Optimal Sharecropping with Transaction Costs: An Application to Seafood Sector

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Abstract This paper challenges the long-standing view that sharecropping is not an inefficient contractual arrangement by introducing the concept of transaction costs. It develops a theoretical framework to analyze how these costs affect the optimal share rate in sharecropping and applies it to "share-fishing" in the seafood industry. The model shows that a landlord's share rate is influenced by factors such as transaction costs, land use returns, the number of tenants, and agri-food prices. When extending the model to account for climate uncertainty, it finds that tenants' specific investments and market structures further impact the landlord's share rate, with a similar investigation in share-fishing practices.

Keyword Crop Share, Optimal Price Control, Climate Risk Uncertainty, Share-fishing

1 Introduction

A large body of literature in economics holds that transaction costs are a major determinant of contractual choice (Coase, 1937; Coase, 1998). This literature provides evidence that transaction costs influence the efficient allocation of resources through various contractual arrangements (Williamson, 1985; Williamson, 2000). This paper revises the traditional sharecropping model by introducing transaction costs. I am particularly interested in how transaction costs determine the share rate under share contracts, and thus affect social welfare. For two decades I have been concerned with how transaction costs determine the choice of share contracts and how sharecropping affects efficient resource allocation. Transaction costs in share tenancy are tremendously critical (Cheung, 1969a), yet obviously the standard sharecropping model has largely overlooked the impacts of transaction costs in theoretical literature, with a few exceptions (Alston et al., 1984; Datta et al.,1986; Allen and Lueck, 1993). My model will address the following questions: (i) How are optimal share rates determined by transaction costs, and how do they affect social welfare? (ii) How does market structure affect the socially optimal share rate under climate uncertainty, and does climate change affect social welfare when transaction costs are considered? (iii) What are the theoretical implications for the share-harvesting and open-access equilibria?

The motivation of this paper is twofold: (i) To develop a simple sharecropping model with transaction costs, which corrects the fallacies in the traditional theory of share tenancy, including some theoretical extensions related to risk-sharing and incentives in sharecropping. (ii) To apply the model to seafood share-fishing, as it not only for extends our understanding of the impacts of transaction costs on share contracts but also provides deep insight into the far more complex conditions of share-harvesting in fisheries. My focus

is on the determinants of the optimal share rate and the comparison of social welfare under different constraints.

To be sure, "sharecropping economics" is an old but challenging field. In this paper, I provide a perspective centered on transaction costs. Since I emphasize transaction costs, there are parallels between how transaction costs appear in my model and how individually and socially optimal share rates appear in the sharecropping literature. Compared to the existing literature, the most distinctive characteristics of my model are:

(1) Transaction costs: in neoclassical models (Stiglitz, 1974; Burchardi et al, 2019), transaction costs in sharecropping are typically ignored; in my setup, introducing the landlord's contracting costs and tenant's specific investment is fundamental.

(2) Rent dissipation from share rate regulation and price control: my setup features share rate regulation and price control, which generate rent dissipation; in Ghatak and Mookherjee (2024) style model, typically the regulator only imposes a minimum share rate of the tenant as an exogeneous variable, while Basu and Emerson (2000) and Basu and Emerson (2003) style models do not consider rent dissipation in the tenancy rent control.

(3) Calibration of the social welfare function: my model that is applied to share-fishing contracts compares social welfare under a perfectly competitive market versus a non-competitive market; in Hämäläinen et al.(1986) and Hämäläinen et al.(1990) style share-fishing models, they just solve the social optimum harvest.

This paper proceeds by placing the theory into the literature and discussing its application in Section 2. In Section 3, I develop a simple transaction cost theoretical framework of sharecropping. The model is extended in Section 4, followed by the application of the theoretical models in Section 5. Section 6 concludes.

2 Literature Review

Apart from the empirical literature on the efficiency of sharecropping (Otsuka and Hayami, 1988), my paper builds more specifically on the theoretical literature in microeconomics that derives the optimal share rate from transaction cost theory. I explicitly assume that both landlords and tenants consider their transaction costs. In New Institutional Economics (NIE), share tenancy and fixed-rent contracts are alternative arrangements designed to save transaction costs. The choice between these alternatives has been discussed in the economic literature on contract theory (Taslim, 1992), from the classic theory of share tenancy by Johnson (1950) to a recent synthesis by Ghatak and Mookherjee (2024) that revises the traditional E-K model (Eswaran and Kotwal, 1985). Yet, in formal economic models, the determinants of contracting costs have been largely ignored. The prolific literature on contractual relations of governance has focused on the dimensions of transaction costs (Williamson, 1985), while studies on efficient contractual choice have mainly emphasized on trust in risk sharing (Cole et al., 2024) or the choice of equilibrium between vertical integration and non-integration (Acemoglu et al., 2009). However, the separate analyses on share contract choice and the determinants of transaction costs have resulted in both theoretical confusion and questionable interpretations of empirical data for fixed-rent contract (Jacoby, 2008), especially those drawing causal inferences about the ratchet effect on share contract choice (Allen and Lueck, 1999). This study aims to contribute to a "transaction cost theory of sharecropping", where the determinants of the optimal share rate are modelled together as substitutes along a continuous spectrum of contract choices within transaction cost theory, through my own theoretical analyses.

2.1 Relation to the Sharecropping Literature

My study relates and contributes to at least two strands of literature. The first pertains to the share tenancy theory in agricultural economics. For instance, Cheung (1968) argues that

under the condition of zero transaction costs, a share contract, like other forms of contracts, can result in efficient resource allocation. Cheung (1968) also demonstrates that the resource allocation implied by private property rights in a free market is the same whether the landlord leases his holdings on a fixed-rent basis, or shares the actual yield with his tenant. Specifically, "the rent per acre of land equals the marginal product of land in equilibrium under share contract, which is a condition identical to that of a fixed-rent contract" (Cheung, 1969b). Under a fixed-rent contract, the tenant alone controls the residual claim, while under a share contract, the residual claim is mutually shared by the landlord and tenant. From the perspective of the Coase Theorem, given zero transaction costs, the efficient outcome will be achieved by private negotiation, regardless of who controls the property rights (Coase, 1960). Hence, Cheung's sharecropping theory contains the first formal demonstration of the Coase Theorem (Allen and Lueck, 2018).

Furthermore, Cheung (1969b) argues that with positive transaction costs, the choice of contractual arrangement aims to maximize gains from risk dispersion, making share tenancy a device for risk dispersion. Based on it, Stiglitz (1974) points out that sharecropping is adopted for its incentive effects and risk-sharing features. While these studies focus on the risk-sharing function of share contracts, they overlook the impacts of transaction costs on optimal share rate. Although Cheung (1969b) acknowledges the importance of transaction costs in choosing share contracts, he does not incorporate them into the standard model. In the context of my paper, landlords' contracting costs and tenants' specific investments affect the optimal share rates. Thus, one of the contributions of my paper to this literature is to provide a theoretical framework for sharecropping that incorporates transaction costs.

My paper is most closely related to the efficiency and dynamics of sharecropping literature under specific conditions. Rao (1971) assumes that share contracts may be

influenced by the relative significance of entrepreneurial functions. In Bardhan and Srinivasan (1971), the rental share is regarded as a price-like variable, parametrically given to landlords and tenants. Contrary to my paper, Hsiao (1975) illustrates that share tenancy does not result in inefficient resource allocation. However, in a competitive world with no uncertainty, sharecropping is found to be Pareto efficient (Reid, 1976). Building on it, Reid (1977) shows that sharecropping provides sufficient incentives, leading to an efficient outcome. But Shaban (1987) supports the notion of the "Marshallian productive inefficiency" of sharecropping. In these papers, transaction costs are not considered, unlike in mine. In fact, the rental share can be derived through a bargaining game between landlords and tenants (Bell and Zusman, 1976). My paper introduces a non-cooperative game among tenants with transaction costs and has not yet to address the Nash bargaining solutions. Similar to my treatment of transaction costs, Lucas (1979) hypothesizes that monitoring costs are one of the determinants of sharecropping. Nevertheless, under zero transaction costs, the contracting solution aligns with the market solution where a market for the externality exists (Roumasset, 1979).

My findings follow the original treatment of uncertainty from Stiglitz (1974). Under the same framework of uncertainty, the studies explore various aspects of sharecropping and interlinked contracts, including the link between credit and sharecropping (Braverman and Stiglitz, 1981), the impact of sharecropping on efficiency, tenant behavior, and landlord profits (Braverman and Stiglitz, 1982), landlords' resistance to innovations under sharecropping (Braverman and Stiglitz, 1986), the inefficiency of equalizing cost and output shares under difficult labor monitoring conditions (Braverman and Stiglitz, 1986). In addition, other studies discuss the dominance of share tenancy under limited liability (Basu, 1992), kinship effect on sharecropping (Sadoulet et al., 1997), trade-offs between production and land quality under optimal contract (Dubois, 2002), variation in contract

terms based on tenant's unreported output (Janvry and Sadoulet, 2007), the effect of competing agents on Pareto-improving subset of sharecropping (Sen, 2011), inefficiency of sharecropping exacerbated by disincentives to investment in soil fertility and irrigation (Deininger et al., 2013), the effects of liquidity constraints (Das et al., 2019), the impact of information structure (He and Collins, 2021), and the effect of positional advantage of the first proposers (Bruno Gabriel Witzel de Souza, 2024).

2.2 Relation to the Applied Literature in Share-fishing and Seafood Harvesting

The main application of sharecropping is in fisheries, because share remuneration systems dominate the fishing industry (van Ginkel, 2013). For instance, building on his own sharecropping model (Sutinen, 1975), Sutinen (1979) illustrates that such systems are preferred by industry participants in fisheries, while Anderson (1982) argues that the share contract does not impact the economically efficient level of output. At the same time, Ferris and Plourde (1982) find that the effects of share-fishing remain ambiguous, though later, Plourde and Smith (1989) show that share contracts are optimal when both boat owners and crew are risk-averse. In addition, Platteau and Nugent (1992) analyze the prevalence of share contracts in fisheries, emphasizing their dominance in this sector. Also, McConnell and Price (2006) suggest that share contracts help mitigate team agency problems by providing effective incentives.

My predictions for optimal share rate of ship owner are consistent with the results from Nguyen and Leung (2009) indicating that the risk behavior of crew and owners plays a key role in determining the optimal share rate, while Abbott et al. (2010) find little evidence of significant changes in the share contracts used to compensate fishermen over time. Based on the empirical results, Nguyen and Leung (2009) demonstrate that share contracts benefit both vessel owners and crew members economically. Additionally, Thuy et al. (2013) highlight that allocative efficiency is preserved if share contracts are applied consistently

across fleets, using a Vietnamese small-scale purse seine fisheries dataset. Espinoza (2015) demonstrates that share contract choices depend on factors like bargaining power, monitoring costs, and external conditions in Chilean artisanal fisheries.

The papers above provided supportive empirical evidence for the predictions of my model regarding seafood share-harvesting. Although this model provides insight into analyzing the role of transaction cost in a share contract fishery, its focus on optimal share rate and maximizing social welfare is limited. Guillen et al. (2015) find that crew salaries increase in line with vessel performance under share contracts, but later, Guillen et al. (2017) indicate that overexploitation and reduced productivity have diminished the attractiveness of these agreements. Pinello et al. (2018) point out that share contracts do not fully capture the real value of fishing income, while Salazar and Dresdner (2020) find that net income shares are, on average, higher in fishing communities regulated by an organizational regime compared to those regulated by an area-based regime, through a difference-in-differences approach. I now build, in the next section, a simple theoretical framework of transaction costs in share contracts to derive the determinants of optimal share rate.

3 Framework

In this section, I develop a theoretical framework to refine the traditional sharecropping theory, as well as to introduce the concept of transaction costs and build a dynamic equilibrium model of social optimum regulations in what I think is the simplest possible institutional environment.

3.1 A Simple Model

In this subsection I highlight the fallacies in Cheung's theory of share tenancy and refute his claim that share tenancy is efficient.

3.1.1 The Fallacies of Traditional Theory of Share Tenancy

To refine traditional theory of share tenancy (Cheung, 1968b), I consider a tenant has a homogeneous agri-food production function be $q = q(L, h)$, where L denotes the amount of tenant labor and h denotes the amount of farmland leased by the tenant. Let $h = \frac{H}{m}$, where the amount of farmland per tenant is equal to the total farmland owned by the landlord, H, divided by the number of tenants, m. Note that I assume that all farmland is owned by landlord, following Cheung (1968)'s assumption. However, in section 3.3.2, I revise this assumption.

Second, Cheung (1968) assumes each tenant signs a wage contract with landlord, and then landlord can choose the amount of tenant's labor with the market wage rate, W. In addition, his model assumes that, under perfect competition between tenants, $WL = (1 - r)q(L, h)$. But this constraint assumes that tenant's total cost (the sum of production and transaction costs) is zero, and it assumes tenant's total wage is identical to the tenant's share of the quantities produced. Actually, Cheung (1968) overlooks the inclusion of the agri-food price (P) on the right-hand side of the constraint. Obviously, this omission makes his assumption unrealistic and incorrect. To accurately describe tenant behavior in a perfectly competitive market equilibrium, tenant profit is assumed to be at least zero: $\pi_w = (1 - r)q(L, h)P - WL - c_t(s, L) \geq 0$, where $c_w(s, L)$ represents tenant's total cost function and s denotes tenant's specific investment. Actually, WL should be explained as the tenant's opportunity cost of giving up farming, which is identical to the total non-farming income he earns in the non-agricultural market. So, Cheung (1968)'s assumption of a labor wage contract between the landlord and tenant is both unnecessary and unrealistic. Indeed, in a pure sharecropping system, the landlord only signs a share contract for the agri-food quantities produced with tenant. In such case, landlord can not control tenant's labor investment. It implies that the amount tenant's labor cannot become a choice variable

for the landlord in the profit maximization problem. In addition, I assume that the tenant's marginal transaction cost is positive ($\frac{\partial c_w}{\partial s} > 0$) and the marginal production cost is positive ($\frac{\partial c_w}{\partial L} > 0$). Thus, the landlord's constraint should be $WL + c_w(s, L) \leq (1 - r)q(L, h)P$.

On the other hand, Cheung's model (1968) does not account for the landlord's opportunity cost. For instance, farmland could be explored and used as an agritourism resource (Ferreira and Sánchez-Martín, 2022). So, I let v denote the return rate from non-agricultural exploration of farmland and vH represent the landlord's opportunity cost of renting out his or her farmland to tenants. Besides, Cheung (1968) assumes the landlord is able to induce tenants' investments in their labor inputs, treating each tenant's labor (L) as one of the choice variables in his model. However, in the real world, a principal-agent problem arises due to asymmetric information and information costs. Hence, the landlord cannot control the tenant's labor investment, and the amount of tenant labor cannot be treated as a choice variable for the landlord. Consequently, in my model, the landlord's profit function under a share contract ($r > 0$) is π_l, which depends on the choices of m, r, and H, subject to the constraint of perfect competition among tenants. The landlord's payoff, therefore, also accounts for the opportunity cost of the alternative option. The representative landlord solves

$$\underset{\{m,r,H\}}{\text{Max}}\ \pi_l = mrPq(L, h) - vH,$$

subject to

$$WL + c_w(s, L) \leq (1 - r)Pq(L, h).$$

Therefore, the landlord's optimal share rate is

$$r^* = \frac{\frac{\partial q}{\partial h}}{\frac{q}{h}} \quad (1).$$

The implication of equation (1) is that, the landlord's optimal share rate is equal to the tenant's elasticity of output with respect to farmland, $\frac{\frac{\partial q}{\partial h}}{\frac{q}{h}}$. It also implies that the rent per acre of farmland is equal to the marginal product of farmland in equilibrium under the share contract, which is consistent with Cheung (1968)'s Pareto optimality solution under the share contract (Cheung's optimal share rate $r_{Pareto} = \frac{\frac{\partial q}{\partial h}}{\frac{q}{h}} = r^*$). In addition, $\pi_l = 0$.

The next proposition describes the key property of the equilibrium (proof in Appendix A).

PROPOSITION 1: *Suppose that contracting is costless and each tenant's economic profits are zero. The landlord's optimal share rate under the share contract is identical to the tenant's elasticity of output with respect to farmland, and landlord's economic profits are zero.*

3.1.2 Non-Alternative Option and Landlord's Costs

I now consider the same problem with the inclusion of the landlord's production cost and transaction cost. Actually, the solutions thus far have been primarily based on the condition that the landlord's transaction and production costs are zero. However, these costs are positive in the real world. So, the purpose of this section is to extend the model to incorporate nonzero production and transaction costs, without considering the opportunity cost of using farmland, to explain the choice of share contracts. In the next section, I will address the landlord's opportunity cost, production cost, and transaction cost.

Let c_l represent the landlord's total cost. The landlord's marginal transaction cost for contracting should be positive ($\frac{\partial c_l}{\partial m} > 0$), and the marginal production cost for maintaining land improvements should be positive ($\frac{\partial c_l}{\partial H} > 0$). So, the maximization problem of the landlord under a share contract ($r > 0$), subject to the constraint competition between tenants, can then be written as

$$\underset{\{m,r,H\}}{\text{Max}}\ \pi_l = mrPq(L,h) - c_l(m,H),$$

subject to

$$WL + c_w(s,L) \leq (1-r)Pq(L,h).$$

Thus, the landlord's optimal share rate becomes

$$r^{**} = \frac{\frac{\partial c_l}{\partial m}}{Pq} + \frac{\partial q/\partial h}{q/h}.$$

Hence:

$$r^{**} > r^{*} \ (2).$$

Equation (2) implies that if the landlord's transaction costs are positive, his optimal share rate is greater than the tenant's elasticity of output with respect to farmland, under the share contract. I therefore obtain the following lemma (see Appendix B for the formal proof).

LEMMA 1: *With positive transaction costs under a share contract, the landlord's optimal share rate is greater than the tenant's elasticity of output with respect to farmland.*

The implication of Lemma 1 is that, the allocation of resources with positive transaction and production costs is inefficient, under share tenancy. So, when the landlord's production and transaction costs are nonzero, sharecropping becomes inefficient.

3.1.3 Landlord's Optimal Share Rate

In this section, I consider the landlord's opportunity cost, production cost, and transaction cost. Therefore, the landlord's profit maximization problem under a share contract ($r > 0$), subject to the constraint competition between tenants, can then be rewritten as

$$\underset{\{m,r,H\}}{\text{Max}}\ \pi_l = mrPq(L,h) - vH - c_l(m,H),$$

subject to

$$WL + c_w(s,L) \leq (1-r)Pq(L,h).$$

Finally, the landlord's optimal share rate is

$$r^{***} = \frac{\frac{\partial c_l}{\partial m} + h(v + \frac{\partial c_l}{\partial H})}{Pq} > r^{**} > r_{Pareto} \quad (3).$$

Therefore, I obtain the second lemma (also see Appendix C for the formal proof).

LEMMA 2: *Given that the landlord's transaction cost and production cost are positive, these costs cause the landlord's optimal share rate to deviate from the equilibrium where share tenancy is efficient.*

The theoretical implication of Lemma 2 is that if the landlord's transaction cost, production cost, and opportunity cost of using farmland are zero, share tenancy should satisfy the Pareto condition. Cheung (1968) proves that when share rate equals to the elasticity of output with respect to farmland, share tenancy meets the Pareto condition. However, his proof is based on the assumptions that the landlord's transaction and production costs are zero and that farmland cannot be explored or used in alternative ways. Indeed, in the real world, the landlord's transaction costs, production costs, and opportunity cost of using farmland are positive. I have shown that optimal share rate will not equal the elasticity of output with respect to farmland, if these costs are positive. Hence, positive transaction and production costs lead to inefficient resource allocation under share tenancy in equilibrium. In other words, share tenancy may imply inefficiency in resource allocation as long as the landlord's production and transaction costs are considered.

3.2 Socially Optimal Regulations

I now explore a possible approach for a social planner to allocate resources through regulations, including a minimum share rate policy and agri-food price control.

3.2.1 Minimum Share Rate Policy

In this section, I consider a set of government regulations concerning share rate, including the minimum share rate policy. Suppose that share rate is increased from r^{***} in equation (3) to $\bar{r}$ ($> r^{***}$). On the other hand, R(r) represents the dissipated rent due to the implementation of the minimum share rate policy (Cheung, 1969b). Assume the regulator

can select the regulated share rate and amount of farmland assigned to tenants who stay in a long-run perfectly competitive market to maximize social welfare:

$$\underset{\{\bar{r}.H\}}{\text{Max}}\, SW = m[Pq(L,h) - WL - c_w(s,L)] - vH - c_l(m,H) - \int_{r^{***}}^{\bar{r}} \mathrm{R}(r)dr,$$

subject to

$$WL + c_w(s,L) = (1-\bar{r})Pq(L,h).$$

Now, substituting the constraint into the social welfare function (SW), I obtain the following simpler unconstrained maximization problem for the regulator:

$$\underset{\{\bar{r}.H\}}{\text{Max}}\, SW = m\bar{r}Pq(L,h) - vH - c_l(m,H) - \int_{r^{***}}^{\bar{r}} \mathrm{R}(r)dr.$$

The first-order conditions are

$$\frac{\partial SW}{\partial \bar{r}} = \mathrm{mP}q\left(L,\frac{H}{m}\right) - R(\bar{r}) = 0 \quad (4)$$

$$\frac{\partial SW}{\partial H} = \bar{r}P\frac{\partial q}{\partial h} - v - \frac{\partial c_l}{\partial H} = 0 \quad (5)$$

I now present two comparative static results derived from equations (4) and (5). These results are facilitated by the general solutions of the equilibrium, showing the effects of the return rate from non-agricultural exploration of farmland (v), the number of tenants (m) and the price of agri-food (P) on the regulated share rate ($\bar{r}$). The main results are provided in the next proposition (proof in Appendix D).

PROPOSITION 2: *When tenants stay in a long-run perfectly competitive market, the introduction (or increase) of the return rate from non-agricultural exploration of farmland leads to an increase in the regulated share rate; The introduction (or increase) of the number of tenants leads to a rise in the regulated share rate; The introduction (or increase) of the price of agri-food leads to a decrease in the regulated share rate.*

3.2.2 Optimum Social Welfare with Agri-food Price Control

I now discuss how the dissipation of rent can be endogenized with the social welfare maximization problem. Suppose regulators set a maximum price of agri-food, $\bar{P}$, which

generates dissipated rents denoted by $S(\bar{P})$ (Cheung, 1974). Suppose that S is an increasing and concave function of $\bar{P}$, with $S'(\bar{P}) > 0$ and $S''(\bar{P}) < 0$.The social welfare optimization problem can be expressed as

$$\underset{\{\bar{P}.H\}}{\text{Max}}\, SW = m[\bar{P}q(L,h) - WL - c_w(s,L)] - vH - c_l(m,H) - S(\bar{P}),$$

subject to

$$WL + c_w(s,L) = (1-\bar{r})\bar{P}q(L,h).$$

The first-order conditions are

$$\frac{\partial SW}{\partial \bar{P}} = \text{mr}q\left(L, \frac{H}{m}\right) - S'(\bar{P}) = 0 \quad (7)$$

$$\frac{\partial SW}{\partial H} = \bar{P}r\frac{\partial q}{\partial h} - v - \frac{\partial c_l}{\partial H} = 0 \quad (8)$$

The second-order conditions of the maximization yield the following proposition (proof in Appendix E).

LEMMA 3: *When tenants stay in a long-run perfectly competitive market, the introduction (or increase) of the return rate from non-agricultural exploration of farmland leads to a higher maximum price for agri-food.*

3.2.3 Dynamic Socially Optimal Regulations

The agronomy environment that I work with throughout this subsection builds on a simple dynamic model. There are m_t tenants in the agronomy and their population grows exogenously over time: $m_t = m_0 e^t$. Furthermore, suppose agri-food production function, q_t, is taking Cobb-Douglas form, I have

$$q_t = h_t^{1-\sigma} L_{pt}^{\sigma},$$

where he parameter σ will be the elasticity of output with respect to of L_{pt} in the agri-food production. Furthermore, I assume that $0 < \sigma < 1$.

Landlord's total cost function is supposed to be $c_l(m_t, H) = km_t + \frac{\tilde{k}}{2}H^2$ (k and $\tilde{k}$ capture the weight on transaction costs and production costs), and rent dissipation function due to agri-food price control is assumed to be $\mathrm{S}(\bar{P}_t) = ln\bar{P}_t$. In addition, I assume the marginal rent dissipation function with respect to regulated share rate is assumed to be $\mathrm{R}(r_t) = \eta {r_t}^{-\eta}$ and $0 < \eta < 1$.

Now, I state the intertemporal social welfare maximization problem concisely. The dynamic optimal allocation solves

$$U_0 = \underset{\{\bar{P}_t, \bar{r}_t, L_{pt}\}}{\mathrm{Max}} \int_0^{\infty} e^{-\rho t} \Big\{ m_t \big[\bar{P}_t h_t^{1-\sigma} L_{pt}^{\sigma} - W_t L_{pt} - c_{wt}\big(s, L_{pt}\big)\big] - vH - km_t - \frac{\tilde{k}}{2}H^2 - ln\bar{P}_t - \int_{r^{***}}^{\bar{r}_t} \eta {r_t}^{-\eta} dr_t \Big\} \mathrm{dt},$$

subject to

$$W_t L_{pt} + c_{wt}\big(s, L_{pt}\big) = (1 - \bar{r}_t)\bar{P}_t h_t^{1-\sigma} L_{pt}^{\sigma},$$

$$\dot{m}_t = \frac{1}{\chi}(L_t - L_{pt}),$$

$$m_t = m_0 e^t.$$

The second constraint is straightforward: χ units of tenant labor are required to produce a new agri-food. The tenants' labor allocated to agri-food production is equal to the total tenants' labor available in the agronomy (L_t) minus their labor used in the agri-food production process (L_{pt}). Suppose the regulator wants to control both the agri-food price and the share rate. Additionally, the regulator determines the allocation of tenants' labor between production and entry. The optimal allocation is provided in Proposition 3.

PROPOSITION 3: *When tenants stay in a long-run perfectly competitive market, the optimal allocation is*

$\bar{r}_t^{\,*} = \eta^{\frac{1}{\eta - 1}}$ (9),

$$L_{pt}^{*}=\frac{\sigma\chi}{\left(\mu_0+\frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right)e^{\rho t}-\frac{k-\frac{\sigma}{m_0e^t}}{\rho}} \quad (10),$$

$$\bar{P}_t^{\,*}=\frac{\left[\frac{\sigma\chi}{\left(\mu_0+\frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right)e^{\rho t}-\frac{k-\frac{\sigma}{m_0e^t}}{\rho}}\right]^{-\sigma}}{(m_0e^t)^{\sigma}\eta^{\frac{1}{\eta-1}}\mathrm{H}^{1-\sigma}} \quad (11),$$

$$U_0^{*}=\frac{1-vH-\frac{\tilde{k}}{2}H^2}{\rho}+\frac{km_0}{1-\rho}+\frac{1}{\eta-1}\left[\frac{1}{\rho}-\frac{(km_0)^{1-\eta}}{1-\eta-\rho}\right]-\frac{\sigma}{\rho}\left[1+ln\left(\mu_0+\frac{k}{\rho}\right)\right]+\frac{\sigma}{\rho^2}+\frac{\sigma ln\sigma\chi+\sigma lnm_0+\frac{ln\eta}{\eta}+(1-\sigma)lnH}{\rho} \quad (12).$$

PROOF:

See Appendix Section F.

The most important result in the proposition is the solution for the landlord's optimal regulated share rate in equation (9). In particular, this solution shows that the landlord's optimal share rate, as controlled by the social planner, remains constant over time in equilibrium.

3.3 Climate Change Risk and Farmland Ownership Restrictions

On one hand, agri-food production is always threatened by climate change. On the other hand, I assume farmland is owned by the landlord. However, property rights to farmland are not always held by individuals. They can be occupied and controlled by the state. Therefore, I investigate the impacts of climate change and farmland ownership arrangements on sharecropping and social optimum.

3.3.1 Climate Change with Transaction Costs

Following the definition of climate risk from Stiglitz (1974), I assume the probability density function of natural factors of production (e.g., weather or biological stock) is $g(\theta)$, where θ represents climate change. Furthermore, I assume $0 < g(\theta) \leq 1$, and $g'(\theta)$<0.Therefore, the landlord's profit maximization problem can then be rewritten as

$$\underset{\{m,r,H\}}{\text{Max}}\ \pi_l = mrPg(\theta)q(L,h) - vH - c_l(m,H),$$

subject to

$$WL + c_w(s,L) \leq (1-r)Pg(\theta)q(L,h).$$

Finally, the landlord's optimal share rate is

$$r^{****} = \frac{\frac{\partial c_l}{\partial m} + h(v + \frac{\partial c_l}{\partial H})}{Pqg(\theta)} \quad (13).$$

Therefore, I obtain Proposition 4 (also see Appendix G for the formal proof).

PROPOSITION 4: *Given that the landlord's transaction cost, production cost, and opportunity cost of using farmland are positive, climate change encourages an increase in the landlord's optimal share rate.*

3.3.2 Optimal Control of Farmland Ownership

Indeed, H is assumed to be the total farmland owned by the landlord. However, if λ ($0 \leq \lambda \leq 1$) denotes the degree of privatization of farmland, $H = \lambda Z$, so $h_t = \frac{\lambda Z}{m_t}$ and Z represents the size of farmland. When $\lambda = 0$, the farmland becomes public domain, and the landlord has no exclusive rights (Cheung, 1970). Increasing λ implies that the landlord owns and controls a greater portion of farmland, representing a shift from a non-private ownership arrangement to private ownership arrangement. When $\lambda = 1$, the landlord has complete property rights to the farmland (H=Z). In addition, I assume the social planner is able to control the degree of privatization of farmland and tenants stay in a long-run perfectly competitive market. Therefore, λ becomes one of the choice variables in the intertemporal social welfare maximization problem. In this case, the optimal allocation solves

$$U_1 = \underset{\{\bar{P}_t, \bar{r}_t, L_{pt}, \lambda\}}{\text{Max}} \int_0^{\infty} e^{-\rho t} \Big\{ m_t \Big[\bar{P}_t g(\theta_t) (\frac{\lambda Z}{m_t})^{1-\sigma} L_{pt}^{\sigma} - W_t L_{pt} - c_{wt}(s, L_{pt}) \Big] - v\lambda Z - km_t -$$

$$\frac{\tilde{k}}{2}(\lambda Z)^2 - ln\bar{P}_t - \int_{r^{****}}^{\bar{r}_t} \eta r_t^{-\eta} dr_t \Big\} \text{dt},$$

subject to

$$W_t L_{pt} + c_{wt}(s, L_{pt}) = (1 - \bar{r}_t)\bar{P}_t g(\theta_t)(\frac{\lambda Z}{m_t})^{1-\sigma} L_{pt}^{\sigma},$$

$$\dot{m}_t = \frac{1}{\chi}(L_t - L_{pt}),$$

$$m_t = m_0 e^t.$$

The optimal allocation is given in Lemma 4.

Lemma 4: *When tenants stay in a long-run perfectly competitive market, farmland privatization discourages the socially optimal price of agri-food. In addition, the socially optimal share rate for the landlord is not determined by the farmland ownership arrangement or climate change.*

PROOF:

See Appendix Section H.

The most important implication of Lemma 4 is that the solution for the landlord's optimal regulated share rate in equilibrium remains unaffected by the farmland ownership arrangement or climate change. Specifically, this solution demonstrates that the landlord's optimal share rate, as controlled by social planner, does not depend on whether farmland is privately owned or on the impacts of climate change. Indeed, Cheung (1968) assumes that farmland is privately owned by the landlord. However, Lemma 4 shows that the landlord's optimal regulated share rate does not rely on this assumption of private property rights. And it is not related to the farmland ownership arrangement. In addition, the theoretical implication of Lemma 4 is that share tenancy cannot serve as a mechanism for climate risk sharing. Cheung (1969a) hypothesizes that the choice of share tenancy is made so as to maximize the gains from risk dispersion, so share tenancy may then be regarded as a device for risk dispersion; that is, given the postulate of risk aversion, a share contract will be mutually preferred by the landowner and the tenant. Nevertheless, according to Lemma 4, climate change does not affect the landlord's optimal regulated share rate. Hence, share

tenancy cannot be served as a device for climate risk sharing, which is also inconsistent with Stiglitz (1974).

3.3.3 Non-Competitive Market of Agri-food

Previously, I assume tenants stay in a long-run perfectly competitive market, so their economic profits are supposed to be zero. Nevertheless, if the market structure changes to a non-competitive one, will the optimal solutions be revised? Let us first look at the profit maximization problem from the point of view of the tenants who stay in a non-competitive agri-food market. That is, the representative tenant solves

$$\underset{\{L\}}{\text{Max}}\, \pi_w = (1-r)Pg(\theta)q(L,h) - WL - c_w(s,L)$$

or, $(1-r)Pg(\theta)\frac{\partial q}{\partial L} = W + \frac{\partial c_w}{\partial L}$.

In this case, the landlord's profit maximization problem can then be rewritten as

$$\underset{\{m,r,H\}}{\text{Max}}\, \pi_l = mrPg(\theta)q(L,h) - vH - c_l(m,H),$$

subject to

$$W + \frac{\partial c_w}{\partial L} = (1-r)Pg(\theta)\frac{\partial q}{\partial L}.$$

So, the landlord's optimal share rate is

$$r^{nc} = r^{****} \text{ (14)}.$$

Therefore, I obtain Proposition 5 (also see Appendix I for the formal proof).

PROPOSITION 5: *Given that the landlord's transaction cost, production cost, and opportunity cost of using farmland are positive, market structure of agri-food does not affect the landlord's optimal share rate.*

Finally, I investigate the effect of market structure on socially optimal allocation over time. Here, let $c_{wt}(s, L_{pt}) = \alpha s L_{pt}$, where α is a constant. The optimal allocation solves:

$$U_2 = \underset{\{\bar{P}_t, \bar{r}_t, L_{pt}, \lambda\}}{\text{Max}} \int_0^{\infty} e^{-\rho t} \left\{ m_t \left[\bar{P}_t g(\theta_t) \left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} - W_t L_{pt} - \alpha s L_{pt} \right] - v\lambda Z - k m_t - \right.$$

$$\left. \frac{\tilde{k}}{2}(\lambda Z)^2 - ln\bar{P}_t - \int_{r^{nc}}^{\bar{r}_t} \eta r_t^{-\eta} dr_t \right\} \text{dt},$$

subject to

$$W_t + \alpha s = \sigma(1 - \bar{r}_t)\bar{P}_t g(\theta_t) \left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma-1},$$

$$\dot{m}_t = \frac{1}{\chi}(L_t - L_{pt}),$$

$$m_t = m_0 e^t.$$

The optimal allocation is given in Lemma 5.

Lemma 5: *The optimal allocation is*

$$\bar{r}_t^{***} = \frac{3\sigma - 1 \pm 2\sqrt{(2\sigma - 1)\sigma}}{\sigma} \quad (15),$$

$$L_{pt}^{***} = \frac{\sigma\chi}{\left(\mu_0 + \frac{k - \frac{\sigma}{m_0 e^t}}{\rho}\right) e^{\rho t} - \frac{k - \frac{\sigma}{m_0 e^t}}{\rho}} \quad (16),$$

$$\bar{P}_t^{***} = \frac{\left[\frac{\sigma\chi}{\left(\mu_0 + \frac{k - \frac{\sigma}{m_0 e^t}}{\rho}\right) e^{\rho t} - \frac{k - \frac{\sigma}{m_0 e^t}}{\rho}} \right]^{-\sigma}}{(m_0 e^t)^{\sigma} [2\sigma \pm 2\sqrt{(2\sigma - 1)\sigma}] \left[\frac{-vZ + \sqrt{(vZ)^2 + 4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z} \right]^{1-\sigma} g(\theta_t)} \quad (17),$$

$$U_2^* = \frac{1 - v\frac{-vZ + \sqrt{(vZ)^2 + 4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z} - \frac{\tilde{k}}{2}\left[\frac{-vZ + \sqrt{(vZ)^2 + 4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}\right]^2}{\rho} + \frac{k m_0}{1-\rho} + \frac{\eta}{\eta - 1}\left[\frac{3\sigma - 1 \pm 2\sqrt{(2\sigma-1)\sigma}}{\sigma}\right]^{1-\eta} \left[\frac{1}{\rho} - \frac{(k m_0)^{1-\eta}}{1 - \eta - \rho}\right] -$$

$$\frac{\sigma}{\rho}\left[1 + ln\left(\mu_0 + \frac{k}{\rho}\right)\right] + \frac{\sigma}{\rho^2} + \frac{\sigma ln\sigma\chi + \sigma ln m_0 + \ln[2\sigma \pm 2\sqrt{(2\sigma-1)\sigma}] + (1-\sigma)\ln\left[\frac{-vZ + \sqrt{(vZ)^2 + 4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}\right]}{\rho} + \ln\left(\frac{1}{\sqrt{2\pi}\sigma_\theta}\right)\frac{1}{\rho} -$$

$$\frac{1}{2\sigma_\theta^2}\left(\frac{\theta_0^2}{2\beta + \rho} - \frac{2\mu_\theta\theta_0}{\beta + \rho} + \frac{\mu_\theta^2}{\rho}\right) \quad (18).$$

PROOF:

See Appendix Section J.

3.4 Visualizations of Calibrations: Comparative Statics and Numerical Examples

I now provide a set of calibrations to demonstrate the visualizations of the models above. In subsection 3.4.1, I present numerical examples to illustrate how the socially optimal share rate of the landlord changes with parameter η, and, how the socially optimal price of agri-food changes with the degree of farmland privatization over time.

3.4.1 Comparative Static Results

In this subsection, I present some comparative static results. Three comparative static exercises are given: (a) changes in the parameter that determines the sensitivity of the rent dissipation (i.e., the effect of variations in η on socially optimal share rate); (b) changes in the elasticity of output with respect to tenant's labor (i.e., the effect of variations in σ on socially optimal share rate), and (c) changes in the optimal degree of farmland privatization (i.e., the effect of variations in λ^* on socially optimal price of agri-food).

A. Changes in η

From equation (9), the effect of changes in the parameter that determines the sensitivity of the rent dissipation can change the socially optimal share rate of the landlord, $\bar{r}_t^*$. Figure A3 displays such effects when η moves from zero to one. When η increases, $\bar{r}_t^*$ rises.

B. Changes in σ

This comparative static result shows that from equation (15) the effect of changes in the elasticity of output with respect to tenant's labor will affect socially optimal share rate of the landlord. Figure A4 shows such an effect when σ changes to one. The threshold level of σ at which $\bar{r}_{t+} = \frac{3\sigma-1+2\sqrt{(2\sigma-1)\sigma}}{\sigma}$ shifts to $\bar{r}_{t-} = \frac{3\sigma-1-2\sqrt{(2\sigma-1)\sigma}}{\sigma}$ is about 0.48. It implies that when σ is less than the threshold level, $\bar{r}_{t+}$ and $\bar{r}_{t-}$ overlap. In this case, an increase in σ will increase $\bar{r}_t^{***}$. When σ is greater than the threshold level, $\bar{r}_{t+}$ disappears but $\bar{r}_{t-}$ still exists. In this case, an increase in σ will decrease $\bar{r}_t^{***}$. Figure A1 and Figure A2 in the appendix show detailed visualizations of $\bar{r}_{t+}$ and $\bar{r}_{t-}$, respectively.

C. Changes in λ^*

According to Lemma 4, this comparative static result illustrates that changes in the optimal degree of farmland privatization can affect the socially optimal price of agri-food. Figure A5 shows such changes for some given values of the other parameters ($\mu_0 = 10$, $k = 0.001$, $m_0 = 10$, $\rho = 0.2$, $\eta = 0.5$, $\chi = 1$, Z=1, $\sigma = 0.9$, $\theta = 0.5$). When the optimal degree of farmland privatization increases, the socially optimal price of agri-food declines, and thus deregulation of property rights of farmland leads to greater farmland privatization, which, in turn, results in a lower socially optimal price of agri-food.

3.4.2 Quantitative Analysis of Social Welfare and Agri-food Market Structure

In this subsection I compare social welfare for a competitive market and a non-competitive market. First of all, I define the difference in social welfare between a competitive market and a non-competitive market as below: $D = U_2^* - U_1^* = \frac{\eta}{\eta-1}\left[\frac{3\sigma-1\pm2\sqrt{(2\sigma-1)\sigma}}{\sigma}\right]^{1-\eta}\left[\frac{1}{\rho} - \frac{(km_0)^{1-\eta}}{1-\eta-\rho}\right] + \frac{\sigma ln\sigma\chi+\sigma lnm_0+\ln\left[2\sigma\pm2\sqrt{(2\sigma-1)\sigma}\right]+(1-\sigma)\ln\left[\frac{-vZ+\sqrt{(vZ)^2+4\bar{k}Z^2(1-\sigma)}}{2\bar{k}Z}\right]}{\rho} - \frac{1}{\eta-1}\left[\frac{1}{\rho} - \frac{(km_0)^{1-\eta}}{1-\eta-\rho}\right] - \frac{\sigma ln\sigma\chi+\sigma lnm_0+\frac{ln\eta}{\eta-1}+(1-\sigma)\ln\left[\frac{-vZ+\sqrt{(vZ)^2+4\bar{k}Z^2(1-\sigma)}}{2\bar{k}Z}\right]}{\rho} = \left[\frac{1}{\rho} - \frac{(km_0)^{1-\eta}}{1-\eta-\rho}\right]\left\{\frac{\eta}{\eta-1}\left[\frac{3\sigma-1\pm2\sqrt{(2\sigma-1)\sigma}}{\sigma}\right]^{1-\eta} - \frac{1}{\eta-1}\right\} + \frac{\ln\left[2\sigma\pm2\sqrt{(2\sigma-1)\sigma}\right]-\frac{ln\eta}{\eta-1}}{\rho}$. The parameters that I calibrate are k and m_0, so I hold all other parameters at the baseline simulation and explore the behavior of the model across a wide range values for them. If $\eta = 0.5$, then $D(k, m_0, \rho, \sigma, \bar{r}_{t+}; \eta = 0.5) = \left[\frac{1}{\rho} - \frac{(km_0)^{0.5}}{0.5-\rho}\right]\left\{-\left[\frac{3\sigma-1+2\sqrt{(2\sigma-1)\sigma}}{\sigma}\right]^{0.5} + 2\right\} + \frac{\ln\left[2\sigma+2\sqrt{(2\sigma-1)\sigma}\right]-2ln2}{\rho}$ and $D(k, m_0, \rho, \sigma, \bar{r}_{t-}; \eta = 0.5) = \left[\frac{1}{\rho} - \frac{(km_0)^{0.5}}{0.5-\rho}\right]\left\{-\left[\frac{3\sigma-1+2\sqrt{(2\sigma-1)\sigma}}{\sigma}\right]^{0.5} + 2\right\} + \frac{\ln\left[2\sigma-2\sqrt{(2\sigma-1)\sigma}\right]-2ln2}{\rho}$.

