Labor Allocation and Public Sector Performance: Evidence from Delayed Turnover in Fire Departments*

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Abstract

Using data on the universe of interventions and personnel of the Italian Fire and Rescue Service in 2014-2022, we investigate the impact of labor allocation on public sector performance. First, we document the presence of substantial delays in the turnover of firefighters in all positions: it takes multiple months to replace retirees in a fire department. Second, employing a difference-in-differences approach, we find that the shortage of lower-level operative leaders significantly slow down interventions, while the absence of higher-level managers has no effect in the short term. Third, we develop a model for public-sector allocation and estimate that the actual labor allocation has an average intervention time that is 16.2 percent longer than the efficient allocation. While misallocations across job positions, space and time are of similar magnitude, half of the efficiency gains can be obtained by shuffling workers across departments without affecting the total government expenditures in a month, department or job position.

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1 Introduction

The public sector is the largest employer in industrialized countries, accounting for an average of 18.6 percent of total workers [OECD, 2023]. Since most public organizations operate as monopolies in service provision and lack competitive incentives, this significant portion of the labor force may not be efficiently allocated [Dixit, 2002]. Due to its size, inefficiencies in the public sector can severely hinder economic performance by imposing an excessive tax burden and delivering suboptimal levels of public goods [Hall and Jones, 1999; Acemoglu et al., 2001].

Misallocation of public employees can arise along several dimensions. Over time, this occurs because public-sector jobs are generally permanent, while demand for public service can be volatile or seasonal. Additionally, public employees retire continuously while hiring happens at discrete and irregular intervals [Giorgiantonio et al., 2016]. Over job positions, inefficiencies can stem from the fact that career advancement in the public sector is often based on seniority rather than merit [Bertrand et al., 2020]. Across space, misallocation can result from mismatches between heterogeneous demand (e.g. local natural hazard events) and heterogeneous supply (e.g. variation in public-sector wage premia), exacerbated by mobility frictions [Guillouzouic et al., 2024; Schmutz and Sidibé, 2019].

Despite the economic relevance of public employment, there is limited knowledge about the extent, nature and consequences of its allocative inefficiencies. This paper provides empirical evidence of labor misallocation in the public sector and quantifies to what extent labor misallocation reduces public-sector performance. For this, we analyze the case study of the Italian Fire and Rescue Service. This is an ideal setting because fire departments constantly evaluate their own performance measuring their intervention time, a crucial determinant of property damages, injuries and casualties [Wrack, 2008; Corpo Nazionale dei Vigili del Fuoco, 2023a]. Moreover, the Fire Service is characterized by a well-defined job-level hierarchy, which allows to analyze misallocation across job positions and the impact

of different levels of management on public-sector performance.

The empirical approach leverages a unique dataset composed of the reports of all 7 million interventions undertaken by the Italian Fire and Rescue Service in 2014-2022. Reports are compiled by squad leaders and contain the exact time of the dispatch call, the departure from the fire station, the arrival on scene, the departure, and the return to the station. The report specifies the fire station of the squad, the type of intervention, the probable cause, the hectars burned, whether a civilian or a firefighter was injured or died. We combine intervention data with monthly matched employer-employee data on public-sector workers provided by the national social security institute. This contains demographic characteristics, monthly income, exit cause (e.g. age-related-retirement, death), type of contract, rank and fire department for each firefighter between 2014 and 2022.

First, we document the presence of substantial delays in the turnover of firefighters in all positions along the hierarchical structure: it takes more than one year to completely replace retirees in a fire department. Firefighters are forced to retire upon reaching a certain age, creating an exogenous variation that we leverage to estimate whether interventions slow down in the months following age-based retirements. Using a difference-in-differences design, the paper estimates how temporary personnel shortages affect public-sector performance, analyzing the effects at each rank of the fire department's hierarchy, from basic firefighters to executives.

The results indicate that delayed turnover among operative personnel, particularly of squad and shift leaders, significantly increases intervention times. A 10 percentage-point rise in the retirement rate of operative firefighters leads to a 6.7 percent increase in intervention time, driven by longer on-scene times while response or return times are not affected. In contrast, retirements of higher-level officers, such as directors and executives, have an impact on intervention duration that is not statistically different from zero. These results suggest that the leadership of low and middle managers is critical in determining public sector performance,

whereas the lack of higher-level management is less consequential in the short-term.

The model developed in this study simulates the optimal allocation of firefighters across different departments, job positions, and time periods. It is calibrated using the estimated elasticities of intervention time with respect to labor across different positions. The simulations reveal that the actual labor allocation has an average intervention time that is 16.2 percent longer than the efficient allocation.

The simulation results highlight that the largest source of inefficiency stems from the misallocation of labor across different job positions, followed by misallocation across fire departments and over time. Half of the efficiency gains can be obtained by reallocating workers across departments without changing government expenditures across months, departments or job positions.

This paper contributes to the growing literature on the impact of managerial skills and workforce allocation on public sector performance. Janke et al. [2019] do not find evidence that CEOs have any impact on the financial performance or quality of service of complex public-sector organizations (hospitals). On the other hand, Fenizia [2022], Tsai et al. [2015] and Bloom et al. [2015] find that the managerial skills of middle and frontline managers can have a positive effect on the performance of smaller public-sector units under their supervision. This could suggest that focusing on the skill development of frontline managers rather than seeking the best top managers might be optimal for policymakers. However, the heterogeneity in treatment effects estimated across levels of management may be due to the different identification strategies and settings analyzed in the different papers. Our study allows to effectively compare the impact of workers of different positions along the hierarchy of the same public-sector organization, from entry-level workers to top managers.

Closer to this paper, Bennedsen et al. [2024] studies whether some aspects of leadership (demographics, the incentives structures and organization of the entity the lead) correlate with employees' absenteeism in the public sector. While

Bennedsen et al. [2024] exploit the hospitalizations of CEOs to estimate their impact on employees' absenteeism in the private sector, we exploit forced retirements of public-sector managers at each hierarchical level to analyze the impact of leadership gaps on public-sector productivity. While they restrict their definition of leaders to the top managers in each public sector unit, we extend the definition of leaders to any public sector employee that supervise at least four other workers and analyze the impact that leaders at different levels in the chain of command have on workers' productivity and service quality.

In contrast to the private sector, where resource misallocation has been extensively studied [Restuccia and Rogerson, 2017; Hsieh and Klenow, 2009], this paper addresses a gap in the literature by quantifying the effects of labor misallocation in the public sector. Early studies by Ahlbrandt [1973] and Duncombe and Yinger [1993] analyze the presence of inefficiencies and returns to scale in the fire protection service, focusing on the parametric estimation of a public cost function.[1] Our framework is closest in spirit to the one used by Walter [2020] to measure misallocation of teachers in a cross-section of schools. We expand it by allowing for multiple time periods and multiple positions of workers, with heterogeneous real wages and elasticities of production that are empirically estimated. In this way we can quantify the extent of three different types of misallocation: misallocation of firefighters across fire departments, across job positions and over time.

Our findings have relevant policy implications for public sector management. Policymakers should consider adopting more flexible workforce allocation to make public sector more resilient to variations in labor supply and service demand. For instance, allowing for more decentralized hiring practices or planning future hiring based on the local age distribution of public workers. Introducing temporary staffing solutions during high-demand periods could mitigate the adverse effects of workforce shortages, ultimately improving the provision of public goods.

[1] Ahlbrandt [1973] finds that a private company in Scottsdale was able to provide fire protection services at a cost that was 47 percent lower than the estimated cost of the public provider.

The rest of the paper is organized as follows. Section 2 describes the Fire and Rescue Service, as well its hiring and retirement patterns. Section 3 presents the data and the summary statistics of the outcomes of interest. Section 4 lays out the empirical strategy and presents evidence of delayed turnover in fire departments. Section 5 shows the impact of delayed turnover on firefighters' performance. Section 6 outlines a model of labor allocation, quantifies the extent of inefficiency and analyzes the main sources of misallocation. Section 7 concludes.

2 Institutional background

2.1 The Fire and Rescue Service

As of 2022, the Italian Fire and Rescue Service is composed by 851 fire stations coordinated by one fire department in each province (Figure D1).[2] It covers the entire country except for the autonomous provinces of Aosta, Bolzano and Trento which have independent fire services [Corpo Nazionale dei Vigili del Fuoco, 2023c]. The Service has the duty to prevent and extinguish fires as well as to perform rescue and safety operations in a wide range of scenarios from domestic accidents to public calamities. There are 113 different types of intervention in the data provided by the Fire Service. Figure 1 reports the nine most frequent types of interventions (all other types are grouped in 'Other') and their monthly numbers from 2014 to 2022. The figure shows large swings in demand due to natural hazard events, such as normal and vegetation fires in summers (in red and green, respectively) or the earthquakes that caused structural collapses at the end of 2016 (in pink).

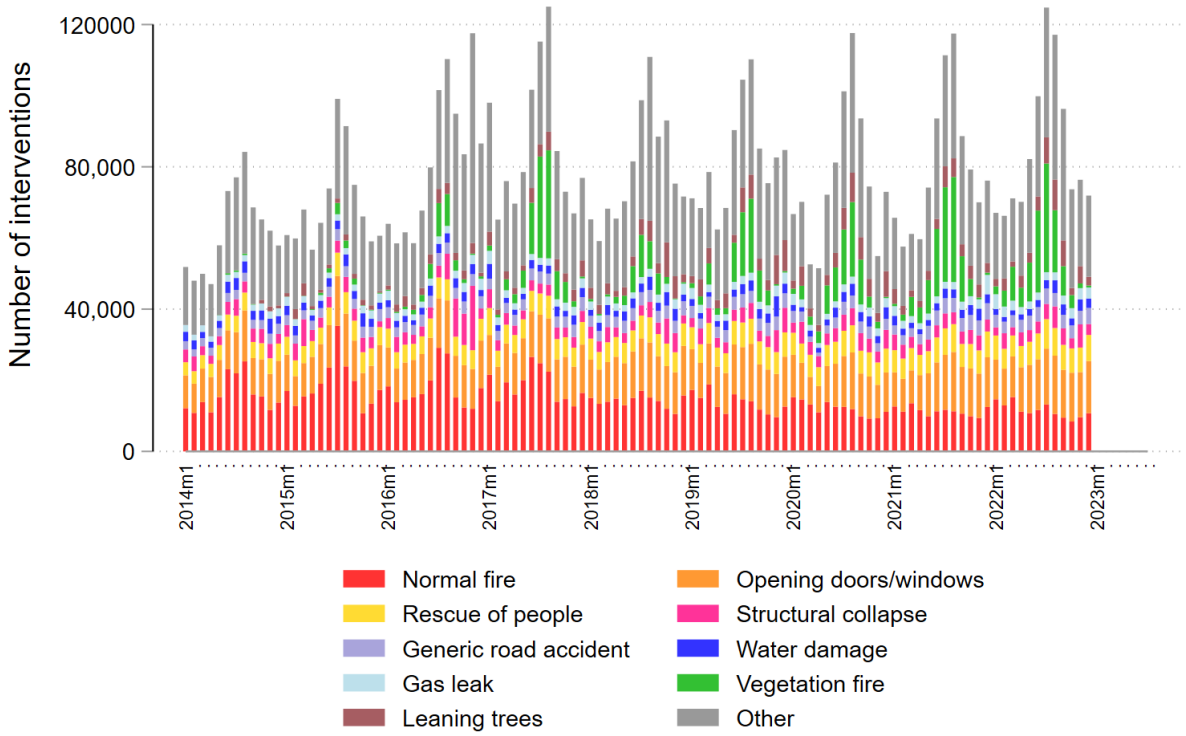
Firefighters are ranked in a hierarchical structure in the following ascending order: Basic Firefighters, Squad Leaders, Shift Leaders, Inspectors, Directors, Executives. The first three are 'operatives' that work on day-to-day operations on the ground, the last three are 'officers' that inspect, train new recruits, manage

[2]There are 159 additional fire stations run by volunteers, but we exclude them from the analysis.

resources and plan large-scale strategies. Firefighters are promoted to a higher position only if they reach certain seniority and educational levels and succeed in a competitive exam. In Section A, we provide a detailed explanation of the responsibilities of each position and the requirements to obtain a promotion.

During an intervention, four Basic Firefighters and a Squad Leader are grouped in teams of five (squads) and equipped with a firetruck. Generally, in a fire department there are eight firefighters assigned per squad (six basic firefighters and two squad leaders) so that the squad can function even when three basic firefighters or one squad leader are absent due to days of rest or sick leaves. Note that a squad cannot intervene if the squad leader is absent. Each provincial fire department has an operations center led by a shift leader that receives intervention requests by phone, assigns the intervention to one or more fire stations in the province and decides how many squads to send.

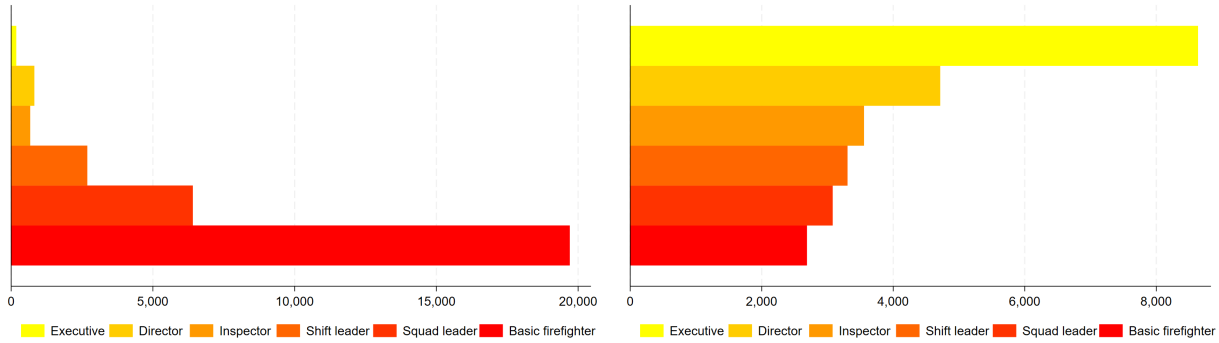
Figure 1: Monthly number of firefighters' interventions by type



Note: This histogram show the monthly number of firefighters' interventions by type of interventions from 2014 to 2022. Data provided by the Italian Fire and Rescue Service. The figure reports the nine most frequent types of interventions, all the other types are grouped in the category 'Other'.

The labor force size decreases and the gross income increases with ranking, as we can see in Panel (a) and (b) of Figure 2. Basic firefighters represent 65 percent of the labor force, squad leaders represent 21 percent of the labor force, shift leaders represent 9 percent and the officer personnel (inspectors, directors and executives) counts for the remaining 5 percent. On average, firefighters earn 2.7 thousand Euros, Squad Leaders 3.1, Shift Leaders 3.3, Inspectors 3.6, Directors 4.7, and Executives 8.6. The number of firefighters slightly increased over 2014-2022 from around 29,600 to 31,600 (Figure D2), with an increase in shift leaders and a reduction in squad leaders at the end of 2019. The real income of each position remained relatively stable over the years (Figure D3).

Figure 2: Average number of firefighters and income by hierarchical position



(a) Number of workers

(b) Monthly gross income

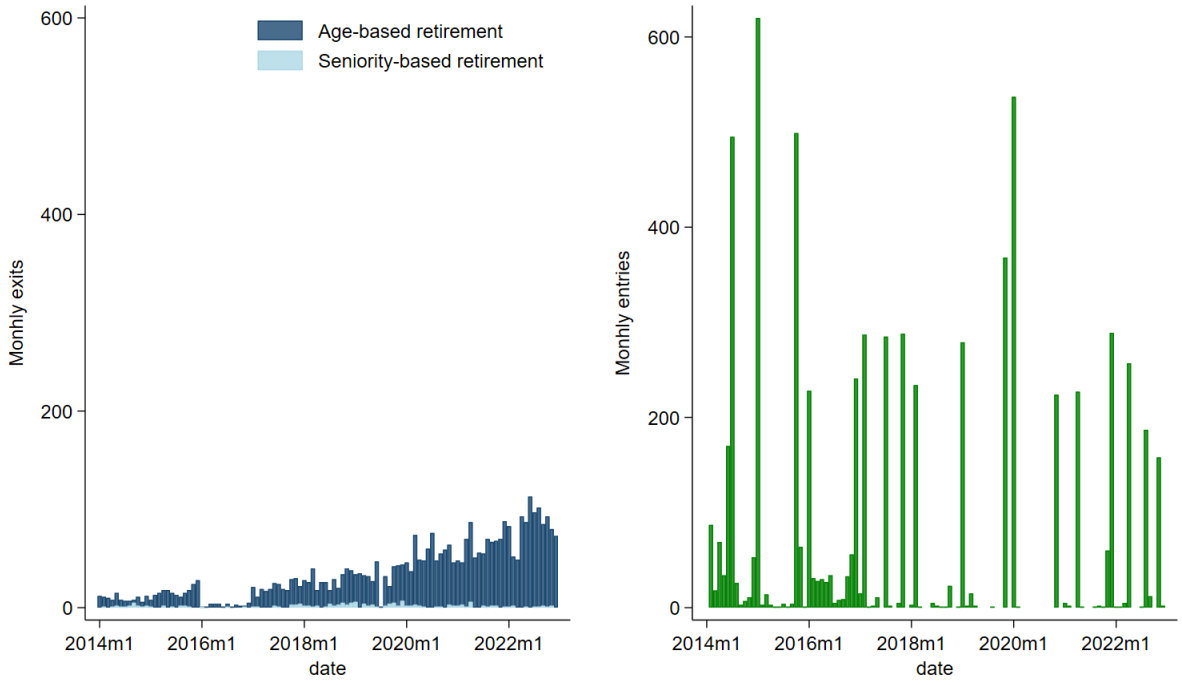
Note: These figures show the average number of firefighters and their average monthly gross income (in 2015 Euros) by hierarchical position in 2014-2022. Data provided by INPS.

2.2 Retirements and hires

There are two criteria that effectively constitute a lower and an upper bound for retirement. First, there is a minimum seniority at which firefighters are *eligible* to retire. As we can see in Table C2, firefighters could retire after 41 years and 6 months of service in 2014, a threshold that gradually increased by nine months in the analyzed period. Second, there is a maximum age at which firefighters are effectively *forced* to retire, independently of their seniority (Decree-law 165/1997). As we can see in Table C2, operatives had to retire at 61 years and 3 months of age in 2014, a threshold that gradually increased by nine months in the analyzed period. According to INPS data, only 0.03 percent of retirements occur after the maximum retirement age.

Since the maximum retirement age is relatively low and the pension amount is lower for firefighters that retire earlier, around 96 percent retire when they reach the maximum retirement age and only 4 percent retire due to seniority (Table C3 and Figure 3). Therefore, the timing of most retirements depend on an exogenous characteristic (age), rather than on an individual decision.

Figure 3: Monthly exits and entries in the Fire and Rescue Service



Note: The left panel shows the number of monthly exits from the Fire and Rescue service, divided by type. The right panel shows the number of monthly entries in the Fire and Rescue Service. A monthly entry is defined as the first time a person is observed working in the Fire and Rescue Service according to the data obtained from INPS (except for January 2014, the first month in the panel).

The age distribution of firefighters is heterogeneous across fire departments (Figure D4a). Accordingly, the average age-related retirement rate over 2014-2022 is also very heterogeneous across provinces, varying from 1 to 24.8 percent (Figure D4b).

While monthly retirements are continuous over the months, the number of new hires is lumpy and not synchronized with retirements (Figure 3).[3] As outlined in Section A, firefighters are recruited and promoted through national competitive exams held at irregular intervals. These exams can span over a year and are followed by several additional months of centralized training [Corpo Nazionale dei Vigili del Fuoco, 2024]. New hires are heterogeneous across provinces varying from 0 to 15.3 percent, but not geographically correlated with retirement rates (Figure D5).

[3]The number of retirees falls for some months in 2016 and 2019 when the retirement-age limit was increased.

3 Data

3.1 Fire and Rescue interventions

The Fire and Rescue Service provided data on all reports compiled by squad leaders for all interventions occurred in Italy from 2008 to 2022. There have been around 12.7 million interventions in this period. We restrict the analysis to the 7.3 million interventions occurred from 2014 to 2022 because personnel data from INPS is only available for this time span.

Each report contains the exact time and date in which: 1) the operation centre of the fire department called the fire station to dispatch the squad; 2) the squad left the fire station; 3) the squad arrived at the place of intervention; 4) the squad left the place of intervention; 5) the squad returned to the fire station. For the same intervention there can be multiple reports because each squad leader has to fill in a report, so we are able to track the number of squads involved in each intervention and the amount of time each squad is engaged in that intervention.

