How Do Heterogeneous Beliefs About Rare Disaster Probability Affect the Equity Term Structure?

Abstract

We show that belief heterogeneity regarding disaster probability influences equity yields, with its impact being amplified when the economy persistently deviates from its long-run mean. Furthermore, when agents also differ in their risk aversion, the effect of belief heterogeneity on equity yields is further strengthened. Additionally, we demonstrate that the disaster risk premium is a nonlinear function of maturity, heterogeneity in beliefs and preferences, the dynamics of disaster probability, and disaster size. These findings reveal new mechanisms that link equity yields, disaster risk, and investor behavior. Moreover, our heterogeneous-agent model successfully generates a procyclical slope of the equity term structure, consistent with empirical evidence.

Keywords: heterogeneous agents; external habit consumption; equity term structure; rare disaster; time-varying disaster probability
JEL: G11, G12, G51, C63

1 Introduction

Recent survey evidence shows that investors who perceive a higher probability of a major market downturn expect lower stock returns (Giglio et al., 2021). This finding highlights the link between perceived disaster probabilities and expected stock returns, aligning with representative-agent models (e.g., Gabaix, 2012; Wachter, 2013). However, as documented by Giglio et al. (2021), this relationship persists at the individual investor level, demonstrating the heterogeneity in investors' disaster probability beliefs. This heterogeneity is pervasive, as investors face challenges in estimating disaster probability due to limited data, the infrequent occurrence of rare disasters, and the potential time variation in disaster probability.

The literature on heterogeneous beliefs has primarily focused on aggregate quantities, such as differences in expected aggregate consumption growth (e.g., Basak, 2005; Atmaz and Basak, 2018), while largely neglecting heterogeneity in disaster probability beliefs. An exception is Chen et al. (2012), who examine the effects of disagreement about disaster probability on the conditional equity premium. However, less is known about how such heterogeneity affects assets with different maturities. How does disagreement over disaster probability influence the yields of short-term versus long-term assets? Does its impact vary over the business cycle? Furthermore, when investors also differ in their risk aversion, how does this risk-aversion heterogeneity interact with belief heterogeneity in shaping asset prices across maturities?

We address these open questions by developing a model of an endowment economy with disaster events, featuring two agents who differ in their risk attitudes and beliefs about the long-run mean of disaster probability, and who exhibit external habit consumption preferences. We use this model to study the effects of disaster probability belief heterogeneity on the term structure of equity yields—a sequence of equity yields for assets with different maturities. Our key findings are as follows:

First, Belief disagreement regarding disaster probability has a stronger effect on the equity yields of long-term assets than on short-term assets. Since investors anticipate disasters to occur infrequently in the future, they expect the accumulated return of long-term maturities to be higher. Moreover, this effect intensifies as belief disagreement widens. For instance, when the optimistic agent becomes even more optimistic, disagreement increases, further increasing equity yields across maturities.

This occurs because optimistic agents expect both short- and long-term cash flows to remain stable and grow, given their perceived low probability of disaster events. As a result, yields across maturities rise, reflecting higher expected returns in the absence of significant downside risk.

Second, The effect of belief disagreement is amplified by the persistence of relative (log) consumption in reverting to its long-term average.[1] Relative (log) consumption, defined as the ratio of aggregate consumption to the standard of living, captures habit formation in consumption. As the standard of living rises, agents adjust their consumption to keep pace with their peers, reflecting their *catching up with the Joneses* preferences.

When an optimistic agent believes that the disaster probability is low and observes that the economy is slow to return to its long-term mean, this persistence reinforces his beliefs. For instance, during expansions, if the agent perceives that the economy will remain in this state for an extended period, his confidence in a low disaster probability strengthens. Consequently, he will anticipate higher yields across maturities.

Third, risk aversion and belief heterogeneity reinforce each other. When the optimistic agent is also less risk-averse, his risk-sharing capacity increases, leading him to allocate a larger share of his portfolio to long-term assets.

Our model provides new insights into how disaster risk influences the yield dynamics of assets with different maturities. The disaster risk premium varies with maturity and depends nonlinearly on heterogeneity in risk aversion and beliefs, the dynamics of disaster probability, the persistence of relative (log) consumption, and disaster size. This framework establishes a rich relationship between disaster risk and the key parameters that capture investors' behavior.

Finally, our model opens a promising avenue for future research on the equity term structure. We assume that the persistence of relative (log) consumption is the same in recessions and expansions; however, it could be business-cycle dependent—higher during recessions, as policymakers aim to accelerate economic recovery, and lower during expansions, when the economy enjoys higher welfare. These persistence dynamics would influence the disaster risk premium and, consequently, equity yields. Another potential extension of our model is to explicitly link disaster probability to the business cycle. In recessions, the disaster probability increases, while in expansions, it decreases. Such an extension could help capture both the procyclical slope of the equity yield term structure and the countercyclical slope of the term structure of one-period returns.

