Ideas and Firm Dynamics

When It Takes Two to Tango

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Abstract

In this paper, we study how idea production has evolved over time and how it interacts with firm dynamics. Using the USPTO patent database linked to Compustat, we document stylized facts about R&D team formation and characteristics: i) the fraction and size of R&D teams have increased over time, with the share of team patents rising from 50% in 1980 to 80% in 2020, ii) R&D teams have become more diverse, with co-inventors increasingly coming from different fields of knowledge, iii) these trends hold across different firm groups, with the patterns being more pronounced for larger, older, or more productive firms. While firms play a central role in the formation of these teams, the interaction between firms and R&D teams is not well understood. We propose a model to explore how rising complementarities between knowledge workers can account for these trends and impact firm dynamics, worker allocation, as well as overall business dynamism and economic growth.

Keywords: innovation, R&D team, inventor allocation, firm dynamics, economic growth

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1 Introduction

The development of new ideas is the driving force behind innovation and economic growth. Ideas come either from individual inventors or from collaborations among teams of inventors. Teams have become increasingly important in the production of new ideas (Jones 2021; Pearce 2023), and firms play a critical role in determining collaborations between inventors. However, little is known about the process of inventor team formation within firms nor about its implications for firm dynamics and macroeconomic outcome. In this paper, we aim to fill this gap by studying the two-way interaction between inventor team formation and firm heterogeneity and dynamics.

We begin by using the USPTO patent database linked to Compustat and documenting stylized facts about inventor teams. First, patents developed by teams now account for 80% of all private patents in the U.S., compared to just 50% in 1980. Second, the diversity of expertise within inventor teams has increased over time. Third, we apply the framework proposed by (Bonhomme 2021) to estimate the impact of team size on output, and compare pre- and post-2000 periods. We find that that teams have become more productive since 2000. These facts point to a strengthening of complementarities between team members over time. Additionally, we observe that team characteristics vary across the firm size, productivity, and age distributions: small, less productive, or young firms are more likely to patent through solo inventors or small teams, and team size rises with firm size and age.

Next, we develop a simple model to analyze a firm's decision to hire R&D workers and form inventor teams. Large, productive firms have sufficient scale to pay the fixed costs required to form a team. Consistent with the empirical evidence, we examine the impact of rising knowledge worker complementarity on the scale of R&D activity in incumbent firms, the entry of firms into team-based R&D production, as well as the entry into R&D. We find that a reduction in the cost of hiring R&D workers does not explain the patterns of team formation and firm entry as effectively as an increase in complementarities. Our model offers an explanation for the rising concentration of

both R&D workers and patents at larger firms.

Related Literature A growing body of literature examines the match between inventors and firms (Akcigit and Goldschlag 2023a; Akcigit and Goldschlag 2023b; Babalievsky 2023; Manera 2022; Liu 2023). A separate strand of literature models firms as collections of coworkers or co-inventors, without incorporating additional dimensions of firm heterogeneity (Herkenhoff et al. 2024; Pearce 2023; Freund 2024). This paper bridges the gap between these two approaches by exploring how inventor team formation varies with firm-level characteristics and connect to firm dynamics.[1]

Our paper is also related to the literature on how interactions between individuals shape economic growth (Lucas and Moll 2014; Akcigit et al. 2018; Prato 2022). Relative to these papers, we abstract from the dynamics of individual-level knowledge accumulation and instead focus on the organizational features that may determine different forms of knowledge production at the firm level (Garicano and Rossi-Hansberg 2015). Our model predicts that R&D workers will increasingly concentrate at large firms, which highlights the importance of understanding how human capital accumulation among R&D workers occurs. If variety in employers and team members is a key source of knowledge accumulation, the trend toward greater worker concentration at large firms could pose challenges. On the other hand, the shift from solo inventions to more collaborative efforts may accelerate on-the-job human capital accumulation for knowledge workers.

Several empirical papers highlight the importance of inventor teams within firms. Jaravel, Petkova, and Bell (2018) find that the death of a co-inventor negatively impacts subsequent inventor productivity. Bhaskarabhatla et al. (2021) show that a significant share of firm-level heterogeneity in research productivity is driven by differences in inventor quality. Kline et al. (2019) demonstrate that firms share around 30% of the rents from patenting with incumbent, high-paid employees, implying that a large share

[1] Jarosch, Oberfield, and Rossi-Hansberg (2021) and Boerma, Tsyvinski, and Zimin (2023) study models of sorting into teams within firms, but not specifically for inventors.

of the rents is retained by firm owners.

Finally, our findings contribute to understanding trends in productivity differences between large and small firms, particularly in the context of research productivity (Andrews, Criscuolo, and Gal 2016; Bessen et al. 2020; Pugsley, Sedlacek, and Sterk 2021; Cavenaile, Celik, and Tian 2020; Olmstead-Rumsey 2022; Akcigit and Ates 2023).

2 Empirics

2.1 Data

Our primary data sources are the USPTO PatentsView and S&P's Compustat. The USPTO PatentsView tracks all patents granted by the U.S. Patent and Trademark Office (USPTO) from 1976 onward. This database provides detailed information, including application and grant dates, technology class classifications, patent inventors, citation data, and the names and addresses of patent assignees. Compustat is a comprehensive database maintained by S&P Global and provides detailed firm-level data for publicly traded companies. It includes information such as firm size, industry scope, balance sheets, income and cash flow statements, and stock prices.

We collect utility patents filed between 1976 and 2018, granted by the USPTO. To identify the patents assigned to US public firms and track them by different firm characteristics, we link the USPTO assignee identifier to the Compustat GVKEY using the bridge provided by Ding, Jo, and Kim (2022) and Braguinsky et al. (2023).[2] When a patent is assigned to multiple entities, we only keep the first assignee following the sequence order recorded by USPTO.

To measure the importance of a patent, we calculate five-year forward citations with vintage fixed effects removed, and obtain real patent values from Kogan et al. (2017) (KPSS values, hereafter). To obtain an accurate estimate of firm age, we also link

[2]This bridge is constructed using the standard name and address matching process, complemented by an internet search-aided algorithm following Autor et al. (2020) as well as additional manual matching. The detailed methodology is described in Ding, Jo, and Kim (2022) and Braguinsky et al. (2023).

PatentsView with the dataset provided by Ewens and Marx (2024) to obtain the firm's founding year.[3]

2.2 Stylized Facts

With this dataset, we have documented the following facts on team formation and its properties.

Fact 1: Team patents have increased from 50% in 1976 to 80% in 2018

Teams have played an increasingly dominant role in innovations over the past five decades. A patent is identified as a team patent if it has multiple inventors, and the group of inventors filing the same patent is identified as a team. Among all patents in the dataset, the share of team patents has risen from below 50% in 1976 to above 80% in 2018. This trend holds either measured in patent counts, KPSS values, or five-year forward citations, as shown in Figure 1.[4]

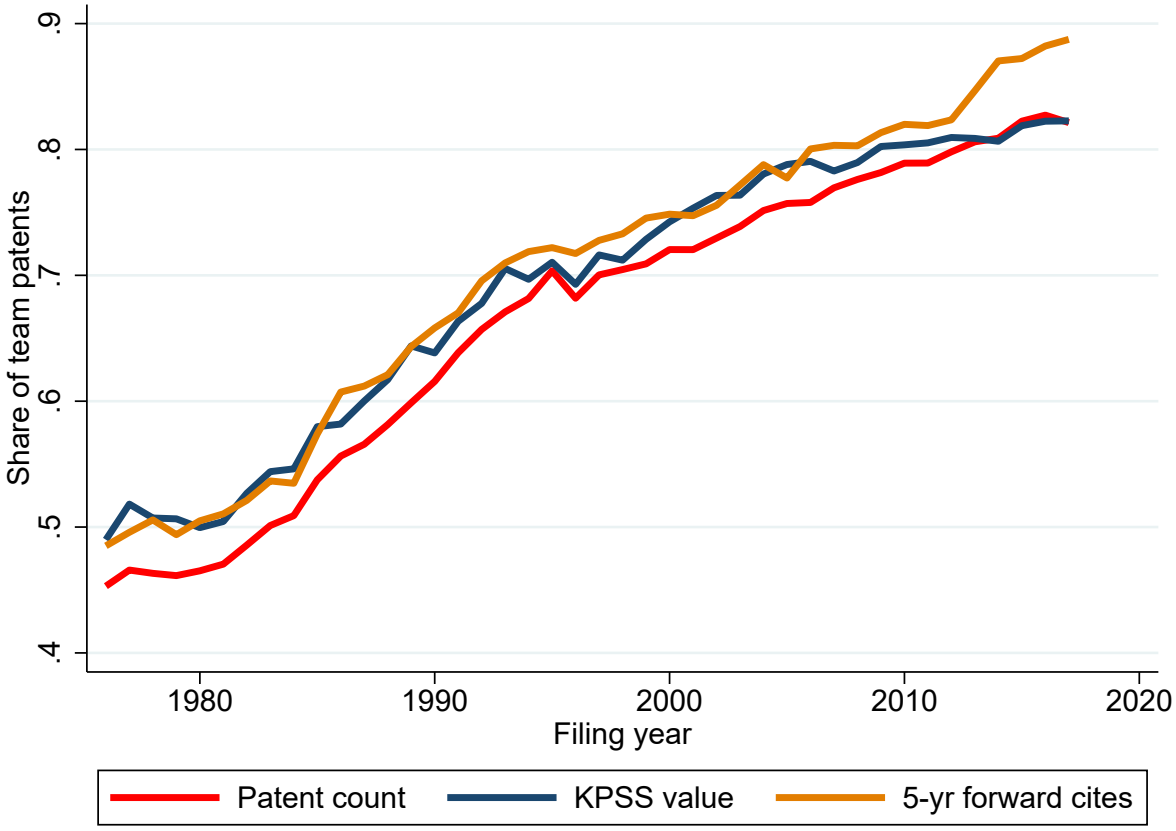
Fact 2: Firms shape teams: 90% of team patents are by inventors at the same firm

