When Does Idiosyncratic Risk Create Asset Bubbles?

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Abstract

Heterogeneous-agent economies with idiosyncratic risk constitute a realistic environment for valuation. This paper asks whether and when idiosyncratic risk lowers the discount rate sufficiently to create asset bubbles. We find that it depends crucially on the degree of inequality in the wealth distribution. The transversality condition is violated and bubbles arise only when wealth inequality is so large that agents expect to become very poor in some states of the world, in which case the asset will appear infinitely large from their perspectives, whereas aggregate uncertainty and growth are irrelevant. This intuition is different from the classic rational bubble in overlapping-generation models, and has important implications for how heterogeneous-agent economies price long-lived assets like government debt and aggregate consumption claim.

Keywords: Transversality Condition, Heterogeneous Agents, Wealth Distribution, Public Debt Valuation

JEL Codes: E44, E62, G12

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1 Introduction

Heterogeneous-agent economies with idiosyncratic risk constitute an important class of workhorse models in macroeconomics and finance. It is well understood that uninsurable idiosyncratic risk lowers the risk-free discount rate due to the precautionary saving motive. However, whether this motive lowers the discount rate enough to violate the no-bubble condition, also known as the transversality condition (TVC) is less well understood. In this paper, we characterize conditions under which idiosyncratic risk breaks the TVC for long-lived assets, a critical issue in asset valuation.

We find that the TVC depends crucially on the degree of inequality in the wealth distribution, which is in turn driven by properties of the idiosyncratic risk. We uncover a new intuition for TVC violation in heterogeneous-agent economies, which is completely different from the classic rational bubble in overlapping-generation models: in our setting, TVC violation requires the wealth of a significant fraction of agents to become sufficiently small relative to the average wealth in the economy. When they evaluate an asset whose cash flow is unaffected by their idiosyncratic risk, this asset will appear infinitely large relative to their own sizes in the limit. As a result, they find this asset very valuable, leading to TVC violation even in the absence of aggregate uncertainty and growth. Through the lens of this new intuition, we re-examine how assets are priced in models like Constantinides and Duffie (1996); Krueger and Lustig (2010), and show that some prior analysis may be incomplete due to the possibility of TVC violation.

We begin our analysis with the model proposed by Brunnermeier, Merkel, and Sannikov (2022), who provide a tractable framework for modeling households facing uninsurable idiosyncratic risk. We note that in this setting, the idiosyncratic risk is *permanent* and follows a random walk. As the wealth distribution evolves over time, almost all households will become very poor, while a vanishing share of households will become very rich and own most of the wealth. In other words, this economy features an exploding degree of inequality in the wealth distribution.

Let us consider how a household in this economy prices a long-lived asset such as government debt. The asset's valuation depends on its cash flow and discount rate, which is a function of the

household's marginal utility growth and subjective discount rate. In usual cases including most representative-agent models, the asset's cash flow and the household's consumption level are cointegrated, and their covariance describing the cash flow's riskiness is the first-order consideration. In our economy, as the household expects its wealth and consumption to fall far below the population average in almost all states of the world, the asset's cash flow will become very large relative to the household's size. In particular, when the magnitude of the idiosyncratic risk is sufficiently large, the household expects its wealth to decline and its marginal utility to grow faster than its subjective discount rate.

As a result of this divergence in size between the household and the asset, while the household's marginal utility growth correctly prices the asset's one-period return, the asset value in the long-run future discounted by the household's marginal utility growth tends to infinity. However, this TVC violation does not mean that the household will have an infinite amount of wealth, and therefore demand more consumption goods than available in the economy. While the household assigns an infinite valuation to aggregate cash flows, its valuation of its own consumption stream is finite because the household discounts assets exposed to its own idiosyncratic risk at higher rates. As a result, each household's perception of its own wealth is finite in this economy.

Having said that, permanent idiosyncratic risk does not always lead to TVC violation. When the idiosyncratic risk is permanent but sufficiently small, while the household still expects its wealth to eventually vanish relative to the average wealth, this process is slow and dominated by the subjective discount rate. This case is similar to the standard scenario in which the cash flow grows exponentially but the growth rate is dominated by the household's discount rate, so that valuation is finite and the TVC holds.

We can also examine the condition for TVC violation by comparing the discount rate r with the growth rate g. Since households are risk-averse, the relevant discount rate r for any asset is not the risk-free rate but instead adjusts for its risk premium. We show that r minus g is decreasing in the magnitude of the idiosyncratic risk, which is due to the precautionary saving motive. Moreover, r is lower than g precisely when the magnitude of the idiosyncratic risk is large enough to generate

TVC violation. Therefore, $r > g$ remains sufficient and necessary for the TVC to hold in this setting. The relative magnitude of r vs. g depends on the relative magnitude of the idiosyncratic risk and the subjective discount rate, whereas the magnitude of the aggregate risk and growth rate, and the absolute sizes of wealth and asset are irrelevant.

The above analysis implies that TVC violation occurs if there is sufficiently large permanent idiosyncratic risk, which leads to an exploding amount of wealth inequality. A natural question is whether TVC violation can still happen when households face transitory idiosyncratic risk and the wealth distribution is stationary, which is perhaps a more realistic setting for studying asset prices.

To answer this question, we consider two additional model ingredients: wealth reset and death. In the first model extension, a redistribution shock resets a fixed fraction of households' wealth back to the average wealth level in each period, which preserves the total amount of resources in the economy. This setting generates a stationary wealth distribution while remaining tractable. To our surprise, even when the amount of wealth inequality is not exploding, the TVC violation for aggregate quantities can still occur. Again, the key is to understand the properties of the wealth distribution.

Specifically, when the intensity of the wealth reset shock is small, each household's wealth can deviate from the average level for a long time before reverting to the mean. Despite the wealth distribution being stationary, a household's wealth level can be much lower than the population average in a non-trivial fraction of states. When this left tail of the wealth distribution is sufficiently heavy, the household expects its average marginal utility growth to be infinitely large, which leads to TVC violation just like the case with permanent idiosyncratic risk.

Conversely, when the intensity of the wealth reset shock is sufficiently large, the wealth distribution is more concentrated around its average. Households expect their wealth levels to be comparable to the size of the economy in most states of the world, and their marginal utility growth is expected to be finite. In this case, the TVC for long-lived assets holds. This property of wealth distribution is again reflected in the r minus g condition: the magnitude of the risk-adjusted r relative to the growth rate g depends on the wealth distribution. $r > g$ arises as a sufficient and

necessary condition for the TVC condition to hold.

These results show that TVC violation for aggregate assets occurs not only when wealth inequality explodes, but also when the wealth distribution is stationary but sufficiently left-skewed. A bounded wealth distribution, on the other hand, is sufficient to guarantee that the TVC holds. In this case, while the uninsurable idiosyncratic shocks can still lower the risk-free rate, there is a limit to how much they can lower the discount rate because the households' marginal utility growth is bounded.

Figure 1 compares these model variants with the data. We use the Gini coefficient based on income data to measure the degree of inequality in the U.S. because the wealth in the model captures not only financial wealth, but also human capital.[1] From 1963 to 2022, the Gini coefficient has been quite stable, fluctuating around 0.4. If we start with the same degree of inequality in the baseline model with permanent idiosyncratic risk, the wealth distribution will diverge over time. Under our calibration which is motivated by the empirical moments considered in Brunnermeier, Merkel, and Sannikov (2022), we expect the Gini coefficient to increase to 0.6 around 2020, and approach 0.7 after 2050.

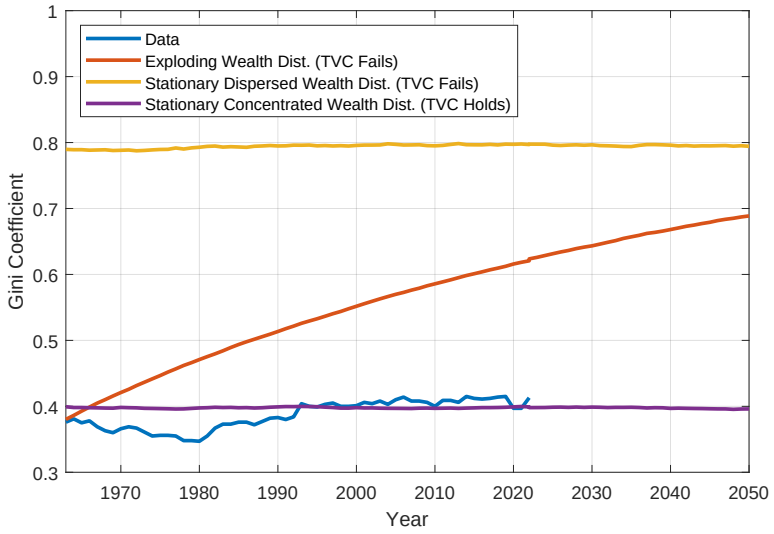


Figure 1: Gini Coefficient of Consumption Inequality.

[1] Greenwald et al. (2021)'s estimates show that the financial wealth Gini is about 0.74 in the U.S., whereas the adjusted total wealth Gini is 0.41, which is also close to the numbers we use here.

In comparison, in models with transitory idiosyncratic risk and stationary wealth distribution, the Gini coefficient remains stable over time. We consider two cases. With a low intensity of the wealth reset shock, the distribution of wealth is heavily left-skewed, implying a Gini coefficient of 0.8. With a higher intensity of the wealth reset shock, the distribution of wealth is more concentrated, and the Gini coefficient is around 0.4. The data is more consistent with the latter case, in which the wealth distribution is not so heavily left-skewed. In this case, the TVC for long-lived assets holds, which allows us to continue to use the discounted present value approach to evaluate asset prices.

In the second model extension, we study how our conclusions change when we introduce death to our model. We assume that households die at a constant rate, and new households are born to maintain a constant population. This ingredient always makes the households more impatient, so that they consume a higher fraction of their wealth and discount future cash flows more heavily.

We consider two variants of the model with death. First, if each newborn household inherits the wealth of a specific dying household, then, the wealth distribution is going to diverge over time like our baseline case with permanent idiosyncratic risk. In this case, we obtain identical results as in our baseline case: if the magnitude of the idiosyncratic risk is large enough relative to the subjective discount rate, the households are still concerned about their vanishing wealth shares in scenarios in which they survive, which leads to TVC violation. The only difference is that the subjective discount rate we use needs to be adjusted upwards by the death rate.

Second, if each newborn household starts with the average wealth of the dying households, the resulting model behaves almost the same as the case with wealth reset shock. In this case, the wealth distribution is stationary, but whether the TVC holds still depends on the degree of inequality in the wealth distribution, with death acting like the wealth reset shock. These results show that introducing death does not change our main conclusions.

In sum, our results show that the degree of inequality has surprisingly important implications for asset valuation in heterogeneous-agent models. For example, through the lens of our characterization, the baseline model in Constantinides and Duffie (1996) belongs to the case with permanent

idiosyncratic risk, which implies a diverging wealth distribution. If the magnitude of the idiosyncratic risk is large enough, then, the TVC for aggregate assets may fail even when the model has well-defined pricing relationships for one-period returns. Constantinides and Duffie (1996) also consider a model variant which is similar to our case with wealth reset shock. Even in this setting, TVC violation can still occur if the wealth distribution is sufficiently left-skewed. As a result, we cannot take TVC for granted when developing and applying heterogeneous-agent models.

In addition, our characterization of the TVC can be applied to any aggregate assets that are cointegrated with the output, which sheds light on two important questions in macroeconomics and finance: the valuation of government debt and the valuation of the households' wealth portfolio. In the former case, the TVC is a precondition for deriving the valuation of the government bond portfolio based on the discounted present value of primary surpluses. If the TVC is violated, the government may be able to issue a lot more debt than justified by its fiscal backing, at least in theory. In the latter case, TVC violation implies that the valuation of the households' wealth portfolio could be a lot higher than previously thought, which affects how we think about the magnitude of human capital.

Our paper proceeds as follows. Section 2 studies the TVC in a baseline model with permanent idiosyncratic risk. Section 3 introduces wealth reset to model transitory idiosyncratic risk. Section 4 studies models with death. Section 5 discusses the implications of our results for workhorse heterogeneous-agent models. Section 6 concludes. All proofs are in the Appendix.

Literature.

Our results parallel the literature on bubbles and TVC violations in overlapping-generation (OLG) models (Samuelson, 1958; Diamond, 1965; Blanchard and Watson, 1982). Just as Tirole (1985) characterizes conditions under which bubbly equilibrium exists in OLG models, we characterize conditions under which the TVC is violated in heterogeneous-agent models. We show that TVC violation depends on the degree of inequality in the wealth distribution. As a result, bubbles can arise even in some models with stationary wealth distribution, while the TVC can hold in some models with exploding wealth inequality.

Our paper is related to Brunnermeier, Merkel, and Sannikov (2022), who study the pricing of government debt in a model with idiosyncratic risk. Their *dynamic trading perspective* prices the cash flows each individual receives from the government bond market, which, due to their trading and rebalancing, are not the same as the aggregate cash flows paid out by the government. They show that this perspective leads to a well-defined TVC *for individual wealth and asset holdings*, which offers important insights about the valuation of government debt. Our results do not contradict with this perspective, as it also applies in all model variants we study. Instead, we focus on the *buy and hold perspective* in Brunnermeier et al. (2022)'s terminology. Brunnermeier et al. (2022) point out that this perspective, which prices the aggregate government cash flows discounted by households' marginal utilities, may or may not lead to well-defined TVC *for aggregate cash flows*. Our paper develops a precise characterization of this valuation question, which is central to standard asset pricing approaches.

In particular, when applied to the context of government debt, our results show that we can still relate the market value of government debt to the present value of government surpluses when the aggregate TVC holds. This approach survives in heterogeneous-agent economies when inequality in the wealth distribution is not exploding and not too large. In this case, it is still valid to estimate any household's or the average household's stochastic discount factor (SDF) and use it to price the aggregate government cash flows, as in Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2024, 2020). To the extent that Jiang et al. (2024) find a gap between the market value of government debt and the present value of aggregate surpluses, which is based on an estimated SDF that prices tradable assets, this puzzle still requires a resolution.

2 Transversality Condition with Permanent Idiosyncratic Risk

2.1 Model Set-Up

We begin with the set-up developed by Brunnermeier et al. (2022), which provides a tractable example of heterogeneous-agent economies with idiosyncratic risk. The details of the model are

presented in Appendix A.1, and we will consider variations and generalizations in the next section.

In this set-up, there is a continuum of households indexed by $i \in [0,1]$, who are exposed to idiosyncratic risk that they cannot fully insure. The household preference is given by

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}\log c_t^i dt\right].$$

Each household manages its own capital, whose growth is driven by investment ι_t^i, depreciation δ, trading in capital market $d\Delta_t^{k,i}$, and an idiosyncratic risk $d\tilde{Z}_t^i$ with exogenously specified volatility $\tilde{\sigma}_t$:

$$\frac{dk^i}{k^i} = \left(\Phi(\iota_t^i) - \delta\right)dt + d\Delta_t^{k,i} + \tilde{\sigma}_t d\tilde{Z}_t^i. \tag{1}$$

In this set-up, each household has to hold a fraction $\bar{\chi}$ of its own capital, whose risk remains undiversifiable. They can, and do, sell the remaining shares in exchange for aggregate bond and equity assets that are not exposed to their idiosyncratic risk. In equilibrium, the law of motion for individual household's wealth n_t^i is given by

$$\frac{dn_t^i}{n_t^i} = g_t dt + \sigma_t^m dZ_t + (1-\vartheta_t)\bar{\chi}\tilde{\sigma}_t d\tilde{Z}_t^i, \tag{2}$$

which has a common drift term g_t, and is exposed to an aggregate shock dZ_t with loading σ_t^m and to the idiosyncratic shock $d\tilde{Z}_t^i$ with loading $(1-\vartheta_t)\bar{\chi}\tilde{\sigma}_t$. This loading captures the amount of idiosyncratic risk that the household has to bear in equilibrium, which is the product of its own capital's risk exposure $\bar{\chi}\tilde{\sigma}_t$ and the share $(1-\vartheta_t)$ of this capital in its total wealth.