This article contributes to the literature on the effects of belief and preference heterogeneity on asset prices.[2] The closest work to ours is Chen et al. (2012), which examines belief hetero-

[1] When relative (log) consumption is highly persistent, the mean reversion process is slow, implying that deviations from the long-run mean persist for an extended period.

[2] Recent developments in belief heterogeneity include Basak (2005); Bhamra and Uppal (2014); Han et al. (2019); Gao et al. (2020); Giglio et al. (2021); Buraschi and Whelan (2022); Baron et al. (2024); Martin and Papadimitriou (2022); Caramp and Silva (2024), and Bauer and Chernov (2024), among others. Studies on

geneity in disaster probability and its effects on risk-sharing rules. While their study primarily focuses on the equity premium, our work centers on the equity term structure. Moreover, we incorporate risk-aversion heterogeneity and external consumption habit preferences, which allow our model to generate a procyclical slope of the equity term structure—an aspect absent in Chen et al. (2012).[3]

We also contribute to the equity term structure literature.[4] Models in this literature have sought to capture the procyclical behavior of the slope of the term structure of equity yields—negative in recessions and positive in expansions. We complement this strand of research by demonstrating that heterogeneity in beliefs about disaster probability plays a crucial role in explaining the level of equity yields. Furthermore, we show that disaster risk, as priced in equity yields, exhibits a highly nonlinear relationship with belief disagreement and risk aversion. This highlights new channels through which investors' characteristics influence equity yields in equilibrium.

The remainder of this paper is organized as follows. Section 2 describes the two-agent heterogeneous model with rare disaster and time-varying disaster intensity. Section 3 outlines the strategy for determining the equilibrium. Section 4 examines the equity term structure, and we conclude in Section 5. Online Appendix provides proofs and derivations.

risk-aversion heterogeneity include Wang (1996); Chan and Kogan (2002); Longstaff and Wang (2012); Gârleanu and Panageas (2015); Schneider (2022); Gârleanu and Panageas (2023).

[3]Du (2011) is also related to our work. He develops a representative-agent model with habit formation and rare disasters to study S&P 500 index options. Although his model shares two key features with ours, we differ in our object of study—focusing on the equity term structure—and in our modeling approach, which incorporates heterogeneity and a time-varying disaster probability.

[4]This literature has expanded significantly over the last decade, with key contributions from Wachter (2006, 2013); Marfè (2017); Curatola (2015); Croce et al. (2015); Van Binsbergen and Koijen (2017); Backus et al. (2018); Gormsen (2021); Gonçalves (2021); Boguth et al. (2023), among others.

2 Model

This section describes an economy with external habit preferences and rare disasters, where agents differ in risk tolerance and beliefs about the long-run average disaster probability. Proofs of lemmas related to our heterogeneous-agent model and equity term structure calculations are provided in the Online Appendix.

2.1 The Aggregate Endowment and Rare Disaster

We consider an endowment economy where the aggregate endowment, Y_t, consists of two components: a diffusion component that describes its dynamics in normal times and a disaster component that captures its behavior during disaster events. To incorporate both effects, we model the aggregate endowment as an affine jump-diffusion process:

$$dY_t = \underbrace{\mu Y_t dt + \sigma Y_t dZ_t}_{\text{Diffusion component}} + \underbrace{\left(e^B - 1\right) Y_t dN_t}_{\text{Disaster component}}, \tag{1}$$

where μ and σ are positive constants, Z_t is a standard Brownian motion, $B \in (-1, 0)$ is a negative constant representing the instantaneous percentage drop in aggregate consumption when a disaster occurs.[5] Disaster events are captured by a Poisson process, N_t, with a time-varying intensity λ_t. As noted by Wachter (2013), λ_t can be interpreted as the disaster probability. To model λ_t, we employ the Cox-Ingersoll-Ross (CIR) process (Cox et al., 1985), which features mean reversion and stochastic volatility:

$$d\lambda_t = \kappa\left(\bar{\lambda} - \lambda_t\right) dt + \sigma_\lambda \sqrt{\lambda_t} dZ_{\lambda,t}, \tag{2}$$

where $Z_{\lambda,t}$ is a independent standard Brownian motion, κ denotes the speed of mean reversion, and $\bar{\lambda}$ is the the long-run mean of λ_t. Assuming $\kappa > 0$ and $\bar{\lambda} > 0$, Cox et al. (1985) show that λ_t has a stationary Gamma distribution with shape parameter $2\kappa\bar{\lambda}/\sigma_\lambda^2$ and scale parameter $\sigma_\lambda^2/(2\kappa)$. To assure that λ_t is non-negative, we calibrate σ_λ, $\bar{\lambda}$, and κ such that the Feller condition holds: $2\kappa\bar{\lambda}/\sigma_\lambda^2 > 1$.

