Banks' skin-in-the-game and hold-up by illiquid firms: strategic bargaining, dynamic inconsistency, and credit constraints[1]

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Abstract

The bank-firm relationship features a bilateral monopoly. The bank acquires monopolistic information on the firm: this is known to induce hold-up by banks at loan renewal. Conversely, if assets are firm-specific, only the firm can fully repay the bank. Both features together give rise to strategic bargaining at loan renewal. Bargaining powers are determined only by discount rates and interest flows. Outside options (which are unrelated to bargaining powers due to the outside option principle) are affected by financial frictions such as asset specificity. Credit constraints affect the first-period loan, that is, the bank declines to lend initially, if (1) the perfect equilibrium partition determined by bargaining powers does not allow the bank to break even on the second-period loan and (2) when the initial bank declines to roll over, the firm benefits more by defaulting than by borrowing from another bank. Condition (2) means that the bank cannot break even by declining to roll over. Such hold-up by illiquid firms provides a new foundation for long-term lending to finance long-term projects.

Keywords

Hold-up, asset specificity, liquidity, subgame perfection,
non-cooperative bargaining, strategic default, credit constraints

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1. Introduction

Why are long-term loans needed to finance long-term investment[3]? I propose a new answer: *ex-post,* illiquid firms can hold up banks, which therefore decline to lend *ex-ante*. The mechanism is as follows: if bank b lends short, she has skin-in-the-game (SITG) and may be forced to grant a new loan at abnormally favorable rates to have the initial loan repaid and the project completed[4]. That is, firm f may hold up b. Then, *ex-ante*, b declines to lend short.

It is known that b has monopolistic information on f and has hold-up power. But if f's asset is specific, and so is worth less than the loan, only f can fully repay b: f also has monopoly power. When the loan must be rolled over, a bilateral monopoly exists so that b and f must bargain to share the surplus.

Agents are self-interested, and this bargaining process is non-cooperative. The game is not repeated, ruling out "Folk theorem" solutions: b and f cannot implement threats that are not optimal *ex-post*. That is, their actions must be supported by subgame-perfect strategies. Rubinstein (1982) analyses such situations in an alternating offer model and finds a unique Perfect Equilibrium Partition (PEP).

The PEP corresponds to an agreement between b and f on a new loan. Negotiating powers, which determine the PEP, are derived from discount factors, regardless of who plays first (Binmore, 1986, Sutton, 1986). Additionally, b's power is weakened since she must pay interest on what she borrowed from financial markets to lend to f, whereas what f pays b is part of the negotiation: it is an outcome of the negotiation, not an input.

This PEP is bracketed by outside options that b or f calls if this is more profitable than the PEP. Conversely, these outside options do not affect the PEP (outside option principle, Binmore *et al.* 1989). b has two outside options: declining to roll over the loan or seizing collateral (i.e., forcing f's bankruptcy). f has two outside options: borrowing from another bank or filing for bankruptcy.

If, when b declines to roll over, f is better off defaulting than borrowing from another bank, then and only then will b have a weak outside option: both her options lead to f's bankruptcy and since by assumption f's asset is worth less than the loan value, b cannot break even by calling either option.

To analyze this situation, I extend earlier PEP results (Binmore, 1986, Sutton, 1986) to include interest-bearing or interest-costing assets. I combine macroeconomic practice, which routinely considers income flows, and the game-theoretic literature which provides microeconomic foundations. I build on Sutton, (1986) rather than Binmore *et al.* (1986). To combine the PEP and outside options, I extend

[3] In France, the median initial maturity of construction investment loans is 82 months compared with only 2.5 months for export/import credit and 4 months for working capital facility lending. Computations are based on ANACREDIT data for 2023, which do not single out non-construction investment loans.

[4] Throughout the analysis, I use "she" for the initial bank b and "he" for the firm/entrepreneur f to ensure clarity, conciseness and neutrality.

the outside-option principle (hereafter OOP) to options that are binding (*i.e.,* better than the PEP for a player) and/or include a loss threat: even then, the option cannot be leveraged to increase the holder's income (or only infinitesimally).

Overall, b declines to lend short *ex-ante* if two conditions are satisfied: (1) the PEP does not grant b a break-even income (this must hold regardless of f's quality over the second period, which prevents b from compensating losses in bad states with profits in good states); (2) b has a weak outside option. Noteworthily, b declining to lend *ex-ante* means that credit constraints occur, but also that the *ex-post* bargaining mechanism identified in the present study is off-equilibrium path.

The mechanism relies on the initial bank having **skin-in-the-game** (b has a vested interest in f's ability to repay the loan and may accept to roll over at a loss) and three **frictions**: f's *illiquidity*, the *unverifiability* of f's probability of default (PD) by a court, and the possibility of *strategic default*. They form a minimum set of frictions likely to induce hold-up by firms, while for instance, information asymmetries between the bank and the firm or moral hazard are not required. SITG and illiquidity create the conditions for a bilateral monopoly (and therefore hold-up) and the three frictions affect outside options, as explained in the following paragraphs.

Banks' SITG is pervasive. Caballero *et al.* (2008) show that during the Japanese 1990s crisis, large banks financed insolvent borrowers in the hope of recovery. This depressed job dynamics, dampened productivity, and prolonged stagnation. SITG also generalizes the soft-budget-constraint analyzed by Dewatripont and Maskin (1995) in which a bank must roll over without another bank's support. From the viewpoint of the property-rights theory (Grossman and Hart, 1986, Hart and Moore, 1990), SITG corresponds to a specific investment made by b by lending to f, giving f hold-up power.

Illiquidity has two components: timing and asset specificity. The timing component is the most obvious: f is illiquid after building the productive asset because he has not produced and sold yet, and he needs a new loan to repay the first one. At that time, the asset is illiquid because it is f-specific. Indeed, an asset is liquid *iff* it has the same value for all agents (Kiyotaki and Moore, 2001).

After the short-term loan, f's asset is specific and b incurs a loss if she seizes this asset. Conversely, over the second period, f-specific productive asset is turned into fully versatile sales incomes and b can seize these incomes to be fully repaid if f attempts to hold her up. Thus, asset specificity (documented, e.g., by Kermani and Ma, 2023) provides an alternative foundation to the Diamond and Dybvig (1983) assumption that long-term projects are productive but are inefficient if liquidated early.

Therefore, asset specificity has two effects at the end of the first period. First, it gives monopoly power to f, causing bilateral monopoly. Second, it limits the value of one of b's options (seizing f's asset).

The *unverifiability* of the firm's situation by a court is a common assumption. I assume more specifically that f's PD is unverifiable. This is corroborated by institutional evidence and is realistic because, even *ex-post*, neither default nor survival provides compelling evidence on *ex-ante* risk. Here, unverifiability relates directly to the outside option values for b and f of f's bankruptcy. Indeed, could a court verify f's PD, b would seize the asset and ask the court to impose a fair rate over the second period-loan, thereby preventing hold-up by the firm. This cause of incompleteness of loan contracts complements the views of Hart and Moore (1998) who rather assume income' unverifiability (see § 5.3). Pending further investigation, it seems immune to the critique of Maskin and Tirole (1999).

Finally, firms can choose *strategic default*, although moral hazard is assumed away. I provide novel empirical results that entrepreneurs quickly default after a positive shock to private benefits. This is consistent with benefits extraction at the time of bankruptcy, while Giroud *et al.* (2012) find effects over the years before default. This is also consistent with a direct effect, while Schoenherr and Starmans (2022) find that private benefits affect risk-taking, which causes default only after a while. This possibility for f to default strategically is key in the mechanism causing the weak option case, which in turn makes option values of f's bankruptcy relevant.

These frictions are instrumental in explaining why long-term loans are not affected by this new credit constraints mechanism. After a long-term loan, f has produced, sold, and has liquid assets that a court can verify to force f to repay b. Alternatively, f may want to default in advance to force renegotiation on the long-term loan, but the court can verify that f does not need to repay in advance, and can prevent default. Even if f were able to default in advance, such default would inform the court that f is either unlikely to pay his future debt (making him unable to extort a low rate on a new loan) or attempting to hold up b. That is, long-term debt prevents illiquidity and limits unverifiability of f's PD and (directly or not) his ability to default strategically.

This new mechanism explains long-term lending even when institutional features counteract **traditional answers**. *Banks' inability to roll over short-term loans* (Diamond and Dybvig, 1983) has pushed governments and central banks to establish deposit insurance and loan refinancing, strongly limiting banks' default risk, and public resolution mechanisms. Second, *hedging by firms against interest rate changes* explains the preference for long-term lending; the share of variable-rate loans drops from 54% for maturities below one year to 9% for maturities between three and four years (French ANACREDIT data, 2023). But proper derivatives may contribute to hedging against macroeconomic-related rate changes, limiting the need for long-term loans. Finally, *firms may hedge against hold-ups*: banks accumulates inside information and may extract a monopoly rate at loan renewal (Rajan, 1992, Houston and James, 1996, Farinha and Santos, 2002, Santos and Winton, 2008, Hale and Santos, 2009, Schenone, 2010, Bird *et al.*, 2019), leading firms to favor long-term loans to

avoid such an outcome. Still, hold-up by banks relies on costly information acquisition. Conversely, the French Central bank rated 300.000 firms in 2023, limiting banks' informational monopoly and their ability to hold up firms. Many European economies exhibit similar conditions.

Overall, the mechanism identified here is consistent with institutional features put in place to tackle the problems underlying some of the traditional answers, and explains why long-term projects cannot be funded without long-term financing (Choudhary and Limodio, 2022).

The findings present several policy implications including fostering long-term bank lending (as for example Diamond and Dybvig, 1983, but for very different reasons), and providing courts with independent external ratings to alleviate unverifiability frictions.

In addition, the contracts proposed by von Thadden (1995) to protect firms from hold-up by banks (with a rate fixed *ex-ante* but a decision to roll over by the bank) could be adapted to protect banks from hold-up by firms. Interestingly, such contracts would provide for low interest rates in the first period and high interest rates in the second period, to maximize the probability of rollover. Increasing rates over the course of the loan relationship are often used to provide empirical support for the importance of asymmetric information in banking. This result shows that more precise research on this important topic is needed.

The rest of this study is organized as follows. Section 2 presents the model. Section 3 extends the results of Binmore (1986) and Sutton (1986) by including interest-bearing assets. It also extends the OOP to the binding option and loss threat cases. Section 4 solves the model backwards using the tools developed in Section 3 and identifies the conditions for hold-up by f. Section 5 identifies the financial frictions supporting these results. Section 6 concludes.

2. Structure of the economy

In a two-period production economy, firm f builds productive capacity over the first period (from T^0 to T^1) and produces over the second period (from the first-period loan repayment to T^2). A loan negotiation game between f and bank b starts at T^1, at the juncture of both periods. Following Diamond and Dybvig (1983) long-term production is profitable but liquidation induces a loss related to the lack of liquidity of the asset built over the first period.

2.1 Firms

Production process. Firm f builds a factory over period 1 and produces over period 2. If f survives until T^2, he produces y (raw value at T^2).

Capital misallocation is primarily due to the extensive margin (Buera *et al.*, 2011, Midrigan and Xu, 2014). Accordingly, the project has unit size and is financed or not binarily (by a single bank), which

avoids issues related to scale effects. Specifically, this avoids the discrepancy between productivity and profitability induced by downsizing production (Agrell and West, 2001). The investment decision is based on project characteristics but not on other firm characteristics, that do not influence yield. That is, the firm is equivalent to the project (and besides, equivalent to the entrepreneur).

