its termination. If, upon termination, the remaining loan balance exceeds the house's value, neither the retirees nor their heirs are liable for the shortfall. The government bears the losses if house value declines. However, the structural complexity of RMs makes it difficult to analyze their impacts on household decisions, balance sheet dynamics, mortgage and housing markets, as well as social welfare.

In this paper, we measure RM's economic impacts through various channels using US loan-level data from CoreLogic and Black Knight. We set up a general equilibrium model with heterogeneous households subject to multiple risks, competitive lenders, and endogenous house prices, with the presence of conventional mortgages and RMs, accounting for the transition of macroeconomic states among booms, recessions, and crises with cyclical interest rates. Mortgage and RM spreads

[1] The Geneva Association (2024) reports that the total world's pension gap amounts to be $41 trillion. There exists about $10-15 trillion projected shortfall in the US pension system.

[2] According to Chen et al. (2020), on US the household balance sheet, housing estates are 10 times the size of liquid assets, such as cash and equivalents (financial securities (equities, and bonds). Similarly, Nakajima and Telyukova (2020) states that excluding housing wealth, retirees' net worth would be 28% - 44% lower, depending on age groups. Iacoviello (2011) also estimates that housing wealth is about one-half of total household net worth with the U.S. data.

[3] In the US, RMs are commonly Federal Housing Administration (FHA) - insured Home Equity Conversion Mortgages (HECMs), available to individuals aged 62 and older and are primarily used to finance retirement consumption.

[4] See Nakajima and Telvukova (2017, 2020), and Chen et al. (2020).

are set to break even by lenders. Households' decisions influence the equilibrium house prices and spreads of mortgages and RMs, which then feed back to them. Based on their expectations about equilibrium house prices, mortgage and RM rates, households make decisions on consumption, saving, homeownership, default, and (re)financing through conventional mortgages, as well as whether to maintain their property and borrow RMs.

Our calibrated model shows that including RMs in the credit market generates the loan-to-value (LTV) ratio aligned with the Survey of Consumer Finances (SCF), while matching the homeownership rate. Since RMs can be used to repay conventional mortgages within LTV limits, this feature makes them an attractive alternative to refinancing because RMs relieve borrowers from regular repayments, thereby reducing liquidity concerns, which can be observed from CoreLogic and Black Knight. Thus, calibrating our model using such datasets enables us to better capture RMs' impacts in a realistic setting.[5] Most papers modelling RMs used public datasets such as the Panel Study of Income Dynamics (PSID) and Health and Retirement Study (HRS), which are survey-based datasets dependent on estimation and weighting methodologies. As RM take-up is relatively small compared to the general eligible population, the loan-level granular datasets add significant value to the accuracy of our analysis.

In the presence of RMs, our model also matches the Payment-to-Income (PTI) ratio with the empirical evidence very well. In comparison, without RMs, Guren et al. (2021) generates a PTI ratio that grows exponentially throughout retirement, a finding that contradicts empirical evidence. Furthermore, the refinancing rate of conventional mortgages in our model remains relatively stable and moderate during the retirement period. These findings also differ from the study by Guren et al. (2021), whose findings indicate a low refinancing rate during the working period followed by a sudden spike after retirement. Introducing RMs into our model, such that households can use RMs as a substitution for refinancing with the additional benefit of no monthly repayments and an embedded put option of housing price, results in this moderate refinancing rate during retirement.

Meanwhile, using the insurance premium and other cost information from the HECM guide-

[5]For example, both CoreLogic and Black Knight can be linked to credit information and American Community Survey (ACS) that provides household demographics, socioeconomic characteristics, household consumption and investment behaviors, alongside their housing condition, house prices, first liens (conventional mortgages and RMs), and second or junior lien mortgage (if any) information.

lines, our model generates a relatively low RM take-up rate of 4.5% among eligible homeowners with a hump-shaped pattern over the life cycle, aligned with empirical observation. In an experiment, we show that reducing the initial RM setup and product costs increases the take-up rate up to 10% among eligible homeowners aged 62 to 65, which is significantly higher than empirical evidence. Our results suggest that high set-up and product costs lead to a low take-up rate, which is consistent with those reported in the literature, such as Cocco and Lopes (2020) and Nakajima and Telyukova (2017).

More importantly, our main findings indicate that RMs effectively smooth household consumption over the life cycle. RMs allow households 62 and older to extract home equity and address the reduction in income that typically occurs after retirement. The consumption growth volatility of households with RMs is reduced by 17.04% (working) and 8.36% (retiring). In addition, RMs provide liquidation for senior households, and, in the earlier years of the loans, these households save slightly less. This enhanced flexibility smooths their consumption throughout their working and retirement periods.

To assess the impacts of RMs on the housing market and the welfare of households, we evaluate an alternative economy without RMs in the credit market, with all other parameters held constant as in our economy with RMs. We find that given the same housing supply, house prices increase significantly (from 0.66% to 1.90%) in the economy with RMs. Meanwhile, homeowners attain substantial welfare gains measured by the consumption certainty equivalent variation as 1.29% (age <62), 4.77% (ages 62-75), and 7.47% (age >75). Renters, on the other hand, enjoy marginal welfare improvement. Moreover, the volatility of consumption growth decreases significantly for households in the economy with RMs. Therefore, these findings suggest that introducing RMs as a financial product for trading liquidations enhances the efficiency of the housing market and benefits households.

Our paper is related to existing RM literature. Case and Schnare (1994) investigate RM's growth in mid-2000s, including loan and property characteristics; borrowers' socioeconomic and demographic aspects. They point out regulatory and legal issues may hamper growth. Rasmussen et al. (1995) and Merrill et al. (1994) define RM's potential beneficiaries by age, income, and home equity and identify geographical locations with a high concentration of such target groups for

market development. Davidoff (2015) examines the situations in which reverse mortgages might be beneficial for homeowners. Recently, Nakajima and Telyukova (2017) suggest that bequest motives, uncertainty about health and expenses, and loan costs account for low demand. Cocco and Lopes (2020) propose *aging-in-place*, and suggest maintenance cost reduces the benefits of the RMs for borrowers.[6] Both Nakajima and Telyukova (2017) and Cocco and Lopes (2020) predict RMs' low demand, aligned with empirical data, assuming exogenous home prices and RM rates.[7] These papers employ a partial equilibrium life-cycle model that considers only the household balance sheet, not interactions between the credit market and the housing market.

In contrast to them, we endogenize house prices, mortgage rate, and RM rate, which improves the assessment of RMs' impacts on household balance sheets, decisions, and welfare for both retired and working households. To the best of our knowledge, our model is one of the first to clear the housing and credit markets simultaneously, including conventional mortgages and RMs. Meanwhile, our model allows RM borrowers endogenously choose a lump sum or drawdown, in contrast to the common assumption in existing literature that they would only take a lump sum or drawdown. This endogenous choice adds a realistic dimension to our analysis, reflecting the actual decision-making process faced by homeowners considering RMs.

Our paper also contributes to the integrated mortgage-housing market literature. Earlier attempts to integrate the mortgage-housing market into the same framework treat either the mortgage or housing market as exogenous. For example, Ortalo-Magne and Rady (2006), Favilukis et al. (2017), and Tim et al. (2014) study equilibrium housing markets, treating mortgages as exogenous or stylized; Campbell and Cocco (2015) and Corbae and Quintin (2015) consider equilibrium mortgage market, taking housing market and house prices as given. On the other hand, the real estate's unique characteristics, such as illiquidity, borrowing constraints for low-wealth households, and households' heterogeneous preferences, hinder immediate demand-supply equilibrium; fluctuations in the housing market affect housing prices, which feed back on all types of mortgages, including RMs (see Case and Shiller (1988), Mankiw et al. (1989), DiPasquale and

[6] *Aging-in-place* was originally raised in environmental psychology, referring to the emotional attachment that binds a person to a particular place.

[7] Campbell and Cocco (2003) estimate only 2%–3% senior homeowners among eligible older homeowners borrow RM, as the US Census Bureau reports. On the other hand, Nakajima and Telyukova (2017) estimate this ratio to be 1.9% in 2013, down from 2.1% in 2011.

Wheaton (1994), Riddel (2004) and Hwang and Quigley (2006)). Closer to our model, Kung (2014) and Guren et al. (2021) create a general equilibrium housing and mortgage model with endogenous house prices and mortgage rates. Building on this foundation, our study offers a deeper insight into credit market dynamics and complexities by incorporating the interplay between mortgages and reverse mortgages.

Within the mortgage-housing literature, refinancing and default activities are discussed in detail. Empirical studies, such as Campbell (1983) and Deng et al. (2000), suggest that borrowers delay default until it is irreversible and their option to default is deep in the money. More recent studies find that with the presence of precautionary savings, there exist inconsistent correlations between mortgage default and unemployment, and these correlations vary depending on the research period and location. (see Elul et al. (2010)); Quercia et al. (2012); Ghent and Kudlyak (2011); Archer and Smith (2013); Tian et al. (2015)). Chen et al. (2020) follow Campbell and Cocco (2003) and integrate mortgage refinancing and home equity-based borrowing (HELOC) using a partial equilibrium approach. They show that during the recession, liquidity-constraint households refinance even with heightened borrowing costs due to adverse income shocks, contradicting the conventional belief that low interest rates drive refinancing. Our paper strengthens their observations by demonstrating that even when the RM rate is higher than the mortgage rate, households widely use RMs as an alternative to refinancing. Furthermore, we allow homeowners to strategically default to maximize utility, as well as facing forced default. Overall, our model generates a stable refinancing rate and aligns default rates with empirical data across households of all ages.

The remainder of the paper is organized as follows. Section 2 highlights the importance of RMs based on empirical evidence. Section 3 introduces our main model setup. Section 4 demonstrates our calibration methodologies, and Section 5 presents the quantitative analysis. Section 6 evaluates an alternative economy without RMs in the credit market, with all other parameters held constant as in the original economy with RMs. Section 7 concludes.

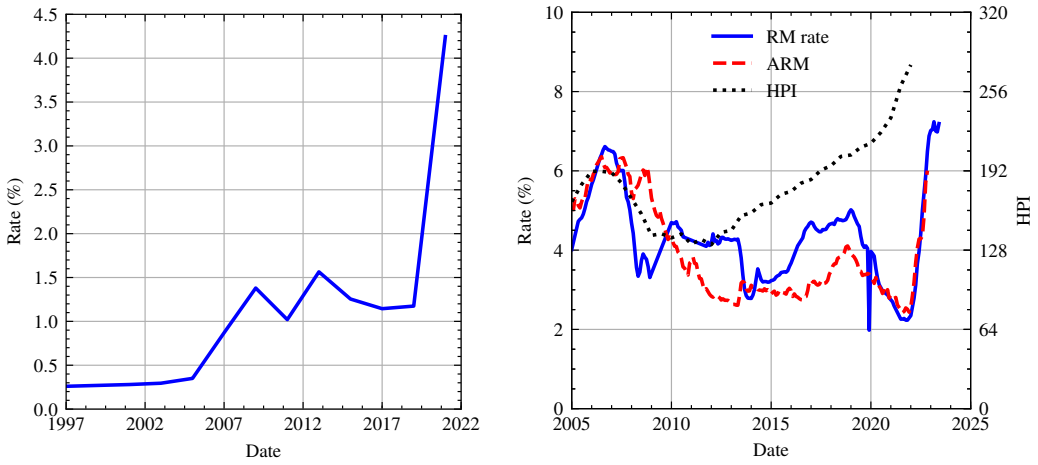
# 2 Empirical Motivation

Using proprietary data from CoreLogic and Black Knight, we compile a complete and granular dataset on household mortgage open liens up to 2021. The advantage of our dataset is that it covers the actual mortgage and RM borrowers' property information, such as their historical and market value, housing type, and condition. All mortgages and RMs (initial balance, interest rate, terms, etc.) were secured on the properties in the entire country during our research period, which covers almost all of the properties secured on such mortgages. Other datasets used in previous literature, such as PSID and HRS, cover only a subset of the population through surveys that can sometimes bias the model results because RM is a relatively small market compared to conventional mortgages and other more developed financial products.