To visualize both processes, we simulate them in Figure 1. The left-hand panel illustrates the behavior of the aggregate endowment, modeled as a jump-diffusion process with a time-

[5] When a disaster event occurs ($dN_t = 1$), the endowment process Y_t experiences a jump of size:

$$dY_t = \left(e^B - 1\right) Y_t dN_t \rightarrow dY_t = \left(e^B - 1\right) Y_t.$$

Dividing through by Y_t, the relative change in the endowment process is $dY_t/Y_t = e^B - 1$. For small values of B, the exponential term can be approximated using a first-order Taylor expansion ($e^B \approx 1 + B$). Substituting this approximation yields: $dY_t/Y_t \approx 1 + B - 1 = B$. Thus, for small B, the relative jump in the endowment process is approximately equal to B.

varying disaster probability, as defined by equations (1) and (2). It is evident that disaster events significantly reduce the aggregate endowment. The right-hand panel depicts the disaster probability, featuring mean reversion and stochastic volatility.

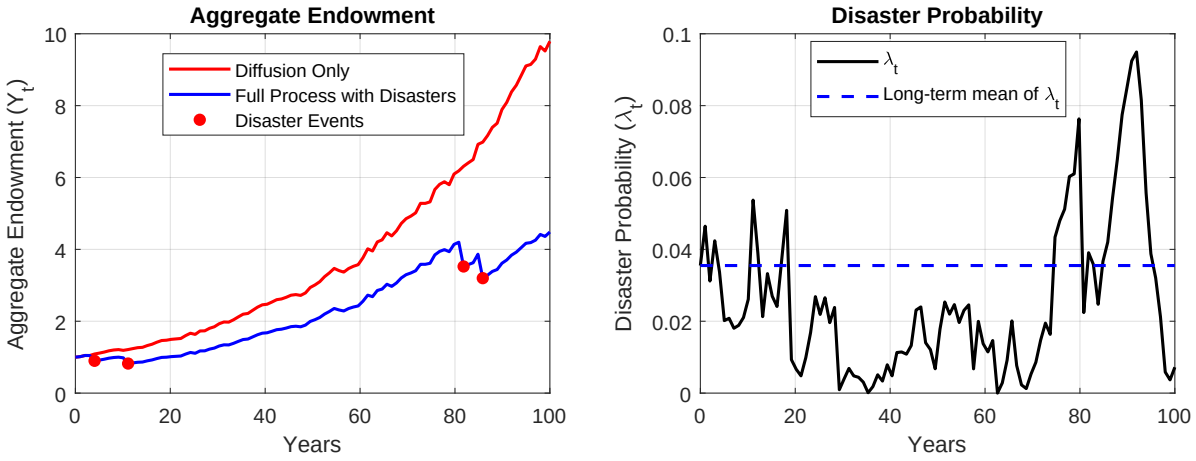


Figure 1: **Aggregate Endowment and Disaster Probability.** The left panel illustrates the endowment process Y_t, modeled as a jump-diffusion process (blue line) and as a pure diffusion process (red line), with parameters $\mu = 0.0252$, $\sigma = 0.020$, and $B = -0.2357$. The right panel presents the disaster probability, modeled as a CIR process, with parameters $\kappa = 0.080$, $\bar{\lambda} = 0.0355$, and $\sigma_\lambda = 0.067$. This calibration follows Wachter (2013) and Fernández-Villaverde and Levintal (2024).

2.2 Standard of Living

We assume that agents derive welfare not only from their own consumption but also from past aggregate consumption, represented by X_t, which we refer to as the standard of living in the economy. Following Chan and Kogan (2002), we define X_t as the weighted geometric average of the historical realizations of the log aggregate endowment, $y_s = \log Y_s$, given by:

$$X_t = \exp\left(\frac{\int_0^t \chi_s y_s ds}{\int_0^t \chi_s ds}\right), \tag{3}$$

where χ_s denotes the weighting function, defined as:

$$\chi_s = \lambda_x e^{-\lambda_x (t-s)}, \quad s < t, \quad \lambda_x \in [0, 1]. \tag{4}$$

Fixing the current period t, the weighting function is increasing in s, indicating that more recent values of the aggregate endowment exert a greater influence on X_t than older values.

Additionally, λ_x captures the persistence of the history of y_s on X_t. Specifically, higher values of λ_x significantly increase the weight of recent realizations of y_s, whereas lower values of λ_x result in past realizations of y_s having a relatively uniform influence. The shape of the weighting function and the influence of λ_x are illustrated in Figure 2.