At the same time, most team collaboration in innovation occurs within firms rather than between them, highlighting the role of firms as a critical platform for team collaboration. We identify an inventor's affiliation in a given year as the firm with the largest number of patents filed by that inventor during that year. In the event of a tie, we assign the affiliation to the firm associated with the patent having the latest filing date. In the dataset, 89.5% of the team patents are filed by inventors affiliated with the same firm.

[3] Eventually, we aim to incorporate administrative data from the U.S. Census Bureau to extend our analysis, which is currently in progress.

[4] In this figure, we restrict our sample to Compustat firms with patent filings due to the use of KPSS values and to maintain consistency of the sample across series. The result is robust when using the full set of USPTO assignees.

Figure 1: Team Patents Shares



Note: The figure represents the share of team patents at Compustat firms from 1976 to 2018. A team patent is defined as one assigned by multiple inventors. The number of patents is counted unweighted (red line), weighted in KPSS values (navy line) extracted from Kogan et al. (2017), and weighted in five-year forward citations with vintage fixed effects removed (orange line).

Fact 3: Knowledge diversity within teams rises, suggesting complementarities

To measure knowledge diversity, we first identify the "core" CPC subgroup associated with an inventor in a given year as a proxy for the inventor's type or area of expertise.[5,6] The core CPC subgroup of an inventor is defined as the subgroup with the highest

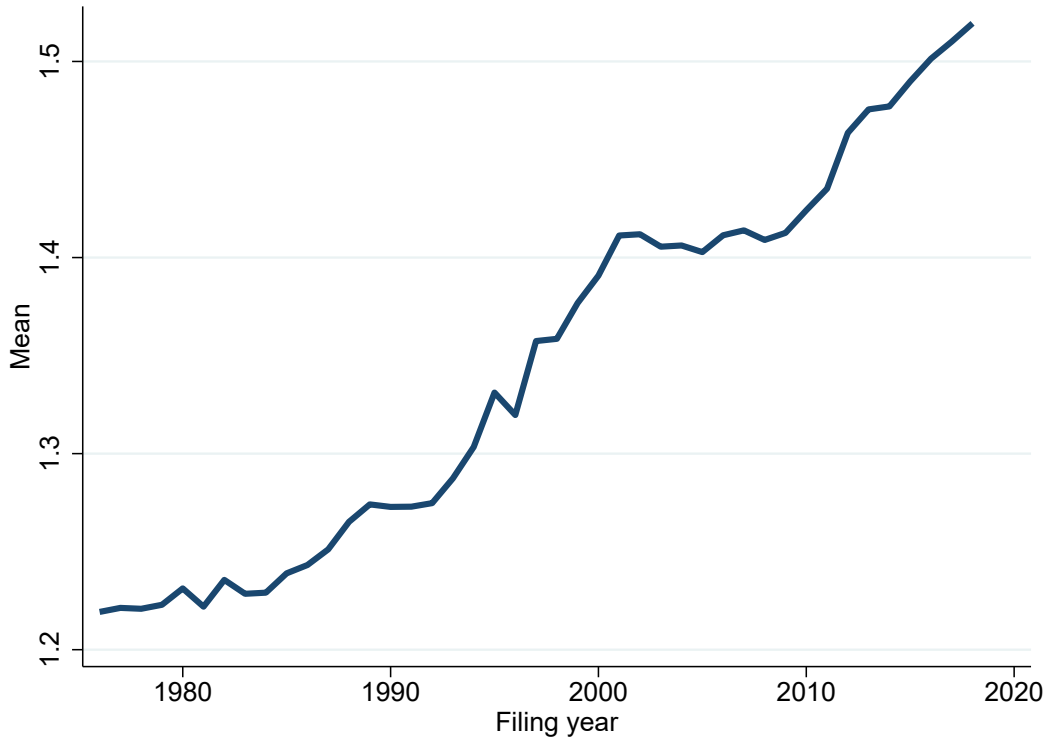
[5]For example, G06N 20/00 is "Machine learning" or A23B 11/00 is "Preservation of milk or dairy products."

[6]For a patent associated with multiple CPC subgroups, we pick the main CPC subgroups based on the sequence variable provided by the USPTO.

number of patents filed by that inventor up to that year.[7] For each team, we calculate the number of core CPC subgroups associated with its inventors, which serves as a measure of the team's knowledge diversity in that year. We then average this knowledge diversity measure across all teams within a firm for that year. Finally, we compute the average of these firm-level measures across all firms in a given year.

The result is plotted in Figure 2, which shows an upward trend and an increasing likelihood of the average team working with inventors with different specialties.

Figure 2: Average Number of Inventor CPC Subgroups in Teams in a Firm



Note: The figure illustrates the average number of core CPC subgroups associated with inventors on team patents in USPTO. A core CPC subgroup is defined as the modal CPC subgroup in an inventor's patenting history up to a given year. For each firm, we calculate the average number of core CPC subgroups for inventors on team patents and then compute the mean across all firms in a given year.

[7]Eventually, we aim to leverage the Longitudinal Employer-Household Dynamics (LEHD) dataset, which contains detailed demographic and educational information for workers covered by the U.S. Unemployment Insurance (UI) system. By linking it to the Longitudinal Business Database (LBD) and USPTO inventor data, we plan to track workers' patenting activities within and across firms. This is in progress.

Fact 4: Team size rises with firm productivity and age

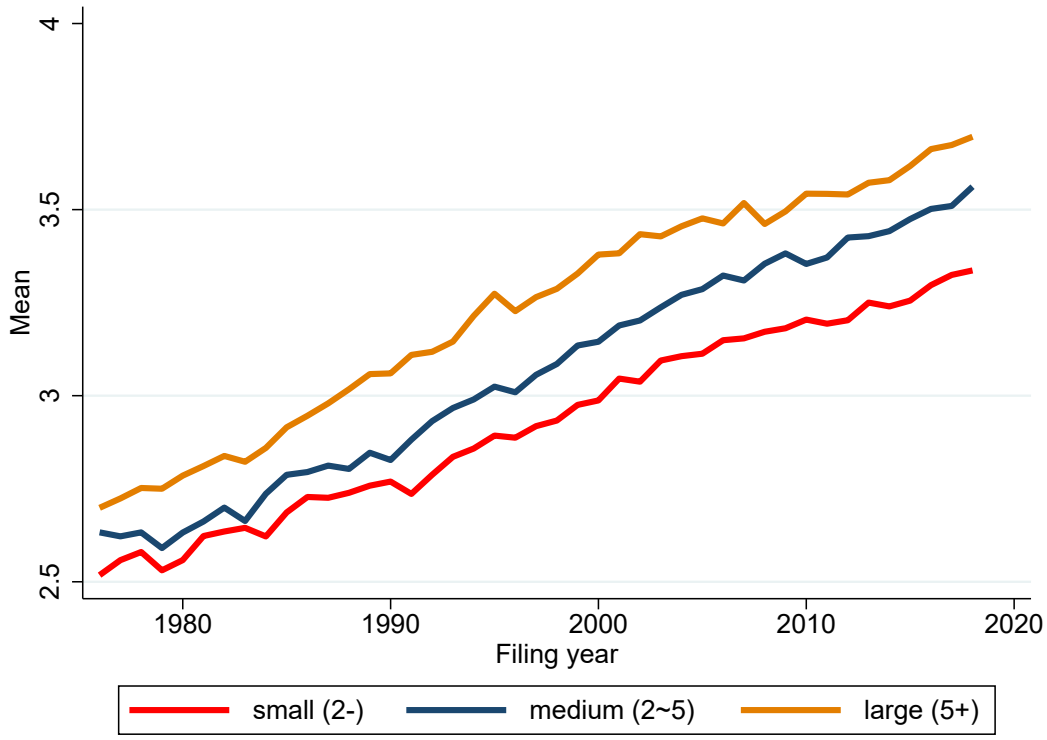
There is heterogeneity in team size across firms based on various characteristics. To analyze this, we classify firms into three groups using the number of patents filed or the number of inventors in a year as a proxy for firm productivity or size in the USPTO. Firms filing a single patent are categorized as small (or low-productivity) firms, those filing between two and five patents are classified as moderately sized (or moderately productive) firms, and firms filing more than five patents are designated as large (or highly productive) firms. Similarly, we define firms as small if they have fewer than five inventors, medium-sized if they have between five and ten inventors, and large if they have more than ten inventors.