As depicted in graph A of Figure 1, we first notice that the RM take-up rate in the US has increased more than tenfold, from approximately 0.3% in 1997 to 4.3% in 2021.[8] We also find that younger retirees (those aged 62-70) and unmarried retirees are more likely to take up RMs from Figure 2. Younger retirees, who have more years of retirement ahead of them, have opted for RMs to secure additional funds for their extended retirement period and their growing consumption needs. On the other hand, unmarried retirees, who lack a spouse to share financial risks and typically have lower average wealth and retirement income than their married counterparts, may seek RMs to supplement their available funds and enhance their consumption capacity.

Moreover, graph A of Figure 1 shows a noticeable drop (from 1.4% to 1%) from 2008 to 2011. Since the Global Financial Crisis (GFC) in 2008, some lenders have left the market. This results in RM plan issuance in the US being reduced following the GFC (see Cocco and Lopes (2020)). Moreover, the RM market suffered a more severe blow due to the correction in the housing market following the artificial housing boom from the 1990s to 2006. This was intensified by the fact that lending criteria heavily relies on LTV, which is intricately connected to housing market performance. The market gradually recovered in 2011 and has significantly increased over the past decade. Additionally, the 2015 RM reform allowing RMs to be used as a down payment for a property of an eligible senior homeowners' offspring has further contributed to the growth

[8] The consistent growth pattern has also been observed in several developed economies, such as the UK, Australia, and Hong Kong, which is captured in Figure 7 in Appendix A.




(a) (b)

**Figure 1: RM Market in the US**

This figure shows the current RM market in the US. Graph A plots the RM take-up rate in the US, and graph B plots mortgage, RM rates, and HPI in the US.

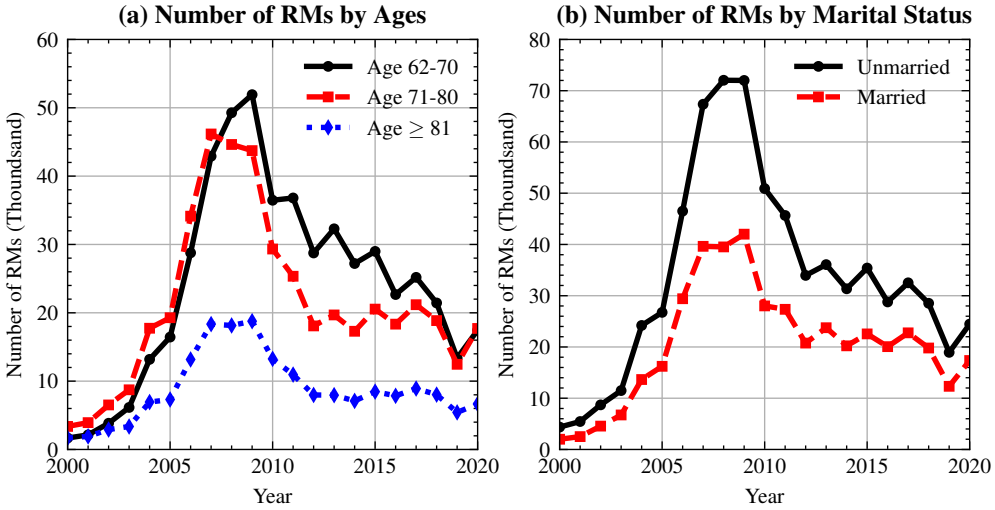



**Figure 2: Number of RMs by Borrower Age and Marital Status in the US**

This figure shows the RM borrowers' age groups and marital status in the US. Graph A plots three age groups: (1) just retired aged 62-70, (2) mid-retirement (aged 71-80), and (3) old age retirees (aged >81) of RM borrowers in the US. Graph B plots two curves representing unmarried and married RM borrowers' numbers, respectively.

of this market.

Graph A of Figure 1 also witnesses uninterrupted growth in the RM market following the COVID-19 crisis, in contrast to the market contraction following the GFC. The post-COVID-19 period saw high interest rates as the central banks increased the base rate multiple times in a short period of time in order to cool inflation caused by the prolonged ultra-low interest rate environment. At first glance, the rapid growth in the RM market following the COVID-19 crisis seems to contradict some predictions that suggest higher interest rates usually cool down the housing market and mortgage market. However, this surge in RMs in the post-COVID-19 crisis supports the conclusion by Hurst and Stafford (2004) that enabling households to convert home equity into liquid assets re-adjusts households' decision to refinance, even at the costs of higher borrowing costs. This is one of the reasons that RMs have become increasingly important in the household finance and credit literature, such that RMs provide a direct way to borrow against home value as an alternative to mortgage refinancing.

In summary, the two crisis periods have caused opposite RM market movements because of complex compounding effects of central bank base rates, housing market performances, and labor income instability. These dynamics on the household balance sheet and choices integrate LTV, LTI constraints, and economic states, which are essential and highlight our contribution to this end.

Finally, graph B of Figure 1 shows the growing trends of the conventional mortgage rates, RM rates, and the housing price index (HPI) in the US from 2005 to 2023. HPI is taken from CoreLogic HPI. Recently, a remarkable increase in the HPI has been accompanied by significant mortgage and RM rates increase. It reveals that, as anticipated, RM rates generally exceed conventional mortgage rates to compensate for the riskiness of such instruments. Although fluctuating, declines have been notable during the past two decades, mainly because of increased lenders' appetite to lend, improved regulation and consumer knowledge of the product. In some periods, RM rates fall below conventional mortgage rates during housing and credit market frictions, for example, the period following the GFC in 2008, during which HPI dropped significantly, coupled with extra low interest rates. The inherent reason for the lowered RM rate is driven by the low central bank base rate and the lack of appetite by consumers for borrowing and lenders for lending. RM rates

recovered as the HPI rose in 2010 and stayed above conventional mortgage rates. Another period when RM rates experienced a sharp drop below conventional mortgage rates was the immediate aftermath of the COVID-19 crisis in late 2019, but it has quickly picked up and followed the pattern of mortgage rates since then. This is a short-term negative shock on the RM rates, possibly because of the unstable market activities due to the COVID-19 crisis.

These observations emphasize the dynamic nature of these variables and call for an integrated framework when considering RMs. Such a framework encourages us to consider the interplay between the credit and housing markets, recognizing their simultaneous influence on RM market dynamics, ultimately impacting household balance sheets and decisions.

# 3 The Model

We consider a general equilibrium model that includes heterogeneous households and competitive mortgage lenders, within which house prices, mortgage rates and RM rates are determined at equilibrium. Households have a finite living span divided into two phases: the working period and the retirement period. During the working period, households earn labor income and pay taxes. During retirement, households receive retirement income. In each period, households decide to rent or buy houses. Renters make decisions on consumption and saving. Renters can buy houses and finance housing through conventional mortgages. On the other hand, homeowners choose how much to consume and save, whether to stay in their houses or sell and become renters. Additionally, they can borrow against their home equity using RMs during retirement. However, homeowners who use RMs cannot simultaneously borrow using conventional mortgages.

Mortgage lenders price conventional mortgages and RMs. They receive the mortgage payments until households decide to default or to refinance. In the case of RMs, lenders recover the collateral when households die or move out of the property to long-term care facilities. Since our model abstracts from inflation dynamics, we consider a real economy in which both mortgages and RMs are inflation-indexed.

## 3.1 Economy

Our model is discrete, and each period corresponds to 1 year. In each time period $t$, the economy ($\theta_t$) can be in crisis, recession, and expansion. We employ an exogenous transition probability matrix ($P_\theta$) to describe the evolution between these macroeconomic conditions.

## 3.2 Households

Following the convention in life-cycle models, we assume households start working at age 20, retire at 62, and can live up to 100 ($T = 80$). The probability that households are alive at age $(a + 1)$ conditional on being alive at age $a$ is denoted by $p_a$ ($p_{80}$ equal to 0).

**Preferences** Following Campbell and Cocco (2015), we assume household $i$ derive utility from housing $H_{it}$, from nondurable consumption $C_{it}$ and from leaving a bequest motive. Since house size remains fixed over time, we can drop housing from the preference specification. On the other hand, household $i$ is penalized for default. The recursive utility function of household $i$ in period $t$ can be written as

$$U_{it} = \frac{C_{it}^{1-\gamma_i}}{1-\gamma_i} + u_o D_{it}^o - u_d D_{it}^d + \beta_i \left[ p_{a_{it}} \mathbb{E}_t[U_{it+1}] + (1 - p_{a_{it}}) \psi^{bq} \frac{(Q_{it+1} + \xi)^{1-\gamma_i}}{1-\gamma_i} \right] \tag{1}$$

where $\gamma_i$ is the coefficient of relative risk aversion, $\beta_i$ is the discount factor, $u_o$ is the utility benefit from owning a house, $u_d$ is the utility penalty from defaulting on mortgages, $D_{it}^o$ and $D_{it}^d$ are indicator functions for owning a house and default on mortgages, respectively. $Q_{it}$ is the terminal wealth, $\xi$ is a bequest motive shift parameter, and $\psi^{bq}$ is the bequest motive weight.

**Labor Income** Before retirement, households' labor income is exposed to both aggregate shocks from the economic state and idiosyncratic shocks. Specifically, the labor income at age $t$ ($Y_{it}$) is given in logs ($y_{it} = log(Y_{it})$) by

$$y_{it} = v_t + \epsilon_{it}, \quad t < R, \tag{2}$$

where $v_t$ is the aggregate labor income determined by the economics state $\theta_t$ and the idiosyncratic labor income shock $\epsilon_{it}$ follows a Markov process with transition matrix $P_y(\theta_t)$ as a function of the economic state $\theta_t$.

Moreover, we assume a linear taxation rule. Households' labor income during the working period is taxed at the constant tax rate $\tau$. For simplicity, income during retirement is assumed to be exogenous and deterministic. Income is specified as a constant fraction $\lambda$ of the labor income in the last working year,

$$y_{it} = log(\lambda) + y_{iR} \quad t > R, \tag{3}$$

where $R = 43$, corresponding to the retirement age 62.

**Financial Market** We assume the market has only one asset, a one-period riskless bond with return $r^f(\theta_t)$, in which households can invest. The risk-free real interest rate is exogenous and depends on the economy. In expansions, the risk-free real interest rate is higher, and it hits the lowest point during crises.

**Housing and Rental Markets** Our model assumes there is no housing construction, so the housing supply is fixed. The date $t$ real housing price $P_t^h$ is endogenously determined by the equilibrium of each period and depends on the economic state. In each time period, households can decide to be homeowners (denoted as $D_{it}^o = 1$) or renters (denoted as $D_{it}^o = 0$), and housing size $H_{it}$ is fixed for all homeowners.

Homeowners must pay per-period maintenance cost $C_{ma}(H_{it}, P_t^h)$, proportional to the value of the property, to keep the house from depreciating

$$C_{ma}(H_{it}, P_t^h) = c_{ma} H_{it} P_t^h, \tag{4}$$

where $c_{ma}$ measures the maintenance cost per unit of total property value. Borrowers unable to pay the maintenance cost will be forced to sell their houses and become renters.

We assume that homeowners face moving shocks, which forces the homeowners to move out from their houses, capturing the probability $p_m$ of moving involuntarily with moving cost

$C_{ms}(H_{it}, P_t^h)$

$$C_{ms}(H_{it}, P_t^h) = k_{ms} + c_{ms} H_{it} P_t^h, \tag{5}$$

where $k_{ms}$ captures the fixed cost and $c_{ms}$ quantifies the variable moving cost per unit of the total property value.

Existing literature has shown that the rental market and owner-occupied markets are segmented. Rental housing can be produced and destroyed at a variable cost $q$, so in equilibrium, renters face a renting cost $C_r = q$. We do not differentiate the owner-occupied house price and the rental property price and only consider the equilibrium of the owner-occupied housing stock. This implies that in our model, the price-to-rent ratio varies with the house price.

**Mortgages, Refinancing and Default** Households pay a downpayment to buy houses. Let $\zeta_t^m$ denote the downpayment ratio. It is subject to the maximum LTV constraint ($\phi_t^m$). The initial balance $M_{it}^m$ for the household $i$ at the beginning of the period $t$ satisfies

$$M_{it}^m = (1 - \zeta_t^m) P_t H_{it}. \tag{6}$$

A lower downpayment ratio leads to a higher LTV ratio, but it can not exceed $\phi_t^m$:

$$\zeta_t^m \geq 1 - \phi_t^m. \tag{7}$$

We assume that $\phi_t^m$ varies exogenously and depends on the macroeconomic states.