Figure A6 illustrates the social welfare difference, D, for various combinations of ρ, σ, $\eta = 0.5$, $\bar{r}_{t+} = \frac{3\sigma-1+2\sqrt{(2\sigma-1)\sigma}}{\sigma}$ and $km_0 = 0.1$. Actually, the sign of D depends upon those combinations of parameters. Figure A7 and Figure A8 present the calibrations of D for $km_0 = 1$ and $km_0 = 10$, respectively.

Figure A9 illustrates the social welfare difference, D, for various combinations of ρ, σ, $\eta = 0.5$, $\bar{r}_{t-} = \frac{3\sigma-1-2\sqrt{(2\sigma-1)\sigma}}{\sigma}$ and $km_0 = 0.1$. Actually, the sign of D depends upon those combinations of parameters. Figure A10 and Figure A11 present the calibrations of D for $km_0 = 1$ and $km_0 = 10$, respectively.

4 Extensions

In the previous section, I examined the impact of climate change on sharecropping. Building on that, this section develops a general equilibrium model incorporating uncertainty from climate change. The analysis is structured into four parts: In the first, tenants are price takers in a perfectly competitive agri-food market. In the second part of this section, the tenant assumes a monopoly position, but the agri-food price is regulated by the state. In the third part, tenants stay in monopoly and oligopoly markets without intervention in agri-food pricing. In the last part, tenants' specific investments are assumed to be variable.

4.1 Tenants in a Perfectly Comparative Market (m>2)

Here, I follow the assumption of the mixed sharecropping system from Stiglitz (1974) with additional parameter $\beta(x)\begin{cases} > 0, if\ the\ tenent\ receives\ a\ payment \\ = 0, it\ is\ a\ pure\ sharecropping\ system \\ < 0, if\ the\ tenent\ pays\ the\ landlord \end{cases}$, where $x \equiv (1-r)Pq(L,h)$.

Suppose that both landlords and tenants are expected utility maximizers. Let U_l represent the utility function of landlord, and U_w present the utility function of tenant.

On one hand, the tenant solves

$$\underset{\{L\}}{\text{Max}}\ \text{E}U_w[\pi_w(\theta)] = \text{E}U_w[(1-r)Pq(L,h)g(\theta) - WL - c_w(s,L) + \beta(x)].$$

On the other hand, the landlord wishes to solve

$$\underset{\{m,x\}}{\text{Max}}\, \text{E}U_l[\pi_l(\theta)] = \text{E}U_l[mrPg(\theta)q\left(L,\frac{H}{m}\right) - vH - c_l(m,H) - m\beta(x)].$$

Therefore, the landlord's optimal share rate is

$$r^{PC} = \frac{-P\frac{\partial q}{\partial L}\left(\frac{\partial c_l}{\partial m}+\beta\right)+\left(W+\frac{\partial c_w}{\partial L}\right)Pq+\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}P^2q\frac{\partial q}{\partial L}\left(\frac{\partial q}{\partial h}\frac{h}{q}+1\right)+\Delta}{2\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}P^2q\frac{\partial q}{\partial L}} \quad (19),$$

where $\Delta = \sqrt{[P\frac{\partial q}{\partial L}\left(\frac{\partial c_l}{\partial m}+\beta\right)-\left(W+\frac{\partial c_w}{\partial L}\right)Pq-\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}P^2q\frac{\partial q}{\partial L}\left(\frac{\partial q}{\partial h}\frac{h}{q}+1\right)]^2-4\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}P^2q\frac{\partial q}{\partial L}[\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}P^2q\frac{\partial q}{\partial L}\frac{\partial q}{\partial h}\frac{h}{q}-P\frac{\partial q}{\partial L}\left(\frac{\partial c_l}{\partial m}+\beta\right)]}$

(also see Appendix K for the formal proof).

Equation (19) implies that the landlord's optimal share rate is not determined by his or her risk attitude but depends on tenants' risk attitudes. The following propositions and lemmas also provide such economic implications. Hence, the landlord's risk attitude does not affect sharecropping, which is not consistent with Stiglitz (1974).

4.2 Monopoly Tenant under Agri-food Price Control (m=1)

I now turn to an examination of sharecropping, for the monopoly tenant's optimal decision under agri-food price control. In this case, the monopoly tenant is still not able to influence the agri-food price. Therefore, the equilibrium condition for the monopoly tenant remains unchanged. I focus on the landlord's expected utility maximization problem. Assume that the landlord would choose a particular share contract in which he or she can determine the total farmland ownership (H). The landlord wishes to solve

$$\underset{\{H,x\}}{\text{Max}}\text{E}U_l[\pi_l(\theta)] = \text{E}U_l\{[\bar{P}q(L,H) - x]g(\theta) - vH - c_l(1,H) - \beta(x)\}.$$

Therefore, the solution of this problem is

$$r^{MPC} = 1 - \frac{W+\frac{\partial c_w}{\partial L}}{[\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}+\frac{v+\frac{\partial c_l}{\partial H}}{\bar{P}\frac{\partial q}{\partial H}}]\bar{P}\frac{\partial q}{\partial L}} \quad (20).$$

From equation (20), it is clear that (also see Appendix L for the formal proof)

PROPOSITION 6: *An increase in the tenant's wage rate (W) and the tenant's marginal production cost with respect to labor ($\frac{\partial c_w}{\partial L}$) both lower the landlord's optimal share rate (r^{PC}). r^{PC} decreases (increases) when v, $\frac{\partial c_l}{\partial H}$, or $\bar{P}$ decrease (increase).*

4.3 Monopoly and Oligopoly Tenants without Agri-food Price Control

In this subsection, assume tenants have market power to influence agri-food prices.

4.3.1 Monopoly (m=1)

Now, suppose that a monopoly tenant faces a linear demand function $P = a - bq(L, H)$. The tenant then solves

$$\underset{\{L\}}{\text{Max}}\ \text{E}U_w[\pi_w(\theta)] = \text{E}U_w\{(1-r)[a - bq(L,H)]q(L,H)g(\theta) - WL - c_w(s,L) + \beta(x)\}.$$

On the other hand, the landlord wishes to solve

$$\underset{\{H,x\}}{\text{Max}}\ \text{E}U_l[\pi_l(\theta)] = \text{E}U_l\{[(a - bq(L,H))q(L,H) - x]g(\theta) - vH - c_l(1,H) - \beta(x)$$

Therefore, the landlord's optimal share rate is

$$r^M = 1 - \frac{W + \frac{\partial c_w}{\partial L}}{[\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta} - \frac{v + \frac{\partial c_l}{\partial H}}{(a-2bq)\frac{\partial q}{\partial H}}](a-2bq)\frac{\partial q}{\partial L}} \quad (21).$$

From equation (20) and (21), I obtain the following lemma (also see Appendix M for the formal proof).

LEMMA 6: The landlord's optimal share rate under agri-food price control is greater than it is without agri-food price control when the tenant stays in a monopoly market for agri-food.

4.3.2 Cournot Game with Complete Information (m=2)

Here, I consider a Cournot duopoly game with complete information between two tenants. Let q_i and q_j denote the quantities of a homogeneous agri-food produced by

tenants i and j, respectively. Let $P(q_i, q_j) = a - b[q_i\left(L_i, \frac{H}{2}\right) + q_j\left(L_j, \frac{H}{2}\right)]$ be the market-clearing price when the aggregate quantity on the market is $Q = q_i\left(L_i, \frac{H}{2}\right) + q_j\left(L_j, \frac{H}{2}\right)$.

In the Cournot duopoly model, the tenants' labor inputs pair $\left(L_i^*, L_j^*\right)$ is a Nash equilibrium if, for each tenant i, tenant i solves:

$$\underset{\{L_i\}}{\text{Max}}\, \text{E}U_w^i\{\pi_w^i[q_i\left(L_i, \frac{H}{2}\right), q_j^*(L_j^*, \frac{H}{2})]\} = \text{E}U_w^i\{(1-r)[a - b\left(q_i\left(L_i, \frac{H}{2}\right) + q_j^*\left(L_j^*, \frac{H}{2}\right)\right)]q_i\left(L_i, \frac{H}{2}\right)g(\theta) - WL_i - c_w^i(s, L_i) + \beta(x)\}.$$

The landlord wishes to solve

$$\underset{\{H,x\}}{\text{Max}}\, \text{E}U_l[\pi_l(\theta)] = \text{E}U_l\{2[(a - b\left(q_i^*\left(L_i^*, \frac{H}{2}\right) + q_j^*\left(L_j^*, \frac{H}{2}\right)\right))q_i^*\left(L_i^*, \frac{H}{2}\right) - x]g(\theta) - vH - c_l(2, H) - 2\beta(x)\}.$$

Therefore, the landlord's optimal share rate in a duopoly market for agri-food is

$$r^{CGCI} = 1 - \frac{W + \frac{\partial c_w^i}{\partial L_i}}{\left[\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta} - \frac{v + \frac{\partial c_l}{\partial H}}{\left(a - 4bq_i^*\right)\frac{\partial q_i^*}{\partial h}}\right]\left(a - 3bq_i^*\right)\frac{\partial q_i}{\partial L_i}} < r^M \quad (22).$$

So, I obtain the following lemma (also see Appendix N for the formal proof).

LEMMA 7: The landlord's optimal share rate in a duopoly market for agri-food is less than that in a monopoly market for agri-food.

4.4 Tenants' Specific Investments under Duopoly Game with Complete Information (m=2)

Here, I investigate the impact of tenants' specific investments on sharecropping in the Cournot game with complete information. Let s_i and s_j denote the specific investments from tenants i and j in the process of agri-food production, respectively. Let $P(q_i, q_j) = P = a - b[q_i\left(s_i L, \frac{H}{2}\right) + q_j\left(s_j L, \frac{H}{2}\right)]$ be the market-clearing price. In the Cournot duopoly model, the tenants' specific investments pair $\left(s_i^*, s_j^*\right)$ is a Nash equilibrium if, for each tenant i, tenant i solves:

$$\underset{\{s_i\}}{\text{Max}}\text{E}U_w^i\{\pi_w^i\left[q_i\left(s_iL,\frac{H}{2}\right),q_j^*\left(s_j^*L,\frac{H}{2}\right)\right]\}=\text{E}U_w^i\{(1-r)\left[a-b\left(q_i\left(s_iL,\frac{H}{2}\right)+\right.\right.$$

$$\left.\left.q_j^*\left(s_j^*L,\frac{H}{2}\right)\right)\right]q_i\left(s_iL,\frac{H}{2}\right)g(\theta)-WL-c_w^i(s_i,L)+\beta(x)\}.$$

The landlord wishes to solve

$$\underset{\{H,x\}}{\text{Max}}\,\text{E}U_l[\pi_l(\theta)]=\text{E}U_l\{2[(a-b\left(q_i^*\left(s_i^*L,\frac{H}{2}\right)+q_j^*\left(s_j^*L,\frac{H}{2}\right)\right))q_i^*\left(s_i^*L,\frac{H}{2}\right)-x]g(\theta)-$$

$$vH-c_l(2,H)-2\beta(x).$$

Therefore, the landlord's optimal share rate is

$$r^{SS}=1-\frac{W+\frac{\partial c_w^i}{\partial s_i}}{\left[\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}-\frac{v+\frac{\partial c_l}{\partial H}}{\left(a-4bq_i^*\right)\frac{\partial q_i^*}{\partial h}}\right]\left(a-3bq_i^*\right)\frac{\partial q_i}{\partial l_i}L}\quad(23).$$

So, I obtain the following lemma (also see Appendix O for the formal proof).

PROPOSITION 7: *The impact of tenants' marginal transaction costs ($\frac{\partial c_w^i}{\partial s_i}$) on the landlord's optimal share rate (r^{SS}) is ambiguous, but r^{SS} is the function of $\frac{\partial c_w^i}{\partial s_i}$, given that two tenants invest in specific technologies in the agri-food production process.*

4.5 Optimum Price Control under Uncertainty

Finally, I consider a socially optimal allocation in which the social planner is risk-neutral (e.g., the social welfare function is U_s, and then $U_s'=1$)and tenants who stay in a non-competitive market invest in specific technologies in the agri-food production process. The optimal allocation solves

$$U_3=\underset{\{\bar{P}_t,\bar{r}_t,L_{pt},\lambda\}}{\text{Max}}\int_0^\infty e^{-\rho t}\{\int_{\underline{\theta_t}}^{\overline{\theta_t}}U_s\{m_t[\bar{P}_t\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma}\left(sL_{pt}\right)^\sigma g(\theta_t)-W_tL_{pt}-\alpha sL_{pt}]-v\lambda Z-$$

$$km_t-\frac{\tilde{k}}{2}(\lambda Z)^2-ln\bar{P}_t-\int_{r^{nc}}^{\bar{r}_t}\eta r_t^{-\eta}dr_t\}f(\theta_t)d\theta_t\,\text{dt},$$

subject to

$$W_t+\alpha s_t=\sigma(1-\bar{r}_t)\bar{P}_tg(\theta_t)(\frac{\lambda Z}{m_t})^{1-\sigma}s^\sigma L_{pt}^{\sigma-1},$$

$$\dot{m}_t = \frac{1}{\chi}(L_t - L_{pt}),$$

$$m_t = m_0 e^t.$$

The optimal allocation is given in Lemma 8 (also see Appendix P for the formal proof).

LEMMA 8: *The optimal allocation is*

$$\bar{r}_t^{****} = \frac{3\sigma - 1 \pm 2\sqrt{(2\sigma-1)\sigma}}{\sigma} \quad (24),$$

$$L_{pt}^{****} = \frac{\sigma\chi}{\left[\mu_0 + \frac{U_s'\left(k - \frac{\sigma}{m_0 e^t}\right)}{\rho}\right] e^{\rho t} - \frac{U_s'\left(k - \frac{\sigma}{m_0 e^t}\right)}{\rho}} \quad (25),$$

$$\bar{P}_t^{****} = \frac{\left[\frac{\sigma\chi}{\left[\mu_0 + \frac{U_s'\left(k - \frac{\sigma}{m_0 e^t}\right)}{\rho}\right] e^{\rho t} - \frac{U_s'\left(k - \frac{\sigma}{m_0 e^t}\right)}{\rho}}\right]^{-\sigma}}{(m_0 e^t)^{\sigma} \int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s'\left[\frac{2-3\sigma \pm 2\sqrt{2\sigma^2 - 3\sigma + 1}}{1-\sigma}(1-\sigma) + \sigma\right]\left[\frac{-vZ + \sqrt{(vZ)^2 + 4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}\right]^{1-\sigma} s^{\sigma} g(\theta_t) f(\theta_t) d\theta_t} \quad (26).$$

5 Application

Indeed, around the world, labor in fisheries is typically compensated through a share contract rather than a fixed rent contract or flat wage contract (van Ginkel, 2013). Therefore, this section has two purposes. One is to apply the transaction cost theory of sharecropping above to seafood harvesting and formally derive the determinants of the optimal share rate. The other is to examine the policy implications for the seafood harvesting industry under share contracts.

5.1 Share-Fishing Model

In this subsection, a model of share-fishing is developed based on the previous section. In what follows, I suppose that a boat is considered to be owned and controlled by a captain (typically the boat-owner). So, each boat consists of an owner and a small crew (a group of fishermen) who work under the direction of the captain and share the rent produced by

seafood harvesting. The purpose of this subsection is to illustrate the determinants of the optimal share rate for the boat-owner. To apply and simplify the transaction cost theory of sharecropping, following (Sutinen, 1975), I assume the boat-owner wishes to solve

$$\underset{\{\beta,r\}}{\text{Max}}\ \text{E}U_l[\pi_l(\theta)] = \text{E}\{-\exp[-\epsilon_l\pi_l(\theta)]\} = -\frac{\wedge^z \exp\left[\epsilon_l\left(vH+km+\frac{\tilde{k}}{2}H^2+m\beta\right)\right]}{(\epsilon_l mrPq+\wedge)^z}.$$

subject to

$$\text{E}U_w[\pi_w(\theta)] = \text{E}\{-\exp[-\epsilon_w\pi_w(\theta)]\} = -\frac{\wedge^z \exp[\epsilon_w(WL+\alpha sL-\beta)]}{[\epsilon_w(1-r)Pq+\wedge]^z} = \overline{U}_w,$$

where ϵ_l and ϵ_w are measures of absolute risk aversion of the boat-owner and the fisherman, respectively. Here, I introduce Stiglitz (1974)'s "utility equivalent contracts" that yield the same level of expected utility, $\overline{U}_w$, to the fisherman.

The boat-owner's optimal share rate is

$$r_0 = \frac{\epsilon_w}{\epsilon_l m+\epsilon_w} \quad (27).$$

So, I obtain the following proposition (also see Appendix Q for the formal proof).

PROPOSITION 8: *An increase in the number of fishermen (m), the fishermen's absolute risk aversion (ϵ_w), and the boat-owner's absolute risk aversion (ϵ_l) lowers the boat-owner's optimal share rate (r_0).*

5.2 Open-Access Harvest in Share-Fishing Equilibrium

Fishing boats will continue to enter an open-access fishery until the boat-owners' economic profits reach zero. These economic profits are expressed as follows:

$$\pi_l = mrPq\left(L,\frac{H}{m}\right)g(\theta) - vH - km - \frac{\tilde{k}}{2}H^2 - m\beta = 0.$$

At the same time, suppose fishermen will continue to work until their total profits reach zero, i.e.:

$$\pi_w = (1-r)Pq\left(L,\frac{H}{m}\right)g(\theta) - WL - \alpha sL + \beta = 0.$$

Thus, the open-access equilibrium share rate can be expressed as:

$$r_1 = \frac{vH + km + \frac{\tilde{k}}{2}H^2 + m\beta}{mWL + m\alpha sL + vH + km + \frac{\tilde{k}}{2}H^2} \quad (28).$$

So, I obtain the following lemma (also see Appendix R for the formal proof).

LEMMA 9: *An increase in the fishermen's specific investments (s) and labor inputs (L) both lowers the boat-owner's optimal share rate (r_1) in an open-access fishery. r_1 is lowered (or raised) when β increases (or decreases).*

5.3 Dynamic Optimal Regulation of Seafood Share-Harvesting

To study the allocation of the socially optimal share-harvesting policy, I consider seafood share-fishing arrangements involving N boat-owners and m fishermen under different market structures.

5.3.1 Perfectly Competitive Market

First of all, I consider a single stock fishery ($g(\theta_t)$) harvested by N identical vessels, with m fishermen per vessel, operating in a long-run perfectly competitive market. The optimal allocation solves

$$U_4 = \underset{\{\bar{P}_t, \bar{r}_t, L_{pt}\}}{\text{Max}} \int_0^\infty e^{-\rho t} N \left\{ m_t \left[\bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} (sL_{pt})^\sigma - W_t L_{pt} - c_{wt}(s, L_{pt}) \right] - vH - km_t - \frac{\tilde{k}}{2}H^2 - ln\bar{P}_t - \int_{r_0}^{\bar{r}_t} \eta r_t^{-\eta} dr_t \right\} \text{dt},$$

subject to

$$W_t L_{pt} + c_{wt}(s, L_{pt}) = (1 - \bar{r}_t)\bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} (sL_{pt})^\sigma,$$

$$\dot{m}_t = \frac{1}{\chi}(L_t - L_{pt}),$$

$$m_t = m_0 e^t,$$

$$\dot{g}(\theta_t) = F[g(\theta_t)] - Nm_t g(\theta_t) = g(\theta_t) - [g(\theta_t)]^2 - Nm_t g(\theta_t),$$

where $g(\theta_t)$ represents stock of seafood at time t, θ_t represents marine climate change, $F[g(\theta_t)] = g(\theta_t) - [g(\theta_t)]^2$ represents biological growth function of seafood (e.g., shrimp, lobster, Tuna, or Cod), and $Nm_t g(\theta_t)$ is assumed to be harvest function of seafood.

The optimal allocation is given in Lemma 10 (also see Appendix S for the formal proof).

Lemma 10: *When fishermen stay in a long-run perfectly competitive market, the optimal allocation is*

$$\bar{r}_t^{*****} = \eta^{\frac{1}{\eta-1}} \quad (29),$$

$$L_{pt}^{*****} = \frac{\sigma\chi}{\left[\frac{\left(k-\frac{\sigma-e^{\rho t_{-1}}}{m_0 e^t}\right)}{\rho}\right] e^{\rho t} - \frac{\left(k-\frac{\sigma-e^{\rho t_{-1}}}{m_0 e^t}\right)}{\rho}} \quad (30),$$

$$\bar{P}_t^{*****} = \frac{\left[\frac{\sigma\chi}{\left[\frac{\left(k-\frac{\sigma-e^{\rho t_{-1}}}{m_0 e^t}\right)}{\rho}\right] e^{\rho t} - \frac{\left(k-\frac{\sigma-e^{\rho t_{-1}}}{m_0 e^t}\right)}{\rho}}\right]^{-\sigma}}{(m_0 e^t)^{\sigma} \eta^{\frac{1}{\eta-1}} \mathrm{H}^{1-\sigma} \mathrm{s}^{\sigma} g(\theta_t)} \quad (31),$$

$$U_4^* = N \left\{ \frac{1 - vH - \frac{\tilde{k}}{2}H^2 - \frac{1-\eta\left(\frac{\epsilon_w}{\epsilon_l m + \epsilon_w}\right)^{1-\eta}}{1-\eta}}{\rho} + \frac{km_0}{1-\rho} + \frac{1}{\eta-1}\left[\frac{1}{\rho} - \frac{(km_0)^{1-\eta}}{1-\eta-\rho}\right] \right.$$

$$- \frac{\sigma}{\rho}\left[1 + ln\left(\frac{k}{\rho}\right)\right] + \frac{\sigma}{\rho^2}$$

$$\left. + \frac{\sigma ln\sigma\chi + \sigma ln m_0 + \frac{ln\eta}{\eta-1} + (1-\sigma)lnH + \sigma lns + \ln[g(\theta_t)]}{\rho} \right\} \quad (32).$$

5.3.2 Non-Competitive Market

Let me consider a scenario where fishermen stay in a non-competitive market. In this case, the social planner maximizes

$$U_5 = \underset{\{\bar{P}_t, \bar{r}_t, L_{pt}\}}{\text{Max}} \int_0^{\infty} e^{-\rho t} N \left\{ m_t \left[\bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} (sL_{pt})^{\sigma} - W_t L_{pt} - c_{wt}(s, L_{pt}) \right] - vH - km_t - \frac{\tilde{k}}{2} H^2 - ln\bar{P}_t - \int_{r_0}^{\bar{r}_t} \eta r_t^{-\eta} dr_t \right\} \text{dt},$$

subject to

$$W_t + \alpha s = \sigma(1 - \bar{r}_t)\bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma-1},$$

$$\dot{m}_t = \frac{1}{\chi}(L_t - L_{pt}),$$

$$m_t = m_0 e^t,$$

$$\dot{g}(\theta_t) = g(\theta_t) - [g(\theta_t)]^2 - N m_t g(\theta_t).$$

The optimal allocation is given in Lemma 11(also see Appendix T for the formal proof).

Lemma 11: *The optimal allocation is*

$$\bar{r}_t^{******} = \frac{3\sigma - 1 \pm 2\sqrt{(2\sigma - 1)\sigma}}{\sigma} \quad (33),$$

$$L_{pt}^{******} = L_{pt}^{*****} \quad (34),$$

$$\bar{P}_t^{******} = \frac{\left[\frac{\sigma\chi}{\left[\frac{\left(k - \frac{\sigma - e^{\rho t} - 1}{m_0 e^t} \right)}{\rho} \right] e^{\rho t} - \frac{\left(k - \frac{\sigma - e^{\rho t} - 1}{m_0 e^t} \right)}{\rho}} \right]^{-\sigma}}{(m_0 e^t)^{\sigma} [2\sigma \pm 2\sqrt{(2\sigma - 1)\sigma}] \mathrm{H}^{1-\sigma} \mathrm{s}^{\sigma} g(\theta_t)} \quad (35),$$

$$U_5^* = N \left\{ \frac{1 - vH - \frac{\tilde{k}}{2} H^2 + \frac{\eta}{\eta - 1} \left\{ \left[\frac{3\sigma - 1 \pm 2\sqrt{(2\sigma - 1)\sigma}}{\sigma} \right]^{1-\eta} - \left(\frac{\epsilon_w}{\epsilon_l m + \epsilon_w} \right)^{1-\eta} \right\}}{\rho} + \frac{km_0}{1 - \rho} + \frac{1}{\eta - 1} \left[\frac{1}{\rho} - \frac{(km_0)^{1-\eta}}{1 - \eta - \rho} \right] - \frac{\sigma}{\rho} \left[1 + ln\left(\frac{k}{\rho}\right) \right] + \frac{\sigma}{\rho^2} + \frac{\sigma ln\sigma\chi + \sigma ln m_0 + \ln[2\sigma \pm 2\sqrt{(2\sigma - 1)\sigma}] + (1 - \sigma) lnH + \sigma lns + \ln[g(\theta_t)]}{\rho} \right\} \quad (36).$$

5.3.3 Calibration

In this subsection, I calculate the difference in social welfare between the competitive market and the non-competitive market as follows: $D_1 = U_5^* - U_4^* =$

$\frac{N\eta\{\left[\frac{3\sigma-1\pm2\sqrt{(2\sigma-1)\sigma}}{\sigma}\right]^{1-\eta}-\left(\frac{\epsilon_w}{\epsilon_l m+\epsilon_w}\right)^{1-\eta}\}+1-\eta\left(\frac{\epsilon_w}{\epsilon_l m+\epsilon_w}\right)^{1-\eta}}{\rho(\eta-1)}+\frac{N\{\ln\left[2\sigma\pm2\sqrt{(2\sigma-1)\sigma}\right]-\frac{ln\eta}{\eta}\}}{\rho}$. Figure A12 and Figure A13 present the calibrations of D_1 for ($\bar{r}_t^{******}=\bar{r}_{t+}$ and $\bar{r}_t^{******}=\bar{r}_{t-}$, respectively (also see the details of D_1 with different parameters in Figure A14 and Figure A15 of appendix). It implies that the market structures of fishermen's labor may indirectly affect social welfare.

6 Conclusion

For half a century, economists have argued that sharecropping is not an inefficient contractual arrangement. In contrast, I illustrate that if transaction costs are considered, share contracts cannot lead to Pareto efficiency. In this paper, I began with two main purposes. One is to develop a theoretical framework for analyzing the impact of transaction costs on optimal share rate. The other is to apply the transaction cost theory of sharecropping to "share-fishing" in the seafood harvest industry.

In the basic model, with positive transaction costs under share contracts, the landlord's optimal share rate exceeds the tenant's elasticity of output with respect to farmland. Additionally, the landlord's marginal transaction and production costs cause the optimal share rate to deviate from the equilibrium where share tenancy is efficient. Furthermore, I have proposed a model in which the return rate from non-agricultural exploration of farmland is positively associated with the regulated share rate; the number of tenants is positively associated with regulated share rate; and the price of agri-food is negatively associated with the regulated share rate, when tenants stay in a long-run perfectly competitive market. Finally, while climate change is positively associated with the landlord's optimal share rate, the socially optimal share rate of the landlord is not determined by farmland ownership arrangements or climate change. Meanwhile, when

tenants stay in a long-run perfectly competitive market, the market structure of agri-food does not affect the landlord's optimal share rate.

I extend the model to account for climate uncertainty, in which the tenant's wage rate and marginal production cost with respect to labor both lower the landlord's optimal share rate. Conversely, the return rate from non-agricultural exploration of farmland, the landlord's marginal production cost with respect to farmland size, and the regulated price of agri-food all encourage a higher landlord's optimal share rate. Moreover, the landlord's optimal share rate under agri-food price control is higher than that without agri-food price control, when the tenant stays in a monopoly market for agri-food. The landlord's optimal share rate in a duopoly market for agri-food is lower than in a monopoly market. Given that two tenants invest in specific technologies for agri-food production, the landlord's optimal share rate is the function of the impact of tenants' marginal transaction costs.

Finally, I apply the transaction cost theory of sharecropping to seafood share-harvesting and show that the number of fishermen, the fishermen's absolute risk aversion, and the boat-owner's absolute risk aversion are negatively related to the boat-owner's optimal share rate. In an open-access fishery, both the fishermen's specific investments and labor inputs lower the boat-owner's optimal share rate.

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A. PROOF OF PROPOSITION 1

The landlord problem is:

$$\underset{\{m,r,H\}}{\text{Max}}\ \pi_l = mrPq(L,h) - vH,$$

$$s.t.\ WL + c_w(s,L) \leq (1-r)Pq(L,h).$$

To solve this problem, write the Lagrangean:

$$\mathcal{L} = mrq(L,h)P - vH - \lambda[WL + c_w(s,L) - (1-r)q(L,h)P]$$

$$= mrq\left(L,\frac{H}{m}\right)P - vH - \lambda\left[WL + c_w(s,L) - (1-r)q(L,\frac{H}{m})P\right].$$

Now take the first order conditions (FOCs).

1) Start with the FOC w.r.t. m:

$\frac{\partial \mathcal{L}}{\partial m} = rq\left(L,\frac{H}{m}\right)P + mrP\frac{\partial q}{\partial h}\left(-\frac{H}{m^2}\right) + \lambda(1-\mathrm{r})P\frac{\partial q}{\partial h}(-\frac{H}{m^2}) \leq 0$, with complementary slackness

$\frac{\partial \mathcal{L}}{\partial m}m = 0$ and $m > 0$. It implies that: $\frac{\partial \mathcal{L}}{\partial m} = rPq\left(L,\frac{H}{m}\right) - rhP\frac{\partial q}{\partial h} - \lambda(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{h}{m} = 0\ \ (A1)$.

2) Compute the FOC w.r.t. r:

$\frac{\partial \mathcal{L}}{\partial r} = mPq\left(L,\frac{H}{m}\right) - \lambda Pq(L,\frac{H}{m}) \leq 0$, with complementary slackness $\frac{\partial \mathcal{L}}{\partial r}r = 0$ and $r > 0$. It

implies that: $\frac{\partial \mathcal{L}}{\partial r} = mqP\left(L,\frac{H}{m}\right) - \lambda Pq(L,\frac{H}{m}) = 0\ \ (A2)$.

3) Compute the FOC w.r.t. H:

$\frac{\partial \mathcal{L}}{\partial H} = mrP\frac{\partial q}{\partial h}\frac{1}{m} - v + \lambda(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{1}{m} \leq 0$, with complementary slackness $\frac{\partial \mathcal{L}}{\partial H}H = 0$ and $H > 0$.

It implies that: $\frac{\partial \mathcal{L}}{\partial H} = rP\frac{\partial q}{\partial h} - v + \lambda(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{1}{m} = 0\ \ (A3)$.

4) Compute the FOC w.r.t. λ:

$\frac{\partial \mathcal{L}}{\partial \lambda} = Wt + c_t(s,L) - (1-r)Pq(L,\frac{H}{m}) \geq 0$, with complementary slackness $\frac{\partial \mathcal{L}}{\partial \lambda}\lambda = 0$ and $\lambda \geq 0$.

It implies that if $\lambda = 0$, then $\frac{\partial \mathcal{L}}{\partial r} = mPq\left(L,\frac{H}{m}\right) > 0$, which violates with $\frac{\partial \mathcal{L}}{\partial r} = 0$ in equation (A2);

if $\lambda > 0$, then $\frac{\partial \mathcal{L}}{\partial \lambda} = WL + c_w(s,L) - (1-r)Pq\left(L,\frac{H}{m}\right) = 0\ \ (A4)$. Hence, there is no corner

solution in this optimization problem.

Finally, the 4 FOCs of the landlord problem are:

$$\begin{cases} rPq\left(L,\frac{H}{m}\right) - rhP\frac{\partial q}{\partial h} - \lambda(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{h}{m} = 0 \ \ (A1) \\ mqP\left(L,\frac{H}{m}\right) - \lambda Pq\left(L,\frac{H}{m}\right) = 0 \ \ (A2) \\ rP\frac{\partial q}{\partial h} - v + \lambda(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{1}{m} = 0 \ \ (A3) \\ WL + c_w(s,L) - (1-r)Pq\left(L,\frac{H}{m}\right) = 0 \ \ (A4) \end{cases}$$

5) Solution in terms of landlord's optimal share rate

From equation (A2),

$$m = \lambda \text{ (A5).}$$

Next, substitute in (A1):

$$rPq - rhP\frac{\partial q}{\partial h} - m(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{h}{m} = 0$$

Hence:

$$rPq - rhP\frac{\partial q}{\partial h} - \mathrm{P}\frac{\partial q}{\partial h}h + \mathrm{rP}\frac{\partial q}{\partial h}h = 0.$$

Then:

$$r = \frac{\partial q/\partial h}{q/h} \text{ (A6).}$$

This indicates that the optimal share rate is consistent with Cheung (1968)'s solution, because Cheung (1968) argues that $r_{Cheung} = \frac{\partial q/\partial h}{q/h}$ is the equilibrium condition of a share contract.

Plugging (A1) in (A3):

$$rP\frac{\partial q}{\partial h} - v + m(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{1}{m} = 0,$$

or

$$v = P\frac{\partial q}{\partial h} \quad \text{(A7).}$$

That is, in equilibrium, the optimal return rate of farmland exploration equals the value of the marginal product of farmland.

Finally, plug (A6) in (A7):

$$r^* = \frac{\frac{v}{P}}{\frac{q}{h}} = r_{Pareto},$$

which is stated as in equation (1) in the text.

In this case,

$$\pi_l = mr^* Pq - Hv = 0.$$

B. PROOF OF LEMMA 1

The proof is similar to that of Proposition 1. The landlord problem is:

$$\underset{\{m,r,H\}}{\text{Max}}\ \pi_l = mrPq(L,h) - c_l(m,H),$$

$$s.t.\ WL + c_w(s,L) \leq (1-r)Pq(L,h).$$

To solve this problem, write the Lagrangean:

$$\mathcal{L}1 = mrPq\left(L, \frac{H}{m}\right) - c_l(m, H) - \mu\left[WL + c_w(s, L) - (1-r)Pq(L, \frac{H}{m})\right].$$

Now take the FOCs.

1) Start with the FOC w.r.t. m:

$\frac{\partial \mathcal{L}1}{\partial m} = rPq\left(L, \frac{H}{m}\right) + mrP\frac{\partial q}{\partial h}\left(-\frac{H}{m^2}\right) - \frac{\partial c_l}{\partial m} + \mu(1-\mathrm{r})P\frac{\partial q}{\partial h}(-\frac{H}{m^2}) \leq 0$, with complementary slackness $\frac{\partial \mathcal{L}1}{\partial m}m = 0$ and $m > 0$. It implies that:$\frac{\partial \mathcal{L}1}{\partial m} = rPq\left(L, \frac{H}{m}\right) - rhP\frac{\partial q}{\partial h} - \frac{\partial c_l}{\partial m} -$

$\mu(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{h}{m} = 0 \ \ (A8)$.

2) Compute the FOC w.r.t. r:

$\frac{\partial \mathcal{L}1}{\partial r} = mPq\left(L, \frac{H}{m}\right) - \mu Pq(L, \frac{H}{m}) \leq 0$, with complementary slackness $\frac{\partial \mathcal{L}1}{\partial r}r = 0$ and $r > 0$. It

implies that:$\frac{\partial \mathcal{L}1}{\partial r} = mPq\left(L, \frac{H}{m}\right) - \mu Pq(L, \frac{H}{m}) = 0 \ \ (A9)$.

3) Compute the FOC w.r.t. H:

$\frac{\partial \mathcal{L}1}{\partial H} = mrP\frac{\partial q}{\partial h}\frac{1}{m} - \frac{\partial c_l}{\partial H} + \mu(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{1}{m} \leq 0$, with complementary slackness $\frac{\partial \mathcal{L}1}{\partial H}H = 0$ and $H >$

0. It implies that:$\frac{\partial \mathcal{L}1}{\partial H} = mrP\frac{\partial q}{\partial h}\frac{1}{m} - \frac{\partial c_l}{\partial H} + \mu(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{1}{m} = 0 \ \ (A10)$.

4) Compute the FOC w.r.t. μ:

$\frac{\partial \mathcal{L}1}{\partial \mu} = WL + c_t(s, L) - (1-r)q(L, \frac{H}{m}) \geq 0$, with complementary slackness $\frac{\partial \mathcal{L}1}{\partial \mu}\mu = 0$ and $\mu \geq 0$.

It implies that if $\mu = 0$, then $\frac{\partial \mathcal{L}1}{\partial r} = mPq\left(L, \frac{H}{m}\right) > 0$, which violates with $\frac{\partial \mathcal{L}1}{\partial r} = 0$ in (A9); if $\mu >$

0, then $\frac{\partial \mathcal{L}1}{\partial \mu} = WL + c_w(s, L) - (1-r)Pq\left(L, \frac{H}{m}\right) = 0 \ \ (A11)$. Hence, there is no corner solution

in this optimization problem.

Finally, the 4 FOCs of the landlord problem are:

$$\begin{cases} rPq\left(L,\frac{H}{m}\right) - rhP\frac{\partial q}{\partial h} - \frac{\partial c_l}{\partial m} - \mu(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{h}{m} = 0 \ \ (A8) \\ mPq\left(L,\frac{H}{m}\right) - \mu Pq(L,\frac{H}{m}) = 0 \ \ (A9) \\ mrP\frac{\partial q}{\partial h}\frac{1}{m} - \frac{\partial c_l}{\partial H} + \mu(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{1}{m} = 0 \ \ (A10) \\ WL + c_w(s,L) - (1-r)Pq\left(L,\frac{H}{m}\right) = 0 \ \ (A11) \end{cases}$$

5) Solution in terms of landlord's optimal share rate

From equation (A9), I have

$$m = \mu \text{ (A12).}$$

Next, substitute in (A8):

$$rPq - rhP\frac{\partial q}{\partial h} - \frac{\partial c_l}{\partial m} - m(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{h}{m} = 0.$$

Thus:

$$rPq - rhP\frac{\partial q}{\partial h} - \frac{\partial c_l}{\partial m} - \mathrm{P}\frac{\partial q}{\partial h}h + \mathrm{rP}\frac{\partial q}{\partial h}h = 0,$$

or

$$r = \frac{\frac{\partial c_l}{\partial m}}{Pq} + \frac{\partial q/\partial h}{q/h} \text{ (A13).}$$

Plugging (A9) in (A10):

$$rP\frac{\partial q}{\partial h} - \frac{\partial c_l}{\partial H} + (1-\mathrm{r})P\frac{\partial q}{\partial h} = 0.$$

Then:

$$P\frac{\partial q}{\partial h} = \frac{\partial c_l}{\partial H} \text{ (A14).}$$

Therefore, substitute in (A13):

$r^{**} = \frac{\frac{\partial c_l}{\partial m}}{Pq} + \frac{\partial q/\partial h}{q/h} > \frac{\partial q/\partial h}{q/h} = r_{Cheung}$, which is stated as equation (2) in the text.

C. PROOF OF LEMMA 2

The proof is similar to that of Lemma 1. The landlord problem is:

$$\underset{\{m,r,H\}}{\text{Max}}\ \pi_l = mrPq(L,h) - vH - c_l(m,H),$$

$$s.t.\ WL + c_w(s,L) \leq (1-r)Pq(L,h).$$

To solve this problem, write the Lagrangean:

$$\mathcal{L}2 = mrPq\left(L,\frac{H}{m}\right) - vH - c_l(m,H) - \varphi\left[WL + c_w(s,L) - (1-r)Pq(L,\frac{H}{m})\right].$$

Now take the FOCs.

1) Start with the FOC w.r.t. m:

$\frac{\partial \mathcal{L}2}{\partial m} = rPq\left(L, \frac{H}{m}\right) + mrP\frac{\partial q}{\partial h}\left(-\frac{H}{m^2}\right) - \frac{\partial c_l}{\partial m} + \varphi(1-\mathrm{r})P\frac{\partial q}{\partial h}(-\frac{H}{m^2}) \leq 0$, with complementary slackness $\frac{\partial \mathcal{L}2}{\partial m}m = 0$ and $m > 0$. It implies that:$\frac{\partial \mathcal{L}2}{\partial m} = rPq\left(L, \frac{H}{m}\right) - rhP\frac{\partial q}{\partial h} - \frac{\partial c_l}{\partial m} -$

$\varphi(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{h}{m} = 0 \ \ (A15)$.