Prices are constant and fixed at unity, eliminating the discrepancy between productivity and profitability documented by Foster *et al.* (2008) when prices vary by demand and market power; therefore, we can focus on productivity issues. Projects atomisticity may rationalize constant prices.

Default risk. f is risk neutral. Over period 1, f's PD (p_X^1) is null. Over period 2, with probability q, f is in the "good" state and has PD equal to p_G^2**;** with probability $1-q$, f is in the "bad" state and has the PD equal to p_B^2 . p_B^2 and p_G^2 are the exogenous or "fundamental" *failure rates* of f. Additionally, f can choose bankruptcy, but this outcome is out-of-equilibrium. In other words:

[1] $$\begin{bmatrix} P(p_G^2) \\ P(p_B^2) \end{bmatrix} = \begin{bmatrix} q \\ 1-q \end{bmatrix} \text{ with } p_G^2 < p_B^2$$

The mechanism examined in this study would also hold true for constant or even null PDs. However, PDs are naturally stochastic, justifying the unverifiability assumption detailed in § 5.3.

2.2 Banks

Banks are risk-neutral. As in Petersen and Rajan (1995) and von Thadden (1995), they are the only source of external finance. At T^0, banks are able to lend short-term (one-period) for projects with a positive NPV. At T^1, each bank is able to roll over the first-period loan.

All banks have the same cost structure. Cashless banks fully refinance on financial markets at the positive but negligible risk-free rate r_i over period i (the change from r_1 to r_2 occurs when the first loan is repaid, i.e., when the new loan starts). The first time a bank lends to f, she incurs the positive information cost c_s. Banks never default, due to regulations and supervision to prevent bank failures.

2.3 Information

To identify a bare minimum set of frictions necessary to induce credit constraints, I rule out information asymmetries at T^0: all the banks and f know p_X^1, q, p_B^2, p_G^2 and y. Over period 1, b (the bank that granted the first-period loan) bears the information cost c_s over and learn f' second-period state (G or B). Other banks can pay c_s to learn it at T^1, so that perfect competition corresponds to c_s=0, which is also symmetric information between b and f.

The mechanism identified in the present paper can hold with c_s=0. Still, for the sake of generality and comparability with the literature, it is interesting to also consider positive information costs. The literature generally assumes that if $c_s > 0$, a firm that turns to a new bank must pay c_s. To get such a result in a rigorous framework, I assume that banks (other than b) form a competitive margin; I adapt

the framework of Mazet-Sonilhac, (2022) and the firm that wants to borrow from a new bank faces a one-shot cost c_f to transmit information to all these banks. In the rest of the analysis, when considering the situation at T^1, the information cost c_s (which is the cost born by b over period 1) actually stands for the (identical-valued) c_f and is borne by f if he decides to borrow from another bank. Finally, I assume that f does not need to borrow to pay c_f and does not pay or receive related interests.

In line with (almost) symmetric information, I rule out costly state verification (Townsend, 1979, Gale and Hellwig, 1985); the final production y is costlessly observed by banks and courts.

Conversely, the court ignores and cannot verify stochastic elements, namely p_X^1, q, p_B^2, p_G^2, G and B. this unverifiability assumption is discussed in § 5.3.

Also by the symmetry assumption, I rule out moral hazard. Firms always exert effort (this is a crucial difference from, e.g., von Thadden, 1995) and cannot increase risk to increase the expected private benefits. However, in the short-term loan case, the firm can (threaten to) decline to borrow from another bank if the existing bank declines to roll over. This enables the firm to extort low rates from this initial bank.

2.4 Firm failures

The project requires an investment of size 1 to build an asset. In the case of failure over period 1, the bank recovers only Γ^1 with $0 < \Gamma^1 \leq 1$. $1 - \Gamma^1$ measures asset specificity (Kermani and Ma, 2023), providing new foundation for the liquidation-related loss assumption (Diamond and Dybvig, 1983).

Alternatively, $1 - \Gamma^1$ may represent profits diverted by the firm, which may look surprising with a well-functioning legal system given the symmetric information assumption. However, an entrepreneur can legally appropriate private benefits (e.g., when including his wage in these private benefits). Moreover, in § 5.1, I show that the net private benefits *at the time of default* matter. To better distinguish between these mechanisms, private benefits are identified as such and labeled PB^1 with:

[2] $$0 \leq PB^1 \leq 1 - \Gamma^1$$

The right-hand-side constraint derives from the assumption that no production occurs over period 1. Similarly, I define Γ^2 (what the bank receives in the case of failure over period 2) and PB^2 with:

[3] $$0 \leq PB^2 \leq y - \Gamma^2$$

Notably, the interpretation of Γ^2 may differ from that of Γ^1. Γ^1 is the asset's collateral value before production starts. But the asset is exhausted at T^2 (if not, production continues after T^2). Γ^2 is rather the value of a blanket lien providing b with a guarantee based on f's overall production. Thus, at T^1, Γ^1 may be interpreted as the liquidation value of the firm and Γ^2, at least to some extent, as its going-concern value (Kermani and Ma, 2020). Finally, f bears a fixed failure cost F in the case of default.

3. Method

Here, I explain why bank b and firm f implement non-cooperative bargaining with subgame-perfect strategies when the loan must be renewed and build the tools necessary to analyze it.

3.1 Non-cooperative bargaining, subgame-perfection, and their consequences

b and f are rational and self-interested. They implement non-cooperative bargaining since they cannot get more by cooperating: the surplus (defined by [24]) cannot be increased through cooperation.

At T^0, f has committed to repay the loan, but at T^1 he prefers to renegotiate if he finds it more profitable, indicating dynamic inconsistency. If b and f agreed on an unconditional second-period loan, that would be a long-term loan. A conditional contract for the second-period loan would not be enforceable because a crucial aspect of such a contract, i.e., PD, cannot be observed by a court (§ 5.3).

Bargaining is not repeated, preventing bargainers from using "Folk theorem" strategies (*i.e.,* implementing loss-loss threats to affect their opponent's strategy)[5]. Thus, rational agents with common knowledge of their rationality implement subgame-perfect strategies, the model must be solved backward, and even off-equilibrium outcomes must be considered carefully. Consequently, I consider what would happen after f's bankruptcy, even if b and f actually avoid such a costly outcome. Under the usual conditions in the bargaining literature, a unique subgame-perfect equilibrium exists.

Rubinstein (1982) introduces models of non-cooperative (strategic) bargaining with alternating offers and shows that only two subgame-perfect equilibria exist, depending on the time preference of both agents and who makes the first offer. Here, a player can immediately make a counteroffer after rejecting an offer: Δ, the time-interval between two offers, tends towards zero. This makes the first player's identity irrelevant: a unique subgame-perfect equilibrium exists (Binmore, 1986, Sutton, 1986, Binmore *et al.*, 1986, Binmore, 1987, Binmore *et al.*, 1989, Osborne and Rubinstein, 1990, prop. 4.5).

When δ_f (resp. δ_b) is the discount factor for player *f* (resp. *b*), the share of player *f* (resp. *b*) is $s_f^* = \frac{\ln \delta_b}{\ln \delta_f + \ln \delta_b}$ (resp. $s_b^* = \frac{\ln \delta_f}{\ln \delta_f + \ln \delta_b}$) (Sutton, 1986, p. 711, Binmore, 1987, p. 73, Osborne and Rubinstein, 1990, p. 84). If r_b (resp. r_f) is the rate facing *b* (resp. *f*), then $\delta_b = \frac{1}{1+r_b}$; $\delta_f = \frac{1}{1+r_f}$. Noteworthily, rates play a role, although no interest is paid or received during negotiations in these baseline models.

By the outside option principle (Binmore *et al.*, 1989), outside options do not affect the computation of this equilibrium, but a bargainer replaces this equilibrium partition with the outside option if it is more profitable. Binmore *et al.* (1989) write (p. 756): "how can a bargainer use his outside option to gain leverage? By threatening to play the deal-me-out card [i.e., calling the outside option]. When is

[5] I thank Gisèle Umbhauer for underlying this point.

such a threat credible? Only when dealing himself out gives the bargainer a bigger payoff than dealing himself in. It follows that the agreement that would be reached *without* outside options is *immune* to deal-me-out threats, unless the deal assigns one of the bargainers less than he can get elsewhere". That is, outside options do not affect the bargaining solution but constrain its validity range.

A unique result ensues: either the perfect equilibrium or the outside option, depending on what is more profitable (Binmore *et al*, 1986, prop. 6, Sutton, 1986, § 3, Binmore, 1986, Binmore *et al.*, 1989).

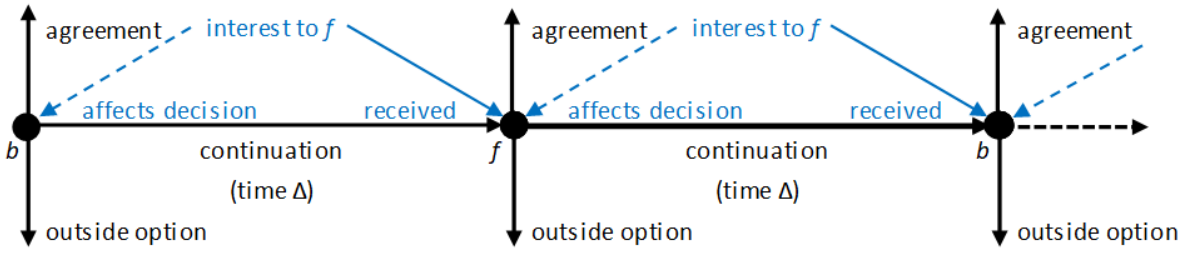
In § 3.2, I analyze the perfect equilibrium in a strategic (non-cooperative, sequential) bargaining with interest-bearing or interest-costing assets. In § 3.3, I provide extensions of the outside option principle.

3.2 Bargaining with interest-bearing or interest-costing assets

Classical game-theoretic models of alternative bargaining rely on three components: the surplus to be shared and discounting factors, which together determine a central equilibrium, and outside options. Crucially, this surplus is not delimited by the outside options (Binmore *et al.*, 1986, prop. 6, Sutton, 1986, § 3, Binmore, 1986, Binmore *et al.* 1989).

Macroeconomists rightly connect disagreement points to income flows during negotiations, but often resort to cooperative bargaining (Hall and Milgrom, 2008, being an exception) and do not distinguish between disagreement points and outside options. Also, they do not consider the subgame-perfection of supporting strategies. On the game-theoretic side, Binmore, Shaked and Sutton (1989) correctly define the disagreement point as "income flows during a [negotiation]" and outside options as the best income flows if collaboration stops. Still, to the best of my knowledge, the game-theoretic literature has not formalized the effect of interest- flows during negotiations yet (Binmore *et al.*, 1986, p. 182, fix income flows to zero in the -relevant- time-preference case). I provide such analysis here.

Graph 1: Bargaining game of alternating offers with interest income during the negotiation



Let S be the total surplus (including the value of interest-bearing assets) to be shared between *b* and *f*. The bargaining process consists of alternating offers with immediate responses. The time between two offers is *Δ*, which tends toward zero. Discounting factors are determined by the interest rate to which each player is exposed: $\delta_f = \frac{1}{1+r_f}$ and $\delta_b = \frac{1}{1+r_b}$. s_f is the share of the surplus that *f* offers to keep and s_b is the share that *b* offers to keep. *f* has an asset *P* that pays interest at rate r_f.