The report specifies both the fire station and the department of the squad, the nature of the intervention (e.g. wildfire, water damages, elevator rescue), the probable cause (e.g. arson, explosion, electrical malfunction), the number of hectars burned, whether a civilian was injured or died, whether a firefighter was injured or died. In our dataset each observation is identified by the year, the province, the file number (which is an integer number starting from 1 in each province at the beginning of a year) and the ordinal number of the squad involved in the intervention. Therefore a single observation is a squad-intervention pair, which we denote by i. Note that squads from different fire stations in the same province can be called for the same intervention.

3.2 Personnel

For data on personnel employed by the Italian Fire and Rescue Service, we use matched employeer-employee monthly data on public-sector workers collected

by INPS. This contains demographic characteristics, monthly income, pension contributions, exit cause (e.g. seniority-related retirement, age-related-retirement, death), type of contract (permanent, volunteer), level/position and fire department for each firefighter in each month between 2014 and 2022.[4] We restrict the sample to firefighters employed in the national Fire and Rescue Service, which excludes the autonomous provinces of Aosta, Bolzano and Trento. Excluding autonomous provinces, INPS categorizes firefighters into 100 provincial fire departments, according to the province subdivision system in place between 1995 and 2005.[5] We link personnel and interventions data at fire-department level.

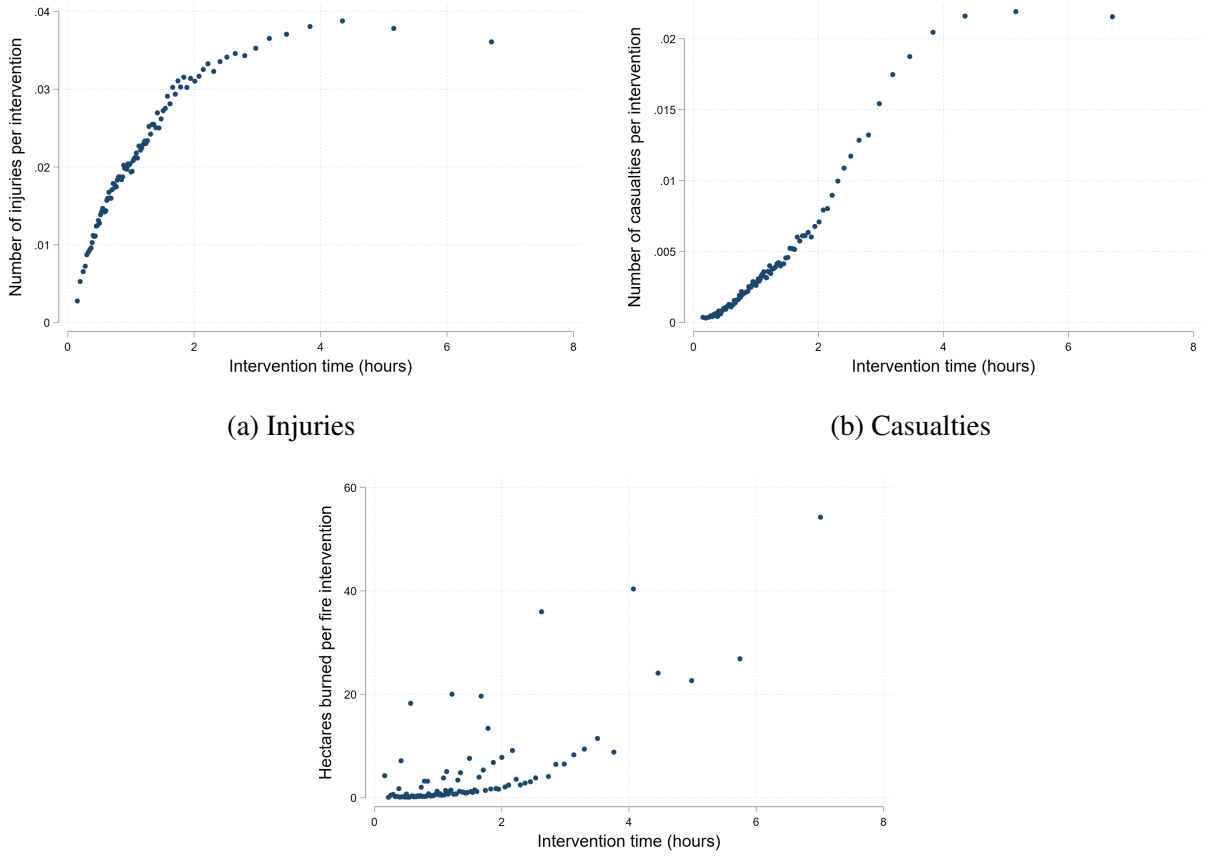
3.3 Summary statistics

To assess the effectiveness of firefighters' performance we mainly focus on the intervention time: the number of hours from the emergency call to the time the squad returns to the fire station. This is the sum of the dispatch time (from when the operation room calls the fire station to when the squad leaves the fire station), the response time (from when the squad exits the station to when it arrives at the intervention site), the on-scene time (from when the squad arrives at the intervention site to when it leaves), and the return time (from when the squad leaves the intervention site to when it arrives at the fire station). For 20 percent of the observations, the return time is missing. This usually occurs when a fire squad goes to one intervention site to another without stopping at the fire station. For the intervention time of these observations, we use the number of minutes between the emergency call and the squad departure from the intervention site, and we add an indicator for these observations as a control in the regressions.

[4]There are also *professionals* that have technical roles such as accountants, administrators and computer engineers. We exclude them from the analysis.

[5]Fire and Rescue interventions data categorizes firefighters into 103 provincial fire departments, with three additional provinces introduced in 2009 (Barletta-Andria-Trani, Fermo, Monza-Brianza) carved out from the provinces of Bari, Ascoli Piceno and Milan, respectively. For consistency with INPS data, we keep the old classification with 100 provinces.

Figure 4: Injuries, casualties and hectares burned against intervention time



(a) Injuries

(b) Casualties

(c) Hectares burned

Note: These figures show the average number of injuries per intervention (Panel a), casualties per intervention (Panel b) and hectares burned per intervention against fires (Panel c) in 100 equal-sized bins. Each bin corresponds to a percentile of the distribution of intervention times in hours.

Intervention times are commonly used by Fire Services of most countries to evaluate the performance of their fire departments [Wrack, 2008; Corpo Nazionale dei Vigili del Fuoco, 2023b]. Higher intervention times are associated with higher casualties, injuries, property loss and environmental damages [Wrack, 2008]. The probability of a flashover, the instant all flammable material in a room ignites, crucially depends on time from ignition [Wrack, 2008]. Despite being correlational evidence, Panels (a) and (b) of Figure 4 confirm in our data that injuries and casualties steeply rises with intervention time, at least in the first two hours after the emergency call. Panel (c) shows that the number of hectares burned grows with in-

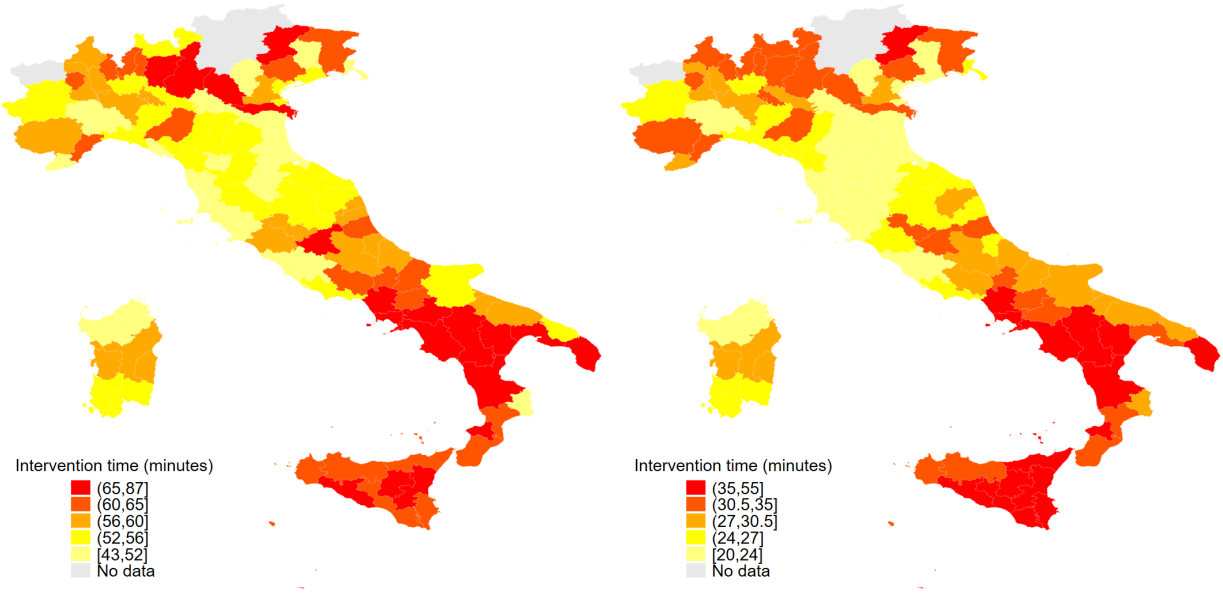
tervention time at an increasing rate, in line with fire growth models and previous findings [Ramachandran, 1986].

Table 1 shows the average, standard deviation, minimum, maximum and non-missing observations for each of the main variables used in the analysis. Interventions last around one hour and a half, on average. It takes only one minute for a fire squad to leave the station after receiving a dispatch call. Average response times are shorter (16 minutes) than return times (21 minutes) and most of the intervention is spent on-scene (53 minutes). The distribution of intervention times is right-skewed with few interventions lasting more than two hours (Figure D6).

The type of the intervention affects how time-sensitive the intervention is. To account for this, we split the sample in urgent and non-urgent interventions. We define urgent interventions as those involving fires, explosions, gas leaks, transport accidents, landslides, earthquakes, floods, people searches and rescues. Non-urgent interventions include recovery of goods, removal of debris, leaning trees, pest control, animal rescues, door/window openings, elevator rescues, and safety checks. Given their higher complexity, urgent interventions take 25 minutes longer than non-urgent interventions, on average.

As we can see in Panel (a) of Figure 5, the median intervention time is higher in the South of Italy where most interventions take at least one hour. Panel (b) of Figure 5 shows a similar pattern for on-scene time, which excludes the quality of roads as a potential explanation. The spatial heterogeneity in intervention times does not seem to depend on the urgency of the interventions (Figure D7), the type of interventions (Figure D8), or the density of fire stations (Figure D1). We view this as additional evidence of spatial disparities in performance across Italian public sector services, with the South experiencing longer disposition times in civil courts [Cugno et al., 2022], lower punctuality in local public transport [Mocetti and Roma, 2021], greater delays in public administration payments to firms [Cristina Da Rold, 2019], and slower implementation of public works [Baltrunaite et al., 2021].

Figure 5: Median intervention time and on-scene time by province



(a) Intervention time

(b) On-scene time

Note: Panels (a) shows the median intervention time (in minutes) by province for all interventions. Panels (b) shows the median on-scene time (in minutes) by province for all interventions.

Table 1 shows that civilian injuries and casualties are relatively common, occurring in 2.1 and 0.5 percent of the interventions, respectively. Firefighters' injuries are more rare (0.04 percent) and there are only 6 observations in the entire sample with a firefighter's casualty. On average, 0.1 hectares burn in an intervention with most interventions having zero hectares burned. This is expected considering that only 30 percent of interventions involve fires.

Fire departments do not usually work at full capacity. 33.6 percent of the working firefighters are actively employed in interventions on average, but this hides considerable heterogeneity across hours of the day, seasons and space. Large swings in demand, such as summer wildfires or the earthquakes of central Italy in 2016-17 (in green and pink in Figure 1) can increase capacity utilization over 100 percent, suggesting the presence of aid from squads of other provincial fire departments. More than two thirds of the interventions involve a single squad but few interventions necessitate many, leading to an average of 2 squads per inter-

vention.

Table 1: Summary statistics

	Obs.	Mean	S.D.	Min.	Max.
		A. Fire and Rescue intervention data			
Intervention time	7256058	1.448	1.630	0.117	11.250
Dispatch time	6120729	0.017	0.000	0.017	0.017
Response time	5690139	0.273	0.247	0.017	1.500
On-scene time	6723120	0.875	1.252	0.017	8.567
Return time	5508327	0.347	0.341	0.017	2.067
Urgent intervention time	4091677	1.627	1.764	0.117	11.250
Non-urgent intervention time	3164381	1.217	1.406	0.117	11.250
Civilian injury (%)	7387335	2.109	14.369	0.000	100.000
Civilian casualty (%)	7387335	0.480	6.913	0.000	100.000
Firefighter injury (%)	7387335	0.035	1.873	0.000	100.000
Firefighter casualty (%)	7387335	0.000	0.090	0.000	100.000
Hectares burned	7387335	0.148	0.931	0.000	8.000
Squads per intervention	7387335	1.993	8.791	1.000	480.000
Capacity utilization (%)	7387335	33.643	59.792	0.000	1382.820
		B. Personnel data			
Number of basic firefighters	10800	197	224	44	3013
Number of squad leaders	10800	64	63	5	728
Number of shift leaders	10800	27	32	0	402
Number of all operatives	10800	288	304	66	3550
Number of inspectors	10800	7	18	0	469
Number of directors	10800	8	17	0	211
Number of executives	10800	2	5	0	66
Number of all officers	10800	17	37	1	732
Number of all	10800	305	335	72	3777
Retirement rate (%) basic firefighters	10800	0.008	0.076	0.000	1.887
Retirement rate (%) squad leaders	10800	0.061	0.348	0.000	5.255
Retirement rate (%) shift leaders	10800	0.932	2.410	0.000	28.235
Retirement rate (%) all operatives	10800	0.109	0.255	0.000	2.705
Retirement rate (%) inspectors	10800	0.412	3.369	0.000	91.525
Retirement rate (%) directors	10800	0.086	1.243	0.000	45.957
Retirement rate (%) executives	10800	0.250	4.344	0.000	121.348
Retirement rate (%) all officers	10800	0.236	1.505	0.000	27.481
Retirement rate (%) all	10800	0.115	0.256	0.000	2.689

Notes: This table reports the number of observations, mean, standard deviations, minimum and maximum of the outcome variables in 2014-2022. Panel A shows data on the interventions of the Fire and Rescue Service at squad-intervention level. Panel B show personnel data on the labor force and the retirement rates by position at province-month level. Times are in hours. Squads per intervention are the number of squads involved in an intervention. Capacity utilization is the percentage of total operative firefighters at work involved in interventions in a province-hour. Retirement rates are monthly retirement rates (in percent) obtained by dividing the number of monthly retirements by the average number of firefighters of the same position in the fire department during the period 2014-2022. Injuries and casualties are the percentage of interventions that involved an injury or a casualty. Times, hectares burned and capacity utilization are windsorized at the 1^{st} and 99^{th} percentiles.

Regarding the amount of personnel (Panel B of Table 1), a provincial fire department has on average 197 basic firefighters, 64 squad leaders, 27 shift leaders, 7 inspectors, 8 directors and 2 executives. The number of basic firefighters is around three times the number of squad leaders, which correspond to the proportion of six

basic firefighters to two squad leaders assigned to each squad.

The number of retirements are heterogeneous across positions (Table C4). Panel B of Table 1 shows that shift leaders have the largest monthly age-dependent retirement rate on average (0.93 percent), followed by inspectors (0.41), executives (0.25), directors (0.09), squad leaders (0.06), and basic firefighters (0.01). We can attribute the elevated retirement rate among Shift Leaders to being the highest position attainable solely through seniority and an internal exam, without requiring a degree or passing a public competitive exam (Table C1).

4 Empirical strategy

We start by documenting the presence of delayed turnover in all positions of the Fire Service. Then, we show how we can exploit delayed turnover to estimate the impact of personnel shortage on public-sector performance and the labor elasticities of performance.

4.1 Delayed turnover

We focus on age-dependent retirements, which occur when firefighters reach an age at which they are forced to retire. This reduces endogeneity concerns due to firefighters postponing or anticipating their retirement depending on the performance of the department or the lack of potential substitutes.

Let L_{pdt} denote the number of permanent firefighters in position p in fire department d and month t, R_{pdt} the number of age-dependent retirements and E_{pdt} the number of entries (hires, promotions, transfers) in the same position, department and month. First, we are interested in estimating how quickly fire departments can recover the number of permanent firefighters in position p when some of their firefighters in position p retire.

We normalize all variables by the average number of permanent firefighters of position p working for fire department d across all months of available data ($\overline{L_{pd}}$)

and denote them with lower capital letters $l_{pdt} = L_{pdt}/\overline{L_{pd}}$, $r_{pdt} = R_{pdt}/\overline{L_{pd}}$ and $e_{pdt} = E_{pdt}/\overline{L_{pd}}$. This normalization accounts for fire department size and allows for departments with zero firefighters, hires or retirees of position p in period t. Define the growth rate of p personnel as $g_{pdt} = l_{pdt} - l_{pdt-1}$.

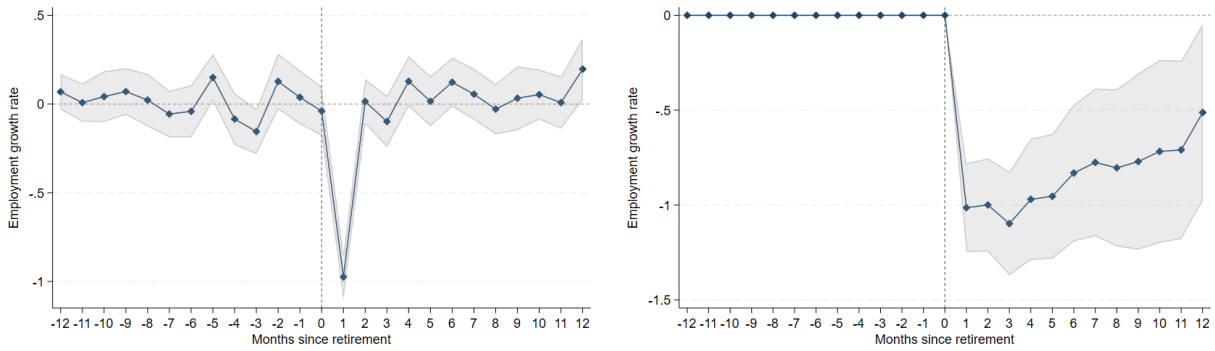
We estimate the following:

$$g_{pdt} = \sum_{h=\underline{h}}^{\overline{h}} \beta_h r_{pdt-h} + \eta_d + \gamma_t + \varepsilon_{dt} \tag{1}$$

This specification includes month (γ_t) and fire-department (η_d) fixed effects and allows for lags to show the dynamic impacts of retirements on the labor force and leads to detect pre-trends. The baseline specification includes twelve lags and twelve leads ($\overline{h} = \underline{h} = 12$) and we cluster the standard errors at fire-department level.

If turnover is perfectly timed, no coefficient β_h for any $h \in [\underline{h}, \overline{h}]$ would be significantly different from 0. The number of new retirees would be immediately compensated by the number of new hires in the same position. If there is some anticipation effect, we would expect a significantly positive coefficient $\beta_h > 0$ for some $h \leq 0$. This could happen if fire departments plan in advance to prepare the new recruits for some months before letting the old firefighters retire. If instead the turnover is lagged, we would expect the estimate of the coefficient β_1 to be close to -1 and the sum of the coefficients $\sum_{h=1}^{\tilde{h}} \beta_h$ to approach 0 after some time $\tilde{h} > 0$. The higher $\tilde{h}$, the longer the turnover lag. Similarly to Cellini et al. [2010], this reasoning holds under the assumption that the effects of a retirement depend only on the time elapsed since retirement and not on the period in which retirement occurred or on the history of retirements (see Appendix Section B for a proof).

Figure 6: Effect of age-related retirements on employment growth



(a) Temporary (12 months)

(b) Cumulative (12 months)

Note: We restrict the sample to firefighters that have a permanent contract. Data provided by INPS.

Figure 6 shows the estimates of β_h for $h \in [-12, 12]$ in Equation (1) and confirms the presence of lagged turnover in fire departments. The estimate for β_1 is negative and not statistically different from -1. The estimates for β_h with $h < 0$ oscillates around 0 with no positive trend. The sum of the coefficients $\sum_{h=0}^{\tilde{h}} \beta_h$ is significantly negative up to $\tilde{h} = 12$ months after the retirement shock. Only half of age-related retirements are replaced within a year. This already suggests the presence of misallocation over time: if the allocation of workers were optimal before the retirement, the retiree should be immediately replaced by another firefighter.