For each year, we calculate the average team sizes for these three groups and examine their variations over time, as shown in Figures 3 and 4. The results indicate that medium-sized and large (or moderately and highly productive) firms consistently have larger teams than small (or low-productivity) firms, both in terms of the number of patents and inventors.

Furthermore, this pattern also holds among Compustat firms, where more precise firm productivity and size is available. Figure 5 illustrates this pattern for firms grouped into three categories based on their percentiles in the within-industry (NAICS4) cross-sectional productivity distribution for each year. The results also indicate that more productive firms tend to have larger teams compared to less productive firms, with productivity measured by revenue productivity in Compustat. Similarly, Figure 6 shows this pattern across firms of different employment size. The results suggest that larger firms have larger teams than smaller firms in Compustat.

In addition, a similar trend is observed with respect to firm age. Firm age is calculated by subtracting the founding year from Ewens and Marx (2024) from the current year. We then classify firms into three groups based on their age: those younger than six years are considered young firms, those between six and twenty years are medium-aged firms, and those older than twenty years are categorized as old firms. The trends

Figure 3: Average Team Sizes in a Firm (By Number of Patents)



Note: The figure illustrates the average team size in firms, categorized by firm size groups based on the number of patents filed. Small firms are those with one patent (red line), medium-sized firms are those filing between two and five patents (navy line), and large firms are those filing more than five patents (orange line) in the USPTO data.

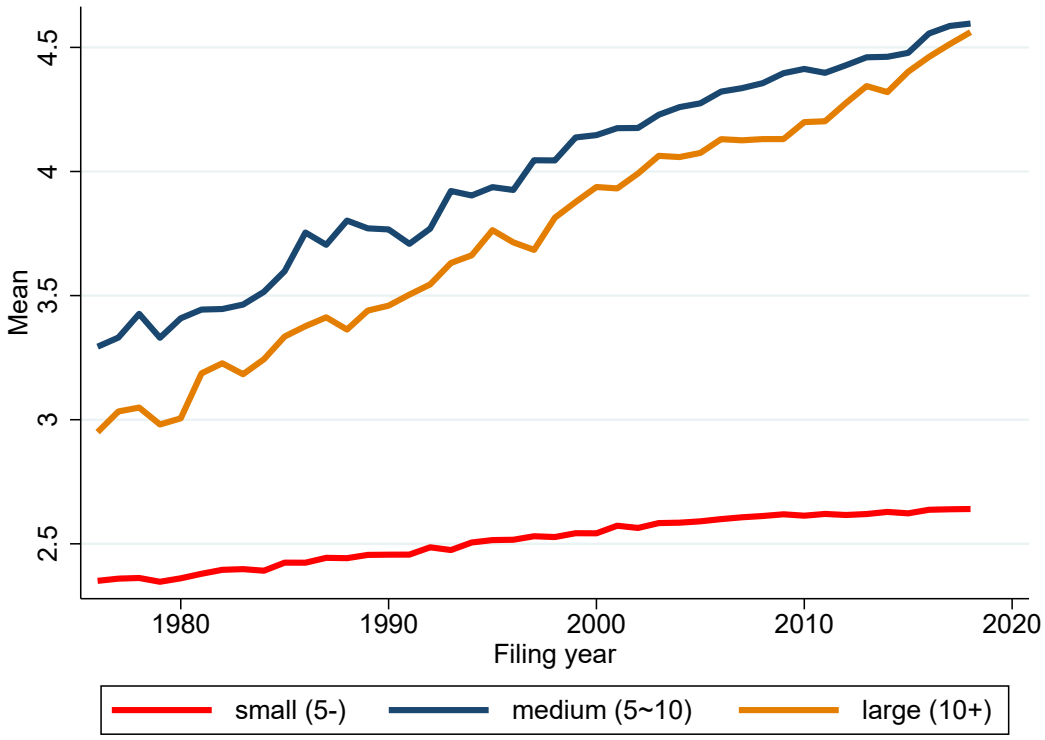
in team sizes across these age groups are plotted in Figure 7. After the year 2000, young firms tend to have smaller team sizes compared to older firms.

Fact 5: Firms are less likely to be single-inventor firms now than in the past

Our model focuses on the boundary between invention by single inventors and teams. To explore how this boundary has evolved over time, we compute the fraction of firms with a single inventor in a given year. Figure 8 illustrates the increasing trend of this ratio over time.

Furthermore, young firms are particularly likely to be single-inventor firms. To see the trend of the likelihood for young firms over time, we calculate the proportion of

Figure 4: Average Team Sizes in a Firm (By Number of Inventors)



Note: The figure illustrates the average team size in firms, categorized by firm size groups based on the number of inventors. Small firms are those with five or fewer inventors (red line), medium-sized firms are those filing between five and ten inventors (navy line), and large firms are those with more than ten inventors (orange line) in the USPTO data.

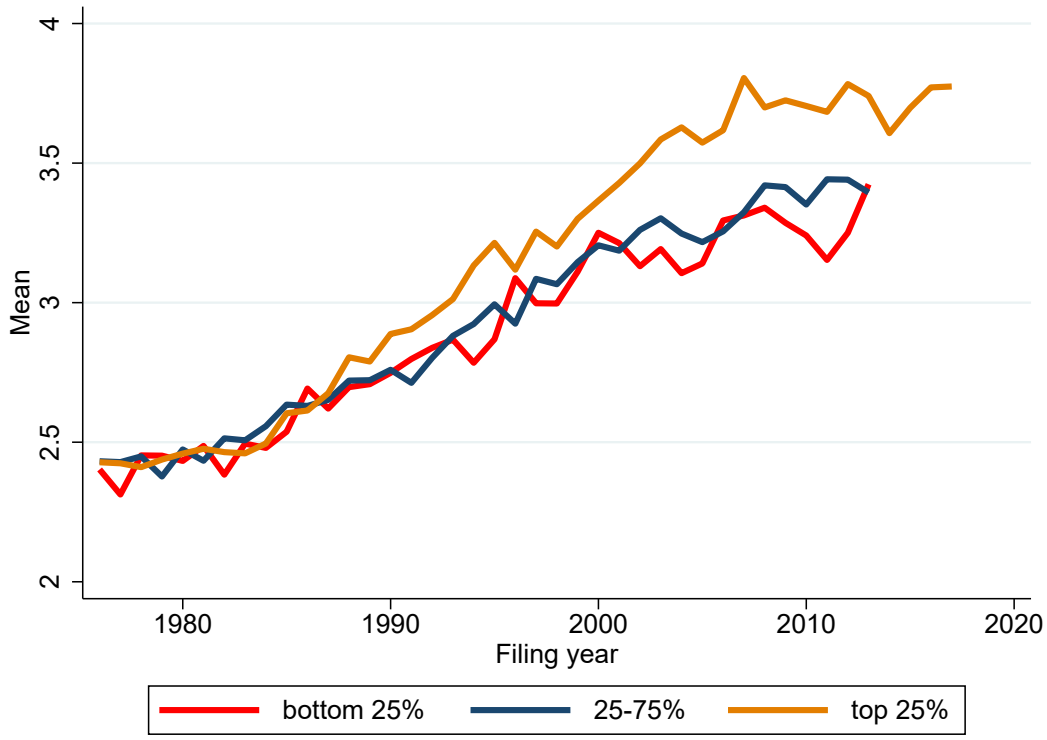
single-inventor firms among all young firms that have ever filed a patent in a given year. This is plotted in Figure 9, showing a declining trend of the fraction of single-inventor young firms over time.

Fact 6: Team complementarities increase

Finally, we aim to directly assess differences in complementarities among patenting team members across firms of varying sizes and ages, and to explore how these differences have evolved over time.

To do this, we estimate the linear model of team complementarities proposed by

Figure 5: Average Team Sizes in a Firm over Years (By Firm TFPR)



Note: The figure illustrates the average team size in firms, categorized by firm productivity. Firms are classified by their percentile of the cross-sectional productivity distribution within a pair of industry (NAICS4) and year. Top 25% (red line) indicates the top 25 percentile firms, 25-75% (navy line) indicates firms between 25 and 75 percentiles, and bottom 25% (orange line) indicates those in the bottom 25 percentile in productivity distribution. Firm productivity is measured by the revenue productivity in Compustat.