I first consider the case in which *b* offers first. We get:

[4] $$s_b = \frac{\left(1-\delta_f^{\Delta}\right)(S-P)}{S\left(1-\delta_f^{\Delta}\delta_b^{\Delta}\right)}$$

Proof: see Appendix 1. Letting $\Delta \to 0$, and using $1 - x^{\Delta} \underset{\Delta \to 0}{\longrightarrow} -\ln\ x$, we get:

[5] $$s_b = \frac{ln\delta_f}{ln\delta_f + ln\delta_b}\frac{(S-P)}{S}$$

Then I consider the case in which *f* offers first. We get:

[6] $$s_f = \frac{S - \delta_b^{\Delta}\left[S - P\{(1+r_f)^{\Delta} - 1\}\right]}{S\left(1-\delta_b^{\Delta}\delta_f^{\Delta}\right)}$$

Proof: see Appendix 1. Letting $\Delta \to 0$, and using $1 - x^{\Delta} \underset{\Delta \to 0}{\longrightarrow} -\ln\ x$, we get:

[7] $$s_f S = (S+P)\frac{\ln \delta_b}{\ln \delta_f + \ln \delta_b} - P\frac{\ln \delta_b - \ln \delta_f}{\ln \delta_f + \ln \delta_b} = (S+P)s_f^* - P\frac{\ln \delta_b - \ln \delta_f}{\ln \delta_f + \ln \delta_b}$$

If $\delta_f = \delta_b$ (symmetric interest rates case), then $\frac{\ln \delta_b - \ln \delta_f}{\ln \delta_f + \ln \delta_b} = 0$, $s_f = \left(1 + \frac{P}{Z}\right)\frac{1}{2}$ and $s_b = \left(1 - \frac{P}{Z}\right)\frac{1}{2}$. The existence of the asset distorts the partition of surplus S in favor of interest-receiving player *f* at the expense of *b*. More generally, when $\delta_f \neq \delta_b$, we have:

[8] $$s_b S + s_f S = S + P\frac{\ln \delta_b - \ln \delta_f}{\ln \delta_f + \ln \delta_b} - P\frac{\ln \delta_b - \ln \delta_f}{\ln \delta_f + \ln \delta_b} = S$$

3.3 Extensions of the outside option principle

Hereafter, I consider extensions of the outside option principle, which cannot be "proven": Binmore *et al.* (1989) do not prove it but rather show its empirical relevance. Still, it is clear that the underlying reason for the outside option principle is the lack of a subgame-perfect strategy supporting the threat to use the option when it is not "binding" (i.e., when the PEP provides more to the option holder).

First, I consider a special case of the outside option principle: what happens when calling the outside option induces a loss to the other bargainer? Such a situation typically occurs when the outside option of a creditor leads to the debtor's bankruptcy, since the debtor then incurs (at least) the administrative costs of bankruptcy. The answer is that the outside option principle fully applies and if the outside option is not binding, a bargainer cannot get more by threatening to deal himself out.

I consider the case in which the PEP provides bargainer *b* with the utility (or amount) P^b and bargainer *f* with the utility $S_f = S - S_b$ where S is the surplus to be shared between *b* and *f*. I assume that P^b is higher than the utility *b* can get by playing the "deal-me-out" card:

[9] $$Out_b < S_b$$

When *b* chooses the outside option, he incurs a specific positive loss F on *f*:

[10] $$Out_f = S_f - F$$

To understand whether *b* can exert pressure on f and extort more than S_b by threatening f to implement the outside option, I refer to Binmore *et al.* (1989, p. 756; "Deal-me out" means using the outside option): "how can a bargainer use his outside option to gain leverage? By threatening to play the deal-me-out card. When is such a threat credible? Only dealing himself out gives the bargainer a bigger payoff than dealing himself in. It follows that the agreement that would be reached *without* outside options is *immune* to deal-me-out threats, unless the deal assigns one of the bargainers less than he can get elsewhere."

Principle A: A player with a non-binding outside option including the threat of loss to the other player can neither extort strictly more than the option nor affect the other player's actions.

When the outside option is not binding, the maximum of this option and the PEP is the PEP, so the outside option principle means that the option cannot be leveraged to increase this maximum utility. Now, I extend the analysis to the case in which the outside option is binding, and I show that even in this case, the option cannot be leveraged to increase this maximum. This is relevant here since, as will be clear from Section 4, the bank pays interest during the negotiation, which from § 3.2 distorts the perfect-equilibrium against the bank, making the binding case likely. Also, this binding case corresponds to "traditional" situations in which the collateral value matters for the bank.

Thus, I consider the case in which the outside option provides *b* with more utility than the PEP determined by the surplus, discount factors, and interest-bearing or interest-costing assets. Such an option can be labeled "binding" by making an analogy between the option and a condition. The questions are: can the bargainer who has this option gain more than this option and can he affect his opponent? The answer is no because

[11] $$S_b < Out_b$$

Can *b* obtain more than Out_b? We must distinguish the case in which *f* makes the first offer from that in which *b* makes the first offer. For simplicity, I assume that the surplus is 1, so that $s_f = S_f$ and $s_b = S_b$ with $s_f = 1 - s_b$

Principle B: Even when an outside option is binding, the player who has this outside option can neither obtain more than the outside option nor affect the other player's actions.

A distinct question is: what happens when an outside option is binding *and* induces a loss to the other player? When *b* chooses the outside option, it results in a positive loss F for *f*:

[12] $$(S^f | Out_b) = P^f - F$$

Additionally, *b*'s option is binding:

[13] $$S_b \leq Out_b$$

Principle C: A player who has a binding outside option that includes a positive loss threat can affect the other player's actions and extract strictly more than this outside option. However, while the additional amount is positive, it can be made arbitrarily small by the other player.

4. Results

To solve the model backward, I consider two cases that constitute the outside options for *f* and *b* during the second-period loan negotiation: default (§ 4.1) and repayment with a loan from a new bank (§ 4.2). Then, I study the negotiation (§ 4.3) and identify sufficient conditions for hold-up by *f* (§ 4.4).

To simplify the computations and facilitate a comparison with most of the bank-firm literature, I assume that *f's*' information costs are zero ($c_f = 0$). Screening/monitoring costs are non-negative and perfect bank competition/symmetric information corresponds to $c_s = 0$. I must compute various "fair" rates that enable *b* to break even depending on the fixed costs and loan size.

4.1 Post-default operations

After a default, f no longer has to repay the loan. b has seized the collateral asset but cannot use it: only an entrepreneur/firm can. Since the asset is f-specific, what another firm would produce with this asset is lower than y (what *f* produces), opening the way to a bilateral negotiation between f and b; then, the collateral value would not be defined *ex-ante* as Γ^1 but would be the negotiated value.

This case would rely on the assumption that after a petition had been filed, the court would not do the job properly, and *f* and *b* would continue to bargain directly. Here, I consider a more traditional case: once default occurs, a court takes control of the process and organizes an auction, preventing direct bargaining between f and b and ensuring a competitive outcome in which b always receives Γ^1 [6].

If the project fails in the first period, the firm retains only the net private benefits of default, $PB^1 - F$. Conversely, if the default is strategic (implying that the project was successful over the first period) and the asset is f-specific, f values the asset more than other potential buyers do, and can buy it back[7] at the competitive price Γ^1 during the auction process[8].

[6] It is usually considered to be optimal that the court organize a second-price auction so that f reveals his true value (but pays only Γ^1, the value of the asset to other entrepreneurs). Alternatively, here, in a first-price auction one may assume that *f* knows his competitors' valuation and f bids $\Gamma^1 + \epsilon$ (with ϵ positive but arbitrarily small).
[7] especially if the bank had declined to roll over the loan (or to offer an unacceptable rate) and instead seized the collateral. The firm may disagree and attempt to buy back the asset (e.g., by borrowing from another bank).
[8] One may wonder if a third party would buy the asset (at a price higher than the competitive price) to negotiate with f. Actually, a third party would refuse to pay more than the competitive (i.e., non-specific user) price. Indeed, at equilibrium, most defaults may be non-strategic. Crucially, the "illiquid firm hold-up" mechanism

Buying back the asset requires borrowing $\Gamma^1 - PB^1 + F$ [9] at the "fair" rate $r_X^{2,f}$. Then, f obtains the net income:

[14] $$Z_f = PB^1 - F + \frac{1}{1+r_2}\left\{(1-p_X^2)\left(y - (\Gamma^1 - PB^1 + F)r_X^{2,f}\right) + p_X^2(PB^2 - F)\right\} - \Gamma^1$$

where $\frac{1}{1+r_2}$ recalls that if r_2 (the second-period riskless rate in financial markets) were not almost null, revenues at T^2 should be discounted[10]. For simplicity, I assume that r_2 is arbitrarily small. Then[11]:

[15] $$r_X^{2,f} = \frac{p_X^2(\Gamma^1 - PB^1 + F - \Gamma^2)}{(\Gamma^1 - PB^1 + F)(1-p_X^2)} + \frac{c_s}{(\Gamma^1 - PB^1 + F)(1-p_X^2)}$$

The last term means that the fixed screening cost is distributed over the second-period loan $(\Gamma^1 - PB^2 + F)$ and that the bank repays her own credit even if the firm defaults $(1-p_X^2)$. Finally:

[16] $$Z_f = PB^1 - F + (1-p_X^2)y - (\Gamma^1 - PB^1 + F)p_X^2(\Gamma^1 - PB^1 + F - \Gamma^2) - c_s + p_X^2(PB^2 - F) - \Gamma^1$$

The firm must borrow from a bank to pay the initial bank. However, the initial bank has a price advantage for such a loan because it has already borne the screening costs. *Ex-post*, it is suboptimal for the firm to borrow from another bank or for the bank to decline lending. Thus:

Result A: If the firm must borrow to buy back the asset, it borrows from the initial bank at rate $r_X^{2,f}$, and the bank obtains the collateral value Γ^1 and the profit on the second-period loan c_S:

[17] $$Z_b = \Gamma^1 + c_s$$

4.2 The other bank option

A^2 is the amount borrowed over period 2, *i.e.* from T^1 to T^2, if f repays the first-period loan. As no production occurs over period 1, A^2 covers the repayment of the (unit) principal and period-1 interest:

[18] $$A^2 = 1 + r_W^1$$

where r_W^1 is the first-period interest rate with $W = G$ if f was *ex-ante* relatively safe over the first period and $W = B$ if f was relatively risky.

f has the option to borrow from another bank. Depending on whether being good or bad, f could borrow at the *ex-post* "fair" rate:

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described in this paper is off-equilibrium path, and many banks would have to diverge from the equilibrium path simultaneously for the situation described here to become relevant.

[9] The firm must pay Γ^1 to buy back the asset and has received $PB^1 - F$ from the default.

[10] If defaults occurred uniformly over period 2 instead, the discount factor would rather be close to $\frac{1}{1+r_2/2}$.

[11] Here, $r_X^{2,f}$ is computed for a loan of size $\Gamma^1 - PB^1 + F$, collateral value Γ^2, and information costs c_S.