Figures D10 and D11 show the estimates of β_h for $h \in [-12, 12]$ in Equation (1), separately for each position. Turnover appears to be lagged in all positions by several months, even though the lag appears to be shorter for basic firefighters than for the other higher positions. Retirements of most positions are not fully replaced within one year.

One way fire departments could compensate for the loss of a permanent firefighter is to increase the employment of volunteers. This should not occur as volunteers operate in separate fire stations and do not replace retired firefighters unless they are permanently hired through national competitive exams. Let $v_{dt} = \frac{V_{dt}}{\overline{L}}$ denote the full-time-equivalent volunteering rate, where V_{dt} is the number of days of a volunteer's work in the department-month divided by the average

monthly working days of a permanent firefighter.[6] Figure D9 shows the estimates of β_h using v_{dt} as the dependent variable. The employment of volunteers does not increase after the retirement of a permanent firefighters.

4.2 Identification of the impact of delayed turnover

After having shown that there is lagged turnover in every position after a firefighter retires, we can estimate the impact of the temporary drop in the labor force on the outcome variables, using the following 'reduced-form' regression:

$$log(h_{isdt}) = \sum_{p \in \mathcal{P}} \alpha_p r_{pdt-1} + \zeta_s + \xi_t + \lambda_{j(i)} + \theta D_{isdt} + u_{isdt} \tag{2}$$

For intervention-squad i from fire station s in month t, we regress the logarithm of intervention time $log(h_{isdt})$ on the lagged retirement rates of position-p firefighters in the fire department d to which the fire station belongs. We include fire-station (ζ_s) and month (ξ_t) fixed effects to control for time-invariant heterogeneity and common shocks. For interventions we can identify not only the fire department, but also the fire station. Therefore, we include the more granular fire-station fixed effects, which make fire-department fixed effects redundant. We also add type-of-intervention fixed effects (λ_j) to account for the higher complexity of certain interventions (e.g. wildfires) relative to others (e.g. elevator rescues). Finally, we include an indicator D_{isdt} for squad-interventions with missing return time, whose intervention time is up to on-scene time. In all the regressions, we cluster standard errors at fire-station level.

The parameters of interest are α_p for each firefighter position p. α_p can be interpreted as the percentage change in the expected intervention time for a one percentage-point increase in the retirement rate of position-p firefighters.[7] The

[6] We assume the working day of a volunteer coincides with the 12-hour shift of a permanent firefighter. Permanent firefighters work 2 shifts every 4 days, for a total of 15 shifts per month [Comando Vigili del Fuoco Milano, 2023]. To obtain V_{dt}, we divide the observed working days of all volunteers in the department-month by 15.

[7] For small α_p, a one percentage-point increase in the retirement rate multiplies the expected intervention time by $e^{\alpha_p/100} \approx 1 + \alpha_p/100$.

identifying assumptions are that in absence of retirements, the average intervention time in different fire departments would have evolved in parallel, and that treatment effects are homogeneous over months and across departments [de Chaisemartin and D'Haultfœuille, 2020]. As a robustness check, we relax the second assumption using the estimator proposed by de Chaisemartin et al. [2024], which allows for heterogeneous treatment effects and can be used when the treatment is continuous and not absorbing, as in this case.

We can also estimate the elasticities of intervention time with respect to position-p labor force using the following two-stage instrumental variable regression:

$$log(L_{pdt}) = \sum_{\tilde{p} \in \mathcal{P}} \pi_{\tilde{p}} r_{\tilde{p}dt-1} + \mu_s + \phi_t + \delta_{j(i)} + \upsilon D_{isdt} + \iota_{dt}, \quad \forall\, p \in \mathcal{P} \tag{3}$$

$$log(h_{isdt}) = -\sum_{p} \psi_p log(\hat{L}_{pdt}) + \sigma_s + \xi_t + \chi_{j(i)} + \rho D_{isdt} + u_{isdt} \tag{4}$$

We regress the logarithm of intervention time on the logarithm of the labor force of each position, instrumented by their retirement rates in the previous period. The parameter of interest is $-\psi_p$, the labor elasticity of intervention time which we use in the model to measure misallocation. We will plug these estimated elasticities in the model to obtain the optimal labor allocations. The additional identifying assumptions required for the instrumented difference-in-differences are: 1) the labor force in different fire departments would have evolved in parallel in absence of retirements; 2) the effect of retirements on the labor force is always negative (monotonicity); 3) retirements affect intervention time only through the labor force reduction and do not affect past intervention times or past labor force [Hudson et al., 2017].

We report the impact of the lagged retirement rate on the logarithm of the workforce by position in Table C5. The lagged retirement rate has a significantly negative effect on the logarithm of operatives and officers. Lagged retirements are

a sufficiently strong instrument for operatives and officers according to standard criteria [Stock and Yogo, 2002], with an F-statistic of 26.5 and 22.7 respectively. When we disaggregate within operatives and within officers, the instrument remains relatively strong for most positions, except for basic firefighters and squad leaders. For this reason, we replace the logarithm of the workforce with the growth rate of personnel in Equation (3) when we estimate the elasticities by single position. As we can see in Table (Table C6), lagged retirements of firefighters of each position have a negative effect close to -1 on the growth rate of personnel working in that position in the same fire department, with F-statistics ranging from 17 to 1142.[8]

5 Results

5.1 Explained variation from fixed effect regressions

To analyze the variation in productivity that is explained by leaders of different levels in the chain of command, we restrict our sample to the subset of fire departments for which at least one leader can be observed in at least one other fire department. We also impose that leaders have to be in each fire department for at least three years to allow enough time for the leader to have an impact on the productivity of the fire department [Bertrand and Schoar, 2003].

For each outcome of interest y_{dt} estimate the following regression:

$$log(h_{isdt}) = \nu_s + \tau_t + \omega_{j(i)} + \rho D_{isdt} + \epsilon_{isdt} \tag{5}$$

This analysis is based on the assumption that leaders' moves are as-good-as random, conditional on fire department and month fixed effects. This assumption would be violated if leaders are sent to fire departments on the basis of their comparative advantage or in response to fire-department idiosyncratic trends in

[8] On the other hand, Table C7 shows that the number of FTE volunteers employed does not significantly increase when there is a rise in the retirement rate of any position of permanent firefighters.

productivity and transitory shocks [Fenizia, 2022]. To circumvent these issues, we use leadership gaps in the aftermath of forced age-dependent retirements to analyze the causal impact of leaders' absence on firefighters' productivity.

To obtain a promotion from first-level executive (Primi dirigenti) to superior executive (Dirigenti superiori), the manager must have worked for at least one year in three different departments and at least once as a first-level executive [Gazzetta Ufficiale, 2017].

To have a sense of the magnitude of the estimates, Bertrand and Schoar [2003] find that adding the fixed effects to models of corporate practices that already account for observable and unobservable firm characteristics results in increases in adjusted R2's of more than four percentage points.

Following the approach of Bertrand and Schoar [2003], we limit our sample to departments where at least one top executive can be observed in at least one other department. Additionally, we require that executives remain in each department for a minimum of three years. For departments meeting these criteria, we retain all observations, including years where the department has executives not observed in multiple departments. The final sample consists of XXX departments and XXX individual executives who can be tracked across at least two different departments. On average, executives stay in a given department for XXX years in our dataset. The three-year requirement ensures that executives have sufficient time to leave a measurable impact on their department. While relaxing this requirement yielded qualitatively similar results, they were, as expected, statistically weaker. We provide summary statistics for the 'Executive-department matched sample' in Table XXX. This refers to the set of department-month observations for departments that have at least one executive observed in multiple departments with at least a three-year stay at each department.]

I construct the sample as follows. For each department and month I select the highest-ranking executive. If multiple executives share the same rank, I select the one with the longest tenure, the oldest, the one with the highest salary (in this order

of priority).

5.2 The Impact of Delayed Turnover on Public Service Performance

Column 1 of Table 2 shows that delayed turnover among operative firefighters leads to slower intervention times. The geometric mean of intervention time is 61 minutes ($e^{0.016} \cdot 60$). A 10 percentage-point rise in the retirement rate of operative firefighters prolongs interventions by 6.7 percent (4.1 minutes) in the following month. This effect is entirely driven by a 12.9 percent increase of on-scene time, prolonging it by 3.6 minutes from a geometric mean of 28 minutes. Retirement rates of operatives do not affect response or return times (columns 2-4 of Table 2). In terms of labor elasticities, increasing the workforce of operatives by 10 percent reduces intervention time by 5.0 percent and on-scene time by 10.3 percent. Conversely, the absence of officers does not have a significant impact on intervention or on-scene times and only a small, marginally significant effect on return times.

Table 2: Effect of age-based retirement rates of operatives and officers on intervention time.

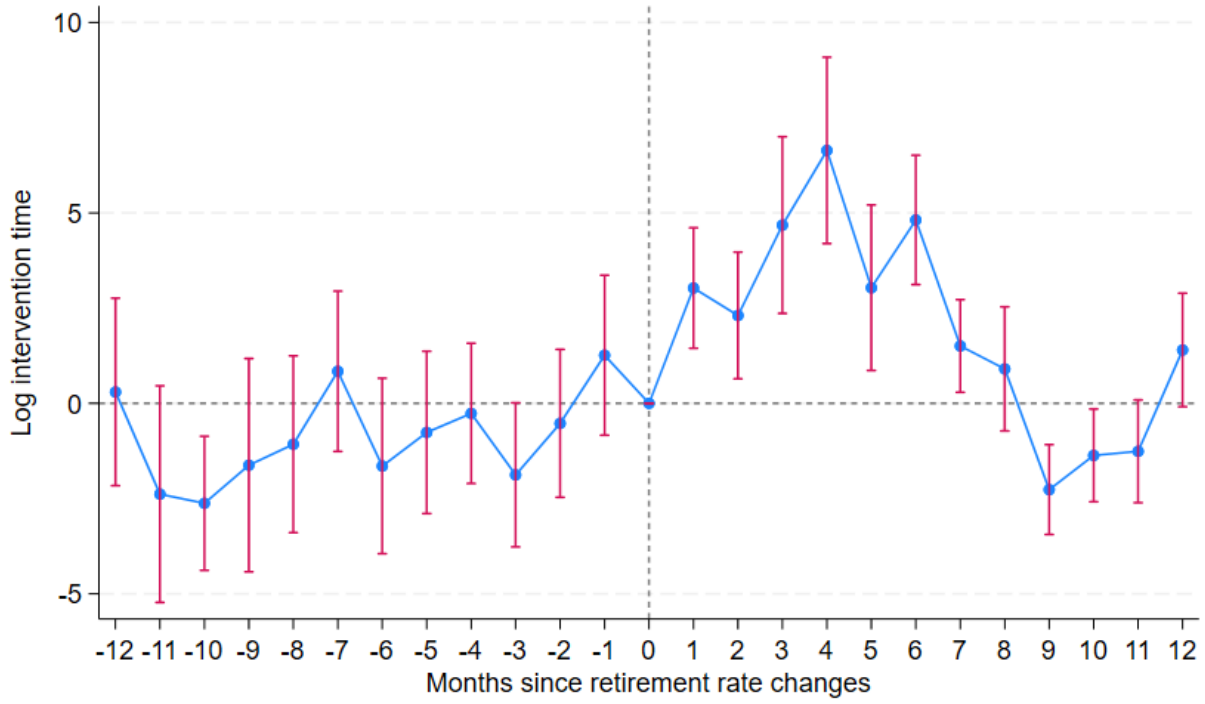
	Reduced form			
	Log intervention time	Log response time	Log on-scene time	Log return time
Operative retirement rate	0.667**	-0.002	1.292***	-0.032
	(0.325)	(0.357)	(0.386)	(0.329)
Officer retirement rate	0.004	-0.024	-0.035	0.083*
	(0.042)	(0.044)	(0.055)	(0.046)
	Two-stage least squares			
	Log intervention time	Log response time	Log on-scene time	Log return time
Log(operatives)	-0.498*	-0.035	-1.029***	0.148
	(0.264)	(0.274)	(0.341)	(0.253)
Log(officers)	0.040	0.044	0.157	-0.156*
	(0.088)	(0.086)	(0.123)	(0.091)
Observations	7211063	5649755	6679058	5469145
Mean	0.016	-1.628	-0.722	-1.431

Notes: The panel 'Reduced form' reports the estimates for the effect of the lagged monthly retirement rate of operative and officer firefighters on the outcome variables indicated by the column headings (Equation 2). The panel 'Two-stage least squares' reports the estimates for the effect of the logarithm of operative and officer firefighters, instrumented by the lagged retirement rates, on the outcome variables indicated by the column headings (Equation 4). We exclude false alarms, interventions that were no longer necessary, and interventions by volunteers. All regressions include an indicator for missing return time, as well as fire station, type of intervention and month fixed effects. Standard errors are clustered at fire-station level. *Mean* is the average of the dependent variable in the regression sample.

In Figure 7 we show the dynamic impact of the retirement rate of operatives

on the logarithm of intervention time, using the estimator proposed by de Chaisemartin et al. [2024] that is robust to heterogeneous treatment effects. The estimator compares the evolution of intervention times of departments whose retirement rate changes in a certain period (switchers) to departments whose retirement rate remain the same as switchers' retirement rate in the initial period (stayers). The estimated treatment effects are the weighted average of switchers' slopes of log intervention time with respect to the retirement rate, where switchers receive a weight proportional to the absolute value of their retirement rate change. It allows to estimate dynamic effects and placebo effects after and before retirement rate changes. Figure 7 shows that departments experiencing an increase in operatives' monthly retirement rate report longer intervention times in the following seven months, whereas there is no clear trend in the twelve months before. On the other hand, we do not see a clear increase in intervention time after officers retire, even though the estimates are noisy before and after the change in the retirement rate (Figure D12).

Figure 7: Dynamic effects of retirements of operative firefighters on intervention time



Note: This figure shows the placebo and dynamic effects of retirements of operative firefighters on the logarithm of intervention time in the year prior to and after an increase in the retirement rate. The effects are estimated using the estimator by de Chaisemartin et al. [2024], which is robust to heterogeneous treatment effects. The red lines indicate 95 percent confidence intervals.

Table 3 disaggregates the effects by position. The effect is driven by the retirements of shift leaders and squad leaders, whereas the estimates for basic firefighters noisy and not statistically different from 0. A 10 percentage-point rise in squad and shift leaders' retirement rate prolongs on-scene times by 4.5 and 1.3 percent respectively. Note that a 1 percentage-point increase in the retirement rate of basic firefighters implies a much larger drop of the total labor force than a 1 percentage-point increase in the retirement of shift leaders. In terms of labor elasticities, a 10 percentage-point rise in basic firefighters reduces intervention times by 9.1 percent, a 10 percentage-point increase in squad leaders reduces intervention times by 3.5 percent, a 10 percentage-point rise in shift leaders' labor force reduces intervention times by 0.6 percent. The effects of retirements of all officer positions (inspectors, directors and executives) are very small and not statistically

different from 0.

We can rationalize these findings by considering that operative firefighters are directly involved in day-to-day operations on the ground, whereas officers work on long-term strategies and their absence may be less consequential in the short-term. These results reveal that the absence of low and middle managers can immediately reduce the effectiveness of public service, whereas the temporary leadership gap of high-level managers does not appear to have negative repercussions in the short-term.

Table 3: Effect of age-based retirement rate by position on intervention time.

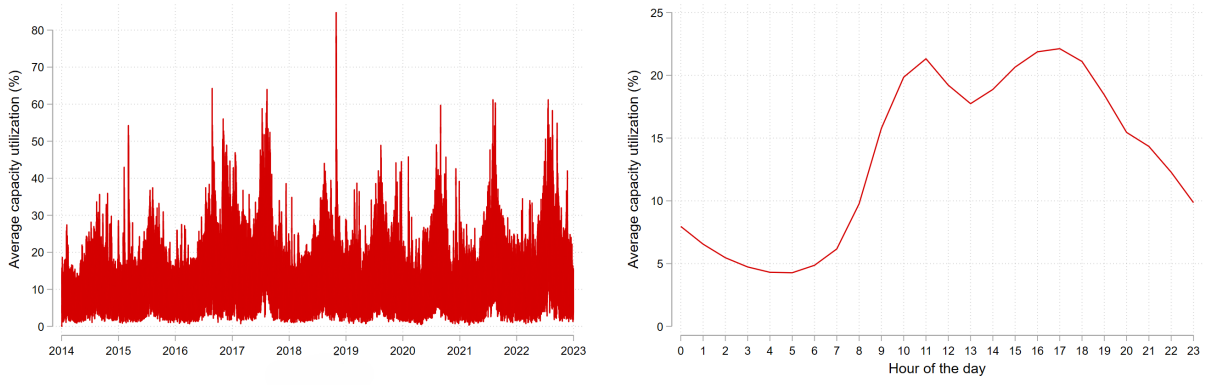
	Reduced form			
	Log intervention time	Log response time	Log on-scene time	Log return time
Basic firefighters retirement rate	0.445	0.913	0.626	-1.193
	(1.006)	(0.956)	(1.286)	(0.854)
Squad leaders retirement rate	0.413*	0.391*	0.454*	0.100
	(0.211)	(0.232)	(0.249)	(0.197)
Shift leaders retirement rate	0.063**	-0.017	0.128***	0.029
	(0.032)	(0.034)	(0.038)	(0.030)
Inspectors retirement rate	0.001	-0.007	-0.005	0.023
	(0.020)	(0.020)	(0.025)	(0.020)
Directors retirement rate	0.005	-0.005	-0.035	0.075
	(0.048)	(0.049)	(0.065)	(0.051)
Executives retirement rate	0.007	0.002	-0.004	0.016
	(0.012)	(0.015)	(0.014)	(0.016)
	Two-stage least squares			
	Log intervention time	Log response time	Log on-scene time	Log return time
Basic firefighters growth rate	-0.908	-1.461	-1.173	1.377
	(1.386)	(1.378)	(1.808)	(1.202)
Squad leaders growth rate	-0.350*	-0.305	-0.401*	-0.156
	(0.202)	(0.234)	(0.242)	(0.198)
Shift leaders growth rate	-0.060*	0.011	-0.121***	-0.016
	(0.033)	(0.037)	(0.039)	(0.032)
Inspectors growth rate	0.005	0.017	0.012	-0.029
	(0.024)	(0.024)	(0.030)	(0.023)
Directors growth rate	-0.003	0.017	0.041	-0.112*
	(0.058)	(0.059)	(0.074)	(0.060)
Executives growth rate	-0.008	-0.004	0.002	-0.015
	(0.012)	(0.014)	(0.013)	(0.015)
Observations	7211063	5649755	6679058	5469145
Mean	0.016	-1.628	-0.722	-1.431

Notes: This table shows the estimate for the effect of the lagged monthly retirement rate and growth rate of firefighters of the position indicated in each row on the outcome variables indicated in each column. We exclude from the sample interventions made by squads of volunteers and false alarms, and interventions that were no longer necessary. All regressions include fire station and day fixed effects. Standard errors are clustered at fire-station level. *Mean* is the average of the dependent variable in the regression sample.

5.3 Heterogeneity analysis by capacity utilization and urgency

The response of fire departments to personnel shortages may depend on capacity utilization and the urgency of the interventions. When most teams in fire stations are involved in interventions, personnel shortages are likely to slow interventions down due to lower availability of potential substitutes. We define capacity utilization of a fire department $C_{d\kappa}$ as the fraction of operatives in fire department d involved in interventions during a certain hour κ. According to their contract, operative firefighters are required to work for 133.5 twelve-hour shifts per year with a thirty-minute lunch break per shift [UIL, 2022]. Therefore, they work an average of $\overline{h} = 133.5 \cdot (12 - 0.5)/(365.25 \cdot 24) = 17.5$ percent of their time. During a certain hour κ of month t, there are $N_{dt} \cdot \overline{h}$ operatives working in fire department d on average. Considering that each squad-intervention i from fire station s during hour κ involves 5 firefighters and lasts $h_{isd\kappa}$, capacity utilization is $C_{d\kappa} \equiv \frac{\sum_s \sum_i 5 \cdot h_{isd\kappa}}{N_{dt} \cdot \overline{h}}$. Average capacity utilization is very heterogeneous depending on the period of the year (Figures 8a) and the hour of the day (8b), with peaks above 60 percent during summer days and rush hours.

Figure 8: Capacity utilization



(a) By date-hour

(b) By hour of the day

Note: Panel (a) shows the national average of capacity utilization in each hour of the analyzed period (2014-22). Panel (b) shows the national average of capacity utilization in each hour of the day. Capacity utilization is defined as the number of working operative firefighters that are employed in interventions during a given hour.

