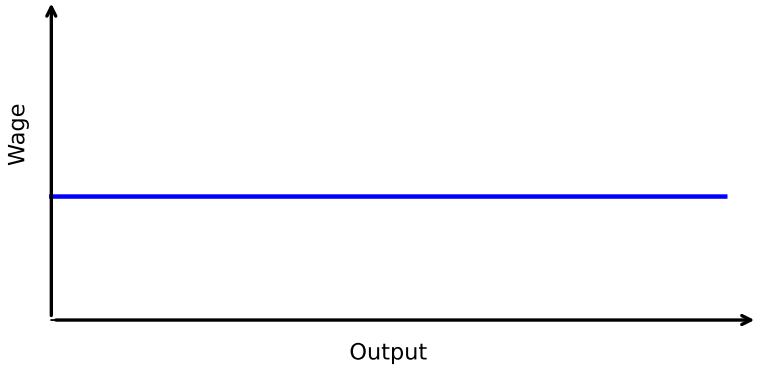
Incentives and Selection

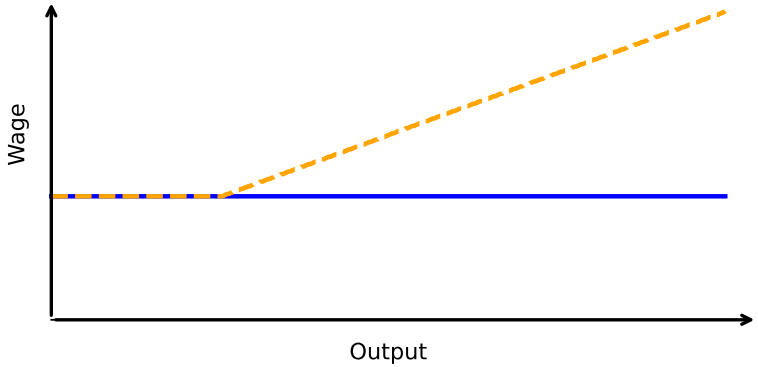
Henrique Castro-Pires
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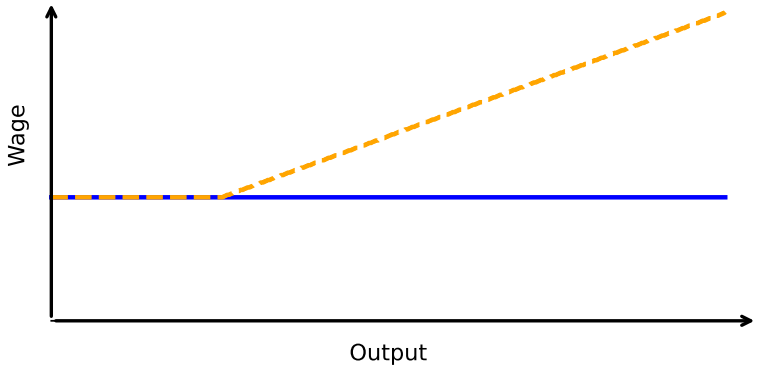
Lazear (2000): Performance Pay and Productivity



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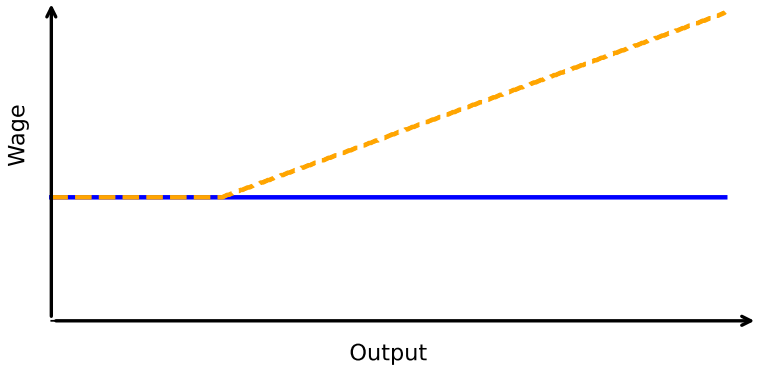


Lazear (2000): Performance Pay and Productivity



Productivity ↑ 44%

Lazear (2000): Performance Pay and Productivity



Productivity ↑ 44% 1/2 due to higher effort 1/2 due to better selection

Conventional wisdom

- Higher-powered incentives lead to better selection
  - Because they are more attractive to high performers
  - *Surveys:* Bandiera et al. (2011), Oyer and Schaefer (2011)...
- If not, "non-standard" preferences must be the source of worsened selection.