The log preference implies the following marginal utility:

$$m_t^i = e^{-\rho t}\frac{1}{c_t^i} = e^{-\rho t}\frac{1}{\rho n_t^i}. \tag{3}$$

Plugging in the law of motion for wealth into the marginal utility (3), we obtain the following

dynamics

$$\frac{dm_t^i}{m_t^i} = -r_t^f dt - \sigma_t^m dZ_t - (1-\vartheta_t)\bar{\chi}\tilde{\sigma}_t d\tilde{Z}_t^i, \tag{4}$$

where the real risk-free rate is

$$r_t^f = \rho + g_t - (\sigma_t^m)^2 - (1-\vartheta_t)^2\bar{\chi}^2\tilde{\sigma}_t^2. \tag{5}$$

In this economy, the risk-free rate r_t^f is given by the growth rate g_t plus the subjective discount rate ρ minus the sum of the aggregate and idiosyncratic risks' variances. A more volatile idiosyncratic risk lowers the interest rate due to the precautionary saving motive, which is standard in a large class of heterogeneous-agent models.

2.2 Transversality Condition for Individual Wealth

Iterating forward the households' budget constraint, we obtain that the household's wealth is equal to the present discounted value of its future consumption plus a transversality term:

$$n_t^i = \mathbb{E}_t\left[\int_0^\infty m_{t+j}^i c_{t+j}^i dj\right] + \lim_{T\to\infty}\mathbb{E}_t\left[m_T^i n_T^i\right]. \tag{6}$$

The transversality condition (TVC) is given by

$$\lim_{T\to\infty}\mathbb{E}_t\left[m_T^i n_T^i\right] = 0.$$

If this TVC holds, we can evaluate the household's wealth portfolio n_t^i based on the present discounted value of consumption. If the TVC is violated, the household's net worth n_t^i can exceed the present value of consumption.

In this setting, the individual level TVC always holds for all households i. To see this, simply

plug Eq. (3) into the TVC:

$$\lim_{T\to\infty} \mathbb{E}_0\left[m_T^i n_T^i\right] = \lim_{T\to\infty} \mathbb{E}_0\left[e^{-\rho T}\frac{1}{\rho n_T^i} n_T^i\right] = 0. \tag{7}$$

Intuitively, this TVC is part of each household's optimality condition. It holds since households leave no money on the table when they optimize their consumption and saving choices.

We can also frame this TVC in light of the "r minus g" argument. Given the law of motion (2) and (4) for individual wealth and marginal utility, we obtain

$$\frac{d(m_t^i n_t^i)}{m_t^i n_t^i} = -[\underbrace{r_t^f}_{\text{risk-free rate}} + \underbrace{(\sigma_t^m)^2}_{\text{risk premium for aggregate risk}} + \underbrace{\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2}_{\text{risk premium for idiosyncratic risk}} - \underbrace{g_t}_{\text{growth rate}}]dt, \tag{8}$$

which expresses the dynamics of the transversality term $m_t^i n_t^i$ in terms of the discount rate and the growth rate. Plugging in the expression (5) for the risk-free rate, we have

$$(r_t^f + (\sigma_t^m)^2 + (1-\vartheta_t)^2\bar{\chi}^2\tilde{\sigma}_t^2) - g_t = \rho > 0,$$

which implies that the appropriate discount rate for individual wealth, which accounts for the risk premia for aggregate and idiosyncratic risks, is higher than the growth rate: r is always above g. This guarantees that the discounted wealth $m_t^i n_t^i$ has a negative drift, and therefore it tends to zero in the limit.

2.3 Transversality Condition for Aggregate Wealth

Next, we evaluate the TVC associated with the aggregate wealth $\bar{n}_t = \int_0^1 n_t^i di$. Its law of motion is given by

$$\frac{d\bar{n}_t}{\bar{n}_t} = g_t dt + \sigma_t^m dZ_t,$$

which, compared to the law of motion (2) for individual wealth, does not load on any idiosyncratic risk.

A claim to this aggregate wealth $\bar{n}_t$ pays it out at time t. We are interested in studying the value of such a claim in the distant future. If we evaluate this claim using an individual household's marginal utility m_t^i, we obtain

$$\lim_{T\to\infty} \mathbb{E}_0 \left[m_T^i \bar{n}_T \right], \tag{9}$$

which answers how much the household would pay today in exchange for receiving the aggregate wealth in the distant future.

Why should the households care about this quantity? Consider, for example, their valuation of government risk-free debt. Just like the individual household's budget constraint (6), the government faces the following intertemporal budget condition:

$$b_t = \mathbb{E}_t \left[\int_0^\infty m_{t+j}^i s_{t+j} dj \right] + \lim_{T\to\infty} \mathbb{E}_t \left[m_T^i b_T \right],$$

where b_t denotes the market value of government debt and s_t denotes the primary surplus. In this set-up, each household allocates ϑ_t fraction of its wealth to government debt. So, in aggregate, $b_t = \vartheta_t \bar{n}_t$. The government debt's transversality condition is given by

$$\lim_{T\to\infty} \mathbb{E}_t \left[m_T^i \vartheta_T \bar{n}_T \right].$$

Since $\vartheta_t \in (0,1)$, $0 < \mathbb{E}_t \left[m_T^i \vartheta_T \bar{n}_T \right] < \mathbb{E}_t \left[m_T^i \bar{n}_T \right]$ for all time T. So, the TVC (9) for aggregate wealth is also a sufficient condition for the TVC for government debt to hold.[2] More broadly, for any asset whose cash flow is tied to aggregate quantities, the TVC (9) for aggregate wealth also sheds light on its TVC.

Equivalently, we can consider this TVC from the perspective of an average household. We use

[2] If ϑ_t is a constant, so that the market value of debt is cointegrated with the aggregate wealth, then, Eq. (9) is also a necessary condition.

the equal-weighted average marginal utility growth $\bar{m}_t = \int_0^1 (m_t^i / m_0^i) di$, which has the following law of motion:

$$\frac{d\bar{m}_t}{\bar{m}_t} = -r_t^f dt - \sigma_t^m dZ_t.$$

Compared with the individual household's stochastic discount factor (SDF) m_t^i, the average SDF $\bar{m}_t$ has the same drift and the same loading on the aggregate risk, but does not load on the idiosyncratic risk. Because the aggregate wealth $\bar{n}_T$ does not load on the idiosyncratic risk, either, the two TVCs are identical:

$$\lim_{T\to\infty} \mathbb{E}_0 \left[m_T^i \bar{n}_T \right] = \lim_{T\to\infty} \mathbb{E}_0 \left[\bar{m}_T \bar{n}_T \right].$$

More precisely, if one is zero, the other is zero as well. If one is infinite, the other is infinite as well. These are the two possibilities in our set-up. Since it is more convenient to consider the average household's valuation, we will focus on the TVC discounted by the average SDF $\bar{m}_t$.

Unlike the TVC (7) for individual wealth n_t^i, this TVC for aggregate wealth $\bar{n}_t$ does not necessarily hold. Below, we provide two complementary perspectives to characterize this TVC, one focusing on the inequality in the wealth distribution and the other focusing on the comparison between the discount rate (r) and the growth rate (g).

The Wealth Distribution Perspective. Let $\eta_t^i = n_t^i / \bar{n}_t$ denote the wealth share of individual i. By definition, the wealth share has an average of 1: $\int_0^1 \eta_t^i di = 1$. We have the following result.

Proposition 1. *The transversality condition for aggregate wealth can be expressed as*

$$\lim_{T\to\infty} \mathbb{E}_0 \left[\bar{m}_T \bar{n}_T \right] = \bar{n}_0 \lim_{T\to\infty} \mathbb{E}_0 \left[e^{-\rho T} \int_0^1 \frac{\eta_0^i}{\eta_T^i} di \right]. \tag{10}$$

The proof is presented in Appendix A.2. Eq. (10) is the central equation of our paper: whether the aggregate TVC holds or not depends on the integral of inverse wealth share growth, i.e.,

$\int_0^1(\eta_0^i/\eta_T^i)di$, which is a measure of inequality in the wealth distribution. If inequality is exploding, some households' wealth share η_T^i will become infinitely small and $1/\eta_T^i$ infinitely large in the limit. In this case, Eq. (10) may diverge to infinity. On the other hand, if the wealth share η_T^i is bounded from below, then, the TVC always holds.

As in Brunnermeier et al. (2022)'s original set-up, Eq. (2) implies that idiosyncratic risk is permanent, and the wealth share follows a random walk:

$$\frac{d\eta_t^i}{\eta_t^i} = (1-\vartheta_t)\bar{\chi}\tilde{\sigma}_t d\tilde{Z}_t^i.$$

In this setting, as time passes, almost everyone becomes relatively poor, and wealth is concentrated in a vanishing measure of rich households. As a result, the share of households with above average wealth approaches zero: $\mathbb{P}(\eta_T^i \geq 1) \to 0$, which leads to an exploding amount of inequity and hence violation of the TVC for aggregate wealth.

To illustrate this point graphically, we simulate the model under a simple calibration. We report the parameter values in Appendix Table A.1. We make the following simplifying assumptions:

Assumption 1. *(a) There is no aggregate risk:* $\sigma_t^m = 0$.

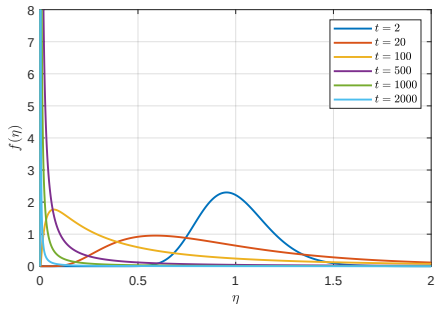
(b) The idiosyncratic risk has a constant volatility: $\tilde{\sigma}_t = \tilde{\sigma}$.

(c) The economy starts at the steady state: $\vartheta_t = \vartheta$.

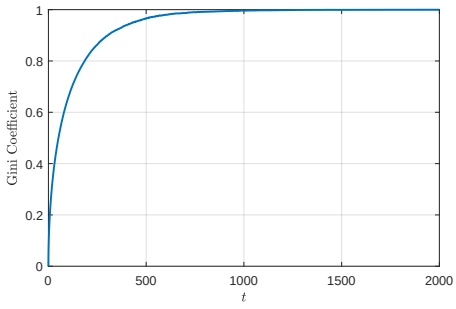
(c) All households start with equal wealth shares: $\eta_0^i = 1$.

Figure 2(a) reports the distribution of the wealth share η_t^i for different time t. As time passes, the average wealth share remains at 1, but its distribution becomes increasingly dispersed. The figure shows that the median and the mode of the wealth distribution shift to the left, indicating that most households become poorer over time, while a vanishing share of households become richer.

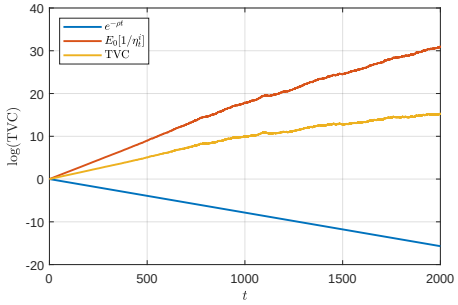
This result implies that, when we ask all households to price the claim to aggregate wealth $\bar{n}_T$, all but a vanishing share of the households will become infinitesimally small relative to the size of



(a) Wealth Share Distribution.



(b) Gini Coefficient.



(c) TVC for Aggregate Wealth.

Figure 2: Dynamics with Permanent Idiosyncratic Risk.

this asset, thereby imputing an infinite valuation for the aggregate wealth claim. Algebraically,

$$\lim_{T\to\infty} \mathbb{E}_0\left[m_T^i \bar{n}_T\right] = \lim_{T\to\infty} \mathbb{E}_0\left[m_T^i n_T^i \frac{\bar{n}_T}{n_T^i}\right] = \lim_{T\to\infty} \mathbb{E}_0\left[\exp(-\rho T)\frac{1}{\rho}\frac{1}{\eta_T^i}\right].$$

On the right-hand side, $\exp(-\rho T)$ captures the subjective discount factor, whereas the expected inverse wealth share $1/\eta_T^i$ captures the growth of the marginal utility. When the marginal utility grows to infinity faster than the subjective discount factor approaching zero, this transversality term diverges to infinity.

Figure 2(b) reports the expected Gini coefficient of the wealth distribution over time, which measures the degree of inequality in the wealth distribution. As we discussed in the introduction, the Gini coefficient is fairly stable in the post-war U.S. data, while in this model it approaches 1 as time passes, indicating an exploding amount of inequality.

Figure 2(c) reports the average path of the discounted aggregate wealth in Eq. (10) over time,

which has two terms: while the subjective discount factor $e^{-\rho T}$ declines over time, the integral of inverse wealth share $\int_0^1 \frac{1}{\eta_T^i} di$ diverges to infinity. In our calibration, the magnitude of the idiosyncratic risk $\tilde{\sigma}$ is sufficiently large, so that the term capturing wealth inequality increases faster than the time discount, leading to an infinite value of the discounted wealth in the limit.

It is worth noting that this argument is not driven by a force which, as a household becomes poorer, increases the magnitude of its idiosyncratic risk and thereby strengthens the precautionary saving motive. In our setting, poor and rich households hold the same portfolio, and their main difference is the size of their wealth, which determines how large any aggregate asset appears to them. The transversality condition is violated because all but a vanishing share of the households become infinitesimally small relative to the size of the aggregate asset, which is different from the mechanism due to risk premium.

The r Minus g Perspective. We can also understand our main result in light of the "r minus g" argument. Note that the discounted aggregate wealth has the following dynamics:

$$\frac{d(\bar{m}_t \bar{n}_t)}{\bar{m}_t \bar{n}_t} = [- \underbrace{r_t^f}_{\text{risk-free rate}} - \underbrace{(\sigma_t^m)^2}_{\text{risk premium for aggregate risk}} + \underbrace{g_t}_{\text{growth rate}}]dt.$$

Compared to the discounted individual wealth in Eq. (8), the discounted aggregate wealth does not contain the risk premium for idiosyncratic risk. The relevant $r - g$ condition becomes

$$(r_t^f + (\sigma_t^m)^2) - g_t = \rho - \bar{\chi}^2 (1 - \vartheta_t)^2 \tilde{\sigma}_t^2,$$

which is no longer guaranteed to be positive. This condition determines the aggregate TVC, which holds if and only if

$$\lim_{T \to \infty} \mathbb{E}_0 \left[\bar{m}_T \bar{n}_T \right] = \lim_{T \to \infty} \bar{n}_0 \mathbb{E}_0 \left[\exp \left(- \int_0^T [\rho - \bar{\chi}^2 (1 - \vartheta_t)^2 \tilde{\sigma}_t^2] \, dt \right) \right] = 0.$$

In other words, the aggregate TVC is violated when the magnitude of the idiosyncratic risk $\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2$ is sufficiently large relative to the subjective discount rate ρ. In this case, because inequality explodes fast enough, households lower their discount rate for the aggregate wealth, leading to $r < g$.