2) Compute the FOC w.r.t. r:

$\frac{\partial \mathcal{L}2}{\partial r} = mPq\left(L, \frac{H}{m}\right) - \varphi Pq(L, \frac{H}{m}) \leq 0$, with complementary slackness $\frac{\partial \mathcal{L}2}{\partial r}r = 0$ and $r > 0$. It

implies that:$\frac{\partial \mathcal{L}2}{\partial r} = mPq\left(L, \frac{H}{m}\right) - \varphi Pq(L, \frac{H}{m}) = 0 \ \ (A16)$.

3) Compute the FOC w.r.t. H:

$\frac{\partial \mathcal{L}2}{\partial H} = mrP\frac{\partial q}{\partial h}\frac{1}{m} - v - \frac{\partial c_l}{\partial H} + \varphi(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{1}{m} \leq 0$, with complementary slackness $\frac{\partial \mathcal{L}2}{\partial H}H = 0$ and

$H > 0$. It implies that: $\frac{\partial \mathcal{L}2}{\partial H} = mrP\frac{\partial q}{\partial h}\frac{1}{m} - v - \frac{\partial c_l}{\partial H} + \varphi(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{1}{m} = 0 \ \ (A17)$.

4) Compute the FOC w.r.t. φ:

$\frac{\partial \mathcal{L}2}{\partial \varphi} = WL + c_w(s, L) - (1-r)q(L, \frac{H}{m}) \geq 0$, with complementary slackness $\frac{\partial \mathcal{L}2}{\partial \varphi}\varphi = 0$ and $\varphi \geq$

0. It implies that if $\varphi = 0$, then $\frac{\partial \mathcal{L}2}{\partial r} = mPq\left(L, \frac{H}{m}\right) > 0$, which violates with $\frac{\partial \mathcal{L}2}{\partial r} = 0$ in (A16); if

$\varphi > 0$, then $\frac{\partial \mathcal{L}2}{\partial \varphi} = WL + c_w(s, L) - (1-r)Pq\left(L, \frac{H}{m}\right) = 0 \ \ (A18)$. Hence, there is no corner

solution in this optimization problem.

Finally, the 4 FOCs of the landlord problem are:

$$\begin{cases} rPq\left(L,\frac{H}{m}\right) - rhP\frac{\partial q}{\partial h} - \frac{\partial c_l}{\partial m} - \varphi(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{h}{m} = 0 \ \ (A15) \\ mPq\left(L,\frac{H}{m}\right) - \varphi Pq(L,\frac{H}{m}) = 0 \ \ (A16) \\ mrP\frac{\partial q}{\partial h}\frac{1}{m} - v - \frac{\partial c_l}{\partial H} + \varphi(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{1}{m} = 0 \ \ (A17) \\ WL + c_w(s,L) - (1-r)Pq\left(L,\frac{H}{m}\right) = 0 \ \ (A18) \end{cases}$$

5) Solution in terms of landlord's optimal share rate

From equation (A16),

$$m = \varphi \text{ (A19).}$$

Next, substitute in (A15),

$$rPq - rhP\frac{\partial q}{\partial h} - \frac{\partial c_l}{\partial m} - m(1-\mathrm{r})P\frac{\partial q}{\partial h}\frac{h}{m} = 0.$$

Hence:

$$rPq - rhP\frac{\partial q}{\partial h} - \frac{\partial c_l}{\partial m} - \mathrm{P}\frac{\partial q}{\partial h}h + \mathrm{rP}\frac{\partial q}{\partial h}h = 0.$$

Thus:

$$r = \frac{\frac{\partial c_l}{\partial m}}{Pq} + \frac{\partial q/\partial h}{q/h} \text{ (A20).}$$

Plugging (A19) in (A17):

$$rP\frac{\partial q}{\partial h} - v - \frac{\partial c_l}{\partial H} + (1-\mathrm{r})P\frac{\partial q}{\partial h} = 0,$$

or

$$P\frac{\partial q}{\partial h} = v + \frac{\partial c_l}{\partial H} \text{ (A21).}$$

Therefore, plug (A21) in (A20):

$r^{***} = \frac{\frac{\partial c_l}{\partial m} + h\left(v + \frac{\partial c_l}{\partial H}\right)}{Pq} > r^{**} = \frac{\frac{\partial c_l}{\partial m} + h\frac{\partial c_l}{\partial H}}{Pq} > r_{Cheung}$, which is stated as equation (3) in the text.

D. PROOF OF PROPROSITION 2

First, I verify that the second-order conditions are satisfied. To see this, using the first-order conditions (4) and (5), the second-order conditions can be expressed as:

$$|Z_{11}| = \frac{\partial^2 SW}{\partial \bar{r}^2} = -\frac{\partial R}{\partial \bar{r}} > 0,$$

$$|Z_{22}| = \frac{\partial^2 SW}{\partial H^2} = -\frac{\partial^2 c_l}{\partial H^2} < 0,$$

$$|Z_{12}| = \frac{\partial}{\partial H}\left(\frac{\partial SW}{\partial \bar{r}}\right) = P\frac{\partial q}{\partial h} > 0,$$

$$|Z_{21}| = \frac{\partial}{\partial \bar{r}}\left(\frac{\partial SW}{\partial H}\right) = P\frac{\partial q}{\partial h} > 0,$$

where by assumption, $\frac{\partial R}{\partial \bar{r}} < 0$ implies that an decrease in regulated share rate leads to an increase in dissipated rent (Cheung, 1974). Because when share rate is regulated at a lower level than the market equilibrium level, public domain creating rent dissipation expands more (Barzel, 1997). In addition, assume $\frac{\partial^2 c_l}{\partial H^2} > 0$, which illustrates the law of increasing marginal transaction cost with respect to total farmland owned by the landlord. Note that $\Delta = Z_{11}Z_{22} - Z_{12}Z_{21} < 0$, and the second-order conditions are satisfied.

Next, the comparative static results follow from the matrix:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} \frac{\partial \bar{r}}{\partial m} \\ \frac{\partial H}{\partial m} \end{bmatrix} = \begin{bmatrix} -\frac{\partial}{\partial m}\left(\frac{\partial SW}{\partial \bar{r}}\right) \\ -\frac{\partial}{\partial m}\left(\frac{\partial SW}{\partial H}\right) \end{bmatrix} = \begin{bmatrix} \frac{\partial q}{\partial h}h \\ 0 \end{bmatrix},$$

where

$$\frac{\partial \bar{r}}{\partial m} = \frac{\begin{vmatrix} \frac{\partial q}{\partial h}h & Z_{12} \\ 0 & Z_{22} \end{vmatrix}}{Z_{11}Z_{22} - Z_{12}Z_{21}} = \frac{\frac{\partial q}{\partial h}hZ_{22}}{Z_{11}Z_{22} - Z_{12}Z_{21}} > 0 \qquad \text{(A22)}.$$

Equation (A22) indicates the extent to which regulated share rate can be increased by introducing return rate from non-agricultural exploration of farmland.

Similarly, with regard to the impact of the return rate from non-agricultural exploration of farmland on regulated share rate, I obtain the following matrix:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} \frac{\partial \bar{r}}{\partial v} \\ \frac{\partial H}{\partial v} \end{bmatrix} = \begin{bmatrix} -\frac{\partial}{\partial v}\left(\frac{\partial SW}{\partial \bar{r}}\right) \\ -\frac{\partial}{\partial v}\left(\frac{\partial SW}{\partial H}\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

where

$$\frac{\partial \bar{r}}{\partial v} = \frac{\begin{vmatrix} 0 & Z_{12} \\ 1 & Z_{22} \end{vmatrix}}{Z_{11}Z_{22} - Z_{12}Z_{21}} = \frac{-Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}} > 0 \quad \text{(A23)}.$$

Equation (A23) indicates the extent to which regulated share rate can be reduced by introducing the return rate from non-agricultural exploration of farmland.

Finally, I derive marginal relationships between $\bar{r}$ and P from the following matrix:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} \frac{\partial \bar{r}}{\partial P} \\ \frac{\partial H}{\partial P} \end{bmatrix} = \begin{bmatrix} -\frac{\partial}{\partial P}\left(\frac{\partial SW}{\partial \bar{r}}\right) \\ -\frac{\partial}{\partial P}\left(\frac{\partial SW}{\partial H}\right) \end{bmatrix} = \begin{bmatrix} -\mathrm{mq} \\ -\bar{r}\frac{\partial q}{\partial h} \end{bmatrix},$$

where

$$\frac{\partial \bar{r}}{\partial P} = \frac{\begin{vmatrix} -\mathrm{mq} & Z_{12} \\ -\bar{r}\frac{\partial q}{\partial h} & Z_{22} \end{vmatrix}}{Z_{11}Z_{22} - Z_{12}Z_{21}} = \frac{-Z_{22}\mathrm{mq} + Z_{12}\bar{r}\frac{\partial q}{\partial h}}{Z_{11}Z_{22} - Z_{12}Z_{21}} < 0 \quad (A24).$$

Equation (A24) indicates the extent to which regulated share rate can be reduced by increasing the price of agri-food.

E. PROOF OF LEMMA 3

First, from the first-order conditions (7) and (8), I obtain

$$\frac{\partial SW}{\partial \bar{P}} = mrq\left(L, \frac{H}{m}\right) - \frac{\partial S}{\partial \bar{P}} = 0 \quad (A25),$$

and

$$\frac{\partial SW}{\partial H} = r\bar{P}\frac{\partial q}{\partial h} - v - \frac{\partial c_l}{\partial H} = 0 \quad (A26).$$

Using equations (A25) and (A26), the second order conditions can be expressed as:

$$|J_{11}| = \frac{\partial^2 SW}{\partial \bar{P}^2} = -\frac{\partial^2 SW}{\partial \bar{P}^2} > 0,$$

$$|J_{22}| = \frac{\partial^2 SW}{\partial H^2} = -\frac{\partial^2 c_l}{\partial H^2} < 0,$$

$$|J_{12}| = \frac{\partial}{\partial H}\left(\frac{\partial SW}{\partial \bar{P}}\right) = r\frac{\partial q}{\partial h} > 0,$$

$$|J_{21}| = \frac{\partial}{\partial \bar{P}}\left(\frac{\partial SW}{\partial H}\right) = r\frac{\partial q}{\partial h} > 0.$$

Since $\Delta' = J_{11}J_{22} - J_{12}J_{21} < 0$, the second-order conditions are satisfied for the requirement of social welfare maximization.

Next, I derive marginal relationship between $\bar{P}$ and v as the following matrix:

$$\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \frac{\partial \bar{P}}{\partial v} \\ \frac{\partial H}{\partial v} \end{bmatrix} = \begin{bmatrix} -\frac{\partial}{\partial v}\left(\frac{\partial SW}{\partial \bar{P}}\right) \\ -\frac{\partial}{\partial v}\left(\frac{\partial SW}{\partial H}\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

where

$$\frac{\partial \bar{P}}{\partial v} = \frac{\begin{vmatrix} 0 & J_{12} \\ 1 & J_{22} \end{vmatrix}}{J_{11}J_{22} - J_{12}J_{21}} = \frac{-J_{12}}{J_{11}J_{22} - J_{12}J_{21}} > 0 \quad (A27).$$

Equation (A27) indicates the extent to which regulated price of agri-food can be reduced by decreasing the return rate from non-agricultural exploration of farmland.

F. PROOF OF PROPROSITION 3

The social planner problem is:

$$U_0 = \underset{\{\bar{P}_t, \bar{r}_t, L_{pt}\}}{\text{Max}} \int_0^{\infty} e^{-\rho t} \left\{ m_t\left[\bar{P}_t h_t^{1-\sigma} L_{pt}^{\sigma} - W_t L_{pt} - c_{wt}(s, L_{pt})\right] - vH - km_t - \frac{\tilde{k}}{2}H^2 - ln\bar{P}_t - \int_{r^{***}}^{\bar{r}_t} \eta r_t^{-\eta} dr_t \right\} \text{dt},$$

s.t.

$$W_t L_{pt} + c_{wt}(s, L_{pt}) = (1 - \bar{r}_t)\bar{P}_t h_t^{1-\sigma} L_{pt}^{\sigma},$$

$$\dot{m}_t = \frac{1}{\chi}(L_t - L_{pt}),$$

$$m_t = m_0 e^t.$$

Next, define Hamiltonian with state variable m_t, control variables $\{\bar{P}_t, \bar{r}_t, L_{pt}\}$ and co-state variable μ_t:

$$\mathcal{H}(\bar{P}_t, \bar{r}_t, L_{pt}, m_t, \mu_t) = m_t\left[\bar{P}_t h_t^{1-\sigma} L_{pt}^{\sigma} - W_t L_{pt} - c_{wt}(s, L_{pt})\right] - vH - km_t - \frac{\tilde{k}}{2}H^2 - ln\bar{P}_t -$$

$$\int_{r^{***}}^{\bar{r}_t} \eta r_t^{-\eta} dr_t + \frac{\mu_t}{\chi}(L_t - L_{pt}) = m_t\left[\bar{P}_t h_t^{1-\sigma} L_{pt}^{\sigma} - (1-\bar{r}_t)\bar{P}_t h_t^{1-\sigma} L_{pt}^{\sigma}\right] - vH - km_t - \frac{\tilde{k}}{2}H^2 -$$

$$ln\bar{P}_t - \eta\frac{\bar{r}_t^{1-\eta} - (r^{***})^{1-\eta}}{1-\eta} + \frac{\mu_t}{\chi}(L_t - L_{pt}) = m_t\bar{r}_t\bar{P}_t h_t^{1-\sigma} L_{pt}^{\sigma} - vH - km_t - \frac{\tilde{k}}{2}H^2 - ln\bar{P}_t -$$

$$\eta\frac{\bar{r}_t^{1-\eta} - (r^{***})^{1-\eta}}{1-\eta} + \frac{\mu_t}{\chi}(L_t - L_{pt}).$$

The FOCs are:

$$\begin{cases} \dfrac{\partial \mathcal{H}}{\partial L_{pt}} = 0 \\ \dfrac{\partial \mathcal{H}}{\partial \bar{P}_t} = 0 \\ \dfrac{\partial \mathcal{H}}{\partial \bar{r}_t} = 0 \\ \rho = \dfrac{\partial \mathcal{H}/\partial m_t}{\mu_t} + \dfrac{\dot{\mu}_t}{\mu_t} \end{cases}$$

Start with $\frac{\partial \mathcal{H}}{\partial L_{pt}} = 0$, which implies

$$\sigma m_t \bar{r}_t \bar{P}_t h_t^{1-\sigma} L_{pt}^{\sigma-1} = \frac{\mu_t}{\chi},$$

and therefore

$$L_{pt} = \left(\frac{\mu_t}{\chi\sigma m_t \bar{r}_t \bar{P}_t h_t^{1-\sigma}}\right)^{\frac{1}{\sigma-1}} \quad (A30).$$

Next, consider $\frac{\partial \mathcal{H}}{\partial \bar{P}_t} = 0$:

$$m_t \bar{r}_t h_t^{1-\sigma} L_{pt}^{\sigma} = \frac{1}{\bar{P}_t}.$$

Substituting gives

$$\bar{P}_t = \frac{1}{m_t \bar{r}_t h_t^{1-\sigma} L_{pt}^{\sigma}} \ (A31).$$

The next FOC is similar, $\frac{\partial \mathcal{H}}{\partial \bar{r}_t} = 0$:

$$m_t \bar{P}_t h_t^{1-\sigma} L_{pt}^{\sigma} = \eta \bar{r}_t^{-\eta}.$$

Substituting gives

$$\bar{r}_t^{\,*} = \eta^{\frac{1}{\eta-1}} \ (A32).$$

Finally, the last FOC is $\rho = \frac{\partial \mathcal{H}/\partial m_t}{\mu_t} + \frac{\dot{\mu}_t}{\mu_t}$. The derivative of the Hamiltonian $\frac{\partial \mathcal{H}}{\partial m_t}$ is:

$$\frac{\partial \mathcal{H}}{\partial m_t} = \bar{r}_t \bar{P}_t h_t^{1-\sigma} L_{pt}^{\sigma} + (\sigma - 1)\bar{r}_t \bar{P}_t h_t^{1-\sigma} L_{pt}^{\sigma} - k = \frac{\sigma}{m_t} - k.$$

Substituting this into the FOC and rearranging to solve for μ_t gives

$$\dot{\mu_t} - \rho\mu_t = k - \frac{\sigma}{m_t}.$$

Solving for μ_t^*:

$$\mu_t^* = \left(\mu_0 - \frac{k - \frac{\sigma}{m_t}}{-\rho}\right) e^{\rho t} + \frac{k - \frac{\sigma}{m_t}}{-\rho} = \left(\mu_0 + \frac{k - \frac{\sigma}{m_0 e^t}}{\rho}\right) e^{\rho t} - \frac{k - \frac{\sigma}{m_0 e^t}}{\rho} \ (A33).$$

(1) Solving the Optimal Allocation

The 4 FOCs are:

$$\begin{cases} L_{pt} = \left(\frac{\mu_t}{\chi \sigma m_t \bar{r}_t \bar{P}_t h_t^{1-\sigma}}\right)^{\frac{1}{\sigma-1}} \ (A30) \\ \bar{P}_t = \frac{1}{m_t \bar{r}_t h_t^{1-\sigma} L_{pt}^{\sigma}} \ (A31) \\ \bar{r}_t^{\,*} = \eta^{\frac{1}{\eta-1}} \ (A32) \\ \mu_t^* = \left(\mu_0 + \frac{k - \frac{\sigma}{m_0 e^t}}{\rho}\right) e^{\rho t} - \frac{k - \frac{\sigma}{m_0 e^t}}{\rho} \ (A33) \end{cases}$$

1) Solving for L_{pt} and $\bar{P}_t$(eqs. A30 and A31).

Combining (A30), (A31), and (A33) gives

$$L_{pt}^{*}=\frac{\sigma\chi}{\mu_t^{*}}=\frac{\sigma\chi}{\left(\mu_0+\frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right)e^{\rho t}-\frac{k-\frac{\sigma}{m_0e^t}}{\rho}}\quad(A34).$$

Again, combining (A31), (A32), and (A34) gives

$$\bar{P}_t^{\,*}=\frac{1}{m_t\bar{r}_t^{\,*}h_t^{1-\sigma}L_{pt}^{*\sigma}}=\frac{\left[\frac{\sigma\chi}{\left(\mu_0+\frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right)e^{\rho t}-\frac{k-\frac{\sigma}{m_0e^t}}{\rho}}\right]^{-\sigma}}{(m_0e^t)^{\sigma}\eta^{\frac{1}{\eta-1}}\mathrm{H}^{1-\sigma}}\quad(A35).$$

2) Solving for U_0.

According to equation (3), $r^{***}=\frac{\frac{\partial c_l}{\partial m}+h(v+\frac{\partial c_l}{\partial H})}{Pq}$. Plugging equations (A34) and (A35) yields:

$$r^{***}=\frac{\frac{\partial c_l}{\partial m}+h(v+\frac{\partial c_l}{\partial H})}{Pq}=\frac{k+\frac{H}{m_t}(v+\tilde{k}H)}{\bar{P}_t^{\,*}h_t^{1-\sigma}L_{pt}^{*\sigma}}=\frac{k+\frac{H}{m_t}(v+\tilde{k}H)}{\frac{1}{m_t\bar{r}_t^{\,*}}}=km_t\bar{r}_t^{\,*}+\bar{r}_t^{\,*}H\left(v+\tilde{k}H\right)=\bar{r}_t^{\,*}\left[km_t+H\left(v+\tilde{k}H\right)\right]=\eta^{\frac{1}{\eta-1}}(km_0e^t+Hv+\tilde{k}H^2)\ .$$

Next, by definition $U_0=\int_0^{\infty}e^{-\rho t}\left\{m_t\left[\bar{P}_th_t^{1-\sigma}L_{pt}^{\sigma}-W_tL_{pt}-c_{wt}\left(s,L_{pt}\right)\right]-vH-km_t-\frac{\tilde{k}}{2}H^2-ln\bar{P}_t-\int_{r^{***}}^{\bar{r}_t}\eta r_t^{-\eta}dr_t\right\}\mathrm{dt}$. Then:

$$U_0=\int_0^{\infty}e^{-\rho t}\left[m_t\bar{r}_t^{\,*}\bar{P}_t^{\,*}h_t^{1-\sigma}L_{pt}^{*\sigma}-vH-km_t-\frac{\tilde{k}}{2}H^2-ln\bar{P}_t^{\,*}-\eta\frac{(\bar{r}_t^{\,*})^{1-\eta}-(r^{***})^{1-\eta}}{1-\eta}\right]\mathrm{dt}$$

$$=\int_0^{\infty}e^{-\rho t}\left[1-vH-km_0e^t-\frac{\tilde{k}}{2}H^2-ln\bar{P}_t^{\,*}-\eta\frac{\eta^{\frac{1-\eta}{\eta-1}}-\left[\eta^{\frac{1}{\eta-1}}\left(km_0e^t+Hv+\tilde{k}H^2\right)\right]^{1-\eta}}{1-\eta}\right]\mathrm{dt}$$

$$=\int_0^{\infty}e^{-\rho t}\left\{\left(1-vH-\frac{\tilde{k}}{2}H^2\right)-km_0e^t-\frac{\eta}{1-\eta}\left[\frac{1}{\eta}-\frac{1}{\eta}\left(km_0e^t+Hv+\tilde{k}H^2\right)^{1-\eta}\right]-ln\bar{P}_t^{\,*}\right\}dt$$

$$=\int_0^{\infty}e^{-\rho t}\left\{\left(1-vH-\frac{\tilde{k}}{2}H^2\right)-km_0e^t-\frac{1-(km_0e^t+vH+\tilde{k}H^2)^{1-\eta}}{1-\eta}-ln\bar{P}_t^{\,*}\right\}dt$$

$$= \int_0^{\infty} \left(1 - vH - \frac{\tilde{k}}{2}H^2\right) e^{-\rho t}\, dt + \int_0^{\infty} (-km_0 e^t) e^{-\rho t}\, dt +$$

$$\int_0^{\infty} \left(-\frac{1-(km_0 e^t + vH + \tilde{k}H^2)^{1-\eta}}{1-\eta}\right) e^{-\rho t}\, dt + \int_0^{\infty} \left(-ln\bar{P}_t^{\,*}\right) e^{-\rho t}\, dt = \frac{1 - vH - \frac{\tilde{k}}{2}H^2}{\rho} + \frac{km_0}{1-\rho} +$$

$$\frac{1}{\eta - 1}\left[\frac{1}{\rho} - \frac{(km_0)^{1-\eta}}{1-\eta-\rho}\right] - \frac{\sigma}{\rho}\left[1 + ln\left(\mu_0 + \frac{k}{\rho}\right)\right] + \frac{\sigma}{\rho^2} + \frac{\sigma ln\sigma\chi + \sigma ln m_0 + \frac{ln\eta}{\eta} + (1-\sigma)lnH}{\rho}.$$

Note that:

(i) $\int_0^{\infty} \left(1 - vH - \frac{\tilde{k}}{2}H^2\right) e^{-\rho t}\, dt = \left(1 - vH - \frac{\tilde{k}}{2}H^2\right) \int_0^{\infty} e^{-\rho t}\, dt = \frac{1 - vH - \frac{\tilde{k}}{2}H^2}{\rho}$.

(ii) $\int_0^{\infty} (-km_0 e^t) e^{-\rho t}\, dt = -km_0 \int_0^{\infty} e^{(1-\rho)t}\, dt = \frac{-km_0}{1-\rho}\left[e^{(1-\rho)t}\right]_0^{\infty} = \frac{km_0}{1-\rho}$, because

$\lim_{t\to\infty} e^{(1-\rho)t} = \lim_{t\to\infty} \frac{1}{e^{(\rho-1)t}} = 0$ and $\lim_{t\to 0} e^{(1-\rho)t} = 1$ $(\rho > 1)$.

(iii) $\int_0^{\infty} \left(-\frac{1-(km_0 e^t + vH + \tilde{k}H^2)^{1-\eta}}{1-\eta}\right) e^{-\rho t}\, dt = \frac{1}{\eta-1}\int_0^{\infty} [1 - \left(km_0 e^t + vH + \tilde{k}H^2\right)^{1-\eta}] e^{-\rho t}\, dt =$

$$\frac{1}{\eta-1}\left[\int_0^{\infty} e^{-\rho t}\, dt - \int_0^{\infty} \left(km_0 e^t + vH + \tilde{k}H^2\right)^{1-\eta} e^{-\rho t} dt\right] = \frac{1}{\eta-1}\left[\frac{1}{\rho} - \int_0^{\infty} \left(km_0 e^t (1 + \right.\right.$$

$$\left.\left.\frac{vH + \tilde{k}H^2}{km_0 e^t}\right)^{1-\eta} e^{-\rho t} dt\right] \approx \frac{1}{\eta-1}\left[\frac{1}{\rho} - \int_0^{\infty} (km_0 e^t)^{1-\eta} e^{-\rho t} dt\right] = \frac{1}{\eta-1}\left[\frac{1}{\rho} -\right.$$

$$\left.(km_0)^{1-\eta} \int_0^{\infty} e^{t(1-\eta)} e^{-\rho t} dt\right] = \frac{1}{\eta-1}\left[\frac{1}{\rho} - (km_0)^{1-\eta} \int_0^{\infty} e^{t(1-\eta-\rho)}\, dt\right] = \frac{1}{\eta-1}\left[\frac{1}{\rho} - \frac{(km_0)^{1-\eta}}{1-\eta-\rho}\right].$$

$$(iv)\ \int_0^{\infty} \left(-ln\bar{P}_t^{\,*}\right) e^{-\rho t}\, dt = -\int_0^{\infty} e^{-\rho t} \ln \frac{\left[\left(\mu_0 + \frac{k - \frac{\sigma}{m_0 e^t}}{\rho}\right) e^{\rho t} - \frac{k - \frac{\sigma}{m_0 e^t}}{\rho}\right]^{-\sigma}}{(m_0 e^t)^{\sigma} \eta^{\frac{1}{\eta-1}} \mathrm{H}^{1-\sigma}} dt = -\int_0^{\infty} e^{-\rho t} \{\sigma \ln \left[\left(\mu_0 + \right.\right.$$

$$\left.\left.\frac{k - \frac{\sigma}{m_0 e^t}}{\rho}\right) e^{\rho t} - \frac{k - \frac{\sigma}{m_0 e^t}}{\rho}\right] - \sigma ln\sigma\chi - \sigma ln m_0 - \sigma t - \frac{ln\eta}{\eta} - (1-\sigma)lnH\} dt =$$

$$-\int_0^{\infty} e^{-\rho t} \{\sigma \ln \left[\left(\mu_0 + \frac{k - \frac{\sigma}{m_0 e^t}}{\rho}\right) e^{\rho t} - \frac{k - \frac{\sigma}{m_0 e^t}}{\rho}\right] - \sigma t\} dt - \int_0^{\infty} e^{-\rho t} \{ - \sigma ln\sigma\chi - \sigma ln m_0 - \frac{ln\eta}{\eta - 1} -$$

$(1-\sigma)lnH\}dt = -\int_0^\infty e^{-\rho t}\{\sigma \ln\left[\left(\mu_0 + \frac{k-\frac{\sigma}{m_0 e^t}}{\rho}\right)e^{\rho t} - \frac{k-\frac{\sigma}{m_0 e^t}}{\rho}\right]\}dt - \int_0^\infty -\sigma t e^{-\rho t}\,dt -$

$[-\sigma ln\sigma\chi - \sigma lnm_0 - \frac{ln\eta}{\eta-1} - (1-\sigma)lnH]\int_0^\infty e^{-\rho t}\,dt = A + \sigma\int_0^\infty te^{-\rho t}\,dt + [\sigma ln\sigma\chi +$

$\sigma lnm_0 + \frac{ln\eta}{\eta-1} + (1-\sigma)lnH]\int_0^\infty e^{-\rho t}\,dt = -\frac{\sigma}{\rho}[1 + ln\left(\mu_0 + \frac{k}{\rho}\right)] + \frac{\sigma}{\rho^2} +$

$\frac{\sigma ln\sigma\chi + \sigma lnm_0 + \frac{ln\eta}{\eta-1} + (1-\sigma)lnH}{\rho}$,

where $A = -\int_0^\infty e^{-\rho t}\{\sigma \ln\left[\left(\mu_0 + \frac{k-\frac{\sigma}{m_0 e^t}}{\rho}\right)e^{\rho t} - \frac{k-\frac{\sigma}{m_0 e^t}}{\rho}\right]\}dt = -\int_0^\infty e^{-\rho t}\{\sigma \ln\left[\mu_0 e^{\rho t} +\right.$

$\left.(\frac{k-\frac{\sigma}{m_0 e^t}}{\rho})e^{\rho t} - \frac{k-\frac{\sigma}{m_0 e^t}}{\rho}\right]\}dt = -\int_0^\infty e^{-\rho t}\{\sigma \ln\left[\mu_0 e^{\rho t} + \frac{ke^{\rho t}}{\rho} - \frac{\sigma}{\rho m_0}e^{(\rho-1)t} - \frac{k}{\rho} + \frac{\sigma}{\rho m_0 e^t}\right]\}dt.$

Because $\lim_{t\to\infty}\frac{\sigma}{\rho m_0 e^t} = 0$, then: $A = -\int_0^\infty e^{-\rho t}\{\sigma \ln\left[\mu_0 e^{\rho t} + \frac{ke^{\rho t}}{\rho} - \frac{\sigma}{\rho m_0}e^{(\rho-1)t} - \frac{k}{\rho}\right]\}dt.$

For large t, if $\rho > 1$, then $e^{\rho t}$ grows much faster than $e^{(\rho-1)t}$. The term $\mu_0 e^{\rho t}$ and $\frac{ke^{\rho t}}{\rho}$ dominate over $\frac{\sigma}{\rho m_0}e^{(\rho-1)t}$ and $\frac{k}{\rho}$.

Therefore:

$A \approx -\sigma\int_0^\infty e^{-\rho t}\ln\left(\mu_0 e^{\rho t} + \frac{ke^{\rho t}}{\rho}\right)dt = -\sigma\int_0^\infty e^{-\rho t}\ln\left[\left(\mu_0 + \frac{k}{\rho}\right)e^{\rho t}\right]dt =$

$-\sigma\int_0^\infty e^{-\rho t}\ln[\left(\mu_0 + \frac{k}{\rho}\right) + lne^{\rho t}]dt = -\sigma\int_0^\infty e^{-\rho t}\ln\left[\left(\mu_0 + \frac{k}{\rho}\right) + \rho t\right]dt =$

$-\sigma\int_0^\infty e^{-\rho t}\ln\left(\mu_0 + \frac{k}{\rho}\right)dt - \sigma\int_0^\infty e^{-\rho t}\rho t\,dt = -\sigma ln\left(\mu_0 + \frac{k}{\rho}\right)\int_0^\infty e^{-\rho t}\,dt - \sigma\rho\int_0^\infty e^{-\rho t}t\,dt =$

$\frac{-\sigma ln\left(\mu_0 + \frac{k}{\rho}\right)}{\rho} - \frac{\sigma\rho}{\rho^2} = -\frac{\sigma}{\rho}[1 + ln\left(\mu_0 + \frac{k}{\rho}\right)].$

G. PROOF OF PROPROSITION 4

The landlord problem is:

$$\underset{\{m,r,H\}}{\text{Max}}\ \pi_l = mrPg(\theta)q(L,h) - vH - c_l(m,H),$$

$$s.t.\ WL + c_w(s,L) \leq (1-r)Pg(\theta)q(L,h).$$

To solve this problem, write the Lagrangean:

$$\mathcal{L}3 = mrPg(\theta)q\left(L,\frac{H}{m}\right) - vH - c_l(m,H) - \varphi\left[WL + c_w(s,L) - (1-r)Pg(\theta)q(L,\frac{H}{m})\right].$$

Now take the FOCs.

1) Start with the FOC w.r.t. m:

$\frac{\partial \mathcal{L}3}{\partial m} = rPg(\theta)q\left(L, \frac{H}{m}\right) + mrPg(\theta)\frac{\partial q}{\partial h}\left(-\frac{H}{m^2}\right) - \frac{\partial c_l}{\partial m} + \varphi(1-\mathrm{r})Pg(\theta)\frac{\partial q}{\partial h}(-\frac{H}{m^2}) \leq 0$, with complementary slackness $\frac{\partial \mathcal{L}3}{\partial m}m = 0$ and $m > 0$. It implies that:$\frac{\partial \mathcal{L}3}{\partial m} = rPg(\theta)q\left(L, \frac{H}{m}\right) -$ $rhPg(\theta)\frac{\partial q}{\partial h} - \frac{\partial c_l}{\partial m} - \varphi(1-\mathrm{r})Pg(\theta)\frac{\partial q}{\partial h}\frac{h}{m} = 0 \ \ (A36)$.

2) Compute the FOC w.r.t. r:

$\frac{\partial \mathcal{L}3}{\partial r} = mPg(\theta)q\left(L, \frac{H}{m}\right) - \varphi Pg(\theta)q(L, \frac{H}{m}) \leq 0$, with complementary slackness $\frac{\partial \mathcal{L}2}{\partial r}r = 0$ and $r > 0$. It implies that: $\frac{\partial \mathcal{L}3}{\partial r} = mPg(\theta)q\left(L, \frac{H}{m}\right) - \varphi Pg(\theta)q(L, \frac{H}{m}) = 0 \ \ (A37)$.

3) Compute the FOC w.r.t. H:

$\frac{\partial \mathcal{L}3}{\partial H} = mrPg(\theta)\frac{\partial q}{\partial h}\frac{1}{m} - \frac{\partial c_l}{\partial H} - v + \varphi(1-\mathrm{r})Pg(\theta)\frac{\partial q}{\partial h}\frac{1}{m} \leq 0$, with complementary slackness $\frac{\partial \mathcal{L}3}{\partial H}H = 0$ and $H > 0$. It implies that: $\frac{\partial \mathcal{L}3}{\partial H} = mrPg(\theta)\frac{\partial q}{\partial h}\frac{1}{m} - v - \frac{\partial c_l}{\partial H} + \varphi(1-\mathrm{r})Pg(\theta)\frac{\partial q}{\partial h}\frac{1}{m} =$ $0 \ \ (A38)$.

4) Compute the FOC w.r.t. φ:

$\frac{\partial \mathcal{L}3}{\partial \varphi} = WL + c_w(s, L) - (1-r)g(\theta)q(L, \frac{H}{m}) \geq 0$, with complementary slackness $\frac{\partial \mathcal{L}3}{\partial \varphi}\varphi = 0$ and $\varphi \geq 0$. It implies that if $\varphi = 0$, then $\frac{\partial \mathcal{L}3}{\partial r} = mPg(\theta)q\left(L, \frac{H}{m}\right) > 0$, which violates with $\frac{\partial \mathcal{L}3}{\partial r} = 0$ in (A37); if $\varphi > 0$, then $\frac{\partial \mathcal{L}3}{\partial \varphi} = WL + c_w(s, L) - (1-r)Pg(\theta)q\left(L, \frac{H}{m}\right) = 0 \ \ (A39)$. Hence, there is no corner solution in this optimization problem.

Finally, the 4 FOCs of the landlord problem are:

$$\begin{cases} rPg(\theta)q\left(L,\frac{H}{m}\right) - rhPg(\theta)\frac{\partial q}{\partial h} - \frac{\partial c_l}{\partial m} - \varphi(1-\mathrm{r})Pg(\theta)\frac{\partial q}{\partial h}\frac{h}{m} = 0 \ \ (A36) \\ mPg(\theta)q\left(L,\frac{H}{m}\right) - \varphi Pg(\theta)q(L,\frac{H}{m}) = 0 \ \ (A37) \\ mrPg(\theta)\frac{\partial q}{\partial h}\frac{1}{m} - v - \frac{\partial c_l}{\partial H} + \varphi(1-\mathrm{r})Pg(\theta)\frac{\partial q}{\partial h}\frac{1}{m} = 0 \ \ (A38) \\ WL + c_w(s,L) - (1-r)Pg(\theta)q\left(L,\frac{H}{m}\right) = 0 \ \ (A39) \end{cases}$$

5) Solution in terms of landlord's optimal share rate

From equation (A37),

$$m = \varphi \text{ (A40).}$$

Next, Substitute in (A36),

$$rPg(\theta)q - rhPg(\theta)\frac{\partial q}{\partial h} - \frac{\partial c_l}{\partial m} - m(1-\mathrm{r})Pg(\theta)\frac{\partial q}{\partial h}\frac{h}{m} = 0.$$

That is,

$$rPg(\theta)q - rhPg(\theta)\frac{\partial q}{\partial h} - \frac{\partial c_l}{\partial m} - \mathrm{P}g(\theta)\frac{\partial q}{\partial h}h + \mathrm{r}\mathrm{P}g(\theta)\frac{\partial q}{\partial h}h = 0.$$

Hence:

$$r = \frac{\frac{\partial c_l}{\partial m}}{Pqg(\theta)} + \frac{\partial q/\partial h}{q/h} \text{ (A41).}$$

Plugging (A40) in (A38),

$$rPg(\theta)\frac{\partial q}{\partial h} - v - \frac{\partial c_l}{\partial H} + (1-\mathrm{r})Pg(\theta)\frac{\partial q}{\partial h} = 0.$$

Thus:

$$Pg(\theta)\frac{\partial q}{\partial h} = v + \frac{\partial c_l}{\partial H} \text{ (A42).}$$

Therefore, substitute (A42) in (A41):

$r^{****} = \frac{\frac{\partial c_l}{\partial m} + h(v + \frac{\partial c_l}{\partial H})}{Pqg(\theta)} > \frac{\partial q/\partial h}{q/h} = r_{Cheung}$, which is stated as equation (13) in the text.

In this case,

$$\frac{\partial r^{****}}{\partial \theta} = \frac{\partial r^{****}}{\partial g(\theta_t)} g'(\theta) = -\frac{\frac{\partial c_l}{\partial m}}{Pq[g(\theta)]^2} g'(\theta) > 0.$$

H. PROOF OF LEMMA 4

The social planner problem is:

$$U_1 = \underset{\{\bar{P}_t, \bar{r}_t, L_{pt}, \lambda\}}{\text{Max}} \int_0^{\infty} e^{-\rho t} \left\{ m_t \left[\bar{P}_t g(\theta_t) \left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} - W_t L_{pt} - c_{wt}\left(s, L_{pt}\right) \right] - v\lambda Z - km_t - \right.$$

$$\left. \frac{\tilde{k}}{2} (\lambda Z)^2 - ln\bar{P}_t - \int_{r^{****}}^{\bar{r}_t} \eta r_t^{-\eta} dr_t \right\} \text{dt},$$

s.t.

$$W_t L_{pt} + c_{wt}(s, L_{pt}) = (1 - \bar{r}_t)\bar{P}_t g(\theta_t)(\frac{\lambda Z}{m_t})^{1-\sigma} L_{pt}^{\sigma},$$

$$\dot{m}_t = \frac{1}{\chi}(L_t - L_{pt}),$$

$$m_t = m_0 e^t.$$

Next, define Hamiltonian with state variable m_t, control variables $\{\bar{P}_t, \bar{r}_t, L_{pt}, \lambda\}$ and co-state variable μ_t:

$$\mathcal{H}_1(\bar{P}_t, \bar{r}_t, L_{pt}, m_t, \lambda, \mu_t) = m_t\left[\bar{P}_t g(\theta_t)(\frac{\lambda Z}{m_t})^{1-\sigma} L_{pt}^{\sigma} - W_t L_{pt} - c_{wt}(s, L_{pt})\right] - v\lambda Z - km_t -$$

$$\frac{\tilde{k}}{2}(\lambda Z)^2 - ln\bar{P}_t - \int_{r^{****}}^{\bar{r}_t} \eta r_t^{-\eta} dr_t + \frac{\mu_t}{\chi}(L_t - L_{pt}) = m_t\left[\bar{P}_t g(\theta_t)(\frac{\lambda Z}{m_t})^{1-\sigma} L_{pt}^{\sigma} - (1 -\right.$$

$$\left.\bar{r}_t)\bar{P}_t g(\theta_t)(\frac{\lambda Z}{m_t})^{1-\sigma} L_{pt}^{\sigma}\right] - v\lambda Z - km_t - \frac{\tilde{k}}{2}(\lambda Z)^2 - ln\bar{P}_t - \eta\frac{\bar{r}_t^{1-\eta} - (r^{****})^{1-\eta}}{1-\eta} + \frac{\mu_t}{\chi}(L_t - L_{pt}) =$$

$$m_t \bar{r}_t \bar{P}_t g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} - v\lambda Z - km_t - \frac{\tilde{k}}{2}(\lambda Z)^2 - ln\bar{P}_t - \eta\frac{\bar{r}_t^{1-\eta} - (r^{****})^{1-\eta}}{1-\eta} + \frac{\mu_t}{\chi}(L_t - L_{pt}).$$

The FOCs are:

$$\begin{cases} \dfrac{\partial \mathcal{H}_1}{\partial L_{pt}} = 0 \\ \dfrac{\partial \mathcal{H}_1}{\partial \bar{P}_t} = 0 \\ \dfrac{\partial \mathcal{H}_1}{\partial \bar{r}_t} = 0 \\ \dfrac{\partial \mathcal{H}_1}{\partial \lambda} = 0 \\ \rho = \dfrac{\partial \mathcal{H}_1 / \partial m_t}{\mu_t} + \dfrac{\dot{\mu}_t}{\mu_t} \end{cases}$$

Start with $\frac{\partial \mathcal{H}_1}{\partial L_{pt}} = 0$, which implies

$$\sigma m_t \bar{r}_t \bar{P}_t g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma-1} = \frac{\mu_t}{\chi}$$

and therefore

$$L_{pt} = \left(\frac{\mu_t}{\chi\sigma m_t \bar{r}_t \bar{P}_t g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma}}\right)^{\frac{1}{\sigma-1}} \quad (A43).$$

Next, consider $\frac{\partial \mathcal{H}_1}{\partial \bar{P}_t} = 0$:

$$m_t \bar{r}_t g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} = \frac{1}{\bar{P}_t}.$$

Substituting gives

$$\bar{P}_t = \frac{1}{m_t \bar{r}_t g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma}} \quad (A44).$$

The next FOC is similar, $\frac{\partial \mathcal{H}_1}{\partial \bar{r}_t} = 0$:

$$m_t \bar{P}_t g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} = \eta \bar{r}_t^{-\eta}.$$

Substituting gives

$$\bar{r}_t^{**} = \eta^{\frac{1}{\eta-1}} \quad (A45).$$

The next FOC is similar, $\frac{\partial \mathcal{H}_1}{\partial \lambda} = 0$:

$$(1-\sigma) m_t \bar{r}_t \bar{P}_t g(\theta_t) \lambda^{-\sigma}\left(\frac{Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} - vZ - \tilde{k}\lambda Z^2 = 0.$$

$$\Longrightarrow \frac{(1-\sigma) m_t \bar{r}_t \bar{P}_t g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma}}{\lambda} - vZ - \lambda\tilde{k}Z^2 = 0$$

$$\Longrightarrow \frac{1-\sigma}{\lambda} - vZ - \lambda\tilde{k}Z^2 = 0 \Longrightarrow 1 - \sigma - \lambda vZ - \lambda^2\tilde{k}Z^2 = 0$$

$$\Longrightarrow (\tilde{k}Z^2)\lambda^2 + (vZ)\lambda + (\sigma - 1) = 0$$

Substituting gives

$$\lambda^* = \frac{-vZ + \sqrt{(vZ)^2 + 4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z^2} \quad (A46).$$

Finally, the last FOC is $\rho = \frac{\partial \mathcal{H}_1 / \partial m_t}{\mu_t} + \frac{\dot{\mu}_t}{\mu_t}$. The derivative of the Hamiltonian $\frac{\partial \mathcal{H}_1}{\partial m_t}$ is:

$$\frac{\partial \mathcal{H}_1}{\partial m_t} = \bar{r}_t \bar{P}_t g(\theta_t) \left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} + (\sigma - 1)\bar{r}_t \bar{P}_t g(\theta_t) \left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} - k = \frac{\sigma}{m_t} - k.$$

Substituting this into the FOC and rearranging to solve for μ_t gives

$$\dot{\mu_t} - \rho\mu_t = k - \frac{\sigma}{m_t}.$$

Solving for μ_t^{**}:

$$\mu_t^{**} = \left(\mu_0 + \frac{k - \frac{\sigma}{m_0 e^t}}{\rho}\right) e^{\rho t} - \frac{k - \frac{\sigma}{m_0 e^t}}{\rho} = \mu_t^{*} \ (A47).$$

(1) Solving the Optimal Allocation

The 5 FOCs are:

$$\begin{cases} L_{pt} = \left(\dfrac{\mu_t}{\chi\sigma m_t \bar{r}_t \bar{P}_t g(\theta_t) \left(\frac{\lambda Z}{m_t}\right)^{1-\sigma}}\right)^{\frac{1}{\sigma-1}} \ (A43) \\ \bar{P}_t = \dfrac{1}{m_t \bar{r}_t g(\theta_t) \left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma}} \ (A44) \\ \bar{r}_t^{**} = \eta^{\frac{1}{\eta-1}} \ (A45) \\ \lambda^{*} = \dfrac{-vZ + \sqrt{(vZ)^2 + 4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z^2} \ (A46) \\ \mu_t^{**} = \left(\mu_0 + \dfrac{k - \frac{\sigma}{m_0 e^t}}{\rho}\right) e^{\rho t} - \dfrac{k - \frac{\sigma}{m_0 e^t}}{\rho} \ (A47) \end{cases}$$

1) Solving for L_{pt} and $\bar{P}_t$(eqs. A43 and A44).

