Social Structure and Political Dynamics in The Long Run

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Abstract

Why do some societies succeed in breaking away from historical traditions while others remain trapped? This paper proposes a formal model to distinguish different types of authoritarian regimes and investigate their long-term trajectories, highlighting a co-evolution of political institutions and social structures. I model society as comprising three key groups—the ruler, the elites, and the commoners. Different coalitions among these groups determine not only the regime types but also redistribution policies. In the long run, these policies shape the accumulation of wealth and power, where small initial differences can lead to divergent paths of development. Finally, the paper examines several historical case studies to illustrate potential pathways of long-run political development.

1 Introduction

History matters. Contemporary political institutions are often deeply rooted in historical precedents. Many argue that today's authoritarian regimes are products of long-standing non-democratic traditions. However, medieval Europe was also far from democratic, and yet many European societies eventually transitioned to democracies. On the other hand, more recent literature shows that even some authoritarian regimes, such as late imperial China, demonstrated certain "inclusive" features (Jia et al., 2023;

Wang and Yang, 2025). This raises a fundamental puzzle: Why have some societies been able to break away from their historical constraints while others have not?

Historical regimes, though broadly non-democratic by modern standards, exhibit significant variations, ranging from absolute deified rulers in ancient Egypt to kingship-based chiefdoms from the Eurasian Steppe (Anderson, 1974; Blaydes and Chaney, 2013; Boix, 2015). In medieval Europe, monarchs depended heavily on the feudal lords for both military and financial support. By the mid-ninth century, "the freemen stayed at home, and the fighting was done nearly exclusively by the leaders and their vassals" (Boix, 2015). The landed aristocracy remained an important player in European politics and eventually had their positions formalized through the establishment of parliaments (Blaydes and Chaney, 2013). In comparison, the landed aristocracy disappeared almost completely by the eleventh century in China (Tackett, 2014; Wen et al., 2024). Late Imperial Chinese states, instead, cultivated close cooperation between the monarchy and the literate middle class, commonly referred to as the "gentry" (Huang and Yang, 2022). The state relied on the gentry to staff its bureaucracy, and local gentries played increasingly prominent roles in providing state services, such as famine relief, public education, tax collection, and charitable activities (Wang, 2022; Will et al., 2020).

The evidence here not only necessitates a more nuanced analysis of historical regimes but also suggests the possibility that small differences in these historical institutions might have contributed to divergent long-term political outcomes.

This paper proposes a formal model to investigate the long-term evolution of authoritarian regimes, highlighting a co-evolution of political institutions and social structures. A society is assumed to consist of three social groups: the ruler, the elites, and the commoners. Different coalitions among these groups give rise to four distinct types of authoritarian regimes. In a *Feudal Monarchy*, the ruler forms a coalition with the elites to suppress the commoners. A *Populist Autocrat*, on the other hand, co-opts the commoners against the elites. An *Absolute Dictatorship* occurs when the ruler is powerful enough to suppress both the elites and the commoners without needing support from either group. Lastly, in an *Aristocratic Oligarchy*, the elites hold dominance and rule collectively as a group.

More importantly, the model underscores a potential divergence in political development. In a society with a relatively weak ruler, the balance of power between the ruler and the elites can be maintained by appropriate redistribution policies, resulting

in a stable long-run equilibrium of Feudal Monarchy. Conversely, a slightly stronger ruler may adopt a populist approach, gradually encroaching on the wealth and power of the elites. Over time, a Populist Autocracy equilibrium can evolve into an Absolute Dictatorship, while a Feudal Monarchy equilibrium may shift in the direction of ever-increasing elite dominance, depending on the parameters. In other words, small initial differences in the power distribution among social groups can lead to vastly contrasting long-run outcomes.

This divergence stems from a key assumption in the model, which captures different levels of productivity among social groups. Productivity, in this context, can be understood as efficiency in economic production or military capacity; as in the pre-modern world, the ability to defend oneself was closely tied to the ability to generate wealth. Drawing on the extensive literature on institutional decay and principal-agent problems in governance, I assume that the elites have a higher level of productivity than the ruler. In an Absolute Dictatorship or a Populist Autocracy equilibrium, the ruler can exploit his strong position to impose high taxes on the elites, curbing the future growth of the elites' wealth and power. In contrast, in a Feudal Monarchy or an Aristocratic Oligarchy, taxation is insufficient to offset the growth of the elites' wealth, resulting in the society being permanently locked in elite dominance.

I tentatively borrow a term from the literature on populism to name one of the equilibrium types in my model. In the model's framework, a *Populist Autocracy* does not need to be "populist" in the modern sense as involving anti-establishment movements, nor does it require the leader to cater to the general populace (Berman, 2021). I borrow the characterization of populism from Funke et al. (2023) as embodying both an "anti-elite" element and a means of "compensating commoners." An equilibrium outcome qualifies as a *Populist Autocracy* if it meets two criteria: (a) it is characterized by a "coalition of the weak," where the ruler deliberately co-opts a weaker segment of society to sustain his rule, and (b) it establishes complementary institutions to inhibit power accumulation by the elites, in order to prevent the emergence of independent power bases. The idea of a *Populist Autocracy* can very well be applied to contemporary authoritarian politics (Shih, 2022; Egorov and Sonin, 2011; Svolik, 2012).

The paper concludes with three historical case studies to illustrate potential pathways of political development. Feudal Europe demonstrates dynamics in a Feudal Monarchy equilibrium, where weak rulers made policy compromises with the landed

aristocracy, which further weakened their future positions. Imperial China serves as an example of pre-modern Populist Autocracy, where rulers leveraged exam-based bureaucratic recruitment to co-opt the literate middle class and maintain political control of the elites. This exam system functioned, in effect, as an "inheritance tax" on the intergenerational transfer of political power. In the medieval Muslim world, the rulers relied on the *Mamluk* slave-soldier institution to bypass the Arab elites, co-opting a foreign element to sustain their rule. In the long run, these institutions illustrated interesting divergent dynamics where social structures and political institutions reinforced each other.

The remainder of the paper is structured as follows. Section 2 reviews the related literature. Section 3 provides the conceptual framework and a brief overview of various types of historical authoritarian regimes. Section 4 introduces the theoretical model, initially as a stationary dynamic game, followed by a dynamic model with non-overlapping generations that incorporates wealth production and power accumulation. Section 5 explores three historical case studies on Feudal Europe, Imperial China, and the medieval Muslim World to illustrate potential pathways of political development. Finally, Section 6 concludes the paper and discusses the broader implications of the findings. The proofs for the propositions are provided in the Online Appendix.

2 Literature

An institutional change can be induced by at least two forces: an exogenous shock to the environment, which renders a disruption to the existing equilibrium, or a gradual, incremental, and endogenous process of institutional evolution, driven by behaviors of the participants within the institution. While most institutional changes are probably caused by a combination of exogenous and endogenous factors, the literature has long put more emphasis on events that are discontinuous and dramatic (Mahoney and Thelen, 2010). We have ample studies to explain crucial moments when existing institutions were upended and replaced with fundamentally new ones. However, equally useful tools are still lacking for explaining the more gradual evolution of institutions once they have been established (Greif and Laitin, 2004; Pierson, 2004; Mahoney and Thelen, 2010).

By adopting a dynamic model with power accumulation, this paper strives to explain both institutional continuity and changes at the same time. Joining a recent

strand of literature in formal theory, such as Greif and Laitin (2004), Svolik (2009) and Acemoglu and Robinson (2006, 2023); Acemoglu et al. (2015), this paper hopes to bridge the literature that emphasizes "history dependence" and "institutional lock-in," on the one hand, and gradual institutional changes on the other (for a survey on this literature, see Gailmard, 2023). Similar to many works in the literature, this paper puts emphasis on endogenous sources of change due to power accumulation rather than exogenous sources of change, such as technological advancement or natural disasters.

The literature on institutional development often categorizes societies along a one-dimensional spectrum, with inclusive, open-access, and egalitarian systems on one end and extractive, limited-access, and unequal systems on the other (North and Weingast, 1989; North et al., 2009; Acemoglu and Robinson, 2001; Acemoglu et al., 2015). However, comparative history scholarship shows that some regimes, such as Imperial China, exhibited both repressive and inclusive elements (for a discussion of empirical evidence, see Jia et al., 2023). To address this puzzle, this paper introduces a novel framework to differentiate between various types of authoritarian regimes and capture a broader range of political dynamics. Importantly, I do not require the political regimes to be representable by an ordered set as in Acemoglu et al. (2015).

This paper studies political development in the very long run, adopting a time frame spanning centuries, even millennia. When viewed from such a long durée, institutions can no longer be treated as "the rules of the game" (North and Weingast, 1989). Rather, institutions have to be considered endogenous and dependent on other slow-moving factors. This paper identifies two such factors: social structure, or power distribution among social groups (Mahoney and Thelen, 2010; Acemoglu et al., 2015), and the level of productivity (Galor and Weil, 2000). Studies that examine the interaction between social structure and political development have a long history, such as Skocpol (1979); Mann (1986); Moore (1993); Goldstone (1991); Acemoglu and Robinson (2001); Boix (2015); Wang (2022), not to mention classical theories by Karl Marx and Max Weber. This paper builds on the insights of this large literature. However, I do not intend to make much contribution to social theories of political development. Instead, I take a highly stylized and reduced-form characterization of social structure and focus on the issue of political dynamics. I aim to explain not only how social structures influence institutional evolution but also how societies developed different social structures in the first place.

Moreover, because of the unusually long time horizon, this paper utilizes a non-

overlapping generation model with short-lived individuals, rather than a model with perfectly forward-looking rationality. In other words, it is assumed that individuals understand the strategic implications of their behavior in the short run, but they are unable to foresee the gradual build-up of institutions in the long run. I believe this assumption provides an appropriate framework for the analysis of institutions in the very long run.

This paper is closely related to the literature on authoritarian politics. The literature has long identified two fundamental threats to autocrats' survival: elite defection and mass rebellion (De Mesquita et al., 2005; Svolik, 2012). Various studies examine autocratic institutions that facilitate power-sharing with the elites, including ruling parties, legislature, cabinet, and coercive apparatus (Boix and Svolik, 2013; Myerson, 2008; Geddes et al., 2018; Gandhi, 2008; Magaloni, 2008; Francois et al., 2015; Meng and Paine, 2022; Meng et al., 2023). Nevertheless, most of these institutions function better when the power distribution of the ruler vis-a-vis the elites is more balanced and break down easily when the power distribution gets lopsided. This theoretical implication stands in sharp contrast with the historical evidence of long-lasting authoritarian regimes.

To explain this puzzle, this paper distinguishes between different types of historical authoritarian regimes. Importantly, I do not make the assumption that the ruler has to select members of his coalition from the elites. Instead, I allow the ruler to form a "coalition of the weak" by co-opting commoners or members of other weaker segments of society. These institutions are self-enforcing and re-enforcing, and they mitigate both threats of elite defection and mass rebellion at the same time.[1] The theoretical predictions and historical case studies suggest that the concept of a *Populist Autocracy* can be a useful candidate to explaining the puzzling stability of some of the famous historical authoritarian regimes.

Lastly, this study contributes to the literature on Political Divergence. A growing body of work investigates the divergent paths of political development across different regions. Historically, different regions displayed vastly contrasting long-run outcomes, such as political unification vs. fragmentation, centralization vs. decentralization, and consolidation of absolutism vs. the rise of democracies. Factors that have been proposed to explain this divergence include geography, agriculture, war and conflict,

[1]By allowing the ruler to co-opt the commoners, this paper is also related to the literature on authoritarian control, such as Geddes (1999); Magaloni and Kricheli (2010); Svolik (2012); Gandhi and Przeworski (2007); Mattingly (2019); Liu (2023), among others.

critical junctures, colonialism, culture, social organizations, economic institutions, and many more (Diamond, 1997; Greif and Tabellini, 2010; Kuran, 2011; Blaydes and Chaney, 2013; Stasavage, 2016; Fukuyama, 2011; Hariri, 2012, 2015; Dincecco and Wang, 2018; Ko et al., 2018; Ma and Rubin, 2019; Fernández-Villaverde et al., 2023; Greif et al., ming; Huang and Yang, 2022; Olsson and Paik, 2016, 2020; Cox, 2017). Most closely related to this paper, Stasavage (2016) and Ahmed and Stasavage (2020) contrasts the bureaucratic path of political development in China with the assembly-based path in Europe from the perspective of information asymmetry and administrative cost. Jia et al. (2023) attribute autocratic stability in historical China to a more equal power distribution between the elites and the people as compared to Europe.