On the other hand, if the variance of the permanent idiosyncratic risk is small enough, i.e., $\bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 < \rho$, the discount rate r can be above the growth rate g. In this case, the aggregate TVC holds: $\lim_{T\to\infty} \mathbb{E}_0\left[\bar{m}_T\bar{n}_T\right] = 0$. This result is also consistent with the wealth distribution perspective. In this case, while the wealth distribution is still diverging, it is slow enough that this risk premium effect is dominated by the household's subjective discount rate ρ.

This discussion shows that the TVC for aggregate wealth is violated only when wealth inequality explodes sufficiently quickly, which drives the (risk-adjusted) discount rate below the growth rate, i.e., $r < g$. Conversely, what r minus g captures in this setting is how fast permanent idiosyncratic risk drives the wealth distribution to explode.

3 Transversality Condition with Transitory Idiosyncratic Risk

The previous section analyzes the transversality condition in the presence of permanent idiosyncratic risk. In this case, an exploding amount of wealth inequality is required to generate TVC violations for aggregate quantities. In reality, it is perhaps more plausible to think about wealth distribution that does not diverge over time, which requires the idiosyncratic risk to be transitory. In this section, we explore the implications for the transversality condition under this condition.

3.1 Model Set-Up

We modify one assumption in the previous section to generate transitory idiosyncratic risk. For each household, their wealth n_t^i jumps back to the average wealth level $\bar{n}_t$ with Poisson intensity ζ. We denote the jump process as $\tilde{J}_t^i$, which is i.i.d. across households.[3] Since this jump occurs

[3]Modeling mean-reversion in this fashion allows us to generate a stationary wealth distribution in a tractable way. If, instead, we assume that the individual wealth faces a proportional tax that depends on the wealth level, the solution

with equal intensity to all households, it preserves the total amount of wealth in this economy, and can thus be understood as a wealth redistribution.

We present detailed derivations of this model variant in Appendix A.3. In this set-up, individual wealth satisfies

$$\frac{dn_t^i}{n_t^i} = g_t dt + \sigma_t^m dZ_t + (1-\vartheta_t)\tilde{\sigma}_t \bar{\chi} d\tilde{Z}_t^i + \frac{\bar{n}_t - n_t^i}{n_t^i} d\tilde{J}_t^i,$$

which differs from Eq. (2) in the baseline model with no jumps in three ways. First, due to the household's exposure to the jump risk, the wealth n_t^i reverts to the mean level $\bar{n}_t$ when the jump $\tilde{J}_t^i$ occurs.

Second, the household internalizes the jump risk in its consumption decision. Because the household may lose its savings when the jump occurs, it becomes effectively more impatient and consumes a higher share of its wealth: $c_t^i = (\rho + \zeta) n_t^i$. Since the household consumes more and saves less, the growth rate in this economy can also be different from the baseline model.

Third, the risk-free rate is given by

$$r_t^f = \rho + \zeta + g_t - (\sigma_t^m)^2 - (1-\vartheta_t)^2 \bar{\chi}^2 \tilde{\sigma}_t^2.$$

Compared to Eq. (5) the baseline case with permanent idiosyncratic risk, this formula additionally contains the jump intensity ζ to compensate for the jump risk, again reflecting the household's higher impatience.

Throughout this section, we assume that the magnitude of the idiosyncratic risk is large enough to dominate the subjective discount rate, i.e., $\rho \leq \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2$, so that the TVC fails when the jump risk turns zero. Otherwise, $\rho > \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2$ directly implies that the TVC holds, even when there is no jump risk that shrinks the wealth distribution.

is much more complicated even with log utility.

3.2 Transversality Condition for Individual Wealth

Since $m_t^i = e^{-\rho t}/(\rho n_t^i)$, the law of motion for wealth implies the following SDF dynamics:

$$\begin{aligned}\frac{dm_t^i}{m_{t-}^i} &= -[\rho + g_t - (\sigma_t^m)^2 - (1-\vartheta_t)^2\bar{\chi}^2\tilde{\sigma}_t^2]dt - \sigma_t^m dZ_t - \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t d\tilde{Z}_t^i \\ &\quad + \frac{1}{m_t^i}\left(e^{-\rho t}\frac{1}{(\rho+\zeta)\bar{n}_t} - e^{-\rho t}\frac{1}{(\rho+\zeta)n_{t-}^i}\right)d\tilde{J}_t^i\end{aligned} \tag{11}$$

The dynamics of the transversality term $m_t^i n_t^i$ for individual wealth is identical to the case of permanent idiosyncratic risk:

$$\frac{d(m_t^i n_t^i)}{m_t^i n_t^i} = -\rho dt < 0.$$

In other words, the discount rate on the individual wealth is always above the growth rate, and the TVC for individual wealth holds.

3.3 Transversality Condition for Aggregate Wealth

The jump in the wealth process requires us to reconsider the TVC for aggregate assets. Take the government debt as an example. Given the household SDF dynamics (11) and letting $b_t = B_t/P_t$ denote the aggregate market value of government debt in real terms, we obtain

$$d(m_t^i b_t) = -m_{t-}^i s_t dt + \zeta e^{-\rho t}\frac{1}{\rho\bar{n}_t}b_t dt + b_t\left(e^{-\rho t}\frac{1}{(\rho+\zeta)\bar{n}_t} - m_{t-}^i\right)(d\tilde{J}_t^i - \zeta dt) + (dZ_t \text{ and } d\tilde{Z}_t^i \text{ terms}).$$

Integrate both sides over time t to obtain

$$b_0 = \mathbb{E}_0\left[\int_0^T \frac{m_t^i}{m_0^i}s_t\, dt\right] + \mathbb{E}_0\left[\frac{m_T^i}{m_0^i}b_T\right] - \zeta\mathbb{E}_0\left[\int_0^T e^{-\rho t}\frac{\bar{n}_0}{\bar{n}_t}b_t\, dt\right]. \tag{12}$$

This result shows that the government debt b_0 is backed by the present value of future surpluses (the first term on the right-hand side), the standard TVC term (the second term), and an additional term that reflects the jump risk in the wealth share. From an individual household's perspective,

when the jump happens, its own wealth is reset to the average level regardless of how much and what assets it holds. This means that the household effectively loses any assets it holds when the jump happens. The last term reflects this loss of value, which is equal to the jump intensity ζ times the present value of the asset holding b_t. This loss of value lowers the household's valuation b_0 of the government debt below the entire present discounted value of government surpluses s_t and bond terminal value b_T.

It is more convenient to consider the average SDF $\bar{m}_t = \int_0^1 (m_t^i/m_0^i)di$. In this setting, the dynamics of the average SDF is given by

$$d\bar{m}_t = -[\rho + g_t - (\sigma_t^m)^2 - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2]\bar{m}_t dt - \sigma_t^m \bar{m}_t dZ_t + \int_0^1 e^{-\rho t}\frac{1}{\hat{\rho}}\left(\frac{1}{\bar{n}_t} - \frac{1}{n_t^i}\right)\frac{1}{m_0^i}d\tilde{J}_t^i di,$$

where the last term reflects the effect of the wealth reset shock. While this shock has no effect on the aggregate wealth $\bar{n}_t$, it affects the average marginal utility $\bar{m}_t$ since individual marginal utilities are non-linear in wealth.

Integrating Eq. (12) over households i, we obtain the following result.

Proposition 2. *In the setting with jump risk in wealth, the intertemporal government budget condition is given by*

$$b_0 = \mathbb{E}_0\left[\int_0^T \bar{m}_t s_t dt\right] + \mathbb{E}_0\left[\bar{m}_T b_T\right] - \zeta\mathbb{E}_t\left[\int_0^T e^{-\rho t}\frac{\bar{n}_0}{\bar{n}_t}b_t dt\right].$$

The proof is presented in Appendix A.4. This proposition characterizes the intertemporal government budget condition from the perspective of the average household, which has a similar interpretation as Eq. (12) from the perspective of an individual household: the government debt is backed by future surpluses, the standard TVC term, and an additional term capturing the effect of wealth reset.

Since $b_t < \bar{n}_t$, the last term is bounded. Proposition 2 implies

$$b_0 \leq \mathbb{E}_0 \left[\int_0^T \bar{m}_t s_t dt \right] + \mathbb{E}_0 \left[\bar{m}_T \bar{n}_T \right],$$

which shows that the discounted aggregate wealth $\bar{m}_t \bar{n}_t$ again arises as the key object determining the TVC for aggregate assets. Below, we offer two perspectives to understand its property.

The Wealth Distribution Perspective. For simplicity of exposition, we maintain the same assumption that the households begin with identical wealth: $\eta_0^i = 1$. Then, the TVC term (10), reproduced below, also arises in this model variant:

$$\lim_{T \to \infty} \mathbb{E}_0 \left[\bar{m}_T \bar{n}_T \right] = \lim_{T \to \infty} e^{-\rho T} \bar{n}_0 \mathbb{E}_0 \left[\int_0^1 \frac{1}{\eta_T^i} di \right]. \tag{13}$$

Like the baseline case with permanent idiosyncratic risk, whether the TVC holds for the aggregate wealth depends on how the integral of inverse wealth share $\int_0^1 (1/\eta_T^i) di$ evolves over time and compares to the subjective discount rate. The difference is that transitory idiosyncratic risk makes wealth dynamics mean-reverting, whereas permanent idiosyncratic risk leads to a diverging wealth distribution.

For a concrete example, we again consider a simple calibration under Assumption 1. In this case, the government commits to a constant market value share of debt: $\vartheta_t = \vartheta$. Starting from any initial distribution, the wealth distribution converges to the distribution with density function f that satisfies the following Kolmogorov forward equation:

$$0 = \frac{1}{2} \bar{\chi}^2 (1 - \vartheta)^2 \tilde{\sigma}^2 (\eta^2 f''(\eta) + 4\eta f'(\eta) + 2f(\eta)) - \zeta f(\eta). \tag{14}$$

We further derive closed-form solutions to this Kolmogorov forward equation, which we

present in Appendix A.5. We obtain two distinct cases. First, if

$$\zeta > \bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2, \tag{15}$$

i.e., the jump intensity is sufficiently high relative to the magnitude of the idiosyncratic risk, then, the integral of inverse wealth share is well-defined:

$$\lim_{T\to\infty} \mathbb{E}_0\left[\int_0^1 \frac{1}{\eta_T^i} di\right] < \infty,$$

which implies that the TVC (13) for aggregate wealth holds.

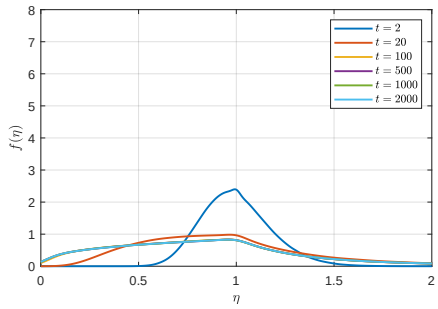
Figure 3 presents such an example, assuming a high jump intensity of $\zeta = 2.5\%$. Panel (a) plots the distribution of wealth share η_t^i for different time t. Starting with identical wealth share for all households, i.e., $\eta_0^i = 1$, it quickly converges to a well-defined wealth distribution after 20 periods. Consistent with this convergent behavior, the Gini coefficient of the wealth distribution in Panel (b) quickly converges to around 0.4.

Panel (c) reports the average path of the TVC term. Since the integral of inverse wealth share $\mathbb{E}_0\left[\int_0^1 \frac{1}{\eta_t^i} di\right]$ converges to a finite value, while the subjective discount factor $\exp(-\rho T)$ decreases over time, the TVC for aggregate wealth converges to zero.

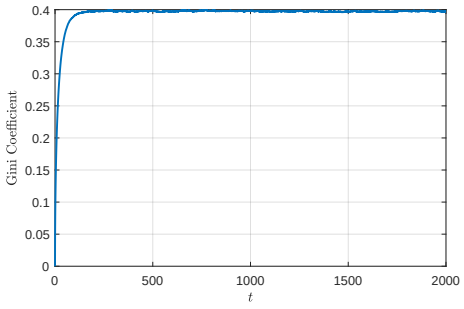
Second, consider the case in which $\zeta \leq \bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$, i.e., the jump intensity is lower than the magnitude of the idiosyncratic risk. In this case, while we still obtain a stationary wealth distribution in the limit, its left tail is too heavy so that the integral of inverse wealth is not well-defined:

$$\lim_{T\to\infty} \mathbb{E}_0\left[\int_0^1 \frac{1}{\eta_T^i} di\right] = \infty.$$

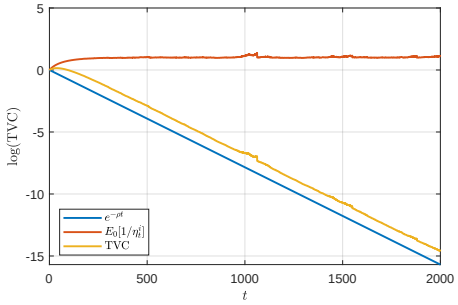
Figure 4 presents such an example, assuming a low jump intensity of $\zeta = 0.36\%$. Panel (a) shows the wealth distribution. Unlike the baseline case with permanent idiosyncratic risk, the wealth distribution slowly converges to a well-defined distribution. However, the left tail of this



(a) Wealth Share Distribution.

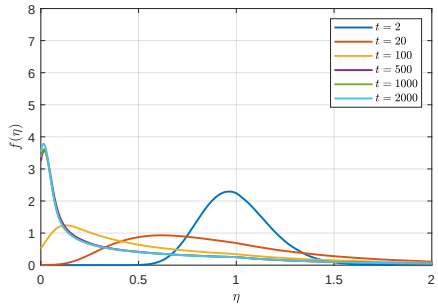


(b) Gini Coefficient.

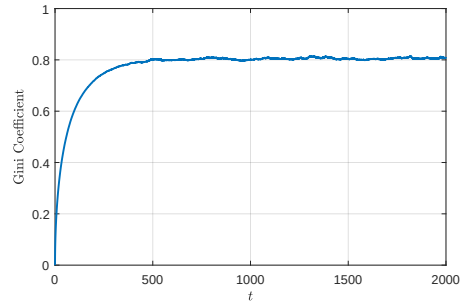


(c) TVC for Aggregate Wealth.

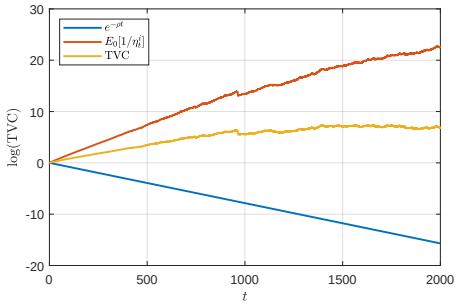
Figure 3: Dynamics with Transitory Idiosyncratic Risk, High Wealth Reset Intensity.



(a) Wealth Share Distribution.



(b) Gini Coefficient.



(c) TVC for Aggregate Wealth.

Figure 4: Dynamics with Transitory Idiosyncratic Risk, Low Wealth Reset Intensity.

distribution is much heavier than the case presented in Figure 3.