Combining (A43), (A44), and (A47) gives

$$L_{pt}^{**} = \frac{\sigma\chi}{\left(\mu_0 + \frac{k - \frac{\sigma}{m_0 e^t}}{\rho}\right) e^{\rho t} - \frac{k - \frac{\sigma}{m_0 e^t}}{\rho}} = L_{pt}^{*} \ (A48).$$

Again, combining (A44), (A45), (A46), and (A48) gives

$$\bar{P}_t^{**} = \frac{\left[\frac{\sigma\chi}{\left(\mu_0+\frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right)e^{\rho t}-\frac{k-\frac{\sigma}{m_0e^t}}{\rho}}\right]^{-\sigma}}{(m_0e^t)^{\sigma}\eta^{\frac{1}{\eta-1}}(\lambda^*\mathrm{Z})^{1-\sigma}g(\theta_t)} = \frac{\left[\frac{\sigma\chi}{\left(\mu_0+\frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right)e^{\rho t}-\frac{k-\frac{\sigma}{m_0e^t}}{\rho}}\right]^{-\sigma}}{(m_0e^t)^{\sigma}\eta^{\frac{1}{\eta-1}}[\frac{-vZ+\sqrt{(vZ)^2+4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}]^{1-\sigma}g(\theta_t)} \quad (A49).$$

Therefore, compute the derivative $\frac{\partial \bar{P}_t^{**}}{\partial \lambda^*}$:

$$\frac{\partial \bar{P}_t^{**}}{\partial \lambda^*} = (\sigma-1)(\lambda^*)^{\sigma-2}\frac{\left[\frac{\sigma\chi}{\left(\mu_0+\frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right)e^{\rho t}-\frac{k-\frac{\sigma}{m_0e^t}}{\rho}}\right]^{-\sigma}}{(m_0e^t)^{\sigma}\eta^{\frac{1}{\eta-1}}\mathrm{Z}^{1-\sigma}g(\theta_t)} < 0.$$

2) Solving for U_1.

According to equation (13), $r^{****} = \frac{\frac{\partial c_l}{\partial m}+h(v+\frac{\partial c_l}{\partial H})}{Pqg(\theta)}$. Plugging equations (A48) and (A49) yields:

$$r^{****} = \frac{\frac{\partial c_l}{\partial m}+h(v+\frac{\partial c_l}{\partial H})}{Pqg(\theta)} = \frac{k+\frac{H}{m_t}(v+\tilde{k}H)}{\bar{P}_t^{**}h_t^{1-\sigma}L_{pt}^{*\sigma}g(\theta)} = \frac{k+\frac{H}{m_t}(v+\tilde{k}H)}{\frac{1}{m_t\bar{r}_t^{**}}} = km_t\bar{r}_t^{**} + \bar{r}_t^{**}H\big(v+\tilde{k}H\big) = \eta^{\frac{1}{\eta-1}}\big[km_t + \lambda Z\big(v + \tilde{k}\lambda Z\big)\big] = \eta^{\frac{1}{\eta-1}}[km_0e^t + \lambda Zv + \tilde{k}(\lambda Z)^2].$$

Next, by definition $U_1 = \int_0^{\infty} e^{-\rho t}\Big\{m_t\Big[\bar{P}_t g(\theta_t)(\frac{\lambda Z}{m_t})^{1-\sigma}L_{pt}^{\sigma} - W_tL_{pt} - c_{wt}\big(s, L_{pt}\big)\Big] - v\lambda Z - km_t - \frac{\tilde{k}}{2}(\lambda Z)^2 - ln\bar{P}_t - \int_{r^{****}}^{\bar{r}_t}\eta r_t^{-\eta}dr_t\Big\}\mathrm{dt}$. Then:

$$U_1 = \int_0^{\infty} e^{-\rho t}\left[m_t\bar{r}_t^{**}\bar{P}_t^{**}g(\theta_t)\left(\frac{\lambda^*Z}{m_t}\right)^{1-\sigma}L_{pt}^{\sigma} - v\lambda^*Z - km_t - \frac{\tilde{k}}{2}(\lambda^*Z)^2 - ln\bar{P}_t^{**} - \eta\frac{(\bar{r}_t^{**})^{1-\eta}-(r^{****})^{1-\eta}}{1-\eta}\right]\mathrm{dt}$$

$$= \int_0^{\infty} e^{-\rho t} \left[1 - v\lambda^* Z - km_0 e^t - \frac{\tilde{k}}{2}(\lambda^* Z)^2 - ln\bar{P}_t^{**} \right.$$

$$\left. - \eta \frac{\eta^{\frac{1-\eta}{\eta-1}} - \left[\eta^{\frac{1}{\eta-1}}(km_0 e^t + \lambda^* Z v + \tilde{k}(\lambda^* Z)^2)\right]^{1-\eta}}{1-\eta} \right] \mathrm{dt}$$

$$= \int_0^{\infty} e^{-\rho t} \left\{ [1 - v\lambda^* Z - \frac{\tilde{k}}{2}(\lambda^* Z)^2] - km_0 e^t - \frac{\eta}{1-\eta}\left[\frac{1}{\eta} - \frac{1}{\eta}\left(km_0 e^t + \lambda^* Z v + \tilde{k}(\lambda^* Z)^2\right)^{1-\eta}\right] \right.$$

$$\left. - ln\bar{P}_t^{**} \right\} dt$$

$$= \int_0^{\infty} e^{-\rho t} \left\{ [1 - v\lambda^* Z - \frac{\tilde{k}}{2}(\lambda^* Z)^2] - km_0 e^t - \frac{1 - \left(km_0 e^t + \lambda^* Z v + \tilde{k}(\lambda^* Z)^2\right)^{1-\eta}}{1-\eta} \right.$$

$$\left. - ln\bar{P}_t^{**} \right\} dt$$

$$= \int_0^{\infty} [1 - v\lambda^* Z - \tfrac{\tilde{k}}{2}(\lambda^* Z)^2] e^{-\rho t}\, dt + \int_0^{\infty} (-km_0 e^t) e^{-\rho t}\, dt +$$

$$\int_0^{\infty} [-\tfrac{1-\left(km_0 e^t + \lambda^* Z v + \tilde{k}(\lambda^* Z)^2\right)^{1-\eta}}{1-\eta}] e^{-\rho t}\, dt + \int_0^{\infty} \left(-ln\bar{P}_t^{**}\right) e^{-\rho t}\, dt =$$

$$\frac{1 - v\frac{-vZ + \sqrt{(vZ)^2 + 4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z} - \frac{\tilde{k}}{2}[\frac{-vZ + \sqrt{(vZ)^2 + 4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}]^2}{\rho} + \frac{km_0}{1-\rho} + \frac{1}{\eta - 1}\left[\frac{1}{\rho} - \frac{(km_0)^{1-\eta}}{1-\eta-\rho}\right] - \frac{\sigma}{\rho}\left[1 + ln\left(\mu_0 + \frac{k}{\rho}\right)\right] +$$

$$\frac{\sigma}{\rho^2} + \frac{\sigma ln\sigma\chi + \sigma ln m_0 + \frac{ln\eta}{\eta-1} + (1-\sigma)\ln[\frac{-vZ + \sqrt{(vZ)^2 + 4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}]}{\rho} + \ln\left(\frac{1}{\sqrt{2\pi}\sigma_\theta}\right)\frac{1}{\rho} - \frac{1}{2\sigma_\theta^2}\left(\frac{\theta_0^2}{2\beta+\rho} - \frac{2\mu_\theta\theta_0}{\beta+\rho} + \frac{\mu_\theta^2}{\rho}\right).$$

Note that:

(i) $\int_0^{\infty} [1 - v\lambda Z - \frac{\tilde{k}}{2}(\lambda Z)^2] e^{-\rho t}\, dt = \left[1 - v\lambda Z - \frac{\tilde{k}}{2}(\lambda Z)^2\right] \int_0^{\infty} e^{-\rho t}\, dt = \frac{\left[1 - v\lambda Z - \frac{\tilde{k}}{2}(\lambda Z)^2\right]}{\rho} =$

$$\frac{1 - v\frac{-vZ + \sqrt{(vZ)^2 + 4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z} - \frac{\tilde{k}}{2}[\frac{-vZ + \sqrt{(vZ)^2 + 4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}]^2}{\rho}.$$

(ii) $\int_0^{\infty}(-km_0e^t)e^{-\rho t}\,dt = -km_0\int_0^{\infty}e^{(1-\rho)t}\,dt = \frac{-km_0}{1-\rho}\left[e^{(1-\rho)t}\right]_0^{\infty} = \frac{km_0}{1-\rho}$, because

$\lim_{t\to\infty} e^{(1-\rho)t} = \lim_{t\to\infty}\frac{1}{e^{(\rho-1)t}} = 0$ and $\lim_{t\to 0} e^{(1-\rho)t} = 1$ $(\rho > 1)$.

(iii)$\int_0^{\infty}[-\frac{1-\left(km_0e^t+\lambda^*Zv+\tilde{k}(\lambda^*Z)^2\right)^{1-\eta}}{1-\eta}]e^{-\rho t}\,dt = \frac{1}{\eta-1}\int_0^{\infty}[1-\left(km_0e^t + v\lambda^*Z + \tilde{k}(\lambda^*Z)^2\right)^{1-\eta}]e^{-\rho t}\,dt = \frac{1}{\eta-1}\left[\int_0^{\infty}e^{-\rho t}\,dt - \int_0^{\infty}\left(km_0e^t + v\lambda^*Z + \tilde{k}(\lambda^*Z)^2\right)^{1-\eta}e^{-\rho t}dt\right] =$

$\frac{1}{\eta-1}\left[\frac{1}{\rho} - \int_0^{\infty}\left(km_0e^t(1+\frac{v\lambda^*Z+\tilde{k}(\lambda^*Z)^2}{km_0e^t})\right)^{1-\eta}e^{-\rho t}dt\right] \approx \frac{1}{\eta-1}\left[\frac{1}{\rho} - \int_0^{\infty}(km_0e^t)^{1-\eta}e^{-\rho t}dt\right] =$

$\frac{1}{\eta-1}\left[\frac{1}{\rho} - (km_0)^{1-\eta}\int_0^{\infty}e^{t(1-\eta)}e^{-\rho t}dt\right] = \frac{1}{\eta-1}\left[\frac{1}{\rho} - (km_0)^{1-\eta}\int_0^{\infty}e^{t(1-\eta-\rho)}\,dt\right] =$

$\frac{1}{\eta-1}\left[\frac{1}{\rho} - \frac{(km_0)^{1-\eta}}{1-\eta-\rho}\right]$.

$(iv)\int_0^{\infty}\left(-ln\bar{P}_t^{**}\right)e^{-\rho t}\,dt = -\int_0^{\infty}e^{-\rho t}\ln\frac{\left[\frac{\sigma\chi}{\left(\mu_0+\frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right)e^{\rho t}-\frac{k-\frac{\sigma}{m_0e^t}}{\rho}}\right]^{-\sigma}}{(m_0e^t)^{\sigma}\eta^{\frac{1}{\eta-1}}(\lambda^*\mathrm{Z})^{1-\sigma}g(\theta_t)}dt = -\int_0^{\infty}e^{-\rho t}\{\sigma\ln\left[\left(\mu_0 + \frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right)e^{\rho t} - \frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right] - \sigma ln\sigma\chi - \sigma lnm_0 - \sigma t - \frac{ln\eta}{\eta-1} - (1-\sigma)\ln(\lambda^*\mathrm{Z}) - \ln g(\theta_t)\}dt =$

$-\int_0^{\infty}e^{-\rho t}\{\sigma\ln\left[\left(\mu_0 + \frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right)e^{\rho t} - \frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right] - \sigma t\}dt - \int_0^{\infty}e^{-\rho t}\{-\sigma ln\sigma\chi - \sigma lnm_0 - \frac{ln\eta}{\eta-1} -$

$(1-\sigma)\ln[\frac{-vZ+\sqrt{(vZ)^2+4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}] - \ln g(\theta_t)\}dt = -\int_0^{\infty}e^{-\rho t}\{\sigma\ln\left[\left(\mu_0 + \frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right)e^{\rho t} -$

$\frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right]\}dt - \int_0^{\infty}-\sigma te^{-\rho t}\,dt - \{-\sigma ln\sigma\chi - \sigma lnm_0 - \frac{ln\eta}{\eta-1} - (1-$

$\sigma)\ln[\frac{-vZ+\sqrt{(vZ)^2+4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}]\}\int_0^{\infty}e^{-\rho t}\,dt + \int_0^{\infty}\ln g(\theta_t)e^{-\rho t}\,dt = -\frac{\sigma}{\rho}[1+ln\left(\mu_0+\frac{k}{\rho}\right)] +$

$\sigma\int_0^{\infty}te^{-\rho t}\,dt + \{\sigma ln\sigma\chi + \sigma lnm_0 + \frac{ln\eta}{\eta-1} + (1-\sigma)\ln[\frac{-vZ+\sqrt{(vZ)^2+4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}]\int_0^{\infty}e^{-\rho t}\,dt +$

$$\int_0^{\infty}[\ln g(\theta_t)]e^{-\rho t}\,dt = -\frac{\sigma}{\rho}\left[1 + ln\left(\mu_0 + \frac{k}{\rho}\right)\right] + \frac{\sigma}{\rho^2} +$$

$$\frac{\sigma ln\sigma\chi + \sigma lnm_0 + \frac{ln\eta}{\eta-1} + (1-\sigma)\ln\left[\frac{-vZ+\sqrt{(vZ)^2+4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}\right]}{\rho} + \ln\left(\frac{1}{\sqrt{2\pi}\sigma_\theta}\right)\frac{1}{\rho} - \frac{1}{2\sigma_\theta^2}\left(\frac{\theta_0^2}{2\beta+\rho} - \frac{2\mu_\theta\theta_0}{\beta+\rho} + \frac{\mu_\theta^2}{\rho}\right).$$

When $g(\theta_t) = \frac{1}{\sqrt{2\pi}\sigma_\theta}\exp\left[-\frac{(\theta_t-\mu_\theta)^2}{2\sigma_\theta^2}\right]$ and $\theta_t = \theta_0 e^{-\beta t}$, $\int_0^{\infty}[\ln g(\theta_t)]e^{-\rho t}\,dt =$

$$\int_0^{\infty} e^{-\rho t}\ln\left\{\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp\left[-\frac{(\theta_t-\mu_\theta)^2}{2\sigma_\theta^2}\right]\right\}dt = \int_0^{\infty} e^{-\rho t}\ln\left\{\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp\left[-\frac{(\theta_t-\mu_\theta)^2}{2\sigma_\theta^2}\right]\right\}dt =$$

$$\int_0^{\infty} e^{-\rho t}\left\{\ln\left(\frac{1}{\sqrt{2\pi}\sigma_\theta}\right) + \ln\exp\left[-\frac{(\theta_t-\mu_\theta)^2}{2\sigma_\theta^2}\right]\right\}dt = \int_0^{\infty} e^{-\rho t}\ln\left(\frac{1}{\sqrt{2\pi}\sigma_\theta}\right)dt +$$

$$\int_0^{\infty}\left[-\frac{(\theta_t-\mu_\theta)^2}{2\sigma_\theta^2}\right]e^{-\rho t}\,dt = \ln\left(\frac{1}{\sqrt{2\pi}\sigma_\theta}\right)\int_0^{\infty} e^{-\rho t}dt - \frac{1}{2\sigma_\theta^2}\int_0^{\infty}(\theta_t-\mu_\theta)^2 e^{-\rho t}\,dt = \ln\left(\frac{1}{\sqrt{2\pi}\sigma_\theta}\right)\frac{1}{\rho} -$$

$$\frac{1}{2\sigma_\theta^2}\int_0^{\infty}\left(\theta_0 e^{-\beta t} - \mu_\theta\right)^2 e^{-\rho t}dt = \ln\left(\frac{1}{\sqrt{2\pi}\sigma_\theta}\right)\frac{1}{\rho} - \frac{1}{2\sigma_\theta^2}\int_0^{\infty}(\theta_0^2 e^{-2\beta t} - 2\mu_\theta\theta_0 e^{-\beta t} + \mu_\theta^2)e^{-\rho t}\,dt =$$

$$\ln\left(\frac{1}{\sqrt{2\pi}\sigma_\theta}\right)\frac{1}{\rho} - \frac{1}{2\sigma_\theta^2}\int_0^{\infty}[\theta_0^2 e^{-(2\beta+\rho)t} - 2\mu_\theta\theta_0 e^{-(\beta+\rho)t} + \mu_\theta^2 e^{-\rho t}]dt = \ln\left(\frac{1}{\sqrt{2\pi}\sigma_\theta}\right)\frac{1}{\rho} -$$

$$\frac{1}{2\sigma_\theta^2}\left[\theta_0^2\int_0^{\infty} e^{-(2\beta+\rho)t}dt - 2\mu_\theta\theta_0\int_0^{\infty} e^{-(\beta+\rho)t}\,dt + \mu_\theta^2\int_0^{\infty} e^{-\rho t}\,dt\right] = \ln\left(\frac{1}{\sqrt{2\pi}\sigma_\theta}\right)\frac{1}{\rho} - \frac{1}{2\sigma_\theta^2}\left(\frac{\theta_0^2}{2\beta+\rho} -\right.$$

$$\left.\frac{2\mu_\theta\theta_0}{\beta+\rho} + \frac{\mu_\theta^2}{\rho}\right).$$

I. PROOF OF PROPROSITION 5

The landlord problem is:

$$\underset{\{m,r,H\}}{\text{Max}}\ \pi_l = mrPg(\theta)q(L,h) - vH - c_l(m,H),$$

$$s.t.\ W + \frac{\partial c_w}{\partial L} = (1-r)Pg(\theta)\frac{\partial q}{\partial L}.$$

To solve this problem, write the Lagrangean:

$$\mathcal{L}4 = mrPg(\theta)q\left(L,\frac{H}{m}\right) - vH - c_l(m,H) - \varphi\left[W + \frac{\partial c_w}{\partial L} - (1-r)Pg(\theta)\frac{\partial q}{\partial L}\right].$$

Now take the FOCs.

1) Start with the FOC w.r.t. m:

$$\frac{\partial \mathcal{L}4}{\partial m} = rPg(\theta)q\left(L,\frac{H}{m}\right) - rhPg(\theta)\frac{\partial q}{\partial h} - \frac{\partial c_l}{\partial m} - \varphi(1-\mathrm{r})Pg(\theta)\frac{\partial q}{\partial L \partial h}\frac{h}{m} = 0\ \ (A50).$$

2) Compute the FOC w.r.t. r:

$$\frac{\partial \mathcal{L}4}{\partial r} = mPg(\theta)q\left(L,\frac{H}{m}\right) - \varphi Pg(\theta)\frac{\partial q}{\partial L} = 0\ \ (A51).$$

3) Compute the FOC w.r.t. H:

$$\frac{\partial \mathcal{L}4}{\partial H} = mrPg(\theta)\frac{\partial q}{\partial h}\frac{1}{m} - v - \frac{\partial c_l}{\partial H} + \varphi(1-\mathrm{r})Pg(\theta)\frac{\partial\left(\frac{\partial q}{\partial L}\right)}{\partial h}\frac{1}{m} = 0 \ \ (A52).$$

4) Compute the FOC w.r.t. φ:

$$\frac{\partial \mathcal{L}4}{\partial \varphi} = W + \frac{\partial c_w}{\partial L} - (1-r)Pg(\theta)\frac{\partial q}{\partial L} = 0 \ \ (A53).$$

Finally, the 4 FOCs of the landlord problem are:

$$\begin{cases} rPg(\theta)q\left(L,\dfrac{H}{m}\right) - rhPg(\theta)\dfrac{\partial q}{\partial h} - \dfrac{\partial c_l}{\partial m} - \varphi(1-\mathrm{r})Pg(\theta)\dfrac{\partial q}{\partial L\partial h}\dfrac{h}{m} = 0 \ \ (A50) \\ mPg(\theta)q\left(L,\dfrac{H}{m}\right) - \varphi Pg(\theta)\dfrac{\partial q}{\partial L} = 0 \ \ (A51) \\ mrPg(\theta)\dfrac{\partial q}{\partial h}\dfrac{1}{m} - v - \dfrac{\partial c_l}{\partial H} + \varphi(1-\mathrm{r})Pg(\theta)\dfrac{\partial\left(\frac{\partial q}{\partial L}\right)}{\partial h}\dfrac{1}{m} = 0 \ \ (A52) \\ W + \dfrac{\partial c_w}{\partial L} - (1-r)Pg(\theta)\dfrac{\partial q}{\partial L} = 0 \ \ (A53) \end{cases}$$

5) Solution in terms of landlord's optimal share rate

From equation (A51),

$$m = \frac{\varphi}{q}\frac{\partial q}{\partial L} \text{ (A54)}.$$

Next, substitute in (A50),

$$rPg(\theta)q - rhPg(\theta)\frac{\partial q}{\partial h} - \frac{\partial c_l}{\partial m} - \frac{mq}{\frac{\partial q}{\partial L}}(1-\mathrm{r})Pg(\theta)\frac{\partial q}{\partial L\partial h}\frac{h}{m} = 0.$$

Hence:

$$rPg(\theta)q - rhPg(\theta)\frac{\partial q}{\partial h} - \frac{\partial c_l}{\partial m} - \frac{q}{\frac{\partial q}{\partial L}}(1-\mathrm{r})Pg(\theta)\frac{\partial q}{\partial L\partial h}h = 0 \text{ (A55)}.$$

Substitute (A54) in (A52),

$$rPg(\theta)\frac{\partial q}{\partial h} - v - \frac{\partial c_l}{\partial H} + \frac{q}{\frac{\partial q}{\partial L}}(1-\mathrm{r})Pg(\theta)\frac{\partial q}{\partial L\partial h} = 0.$$

Then:

$$-rhPg(\theta)\frac{\partial q}{\partial h}+vh+\frac{\partial c_l}{\partial H}\mathrm{h}=\frac{q}{\frac{\partial q}{\partial L}}(1-\mathrm{r})Pg(\theta)\frac{\partial q}{\partial L\partial h}h \text{ (A56).}$$

Therefore, substitute in (A55),

$r^{nc}=\frac{\frac{\partial c_l}{\partial m}+h(v+\frac{\partial c_l}{\partial H})}{Pqg(\theta)}=r^{****}$, which is stated as equation (14) in the text.

J. PROOF OF LEMMA 5

The social planner problem is:

$$U_2=\underset{\{\bar{P}_t,\bar{r}_t,L_{pt},\lambda\}}{\mathrm{Max}}\int_0^{\infty}e^{-\rho t}\left\{m_t\left[\bar{P}_t g(\theta_t)(\frac{\lambda Z}{m_t})^{1-\sigma}L_{pt}^{\sigma}-W_t L_{pt}-\alpha s L_{pt}\right]-v\lambda Z-km_t-\frac{\tilde{k}}{2}(\lambda Z)^2-\right.$$

$$\left.ln\bar{P}_t-\int_{r^{nc}}^{\bar{r}_t}\eta r_t^{-\eta}dr_t\right\}\mathrm{dt},$$

s.t.

$$W_t+\alpha s=\sigma(1-\bar{r}_t)\bar{P}_t g(\theta_t)(\frac{\lambda Z}{m_t})^{1-\sigma}L_{pt}^{\sigma-1},$$

$$\dot{m}_t=\frac{1}{\chi}(L_t-L_{pt}),$$

$$m_t=m_0e^t.$$

Next, define Hamiltonian with state variable m_t, control variables $\{\bar{P}_t,\bar{r}_t,L_{pt},\lambda\}$ and co-state variable μ_t:

$$\mathcal{H}_2\left(\bar{P}_t, \bar{r}_t, L_{pt}, m_t, \lambda, \mu_t\right)$$

$$= m_t\left[\bar{P}_t g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} - W_t L_{pt} - \alpha s L_{pt}\right] - v\lambda Z - k m_t - \frac{\tilde{k}}{2}(\lambda Z)^2 - ln\bar{P}_t$$

$$- \int_{r^{nc}}^{\bar{r}_t} \eta {r_t}^{-\eta} dr_t + \frac{\mu_t}{\chi}\left(L_t - L_{pt}\right)$$

$$= m_t\left[\bar{P}_t g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} - \sigma(1-\bar{r}_t)\bar{P}_t g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma}\right] - v\lambda Z - k m_t$$

$$- \frac{\tilde{k}}{2}(\lambda Z)^2 - ln\bar{P}_t - \int_{r^{nc}}^{\bar{r}_t} \eta {r_t}^{-\eta} dr_t + \frac{\mu_t}{\chi}\left(L_t - L_{pt}\right)$$

$$= m_t(1 - \sigma + \sigma\bar{r}_t)\bar{P}_t g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} - v\lambda Z - k m_t - \frac{\tilde{k}}{2}(\lambda Z)^2 - ln\bar{P}_t$$

$$- \eta\frac{{\bar{r}_t}^{1-\eta} - (r^{nc})^{1-\eta}}{1-\eta} + \frac{\mu_t}{\chi}(L_t - L_{pt})$$

The FOCs are:

$$\begin{cases} \dfrac{\partial \mathcal{H}_2}{\partial L_{pt}} = 0 \\ \dfrac{\partial \mathcal{H}_2}{\partial \bar{P}_t} = 0 \\ \dfrac{\partial \mathcal{H}_2}{\partial \bar{r}_t} = 0 \\ \dfrac{\partial \mathcal{H}_2}{\partial \lambda} = 0 \\ \rho = \dfrac{\partial \mathcal{H}_2 / \partial m_t}{\mu_t} + \dfrac{\dot{\mu}_t}{\mu_t} \end{cases}$$

Start with $\frac{\partial \mathcal{H}_2}{\partial L_{pt}} = 0$, which implies

$$\sigma m_t(1 - \sigma + \sigma\bar{r}_t)\bar{P}_t g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma-1} = \frac{\mu_t}{\chi},$$

and therefore

$$L_{pt} = \left[\frac{\mu_t}{\chi\sigma m_t(1-\sigma+\sigma\bar{r}_t)\bar{P}_t g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma}}\right]^{\frac{1}{\sigma-1}} \quad (A57).$$

Next, consider $\frac{\partial \mathcal{H}_2}{\partial \bar{P}_t} = 0$:

$$m_t(1-\sigma+\sigma\bar{r}_t)g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} = \frac{1}{\bar{P}_t}.$$

Substituting gives

$$\bar{P}_t = \frac{1}{m_t(1-\sigma+\sigma\bar{r}_t)g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma}} \quad (A58).$$

The next FOC is similar, $\frac{\partial \mathcal{H}_2}{\partial \bar{r}_t} = 0$:

$$m_t\sigma\bar{P}_t g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} = \eta\bar{r}_t{}^{-\eta}.$$

Substituting gives

$$\frac{\sigma}{1-\sigma+\sigma\bar{r}_t} = \eta\bar{r}_t{}^{-\eta}.$$

$$\Rightarrow \eta = \frac{\sigma\bar{r}_t{}^{\eta}}{1-\sigma+\sigma\bar{r}_t} \Rightarrow \eta(1-\sigma+\sigma\bar{r}_t) = \sigma\bar{r}_t{}^{\eta} \Rightarrow \eta(1-\sigma)+\eta\sigma\bar{r}_t = \sigma\bar{r}_t{}^{\eta} \Rightarrow \sigma\bar{r}_t{}^{\eta}$$

$$= \eta(1-\sigma)+\eta\sigma\bar{r}_t \Rightarrow \bar{r}_t{}^{\eta} = \frac{\eta(1-\sigma)}{\sigma} + \eta\bar{r}_t$$

While an explicit analytical solution for $\bar{r}_t$ in terms of η and σ is not easily attainable due to the complexity of the equation, if $\eta = 0.5$, I can use this specific value to simplify the expression for $\bar{r}_t$ in terms of σ. Let's proceed with substituting $\eta = 0.5$ into the equation:

$$\bar{r}_t{}^{0.5} = 0.5\bar{r}_t + \frac{1-\sigma}{2\sigma}.$$

Square both sides and expand the right-hand side, then rearrange the equation to form a quadratic equation:

$$\bar{r}_t = 0.25\bar{r}_t{}^2 + \frac{1-\sigma}{2\sigma}\bar{r}_t + (\frac{1-\sigma}{2\sigma})^2 \Rightarrow 0.25\bar{r}_t{}^2 + \bar{r}_t\left(\frac{1-3\sigma}{2\sigma}\right) + \frac{(1-\sigma)^2}{4\sigma^2} = 0.$$

So, the solutions for $\bar{r}_t$ in terms of σ are:

$$\bar{r}_t{}^{***} = \frac{\frac{3\sigma-1}{2\sigma} \pm \sqrt{\left(\frac{1-3\sigma}{2\sigma}\right)^2 - 4\times 0.25\frac{(1-\sigma)^2}{4\sigma^2}}}{2\times 0.25} = \frac{3\sigma-1\pm 2\sqrt{(2\sigma-1)\sigma}}{\sigma} \quad (A59).$$

The next FOC is similar, $\frac{\partial \mathcal{H}_2}{\partial \lambda} = 0$:

$$(1-\sigma)m_t(1-\sigma+\sigma\bar{r}_t)\bar{P}_t g(\theta_t)\lambda^{-\sigma}\left(\frac{Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} - vZ - \tilde{k}\lambda Z^2 = 0$$

$$\Longrightarrow \frac{(1-\sigma)m_t(1-\sigma+\sigma\bar{r}_t)\bar{P}_t g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma}}{\lambda} - vZ - \lambda\tilde{k}Z^2 = 0$$

$$\Longrightarrow \frac{1-\sigma}{\lambda} - vZ - \lambda\tilde{k}Z^2 = 0 \Longrightarrow 1-\sigma-\lambda vZ - \lambda^2\tilde{k}Z^2 = 0$$

$$(\tilde{k}Z^2)\lambda^2 + (vZ)\lambda + (\sigma-1) = 0$$

Substituting gives

$$\lambda^{**} = \frac{-vZ+\sqrt{(vZ)^2+4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z^2} \ (A60).$$

Finally, the last FOC is $\rho = \frac{\partial \mathcal{H}_2/\partial m_t}{\mu_t} + \frac{\dot{\mu_t}}{\mu_t}$. The derivative of the Hamiltonian $\frac{\partial \mathcal{H}_2}{\partial m_t}$ is:

$$\frac{\partial \mathcal{H}_2}{\partial m_t} = (1-\sigma+\sigma\bar{r}_t)\bar{P}_t g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} + (\sigma-1)(1-\sigma+\sigma\bar{r}_t)\bar{P}_t g(\theta_t)\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} - k$$

$$= \frac{\sigma}{m_t} - k.$$

Substituting this into the FOC and rearranging to solve for μ_t gives

$$\dot{\mu_t} - \rho\mu_t = k - \frac{\sigma}{m_t}$$

Solving for μ_t^{***}:

$$\mu_t^{***} = \mu_t^{**} \ (A61).$$

(1) Solving the Optimal Allocation

The 5 FOCs are:

$$\begin{cases} L_{pt} = \left[\dfrac{\mu_t}{\chi\sigma m_t(1-\sigma+\sigma\bar{r}_t)\bar{P}_t g(\theta_t)\left(\dfrac{\lambda Z}{m_t}\right)^{1-\sigma}}\right]^{\frac{1}{\sigma-1}} & (A57) \\ \bar{P}_t = \dfrac{1}{m_t(1-\sigma+\sigma\bar{r}_t)g(\theta_t)\left(\dfrac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma}} & (A58) \\ \bar{r}_t^{***} = \dfrac{3\sigma-1\pm 2\sqrt{(2\sigma-1)\sigma}}{\sigma} & (A59) \\ \lambda^{**} = \dfrac{-vZ+\sqrt{(vZ)^2+4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z^2} & (A60) \\ \mu_t^{***} = \left(\mu_0 + \dfrac{k-\dfrac{\sigma}{m_0 e^t}}{\rho}\right)e^{\rho t} - \dfrac{k-\dfrac{\sigma}{m_0 e^t}}{\rho} & (A61) \end{cases}$$

1) Solving for L_{pt} and $\bar{P}_t$(eqs. A57 and A58).

Combining (A57), (A58), and (A61) gives

$$L_{pt}^{***} = \frac{\sigma\chi}{\left(\mu_0+\frac{k-\frac{\sigma}{m_0 e^t}}{\rho}\right)e^{\rho t}-\frac{k-\frac{\sigma}{m_0 e^t}}{\rho}} = L_{pt}^{**} \quad (A62)$$

In addition, combining (A58), (A59), (A60), and (A62) gives

$$\bar{P}_t^{***} = \frac{\left[\frac{\sigma\chi}{\left(\mu_0+\frac{k-\frac{\sigma}{m_0 e^t}}{\rho}\right)e^{\rho t}-\frac{k-\frac{\sigma}{m_0 e^t}}{\rho}}\right]^{-\sigma}}{(m_0 e^t)^{\sigma}(1-\sigma+\sigma\bar{r}_t)[\frac{3\sigma-1\pm 2\sqrt{(2\sigma-1)\sigma}}{\sigma}](\lambda^{**}\mathrm{Z})^{1-\sigma}g(\theta_t)} =$$

$$\frac{\left[\frac{\sigma\chi}{\left(\mu_0+\frac{k-\frac{\sigma}{m_0 e^t}}{\rho}\right)e^{\rho t}-\frac{k-\frac{\sigma}{m_0 e^t}}{\rho}}\right]^{-\sigma}}{(m_0 e^t)^{\sigma}[2\sigma\pm 2\sqrt{(2\sigma-1)\sigma}][\frac{-vZ+\sqrt{(vZ)^2+4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}]^{1-\sigma}g(\theta_t)} \quad (A63).$$

2) Solving for U_2.

According to equation (14), $r^{nc} = r^{****} = \frac{\frac{\partial c_l}{\partial m} + h(v + \frac{\partial c_l}{\partial H})}{Pqg(\theta)}$. Plugging previous results yields: $r^{nc} = km_t\bar{r}_t^{***} + \bar{r}_t^{***}H(v + \tilde{k}H) = [\frac{3\sigma - 1 \pm 2\sqrt{(2\sigma-1)\sigma}}{\sigma}][km_t + \lambda^{**}Z(v + \tilde{k}\lambda^{**}Z)] = [\frac{3\sigma - 1 \pm 2\sqrt{(2\sigma-1)\sigma}}{\sigma}][km_0e^t + \lambda^{**}Zv + \tilde{k}(\lambda^{**}Z)^2]$.

Next, by definition

$U_2 = \int_0^\infty e^{-\rho t}\left\{m_t\left[\bar{P}_t g(\theta_t)(\frac{\lambda Z}{m_t})^{1-\sigma}L_{pt}^{\sigma} - W_t L_{pt} - \alpha s L_{pt}\right] - v\lambda Z - km_t - \frac{\tilde{k}}{2}(\lambda Z)^2 - ln\bar{P}_t - \int_{r^{nc}}^{\bar{r}_t} \eta r_t^{-\eta} dr_t\right\}$ dt. Then:

$$U_2 = \int_0^\infty e^{-\rho t}\left[m_t(1 - \sigma + \sigma\bar{r}_t)\bar{P}_t^{***} g(\theta_t)\left(\frac{\lambda^{**}Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} - v\lambda^{**}Z - km_t - \frac{\tilde{k}}{2}(\lambda^{**}Z)^2 - ln\bar{P}_t^{***} - \eta\frac{(\bar{r}_t^{***})^{1-\eta} - (r^{nc})^{1-\eta}}{1-\eta}\right]\text{dt}$$

$$= \int_0^\infty e^{-\rho t}\left[1 - v\lambda^{**}Z - km_t - \frac{\tilde{k}}{2}(\lambda^{**}Z)^2 - ln\bar{P}_t^{***} + \frac{\eta}{\eta - 1}[\frac{3\sigma - 1 \pm 2\sqrt{(2\sigma-1)\sigma}}{\sigma}]^{1-\eta}\{1 - \left[km_0e^t + \lambda^{**}Zv + \tilde{k}(\lambda^{**}Z)^2\right]^{1-\eta}\}\right]\text{dt}$$

$$= \int_0^\infty e^{-\rho t}\left\{[1 - v\lambda^{**}Z - \frac{\tilde{k}}{2}(\lambda^{**}Z)^2 - \lambda^{**-\alpha}] - km_0e^t + \frac{\eta}{\eta - 1}[\frac{3\sigma - 1 \pm 2\sqrt{(2\sigma-1)\sigma}}{\sigma}]^{1-\eta}\{1 - \left[km_0e^t + \lambda^{**}Zv + \tilde{k}(\lambda^{**}Z)^2\right]^{1-\eta}\} - ln\bar{P}_t^{***}\right\}dt$$

$$= \int_0^\infty [1 - v\lambda^{**}Z - \frac{\tilde{k}}{2}(\lambda^{**}Z)^2 - \lambda^{**-\alpha}]e^{-\rho t}\,dt + \int_0^\infty(-km_0e^t)e^{-\rho t}\,dt + \int_0^\infty \frac{\eta}{\eta-1}[\frac{3\sigma-1\pm2\sqrt{(2\sigma-1)\sigma}}{\sigma}]^{1-\eta}\{1 - \left[km_0e^t + \lambda^{**}Zv + \tilde{k}(\lambda^{**}Z)^2\right]^{1-\eta}\}e^{-\rho t}\,dt +$$

$$\int_0^\infty(-ln\bar{P}_t^{***})e^{-\rho t}\,dt = \frac{1 - v\frac{-vZ + \sqrt{(vZ)^2 + 4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z} - \frac{\tilde{k}}{2}[\frac{-vZ + \sqrt{(vZ)^2 + 4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}]^2}{\rho} + \frac{km_0}{1-\rho} +$$

$\frac{\eta}{\eta-1}[\frac{3\sigma-1\pm2\sqrt{(2\sigma-1)\sigma}}{\sigma}]^{1-\eta}\left[\frac{1}{\rho}-\frac{(km_0)^{1-\eta}}{1-\eta-\rho}\right]-\frac{\sigma}{\rho}[1+ln\left(\mu_0+\frac{k}{\rho}\right)]+\frac{\sigma}{\rho^2}+$

$\frac{\sigma ln\sigma\chi+\sigma lnm_0+\ln[2\sigma\pm2\sqrt{(2\sigma-1)\sigma}]+(1-\sigma)\ln[\frac{-vZ+\sqrt{(vZ)^2+4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}]}{\rho}+\ln\left(\frac{1}{\sqrt{2\pi}\sigma_\theta}\right)\frac{1}{\rho}-\frac{1}{2\sigma_\theta^2}(\frac{\theta_0^2}{2\beta+\rho}-\frac{2\mu_\theta\theta_0}{\beta+\rho}+$

$\frac{\mu_\theta^2}{\rho})$.