Two main types of mortgage contracts are available on the conventional mortgage market: fixed-rate mortgages (FRM) and adjustable-rate mortgages (ARM). For simplicity, we only model one type of the main residential mortgage: the long-term FRM, which represents more than 90% of mortgages originated in the US. Households with an FRM bear the same mortgage rate $r^m(\theta_t)$ determined by the economic state $\theta_t$ at origination. We assume that FRM rates are the same for all households in the same economic states.

With the timing assumption that households pay their interest between periods $t$ and $t+1$ in advance at time $t$, the minimum payment on a mortgage for an agent who does not move or

refinance at age $a_{it}$ is given by:

$$M^m_{it-1} - M^m_{it}(1 - r^m(\theta_t)) \geq M^m_{it-1} \frac{r^m(\theta_t)\left(1 + \left(\frac{r^m(\theta_t)}{1 - r^m(\theta_t)}\right)^{T-a_{it}+1}\right)}{\left(\frac{r^m(\theta_t)}{1 - r^m(\theta_t)}\right)^{T-a_{it}+1} - 1}. \tag{8}$$

When mortgage rates drop below the initial mortgage rates, households can choose to refinance. If households decide to refinance, they face LTV constraint (7) and need to pay for refinancing costs

$$C_{re}(M^m_{it+1}) = k_{re} + c_{re} M^m_{it+1}, \tag{9}$$

where $k_{re}$ is the fixed refinancing cost and $c_{re}$ is the variable refinancing cost per unit of the mortgage balance. Households can refinance their mortgage and pay the refinancing cost at the beginning of each period.

If households default, they suffer a utility penalty of $u_d$ and lose their house today. Moreover, defaulting households are temporarily barred from the credit markets. However, their default status may be lifted, enabling them to re-enter the housing market using mortgages.

**Reverse Mortgages** RMs allow older homeowners to borrow against their housing wealth without moving out of the house. This option also offers insurance against substantial declines in house prices. However, in order to borrow RMs, homeowners must have paid off a significant part of their conventional mortgages. In our model, we incorporate this feature, following the guidelines established by the HECM program, the most popular RM program in the US.

To borrow against the home value, households must be at least 62 years old and have more than $\phi^{rm}_{own}(\theta_t)$ home equity. $\phi^{rm}_{own}(\theta_t)$ is the minimum home equity requirement for RMs determined by the economic state $\theta_t$. Therefore, they must satisfy the following conditions when initiating RMs:

$$1 - \frac{M^m_{it}}{P^h_t H_{it}} \geq \phi^{rm}_{own}(\theta_t). \tag{10}$$

Households are also constrained by the maximum amount of RMs they can borrow, which is determined by the lower value between a constant maximum claim amount ($Q^{rm}$) and the principal limit based on the home value. According to HECM, the principal limit factor (PLF) $\phi^{rm}$ is determined by the age of the borrower $a_{it}$ and the RM rate $r^{rm}(\theta_t)$. Let $M_{it}^{rm}$ denote the amount a household chooses to claim. Therefore, $M_{it}^{rm}$ should satisfy

$$M_{it}^{rm} \leq \min\left\{Q^{rm}, \phi^{rm}(a_{it}, r^{rm}(\theta_t))P_t^h H_{it}\right\}. \tag{11}$$

Although HECM provides several types of repayment options for borrowers to choose from, we only focus on the two most common types of RMs: 1) the lump sum RM, which is a single large payout at the start of the loan, and 2) the drawdown RM, which is a stream of unscheduled payments at the borrower's request until the line of credit is exhausted. In addition, households can choose from fixed-rate RMs (FRRM) or adjustable-rate RMs (ARRM). Only lump sum RM is available for FRRM.

If households decide to take a drawdown, they can determine the proportion of the total amount they wish to receive at the start of the loan, denoted by $\alpha_{it}^{rm} M_{it}^{rm}$ at origination, while the remaining amount will be available in the form of a line of credit. Given that the amount $M_{it}^{rm}$ includes the cost $c_0^{rm}$, which comprises the initial mortgage insurance premium and origination fee, households will receive $(\alpha_{it}^{rm} - c_0^{rm})M_{it}^{rm}$ at closing of the origination. In each time period, households make decisions about whether to draw down the credit line if the credit line is not exhausted ($\alpha_{it}^{rm} < 1$). They are also allowed to prepay the RM balance. The remaining RM balance and credit line both accumulate with the RM rate $r^{rm}(\theta_t)$ and the annual loan fee rate $c_a^{rm}$, including annual insurance cost and annual service cost. Specifically, after origination, households face the following constraints:

$$M_{it}^{rm} \leq M_{it-1}^{rm}(1 + r^{rm}(\theta_{t-1}) + c_a^{rm}), \tag{12}$$

$$(1 - \alpha_{it}^{rm})M_{it}^{rm} \leq (1 - \alpha_{it-1}^{rm})M_{it-1}^{rm}(1 + r^{rm}(\theta_{t-1}) + c_a^{rm}). \tag{13}$$

where the first constraint indicates how the RM balance accumulates and the second constraint implies that the remaining line of credit cannot be increased as households prepay the RM balance.

If the household chooses to take a lump sum, $\alpha_{it}^{rm}$ is fixed to be 1, indicating that the entire amount, $M_{it}^{rm}$, will be received at the beginning. Therefore, these borrowers only face constraint 12.

RMs terminate whenever the borrowers move out to long-term care facilities or decease, and RM lenders get paid for the roll-up RM balance or the home value, whichever is lower. Usually, lenders will sell the house to recover the loan balance, and the remaining residue home value is inherited by the borrowers' heirs or returned to the borrowers. There is a possibility that a housing market slump could result in a decrease in the property portfolio value of lenders, leading to potential losses on RMs upon termination. However, as borrowers pay insurance premiums annually, HUD insures against these potential losses.

**Health Risk and Medical Cost** During retirement, households are exposed to health risks. We introduce a health status indicator, $D_{it}^{med}$, to capture this risk. It takes a value of 1 if households are in good health and a value of 0 if they are sick. Once households become sick, they will remain in that state for the rest of their lives. We assume that the health state $D_{it}^{med}$ of household $i$ in period $t$ follows a Markov process independent of the economic and idiosyncratic states.

A senior household $i$ pays medical expenses of $c_{med}(a_{it}, D_{it}^{med})Y_{it}$, where $c_{med}$ represents the proportion of income and is determined by the health state $D_{it}^{med}$ and the age $a_{it}$.

**Government Transfers** Households are guaranteed a lower bound per-period cash-on-hand, which can be viewed as a subsistence level. This assumption is motivated by the existence of social welfare programs such as means-tested income support. When households' wealth and housing value drops below this lower bound per-period cash-on-hand, the government provides a consumption floor, which enables them to consume non-durable goods and to rent a house.

**Households Optimization Problem** Households are price takers who maximize their lifetime utility, given their expectations about future economic state and house prices. We denote the aggregate state as a vector $\Theta_t = \{\theta_t, r^f(\theta_t), P_t^h, r^m(\theta_t), r^{rm}(\theta_t), \Pi_t\}$, where $\Pi_t$ is the distribution of households.[9] Household $i$'s idiosyncratic state at the start of each period $t$ before making

[9]The distribution of households affects the equilibrium and, thus, should be included in the aggregate state. The distribution is over the idiosyncratic states of households.

decisions is $S_{it} = (a_{it}, Y_{it}^{id}, W_{it}, D_{it}^{o}, D_{it}^{mov}, D_{it}^{med}, M_{it}^{m}, r_{it}^{m}, D_{it}^{d}, D_{it}^{rm}, M_{it}^{rm}, D_{it}^{fix}, r_{it}^{rm}, \alpha_{it}^{rm})'$, a vector of age, idiosyncratic labor income, saving, homeownership status, moving shocks, health state, mortgage balance, the mortgage accumulated rate determined by the economic state when the mortgage originated, default status, RM availability, RM claim amount, indicators for FRRM, RM accumulated rate ($r_{it}^{rm}$),[10] and the share of the RM claim amount retrieved.

In each period $t$, household $i$ makes decisions $A_{it} = (D_{it}^{H}, W_{it+1}, M_{it+1}^{m}, M_{it+1}^{rm}, D_{it+1}^{fix}, \alpha_{it+1}^{rm})'$ of housing ($D_{it}^{H}$), saving ($W_{it+1}$), mortgage and RM ($M_{it+1}^{m}$, $M_{it+1}^{rm}$, $D_{it+1}^{fix}, \alpha_{it+1}^{rm}$), to maximize the lifetime utility. The housing decision, represented as $D_{it}^{H}$, is characterized by a range of values from -2 to 4, each representing a distinct decision. Renters have the option to continue renting ($D_{it}^{H} = 0$), while those without a default flag can choose to purchase a house ($D_{it}^{H} = 3$). On the other hand, homeowners have the choice to retain ownership ($D_{it}^{H} = 1$), default on their mortgage ($D_{it}^{H} = -1$), refinance the mortgage ($D_{it}^{H} = 4$), or become renters ($D_{it}^{H} = -2$), or move to a new house ($D_{it}^{H} = 2$). However, homeowners who face a moving shock have only two options: either to move or to default.

The optimization problem is then

$$V(S_{it}, \Theta_t) = \max_{A_{it}} U_{it}(S_{it}, A_{it}, \Theta_t), \tag{14}$$

subject to

$$Q_{it}(S_{it}, \Theta_t) = W_{it} - M_{it}^{m} + D_{it}^{o} \max(H_{it} P_t^{h} - \alpha_{it}^{rm} M_{it}^{rm}, 0), \tag{15}$$

$$\begin{aligned} C_{it}(S_{it}, A_{it}, \Theta_t) = &\left[Y_{it} - \tau(Y_{it}) - \phi_c(S_{it}, A_{it}, \Theta_t) - \left(M_{it}^{m} - (1 - r_{it}^{m}) M_{it+1}^{m}\right)\right] \\ &+ \left[\frac{W_{it} - W_{it+1}}{1 + r^{f}(\theta_t)}\right] + (D_{it+1}^{o} - D_{it}^{o}) P_t^{h} \\ &+ \left[\alpha_{it+1}^{rm} M_{it+1}^{rm} - \alpha_{it}^{rm}(1 + r_t^{rm}) M_{it}^{rm}\right], \end{aligned} \tag{16}$$

$$\phi_c(S_{it}, A_{it}, \Theta_t) = D_{it+1}^{o} C_{ma} + (1 - D_{it+1}^{o}) q + D_{it}^{m} C_{ms} + \mathbb{1}(D_{it}^{H} = 4) C_{re}, \tag{17}$$

$$C_{it}(S_{it}, A_{it}, \Theta_t) \geq C_m, \tag{18}$$

[10] For FRRM, $r_{it}^{rm}$ depends on the economic state when the RM originated and remains the same until the mortgage is paid off or refinanced; For ARRM, $r_{it}^{rm}$ varies across the mortgage term and only depends on the current rate.

$$D^o_{it+1} = \begin{cases} 1, & \text{if} \quad D^H_{it} > 0 \\ 0, & \text{if} \quad D^H_{it} \leq 0 \end{cases}, \tag{19}$$

$$D^d_{it+1} = \begin{cases} 1, & \text{if} \quad D^H_{it} = -1 \\ 0, & \text{if} \quad D^H_{it} \neq -1, D^d_{it} = 0 \\ 0 \quad \text{with probability } p_{md}, & \text{if} \quad D^H_{it} \neq -1, D^d_{it} = 1 \end{cases}, \tag{20}$$

$$M^m_{it+1} = 0, \quad \text{if} \quad D^H_{it} \leq 0, \tag{21}$$

$$\text{LTV constraint} \quad (7), \quad \text{if} \quad D^H_{it} = 2, 3, 4, \tag{22}$$

$$\text{Minimum payment} \quad (8), \quad \text{if} \quad D^H_{it} = 1, \tag{23}$$

$$M^{rm}_{it+1} = 0, \quad \text{if} \quad D^H_{it} \neq 1, \tag{24}$$

$$\text{RM maximum claim} \quad (11), \quad \text{if} \quad D^H_{it} = 1, a_{it} \geq R, M^{rm}_{it} = 0 \text{ and } (10) \tag{25}$$

$$\alpha^{rm}_{t+1} = 1, \quad \text{if} \quad D^{fix}_{it} = 1 \text{ or } D^{fix}_{it+1} = 1 \tag{26}$$

$$\text{RM balance accumulation} \quad (12), \quad \text{if} \quad D^H_{it} = 1, M^{rm}_{it} > 0 \tag{27}$$

$$\text{RM credit accumulation} \quad (13), \quad \text{if} \quad D^H_{it} = 1, M^{rm}_{it} > 0. \tag{28}$$

where $\phi_c(S_{it}, A_{it}, \Theta_t)$ is the total cost of housing (maintenance), moving and financial services (refinance), and we impose the budget constraint and a minimum level of consumption $C_m$.