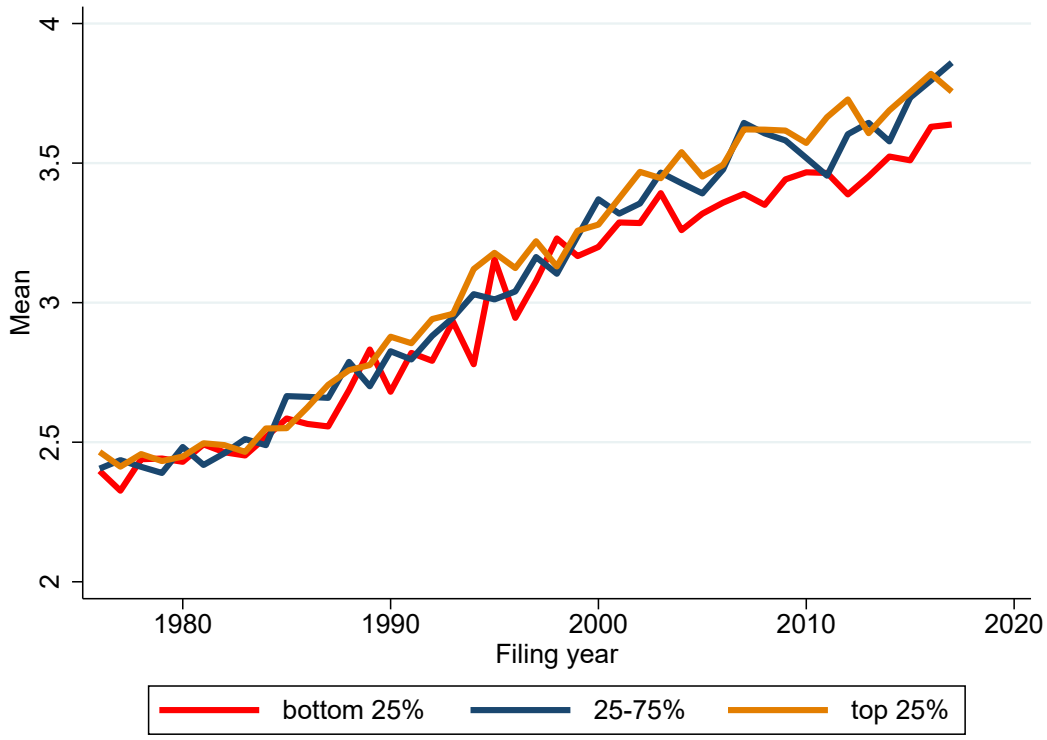
Bonhomme (2021):

$$Y_j = \lambda_n(\alpha_{i1(j)} + \cdots + \alpha_{in(j)}) + \epsilon_j,$$

where Y_{nj} is the 5-year forward citations of patents produced by team j of size n (including single inventor patents), α_i is individual i's fixed effect, and λ_n is the team-size scaling factor for size n, which takes value one for single inventor teams.

The procedure of model estimation follows Section 3 in Bonhomme (2021). First, we use the identification condition to iteratively determine the subset of inventors for which the inventor fixed effects can be determined, and restrict the sample to this subset. Next, we estimate the complementarity coefficients from a linear moment condition.

Figure 6: Average Team Sizes in a Firm over Years (By Firm Employment)

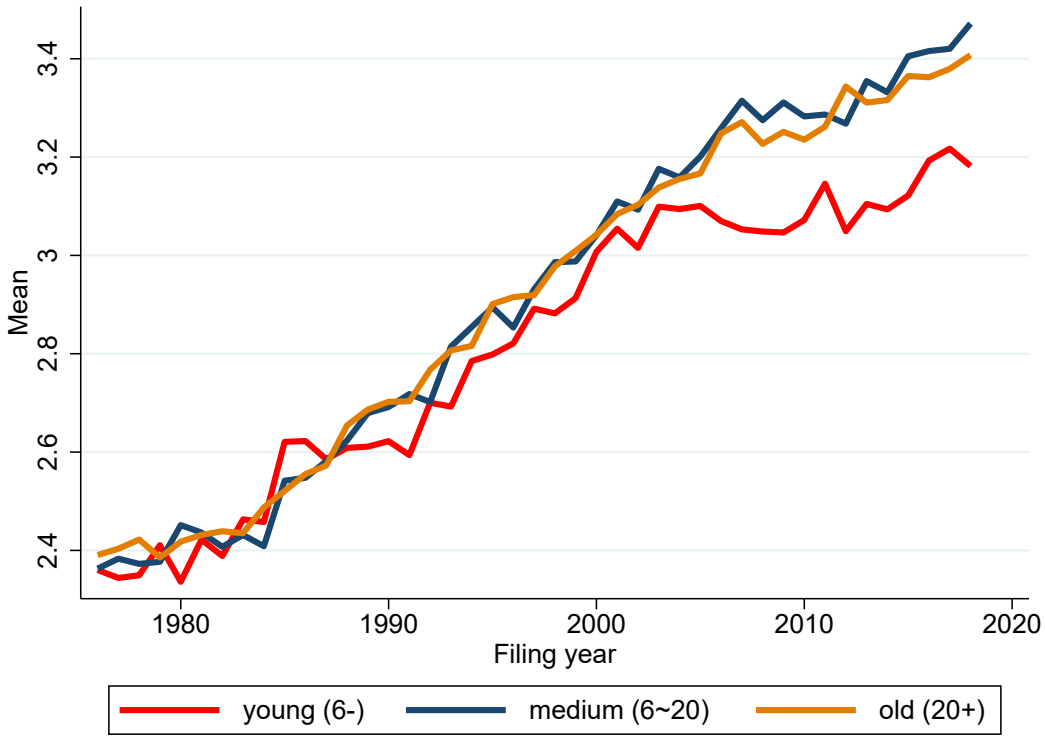


Note: The figure illustrates the average team size in firms, categorized by firm employment size. Firms are classified by their percentile of the cross-sectional size distribution within a pair of industry (NAICS4) and year. Top 25% (red line) indicates the top 25 percentile firms, 25-75% (navy line) indicates firms between 25 and 75 percentiles, and bottom 25% (orange line) indicates those in the bottom 25 percentile in employment size distribution. The solid lines represent the mean team size across firms, while the short-dashed lines indicate the median team size for firms in each group in a given year.

Last, the inventor fixed effects can be estimated in a standard OLS regression.

The interpretation is as follows. Output (citations) is a linear function of the team members' fixed effects. λ_n captures the effects of working in a team of size n. If $\lambda_n = 1$ for $n > 1$, the citations produced from this collaboration are the same as the sum of what the team members could produce individually. If $\lambda_n > 1$ it indicates that the team produces more citations than the members could generate on their own. An increase in λ_n over time suggests increasing complementarities for teams of size n. However, this model does not account for selection into teams or the impact of teams on the probability of securing a patent, among other factors.

Figure 7: Average Team Sizes in a Firm over Years (By Firm Age)



Note: The figure illustrates the average team size in firms, categorized by firm age groups. Firm age data is sourced from Ewens and Marx (2024). Firms are classified as young if their age is five years or less (red line), medium-aged if their age is between six and twenty years (navy line), and old if their age is above twenty years (orange line).