Note that:

(i) $\int_0^\infty[1-v\lambda^{**}Z-\frac{\tilde{k}}{2}(\lambda^{**}Z)^2]e^{-\rho t}\,dt=\left[1-v\lambda^{**}Z-\frac{\tilde{k}}{2}(\lambda^{**}Z)^2\right]\int_0^\infty e^{-\rho t}\,dt=$

$\frac{\left[1-v\lambda^{**}Z-\frac{\tilde{k}}{2}(\lambda^{**}Z)^2\right]}{\rho}=\frac{1-v\frac{-vZ+\sqrt{(vZ)^2+4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}-\frac{\tilde{k}}{2}[\frac{-vZ+\sqrt{(vZ)^2+4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}]^2}{\rho}$.

(ii) $\int_0^\infty(-km_0e^t)e^{-\rho t}\,dt=-km_0\int_0^\infty e^{(1-\rho)t}\,dt=\frac{-km_0}{1-\rho}\left[e^{(1-\rho)t}\right]_0^\infty=\frac{km_0}{1-\rho}$, because

$\lim_{t\to\infty}e^{(1-\rho)t}=\lim_{t\to\infty}\frac{1}{e^{(\rho-1)t}}=0$ and $\lim_{t\to0}e^{(1-\rho)t}=1$ $(\rho>1)$.

(iii)$\int_0^\infty\frac{\eta}{\eta-1}[\frac{3\sigma-1\pm2\sqrt{(2\sigma-1)\sigma}}{\sigma}]^{1-\eta}\{1-\left[km_0e^t+\lambda^{**}Zv+\tilde{k}(\lambda^{**}Z)^2\right]^{1-\eta}\}e^{-\rho t}\,dt=$

$\frac{\eta}{\eta-1}[\frac{3\sigma-1\pm2\sqrt{(2\sigma-1)\sigma}}{\sigma}]^{1-\eta}\int_0^\infty\{1-\left[km_0e^t+\lambda^{**}Zv+\tilde{k}(\lambda^{**}Z)^2\right]^{1-\eta}\}e^{-\rho t}\,dt=$

$\frac{\eta}{\eta-1}[\frac{3\sigma-1\pm2\sqrt{(2\sigma-1)\sigma}}{\sigma}]^{1-\eta}\{\int_0^\infty e^{-\rho t}\,dt-\int_0^\infty\left[km_0e^t+\lambda^{**}Zv+\tilde{k}(\lambda^{**}Z)^2\right]^{1-\eta}e^{-\rho t}dt\}=$

$\frac{\eta}{\eta-1}[\frac{3\sigma-1\pm2\sqrt{(2\sigma-1)\sigma}}{\sigma}]^{1-\eta}\left[\frac{1}{\rho}-\int_0^\infty\left(km_0e^t(1+\frac{v\lambda^*Z+\tilde{k}(\lambda^*Z)^2}{km_0e^t}\right)^{1-\eta}e^{-\rho t}dt\right]\approx$

$\frac{\eta}{\eta-1}[\frac{3\sigma-1\pm2\sqrt{(2\sigma-1)\sigma}}{\sigma}]^{1-\eta}\left[\frac{1}{\rho}-\int_0^\infty(km_0e^t)^{1-\eta}e^{-\rho t}dt\right]=\frac{1}{\eta-1}\left[\frac{1}{\rho}-\right.$

$\left.(km_0)^{1-\eta}\int_0^\infty e^{t(1-\eta)}e^{-\rho t}dt\right]=\frac{\eta}{\eta-1}[\frac{3\sigma-1\pm2\sqrt{(2\sigma-1)\sigma}}{\sigma}]^{1-\eta}\left[\frac{1}{\rho}-(km_0)^{1-\eta}\int_0^\infty e^{t(1-\eta-\rho)}\,dt\right]=$

$\frac{\eta}{\eta-1}[\frac{3\sigma-1\pm2\sqrt{(2\sigma-1)\sigma}}{\sigma}]^{1-\eta}\left[\frac{1}{\rho}-\frac{(km_0)^{1-\eta}}{1-\eta-\rho}\right]$.

$$(iv)\int_0^{\infty}\left(-ln\bar{P}_t^{***}\right)e^{-\rho t}\,dt = -\int_0^{\infty} e^{-\rho t}\ln\frac{\left[\left(\mu_0+\frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right)e^{\rho t}-\frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right]^{-\sigma\chi}}{(m_0e^t)^{\sigma\left[2\sigma\pm2\sqrt{(2\sigma-1)\sigma}\right]}(\lambda^{**}\mathrm{Z})^{1-\sigma}g(\theta_t)}dt =$$

$$-\int_0^{\infty} e^{-\rho t}\{\sigma\ln\left[\left(\mu_0+\frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right)e^{\rho t}-\frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right]-\sigma ln\sigma\chi-\sigma lnm_0-\sigma t-ln[2\sigma\pm$$

$$2\sqrt{(2\sigma-1)\sigma}]-(1-\sigma)\ln(\lambda^{**}\mathrm{Z})-\ln g(\theta_t)\}dt = -\int_0^{\infty} e^{-\rho t}\{\sigma\ln\left[\left(\mu_0+\frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right)e^{\rho t}-$$

$$\frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right]-\sigma t\}dt-\int_0^{\infty} e^{-\rho t}\{-\sigma ln\sigma\chi-\sigma lnm_0-\ln[2\sigma\pm2\sqrt{(2\sigma-1)\sigma}]-(1-$$

$$\sigma)\ln[\frac{-vZ+\sqrt{(vZ)^2+4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}]-\ln g(\theta_t)\}dt = -\int_0^{\infty} e^{-\rho t}\{\sigma\ln\left[\left(\mu_0+\frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right)e^{\rho t}-\frac{k-\frac{\sigma}{m_0e^t}}{\rho}\right]\}dt-$$

$$\int_0^{\infty}-\sigma te^{-\rho t}\,dt-\{-\sigma ln\sigma\chi-\sigma lnm_0-\ln[2\sigma\pm2\sqrt{(2\sigma-1)\sigma}]-(1-$$

$$\sigma)\ln[\frac{-vZ+\sqrt{(vZ)^2+4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}]\}\int_0^{\infty} e^{-\rho t}\,dt+\int_0^{\infty}\ln g(\theta_t)e^{-\rho t}\,dt = -\frac{\sigma}{\rho}[1+ln\left(\mu_0+\frac{k}{\rho}\right)]+$$

$$\sigma\int_0^{\infty} te^{-\rho t}\,dt+\{\sigma ln\sigma\chi+\sigma lnm_0+\ln[2\sigma\pm2\sqrt{(2\sigma-1)\sigma}]+(1-$$

$$\sigma)\ln[\frac{-vZ+\sqrt{(vZ)^2+4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}]\int_0^{\infty} e^{-\rho t}\,dt+\int_0^{\infty}[\ln g(\theta_t)]e^{-\rho t}\,dt = -\frac{\sigma}{\rho}[1+ln\left(\mu_0+\frac{k}{\rho}\right)]+\frac{\sigma}{\rho^2}+$$

$$\frac{\sigma ln\sigma\chi+\sigma lnm_0+\ln[2\sigma\pm2\sqrt{(2\sigma-1)\sigma}]+(1-\sigma)\ln[\frac{-vZ+\sqrt{(vZ)^2+4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}]}{\rho}+\ln\left(\frac{1}{\sqrt{2\pi}\sigma_\theta}\right)\frac{1}{\rho}-\frac{1}{2\sigma_\theta^2}(\frac{\theta_0^2}{2\beta+\rho}-\frac{2\mu_\theta\theta_0}{\beta+\rho}+$$

$$\frac{\mu_\theta^2}{\rho}).$$

When $g(\theta_t)=\frac{1}{\sqrt{2\pi}\sigma_\theta}\exp[-\frac{(\theta_t-\mu_\theta)^2}{2\sigma_\theta^2}]$ and $\theta_t=\theta_0e^{-\beta t}$, $\int_0^{\infty}[\ln g(\theta_t)]e^{-\rho t}\,dt =$

$$\ln\left(\frac{1}{\sqrt{2\pi}\sigma_\theta}\right)\frac{1}{\rho}-\frac{1}{2\sigma_\theta^2}(\frac{\theta_0^2}{2\beta+\rho}-\frac{2\mu_\theta\theta_0}{\beta+\rho}+\frac{\mu_\theta^2}{\rho}).$$

K. PROOF OF EQUATION (19)

First, the tenant problem is given by:

$$\underset{\{L\}}{\text{Max}}\, \text{E}U_w[\pi_w(\theta)] = \text{E}U_w[(1-r)Pq(L,h)g(\theta) - WL - c_w(s,L) + \beta(x)] = \text{E}U_w[xg(\theta) -$$

$$WL - c_w(s,L) + \beta(x)] = \int_{\underline{\theta}}^{\overline{\theta}} U_w[xg(\theta) - WL - c_w(s,L) + \beta(x)]f(\theta)d\theta,$$

where $f(\theta)$ denotes a probability density function of random variable θ defined over the range $[\underline{\theta}, \overline{\theta}]$.

Start with the FOC w.r.t. L:

$$\frac{\partial \mathrm{E}U_w}{\partial \mathrm{L}} = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w}{\partial \pi_w}\left[\frac{\partial x}{\partial L} g(\theta) - W - \frac{\partial c_w}{\partial L} + \beta'(x)\frac{\partial x}{\partial L}\right] f(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w}{\partial \pi_w}\Big[(1-r)P\frac{\partial q}{\partial L} g(\theta) - W - \frac{\partial c_w}{\partial L} +$$

$$\beta'(x)(1-r)P\frac{\partial q}{\partial L}\Big] f(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \{[\frac{\partial U_w}{\partial \pi_w}(1-r)P\frac{\partial q}{\partial L} g(\theta)f(\theta) + \frac{\partial U_w}{\partial \pi_w}[\Big(-W - \frac{\partial c_w}{\partial L}\Big) + \beta'(x)(1 -$$

$$r)P\frac{\partial q}{\partial L} f(\theta)]\}d\theta = (1-r)P\frac{\partial q}{\partial L}\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta)f(\theta)d\theta - [\Big(W + \frac{\partial c_w}{\partial L}\Big) - \beta'(x)(1 -$$

$$r)P\frac{\partial q}{\partial L}] \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta)d\theta = 0.$$

$$\Longrightarrow (1-r)P\frac{\partial q}{\partial L}\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta)f(\theta)d\theta = [\Big(W + \frac{\partial c_w}{\partial L}\Big) - \beta'(x)(1-r)P\frac{\partial q}{\partial L}]\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta)d\theta$$

$$\Longrightarrow \frac{\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta)d\theta} = \frac{\Big(W + \frac{\partial c_w}{\partial L}\Big) - \beta'(x)(1-r)P\frac{\partial q}{\partial L}}{(1-r)P\frac{\partial q}{\partial L}} = \frac{\Big(W + \frac{\partial c_w}{\partial L}\Big)}{(1-r)P\frac{\partial q}{\partial L}} - \beta'(x) \text{ (A64).}$$

Next, the problem faced by the landlord is:

$$\underset{\{m,x\}}{\mathrm{Max}}\, \mathrm{E}U_l[\pi_l(\theta)] = \mathrm{E}U_l[mrPg(\theta)q\left(L, \frac{H}{m}\right) - vH - c_l(m,H) - m\beta(x)] = \mathrm{E}U_l\{m[Pq\left(L, \frac{H}{m}\right) -$$

$$x]g(\theta) - vH - c_l(m,H) - m\beta(x)\} = \int_{\underline{\theta}}^{\overline{\theta}} U_l\{m[Pq\left(L, \frac{H}{m}\right) - x]g(\theta) - vH - c_l(m,H) -$$

$$m\beta(x)\}f(\theta)d\theta.$$

The FOC w.r.t x:

$$\frac{\partial \mathrm{E}U_l}{\partial \mathrm{x}} = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}[-mg(\theta) - m\beta'(x)]f(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} [\frac{\partial U_l}{\partial \pi_l}(-m)g(\theta)f(\theta) - \frac{\partial U_l}{\partial \pi_l} m\beta'(x)f(\theta)]d\theta =$$

$$-m\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta)f(\theta)d\theta - m\beta'(x)\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta)d\theta = 0.$$

Hence:

$$\Longrightarrow -m\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta)f(\theta)d\theta = m\beta'(x)\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta)d\theta$$

$$\Longrightarrow \frac{\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta)d\theta} = -\beta'(x) \text{ (A65).}$$

On the other hand, the FOC w.r.t. m is:

$$\frac{\partial \mathrm{E}U_l}{\partial \mathrm{m}} = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}\left[Pq\left(L,\frac{H}{m}\right)g(\theta) + mP\frac{\partial q}{\partial h}\frac{\partial h}{\partial m}g(\theta) - xg(\theta) - m\frac{\partial x}{\partial m}g(\theta) - \frac{\partial c_l}{\partial m} - \beta - m\beta'(x)\frac{\partial x}{\partial m}\right]f(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}\left[Pq\left(L,\frac{H}{m}\right)g(\theta) + mP\frac{\partial q}{\partial h}(-\frac{H}{m^2})g(\theta) - xg(\theta) - m(r-1)P\frac{\partial q}{\partial h}(\frac{H}{m^2})g(\theta) - \frac{\partial c_l}{\partial m} - \beta - m\beta'(x)(r-1)P\frac{\partial q}{\partial h}(\frac{H}{m^2})\right]f(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}\{[Pq\left(L,\frac{H}{m}\right) - P\frac{\partial q}{\partial h}h - x - (r-1)P\frac{\partial q}{\partial h}h]g(\theta) - \frac{\partial c_l}{\partial m} - \beta - \beta'(x)(r-1)P\frac{\partial q}{\partial h}h\}f(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}\left[Pq\left(L,\frac{H}{m}\right) - P\frac{\partial q}{\partial h}h - x - (r-1)P\frac{\partial q}{\partial h}h\right]g(\theta)f(\theta)d\theta + \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}[-\frac{\partial c_l}{\partial m} - \beta - \beta'(x)(r-1)P\frac{\partial q}{\partial h}h]f(\theta)d\theta =$$

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}\left[Pq\left(L,\frac{H}{m}\right) - x - rP\frac{\partial q}{\partial h}h\right]g(\theta)f(\theta)d\theta - \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}\left[\frac{\partial c_l}{\partial m} + \beta + \beta'(x)(r-1)P\frac{\partial q}{\partial h}h\right]f(\theta)d\theta =$$

$$\left[rPq\left(L,\frac{H}{m}\right) - rP\frac{\partial q}{\partial h}h\right]\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}g(\theta)f(\theta)d\theta - \left[\frac{\partial c_l}{\partial m} + \beta + \beta'(x)(r-1)P\frac{\partial q}{\partial h}h\right]\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}f(\theta)d\theta = 0.$$

Thus:

$$\Longrightarrow \left[rPq\left(L,\frac{H}{m}\right) - rP\frac{\partial q}{\partial h}h\right]\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}g(\theta)f(\theta)d\theta$$

$$= \left[\frac{\partial c_l}{\partial m} + \beta + \beta'(x)(r-1)P\frac{\partial q}{\partial h}h\right]\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}f(\theta)d\theta$$

$$\Longrightarrow \frac{\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}f(\theta)d\theta} = \frac{\frac{\partial c_l}{\partial m} + \beta + \beta'^{(x)(r-1)}P\frac{\partial q}{\partial h}h}{rPq\left(L,\frac{H}{m}\right)\left(1 - \frac{\partial q}{\partial h}\frac{h}{q}\right)} \quad (A66).$$

Therefore, plug (A66) in (A65):

$$-\beta'(x) = \frac{\frac{\partial c_l}{\partial m} + \beta + \beta\prime(x)(r-1)P\frac{\partial q}{\partial h}h}{rPq\left(L,\frac{H}{m}\right)(1-\frac{\partial q h}{\partial h q})}.$$

Hence:

$$\Longrightarrow -\beta'(x)rPq\left(L,\frac{H}{m}\right)(1 - \frac{\partial q}{\partial h}\frac{h}{q}) = \frac{\partial c_l}{\partial m} + \beta + \beta'(x)(r-1)P\frac{\partial q}{\partial h}h$$

$$\Longrightarrow -\beta'(x)Pq[r(1-\frac{\partial q}{\partial h}\frac{h}{q})+(r-1)\frac{\partial q}{\partial h}\frac{h}{q}]=\frac{\partial c_l}{\partial m}+\beta$$

$$\Longrightarrow -\beta'(x)Pq\left(r-\frac{\partial q}{\partial h}\frac{h}{q}\right)=\frac{\partial c_l}{\partial m}+\beta \;\Longrightarrow r=\frac{\partial q}{\partial h}\frac{h}{q}+\frac{\frac{\partial c_l}{\partial m}+\beta}{-\beta'(x)Pq} \quad (A67).$$

Plug (A64) in (A67):

$$r=\frac{\partial q}{\partial h}\frac{h}{q}+\frac{\frac{\partial c_l}{\partial m}+\beta}{[\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}-\frac{\left(W+\frac{\partial c_w}{\partial L}\right)}{(1-r)P\frac{\partial q}{\partial L}}]Pq}=\frac{\partial q}{\partial h}\frac{h}{q}-\frac{(\frac{\partial c_l}{\partial m}+\beta)(1-r)P\frac{\partial q}{\partial L}}{[\left(W+\frac{\partial c_w}{\partial L}\right)-\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}(1-r)P\frac{\partial q}{\partial L}]Pq}.$$

Thus:

$$\Longrightarrow \left(\frac{\partial c_l}{\partial m}+\beta\right)(1-r)P\frac{\partial q}{\partial L}=(\frac{\partial q}{\partial h}\frac{h}{q}-r)\left[\left(W+\frac{\partial c_w}{\partial L}\right)-\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}(1-r)P\frac{\partial q}{\partial L}\right]Pq$$

$$\Longrightarrow \left[\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}P^2q\frac{\partial q}{\partial L}\right]r^2+\left[P\frac{\partial q}{\partial L}\left(\frac{\partial c_l}{\partial m}+\beta\right)-\left(W+\frac{\partial c_w}{\partial L}\right)Pq-\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}P^2q\frac{\partial q}{\partial L}\left(\frac{\partial q}{\partial h}\frac{h}{q}+1\right)\right]r+\left[\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}P^2q\frac{\partial q}{\partial L}\frac{\partial q}{\partial h}\frac{h}{q}-P\frac{\partial q}{\partial L}\left(\frac{\partial c_l}{\partial m}+\beta\right)\right]=0.$$

Let $A=\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}P^2q\frac{\partial q}{\partial L}$, $B=P\frac{\partial q}{\partial L}\left(\frac{\partial c_l}{\partial m}+\beta\right)-\left(W+\frac{\partial c_w}{\partial L}\right)Pq-\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}P^2q\frac{\partial q}{\partial L}\left(\frac{\partial q}{\partial h}\frac{h}{q}+1\right)$, $C=\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}P^2q\frac{\partial q}{\partial L}\frac{\partial q}{\partial h}\frac{h}{q}-P\frac{\partial q}{\partial L}\left(\frac{\partial c_l}{\partial m}+\beta\right)$, and $\Delta=\sqrt{B^2-4AC}=$

$$\sqrt{[P\frac{\partial q}{\partial L}\left(\frac{\partial c_l}{\partial m}+\beta\right)-\left(W+\frac{\partial c_w}{\partial L}\right)Pq-\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}P^2q\frac{\partial q}{\partial L}\left(\frac{\partial q}{\partial h}\frac{h}{q}+1\right)]^2-4\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}P^2q\frac{\partial q}{\partial L}[\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}P^2q\frac{\partial q}{\partial L}\frac{\partial q}{\partial h}\frac{h}{q}-P\frac{\partial q}{\partial L}\left(\frac{\partial c_l}{\partial m}+\beta\right)]}.$$

Thus:

$r^{PC} = \frac{-B+\Delta}{2A} = \frac{-P\frac{\partial q}{\partial L}\left(\frac{\partial c_l}{\partial m}+\beta\right)+\left(W+\frac{\partial c_W}{\partial L}\right)Pq+\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_W}{\partial \pi_W}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_W}{\partial \pi_W}f(\theta)d\theta}P^2q\frac{\partial q}{\partial L}\left(\frac{\partial qh}{\partial hq}+1\right)+\Delta}{2\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_W}{\partial \pi_W}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_W}{\partial \pi_W}f(\theta)d\theta}P^2q\frac{\partial q}{\partial L}}$, which is stated as equation (19) in the text.

L. PROOF OF PROPOSITION 6

The landlord problem is:

$$\underset{\{H,x\}}{\text{Max}}\text{E}U_l[\pi_l(\theta)] = \text{E}U_l\{[\bar{P}q(L,H)-x]g(\theta)-vH-c_l(1,H)-\beta(x)\} = \int_{\underline{\theta}}^{\bar{\theta}} U_l\{[\bar{P}q(L,H)-$$

$$x]g(\theta)-vH-c_l(1,H)-\beta(x)\}f(\theta)d\theta.$$

Start with the FOC w.r.t. x:

$\frac{\partial \mathrm{E}U_l}{\partial \mathrm{x}} = \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l}[-g(\theta) - \beta'(x)]f(\theta)d\theta = \int_{\underline{\theta}}^{\bar{\theta}} [\frac{\partial U_l}{\partial \pi_l}(-1)g(\theta)f(\theta) - \frac{\partial U_l}{\partial \pi_l}\beta'(x)f(\theta)]d\theta =$

$-\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta)f(\theta)d\theta - \beta'(x)\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta)d\theta = 0.$

Hence:

$$\Longrightarrow -\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta)f(\theta)d\theta = \beta'(x)\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta)d\theta$$

$$\Longrightarrow \frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta)d\theta} = -\beta'(x) \ \ (A68).$$

Next, compute the FOC w.r.t. H:

$\frac{\partial \mathrm{E}U_l}{\partial \mathrm{H}} = \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l}\left[r\bar{P}\frac{\partial q}{\partial H}g(\theta) - v - \frac{\partial c_l}{\partial H} - \beta'(x)\frac{\partial x}{\partial H}\right]f(\theta)d\theta = \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l}\left[r\bar{P}\frac{\partial q}{\partial H}g(\theta) - v - \frac{\partial c_l}{\partial H} -\right.$

$\left.\beta'(x)(1-r)\bar{P}\frac{\partial q}{\partial H}\right]f(\theta)d\theta = r\bar{P}\frac{\partial q}{\partial H}\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta)f(\theta)d\theta - \left[v + \frac{\partial c_l}{\partial H} + \beta'(x)(1 -\right.$

$\left.r)\bar{P}\frac{\partial q}{\partial H}\right]\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta)d\theta = 0.$

Thus:

$$\Longrightarrow r\bar{P}\frac{\partial q}{\partial H}\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta)f(\theta)d\theta = \left[v + \frac{\partial c_l}{\partial H} + \beta'(x)(1-r)\bar{P}\frac{\partial q}{\partial H}\right]\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta)d\theta$$

$$\Longrightarrow \frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta)d\theta} = \frac{v + \frac{\partial c_l}{\partial H} + \beta'(x)(1-r)\bar{P}\frac{\partial q}{\partial H}}{r\bar{P}\frac{\partial q}{\partial H}} \ (A69).$$

Therefore, plug (A68) in (A69),

$$-\beta'(x) = \frac{v + \frac{\partial c_l}{\partial H} + \beta'(x)(1-r)\bar{P}\frac{\partial q}{\partial H}}{r\bar{P}\frac{\partial q}{\partial H}}.$$

Hence:

$$\Longrightarrow -\beta'(x)r\bar{P}\frac{\partial q}{\partial H} = v + \frac{\partial c_l}{\partial H} + \beta'(x)(1-r)\bar{P}\frac{\partial q}{\partial H} = v + \frac{\partial c_l}{\partial H} + \beta'(x)\bar{P}\frac{\partial q}{\partial H} - \beta'(x)r\bar{P}\frac{\partial q}{\partial H}$$

$$\Longrightarrow \beta'(x) = \frac{v + \frac{\partial c_l}{\partial H}}{\bar{P}\frac{\partial q}{\partial H}}$$

$$\Longrightarrow \frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta} = \frac{\left(W + \frac{\partial c_w}{\partial L}\right)}{(1-r)\bar{P}\frac{\partial q}{\partial L}} - \frac{v + \frac{\partial c_l}{\partial H}}{\bar{P}\frac{\partial q}{\partial H}}$$

$$\Longrightarrow \frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta} + \frac{v + \frac{\partial c_l}{\partial H}}{P\frac{\partial q}{\partial H}} = \frac{W + \frac{\partial c_w}{\partial L}}{(1-r)\bar{P}\frac{\partial q}{\partial L}}$$

$$\Longrightarrow \frac{W + \frac{\partial c_w}{\partial L}}{\left[\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta} + \frac{v + \frac{\partial c_l}{\partial H}}{\bar{P}\frac{\partial q}{\partial H}}\right]\bar{P}\frac{\partial q}{\partial L}} = 1 - r$$

So, the landlord's optimal share rate is:

$r^{MPC} = 1 - \frac{W+\frac{\partial c_w}{\partial L}}{\left[\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta} + \frac{v+\frac{\partial c_l}{\partial H}}{\bar{P}\frac{\partial q}{\partial H}}\right]\bar{P}\frac{\partial q}{\partial L}} = 1 - \frac{W+\frac{\partial c_w}{\partial L}}{\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta}\bar{P}\frac{\partial q}{\partial L} + \frac{(v+\frac{\partial c_l}{\partial H})\frac{\partial q}{\partial L}}{\frac{\partial q}{\partial H}}}$, which is stated as

equation (20) in the text.

Compute the derivative $\frac{\partial r^{MPC}}{\partial W}$:

$$\frac{\partial r^{MPC}}{\partial W} = -\frac{1}{\left[\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta} + \frac{v+\frac{\partial c_l}{\partial H}}{\bar{P}\frac{\partial q}{\partial H}}\right]\bar{P}\frac{\partial q}{\partial L}} < 0.$$

Compute the derivative $\frac{\partial r^{MPC}}{\partial(\frac{\partial c_w}{\partial L})}$:

$$\frac{\partial r^{MPC}}{\partial(\frac{\partial c_w}{\partial L})} = -\frac{1}{\left[\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta} + \frac{v+\frac{\partial c_l}{\partial H}}{\bar{P}\frac{\partial q}{\partial H}}\right]\bar{P}\frac{\partial q}{\partial L}} < 0.$$

Compute the derivative $\frac{\partial r^{MPC}}{\partial v}$:

$$\frac{\partial r^{MPC}}{\partial v} = \frac{(W+\frac{\partial c_W}{\partial L})}{\left[\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W}{\partial \pi_W} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W}{\partial \pi_W} f(\theta) d\theta} + \frac{v+\frac{\partial c_I}{\partial H}}{\bar{P}\frac{\partial q}{\partial H}}\right]^2 \frac{\partial q}{\partial H}\bar{P}^2\frac{\partial q}{\partial L}} > 0.$$

Compute the derivative $\frac{\partial r^{MPC}}{\partial(\frac{\partial c_I}{\partial H})}$:

$$\frac{\partial r^{MPC}}{\partial(\frac{\partial c_I}{\partial H})} = \frac{(W+\frac{\partial c_W}{\partial L})}{\left[\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W}{\partial \pi_W} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W}{\partial \pi_W} f(\theta) d\theta} + \frac{v+\frac{\partial c_I}{\partial H}}{\bar{P}\frac{\partial q}{\partial H}}\right]^2 \frac{\partial q}{\partial H}\bar{P}^2\frac{\partial q}{\partial L}} > 0.$$

Compute the derivative $\frac{\partial r^{MPC}}{\partial(\bar{P})}$:

$$\frac{\partial r^{MPC}}{\partial(\bar{P})} = \frac{(W+\frac{\partial c_W}{\partial L})\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W}{\partial \pi_W} g(\theta) f(\theta) d\theta}{\partial L}}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W}{\partial \pi_W} f(\theta) d\theta}}{\left[\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W}{\partial \pi_W} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W}{\partial \pi_W} f(\theta) d\theta}\bar{P}\frac{\partial q}{\partial L} + \frac{(v+\frac{\partial c_I}{\partial H})\frac{\partial q}{\partial L}}{\frac{\partial q}{\partial H}}\right]^2} > 0.$$

M. PROOF OF LEMMA 6

The tenant problem is:

$$\underset{\{L\}}{\text{Max}} \text{E}U_w[\pi_w(\theta)] = \text{E}U_w\{(1-r)[a-bq(L,H)]q(L,H)g(\theta) - WL - c_w(s,L) + \beta(x)\} =$$

$$\int_{\underline{\theta}}^{\bar{\theta}} U_w\{(1-r)[aq(L,H) - bq(L,H)^2]g(\theta) - WL - c_w(s,L) + \beta(x)\}f(\theta)d\theta.$$

The FOC w.r.t. L is:

$$\frac{\partial \mathrm{E}U_w}{\partial \mathrm{L}} = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w}{\partial \pi_w}\left[(1-r)(a-2bq)\frac{\partial q}{\partial L}g(\theta) - W - \frac{\partial c_w}{\partial L} + \beta'(x)\frac{\partial x}{\partial L}\right] f(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w}{\partial \pi_w}\Big[(1-r)(a-$$

$$2bq)\frac{\partial q}{\partial L}g(\theta) - W - \frac{\partial c_w}{\partial L} + \beta'(x)(1-r)(a-2bq)\frac{\partial q}{\partial L}\Big] f(\theta)d\theta = (1-r)(a-$$

$$2bq)\frac{\partial q}{\partial L}\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta - [\left(W + \frac{\partial c_w}{\partial L}\right) - \beta'(x)(1-r)(a-2bq)\frac{\partial q}{\partial L}]\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta = 0.$$

Thus:

$$\Longrightarrow (1-r)(a-2bq)\frac{\partial q}{\partial L}\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta$$

$$= [\left(W + \frac{\partial c_w}{\partial L}\right) - \beta'(x)(1-r)(a-2bq)\frac{\partial q}{\partial L}]\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta$$

$$\Longrightarrow \frac{\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta} = \frac{\left(W + \frac{\partial c_w}{\partial L}\right)}{(1-r)(a-2bq)\frac{\partial q}{\partial L}} - \beta'(x)\ (A70).$$

Next, the landlord's problem is given by:

$$\underset{\{H,x\}}{\mathrm{Max}}\ \mathrm{E}U_l[\pi_l(\theta)] = \mathrm{E}U_l\{[(a-bq(L,H))q(L,H)-x]g(\theta) - vH - c_l(1,H) - \beta(x)\} =$$

$$\int_{\underline{\theta}}^{\overline{\theta}} U_l\{[aq(L,H) - bq(L,H)^2 - x]g(\theta) - vH - c_l(1,H) - \beta(x)\}f(\theta)d\theta.$$

The FOC w.r.t. x:

$$\frac{\partial \mathrm{E}U_l}{\partial \mathrm{x}} = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}[-g(\theta) - \beta'(x)]f(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} [\frac{\partial U_l}{\partial \pi_l}(-1)g(\theta)f(\theta) - \frac{\partial U_l}{\partial \pi_l}\beta'(x)f(\theta)]d\theta =$$

$$-\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}g(\theta)f(\theta)d\theta - \beta'(x)\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}f(\theta)d\theta = 0.$$

Hence:

$$\Longrightarrow -\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}g(\theta)f(\theta)d\theta = \beta'(x)\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}f(\theta)d\theta$$

$$\Longrightarrow \frac{\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}f(\theta)d\theta} = -\beta'(x)\ \ (A71).$$

$\frac{\partial \mathrm{E}U_l}{\partial \mathrm{H}} = \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l}\left[r(a\frac{\partial q}{\partial H} - 2bq\frac{\partial q}{\partial H})g(\theta) - v - \frac{\partial c_l}{\partial H} - \beta'(x)\frac{\partial x}{\partial H}\right] f(\theta)d\theta = \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l}\Big[r(a -$

$2bq)\frac{\partial q}{\partial H}g(\theta) - v - \frac{\partial c_l}{\partial H} - \beta'(x)(1-r)(a-2bq)\frac{\partial q}{\partial H}\Big] f(\theta)d\theta = r(a -$

$2bq)\frac{\partial q}{\partial H}\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta)f(\theta)d\theta - \left[v + \frac{\partial c_l}{\partial H} + \beta'(x)(1-r)(a-2bq)\frac{\partial q}{\partial H}\right]\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta)d\theta = 0.$

Thus:

$$\Longrightarrow r(a-2bq)\frac{\partial q}{\partial H}\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta)f(\theta)d\theta$$

$$= \left[v + \frac{\partial c_l}{\partial H} + \beta'(x)(1-r)(a-2bq)\frac{\partial q}{\partial H}\right]\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta)d\theta$$

$$\Longrightarrow \frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta)d\theta} = \frac{v + \frac{\partial c_l}{\partial H} + \beta'(x)(1-r)(a-2bq)\frac{\partial q}{\partial H}}{r(a-2bq)\frac{\partial q}{\partial H}} \quad (A72).$$

Therefore, plug (A71) in (A72),

$$-\beta'(x) = \frac{v + \frac{\partial c_l}{\partial H} + \beta\prime(x)(1-r)(a-2bq)\frac{\partial q}{\partial H}}{r(a-2bq)\frac{\partial q}{\partial H}}.$$

Hence:

$$\Longrightarrow \beta'(x) = -\frac{v + \frac{\partial c_l}{\partial H}}{(a-2bq)\frac{\partial q}{\partial H}}$$

$$\Longrightarrow \frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta)d\theta} - \frac{v + \frac{\partial c_l}{\partial H}}{(a-2bq)\frac{\partial q}{\partial H}} = \frac{W + \frac{\partial c_w}{\partial L}}{(1-r)(a-2bq)\frac{\partial q}{\partial L}}$$

$\Longrightarrow r^M = 1 - \frac{W + \frac{\partial c_w}{\partial L}}{\left[\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta)d\theta} - \frac{v + \frac{\partial c_l}{\partial H}}{(a-2bq)\frac{\partial q}{\partial H}}\right](a-2bq)\frac{\partial q}{\partial L}} = 1 - \frac{W + \frac{\partial c_w}{\partial L}}{\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta)d\theta}(a-2bq)\frac{\partial q}{\partial L} - \frac{(v + \frac{\partial c_l}{\partial H})\frac{\partial q}{\partial L}}{\frac{\partial q}{\partial H}}}$, which is

stated as equation (20) in the text.

Assume $\bar{P} = P = a - bq$, so the landlord's optimal share rate is:

$$r^{MPC}(\bar{P}) = 1 - \frac{W+\frac{\partial c_W}{\partial L}}{\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W}{\partial \pi_W} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W}{\partial \pi_W} f(\theta) d\theta} \bar{P} \frac{\partial q}{\partial L} + \frac{(v+\frac{\partial c_I}{\partial H})\frac{\partial q}{\partial L}}{\frac{\partial q}{\partial H}}} = 1 - \frac{W+\frac{\partial c_W}{\partial L}}{\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W}{\partial \pi_W} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W}{\partial \pi_W} f(\theta) d\theta} (a-bq) \frac{\partial q}{\partial L} + \frac{(v+\frac{\partial c_I}{\partial H})\frac{\partial q}{\partial L}}{\frac{\partial q}{\partial H}}}.$$

Again, let $P^M = a - 2bq$, then the landlord's optimal share rate becomes:

$r^M(P^M) = 1 - \frac{W+\frac{\partial c_W}{\partial L}}{\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W}{\partial \pi_W} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W}{\partial \pi_W} f(\theta) d\theta} (a-2bq) \frac{\partial q}{\partial L} - \frac{(v+\frac{\partial c_I}{\partial H})\frac{\partial q}{\partial L}}{\frac{\partial q}{\partial H}}}$, which is stated as equation (21) in the text.

Define $\bar{P} - P^M = a - bq - (a - 2bq) = bq > 0$ and $r'(P) > 0$.