Building on the literature, I propose an explanation for political divergence — through the mediating effect of social structures, which can be understood as quasi-parameters in Greif and Laitin (2004). Since a society's social structure has its own dynamics and can persist long after formal institutions collapse, a positive feedback loop could emerge between social structures and political regimes. This paper shows how small initial differences in the ruler-elite power balance could lead to the entrenchment of autocracies in East and Central Asia and the consolidation of aristocracy in Europe, with the latter eventually establishing constitutional monarchy and parliamentary democracies. Small initial differences were amplified by self-reinforcing institutions. Since political institutions can have a profound influence on economic development, it is natural to hypothesize that the political divergence studied in this paper may have contributed to long-run economic divergence (Jones, 2003; Diamond, 1997; Pomeranz, 2009; Rosenthal and Wong, 2011; Brandt et al., 2014), an issue that is beyond the scope of this paper and calls for future studies.

3 Social Structure and Regime Types: A Conceptual Framework

Throughout history, human societies are organized through networks of power. These power bases could be derived from multiple sources; as Mann (1986) famously argued, they could be ideological, political, economic, or military. These sources of power were rarely distributed evenly among the general population. Instead, they tend to concentrate within specific individuals or groups. Such elites derive their influence either from inherited status—what Weber (1946) refers to as "traditional authority"

—or through various forms of "merit," such as military prowess, business expertise, personal charisma, and more (Weber, 1946; Mann, 1986; Tilly, 1992).

These unevenly distributed powers gave rise to different social structures. Tribes and chiefdoms had low levels of inequality, while established states were usually accompanied by entrenched elites (Boix, 2015). In regions with early development of bureaucratic states, such as China, Egypt, and the Middle East, the capacity of the central states, and by extension, the power of the ruler, was relatively strong compared to other groups of society (Blaydes, 2017; Fukuyama, 2011). There could also be changes over time. The early Roman Republic, for example, started with a relatively low level of wealth inequality. The quick expansion of the empire greatly boosted the wealth and power of the elites and gave rise to the strong dominance of the aristocracy (Hopkins, 2012).

In a highly stylized fashion, the following Figure 1 captures variations across societies with three social groups: the ruler, the elites, and the commoners. Some societies were dominated by the elites, as shown in Figure 1(a). Examples of such societies include the late Roman Republic and medieval Italian city-states. Some had a balance of power between the ruler and the elites, such as in medieval England or France. Others had supreme rulers with concentrated power, as depicted in Figure 1(b). Examples of such societies include ancient Egypt and late Imperial China.

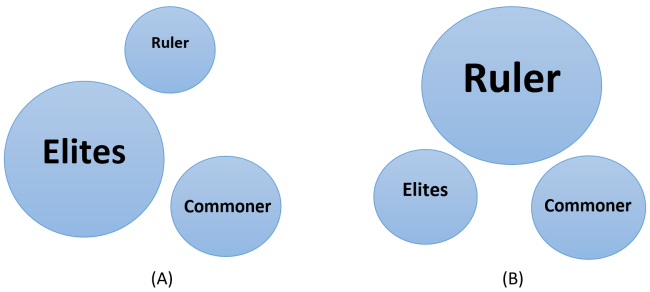


Figure 1: Different Social Structures

In the long run, political regimes emerge endogenously through interactions, bargaining, and compromises among various social groups. Different social structures often give rise to distinct political regimes.

Observation 3.1 (**Absolute Dictatorship**). *If the ruler is the dominant player in the society, he can suppress all other social groups without worrying about the security of his rule. In this case, the ruler can tax both the elites and the commoners heavily.*

Arguably, it is hard to imagine a ruler who would be powerful enough to rule without any additional help. De Mesquita et al. (2005) argues that all rulers need supporters; they only differ in the sizes of their coalitions and the identity of their supporters. For instance, most medieval European monarchs formed coalitions with the landed aristocracy.

Observation 3.2 (**Feudal Monarchy**). *If the elites are strong, they can threaten to rebel against the ruler and demand favorable concessions. In this case, it is optimal for the ruler to tax the commoners and ally with the elites.*

Observation 3.3 (**Aristocratic Oligarchy**). *If the elites are the dominant group in the society, they can overthrow the ruler and establish a rule of the oligarchs.*

The two types of regimes described above are probably the most prevalent in the pre-modern world. European monarchs relied heavily on the landed aristocracy for financial revenue and military support, and the majority of military conflicts prior to the seventeenth century in Europe stemmed from disputes among members of the elite class. However, this may not be the case in some other societies. In Fukuyama's words, "*we tend to see monarchs in agrarian societies as just other members of a predatory elite, ..., In fact, there was almost always a three-sided struggle going on in these societies, among the king, an aristocratic or oligarchic elite, and non-elite actors like peasants or townsmen. The king often took the side of the non-elite actors against the oligarchy, both to weaken potential political challenges and to claim his share of tax revenues*" (Fukuyama, 2011, p.194). Both Persian and Chinese political theories considered it the responsibility of an ideal monarch to protect the peasantry from the rapacious behavior of landlords and other elites, in the name of justice and political stability (Fukuyama, 2011). I summarize this phenomenon with the following observation.

Observation 3.4 (**Populist Autocracy**). *If there is a balance of power between the ruler and the elites, the ruler may choose to co-opt other social groups to sustain his rule. Since the commoners or other weaker segments of society are cheaper to co-opt than the elites, the ruler may find it more beneficial to form a "coalition of the weak," co-opting the commoners and taxing the elites.*

This last type of historical regime warrants some discussion. Given the low level of human capital and the high cost of coordination involved in a popular uprising,

the power of the elites is usually stronger than that of the commoners in pre-modern societies. Therefore, once the elites and the commoners overthrow the ruler, either a new ruler will be created, or the elites will dominate future periods collectively as a group and be able to set the maximum tax rate on the commoners. Foreseeing this future outcome, the commoners would be inclined to ally with the ruler. This idea is similar to the "slippery slope" dynamics highlighted by Acemoglu et al. (2015), and it makes the commoners cheaper to co-opt than the elites. In this case, the ruler can take the initiative and make an enticing offer to the commoners to enlist their support, taxing the elites and compensating the commoners.

Moreover, in a dynamic setting, exogenous shocks that disrupt ruler-elite power balance can trigger cascading effects with long-term consequences. I will formalize these discussions in the following section with a model of dynamic game theory.

4 A Formal Theory

4.1 Model Setup

There are three social groups in the society: one ruler, N elites, and M commoners. N and M are positive integers. Each individual is endowed with an amount of wealth. This wealth should be understood as encompassing various forms of assets, including human capital, physical capital, and socio-political capital. It represents both the economic wealth and the political power of an individual. In the following, I will use wealth or asset interchangeably when referring to the endowment of the individuals. The ruler's wealth is denoted by w_r. Each member of the elites has an asset of w_e, and each member of the commoners has an asset of w_c. W_e and W_c indicate the total wealth of the elites and that of the commoners, respectively. In other words, $W_e = Nw_e$, and $W_c = Mw_c$. I assume $w_r > W_c$ and $W_e > W_c$, but the relationship between w_r and W_e is un-specified.

Consider an infinite-stage game. In period t, the society has a wealth endowment of (w_{rt}, w_{et}, w_{ct}). A stationary economy with no asset accumulation best captures pre-modern agricultural societies (Galor and Weil, 2000). Therefore, in this section, I will focus on a stationary economy with no wealth accumulation. A dynamic setting with wealth accumulation will be studied in Section 4.3. Since each individual's wealth will remain the same in this section unless a rebellion happens, I can drop the time subscript and refer to wealth endowments as (w_r, w_e, w_c). The assets will be used to

generate a per-period income, which is then subject to taxation and can be used for consumption.

In period t, the ruler proposes two tax rates, one for the elites and one for the commoners (τ_{ct}, τ_{et}), and each individual chooses to accept the proposal or to stage a rebellion. I assume throughout the paper that tax rates can be group-specific but not individual-specific. The tax rate can be positive or negative. If negative, it indicates redistribution. I assume that the ruler will stay in power if and only if his supporting coalition has more than the majority of the societal wealth. Whenever a rebellion is staged, $1 - \theta$ of the societal wealth is destroyed in the conflict. As a standard assumption, $\theta \in (0, 1)$. The winning coalition will split the losing group's wealth among them, and each individual gets a share that is proportional to his personal wealth as compared to the total wealth of the winning coalition. For simplicity, I assume that once a rebellion breaks out, everyone in the society has to take a side.

In the period following a rebellion, the social group with the largest amount of wealth from the winning coalition will become the next ruler. There is no information asymmetry in the economy. All players will be able to see others' strategies.

There is a unique consumption good Y in the society. The consumption good can be produced by the assets, and its price is normalized to one. I assume that there are two methods of producing the final good, both linear in capital. The first is a formal or market technology, $Y_{it} = a_i h_{it}$, where h_{it} is the amount of asset devoted to formal production by individual i at time t. Individuals from different social groups may have different levels of productivity, as captured by the parameter $a_i \in \{a_e, a_r, a_c\}$. This production by formal technology is monitored by the state and subject to taxation. The second is an "informal" or home production technology, $Y_{it} = b(w_i - h_{it})$, where $w_i - h_{it}$ is the amount of capital left for home production by individual i at time t. I assume that $min\{a_e, a_r, a_c\} > b$; thus, market production is always more productive. The only role of home production in my analysis is to ensure an equilibrium tax rate of less than 100 percent; because while taxes can be imposed on market production, home production is not taxable. A high value of b implies that only limited taxes can be levied on income.

The analysis can be simplified by exploiting two features of the model. First, the capital allocation decision takes a simple form: if the tax rate $\tau_{it} > (a_i - b)/a_i$, then members of the social group i will allocate all their capital to home use, and the ruler loses all tax revenue from this social group in time period t. On the other hand, if

$\tau_{it} \leq (a_i - b)/a_i$, then members of social group i allocate all their capital to market use. No ruler would ever choose $\tau_{it} > (a_i - b)/a_i$, so we can restrict attention to $\tau_{it} \leq (a_i - b)/a_i$ and $h_{it} = w_i$, which greatly simplifies my analysis. Let $\bar{\tau}_i = (a_i - b)/a_i$ denote the maximum possible tax rate for social group i, $i \in \{e, c\}$. The maximum level tax rate is different for each social group, with $\bar{\tau}_e = \frac{a_e - b}{a_e}, \bar{\tau}_c = \frac{a_c - b}{a_c}$.

Second, all members of the elites have identical preferences, and all commoners have the same preferences. When it comes to the rebellion decision, there is no "free-rider problem" because if an individual chooses not to take part in the rebellion, he will be excluded from the resulting redistribution. Therefore, I can treat all commoners as one player and all elites as one player. This economy can be simplified as a dynamic game among three players/social groups: the ruler, the elites, and the commoners.

All individuals have identical preferences represented by a linear indirect utility function over net income and a discount factor $\beta \in (0, 1)$. All individuals care about their discounted sums of future utilities. Post-tax utility in period t for individual i is given by $u_{it} = (1 - \tau_{it}) a_i w_i$, for $i = e, c$, where τ_{it} is the tax rate imposed on social group i at time t. The social budget constraint implies that the ruler's utility is given by $u_{rt} = a_r w_r + \tau_{et} a_e W_e + \tau_{ct} a_c W_c$, and it has to be non-negative for each period. In other words, if the ruler decides to reward the elites or the commoners with redistribution, he will have to use his own income to make up for any difference between redistribution and taxation.

The timing of events within each period can be summarized as follows.

(1) The ruler announces the tax rates (τ_{et}, τ_{ct}).

(2) The elites decide whether or not to initiate a rebellion.

(3) The commoners decide whether to join the elites' rebellion (if there is one), to start their own rebellion, or to accept the proposal.

(4) The capital stock is allocated between market and home production, and incomes are realized.

I will solve for the equilibrium in the next section.

4.2 Stationary Equilibrium

I impose a few parametric assumptions.

Assumption 1. $W_e \geq W_c$

Assumption 2. $\theta \geq \frac{b}{a_c - a_c\beta + b\beta}$

Assumption 3. *The productivity of the social groups satisfies* $a_e > a_r \geq a_c \geq b > 0$.

Assumption 1 characterizes the power distribution of a pre-modern society. In a pre-modern society, even if the commoners had sufficient power to overthrow the elites, it would be difficult to obtain large-scale coordination. Hence, I assume that the power of the elites is always stronger than that of the commoners. The goal of this assumption is to simplify the discussion and allow us to focus on theoretical implications more relevant for historical political development. In a companion paper of this, I relax this assumption and examine the case when the domain of W_c is unbounded.

Assumption 2 guarantees that the ruler cannot co-opt the commoners with the maximum level of taxation. This assumption is not necessary, but it enables us to focus on the more interesting cases of differential tax rates without considering the mundane scenario where no rebellion ever breaks out even with the maximum level of taxation.