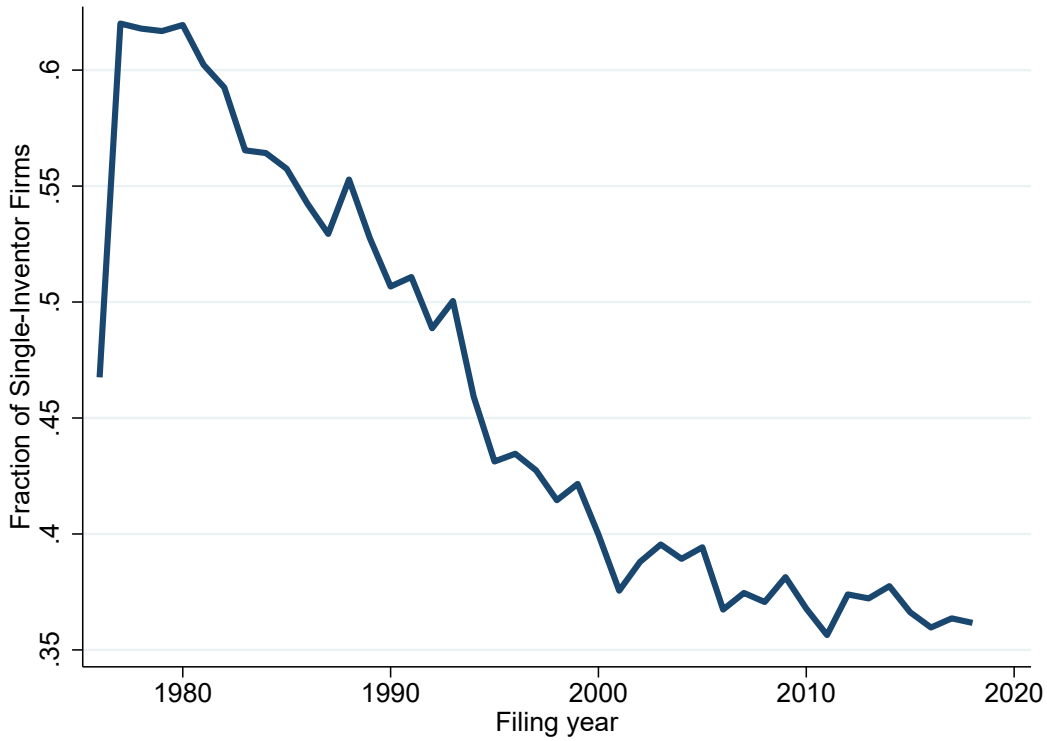
As shown by the coefficients listed in Table 1, firms in the post-2000 period usually exhibit larger team complementarities than firms in the pre-2000 period, especially for teams of size two. Notably, complementarities for teams of two grew significantly for older firms after 2000.

3 Model

In this section, we construct a simple static model to explain the stylized facts observed in the data.[8] The model features two key types of agents: final goods producers and

[8]For this version, we prioritize parsimony to enhance expositional clarity.

Figure 8: Fraction of Single-Inventor Firms



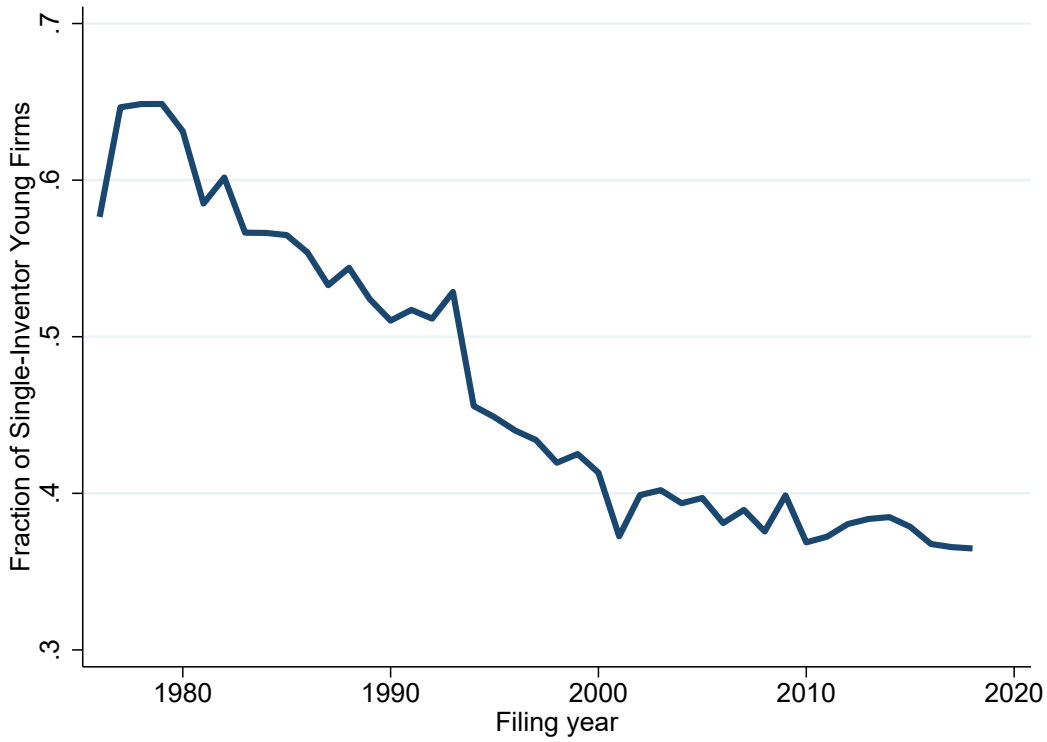
Note: The figure shows the fraction of firms with a single inventor in a given year.

intermediate goods producers. Intermediate good producers vary in their product quality (equivalently, productivity). Quality- (or productivity-)enhancing innovation can be produced using multiple types of human capital. For simplicity, we assume two knowledge worker types (for example, workers in two fields on knowledge measured by the CPC subgroup). Firms incur a search cost, c, to hire each type of knowledge worker. Production labor is hired in a frictionless labor market.

3.1 Final Goods Producer

The final goods producer manufactures the final good using production workers (L_t) and a continuum of differentiated intermediate goods indexed by $j \in [0, 1]$. The pro-

Figure 9: Fraction of Single-Inventor Young Firms



Note: The figure shows the fraction of young firms (aged five or less) with a single inventor in a given year. Firm age is sourced from Ewens and Marx (2024).

duction function exhibits constant returns to scale and is specified as follows:

$$Y_t = \frac{L_t^\theta}{1-\theta} \int_0^1 A_{jt}^\theta y_{jt}^{1-\theta} dj, \tag{3.1}$$

where y_{jt} and A_{jt} are the quantity and quality of good j, respectively. The market is competitive, with the price normalized to one, and producers take input prices as given.

3.2 Intermediate Goods Producers

The intermediate goods producers produce and sell differentiated intermediate goods to final good producers. Each producer operates as a single-product firm, exerting monopolistic market power within its own product market. Intermediate goods are

Table 1: Complementarity estimates for periods and subgroups of firms

Period	Group	λ_1	λ_2	λ_3	$\lambda_{\geq 4}$	# of Patents	# of Inventors
≤ 2000	All	1.0000	0.5674	0.4147	0.3116	1,022,209	206,849
≤ 2000	Small	1.0000	0.5699	0.4089	0.3209	48,392	10,697
≤ 2000	Large	1.0000	0.5627	0.4116	0.3088	885,695	181,694
≤ 2000	Young	1.0000	0.5919	0.4424	0.3581	12,982	3,234
≤ 2000	Old	1.0000	0.5587	0.4173	0.3149	323,371	65,928
> 2000	All	1.0000	0.6607	0.4803	0.3543	3,103,493	590,872
> 2000	Small	1.0000	0.6447	0.5027	0.3949	308,769	71,193
> 2000	Large	1.0000	0.6569	0.4750	0.3492	2,652,074	503,741
> 2000	Young	1.0000	0.5870	0.4526	0.3489	49,209	12,803
> 2000	Old	1.0000	0.6417	0.4631	0.3384	906,673	181,569

Note: This table lists the complementarity estimates for various periods and subgroups of firms. The dataset consists of all USPTO patents filed between 1976 and 2018. The sample of patents is divided into two periods based on their filing years: the first period includes patents filed no later than 2000, and the second period includes patents filed after 2000. Within each period, the sample is further divided by firm size and firm age. The group of small firms consists of those classified as small or micro entities in the USPTO dataset, while the group of large firms includes the remaining firms. The group of young firms includes those with an age of five years or less, where firm age is defined as the difference between the current year and the founding year from Ewens and Marx (2024). The group of old firms includes those with an age greater than five years.

produced at a constant unit marginal cost, measured in terms of final goods. The profit-maximization problem for these producers is given by:

$$\pi_{jt} = \max_{y_{jt}} L_t^{\theta} A_{jt}^{\theta} y_{jt}^{1-\theta} - y_{jt}. \tag{3.2}$$

Intermediate goods producers can undertake R&D activities to innovate. By hiring R&D workers, they enhance the quality of their products. Successful innovation increases product quality by the ratio of $\lambda > 1$. The quality of product j evolves according to the following equation:

$$A_{jt} = \begin{cases} \lambda A_{jt} \text{ with } \phi_{jt} \\ A_{jt} \text{ with } 1 - \phi_{jt} \end{cases},$$

where

$$\phi_{jt} = \left(\frac{R_{jt}}{\chi A_{jt}} \right)^{\frac{1}{\psi}} \tag{3.3}$$

$$R_{jt} = \left(h_{1jt}^{\rho} + h_{2jt}^{\rho} \right)^{\frac{1}{\rho}}. \tag{3.4}$$

The success probability of innovation, ϕ_{jt}, depends on the level of R&D investment, R_{jt}.[9] The R&D investment, R_{jt}, requires hiring R&D workers, h_{1jt} and/or h_{2jt}. This formalizes the process of forming a team of different workers to produce innovation. Although the two types of R&D workers have no inherent heterogeneity in skills or productivity, they contribute to R&D production through a CES function that captures complementarity.[10] The parameter ρ determines the degree of complementarity.[11] Note that if firms hire only one type of worker, the output is linear in the number of workers. However, if firms hire both types of workers, the outcome reflects the complementarity between them, which is nonlinear in the amount of workers.