Table 4 shows that the worsening of performance is entirely due to shortages of operative personnel when capacity utilization is above the median. A 10 percentage-point increase in retirement rates of operatives raises intervention and on-scene times by 9 and 19 percent respectively when departments are working at high capacity. Again, the impact is driven by the retirements of squad leaders and shift leaders (Table C8). We do not find any significant impact of retirements of operatives or officers on times of intervention when capacity is utilized less than the median level. On the contrary, we do not find substantial differences when we separate urgent and non-urgent interventions (Tables C9-C10).

Table 4: Effect of age-based retirement rate by position on intervention time, by operative and directive personnel. Heterogeneity by capacity utilization.

	Log intervention time	Log response time	Log on-scene time	Log return time
	Below median capacity utilization			
		Reduced form		
Operative retirement rate	0.051	0.041	0.220	-0.174
	(0.294)	(0.372)	(0.387)	(0.362)
Directive retirement rate	-0.015	-0.054	-0.085	0.045
	(0.041)	(0.047)	(0.053)	(0.053)
		Two-stage least squares		
Log(operatives)	-0.084	-0.141	-0.406	0.246
	(0.310)	(0.356)	(0.430)	(0.351)
Log(officers)	0.031	0.100	0.172	-0.093
	(0.081)	(0.089)	(0.112)	(0.104)
Observations	3610906	3065906	3364499	2895156
Mean	-0.086	-1.702	-0.852	-1.527
	Above median capacity utilization			
		Reduced form		
Operative retirement rate	0.894**	-0.210	1.871***	-0.049
	(0.416)	(0.426)	(0.506)	(0.400)
Directive retirement rate	0.028	0.014	0.013	0.122**
	(0.056)	(0.056)	(0.075)	(0.055)
		Two-stage least squares		
Log(operatives)	-0.540*	0.153	-1.185***	0.184
	(0.287)	(0.285)	(0.372)	(0.276)
Log(officers)	0.007	-0.040	0.099	-0.240**
	(0.123)	(0.112)	(0.171)	(0.114)
Observations	3600135	2583824	3314534	2573967
Mean	0.119	-1.541	-0.591	-1.324

Notes: This table shows the estimate for the effect of the lagged monthly retirement rate of firefighters of the position indicated in each row on the outcome variables indicated in each column, by below and above median capacity utilization. We exclude for the sample interventions made by squads of volunteers and false alarms, and interventions that were no longer necessary. All regressions include fire station and month fixed effects. Standard errors are clustered at fire-station level. *Mean* is the average of the dependent variable in the regression sample.

6 The model

6.1 The model setup

In this section we build a model to quantify the reduction in total intervention time if firefighters were allocated optimally across fire departments. We aim to find the constrained efficient allocation of workers, holding constant the total factor productivity of each intervention, wages and government expenditures.

We focus on the allocation of operatives only, as variation in their labor supply significantly affects intervention times. Let $\mathcal{P}$ denote the set of operative positions: basic firefighter, squad leader and shift leader. Let $\mathcal{D}$ denote the set of fire departments in the Fire and Rescue Service. Let $\mathcal{T}$ denote the set of months from 2014 to 2022. The social planner chooses the number of firefighters L_{pdt} for each operative job position, fire department and month to minimize the total intervention time:

$$\begin{aligned} \min_{\{L_{pdt}\}_{p\in\mathcal{P},d\in\mathcal{D},t\in\mathcal{T}}} & \sum_t \sum_d \sum_i h_{idt} \\ s.t. \quad & \frac{1}{h_{idt}} = A_{idt} \prod_p L_{pdt}^{\psi_p} \quad \forall\, i,d,t \\ & \sum_t \sum_d \sum_p w_{pt} L_{pdt} \leq \sum_t \sum_d \sum_p G_{pdt} \\ & \psi_p > 0 \;\forall\, p \end{aligned} \tag{6}$$

The output of a squad i is the number of interventions completed by the squad in an hour (the inverse of the intervention time). This is modelled as a Cobb-Douglas production function in which the factors of production are the number of firefighters in each position in the department in that time period. Each squad has a idiosyncratic total factor productivity (TFP) A_{idt}, which captures the productivity of department d and of the squad involved in the intervention i, but also the type of intervention i, spatial characteristics (e.g. the quality of infrastructures) and temporal shocks (e.g. weather conditions). In the reduced-form analysis (Equation

4), TFP is modelled as the sum of fire-station, month and type of intervention fixed effects, the controls and the regression residual. Since y_{dt} and L_{pdt} are observed and ψ_p is estimated outside of the model, we can recover the idiosyncratic TFP of each intervention as $A_{idt} = \frac{1}{h^o_{idt} \prod_p (L^o_{pdt})^{\psi_p}}$ where h^o_{idt} and L^o_{pdt} are the observed intervention duration for intervention i and the observed position-p number of firefighters in department d and month t, respectively. Figure D13 shows that on average the Centre and North of Italy have a larger TFP than the South, in line with the shorter average intervention duration observed in Figure 5.

The factors of production are the different types of labor classified according to firefighters' positions. ψ_p is the elasticity of output y_{dt} with respect to the number of firefighters in position p. As long as there is a positive marginal product for each type of labor ($\psi_p > 0$ for all p) the problem has a unique global minimum.[9] As explained in Sections 4 and 5, we estimate each elasticity of output with respect to labor position ψ_p by exploiting the retirements of firefighters in each position which caused a temporary drop in personnel in that position. The estimates are $\psi_f = 0.934$, $\psi_s = 0.351$ and $\psi_r = 0.060$ for basic firefighters, squad leaders and shift leaders, respectively (Table 3).

Our framework is close in spirit to the one used by Walter [2020] to measure misallocation of teachers in a cross-section of schools. We expand this model by allowing for multiple time periods and multiple positions of workers, with different elasticities of production and real wages w_{pt}. In this way we can quantify the extent of three different types of misallocation: misallocation of firefighters across fire departments, across job positions and over time. The real wage w_{pt} is the observed monthly average wage deflated to 2015 Euros earned by position-p firefighters in the year of month t. Observing wages and workforce, we can measure government expenditures for all firefighters of the same position in each fire department and month as $G_{pdt} = w_{pt} L^o_{pdt}$.

[9] After plugging the equality constraint, the objective function becomes $A^{-1}_{idt} \prod_p L^{-\psi_p}_{pdt}$. If $\psi_p > 0$ for all p, the objective function is strictly convex as all the leading principal minors of its Hessian are strictly positive. Given the linear budget constraint, the optimization problem is a convex optimization problem with a unique global minimum.

6.2 Model solution

Define $A_{dt}^{-1} \equiv \sum_i A_{idt}^{-1}$ and let λ denote the Lagrange multiplier of the government budget constraint. The first-order conditions are:

$$\psi_p A_{dt}^{-1} L_{pdt}^{-1} \prod_{\tilde{p}} L_{\tilde{p}dt}^{-\psi_{\tilde{p}}} = \lambda w_{pt} \quad \forall\, p, d, t \tag{7}$$

We can find the $T \cdot P \cdot D$ optimal labor allocations $\{L_{pdt}\}$ by solving a system of linear equations composed by the budget constraint and by the $T \cdot P \cdot D - 1$ linearly independent ratios of first-order conditions:

$$\frac{\psi_p L_{pdt}^{-1} A_{dt}^{-1} \prod_{\tilde{p}} L_{\tilde{p}dt}^{-\psi_{\tilde{p}}}}{\psi_1 A_{11}^{-1} L_{111}^{-1} \prod_{\tilde{p}} L_{\tilde{p}11}^{-\psi_{\tilde{p}}}} = \frac{w_{pt}}{w_{11}} \quad \forall (p, d, t) \neq (1, 1, 1) \tag{8}$$

which equalize the ratio of marginal products to the ratio of factor prices. The solution of this system is:

$$L_{pdt}^{*} = \frac{\sum_t \sum_d \sum_p G_{pdt}}{\sum_s \sum_e \sum_q \left[\frac{A_{dt}}{A_{es}} \prod_{\tilde{p}} \left(\frac{w_{\tilde{p}s}}{w_{\tilde{p}t}} \right)^{\psi_{\tilde{p}}} \right]^{\frac{1}{(1+\sum_{\tilde{p}} \psi_{\tilde{p}})}} \frac{\psi_q}{\psi_p} w_{pt}} \quad \forall\, p, d, t \tag{9}$$

The more productive is a department relative to the others, the less workers should be allocated to that department. The more elastic is intervention time with respect to the supply of position-p firefighters (relative to the other positions) the more firefighters of this position should be employed. The higher the wage of firefighters relative to other positions, times and departments, the lower should be their employment.

L_{pdt}^{*} is the efficient allocation of firefighters over space, time and positions, which achieves the minimum intervention time $H^{*} = \sum_t \sum_d \sum_i h_{idt}(L_{1dt}^{*}, ..., L_{pdt}^{*})$. The percentage increase in observed total intervention time with respect to the unconstrained efficient level is $\mu^{o} = \frac{H^{o} - H^{*}}{H^{*}}$.

6.3 Which type of misallocation is more relevant?

For an optimal allocation of workers, their marginal products must be equal to their price ratio across space, time and job position.[10] To measure the extent of misallocation across space, we add the following constraints to the model: $\sum_t \sum_p w_{pt} L_{pdt} = \sum_t \sum_p G_{pdt}$ for all d. In this scenario, the government cannot change the share of total national expenditures spent for each fire department that is observed in the data. Solving problem (6) with this additional constraint, we obtain the constrained-efficient allocation $\{L^{CS}_{pdt}\}$:

$$L^{CS}_{pdt} = \frac{\sum_p \sum_t G_{pdt}}{\sum_q \sum_s \left[\frac{A_{dt}}{A_{ds}} \prod_{\tilde{p}} \left(\frac{w_{\tilde{p}s}}{w_{\tilde{p}t}} \right)^{\psi_{\tilde{p}}} \right]^{1/(1+\sum_{\tilde{p}} \psi_{\tilde{p}})} \frac{\psi_q}{\psi_p} w_{pt}} \quad \forall\, p, d, t \tag{13}$$

The total intervention time is $H^{CS} \geq H^*$ and the percentage increase in total intervention time with respect to the unconstrained efficient level $\mu^{CS} = \frac{H^{CS} - H^*}{H^*}$ is our measure of misallocation over space.

To measure the extent of misallocation over time, we add the following constraints to the model: $\sum_d \sum_p w_{pt} L_{pdt} = \sum_d \sum_p G_{pdt}$ for all t. In this scenario, the government cannot change the share of total national expenditures spent in each time period that is observed in the data (e.g. for instance due to electoral or budget concerns). Solving problem (6) with this additional constraint, we obtain

[10] Three subsets of the set of equations (8) have to be satisfied to obtain an optimal allocation across space, time and job position, respectively:

$$A_{dt} L_{pdt} \prod_{\tilde{p}} L^{\psi_{\tilde{p}}}_{\tilde{p}dt} = A_{\tilde{d}t} L_{p\tilde{d}t} \prod_{\tilde{p}} L^{\psi_{\tilde{p}}}_{\tilde{p}\tilde{d}t} \quad \forall p, d, \tilde{d}, t \tag{10}$$

$$\frac{L_{pdt} \prod_{\tilde{p}} L^{\psi_{\tilde{p}}}_{pd\tilde{t}} A_{d\tilde{t}}}{L_{pdt} \prod_{\tilde{p}} L^{\psi_{\tilde{p}}}_{\tilde{p}dt} A_{dt}} = \frac{w_{pt}}{w_{p\tilde{t}}} \quad \forall p, d, t, \tilde{t} \tag{11}$$

$$\frac{\psi_p}{\psi_{\tilde{p}}} \frac{L_{\tilde{p}dt}}{L_{pdt}} = \frac{w_{pt}}{w_{\tilde{p}t}} \quad \forall p, \tilde{p}, d, t \tag{12}$$

the constrained-efficient allocation $\{L_{pdt}^{CT}\}$:

$$L_{pdt}^{CT} = \frac{\sum_p \sum_d G_{pdt}}{\sum_q \sum_e \left[\frac{A_{dt}}{A_{et}}\right]^{1/(1+\sum_{\tilde{p}} \psi_{\tilde{p}})} \frac{\psi_q}{\psi_p} w_{pt}} \quad \forall\, p, d, t \tag{14}$$

The total intervention time is $H^{CT} \geq H^*$ and the percentage increase in total intervention time with respect to the unconstrained efficient level $\mu^{CT} = \frac{H^{CT} - H^*}{H^*}$ is our measure of misallocation over time.

To measure the extent of misallocation across job positions, we add the following constraints to the model: $\sum_t \sum_d w_{pt} L_{pdt} = \sum_t \sum_d G_{pdt}$ for all p. In this scenario, the government cannot change the share of total national expenditures over the years spent on each firefighter position that is observed in the data. The same share of public funds will go to basic firefighters, to Squad Leaders and to Shift Leaders as observed in the data. Solving problem (6) with this additional constraint, we obtain the constrained-efficient allocation $\{L_{pdt}^{CP}\}$:

$$L_{pdt}^{CP} = \frac{\sum_d \sum_t G_{pdt}}{\sum_e \sum_s \left[\frac{A_{dt}}{A_{es}} \prod_{\tilde{p}} \left(\frac{w_{\tilde{p}s}}{w_{\tilde{p}t}}\right)^{\psi_{\tilde{p}}}\right]^{1/(1+\sum_{\tilde{p}} \psi_{\tilde{p}})} w_{pt}} \quad \forall\, p, d, t \tag{15}$$

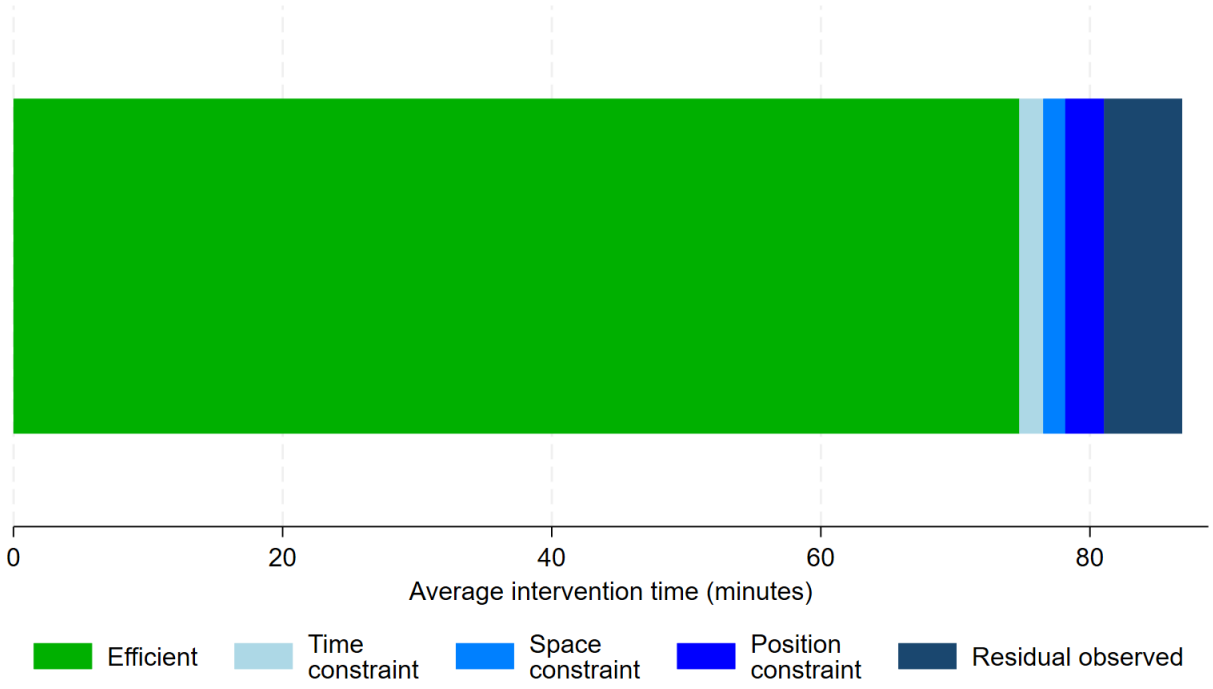
The total intervention time is $H^{CP} \geq H^*$ and the percentage increase in total intervention time with respect to the unconstrained efficient level $\mu^{CP} = \frac{H^{CP} - H^*}{H^*}$ is our measure of misallocation over job positions.

The system of equations (10)-(12) contains $P \cdot D \cdot T$ linearly independent equations that pin down $P \cdot D \cdot T$ unknowns $\{L_{pdt}\}$. The first counterfactual imposes D additional constraints and the set of equations (10) do not generally hold anymore. The second counterfactual imposes T additional constraints and the set of equations (11) do not generally hold anymore. The third counterfactual imposes P additional constraints and the set of equations (12) do not generally hold anymore. Note that the other two sets of equations continue to hold.

6.4 Simulations

The efficient allocation of labor reduces average intervention time from 87 to 75 minutes (Table C15 and Figure 9), which makes the observed average intervention time 16.2 percent longer than optimal. The largest source of inefficiency is misallocation over job positions (+3.9 percent higher intervention time with respect to the efficient allocation), followed by space (+2.7 percent) and time misallocation (+2.4 percent).

Figure 9: Average intervention time, observed and counterfactuals

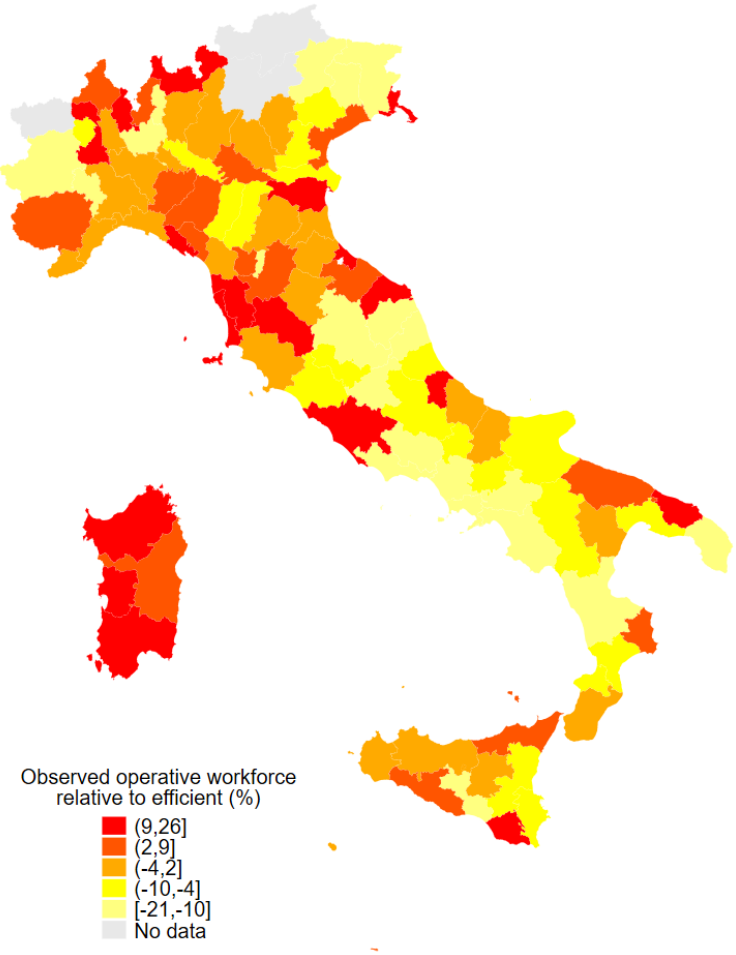


Note: This figure shows the average intervention time (in minutes) that is: simulated under an efficient labor allocation (in green); simulated under a constrained efficient labor allocation adding time constraints (light blue); adding space constraints (mid-blue); adding position constraints (dark blue); adding the residual time necessary to obtain the observed average time (navy).

The three constraints together do not explain the entire amount of inefficiency, as they correspond to $T + D + S$ restrictions while the first-order conditions are $T \cdot D \cdot S$. The social planner can obtain a better labor allocation, even holding constant the total amount of resources allocated to each job position, fire depart-

ment, or month. For instance, suppose department A lacks basic firefighters and has an abundance of squad leaders in a certain month, while department B has an abundance of basic firefighters and lacks squad leaders in the same month, as presented in Figure D15. The planner could optimally reallocate basic firefighters from B to A and squad leaders from A to B in correct proportions according to their wages, without affecting the total amount of resources devoted to each position, to each department and to each month. Therefore, even without changing the government budget over time, space, or job positions, the public administration can substantially improve its performance, simply by transferring workers across departments.