[19] $$r_X^{2,O} = \frac{A^2 r + p_X^2(A^2 - \Gamma^2) + c_s(1+r)}{(1-p_X^2)A^2} = \frac{(1+r_W^1)r + p_X^2(1+r_W^1 - \Gamma^2) + c_s(1+r)}{(1-p_X^2)(1+r_W^1)}$$

Where the superscript *o* denotes the outside option, $X = G$ if the firm is relatively safe in the second period, and $X = B$ if it is risky.

Proof:

The expected income of the new bank at T^2 *must cover the amount this bank must repay, that is* $(A^2 + c_s)(1+r)$. *Given bank competition, it cannot be higher. Thus,*

$$(A^2 + c_s)(1+r) = A^2(1-p_X^2)(1+r_X^{2,O}) + p_X^2\Gamma^2$$

$$\left(1+r_X^{2,O}\right) = \frac{(A^2 + c_s)(1+r) - p_X^2\Gamma^2}{A^2(1-p_X^2)}$$

$$r_X^{2,O} = \frac{(A^2 + c_s)(1+r) - p_X^2\Gamma^2 - A^2(1-p_X^2)}{A^2(1-p_X^2)} = \frac{A^2 r + p_X^2(A^2 - \Gamma^2) + c_s(1+r)}{A^2(1-p_X^2)}$$

QED.

Since r_2 is arbitrarily small to simplify the analysis, $r_X^{2,O} = \frac{p_X^2(A^2-\Gamma^2)+c_s}{A^2(1-p_X^2)}$ where $p_X^2(A^2 - \Gamma^2)$ is the expected cost of the firm's default incurred by the bank. In the denominator, $(1-p_X^2)$, which inflates $r_X^{S,O}$, reflects bank refunding its own creditor(s) even if the firm defaults.

The second period "fair" interest rate *for the initial bank*, $r_X^{2,F}$, corresponds to the case in which $c_s = 0$. It can be deduced directly from the second-period zero profit condition for the initial bank: $0 = (1-p_X^2)A^2 r_X^{2,F} + p_X^2(\Gamma^2 - A^2)$, which gives $r_X^{2,F} = \frac{p_X^2(A^2-\Gamma^2)}{A^2(1-p_X^2)}$

Thus:

[20] $$r_X^{2,O} \geq r_X^{2,F}$$

If *f* borrows from another bank to repay the loan, he gets:

[21] $$Y_f = \frac{1}{1+r_2}\left\{(1-p_X^2)\left(y - (1+r_W^1)r_X^{2,O}\right) + p_X^2(PB^2 - F)\right\}$$

Assuming, as before, that r_2 is null (or positive but arbitrarily small), we get:

[22] $$Y_f = (1-p_X^2)y - p_X^2(1+r_W^1 - \Gamma^2) - c_s + p_X^2(PB^2 - F)$$

4.3 Second-period loan negotiation: perfect equilibrium partition and outside options

The bank and firm must bargain for the potential surplus given a bilateral monopoly at T^1. Agents are forward-looking and fully rational, and *ex-ante* contracts are not enforceable (§ 5.3); thus, a dynamic inconsistency exists, and agents bargain rather than implement *ex-ante* contracts.

Here, I consider the negotiation of the second-period loan at T^1. At T^1, both bargainers learn if the first-period investment is successful (no exogenous failure) and know the second-period PD. During the negotiation, failure is either the bank's or the firm's decision because, under bankruptcy law, either a creditor or the debtor must file the petition for bankruptcy (details vary depending on national regulations; interestingly, even the possibility for the bank to file for the firm's bankruptcy, considered here, does not protect the bank from hold-up by the firm). This simplifies the analysis. Real-life characteristics correspond to a classical bargaining framework. For simplicity, I assume that the firm has fully paid the other creditors (suppliers), so that these other creditors do not interfere in the negotiation process. If they did, b and f would face an exogenous risk of breakdown, which would complicate the analysis slightly (Binmore *et al.*, 1986).

I assume alternative offers bargaining (Rubinstein, 1982). Bargainers can opt out after rejecting their opponent's offer. In the present framework of loan negotiation, it is realistic to assume that the time Δ between a rejection and a counteroffer tends toward zero. Either f and b both agree on the interest rate for the new loan, or one of them exercises an outside option.

Surplus in the second-period loan negotiation

Since the project is profitable, the surplus becomes:

[23] $$S_{raw}^{nego} = (1 - p_X^2)y + p_X^2(\Gamma^2 + PB^2 - F)$$

This surplus is obtained at the end of the second period, that is over a period of length "one" after an agreement. S_{raw}^{nego} is the certainty equivalent to the surplus at that time. That is, by committing to repay the actual end-of-period income (y or $\Gamma^2 + PB^2 - F$), the bank and firm could borrow $S_{raw}^{nego}/(1 + r_2)$ at the time of the agreement from risk averse financial markets. Moreover, since r_2 is arbitrarily small, S_{raw}^{nego} is the surplus at the time of the agreement.

From this raw surplus, the bank and firm must deduct what is paid to the outside financial system: $S_{net}^{nego} = S_{raw}^{nego} - NFP^{nego}$ where NFP denotes net payments to the outside financial system (other banks and financial markets). The objective is to ensure that these various components are constant so that S_{net}^{nego} is constant, and the classical bargaining framework applies.

From **Result A**, the amount paid to the outside financial system is the (limited) amount paid by *b* to financial markets at the riskless rate, which is arbitrarily small. Therefore, $NFP^{nego} \approx 0$ and:

[24] $$S_{net}^{nego} \approx S_{raw}^{nego} = (1 - p_X^2)y + p_X^2(\Gamma^2 + PB^2 - F)$$

Discounting factors

As stated in Section 2, for simplicity, I assume that the refinancing rate r_2 is positive but arbitrarily small. Thus, when both *b* and *f* have access to financial markets at the riskless rate, $\delta_b = \delta_f$ and, therefore, $s_b = s_f = 0.5$. *b*, which never defaults, always has access to financial markets at the riskless

interest rate. This is also the case for *f* when he has excess liquidity to invest in financial markets or when a positive profit is discounted.

During the bargaining from T^1, b and f face the same risk-free interest rate r_2. This result is clear as regards b, who borrows at this rate in financial markets.

Since the bargaining process has a stationary structure, f receives the same equilibrium profit whenever a deal is struck. This profit will be obtained at the end of the second period, which lasts a period of length one after the deal. But under the simplifying assumption that the second-period riskless interest rate is constant (actually, stationary would suffice), the discounted value of this profit at the time of the deal is independent of this time. Finally, the value of this profit at agreement time is discounted at T^1 (the start of the bargaining) at this same rate r_2, which is what was to be shown. Noteworthily, the interest rate that f pays b over the second period, after the negotiation, is an outcome of this negotiation and not an input that affects the choice of f. Finally, the risk margin does not enter the discounting factor because what matters for f is the certainty equivalent.

Interest-bearing or interest-costing assets in the second-period loan negotiation

f has neither interest-bearing nor interest-costing assets during the negotiation. The absence of interest-costing assets is unintuitive as f is b's debtor. But any amount that f pays b is part of the negotiation, and agents are forward-looking, so that any previous commitment is irrelevant. In other words, what f pays b is an outcome of the bargaining process and not an input in the process.

Since f does not pay interest on his debt to b, b has no interest-bearing assets. Conversely, b must pay interest at the risk-free interest rate r_2 (although b initially borrowed at the first-period risk-free rate r_1) on the first-period loan principal (which is 1), since b borrows from financial markets to lend to f.

Perfect-Equilibrium Partition

Since the discounting factors δ_b and δ_f are equal, the formula in § 3.3 simplifies, so that at the PEP, *b* and *f* respectively obtain S_b and S_f with:

[25] $$S_b = 0.5\left[S_{net}^{nego} - 1\right] = 0.5[(1 - p_X^2)y + p_X^2(\Gamma^2 + PB^2 - F) - 1]$$

[26] $$S_f = 0.5\left[S_{net}^{nego} + 1\right] = 0.5[(1 - p_X^2)y + p_X^2(\Gamma^2 + PB^2 - F) + 1]$$

Determination of the outside option of the firm in the second-period loan negotiation

f may either file for bankruptcy and obtain Z_f or borrow from another bank to repay the initial bank and get Y_f[12]. The potentially binding option (i.e., the most profitable for *f*) must be determined first.

[12] *Ex-post, f* actually borrows from the initial bank since, if *f* can borrow from another bank at rate $r_X^{2,f}$, it is mutually beneficial for *f* and *b* to make a loan contract at rate $r_X^{2,f} - \varepsilon_X^{2,f}$ with $\varepsilon_X^{2,f}$ positive but arbitrarily small.

$$Z_f > Y_f \leftrightarrow$$

[27] $$PB^1 - F + (1 - p_X^2)y - (\Gamma^1 - PB^1 + F)p_X^2(\Gamma^1 - PB^1 + F - \Gamma^2) - c_S + p_X^2(PB^2 - F) - \Gamma^1 > (1 - p_X^2)y - p_X^2(1 + r_W^1 - \Gamma^2) - c_s + p_X^2(PB^2 - F)$$

i.e.

[28] $$Z_f > Y_f \leftrightarrow (PB^1 - F - \Gamma^1)(1 + p_X^2(\Gamma^1 - PB^1 + F - \Gamma^2)) + p_X^2(1 - \Gamma^2) + p_X^2 r_W^1 > 0$$

f prefers default to repayment if:

- Private benefits over the first period (PB^1) increase (the firm is incentivized to default);
- Failure costs decrease;
- The first-period rate r_W^1 is higher. Owing to liquidity constraints (no production over period 1), it increases the amount that the firm must borrow for the second period;
- The first-period collateral value Γ^1 is lower, as *f* buys this asset back at that price in the case of default;
- The second-period collateral value Γ^2 is lower. Indeed, lower collateral increases the other bank's option interest rate, $r_X^{S,O}$, which makes this option less interesting.

Information costs do not affect a firm's choice of the optimal outside option. It is clear that these costs are paid by *f* if he borrows from another bank to repay the initial bank. But if he files for bankruptcy, he will eventually buy back the asset from the initial bank, who will extract the information costs as a profit; see § 4.1).

Interestingly, one could imagine that a higher y would make the "other bank" option comparatively more attractive for f. This is actually not the case since in the case of default, if y is sufficiently high, f will buy back the asset and produce (with probability $1 - p_X^2$, as in the "other bank" case).

Determination of the outside option of the bank in the second-period loan negotiation

For b, two potential outside options exist:

(1) Seizing the collateral, which induces f's bankruptcy, and obtaining Γ^1;

(2) Declining to roll over the first period loan.

What b obtains in the second case depends on f's response. If f chooses to default, then b gets Γ^1. Conversely, if f is better off borrowing from another bank, then the initial loan is repaid and b obtains $1 + r_W^1$. Thus, the outcome for b of declining to roll over depends on whether [28] holds. If [28] holds, b has a weak outside option (2) which amounts to filing for bankruptcy (i.e., option (1) above). Conversely, if [28] does not hold and the firm is better off borrowing from another bank than defaulting, b has a strong outside option and is always fully repaid, even if the PEP does not provide so. Thus, [28] is labeled the "weak option condition".