  e.g., attracting less prosocial / intrinsicaly motivated workers

This paper in a nutshell

Positive:

1. Do steeper incentives (always) improve selection?

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   - Characterize model primitives under which selection improves or worsens
   - Sufficient statistic for improved (harmed) selection

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2. When do steeper incentives improve or harm selection?
   - Characterize model primitives under which selection improves or worsens
   - Sufficient statistic for improved (harmed) selection

Prescriptive:

3. How to optimize contract accounting for selection?
   - Find the best direction of improvement subject to not hurting selection
   - *Trade-off:* insurance, incentives, and shifting payments to improve selection

Model: Players & Timing

- There is a principal and a unit mass of agents
- The principal wants to hire a fixed number of agents and motivate them to exert effort
- Each agent has type $t \in \{l, h\}$, decides whether to apply and, if hired, chooses effort

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Timing:

1- The principal posts a wage scheme $w(x)$ *(not a menu)*

2- Each agent draws his outside option and decides whether to apply for job

3- The principal screens applicants and hires at random among those who pass the test

4- Each hired agent choose effort a

5- Each worker's output $x \sim f(\cdot|a)$ and payoffs are realized

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$$u_t(w) := \max_a \int \underbrace{v(w(x))}_{\text{utility}} \overbrace{f(x|a)}^{\text{output dist.}} \, dx - \underbrace{c_t(a)}_{\text{effort cost}}$$

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- Each type-t agent draws outside option $\overline{u} \sim G_t(\cdot)$ and applies iff $u_t(w) \geq \overline{u}$
  - *Assumption:* High types have better outside options; i.e., $G_h(\cdot) \succeq^{fosd} G_l(\cdot)$

Model: The Principal

- Ex-ante, principal the share of high types in the population is p

Screening test:

- Each type-t agent passes the test with probability $1 - r_t$. (*Assume:* $r_h < r_l$)
- The principal hires at random among the applicants who pass the test

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Principal's payoff (per worker)**:**

$$\pi(w) = \int \left[x - w(x)\right] \left[q(w) f\left(x|a_h(w)\right) + (1 - q(w)) f\left(x|a_l(w)\right)\right] dx,$$

where $q(w)$ is probability that each worker is a high type

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Main object of interest: How does $q(w)$ change with w?

Building Blocks

Given contract w:

- Each type-t agent applies with probability $G_t := G_t(u_t(w))$
- The probability that each agent is a high type is

$$q(w) := \frac{p(1-r_h)G_h}{p(1-r_h)G_h + (1-p)(1-r_l)G_l}$$

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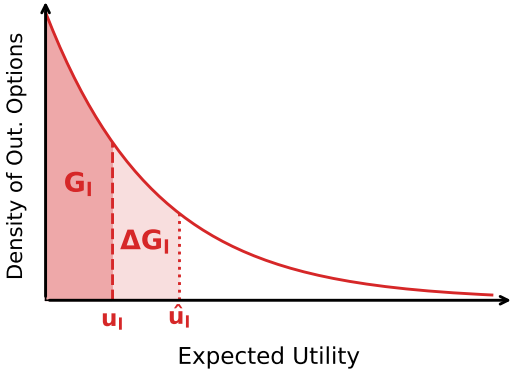
Definition

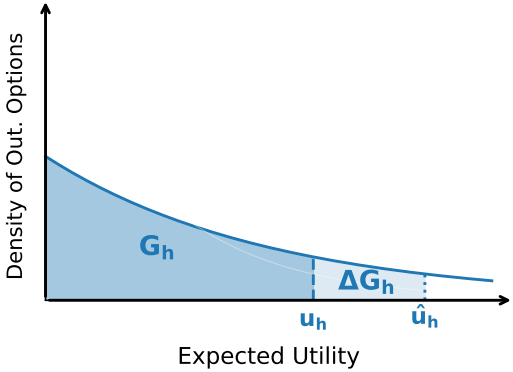
A change in w improves selection if it causes $q(w)$ to rise (and vice versa)

Remark 1

A change in w improves selection if and only if it causes G_h/G_l to rise.