Assumption 3 captures the different levels of productivity across social groups. Most importantly, I assume $a_e > a_r$ to reflect the principle-agent problems in governance. The ruler usually governs a larger area than the elites and has to rely on others to manage the properties for him; this creates the opportunity for others to encroach on the productivity of the ruler's assets. This principle-agent problem would exacerbate as the size of the property increases. Hence, the productivity of the ruler's assets would be lower than that of the elites.

Let $V_{it}(w_e, w_r, w_c)$ denote the discounted continuum utility for an individual from the social group i at time t. In each period, a per-period income is generated for all individuals, which is subject to taxation. As a reminder, θ is the conflict destruction parameter, and β is the discounting factor.

Proposition 1 (**Absolute Dictatorship Equilibrium**). *If* $w_r \geq W_e + W_c$, *the ruler can survive with no additional support. He represses all other social groups and sets the maximum tax rate on both the elites and the commoners,* $\tau_{ct}^{Abs} = \bar{\tau}_c, \tau_{et}^{Abs} = \bar{\tau}_e$.

Proposition 2 (**Populist Autocracy Equilibrium**). *If* $W_e - W_c \leq w_r < W_e + W_c$,

the ruler sets the maximum tax rate on the elites and co-opts the commoners:

$$\tau_{ct}^{Pop} = 1 - \theta(1 + \frac{w_r}{W_c + W_e})(1 - \frac{a_c - b}{a_c}\beta) \leq \bar{\tau}_c, \qquad \tau_{et}^{Pop} = \bar{\tau}_e \tag{1}$$

As a result, the commoners ally with the ruler against the elites.

In a *Populist Autocracy* equilibrium, if the commoners participate in a successful rebellion led by the elites, they will be taxed by the elites at the maximum rate in subsequent periods. This expectation of unfavorable institutional dynamics in the future makes the commoners sympathetic to the ruler and relatively cheap to co-opt. This is the critical dynamics behind Proposition 2.

Proposition 3 (**Feudal Monarchy Equilibrium**). *If $W_e > W_c + w_r$ and θ is sufficiently small, the ruler sets the maximum tax rate on the commoners and co-opts the elites:*

$$\tau_{ct}^{Feud} = \bar{\tau}_c, \qquad \tau_{et}^{Feud} = 1 - \theta(1 + \frac{w_r}{W_c + W_e})(1 + \beta\frac{a_c - b}{a_e}\frac{W_c}{W_e}) < \bar{\tau}_e \tag{2}$$

As a result, the elites ally with the ruler against the commoners.

To make the Feudal Monarchy equilibrium feasible, θ has to satisfy the following condition: $\theta \leq \frac{a_r w_r + a_e W_e + (a_c - b)W_c}{a_e W_e + (a_c - b)W_c \beta}\frac{W_c + W_e}{w_r + W_c + W_e}$. In other words, rebellions have to be sufficiently destructive. Otherwise, the ruler will not be able to afford a redistribution that is large enough to pacify the elites. In this case, the elites will rebel regardless of the distribution.

Corollary 3.1 (**Aristocratic Oligarchy Equilibrium**). *If $W_e > w_r + W_c$ and θ is sufficiently large, the elites will rebel and rule collectively as a group in subsequent periods.*

For a formal statement of the propositions and their proofs, please see the Online Appendix.

The following Figure 2 plots the possible types of equilibrium regimes under a simulation. When the ruler is the dominant player in the society, he can govern unilaterally and suppress all social groups under an *Absolute Dictatorship*, as highlighted in the top left-corner region in Figure 2. Conversely, when the elites hold greater power, the ruler must appease them through reduced taxes or even redistribution, resulting in a

Feudal Monarchy, as highlighted in the bottom right-corner region in Figure 2. When a conflict is not too destructive (with a sufficiently large θ), the compensation from the ruler may fail to placate the elites. In this case, an elite rebellion is inevitable, and an *Aristocratic Oligarchy* will be established in the meshed blue region in Figure 2. Lastly and most interestingly, when power is balanced between the ruler and the elites, as indicated by the narrow stripe of parametric space in the middle of Figure 2, the ruler may choose to co-opt the commoners instead. In this case, the ruler will form a coalition with the commoners in a *Populist Autocracy* and tax the elites heavily.

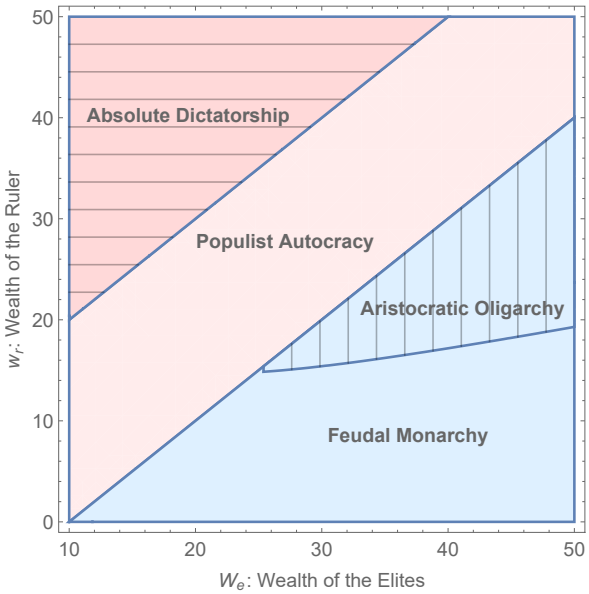


Figure 2: Simulation of Equilibrium Political Regimes

Note: The parametric setting for the simulation: $W_c = 10, \theta = 0.9, \beta = 0.9, a_e = 0.4, a_c = a_r = 0.2$, and $b = 0.1$.

4.3 Political Dynamics

A crucial driver of political dynamics is wealth and power accumulation. Numerous empires collapsed under internal pressure of instability because the central states were unable to control the regional power-holders. This institutional decay is among the leading factors that explain the fall of the Roman Empire (Scheidel, 2019), the Han Dynasty (Fukuyama, 2011), and many other empires of large landmasses. In this section, I maintain the major setting of the previous stationary model with one modification: a member of the elites or the ruler can save a portion of their disposable

income and invest it into their wealth in the next period.

Specifically, at time t, an individual from the social group i receives a per-period income of $a_i w_{it}$. Again, the parameter a_i captures the productivity level of the social group i, and w_{it} indicates the wealth of a member from group i at time t. After paying the tax at a rate of τ_{it}, the individual will consume a $1-s$ portion of the post-tax disposable income and save the rest of it as an investment into his wealth for the next period. If no investment is made, the individual's wealth will depreciate at a rate of $\delta \in (0,1)$. Following the tradition of the literature, I assume that commoners cannot save. Their output level is too low to meet the minimum threshold of investment. Their wealth is also not subject to depreciation. As a result, the wealth of the commoners will not change over time unless a rebellion is staged. On the other hand, the ruler and the elites will automatically save a portion s of their disposable income.

Assumption 4. *The ruler's and the elites' wealth depreciates over time, with the depreciation rate $\delta \in (0,1)$ satisfying $sa_e > sa_r \geq sa_r > \delta > sb$.*

If an individual's savings are lower than his wealth depreciation, his wealth will shrink over time. Otherwise, his wealth will grow. To make things interesting, I assume that $sa_e > sa_r > \delta > sb$. In other words, in the absence of taxation, the wealth of the elites will grow at a speed faster than that of the ruler. The assumption that $sa_r > \delta$ is for simplicity only.[2] Since only relative wealth matters, the critical component of the assumption is that the productivity of the elites surpasses that of the ruler.

I consider a non-overlapping generation model with myopic players. Each player will live for two periods, and he cares about his utility only during these two periods. The tax rates will be set at the beginning of the first period. Once set, the tax rates will persist for two periods and be reset in the third period. I assume there is no commitment problem; once a strategy is chosen, the player will follow through over the two periods.

For simplicity, I assume that the initial wealth of the social groups satisfies $W_{e0} > W_{c0}$ and $w_{r0} > W_{c0}$, but the relationship between w_{r0} and W_{e0} is un-specified. Moreover, the shrinking of W_{et} will stop once it reaches the level of W_{ct}. This assumption better captures a pre-modern society where the power of the elites was stronger than that of the commoners.

[2]This assumption guarantees that either the ruler or the elites will have a wealth greater than the commoners so that we do not need to consider the case when $W_c > W_e + w_r$.

Proposition 4 (**Types of Regimes**). *When $w_{rt} > W_{et} + W_{ct}$, an Absolute Dictatorship equilibrium arises, and the ruler suppresses all other social groups. When $W_{et} - W_{ct} < w_{rt} < W_{et} + W_{ct}$, a Populist Autocracy equilibrium arises, and the ruler co-opts the commoners against the elites. When $W_{et} - W_{ct} > w_{rt}$, either a Feudal Monarchy equilibrium or an Aristocratic Oligarchy equilibrium arises.*

This proposition is similar to the ones in Section 4.2. The only difference is that equilibrium tax rates are slightly revised here since the individuals in this section only care about their utility over two periods. For a formal statement of the proposition and its proof, please see the Online Appendix.

Corollary 4.1 (**Directions of Political Dynamics**). *In an Absolute Dictatorship equilibrium, we have $W_{et+1} < W_{et}$, and $w_{rt+1} > w_{rt}$. In a Populist Autocracy equilibrium, we have $W_{et+1} < W_{et}$. In a Feudal Monarchy equilibrium, if rebellions are not very destructive or the productivity of the elites sufficiently high (i.e. $sa_e\theta \geq \delta$), we have $W_{et+1} > W_{et}$. Otherwise, the evolution of W_{et} is unclear.*

The following Figure 3 provides two cases of simulation for the political dynamics. To facilitate the discussion, I first define the following terms.

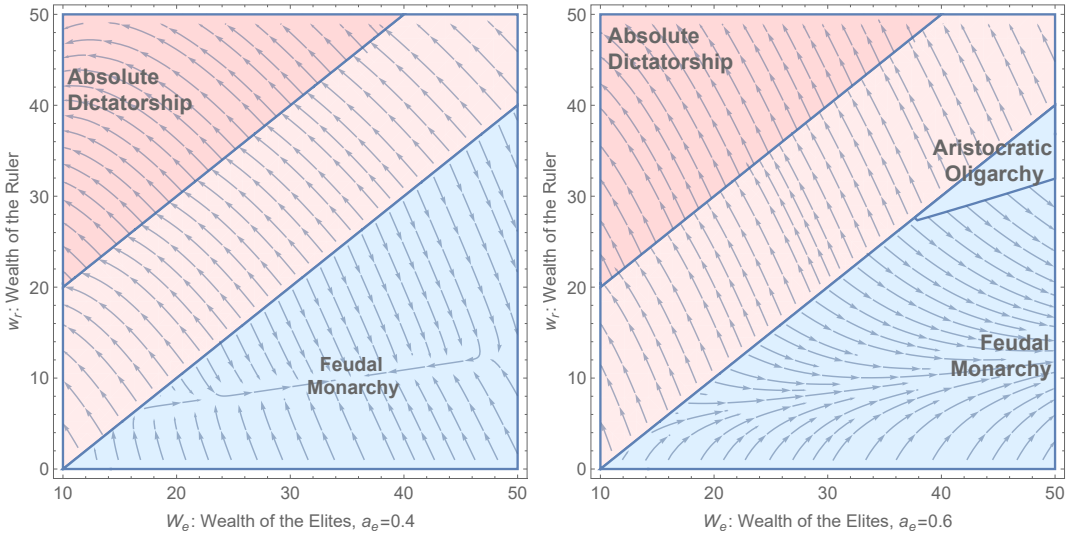
Definition 1 (**Political Divergence**). *I define two societies to be in a Political Divergence if the dynamic evolutions of their state variables (w_{rt}, W_{et}, W_{ct}) lead to different types of equilibrium regimes in the long run.*

Definition 2 (**Political Convergence**). *I define two societies to be in a Political Convergence if the dynamic evolutions of their state variables (w_{rt}, W_{et}, W_{ct}) lead to the same type of equilibrium regmines in the long run.*

Figure 3 illustrates two potential pathways of political divergence. In Figure 3(a), two long-run equilibrium outcomes are possible. When the initial wealth of the ruler is large compared to that of the elites, the society will evolve in the direction of absolutism, where the power balance between the ruler and the elites gets more and more lopsided over time, as highlighted by the red region in Figure 3(a). When the initial wealth of the ruler is small compared to that of the elites, the society will eventually reach a long-run stable *Feudal Monarchy* equilibrium, where taxation keeps the wealth of the elites constant over time, as highlighted by the blue region in Figure 3(a).