4.4 Hold-up by the firm

In this subsection, I identify the sufficient conditions for hold-up by the firm. I focus on the case in which $S_b < \bar{L}$, where S_b is the amount (or utility) accruing to *b* at the PEP in the negotiation that occurs at T^1, and $\bar{L}$ corresponds to the value of repaying the first-period debt (capital and "fair" interest), borrowing from the initial bank at a "fair" second-period rate and repaying at the second period. In total, this means repaying the capital and interest of the first-period loan as well as the interest of the second-period loan,

[29] $$\bar{L} = 1 + r_W^{1,F} + (1 - p_X^2)(1 + r_W^1) r_G^{2,f}$$

where $r_W^{1,F}$ is the "fair" first-period rate that covers the expected expenses borne by b over the first period, including screening costs:

[30] $$r_W^{1,F} = \frac{p_W^1(\Gamma^1 - 1) + c_S}{1 - p_W^1}$$

and $r_G^{2,f}$ is the "fair" second-period rate that covers b's expected expenses incurred when f is good over the second period:

[31] $$r_G^{2,f} = \frac{p_G^2\left(1 + r_W^{1,F} - \Gamma^2\right)}{(1 - p_G^2)(1 + r_W^{1,F})}$$

Since $r_G^{2,f} < r_B^{2,f}$, $S_b < \bar{L}$ and [29] ensure that *ex-post*, irrespective of whether f is in state G or B, f will repay the initial loan only if b grants a second-period rate at which she suffers a loss.

Hereafter, I assume that $c_S = 0$, both for simplicity and to show that the mechanism described here does not rely on screening costs. Thus, I define the "fair" first-period rate without screening costs:

[32] $$r_W^{1,f} = \frac{p_W^1(\Gamma^1 - 1)}{1 - p_W^1}$$

Again, I consider what happens when the PEP provides b with a share S_b lower than $\bar{L}$, i.e.:

[33] $$0.5\left[\frac{(1 - p_X^2)y + p_X^2(\Gamma^2 + PB^2 - F)}{1 + r_X^2} - 1\right] \leq 1 + r_W^{1,F} + (1 - p_G^2)(1 + r_W^1) r_G^{2,f}$$

Below, I show that when [33] holds, f repays the first-period loan but forces b to grant an abnormally favorable rate on the second-period loan. Recall that b can neither force f to repay the first-period loan nor fix the conditions for the second-period loan. From a game-theory perspective, b's negotiating power is not sufficiently high. From a legal and economic viewpoint, the first-period contract is incomplete: f can (falsely) pretend that the first-period interest rate was "too high" and the current

risk is low to justify a low second-period rate. A discussed in § 5.2 and 5.3, while loan contracts are not fully enforceable at T^1, they would be fully enforceable at T^2 because f is illiquid at T^1 but not at T^2.

If b played the outside option and seized the asset, f would incur a loss F due to the administrative costs of bankruptcy (even if f buys back the asset at the price Γ^1). Can b leverage her outside options to affect f's actions? I consider two subcases of the "$S_b < \bar{L}$" situation.

First, I consider the case $\boldsymbol{Out_b < S_b < \bar{L}}$. In this case, b's outside option (filing for f's bankruptcy) affords b less than the PEP, S_b, induced by b's and f's negotiating powers, which itself brings less than the "fair" amount necessary for b to cash in on. Then, from **Principle A** (§ 3.4), b cannot affect f's actions. In other words, during the negotiation that starts at Γ^1, b cannot credibly threaten to implement her outside option and will rather roll over the loan at a loss.

The intuition is as follows: at the PEP determined by b's and f's negotiating powers, b accepts an expected loss on the second-period loan to avoid a larger loss on the first-period loan.

This case constitutes a hold-up by the firm, which profits from condition [33] and repays the first-period loan but extorts a second-period interest rate below the fair interest rate which would cover the expected expenses of the bank. Consequently, *ex-ante*, the bank will decline to lend and productive projects are not financed; in other words, firm's hold-up induces credit constraints if the bank is unable to lend long and can only offer one-period loans.

Now, I consider the case $\boldsymbol{S_b < Out_b < \bar{L}}$. I focus on the case in which b has a **weak outside option** because f is better off defaulting when b declines to roll over (weak option condition).

Since $\boldsymbol{S_b < Out_b}$, b's outside option is binding and induces a specific loss. From § 3.6, b can therefore affect f's actions, so we must consider the possibility that b affects the first- and/or second-period interest rate to recoup her costs. This outside option also relates to the financial literature, which relies on the possibility of the bank seizing assets to force the firm to repay the first-period loan.

Then, from **Principle C**, b forces f to take out a second-period loan at a rate such that b gets $Out_b + \epsilon$, where f can make ϵ arbitrarily small. That is, b obtains only (ϵ more than) the two-period income corresponding to the weak outside option (which amounts to filing for the firm's bankruptcy). Then, unless the first-period collateral value fully covers the first-period loan ($\Gamma^1 = 1$), b incurs a loss. Since this is fully predictable, b originally declines to lend short. Once again, if b is unable to lend long, it induces credit constraints, and some productive projects are not financed.

In other words, for a series of productive projects defined by [33], hold-up by firms may occur and prevent banks from lending short to finance long-term projects. This occurs either because the bank's outside option is not binding (outside option principle) or, if the bank's outside option is binding (and

includes a loss threat to the firm, namely, failure costs), because the weak option condition holds and the collateral value does not fully cover the first-period loan.

Discussion. One may wonder whether the bank may actually accept losing money on the new loan to ensure that the initial loan is repaid. A (wrong) objection would be that the bank's option to file for the firm's bankruptcy would increase her negotiating power and avoid the $S_b < \bar{L}$ situation.

From a game-theoretic perspective, such a misleading view confuses an outside option with a no-deal situation, itself considered as a lower bound on the computation of the equilibrium partition in the "split-the-difference" approach of cooperative bargaining. But from the non-cooperative game-theoretic literature presented in § 3.2, an outside option (here, recovering the collateral value by filing for f's bankruptcy) is **not** the lower bound that determines what b would obtain at the PEP if f had all the bargaining power. This lower bound is rather zero, which explains why b does not break even if f has sufficient negotiating power. A related but different misconception is that b could leverage her outside option to bargain and eventually obtain more than this outside option. Actually, that would directly contradict the outside option principle and its extensions described in Section 3.

From an economic perspective, agents are rational and forward-looking. Thus, the agreement at T^0 to repay the loan has no value at T^1. f will not repay if refusing is optimal, and, more generally, his actions will be determined by optimal choice of future actions rather than his past commitment. In the words of property-rights theory (Grossman and Hart, 1986, Hart and Moore, 1990) and its application to credit (Hart and Moore, 1998), the loan is, for the bank, a sunk investment cost. Additionally, the lending bank has SITG when the loan must be rolled over. The empirical literature (Caballero *et al.*, 2008) shows that banks are ready to lose money on new loans to avoid losses on existing loans. That is, in various bargaining processes that occur during loan renewal, a bank's no-loss condition on new loans is definitely not a lower bound.

One may also wonder whether the bank can avoid a loss by playing on the first-period rate. By reducing r_W^1, b may indeed reduce the right-hand side of [33], potentially avoiding the dynamic loss case. However, if b reduces r_W^1 below $r_W^{1,F}$, she suffers an expected loss over the first period and must require a rate higher than the competitive one over the second period. If screening costs are sufficiently low, another bank will then make a better offer and b will not recoup this loss with a second-period profit. Conversely, if b requires a higher r_W^1, the incentive for f to default at T^1 increases, and, more generally, [33] is more likely to hold so that *ex-ante*, b is even more willing to decline to lend short[13].

[13] The fact that measures taken at T^0 to compensate for future losses actually backfire is a form of dynamic inconsistency. Moreover, I studied the interaction of first and second-period rates in a previous draft and showed that, unless the first-period loan was fully covered, there was no equilibrium combination of both rates. This is no longer necessary in the present draft, based on non-cooperative bargaining which makes it possible to directly

5. The nature of frictions

I use a straightforward model to identify a bare minimum set of frictions that can induce hold-up by firms. Usual components of credit constraints models such as risk aversion, liquidity shocks, debt overhang, creditors' coordination issues, implicit options, adverse selection, and moral hazard, are assumed away. Regarding the relevant frictions, hold-up can occur at T^1 but not at T^2 because the firm's liquidity improves over period 2 (sales income accumulated over period 2 is available and versatile at T^2). **Illiquidity** at T^1 flows not only from delayed funds availability but also from asset specificity at T^1. Still, as noted by Caballero and Hammour (1998), "specificity acquires a more troublesome dimension when combined with contracting difficulties". Here, the **incompleteness** of short-term loan contracts flows from the unverifiability of credit risk by a court; this is unusual as a foundation for incompleteness, but is supported by the literature on banks' specificity and by institutional evidence. But first, hold-up is based on the possibility of **strategic default**, and I provide empirical evidence for such defaults, even when risk-inducing moral hazard is assumed away. Finally, I discuss the respective roles of information asymmetries and SITG.

5.1 Strategic defaults: net bankruptcy benefits rather than risk-taking

The hold-up mechanism identified above relies on the possibility of firms strategically defaulting. However, I assume away moral hazard and risk-taking because, strategic defaults apart, the credit risk is fully exogenous. In addition, according to the weak option condition [28], the strategic default decision depends directly on the net private benefits in the case (and only in the case) of bankruptcy.

LoPucki (2010) provides examples of private benefits extraction related to the strategic choice of bankruptcy courts. Such forum shopping is a pervasive feature of US bankruptcies, as shown by Antill and Bellon (2024), who rather document its positive effects.

Schoenherr and Starmans (2022) find that lower failure costs (*i.e.,* higher net bankruptcy benefits) induce more risk-taking and eventually more failures[14], but their findings fail to support the present model because I assume away risk-taking. Giroud et al. (2012) show that entrepreneurs can divert assets over several years. While this provides evidence for private benefits, they fail to differentiate between default and non-default cases; such a distinction is crucial for the weak option condition [28].

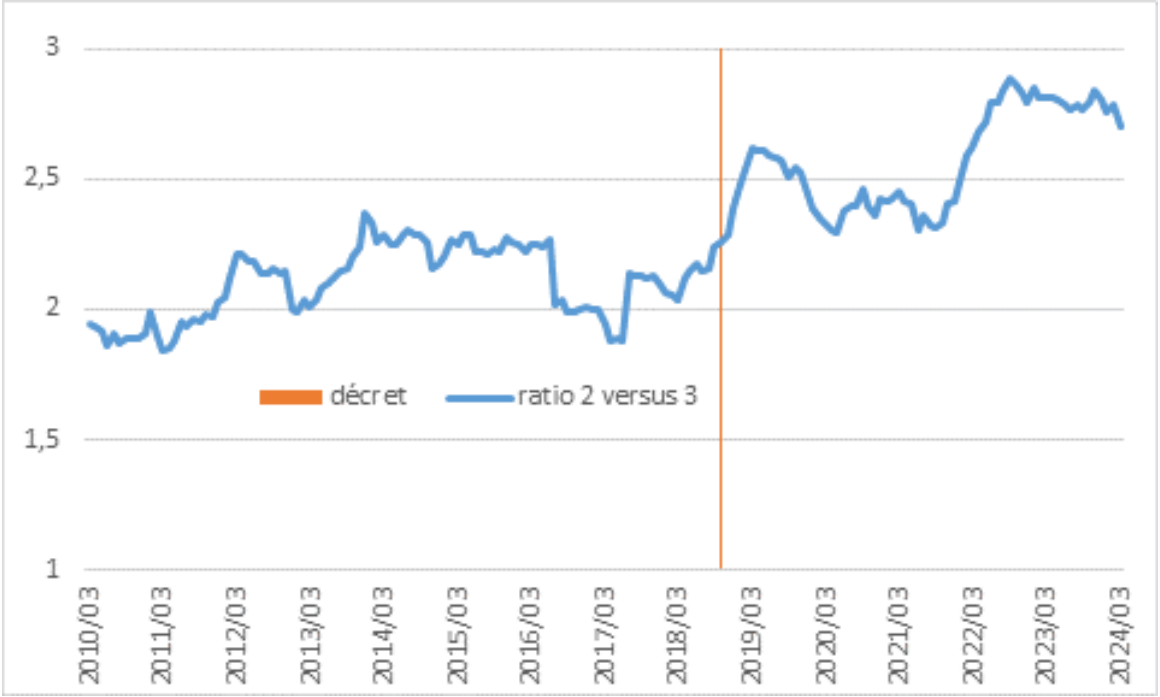
I consider a natural experiment that affects managers' failure costs and private benefits. Until September 2018, managers with two failures over the previous five years were "flagged" by the French

identify the conditions for inconsistency. Still, studying the interactions of first and second-period rates could be interesting when the firm repays the bank if "good" over period two.