Figure 10: Observed vs efficient distribution of operative firefighters by fire department

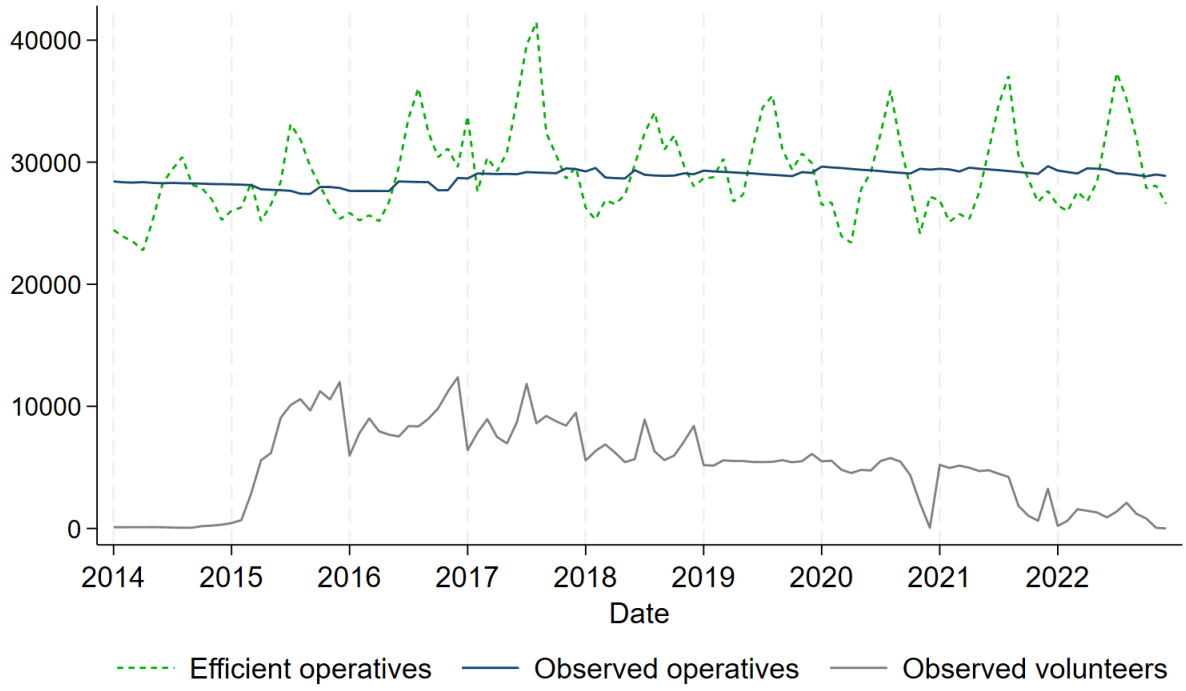


Note: This figure shows the percentage change in the fire department labor force in the observed data relative to the efficient labor allocation. Red (yellow) means the observed labor force is higher (lower) than efficient. Data provided by INPS.

Figure D14 shows that the number of basic firefighters and squad leaders should increase (+6.7 and +7.8 percent), while the number of shift leaders should decrease (-58 percent). Operatives should be reallocated from the more productive Center-North to the less productive South, in order to speed up the longer intervention times of Southern departments (Figures 5 and 10). This model implication hinges on the strong assumption that the government cannot improve total factor productivity of Southern fire departments in order to improve their performance. Finally, Figure 11 shows that the efficient allocation (the dashed green line) is more volatile over time than the observed allocation (the solid blue line). The

number of observed operatives remains stable just below thirty thousands in 2014-2022, whereas it would be optimal to have more of them during summers when wildfires are more frequent and after natural disasters (e.g. the earthquakes in the Centre of Italy from August 2016 to January 2017). Volunteers that are called and paid for single days of work can help achieve a more efficient allocation. Yet, their observed employment (the gray solid line) does not seem to fully follow the seasonality of demand.

Figure 11: Observed vs efficient workforce over time



Note: This figure shows the number of operative firefighters in the observed data (the blue solid line) relative to the efficient labor allocation (the green dashed line). The number of observed volunteers employeed is shows by the gray solid line. Data provided by INPS.

7 Conclusions

This paper explores the presence of labor misallocation in the public sector, analyzing the case of the Italian Fire and Rescue Service. We document the presence of delayed turnover across all positions along the hierarchical structure of the

service, with negative repercussions on public-service performance. Temporary shortages of operatives, in particular of low and middle managers, significantly slow interventions down. We estimate a 6.7 percent prolongation in intervention times when retirement rates of operatives increase by 10 percentage points. On the contrary, the absence of higher-level managers has no impact on public sector performance, at least in the short-term.

We also leverage the impact of delayed turnover on public sector performance in order to estimate labor elasticities of intervention time by position, which we then use to solve a model of labor allocation. Efficient labor allocation, achieved through transferring workers from North to South, from low to high-demand seasons, and from high to low ranks, improves public-service performance by 16.2 percent. Notably, half of these efficiency gains can be realized by reallocating workers while maintaining a constant total budget across time, space, and job positions.

To enhance the resilience of the public sector against supply and demand variations, policymakers could consider several policies. Planning new hires based on the local age distribution of public workers would anticipate retirement flows, ensure a steady workforce and avoid leadership gaps over time. Decentralizing the hiring process to local public-sector units would lead to more responsive staffing, tailored to the idiosyncratic needs of different regions. Hiring seasonal workers in public-sector organizations affected by seasonal demand could provide the necessary flexibility to meet short-term demand surges without committing to costly permanent hires. By implementing these strategies, the public sector could better manage workforce challenges and maintain an adequate service level in the face of fluctuating and heterogeneous supply and demand.

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Appendix A Firefighters' tasks and career

According to their position, firefighters are responsible for different tasks. Basic Firefighters perform rescue, prevention, and surveillance activities on the ground. They are also in charge of driving vehicles, engaging in radio and telecommunications activities, as well as basic equipment maintenance.

Each squad is composed by four Basic Firefighters (including the driver) and a Squad Leader, who coordinates the squad members and drafts a report at the end of each intervention. Shift Leaders coordinate multiple squads of the same station during a work shift, taking operational responsibility on the ground and optimizing resources during interventions.

Inspectors perform technical-inspection activities, draft office acts, and manage contractual procedures for work, services, and supplies. They assist with general management activities of their assigned structure, contribute to intervention plans for urgent rescue, civil defense, and civil protection activities, and draft detailed projects for their implementation.

Directors are managers in charge of fire stations and offices not reserved for Executives. They propose intervention plans for rescue and civil defense activities and autonomously carry out interventions within their competence area. In the event of civil protection emergencies, they may be entrusted with the responsibility of managing complex and articulated operational groups and engage in study and research activities.

Executives are in charge of central directorates, regional offices, provincial fire departments and training centres. They manage the internal organization, the human and instrumental resources of the office and follow the training of the staff. They direct the activities of urgent technical rescue and civil protection, and can exercise spending powers within the limits of the funds assigned to them [Corpo Nazionale dei Vigili del Fuoco, 2023c].

Operatives have work shifts that span 12 hours, commencing either at 8 a.m.

for the day shift or at 8 p.m. for the night shift. The recurring work schedule is structured as follows: one day shift is followed by a 24-hour rest period, succeeded by one night shift, which is then followed by a 48-hour rest period. Officers usually have work shifts from 8 a.m to 6 p.m five days a week.

Competitive exams to become Basic Firefighter, Inspector or Director are open to the general population, whereas exams for the other four positions are accessible to internal candidates only. These exams are conducted at irregular intervals and can extend over several months. For example, between 2008 and 2024, there were four exams held to recruit basic firefighters. Each exam included a preliminary screening test, a physical aptitude assessment, an oral examination, and medical evaluations. The entire process—from the exam announcement to the completion of medical checks—lasted over a year. [Corpo Nazionale dei Vigili del Fuoco, 2024]. Before starting to work as a Basic Firefighter, Inspector or Director, the new appointee has to attend a six-month practical and theoretical course in a fire training centre (nine-month for Directors) and a three-month operative traineeship in a local fire department [Gazzetta Ufficiale, 2017]. The six positions are further divided according to more granular levels, reached after meeting specified seniority requirements [Gazzetta Ufficiale, 2017]. All positions and levels as well the exams, the seniority and education requirements to access each one of them are listed in Table C1.

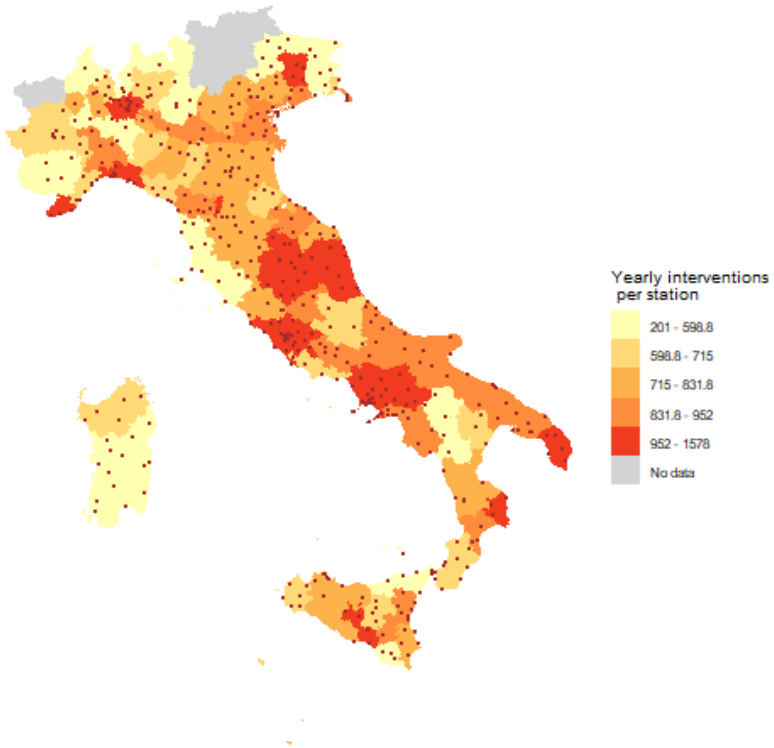
Appendix B Estimation of turnover delay

As in Cellini et al. [2010], assume that the effects of a retirement on the labor force depend only on the time elapsed since retirement and not on the time period in which retirement occurred or on the history of retirements. That is, $\frac{de_{pdt}}{dr_{pdt-k}}$ and $\frac{dr_{pdt}}{dr_{pdt-k}}$ can depend on k but not on t or on $\{r_{pd1}, ..., r_{pdt-1}\}$. Note that the labor flow equation is $l_{pdt} = l_{pdt-1} + e_{pdt-1} - r_{pdt-1}$. Iterating, we obtain $l_{pdt} = l_{pdt-\tilde{h}} + \sum_{h=1}^{\tilde{h}} e_{pdt-h} - \sum_{h=1}^{\tilde{h}} r_{pdt-h}$. If turnover is perfectly timed,

we have $\frac{\partial e_{pdt-1}}{\partial r_{pdt-1}} = 1$ and $\beta_1 = \frac{d(l_{pdt}-l_{pdt-1})}{dr_{pdt-1}} = 0$. If turnover is lagged, we have $\frac{\partial e_{pdt-1}}{\partial r_{pdt-1}} = 0$ and $\beta_1 = \frac{d(l_{pdt}-l_{pdt-1})}{dr_{pdt-1}} = -1$. If the labor force is restored after $\tilde{h}$ periods: $l_{pdt} - l_{pdt-\tilde{h}} = 0$ and $\sum_{h=1}^{\tilde{h}} e_{pdt-h} = \sum_{h=1}^{\tilde{h}} r_{pdt-h}$. Differentiating both sides, $\sum_{h=1}^{\tilde{h}} \frac{\partial e_{pdt-h}}{\partial r_{pdt-\tilde{h}}} - \sum_{h=1}^{\tilde{h}-1} \frac{\partial r_{pdt-h}}{\partial r_{pdt-\tilde{h}}} = 1$. Then, under the stated assumption we have $\sum_{h=1}^{\tilde{h}} \beta_h = \sum_{h=1}^{\tilde{h}} \frac{d(l_{pdt}-l_{pdt-1})}{dr_{pdt-h}} = \sum_{h=1}^{\tilde{h}} \frac{\partial e_{pdt-1}}{\partial r_{pdt-h}} - \sum_{h=2}^{\tilde{h}} \frac{\partial r_{pdt-1}}{\partial r_{pdt-h}} - 1 = \sum_{h=1}^{\tilde{h}} \frac{\partial e_{pdt-h}}{\partial r_{pdt-\tilde{h}}} - \sum_{h=1}^{\tilde{h}-1} \frac{\partial r_{pdt-h}}{\partial r_{pdt-\tilde{h}}} - 1 = 0$.[11]

Appendix C Additional Figures

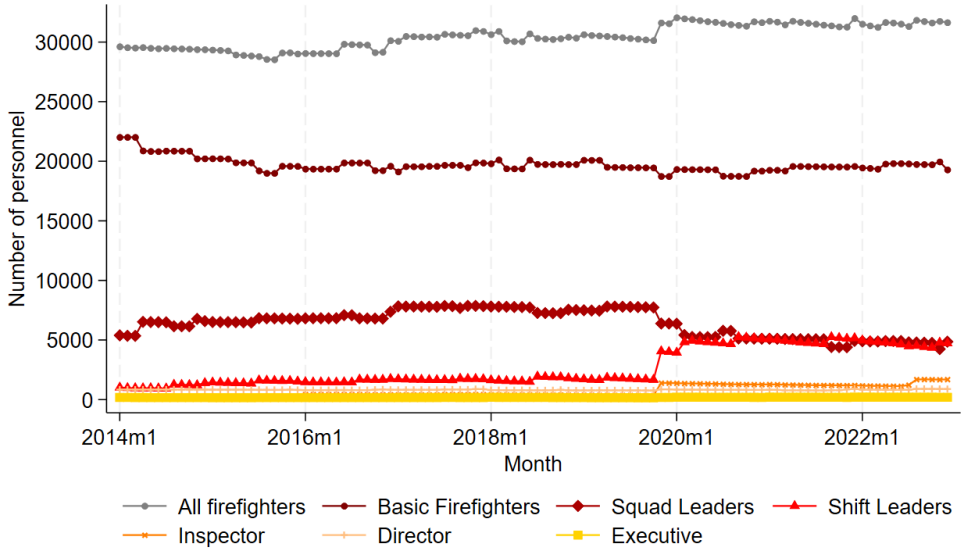
Figure D1: Fires station location and density of interventions



Note: This map shows the average yearly number of interventions per fire station by province (from yellow to red). A brown dot indicates the location of a fire station.

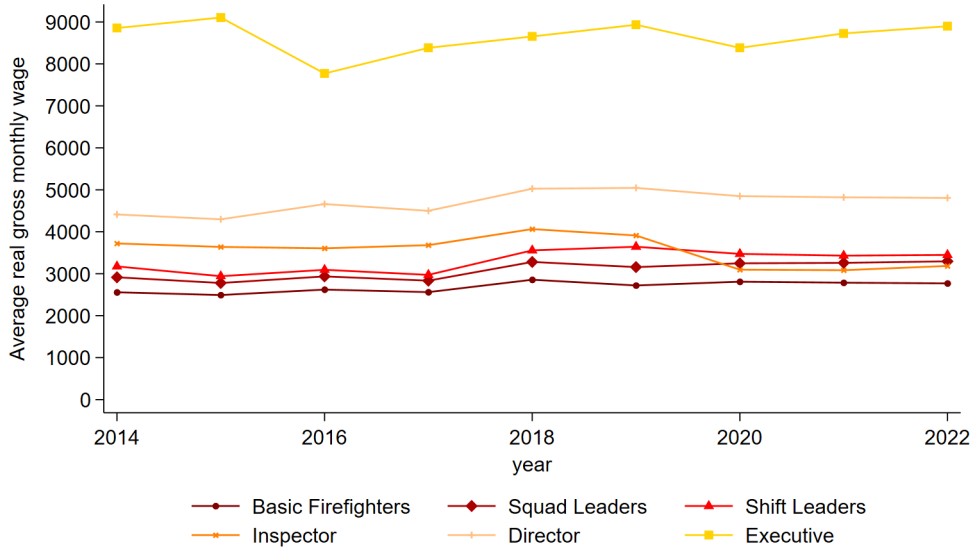
[11] Note that normalizing by the *average* labor force in each department, which is constant over time, allows to interpret the sum of the coefficients β_h as the cumulative effect of retirements on the labor force. A sum of the coefficients equal to 0 implies a return to the original labor force. This would not generally be true if we normalize by the labor force in the previous period, which varies over time.

Figure D2: Number of personnel over time by hierarchial position



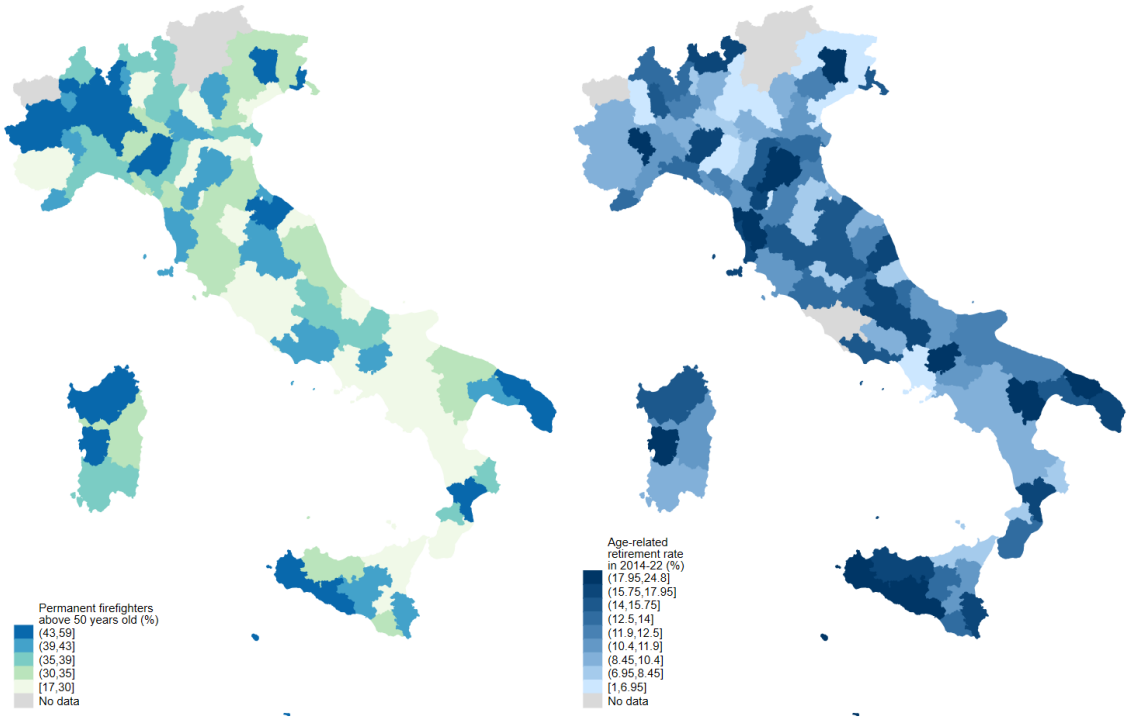
Note: The figure shows the number of personnel in each month from 2014 to 2022 by hierarchical position. Data provided by INPS

Figure D3: Average monthly gross income over time by hierarchical position



Note: The figure shows the yearly average of monthly gross income (in 2015 Euros) by hierarchical position from 2014 to 2022. Data provided by INPS.