The economy in Figure 3(a) has a moderate level of elite productivity, with $a_e = 0.4$. In comparison, Figure 3(b) illustrates an economy where the elites enjoy a substantial

advantage in productivity, with $a_e = 0.6$ and $a_r = a_c = 0.2$. Everything else is identical between the two economies. Interestingly, this one small change is able to induce different long-run outcomes in Figure 3(b). Figure 3(b) has three possible long-run outcomes. When the initial wealth of the ruler is large compared to that of the elites, the society evolves in the direction of absolutism, which is similar to Figure 3(a). However, when the initial wealth of the ruler is small, the society in Figure 3(b) may develop either an *Aristocratic Oligarchy* equilibrium or a *Feudal Monarchy* equilibrium. In both cases, the society evolves in the direction of perpetual elite dominance, where the wealth of the elites continues to grow over time.



(a) (b)

Figure 3: Potential Pathways of Political Divergence

Note: The parametric setting for the above simulations satisfies $\theta = 0.8, W_c = 10, \beta = 0.5, \delta = 0.2, s = 0.5, a_c = a_r = 0.2, b = 0.1$. The difference between the two graphs is due to different levels of elite productivity a_e.

What is depicted in Figure 3 resonates with the development trajectories of many regions of the world. In European states that relied primarily on agriculture, such as medieval England, the power of the elites was entrenched. The rulers had to rely on the elites to sustain their rule both militarily and financially. This balance of power eventually led to a formalized system to coordinate and resolve conflicts between the elites and the crown, as depicted in the blue region in Figure 3(a). Constitutional monarchy emerged as a compromise between the ruler and the elites. In this "grand bargain," the propertied class and hereditary nobility secured property rights and a

degree of autonomy, while the sovereign gained personal security and the right to tax, albeit at a constrained level. Over time, feudalism formalized into parliamentary democracy.

In European states that relied more on long-distance trade, such as the Italian city-states, the elite enjoyed a much higher level of productivity compared to the ruler or the general population. This productivity advantage pushed their society in the direction of ever-strengthening elite dominance, as depicted in the blue region in Figure 3(b).

In comparison, in East Asia, the geography and the early establishment of centralized states provided the rulers with slightly stronger positions to begin with. This mild advantage enabled the rulers to adopt populist institutions, such as the *Civil Service Examination System* (more details in Section 5). These populist institutions allowed the rulers to form "coalitions of the weak," co-opting weaker segments of the population and bypassing the elites. These institutions also helped inhibit the inter-generational transition of power within the elites and worked similarly to an "inheritance tax" on political power. In the long run, these populist institutions curbed the power accumulation of the elites and moved the society in the direction of absolutism, as depicted in the red regions in Figure 3(a) and (b).

One issue is worth noting in Figure 3: what is efficient for the society as a whole may not necessarily be reflected in the political regime that emerges out of it. In both Figure 3(a) and (b), the elites have a higher level of productivity. From a socially optimal point of view, it may be beneficial to leave more productive assets in the hands of the elites. But less efficient *Populist Autocracy* equilibrium and *Absolute Dictatorship* equilibrium can still emerge in both cases as a result of the political dynamics among social groups.

Lastly, not all societies will necessarily diverge. Essentially, either a sufficiently destructive rebellion parameter θ or a sufficiently low level of elite productivity a_e can produce a political convergence towards Absolute Dictatorship. When rebellions are destructive (i.e., small θ), the ruler obtains more leverage over the other social groups and, hence, is able to demand higher taxes in any type of equilibrium. This higher tax rate may prevent the growth of the elites' wealth even in a Feudal Monarchy equilibrium. The Online Appendix Secion C provides several simulations to highlight these possibilities.

4.4 Contingent Institutional Change

The idea of contingent institutional change is not new in political science. Contingent institutional change refers to a type of institutional transformation that is not predetermined by structural factors alone but, rather, depends on specific contextual conditions. This concept contrasts with deterministic views of institutional evolution, which emphasize long-term structural trends, such as economic development or modernization, as the primary drivers of institutional change.

My framework allows us to formalize one type of contingent institutional change, which still depends on the structural factors but not in a uniform way. I define the following term:

Definition 3 (**Contingent Institutional Change**). *I define a society to demonstrate contingent institutional change in response to an exogenous event if the post-event evolutionary direction of the society's state variable* (w_{rt}, W_{et}, W_{ct}) *depends on the society's specific location in the parametric space at the time of the event.*

Figure 4 illustrates two possible pathways of institutional change; yet, Figure 4(a) is not contingent while Figure 4(b) is. In both cases, an exogenous event strengthens the power of the ruler. This event may take the form of a ruler forming a marriage alliance with a powerful ally or simply succession by a more capable heir. Regardless of the specific cause, societies A and B move upward in both Figure 4(a) and 4(b).

However, in Figure 4(a), all social transitions occur within the boundaries of the Feudal Monarchy region. In other words, the type of equilibrium that emerges before and after the shock remains unchanged. After succession by a more capable ruler, societies A and B continue to move in the direction of increasingly powerful elites. Despite having different starting points, A and B will ultimately converge on the same long-term outcome.

In contrast, Figure 4(b) illustrates a case of contingent institutional change. While both societies A and B move upward in Figure 4(b), this shift allows A to break free from its long-term constraints. The exogenous event moves A from the Feudal Monarchy equilibrium region to the Populist Autocracy region, setting it on an entirely different long-term trajectory.

This type of contingent institutional change is caused by societies residing in different basins of attraction. While two societies may appear similar on the surface, subtle differences in their state parameters can lead to the emergence of distinct polit-

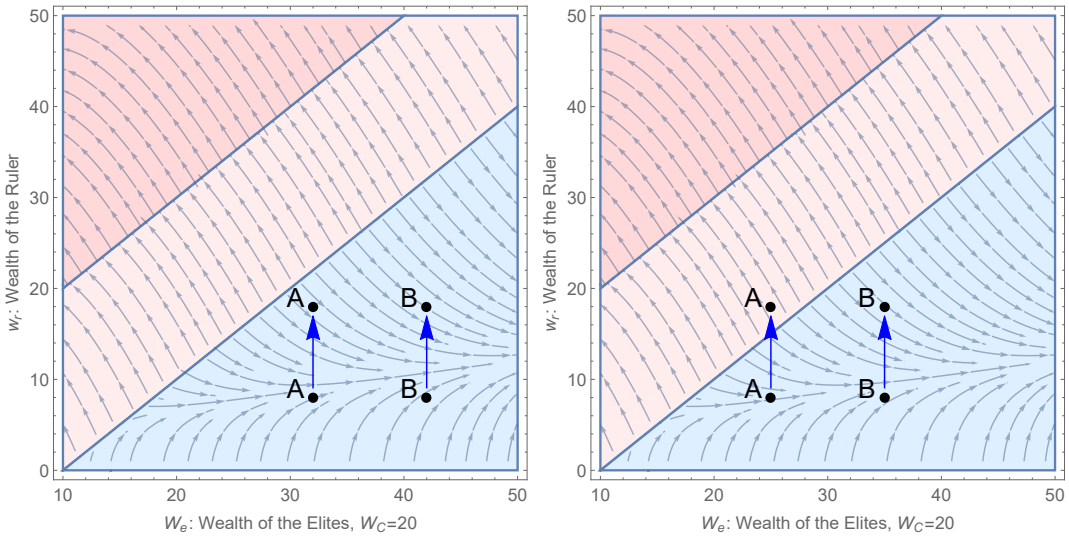


Figure 4: Potential Pathways of Contingent Institutional Changes

Note: The parametric setting for the above simulations satisfies $\theta = 0.8, W_c = 10, \beta = 0.5, \delta = 0.2, s = 0.5, a_c = a_r = 0.2, a_e = 0.5, b = 0.1$.

ical institutions. These institutions, in turn, shape the dynamics of wealth and power accumulation, ultimately setting the societies on divergent trajectories.

5 Historical Case Studies

Until now, I have employed the concept of a *Feudal Monarchy* or a *Populist Autocracy* without much substantive justification. In the following, I provide three historical case studies to illustrate an application of the model to reality. Specifically, in the theoretical framework of this paper, a Populist Autocracy does not need to be "populist" in the modern sense as involving anti-establishment movements. A polity qualifies as a Populist Autocracy if it meets two criteria: (a) it is characterized by a "coalition of the weak," where the ruler deliberately co-opts weaker segments of society to sustain his rule, and (b) it establishes complementary institutions aimed at inhibiting the accumulation of power by anyone other than the ruler, in order to prevent the emergence of independent power bases.

5.1 Feudal Europe

It should be noted that for a society to reside in a Feudal Monarchy equilibrium, it does not mean that the same family stays prominent forever. In fact, membership in the elite group can be quite volatile even for societies that are considered "aristocratic" and "stable." For example, among the 63 English noble families of 1559, 26 had gone extinct in a century (Stone, 1967, p.79). In the French county of Forez, about 13 percent of the 215 noble families in the 13th century had disappeared by 1300. Another 53.6 percent went extinct in the next century, and another 55 percent disappeared in the one after the next (Perroy, 1962, p.31).

Despite the ups and downs experienced by individual families, the broader social structure can remain relatively stable as long as prevailing institutions uphold it. A case in point is the Earl of Pembroke, a prominent title in the Peerage of England. Originally created in the 12th century by King Stephen of England, the title has been recreated ten times in history. The title was forfeited on three occasions when its holders rebelled or plotted against the king and reverted back to the crown multiple times due to the extinction of the male line. Yet each time, the title was reestablished—sometimes for a family relative of the previous holder, at other times for an entirely new family as a reward for their loyalty to the crown. Although the title switched hands from one family to another, the feudal rights associated with the title remained relatively stable.

How did such social-structure-preserving institution first emerge? Scholars generally agree that after the collapse of the Roman Empire and subsequent economic reversion, the Carolingian kings lacked both a bureaucratic administration to manage their lands and the financial resources to maintain a standing army. As a result, they depended on the nobility to provide soldiers and lead their armies as generals. The weak medieval monarchs were forced to compromise with the landowners, granting land rights in exchange for military service, which led to the establishment of feudalism.

The terms of these feudal agreements reflected the prevailing balance of power. Beeler (1971) observed, amusingly, that contributions to the feudal hosts often bore little relation to the actual military capacity of the nobility. For instance, by the 12th century, the Count of Champagne commanded the service of about 2,300 knights, yet he fulfilled his obligations to the crown by sending a mere ten knights. Similarly, the Duke of Normandy fulfilled his military obligation with ten knights, albeit his own vassals were responsible for the service of about 1,500 (Beeler, 1971, p.37). Clearly, the balance of power favored the nobility rather than the crown.

This dynamic persisted for centuries. In some parts of Europe, such as England, the monarchs repeatedly resorted to selling their own land to finance the army and sustain the government (North and Weingast, 1989). Over time, a series of events culminated in the establishment of a constitutional monarchy. In some parts of Europe, it would take a range of new developments, including the gradual development of bureaucratic capacity (Grzymała-Busse, 2023), the Military Revolution (Gennaioli and Voth, 2015), and, in some cases, a series of resourceful and strong-minded rulers (Fukuyama, 2014) to eventually reshape the developmental trajectory towards centralization. However, despite these developments, European rulers never achieved the level of despotic authority seen among their Eastern counterparts.

Social structures and political institutions can reinforce each other. This positive feedback loop can firmly trap a society with a weak ruler in a Feudal Monarchy equilibrium. The ruler made policy compromises with the elites to secure their support. These compromises, combined with the elites' natural advantage in productivity, ensured that their power grew at a faster rate on average than that of the rulers. In the absence of exogenous shocks—such as the introduction of new technology or the emergence of a new enemy—a society in a Feudal Monarchy equilibrium could not easily break free from its historical constraints.

5.2 Imperial China

Imperial China has long been considered a prime example of *oriental despotism*, a highly centralized and stable authoritarian regime ruled by supreme dictators (Wittfogel, 1957). What is less known, however, is that from the second until the seventh century CE, China was in fact closer to a model of Feudal Monarchy rather than that of Absolute Dictatorship.

Aristocratic families monopolized political offices at the time. This system enabled the elites to secure power transitions within their families for centuries, leading to the emergence of "great aristocratic clans" which, in many respects, resembled the landed aristocracy of medieval Europe (Johnson, 1970). Aristocratic strongholds and fortresses dotted the landscape (Chen et al., 2024); and private armies were pervasive, funded by private wealth in the form of "grand estates" (Ebrey, 1978). As shown in Section 4, the ruler in a Feudal Monarchy equilibrium may find it challenging to contain the growth of local powerbases. The gradual decline of the central government, the rise of regional powerbases, and external pressures from the nomadic groups eventually led

to the collapse of the Eastern Han dynasty in the third century, mirroring the fall of the Roman Empire.