## 3.3 Mortgage Lenders

Lenders are perfectly competitive and discount the pay-offs from loans using a projected pricing kernel. Following Rosenberg and Engle (2002), we assume that this projected pricing kernel depends on the current macroeconomic state and tomorrow's state, and is required to satisfy

$$E_t \left[\Lambda^a(\theta_t, \theta_{t+1})\right] = \frac{1}{1 + r^f(\theta_t) + \kappa^a}, \quad a = m, rm, \tag{29}$$

where $\kappa^m$ and $\kappa^{rm}$ are the expected risk premium for mortgages and RMs, respectively.[11]

Lenders hold the mortgages and RMs portfolio and receive the payments from borrowers in each

[11] Rosenberg and Engle (2002) show that a projected pricing kernel can be derived from a unique pricing kernel conditional on the specific asset classes' payoff.

period $t$. These payments depend on the borrowers' decisions ($A_{it}$) and the aggregate state ($\Theta_t$). Given the multi-period nature of the loan, lenders need to recursively compute the net present value of loans. The net present value of the expected payments from the mortgage originated to household $i$ is given by

$$\begin{aligned}\Pi^m(S_{it}, A_{it}, \Theta_t) =& \mathbb{1}(D_{it}^H = -2, 2, 4) M_{it}^m + \mathbb{1}(D_{it}^H = -1) \Upsilon H_{it} P_t^h \\ &+ \mathbb{1}(D_{it}^H = 1) \Big[ M_{it}^m - M_{it+1}^m (1 - r_{it+1}^m) + E_t \big[ \Lambda_{t,t+1}^m p_{a_{it}} \Pi^m(S_{it+1}, A_{it+1}, \Theta_{t+1}) \\ &+ \Lambda_{t,t+1}^m (1 - p_{a_{it}}) \min(M_{it+1}^m, W_{it+1} + H_{it+1} P_{t+1}^h) \big] \Big] \end{aligned} \tag{30}$$

where $\Upsilon$ is the foreclosure sale recovery rate in case the household defaults on the mortgage. Equation (30) consists of the terms that represent the expected benefits in different scenarios. The first two terms represent the payment received in period $t$, if the households decide not to retain their ownership. The next term represents the payment received, if the households choose to remain in their homes, including the mortgage payment in period $t$ and the expected payoff in the next period.

Similarly, lenders calculate the net present value $\Pi^{rm}(S_{it}, A_{it}, \Theta_t)$ of the expected payments from the RM originated to household $i$,

$$\begin{aligned}\Pi^{rm}(S_{it}, A_{it}, \Theta_t) =& \mathbb{1}(D_{it}^H = -2, 2) \min(\alpha_{it}^{rm} M_{it}^{rm}, H_{it} P_t^h) + \mathbb{1}(D_{it}^H = 1) \Big[ -\alpha_{it+1}^{rm} M_{it+1}^{rm} \\ &+ \alpha_{it}^{rm} M_{it}^{rm} (1 + r_{it}^{rm} + c_a^{rm}) + E_t \big[ \Lambda_{t,t+1}^{rm} p_{a_{it}} \Pi^{rm}(S_{it+1}, A_{it+1}, \Theta_{t+1}) \\ &+ \Lambda_{t,t+1}^{rm} (1 - p_{a_{it}}) \min(\alpha_{it+1}^{rm} M_{it+1}^{rm}, H_{it+1} P_{t+1}^h) \big] \Big]. \end{aligned} \tag{31}$$

Equation (31) also considers two scenarios: one where households choose not to retain ownership, and the other where households continue to live in their homes.

## 3.4 Equilibrium

The economy is characterized by the law of motion for the aggregate state $\Theta_t = \{\theta_t, r^f(\theta_t), P_t^h, r^m(\theta_t), r^{rm}(\theta_t), \Pi_t\}$. Households choose their consumption and housing decisions ($A_{it}$) to maximize their

expected lifetime utility, given their expectations on future housing prices.

Solving the equilibrium requires households to forecast the law of motion for $\Theta_t$, which includes an infinite-dimensional object $\Pi_t$. Except for housing price, the law of motion for the other components of $\Theta_t$ depends on $\theta_t$, which follows a Markov process with exogenous transition probabilities. For the law of motion for house prices, we follow Krusell and Smith (1998) and consider a forecasting model where households use a simple AR(1) forecast rule based on the current and next-period economic states:

$$\ln P^h_{t+1} = f_{(\theta_t,\theta_{t+1})}(\ln P^h_t). \tag{32}$$

We parameterize $f$ as a linear spline:

$$f_{(\theta_t,\theta_{t+1})}(x) = b_h + \sum_{i=1}^{n_h} k_{ih}(x - x_i), \tag{33}$$

where $b_h, k_{ih}$ depends on $(\theta_t, \theta_{t+1})$, and $x_i$ are the knots of the spline.

Meanwhile, mortgage lenders reach a break-even point with the mortgage and RM rates:

$$E_{\theta_t=\theta_i}\left[E_{S_{jt}\sim\Pi^{orig}_{m,t}}\left[\Lambda^m_{t,t+1}\Pi^m(S_{jt+1}, A_{jt+1}, \Theta_{t+1}) - (1 - r^m(\theta_t))M^m_{jt+1}\right]\right] = 0, \forall\theta_i, \tag{34}$$

$$E_{\theta_t=\theta_i}\left[E_{S_{jt}\sim\Pi^{orig}_{rm,t}}\left[\Lambda^{rm}_{t,t+1}\Pi^{rm}(S_{jt+1}, A_{jt+1}, \Theta_{t+1}) - \frac{\alpha^{rm}_{jt+1}M^{rm}_{jt+1}}{1 + r^{rm}(\theta_t) + c^{rm}_a}\right]\right] = 0, \forall\theta_i, \tag{35}$$

where $\Pi^{orig}_{m,t}$ and $\Pi^{orig}_{rm,t}$ are the distribution of newly originated mortgages and RMs in period $t$, respectively.

Given the optimal decisions, the housing market clears at $P^h_t$. Since the supply of houses is fixed, the equilibrium is achieved when the supply from movers, defaulters, and investors who bought last period equals the demand from renters, homeowners who are moving, and investors:

$$\int D^o_{it} d\Pi_t = \int D^o_{it+1} d\Pi_{t+1}. \tag{36}$$

Household expectations for housing prices can be verified in equilibrium.

We solve the equilibrium numerically. Let the endogenous parameters of the equilibrium be denoted as $\Xi = (b_h, k_{ih}, r^m, r^{rm})^T$. We follow the following steps:

i) Initialize $\Xi$ as $\Xi_0$.

ii) Solve the household's optimization problem under $\Xi_{n-1}$.

iii) Simulate an economy with a population size of 100,000 for 19,000 periods with the home price decided by (36) each period.[12]

iv) Estimate $\Xi$ as $\Xi_n$, for $n = 1, 2, \cdots$, by (34) , (35) and (33) based on the simulation results.

v) Terminate if:

$$||\Xi_n - \Xi_{n-1}||_\infty < \epsilon. \tag{37}$$

# 4 Calibration

We use several data sources to calibrate our baseline model. First, we calibrate the stand-alone parameters that can be clearly identified without explicitly referring to our model setting. For example, mortality and survival probabilities are exogenous variables that do not depend on our model. Therefore, we can adopt statistics published by the National Center for Health Statistics to parameterize the conditional survival probabilities. Second, we calibrate a number of parameters to standard values from existing household finance literature, such as the labor income process. Lastly, we estimate the remaining parameters to align with specific moments of the empirical data, ensuring an overall goodness-of-fit between the model and empirical observations. Table 1 summarizes the calibrated parameter values.

**Aggregate State Variables** We first choose aggregate and idiosyncratic shocks that mimic the dynamics of modern business cycles in the US. Then we exogenously calibrate a set of parameters to widely accepted values in the macroeconomic and housing literature.

Following Guren et al. (2021), we calibrate the Markov transition matrix between macroeconomic states such as crisis, recession, and expansion based on the frequency and duration of such events defined by NBER.[13] If the NBER peak of the previous expansion takes place in the first

[12]If a household dies, a new household aged 20 with no assets will enter the economy.

[13]Please see Appendix B for the full transition matrix.

## Table 1: Baseline Calibration Parameters

The table shows calibrated parameters for the baseline model. Panel A shows the preference parameters. Panel B shows the mortgage-related parameters, and panel C shows the parameters associated with reverse mortgages. Panel D shows the parameters of labor income process. Panel E presents the remaining parameters.

| **Description** | **Parameter value** |
|---|---|
| *Panel A. Preference parameters* | |
| Years in life ($T$) | 80 |
| Risk Aversion ($\gamma$) | 3.0 |
| Bequest motive shifter ($\xi$) | 0.5 |
| Bequest motive multiplier ($\psi^{bq}$) | 250 |
| Utility from homeownership ($u_o$) | 10 |
| Discount factor ($\beta$) | 0.98 |
| Default penalty ($u_d$) | 10 |
| *Panel B. Mortgages* | |
| Max LTV, tight credit ($\phi^m$) | 0.8 |
| Max LTV, loose credit ($\phi^m$) | 0.95 |
| Variable refinance cost rate ($c_{re}$) | 1.0% |
| Fixed refinance cost ($k_{re}$) | 0.04 |
| Prob. of default flag removed ($p_{md}$) | 0.1 |
| Foreclosure sale recovery ($\Upsilon$) | 0.654 |
| *Panel C. Reverse mortgages* | |
| Min Equity Req. in crisis/recession, RM ($\phi^{rm}_{own}$) | 0.60 |
| Min Equity Req. in expansion, RM ($\phi^{rm}_{own}$) | 0.50 |
| Initial fee of RM ($c^{rm}_0$) | 0.02 |
| Annual fee of RM ($c^{rm}_a$) | 0.01 |
| *Panel D. Labor income* | |
| Retirement age ($R$) | 62 |
| Log income decline in retirement ($\lambda$) | 0.35 |
| Tax rate ($\tau$) | 0.2 |
| *Panel E. Other parameters* | |
| Homeownership rate ($H_r$) | 0.65 |
| Risk-free rate ($r^f$) | $[1.26\%, 2.32\%, 4.26\%]$ |
| Medical cost rate ($c_{med}$) (Age $[62, 85)$) | 0.15 (good), 0.2 (bad) |
| Medical cost rate ($c_{med}$) (Age $[85, 100)$) | 0.25 (good), 0.35 (bad) |
| Rent cost ($q$) | 0.20 |
| Maintenance cost rate ($c_{ma}$) | 0.025 |
| Variable moving cost rate ($c_{ms}$) | 3.0% |
| Fixed moving cost ($k_{ms}$) | 0.1 |
| Prob. of moving, working ($p_{ms}$) | 0.1 |
| Prob. of moving, retiring ($p_{ms}$) | 0.02 |
| Lower bound per-period cash-on-hand | 0.1 |

half of a given year, that year is classified as the first year of the new recession, whereas if the peak happens during the second half of a year, the recession follows in the subsequent year. The ending date of a recession is defined as the year following the start year of the expansion announced by the NBER because the unemployment rate is a lagging variable and does not immediately fall after NBER troughs. Moreover, we assume crises happen every 75 years, and all other NBER recessions are regular, cyclical recessions.

**Projected Pricing Kernel, Mortgage Spread and RM Spread** We calibrate the lender's projected pricing kernel to match the interest rate, mortgage spread and RM spread. Following Backus et al. (2011) and Guren et al. (2021) that calibrate the projected pricing kernel using options market data, we set the risk price for the crisis state to be 6.1 times that of non-crisis states. Additionally, we assume the projected pricing kernel remains constant when the economy switches from non-expansion to any other state and is the same as when the economy transitions from expansion to any other non-crisis state.[14] By doing so, we ensure lenders charge a fair premium for insurance against crisis states.