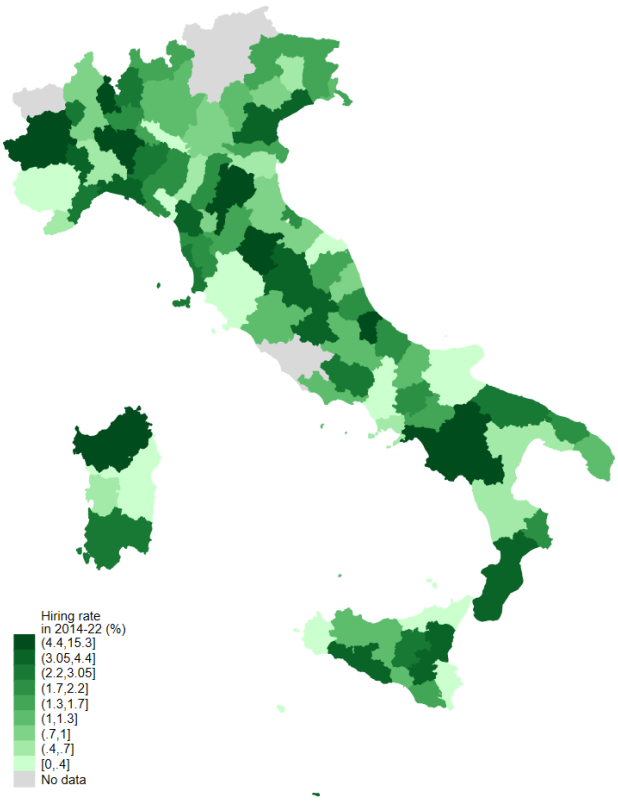
Figure D4: Shares of firefighters above 50 and age-related retirement rates by province



(a) Fraction above 50 years old in January 2014 (b) Retirement rate in 2014-2022

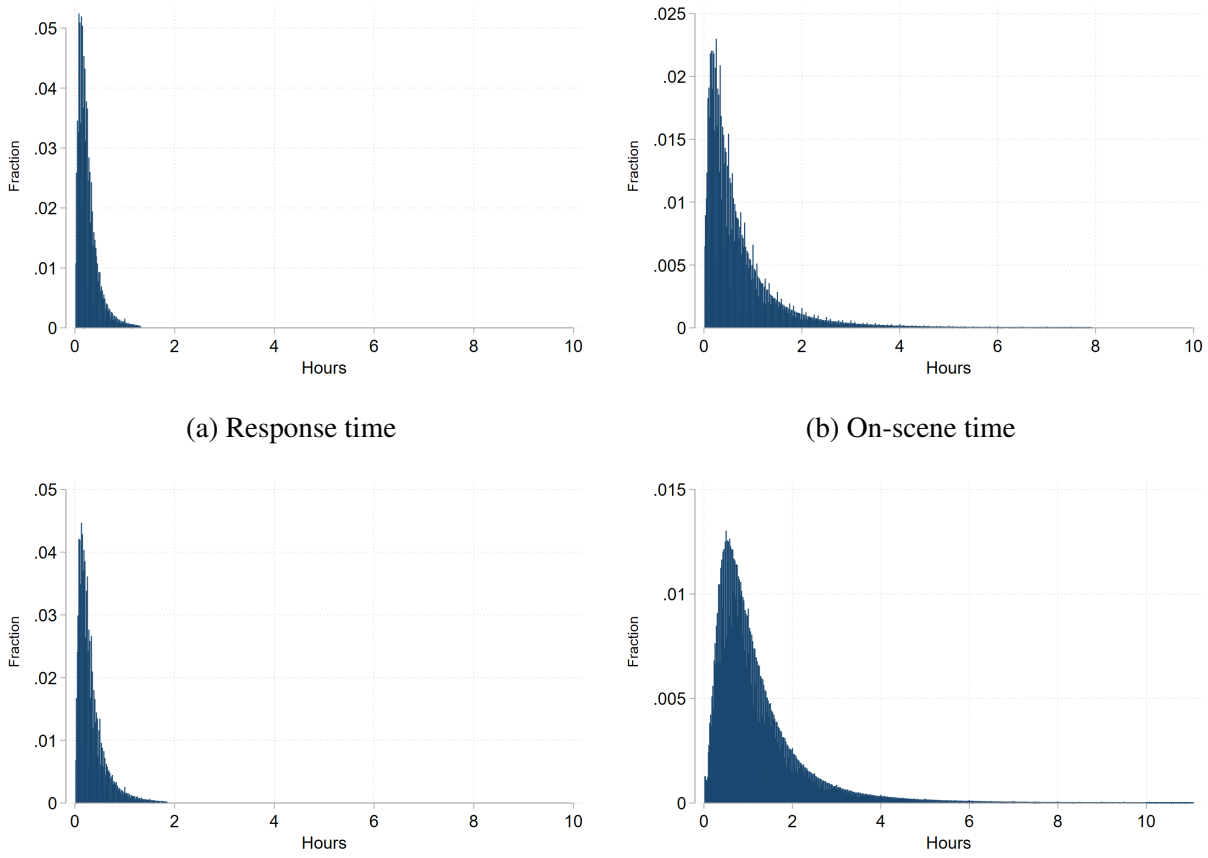
Note: Panel (a) shows the average fraction of firefighters above 50 years of age in each province over 2014-2022. Panel (b) shows the average age-related retirement rate of firefighters in each province over 2014-2022. The average age-related retirement rate is the ratio between the total number of age-related retirees in 2014-2022 divided by the total number of firefighters that worked for at least one month in 2014-2022. Data provided by INPS.

Figure D5: Hiring rates by province in 2014-2022



Note: The figure shows the number of permanent firefighters appearing in the data for the first time (interpreted as new hires) divided by the number of permanent firefighters in the same month. Data provided by INPS.

Figure D6: Distribution of intervention and intervention segment times



(a) Response time

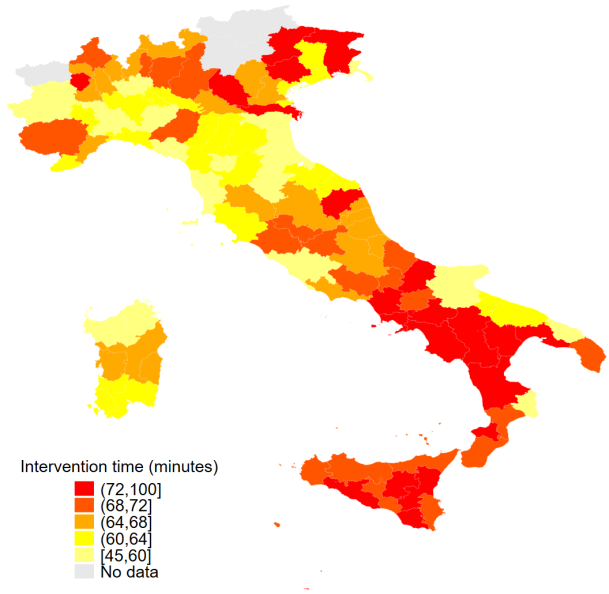
(b) On-scene time

(c) Return time

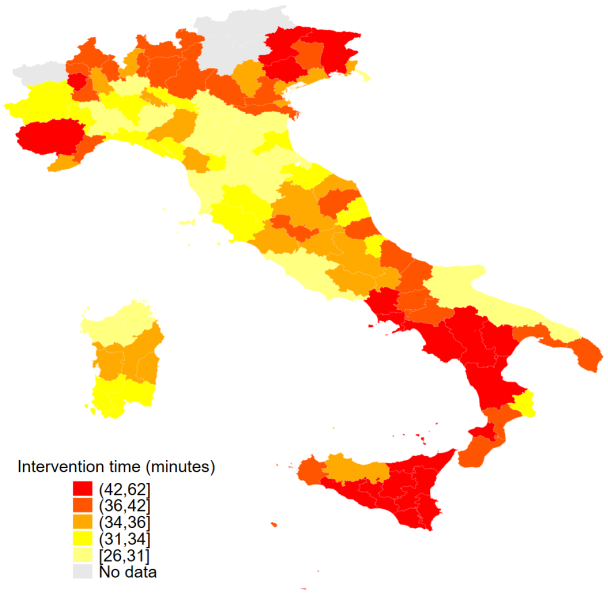
(d) Intervention time

Note: Panel (a) shows the distribution of the arrival times: the number of hours from when the squad exits the fire station to when it arrives at the intervention site. Panel (b) shows the distribution of on-scene time: the number of hours from when the squad arrives at the intervention site to when it leaves the intervention site. Panel (c) shows the distribution of return times: the number of hours from when the squad leaves the intervention site to when it returns to the fire station. Panel (d) shows the distribution of intervention times: the number of hours from when the squad exits the fire station to when it returns to the fire station.

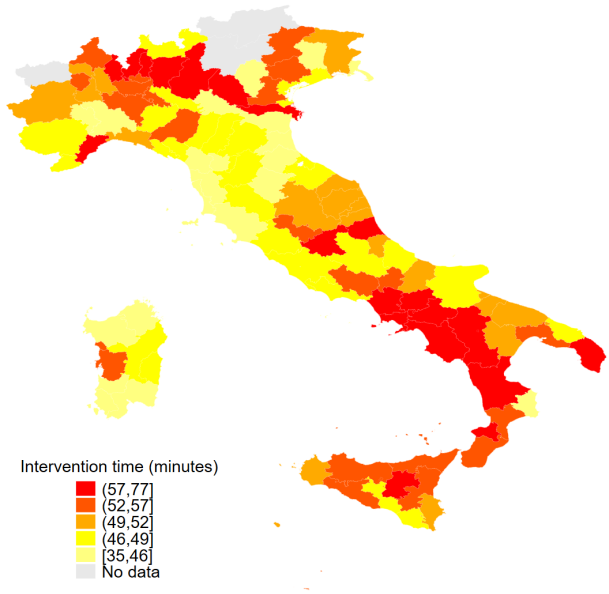
Figure D7: Median intervention time and on-scene time by province, urgent and non-urgent interventions



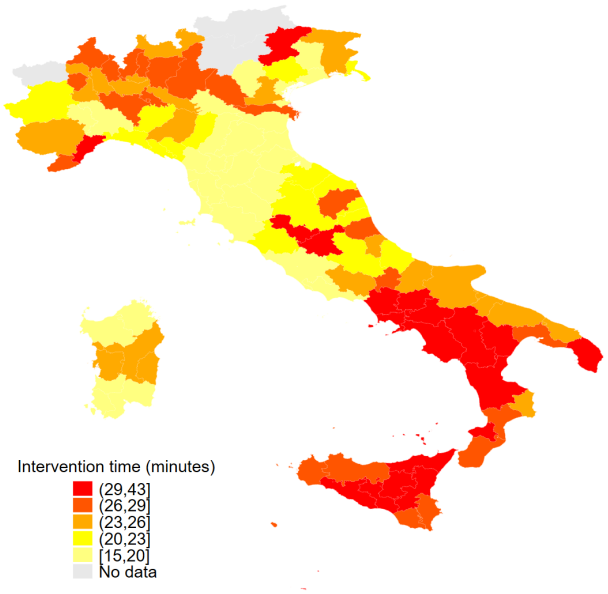
(a) Urgent intervention time



(b) Urgent on-scene time



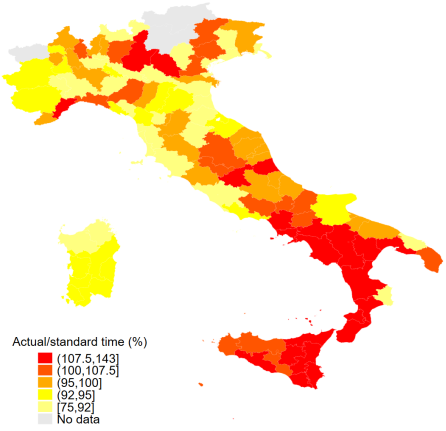
(c) Non-urgent intervention time



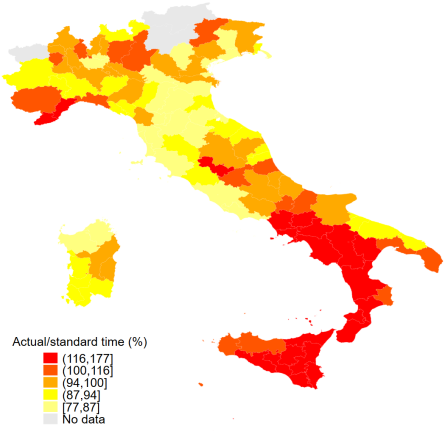
(d) Non-urgent on-scene time

Note: Panels (a) and (c) show the median intervention time (in minutes) by province for urgent interventions and non-urgent interventions in 2014-2022. Panels (b) and (d) show the median on-scene time (in minutes) by province for urgent interventions and non-urgent interventions in 2014-2022. Urgent interventions include fires, explosions, gas leaks, transport accidents, landslides, earthquakes, floods, people searches and rescues. Non-urgent interventions include recovery of goods, removal of debris, leaning trees, pest control, animal rescues, elevator rescues, door/window openings, safety checks.

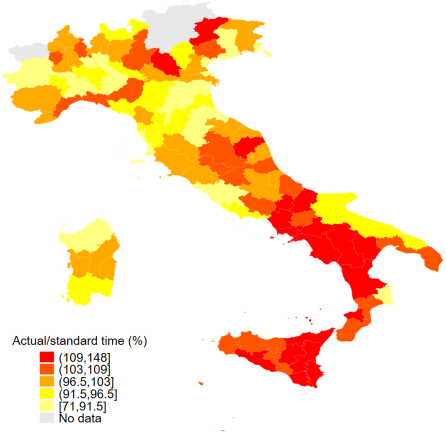
Figure D8: Median intervention and on-scene time over standard time by province



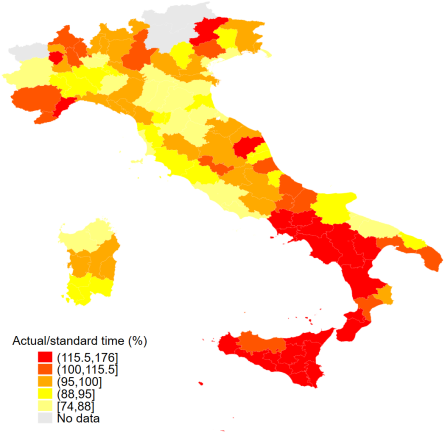
(a) Intervention time, all



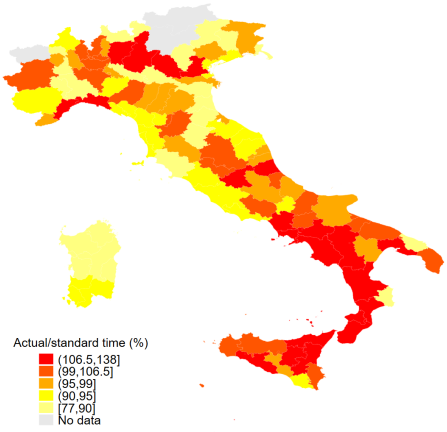
(b) On-scene time, all



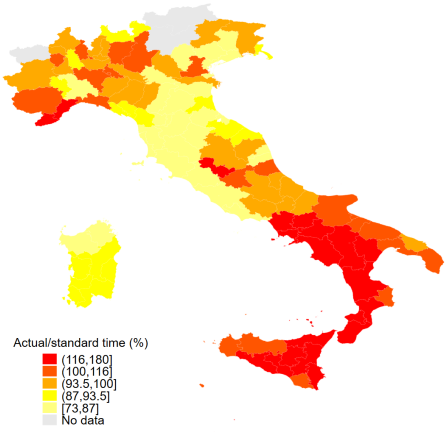
(c) Intervention time, urgent



(d) On-scene time, urgent



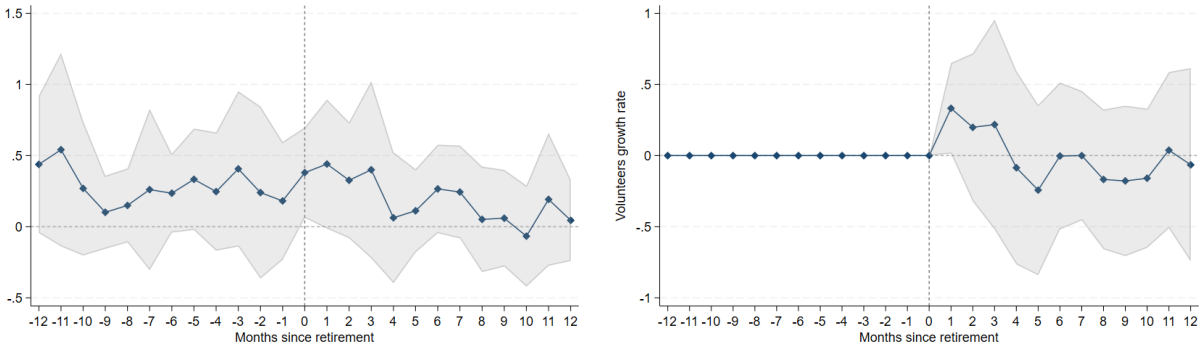
(e) Intervention time, non-urgent



(f) On-scene time, non-urgent

Note: Panels (a), (c) and (e) show the median intervention time (in minutes) divided by the standard intervention time for each type of intervention by province for all interventions, urgent interventions and non-urgent interventions in 2014-2022. Panels (b), (d) and (f) show the median on-scene time (in minutes) divided by the standard on-scene time for each type of intervention by province for all interventions, urgent interventions and non-urgent interventions in 2014-2022. The standard intervention time is the national median intervention time in 2014-2022 for that type of intervention.

Figure D9: Effect of age-related retirements on growth of FTE volunteers

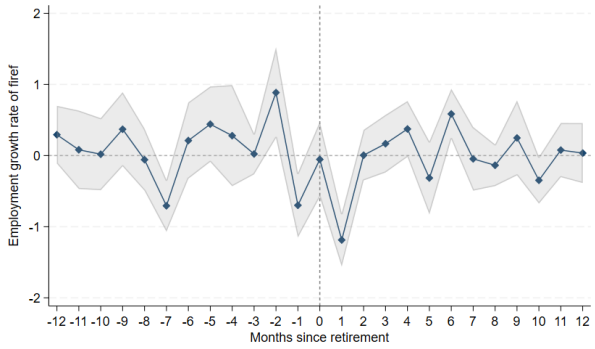


(a) Temporary (12 months)

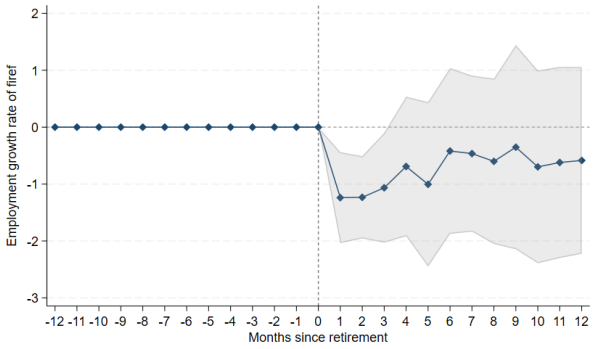
(b) Cumulative (12 months)

Note: FTE volunteer is equal to the number of days worked by the volunteer in the month divided by the standard number of days worked in a month by a permanent firefighter (15). Data provided by INPS.

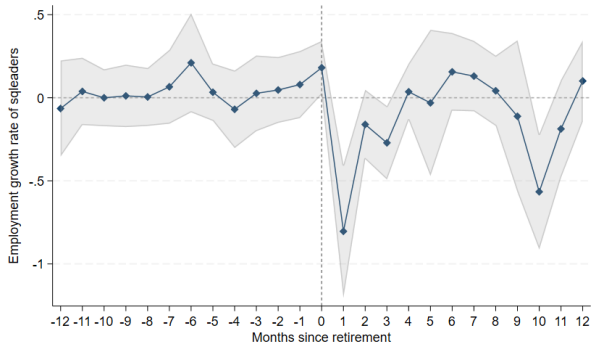
Figure D10: Effect of age-related retirements on employment growth by position, non-officers



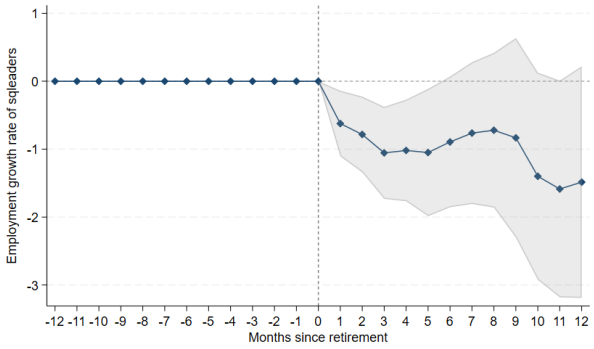
(a) Basic Firefighters, temporary (12 months)



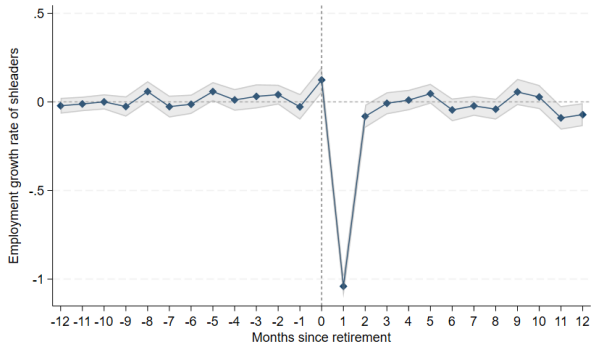
(b) Basic Firefighters, cumulative (12 months)



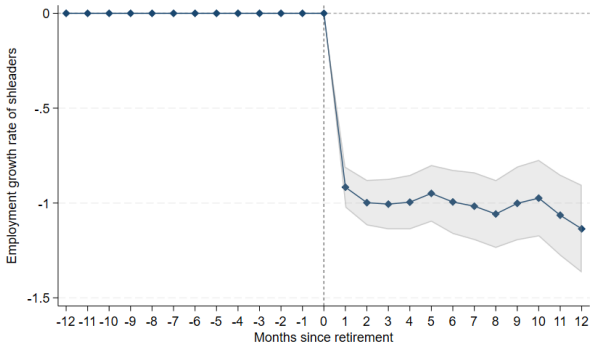
(c) Squad leaders, temporary (12 months)



(d) Squad leaders, Cumulative (12 months)



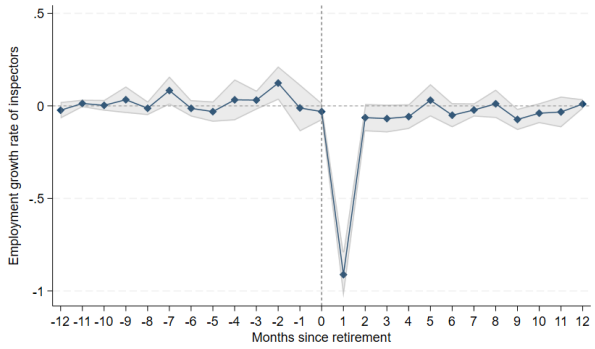
(e) Shift leaders, temporary (12 months)



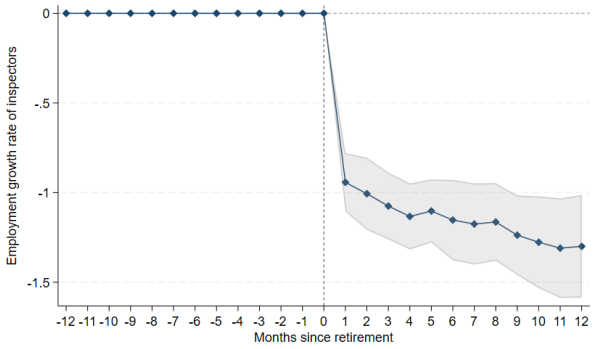
(f) Shift leaders, Cumulative (12 months)

Note: We restrict the sample to non-officer firefighters that have a permanent contract. Data provided by INPS.