An Illustrative Example

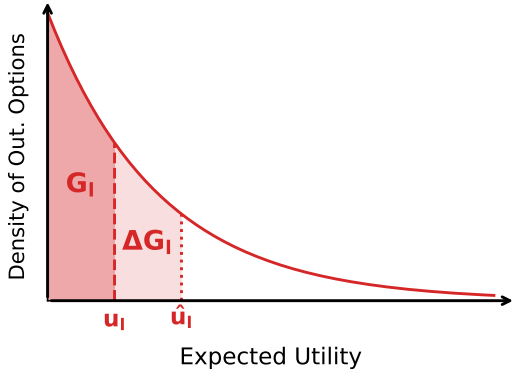


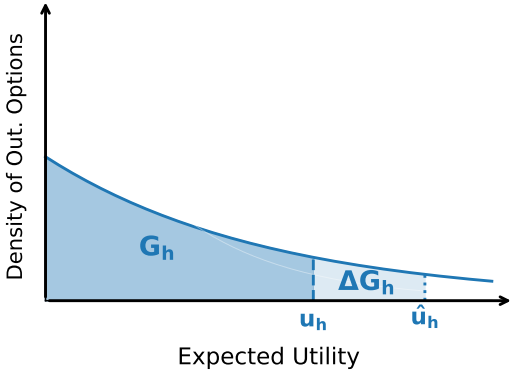


Given status quo contract w, each low and high type applies w.p G_l and G_h, respectively

If we replace w with $\hat{w}$, both types' payoffs and their probabilities of applying will change

An Illustrative Example





Selection improves if and only if

$$\frac{\Delta G_h}{G_h} > \frac{\Delta G_l}{G_l} \textit{ i.e., iff } \%\Delta \Pr\{\text{high type applies}\} > \%\Delta \Pr\{\text{low type applies}\}$$

Example: Linear contracts

Assume:

- Linear contracts: $w(x) = \alpha x$
- Risk-neutral agents with isoelastic effort costs and type-independent outside options

Raising the slope α harms selection if

$$\frac{d}{d\alpha}\frac{G(u_h(w))}{G(u_l(w))} =^s u_h\rho(u_h) - u_l\rho(u_l) < 0,$$

where $\rho := g/G$. That is, raising α harms selection if ρ has elasticity < -1 (and vice versa)

- ρ measures the increase in the probability of applying if an agent gets an extra util
- *Distributions with this property:* Exponential, Pareto (shape> 1), Gaussian (right tail)

Beyond linear contracts

- We evaluate the effects of small contract changes in arbitrary directions.

  i.e., we replace $w(x)$ by $w(x) + \varepsilon\ell(x)$ for some small ε.

- Use notion of a directional derivative. Define the Gateaux differential in direction ℓ:

$$\mathcal{D}h(w, \ell) := \lim_{\varepsilon \downarrow 0} \frac{h(w + \varepsilon\ell) - h(w)}{\varepsilon}$$

Definition

Modifying w in direction ℓ improves selection if $\mathcal{D}q(w, \ell) > 0$ (and harms selection if < 0).

Key Lemma

Lemma 1

Modifying the contract w in direction ℓ improves selection if and only if

$$\mathcal{D}q(w,\ell) =^s \rho_h(u_h) \times \mathcal{D}u_h(w,\ell) - \rho_l(u_l) \times \mathcal{D}u_l(w,\ell) > 0,$$

The key determinants of selection are:

1. The reverse hazard rates $\rho_h := g_h/G_h$ and $\rho_l := g_l/G_l$

   Recall that ρ_t measures the % increase in type-t applicants if they get an extra util

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1. The reverse hazard rates $\rho_h := g_h/G_h$ and $\rho_l := g_l/G_l$

   Recall that ρ_t measures the % increase in type-t applicants if they get an extra util

2. The payoff gains $\mathcal{D}u_h$ and $\mathcal{D}u_l$

"Rotations" – the trivial cases

Recall that selection **improves** if $\rho_h(u_h) \times \mathcal{D}u_h(w, \ell) > \rho_l(u_l) \times \mathcal{D}u_l(w, \ell)$ *and vice versa*

Theorem 1

- *If* $\mathcal{D}u_h(w, \ell) \geq 0 \geq \mathcal{D}u_l(w, \ell)$*, then selection* **improves**
- *If* $\mathcal{D}u_h(w, \ell) \leq 0 \leq \mathcal{D}u_l(w, \ell)$*, then selection is* **harmed**

If high types are made better off and low types worse off, then selection is obviously improved

e.g., with affine contracts, increase the slope while decreasing fixed wage

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e.g., with affine contracts, increase the slope while decreasing fixed wage

Challenge: Need model primitives to know *how much* to decrease the fixed wage

"Steepening" incentives may harm selection

Def: Modifying w in direction ℓ "steepens" incentives if both types obtain stronger incentives

Theorem 2

- *Consider a steepening of w in direction ℓ such that $\mathcal{D}u_h \times \mathcal{D}u_l > 0$.*
- *There exist G_h and $G_l \prec_{fosd} G_h$ such that this modification harms selection.*

Takeaway: For any marginal modification for which the selection effect is nontrivial, there exist outside option distributions such that this modification *harms* selection.