The collapse of the Eastern Han dynasty ushered in a period of fragmentation that lasted almost four centuries. During this period of disunion, various local power-holders vied for supremacy, with the Northern Wei dynasty eventually re-obtaining state-building via strategically compensating the local aristocrats with offices in the central bureaucracy (Chen et al., 2024). However, the Northern Wei, as well as its successor dynasties such as the Sui and the Tang, continued to rely on the support of the aristocratic clans; and when they lost it, internal instabilities quickly followed.

The exam system both marked and contributed to China's transition from a Feudal Monarchy equilibrium to a Populist Autocracy. In East Asia, an important institution was the *Civil Service Examination System*, hereafter referred to as the "exam system" or "*Keju*." *Keju* was a method of recruiting officials for the imperial government on a large scale. It was based on standardized written tests on Confucian classics and, in its fully developed form, was open to the great majority of the male population (for an overview of the history of the exam system, see Wang and Yang (2025)).

Keju began to take shape in the late sixth century as an outgrowth of previous recruitment methods. It started on a small scale. Over the following centuries, several exogenous events steadily weakened the aristocratic families, including a destructive peasant rebellion and continued civil wars (Tackett, 2014). Many scholars believe that, in Wang's (2022) words, "the demise of the medieval aristocracy marks the crossing of one of the most important watersheds in Chinese history." What is interesting, however, is not the destruction of one aristocratic generation but the almost perpetual absence of another generation emerging. An altered social structure in the 10th century, together with other changes in the economy and technology, enabled the expansion and formalization of the exam system, very much like an upward shift of society A in Figure 4(b) allowed its policy to break free from historical constraints.[3]

To make the exam system truly accessible to a broad section of the male population, the imperial government established what could be considered the world's first public school system, extending to the prefectural level by the eleventh century and down

[3]Several other events or developments at the time were also believed to have contributed to this transition, such as the Huang-chao rebellion, the invention of printing, the spread of literacy, and the economic development of the South. These issues have been studied extensively in the famous "Tang-Song transition" literature. Many historians have contributed to this literature, such as Naitō Konan, Peter Bol, Patricia Ebrey, Robert Hartwell, Paul Smith, Denis Twitchett, Chen Yinke, and many others.

to the county level by the fifteenth century (Wang and Yang, 2025). Aided by the widespread of printing technology and state-sponsored education, *Keju* was able to attract large-scale participation. Ho (1962) shows that for the entire 14th to 19th century period, more than 30 percent of the candidates that successfully passed the exam system came from families with no records of any degrees or official positions in the three preceding generations.

The rulers of Imperial China used this institution to co-opt the literate middle class and to form a "coalition of the weak." Compared to other pre-modern states, late Imperial China was able to orchestra close collaboration between the state and the broader society. The state relied on the gentries to staff its bureaucracy, facilitate tax collection, and aid in various state functions, such as community education, charities, famine relief, and the promotion of state ideology (Elman et al., 2000; Wang, 2022; Will et al., 2020). Bai and Jia (2016) further demonstrate the impact of *Keju* in pacifying the general populace, as its abolition triggered widespread popular uprisings in the early twentieth century.

In addition to being an effective tool for co-opting commoners, the competitive nature of the exam system also curbed power accumulation within the elites. Various measures were implemented to maintain the fairness and competitiveness of *Keju*.[4] The exam system was also aided by other institutional arrangements, such as the absence of primogeniture, the equal division of inheritance, and the gradual decline of Yin privilege, which used to enable descendants of elite families to enter government automatically. Wang and Yang (2025) show that the aristocratic advantage in political selection and promotion continued to decline over the millennium following the establishment of *Keju*. In the seventh century, approximately 60 to 80 percent of ministers had parental ties to the government.[5] After the introduction of *Keju*, this trend began to wane progressively, eventually reaching a low level of below 20 percent after the seventeenth century.

Viewed from the theoretical framework of this paper, the exam system fulfilled two political functions simultaneously: it limited the power accumulation of the elites, and it compensated the commoners with potential opportunities for social mobility. It embodied both an "anti-elite" mechanism and a means of "compensating commoners."

[4]For instance, exam papers were strictly anonymized and even transcribed to prevent the identification of candidates through handwriting. Cheating and clientelism were strictly prohibited, with punishments as severe as death.

[5]Parental ties to the government here include all kinds of government positions and titles, inheritable or not.

Keju demonstrated a remarkable level of resilience and showed no signs of institutional decay over a millennium (Wang and Yang, 2025). In the millennium following the eleventh century, China never witnessed again the rise of a landed aristocracy with influence comparable to that existed before the sixth century. Once transitioned into a Populist Autocracy equilibrium, China was trapped.

The exam system also spread to other East Asian countries through traveling monks and diplomats. Comparable exam systems were established in Korea (958–1894), Vietnam (1075–1913), Japan (briefly during the Heian period), and Ryukyu (early fifteenth century until the Japanese occupation in 1609). In Korea and Vietnam, *Keju* evolved into an important pathway to political offices, profoundly shaping their subsequent political and social development.

Japan, however, presents an interesting case in point. Despite sharing cultural similarities with China, the political system of medieval Japan was fundamentally different. Many scholars emphasize the contrast between Japan's feudal, decentralized political regime and China's centralized bureaucratic state in the seventeenth to eighteenth centuries (Duus, 1969; Koyama et al., 2018; Ma and Rubin, 2024). Pre-modern Japan, to some extent, resembled the European model of feudalism more closely than the East Asian model of centralized absolutism. Such a power structure that favored regional powerbases would likely place Japan within the domain of a Feudal Monarchy equilibrium, as defined in the theoretical framework of the previous section. Consistent with the model predictions, the eighth-century reform trying to introduce *Keju* to Japan quickly failed in the face of aristocratic opposition. This failed attempt in Japan stood in sharp contrast with the wide adoption of *Keju* in other East Asian countries, and it underscores a key connection between the social structure of a society and its institutional development.

5.3 Medieval Muslim World and Mumlukism

The medieval Muslim world presents an interesting case in point. Blaydes (2017) argues that the Middle East's early shift to settled agriculture positioned the region as home to some of the most advanced and well-developed states in the ancient world. As these states emerged from Late Antiquity, their strong fiscal and bureaucratic capacities allowed for institutional developments that were not feasible in Europe. This included the use of slave soldiers as the military elite and state control over land, which could be allocated to these soldiers through temporary, revocable land grants.

Throughout the medieval Muslim world, slave soldiers, known as Mamluks, played a significant role in the political systems. In Fukuyama (2011)'s words, "*the Mamluks saved Islam.*" It is believed that the large-scale utilization of a military unit composed of slave soldiers originated in the ninth century under the Abbasid Caliphate in Baghdad (Crone, 1980). Over time, the Mamluks evolved into a formidable knightly class within various Muslim societies, particularly in Egypt and Syria, as well as in the Ottoman Empire, the Levant, and Mesopotamia (Fukuyama, 2011). The system endured for nearly 1,000 years, from the ninth century to the early nineteenth century.

During the early phase under the Abbasid Caliphate, most slave soldiers were captured or acquired through the slave trade, primarily from Turkic tribes in Transoxania (Crone, 1980). In later Muslim empires, Christian populations under their control often became the source of Mamluks. For instance, in the Ottoman Empire, these slave soldiers, known as Janissaries, were conscripted through the so-called "levy of Christian youths" in the Balkans. Young Christian boys were taken, converted to Islam, and trained for service in the Ottoman military (Fukuyama, 2011). Even the Turkic regimes of the Middle East, too, became dependent on slave soldiers (Blaydes, 2017). Although practices varied across polities, one principle remained the same: Mamluks were drawn either from foreigners or minority religious or ethnic groups. And because of their minority origin, the "uprootedness" of their experience, and their isolation from the general population, Mamluks were considered more loyal to the monarchs and often used as a counterbalance to the other tribal or kinship-based interests in the polity (Fukuyama, 2011; Petry, 2022).

Although Mamluks were purchased or taken as property, their status was elevated above that of ordinary slaves. They were regarded as an "elite military class" and often held social standing above the general population. In many cases, Mamluks could ascend the political hierarchy, attaining positions such as high-ranking government officials, governors, amirs, and even viziers. In Egypt, their power grew to such an extent that they seized control of the sultanate in 1250, establishing the Mamluk Sultanate (Petry, 2022).

However, despite the relatively high status and payment they enjoyed during their tenure, Mamluks were generally not allowed to pass their power to their descendants. When studying the Mamluk Sultanate in Egypt, historian David Ayalon proposed the concept of "one-generation aristocracy" because the descendants of the Mamluks were forbidden to join the ranks. Fukuyama (2011) invokes the same concept of "one-

generation aristocracy" when describing the Janissaries of the Ottoman Empire. Haarmann (1998) used the phrase "Joseph's Law," referring to the biblical figure Joseph, to summarize the principle of Mamlukism: a perpetual renewal with new blood from the outside.

In practice, however, it is human nature to want to preserve their privileges once granted power. Many Mamluks did find ways to provide for their descendants. Some obtained outstanding achievements or enjoyed exceptional trust from the Sultans. These exceptional Mamluk officials could, at times, manage to secure prominent positions for their relatives, as seen in the case of Grand Vizier Sokollu Mehmed Pasha of the Ottoman Empire. The majority of the Mamluks, the middle- and low-ranking officials, sought alternative ways to ensure the transfer of at least some wealth to their descendants. Mamluk officials were rewarded with salaries and the *iqta's*, i.e., land grants entitling them to the revenue from a tax district, allocated according to their ranks and duties.[6] As political positions were not inheritable, these grants could not be passed onto heirs either. As a result, many Mamluks had to resort to unconventional methods to secure the transfer of wealth. One such method involved the establishment of charitable endowments (*waqf*) (Blaydes, 2017). These foundations could be used to support the upkeep of mosques, colleges, or other charities, as well as to support the donor's family (the so-called family *awqaf*). Historians have noted, with some irony, the eagerness of Mamluk generals to invest in such pious foundations, which "not only enhanced their status in this and in the other world but also offered provisions for their families beyond the limited tenure of their employment" (Haarmann, 1998). It also became a dubious custom to appoint one's own descendants as administrators of such pious foundations, a practice that must sound familiar to modern-day super-wealthy families (Haarmann, 1998, p.71).

Moreover, the Mamluks continued to push for a relaxation of the rule. Certain Muslim states—such as the Mamluk Sultanate of Egypt after the 14th century and the Ottoman Empire after the late 16th century—eventually gave in and permitted sons of Mamluk officials to enter into the hierarchy, albeit usually at the middle or low ranks (Haarmann, 1998; Fukuyama, 2011).

[6]Even before the advent of Mamlukism, the Islamic military had been paid by salaries rather than land rights. The land tax and poll tax of the Sasanian and Byzantine regimes continued to function under Islamic rule (Kennedy, 2001). According to Kennedy (2001, p.78), the concept of *fay'*, as a hereditary resource to the military, was effectively dead by Abbasid period. The *iqta* system was first introduced in mid-tenth century Iraq, eventually spreading to Iran, Afghanistan, Syria, and Egypt (Blaydes, 2017).

Overall, despite the institutional erosion and the innovative strategies employed by the individuals, the Mamluk institutions did create substantial obstacles to the transfer and accumulation of power within the military elites. For instance, the wealth of Mamluk descendants in the Mamluk Sultanate of Egypt, where they enjoyed arguably the highest social status in the Muslim world, remained modest at the family level, as shown by three cadastral surveys conducted between the fourteenth and fifteenth centuries (Richards, 1998). This stands in stark contrast to the European aristocracy, which enjoyed centuries of prominence.[7]

From this perspective, the institutions of the medieval Muslim world did have some elements that resonate with the concept of a Populist Autocracy. The monarchies did not depend on the elites for military support. Instead, they forged a "coalition of the weak," which incorporated slave soldiers drawn from minority groups or outsiders. Moreover, these regimes implemented rules to prevent, or at least limit, the inheritance of political power within these newly created military elites. This approach seemed to be somewhat successful. Blaydes and Chaney (2013) argue that because a landed aristocracy did not emerge as an influence-wielding social class until relatively late, power-sharing institutions were not established in the Middle East, and an autocratic form of governance prevailed.

However, it is important to note that the Muslim world differed substantially from a typical Populist Autocracy because, unlike the exam system in Imperial China, the Mamluk soldiers were not drawn from the broader society. In many cases, they were not even a natural component of the society but a foreign element. The Mamluk institution tried to maintain elite control without co-opting the general population. As a result, it was not able to establish the type of broad political coalition that the exam system was able to achieve in Imperial China. Hence, rulers in the medieval Muslim world enjoyed a strong but unstable form of autocratic rule.[8]

[7] In some Muslim states, the Mamluks did manage to consolidate power not along family lines but through ethnic or fractional ties (Fukuyama, 2011). However, these power consolidations were not easy to maintain for long due to the rule of recruitment from outside.