Panel (b) plots the Gini coefficient. While it also converges to a constant value under this stationary distribution, the Gini coefficient, which is around 0.8, is much higher than the case with a high jump intensity, which reflects a much heavier left tail in the wealth distribution.

Panel (c) reports the average path of the TVC term. The integral of inverse wealth share $\mathbb{E}_0\left[\int_0^1 \frac{1}{\eta_t^i} di\right]$ diverges to infinity, while the subjective discount factor $e^{-\rho T}$ decreases over time. Under this calibration, the integral of inverse wealth share diverges faster, which leads the TVC for aggregate wealth to diverge to infinity.

These two cases show that a stationary wealth distribution does not guarantee the TVC for aggregate assets. TVC violation can still occur when the left tail of the wealth distribution is heavy enough, so that the households expect to experience a very high marginal utility upon negative realizations of the idiosyncratic shock. In this case, the integral of inverse wealth share, which measures the degree of inequality in the wealth distribution, still diverges to infinity just like the case with permanent idiosyncratic risk.

The r Minus g Perspective. We also obtain a more general result characterizing the dynamics of $\bar{m}_t \bar{n}_t$, which does not rely on Assumption 1 and the specific wealth distribution.

Proposition 3. *The TVC term for aggregate wealth has the following dynamics:*

$$d[\bar{m}_t \bar{n}_t] = -[\rho - \bar{\chi}^2(1-\vartheta_t)^2 \tilde{\sigma}_t^2 + \zeta]\bar{m}_t \bar{n}_t dt + \zeta e^{-\rho t} \bar{n}_0 dt$$

The proof is presented in Appendix A.6. This proposition shows that the sign of $\rho - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 + \zeta$ plays a crucial role in determining the TVC for aggregate wealth. This term corresponds to the relevant r minus g condition, which is

$$(r_t^f + (\sigma_t^m)^2) - g_t = \rho - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 + \zeta.$$

Thus, this proposition shows that the relevant condition for the TVC is again "$r - g > 0$." To satisfy

the TVC for aggregate wealth, we need either a sufficiently high discount rate, which requires a small enough volatility $\tilde{\sigma}_t$ of the idiosyncratic risk, or a sufficiently high jump risk ζ so that the households' wealth tends to revert to the mean. This condition rules out the cases in which the left-tail of wealth distribution explodes fast enough to dominate the subjective discount rate in the expression (13) from the wealth distribution perspective.

Returning to the example under Assumption 1, we obtain the following result:

$$\bar{m}_t \bar{n}_t = \bar{n}_0 e^{-[\rho - \bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + \zeta]t} + \frac{\zeta \bar{n}_0}{[-\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + \zeta]}(e^{-\rho t} - e^{-[\rho - \bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + \zeta]t}).$$

The last term shrinks to zero as t increases and therefore does not matter for the aggregate TVC. Hence, the aggregate TVC holds if and only if

$$\zeta > \bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 - \rho. \tag{16}$$

Interestingly, this condition is not the same as the condition (15) which guarantees a finite integral of inverse wealth share under the stationary wealth distribution in the limit. We obtain three cases summarized by Table 1, Panel (b).

First, if the jump intensity in the wealth process is sufficiently high, i.e., $\zeta > \bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$, then, both condition (15) for a finite integral of inverse wealth share and condition (16) for the aggregate TVC are satisfied.

Table 1: Summary of the TVC for Aggregate Wealth.

(a) Without Wealth Reset, Permanent Idiosyncratic Risk				
Idiosyncratic Volatility	Parameter Region	TVC holds?	$\mathbb{E}[\int(1/\eta^i_\infty)di] = \infty$?	$r > g$?
Low	$\rho > \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2$	Yes	Yes	Yes
High	$\rho \leq \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2$	No	Yes	No
(b) With Wealth Reset, Transitory Idiosyncratic Risk				
Jump Intensity	Parameter Region	TVC holds?	$\mathbb{E}[\int(1/\eta^i_\infty)di] = \infty$?	$r > g$?
High	$\zeta > \bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$	Yes	No	Yes
Moderate	$\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 - \rho < \zeta < \bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$	Yes	Yes	Yes
Low	$0 < \zeta \leq \bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 - \rho$	No	Yes	No

Second, if the jump intensity is moderate, i.e., $\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 - \rho < \zeta < \bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2$, then, condition (15) for a finite integral of inverse wealth share is violated, but condition (16) for the aggregate TVC is satisfied. In this interesting case, while the inequality in the wealth distribution as measured by the integral of inverse wealth share is not bounded in the limit, it approaches infinity at a slower rate than the subjective discount factor, which allows the TVC for aggregate wealth to hold.

Finally, if the jump intensity is low, i.e., $0 < \zeta \leq \bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 - \rho$, then, both condition (15) for a finite integral of inverse wealth share and condition (16) for the aggregate TVC are violated. In this case, the left tail of the wealth distribution is too large, and the TVC for aggregate wealth is violated.

3.4 Summary

Table 1, Panel (a) also summarizes the case with permanent idiosyncratic risk. In this case, breaking the TVC for aggregate wealth and other long-lived assets requires a large enough magnitude for the permanent idiosyncratic risk, which makes the inequality in the wealth distribution diverge fast enough and impoverishes all but a vanishing share of households in the limit.

Summarizing our discussions of the cases with permanent and transitory idiosyncratic risk, we arrive at the following conclusions from our theoretical characterizations:

First, non-stationary wealth distribution does not necessarily imply aggregate TVC fails. When the wealth distribution explodes slower than the subjective discount rate, the TVC still holds.

Second, stationary wealth distribution does not necessarily imply aggregate TVC holds. The left tail of the wealth distribution matters. When the left tail is heavy enough, the TVC is violated.

Third, there is one certain argument we can make: the TVC for aggregate assets requires that the left tail of the wealth distribution is not too heavy. "Not too heavy" is in the sense that the left tail does not become infinitely large fast enough as the wealth distribution diverges over time, nor have a large mass in the case of a stationary wealth distribution. When the left tail is too heavy, enough households expect to become infinitesimally small relative to the size of the economy, and

aggregate assets will appear infinitely large to them, which is the source of TVC violation.

4 Transversality Condition with Stochastic Death

Moreover, our characterizations of the TVC remain valid in a model where the households die with a constant probability ζ. For concreteness, let us first consider the following argument in discrete time. Let c_t^i denote the household's consumption at time t if it survives till this time, and suppose the household's utility is 0 in periods after death. Consider a long-lived asset with price p_t and dividend d_t at time t. The household alive at time t has the following first-order condition

$$u'(c_t^i)p_t = (1-\zeta)\mathbb{E}_t\left[\delta u'(c_{t+1}^i)(p_{t+1}+d_{t+1})\right] + \zeta \times 0,$$

which means that the asset is valuable to the household in period t only because there is a chance that the household survives in period $t+1$ and receives the payoff.

Iterating this equation forward, we obtain

$$u'(c_t^i)p_t = \mathbb{E}_t\left[\sum_{k=1}^{\infty}((1-\zeta)\delta)^k u'(c_{t+k}^i)d_{t+k}\right] + \lim_{k\to\infty}\mathbb{E}_t\left[((1-\zeta)\delta)^k u'(c_{t+k}^i)p_{t+k}\right],$$

which is very similar to the case without death, except that the subjective discount rate δ is augmented with the survival probability $(1-\zeta)$. This is very intuitive: death increases the household's effective discount rate, making them act more impatiently.

Other than this difference, the valuation of this long-lived asset still depends on the distribution of the household's marginal utility growth condition on survival. If the wealth and consumption distributions are sufficiently left-skewed, then the TVC can still fail. Thus, we can recover our key result that the TVC for aggregate assets depends on the thickness of the left tail of the wealth distribution in this setting.

In Appendix A.7, we make this point more rigorously in a continuous-time model that is similar to our previous setting. Households die at a rate of ζ, while a fraction ζ of new households

are born and receive the wealth of the deceased households, so that the total amounts of wealth and financial assets are preserved. We consider two possibilities. First, if each new household simply inherits the wealth from a deceased household, then, this setting is identical to the baseline case without the jump risk as presented in Section 2. There is an exploding degree of inequality, and the TVC for aggregate assets can be violated if the magnitude of the idiosyncratic risk is large enough. The only difference is that the subjective discount rate ρ is augmented by the death rate ζ just like the discrete-time example above. Specifically, the sufficient and necessary condition for the TVC to hold is

$$\rho + \zeta - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 > 0,$$

which is identical to the baseline case with permanent idiosyncratic risk except the subject discount rate ρ is augmented by the death rate ζ.

Second, if the dying households' wealth is pooled together and distributed equally to newborn households, which also preserves the total amount of wealth, then, this setting is identical to our case with the wealth reset shock in Section 3. The wealth distribution is stationary, but the TVC for aggregate assets still depends on the thickness of its left tail. Similarly, the only difference is that the subjective discount rate ρ is augmented by the death rate ζ. The sufficient and necessary condition for the TVC to hold is again

$$\rho + \zeta - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 > 0,$$

which guarantees that the left tail of the stationary wealth distribution is thin enough so that the households do not expect to become infinitely smaller than the aggregate asset.

This discussion shows that introducing stochastic death is not a quick fix to the issue of TVC for long-lived assets. Whether the TVC holds or not still depends on the relative magnitude of the subjective discount rate and the idiosyncratic risk. Introducing stochastic death only increases the effective subjective discount rate, which makes the TVC more likely to hold if the model has

the same values for other parameters. The key determinant is still the degree of inequality in the wealth distribution.

5 Implications for Heterogeneous-Agent Models

Our results show that the inequality in the wealth distribution is the key to understand TVCs in heterogeneous-agent models. In this section, we apply this insight to several workhorse models in macroeconomic and finance literature, and evaluate its implications for asset valuation.

5.1 Constantinides-Duffie

Constantinides and Duffie (1996) study an economy with heterogeneous agents and idiosyncratic risk, which features a no-trade equilibrium. In their baseline model, each agent's labor income is subject to permanent idiosyncratic risk. This risk can generate negative labor income for some households, and reduce their total wealth and consumption to zero in the limit. Specifically, household i's one-period pricing kernel is

$$m_{t+1}^i = \exp(-\rho - \alpha \Delta c_{t+1}^i),$$

where the consumption growth is given by

$$\Delta c_{t+1}^i = \Delta c_{t+1} + y_{t+1}\eta_{t+1}^i - \frac{y_{t+1}^2}{2}.$$

As the idiosyncratic shock η_{t+1}^i is i.i.d., the household's consumption share $c_t^i/\bar{c}_t$ follows a random walk. As a result, the distribution of consumption and wealth features an exploding degree of inequality.

This model is very similar to our baseline model without jump risk. If the permanent idiosyncratic risk is large enough, then the TVC is violated for the long-lived asset such as the equity

market.[4] In this case, while the households' SDF correctly prices the one-period return of a long-lived asset like equity with price p_t and dividend d_t, i.e.,

$$\mathbb{E}_t[m^i_{t+1}(p_{t+1} + d_{t+1})] = p_t,$$

the equity market will be infinitely valuable as the transversality condition is violated, i.e.,

$$\lim_{T\to\infty} \mathbb{E}_0\left[m^i_T p_T\right] = \infty.$$

In this sense, the characterization of the one-period return is an incomplete characterization of the asset's value.

Constantinides and Duffie (1996) also consider a model variant in their appendix which features wealth reset. This model can be replicated by our model with wealth reset. As we showed in Section 3, even though the wealth distribution is stationary, the TVC for aggregate wealth still depends on the thickness of the left tail of the wealth distribution. As a result, we cannot take the TVC for granted in this model.

5.2 Krueger-Lustig

Krueger and Lustig (2010) propose a heterogeneous-agent model in which a fraction of the total endowment is distributed as equity dividend while the rest is labor income in each period. On the equity side, every household holds the market index, although they differ in their positions. On the labor income side, each household faces an idiosyncratic risk driven by a Markov process. This framework is general, and the TVC may or may not hold depending on how the labor income is distributed. For example, Constantinides and Duffie (1996)'s model is within the family of model

[4]One detail is that Constantinides and Duffie (1996) impose an assumption that the cumulative SDF M_t satisfies $\lim_{t\to\infty} \mathbb{E}[M_t] = 0$ (see Eq. (5) in the paper). However, this condition is not enough to rule out TVC violation. If the aggregate wealth grows at a constant rate $g > 0$, which is sufficiently large relative to the magnitude of the idiosyncratic risk, then, the cumulative SDF $\bar{m}_t$ in our baseline model will converge to zero. However, the rate at which the SDF converges to zero can be dominated by the rate at which the cash flow grows, so that the discounted present value of the asset is infinite, which leads to TVC violation.

studied by Krueger and Lustig (2010), where the aggregate TVC can blow up.

Krueger and Lustig (2010) study a baseline Bewley model. In this model, household i's total income can be written as

$$(1-\alpha)\eta_t^i + \sigma_{t-1}^i(p_t+\alpha),$$

where $(1-\alpha)$ is the aggregate labor income and η_t^i is the share that belongs to household i; σ_{t-1}^i is the household's holding of the aggregate equity, p_t is the price of the equity, and α is the aggregate dividend. In this setting, if labor income share η_t^i is bounded from below by a positive number, and households are subject to short-selling constraint, i.e., $\sigma_{t-1}^i > 0$, each household's income and wealth shares are bounded by a positive number from below. In this case, the households' marginal utility growth is bounded from above, which allows the TVC for aggregate assets to satisfy trivially.

5.3 Brunnermeier-Sannikov-Merkel

Our results also clarify the characterizations of the intertemporal government budget condition in Brunnermeier et al. (2022). As we discussed in the literature review, Brunnermeier et al. (2022) propose a *dynamic trading perspective* which focuses on an individual household's wealth process and its time-varying holding of the Treasury portfolio. As the TVC for individual wealth always holds, this perspective is always valid, and offers a novel way of understanding how individual households value government debt. Our analysis does not contradict with this perspective. In fact, since the TVC for individual wealth always holds in our settings, this perspective is always applicable.

In comparison, the *buy and hold perspective* evaluates the aggregate government budget. As Brunnermeier et al. (2022) point out, the TVC for individual wealth process does not necessarily imply that the TVC for aggregate government debt portfolio holds, because households do not buy and hold a fixed fraction of outstanding government debt. However, since many prior works examine the government debt, the wealth portfolio, and other aggregate assets such as the equity

market using this aggregate perspective, it is still important to understand the conditions under which the TVC for aggregate wealth holds. Our results provide a general framework to answer this question. Specifically, we find that the key is to understand the properties of the wealth distribution, and its left tail in particular. When the left tail is not too heavy, the TVC for aggregate assets hold, so that we can evaluate these assets based on their aggregate cash flows using the standard discounted present value formula.

5.4 HANK Models

HANK models also feature heterogeneous agents facing idiosyncratic risk, which means that the TVC for long-lived assets also depends on the wealth distribution. For example, in McKay, Nakamura, and Steinsson (2016), each household receives an equal share of the dividend from monopolistically competitive firms, and faces a non-negative borrowing constraint. As a result, household wealth is bounded from below by the dividend-output ratio, which limits the degree of wealth inequality and hence the expected growth rate of household marginal utility.

Similarly, in Kaplan, Moll, and Violante (2018), households receive lump-sum government transfer of constant proportion to the output, and also face a borrowing constraint. In the steady state, household income is bounded from below by the government transfer minus borrowing limit times the exogenous borrowing rate, which is positive under the paper's calibration.[5] As a result, household wealth share is bounded from below despite an extremely tailed financial asset distribution. Through the lens of our characterization, we can see that bounded wealth distribution implies that the TVC for aggregate assets holds in these models.