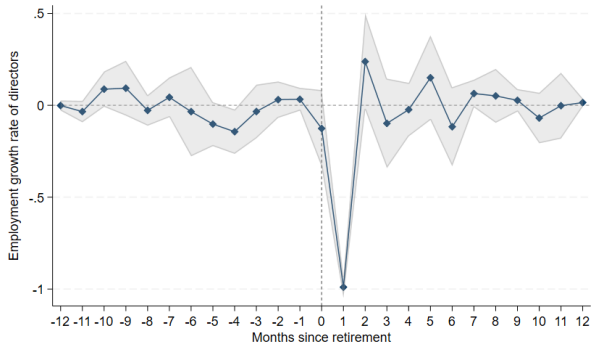
Figure D11: Effect of age-related retirements on employment growth by position, officers



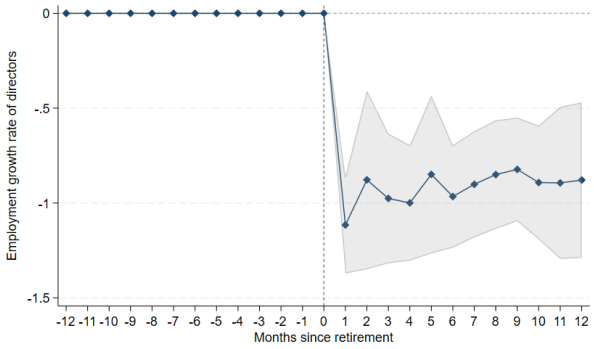
(a) Inspectors, temporary (12 months)



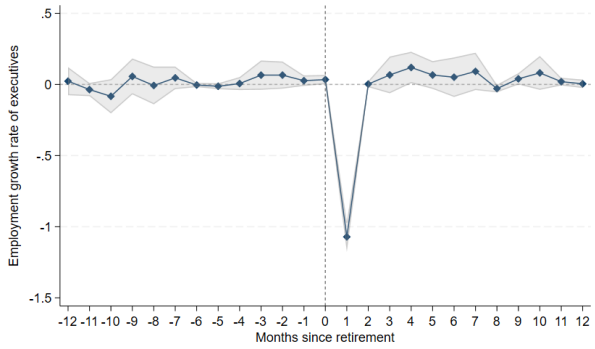
(b) Inspectors, cumulative (12 months)



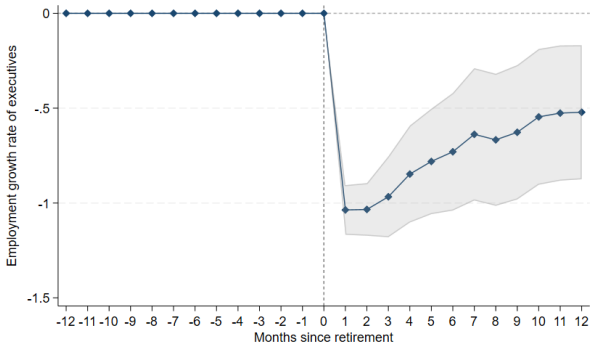
(c) Directors, temporary (12 months)



(d) Directors, Cumulative (12 months)



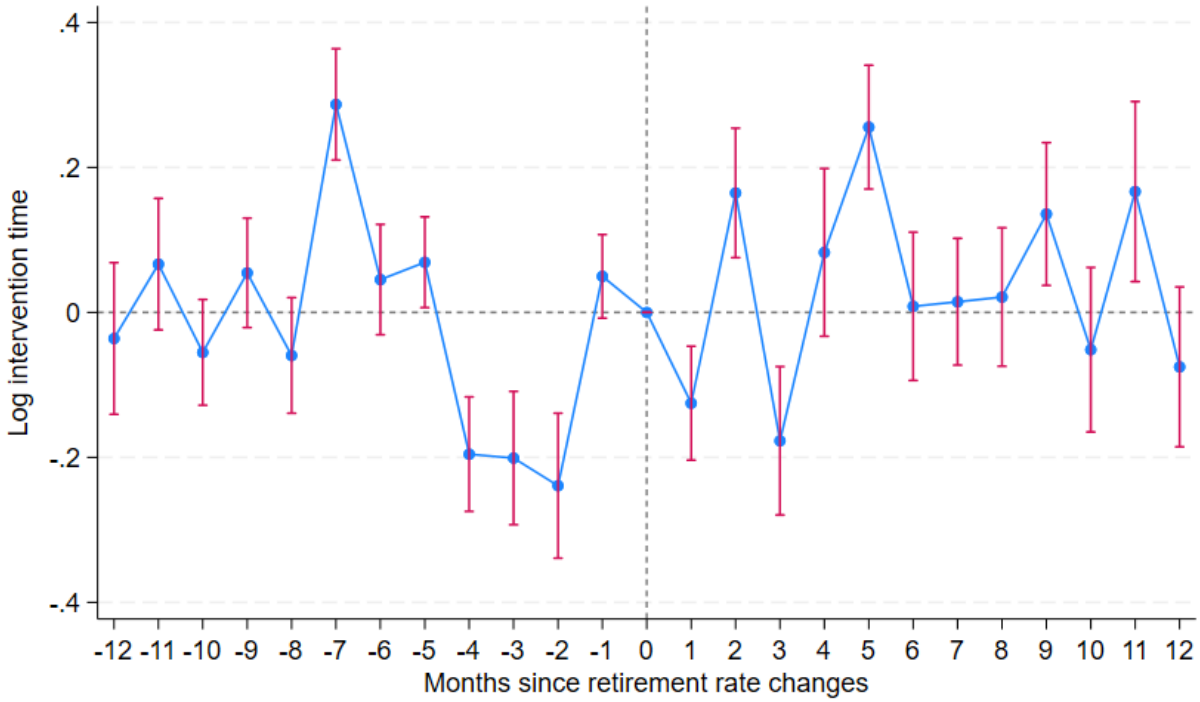
(e) Executives, temporary (12 months)



(f) Executives, Cumulative (12 months)

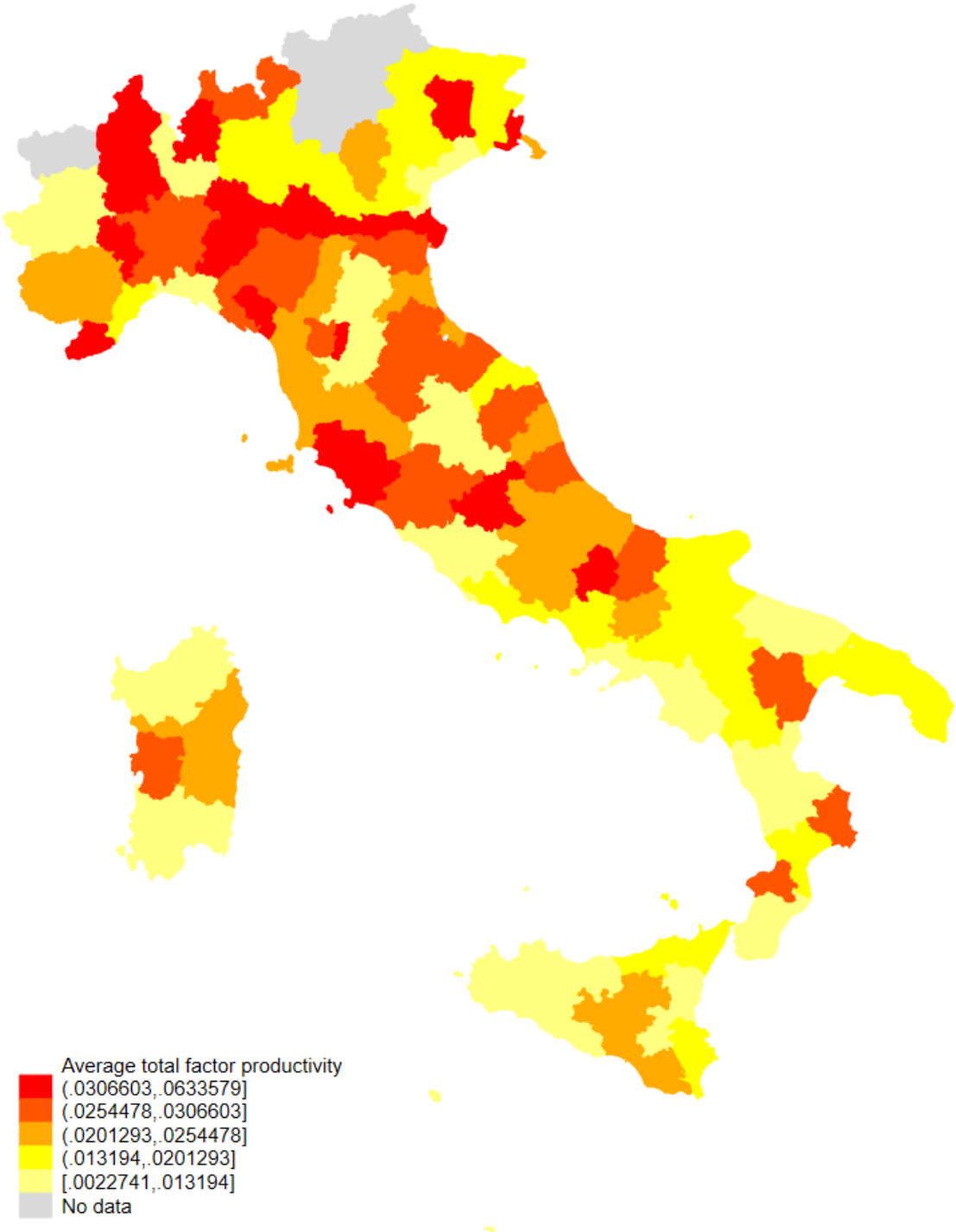
Note: We restrict the sample to officer-firefighters that have a permanent contract. Data provided by INPS.

Figure D12: Dynamic effects of retirements of officers on intervention time



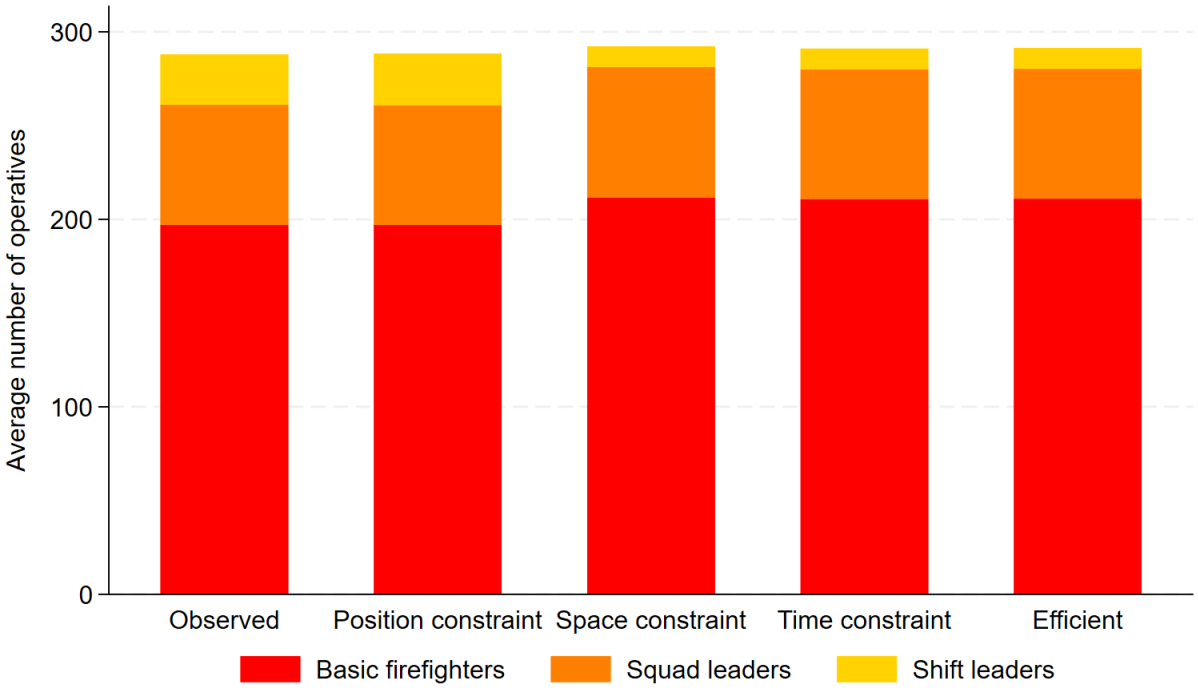
Note: This figure shows the placebo and dynamic effects of retirements of officers on the logarithm of intervention time in the year prior to and after an increase in the retirement rate. The effects are estimated using the estimator by de Chaisemartin et al. [2024], which is robust to heterogeneous treatment effects. The treatment effects are the weighted average of switchers' slopes, where switchers receive a weight proportional to the absolute value of their treatment change. The red lines indicate 95 percent confidence intervals.

Figure D13: Average total factor productivity by province



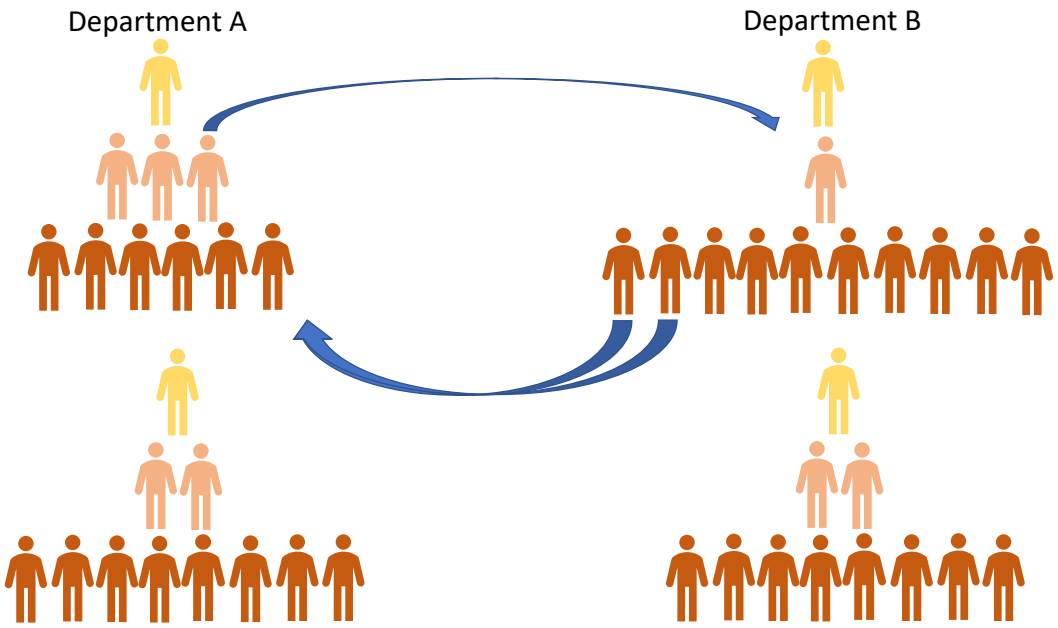
Note: The figure shows the average TFP for the interventions occurred in each province from 2014 to 2022. The TFP of each intervention is obtained by inverting the model production function $A_{idt} = \frac{1}{h^o_{idt} \prod_p (L^o_{pdt})^{\psi_p}}$ where h^o_{idt} and plugging the observed hours of intervention, the observed labor force of each position and the estimated ψ_p.

Figure D14: Average labor force by position, observed and counterfactuals



Note: This figure shows the average number of firefighters by position that are: observed in the data; simulated under a constrained efficient labor allocation with position, space and time constraints; simulated under an efficient labor allocation. Data provided by INPS.

Figure D15: Example of residual misallocation



Note: This figure shows an example of residual misallocation while holding the amount of resources allocated to each department, month and job position constant. Basic firefighters are in red, squad leaders are in orange and shift leaders in yellow. The efficient allocation is presented in the second row. In this example we assume that the wage of a squad leader is twice as much as the wage of a basic firefighter. The reallocation does not affect the total amount of resources over space, because each department continues to have the same wage expenditures. The reallocation does not affect the total amount of resources over time, because it simply moves workers across departments within the same period. The reallocation does not affect the total amount of resources across job positions, because the total number of basic firefighters and the total number of squad leaders are unchanged.

Appendix D Additional Tables

Table C1: Seniority, education requirements and exams by position and level

Position	Level	Seniority	Education	Exam
Basic Firefighter	Firefighter	Being under 37 years of age	High school diploma	Yes, open
	Expert Firefighter	4 years as Firefighter		No
	Coordinator Firefighter	8 years as Expert Firefighter		No
Squad Leader	Squad Leader	Being a Coordinator Firefighter		Yes, internal
	Expert Squad Leader	5 years as Squad Leader		No
Shift Leader	Shift Leader	5 years as Expert Squad Leader		Yes, internal
	Expert Shift Leader	5 years as Shift Leader		No
Inspector	Inspector	15 years as firefighter or being a Platoon or Shift Leader	BSc Engineering or Architecture	Yes, 50% open
	Expert Inspector	7 years as Inspector		No
	Coordinator Inspector	16 years as Expert Inspector		No
Director	Vice-Director		MSc Economics or Master in Law	Yes, 75% open
	Director	2 years as Vice-Director		No
	Director Vice-Executive	5.5 years as Director		No
Executive	First Executive	2 years as Director Vice-Executive		Yes, internal
	Superior Executive	3 years as First Executive		No
	General Executive	Chosen among Superior Executives by the Minister of Interior		No

Note: This table shows the list of positions and levels for firefighters working in the Fire and Rescue Service. For each position-level it reports the required seniority, level of education and whether the promotion is conditional on successfully passing a competitive exam. Firefighters are also required to have political rights and a certified level of physical and mental fitness. Promotions are blocked if the firefighter has incurred in a disciplinary sanction equal to or more severe than a pecuniary sanction in the previous three years.