3.3 Equilibrium

Hereafter, the time subscript is suppressed given the static structure of the model.

3.3.1 Optimal Production

Final goods producers determine their optimal levels of labor and intermediate goods inputs. Letting p_j denote the price of product j, the inverse demand function for intermediate good j is given by:

$$p_j = L^{\theta} A_j y_j^{-\theta}. \tag{3.5}$$

[9] For simplicity, we assume that R&D investment can instantaneously improve product quality. This assumption gives the model a static structure, simplifying the analytical derivation of decision rules.

[10] Note that the worker types here do not necessarily capture inherent heterogeneity in skills or productivity. Instead, they may serve as a proxy for different areas of expertise, reflecting workers with varying backgrounds, as observed in the data.

[11] Note that $\rho = 1$ represents perfect substitution, $\rho = -\infty$ indicates perfect complements, and $\rho = 0$ implies unit elasticity between the two types of inputs. Defining the elasticity of substitution as $\sigma \equiv \frac{1}{1-\rho}$, this corresponds to $\sigma = \infty$ for $\rho = 1$, $\sigma = 0$ for $\rho = -\infty$, and $\sigma = 1$ for $\rho = 0$.

Intermediate goods producers take the inverse demand function (3.5) as given and maximize their profit function (3.2). The optimal production quantity and pricing decisions are derived as follows:

$$y_j = (1-\theta)^{\frac{1}{\theta}} L A_j \tag{3.6}$$

$$p_j = \frac{1}{1-\theta}. \tag{3.7}$$

These equations simplify the equilibrium profit to:

$$\pi_j = \theta(1-\theta)^{\frac{1-\theta}{\theta}} L A_j \equiv \pi A_j. \tag{3.8}$$

3.3.2 Optimal Innovation

Intermediate goods producers determine their optimal level of innovation by solving the following problem:

$$\max_{\{h_{1j}, h_{2j}\}} \phi_j \pi \lambda A_j + (1-\phi_j)\pi A_j - c\mathrm{I}_{h_{1j}>0} - c\mathrm{I}_{h_{2j}>0} - w(h_{1j}+h_{2j}) \tag{3.9}$$

subject to the constraints in (3.3) and (3.4). Due to search costs, firms must decide whether to hire at all, and, if they do, whether to hire one or both types of human capital, and how much of each type to hire.

Solving (3.9), the demand for human capital is derived as follows:

If no human capital is hired, the firm gains nothing:

$$h_{1j}^{no}, h_{2j}^{no}, \phi_j^{no}, \pi_j^{no} = 0. \tag{3.10}$$

If only one type is hired, and given the symmetry, the firm is indifferent between the two types and hires:

$$h_{ij}^{one} = \left(\frac{\pi(\lambda-1)}{\chi^{\frac{1}{\psi}}\psi w}\right)^{\frac{\psi}{\psi-1}} A_j \text{ for } i = 1 \text{ or } 2. \tag{3.11}$$

This results in the following innovation intensity and profits:

$$\phi_j^{one} = \left(\frac{\pi(\lambda-1)}{\chi\psi w}\right)^{\frac{1}{\psi-1}} \tag{3.12}$$

$$\pi_j^{one} = \left(\frac{\pi(\lambda-1)}{\chi\psi w}\right)^{\frac{1}{\psi-1}} \pi(\lambda-1)A_j + \pi A_j - w\left(\frac{\pi(\lambda-1)}{\chi^{\frac{1}{\psi}}\psi w}\right)^{\frac{\psi}{\psi-1}} A_j - c. \tag{3.13}$$

If both types of human capital are hired, the firm chooses use team production :

$$h_{ij}^{both} = \left(\frac{\pi(\lambda-1)}{2\chi^{\frac{1}{\psi}}\psi w}\right)^{\frac{\psi}{\psi-1}} 2^{\frac{1}{\rho(\psi-1)}} A_j \text{ for } i = 1, 2, \tag{3.14}$$

which results in the following innovation intensity and profits:

$$\phi_j^{both} = 2^{\frac{1}{\rho(\psi-1)}}\left(\frac{\pi(\lambda-1)}{2\chi\psi w}\right)^{\frac{1}{\psi-1}} \tag{3.15}$$

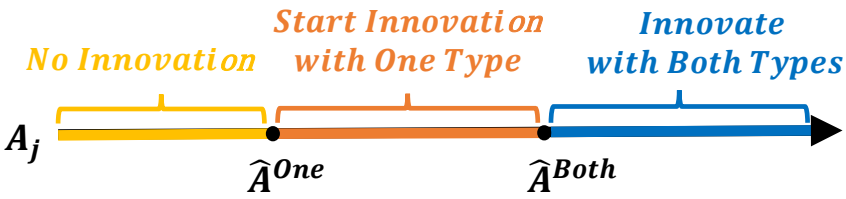
$$\pi_j^{both} = 2^{\frac{1}{\rho(\psi-1)}}\left(\frac{\pi(\lambda-1)}{2\chi\psi w}\right)^{\frac{1}{\psi-1}} \pi(\lambda-1)A_j + \pi A_j - 2w\left(\frac{\pi(\lambda-1)}{2\chi^{\frac{1}{\psi}}\psi w}\right)^{\frac{\psi}{\psi-1}} 2^{\frac{1}{\rho(\psi-1)}} A_j - 2c. \tag{3.16}$$

Firms decide whether to hire both types of R&D workers, only one type, or none at all, based on productivity. This decision is governed by two productivity cutoffs. The first cutoff, denoted $\hat{A}^{one}$, is the threshold above which firms hire at least one type of human capital and engage in innovation. This can be interpreted as an entry cutoff, above which firms enter into innovation. The second cutoff, denoted $\hat{A}^{both}$, determines whether firms hire one or both types of human capital. This is illustrated in Figure 10.

In particular, $\hat{A}^{one}$ is determined by

$$\min[\pi_j^{one}(\hat{A}^{one}), \pi_j^{both}(\hat{A}^{one})] = 0,$$

Figure 10: Productivity Cutoffs



Note: The figure illustrates the two endogenous productivity cutoffs in the model: the higher cutoff, $\hat{A}^{both}$, which determines whether firms hire both types of R&D workers or only one type, and the lower cutoff, $\hat{A}^{one}$, which governs the entry of firms into innovation by hiring only one type of R&D worker.

which gives the following expression for $\hat{A}^{one}$:

$$\hat{A}^{one} = \frac{c}{\left(\frac{(\pi(\lambda-1))^{\frac{\psi}{\psi-1}}}{(\chi\psi w)^{\frac{1}{\psi-1}}}(1-\psi)+\pi\right)}. \tag{3.17}$$

Similarly, $\hat{A}^{both}$ is determined at the point where the following condition holds:

$$\pi_j^{one}(\hat{A}^{both}) = \pi_j^{both}(\hat{A}^{both}) > 0,$$

which gives the following expression for $\hat{A}^{both}$:

$$\hat{A}^{both} = \frac{c}{(1-2^{\frac{1}{(\psi-1)}(\frac{1}{\rho}-1)})\frac{(\pi(\lambda-1))^{\frac{\psi}{\psi-1}}}{(\chi\psi w)^{\frac{1}{\psi-1}}}(\psi-1)}. \tag{3.18}$$

3.3.3 Equilibrium Wage

Suppose the total supply of R&D capital is fixed at $\bar{h} = \bar{h}_1 = \bar{h}_2$. Also, let $F(A_j)$ denote the productivity distribution of firms. The labor market clearing for each R&D worker

type $i = \{1, 2\}$ is given by:

$$\int_{\hat{A}^{one}}^{\hat{A}^{both}} h_{ij}^{one} dF(A_j) + \int_{\hat{A}^{both}}^{\infty} h_{ij}^{both} dF(A_j) = \bar{h}_i \text{ for } i = 1, 2 \tag{3.19}$$

Define $\bar{A}^{one} \equiv \int_{\hat{A}^{one}}^{\hat{A}^{both}} dF(A_j)$ as the mean productivity of firms that hire only one type of R&D worker, and $\bar{A}^{both} \equiv \int_{\hat{A}^{both}}^{\infty} dF(A_j)$ as the mean productivity for firms that hire both types. Rewriting the labor market clearing condition (3.19) with (3.14), (3.11), along with these definitions, we obtain:

$$\left(\frac{1}{w}\right)^{\frac{\psi}{\psi-1}} \left(\frac{\pi(\lambda-1)}{\chi^{\frac{1}{\psi}}\psi}\right)^{\frac{\psi}{\psi-1}} \bar{A}^{one} + \left(\frac{1}{w}\right)^{\frac{\psi}{\psi-1}} \left(\frac{\pi(\lambda-1)}{2\chi^{\frac{1}{\psi}}\psi}\right)^{\frac{\psi}{\psi-1}} 2^{\frac{1}{\rho(\psi-1))}} \bar{A}^{both} = \bar{h}, \tag{3.20}$$

which characterizes the equilibrium wage in the model.