Lenders impose a requirement that guarantees a certain return on investment for a certain payoff of one unit at date $t+1$. This return is determined by adding the cost of making mortgage loans (RMs), denoted as $\kappa^m$ ($\kappa^{rm}$), to the risk-free rate $r^f(\theta_t)$. In this case, we have set $\kappa^m$ to 125 basis points (bps) to ensure that the average difference between the FRM rate and a 10-year bond is 1.65% and $\kappa^{rm}$ to 175 bps to target the RM spread of 2.0%.

To determine the risk-free rate, we calibrate it based on the historical real interest rate observed between 1985 and 2022. Specifically, during recessions, the risk-free rate is set at 2.32%, reflecting the lower economic activity and market conditions. During expansions, when the economy is performing well, the risk-free rate is set at 4.26%. In the case of a crisis, we assume a significantly lower risk-free rate of 1.26% to account for the heightened uncertainty and economic instability.

**Labor Income Process** We set the replacement ratio during retirement to be 0.68, and for income shocks in the working period, we rely on the estimates in Guvenen et al. (2014) and Shen (2023), which calibrate a quantitative labor income model using a large and confidential US

[14]For detailed procedure, please see Backus et al. (2011) and Guren et al. (2021).

dataset. The parameters with respect to the mixture of normal distributions during expansions and recessions can be calibrated. We then follow Guren et al. (2021) to discretize the distributions. The tax rate is assumed to be 20%, which is around the average of the current tax scheme.

**RMs** Except for the RM rates, we calibrate all other RM-related parameters and age-dependent collateral constraints based on the existing RM contracts from Corelogic and HECM. We find 0.5% for the insurance premium and 1% for the annual fee. We also find that the minimum equity requirement during crisis and recession is 60%, which is higher than that during the expansion of 50%. Table 2 shows the PLF for various RM rates taken from HECM.[15] For example, for the age 62 and RM rate of 3%, homeowners can borrow a maximum of 52.4% of their home equity. Unlike conventional mortgages, the loan amounts granted in RMs depend on the borrowers' ages, increasing as they age. The rationale behind this is that as borrowers grow older, the projected interest expenses decrease, and the anticipated future house price appreciation diminishes, reducing the risk for the lenders. Generally, the collateral constraint related to conventional mortgages becomes more stringent as the borrower ages. However, RMs provide a more relaxed collateral constraint that eases with age, resulting in a more favorable borrowing environment across all life stages.

**Table 2: Principal Limit Factor for RMs**

| | Age | | | | | | |
|---|---|---|---|---|---|---|---|
| RM rate | 62 | 70 | 75 | 80 | 85 | 90 | 95 |
| 3% | 52.4% | 57.6% | 60.9% | 64.2% | 68.5% | 73.0% | 75.0% |
| 4% | 47.0% | 52.2% | 54.7% | 58.5% | 63.6% | 69.1% | 75.0% |
| 6% | 35.7% | 41.5% | 44.3% | 48.8% | 54.9% | 61.8% | 70.1% |

This table states the RM rates and principal limit factor (PLF, which is essentially the loan to home equity ratio) that lenders are willing to lend to RM borrowers by various age groups and at three different RM rates levels: 3%, 4%, 6%.

**Health Status and Medical Cost** We calibrate the health status and medical costs using the HRS. More specifically, we estimate transition probabilities of household health status, conditional

[15]Please refer to HECM guidelines for the complete table, and we assume the expected interest rate used in HECM is the same as RM rate, for simplicity.

on age and current health. We find that health status is generally persistent, especially for households that are sick. Therefore, we assume once households are sick, they will stay sick for the rest of their lives. Moreover, the probability of transitioning from a good to a bad state is assumed to be 0.2 in each period. Then, we estimate parameters of the distribution of medical costs, conditional on age and health status. Not surprisingly, medical costs tend to be higher for older and less healthy households.

**Maintenance and Moving** We take the annual maintenance cost from Guren et al. (2021) and assume it is the same for both workers and retirees for simplicity, although we acknowledge that retirees usually face relatively lower maintenance costs.[16] We study the moving decisions of workers using the PSID and of retirees using the HRS data. We estimate probabilities of moving, and we find that workers are often more likely to relocate compared to retirees, which might be due to career opportunities, more flexibility, and less preference for "aging in place". Following Guren et al. (2021), we set the fixed cost of moving to be equivalent to 10% of the average household annual income, approximately amounting to $5,000. Additionally, proportional costs, incurred by both buyers and sellers, are set at 3% of the house value to account for expenses such as closing costs and realtor fees.

**Preferences** We estimate the preference parameters related to households' consumption, financial and housing decisions using the method of simulated moments (MSM). In particular, we use the data from the American Housing Survey (AHS) and the SCF, and we search for the set of parameters that produce simulated life-cycle decision profiles closely aligning with those observed in the empirical data. The mechanics of our MSM approach are fairly standard. We compute life-cycle histories for a large number of artificial households. Each of these households is endowed with a specific value for the state vector, and a series of idiosyncratic shocks are assigned to them in a manner consistent with the stochastic processes explicated in Section 3.

To facilitate the computational process, we discretize the state space and employ value function iteration to solve the model numerically. This iterative procedure enables us to derive a set of

[16]Nakajima and Telyukova (2017) calibrated the annual maintenance cost for retirees to be 1.7%, based on the depreciation rate of residential capital. Cocco and Lopes (2020) propose maintenance expenses among retirees for the different permanent income groups and for different ages.

decision rules that govern the choices made by the households. By combining these decision rules with the simulated endowments and shocks, we are able to simulate various aspects of household behavior, including wealth accumulation, housing decisions, refinancing, and the decision to take up RMs. Subsequently, we compute age profiles based on the artificial household histories, employing the same methodology used to analyse the actual data. We adjust until the difference between the data and simulated profiles is minimized as follows

$$(\hat{m} - m)' W (\hat{m} - m), \tag{38}$$

where $\hat{m}$ refers to the simulated moments, $m$ refers to the targets, and $W$ is the inverse of the covariance matrix of the empirical moments, which is estimated by bootstrapping the true data.

Our target moments are the age profiles of homeownership rates and default rate. We find that the estimated discount factor $\beta$ is 0.98 and the coefficient of relative risk aversion is 3, which both fall within the reasonable range in the literature.[17] Moreover, to match the ratio of total net worth at age 60 to net worth at age 45 in the SCF, we calibrate the bequest motive shifter to 0.5 and the bequest motive multiplier to 250.

**Other Parameters** Household refinancing processes are associated with fixed and variable costs. Specifically, following Guren et al. (2021), refinancing entails a fixed cost equal to 4% of the average annual income, roughly equating to $2,000, along with a variable cost equivalent to 1% of the remaining mortgage balance. This variable cost aligns with the average refinancing costs reported by the Federal Reserve. For renters, a monthly rent payment is set at 20% of their income to maintain a rent-to-income ratio of 20%. To be consistent with the average homeownership of 65% in the AHS, we assign a utility benefit of 10 to owning a home.

Following a foreclosure event, we assume that the bank recovers 65% of the property's market value. Additionally, we consider maximum LTV is 95% under loose credit and 80% under tight credit. To replicate the observed foreclosure rate of 8% in the housing stock between 2006 and 2013, we set the default penalty to 10. We assume that the probability of default flag being

[17] Yogo (2016) uses Epstein-Zin preferences to study the portfolio choice in retirement with health risk and derives the risk aversion coefficient to be 5. Nakajima and Telyukova (2020) only focus on retirees aged 62 and above and calibrate the risk aversion to be 2 to fit the age profiles in the HRS.

removed is 0.1.

# 5 Quantitative Analysis

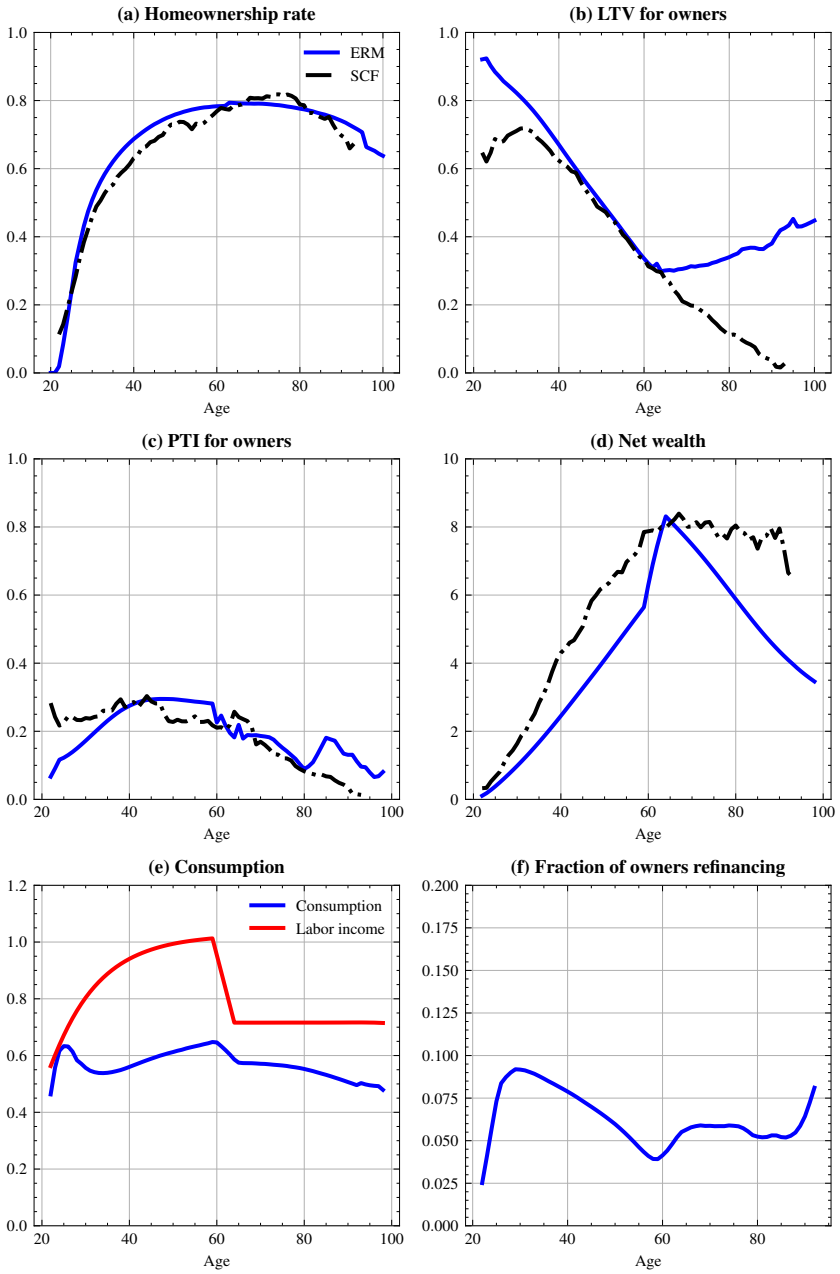
This section investigates whether the model can quantitatively match homeownership, wealth accumulation (decumulation), and other life-cycle patterns. Subsequently, we examine both the volatility of consumption growth and the RM take-up rate. Finally, we conduct an experiment in which the RM-related costs are reduced and explore these effects on the take-up rate.

## 5.1 Life-cycle Profiles

We simulate the decisions of saving, consumption, housing, and RMs of 100,000 households over the life cycle and present the average profiles in Figure 3. For comparison, we also report the average profiles in the SCF from 2007 to 2019. The model matches the life-cycle patterns in the SCF data well.

In graph A of Figure 3, we present the homeownership rate patterns over the life cycle. The model's simulations yield an average homeownership rate of 65%, which aligns with the data. However, we observe that the model tends to underestimate the homeownership rate for very young households and overestimate it for middle-aged households. This is possibly due to the existence of RM in the model: very young households are penalized financially, such that they cannot get on the property ladder, whereas among the rest of the life cycle, homeowners tend to hold on to property homeownership due to the benefit of RMs.