[14] This is especially clear in the first draft, *Schoenherr, 2017, "Managers' Personal Bankruptcy Costs and Risk-Taking"*. In the published draft, this effect is balanced with the beneficial effects of stronger creditors' rights when firms are credit-constrained. Rodano *et al.* (2016) also provide interesting results.

Central Bank (the flag was available to all banks). On October 1, 2018, this flag was suppressed, and, since then, managers have been flagged only if they experienced three failures over the previous five years. If net private benefits are relevant to default, the number of managers with two failures relative to those with three failures should have increased after this regulatory change.

Graph 2: Ratio of managers with two and three-defaults



As shown in Graph 2, this ratio increased quickly, over five months, from October 2018, which is not consistent with a risk-taking channel that would induce default after a while, but rather with strategic failures directly linked to private benefits in the case (and only in the case) of default. Therefore, these empirical results are consistent with the assumption in the present model that firms can decide to default strategically based on private benefits, even if moral hazard issues and risk-taking are assumed away, which is innovative *w.r.t.* Giroud et al. (2012) and Schoenherr and Starmans (2022).

5.2 Liquidity: from funds availability to asset specificity

f's illiquidity is a crucial feature of this analysis. The distinction between liquidity and solvency is common for banks. Here, it is used for firm f, which is productive (hence solvent) if the project is conducted until T^2, but is illiquid at T^1, explaining why hold-up can occur at T^1 but not at T^2.

This differs from the zombie lending literature (e.g. Caballero *et al.*, 2008) which also considers holdup by firms due to banks' SITG. Unproductive zombie firms hold up banks that their involuntary failure would hurt. Here, productive but illiquid firms hold up banks by threatening to default strategically.

Indeed, f declines to borrow from another bank to repay the first-period loan (which would keep the productive project working and allow f to reap the subsequent benefits) because f finds it more

profitable to default if the weak option condition [28] holds (this crucial condition essentially corresponds to high private benefits). That is, f strategically exploits his own illiquidity while he could find liquidity if he wanted to. § 5.3 explains this mechanism in more detail.

Crucially, at T^1, b can seize f's asset, but it does not cover the debt one-for-one because it is affected by asset specificity (Kermani and Ma, 2023) and has a lower value for a new buyer than for the initial firm/entrepreneur f who built it for a specific purpose; the case in which the collateral value is lower than the debt value is the interesting case in which liquidation induces a loss to the bank : (e.g. Diamond and Dybvig, (1983). Conversely, at T^2, b can seize income y, which fully covers the two-period debt (for the distinction between collateral and going-concern values, see Kermani and Ma (2020)). The second-period income y is liquid, that is, it has the same value for all agents. This definition of liquidity, as opposed to specificity, comes from Kiyotaki and Moore (2001).

Overall, because f's asset is worth less than the debt to the bank (owing to asset specificity), only f can fully repay b. In the words of property rights theory (Grossman and Hart, 1986, Hart and Moore, 1990), by lending to f, b has made an f-specific investment. In other words, asset-firm specificity induces bank-firm specificity, which in turn induces a bilateral monopoly that forces negotiations at T^1 (on relationship specificity as a source of bargaining, see e.g. Caballero and Hammour, 1998).

5.3 Exogenous default risk, unverifiability, and incompleteness

The incompleteness of future loan contracts, central to the present analysis, takes two forms. First, it is difficult to establish, *ex-ante,* an unconditional second-period loan contract. Second, the PD over the second period is unverifiable by a third party (typically a court).

The difficulty in drawing up an unconditional second-period loan contract at T^0 has two causes; one is model-specific and the other is more general and realistic. The former is that the information cost c_s is assumed to be a monitoring cost over the first period, such that b obtains information on f's PD only at $T^0 + \varepsilon$. This simplifying assumption prevents negotiation at T^0 and forces the focus onto negotiation at T^1, which is the core concept of the present study. If c_s was instead a screening cost, b would know the first-period PD at T^0, potentially inducing a first negotiation the, but a negotiation would still occur at T^1. Thus, this model-specific feature is not determining.

A more fundamental cause for incompleteness is that at T^1, the second-period PD, p_X^2, is not verifiable by a court. That is, f can pretend that b is trying to hold him up and justify his choice to default if b declines to roll over. Conversely, if a court could observe p_X^2, it could enforce a "fair" loan contract.

The literature on bank specificity supports the PD's unverifiability: if banks need special skills to analyze borrowers' PD, courts, which lack these skills, cannot evaluate the PD. Further, even *ex-post*, failure or success provides incomplete information on *ex-ante* PD. More generally, the unverifiability of *ex-post*

firm outcomes is a central feature of bank-firm relationships. This explains why debt contracts are not contingent on firms' outcomes (Hart and Moore, 1998) and why firms must pledge collateral to borrow (*e.g.* Kiyotaki and Moore, 2001). Overall, the PD's unverifiability assumption is weaker than the rather general unverifiability assumption in Hart and Moore (1998). Pending further investigation, it is also immune to the critique of Maskin and Tirole (1999) for whom the costs of describing future states do not necessarily interfere with optimal contracting[15].

Crucially, if a liquid entrepreneur with a two-period loan contract defaulted at T^1, he could not pretend that the bank had underestimated his risk. Indeed, although *ex-post* outcomes do not provide full information on *ex-ante* risk, a failure may discard the assumption that the bank was overestimating the risk. Thus, a liquid firm could not credibly hold up the bank.

The fact that the second-period PD is known (by f and b) only at T^1 (cf. [1]) is less problematic. This delay induces costs because the agents must list all potential cases (here just two), which is the most common mechanism supporting the incompleteness of contracts. The three-step solutions proposed by Maskin and Tirole (1999) may fully work here. An even simpler solution would be to implement von Thadden (1995) contracts, in which the interest rate is fixed *ex-ante* but the bank decides whether to roll over, and to supplement such contracts by making it obligatory for the firm to take a loan (at pre-specified conditions) if the bank decides to roll over. Interestingly, if these modified von Thadden contracts were feasible, they would stipulate high rates over period 2, hence low rates over period 1 by competition, and would therefore be (partially) observationally similar to hold-up by banks.

5.4 The role (or not) of information asymmetries

The credit constraint literature extensively uses asymmetric information (Diamond, 1984, Freixas and Rochet, 2008), but unverifiability by a court is something else. Hart and Moore (1998, p. 2) assume "symmetric information between the entrepreneur and investor both when the contract is written and once the relationship is under way. However, many of the variables […] are assumed not to be verifiable by outsiders, e.g., a court; hence contracts cannot be conditioned (directly) on these".

While information asymmetries are crucial to support hold-up by banks, they are not crucial in the SITG mechanism. In the present analysis, without information asymmetries, only f would have monopoly (and bargaining) power, so that *ex post*, b would get only the value of her outside option, and f the rest of the surplus. Therefore, under condition (2), b would decline to lend *ex -ante*.

Beyond the mechanism identified in this study, banks' SITG may constitute an alternative to information asymmetries to explain classical empirical results in the relationship banking literature:

[15] More precisely, here, immunity to this critique is unrelated to information perturbations (unlike Aghion et al., 2012). I conjecture it is also unrelated to risk aversion (unlike Segal, 1999 or Hart and Moore, 1999).

why firms with a bank relationship have a higher probability of obtaining credit (Cole, 1998) ; why longer relationships may improve access to credit; why more bank relationships induce higher rates (Petersen and Rajan, 1994); why distressed firms in a group with a bank are more likely to invest and grow than their independent counterparts (Hoshi *et al.,* 1990), or why the failure of (information-sensitive) banks' have a greater effect on the local economy than thrifts' failures (Ashcraft, 2005)[16].

6. Conclusion

This paper provides extensions of the outside option principle and perfect equilibrium partitions in bargaining models with alternating offers. It applies these extensions to the negotiation that occurs when a loan must be rolled over (because the loan relationship is a bilateral monopoly if only the firm can fully repay the bank). It identifies a new mechanism, hold-up by illiquid firms, which discourages banks from lending short to finance long-term projects and may induce credit constraints and capital misallocation when banks are otherwise unable or unwilling to lend long. In addition to banks skin-in-the-game, three types of financial frictions underlie this mechanism: firms' illiquidity after a short-term loan, unverifiability of firms' PD, and the existence of private benefits in the case of a default.

Policy implications include fostering long-term bank loans, providing courts with independent external ratings, and adapting von Thadden (1995) contracts by forcing firms to take loans if banks roll over.

This model is deliberately straightforward to allow the analysis to focus on a new mechanism and identify the minimum required frictions. Combining this new mechanism with classical information asymmetry between banks and firms would be interesting. Grossman and Perry (1986) provide interesting tools[17]. This study assumes that other creditors have been refunded before the negotiations; however, more agents can be considered. The extension is straightforward if these other creditors are not part of the bargaining but can call an outside option (Binmore *et al.*, 1986, p. 181). Conversely, a bargaining process involving these creditors would raise specific technical issues. Compte and Jehiel (2010) provide tools for analyzing such situations. This is left for future research.

The current analysis shows that bargaining is due to occur at loan renewal because the bank and the firm have monopoly power over each other, which alters significantly the analysis of loan renewal. This analysis could be extended to initial loans when banks suffer screening costs and firms suffer cost to provide a new bank with information. Indeed, once a bank and a firm have suffered such informational costs, they find themselves in a bilateral monopoly before the loan contract is drawn up. In other words, the line of research introduced here may alter more generally the bank-firm literature.

[16] For a more detailed discussion, see Bank's skin-in-the-game and hold-up by illiquid firms: strategic bargaining, dynamic inconsistency and credit constraints - Archive ouverte HAL, § 5.4.

[17] Fudenberg and Tirole (1983) examine bargaining with asymmetric incomplete information but with unilateral offers. Rubinstein (1985) extends bargaining to incomplete information on time preferences only.

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Appendix 1: Proofs on the perfect equilibrium partition

Proof of equation [4].