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Takeaway: For any marginal modification for which the selection effect is nontrivial, there exist outside option distributions such that this modification *harms* selection.

Proof idea:

- Selection effect depends on the *utility gains* and the *reverse hazard rates*
- Construct G_h and G_l such that their reverse hazard rates decrease fast enough

A sufficient statistic for improved selection

Denote by $A(w)$ the *fraction* of applicants who pass the screening test

Theorem 4

Modifying the contract w in direction ℓ improves selection if and only if $\mathcal{D}A(w, \ell) > 0$.

Key: High types pass the screening test more often than low types

If pass rate $\uparrow$, modified contract must have attracted proportionally more high than low types

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If pass rate $\uparrow$, modified contract must have attracted proportionally more high than low types

- Valuable information before principal trains the workers and put them to work (costly!)

Finding all directions that *improve* selection

Assumption 1: The principal knows:

- Mass of total and rejected applicants, and screening technology (r_h, r_l)
- Agents' marginal utility $v'(\cdot)$
- Output distributions $f(\cdot|a_l(w))$ and $f(\cdot|a_h(w))$; *i.e.*, can identify high types in workforce

Experiment 1: Principal observes output data from a local modification of w in direction $\hat{\ell}$

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Selection depends on:

- Utility gains $\leftarrow$ Assumption 1
- Reverse hazard rate $\leftarrow$ Assumption 1 + Experiment 1

Optimal local modifications

Principal's profit:

$$\pi(w) = \int \left[x - w(x)\right] \left[q(w) f\left(x|a_h\right) + (1 - q(w)) f(x|a_l)\right] dx$$

Modifying w in direction ℓ has 3 effects:

1- **Direct Effect:** Direct cost of changing payments.

2- **Incentive Effect:** Effect of changes in efforts.

3- **Selection Effect:** Effect of changes in selection.

Optimal local modifications

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Modifying w in direction ℓ has 3 effects:

1- **Direct Effect:** Direct cost of changing payments.

2- **Incentive Effect:** Effect of changes in efforts.

3- **Selection Effect:** Effect of changes in selection.

Ideally, we would like to find the direction ℓ that maximizes the sum of these effects

- Characterizing the magnitude of the selection effect is demanding
- But Assumption 1 + Experiment 1 enable one to infer whether $\mathcal{D}q(w, \ell) > 0$ for every ℓ

Prescriptive problem

$$\max_{\ell:\ \|\ell\| \leq 1} \left\{ (\text{Incentive effect}) + (\text{Direct effect}) \textit{ subject to } \mathcal{D}q(w, \ell) > 0 \right\} \qquad \text{(PP)}$$

Information needed to compute each effect:

- *Positive selection effect:* Per Theorem 4, need Assumption 1 + Experiment 1
- *Direct effect:* Observational data under contract w
- *Incentive effect:* Need one more assumption + one more experiment

Prescription: Replace w with $w(x) + \epsilon \ell^*(x)$ for some small $\varepsilon > 0$

Computing incentives effect: Condition + Experiment

Experiment 2: Post-hiring, offer an unannounced increase in wages in direction ℓ'

- Observe how the output distribution responds holding $q(w)$ constant
- Allows us to identify the incentive effect; *i.e.*, effort response holding selection fixed

Assumption 2: $f(x|a)$ is affine in a

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Theorem 6

Assumptions 1+2 and Experiments 1+2 suffice to solve (PP)

Trade-off: Insurance *vs.* incentives *vs.* shifting payments to outputs that improve selection

- Explicit characterization is in the paper

Limitations

Non-local modifications

- To extrapolate, additional assumptions on $c_t(\cdot)$ and $G_t(\cdot)$ are needed

Endogenous screening

- How to optimize the wage scheme and the screening technology simultaneously?

Miscellaneous

- Outside option distributions are assumed to be exogenous
- Binary types
- Binary screening technology
- Positions are scarce, workers are abundant; *i.e.*, filling positions is never a problem