Therefore:

$$r^{MPC}(\bar{P}) > r^M(P^M).$$

N. PROOF OF LEMMA 7

First, the tenant problem is given by:

$$\underset{\{L_i\}}{\text{Max}}\, \text{E}U_w^i\{\pi_w^i[q_i\left(L_i,\frac{H}{2}\right),q_j^*(L_j^*,\frac{H}{2})]\} = \text{E}U_w^i\{(1-r)[a-b\left(q_i\left(L_i,\frac{H}{2}\right)+$$

$$q_j^*\left(L_j^*,\frac{H}{2}\right)\right)]q_i\left(L_i,\frac{H}{2}\right)g(\theta)-WL_i-c_w^i(s,L_i)+\beta(x)\} = \int_{\underline{\theta}}^{\overline{\theta}} U_w^i\{(1-r)[aq_i\left(L_i,\frac{H}{2}\right)-$$

$$bq_i\left(L_i,\frac{H}{2}\right)^2-bq_i\left(L_i,\frac{H}{2}\right)q_j^*\left(L_j^*,\frac{H}{2}\right)]g(\theta)-WL_i-c_w^i(s,L_i)+\beta(x)\}f(\theta)d\theta.$$

The FOC w.r.t. L_i:

$$\frac{\partial \text{E}U_w^i}{\partial L_i} = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w^i}{\partial \pi_w^i}\left[(1-r)(a-2bq_i-bq_j^*)\frac{\partial q_i}{\partial L_i}g(\theta)-W-\frac{\partial c_w^i}{\partial L_i}+\beta'(x)\frac{\partial x}{\partial L_i}\right]f(\theta)d\theta =$$

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w^i}{\partial \pi_w^i}\left[(1-r)(a-2bq_i-bq_j^*)\frac{\partial q_i}{\partial L_i}g(\theta)-W-\frac{\partial c_w^i}{\partial L_i}+\beta'(x)(1-r)(a-2bq_i-\right.$$

$$\left.bq_j^*)\frac{\partial q_i}{\partial L_i}\right]f(\theta)d\theta = (1-r)(a-2bq_i-bq_j^*)\frac{\partial q_i}{\partial L_i}\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w^i}{\partial \pi_w^i}g(\theta)f(\theta)d\theta-[\left(W+\frac{\partial c_w^i}{\partial L_i}\right)-$$

$$\beta'(x)(1-r)(a-2bq_i-bq_j^*)\frac{\partial q_i}{\partial L_i}]\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w^i}{\partial \pi_w^i}f(\theta)d\theta = 0.$$

Hence:

$$\Longrightarrow (1-r)(a-2bq_i-bq_j^*)\frac{\partial q_i}{\partial L_i}\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w^i}{\partial \pi_w^i}g(\theta)f(\theta)d\theta$$

$$= [\left(W+\frac{\partial c_w^i}{\partial L_i}\right)-\beta'(x)(1-r)(a-2bq_i-bq_j^*)\frac{\partial q_i}{\partial L_i}]\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_w^i}{\partial \pi_w^i}f(\theta)d\theta$$

$$\Longrightarrow \frac{\int_{\underline{\theta}}^{\overline{\theta}}\frac{\partial U_w^i}{\partial \pi_w^i}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\overline{\theta}}\frac{\partial U_w^i}{\partial \pi_w^i}f(\theta)d\theta} = \frac{\left(W+\frac{\partial c_w^i}{\partial L_i}\right)}{(1-r)(a-2bq_i-bq_j^*)\frac{\partial q_i}{\partial L_i}}-\beta'(x).$$

Thus, if the labor input pair $\left(L_i^*,L_j^*\right)$ is to be a Nash Equilibrium, the tenants' agri-food production must satisfy $q_i^*\left(L_i^*,\frac{H}{2}\right) = q_j^*\left(L_j^*,\frac{H}{2}\right)$. Therefore,

$$\frac{\int_{\underline{\theta}}^{\overline{\theta}}\frac{\partial U_w^i}{\partial \pi_w^i}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\overline{\theta}}\frac{\partial U_w^i}{\partial \pi_w^i}f(\theta)d\theta} = \frac{\left(W+\frac{\partial c_w^i}{\partial L_i}\right)}{(1-r)(a-3bq_i^*)\frac{\partial q_i}{\partial L_i}}-\beta'(x)\ \ (A73).$$

Next, the landlord's problem is:

$$\underset{\{H,x\}}{\text{Max}}\ \mathrm{E}U_l[\pi_l(\theta)] = \mathrm{E}U_l\{2[(a - b\left(q_i^*\left(L_i^*, \frac{H}{2}\right) + q_j^*\left(L_j^*, \frac{H}{2}\right)\right))q_i^*\left(L_i^*, \frac{H}{2}\right) - x]g(\theta) - vH -$$

$$c_l(2,H) - 2\beta(x)\} = \int_{\underline{\theta}}^{\overline{\theta}} U_l\{2[aq_i^*\left(L_i^*, \frac{H}{2}\right) - 2bq_i^*\left(L_i^*, \frac{H}{2}\right)^2 - x]g(\theta) - vH - c_l(2,H) -$$

$$2\beta(x)\}f(\theta)d\theta.$$

The FOC w.r.t. x:

$$\frac{\partial \mathrm{E}U_l}{\partial \mathrm{x}} = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}[-2g(\theta) - 2\beta'(x)]f(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} [\frac{\partial U_l}{\partial \pi_l}(-2)g(\theta)f(\theta) - \frac{\partial U_l}{\partial \pi_l}2\beta'(x)f(\theta)]d\theta =$$

$$-2\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta)f(\theta)d\theta - 2\beta'(x)\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta)d\theta = 0.$$

Thus:

$$-2\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta)f(\theta)d\theta = 2\beta'^{(x)\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta)d\theta}$$

$$\Longrightarrow \frac{\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta)d\theta} = -\beta'(x)\ \ (A74).$$

The FOC w.r.t. H:

$$\frac{\partial \mathrm{E}U_l}{\partial \mathrm{H}} = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}\left[2r(a\frac{\partial q_i^*}{\partial h}\frac{1}{2} - 4bq_i^*\frac{\partial q_i^*}{\partial h}\frac{1}{2})g(\theta) - v - \frac{\partial c_l}{\partial H} - 2\beta'(x)\frac{\partial x}{\partial H}\right]f(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l}\left[2r(a\frac{\partial q_i^*}{\partial h}\frac{1}{2} -\right.$$

$$\left. 4bq_i^*\frac{\partial q_i^*}{\partial h}\frac{1}{2})g(\theta) - v - \frac{\partial c_l}{\partial H} - 2\beta'(x)\frac{(1-r)}{2}(a - 4bq_i^*)\frac{\partial q_i^*}{\partial h}\right]f(\theta)d\theta = r(a -$$

$$4bq_i^*)\frac{\partial q_i^*}{\partial h}\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta)f(\theta)d\theta - \left[v + \frac{\partial c_l}{\partial H} + 2\beta'(x)\frac{(1-r)}{2}(a - 4bq_i^*)\frac{\partial q_i^*}{\partial h}\right]\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta)d\theta = 0.$$

Hence:

$$r(a - 4bq_i^*)\frac{\partial q_i^*}{\partial h}\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta)f(\theta)d\theta$$

$$= \left[v + \frac{\partial c_l}{\partial H} + \beta'(x)(1-r)(a - 4bq_i^*)\frac{\partial q_i^*}{\partial h}\right]\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta)d\theta$$

$$\Longrightarrow \frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta) d\theta} = \frac{v + \frac{\partial c_l}{\partial H} + \beta\prime(x)(1-r)(a-4bq_i^*)\frac{\partial q_i^*}{\partial h}}{r(a-4bq_i^*)\frac{\partial q_i^*}{\partial h}} \text{ (A75).}$$

Therefore, plug (A74) in (A75),

$$-\beta'(x) = \frac{v + \frac{\partial c_l}{\partial H} + \beta\prime(x)(1-r)(a-4bq_i^*)\frac{\partial q_i^*}{\partial h}}{r(a-4bq_i^*)\frac{\partial q_i^*}{\partial h}}.$$

Thus:

$$\beta'(x) = -\frac{v + \frac{\partial c_l}{\partial H}}{(a-4bq_i^*)\frac{\partial q_i^*}{\partial h}} \quad (A76).$$

Substitute in (A73):

$$\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta} - \frac{v + \frac{\partial c_l}{\partial H}}{(a - 4bq_i^*)\frac{\partial q_i^*}{\partial h}} = \frac{\left(W + \frac{\partial c_w^i}{\partial L_i}\right)}{(1-r)(a-3bq_i^*)\frac{\partial q_i}{\partial L_i}}$$

Hence, the landlord's optimal share rate in a duopoly market for agri-food is:

$$r^{CGCI} = 1 - \frac{W + \frac{\partial c_w^i}{\partial L_i}}{[\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta} - \frac{v + \frac{\partial c_l}{\partial H}}{(a - 4bq_i^*)\frac{\partial q_i^*}{\partial h}}](a - 3bq_i^*)\frac{\partial q_i}{\partial L_i}}$$

$$= 1 - \frac{W + \frac{\partial c_w^i}{\partial L_i}}{\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta}(a - 3bq_i^*)\frac{\partial q_i}{\partial L_i} - \frac{(v + \frac{\partial c_l}{\partial H})(a - 3bq_i^*)\frac{\partial q_i}{\partial L_i}}{(a - 4bq_i^*)\frac{\partial q_i^*}{\partial h}}}$$

$$= 1 - \frac{W + \frac{\partial c_w^i}{\partial L_i}}{\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta}(a - 3bq_i^*)\frac{\partial q_i}{\partial L_i} - \frac{\left(v + \frac{\partial c_l}{\partial H}\right)\frac{\partial q_i}{\partial L_i}[(a - 4bq_i^*) + bq_i^*]}{\frac{\partial q_i^*}{\partial h}(a - 4bq_i^*)}}$$

$$= 1 - \frac{W + \frac{\partial c_w^i}{\partial L_i}}{\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta} (a - 3bq_i^*) \frac{\partial q_i}{\partial L_i} - \frac{\left(v + \frac{\partial c_l}{\partial H}\right) \frac{\partial q_i}{\partial L_i}}{\frac{\partial q_i^*}{\partial h}} \left(1 + \frac{bq_i^*}{a - 4bq_i^*}\right)}$$

$= 1 - \frac{W + \frac{\partial c_w^i}{\partial L_i}}{\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta} (a - 3bq_i^*) \frac{\partial q_i}{\partial L_i} - \frac{\left(v + \frac{\partial c_l}{\partial H}\right) \frac{\partial q_i}{\partial L_i}}{\frac{\partial q_i^*}{\partial h}} - \frac{\left(v + \frac{\partial c_l}{\partial H}\right) \frac{\partial q_i}{\partial L_i}}{\frac{\partial q_i^*}{\partial h}} \left(\frac{bq_i^*}{a - 4bq_i^*}\right)}$, which is stated as equation (22) in the text.

Define $r^{CGCI1}(a - 3bq_i^*) = 1 - \frac{W + \frac{\partial c_w^i}{\partial L_i}}{\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta} (a - 3bq_i^*) \frac{\partial q_i}{\partial L_i} - \frac{\left(v + \frac{\partial c_l}{\partial H}\right) \frac{\partial q_i}{\partial L_i}}{\frac{\partial q_i^*}{\partial h}}}$, I get: $r^{CGCI} < r^{CGCI1}$.

On the other hand, according to $h = \frac{H}{m}$, for monopoly tenant, m=1. In this case, h=H. In addition, assume $P^M(q) = a - 2bq > a - 3bq$ and $r'(P) > 0$.

Therefore, the landlord's share rate in a monopoly market for agri-food is:

$$\mathrm{r^M(P^M)} = 1 - \frac{W + \frac{\partial c_w}{\partial L}}{\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta} (a - 2bq) \frac{\partial q}{\partial L} - \frac{\left(v + \frac{\partial c_l}{\partial H}\right) \frac{\partial q}{\partial L}}{\frac{\partial q}{\partial h}}}.$$

Thus:

$$\mathrm{r^M(P^M)} > 1 - \frac{W + \frac{\partial c_w^i}{\partial L_i}}{\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_w}{\partial \pi_w} f(\theta) d\theta} (a - 3bq_i^*) \frac{\partial q_i}{\partial L_i} - \frac{\left(v + \frac{\partial c_l}{\partial H}\right) \frac{\partial q_i}{\partial L_i}}{\frac{\partial q_i^*}{\partial h}}} = r^{CGCI1} > r^{CGCI}.$$

O. PROOF OF PROPOSITION 7

First, the tenant problem is:

$$\underset{\{s_i\}}{\text{Max}}\text{E}U_w^i\left\{\pi_w^i\left[q_i\left(s_iL,\frac{H}{2}\right),q_j^*\left(s_j^*L,\frac{H}{2}\right)\right]\right\}$$

$$=\text{E}U_w^i\left\{(1-r)\left[a-b\left(q_i\left(s_iL,\frac{H}{2}\right)+q_j^*\left(s_j^*L,\frac{H}{2}\right)\right)\right]q_i\left(s_iL,\frac{H}{2}\right)g(\theta)-WL\right.$$

$$\left.-c_w^i(s_i,L)+\beta(x)\right\}$$

$$=\text{E}U_w^i\left\{(1-r)\left[aq_i\left(s_iL,\frac{H}{2}\right)-bq_i\left(s_iL,\frac{H}{2}\right)^2-bq_i\left(s_iL,\frac{H}{2}\right)q_j^*\left(s_j^*L,\frac{H}{2}\right)\right]g(\theta)\right.$$

$$\left.-WL-c_w^i(s_i,L)+\beta(x)\right\}$$

$$=\int_{\underline{\theta}}^{\overline{\theta}}U_w^i\left\{(1-r)\left[aq_i\left(s_iL,\frac{H}{2}\right)-bq_i\left(s_iL,\frac{H}{2}\right)^2\right.\right.$$

$$\left.\left.-bq_i\left(s_iL,\frac{H}{2}\right)q_j^*\left(s_j^*L,\frac{H}{2}\right)\right]g(\theta)-WL-c_w^i(s_i,L)+\beta(x)\right\}f(\theta)d\theta$$

The FOC w.r.t s_i is:

$\frac{\partial \text{E}U_w^i}{\partial s_i}=\int_{\underline{\theta}}^{\overline{\theta}}\frac{\partial U_w^i}{\partial \pi_w^i}\left[(1-r)(a-2bq_i-bq_j^*)\frac{\partial q_i}{\partial l_i}Lg(\theta)-\frac{\partial c_w^i}{\partial s_i}+\beta'(x)\frac{\partial x}{\partial s_i}\right]f(\theta)d\theta=\int_{\underline{\theta}}^{\overline{\theta}}\frac{\partial U_w^i}{\partial \pi_w^i}\left[(1-\right.$

$\left.r)(a-2bq_i-bq_j^*)\frac{\partial q_i}{\partial l_i}Lg(\theta)-W-\frac{\partial c_w^i}{\partial s_i}+\beta'(x)(1-r)(a-2bq_i-bq_j^*)\frac{\partial q_i}{\partial l_i}L\right]f(\theta)d\theta=$

$(1-r)(a-2bq_i-bq_j^*)\frac{\partial q_i}{\partial l_i}L\int_{\underline{\theta}}^{\overline{\theta}}\frac{\partial U_w^i}{\partial \pi_w^i}g(\theta)f(\theta)d\theta-[\left(W+\frac{\partial c_w^i}{\partial s_i}\right)-\beta'(x)(1-r)(a-2bq_i-$

$bq_j^*)\frac{\partial q_i}{\partial l_i}L]\int_{\underline{\theta}}^{\overline{\theta}}\frac{\partial U_w^i}{\partial \pi_w^i}f(\theta)d\theta=0.$

Thus:

$$(1-r)(a-2bq_i-bq_j^*)\frac{\partial q_i}{\partial l_i}L\int_{\underline{\theta}}^{\overline{\theta}}\frac{\partial U_w^i}{\partial \pi_w^i}g(\theta)f(\theta)d\theta$$

$$=[\left(W+\frac{\partial c_w^i}{\partial s_i}\right)-\beta'(x)(1-r)(a-2bq_i-bq_j^*)\frac{\partial q_i}{\partial l_i}L]\int_{\underline{\theta}}^{\overline{\theta}}\frac{\partial U_w^i}{\partial \pi_w^i}f(\theta)d\theta$$

$$\Longrightarrow \frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W^i}{\partial \pi_W^i} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W^i}{\partial \pi_W^i} f(\theta) d\theta} = \frac{\left(W + \frac{\partial c_W^i}{\partial s_i}\right)}{(1-r)\left(a - 2bq_i - bq_j^*\right)\frac{\partial q_i}{\partial l_i} L} - \beta'(x).$$

Thus, if the specific investment pair $\left(s_i^*, s_j^*\right)$ is to be a Nash Equilibrium, the tenants' agri-food production must satisfy $q_i^*\left(s_i^* L, \frac{H}{2}\right) = q_j^*\left(s_j^* L, \frac{H}{2}\right)$. Therefore,

$$\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W^i}{\partial \pi_W^i} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_W^i}{\partial \pi_W^i} f(\theta) d\theta} = \frac{\left(W + \frac{\partial c_W^i}{\partial s_i}\right)}{(1-r)(a - 3bq_i^*)\frac{\partial q_i}{\partial l_i} L} - \beta'(x) \quad (A77).$$

Next, the landlord's problem is:

$$\underset{\{H,x\}}{\text{Max}}\, \mathrm{E}U_l[\pi_l(\theta)] = \mathrm{E}U_l\{2[(a - b\left(q_i^*\left(s_i^* L, \frac{H}{2}\right) + q_j^*\left(s_j^* L, \frac{H}{2}\right)\right)q_i^*\left(s_i^* L, \frac{H}{2}\right) - x]g(\theta) - vH -$$

$$c_l(2, H) - 2\beta(x)\} = \int_{\underline{\theta}}^{\bar{\theta}} U_l\{2[aq_i^*\left(s_i^* L, \frac{H}{2}\right) - 2bq_i^*\left(s_i^* L, \frac{H}{2}\right)^2 - x]g(\theta) - vH - c_l(2, H) -$$

$$2\beta(x)\} f(\theta) d\theta.$$

The FOC w.r.t x:

$$\frac{\partial \mathrm{E}U_l}{\partial \mathrm{x}} = \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l}\left[-2g(\theta) - 2\beta'^{(x)}\right] f(\theta) d\theta = -2\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta) f(\theta) d\theta - 2\beta'(x)\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta) d\theta = 0.$$

Hence:

$$\frac{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta) f(\theta) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta) d\theta} = -\beta'(x) > 0 \ (A78).$$

The FOC w.r.t H:

$$\frac{\partial \mathrm{E}U_l}{\partial \mathrm{H}} = \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l}\left[2r(a\frac{\partial q_i^*}{\partial h}\frac{1}{2} - 4bq_i^*\frac{\partial q_i^*}{\partial h}\frac{1}{2})g(\theta) - v - \frac{\partial c_l}{\partial H} - 2\beta'(x)\frac{\partial x}{\partial H}\right] f(\theta) d\theta = \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l}\left[2r(a\frac{\partial q_i^*}{\partial h}\frac{1}{2} -\right.$$

$$\left. 4bq_i^*\frac{\partial q_i^*}{\partial h}\frac{1}{2})g(\theta) - v - \frac{\partial c_l}{\partial H} - 2\beta'(x)\frac{(1-r)}{2}(a - 4bq_i^*)\frac{\partial q_i^*}{\partial h}\right] f(\theta) d\theta = r(a -$$

$$4bq_i^*)\frac{\partial q_i^*}{\partial h}\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} g(\theta) f(\theta) d\theta - \left[v + \frac{\partial c_l}{\partial H} + 2\beta'(x)\frac{(1-r)}{2}(a - 4bq_i^*)\frac{\partial q_i^*}{\partial h}\right]\int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial U_l}{\partial \pi_l} f(\theta) d\theta = 0.$$

Thus:

$$r(a-4bq_i^*)\frac{\partial q_i^*}{\partial h}\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_l}{\partial \pi_l}g(\theta)f(\theta)d\theta$$

$$=\left[v+\frac{\partial c_l}{\partial H}+\beta'(x)(1-r)(a-4bq_i^*)\frac{\partial q_i^*}{\partial h}\right]\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_l}{\partial \pi_l}f(\theta)d\theta$$

$$\Longrightarrow \frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_l}{\partial \pi_l}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_l}{\partial \pi_l}f(\theta)d\theta}=\frac{v+\frac{\partial c_l}{\partial H}+\beta'^{(x)}(1-r)(a-4bq_i^*)\frac{\partial q_i^*}{\partial h}}{r(a-4bq_i^*)\frac{\partial q_i^*}{\partial h}}\ (A79).$$

Therefore, plug (A78) in (A79),

$$-\beta'(x)=\frac{v+\frac{\partial c_l}{\partial H}+\beta'(x)(1-r)(a-4bq_i^*)\frac{\partial q_i^*}{\partial h}}{r(a-4bq_i^*)\frac{\partial q_i^*}{\partial h}}$$

$$\Longrightarrow \beta'(x)=-\frac{v+\frac{\partial c_l}{\partial H}}{(a-4bq_i^*)\frac{\partial q_i^*}{\partial h}}<0 \text{ and then } a-4bq_i^*>0.$$

Substitute in (A77):

$$\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}-\frac{v+\frac{\partial c_l}{\partial H}}{(a-4bq_i^*)\frac{\partial q_i^*}{\partial h}}=\frac{W+\frac{\partial c_w^i}{\partial s_i}}{(1-r)(a-3bq_i^*)\frac{\partial q_i}{\partial l_i}L}.$$

So, the landlord's optimal share rate is:

$$r^{SS}=1-\frac{W+\frac{\partial c_w^i}{\partial s_i}}{\left[\frac{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}g(\theta)f(\theta)d\theta}{\int_{\underline{\theta}}^{\bar{\theta}}\frac{\partial U_w}{\partial \pi_w}f(\theta)d\theta}-\frac{v+\frac{\partial c_l}{\partial H}}{(a-4bq_i^*)\frac{\partial q_i^*}{\partial h}}\right](a-3bq_i^*)\frac{\partial q_i}{\partial l_i}L},$$ which is stated as equation (23) in the text.

P. PROOF OF LEMMA 8

The social planner problem is:

$$U_3=\underset{\{\bar{P}_t,\bar{r}_t,L_{pt},\lambda\}}{\text{Max}}\int_0^{\infty}e^{-\rho t}\{\int_{\underline{\theta_t}}^{\overline{\theta_t}}U_s\{m_t[\bar{P}_t\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma}\left(sL_{pt}\right)^{\sigma}g(\theta_t)-W_tL_{pt}-\alpha sL_{pt}]-v\lambda Z-$$

$$km_t-\frac{\tilde{k}}{2}(\lambda Z)^2-ln\bar{P}_t-\int_{r^{nc}}^{\bar{r}_t}\eta r_t^{-\eta}dr_t\}f(\theta_t)d\theta_t\,\text{dt},$$

s.t.

$$W_t + \alpha s_t = \sigma(1-\bar{r}_t)\bar{P}_t g(\theta_t)(\frac{\lambda Z}{m_t})^{1-\sigma} s^{\sigma} L_{pt}^{\sigma-1},$$

$$\dot{m}_t = \frac{1}{\chi}(L_t - L_{pt}),$$

$$m_t = m_0 e^t.$$

Next, define Hamiltonian with state variable m_t, control variables $\{\bar{P}_t, \bar{r}_t, L_{pt}, \lambda\}$ and co-state variable μ_t:

$$\mathcal{H}_3(\bar{P}_t, \bar{r}_t, L_{pt}, m_t, \lambda, \mu_t) = \int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s\{m_t[\bar{P}_t \left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} \left(sL_{pt}\right)^{\sigma} g(\theta_t) - W_t L_{pt} - \alpha s L_{pt}] - v\lambda Z -$$

$$km_t - \frac{\tilde{k}}{2}(\lambda Z)^2 - ln\bar{P}_t - \int_{r^{nc}}^{\bar{r}_t} \eta r_t^{-\eta} dr_t\} f(\theta_t) d\theta_t + \frac{\mu_t}{\chi}(L_t - L_{pt}) =$$

$$\int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s\{m_t[\bar{P}_t \left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} \left(sL_{pt}\right)^{\sigma} g(\theta_t) - \sigma(1-\bar{r}_t)\bar{P}_t g(\theta_t)(\frac{\lambda Z}{m_t})^{1-\sigma} s^{\sigma} L_{pt}^{\sigma}] - v\lambda Z - km_t - \frac{\tilde{k}}{2}(\lambda Z)^2 -$$

$$ln\bar{P}_t - \int_{r^{nc}}^{\bar{r}_t} \eta r_t^{-\eta} dr_t\} f(\theta_t) d\theta_t + \frac{\mu_t}{\chi}(L_t - L_{pt}) = \int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s\{m_t[1 - \sigma(1 -$$

$$\bar{r}_t)]\bar{P}_t \left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} (sL_{pt})^{\sigma} g(\theta_t) - v\lambda Z - km_t - \frac{\tilde{k}}{2}(\lambda Z)^2 - ln\bar{P}_t - \eta \frac{\bar{r}_t^{1-\eta} - (r^{nc})^{1-\eta}}{1-\eta}\} f(\theta_t) d\theta_t +$$

$$\frac{\mu_t}{\chi}(L_t - L_{pt}).$$

The first order conditions are:

$$\begin{cases} \dfrac{\partial \mathcal{H}_3}{\partial L_{pt}} = 0 \\ \dfrac{\partial \mathcal{H}_3}{\partial \bar{P}_t} = 0 \\ \dfrac{\partial \mathcal{H}_3}{\partial \bar{r}_t} = 0 \\ \dfrac{\partial \mathcal{H}_3}{\partial \lambda} = 0 \\ \rho = \dfrac{\partial \mathcal{H}_3 / \partial m_t}{\mu_t} + \dfrac{\dot{\mu}_t}{\mu_t} \end{cases}$$

Start with $\frac{\partial \mathcal{H}_3}{\partial L_{pt}} = 0$, which implies

$$\int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s{}' \sigma m_t (1-\sigma+\sigma\bar{r}_t)\bar{P}_t \left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma-1} s^{\sigma} g(\theta_t) f(\theta_t) d\theta_t = \frac{\mu_t}{\chi},$$

and therefore

$$L_{pt} = \left(\frac{\mu_t}{\int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s{}' \chi \sigma m_t (1-\sigma+\sigma\bar{r}_t)\bar{P}_t \left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} s^{\sigma} g(\theta_t) f(\theta_t) d\theta_t}\right)^{\frac{1}{\sigma-1}} \quad (A80).$$

Next, consider $\frac{\partial \mathcal{H}_3}{\partial \bar{P}_t} = 0$:

$$\int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s{}' m_t (1-\sigma+\sigma\bar{r}_t) \left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} s^{\sigma} g(\theta_t) f(\theta_t) d\theta_t = \frac{1}{\bar{P}_t}.$$

Substituting gives

$$\bar{P}_t = \frac{1}{\int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s{}' m_t (1-\sigma+\sigma\bar{r}_t) \left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} s^{\sigma} g(\theta_t) f(\theta_t) d\theta_t} \quad (A81).$$

The next FOC is similar, $\frac{\partial \mathcal{H}_3}{\partial \bar{r}_t} = 0$:

$$\int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s{}' m_t \sigma \bar{P}_t \left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} s^{\sigma} g(\theta_t) f(\theta_t) d\theta_t = \eta \bar{r}_t{}^{-\eta}.$$

Substitute in (A81):

$$\frac{\sigma}{1-\sigma+\sigma\bar{r}_t} = \eta \bar{r}_t{}^{-\eta}$$

$$\Rightarrow \frac{1}{\bar{r}_t + \frac{1-\sigma}{\sigma}} = \frac{\eta}{\bar{r}_t{}^{\eta}}$$

$$\Rightarrow \bar{r}_t{}^{\eta} = \eta \bar{r}_t + \frac{\eta(1-\sigma)}{\sigma}$$

$$\Rightarrow \bar{r}_t{}^{\eta} - \eta \bar{r}_t = \frac{\eta(1-\sigma)}{\sigma}.$$

While an explicit analytical solution for $\bar{r}_t$ in terms of η and σ is not easily attainable due to the complexity of the equation, if $\eta = 0.5$, I can use this specific value to simplify the expression for $\bar{r}_t$ in terms of σ. Let's proceed with substituting $\eta = 0.5$ into the equation:

$$\bar{r}_t^{\,0.5} = 0.5\bar{r}_t + \tfrac{1-\sigma}{2\sigma}.$$

Square both sides and expand the right-hand side, then rearrange the equation to form a quadratic equation:

$$\bar{r}_t = 0.25\bar{r}_t^{\,2} + \frac{1-\sigma}{2\sigma}\bar{r}_t + \left(\frac{1-\sigma}{2\sigma}\right)^2$$

$$\Rightarrow 0.25\bar{r}_t^{\,2} + \bar{r}_t\left(\tfrac{1-3\sigma}{2\sigma}\right) + \tfrac{(1-\sigma)^2}{4\sigma^2} = 0.$$

So, the solutions for $\bar{r}_t$ in terms of σ are:

$$\bar{r}_t^{\,****} = \frac{\frac{3\sigma-1}{2\sigma} \pm \sqrt{\left(\frac{1-3\sigma}{2\sigma}\right)^2 - 4\times 0.25\frac{(1-\sigma)^2}{4\sigma^2}}}{2\times 0.25} = \frac{3\sigma-1\pm 2\sqrt{(2\sigma-1)\sigma}}{\sigma} \quad \text{(A82)}.$$

The next FOC is similar, $\frac{\partial \mathcal{H}_3}{\partial \lambda} = 0$:

$$\int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s{}'\,(\sigma-1)m_t(1-\sigma+\sigma\bar{r}_t)\bar{P}_t\lambda^{-\sigma}\left(\frac{Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma}s^{\sigma}g(\theta_t)f(\theta_t)d\theta_t - vZ - \tilde{k}\lambda Z^2 = 0.$$

$$\Rightarrow \frac{\int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s{}'\,(\sigma-1)m_t(1-\sigma+\sigma\bar{r}_t)\bar{P}_t\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma}s^{\sigma}g(\theta_t)f(\theta_t)d\theta_t}{\lambda} - vZ - \lambda\tilde{k}Z^2 = 0$$

$$\Rightarrow \frac{1-\sigma}{\lambda} - vZ - \lambda\tilde{k}Z^2 = 0 \Rightarrow 1-\sigma-\lambda vZ - \lambda^2\tilde{k}Z^2 = 0$$

$$\Rightarrow (\tilde{k}Z^2)\lambda^2 + (vZ)\lambda + (\sigma-1) = 0.$$

Substituting gives

$$\lambda^{***} = \frac{-vZ+\sqrt{(vZ)^2+4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z^2} \quad (A83).$$

Finally, the last FOC is $\rho = \frac{\partial \mathcal{H}_2/\partial m_t}{\mu_t} + \frac{\dot{\mu_t}}{\mu_t}$. The derivative of the Hamiltonian $\frac{\partial \mathcal{H}_2}{\partial m_t}$ is:

$$\frac{\partial \mathcal{H}_2}{\partial m_t} = \int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s{}'\Bigg\{(1-\sigma+\sigma\bar{r}_t)\bar{P}_t\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} + (\sigma-1)(1-\sigma+\sigma\bar{r}_t)\bar{P}_t\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma}$$

$$- k\Bigg\} g(\theta_t)f(\theta_t)d\theta_t = \left(\frac{\sigma}{m_t} - k\right)\int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s{}'g(\theta_t)f(\theta_t)d\theta_t = U_s{}'\left(\frac{\sigma}{m_t} - k\right).$$

Substituting this into the FOC and rearranging to solve for μ_t gives

$$\dot{\mu_t} - \rho\mu_t = U_s'(k - \frac{\sigma}{m_t}).$$

Solving for μ_t^{****}:

$$\mu_t^{****} = \left[\mu_0 + \frac{U_s'\left(k - \frac{\sigma}{m_0 e^t}\right)}{\rho}\right] e^{\rho t} - \frac{U_s'\left(k - \frac{\sigma}{m_0 e^t}\right)}{\rho} \quad (A84).$$

(1) Solving the Optimal Allocation

The 5 FOCs are:

$$\begin{cases} L_{pt} = \left(\dfrac{\mu_t}{\int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s' \chi\sigma m_t[\bar{r}_t(1-\sigma)+\sigma]\bar{P}_t\left(\dfrac{\lambda Z}{m_t}\right)^{1-\sigma} s^\sigma g(\theta_t) f(\theta_t) d\theta_t}\right)^{\frac{1}{\sigma-1}} & (A80) \\ \bar{P}_t = \dfrac{1}{\int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s' m_t[\bar{r}_t(1-\sigma)+\sigma]\left(\dfrac{\lambda Z}{m_t}\right)^{1-\sigma} L_{pt}^\sigma s^\sigma g(\theta_t) f(\theta_t) d\theta_t} & (A81) \\ \bar{r}_t^{****} = \dfrac{3\sigma - 1 \pm 2\sqrt{(2\sigma-1)\sigma}}{\sigma} & (A82) \\ \lambda^{***} = \dfrac{-vZ + \sqrt{(vZ)^2 + 4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z^2} & (A83) \\ \mu_t^{****} = \left[\mu_0 + \dfrac{U_s'\left(k - \dfrac{\sigma}{m_0 e^t}\right)}{\rho}\right] e^{\rho t} - \dfrac{U_s'\left(k - \dfrac{\sigma}{m_0 e^t}\right)}{\rho} & (A84) \end{cases}$$

1) Solving for L_{pt} and $\bar{P}_t$(eqs. A80 and A81).

Combining (A80), (A81), and (A84) gives

$$L_{pt}^{****} = \frac{\sigma\chi}{\left[\mu_0 + \frac{U_s'\left(k - \frac{\sigma}{m_0 e^t}\right)}{\rho}\right] e^{\rho t} - \frac{U_s'\left(k - \frac{\sigma}{m_0 e^t}\right)}{\rho}} \quad (A85).$$

In addition, combining (A81), (A82), (A83), and (A85) gives

$$\bar{P}_t^{****} = \frac{\left[\frac{\sigma\chi}{\left[\mu_0+\frac{U_s'\left(k-\frac{\sigma}{m_0e^t}\right)}{\rho}\right]e^{\rho t}-\frac{U_s'\left(k-\frac{\sigma}{m_0e^t}\right)}{\rho}}\right]^{-\sigma}}{\int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s' m_t[\bar{r}_t(1-\sigma)+\sigma]\left(\frac{\lambda Z}{m_t}\right)^{1-\sigma} s^{\sigma} g(\theta_t) f(\theta_t) d\theta_t} =$$

$$\frac{\left[\frac{\sigma\chi}{\left[\mu_0+\frac{U_s'\left(k-\frac{\sigma}{m_0e^t}\right)}{\rho}\right]e^{\rho t}-\frac{U_s'\left(k-\frac{\sigma}{m_0e^t}\right)}{\rho}}\right]^{-\sigma}}{(m_0e^t)^{\sigma}\int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s'[\bar{r}_t(1-\sigma)+\sigma](Z\lambda^{***})^{1-\sigma}s^{\sigma}g(\theta_t)f(\theta_t)d\theta_t} =$$

$$\frac{\left[\frac{\sigma\chi}{\left[\mu_0+\frac{U_s'\left(k-\frac{\sigma}{m_0e^t}\right)}{\rho}\right]e^{\rho t}-\frac{U_s'\left(k-\frac{\sigma}{m_0e^t}\right)}{\rho}}\right]^{-\sigma}}{(m_0e^t)^{\sigma}\int_{\underline{\theta_t}}^{\overline{\theta_t}} U_s'[\frac{2-3\sigma\pm2\sqrt{2\sigma^2-3\sigma+1}}{1-\sigma}(1-\sigma)+\sigma][\frac{-vZ+\sqrt{(vZ)^2+4\tilde{k}Z^2(1-\sigma)}}{2\tilde{k}Z}]^{1-\sigma}s^{\sigma}g(\theta_t)f(\theta_t)d\theta_t} \quad (A86).$$

Q.PROOF OF PROPOSITION 9

Assume the boat-owner's utility function is $U_l[\pi_l(\theta)] = -\exp[-\epsilon_l\pi_l(\theta)]$, and the fisherman's utility function is $U_w[\pi_w(\theta)] = -\exp[-\epsilon_w\pi_w(\theta)]$, where $\epsilon_l > 0$ and $\epsilon_w > 0$. Let $g(\theta)$ follow the gamma probability law with its probability density function

$f[g(\theta)] = \frac{\Lambda^z}{\Gamma(z)}[g(\theta)]^{z-1}\exp[-\Lambda g(\theta)]$ with $g(\theta) \geq 0$, $z = 1,2,\ldots$, $\Lambda > 0$, and $\Gamma(z) = \int_0^\infty \theta^{z-1}e^{-\theta}d\theta$.

According to Sutinen (1975), assume that

$$\mathrm{E}U_l[\pi_l(\theta)] = \mathrm{E}\{-\exp[-\epsilon_l\pi_l(\theta)]\} = -\int_0^\infty e^{-\epsilon_l\pi_l(\theta)}f[g(\theta)]d[g(\theta)] =$$

$$-\int_0^\infty e^{-\epsilon_l\left[mrPq\left(L,\frac{H}{m}\right)g(\theta)-vH-km-\frac{\tilde{k}}{2}H^2-m\beta\right]}f[g(\theta)]d[g(\theta)] =$$

$$-\int_0^\infty e^{\left[-\epsilon_l mrPq\left(L,\frac{H}{m}\right)g(\theta)+\epsilon_l\left(vH+km+\frac{\tilde{k}}{2}H^2+m\beta\right)\right]}\frac{\Lambda^z}{\Gamma(z)}[g(\theta)]^{z-1}\exp[-\Lambda g(\theta)]\,d[g(\theta)] =$$

$$-\frac{\Lambda^z}{\Gamma(z)}\int_0^\infty e^{-g(\theta)\left[\epsilon_l mrPq\left(L,\frac{H}{m}\right)+\Lambda\right]}e^{\epsilon_l\left(vH+km+\frac{\tilde{k}}{2}H^2+m\beta\right)}[g(\theta)]^{z-1}d[g(\theta)] =$$

$$-\frac{\Lambda^z}{\Gamma(z)}e^{\epsilon_l\left(vH+km+\frac{\tilde{k}}{2}H^2+m\beta\right)}\int_0^\infty e^{-g(\theta)\left[\epsilon_l mrPq\left(L,\frac{H}{m}\right)+\Lambda\right]}[g(\theta)]^{z-1}d[g(\theta)] =$$

$$-\frac{\Lambda^z}{\Gamma(z)}e^{\epsilon_l\left(vH+km+\frac{\tilde{k}}{2}H^2+m\beta\right)}\frac{\Gamma(z)}{\left[\epsilon_l mrPq\left(L,\frac{H}{m}\right)+\Lambda\right]^z} = -\frac{\Lambda^z\exp\left[\epsilon_l\left(vH+km+\frac{\tilde{k}}{2}H^2+m\beta\right)\right]}{(\epsilon_l mrPq+\Lambda)^z}.$$

Similarly,

$$\mathrm{E}U_w[\pi_w(\theta)] = \mathrm{E}\{-\exp[-\epsilon_w\pi_w(\theta)]\} = -\frac{\Lambda^z\exp[\epsilon_w(WL+\alpha sL-\beta)]}{[\epsilon_w(1-r)Pq+\Lambda]^z}.$$

Hence, the boat-owner wishes to maximize

$$\underset{\{\beta,r\}}{\mathrm{Max}}\,\mathrm{E}U_l[\pi_l(\theta)] = \mathrm{E}\{-\exp[-\epsilon_l\pi_l(\theta)]\} = -\frac{\Lambda^z\exp\left[\epsilon_l\left(vH+km+\frac{\tilde{k}}{2}H^2+m\beta\right)\right]}{(\epsilon_l mrPq+\Lambda)^z}.$$

On the other hand, the fisherman's problem is:

$$\mathrm{E}U_w[\pi_w(\theta)] = \mathrm{E}\{-\exp[-\epsilon_w\pi_w(\theta)]\} = -\frac{\Lambda^z\exp[\epsilon_w(WL+\alpha sL-\beta)]}{[\epsilon_w(1-r)Pq+\Lambda]^z} = \overline{U}_w.$$

To solve the boat-owner's problem, write the Lagrangean:

$$\mathcal{L}5 = -\frac{\Lambda^z\exp\left[\epsilon_l\left(vH+km+\frac{\tilde{k}}{2}H^2+m\beta\right)\right]}{(\epsilon_l mrPq+\Lambda)^z} + \lambda\{\frac{\Lambda^z\exp[\epsilon_w(WL+\alpha sL-\beta)]}{[\epsilon_w(1-r)Pq+\Lambda]^z} - \overline{U}_w\}.$$

The first-order conditions are as below:

$\frac{\partial \mathcal{L}5}{\partial \beta} = -\wedge^{z}(\epsilon_l mrPq + \wedge)^{-z}(\epsilon_l m)\exp\left[\epsilon_l\left(vH + km + \frac{\tilde{k}}{2}H^2 + m\beta\right)\right] + \lambda\wedge^{z}[\epsilon_w(1-r)Pq + \wedge]^{-z}(-\epsilon_w)\exp[\epsilon_w(WL + \alpha sL - \beta)] = 0.$

$$\Longrightarrow \lambda\wedge^{z}[\epsilon_w(1-r)Pq + \wedge]^{-z}(-\epsilon_w)\exp[\epsilon_w(WL + \alpha sL - \beta)]$$

$$= \wedge^{z}(\epsilon_l mrPq + \wedge)^{-z}(\epsilon_l m)\exp\left[\epsilon_l\left(vH + km + \frac{\tilde{k}}{2}H^2 + m\beta\right)\right] \quad (A87).$$

and

$\frac{\partial \mathcal{L}5}{\partial r} = -\wedge^{z}(-z)(\epsilon_l mrPq + \wedge)^{-z-1}(\epsilon_l mPq)\exp\left[\epsilon_l\left(vH + km + \frac{\tilde{k}}{2}H^2 + m\beta\right)\right] +$

$\lambda\wedge^{z}(-z)[\epsilon_w(1-r)Pq + \wedge]^{-z-1}(-\epsilon_w Pq)\exp[\epsilon_w(WL + \alpha sL - \beta)] = 0.$

$$\Longrightarrow \lambda\wedge^{z}(-z)[\epsilon_w(1-r)Pq + \wedge]^{-z-1}(-\epsilon_w Pq)\exp[\epsilon_w(WL + \alpha sL - \beta)]$$

$$= \wedge^{z}(-z)(\epsilon_l mrPq + \wedge)^{-z-1}(\epsilon_l mPq)\exp\left[\epsilon_l\left(vH + km + \frac{\tilde{k}}{2}H^2 + m\beta\right)\right] \quad (A88).$$

Substitute in (A87),

$$\frac{(-\epsilon_w)[\epsilon_w(1-r)Pq + \wedge]}{(-z)(-\epsilon_w Pq)} = \frac{(\epsilon_l m)(\epsilon_l mrPq + \wedge)}{(-z)(\epsilon_l mPq)}$$

$$\Longrightarrow \epsilon_w(1-r)Pq + \wedge = \epsilon_l mrPq + \wedge$$

$$\Longrightarrow \epsilon_w(1-r) = \epsilon_l mr$$

$\Longrightarrow r_0 = \frac{\epsilon_w}{\epsilon_l m + \epsilon_w}$, which is stated as equation (27) in the text.

Compute the derivative $\frac{\partial r_0}{\partial m}$:

$$\frac{\partial r_0}{\partial m} = -\frac{\epsilon_w \epsilon_l}{(\epsilon_l m + \epsilon_w)^2} < 0.$$

Compute the derivative $\frac{\partial r_0}{\partial \epsilon_l}$:

$$\frac{\partial r_0}{\partial \epsilon_l} = -\frac{m\epsilon_w}{(\epsilon_l m + \epsilon_w)^2} < 0.$$

Compute the derivative $\frac{\partial r_0}{\partial \epsilon_w}$:

$$\frac{\partial r_0}{\partial \epsilon_w} = -\frac{m\epsilon_l}{(\epsilon_l m+\epsilon_w)^2} < 0.$$

R. PROOF OF LEMMA 9

The boat-owner's problem is:

$$\pi_l = mrPq\left(L, \frac{H}{m}\right) g(\theta) - vH - km - \frac{\tilde{k}}{2}H^2 - m\beta = 0.$$

$$\Longrightarrow mrPq\left(L, \frac{H}{m}\right) g(\theta) = vH + km + \frac{\tilde{k}}{2}H^2 + m\beta \ (A89).$$

The fisherman's problem is:

$$\pi_w = (1-r)Pq\left(L, \frac{H}{m}\right) g(\theta) - WL - \alpha sL + \beta = 0.$$

$$\Longrightarrow (1-r)Pq\left(L, \frac{H}{m}\right) g(\theta) = WL + \alpha sL - \beta \ \ (A90).$$

Substitute in (A89),

$$\frac{mr}{(1-r)} = \frac{vH + km + \frac{\tilde{k}}{2}H^2 + m\beta}{WL + \alpha sL - \beta}.$$

$$\Rightarrow mr(WL + \alpha sL - \beta) = (1-r)\left(vH + km + \frac{\tilde{k}}{2}H^2 + m\beta\right)$$

$$= \left(vH + km + \frac{\tilde{k}}{2}H^2 + m\beta\right) - r\left(vH + km + \frac{\tilde{k}}{2}H^2 + m\beta\right)$$

$$\Rightarrow r(mWL + m\alpha sL - m\beta) + r\left(vH + km + \frac{\tilde{k}}{2}H^2 + m\beta\right) = vH + km + \frac{\tilde{k}}{2}H^2 + m\beta$$

$$\Rightarrow r(mWL + m\alpha sL - m\beta + vH + km + \frac{\tilde{k}}{2}H^2 + m\beta) = vH + km + \frac{\tilde{k}}{2}H^2 + m\beta$$

$\Rightarrow r_1 = \frac{vH + km + \frac{\tilde{k}}{2}H^2 + m\beta}{mWL + m\alpha sL + vH + km + \frac{\tilde{k}}{2}H^2}$, which is stated as equation (28) in the text.