[8] A notable example of unstable autocratic rule is the Mamluk Sultanate of Egypt. While the ruler managed to bypass the local elites of the Ayyubid dynasty by relying on the Mamluks, the Mamluks quickly rose to power and ultimately seized control of the Sultanate. As a result, although the Arab elites no longer posed a significant threat, the Sultans struggled to maintain authority over their own Mamluk followers. The Ottomans later refined this system by placing the military branch firmly under the control of the administrative branch (Fukuyama, 2011). While this adjustment created a somewhat more stable system, it was insufficient to prevent the eventual consolidation of the Janissaries as an independent power base.

6 Conclusion

In this paper, I present a formal model of dynamic game theory to analyze various types of historical authoritarian regimes and explore potential pathways of political dynamics.

Four types of historical regimes are highlighted. In a *Feudal Monarchy*, the ruler forms a coalition with the elites to suppress the commoners. A *Populist Autocrat*, on the other hand, co-opts the commoners against the elites. An *Absolute Dictatorship* occurs when the ruler is powerful enough to suppress both the elites and the commoners without needing support from either group. Lastly, in an *Aristocratic Oligarchy*, the power of the elites is so strong that no compensation from the ruler can prevent an elite rebellion.

Two factors are important for understanding the political dynamics: the social structure, or the distribution of power within society, and the productivity levels of different social groups. In a world where the elite's productivity is relatively high compared to other groups, political divergence can occur. In such cases, a society with a slightly stronger ruler may evolve toward Populist Autocracy, eventually leading to Absolute Dictatorship. Conversely, a society with a slightly weaker ruler may develop into a stable Feudal Monarchy or move in the direction of ever-strengthening elite dominance.

Unlike most existing studies on institutional change, this paper focuses on internal sources of change rather than external shocks or uncertainties. Specifically, I consider social structure as the underlying variable driving long-run political divergence. By doing so, this paper offers a useful framework for analyzing political dynamics over the long run.

However, several prominent issues in political development and authoritarian politics are not addressed in this paper. For example, the model does not incorporate uncertainties or information asymmetry. It also omits exogenous shocks and, therefore, is unable to provide insights into the issue of critical junctures. Future research is needed to examine the interaction between endogenous and exogenous sources of institutional change, such as the interaction between social structure and technologies.

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Online Appendix
For "Social Structure and Political Dynamics in the Long Run"

A. Formal Proofs of the Stationary Game

This section provides proofs for the propositions in main text Section 3.2 Stationary Equilibrium. Since wealth endowments are constant in this section, I can omit the time subscript and refer to the wealth endowments as (w_e, w_r, w_c).

Let V_{it} denote the continuum utility for an individual from social group *i* at time *t*. Assume no change in wealth over time. Each period, a per-period payment is generated to all individuals, which is subject to taxation. θ is the conflict destruction parameter, and β is the discounting factor.

Assumption 1: $W_e > W_c$

Assumption 2: $\theta \geq \frac{b}{a-a\beta+b\beta}$

This assumption guarantees that the ruler cannot coopt any social group with the maximum level of tax rate. This assumption is not necessary, but it enables us to focus on the more interesting cases of differential tax rates without considering the scenario where no rebellion ever breaks out even with the maximum level of taxation. I will discuss the implication of this assumption in the proof of Proposition 2.

Assumption 3: $a_e > a_r \geq a_c \geq b > 0$

This assumption assumes that the elites have a higher level of productivity compared to the other social groups in the society.

Proposition 1 (Absolute Dictatorship): if $w_r > W_e + W_c$, the ruler can survive with no additional support. It represses all other social groups. $\tau_{ct}^{Abs} = \bar{\tau}_c$, $\tau_{et}^{Abs} = \bar{\tau}_e$.

Proof: this is obvious. Proof is omitted.

Proposition 2 (Populist Autocracy): if $W_e - W_c < w_r < W_e + W_c$, the ruler will set the following tax rates, $\tau_{ct}^{Pop} = 1 - \theta\left(1 + \frac{w_r}{W_c+W_e}\right)\left(1 - \frac{a_c-b}{a_c}\beta\right)$, $\tau_{et}^{Pop} = \bar{\tau}_e$.

As a result, the commoners will ally with the ruler against the elites.

Proof: First, at each time, the ruler will coopt at most one social group. He will set the maximum tax rate $\bar{\tau}_i$ on the other group.

If the elites rebel in time period *t* and the commoners join, the subsequent game becomes a two-player game dominated by the elites, with modified wealth $\widetilde{w}_e = \theta w_e \left(1 + \frac{w_r}{W_e+W_c}\right), \widetilde{w}_c = \theta w_c \left(1 + \frac{w_r}{W_e+W_c}\right)$. The elites will set the maximum level of tax rate on the commoners in all future periods. Hence,

$$V_{ct+1}(\widetilde{w}_e, 0, \widetilde{w}_c) = \frac{(1-\bar{\tau}_c)a_c}{1-\beta}\widetilde{w}_c = \frac{bw_c\theta}{1-\beta}\left(1 + \frac{w_r}{W_c+W_e}\right)$$

$$V_{et+1}(\widetilde{w}_e, 0, \widetilde{w}_c) = \frac{a_e\widetilde{w}_e + \frac{\bar{\tau}_c a_c M}{N}\widetilde{w}_c}{1-\beta} = \frac{a_e w_e + \frac{(a_c-b)W_c}{N}}{1-\beta}\theta\left(1 + \frac{w_r}{W_c+W_e}\right)$$

Now, let's consider the commoners' decision to participate in the rebellion or not. Let $\eta_c(w_e, w_r, w_c)$ denote the tax rates that makes the commoners indifferent between rebellion or not. If they join the rebellion in time *t*, they obtain a continuum value of the following

$$V_{ct}(rebel \mid w_e, w_r, w_c) = \theta w_c \left(1 + \frac{w_r}{W_e+W_c}\right)\left(a_c + \frac{b\beta}{1-\beta}\right)$$

Therefore, to make the commoners indifferent between joining a rebellion or not, the ruler needs to set the tax rate such that

$$\frac{(1-\eta_c)a_c}{1-\beta}w_c = \theta w_c \left(\frac{b\beta}{1-\beta} + a_c\right)\left(1 + \frac{w_r}{W_c+W_e}\right)$$

$$\Rightarrow \eta_c = 1 - \theta\left(1 + \frac{w_r}{W_c+W_e}\right)\left(1 - \frac{a_c-b}{a_c}\beta\right)$$

If $\eta_c > \bar{\tau}_c$, then the ruler can set $\tau_{ct} = \bar{\tau}_c$ and the commoners will still ally with the ruler. By Assumption 2, we have that : $\theta\left(1 + \frac{w_r}{W_c+W_e}\right) \geq \theta \geq \frac{b}{a_c - a_c\beta + b\beta} > \frac{b}{a_c}$. Hence, $\eta_c \leq \bar{\tau}_c$ always holds true.

Next, let's consider the elites' rebellion decision. Let's assume that the ruler is trying to coopt the elites. Since the ruler will coopt at most one social group at a time, the commoners will certainly join a rebellion if one is staged. Let $\eta_e(w_e, w_r, w_c)$ denote the tax rate that makes the elites indifferent. If the elites rebel at time *t*, the commoners will join and the elites obtain a continuum value of the following

$$V_{et}(rebel \mid w_r, w_e, w_c) = \theta\left(1 + \frac{w_r}{W_c+W_e}\right)\left[a_e w_e + \frac{\beta}{1-\beta}\left(a_e w_e + \frac{(a_c-b)W_c}{N}\right)\right]$$

Therefore, to make the elites indifferent between starting a rebellion or not, the ruler needs to set the tax rate such that

$$\frac{(1-\eta_e)a_e w_e}{1-\beta} = \theta\left(1 + \frac{w_r}{W_c+W_e}\right)\left[a_e w_e + \frac{\beta}{1-\beta}\left(a_e w_e + \frac{(a_c-b)W_c}{N}\right)\right]$$

$$\Rightarrow \eta_e = 1 - \theta\left(1 + \frac{w_r}{W_c + W_e}\right)\left(1 + \beta\frac{a_c - b}{a_e}\frac{W_c}{W_e}\right)$$

If Assumption 2 is satisfied, we have $\theta\left(1 + \frac{w_r}{W_c+W_e}\right) \geq \theta \geq \frac{b}{a_c - a_c\beta + b\beta} > \frac{b}{a_c} \geq \frac{b}{a_e}$. Hence, $\eta_e \leq \bar{\tau}_e$ always holds true. In time period t, the ruler can choose to coopt either the elites or the commoners as allies and maintain his rule. I consider the following three scenarios.

(A) Under Assumption 2, to determine the ruler's preference over an alliance with the elites or one with the commoners, we evaluate the value of the following

$$a_e\eta_e W_e + a_c\bar{\tau}_c W_c - a_e\bar{\tau}_e W_e - a_c\eta_c W_c$$

$$= b(W_e - W_c) - \theta\left(1 + \frac{w_r}{W_c + W_e}\right)(a_e W_e - a_c W_c) - \theta\beta W_c\left(1 + \frac{w_r}{W_c + W_e}\right)(a_c - b)$$

$$\leq -a_c(W_e - W_c)\left[\theta\left(1 + \frac{w_r}{W_c + W_e}\right) - \frac{b}{a_c}\right] - 2\theta\beta W_c\left(1 + \frac{w_r}{W_c + W_e}\right)(a_c - b)$$

By Assumption 2, we have $\theta\left(1 + \frac{w_r}{W_c+W_e}\right) \geq \frac{b}{a_c - a_c\beta + b\beta} > \frac{b}{a_c}$. Therefore, the above equation is always ≤ 0. In this case, it is always optimal for the ruler to ally with the commoners.

Moreover, consider the ruler's budget constraint

$$a_r w_r + a_e\bar{\tau}_e W_e + a_c\eta_c W_c$$

$$= a_r w_r + (a_e - b)W_e + a_c W_c - a_c W_c\theta\left(1 + \frac{w_r}{W_c + W_e}\right)\left(1 - \frac{a_c - b}{a_c}\beta\right)$$

$$= a_r w_r + (a_e - b)W_e + (1 - \theta)a_c W_c + (a_c - b)W_c\beta\theta\left(1 + \frac{w_r}{W_c + W_e}\right) - a_c W_c\theta\left(\frac{w_r}{W_c + W_e}\right)$$

$$\geq a_r w_r - a_c W_c\theta\left(\frac{w_r}{W_c + W_e}\right)$$

$$\geq a_c w_r\left[1 - \theta\left(\frac{W_c}{W_e + W_c}\right)\right] \geq 0$$

Hence, the ruler's budget constraint is always satisfied.

(B) Here, I examine a weaker version of Assumption 2.

Assumption 2.2: $\theta \geq \frac{b}{a_e}$

When Assumption 2 is not satisfied but Assumption 2.2 is satisfied, we have $\eta_c > \bar{\tau}_c$ but $\eta_e \leq \bar{\tau}_e$. In other words, the commoners can be coopt with $\bar{\tau}_c$. Hence, the ruler's tax revenue difference between the two options is given by the following

$$a_e\eta_e W_e + a_c\bar{\tau}_c W_c - a_e\bar{\tau}_e W_e - a_c\bar{\tau}_c W_c = W_e\left[b - \theta\left(1 + \frac{w_r}{W_c + W_e}\right)\left(a_e + \beta(a_c - b)\frac{W_c}{W_e}\right)\right] < 0$$

Hence, with only Assumption 2.2, we still have the conclusion that it is always optimal for the ruler to ally with the commoners.

(C) If Assumption 2.2 is not satisfied, then the ruler can coopt both the elites and the commoners with the maximum level of taxation $\bar{\tau}_c$ and $\bar{\tau}_e$. In this case, the ruler will coopt both.

Overall, to summarize, under Assumption 2, the ruler will choose to coopt the commoners with a tax rate $\tau_{ct}^{Pop} = \eta_c = 1 - \theta\left(1 + \frac{w_r}{W_c+W_e}\right)\left(1 - \frac{a_c-b}{a_c}\beta\right)$. This assumption is not necessary. Without Assumption 2, the ruler will still choose to coopt the commoners, but the tax rate becomes a corner solution.

Q.E.D.

Proposition 3 (Feudal Monarchy): if $W_e > w_r + W_c$ and $\frac{a_r w_r + a_e W_e + (a_c-b)W_c}{a_e W_e + (a_c-b)W_c\beta} \cdot \frac{W_c+W_e}{w_r+W_c+W_e} \geq \theta$, the ruler taxes the commoners with a maximum tax rate and coopts the elites. $\tau_{ct}^{Feud} = \bar{\tau}_c$, $\tau_{et}^{Feud} = 1 - \theta\left(1 + \frac{w_r}{W_c+W_e}\right)\left(1 + \beta\frac{a_c-b}{a_e}\frac{W_c}{W_e}\right) < \bar{\tau}_e$.