Consider the supremum $M_{b,2\Delta}$ that *b* can get at *T*=2*Δ*. Assuming no deal has been struck, if a deal is struck at *T*=2*Δ*, *b* makes the offer and receives at most $s_b S$. Hence, at *T*=*Δ*, when *f* makes the offer, *b* accepts any offer in which he receives more than $\delta_b^{\Delta} s_b S$. That is, *f* gets at least $S - \delta_b^{\Delta} s_b S$ if a deal is struck at *T*=*Δ*. But a deal struck at *T*=*Δ* means that no deal was struck at *T*=0. Thus, at *T*=*Δ*, *f* also receives (in any case actually) the income flow $P\{(1+r_f)^{\Delta} - 1\}$ from asset P. Thus, at *T*=0, *f* requires an offer (defined as $m_{f,\Delta}$) that brings him at least $\delta_f^{\Delta}\left[S - \delta_b^{\Delta} s_b S + P\{(1+r_f)^{\Delta} - 1\}\right]$. That is, *b* can get at most $S - m_{f,\Delta}$ at *T*=0. In turn, this happens to be the supremum of the *T*=0 proposal, $s_b S$, which would be accepted by *f* since the forthcoming game is the same at *T*=0 and *T*=2*Δ*[18]. Therefore:

[A1-1] $$s_b S \leq M_{b,0} = S - \delta_f^{\Delta}\left[S - \delta_b^{\Delta} s_b S + P\{(1+r_f)^{\Delta} - 1\}\right]$$

Table 1: Summary of the computation of s_b when *b* makes the first offer

T	f	b
0	$m_{f,0} = \delta_f^{\Delta}\left[S - \delta_b^{\Delta} s_b S + P\{(1+r_f)^{\Delta} - 1\}\right]$	$s_b S \leq M_{b,0}$ and $M_{b,0} = S - m_{f,0}$
Δ	$m_{f,\Delta} = S - M_{b,\Delta}$ $= S - \delta_b^{\Delta} s_b S$ $\quad (+P\{(1+r_f)^{\Delta} - 1\})$	$M_{b,\Delta} = \delta_b^{\Delta} s_b S$
2Δ		$M_{b,2\Delta} = s_b S$

This table presents the different steps of the computation of s_b when b makes the first offer. At T=2Δ , I determine the maximal amount b that can get (supremum), which then determines what b can get at most at T=Δ, what f can get at least at that time (infimum), what f can get at least at T=0, and finally what b can get at most at that time. This last amount is in turn equal to $s_b S$. Switching the infimum m and supremum M in all steps proves unicity of s_b when b makes the first offer. The blue component is determined by the choices over the previous periods and does not affect the current-period behavior.

Switching the infimum *m* and supremum *M*, "at least" and "at most", in all the steps, we have:

[A1-2] $$S - \delta_f^{\Delta}\left[S - \delta_b^{\Delta} s_b S + P\{(1+r_f)^{\Delta} - 1\}\right] = m_{b,0} \leq s_b S$$

From [A1-1] and [A1-2], we get: $s_b S\left(1 - \delta_f^{\Delta}\delta_b^{\Delta}\right) = S - \delta_f^{\Delta}\left[S + P\{(1+r_f)^{\Delta} - 1\}\right]$, and from the fact that $\delta_f^{\Delta}(1+r_f)^{\Delta} = 1$, we have $S - \delta_f^{\Delta}\left[S + P\{(1+r_f)^{\Delta} - 1\}\right] = S - \delta_f^{\Delta} S - P\left(1 - \delta_f^{\Delta}\right)$. Therefore:

[4] $$s_b = \frac{\left(1 - \delta_f^{\Delta}\right)(S - P)}{S\left(1 - \delta_f^{\Delta}\delta_b^{\Delta}\right)}$$

[18] When assuming that the forthcoming game is identical at 0 and 2*Δ*, I make the same approximation as Binmore *et al.*, (1986), facilitating the comparison with their analysis. S remains unchanged. But between 2*Δ* and 3*Δ*, *f* receives $P\{(1+r_f)^{3\Delta} - (1+r_f)^{2\Delta}\}$ compared with only $P\{(1+r_f)^{\Delta} - 1\}$ between 0 and 2*Δ*. The difference, $\{(1+r_f)^{2\Delta} - 1\}P\{(1+r_f)^{\Delta} - 1\}$, converges to zero when *Δ* tends toward zero. Therefore, the approximation is good, especially when r_f is very small, which is shown in Section 4 to flow directly from the assumption that the riskless refinancing rate is almost zero. Thus, this "local" analysis suffices here. In a more general setting, the compounding interest of asset *P* could exceed what *f* could obtain by making a deal or calling an outside option. In such an "explosive" case, *b* would call her own outside option. *b* would also call her outside option if she were burdened with debt. In both cases, the results presented in § 4.4 would be reinforced.

This proves the unicity of the partition determined by s_b, conditional on *b* making the first offer. This partition is supported as a perfect equilibrium with the following strategies: at each step, a player makes an offer that corresponds to this perfect equilibrium and the other player accepts any offer that does not exceed that amount. As Sutton (1986) notes, this proof does not assume that the strategies supporting the PEP are stationary, but rather exploits the stationarity of the underlying structure.

Proof of equation [6]. Now, I consider the case in which *f* makes the first offer (Table 2). Assuming no deal has been struck yet, at *T*=2*Δ*, f always receives $P(1+r_f)^{2\Delta} - P(1+r_f)^{\Delta}$ (compound interest; flow between *Δ* and 2*Δ*). If a deal is struck at *T*=2*Δ*, f additionally receives $s_f S$. Hence, f receives at least:

[A1-3] $$m_{f,2\Delta} = s_f S + P(1+r_f)^{\Delta}\{(1+r_f)^{\Delta} - 1\}$$

At *T*=*Δ*, *b* makes the offer. In all cases, *f* receives $P(1+r_f)^{\Delta} - P$ (regardless of whether *f* accepts *b*'s offer; therefore, it is not relevant to the comparison). Furthermore, f obtains at least $m_{f,2\Delta}$ in the following period; thus, *f* will not accept any offer lower than $\delta_f^{\Delta}.m_{f,2\Delta}$ (the value $m_{f,2\Delta}$ actualized at *T*=*Δ*) from *b*. That is, at *T*=*Δ*, *b* can obtain at most the supremum $M_{b,\Delta} = S - \delta_f^{\Delta}.m_{f,2\Delta}$

[A1-4] $$M_{b,\Delta} = Z - \delta_f^{\Delta}\left[s_f Z + P(1+r_f)^{\Delta}\{(1+r_f)^{\Delta} - 1\}\right]$$

[A1-5] $$S - \delta_b^{\Delta}\left[Z - \delta_f^{\Delta}\left[s_f S + P\left(1+r_f\right)^{\Delta}\{(1+r_f)^{\Delta} - 1\}\right]\right] = m_{f,0} \leq s_f S$$

Switching the infimum *m* and supremum *M*, "at least" and "at most", in all steps, we have:

[A1-6] $$s_f S \leq M_{f,0} = S - \delta_b^{\Delta}\left[Z - \delta_f^{\Delta}\left[s_f S + P(1+r_f)^{\Delta}\{(1+r_f)^{\Delta} - 1\}\right]\right]$$

Table 2: Summary of the computation of s_f when *f* makes the first offer

T	f	b
0	$s_f S \geq m_{f,0}$ and $m_{f,0} = S - M_{b,0}$	$M_{b,0} = \delta_b^{\Delta} M_{b,\Delta}$
Δ	$m_{f,\Delta} = \delta_f^{\Delta}.m_{f,2\Delta}$ $= \delta_f^{\Delta}\left[s_f S + P(1+r_f)^{\Delta}\{(1+r_f)^{\Delta} - 1\}\right]$	$M_{b,\Delta} = S - \delta_f^{\Delta}.m_{f,2\Delta}$ $= S - \delta_f^{\Delta}.\left[s_f S + P(1+r_f)^{\Delta}\{(1+r_f)^{\Delta} - 1\}\right]$
2Δ	$m_{f,2\Delta} = s_f S \quad (+P(1+r_f)^{\Delta}\{(1+r_f)^{\Delta} - 1\})$	

This table presents the different steps of the computation of s_f when f makes the first offer. At T=2Δ , I determine the minimal amount that f can get (infimum), which determines what f can get at least at T=Δ, what b can get at most at that time (supremum), what b can get at most at T=0, and finally what f can get at least at T=0. This last amount is in turn equal to $s_f S$. Switching the infimum m and supremum M in all the steps proves the unicity of s_f when f makes the first offer. The blue component is determined by the choices over the previous periods and does not affect the current-period behavior.

From [A1-5] and [A1-6], we have:

[6] $$s_f = \frac{S - \delta_b^{\Delta}\left[S - P\{(1+r_f)^{\Delta} - 1\}\right]}{S\left(1 - \delta_b^{\Delta}\delta_f^{\Delta}\right)}$$

This proves the unicity of the perfect partition determined by s_f, conditional on *f* making the first offer.

Appendix 2
Proofs of extensions of the perfect equilibrium partition

Proof of principle A

By assumption, $Out_b < S_b$, *and* $[S_b, S_f]$ *is the only PEP. If* b *offers a partition* $[\widetilde{S_b}, \widetilde{S_f}]$ *with* $S_b < \widetilde{S_b}$ *(and consequently* $\widetilde{S_f} < S_f$*) and threatens to implement the outside option if* f *rejects this offer, then:*

- f *is willing to reject the offer as* $\widetilde{S_f} < S_b$
- *if* f *rejects the offer, b is worse off implementing the threat than yielding, as* $Out_b < S_b$.

That is, the threat is not subgame-perfect. Thus, b *can neither extort anything nor affect f's actions by threatening to incur a loss on f.* QED.

In other words, the outside option principle fully applies in this case.

Proof of principle B

I assume w.l.o.g. that player b *has the outside option. The other player is* f.

- *When* b *makes the first offer:*

I assume that player b *offers* $s_b = s^*$ *with* $Out_b < s^*$. *If* b *can obtain* s^*, *this means that* $s^* \leq M^*$ *where* M^* *is the supremum of what* b *can obtain at the first step. Thus:*

[A2-1] $$Out_b < M^*$$

First step: f accepts the offer $[s^*, 1 - s^*]$ *iff he cannot obtain more by rejecting it and making a counteroffer at the next step (T=Δ). b accepts this counteroffer iff he cannot obtain more by rejecting it and making a counteroffer at the next step (T=2Δ). As past actions are irrelevant,* M^* *is also the supremum of what* b *can obtain at T=2Δ.*

Second step (Sutton method): As M^* *is the supremum of what* b *can get at T=2Δ. Therefore, at T=Δ,* b *accepts any offer at least equal to* $\delta_b^\Delta M^*$ *so that* f *can obtain at least* $1 - \delta_b^\Delta M^*$. *That is, at T=0,* f *can obtain at least* $\delta_f^\Delta(1 - \delta_b^\Delta M^*)$ *and* b *can obtain at most* $M_{b,0} = 1 - \delta_f^\Delta(1 - \delta_b^\Delta M^*)$. *However, from the definition of* M^*, $M_{b,0} = M^*$. *Thus,*

[A2-2] $$M^* = 1 - \delta_f^\Delta(1 - \delta_b^\Delta M^*)$$

That is,

[A2-3] $$M^* = \frac{1 - \delta_f^\Delta}{1 - \delta_f^\Delta \delta_b^\Delta}$$

with $\Delta \to 0$,

[A2-4] $$M^* = \frac{ln\delta_f^\Delta}{ln\delta_f^\Delta + ln\delta_b^\Delta}$$

From Sutton, (1986), *we recognize* M^* as *the PEP. Then, [19] contradicts the assumption that the outside option is binding. QED.*

- *When* f *makes the first offer:*

*Repeating the second step of the "*b *makes the first offer" case and replacing T=0 (resp. T=Δ, resp T=2Δ) with T=Δ (resp. T=2Δ, resp. T=3Δ) shows that at T=Δ,* b *cannot obtain more than* M^*.