[5]Under the steady state in Kaplan et al. (2018), the borrowing limit is one times quarterly labor income. The labor share of the economy is 60% and the unsecured borrowing rate is 8% annually, which caps households' quarterly interest payment at 1.2% of quarterly output. The government transfer is calibrated to 6% of output. Overall, individual's income is bounded by 4.8% of output.

6 Conclusion

This paper characterizes conditions under which the bubbles arise and TVC fails for long-lived assets in heterogeneous-agent economies. We find that the degree of inequality in the wealth distribution is the key to understanding this question. We show examples in which an exploding amount of inequality, or a very heavy left tail in a stationary wealth distribution leads to TVC violation. In comparison, a stationary, more balanced wealth distribution satisfies the TVC. In heterogeneous-agent models, the r minus g condition remains a useful guide to understand the TVC.

These results have important implications for asset valuation in heterogeneous-agent models. For example, the TVC may fail in settings considered by Constantinides and Duffie (1996) and Brunnermeier et al. (2022), where the wealth distribution explodes over time. In contrast, the TVC holds in settings considered by Krueger and Lustig (2010) and McKay et al. (2016); Kaplan et al. (2018), when the wealth distribution is bounded from below. Our results provide a general framework to understand the TVC in these models, and offer a new perspective on how to evaluate long-lived assets in heterogeneous-agent models.

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A Model Appendix

A.1 Derivation of Model in Section 2

A.1.1 Set-up

Households Household's lifetime utility is given by

$$V_0^i = \mathbb{E}\left[\int_0^\infty e^{-\rho t} \log c_t^i dt\right].$$

The capital evolution follows

$$\frac{d\tilde{k}^i}{\tilde{k}^i} = \left(\Phi(\iota_t^i) - \delta\right) dt + \tilde{\sigma}_t d\tilde{Z}_t^i + d\Delta_t^{k,i},$$

where $d\Delta_t^{k,i}$ is the physical capital transactions between households, and

$$\Phi(\iota) = \frac{1}{\phi}\log(1+\phi\iota).$$

the wealth dynamic is

$$\frac{dn_t^i}{n_t^i} = -\frac{c_t^i}{n_t^i}dt + dr_t^B + \theta_t^{K,i}\left(dr_t^{K,i}(\iota_t^i) - dr_t^B\right) + \theta^{E,i}\left(dr_t^{E,i} - dr_t^B\right) + \theta_t^{\bar{E},i}\left(dr_t^{\bar{E},i} - dr_t^B\right).$$

Households face a skin-in-the-game constraint when issuing equity, i.e.,

$$-\theta_t^{E,i} \leq (1-\bar{\chi})\theta_t^{K,i}.$$

Government The nominal growth of government debt follows

$$dB_t = \mu_t^B B_t dt.$$

Government budget constraint is given by

$$i_t B_t + P_t \mathfrak{g} K_t = \mu_t^B B_t + P_t \tau_t a_t K_t,$$

where nominal rate i_t, spending and tax rate $\mathfrak{g}$, productivity a_t are exogenous processes adapted to the Brownian filtration Z_t. Tax rate τ_t is endogenous. With q_t^B defined below, the budget constraint can be restated as

$$\check{\mu}_t^B q_t^B + s_t = 0, \tag{A.1}$$

where $\check{\mu}_t^B := \mu_t^B - i_t$ is the bond issuance (net of interest payment) rate. $s_t := \tau_t a_t - \mathfrak{g}$ is the real surplus-capital ratio.

A.1.2 Symmetric Equilibrium

Aggregation Under symmetric equilibrium,

$$\forall i \in [0,1], \quad (\theta_t^{K,i}, \theta_t^{E,i}, \theta_t^{\bar{E},i}, \iota_t^i, \hat{c}_t^i) = (\theta_t^K, \theta_t^E, \theta_t^{\bar{E}}, \iota_t, \hat{c}_t),$$

where $\hat{c}_t^i = c_t^i / n_t^i$ is the consumption-wealth ratio.

Under symmetric equilibrium, the returns adjust such that no one trades physical capital with others, i.e., $\Delta_t^{k,i} = 0$. Aggregate capital evolves according to

$$dK_t = \left(\Phi(\iota_t) - \delta\right) K_t dt.$$

Define the real price of physical capital as q_t^K and the real bond-capital ratio as $q_t^B = \frac{B_t/P_t}{K_t}$. The state variable ϑ_t is defined as the share of wealth in government bond, i.e.,

$$\vartheta_t = \frac{q_t^B}{q_t^B + q_t^K}. \tag{A.2}$$

The difference of volatility $\sigma_t^{q,B} - \sigma_t^{q,K}$ is $\frac{\sigma_t^{\vartheta}}{1-\vartheta_t}$. Indeed, by definition,

$$\vartheta_t = \frac{q_t^B}{q_t^B + q_t^K}.$$

By Itô's lemma,

$$\vartheta_t \sigma_t^{\vartheta} = \frac{q_t^B}{q_t^B + q_t^K} \left(\sigma_t^{q,B} - \frac{\sigma_t^{q,B} q_t^B + \sigma_t^{q,K} q_t^K}{q_t^B + q_t^K} \right) = \frac{q_t^B}{q_t^B + q_t^K} \frac{(\sigma_t^{q,B} - \sigma_t^{q,K}) q_t^K}{q_t^B + q_t^K},$$

i.e.,

$$\sigma_t^{\vartheta} = \sigma_t^{q,B} - \frac{\sigma_t^{q,B} q_t^B + \sigma_t^{q,K} q_t^K}{q_t^B + q_t^K} = (\sigma_t^{q,B} - \sigma_t^{q,K})(1 - \vartheta_t). \tag{A.3}$$

The difference of drift $\mu_t^{q,B} - \mu_t^{q,K}$ can be derived similarly. Indeed, by Itô's lemma,

$$\begin{aligned}
\mu_t^{\vartheta} \vartheta_t &= \frac{q_t^B}{q_t^B + q_t^K} \left(\mu_t^{q,B} - \frac{q_t^B \mu_t^{q,B} + q_t^K \mu_t^{q,K}}{q_t^B + q_t^K} + \left(\frac{q_t^B \sigma_t^{q,B} + q_t^K \sigma_t^{q,K}}{q_t^B + q_t^K} \right)^2 - \left(\frac{q_t^B \sigma_t^{q,B} + q_t^K \sigma_t^{q,K}}{q_t^B + q_t^K} \right) \sigma_t^{q,B} \right) \\
&= \frac{q_t^B}{q_t^B + q_t^K} (1 - \vartheta_t) \left((\mu_t^{q,B} - \mu_t^{q,K}) - \left(\frac{q_t^B \sigma_t^{q,B} + q_t^K \sigma_t^{q,K}}{q_t^B + q_t^K} \right) (\sigma_t^{q,B} - \sigma_t^{q,K}) \right),
\end{aligned}$$

i.e.,

$$\frac{\mu_t^{\vartheta}}{1 - \vartheta_t} = \mu_t^{q,B} - \mu_t^{q,K} + (\sigma_t^{q,B} - \sigma_t^{\vartheta})(\sigma_t^{q,K} - \sigma_t^{q,B}). \tag{A.4}$$

Return Processes Here, we express the return process with the drift and volatility terms of q_t^B, q_t^K and ϑ_t.

Inflation:

$$\frac{dP_t}{P_t} = \frac{d(B_t/(q_t^B K_t))}{B_t/(q_t^B K_t)} = \left(\mu_t^B - \mu_t^{q,B} - \Phi(\iota_t) + \delta + (\sigma_t^{q,B})^2 \right) dt - \sigma_t^{q,B} dZ_t$$

Bond return:

$$dr_t^B = i_t dt + \frac{d(1/P_t)}{1/P_t} = i_t dt + \frac{d(q_t^B K_t/B_t)}{q_t^B K_t/B_t} = \left(\Phi(\iota_t) - \delta - \check{\mu}_t^B + \mu_t^{q,B}\right) dt + \sigma_t^{q,B} dZ_t. \quad \text{(A.5)}$$

Capital return:

$$\begin{aligned} dr_t^K &= \frac{(1-\tau_t)a_t - \iota_t^i}{q_t^K} dt + \frac{d(q_t^K \tilde{k}_t^i)}{q_t^K \tilde{k}_t^i} \\ &= \left(\frac{(1-\tau_t)a_t - \iota_t^i}{q_t^K} + \mu_t^{q,K} + (\Phi(\iota_t^i) - \delta)\right) dt + \sigma_t^{q,K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t^i. \end{aligned} \quad \text{(A.6)}$$

Equity has the same risk loading as capital, but the expected return may be different when skin-in-the-game constraint is binding. Outside equity has the same aggregate risk loading but idiosyncratic loadings average out.

Market Clearing Goods market clearing:

$$\hat{c}_t(q_t^B + q_t^K) + \mathfrak{g} + \iota_t = a_t. \quad \text{(A.7)}$$

Asset market clearing:

$$\theta_t^K = \frac{q_t^K K_t}{(q_t^B + q_t^K)K_t} = 1 - \vartheta_t, \quad \text{(A.8)}$$

$$\theta^E = \theta^{\bar{E}}. \quad \text{(A.9)}$$

A.1.3 Solution

Households' Problem We use dynamic programming to solve the households' portfolio choice problem. Guess that the value function takes the form $V(n_t^i, v_t) = v_t + \frac{1}{\rho}\log n_t^i$, where v_t depends on investment opportunity in aggregate level. Plug in the value function to obtain the Hamilton-

Jacobi-Bellman equation

$$\rho v_t + \log n_t^i = \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}\left\{ \log c_t^i + \dot{v}_t + \frac{1}{\rho}\frac{dn_t^i}{n_t^i}\frac{1}{dt} - \frac{1}{2}\frac{1}{\rho}\left\langle \frac{dn_t^i}{n_t^i}, \frac{dn_t^i}{n_t^i} \right\rangle \frac{1}{dt} \right\},$$

i.e.,

$$\begin{aligned} 0 = \max_{c_t^i, \iota_t^i, \theta_t^i} \Bigg\{ & -\rho v_t - \log n_t^i + \dot{v}_t + \log c_t^i - \frac{1}{\rho}\frac{c_t^i}{n_t^i} + \frac{1}{\rho}(1 - \theta_t^{K,i} - \theta_t^{E,i} - \theta_t^{\bar{E},i}) \left(\Phi(\iota_t) - \delta - \check{\mu}_t^B + \mu_t^{q,B} \right) \\ & + \frac{1}{\rho}\theta_t^{K,i} \left(\frac{(1-\tau_t)a_t - \iota_t^i}{q_t^K} + \mu_t^{q,K} + (\Phi(\iota_t^i) - \delta) \right) + \frac{1}{\rho}\theta_t^{E,i}\frac{\mathbb{E}_t[dr_t^{E,i}]}{dt} + \frac{1}{\rho}\theta_t^{\bar{E},i}\frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt} \\ & - \frac{1}{2\rho}\left((1 - \theta_t^{K,i} - \theta_t^{E,i} - \theta_t^{\bar{E},i})\sigma_t^{q,B} + (\theta_t^{K,i} + \theta_t^{E,i} + \theta_t^{\bar{E},i})\sigma_t^{q,K} \right)^2 - \frac{1}{2\rho}(\theta_t^{K,i} + \theta_t^{E,i})^2 \tilde{\sigma}_t^2 \Bigg\}, \end{aligned} \tag{A.10}$$

s.t.

$$-\theta_t^{E,i} \leq (1 - \bar{\chi})\theta_t^{K,i}.$$

The lagrangian is given by

$$\begin{aligned} \mathcal{L}_t^i = & -\rho v_t - \log n_t^i + \dot{v}_t + \log c_t^i - \frac{1}{\rho}\frac{c_t^i}{n_t^i} + \frac{1}{\rho}(1 - \theta_t^{K,i} - \theta_t^{E,i} - \theta_t^{\bar{E},i}) \left(\Phi(\iota_t) - \delta - \check{\mu}_t^B + \mu_t^{q,B} \right) \\ & + \frac{1}{\rho}\theta_t^{K,i} \left(\frac{(1-\tau_t)a_t - \iota_t^i}{q_t^K} + \mu_t^{q,K} + (\Phi(\iota_t^i) - \delta) \right) + \frac{1}{\rho}\theta_t^{E,i}\frac{\mathbb{E}_t[dr_t^{E,i}]}{dt} + \frac{1}{\rho}\theta_t^{\bar{E},i}\frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt} \\ & - \frac{1}{2\rho}\left((1 - \theta_t^{K,i} - \theta_t^{E,i} - \theta_t^{\bar{E},i})\sigma_t^{q,B} + (\theta_t^{K,i} + \theta_t^{E,i} + \theta_t^{\bar{E},i})\sigma_t^{q,K} \right)^2 - \frac{1}{2\rho}(\theta_t^{K,i} + \theta_t^{E,i})^2 \tilde{\sigma}_t^2 \\ & + \lambda_t^i(\theta_t^{E,i} + (1 - \bar{\chi})\theta_t^{K,i}). \end{aligned}$$

The first order conditions w.r.t. c_t^i, ι_t^i, $\theta_t^{K,i}$, $\theta_t^{E,i}$ and $\theta_t^{\bar{E},i}$ are

$$-\frac{1}{\rho n_t^i}+\frac{1}{c_t^i}=0, \tag{A.11}$$

$$-\frac{1}{q_t^K}+\frac{1}{1+\phi\iota_t^i}=0, \tag{A.12}$$

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt}-\frac{\mathbb{E}_t[dr_t^B]}{dt}=\left(\sigma_t^{q,B}-(\theta_t^{K,i}+\theta_t^{E,i}+\theta_t^{\bar{E},i})\frac{\sigma_t^\vartheta}{1-\vartheta_t}\right)(\sigma_t^{q,K}-\sigma_t^{q,B})+(\theta_t^{K,i}+\theta_t^{E,i})\tilde{\sigma}_t^2-(1-\bar{\chi})\lambda_t^i, \tag{A.13}$$

$$\frac{\mathbb{E}_t[dr_t^{E,i}]}{dt}-\frac{\mathbb{E}_t[dr_t^B]}{dt}=\left(\sigma_t^{q,B}-(\theta_t^{K,i}+\theta_t^{E,i}+\theta_t^{\bar{E},i})\frac{\sigma_t^\vartheta}{1-\vartheta_t}\right)(\sigma_t^{q,K}-\sigma_t^{q,B})+(\theta_t^{K,i}+\theta_t^{E,i})\tilde{\sigma}_t^2-\lambda_t^i, \tag{A.14}$$

$$\frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt}-\frac{\mathbb{E}_t[dr_t^B]}{dt}=\left(\sigma_t^{q,B}-(\theta_t^{K,i}+\theta_t^{E,i}+\theta_t^{\bar{E},i})\frac{\sigma_t^\vartheta}{1-\vartheta_t}\right)(\sigma_t^{q,K}-\sigma_t^{q,B}). \tag{A.15}$$

Note that we have plugged in $\sigma_t^{q,B}-\sigma_t^{q,K}=\frac{\sigma_t^\vartheta}{1-\vartheta_t}$.