Proof: Since $W_e > w_r + W_c$, once the elites start a rebellion, the commoners has to join or they will lose everything by allying with the ruler. Hence, if the elites start a rebellion in time period *t*, the society becomes a two-player game in the subsequent periods with modified wealth $\widetilde{w}_e = \theta w_e\left(1 + \frac{w_r}{W_e+W_c}\right)$, $\widetilde{w}_c = \theta w_c\left(1 + \frac{w_r}{W_e+W_c}\right)$. The elites get the following continuum utility in time *t* if they rebel

$$V_{et}(rebel \mid w_e, w_r, w_c) = \theta\left(1 + \frac{w_r}{W_c + W_e}\right)\left[a_e w_e + \frac{\beta}{1-\beta}\left(a_e w_e + \frac{(a_c - b)W_c}{N}\right)\right]$$

Therefore, to make the elites indifferent between starting a rebellion or not, the ruler will have to set the tax rate such that

$$\frac{(1-\eta_e)a_e w_e}{1-\beta} = \theta\left(1 + \frac{w_r}{W_c + W_e}\right)\left[a_e w_e + \frac{\beta}{1-\beta}\left(a_e w_e + \frac{(a_c - b)W_c}{N}\right)\right]$$

$$\Rightarrow \eta_e = 1 - \theta\left(1 + \frac{w_r}{W_c + W_e}\right)\left(1 + \beta\frac{a_c - b}{a_e}\frac{W_c}{W_e}\right)$$

By Assumption 2, the above $\eta_e \leq \bar{\tau}_e$.

Consider the ruler's budget constraint:

$$a_r w_r + \bar{\tau}_c a_c W_c + \eta_e a_e W_e$$

$$= a_r w_r + (a_c - b)W_c + a_e W_e - \theta\left(1 + \frac{w_r}{W_c + W_e}\right)(a_e W_e + \beta(a_c - b)W_c)$$

$$= a_r w_r + a_e W_e\left[1 - \theta\left(1 + \frac{w_r}{W_c + W_e}\right)\right] + (a_c - b)W_c\left[1 - \beta\theta\left(1 + \frac{w_r}{W_c + W_e}\right)\right]$$

Therefore, if $a_r w_r + a_e W_e \left[1 - \theta\left(1 + \frac{w_r}{W_c+W_e}\right)\right] + (a_c - b)W_c \left[1 - \beta\theta\left(1 + \frac{w_r}{W_c+W_e}\right)\right] \geq 0$, the ruler can co-opt the elites with a tax rate $\tau_{et}^{Feud} = \eta_e = 1 - \theta\left(1 + \frac{w_r}{W_c+W_e}\right)\left(1 + \beta\frac{a_c-b}{a_e}\frac{W_c}{W_e}\right)$. Otherwise, no compensation can stop the elites from rebellion.

Q.E.D.

Corollary 3.1 (Aristocratic Oligarchy): if $W_e > w_r + W_c$ and $\frac{a_r w_r + a_e W_e + (a_c - b)W_c}{a_e W_e + (a_c - b)W_c \beta} \cdot \frac{W_c + W_e}{w_r + W_c + W_e} < \theta$, the elites will rebel and rule collectively as a group in subsequent periods.

Proof: it is a natural implication of the above proposition.

B. Formal Proofs of the Dynamic Game

This section provides the proofs of Propositions in main text Section 3.3 Political Dynamics. I consider a non-overlapping generation with myopic players. Each player will live for two periods and only cares about his utility over those two periods. The tax rates will be set at the beginning of the first period, and once set, the tax rates will persist two periods. I assume there is no commitment problem; once a strategy is chosen, the player will follow through over the two periods. Since the wealth endowments in this section are no longer constant, I bring back the time subscript for (w_{et}, w_{rt}, w_{ct}).

Each time period, an individual from the social group *i* receives a per-period income of $a_i w_{it}$. After pay tax τ_{it}, he consumes $(1 - s)$ of the post-tax disposable income, $(1 - s)(1 - \tau_{it})a_i w_{it}$, and saves the rest of it as investment into his wealth for next period, $s(1 - \tau_{it})a_i w_{it}, i = e, r$. Follow the tradition of the literature, we assume that commoners are not able to save. The ruler and the elites will automatically save a portion *s* of their disposable income. Moreover, for simplicity, I assume that the tax revenue in time period *t* will not enter the ruler's consumption at time *t*. Instead, it directly enters the ruler's investment and becomes the ruler's wealth in time *t+1*. This assumption is mainly to simplify the calculation.

Assumption 4: The ruler's and the elites' wealth depreciates over time, with the depreciation rate $\delta \in (0,1)$ satisfying $sa_e > sa_r \geq sa_e > \delta > sb$.

The part of the above assumption that $sa_r > \delta$ is not necessary. It is adopted to guarantee that either the ruler or the elites will have a wealth greater than the commoners, so I do not have to discuss the case when $w_{rt} + W_{et} < W_{ct}$.

Proposition 4 (Type of Regimes)

When $w_{rt} > W_{et} + W_{ct}$, an Absolutist Dictatorship equilibrium arises, and the ruler suppresses all other social groups. When $W_{et} - W_{ct} < w_{rt} < W_{et} + W_{ct}$, a Populist Autocracy equilibrium arises, and the ruler co-opts the commoners against the elites. When $W_{et} - W_{ct} > w_{rt}$, either a Feudal Monarchy equilibrium or an Aristocratic Oligarchy equilibrium arises.

To prove the above Proposition 4, I will first prove the following Lemma 1 and Lemma 2.

Lemma 1 (Populist Autocracy) *When $W_{et} - W_{ct} < w_{rt} < W_{et} + W_{ct}$, Populist Autocracy arises, and the ruler co-opts the commoners against the elites.*

$$\tau_{ct}^{Pop} = 1 - \theta\left(1 + \frac{w_{rt}}{W_{ct} + W_{et}}\right)\frac{a_c + b\beta}{a_c(1+\beta)}, \tau_{et}^{Pop} = \bar{\tau}_e$$

Proof: First, the ruler will coopt at most one social group. He will set the maximum tax rate $\bar{\tau}_i$ on the other group. Each player lives for two periods and cares only about his utility over these two periods.

Consider time period *t* as the first of the two periods that this generation will live. If the elites rebel and the commoners join, the subsequent game becomes a two player game dominated by the elites, with modified wealth $\widetilde{w}_{et+1} = \theta w_{et}\left(1 + \frac{w_{rt}}{W_{et}+W_{ct}}\right), \widetilde{w}_{ct+1} = \theta w_{ct}\left(1 + \frac{w_{rt}}{W_{et}+W_{ct}}\right)$. The elites will set the maximum level of tax rate on the commoners in the second period. Hence,

$$V_{ct+1}(\widetilde{w}_{et+1}, 0, \widetilde{w}_{ct+}\ ) = (1 - \bar{\tau}_c)a_c\widetilde{w}_{ct+1} = \ bw_{ct}\theta\left(1 + \frac{w_{rt}}{W_{ct} + W_{et}}\right)$$

Now, let's consider the commoners' decision to participate in the rebellion or not. Let $\eta_c(w_{et}, w_{rt}, w_{ct})$ denote the tax rates that makes the commoners indifferent between rebellion or not. If they join the rebellion, they obtain a discounted utility over the two periods as

$$V_{ct}(rebel \mid w_{et}, w_{rt}, w_{ct}) = \theta w_{ct}\left(1 + \frac{w_{rt}}{W_{et} + W_{ct}}\right)(a_c + b\beta)$$

Therefore, to make the commoners indifferent between joining a rebellion or not, the ruler will have to set the tax rate such that

$$(1 - \eta_c)a_c w_{ct}(1+\beta) = \theta w_{ct}\left(1 + \frac{w_{rt}}{W_{et} + W_{ct}}\right)(a_c + b\beta)$$

$$\Rightarrow \eta_c = 1 - \theta\left(1 + \frac{w_{rt}}{W_{ct} + W_{et}}\right)\frac{a_c + b\beta}{a_c(1+\beta)}$$

If $\eta_c > \bar{\tau}_c$, then the ruler can set $\tau_c = \bar{\tau}_c$ and the commoners will still ally with the ruler. By Assumption 2, we have that $\eta_c \leq \bar{\tau}_c$ always holds true.

Next, let's consider the elites' rebellion decision. Assume that the ruler is trying to coopt the elites. Let $\eta_e(w_{et}, w_{rt}, w_{ct})$ denote the tax rate that makes the elites indifferent between rebellion or not. If the elites rebel, the commoners will certainly join and the elites obtain a discounted utility over the two periods as the following

$$V_{et}(rebel \mid w_{et}, w_{rt}, w_{ct})$$
$$= (1-s)\theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)\left[a_e w_{et} + \beta\left(a_e(1-\delta+sa_e)w_{et} + \frac{(a_c-b)W_{ct}}{N}\right)\right]$$

Therefore, to make the elites indifferent between starting a rebellion or not, the ruler will have to set the tax rate such that

$$(1-s)(1-\eta_e)a_e w_{et}\left(1+\beta\left(1-\delta+sa_e(1-\eta_e)\right)\right)$$
$$= (1-s)\theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)\left[a_e w_{et} + \beta\left(a_e(1-\delta+sa_e)w_{et} + \frac{(a_c-b)W_{ct}}{N}\right)\right]$$

$$\Rightarrow (1-\eta_e)a_e\left(1+\beta(1-\delta+sa_e)\right) \geq \theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)\left[a_e + \beta\left(a_e(1-\delta+sa_e) + \frac{(a_c-b)W_{ct}}{W_{et}}\right)\right]$$

$$\Rightarrow 1-\eta_e \geq \theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)\left(1+\frac{\beta(a_c-b)}{a_e\left(1+\beta(1-\delta+sa_e)\right)}\frac{W_{ct}}{W_{et}}\right)$$

If Assumption 2 is satisfied, we have $\theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)\left(1+\frac{\beta(a_c-b)}{a_e(1+\beta(1-\delta+sa_e))}\frac{W_{ct}}{W_{et}}\right) \geq \frac{b}{a_e}$. Hence, $\eta_e \leq \bar{\tau}_e$ always holds true. The ruler can choose to coopt either the elites or the commoners as allies and maintain his rule.

Hence, under Assumption 2, to determine the ruler's preference over an alliance with the elites or one with the commoners, we evaluate the value of the following

$$a_e\eta_e W_{et} + a_c\bar{\tau}_c W_{ct} - a_e\bar{\tau}_e W_{et} - a_c\eta_c W_{ct}$$
$$= b(W_{et}-W_{ct}) - \theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)(a_e W_{et} - a_c W_{ct})$$
$$- \theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)\beta W_{ct}(a_c-b)\left(\frac{1}{1+\beta}+\frac{1}{1+\beta(1-\delta+sa_e)}\right)$$
$$\leq -a_c(W_{et}-W_{ct})\left[\theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)-\frac{b}{a_c}\right]$$

By Assumption 2, we have $\theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right) \geq \frac{b}{a_c-a_c\beta+b\beta} > \frac{b}{a_c}$, hence, the above equation is always ≤ 0. In this case, it is always optimal for the ruler to ally with the commoners. Populism arises.

Moreover, consider the ruler's budget constraint

$$a_r w_{rt} + a_e\bar{\tau}_e W_{et} + a_c\eta_{ct}W_{ct}$$
$$= a_r w_{rt} + (a_e-b)W_{et} + a_c W_{ct} - a_c W_{ct}\theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)\frac{a_c+b\beta}{a_c(1+\beta)}$$
$$= a_r w_{rt} + (a_e-b)W_{et} + \left(1-\theta\frac{a_c+b\beta}{a_c(1+\beta)}\right)a_c W_{ct} - a_c W_{ct}\theta\left(\frac{w_{rt}}{W_{ct}+W_{et}}\right)\frac{a_c+b\beta}{a_c(1+\beta)}$$
$$\geq a_r w_{rt} - a_c W_{ct}\theta\left(\frac{w_{rt}}{W_c+W_e}\right)\frac{a_c+b\beta}{a_c(1+\beta)}$$

$$\geq a_c w_{rt}\left[1-\theta\left(\frac{W_{ct}}{W_{et}+W_{ct}}\right)\frac{a_c+b\beta}{a_c(1+\beta)}\right]\geq 0$$

Hence, the ruler's budget constraint is always satisfied.

Q.E.D.