Then, at T=0, f *makes an offer* $\left[\delta_b^{\Delta} M^*, 1-\delta_b^{\Delta} M^*\right]$ *and* b *accepts it. Thus,* b *cannot get more than* $\delta_b^{\Delta} M^*$*, which is lower than* M^**. QED.*

The proof extends directly to cases with interest-bearing or interest-costing assets.

Proof of principle C

I assume w.l.o.g. that $S=1$*,* b *has the option, and* f *is the other player:*

- *When* f *makes the first offer:*

First step (b *can affect f's action and obtain strictly more than* Out_b*)*

I assume that the maximum M^* *that* b *can obtain is the outside option:*

[A2-5] $$M^* = Out_b$$

Under [13] and [24] and since $\delta_b^{\Delta} < 1$*, implementing the following strategy is clearly subgame-perfect for* b*: (1)* b *accepts any offer* $[\widehat{s_b}, 1-\widehat{s_b}]$ *when* $Out_b < \widehat{s_b}$ *and (2)* b *calls the outside option when* $\widehat{s_b} < Out_b$.

However, if b *implements this strategy,* f *suffers the loss* F *if he offers* $\widehat{s_b} = Out_b$ *but can avoid this loss by offering* $Out_b + \varepsilon$ *rather than* $\widehat{s_b}$*, with* $0 < \varepsilon$*. This deviation is profitable for* f *as long as* $0 < \varepsilon < F$*. Thus,* b *can obtain* $Out_b + \varepsilon$*, contradicting assumption [38].*

Second step (f *can make the additional amount granted to* b *arbitrarily small)*

I assume that f *cannot make* ε *arbitrarily small. Then, there exists* $\bar{\varepsilon} > 0$ *such that* b can obtain $Out_b + \bar{\varepsilon}$*. The following strategy is subgame perfect for* b*: (1)* b *accepts any offer* $[\widehat{s_b}, 1-\widehat{s_b}]$ *when* $Out_b + \bar{\varepsilon} \leq \widehat{s_b}$ *and (2)* b *calls the outside option when* $\widehat{s_b} < Out_b + \bar{\varepsilon}$ [19].

If f offers $Out_b + \bar{\varepsilon}/2$, b *is worse off calling the outside option rather than accepting this offer. Thus, the strategy described above is not subgame-perfect for* b [20] (noteworthy, I do not use the condition that Δ→ 0).

[19] *I assume that* $Out_b + \bar{\varepsilon}$ *is a maximum; if it is a supremum instead,* $\leq$ *is replaced with* $<$ *in (1) and* $<$ *is replaced with* $\leq$ *in (2)*

[20] Notably, *b* can also reject the offer and make a counteroffer. However, I only show that the strategy described is not subgame perfect. Under the assumption that $\bar{\varepsilon}$ exists, this strategy would be subgame perfect.

- *When* b *makes the first offer:*

*From the "*f *makes the first offer" case,* f *knows that if he rejects* b*'s offer at* $= 0$ *,* the supremum (which is not a maximum) of what he can obtain at $T = \Delta$ is $1 - Out_b$. *Thus, implementing the following strategy is subgame-perfect for* f*: (1)* f *accepts any offer in which he receives at least* $\delta_f^{\Delta}(1 - Out_b)$ *and (2)* f *rejects any offers in which he receives strictly less than* $\delta_f^{\Delta}(1 - Out_b)$.

Then, b *offers* $\left[1 - \delta_f^{\Delta}(1 - Out_b), \delta_f^{\Delta}(1 - Out_b)\right]$*,* f *accepts and* b gets $1 - \delta_f^{\Delta}(1 - Out_b)$, which is higher than Out_b. Indeed, $1 - \delta_f^{\Delta}(1 - Out_b) - Out_b = \left(1 - \delta_f^{\Delta}\right)(1 - Out_b) > 0$.

When $\Delta \to 0$, $\left(1 - \delta_f^{\Delta}\right) \to 0$ and, therefore, $1 - \delta_f^{\Delta}(1 - Out_b) \to Out_b$*; thus, the additional amount that* b *can obtain tends toward zero.* QED

NOT FOR PUBLICATION
Notations and abbreviations
in alphabetical order (Latin then Greek) - § of first occurrence

A^2: Amount that must be borrowed by the firm when rolling over a short-term loan at T^1. § 4.2

b: Agent *b* in Section 3. Otherwise, *b* denotes the initial lending bank. Section 1

c_s: One-shot screening cost by a new bank. Fixed cost. Over period 1, this cost is modeled as a monitoring cost to avoid bargaining issue at T^0. This specific modeling over period 1 has no effect on bargaining at T^1, which is the focus of this analysis. § 2.3

c_f: One-shot cost for the firm of transmitting information to all banks belonging to the competitive margin. Fixed cost. § 2.3

f: Agent *f* in Section 3. Otherwise, *f* denotes the firm/entrepreneur undertaking the project. Section 1

F: Cost of failure (default, bankruptcy) for firms. § 2.4

$\bar{L}$: Minimal amount paid to *b* over both periods so that she breaks even. § 4.4

M^*: Supremum of what *b* can get at a given step - used in the proof of **principle B**. § 3.5

n: # of periods over which agents are forced to bargain in Shaked and Sutton (1984). Here ***n*** =0. § 3.2.

NFP: Net payments to the external financial system (*i.e.* excluding *b*). § 4.3

OOP: Outside option principle. Section 1.

Out_b : (Best) outside option available to the initial lending bank *b*. § 3.4

Out_f : (Best) outside option available to (the firm) *f*. § 3.4

P: Interest-bearing asset. § 3.3

PB^1: Private benefits in the case of a default over period 1 (including negotiations). § 2.4

PB^2: Private benefits in the case of a default over period 2. § 2.4

PD: Firm's probability of default. § 2.1

PEP: Perfect equilibrium partition. Section 1

P^f: Amount or utility that f would obtain by giving *b* just more than her outside option. § 3.6

p_X^2: (With $X = G$ or B) exogenous probability of default of the firm over period 2. § 2.2

q: Probability of *f* being in the "good" state over period 2. § 2.1

r_b: Generic notation for the interest rate agent *b* pays or receives. Used only to compute δ_b. § 3.2

r_f: Generic notation for the interest rate agent *f* pays or receives. Used only to compute δ_f. § 3.2

r_1: First-period cost of funds for banks (risk-free rate in financial markets). By extension, this is the rate that the bank is supposed to pay during the bargaining that starts at T^1. § 2.2

r_2 : Second-period cost of funds for the bank (risk-free rate in financial markets). § 2.2

r_W^1: (With $W = G$ or B) interest rate required by *b* for a short-term loan to a firm of quality W. r_W^1 is a general notation for the effective rate required by *b* at T^1; this rate determines the amount to be repaid at T^1. Note that with the simplifying assumption of no default risk over period 1, whether the state is G or B is not relevant. Still, W is kept in the notation to underline the difference with r_1. § 4.2

$r_W^{1,F}$: (With $W = G$ or B) fair interest rate necessary for *b* to break even over the first-period loan. § 4.4

$r_X^{2,f}$: (With $X = G$ or B) fair interest rate at which a bank breaks even on the second-period loan when the firm strategically defaults and, therefore, borrows a limited amount to buy the asset back; it includes c_s (although the firm actually borrows from the initial bank). § 4.1

$r_X^{2,F}$: (With $X = G$ or B) fair interest rate at which a bank breaks even in the second period when the firm borrows (the full amount of the initial loan) from the initial bank. § 4.2

$r_X^{2,O}$: (With $X = G$ or B) interest rate on a second-period loan when the firm borrows (the full amount of the initial loan) from a new bank; this includes c_s. § 4.2

SITG: skin-in-the-game. Section 1

s^*: *b*'s offer when he makes the first offer, in the proof of **principle B**. § 3.5

s_b: Share of the surplus accruing to *b* according to the PEP. In § 3.3, this notation is used to *compute* the share accrued to *b* at the PEP. Otherwise, it is used as the share accrued to *b* at the PEP. § 3.3

s_b^*: Share of the surplus accruing to *b* according to the PEP in a model without interest-bearing (or costing) assets, as defined in the existing literature. § 3.2

S: Surplus shared between *b* and *f* in a theoretical framework. § 3.3

S_b: Value of the share of surplus accrued to *b* at a PEP. $S_b = s_b S$. § 3.4

$\widetilde{S_b}$: Value of the share of the surplus accrued to *b* in a partition that would differ from the PEP – used in the proof of **principle A**. § 3.4

s_f: Share of the surplus accruing to *f* according to the PEP. In § 3.3, this notation is used to *compute* the share accruing to *b* at the PEP. Otherwise, it is used as the share accrued to *f* at the perfect equilibrium partition. § 3.3

s_f^*: Share of the surplus accruing to *f* according to the PEP in a model without interest-bearing (or costing) assets, as defined in the existing literature. § 3.2

S_f: Value of the share of the surplus accrued to *f* at the PEP. $S_f = s_f S$. § 3.4

$\widetilde{S_f}$: Value of the share of the surplus accruing to *f* in a partition that would differ from the PEP - used in the proof of **principle A**. § 3.4

S_{raw}^{nego}: Actual raw surplus to be shared between the bank and the firm. §4.3

S_{net}^{nego}: Actual net surplus. Equal to actual raw surplus minus net financial payments to the external financial system. Because the latter tends towards zero, based on the assumption on the refinancing rate, it is approximately equal to the actual raw surplus. §4.3

T^0: Beginning of period 1 (capacity building); beginning of first loan. § 2.1

T^1: End of period 1 (capacity building); time to repay the first-period loan, start of bargaining (because the length of bargaining steps tends towards zero, T^1 is approximately the start of the second loan and of period 2 (the production period) § 2.1

T^2: End of period 2 (production period), which starts at the agreement time; time to repay the second-period loan. § 2.1

W: Denotes G if the firm is "good" over period 1 and for B otherwise. § 2.1

X: Denotes G if the firm is "good" over period 2 and for B otherwise. § 2.1

y: Gross material return, production, or capital productivity (unit-size) of the project. § 2.1

Y_f: Firm income if he turns to another bank to repay the first-period loan. § 4.2

Z_b: Bank income in case of firm's strategic default. § 4.1

Z_f: Firm income in case of strategic default (if $Z_f > Y_f$, then $Out_f = Z_f$). § 4.1

Δ: Time-interval between an offer and a counteroffer. § 3.2

δ_b: Discount factor for *b*. § 3.2

δ_f: Discount factor for *f*. § 3.2

Γ^1: Collateral value recovered by the bank in case of failure over period 1 (including the negotiation). § 2.4

ε: Positive element tending towards zero in mathematical proofs.

$\bar{\varepsilon}$: Additional amount (w.r.t. the outside option) that the holder of a binding option with a loss threat can get - tends towards zero - used in the proof of **principle C**. § 3.6

$\varepsilon_X^{2,f}$: Positive element close to zero that the initial bank would grant to the firm to lend rather than letting another bank lend at the "fair" rate including screening cost.

Γ^2: Collateral value recovered by the bank in case of failure over period 2. § 2.4