Graphs B and C of Figure 3 show the LTV and PTI ratio by age. Our model effectively captures the LTV ratios for young homeowners; however, it tends to overpredict the LTV ratios for older homeowners and increases steadily during retirement. This discrepancy can be primarily attributed to including RMs in our analysis. RMs allow older homeowners to borrow against their home equity, which increases their available funds without the need to sell the property. When older homeowners opt for RMs, they receive funds based on the accumulated value of their homes. This influx of borrowed funds increases the overall value of available resources for these homeowners, resulting in a higher loan amount in the LTV ratio calculation. At the same time, since




## Figure 3: Life-cycle Profiles

This figure plots key variables of model results (blue line) in comparison to empirical evidence (black dashed line) over the household life cycle. Graph A plots homeownership rate results; Graph B plots the results of Loan to Value (LTV) for homeowners; Graph C plots Payment to Income ratios (PTI) for homeowners; Graph D plots net wealth for households results; Graph E plots household consumption (blue line) and labor income (red line); Graph F plots the homeowner refinancing rates over the life cycle.

RMs do not require regular repayments like conventional mortgages, the property value remains relatively unchanged. As a result, the LTV ratio for older homeowners increases, indicating a higher proportion of borrowing relative to the value of their homes.[18] Moreover, we observe a close alignment between the PTI ratios and the data for the elderly population, in contrast to the results presented in the study by Guren et al. (2021). This is because older homeowners have access to RMs, which serve as an additional source of funds for mortgage repayment. Consequently, their PTI ratios decrease as this supplementary income from RMs helps to alleviate the burden of mortgage repayments. This contrast in findings highlights the significance of considering the impact of RMs on the PTI ratios of older homeowners.

Figure 3, graph D, shows the wealth accumulation over the life cycle. We find that households are liquidity-constrained during the first 15 years of their working lives. However, as time progresses, households start accumulating wealth at an accelerated pace. This wealth accumulation serves as a crucial form of insurance, providing a cushion against adverse labor income shocks and uncertainties in the macroeconomic environment. During retirement, we observe a rapid decumulation of wealth. This decumulation reflects the utilization of accumulated assets to support living expenses and maintain the desired standard of living during the retirement phase. Overall, the dynamic patterns of wealth accumulation and decumulation throughout the life cycle emphasize the significance of building a robust financial foundation and managing resources effectively to address income fluctuations and retirement needs.

Figure 3, graph E, shows the profiles of consumption and income. Consumption closely tracks income over the life cycle. During the working years, households permanent income increases as households age, leading to a corresponding rise in consumption. As households approach retirement, the income profile becomes less steep and the consumption profile reaching a point where further increases cease. This pattern suggests that as households advance in age, they become less constrained by liquidity concerns. During retirement, we observe a downward slope in the consumption path, as households mainly rely on their accumulated wealth and retirement income to sustain their desired lifestyle. The reduced income during retirement necessitates a

[18] Another reason why the model overpredicts the LTV ratios during retirement is that it does not consider downsizing. Incorporating downsizing would reduce the LTV ratios, but it would complicate the numerical computation. Therefore, we have not included downsizing in our baseline model.

more prudent approach to consumption.

Figure 3, graph F, shows the fraction of owners refinancing. Refinancing is relatively low across all ages. Compared with Guren et al. (2021), our model does not generate jumps at retirement, as homeowners have additional options to smooth their consumption and manage their financial obligations. The availability of RMs serves as a supplementary avenue for homeowners to access their home equity and alleviate liquidity constraints, reducing the need for traditional refinancing. By considering the impact of RMs on homeowners' financial strategies, our model provides insights into a more nuanced understanding of refinancing behavior, highlighting the role of alternative mechanisms in managing liquidity.

Figure 4 displays the proportion of RM borrowers opting for the FRRMs over age. we show that when borrowers age, fewer and fewer borrowers will opt for the FRRMs. FRRMs generally have slightly higher interest rates compared to ARRMs. Therefore, older borrowers may choose ARRM to take advantage of potentially lower interest rates. Another consideration could be that FRRM typically has a fixed lump sum payment, which may not be the most suitable option for older borrowers who do not require a large upfront sum of money. In contrast, ARRM offers a line of credit option, which provides more flexibility in accessing funds as needed. Moreover, on average, 24.34% choose lump sum, while 75.66% select the line of credit option in our model. However, based on the 2021 data, 89.44% of borrowers choose the line of credit, with only 6.82% opting for the lump sum. The model predicts a higher rate of lump sum selection due to excluding two other payment options: tenure and term.[19] Some borrowers in our model who take lump sum might take tenure or term if these two options are available.

## 5.2 Consumption Smoothing and Take-up Rate

Table 3, Panel A, reports the cross-sectional standard deviations of consumption growth for workers and retirees. On average, households who borrow RMs exhibit lower volatility in consumption growth throughout their life cycle. Interestingly, for workers, which has often been overlooked in previous literature on RMs, the availability of RMs as an option for accessing home equity leads to a substantial reduction in the volatility of consumption growth. Moreover, the presence of RMs

[19]Tenure RMs allow borrowers to receive a fixed monthly payout until households die or move out; Term RMs allow borrowers to receive a fixed amount of money during a fixed length of time.

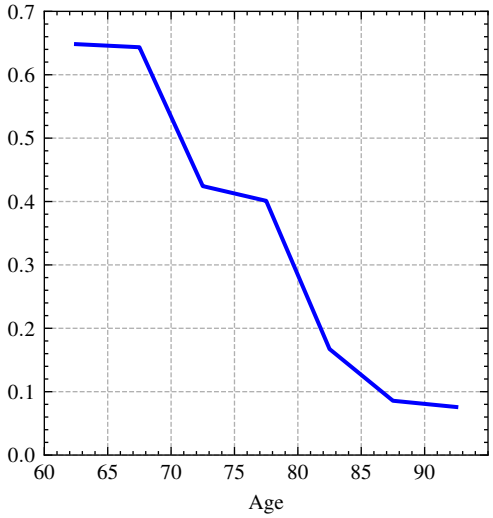



**Figure 4: Fraction of Households taking FRRM**

This figure shows the fractions of households that take FRRM gradually falls as household age, ranging from 65% at age 62 to 8% at age 95. The beginning of the line holds relatively constant for 5 years in the 60s, 70s, and 90s while falling evenly over the rest of the curve.

serves the role to narrow the disparity in consumption growth volatility between working households and retirees. These findings underscore the significance of RMs in improving consumption smoothing and bridging the gap between different household segments.

Table 3, Panel B, shows the take-up rate of RMs across the age groups. The average take-up rate is 4.5%, which is close to the average take-up rate of 4.3% from the data. Moreover, it is not surprising that the take-up is higher among older households, since they are less likely to meet the HELOC or other similar loan constraints, which makes a reverse mortgage attractive.

Figure 5 shows the age distribution histograms of households at the time they signed up for a reverse mortgage, among all households currently holding a reverse mortgage in the model. We find a hump-shaped pattern in RM participation rates throughout the life cycle after an initial spike. Homeowners take up RMs as soon as they are eligible to do so. After that, when households make the decision to originate RMs, they face a trade-off between an immediate benefit of a cash payout and an increasing PLF as they age. Borrowing RMs earlier in life allows households to benefit from it for a longer period during their life cycle; however, they are eligible to extract less home equity due to the lower PLF. These considerations result in the hump-shaped pattern

**Table 3: Volatility of Consumption Growth and Take-up Rates**

| *Panel A: Volatility of Consumption Growth* | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Age Groups | Households without RMs | | | Households with RMs | | | Difference | |
| $< 62$ | 0.851 | | | 0.706 | | | -17.04% | |
| $\geq 62$ | 0.323 | | | 0.296 | | | -8.36% | |
| *Panel B: Take-up Rate of RMs for Owners* | | | | | | | | |
| Age groups | [62,65] | [66,70] | [71,75] | [76,80] | [81,85] | [86,90] | [91,95] | >95 |
| Take-up Rate | 2.21% | 5.04% | 5.16% | 4.87% | 4.92% | 5.03% | 5.54% | 0.00% |

This table shows the model results of two economies (the default economy with RM v.s. alternative economy without RM). Panel A presents volatility consumption growth by two age groups: (1) Pre-retirement (<62), and (2) Post-retirement (>=62), as well as the differences between the two economies. Panel B presents RM take-up rates for homeowners for different age groups.

of RM take-up rates across age groups. Our model generates results consistent with the data. Figure 6 shows empirical histograms of the age distribution of RM borrowers when they first sign RMs, which is taken from Shan (2011). The data suggest that this spike has been more obvious in recent years (graph B). Overall, our model generates the results consistent with the data.

## 5.3 Mortgages and RMs

A significant aspect of our analysis, distinguishing it from previous literature, lies in the equilibrium determination of conventional mortgage rates and RM rates. By considering the equilibrium determination of these factors, we obtain a holistic understanding of the interplay between mortgage rates, RM rates and credit constraints.

Table 4 shows the conventional mortgage rate, RM rate and corresponding spread under different aggregate states. First, mortgage rates are influenced by the prevailing economic conditions and credit constraints. During the crisis, the mortgage rates touch their lowest point, which aligns with our expectations. This is in line with the policy objectives to stimulate the economy and promote borrowing by keeping interest rates low. As the economy transitions from the bottom of the cycle to a recession with loose credit conditions, mortgage rates increase as lenders loosen credit to bolster economic growth. Progressing into the expansion phase with loose credit, mortgage rates

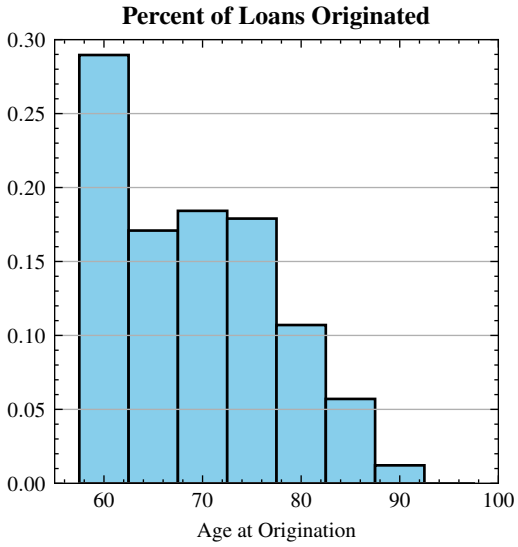



**Figure 5: Model Age Distribution of Borrowers at RM Origination**

This figure shows the histogram of the age distribution of RM borrowers at the time they sign up for RMs from the model.

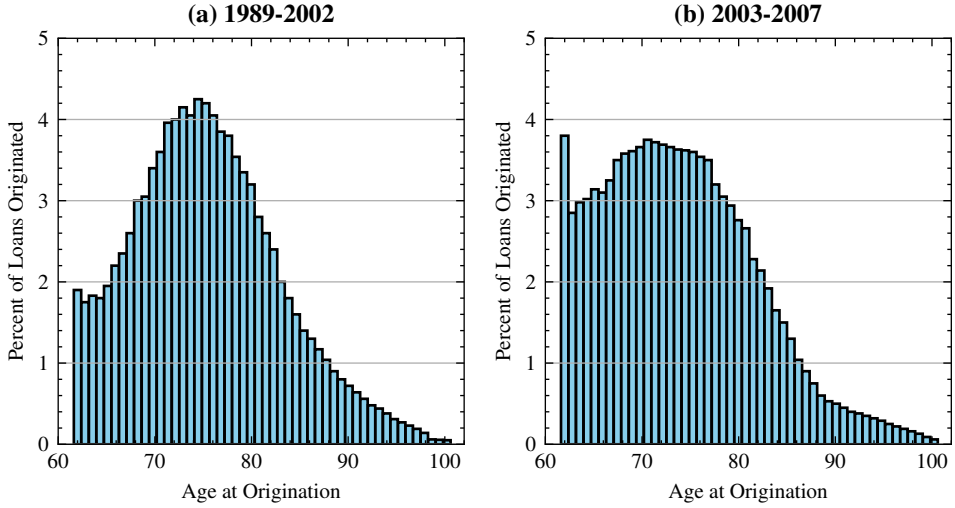



**Figure 6: Empirical Age Distribution of Borrowers at RM Origination**

This figure shows the distribution of borrowers' age at the inception of HECM programs from Shan (2011). Panel (a) shows the patterns of 1989-2002, and panel (b) shows the pattern of 2003-2007.

continue to rise as the economy strengthens. This increase is attributed to heightened demand for loans and increased economic activity, exerting upward pressure on interest rates. Conversely, during periods when credit constraints tighten in both recession and expansion phases, mortgage rates experience hikes. The most significant increase occurs during the expansion with tight credit constraints, indicating a shift towards tighter monetary policy to address economic overheating.