For example, taking the standard value $\psi = 2$ commonly used in the literature (Acemoglu and Cao 2015; Akcigit and Kerr 2018), the equilibrium wage is determined by:

$$w = \left(\frac{\pi(\lambda-1)}{2\chi^{\frac{1}{2}}}\right) \left(\frac{\bar{A}^{one} + \bar{A}^{both} 2^{\frac{1}{\rho}-2}}{\bar{h}}\right)^{\frac{1}{2}}. \tag{3.21}$$

Note from (3.21) that the equilibrium wage is a function of profits, step size, human capital supply, average level of productivity, as well as the complementarity.

3.4 Key Implications

The model has several important implications for understanding firm innovation, R&D labor demand, and the role of input complementarities in shaping hiring decisions. The following key insights emerge from the analysis. For now, we are focusing on a partial equilibrium, where we take the wage w as given.

3.4.1 Extensive Margin: Entry Decision into Innovation and Hiring Both R&D Input Types

First, firms face key decisions at the extensive margin regarding whether to enter the innovation process and whether to hire one or both types of R&D workers. These decisions are based on the equilibrium profits from innovation (with either one or two types of R&D workers) given by (3.13) and (3.16), as well as the productivity cutoffs defined in (3.18) and (3.17).

The firm decisions are influenced by the fixed search costs associated with hiring workers and conducting innovation, c, as well as the degree of complementarity between the two types of R&D inputs ρ. The following propositions characterize this property in more detail.

Proposition 1. *If $0 < \rho \leq 1$ (when inputs are substitutes), then $\pi_j^{one} > \pi_j^{both}$ for any A_j (since $\psi > 1$), and if $\rho < 0$ (when inputs are complements), then $\pi_j^{both} > (<)\pi_j^{one}$ iff $A_j > (<)\hat{A}_j^{hiring}$.*

Proof: Starting with (3.16) *and* (3.13)*, we can rewrite them as follows:*

$$\pi_j^{both} = \left(2^{\frac{1}{(\psi-1)}(\frac{1}{\rho}-1)} \frac{\pi(\lambda-1)^{\frac{\psi}{\psi-1}}}{(\chi\psi w)^{\frac{1}{\psi-1}}}(1-\psi) + \pi \right) A_j - 2c \tag{3.22}$$

$$\pi_j^{one} = \left(\frac{\pi(\lambda-1)^{\frac{\psi}{\psi-1}}}{(\chi\psi w)^{\frac{1}{\psi-1}}}(1-\psi) + \pi \right) A_j - c. \tag{3.23}$$

Comparing (3.23) *and* (3.22) *completes the proof.*

This shows that, depending on the degree of complementarity, firms' hiring decisions vary in an intuitive way. Specifically, if the two types of inputs are substitutes ($0 < \rho \leq 1$), firms will opt to hire only one type of input, regardless of their productivity level, because of the fixed cost to find the second worker type. On the other hand, if the two types of inputs are complements ($\rho < 0$), firms will be willing to employ both types of inputs, provided they are sufficiently productive to cover the search costs.

Following this property, henceforth, we focus on the case of input complementarity,

as firms would have no incentive to employ both types of workers for innovation if the inputs were substitutes. In this scenario, the model would reduce to the standard model with only one type of worker, which is trivial. The results for the case of input substitutes are presented in the Appendix.

Proposition 2. *The hiring cutoff* $\hat{A}_j^{hiring}$ *increases in* $|\rho|$ *if* $\rho < 0$. *Proof: See* (3.18).

Since higher $|\rho|$ in the range of $\rho < 0$ implies a higher degree of complementarity, this suggests that the cutoff rises with more complementarity between input types.[12] Intuitively, when there is more complementarity between inputs, firms need to employ both types of inputs to achieve a given level of innovation. Given the search cost associated with hiring each type of input, this increased need for diverse inputs raises the hiring cutoff, as firms must be sufficiently productive to afford the additional search costs.

Proposition 3. *Both the hiring cutoff* $\hat{A}_j^{hiring}$ *and the entry cutoff* $\hat{A}_j^{entry}$ *increase in* c. *Proof: See* (3.18) *and* (3.17).

This is intuitive because a higher fixed search cost c increases the threshold for firms to be profitable enough to innovate or hire inputs. Consequently, higher search costs lead to a selection in firm entry and hiring decisions, where only more productive firms willing to bear the costs.

3.4.2 Intensive Margin: Deciding the Amount of R&D Inputs and Innovation Intensity

Based on (3.14) and (3.11), the amount of R&D human capital firms hire (the equilibrium size of R&D team) depends on key parameters as well: the degree of complementarity ρ and the fixed search cost c. As a result, the equilibrium innovation intensity of firms, given by (3.15) and (3.12), also depend on these parameters. As before, we

[12] Refer to the Appendix for the case of $\rho > 0$ (substitutable inputs).

focus our analysis on complementarity case $\rho < 0$ and leave the substitute case $\rho > 0$ in the Appendix.[13] Here we assume $\psi > 1$.

Proposition 4. *Suppose $\rho < 0$. Then, both h_{ij} and ϕ_{ij} weakly increase in $|\rho|$.*

Proof: Now suppose $\rho < 0$ and do the same analysis. In this case, we obtain:

$$\frac{\partial h_{ij}^{both}}{\partial |\rho|} = \left(\frac{\pi(\lambda-1)}{2\chi^{\frac{1}{\psi}}\psi w}\right)^{\frac{\psi}{\psi-1}} \frac{2^{\frac{1}{\rho(\psi-1)}}\ln 2}{|\rho|^2(\psi-1)} A_j > 0,$$

$$\frac{\partial \phi_{ij}^{both}}{\partial |\rho|} = \left(\frac{\pi(\lambda-1)}{2\chi\psi w}\right)^{\frac{1}{\psi-1}} \frac{2^{\frac{1}{\rho(\psi-1)}}\ln 2}{|\rho|^2(\psi-1)} > 0,$$

$$\frac{\partial h_{ij}^{one}}{\partial \rho} = 0,$$

$$\frac{\partial \phi_{ij}^{one}}{\partial \rho} = 0.$$

Therefore, as the absolute magnitude of ρ increases (or as ρ becomes more negative, indicating greater complementarity between the two input types), both h_{ij} and ϕ_{ij} increase. When ρ approaches 0, both h_{ij} and ϕ_{ij} approach 0. When ρ approaches $-\infty$, h_{ij} and ϕ_{ij} reach the value $\left(\frac{\pi(\lambda-1)}{\chi^{\frac{1}{\psi}}\psi w}\right)^{\frac{\psi}{\psi-1}} A_j$ and $\left(\frac{\pi(\lambda-1)}{\chi\psi w}\right)^{\frac{1}{\psi-1}}$, respectively. Therefore, within the range of $\rho < 0$ (complementary inputs), firms tend to hire and innovate more if inputs get more complementary to each other (i.e., as $|\rho|$ increases).

This proposition suggests that, all else equal, the size of the R&D team and the intensity of innovation increase when R&D inputs become more complementary to each

[13] Note that (3.14), (3.11), (3.15), and (3.12) depend on ρ through the term $2^{\frac{1}{\rho}}$, which has a discontinuity at $\rho = 0$. The functional form is as follows:

$$\begin{cases} 2^{\frac{1}{\rho}} \to 1 \text{ if } \rho > 0, \rho \to \infty \\ 2^{\frac{1}{\rho}} \to \infty \text{ if } \rho > 0, \rho \to 0 \\ 2^{\frac{1}{\rho}} \to 1 \text{ if } \rho < 0, \rho \to -\infty \\ 2^{\frac{1}{\rho}} \to 0 \text{ if } \rho < 0, \rho \to 0. \end{cases}$$

Thus, we need to consider the derivatives of these functions with respect to ρ by separate cases, $\rho > 0$ and $\rho < 0$.

other. In other words, greater complementarity between input types encourages firms to hire more R&D workers and invest more in innovation.

Lastly, the model also has an implication on equilibrium wage.

Proposition 5. *The equilibrium wage increases in* $|\rho|$ *if* $\rho < 0$. *Proof: See* (3.21).

This is intuitive as the equilibrium wage is adjusted by the changes in labor demand, following Proposition 4.

4 Quantitative Model

We adopt an illustrative set of parameters to show the model can reproduce features of the data and give a sense of how we'll calibrate the complementarity parameter in future versions.