Characterizing Consumption, Investment, q_t^B and q_t^K The first order condition Eq. (A.11) implies constant consumption-wealth ratio

$$c_t^i=\rho n_t^i. \tag{A.16}$$

FOC (A.12) reproduces the Tobin's q condition, and confirms that every household has the same investment ratio

$$q_t^K=1+\phi\iota_t. \tag{A.17}$$

Now we can (jointly) pin down ϑ_t, q_t^B and q_t^K. Recall goods market clearing condition

$$C_t+\mathfrak{g}K_t+\iota_tK_t=a_tK_t.$$

Plug in $C_t = \rho \bar{n}_t = \rho q_t^K K_t/(1-\vartheta_t)$ to obtain

$$\frac{\rho}{1-\vartheta_t} + \frac{\rho\phi}{1-\vartheta_t}\iota_t + \mathfrak{g} + \iota_t = a_t,$$

i.e.,

$$\iota_t = \frac{(a_t - \mathfrak{g})(1-\vartheta_t) - \rho}{1 + \rho\phi - \vartheta_t}. \tag{A.18}$$

Hence, we can express price of capital q_t^K as

$$q_t^K = \frac{1 - \vartheta_t + \phi(a_t - \mathfrak{g})(1-\vartheta_t)}{1 + \rho\phi - \vartheta_t} = (1-\vartheta_t)\frac{1 + \phi(a_t - \mathfrak{g})}{1 + \rho\phi - \vartheta_t}.$$

Since $q_t^B/q_t^K = \vartheta_t/(1-\vartheta_t)$,

$$q_t^B = \vartheta_t \frac{1 + \phi(a_t - \mathfrak{g})}{1 + \rho\phi - \vartheta_t}. \tag{A.19}$$

Portfolio Weights Under symmetric equilibrium,

$$\theta_t^{K,i} = \theta_t^K, \quad \theta_t^{E,i} = \theta_t^E, \quad \theta_t^{\bar{E},i} = \theta_t^{\bar{E}}.$$

Asset Market clearing implies

$$1 - \theta_t^K - \theta_t^E - \theta_t^{\bar{E}} = \vartheta_t$$

and

$$\theta_t^E + \theta_t^{\bar{E}} = 0.$$

When the skin-in-the-game constraint is binding,

$$\theta_t^E + (1-\bar{\chi})\theta_t^K = 0. \tag{A.20}$$

The equations above jointly solve

$$\begin{aligned}
\theta_t^K &= 1 - \vartheta_t, \\
\theta_t^E &= -(1-\bar{\chi})(1-\vartheta_t), \\
\theta_t^{\bar{E}} &= (1-\bar{\chi})(1-\vartheta_t).
\end{aligned}$$

Return Processes Under symmetric equilibrium, the outside equity has the same expected return as individual equity, i.e.,

$$\mathbb{E}_t[dr_t^{E,i}] = \mathbb{E}_t[dr_t^{\bar{E},i}], \tag{A.21}$$

with which we can pin down the lagrangian multiplier from Eq. (A.14) and (A.15)

$$\lambda_t^i = (\theta_t^{K,i} + \theta_t^{E,i})\tilde{\sigma}_t^2 = \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t^2.$$

Note that $\theta_t^{E,i} + \theta_t^{K,i} \geq \chi\theta_t^{K,i}$, hence λ_t^i is always positive and the skin-in-the-game constraint is always binding.

The expected return of equity and outside equity in excess of real bond return can be written as

$$\frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = \left(\sigma_t^{q,B} - \theta_t^K \frac{\sigma_t^\vartheta}{1-\vartheta_t}\right)(\sigma_t^{q,K} - \sigma_t^{q,B}) = \left(\sigma_t^{q,B} - \sigma_t^\vartheta\right)(\sigma_t^{q,K} - \sigma_t^{q,B}).$$

The expected return of capital in excess of equity is

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^{E,i}]}{dt} = \chi\lambda_t^i = (1-\vartheta_t)\bar{\chi}^2\tilde{\sigma}_t^2.$$

Equilibrium: Discipline Drift of the State Variable We discipline μ_t^ϑ as Proposition 1. By Eq. (A.6) and (A.5), the expected return of capital in excess of real bond return is

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^{B}]}{dt} = \frac{(1-\tau_t)a_t - \iota_t}{q_t^K} + \mu_t^{q,K} + \check{\mu}_t^B - \mu_t^{q,B}.$$

Plugging Eq. (A.1) and (A.4) yields

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^{B}]}{dt} = \frac{\check{\mu}_t^B - \mu_t^\vartheta}{1-\vartheta_t} + (\sigma_t^{q,B} - \sigma_t^\vartheta)(\sigma_t^{q,K} - \sigma_t^{q,B}) + \frac{a_t - \mathfrak{g} - \iota_t}{q_t^K}.$$

By Eq. (A.13) and (A.18),

$$(\sigma_t^{q,B} - \sigma_t^\vartheta)(\sigma_t^{q,K} - \sigma_t^{q,B}) + (1-\vartheta_t)\bar{\chi}^2\tilde{\sigma}_t^2 = \frac{\check{\mu}_t^B - \mu_t^\vartheta}{1-\vartheta_t} + (\sigma_t^{q,B} - \sigma_t^\vartheta)(\sigma_t^{q,K} - \sigma_t^{q,B}) + \frac{a_t - \mathfrak{g} - \iota_t}{q_t^K}$$

$$\Rightarrow \mu_t^\vartheta = \check{\mu}_t^B + \rho - (1-\vartheta_t)^2\bar{\chi}^2\tilde{\sigma}_t^2. \tag{A.22}$$

Macro Synthesis Before proceeding, we briefly summarize the equation system here. We suppress the individual superscript for simplicity.

There are 4 exogenous variables:

$$(\mu_t^B, i_t, a_t, \tilde{\sigma}_t),$$

5 endogenous variables:

$$\left(\hat{c}_t, \iota_t, q_t^B, q_t^K, \tau_t\right),$$

and an endogenous state variable ϑ_t.

We have 5 equations: 2 households' first-order conditions (A.11) and (A.12):

$$\hat{c}_t = \rho,$$

$$q_t^K = 1 + \phi\iota_t,$$

1 goods market clearing condition:

$$\hat{c}_t \frac{q_t^K}{1-\vartheta_t} + \mathfrak{g} + \iota_t = a_t,$$

1 wealth share definition:

$$\vartheta_t = \frac{q_t^B}{q_t^B + q_t^K},$$

and 1 government budget constraint:

$$(\mu_t^B - i_t) q_t^B + \tau_t a_t = \mathfrak{g}.$$

Given the state variable ϑ_t, these 5 equations allow us to solve for the 5 endogenous variables.

Besides, the dynamics of state variable ϑ_t are subject to

$$\mu_t^\vartheta = \check{\mu}_t^B + \rho - (1-\vartheta_t)^2 \bar{\chi}^2 \tilde{\sigma}_t^2.$$

In this equilibrium, the expected returns are

$$\begin{aligned}
\frac{\mathbb{E}[dr_t^B]}{dt} &= \Phi(\iota_t) - \delta - \check{\mu}_t^B + \mu_t^{q,B}, \\
\frac{\mathbb{E}[dr_t^K]}{dt} &= \Phi(\iota_t) - \delta - \check{\mu}_t^B + \mu_t^{q,B} + (\vartheta_t \sigma_t^{q,B} + (1-\vartheta_t)\sigma_t^{q,K})(\sigma_t^{q,K} - \sigma_t^{q,B}) + (1-\vartheta_t)\bar{\chi}^2\tilde{\sigma}_t^2, \\
\frac{\mathbb{E}[dr_t^E]}{dt} &= \Phi(\iota_t) - \delta - \check{\mu}_t^B + \mu_t^{q,B} + (\vartheta_t \sigma_t^{q,B} + (1-\vartheta_t)\sigma_t^{q,K})(\sigma_t^{q,K} - \sigma_t^{q,B}), \\
\frac{\mathbb{E}[dr_t^{\bar{E}}]}{dt} &= \Phi(\iota_t) - \delta - \check{\mu}_t^B + \mu_t^{q,B} + (\vartheta_t \sigma_t^{q,B} + (1-\vartheta_t)\sigma_t^{q,K})(\sigma_t^{q,K} - \sigma_t^{q,B}).
\end{aligned}$$

Verifying Value Function I verify the value function by plugging endogenous variables into the HJB Equation (A.24). Indeed,

$$
\begin{aligned}
0 &= -\rho v_t - \log n_t^i + \dot{v}_t + (\log\rho + \log n_t^i) - 1 + \frac{1}{\rho}\left(\Phi(\iota_t) - \delta - \check{\mu}_t^B + \mu_t^{q,B}\right) \\
&+ \frac{1}{\rho}(1-\vartheta_t)\left[(\sigma_t^{q,B} - \sigma_t^{\vartheta})(\sigma_t^{q,K} - \sigma_t^{q,B}) + (1-\vartheta_t)\bar{\chi}^2\tilde{\sigma}_t^2\right] \\
&- \frac{1}{2\rho}\left(\sigma_t^{q,B} - \sigma_t^{\vartheta}\right)^2 - \frac{1}{2\rho}(1-\vartheta_t)^2\bar{\chi}^2\tilde{\sigma}_t^2 \\
&= \dot{v}_t - \rho v_t + h(\vartheta_t, \sigma_t^{\vartheta}, \mu_t^{\vartheta}),
\end{aligned}
$$

which is an linear ODE for v_t. Note that $\vartheta_t, \sigma_t^{\vartheta}, \mu_t^{\vartheta}$ depend on aggregate shock only, so does v_t.

A.2 Proof for Proposition 1

Proof. Recall that

$$
m_t^i = \frac{1}{\rho} e^{-\rho t} \frac{1}{n_t^i}.
$$

We obtain

$$
\begin{aligned}
\lim_{T\to\infty} \mathbb{E}_0\left[\bar{m}_T \bar{n}_T\right] &= \lim_{T\to\infty} \mathbb{E}_0\left[\left(\int_0^1 e^{-\rho T}\frac{n_0^i}{n_T^i}\, di\right)\bar{n}_T\right] \\
&= \lim_{T\to\infty} \mathbb{E}_0\left[e^{-\rho T}\bar{n}_0\left(\int_0^1 \frac{\eta_0^i}{\eta_T^i}\, di\right)\right].
\end{aligned}
$$

□

A.3 Derivation of Model in Section 3

The model set-up follows those in Appendix A.1, except the wealth redistribution shock. We solve the symmetric equilibrium in a similar way. The households' HJB equation is given by

$$(\rho+\zeta)V(n_t^i,v_t)dt=\max_{c_t^i,\iota_t^i,\theta_t^i}\mathbb{E}_t\left[\log c_t^i dt+\zeta V(\bar{n}_t,v_t)dt+V_t dt+V_n(n_t^i,v_t)dn_t^i+\frac{1}{2}V_{nn}(n_t^i,v_t)\langle dn_t^i,dn_t^i\rangle\right].$$

Guess that the value function takes the form $V(n_t^i,v_t)=v_t+\frac{1}{\rho+\zeta}\log n_t^i$, where v_t depends on investment opportunity in aggregate level. We obtain

$$\rho v_t+\log n_t^i-\frac{\zeta}{\rho+\zeta}\log\bar{n}_t=\max_{c_t^i,\iota_t^i,\theta_t^i}\mathbb{E}\left\{\log c_t^i+\dot{v}_t+\frac{1}{\rho+\zeta}\frac{dn_t^i}{n_t^i}\frac{1}{dt}-\frac{1}{2}\frac{1}{\rho+\zeta}\left\langle\frac{dn_t^i}{n_t^i},\frac{dn_t^i}{n_t^i}\right\rangle\frac{1}{dt}\right\}.$$

For simplicity, we denote $\hat{\rho}:=\rho+\zeta$. The HJB equation can be restated as

$$\begin{aligned}0=\max_{c_t^i,\iota_t^i,\theta_t^i}\Bigg\{&-(\hat{\rho}-\zeta)v_t-\log n_t^i+\frac{\zeta}{\hat{\rho}}\log\bar{n}_t+\dot{v}_t+\log c_t^i \qquad \text{(A.23)}\\&-\frac{1}{\hat{\rho}}\frac{c_t^i}{n_t^i}+\frac{1}{\hat{\rho}}(1-\theta_t^{K,i}-\theta_t^{E,i}-\theta_t^{\bar{E},i})\left(\Phi(\iota_t)-\delta-\check{\mu}_t^B+\mu_t^{q,B}\right)\\&+\frac{1}{\hat{\rho}}\theta_t^{K,i}\left(\frac{(1-\tau_t)a_t-\iota_t^i}{q_t^K}+\mu_t^{q,K}+(\Phi(\iota_t^i)-\delta)\right)+\frac{1}{\hat{\rho}}\theta_t^{E,i}\frac{\mathbb{E}_t[dr_t^{E,i}]}{dt}+\frac{1}{\hat{\rho}}\theta_t^{\bar{E},i}\frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt}\\&-\frac{1}{2\hat{\rho}}\left((1-\theta_t^{K,i}-\theta_t^{E,i}-\theta_t^{\bar{E},i})\sigma_t^{q,B}+(\theta_t^{K,i}+\theta_t^{E,i}+\theta_t^{\bar{E},i})\sigma_t^{q,K}\right)^2-\frac{1}{2\hat{\rho}}(\theta_t^{K,i}+\theta_t^{E,i})^2\tilde{\sigma}_t^2\Bigg\}. \qquad \text{(A.24)}\end{aligned}$$

s.t.

$$-\theta_t^{E,i}\leq(1-\bar{\chi})\theta_t^{K,i}$$

The lagrangian is hence given by

$$\begin{aligned}\mathcal{L}_t^i = &-(\hat{\rho}-\zeta)v_t - \log n_t^i + \frac{\zeta}{\hat{\rho}}\log \bar{n}_t + \dot{v}_t + \log c_t^i \\ &- \frac{1}{\hat{\rho}}\frac{c_t^i}{n_t^i} + \frac{1}{\hat{\rho}}(1-\theta_t^{K,i}-\theta_t^{E,i}-\theta_t^{\bar{E},i})\left(\Phi(\iota_t)-\delta-\check{\mu}_t^B+\mu_t^{q,B}\right) \\ &+\frac{1}{\hat{\rho}}\theta_t^{K,i}\left(\frac{(1-\tau_t)a_t-\iota_t^i}{q_t^K}+\mu_t^{q,K}+(\Phi(\iota_t^i)-\delta)\right)+\frac{1}{\hat{\rho}}\theta_t^{E,i}\frac{\mathbb{E}_t[dr_t^{E,i}]}{dt}+\frac{1}{\hat{\rho}}\theta_t^{\bar{E},i}\frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt} \\ &-\frac{1}{2\hat{\rho}}\left((1-\theta_t^{K,i}-\theta_t^{E,i}-\theta_t^{\bar{E},i})\sigma_t^{q,B}+(\theta_t^{K,i}+\theta_t^{E,i}+\theta_t^{\bar{E},i})\sigma_t^{q,K}\right)^2-\frac{1}{2\hat{\rho}}(\theta_t^{K,i}+\theta_t^{E,i})^2\tilde{\sigma}_t^2 \\ &+\lambda_t^i(\theta_t^{E,i}+(1-\bar{\chi})\theta_t^{K,i})\end{aligned}$$

The first order conditions w.r.t. c_t^i, ι_t^i, $\theta_t^{K,i}$, $\theta_t^{E,i}$ and $\theta_t^{\bar{E},i}$ are

$$-\frac{1}{\hat{\rho}n_t^i}+\frac{1}{c_t^i}=0, \tag{A.25}$$

$$-\frac{1}{q_t^K}+\frac{1}{1+\phi\iota_t^i}=0, \tag{A.26}$$

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt}-\frac{\mathbb{E}_t[dr_t^B]}{dt}=\left(\sigma_t^{q,B}-(\theta_t^{K,i}+\theta_t^{E,i}+\theta_t^{\bar{E},i})\frac{\sigma_t^\vartheta}{1-\vartheta_t}\right)(\sigma_t^{q,K}-\sigma_t^{q,B})+(\theta_t^{K,i}+\theta_t^{E,i})\tilde{\sigma}_t^2-(1-\bar{\chi})\lambda_t^i, \tag{A.27}$$

$$\frac{\mathbb{E}_t[dr_t^{E,i}]}{dt}-\frac{\mathbb{E}_t[dr_t^B]}{dt}=\left(\sigma_t^{q,B}-(\theta_t^{K,i}+\theta_t^{E,i}+\theta_t^{\bar{E},i})\frac{\sigma_t^\vartheta}{1-\vartheta_t}\right)(\sigma_t^{q,K}-\sigma_t^{q,B})+(\theta_t^{K,i}+\theta_t^{E,i})\tilde{\sigma}_t^2-\lambda_t^i, \tag{A.28}$$

$$\frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt}-\frac{\mathbb{E}_t[dr_t^B]}{dt}=\left(\sigma_t^{q,B}-(\theta_t^{K,i}+\theta_t^{E,i}+\theta_t^{\bar{E},i})\frac{\sigma_t^\vartheta}{1-\vartheta_t}\right)(\sigma_t^{q,K}-\sigma_t^{q,B}). \tag{A.29}$$

Note that the first order conditions are same as those in appendix A.1 after replacing ρ by $\hat{\rho}$.