Then we find that RM rates consistently remain higher than conventional mortgage rates regardless of the aggregate states. Similar to the conventional mortgage, the RM rates reach their lowest point during the crisis. This aligns with the broader interest rate environment during crises, which aims to facilitate homeowners' access to their home equity at more favourable rates. Moreover, we find that the RM rates only depend on the macroeconomic environment and are irrelevant to credit constraints. This distinction can be attributed to the unique characteristics of RMs. Unlike conventional mortgages, RMs typically do not involve stringent credit checks or income qualifications. Instead, eligibility is primarily determined based on factors such as the homeowner's age, home value, and other specific criteria related to RMs. Furthermore, RMs are structured as non-recourse loans. This means that the borrower's liability is limited to the value of the home. When the loan balance exceeds the home's value upon sale, the insurer (government) absorbs the resulting loss instead of the borrower or their heirs and lenders. This inherent feature of RMs further mitigates the impact of credit constraints on RM rates. Considering these factors, it becomes evident that credit constraints play a minimal role in determining RM rates. Instead, RM rates primarily hinge on the prevailing macroeconomic environment, and the specific attributes and regulations associated with RM programs.

## 5.4 The Impact of RM Costs on RM Demand

In this section, we explore how the take-up rate for RMs is affected by their current terms, in response to the popular claim that the high costs of RMs are to blame. We consider RM take-up rate in a counterfactual model economy where all the costs related to the RMs are halved. Table 5 shows the take-up rate of RMs across the age groups. It is unsurprising to observe a significantly higher take-up rate under this hypothetical condition. This is consistent with previous studies such as Cocco and Lopes (2020) and Nakajima and Telyukova (2017), which have identified the high

**Table 4: Mortgage and RM Rates**

| | Mortgage | | RM | |
|---|---|---|---|---|
| | Rate | Spread | Rate | Spread |
| Crisis | 3.18% | 1.92% | 4.26% | 3.00% |
| Recession and tight | 4.28% | 1.96% | 4.32% | 2.00% |
| Expansion and tight | 5.12% | 0.86% | 5.26% | 1.00% |
| Recession and loose | 4.04% | 1.72% | 4.32% | 2.00% |
| Expansion and loose | 4.59% | 0.33% | 5.26% | 1.00% |

This table presents the equilibrium model results of mortgage and RM rates at 5 economic states (crisis, recession and loose credit, expansion and loose credit, recessions and tight credit, expansion and tight credit). The left column presents the mortgage rates and spreads above the risk-free rates, reflecting varying risk levels. The right column shows the RM rates and spreads, following similar patterns to the mortgage column.

costs associated with RMs as contributing to the low take-up rate. By considering the hypothetical reduction of all costs, our results further support the notion that reducing the financial barriers linked to RMs could lead to an increased adoption rate among homeowners. Additionally, we observe a hump-shaped pattern in the proportion of homeowners opting for RMs during retirement. The take-up rate increases initially but decreases when households reach the age of 71 and above. These older homeowners are more hesitant to access RM than their younger counterparts. This observation is consistent with the findings presented in Table 3.

**Table 5: Take-up Rate of RMs for Owners with Lower Costs**

| Age groups | [62,65] | [66,70] | [71,75] | [76,80] | [81,85] | [86,90] | [91,95] | >95 |
|---|---|---|---|---|---|---|---|---|
| Take-up Rate | 10.71% | 12.44% | 10.24% | 9.80% | 9.64% | 9.12% | 8.62% | 0.00% |

This table presents the RM take-up rates in a scenario with lower initial costs than the empirical evidence. There are eight age groups ranging from early retirement (age 62-65) to old age (>95). Results reflect the impacts of the initial cost of RM on the take-up rate of eligible households within all age groups.

# 6 An Alternative Economy without RMs

To assess the impact of RMs on the housing market and household welfare, we examine an economy without RMs in the credit market while keeping all other parameters constant, as in our RM-

inclusive economy.

## 6.1 House Prices

Exploring the implications of RMs on house prices is crucial, as it sheds light on the broader impact of RM programs on the overall housing market. We find that the inclusion of RMs in the credit market contributes to an overall increase in house prices across all aggregate states, enhancing the competitiveness of the housing market. This inclusion of RMs has both direct and indirect effects on housing prices.

One direct effect is that RMs allow older homeowners to access home equity without selling their homes. These facts reduce the supply of *available* housing for sale and limit the turnover of houses in the market, both exacerbating the scarcity of available homes and pushing prices higher. Besides, RMs can provide homeowners additional financial resources, allowing them to maintain their houses against depreciation. By doing so, they also avoid being forced to sell their houses. On the other hand, the availability of RMs as an additional funding source enhances the attractiveness of homeownership, thereby increasing the demand for houses. Consequently, both the supply and demand for houses shift, leading to a rise in house prices.

However, it is worth noting that there is notable heterogeneity among the aggregate states. Specifically, when credit conditions are tight, the house price experiences a relatively smaller increase compared to other states. This aligns with the expectation that under tight credit conditions, potential home buyers may face greater difficulties in obtaining mortgage financing. This can decrease overall demand for homes, including those financed through RMs. Lower demand alleviates the upward pressure on housing prices, resulting in a smaller price increase. Meanwhile, lenders tend to tighten their lending standards and criteria under tight credit. This means that borrowers, including those seeking RMs, may find it more challenging to qualify for loans. The reduced credit availability can limit the number of potential buyers utilizing RMs, thereby mitigating the impact on house prices.

**Table 6: House Prices**

| Econ state | Without RMs | With RMs | Change | No. of periods |
|---|---|---|---|---|
| Crisis | 3.70 | 3.77 | 1.90% | 60 |
| Recession and loose | 4.71 | 4.80 | 1.91% | 203 |
| Expansion and loose | 4.54 | 4.57 | 0.66% | 1015 |
| Recession and tight | 4.43 | 4.44 | 0.25% | 113 |
| Expansion and tight | 4.25 | 4.27 | 0.48% | 608 |

This table presents modeled real house prices at equilibrium in two economies (default economy with RM v.s. alternative economy without RM, and the difference between the two) in 5 economic states with crisis, recession, and expansion economic states, with varying loose and tight credit conditions. The first two columns present real house prices in the two alternative economies, the third column presents the difference between the first two columns, reflecting the real house price difference between the two economies, and the last column presents the number of periods in our entire simulation exercise.

## 6.2 Welfare Gains

When examining the presence of RMs, the assessment of welfare gain becomes important for comprehending the potential benefits and implications of these financial instruments. Table 7 reports the welfare gains associated with the availability of RMs. In our analysis here, we refer to the economy without RMs as Economy 0 and the economy with RMs as Economy 1. To quantify the household welfare gains, we calculate the consumption certainty equivalent variation. This represents the amount by which households would need to increase their consumption in Economy 0 to achieve the same level of satisfaction as in Economy 1. Specifically, we denote the value functions under Economy 0 and Economy 1 as $V^0$ and $V^1$, respectively. The consumption certainty equivalent variation $\Delta\left(S_{it}, \Sigma_t\right)$ is defined implicitly as

$$\begin{aligned} V^1\left(S_{it}, \Sigma_t\right) =& E_t\Bigg\{\sum_{n=t}^{t+T-a_{it}} \beta^{n-t} \prod_{m=t+1}^{n} p_m(a_{im}) \left[\frac{\left(C_n^0(1+\Delta(S_n,\Sigma_n))\right)^{1-\gamma_i}}{1-\gamma_i} + \alpha H_n^0 + D_{it}^o u_o - D_{it}^d u_d\right] \\ &+ \beta^{n-t}\left(1 - \prod_{m=t+1}^{n} p_m(a_{im})\right) \psi^{bq} \frac{\left(Q_{in}^0 + \xi\right)^{1-\gamma_i}}{1-\gamma_i}\Bigg\} \\ =& V^0\left(S_{it} \mid \Sigma_t\right) + \left[\left(1+\Delta\left(S_n \mid \Sigma_n\right)\right)^{1-\gamma_i} - 1\right] E_t\left\{\sum_{n=t}^{t+T-a_{it}} \beta^{n-t} \prod_{m=t+1}^{n} p_m(a_{im}) \left[\frac{\left(C_n^0\right)^{1-\gamma_i}}{1-\gamma_i}\right]\right\}, \end{aligned} \tag{39}$$

where $C_n^0$ and $D_{it}^h$ are the corresponding policies, and $Q_{in}^0$ is the terminal wealth under Economy 0. Consequently,

$$\Delta\left(S_{it} \mid \Theta_t\right) = \left(1 + \frac{V^1\left(S_{it} \mid \Sigma_t\right) - V^0\left(S_{it} \mid \Sigma_t\right)}{E_t\left\{\sum_{n=t}^{t+T-a_{it}} \beta^{n-t} \prod_{m=t+1}^{n} p_m(a_{im}) \left[\frac{(C_n^0)^{1-\gamma_i}}{1-\gamma_i}\right]\right\}}\right)^{\frac{1}{1-\sigma}} - 1 \tag{40}$$

We report the welfare gains for four groups of households: (1) Young, working homeowners, (2) Young, working renters, (3) Retired homeowners, (4) Retired renters. We find that the retired homeowners experience significant welfare gains. As homeowners age, welfare gains increase. On the other side, we also observe marginal improvement for the renters. While the rising house prices may create additional challenges for renters, particularly for young households who often aspire to become homeowners, an economy equipped with RMs can provide some advantages for renters, albeit modestly. This positive spillover effect on the rental market might come from the fact that renters still have the potential to become homeowners in the future and use RMs as an additional option for accessing home equity. Understanding the potential benefits of RMs allows renters who have the intention to become homeowners to incorporate this option into their long-term financial planning and take advantage of the financial stability that RMs offer.

We gain additional insights into the differences between the two economies from Table 7. Here, we show the cross-sectional standard deviations of consumption growth for households under two economies. In the economy where RMs are accessible, homeowners, regardless of whether they choose to take up RMs, experience lower volatility in consumption growth than in the economy where RMs are unavailable. This suggests that the presence of RMs has a general stabilizing effect on consumption patterns for homeowners. On the other hand, it is unsurprising that the consumption smoothing effect is subtle for renters as their non-participation in homeownership. The decrease in the volatility of consumption growth for both working and retired renters suggests a relatively modest change. Although renters may not directly benefit from the consumption-smoothing advantages attributed to RMs, their choices of homeownership in the later stage of their life cycle could play a role. This contributes to a small but noticeable impact on their volatility of consumption growth. Overall, we conclude that an economy with RMs offers potential benefits

for most population segments, including homeowners and renters.

**Table 7: Comparison between Two Economies**

| *Panel A: Welfare Gains* | | | | | | |
|---|---|---|---|---|---|---|
| | Homeowners | | | Renters | | |
| Age groups | <62 | [62,75] | >75 | <62 | [62,75] | >75 |
| Welfare Gains | 1.29% | 4.77% | 7.47% | 0.18% | 0.31% | 0.76% |
| *Panel B: Volatility of Consumption Growth* | | | | | | |
| | Homeowners | | | Renters | | |
| Age groups | Without RMs | With RMs | Change | Without RMs | With RMs | Change |
| $< 62$ | 0.741 | 0.695 | -6.21% | 0.914 | 0.910 | -0.44% |
| $\geq 62$ | 0.295 | 0.286 | -3.05% | 0.387 | 0.384 | -0.78% |

This table presents follows: welfare gains (shown in Panel A) and volatility consumption growth (Panel B) of homeowners and renters in two economies (the default economy with RM v.s. alternative economy without RM), by 3 age groups: (1) before retirement <62, (2) early retirement (62-75), and (3) well into retirement (>75), as well as the volatility of consumption growth differences between the two economies.

# 7 Conclusion

In this paper, we investigate the effects of RMs using a quantitative model in which households make decisions about saving, consumption, homeownership, and RMs over the life cycle. By incorporating the credit and housing markets into our model, we aim to provide a thorough understanding of how the presence of RMs influences both sectors.

Our model matches the lifecycle profiles observed in the data well, especially for the PTI ratio, the fraction of owners refinancing and the take-up rate. Meanwhile, RM rates consistently remain higher than mortgage rates. We also show that the volatility of consumption growth decreases for older households who opt for RMs compared to those who do not participate in RM programs. Additionally, our model demonstrates that the inclusion of RMs has a significant impact on house prices. As a result, the housing market becomes more competitive, making homeownership more appealing. While these dynamics present challenges for young renters, as they contend with rising

house prices, it is important to acknowledge their potential for future homeownership. In an economy with RMs, young renters have the opportunity to aspire to become homeowners and use RMs as an additional means to access home equity. Therefore, despite facing obstacles, young renters still enjoy marginal benefits in an economy that incorporates RMs. On the other hand, as expected, older homeowners experience significant welfare gains. In summary, an economy that includes RMs demonstrates overall superior total welfare benefits.