4.1 Calibration

The parameterization is summarized in Table 2. Some parameters are taken from the literature. The labor share θ is set to 0.6. The cost curvature of R&D ψ is set to 2 (Acemoglu et al. 2018). We choose a Pareto distribution as the functional form of the productivity distribution $F(A)$ with tail index $\alpha = 1.06$ (Axtell 2001) and the lower bound A_{lb} is normalized to 1.

We also normalize the wage w and the R&D scale parameter χ to be one, taking the wage as exogenous for now. The hiring cost $c = 0.2$ is set so that the hiring cost is 20% of one year's wages. The final parameters to calibrate are λ and ρ. Loosely speaking, conditional on other parameters, λ governs the share of innovating firms since it determines the return to innovation and ρ governs the boundary between hiring one and two types of workers. We've chosen values that match the facts in section 2, namely that the share of single inventor firms (conditional on patenting) in the data is around 50% (Table 3).

	Value	Meaning
θ	0.6	Labor share/markup
λ	2.6	Step size
ψ	2	R&D curvature
χ	1	R&D scale
c	0.2	Search cost
w	1	Wage
α	1.06	Pareto tail index
A_{lb}	1	Pareto lower bound
ρ	-1	Complementarity

Table 2: Model parameters, baseline.

The computation of the share of team patents moment is

$$s_{team} = \frac{s^{both}\phi^{both}}{s^{both}\phi^{both} + s^{one}\phi^{one}}.$$

The model slightly underpredicts the share of team patents because the innovation intensity of the large firms (firms hiring both worker types) is much lower than that of the intensity of the single type firms. Later versions of the calibration should be able to address this, for example by allowing research productivity χ to be correlated with productivity in the goods market A_j rather than constant across firms.

4.2 Results

Figure 11 shows how the model moments vary with changes in worker complementarities. Increasing complementarities raise the innovation intensity of firms hiring both types. By raising the cutoff A^{two} it also increases the share of single inventor firms. The former effect dominates initially so that the share of team patents rises with worker complementarities, but then the latter effect begins to dominate and the share of team patents levels off. This gives a sense of how we can calibrate ρ using data on the share of single inventor firms and the team patent share to match the observed trends in the data.

	PE moments
Innov. intensity ϕ^{one}	0.26
Innov. intensity ϕ^{two}	0.07
share non-patenting firms	0.05
share one type	0.46
share two types	0.49
share single inventor	0.48
share team patents	0.21

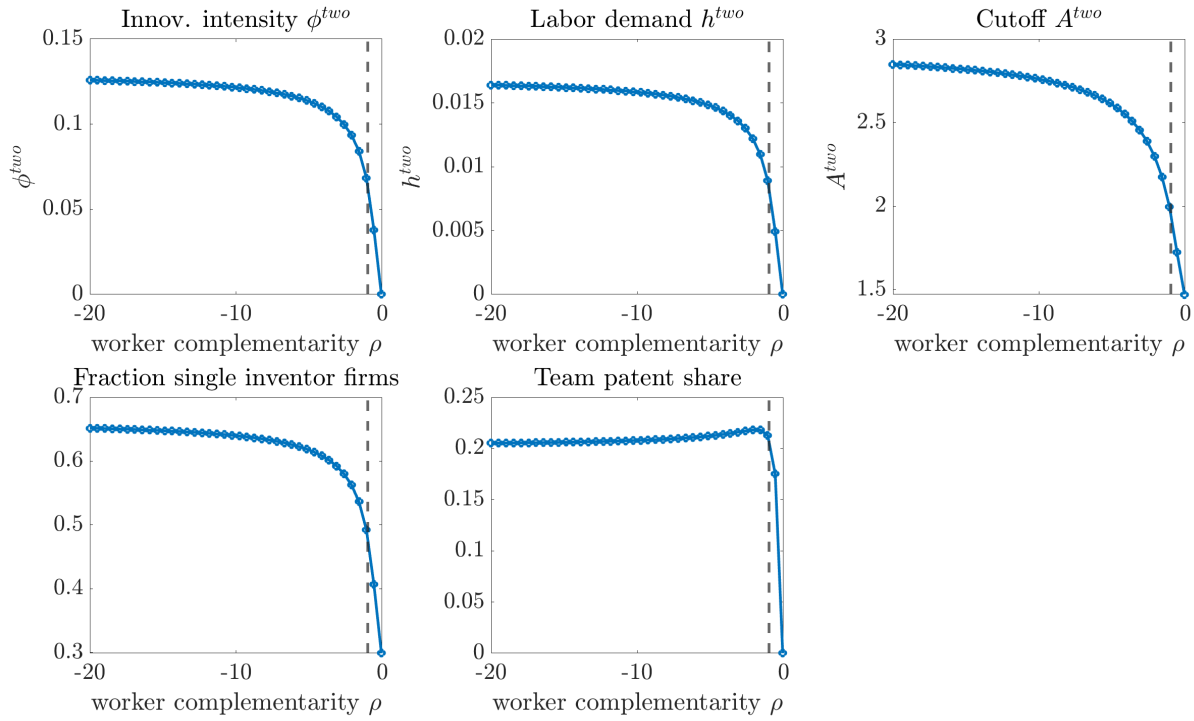
Table 3: Model moments, baseline.

5 Conclusion

Understanding inventor teams has become critical for studying innovation due to the increasing share of ideas produced by teams rather than individuals. Inventors do not collaborate randomly with others, but are more likely to team up with colleagues within the same firm. We provide new evidence showing that larger, more productive, or older firms are more likely to form inventor teams, assemble larger teams, and create teams with more diverse skill sets. In our theoretical framework, this happens because locating and combining workers with different skill types may involve a fixed cost. Our model generates that rising complementarities between knowledge workers lower the threshold for team-based production and enhance the innovation intensity of large or more productive firms, which aligns with the data.

In future versions, we plan to embed the static model into a dynamic setting with an endogenous evolution of the productivity distribution, to explore how rising complementarities impact growth and interact with competition between firms. Also, we would like to quantify its macroeconomic implications on firm entry, business dynamism, as well as market concentration. We also aim to refine our estimates of the increase in complementarities using the administrative employee-employer matched data in the U.S.Census Bureau, linked to the USPTO patent data, on knowledge worker wages. Another direction for future work is to ask how the changing nature of idea production in teams, and within specific firms, will affect the dynamics of knowledge accumulation

Figure 11: Comparative Statics for Worker Complementarities ρ



Note: Model output.

for R&D workers and worker reallocation.

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Appendix

A Propositions for Substitute Case

In this section, we draw the same propositions for the case of $\rho > 0$, when inputs are substitutes.

Proposition A.1. *The hiring cutoff $\hat{A}_j^{hiring}$ decreases in ρ if $\rho > 0$. Proof: See* (3.18).

Since higher ρ in the range of $\rho > 0$ implies a higher degree of substitutability, this suggests that the cutoff rises with less substitutablility between input types.

Proposition A.2. *Suppose $\rho > 0$. Then, both h_{ij} and ϕ_{ij} weakly decrease in ρ.*
Proof: Getting the derivatives of h_{ij}, ϕ_j with respect to ρ in the range of $\rho > 0$, we obtain the following expressions:

$$\frac{\partial h_{ij}^{both}}{\partial \rho} = -\left(\frac{\pi(\lambda-1)}{2\chi^{\frac{1}{\psi}}\psi w}\right)^{\frac{\psi}{\psi-1}} \frac{2^{\frac{1}{\rho(\psi-1)}}\ln 2}{\rho^2(\psi-1)} A_j < 0,$$

$$\frac{\partial \phi_{ij}^{both}}{\partial \rho} = -\left(\frac{\pi(\lambda-1)}{2\chi\psi w}\right)^{\frac{1}{\psi-1}} \frac{2^{\frac{1}{\rho(\psi-1)}}\ln 2}{\rho^2(\psi-1)} < 0,$$

$$\frac{\partial h_{ij}^{one}}{\partial \rho} = 0,$$

$$\frac{\partial \phi_{ij}^{one}}{\partial \rho} = 0.$$

This shows that as ρ increases (with greater substitutability between the two input types), both h_{ij} and ϕ_{ij} decrease. When ρ approaches 0, h_{ij} and ϕ_{ij} approach infinity, and when ρ approaches ∞, h_{ij} and ϕ_{ij} reach the value $\left(\frac{\pi(\lambda-1)}{\chi^{\frac{1}{\psi}}\psi w}\right)^{\frac{\psi}{\psi-1}} A_j$ and $\left(\frac{\pi(\lambda-1)}{\chi\psi w}\right)^{\frac{1}{\psi-1}}$, respectively. Therefore, within the range of $\rho > 0$ (substitutable inputs), firms tend to hire and innovate less as the inputs become more substitutable (i.e., as ρ increases).

Proposition A.3. *The equilibrium wage decreases in* ρ *if* $\rho > 0$. *Proof: See* (3.21).

This is intuitive as the equilibrium wage is adjusted by the changes in labor demand, following Proposition A.2.