Table C2: Retirement criteria

		Firefighters		Professionals	
		Operatives	**Officers**	**Male**	**Female**
Minimum seniority	**2014-2015**	41 years, 6 months. 5 years earlier if 57 years, 3 months old	41 years, 6 months. 5 years earlier if 57 years, 3 months old	42 years, 3 months	41 years, 3 months
	2016-2019	41 years, 10 months. 5 years earlier if 57 years, 7 months old	41 years, 10 months. 5 years earlier if 57 years, 7 months old	42 years, 10 months	41 years, 10 months
	2020-2022	42 years. 5 years earlier if 58 years old	42 years. 5 years earlier if 58 years old	42 years, 10 months	41 years, 10 months
Maximum age	**2014-2015**	61 years, 3 months	65 years, 3 months	65 years, 3 months	65 years, 3 months
	2016-2019	61 years, 7 months	65 years, 7 months	65 years, 7 months	65 years, 7 months
	2020-2022	62 years	66 years	66 years	66 years

Note: This table shows the minimum seniority and the maximum age for firefighters and professionals to obtain a pension. Professionals have technical roles such as accountants, administrators and computer engineers. We exclude them from the analysis.Operatives include the positions of Basic Firefighter, Squad Leader or Shift Leader. Officers include the positions of Inspector, Director or Executive. All professionals can retire 12 months before the age limit without receiving a pension in those 12 months. All firefighters can retire 15 months before the seniority limit without receiving a pension in those 15 months ('finestra mobile').

Table C3: Percentage of age- and seniority-related retirements

	Number	Percentage
Age-related retirement	3778	95.961
Seniority-related retirement	159	4.039

Table C4: Number of operative firefighters' age-related retirements by position

	Freq.	Percentage
Director	92	2.4355
Executive	70	1.852
Firefighter	146	3.864
Inspector	271	7.173
Shift Leader	2754	72.896
Squad Leader	445	11.779

Table C5: Effect of age-based retirement rate on the logarithm of firefighters, by position

	Baisc firefighters	Squad leaders	Shift leaders	All operatives	Inspectors	Directors	Executives	All officers
Lagged	-1.349*	-0.251	-0.549***	-1.552***	-0.365***	-0.819***	-0.491***	-0.547***
retirement rate	(0.792)	(0.678)	(0.197)	(0.302)	(0.099)	(0.274)	(0.136)	(0.115)
F-statistic	2.902	0.137	7.747	26.470	13.513	8.944	13.105	22.737
Obs.	10700	10700	10694	10700	9410	10699	9590	10700
Mean	5.066	3.926	2.866	5.457	1.454	1.725	0.306	2.419

Notes: This table shows the estimate for the effect of the retirement rate of firefighters of the position indicated in the column head lagged by one month on the logarithm of the number of operative firefighters of the same position and fire department. All regressions include province and month fixed effects. Standard errors are clustered at province level. *F-statistic* is the F-statistic testing the null hypothesis that the cofficient for lagged retirement is 0. *Mean* is the average of the dependent variable in the regression sample.

Table C6: Effect of age-based retirement rate on employment growth rate of firefighters, by position

	Basic firefighters	Squad leaders	Shift leaders	All operatives	Inspectors	Directors	Executives	All officers
Lagged retirement rate	-0.754***	-0.969***	-1.048***	-0.995***	-0.911***	-0.949***	-1.085***	-0.936***
	(0.181)	(0.195)	(0.031)	(0.037)	(0.065)	(0.056)	(0.054)	(0.069)
F-statistic	17.373	24.577	1142.249	729.342	198.135	286.307	397.743	183.836
Obs.	10700	10700	10700	10700	10700	10700	10700	10700
Mean	-0.001	-0.001	0.013	0.008	0.019	0.000	-0.000	0.000

Notes: This table shows the estimate for the effect of the retirement rate of firefighters of the position indicated in the column head lagged by one month on the employment growth rate of operative firefighters of the same position and fire department. All regressions include province and month fixed effects. Standard errors are clustered at province level. *F-statistic* is the F-statistic testing the null hypothesis that the cofficient for lagged retirement is 0. *Mean* is the average of the dependent variable in the regression sample.

Table C7: Effect of age-based retirement rate on volunteers growth rate, by position of retirees

	Basic firefighters	Squad leaders	Shift leaders	All operative	Inspectors	Directors	Executives	All directive
Lagged retirement rate	-0.234	0.052	0.008	-0.004	0.001	0.006	-0.003	0.067
	(0.273)	(0.100)	(0.011)	(0.020)	(0.007)	(0.011)	(0.006)	(0.120)
F-statistic	0.738	0.274	0.493	0.035	0.031	0.304	0.168	0.316
Obs.	10700	10700	10700	10700	10700	10700	10700	10700
Mean	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000

Notes: This table shows the estimate for the effect of the retirement rate of firefighters of the position indicated in the column head lagged by one month on the change in the number of volunteers divided by the number of average firefighters in the fire department. The last column uses retirements from all operative firefighters. All regressions include province and month fixed effects. Standard errors are clustered at province level. *Mean* is the average of the dependent variable in the regression sample.

Table C8: Effect of retirement rates on intervention time. Heterogeneity by capacity utilization.

	Log intervention time	Log response time	Log on-scene time	Log return time
		Below median capacity utilization		
		Reduced form		
Basic firefighters retirement rate	1.644	1.863*	1.310	-0.027
	(1.124)	(1.095)	(1.410)	(1.160)
Squad leaders retirement rate	-0.098	0.117	-0.119	-0.034
	(0.153)	(0.199)	(0.214)	(0.214)
Shift leaders retirement rate	0.016	-0.010	0.037	0.001
	(0.029)	(0.034)	(0.039)	(0.035)
Inspectors retirement rate	-0.009	-0.022	-0.022	0.021
	(0.019)	(0.021)	(0.024)	(0.023)
Directors retirement rate	0.023	0.055	-0.061	0.088
	(0.046)	(0.054)	(0.063)	(0.058)
Executives retirement rate	-0.003	-0.006	-0.023	-0.009
	(0.014)	(0.016)	(0.016)	(0.020)
		Two-stage least squares		
Basic firefighters growth rate	-1.995	-2.646*	-1.494	-0.259
	(1.458)	(1.552)	(1.825)	(1.450)
Squad leaders growth rate	0.421	0.211	0.328	0.075
	(0.302)	(0.316)	(0.346)	(0.298)
Shift leaders growth rate	-0.047	-0.028	-0.061	-0.004
	(0.041)	(0.049)	(0.051)	(0.042)
Inspectors growth rate	0.017	0.037	0.028	-0.023
	(0.027)	(0.029)	(0.031)	(0.025)
Directors growth rate	0.077	0.047	0.134	-0.081
	(0.081)	(0.090)	(0.094)	(0.083)
Executives growth rate	0.012	0.014	0.026	0.008
	(0.016)	(0.017)	(0.017)	(0.018)
Observations	3610906	3065906	3364499	2895156
Mean	-0.086	-1.702	-0.852	-1.527
		Above median capacity utilization		
		Reduced form		
Basic firefighters retirement rate	-1.127	-0.382	-0.575	-2.443**
	(1.272)	(1.193)	(1.615)	(1.029)
Squad leaders retirement rate	0.727**	0.606*	0.835**	0.170
	(0.306)	(0.333)	(0.350)	(0.277)
Shift leaders retirement rate	0.089**	-0.032	0.188***	0.047
	(0.042)	(0.043)	(0.050)	(0.039)
Inspectors retirement rate	0.008	0.007	0.008	0.022
	(0.025)	(0.026)	(0.033)	(0.023)
Directors retirement rate	0.013	-0.058	-0.003	0.083
	(0.066)	(0.063)	(0.085)	(0.071)
Executives retirement rate	0.013	0.009	0.008	0.038**
	(0.014)	(0.019)	(0.019)	(0.017)
		Two-stage least squares		
Basic firefighters growth rate	0.971	0.456	-0.012	3.221*
	(1.735)	(1.792)	(2.265)	(1.658)
Squad leaders growth rate	-0.631**	-0.548*	-0.734**	-0.143
	(0.256)	(0.281)	(0.297)	(0.258)
Shift leaders growth rate	-0.071*	0.039	-0.160***	-0.041
	(0.042)	(0.041)	(0.049)	(0.040)
Inspectors growth rate	-0.006	-0.003	-0.004	-0.025
	(0.030)	(0.029)	(0.038)	(0.030)
Directors growth rate	-0.048	0.023	-0.051	-0.062
	(0.083)	(0.079)	(0.104)	(0.093)
Executives growth rate	-0.011	-0.009	-0.010	-0.021
	(0.016)	(0.021)	(0.020)	(0.020)
Observations	3600135	2583824	3314534	2573967
Mean	0.119	-1.541	-0.591	-1.324

Notes: This table shows the estimate for the effect of the lagged monthly retirement rate of firefighters of the position indicated in each row on the outcome variables indicated in each column, by below and above median capacity utilization. We exclude for the sample interventions made by squads of volunteers and false alarms, and interventions that were no longer necessary. All regressions include fire station and month fixed effects. Standard errors are clustered at fire-station level. *Mean* is the average of the dependent variable in the regression sample.

Table C9: Effect of retirements on intervention time. Heterogeneity by urgency of interventions.

	(1) Log intervention time	(2) Log response time	(3) Log on-scene time	(4) Log return time
		Non-urgent interventions		
		Reduced form		
Basic firefighters retirement rate	1.609	2.023*	2.416*	-0.865
	(1.212)	(1.181)	(1.410)	(1.141)
Squad leaders retirement rate	-0.057	0.077	0.042	-0.251
	(0.309)	(0.376)	(0.349)	(0.326)
Shift leaders retirement rate	0.065*	-0.006	0.154***	0.031
	(0.036)	(0.041)	(0.048)	(0.041)
Inspectors retirement rate	0.010	-0.005	-0.020	0.015
	(0.022)	(0.027)	(0.031)	(0.025)
Directors retirement rate	0.085	0.052	-0.021	0.148***
	(0.062)	(0.064)	(0.079)	(0.057)
Executives retirement rate	-0.002	-0.006	-0.022	0.020
	(0.014)	(0.019)	(0.017)	(0.019)
		Two-stage least squares		
Basic firefighters growth rate	-2.574	-3.096*	-3.574	0.857
	(1.873)	(1.865)	(2.197)	(1.638)
Squad leaders growth rate	0.142	0.044	0.034	0.244
	(0.286)	(0.368)	(0.326)	(0.307)
Shift leaders growth rate	-0.079*	-0.017	-0.167***	-0.029
	(0.041)	(0.047)	(0.054)	(0.041)
Inspectors growth rate	0.000	0.022	0.035	-0.023
	(0.030)	(0.037)	(0.041)	(0.030)
Directors growth rate	-0.058	-0.029	0.047	-0.154
	(0.071)	(0.079)	(0.094)	(0.060)
Executives growth rate	0.008	0.012	0.029	-0.019
	(0.017)	(0.021)	(0.021)	(0.018)
Observations	3145403	2443394	2948409	2347006
		Urgent interventions		
		Reduced form		
Basic firefighters retirement rate	-0.307	-0.314	-0.322	-1.737
	(1.396)	(1.316)	(1.883)	(1.185)
Squad leaders retirement rate	0.679**	0.549*	0.673**	0.337
	(0.282)	(0.287)	(0.339)	(0.244)
Shift leaders retirement rate	0.071	-0.020	0.117**	0.027
	(0.046)	(0.042)	(0.056)	(0.040)
Inspectors retirement rate	-0.004	-0.003	0.011	0.029
	(0.026)	(0.025)	(0.033)	(0.024)
Directors retirement rate	-0.023	-0.022	-0.021	0.037
	(0.058)	(0.058)	(0.079)	(0.063)
Executives retirement rate	0.013	0.004	0.013	0.014
	(0.018)	(0.018)	(0.021)	(0.021)
		Two-stage least squares		
Basic firefighters growth rate	0.102	0.186	0.126	2.160
	(1.872)	(1.836)	(2.581)	(1.727)
Squad leaders growth rate	-0.705**	-0.574**	-0.711**	-0.494*
	(0.283)	(0.289)	(0.349)	(0.260)
Shift leaders growth rate	-0.057	0.028	-0.100*	-0.004
	(0.047)	(0.043)	(0.057)	(0.043)
Inspectors growth rate	0.009	0.009	-0.008	-0.032
	(0.029)	(0.027)	(0.037)	(0.029)
Directors growth rate	-0.018	-0.014	-0.020	-0.126
	(0.088)	(0.084)	(0.116)	(0.091)
Executives growth rate	-0.015	-0.007	-0.014	-0.012
	(0.017)	(0.016)	(0.020)	(0.019)
Observations	4065645	3206348	3730632	3122127

Notes: This table shows the estimate for the effect of the lagged monthly retirement rate and the growth rate of the number of firefighters indicated in each row on the outcome variables indicated in each column, by urgent and non-urgent interventions. Urgent interventions include fires, explosions, gas leaks, transport accidents, landslides, earthquakes, floods, people searches and rescues. Non-urgent interventions include recovery of goods, removal of debris, leaning trees, pest control, animal rescues, elevator rescues, door/window openings, safety checks. We exclude for the sample interventions made by squads of volunteers and false alarms, and interventions that were no longer necessary. All regressions include fire station and month fixed effects. Standard errors are clustered at fire-station level. *Mean* is the average of the dependent variable in the regression sample.

Table C10: Effect of retirements on intervention time. Heterogeneity by intervention urgency.

	(1) Log intervention time	(2) Log response time	(3) Log on-scene time	(4) Log return time
		Non-urgent interventions		
		Reduced form		
Operative retirement rate	0.637*	0.067	1.599***	-0.024
	(0.365)	(0.442)	(0.472)	(0.439)
Officer retirement rate	0.025	-0.023	-0.094	0.094*
	(0.047)	(0.059)	(0.064)	(0.056)
		Two-stage least squares		
Log(operatives)	-0.415	-0.088	-1.327***	0.174
	(0.275)	(0.329)	(0.383)	(0.325)
Log(officers)	-0.024	0.044	0.233*	-0.172*
	(0.091)	(0.108)	(0.136)	(0.104)
Observations	3145403	2443394	2948409	2347006
		Urgent interventions		
		Reduced form		
Operative retirement rate	0.769*	-0.086	1.185**	-0.066
	(0.454)	(0.416)	(0.569)	(0.401)
Officer retirement rate	0.003	-0.011	0.039	0.078
	(0.056)	(0.054)	(0.074)	(0.055)
		Two-stage least squares		
Log(operatives)	-0.595	0.050	-0.855*	0.155
	(0.384)	(0.323)	(0.494)	(0.313)
Log(officers)	0.072	0.012	0.042	-0.153
	(0.123)	(0.109)	(0.169)	(0.114)
Observations	4065645	3206348	3730632	3122127
Mean	0.136	-1.618	-0.528	-1.329

Notes: This table shows the estimate for the effect of the lagged monthly retirement rate and the growth rate of the number of firefighters indicated in each row on the outcome variables indicated in each column, by urgent and non-urgent interventions. Urgent interventions include fires, explosions, gas leaks, transport accidents, landslides, earthquakes, floods, people searches and rescues. Non-urgent interventions include recovery of goods, removal of debris, leaning trees, pest control, animal rescues, elevator rescues, door/window openings, safety checks. We exclude for the sample interventions made by squads of volunteers and false alarms, and interventions that were no longer necessary. All regressions include fire station and month fixed effects. Standard errors are clustered at fire-station level. *Mean* is the average of the dependent variable in the regression sample.

Table C11: Effect of age-based retirement rate on injuries and casualties, by operative and directive personnel.

		Reduced form		
	Civilian injury	Firefighter injury	Civilian casualty	Firefighter casualty
Operative retirement rate	-0.026	0.007*	-0.019	0.000
	(0.036)	(0.004)	(0.036)	(0.000)
Directive retirement rate	0.006	-0.000	0.006	-0.000
	(0.006)	(0.001)	(0.006)	(0.000)
		Two-stage least squares		
Operative retirement rate	0.026	-0.007*	0.020	-0.000
	(0.036)	(0.004)	(0.036)	(0.000)
Directive retirement rate	-0.006	0.000	-0.006	0.000
	(0.006)	(0.001)	(0.006)	(0.000)
Obs.	7342030	7342030	7342030	7342030
Mean	0.021	0.000	0.021	0.000

Table C12: Effect of age-based retirement rate by position on injuries and casualties.

	Civilian injury	Firefighter injury	Civilian casualty	Firefighter casualty
		Reduced form		
Basic firefighters retirement rate	-0.002	0.012	-0.008	-0.000
	(0.111)	(0.013)	(0.052)	(0.000)
Squad leaders retirement rate	-0.020	0.002	-0.012	0.000
	(0.023)	(0.003)	(0.009)	(0.000)
Shift leaders retirement rate	0.000	0.001	-0.001	-0.000
	(0.003)	(0.000)	(0.001)	(0.000)
Inspectors retirement rate	0.002	-0.000	0.001	-0.000
	(0.002)	(0.000)	(0.001)	(0.000)
Directors retirement rate	0.004	0.000	-0.000	-0.000
	(0.007)	(0.001)	(0.003)	(0.000)
Executives retirement rate	0.003*	-0.000	0.001	-0.000
	(0.001)	(0.000)	(0.001)	(0.000)
Obs.	7342030	7342030	7342030	7342030
		Two-stage least squares		
Basic firefighters growth rate	0.017	-0.018	0.027	-0.000
	(0.148)	(0.019)	(0.070)	(0.000)
Squad leaders growth rate	0.020	-0.001	0.011	-0.000
	(0.023)	(0.002)	(0.009)	(0.000)
Shift leaders growth rate	-0.001	-0.001*	0.001	0.000
	(0.003)	(0.000)	(0.001)	(0.000)
Inspectors growth rate	-0.002	0.000	-0.001	0.000
	(0.003)	(0.000)	(0.001)	(0.000)
Directors growth rate	-0.003	-0.000	0.000	-0.000
	(0.007)	(0.001)	(0.003)	(0.000)
Executives growth rate	-0.002*	0.000	-0.001	0.000
	(0.001)	(0.000)	(0.001)	(0.000)
Obs.	7342030	7342030	7342030	7342030
Mean	0.021	0.000	0.005	0.000

Notes: This table shows the estimate for the effect of the lagged monthly retirement rate and growth rate of firefighters of the position indicated in each row on the outcome variables indicated in each column. We exclude from the sample interventions made by squads of volunteers and false alarms, and interventions that were no longer necessary. All regressions include fire station and day fixed effects. Standard errors are clustered at fire-station level. *Mean* is the average of the dependent variable in the regression sample.

Table C13: Effect of age-based retirement rate on burned hectares reported by squad leaders, by operative and directive personnel.

	Log hectares burned
	Reduced form
Operative retirement rate	4.979
	(3.323)
Directive retirement rate	-0.400
	(0.493)
Obs.	481952
	Two-stage least squares
Operative retirement rate	-4.706
	(3.203)
Directive retirement rate	0.403
	(0.480)
Obs.	481952
Mean	-0.657

Notes: This table shows the estimate for the effect of the lagged monthly retirement rate and growth rate of operative and directive firefighters on the outcome variable indicated in each column. Operative firefighters include basic firefighters We exclude from the sample interventions made by squads of volunteers and false alarms, and interventions that were no longer necessary. All regressions include fire station and day fixed effects. Standard errors are clustered at fire-station level. *Mean* is the average of the dependent variable in the regression sample.

Table C14: Effect of age-based retirement rate by position on burned hectares reported by squad leaders.

	Log hectares burned
	Reduced form
Basic firefighters retirement rate	-11.898
	(7.878)
Squad leaders retirement rate	-0.948
	(2.151)
Shift leaders retirement rate	0.900***
	(0.332)
Inspectors retirement rate	0.068
	(0.216)
Directors retirement rate	-1.654***
	(0.371)
Executives retirement rate	0.070
	(0.160)
Obs.	481952
Mean	
	Two-stage least squares
	Log hectares burned
Basic firefighters growth rate	15.114
	(10.470)
Squad leaders growth rate	0.298
	(2.800)
Shift leaders growth rate	-0.751**
	(0.348)
Inspectors growth rate	-0.144
	(0.254)
Directors growth rate	1.260**
	(0.548)
Executives growth rate	0.120
	(0.187)
Obs.	481952
Mean	-0.657

Notes: This table shows the estimate for the effect of the lagged monthly retirement rate and growth rate of firefighters of the position indicated in each row on the outcome variables indicated in each column. We exclude from the sample interventions made by squads of volunteers and false alarms, and interventions that were no longer necessary. All regressions include fire station and day fixed effects. Standard errors are clustered at fire-station level. *Mean* is the average of the dependent variable in the regression sample.

Table C15: Average time of interventions, model simulations.

	Mean	S.d.	N
Observed	86.885	97.827	7,253,602
Space constraints	76.820	83.930	7,215,016
Position constraints	77.673	85.846	7,253,602
Time constraints	76.559	84.492	7,253,602
Optimal	74.778	82.647	7,253,602

Notes: This table shows the observed and simulated average intervention time (in minutes) under different allocation constraints.