From a policy perspective, policymakers play a crucial role in promoting the understanding of RMs among households and minimizing the barriers that hinder access to them, all while maintaining an effective risk control framework. Encouraging older households to consider RMs carries multiple advantages, benefiting not only the households themselves but also the broader society. Additionally, such encouragement helps alleviate the strain on public pension schemes, reducing the related pressures and challenges.

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# Appendix A RM Markets in Global Context

Throughout RMs' development in the US, RMs have experienced several major milestones: The first FHA-insured Home Equity Conversion Mortgage (HECM) was issued in 1989, followed by the first major news that HECM would become a permanent product as established by the Housing and Urban Department (HUD) Appropriations Act in 1994. During the 2000s' housing market boom, demand for RM surged, such that homeowners took advantage of the housing value insurance embedded in the RM contract. At the same time, more lenders joined the RM game as HUD increased origination fees and engaged with the American Association of Retired Persons (AARP) to improve counselling services to boost the RM market. More rules were relaxed such that refinancing through existing HECM became possible. Therefore, RMs have become increasingly important for older adults in boosting their retirement incomes.

RM market is growing not only in the US, but also in several developed economies, such as the UK, Australia, and Hong Kong, as shown in Figure 7. Graphs A and C of Figure 7 show the number of new RM plans (including lump sums, draw downs, withdrawals, and home reversion plans) in the UK and Hong Kong, respectively. In comparison, graph B shows the active number of RM plans in any given year in Australia for the same period, allowing expired plans to drop out from the database. In addition, according to Review of Reverse Mortgage Lending Report in Australia, from 2013 to 2017, lenders issued more than 17,000 RM plans. Hence, the market has the potential to reach up to 58,000 active plans in 2017, as represented by the dashed line from 2013 to 2017.

All these economies share similar socio-economic, demographic shifting trend, pension system, as well as financial market advancement and consumer financial literacy level. Among these economies, UK has the most sophisticated and penetrated RM market in the world, defined by the product ranges and optional features such as repayment and withdrawal, number of lenders and consumers, as well as consumer awareness and product reputation and regulation. Since its inception in 1991, the RM market in the UK shows very healthy and significant growth trend, with several turning points. For example, Figure 7 shows that the first downturn of the UK RM market is caused by the GFC rooted by the subprime mortgage crisis in late 2007 in the US, causing a global housing market slump. The RM market gradually recovered since 2008 low point, and has

remained healthy growth, with a threefold size in 2016, due to major new product introduction. Another major turning point for the UK RM market is the COVID-19 crisis, which induced the prevailing extra-low interest rate regime during and after the crisis, boosting the housing market, causing a temporary contraction of the new lump sum plans in 2019, and growth of further advance contracts which can be observed in Figure 8.[20] Lump sum plans pick up the growth trend in 2020 during the living-cost crisis and stays as stable since then.

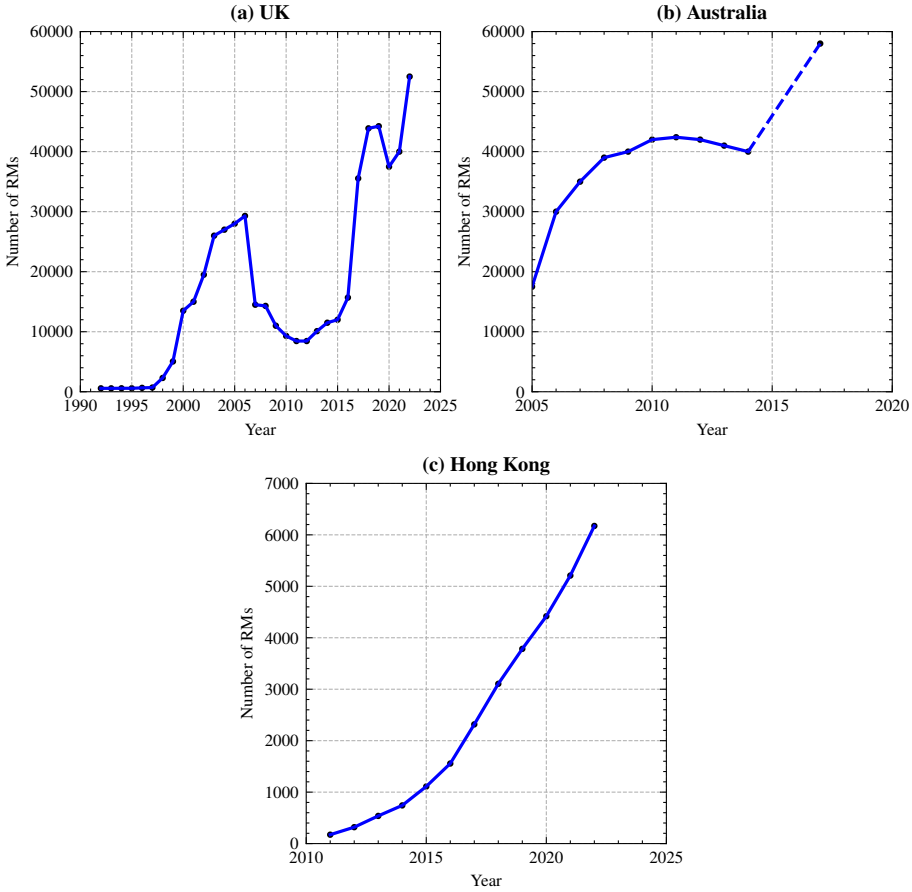



**Figure 7: Number of RMs in Different Markets**

Graph A plots the total number of RM plans in the UK from the early 1990s until now. Graph B plots the number of RM plans from 2005 to 2015 (continuous line) and shows a 5-year projection from 2015 onwards (dashed line). Graph C plots the total number of RM plans in Hong Kong from 2010s (the inception of the RM program in Hong Kong) until now.

Our estimate using the Equity Release Council (ERC) market report and Census data reveals

[20]Before 2005, UK did not report any other detailed information, except the total number of RMs.

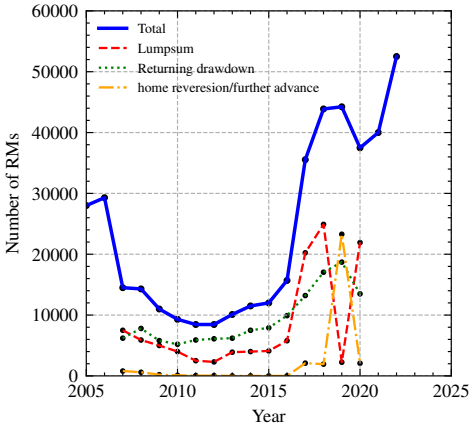



**Figure 8: Number of RMs of Different Types in UK Market**

This figure plots the number of RMs of different types such as (1) Lump sums (red dash line), (2) Returning drawdowns (green dash line), (3) Home reversion plans (yellow dash line). The total number of RM of all types (blue line).

that around 12% of eligible senior households have taken up RM in the UK, which is a relatively high and healthy number for this market and shows strong appetite by consumers and providers, thus a huge potential for this product to succeed in other countries in similar economic and social conditions.[21]

The case of Hong Kong is described in the Hong Kong Monetary Authority market reports. Although still small, the RM market has gained significant interest and growth since its inception in 2011. There are multiple factors contributing to this slow but upward growth trend: Firstly, the lack of social security (public pension plans) in Hong Kong forces senior households to consider alternative methods to increase retirement income. Secondly, the housing market boom in Hong Kong makes housing assets a significant part of a family's wealth. However, the homeownership rate in Hong Kong is low compared to other developed countries such as US and UK, due to the

[21]The recent UK RM take-up rate is estimated by the cumulative number of RM plans issued since its inception (1991-2022), e.g. according to ERC report, there are 592,000 RM plans issued during this period, divided by the number of eligible senior households who own their main house outright. To make this ratio comparable to the case of US and other countries, we use the number of households headed by someone 65+ by the report of Later Life in UK (2019), 6,500,000, with the homeowner ratios of 78%, and the fraction of those still paying mortgages 6%, this makes the RM eligible households to be 4,765,800. Hence the RM participation ratio is estimated to be 12% in 2022. However, if instead the denominator is the population of 65+, the number becomes significantly large, estimated to be 15,500,000 (ERC report 2022), then the RM take-up rate becomes 3.8%.

un-affordability of housing for local citizens, stated by housing price to income ratio of 23.3.[22]

Australian RM regulatory body SEQUAL uses a different estimation method from the US and UK. They calculate the RM participation rate using the population of suitable senior individuals instead of number of eligible households. This makes the estimated RM take-up rate lower and not comparable to those reported by the US and UK regulators or market participants. Deloitte (2021) estimate about half of collective wealth in Australia is tied up in housing, and it is skewed towards the senior population, that is, around AUD$1 trillion. Out of this housing wealth, AUD$300 billion is being accessed by RMs, which amounts to only about 1%-1.5% in 2021, although the market has grown by more than 5 times since its inception. Product ranges, consumer knowledge and brand reputation are not as strong as in the UK and US. The challenges of Australian RM market development is due to major lenders' unwillingness to offer such products, and hence only proprietary and niche lenders are willing to take the risk and offer such products. On the consumer side, the lack of consumer awareness, financial literacy, and product education also contributes to the existing challenges.

# Appendix B Numerical Solution

The model does not offer an analytical solution but is amenable to numerical resolution through a method known as backward induction. Both policy functions and value functions are contingent on the state variables. In the last period, the policy functions are determined by the bequest motive, and the value function corresponds to the bequest function. We use a grid search to optimise the value function. We compute the value associated with different control variables. Then we choose the level of control variables achieving the maximum value, which are saved as the policy rules for the previous period. For every time $t$ prior to $T$, and for each point in the state space, this procedure is iterated backward.

To approximate the distributions of innovations to the permanent labor income shocks, we use

[22] According to Research Office (2021) from Hong Kong, in 2019, homeownership ratio in Hong Kong fell to the lowest point in the past two decades, as 49.8%, following a nearly four-fold growth in flat prices during that period. Although homeownership rate modestly grows to 51.2% during 2020, it is still far below the prevailing 60% level in developed economies. Ernst and Young (2022) states that the median house price to median income ratio of Hong Kong in a 2021 survey is 23.3. Moreover, Cox (2023)) compares house to income ratios over 92 major metropolitan areas and shows that Hong Kong was the highest for consecutive 12 years.

numerical integration techniques. Specifically, our density function for permanent income shocks is reformulated as a combination of Hermite polynomials and the Gaussian kernel. This formulation allows us to leverage Gaussian quadrature points along with adjusted weights to approximate the numerical integrations effectively. In cases where points do not align with the state space grid, we utilize cubic spline interpolation to evaluate the value function. This interpolation method enables us to estimate the value function accurately at points that fall outside the grid. We consistently apply cubic spline interpolation for value function evaluation on the selected grids to ensure a smooth and precise estimation of values across the state space. As for the transition matrix among different economic states, we follow Guren et al. (2021) to set the parameters of economics transition. The transition matrix of the three economic states is given by

$$\mathbb{P}^e = \begin{array}{c} \\ \text{Crisis} \\ \text{Recess (l)} \\ \text{Expans (l)} \\ \text{Recess (t)} \\ \text{Expans (t)} \end{array} \left\| \begin{array}{ccccc} \text{Crisis} & \text{Recess (l)} & \text{Expans(l)} & \text{Recess (t)} & \text{Expans(t)} \\ 0.6364 & 0.0000 & 0.0000 & 0.0000 & 0.3636 \\ 0.0000 & 0.1011 & 0.8989 & 0.0000 & 0.0000 \\ 0.0133 & 0.1832 & 0.8035 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.1011 & 0.8989 \\ 0.0133 & 0.0000 & 0.0200 & 0.1832 & 0.7835 \end{array} \right\| \tag{41}$$

After the optimal policy rules are derived, we start simulating the life-cycle profile for each household. Once households die at age 100, they are dropped from the simulation. New 20-year-old households enter the labor market every year.