Lemma 2: if $W_{et} > w_{rt} + W_{ct}$, we have two possible scenarios:

(A) (Feudal Monarchy): When $a_r w_{rt} + (a_c - b)W_{ct} + a_e W_{et}\left(1+\frac{1+\beta-\beta\delta}{2sa_e\beta}-\frac{\sqrt{(1+\beta-\beta\delta)^2+4sa_e\beta\theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)\left[1+\beta(1-\delta+sa_e)+\frac{\beta(a_c-b)W_{ct}}{a_eW_{et}}\right]}}{2sa_e\beta}\right)\geq 0$, the ruler taxes the commoners with a maximum tax rate and co-opts the elites. We have $\tau_{ct}^{Feud}=\bar{\tau}_c$, $\tau_{et}^{Feud}=1-\theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)\left(1+\frac{\beta(a_c-b)}{a_e(1+\beta)}\frac{W_{ct}}{W_{et}}\right)<\bar{\tau}_e$.

(B) (Aristocratic Oligarchy): When $a_r w_{rt} + (a_c - b)W_{ct} + a_e W_{et}\left(1+\frac{1+\beta-\beta\delta}{2sa_e\beta}-\frac{\sqrt{(1+\beta-\beta\delta)^2+4sa_e\beta\theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)\left[1+\beta(1-\delta+sa_e)+\frac{\beta(a_c-b)W_{ct}}{a_eW_{et}}\right]}}{2sa_e\beta}\right)<0$, the elites always rebel.

Proof: Since $W_{et} > w_{rt} + W_{ct}$, once the elites start a rebellion, the commoners has to join or they will lose everything by allying with the ruler. Hence, if the elites start a rebellion, the society becomes a two-player game in the subsequent periods with $\widetilde{w}_{et+1}=\theta w_e\left(1+\frac{w_{rt}}{W_{et}+W_{ct}}\right)$, $\widetilde{w}_{ct+1}=\theta w_c\left(1+\frac{w_{rt}}{W_{et}+W_{ct}}\right)$. The elites get the following continuum utility

$$V_{et}(rebel \mid w_{et}, w_{rt}, w_{ct}) = (1-s)\theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)\left[a_e w_{et}+\beta\left(a_e w_{et}(1-\delta+a_e s)+\frac{(a_c-b)Mw_{ct}}{N}\right)\right]$$

Therefore, to make the elites indifferent between starting a rebellion or not, the ruler will have to set the tax rate η_e such that

$$(1-s)(1-\eta_{et})a_e w_{et}\left(1+\beta\left(1-\delta+sa_e(1-\eta_e)\right)\right) = (1-s)\theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)\left[a_e w_{et}+\beta\left(a_e w_{et}(1-\delta+a_e s)+\frac{(a_c-b)Mw_{ct}}{N}\right)\right]$$

Let $(1-\eta_{et})=x$

$$\Rightarrow x a_e\left(1+\beta(1-\delta+sa_e x)\right)=\theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)\left[a_e+\beta\left(a_e(1-\delta+sa_e)+\frac{(a_c-b)W_{ct}}{W_{et}}\right)\right]$$

$$\Rightarrow sa_e\beta x^2 + (1+\beta-\beta\delta)x - \theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)\left[1+\beta(1-\delta+sa_e)+\frac{\beta(a_c-b)W_{ct}}{a_eW_{et}}\right] = 0$$

$$\Rightarrow 1-\eta_{et}$$

$$= \frac{-(1+\beta-\beta\delta)+\sqrt{(1+\beta-\beta\delta)^2+4sa_e\beta\theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)\left[1+\beta(1-\delta+sa_e)+\frac{\beta(a_c-b)W_{ct}}{a_eW_{et}}\right]}}{2sa_e\beta}$$

By Assumption 2, the above $\eta_{et} \leq \bar{\tau}_e$.

Now, consider the ruler's budget constraint

$$a_r w_{rt} + a_e\eta_{et}W_{et} + a_c\,\bar{\tau}_c W_{ct}$$

$$= a_r w_{rt} + (a_c-b)W_{ct}$$

$$+ a_eW_{et}\left(1+\frac{1+\beta-\beta\delta}{2sa_e\beta}\right.$$

$$\left.-\frac{\sqrt{(1+\beta-\beta\delta)^2+4sa_e\beta\theta\left(1+\frac{w_{rt}}{W_{ct}+W_{et}}\right)\left[1+\beta(1-\delta+sa_e)+\frac{\beta(a_c-b)W_{ct}}{a_eW_{et}}\right]}}{2sa_e\beta}\right)$$

If the above equation is non-negative, a Feudal Monarchy arises as the equilibrium and the ruler co-opts the elites. Otherwise, a rebellion by the elites is staged and an Aristocratic Oligarchy arises as the equilibrium.

Q.E.D.

Corollary 4.1 (Directions of Political Dynamics)

In an Absolute Dictatorship equilibrium, we have $W_{et+1} < W_{et}, w_{rt+1} > w_{rt}$, and $\frac{w_{rt}}{w_{et}} < \frac{w_{rt+1}}{w_{et+1}}$

In a Populist Autocracy equilibrium, we have $W_{et+1} < W_{et}$.

In a Feudal Monarchy equilibrium, if a rebellion is not very destructive or the productivity of the elites sufficiently high ($i.e. sa_e\theta \geq \delta$), we have $W_{et+1} > W_{et}$. Otherwise, the evolution of W_{et} is unclear.

Proof:

(1) In an Absolute Dictatorship equilibrium, we have the following dynamics

$$w_{et+1} = (1-\delta+sa_e(1-\bar{\tau}))w_{et} = (1-\delta+sb)w_{et} < w_{et}$$

$$w_{rt+1} = (1-\delta+sa_r)w_{rt} + \bar{\tau}_c a_c W_c + \bar{\tau}_e a_e W_e$$

$$= (1-\delta+sa_r)w_{rt} + (a_c-b)W_{ct} + (a_e-b)W_{et} > w_{rt}$$

$$\frac{w_{et+1}}{w_{et}} = 1 - \delta + sb < 1 < 1 - \delta + sa_r < \frac{w_{rt+1}}{w_{rt}}$$

(2) In a Populist Autocracy equilibrium

$$\frac{w_{et+1}}{w_{et}} = (1 - \delta + sa_e(1 - \bar{\tau})) = (1 - \delta + sb) < 1$$

However, for w_{rt},

$$w_{rt+1} = (1 - \delta + sa_r)w_{rt} + \eta_c a_c W_c + \bar{\tau}_e a_e W_e$$
$$= (1 - \delta + sa_r)w_{rt} + \eta_c a_c W_c + (a_e - b)W_{et}$$
$$\eta_c = 1 - \theta\left(1 + \frac{w_{rt}}{W_{ct} + W_{et}}\right)\left(1 - \frac{(a_c - b)\beta}{a_c(1 + \beta)}\right)$$

When $\beta \to 0$, it is possible to get

$$\frac{(a_c - b)\beta}{a_c(1 + \beta)} < \frac{1}{2}$$

Hence,

$$\eta_c = 1 - \theta\left(1 + \frac{w_{rt}}{W_{ct} + W_{et}}\right)\left(1 - \frac{(a_c - b)\beta}{a_c(1 + \beta)}\right) \leq 1 - 2\theta$$

η_c may be negative when θ is large. Hence, the dynamics of $\frac{w_{rt}}{w_{et}}$ is unclear.

(c) In a Feudal Monarchy equilibrium,

$$1 - \eta_e$$
$$= \frac{-(1 + \beta - \beta\delta) + \sqrt{(1 + \beta - \beta\delta)^2 + 4sa_e\beta\theta\left(1 + \frac{w_{rt}}{W_{ct} + W_{et}}\right)\left[1 + \beta(1 - \delta + sa_e) + \frac{\beta(a_c - b)W_{ct}}{a_e W_{et}}\right]}}{2sa_e\beta}$$

To prove $w_{et+1} \geq w_{et}$ is equivalent to proving $1 - \delta + sa_e(1 - \eta_e) \geq 1$

$$\Leftrightarrow \sqrt{(1 + \beta - \beta\delta)^2 + 4sa_e\beta\theta\left(1 + \frac{w_{rt}}{W_{ct} + W_{et}}\right)\left[1 + \beta(1 - \delta + sa_e) + \frac{\beta(a_c - b)W_{ct}}{a_e W_{et}}\right]} \geq 1 + \beta + \delta\beta$$

$$\Leftrightarrow (1 + \beta - \beta\delta)^2 + 4sa_e\beta\theta\left(1 + \frac{w_{rt}}{W_{ct} + W_{et}}\right)\left[1 + \beta(1 - \delta + sa_e) + \frac{\beta(a_c - b)W_{ct}}{a_e W_{et}}\right]$$
$$\geq (1 + \beta + \delta\beta)^2$$

$$\Leftrightarrow sa_e\theta\left(1 + \frac{w_{rt}}{W_{ct} + W_{et}}\right)\left[1 + \beta + \beta(sa_e - \delta) + \frac{\beta(a_c - b)W_{ct}}{a_e W_{et}}\right] \geq \delta(1 + \beta)$$

We already know that $1 + \beta + \beta(sa_e - \delta) + \frac{\beta(a_c - b)W_{ct}}{a_e W_{et}} > 1 + \beta$. If we have $sa_e\theta \geq \delta$, then $w_{et+1} > w_{et}$ is proved. Otherwise, if $sa_e\theta < \delta$, the evolution of w_{et} is unclear.

Q.E.D.

C. Additional Simulations

This section provides additional simulation for Section 4.3 Political Dynamics.

As noted in the main text, not all societies will necessarily diverge. If we relax Assumption 4 and set the depreciation rate to be larger than even the most productive social group, it is possible to obtain political convergence. Following Figure A.1 is a simulation with parameter $sa_e = sa_r = sa_c < \delta$ to highlight this point.

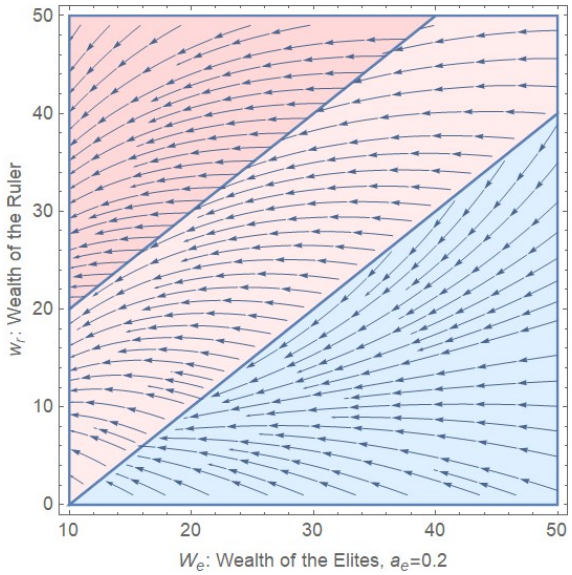


Figure A.1: A pathways of political convergence

Note: The parametric setting for the above simulation satisfies
$\theta = 0.8, W_c = 10, \beta = 0.5, \delta = 0.2, s = 0.5, b = 0.1, a_e = a_r = a_c = 0.2.$

However, this $sa_e < \delta$ is a sufficient but not necessary condition. Other combinations of parameters may also produce a political convergence outcome, as shown in the following Figure A.2.

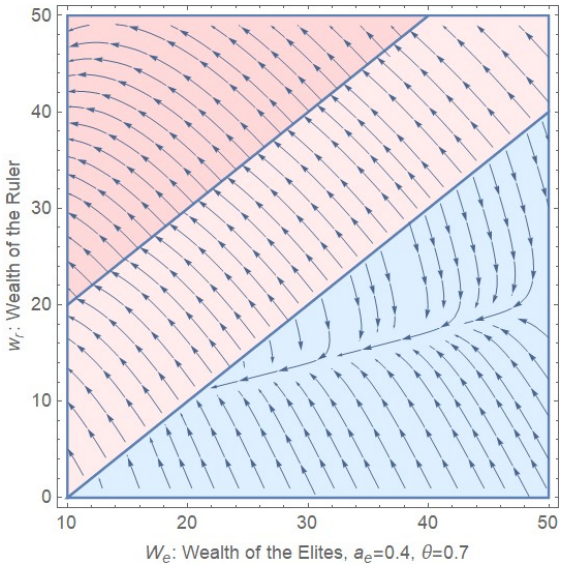


Figure A.2: A pathway of political convergence

Note: The parametric setting for the above simulation satisfies
$\theta = 0.8, W_c = 10, \beta = 0.5, \delta = 0.2, s = 0.5, b = 0.1, a_e = a_r = a_c = 0.2.$

Essentially, either a sufficiently destructive rebellion parameter θ or a sufficiently low level of elite productivity a_e can produce a political convergence towards Absolute Dictatorship. When rebellions are destructive (small θ), the ruler obtains more leverage over the other social groups and is able to demand higher taxes in any type of equilibrium. This prevents the elites from wealth accumulation even in a Feudal Monarchy equilibrium.