Compute the derivative $\frac{\partial r_1}{\partial s}$:

$$\frac{\partial r_1}{\partial s} = -\frac{\left(vH + km + \frac{\tilde{k}}{2}H^2 + m\beta\right)m\alpha L}{\left(mWL + m\alpha sL + vH + km + \frac{\tilde{k}}{2}H^2\right)^2} < 0.$$

Compute the derivative $\frac{\partial r_1}{\partial L}$:

$$\frac{\partial r_1}{\partial L} = -\frac{\left(vH + km + \frac{\tilde{k}}{2}H^2 + m\beta\right)m(W + \alpha s)}{\left(mWL + m\alpha sL + vH + km + \frac{\tilde{k}}{2}H^2\right)^2} < 0.$$

Compute the derivative $\frac{\partial r_1}{\partial \beta}$:

$$\frac{\partial r_1}{\partial \beta} = \frac{m}{mWL+m\alpha sL+vH+km+\frac{\tilde{k}}{2}H^2} > 0.$$

S. PROOF OF LEMMA 10

The social planner problem is:

$$U_4 = \underset{\{\bar{P}_t, \bar{r}_t, L_{pt}\}}{\text{Max}} \int_0^{\infty} e^{-\rho t} N \left\{ m_t \left[\bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} (sL_{pt})^{\sigma} - W_t L_{pt} - c_{wt}\left(s, L_{pt}\right) \right] - vH - km_t - \right.$$

$$\left. \frac{\tilde{k}}{2} H^2 - ln\bar{P}_t - \int_{r_0}^{\bar{r}_t} \eta r_t^{-\eta} dr_t \right\} \text{dt},$$

s.t.

$$W_t L_{pt} + c_{wt}\left(s, L_{pt}\right) = (1 - \bar{r}_t)\bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} (sL_{pt})^{\sigma},$$

$$\dot{m}_t = \frac{1}{\chi}(L_t - L_{pt}),$$

$$m_t = m_0 e^t,$$

$$\dot{g}(\theta_t) = F[g(\theta_t)] - N m_t g(\theta_t) = g(\theta_t) - [g(\theta_t)]^2 - N m_t g(\theta_t).$$

Next, define Hamiltonian with state variable m_t, control variables $\{\bar{P}_t, \bar{r}_t, L_{pt}\}$ and co-state variables μ_t and ν_t:

$$\mathcal{H}_4\left(\bar{P}_t, \bar{r}_t, L_{pt}, m_t, \mu_t, \nu_t\right) = N \left\{ m_t \left[\bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} (sL_{pt})^{\sigma} - W_t L_{pt} - c_{wt}\left(s, L_{pt}\right) \right] - vH - \right.$$

$$\left. km_t - \frac{\tilde{k}}{2} H^2 - ln\bar{P}_t - \int_{r_0}^{\bar{r}_t} \eta r_t^{-\eta} dr_t \right\} = N \left\{ m_t \left[\bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} (sL_{pt})^{\sigma} - (1 - \right.\right.$$

$$\left.\left. \bar{r}_t)\bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} (sL_{pt})^{\sigma} \right] - vH - km_t - \frac{\tilde{k}}{2} H^2 - ln\bar{P}_t - \int_{r_0}^{\bar{r}_t} \eta r_t^{-\eta} dr_t \right\} =$$

$$N m_t \bar{r}_t \bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} (sL_{pt})^{\sigma} - NvH - Nkm_t - N\frac{\tilde{k}}{2} H^2 - \text{N}ln\bar{P}_t - N\eta \frac{\bar{r}_t^{1-\eta} - r_0^{1-\eta}}{1-\eta} +$$

$$\frac{\mu_t}{\chi}\left(L_t - L_{pt}\right) + \nu_t\{(1 - N m_t)g(\theta_t) - [g(\theta_t)]^2\}.$$

The first order conditions are:

$$\begin{cases} \dfrac{\partial \mathcal{H}_4}{\partial L_{pt}} = 0 \\ \dfrac{\partial \mathcal{H}_4}{\partial \bar{P}_t} = 0 \\ \dfrac{\partial \mathcal{H}_4}{\partial \bar{r}_t} = 0 \\ \rho = \dfrac{\partial \mathcal{H}_4 / \partial g(\theta_t)}{\nu_t} + \dfrac{\dot{\nu}_t}{\nu_t} \\ \rho = \dfrac{\partial \mathcal{H}_4 / \partial m_t}{\mu_t} + \dfrac{\dot{\mu}_t}{\mu_t} \end{cases}$$

Start with $\frac{\partial \mathcal{H}_4}{\partial L_{pt}} = 0$, which implies

$$N\sigma m_t \bar{r}_t \bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} \left(L_{pt}\right)^{\sigma-1} s^{\sigma} = \frac{\mu_t}{\chi},$$

and therefore

$$L_{pt} = \left(\frac{\mu_t}{\chi N\sigma m_t \bar{r}_t \bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} s^{\sigma}}\right)^{\frac{1}{\sigma-1}} \text{(A91)}.$$

Next, consider $\frac{\partial \mathcal{H}_4}{\partial \bar{P}_t} = 0$:

$$m_t \bar{r}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} \left(sL_{pt}\right)^{\sigma} = \frac{1}{\bar{P}_t}.$$

Substituting gives

$$\bar{P}_t = \frac{1}{m_t \bar{r}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} \left(sL_{pt}\right)^{\sigma}} \ (A92).$$

The next FOC is similar, $\frac{\partial \mathcal{H}_4}{\partial \bar{r}_t} = 0$:

$$m_t \bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} \left(sL_{pt}\right)^{\sigma} = \eta \bar{r}_t^{\,-\eta}.$$

Substituting gives

$$\bar{r}_t^{\,*****} = \eta^{\frac{1}{\eta-1}} \ (A93) \ .$$

The next FOC is $\rho = \frac{\partial \mathcal{H}_1 / \partial g(\theta_t)}{\nu_t} + \frac{\dot{\nu}_t}{\nu_t}$. The derivative of the Hamiltonian $\frac{\partial \mathcal{H}_4}{\partial g(\theta_t)}$ is:

$$\frac{\partial \mathcal{H}_4}{\partial g(\theta_t)} = N m_t \bar{r}_t \bar{P}_t \left(\frac{H}{m_t}\right)^{1-\sigma} \left(sL_{pt}\right)^{\sigma} + \nu_t[1 - N m_t - 2g(\theta_t)]$$

$$= \frac{N}{g(\theta_t)} + \nu_t[1 - N m_t - 2g(\theta_t)].$$

Substituting this into the FOC and rearranging to solve for ν_t gives

$$\dot{\nu}_t - \rho\nu_t = \nu_t[N m_t - 1 + 2g(\theta_t)] - \frac{N}{g(\theta_t)}.$$

$$\Longrightarrow \dot{\nu}_t + \nu_t[1 - N m_t - 2g(\theta_t) - \rho] = -\frac{N}{g(\theta_t)}$$

Solving for ν_t^*:

$$\nu_t^* = \left[\nu_0 + \frac{\frac{N}{g(\theta_t)}}{1 - N m_t - 2g(\theta_t) - \rho}\right] e^{\rho t} - \frac{\frac{N}{g(\theta_t)}}{1 - N m_t - 2g(\theta_t) - \rho}$$

$$= \lim_{N\to\infty} \left\{\nu_0 + \frac{N}{[1 - N m_0 e^t - 2g(\theta_t) - \rho] g(\theta_t)}\right\} e^{\rho t} - \frac{N}{[1 - N m_0 e^t - 2g(\theta_t) - \rho] g(\theta_t)} \approx -\frac{e^{\rho_t + 1}}{m_t g(\theta_t)} \quad (A94).$$

Note that:

$$\lim_{N\to\infty} \frac{N e^{\rho t}}{[1 - N m_0 e^t - 2g(\theta_t) - \rho] g(\theta_t)} = -\frac{e^{(\rho-1)t}}{m_0 g(\theta_t)}$$

and

$$\lim_{N\to\infty} \frac{N}{[1 - N m_0 e^t - 2g(\theta_t) - \rho] g(\theta_t)} = \frac{e^{-t}}{m_0 g(\theta_t)}.$$

So, if $\nu_0 = 0$, then

$$\nu_t^* = \lim_{N\to\infty} \left\{\nu_0 + \frac{N}{[1 - N m_0 e^t - 2g(\theta_t) - \rho] g(\theta_t)}\right\} e^{\rho t} - \frac{N}{[1 - N m_0 e^t - 2g(\theta_t) - \rho] g(\theta_t)} \approx -\frac{e^{(\rho-1)t}}{m_0 g(\theta_t)} - \frac{e^{-t}}{m_0 g(\theta_t)} =$$

$$-\frac{e^{(\rho-1)t} + e^{-t}}{m_0 g(\theta_t)} = -\frac{(e^{\rho t} + 1) e^{-t}}{m_0 g(\theta_t)}.$$

Finally, the last FOC is $\rho = \frac{\partial \mathcal{H}_4 / \partial m_t}{\mu_t} + \frac{\dot{\mu}_t}{\mu_t}$. The derivative of the Hamiltonian $\frac{\partial \mathcal{H}_4}{\partial m_t}$ is:

$$\frac{\partial \mathcal{H}_4}{\partial m_t} = N\bar{r}_t\bar{P}_t g(\theta_t)\left(\frac{H}{m_t}\right)^{1-\sigma}\left(sL_{pt}\right)^{\sigma} + N(\sigma-1)\bar{r}_t\bar{P}_t g(\theta_t)\left(\frac{H}{m_t}\right)^{1-\sigma}\left(sL_{pt}\right)^{\sigma} - Nk$$

$$-\nu_t N g(\theta_t) = \frac{N\sigma}{m_t} - Nk - \nu_t N g(\theta_t).$$

Substituting this into the FOC and rearranging to solve for μ_t gives

$$\dot{\mu_t} - \rho\mu_t = Nk - \frac{N\sigma}{m_t} + \nu_t^* N g(\theta_t) = N[k - \frac{\sigma}{m_t} + \nu_t^* g(\theta_t)].$$

Solving for μ_t^{*****}:

$$\mu_t^{*****} = \left[\mu_0 + \frac{N\left[k - \frac{\sigma}{m_t} + \nu_t^* g(\theta_t)\right]}{\rho}\right]e^{\rho t} - \frac{N\left[k - \frac{\sigma}{m_t} + \nu_t^* g(\theta_t)\right]}{\rho} = \left[\mu_0 + \frac{N\left(k - \frac{\sigma}{m_t} - \frac{e^{\rho t}+1}{m_t}\right)}{\rho}\right]e^{\rho t} -$$

$$\frac{N\left(k - \frac{\sigma}{m_t} - \frac{e^{\rho t}+1}{m_t}\right)}{\rho} \quad (A95).$$

(1) Solving the Optimal Allocation

The 5 FOCs are:

$$\begin{cases} L_{pt} = \left(\dfrac{\mu_t}{\chi N\sigma m_t\bar{r}_t\bar{P}_t g(\theta_t)\left(\frac{H}{m_t}\right)^{1-\sigma} s^{\sigma}}\right)^{\frac{1}{\sigma-1}} \quad (A91) \\ \bar{P}_t = \dfrac{1}{m_t\bar{r}_t g(\theta_t)\left(\frac{H}{m_t}\right)^{1-\sigma}\left(sL_{pt}\right)^{\sigma}} \quad (A92) \\ \bar{r}_t{}^{*****} = \eta^{\frac{1}{\eta-1}} \quad (A93) \\ \nu_t^* = -\dfrac{(e^{\rho t}+1)e^{-t}}{m_0 g(\theta_t)} \quad (A94) \\ \mu_t^{*****} = \left[\mu_0 + \dfrac{N\left(k - \frac{\sigma}{m_t} - \frac{e^{\rho t}+1}{m_t}\right)}{\rho}\right]e^{\rho t} - \dfrac{N\left(k - \frac{\sigma}{m_t} - \frac{e^{\rho t}+1}{m_t}\right)}{\rho} \quad (A95) \end{cases}$$

1) Solving for L_{pt} and $\bar{P}_t$(eqs. A91 and A92).

Combining (A91), (A92), and (A95) gives

$$L_{pt}^{*****} = \frac{N\sigma\chi}{[\mu_0+\frac{N(k-\frac{\sigma}{m_t}-\frac{e^{\rho t_{+1}}}{m_t})}{\rho}]e^{\rho t}-\frac{N(k-\frac{\sigma}{m_t}-\frac{e^{\rho t_{+1}}}{m_t})}{\rho}} = \frac{\sigma\chi}{[\frac{\mu_0}{N}+\frac{(k-\frac{\sigma}{m_t}-\frac{e^{\rho t_{+1}}}{m_t})}{\rho}]e^{\rho t}-\frac{(k-\frac{\sigma}{m_t}-\frac{e^{\rho t_{+1}}}{m_t})}{\rho}} \approx$$

$$\frac{\sigma\chi}{\left[\frac{\left(k-\frac{\sigma}{m_t}-\frac{e^{\rho t_{+1}}}{m_t}\right)}{\rho}\right]e^{\rho t}-\frac{\left(k-\frac{\sigma}{m_t}-\frac{e^{\rho t_{+1}}}{m_t}\right)}{\rho}} = \frac{\sigma\chi}{\left[\frac{\left(k-\frac{\sigma-e^{\rho t_{-1}}}{m_0 e^t}\right)}{\rho}\right]e^{\rho t}-\frac{\left(k-\frac{\sigma-e^{\rho t_{-1}}}{m_0 e^t}\right)}{\rho}} \quad (A96) \ .$$

Combining (A92), (A96), and (A93) gives

$$\bar{P}_t^{*****} = \frac{\left[\frac{\sigma\chi}{\left[\frac{\left(k-\frac{\sigma-e^{\rho t_{-1}}}{m_0 e^t}\right)}{\rho}\right]e^{\rho t}-\frac{\left(k-\frac{\sigma-e^{\rho t_{-1}}}{m_0 e^t}\right)}{\rho}}\right]^{-\sigma}}{(m_0 e^t)^{\sigma}\eta^{\frac{1}{\eta-1}}\mathrm{H}^{1-\sigma}\mathrm{s}^{\sigma}g(\theta_t)} \quad (A97).$$

2) Solving for U_4.

Next, by definition

$$U_4 = \int_0^{\infty} e^{-\rho t}\, N\left\{m_t\left[\bar{P}_t g(\theta_t)\left(\frac{H}{m_t}\right)^{1-\sigma}(sL_{pt})^{\sigma} - W_t L_{pt} - c_{wt}\left(s, L_{pt}\right)\right] - vH - km_t - \frac{\tilde{k}}{2}H^2 - ln\bar{P}_t - \int_{r_0}^{\bar{r}_t}\eta r_t^{-\eta} dr_t\right\}\mathrm{dt}.$$ Then:

$$U_4 = \int_0^{\infty} e^{-\rho t}\, N\left[m_t \bar{r}_t^{****}\bar{P}_t^{*****}\left(\frac{H}{m_t}\right)^{1-\sigma}\left(sL_{pt}^{*****}\right)^{\sigma} g(\theta_t) - vH - km_t - \frac{\tilde{k}}{2}H^2 - ln\bar{P}_t^{*****} - \eta\frac{(\bar{r}_t^{****})^{1-\eta} - r_0^{1-\eta}}{1-\eta}\right]\mathrm{dt}$$

$$= \int_0^{\infty} e^{-\rho t}\, N\left[1 - vH - km_0 e^t - \frac{\tilde{k}}{2}H^2 - ln\bar{P}_t^{*****} - \eta\frac{\eta^{\frac{1-\eta}{\eta-1}} - (\frac{\epsilon_w}{\epsilon_l m + \epsilon_w})^{1-\eta}}{1-\eta}\right]\mathrm{dt}$$

$$= \int_0^{\infty} e^{-\rho t}\, N\left\{\left(1 - vH - \frac{\tilde{k}}{2}H^2\right) - km_0 e^t - \frac{\eta}{1-\eta}\left[\frac{1}{\eta} - (\frac{\epsilon_w}{\epsilon_l m + \epsilon_w})^{1-\eta}\right] - ln\bar{P}_t^{*****}\right\} dt$$

$$= \int_0^{\infty} e^{-\rho t} N \left\{ [1 - vH - \frac{\tilde{k}}{2} H^2 - \frac{1 - \eta(\frac{\epsilon_w}{\epsilon_l m + \epsilon_w})^{1-\eta}}{1-\eta}] - km_0 e^t - ln\bar{P}_t^{*****} \right\} dt$$

$$= \int_0^{\infty} N[1 - vH - \frac{\tilde{k}}{2} H^2 - \frac{1-\eta(\frac{\epsilon_w}{\epsilon_l m+\epsilon_w})^{1-\eta}}{1-\eta}] e^{-\rho t}\, dt + \int_0^{\infty} (-km_0 e^t) e^{-\rho t}\, dt +$$

$$\int_0^{\infty} \left(-ln\bar{P}_t^{*****}\right) e^{-\rho t}\, dt = N\{\frac{1 - vH - \frac{\tilde{k}}{2}H^2 - \frac{1-\eta\left(\frac{\epsilon_w}{\epsilon_l m+\epsilon_w}\right)^{1-\eta}}{1-\eta}}{\rho} + \frac{km_0}{1-\rho} + \frac{1}{\eta - 1}\left[\frac{1}{\rho} - \frac{(km_0)^{1-\eta}}{1-\eta-\rho}\right] - \frac{\sigma}{\rho}[1 +$$

$$ln\left(\frac{k}{\rho}\right)] + \frac{\sigma}{\rho^2} + \frac{\sigma ln\sigma\chi + \sigma ln m_0 + \frac{ln\eta}{\eta-1} + (1-\sigma) lnH + \sigma lns + \ln[g(\theta_t)]}{\rho}\}.$$

Note that:

(i) $\int_0^{\infty} [1 - vH - \frac{\tilde{k}}{2} H^2 - \frac{1-\eta(\frac{\epsilon_w}{\epsilon_l m+\epsilon_w})^{1-\eta}}{1-\eta}] e^{-\rho t}\, dt = [1 - vH - \frac{\tilde{k}}{2} H^2 -$

$\frac{1-\eta(\frac{\epsilon_w}{\epsilon_l m+\epsilon_w})^{1-\eta}}{1-\eta}] \int_0^{\infty} e^{-\rho t}\, dt = \frac{1 - vH - \frac{\tilde{k}}{2}H^2 - \frac{1-\eta(\frac{\epsilon_w}{\epsilon_l m+\epsilon_w})^{1-\eta}}{1-\eta}}{\rho}$.

(ii) $\int_0^{\infty} (-km_0 e^t) e^{-\rho t}\, dt = -km_0 \int_0^{\infty} e^{(1-\rho)t}\, dt = \frac{-km_0}{1-\rho}\left[e^{(1-\rho)t}\right]_0^{\infty} = \frac{km_0}{1-\rho}$, because

$\lim_{t\to\infty} e^{(1-\rho)t} = \lim_{t\to\infty} \frac{1}{e^{(\rho-1)t}} = 0$ and $\lim_{t\to 0} e^{(1-\rho)t} = 1$ $(\rho > 1)$.

$$(iii) \int_0^{\infty} \left(-ln\bar{P}_t^{*****}\right) e^{-\rho t}\, dt = -\int_0^{\infty} e^{-\rho t} \ln \frac{\left[\frac{\sigma\chi}{\left[\frac{\left(k - \frac{\sigma - e^{\rho t} - 1}{m_0 e^t}\right)}{\rho}\right] e^{\rho t} - \frac{\left(k - \frac{\sigma - e^{\rho t} - 1}{m_0 e^t}\right)}{\rho}}\right]^{-\sigma}}{(m_0 e^t)^{\sigma} \eta^{\frac{1}{\eta-1}} \mathrm{H}^{1-\sigma} \mathrm{s}^{\sigma} g(\theta_t)} dt =$$

$$-\int_0^{\infty} e^{-\rho t}\{\sigma \ln\left\{\left[\frac{k - \frac{\sigma - e^{\rho t} - 1}{m_0 e^t}}{\rho}\right] e^{\rho t} - \frac{k - \frac{\sigma - e^{\rho t} - 1}{m_0 e^t}}{\rho}\right\} - \sigma ln\sigma\chi - \sigma ln m_0 - \sigma t - \frac{ln\eta}{\eta} - (1-\sigma) lnH -$$

$$\sigma lns - \ln[g(\theta_t)]\} dt = -\int_0^{\infty} e^{-\rho t}\{\sigma \ln\left\{\left[\frac{k - \frac{\sigma - e^{\rho t} - 1}{m_0 e^t}}{\rho}\right] e^{\rho t} - \frac{k - \frac{\sigma - e^{\rho t} - 1}{m_0 e^t}}{\rho}\right\} - \sigma t\} dt - \int_0^{\infty} e^{-\rho t}\{ -$$

$$\sigma ln\sigma\chi - \sigma lnm_0 - \frac{ln\eta}{\eta-1} - (1-\sigma)lnH - \sigma lns - \ln[g(\theta_t)]\}dt =$$

$$-\int_0^\infty e^{-\rho t}(\sigma \ln\left\{\left[\frac{k-\frac{\sigma-e^{\rho t}-1}{m_0 e^t}}{\rho}\right]e^{\rho t} - \frac{k-\frac{\sigma-e^{\rho t}-1}{m_0 e^t}}{\rho}\right\})dt - \int_0^\infty -\sigma t e^{-\rho t}\,dt - [-\sigma ln\sigma\chi - \sigma lnm_0 -$$

$$\frac{ln\eta}{\eta-1} - (1-\sigma)lnH - \sigma lns - \ln[g(\theta_t)]]\int_0^\infty e^{-\rho t}\,dt = A' + \sigma\int_0^\infty te^{-\rho t}\,dt + \{\sigma ln\sigma\chi +$$

$$\sigma lnm_0 + \frac{ln\eta}{\eta-1} + (1-\sigma)lnH + \sigma lns + \ln[g(\theta_t)]\}\int_0^\infty e^{-\rho t}\,dt == -\frac{\sigma}{\rho}[1 + ln\left(\frac{k}{\rho}\right)] + \frac{\sigma}{\rho^2} +$$

$$\frac{\sigma ln\sigma\chi+\sigma lnm_0+\frac{ln\eta}{\eta-1}+(1-\sigma)lnH+\sigma lns+\ln[g(\theta_t)]}{\rho},$$

where $A' = -\int_0^\infty e^{-\rho t}(\sigma \ln\left\{\left[\frac{k-\frac{\sigma-e^{\rho t}-1}{m_0 e^t}}{\rho}\right]e^{\rho t} - \frac{k-\frac{\sigma-e^{\rho t}-1}{m_0 e^t}}{\rho}\right\})dt = -\int_0^\infty e^{-\rho t}\{\sigma \ln\left[\frac{ke^{\rho t}}{\rho} -\right.$

$$\left.\frac{(\sigma-1)}{\rho m_0}e^{(\rho-1)t} + \frac{e^{(\rho-1)t}}{\rho m_0} - \frac{k}{\rho} + \frac{\sigma-1}{\rho m_0 e^t} - \frac{e^{(\rho-1)t}}{\rho m_0}\right]\}dt = -\int_0^\infty e^{-\rho t}\{\sigma \ln\left[\frac{ke^{\rho t}}{\rho} - \frac{(\sigma-1)}{\rho m_0}e^{(\rho-1)t} - \frac{k}{\rho} +\right.$$

$$\left.\frac{(\sigma-1)}{\rho m_0}e^{-t}\right]\}dt$$

Because $\lim_{t\to\infty}\frac{\sigma-1}{\rho m_0 e^t} = 0$, then:

$$A' = -\int_0^\infty e^{-\rho t}\{\sigma \ln\left[\frac{ke^{\rho t}}{\rho} - \frac{\sigma-1}{\rho m_0}e^{(\rho-1)t} - \frac{k}{\rho}\right]\}dt.$$

For large t, if $\rho > 1$, then $e^{\rho t}$ grows much faster than $e^{(\rho-1)t}$. The term $\frac{ke^{\rho t}}{\rho}$ dominates over $\frac{\sigma-1}{\rho m_0}e^{(\rho-1)t}$ and $\frac{k}{\rho}$. Therefore,

$$A' \approx -\sigma\int_0^\infty e^{-\rho t}\ln\left(\frac{ke^{\rho t}}{\rho}\right)dt = -\sigma\int_0^\infty e^{-\rho t}\ln\left[\left(\frac{k}{\rho}\right)e^{\rho t}\right]dt = -\sigma\int_0^\infty e^{-\rho t}\ln[\left(\frac{k}{\rho}\right) + lne^{\rho t}]dt =$$

$$-\sigma\int_0^\infty e^{-\rho t}\ln\left[\left(\frac{k}{\rho}\right) + \rho t\right]dt = -\sigma\int_0^\infty e^{-\rho t}\ln\left(\frac{k}{\rho}\right)dt - \sigma\int_0^\infty e^{-\rho t}\rho t\,dt =$$

$$-\sigma ln\left(\frac{k}{\rho}\right)\int_0^\infty e^{-\rho t}\,dt - \sigma\rho\int_0^\infty e^{-\rho t}t\,dt = \frac{-\sigma ln\left(\frac{k}{\rho}\right)}{\rho} - \frac{\sigma\rho}{\rho^2} = -\frac{\sigma}{\rho}[1 + ln\left(\frac{k}{\rho}\right)].$$

T. PROOF OF LEMMA 11

The social planner problem is:

$$U_5 = \underset{\{\bar{P}_t,\bar{r}_t,L_{pt}\}}{\text{Max}} \int_0^{\infty} e^{-\rho t} N \left\{ m_t \left[\bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} (sL_{pt})^{\sigma} - W_t L_{pt} - c_{wt}\left(s, L_{pt}\right) \right] - vH - km_t - \right.$$

$$\left. \frac{\tilde{k}}{2} H^2 - ln\bar{P}_t - \int_{r_0}^{\bar{r}_t} \eta r_t^{-\eta} dr_t \right\} \text{dt},$$

s.t.

$$W_t + \alpha s = \sigma(1 - \bar{r}_t)\bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma-1},$$

$$\dot{m}_t = \frac{1}{\chi}(L_t - L_{pt}),$$

$$m_t = m_0 e^t,$$

$$\dot{g}(\theta_t) = g(\theta_t) - [g(\theta_t)]^2 - N m_t g(\theta_t).$$

Next, define Hamiltonian with state variable m_t, control variables $\{\bar{P}_t, \bar{r}_t, L_{pt}\}$ and co-state variables μ_t and ν_t:

$$\mathcal{H}_5\left(\bar{P}_t, \bar{r}_t, L_{pt}, m_t, \mu_t, \nu_t\right) = N \left\{ m_t \left[\bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} (sL_{pt})^{\sigma} - W_t L_{pt} - c_{wt}\left(s, L_{pt}\right) \right] - vH - \right.$$

$$\left. km_t - \frac{\tilde{k}}{2} H^2 - ln\bar{P}_t - \int_{r_0}^{\bar{r}_t} \eta r_t^{-\eta} dr_t \right\} = N \left\{ m_t \left[\bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} (sL_{pt})^{\sigma} - \sigma(1 - \right.\right.$$

$$\left.\left. \bar{r}_t)\bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} L_{pt}^{\sigma} \right] - vH - km_t - \frac{\tilde{k}}{2} H^2 - ln\bar{P}_t - \int_{r_0}^{\bar{r}_t} \eta r_t^{-\eta} dr_t \right\} = N m_t (1 - \sigma +$$

$$\sigma\bar{r}_t)\bar{P}_t g(\theta_t) \left(\frac{H}{m_t}\right)^{1-\sigma} (sL_{pt})^{\sigma} - NvH - Nkm_t - N\frac{\tilde{k}}{2} H^2 - \text{N}ln\bar{P}_t - N\eta \frac{\bar{r}_t^{1-\eta} - r_0^{1-\eta}}{1-\eta} +$$

$$\frac{\mu_t}{\chi}\left(L_t - L_{pt}\right) + \nu_t\{(1 - Nm_t)g(\theta_t) - [g(\theta_t)]^2\}.$$

The first order conditions are:

$$\begin{cases} \dfrac{\partial \mathcal{H}_5}{\partial L_{pt}} = 0 \\ \dfrac{\partial \mathcal{H}_5}{\partial \bar{P}_t} = 0 \\ \dfrac{\partial \mathcal{H}_5}{\partial \bar{r}_t} = 0 \\ \rho = \dfrac{\partial \mathcal{H}_5 / \partial g(\theta_t)}{\nu_t} + \dfrac{\dot{\nu}_t}{\nu_t} \\ \rho = \dfrac{\partial \mathcal{H}_5 / \partial m_t}{\mu_t} + \dfrac{\dot{\mu}_t}{\mu_t} \end{cases}$$

Start with $\frac{\partial \mathcal{H}_5}{\partial L_{pt}} = 0$, which implies

$$N\sigma m_t(1-\sigma+\sigma\bar{r}_t)\bar{P}_t g(\theta_t)\left(\tfrac{H}{m_t}\right)^{1-\sigma}\left(L_{pt}\right)^{\sigma-1}s^{\sigma} = \tfrac{\mu_t}{\chi},$$

and therefore

$$L_{pt} = \left(\frac{\mu_t}{\chi N\sigma m_t(1-\sigma+\sigma\bar{r}_t)\bar{P}_t g(\theta_t)\left(\frac{H}{m_t}\right)^{1-\sigma}s^{\sigma}}\right)^{\frac{1}{\sigma-1}} \quad (A98).$$

Next, consider $\frac{\partial \mathcal{H}_5}{\partial \bar{P}_t} = 0$:

$$m_t(1-\sigma+\sigma\bar{r}_t)g(\theta_t)\left(\tfrac{H}{m_t}\right)^{1-\sigma}\left(sL_{pt}\right)^{\sigma} = \tfrac{1}{\bar{P}_t}.$$

Substituting gives

$$\bar{P}_t = \frac{1}{m_t(1-\sigma+\sigma\bar{r}_t)g(\theta_t)\left(\frac{H}{m_t}\right)^{1-\sigma}\left(sL_{pt}\right)^{\sigma}} \quad (A99).$$

The next FOC is similar, $\frac{\partial \mathcal{H}_5}{\partial \bar{r}_t} = 0$:

$$\sigma m_t\bar{P}_t g(\theta_t)\left(\tfrac{H}{m_t}\right)^{1-\sigma}\left(sL_{pt}\right)^{\sigma} = \eta\bar{r}_t^{\,-\eta}.$$

Substituting gives

$$\frac{\sigma}{1-\sigma+\sigma\bar{r}_t} = \eta\bar{r}_t^{\,-\eta}$$

$$\Rightarrow \bar{r}_t{}^{\eta} = \frac{\eta(1-\sigma)}{\sigma} + \eta\bar{r}_t.$$

If $\eta = 0.5$, $\bar{r}_t{}^{******} = \frac{3\sigma-1\pm2\sqrt{(2\sigma-1)\sigma}}{\sigma}$ $(A100)$.

The next FOC is $\rho = \frac{\partial\mathcal{H}_1/\partial g(\theta_t)}{\nu_t} + \frac{\dot{\nu}_t}{\nu_t}$. The derivative of the Hamiltonian $\frac{\partial\mathcal{H}_5}{\partial g(\theta_t)}$ is:

$$\frac{\partial\mathcal{H}_5}{\partial g(\theta_t)} = Nm_t(1-\sigma+\sigma\bar{r}_t)\bar{P}_t\left(\frac{H}{m_t}\right)^{1-\sigma}\left(sL_{pt}\right)^{\sigma} + \nu_t[1-Nm_t-2g(\theta_t)]$$

$$= \frac{N}{g(\theta_t)} + \nu_t[1-Nm_t-2g(\theta_t)].$$

Substituting this into the FOC and rearranging to solve for ν_t gives

$$\dot{\nu}_t - \rho\nu_t = \nu_t[Nm_t - 1 + 2g(\theta_t)] - \frac{N}{g(\theta_t)}.$$

$$\Longrightarrow \dot{\nu}_t + \nu_t[1-Nm_t-2g(\theta_t)-\rho] = -\frac{N}{g(\theta_t)}.$$

Solving for ν_t^{**}:

$$\nu_t^{**} = \left[\nu_0 + \frac{\frac{N}{g(\theta_t)}}{1-Nm_t-2g(\theta_t)-\rho}\right]e^{\rho t} - \frac{\frac{N}{g(\theta_t)}}{1-Nm_t-2g(\theta_t)-\rho} = \lim_{N\to\infty}\left\{\nu_0 + \frac{N}{[1-Nm_0e^t-2g(\theta_t)-\rho]g(\theta_t)}\right\}e^{\rho t} -$$

$$\frac{N}{[1-Nm_0e^t-2g(\theta_t)-\rho]g(\theta_t)} \approx -\frac{e^{\rho t}+1}{m_t g(\theta_t)} \ (A101).$$

Note that:

$$\lim_{N\to\infty}\frac{Ne^{\rho t}}{[1-Nm_0e^t-2g(\theta_t)-\rho]g(\theta_t)} = -\frac{e^{(\rho-1)t}}{m_0g(\theta_t)}$$

and

$$\lim_{N\to\infty}\frac{N}{[1-Nm_0e^t-2g(\theta_t)-\rho]g(\theta_t)} = \frac{e^{-t}}{m_0g(\theta_t)}.$$

So, ff $\nu_0 = 0$, then

$$\nu_t^{**} = \lim_{N\to\infty}\left\{\nu_0 + \frac{N}{[1-Nm_0e^t-2g(\theta_t)-\rho]g(\theta_t)}\right\}e^{\rho t} - \frac{N}{[1-Nm_0e^t-2g(\theta_t)-\rho]g(\theta_t)} \approx -\frac{e^{(\rho-1)t}}{m_0 g(\theta_t)} - \frac{e^{-t}}{m_0 g(\theta_t)} =$$

$$-\frac{e^{(\rho-1)t}+e^{-t}}{m_0 g(\theta_t)} = -\frac{(e^{\rho t}+1)e^{-t}}{m_0 g(\theta_t)}.$$

Finally, the last FOC is $\rho = \frac{\partial \mathcal{H}_4/\partial m_t}{\mu_t} + \frac{\dot{\mu_t}}{\mu_t}$. The derivative of the Hamiltonian $\frac{\partial \mathcal{H}_5}{\partial m_t}$ is:

$$\frac{\partial \mathcal{H}_5}{\partial m_t} = N(1-\sigma+\sigma\bar{r}_t)\bar{P}_t g(\theta_t)\left(\frac{H}{m_t}\right)^{1-\sigma}\left(sL_{pt}\right)^{\sigma} + N(\sigma-1)(1-\sigma$$

$$+\sigma\bar{r}_t)\bar{P}_t g(\theta_t)\left(\frac{H}{m_t}\right)^{1-\sigma}\left(sL_{pt}\right)^{\sigma} - Nk - \nu_t N g(\theta_t) = \frac{N\sigma}{m_t} - Nk - \nu_t N g(\theta_t).$$

Substituting this into the FOC and rearranging to solve for μ_t gives

$$\dot{\mu_t} - \rho\mu_t = Nk - \frac{N\sigma}{m_t} + \nu_t^{**} N g(\theta_t) = N[k - \frac{\sigma}{m_t} + \nu_t^{**} g(\theta_t)].$$

Solving for μ_t^{******}:

$$\mu_t^{******} = \left[\mu_0 + \frac{N\left[k - \frac{\sigma}{m_t} + \nu_t^{**} g(\theta_t)\right]}{\rho}\right]e^{\rho t} - \frac{N\left[k - \frac{\sigma}{m_t} + \nu_t^{**} g(\theta_t)\right]}{\rho}$$

$$= \left[\mu_0 + \frac{N\left(k - \frac{\sigma}{m_t} - \frac{e^{\rho t}+1}{m_t}\right)}{\rho}\right]e^{\rho t} - \frac{N\left(k - \frac{\sigma}{m_t} - \frac{e^{\rho t}+1}{m_t}\right)}{\rho} \quad (A102)$$

(1) Solving the Optimal Allocation

The 5 FOCs are:

$$\begin{cases} L_{pt} = \left(\dfrac{\mu_t}{\chi N\sigma m_t(1-\sigma+\sigma\bar{r}_t)\bar{P}_t g(\theta_t)\left(\frac{H}{m_t}\right)^{1-\sigma} s^{\sigma}}\right)^{\frac{1}{\sigma-1}} \quad (A98) \\ \bar{P}_t = \dfrac{1}{m_t(1-\sigma+\sigma\bar{r}_t)g(\theta_t)\left(\frac{H}{m_t}\right)^{1-\sigma}\left(sL_{pt}\right)^{\sigma}} \quad (A99) \\ \bar{r}_t^{******} = \dfrac{3\sigma-1\pm 2\sqrt{(2\sigma-1)\sigma}}{\sigma} \quad (A100) \\ \nu_t^{**} = -\dfrac{e^{\rho t}+1}{m_t g(\theta_t)} \quad (A101) \\ \mu_t^{******} = \left[\mu_0 + \dfrac{N\left(k-\frac{\sigma}{m_t}-\frac{e^{\rho t}+1}{m_t}\right)}{\rho}\right]e^{\rho t} - \dfrac{N\left(k-\frac{\sigma}{m_t}-\frac{e^{\rho t}+1}{m_t}\right)}{\rho} \quad (A102) \end{cases}$$

1) Solving for L_{pt} and $\bar{P}_t$(eqs. A98 and A99).

Combining (A98), (A99), and (A102) gives

$$L_{pt}^{******} = \frac{N\sigma\chi}{\left[\mu_0+\frac{N\left(k-\frac{\sigma}{m_t}-\frac{e^{\rho t_{+1}}}{m_t}\right)}{\rho}\right]e^{\rho t}-\frac{N\left(k-\frac{\sigma}{m_t}-\frac{e^{\rho t_{+1}}}{m_t}\right)}{\rho}} = \frac{\sigma\chi}{\left[\frac{\mu_0}{N}+\frac{\left(k-\frac{\sigma}{m_t}-\frac{e^{\rho t_{+1}}}{m_t}\right)}{\rho}\right]e^{\rho t}-\frac{\left(k-\frac{\sigma}{m_t}-\frac{e^{\rho t_{+1}}}{m_t}\right)}{\rho}} \approx$$

$$\frac{\sigma\chi}{\left[\frac{\left(k-\frac{\sigma}{m_t}-\frac{e^{\rho t_{+1}}}{m_t}\right)}{\rho}\right]e^{\rho t}-\frac{\left(k-\frac{\sigma}{m_t}-\frac{e^{\rho t_{+1}}}{m_t}\right)}{\rho}} = \frac{\sigma\chi}{\left[\frac{\left(k-\frac{\sigma-e^{\rho t_{-1}}}{m_0 e^{t}}\right)}{\rho}\right]e^{\rho t}-\frac{\left(k-\frac{\sigma-e^{\rho t_{-1}}}{m_0 e^{t}}\right)}{\rho}} \quad \text{(A103)}.$$

Combining (A99), (A100), and (A103) gives

$$\bar{P}_t^{******} = \frac{\left[\frac{\sigma\chi}{\left[\frac{\left(k-\frac{\sigma-e^{\rho t_{-1}}}{m_0 e^{t}}\right)}{\rho}\right]e^{\rho t}-\frac{\left(k-\frac{\sigma-e^{\rho t_{-1}}}{m_0 e^{t}}\right)}{\rho}}\right]^{-\sigma}}{(m_0 e^{t})^{\sigma}[2\sigma\pm 2\sqrt{(2\sigma-1)\sigma}]\mathrm{H}^{1-\sigma}\mathrm{s}^{\sigma}g(\theta_t)} \quad (A104).$$

2) Solving for U_5.