Risk-free rate From FOC (A.27)-(A.29), the risk premia for aggregate risk dZ_t and idiosyncratic risk $d\tilde{Z}_t^i$ are not affected by the jump shock. However, the risk-free rate increases since households do not receive payoffs at the jump states. That is, one will not be richer after jump shock if she

saves more before the jump. The stochastic discount factor $m_t^i = e^{-\rho t}\frac{1}{(\rho+\zeta)n_t^i}$ follows

$$dm_t^i = -(\rho + g_t + (\sigma_m^2)^2 + (1-\vartheta_t)^2\bar{\chi}^2\tilde{\sigma}_t^2)m_{t-}^i dt - \sigma_t^m m_{t-}^i dZ_t - (1-\vartheta_t)\bar{\chi}\tilde{\sigma}_t m_{t-}^i d\tilde{Z}_t^i + \left(e^{-\rho t}\frac{1}{(\rho+\zeta)\bar{n}_t} - m_{t-}^i\right) d\tilde{J}_t^i.$$

At time t, consider an risk-free asset that surely delivers one unit of goods at $t+\Delta t$. The time window $\Delta t \to 0$ is short such that jump shock happens once at most. The agent effectively receive this payoff only if jump does not happen. The Euler equation is given by

$$\exp(-r_t^f \Delta t) = \mathbb{E}_t\left[\frac{m_{t+\Delta t}^i}{m_t^i} 1_{\{d\tilde{J}_{t+\Delta t}^i = 0\}}\right],$$

i.e.,

$$\begin{aligned} 1 - r_t^f \Delta t &= \mathbb{E}_t\left[\frac{m_{t+\Delta t}^i}{m_t^i}\middle| d\tilde{J}_{t+\Delta t}^i = 0\right](1-\zeta\Delta t) \\ &= (1-\zeta\Delta t)[1 - (\rho + g_t - (\sigma_t^m)^2 - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2)\Delta t], \end{aligned}$$

which implies

$$r_t^f = \rho + \zeta + g_t - (\sigma_t^m)^2 - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2.$$

A.4 Proof for Proposition 2

Denote the nominal government bond outstanding as B_t and real government bond as b_t. We obtain

$$dB_t = i_t B_t dt - P_t s_t dt.$$

Hence,

$$
\begin{aligned}
d(m_t^i b_t) &= b_t dm_t^i + m_{t-}^i/P_t dB_t + m_{t-}^i B_t d(1/P_t) + B_t dm_t^i d(1/P_t) \\
&= b_t \left(-(r_t^f - \zeta) m_{t-}^i dt - (\sigma_t^m) m_{t-}^i dZ_t - \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t m_{t-}^i d\tilde{Z}_t^i + \left(e^{-\rho t} \frac{1}{(\rho+\zeta)\bar{n}_t} - m_{t-}^i \right) d\tilde{J}_t^i \right) \\
&+ i_t b_t m_{t-}^i dt - s_t m_t^i dt \\
&+ m_{t-}^i b_t (dr_t^B - i_t dt) \\
&- \sigma_t^m m_{t-}^i b_t \sigma_t^{q,B} dt \\
&= -m_{t-}^i s_t dt + m_{t-}^i \zeta b_t dt - m_{t-}^i b_t (\sigma_t^m - \sigma_t^{q,B}) dZ_t - m_{t-}^i b_t \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t d\tilde{Z}_t^i \\
&+ b_t \left(e^{-\rho t} \frac{1}{(\rho+\zeta)\bar{n}_t} - m_{t-}^i \right) d\tilde{J}_t^i \\
&= -m_{t-}^i s_t dt + \zeta e^{-\rho t} \frac{1}{(\rho+\zeta)\bar{n}_t} b_t dt - m_{t-}^i b_t (\sigma_t^m - \sigma_t^{q,B}) dZ_t - m_{t-}^i b_t \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t d\tilde{Z}_t^i \\
&+ b_t \left(e^{-\rho t} \frac{1}{(\rho+\zeta)\bar{n}_t} - m_{t-}^i \right) d(\tilde{J}_t^i - \zeta t),
\end{aligned}
$$

where we plugged in the definition of bond return, i.e.,

$$
dr_t^B = i_t dt + \frac{d(1/P_t)}{1/P_t} = i_t dt + \frac{d(q_t^B K_t/B_t)}{q_t^B K_t/B_t} = (r_t^f + \sigma_t^m \sigma_t^{q,B}) dt + \sigma_t^{q,B} dZ_t.
$$

Integrate $d(m_t^i b_t)$ from $t = 0$ to T and take expectation to obtain[6]

$$
\mathbb{E}_0[m_T^i b_t] - m_0^i b_0 = -\mathbb{E}_0 \left[\int_0^T m_t^i s_t \, dt \right] + \zeta \mathbb{E}_0 \left[\int_0^T e^{-\rho t} \frac{b_t}{(\rho+\zeta)\bar{n}_t} \, dt \right].
$$

Divide both sides by m_0^i to obtain

$$
b_0 = \mathbb{E}_0 \left[\int_0^T \frac{m_t^i}{m_0^i} s_t \, dt \right] - \bar{n}_0 \zeta \mathbb{E}_0 \left[\int_0^T e^{-\rho t} \frac{b_t}{\bar{n}_t} \, dt \right] + \mathbb{E}_0 \left[\frac{m_T^i}{m_0^i} b_T \right].
$$

[6]Note that the set of jump time $\mathcal{T} = \{t | m_t^i \neq m_{t-}^i\}$ has zero measure, and we can replace $t-$ by t in the intergrals. Heuristically, $\int_0^T m_t^i s_t \, dt = \int_0^T m_{t-}^i s_t \, dt + \int_0^T (m_t^i - m_{t-}^i) s_t \, dt = \int_0^T m_{t-}^i s_t \, dt$.

To derive the bond valuation equation from the aggregate SDF, we start by dividing $d(m_t^i B_t/P_t)$ by m_0^i, i.e.,

$$d[(m_t^i/m_0^i)b_t] = -\frac{m_{t-}^i}{m_0^i}s_t dt + \bar{n}_0\zeta e^{-\rho t}\frac{1}{\bar{n}_t}b_t dt - \frac{m_{t-}^i}{m_0^i}b_t(\sigma_t^m - \sigma_t^{q,B})dZ_t - \frac{m_{t-}^i}{m_0^i}b_t\bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t d\tilde{Z}_t^i + b_t\left(\bar{n}_0 e^{-\rho t}\frac{1}{\bar{n}_t} - \frac{m_{t-}^i}{m_0^i}\right)d(\tilde{J}_t^i - \zeta t).$$

Intergrate by i to obtain

$$d[\bar{m}_t b_t] = -\bar{m}_t s_t dt + \bar{n}_0\zeta e^{-\rho t}\frac{1}{\bar{n}_t}b_t dt - \bar{m}_t b_t(\sigma_t^m - \sigma_t^{q,B})dZ_t.$$

Integrate by t and take expectation to obtain

$$\mathbb{E}_0[\bar{m}_T b_T] - b_0 = -\mathbb{E}_0\left[\int_0^T \bar{m}_t s_t dt\right] + \mathbb{E}_t\left[\int_0^T \zeta e^{-\rho t}\frac{\bar{n}_0}{\bar{n}_t}b_t dt\right],$$

i.e.,

$$b_0 = \mathbb{E}_0\left[\int_0^T \bar{m}_t s_t dt\right] - \mathbb{E}_t\left[\int_0^T \zeta e^{-\rho t}\frac{\bar{n}_0}{\bar{n}_t}b_t dt\right] + \mathbb{E}_0\left[\bar{m}_T b_T\right].$$

A.5 Wealth Distribution Dynamics

Recall that the individual wealth satisfies

$$\frac{dn_t^i}{n_t^i} = g_t dt + \sigma_t^m dZ_t + (1-\vartheta_t)\tilde{\sigma}_t\bar{\chi}d\tilde{Z}_t^i + \frac{\bar{n}_t - n_t^i}{n_t^i}d\tilde{J}_t^i.$$

For aggregate wealth,

$$\frac{d\bar{n}_t}{\bar{n}_t} = g_t dt + \sigma_t^m dZ_t.$$

Hence,

$$\frac{d\eta_t^i}{\eta_t^i} = \frac{d(n_t^i/\bar{n}_t)}{n_t^i/\bar{n}_t} = \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t d\tilde{Z}_t^i + \frac{1-\eta_t^i}{\eta_t^i} d\tilde{J}_t^i. \tag{A.30}$$

We obtain the probability density function for η_T^i from the Kolmogorov Forward Equation, i.e.,

$$\frac{\partial g(\eta,t)}{\partial t} = -\frac{\partial}{\partial \eta}(\beta(\eta,t)g(\eta,t)) + \frac{1}{2}\frac{\partial^2}{\partial \eta^2}(\gamma(\eta,t)^2 g(\eta,t)) - \zeta g(\eta,t),$$

where $\beta(\eta,t)=0$, $\gamma(\eta,t) := \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t\eta$. [7] Under Assumption 1, let $\vartheta_t = \vartheta, \tilde{\sigma}_t = \tilde{\sigma}$, we obtain

$$\frac{\partial g(\eta,t)}{\partial t} = \frac{(\bar{\chi}(1-\vartheta)\tilde{\sigma})^2}{2}\frac{\partial^2}{\partial \eta^2}(\eta^2 g(\eta,t)) - \zeta g(\eta,t).$$

The boundary condition is that $g(\eta,t)$ is always a valid pdf at any time t, i.e.,

$$\int_0^\infty g(\eta,t)\, d\eta = 1, \quad \forall t > 0.$$

For initial condition, we require

$$\lim_{t\to 0} g(\eta,t) = \delta(\eta-1),$$

where $\delta(.)$ denotes the Dirac function. Generally, the Kolmogorov Forward Equation PDE has no analytical solution. However, we can obtain the explicit expression of wealth distribution at the steady state. Let $\partial g(\eta,t)/\partial t = 0$, we obtain

$$0 = \frac{1}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2(\eta^2 g''(\eta) + 4\eta g'(\eta) + 2g(\eta)) - \zeta g(\eta),$$

[7] Note that there is a kink at $\eta = 1$, and the KFE holds on $(0,1)$ and $(1,+\infty)$.

i.e.,

$$0 = \eta^2 g''(\eta) + 4\eta g'(\eta) + 2 - \frac{2\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2} g(\eta).$$

The ODE is solved separately on $(0, 1)$ and $(1, +\infty)$. The general solution is

$$g(\eta) = C_1\eta^{\alpha_1} + C_2\eta^{\alpha_2}, \qquad \eta < 1,$$
$$g(\eta) = C_3\eta^{\alpha_1} + C_4\eta^{\alpha_2}, \qquad \eta > 1,$$

where

$$\alpha_1 = -\frac{3}{2} + \frac{\sqrt{1 + \frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{2}, \qquad \alpha_2 = -\frac{3}{2} - \frac{\sqrt{1 + \frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{2}.$$

The steady state probability density function $g(\eta)$ must satisfy

$$\int_0^\infty g(\eta)\, d\eta = 1.$$

Note that $\alpha_1 > -1$ and $\alpha_2 < -1$. Hence, we require $C_2 = 0$ and $C_3 = 0$ to ensure $|\int_0^\infty g(\eta)\, d\eta| < \infty$. Moreover, we require $C_1 = C_4 = C$ such that $g(\eta)$ is continuous at $\eta = 1$. The integral is computed as

$$\begin{aligned}
\int_0^\infty g(\eta)\, d\eta &= \frac{2C}{-1 + \sqrt{1 + \frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}} \eta^{\frac{-1+\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{2}}\Bigg|_0^1 + \frac{2C}{-1 - \sqrt{1 + \frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}} \eta^{\frac{-1-\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{2}}\Bigg|_1^\infty \\
&= \frac{2C}{-1 + \sqrt{1 + \frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}} - \frac{2C}{-1 - \sqrt{1 + \frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}} \\
&= \frac{4\sqrt{1 + \frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}} C,
\end{aligned}$$

which implies $C_1 = C_4 = C = \frac{2\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}$. Note that

$$\begin{aligned}\int_0^\infty \eta g(\eta)\, d\eta &= \frac{2C}{1+\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}\, \eta^{\frac{1+\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{2}}\Bigg|_0^1 + \frac{2C}{1-\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}\, \eta^{\frac{1-\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{2}}\Bigg|_1^\infty \\ &= \frac{2C}{1+\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}} + \frac{2C}{-1+\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}} \\ &= \frac{4\sqrt{1+\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}}}{\frac{8\zeta}{\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2}} C = 1.\end{aligned}$$

Hence we verify that $g(\eta)$ is a valid wealth distribution function, i.e., $\int_0^1 \eta^i\, di = \int_0^\infty \eta g(\eta)\, d\eta = 1$. Now we consider the dynamics of $\mathbb{E}_0\left[\frac{\eta_0^i}{\eta_T^i}\right]$. Applying Itô's lemma to Eq. (A.30), we obtain

$$d\left(\frac{1}{\eta_t^i}\right) = (1-\vartheta)^2\bar{\chi}^2\tilde{\sigma}^2\frac{1}{\eta_t^i}dt + \left(1-\frac{1}{\eta_t^i}\right)\zeta dt - (1-\vartheta)\bar{\chi}\tilde{\sigma}\frac{1}{\eta_t^i}d\tilde{Z}_t^i + \left(1-\frac{1}{\eta_t^i}\right)d(\tilde{J}_t^i - \zeta t).$$

Take expectation to obtain

$$d\mathbb{E}_0\left[\frac{1}{\eta_t^i}\right] = \zeta dt + [(1-\vartheta)^2\bar{\chi}^2\tilde{\sigma}^2 - \zeta]\mathbb{E}_0\left[\frac{1}{\eta_t^i}\right]dt.$$

Note that this is an ODE w.r.t. $\mathbb{E}_0\left[\frac{1}{\eta_t^i}\right]$. The solution is

$$\mathbb{E}_0\left[\frac{1}{\eta_t^i}\right] = \left(\frac{\zeta}{(1-\vartheta)^2\bar{\chi}^2\tilde{\sigma}^2 - \zeta} + \frac{1}{\eta_0^i}\right)e^{[(1-\vartheta)^2\bar{\chi}^2\tilde{\sigma}^2-\zeta]t} - \frac{\zeta}{(1-\vartheta)^2\bar{\chi}^2\tilde{\sigma}^2},$$

i.e.,

$$\mathbb{E}_0\left[\frac{\eta_0^i}{\eta_t^i}\right] = \left(\frac{\zeta\eta_0^i}{(1-\vartheta)^2\bar{\chi}^2\tilde{\sigma}^2 - \zeta} + 1\right)e^{[(1-\vartheta)^2\bar{\chi}^2\tilde{\sigma}^2-\zeta]t} - \frac{\zeta\eta_0^i}{(1-\vartheta)^2\bar{\chi}^2\tilde{\sigma}^2}.$$

Let $\eta_0^i = 1$. By symmetry, $\mathbb{E}_0[\int_0^1 1/\eta_t^i\, di] = \mathbb{E}_0[1/\eta_t^i]$, which is finite if and only if $\zeta > (1-\vartheta)^2\bar{\chi}^2\tilde{\sigma}^2$.

A.6 Proof for Proposition 3

Note that

$$\begin{aligned} d\bar{m}_t &= -[\rho + \mu_t^n - (\sigma_t^m)^2 - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2]\bar{m}_t dt - \sigma_t^m \bar{m}_t dZ_t + \int_0^1 e^{-\rho t}\frac{1}{\hat{\rho}}\left(\frac{1}{\bar{n}_t} - \frac{1}{n_t^i}\right)\frac{1}{m_0^i} d\tilde{J}_t^i di \\ &= -[\rho + \mu_t^n - (\sigma_t^m)^2 - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2]\bar{m}_t dt - \sigma_t^m \bar{m}_t dZ_t + \zeta\left[\int_0^1 e^{-\rho t}\frac{1}{\hat{\rho}}\left(\frac{1}{\bar{n}_t} - \frac{1}{n_t^i}\right)\frac{1}{m_0^i} di\right] dt \\ &= -[\rho + \mu_t^n - (\sigma_t^m)^2 - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 + \zeta]\bar{m}_t dt + \zeta e^{-\rho t}\frac{\bar{n}_0}{\bar{n}_t} dt - \sigma_t^m \bar{m}_t dZ_t. \end{aligned}$$

Hence,

$$d[\bar{m}_t \bar{n}_t] = -[\rho - (\sigma_t^m)^2 - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 + \zeta]\bar{m}_t\bar{n}_t dt + \zeta e^{-\rho t}\bar{n}_0 dt$$

which implies

$$\begin{aligned} \bar{m}_t\bar{n}_t = \bar{n}_0 e^{-[\rho - \bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + \zeta]t} \\ + \frac{\zeta\bar{n}_0}{[-\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + \zeta]}e^{-\rho t} - \frac{\zeta\bar{n}_0}{[-\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + \zeta]}e^{-[\rho - \bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 + \zeta]t}. \end{aligned}$$

A.7 Model with Stochastic Death

We model the stochastic death as follows. Define $\{d_t^i\}$ as a Poisson process with intensity ζ. Death events are independent across agents, i.e., $\{d_t^i\}$ and $\{d_t^j\}$ are stochastically independent for $i \neq j$. Let $d_0^i = 0$. The stochastic death time τ^i is defined as

$$\tau^i := \inf\{t > 0 \mid d_t^i \geq 1\}$$

The death process is hence defined by

$$death_t^i = d^i_{\{t \wedge \tau^i\}},$$

i.e., when agent i is alive, $death_t^i = 0$. Otherwise, $death_t^i = 1$.

Consider household's problem. Heuristically,

$$\begin{aligned} V(n_t^i, v_t) &= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t \left[\log c_t^i dt + e^{-\rho dt} (1_{\text{death}} V(n_{t+dt}^i, v_{t+dt}) + 1_{\text{nodeath}} V(n_{t+dt}^i, v_{t+dt})) \right] \\ &= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t \left[\log c_t^i dt + (1 - \rho dt)(\zeta dt \times 0 + (1 - \zeta dt) V(n_{t+dt}^i, v_{t+dt})) \right] \end{aligned}$$

The Hamilton-Jacobi-Bellman equation is

$$(\rho + \zeta) V(n_t^i, v_t) = \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t \left[\log c_t^i + V_t + V_n(n_t^i, v_t) dn_t^i + \frac{1}{2} V_{nn}(n_t^i, v_t) \langle dn_t^i, dn_t^i \rangle \right]$$

Guess that the value function takes the form

$$V(n_t^i, v_t) = v_t + \frac{1}{\rho + \zeta} \log n_t^i,$$

where v_t depends on investment opportunity in aggregate level. Plug into the HJB equation to obtain

$$(\rho + \zeta) v_t + \log n_t^i = \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E} \left\{ \log c_t^i + \dot{v}_t + \frac{1}{\rho + \zeta} \frac{dn_t^i}{n_t^i} \frac{1}{dt} - \frac{1}{2} \frac{1}{\rho + \zeta} \left\langle \frac{dn_t^i}{n_t^i}, \frac{dn_t^i}{n_t^i} \right\rangle \frac{1}{dt} \right\}.$$

Let $\hat{\rho} = \rho + \zeta$. The first-order conditions have the same formula as in the baseline case, but with $\hat{\rho} = \rho + \zeta$ instead of ρ. They are also very similar to the jump model, except that the utility after death is zero while the utility after wealth reset is $V(\bar{n}_t, v_t)$.

The risk-free rate follows

$$r_t^f = \rho + \zeta + g_t - (\sigma_t^m)^2 - \bar{\chi}^2 (1 - \vartheta_t)^2 \tilde{\sigma}_t^2,$$

which is identical to our case with wealth reset.

Let n_t^i denote the wealth of agent i at time t. For $t > \tau^i$, i.e., after the agent's death, we assume

that n_t^i denotes the wealth following the agent's consumption and investment plan if it were still alive. We denote $\check{m}_t^i = e^{-\rho t}\frac{1}{(\rho+\zeta)n_t^i}$ as the hypothetical marginal utility if the agent were still alive.

We denote m_t^i as the agent's actual marginal utility, which is zero after death:

$$m_t^i := \begin{cases} \check{m}_t^i = e^{-\rho t}\frac{1}{(\rho+\zeta)n_t^i}, & death_t^i = 0, \\ 0, & death_t^i = 1. \end{cases}$$

We obtain

$$dm_t^i = \begin{cases} -[\rho - g_t + (\sigma_t^m)^2 + \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2]m_t^i dt - \sigma_t^m m_t^i dZ_t - \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t m_t^i d\tilde{Z}_t^i, & t < \tau^i, \\ -m_{\tau^i-}^i, & t = \tau, \\ 0 & t > \tau^i. \end{cases}$$

Aggregate TVC For long-lived asset, e.g. government bond at $t < \tau^i$,

$$\begin{aligned} d(m_t^i b_t) &= b_t dm_t^i + m_t^i/P_t dB_t + m_t^i B_t d(1/P_t) + B_t dm_t^i d(1/P_t) \\ &= b_t\left(-(r_t^f - \zeta)m_t^i dt - (\sigma_t^m)m_t^i dZ_t - \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t m_t^i d\tilde{Z}_t^i\right) \\ &+ i_t b_t m_t^i dt - s_t m_t^i dt + m_t^i b_t(dr_t^B - i_t dt) - \sigma_t^m m_t^i b_t \sigma_t^{q,B} dt \\ &= -m_t^i s_t dt + m_t^i \zeta b_t dt - m_t^i b_t(\sigma_t^m - \sigma_t^{q,B})dZ_t - m_t^i b_t \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t d\tilde{Z}_t^i. \end{aligned}$$

At $t = \tau^i$, $d(m_t^i b_t) = -m_{\tau^i-}^i b_{\tau^i}$. At $t \geq \tau^i$, $d(m_t^i b_t) = 0$. Integrate from $t = 0$ to $t = T$ and take expectation. Recall that τ^i is independent of n_t^i and b_t, and

$$\mathbb{E}_0\left[1_{\{\tau^i \leq u\}}\right] = 1 - e^{-\zeta u}.$$

Hence,

$$
\begin{aligned}
m_0^i b_0 &= \mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T\right] \\
&= (1-e^{-\zeta T})\mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T \middle| \tau^i \le T\right] + e^{-\zeta T}\mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T \middle| \tau^i > T\right].
\end{aligned}
$$

The first term is computed as

$$
\begin{aligned}
&\mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T \middle| \tau^i \le T\right] \\
=&\mathbb{E}_0\left[\int_0^{\tau^i} \check{m}_t^i s_t dt \middle| \tau^i \le T\right] - \mathbb{E}_0\left[\int_0^{\tau^i} \zeta \check{m}_t^i b_t dt \middle| \tau^i \le T\right] + \mathbb{E}_0\left[\check{m}_{\tau^i_}^i b_{\tau i} \middle| \tau^i \le T\right],
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbb{E}_0\left[\int_0^{\tau^i} \check{m}_t^i s_t dt \middle| \tau^i \le T\right] &= \mathbb{E}_0\left[\mathbb{E}_0\left[\int_0^{\tau^i} \check{m}_t^i s_t\, dt \middle| \tau^i\right] \middle| \tau^i \le T\right] \\
&= \frac{1}{1-e^{-\zeta T}}\int_0^T \zeta e^{-\zeta u}\mathbb{E}_0\left[\int_0^u \check{m}_t^i s_t\, dt\right] du \\
&= \frac{1}{1-e^{-\zeta T}}\mathbb{E}_0\left[\int_0^T \left(\int_0^u \zeta e^{-\zeta u}\check{m}_t^i s_t\, dt\right) du\right] \\
&= \frac{1}{1-e^{-\zeta T}}\mathbb{E}_0\left[\int_0^T \left(\int_t^T \zeta e^{-\zeta u}\check{m}_t^i s_t\, du\right) dt\right] \\
&= \frac{1}{1-e^{-\zeta T}}\mathbb{E}_0\left[\int_0^T (e^{-\zeta t} - e^{-\zeta T})\check{m}_t^i s_t dt\right],
\end{aligned}
$$

and similarly

$$
\begin{aligned}
\mathbb{E}_0\left[\int_0^{\tau^i} \zeta \check{m}_t^i b_t dt \middle| \tau^i \le T\right] &= \frac{1}{1-e^{-\zeta T}}\mathbb{E}_0\left[\int_0^T \zeta(e^{-\zeta t} - e^{-\zeta T})\check{m}_t^i b_t dt\right], \\
\mathbb{E}_0\left[\check{m}_{\tau^i_}^i b_{\tau i} | \tau^i \le T\right] &= \frac{1}{1-e^{-\zeta T}}\mathbb{E}_0\left[\int_0^T \zeta e^{-\zeta u}\check{m}_u^i b_u du\right].
\end{aligned}
$$

The second term is computed as

$$\begin{aligned}&\mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T \middle| \tau^i > T\right]\\ =&\mathbb{E}_0\left[\int_0^T \check{m}_t^i s_t dt\right] - \mathbb{E}_0\left[\int_0^T \zeta \check{m}_t^i b_t dt\right] + \mathbb{E}_0\left[\check{m}_T^i b_T\right].\end{aligned}$$

Finally,

$$\begin{aligned}m_0^i b_0 &= \mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T\right]\\ &= \mathbb{E}_0\left[\int_0^T (e^{-\zeta t} - e^{-\zeta T})\check{m}_t^i s_t dt\right] + \mathbb{E}_0\left[\int_0^T \zeta e^{-\zeta T}\check{m}_t^i b_t dt\right]\\ &+ e^{-\zeta T}\mathbb{E}_0\left[\int_0^T \check{m}_t^i s_t dt\right] - e^{-\zeta T}\mathbb{E}_0\left[\int_0^T \zeta \check{m}_t^i b_t dt\right] + e^{-\zeta T}\mathbb{E}_0\left[\check{m}_T^i b_T\right]\\ &= \mathbb{E}_0\left[\int_0^T (e^{-\zeta t}\check{m}_t^i) s_t dt\right] + \mathbb{E}_0\left[(e^{-\zeta T}\check{m}_T^i) b_T\right].\end{aligned}$$

Recall that $\check{m}_t^i$ is the marginal utility if the agent were alive at time t. The TVC term for this aggregate asset is the discounted asset value $m_T^i b_T$ times the probability of surviving until T, which is $e^{-\zeta T}$. Following the same steps in A.1, the aggregate TVC holds if and only if $\rho + \zeta - \bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 > 0$.

Individual TVC Similarly, for individual wealth, we obtain

$$\begin{aligned}d(m_t^i n_t^i) &= n_t^i dm_t^i + m_t^i dn_t^i + dm_t^i dn_t^i\\ &= n_t^i(-(r_t^f - \zeta)m_t^i dt - (\sigma_t^m)m_t^i dZ_t - \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t m_t^i d\tilde{Z}_t^i)\\ &+ m_t^i(-c_t^i dt + (r_t^f + (\sigma_t^m)^2 + \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2)n_t^i dt + \sigma_t^m n_t^i dZ_t + \bar{\chi}(1-\vartheta_t)\tilde{\sigma}_t n_t^i d\tilde{Z}_t^i)\\ &- (\sigma_t^m)^2 m_t^i n_t^i dt - \bar{\chi}^2(1-\vartheta_t)^2\tilde{\sigma}_t^2 m_t^i n_t^i dt\\ &= -c_t^i m_t^i dt + \zeta m_t^i n_t^i dt\end{aligned}$$

for $t < \tau^i$, i.e., if the agent were still alive. Following the same steps above, we obtain

$$m_0^i n_0^i = \mathbb{E}_0 \left[\int_0^T (e^{-\zeta t} \check{m}_t^i) c_t^i dt \right] + \mathbb{E}_0 \left[(e^{-\zeta T} \check{m}_T^i) \check{n}_T^i \right].$$

The TVC for individual household's wealth is

$$\lim_{T \to \infty} \mathbb{E}_0 \left[(e^{-\zeta T} \check{m}_T^i \check{n}_T^i) \right] = 0.$$

Again, the analysis is the same as that in A.1, but the subjective discount rate is augmented by ζ.

A.8 Calibration

Table A.1: Parameter Values.

Variable	Notation	Value	Note
Baseline Model with Permanent Idiosyncratic Risk			
real growth rate	g	3%	
subjective discount rate	ρ	0.0063	Target 2% risk-free rate
idiosyncratic risk variance	$\tilde{\sigma}$	0.54	From Brunnermeier et al. (2022)
share of undiversifiable risk	$\bar{\chi}$	0.3	From Brunnermeier et al. (2022)
share of equity	ϑ	0.18	From Brunnermeier et al. (2022)
Model Variant with Transitory Idiosyncratic Risk			
wealth reset intensity, case 1	ζ	2.5%	Target steady state Gini coef. at 0.4
wealth reset intensity, case 2	ζ	0.36%	Target steady state Gini coef. at 0.8