Next, by definition $U_5 = \int_0^{\infty} e^{-\rho t} N\left\{m_t\left[\bar{P}_t g(\theta_t)(\frac{H}{m_t})^{1-\sigma}(sL_{pt})^{\sigma} - W_t L_{pt} - c_{wt}(s, L_{pt})\right] - vH - km_t - \frac{\tilde{k}}{2}H^2 - ln\bar{P}_t - \int_{r_0}^{\bar{r}_t}\eta r_t^{-\eta} dr_t\right\}$ dt. Then:

$$U_5 = \int_0^{\infty} e^{-\rho t} N \left[m_t (1 - \sigma + \sigma \bar{r}_t^{******}) \bar{P}_t^{******} \left(\frac{H}{m_t}\right)^{1-\sigma} \left(sL_{pt}^{******}\right)^{\sigma} g(\theta_t) - vH - km_t - \frac{\tilde{k}}{2} H^2 \right.$$

$$\left. - ln\bar{P}_t^{******} - \eta \frac{(\bar{r}_t^{******})^{1-\eta} - r_0^{1-\eta}}{1-\eta} \right] \mathrm{dt}$$

$$= \int_0^{\infty} e^{-\rho t} N \left[1 - vH - km_0 e^t - \frac{\tilde{k}}{2} H^2 - ln\bar{P}_t^{******} + \frac{\eta}{\eta - 1} \left\{ \left[\frac{3\sigma - 1 \pm 2\sqrt{(2\sigma - 1)\sigma}}{\sigma} \right]^{1-\eta} \right. \right.$$

$$\left. \left. - \left(\frac{\epsilon_w}{\epsilon_l m + \epsilon_w} \right)^{1-\eta} \right\} \right] \mathrm{dt}$$

$$= \int_0^{\infty} e^{-\rho t} N \left\{ \left(1 - vH - \frac{\tilde{k}}{2} H^2 \right) - km_0 e^t + \frac{\eta}{\eta - 1} \left\{ \left[\frac{3\sigma - 1 \pm 2\sqrt{(2\sigma - 1)\sigma}}{\sigma} \right]^{1-\eta} \right. \right.$$

$$\left. \left. - \left(\frac{\epsilon_w}{\epsilon_l m + \epsilon_w} \right)^{1-\eta} \right\} - ln\bar{P}_t^{******} \right\} dt$$

$$= \int_0^{\infty} e^{-\rho t} N \left\{ \left[1 - vH - \frac{\tilde{k}}{2} H^2 + \frac{\eta}{\eta - 1} \left\{ \left[\frac{3\sigma - 1 \pm 2\sqrt{(2\sigma - 1)\sigma}}{\sigma} \right]^{1-\eta} - \left(\frac{\epsilon_w}{\epsilon_l m + \epsilon_w} \right)^{1-\eta} \right\} \right] \right.$$

$$\left. - km_0 e^t - ln\bar{P}_t^{******} \right\} dt$$

$$= \int_0^{\infty} N \left[1 - vH - \frac{\tilde{k}}{2} H^2 + \frac{\eta}{\eta - 1} \left\{ \left[\frac{3\sigma - 1 \pm 2\sqrt{(2\sigma - 1)\sigma}}{\sigma} \right]^{1-\eta} - \left(\frac{\epsilon_w}{\epsilon_l m + \epsilon_w} \right)^{1-\eta} \right\} \right] e^{-\rho t} \, dt +$$

$$\int_0^{\infty} (-km_0 e^t) e^{-\rho t} \, dt + \int_0^{\infty} \left(-ln\bar{P}_t^{******}\right) e^{-\rho t} \, dt =$$

$$N \left\{ \frac{1 - vH - \frac{\tilde{k}}{2} H^2 + \frac{\eta}{\eta - 1} \left\{ \left[\frac{3\sigma - 1 \pm 2\sqrt{(2\sigma - 1)\sigma}}{\sigma} \right]^{1-\eta} - \left(\frac{\epsilon_w}{\epsilon_l m + \epsilon_w} \right)^{1-\eta} \right\}}{\rho} + \frac{km_0}{1 - \rho} + \frac{1}{\eta - 1} \left[\frac{1}{\rho} - \frac{(km_0)^{1-\eta}}{1 - \eta - \rho} \right] - \frac{\sigma}{\rho} \left[1 + ln \left(\frac{k}{\rho} \right) \right] + \right.$$

$$\left. \frac{\sigma}{\rho^2} + \frac{\sigma ln\sigma\chi + \sigma ln m_0 + ln[2\sigma \pm 2\sqrt{(2\sigma - 1)\sigma}] + (1 - \sigma) ln H + \sigma ln s + \ln [g(\theta_t)]}{\rho} \right\}.$$

Note that:

(i) $\int_0^{\infty}[1-vH-\frac{\tilde{k}}{2}H^2+\frac{\eta}{\eta-1}\{\left[\frac{3\sigma-1\pm2\sqrt{(2\sigma-1)\sigma}}{\sigma}\right]^{1-\eta}-\left(\frac{\epsilon_w}{\epsilon_l m+\epsilon_w}\right)^{1-\eta}\}]e^{-\rho t}\,dt=[1-vH-$

$\frac{\tilde{k}}{2}H^2+\frac{\eta}{\eta-1}\{\left[\frac{3\sigma-1\pm2\sqrt{(2\sigma-1)\sigma}}{\sigma}\right]^{1-\eta}-\left(\frac{\epsilon_w}{\epsilon_l m+\epsilon_w}\right)^{1-\eta}\}]\int_0^{\infty}e^{-\rho t}\,dt=$

$\frac{1-vH-\frac{\tilde{k}}{2}H^2+\frac{\eta}{\eta-1}\{\left[\frac{3\sigma-1\pm2\sqrt{(2\sigma-1)\sigma}}{\sigma}\right]^{1-\eta}-\left(\frac{\epsilon_w}{\epsilon_l m+\epsilon_w}\right)^{1-\eta}\}}{\rho}$.

(ii) $\int_0^{\infty}(-km_0e^t)e^{-\rho t}\,dt=-km_0\int_0^{\infty}e^{(1-\rho)t}\,dt=\frac{-km_0}{1-\rho}\left[e^{(1-\rho)t}\right]_0^{\infty}=\frac{km_0}{1-\rho}$, because

$\lim_{t\to\infty}e^{(1-\rho)t}=\lim_{t\to\infty}\frac{1}{e^{(\rho-1)t}}=0$ and $\lim_{t\to0}e^{(1-\rho)t}=1$ $(\rho>1)$.

$(iii)\int_0^{\infty}\left(-ln\bar{P}_t^{******}\right)e^{-\rho t}\,dt=-\int_0^{\infty}e^{-\rho t}\ln\frac{\left[\frac{\sigma\chi}{\left[\frac{\left(k-\frac{\sigma-e^{\rho t}-1}{m_0e^t}\right)}{\rho}\right]e^{\rho t}-\frac{\left(k-\frac{\sigma-e^{\rho t}-1}{m_0e^t}\right)}{\rho}}\right]^{-\sigma}}{(m_0e^t)^{\sigma}[2\sigma\pm2\sqrt{(2\sigma-1)\sigma}]\mathrm{H}^{1-\sigma}\mathrm{s}^{\sigma}g(\theta_t)}dt=$

$-\int_0^{\infty}e^{-\rho t}\{\sigma\ln\left\{\left[\frac{k-\frac{\sigma-e^{\rho t}-1}{m_0e^t}}{\rho}\right]e^{\rho t}-\frac{k-\frac{\sigma-e^{\rho t}-1}{m_0e^t}}{\rho}\right\}-\sigma lnm\sigma\chi-\sigma lnm_0-\sigma t-ln[2\sigma\pm$

$2\sqrt{(2\sigma-1)\sigma}]-(1-\sigma)lnH-\sigma lns-\ln[g(\theta_t)]\}dt=-\int_0^{\infty}e^{-\rho t}\{\sigma\ln\left\{\left[\frac{k-\frac{\sigma-e^{\rho t}-1}{m_0e^t}}{\rho}\right]e^{\rho t}-$

$\frac{k-\frac{\sigma-e^{\rho t}-1}{m_0e^t}}{\rho}\right\}-\sigma t\}dt-\int_0^{\infty}e^{-\rho t}\{-\sigma lnm\sigma\chi-\sigma lnm_0-ln[2\sigma\pm2\sqrt{(2\sigma-1)\sigma}]-(1-\sigma)lnH-$

$\sigma lns-\ln[g(\theta_t)]\}dt=-\int_0^{\infty}e^{-\rho t}(\sigma\ln\left\{\left[\frac{k-\frac{\sigma-e^{\rho t}-1}{m_0e^t}}{\rho}\right]e^{\rho t}-\frac{k-\frac{\sigma-e^{\rho t}-1}{m_0e^t}}{\rho}\right\})dt-\int_0^{\infty}-\sigma te^{-\rho t}\,dt-$

$[-\sigma lnm\sigma\chi-\sigma lnm_0-ln[2\sigma\pm2\sqrt{(2\sigma-1)\sigma}]-(1-\sigma)lnH-\sigma lns-\ln[g(\theta_t)]]\int_0^{\infty}e^{-\rho t}\,dt=$

$A''+\sigma\int_0^{\infty}te^{-\rho t}\,dt+\{\sigma lnm\sigma\chi+\sigma lnm_0+ln[2\sigma\pm2\sqrt{(2\sigma-1)\sigma}]+(1-\sigma)lnH+\sigma lns+$

$\ln[g(\theta_t)]\}\int_0^{\infty} e^{-\rho t}\,dt == -\frac{\sigma}{\rho}[1+ln\left(\frac{k}{\rho}\right)]+\frac{\sigma}{\rho^2}+$

$\frac{\sigma ln\sigma\chi+\sigma lnm_0+ln[2\sigma\pm 2\sqrt{(2\sigma-1)\sigma}]+(1-\sigma)lnH+\sigma lns+\ln[g(\theta_t)]}{\rho}$,

where$A'' = -\int_0^{\infty} e^{-\rho t}(\sigma \ln\left\{\left[\frac{k-\frac{\sigma-e^{\rho t}-1}{m_0 e^t}}{\rho}\right]e^{\rho t}-\frac{k-\frac{\sigma-e^{\rho t}-1}{m_0 e^t}}{\rho}\right\})dt = -\int_0^{\infty} e^{-\rho t}\{\sigma \ln\left[\frac{ke^{\rho t}}{\rho}-\right.$

$\left.\frac{(\sigma-1)}{\rho m_0}e^{(\rho-1)t}+\frac{e^{(\rho-1)t}}{\rho m_0}-\frac{k}{\rho}+\frac{\sigma-1}{\rho m_0 e^t}-\frac{e^{(\rho-1)t}}{\rho m_0}\right]\}dt = -\int_0^{\infty} e^{-\rho t}\{\sigma \ln\left[\frac{ke^{\rho t}}{\rho}-\frac{(\sigma-1)}{\rho m_0}e^{(\rho-1)t}-\frac{k}{\rho}+\right.$

$\left.\frac{(\sigma-1)}{\rho m_0}e^{-t}\right]\}dt.$

Because $\lim_{t\to\infty}\frac{\sigma-1}{\rho m_0 e^t}=0$, then:

$$A' = -\int_0^{\infty} e^{-\rho t}\{\sigma \ln\left[\frac{ke^{\rho t}}{\rho}-\frac{\sigma-1}{\rho m_0}e^{(\rho-1)t}-\frac{k}{\rho}\right]\}dt.$$

For large t, if $\rho > 1$, then $e^{\rho t}$ grows much faster than $e^{(\rho-1)t}$. The term $\frac{ke^{\rho t}}{\rho}$ dominates over $\frac{\sigma-1}{\rho m_0}e^{(\rho-1)t}$ and $\frac{k}{\rho}$. Therefore,

$A' \approx -\sigma\int_0^{\infty} e^{-\rho t}\ln\left(\frac{ke^{\rho t}}{\rho}\right)dt = -\sigma\int_0^{\infty} e^{-\rho t}\ln\left[\left(\frac{k}{\rho}\right)e^{\rho t}\right]dt = -\sigma\int_0^{\infty} e^{-\rho t}\ln[\left(\frac{k}{\rho}\right)+lne^{\rho t}]dt =$

$-\sigma\int_0^{\infty} e^{-\rho t}\ln\left[\left(\frac{k}{\rho}\right)+\rho t\right]dt = -\sigma\int_0^{\infty} e^{-\rho t}\ln\left(\frac{k}{\rho}\right)dt-\sigma\int_0^{\infty} e^{-\rho t}\rho t\,dt =$

$-\sigma ln\left(\frac{k}{\rho}\right)\int_0^{\infty} e^{-\rho t}\,dt-\sigma\rho\int_0^{\infty} e^{-\rho t}t\,dt = \frac{-\sigma ln\left(\frac{k}{\rho}\right)}{\rho}-\frac{\sigma\rho}{\rho^2} = -\frac{\sigma}{\rho}[1+ln\left(\frac{k}{\rho}\right)].$

II Calibrations

1. Effect of σ on Socially Optimal Share Percentage of Landlord $\bar{r}_{t+}$

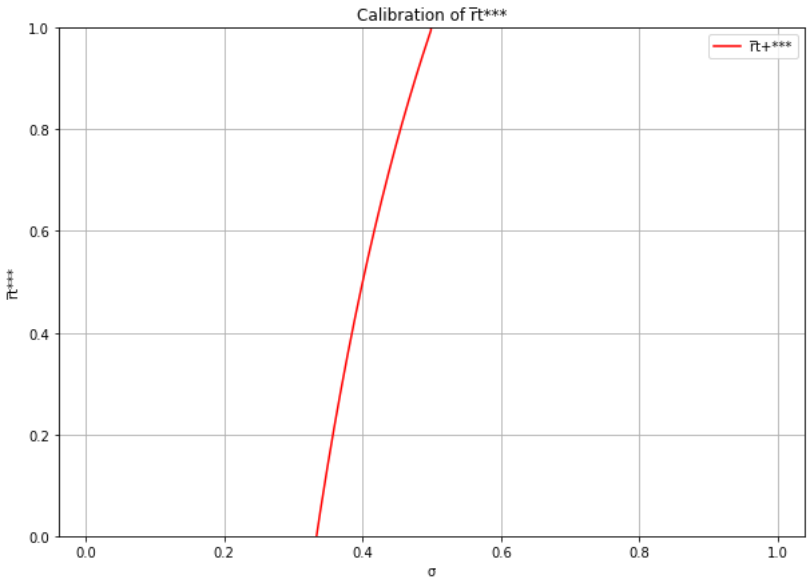


Figure A1: Effect of σ on Socially Optimal Share Percentage of Landlord $\bar{r}_{t+}$

2. Effect of σ on Socially Optimal Share Percentage of Landlord $\bar{r}_{t-}$

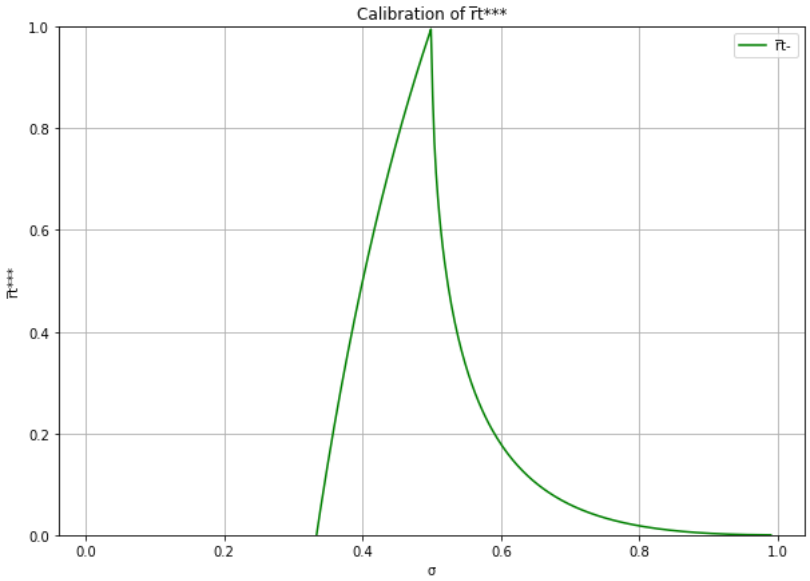


Figure A2: Effect of σ on Socially Optimal Share Percentage of Landlord $\bar{r}_{t-}$

3. Effect η on Socially Optimal Share Rate of Landlord $\bar{r}_t^*$

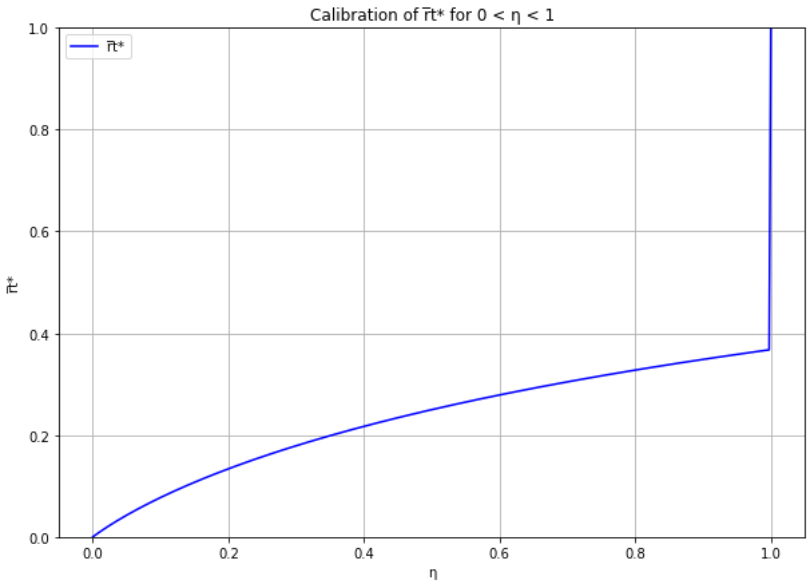


Figure A3: Effect of η on Socially Optimal Share Rate of Landlord $\bar{r}_t^*$

4. Effect of σ on Socially Optimal Share Rate of the Landlord $\bar{r}_t^{***}$

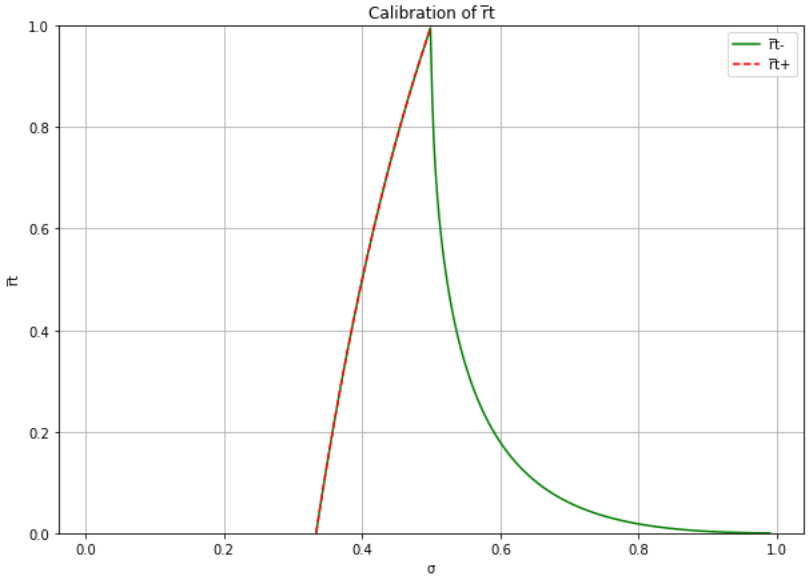


Figure A4: Effect of σ on Socially Optimal Share Rate of the Landlord $\bar{r}_t^{***}$

5. Effect of σ on Socially Optimal Price of Agri-food $\bar{P}_t^{**}$

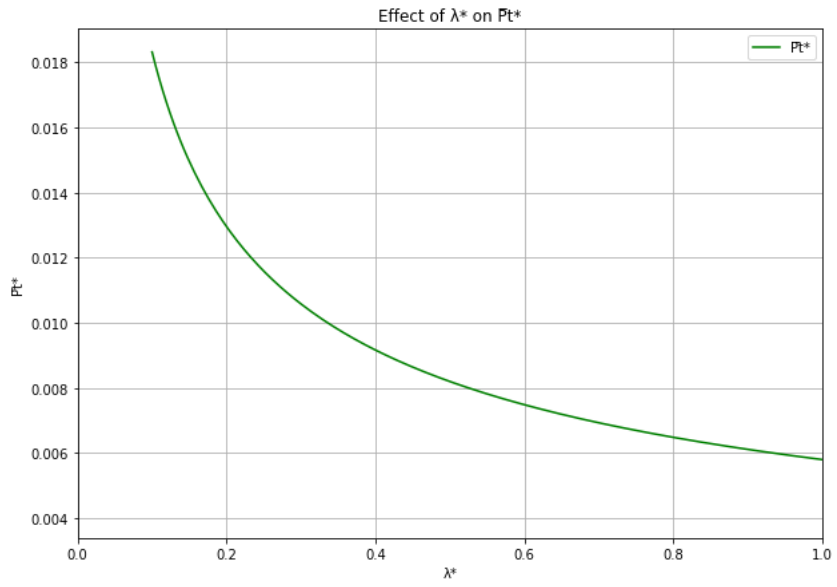


Figure A5: Effect of σ on Socially Optimal Price of Agri-food $\bar{P}_t^{**}$

6. Social Welfare Difference, D, Calibration ($km_0 = 0.1$, $\bar{r}_t^{***} = \bar{r}_{t+}$)

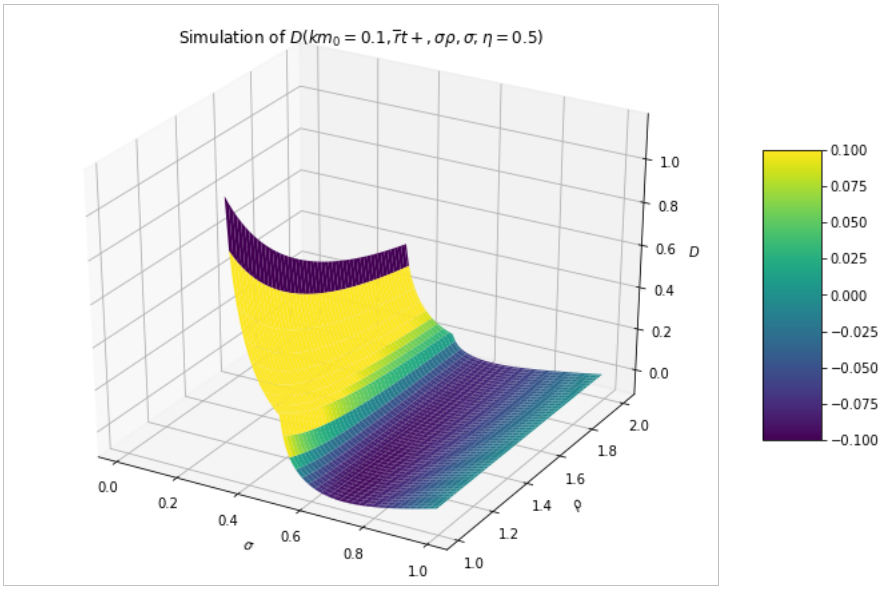


Figure A6: Social Welfare Difference, D, Calibration ($km_0 = 0.1$, $\bar{r}_t^{***} = \bar{r}_{t+}$)

7. Social Welfare Difference, D, Calibration ($km_0 = 1, \bar{r}_t^{***} = \bar{r}_{t+}$)

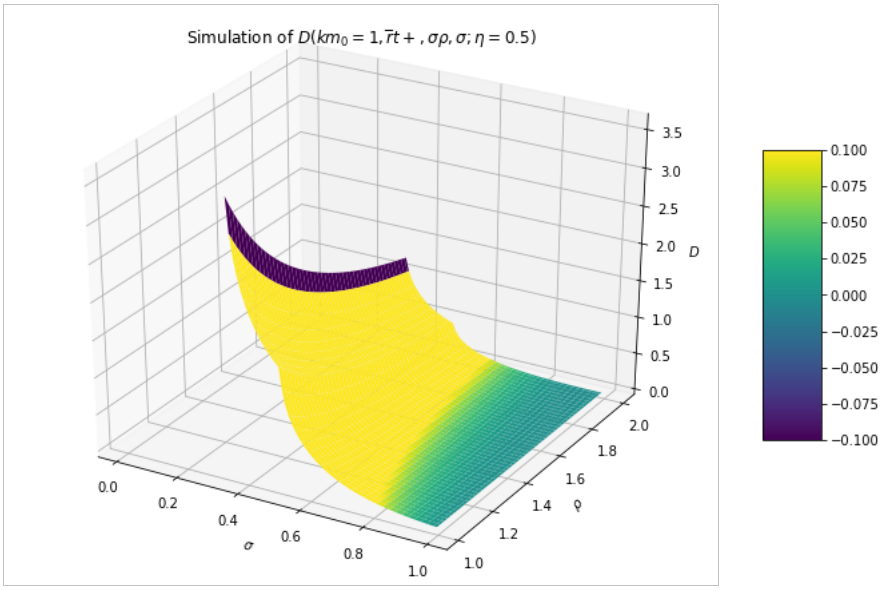


Figure A7: Social Welfare Difference, D, Calibration ($km_0 = 1, \bar{r}_t^{***} = \bar{r}_{t+}$)

8. Social Welfare Difference, D, Calibration ($km_0 = 10, \bar{r}_t^{***} = \bar{r}_{t+}$)

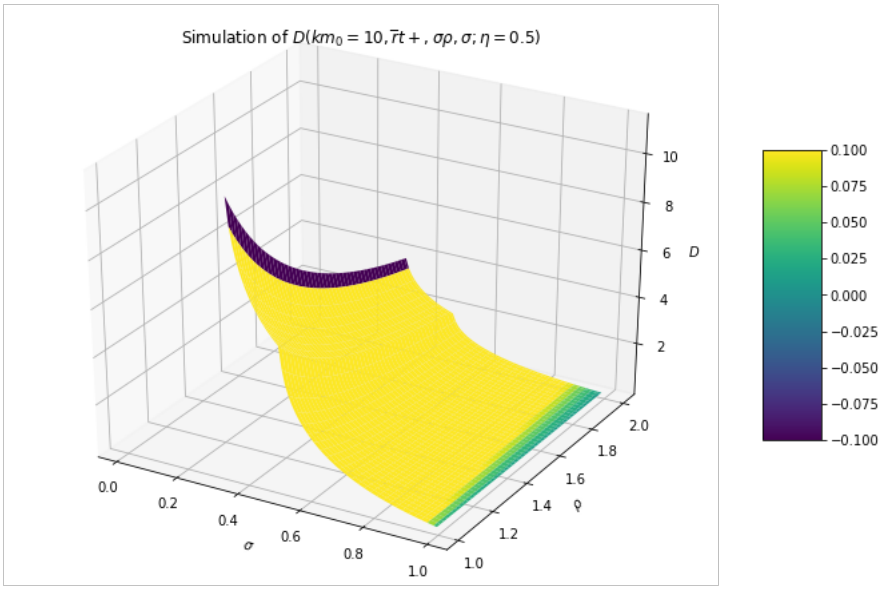


Figure A8: Social Welfare Difference, D, Calibration ($km_0 = 10, \bar{r}_t^{***} = \bar{r}_{t+}$)

9. Social Welfare Difference, D, Calibration ($km_0 = 0.1, \bar{r}_t^{***} = \bar{r}_{t-}$)

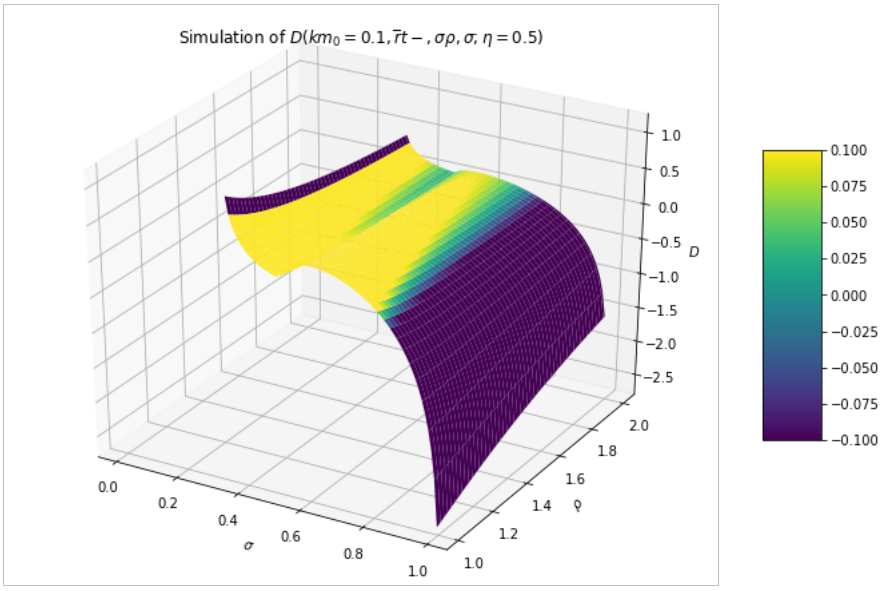


Figure A9: Social Welfare Difference, D, Calibration ($km_0 = 0.1, \bar{r}_t^{***} = \bar{r}_{t-}$)

10. Social Welfare Difference, D, Calibration ($km_0 = 1, \bar{r}_t^{***} = \bar{r}_{t-}$)

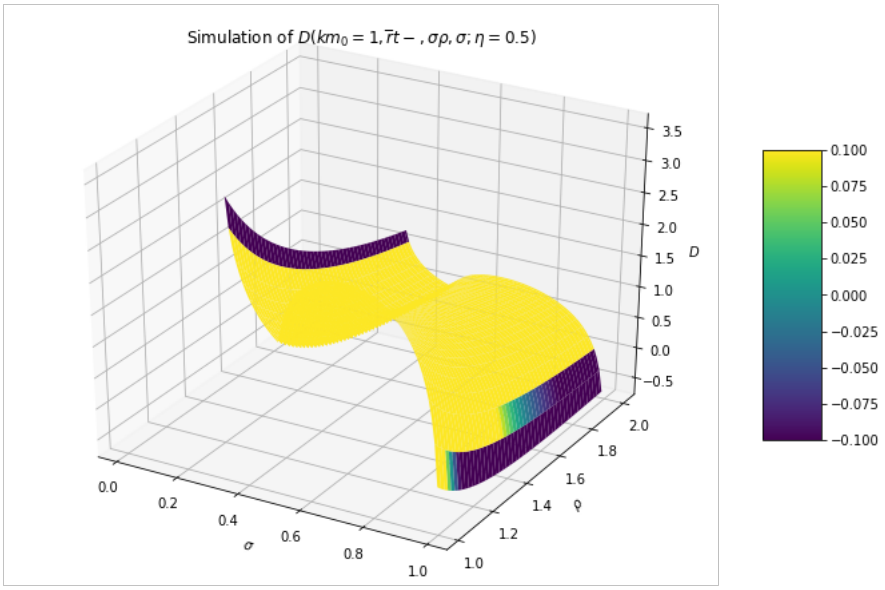


Figure A10: Social Welfare Difference, D, Calibration ($km_0 = 1, \bar{r}_t^{***} = \bar{r}_{t-}$)

11. Social Welfare Difference, D, Calibration ($km_0 = 10, \bar{r}_t^{***} = \bar{r}_{t-}$)

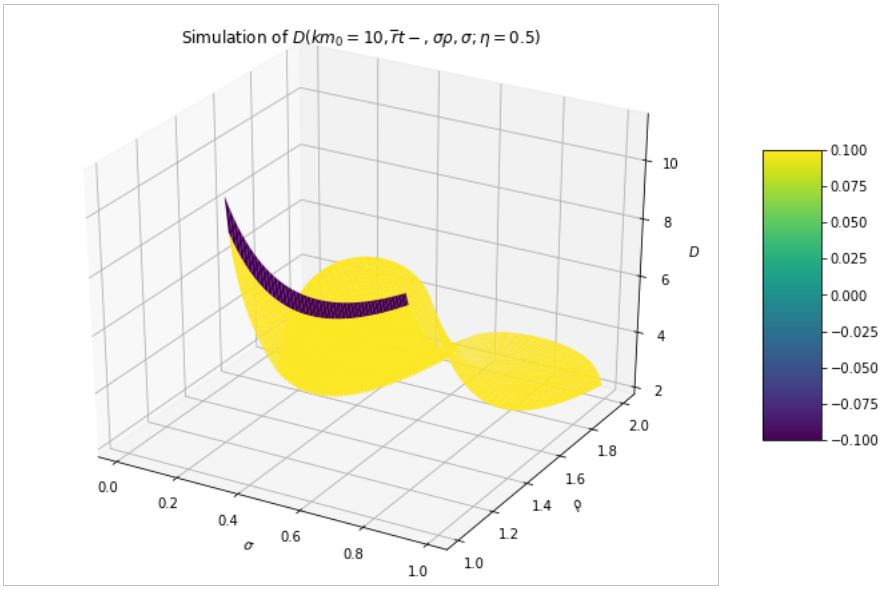


Figure A11: Social Welfare Difference, D, Calibration ($km_0 = 10, \bar{r}_t^{***} = \bar{r}_{t-}$)

12. Social Welfare Difference, D_1, Calibration ($\bar{r}_t^{******} = \bar{r}_{t+}$)

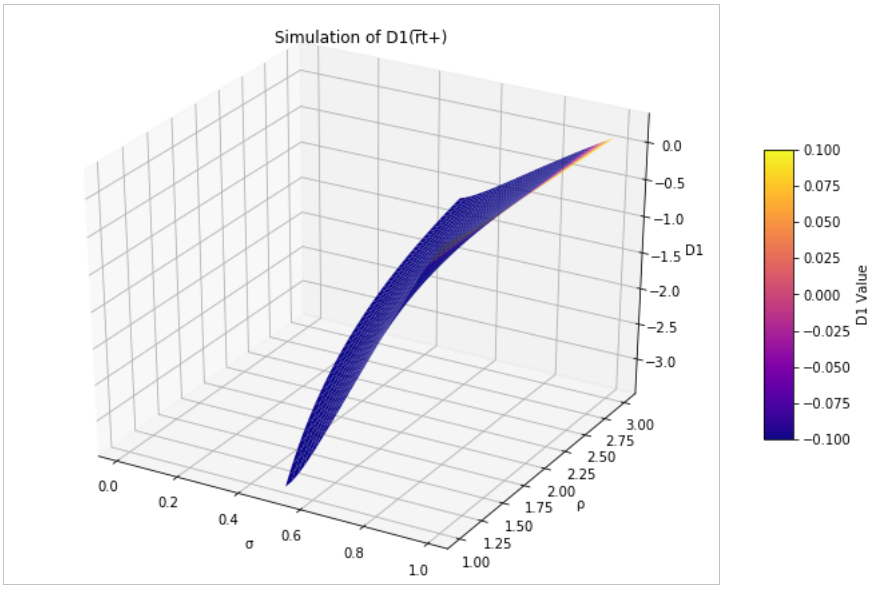


Figure A12: Social Welfare Difference, D_1, Calibration ($\bar{r}_t^{******} = \bar{r}_{t+}$)

13. Social Welfare Difference, D_1, Calibration ($\bar{r}_t^{******} = \bar{r}_{t-}$)

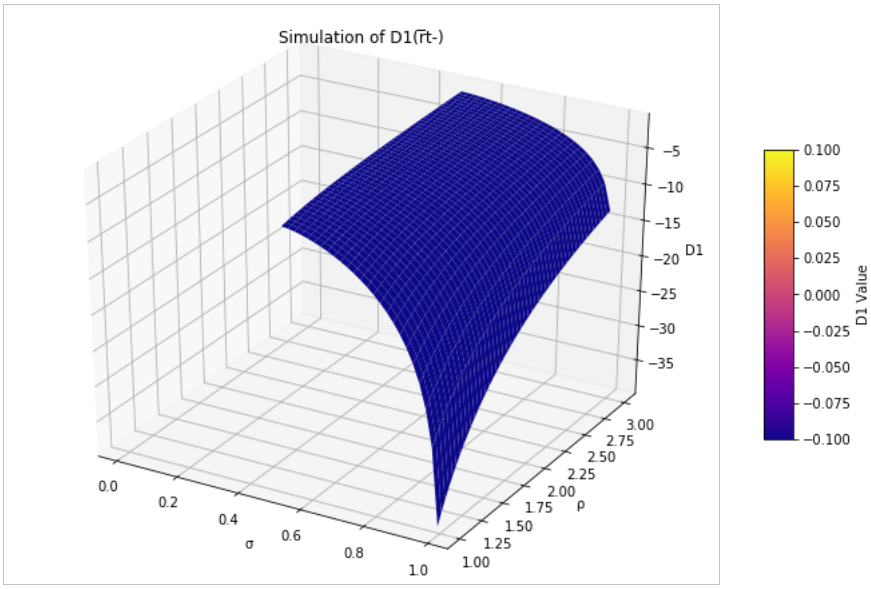


Figure A13: Social Welfare Difference, D_1, Calibration ($\bar{r}_t^{******} = \bar{r}_{t-}$)

14. Social Welfare Difference, D_1, Calibrations with Different Parameters ($\bar{r}_t^{*****} = \bar{r}_{t+}$)

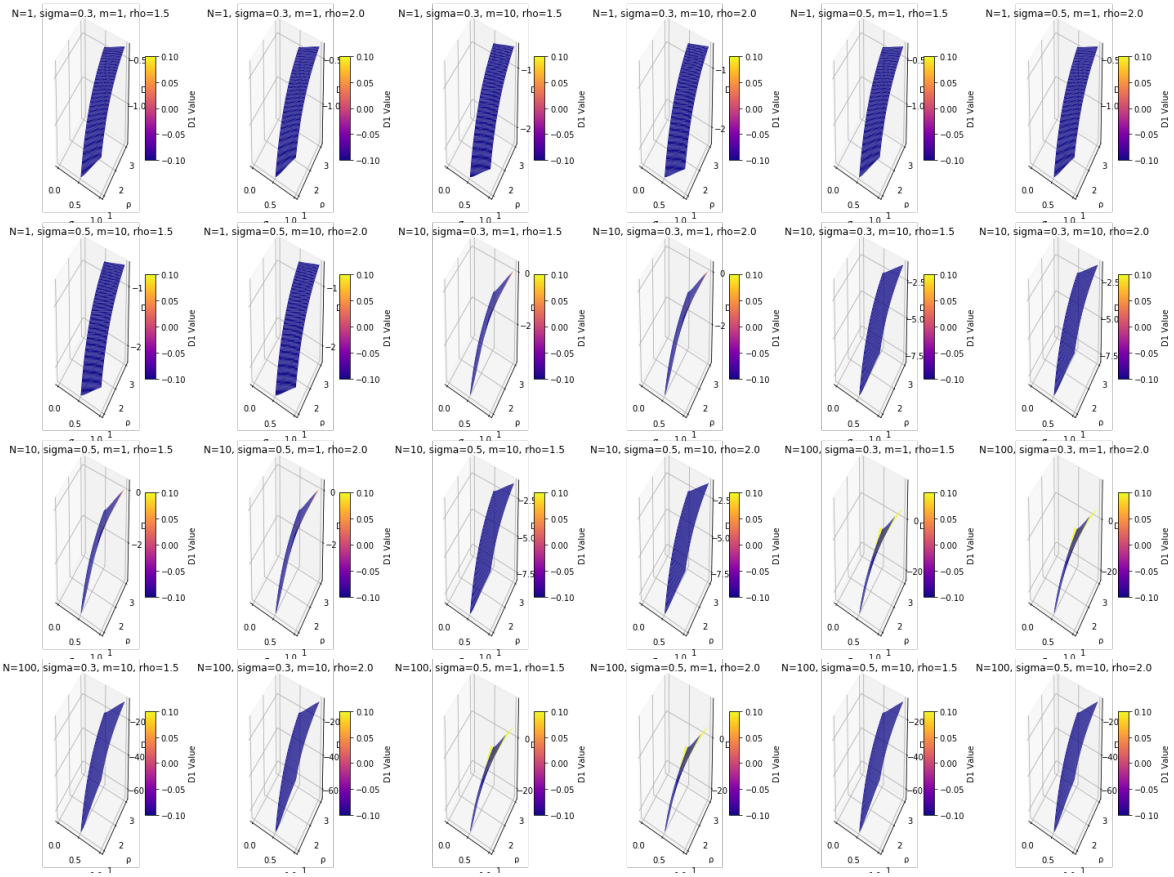


Figure A14: Social Welfare Difference, D_1, Calibrations with Different Parameters ($\bar{r}_t^{*****} = \bar{r}_{t+}$)

15. Social Welfare Difference, D_1, Calibrations with Different Parameters ($\bar{r}_t^{*****} = \bar{r}_{t-}$)

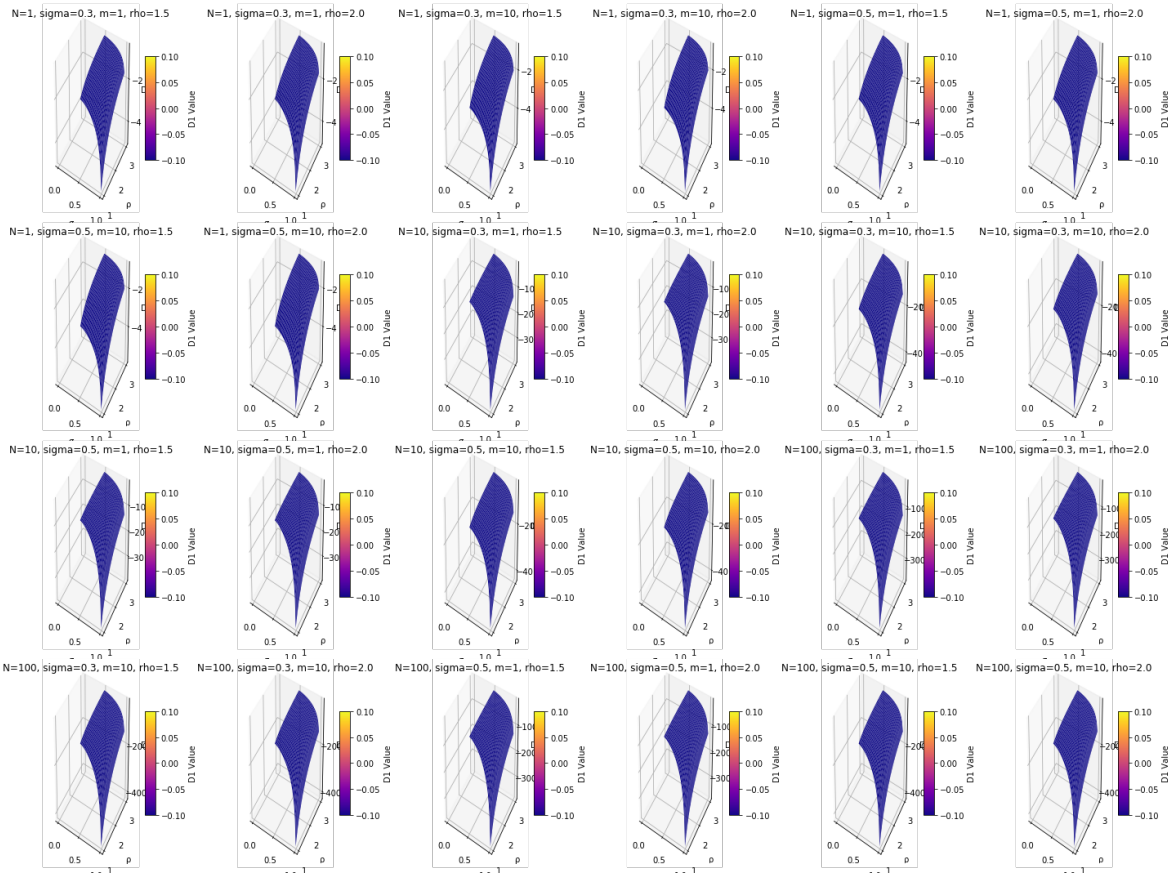


Figure A15: Social Welfare Difference, D_1, Calibrations with Different Parameters ($\bar{r}_t^{*****} = \bar{r}_{t-}$)