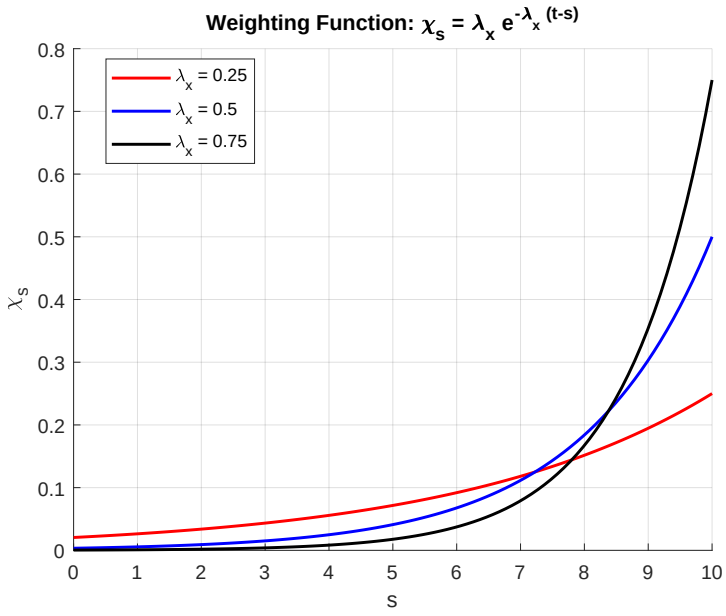


Figure 2: **Weighting Function χ_s and λ_x.** This figure shows the weighting function χ_s for three values of λ_x (0.25, 0.5, and 0.75). The current period t is fixed to ten and χ_s is calculated for $s \leq t$.

We next summarize the two aggregate state variables, Y_t and X_t, into one variable, the relative (log) consumption, given by

$$\omega_t = \log(Y_t/X_t). \tag{5}$$

The variable ω_t measures the aggregate consumption relative to the standard of living and is defined as $\omega_t = \log Y_t - \log X_t$. Since ω_t is correlated with Y_t, we interpret low values of ω_t as indicative of recessions and high values as indicatives of expansions. The following lemma presents the dynamics of y_t, $x_t = \log(X_t)$, and ω_t.

Lemma 2.1. *Given the dynamics of the aggregate endowment Y_t, the dynamics of $y_t = \log Y_t$, shown in equation (6), are derived using Itô's lemma. The dynamics of the log standard of living, x_t (equation 7), follow from applying Itô's lemma to the definition of X_t (equation 3). Finally, applying Itô's lemma to the definition of ω_t yields its dynamics in equation (8). The processes for y_t, x_t, and ω_t are given by:*

$$
\begin{aligned}
dy_t &= \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dZ_t + B dN_t, && (6)\\
dx_t &= \lambda_x (y_t - x_t) dt, && (7)\\
d\omega_t &= \lambda_x \left(\bar{w} - w_t\right) dt + \sigma dZ_t + B dN_t, && (8)
\end{aligned}
$$

where λ_x is the mean-reversion rate, $\bar{\omega} = \left(\mu - \sigma^2/2\right)/\lambda_x$ is the long-run mean to which ω_t reverts in the absent of disaster events, and $\sigma^2/(2\lambda_x) + \bar{\lambda}B^2$ is its long-term variance. Equation (8) implies that ω_t follows a mean-reverting jump-diffusion process.

The dynamics of relative (log) consumption merit further discussion. First, relative consumption is affected by disaster events, reflecting the adjustment in the standard of living when disasters occur. Second, the magnitude of this adjustment depends on the rate of mean reversion (or equivalently, the persistence of the history of the aggregate endowment on the standard of living) λ_x. When λ_x is large, agents place greater value on recent realizations of the aggregate endowment. In the case of a disaster event, a higher λ_x implies a stronger adjustment to the standard of living, closely following the reduction in the aggregate endowment resulting from the disaster. This adjustment is reflected in a lower ω_t compared to the effect of a lower λ_x.

Figure 3 shows the log aggregate endowment y_t, the log of standard of living x_t, and the relative (log) consumption ω_t. The left-hand panel illustrates the behavior of relative (log) consumption ω_t when disaster events are taken into account, while the right-hand panel depicts the effects of λ_x on ω_t: a higher λ_x implies a faster mean reversion of ω_t to its long-run mean, leading to lower volatility.

2.3 Financial Markets

Financial markets are complete, and investment opportunities consist of three types of security: a stock, a risk-free bond, and a set of one-year maturity disaster insurance contracts, as assumed by Chen et al. (2012). The stock price is denoted by S_t, while the bond price is represented by B_t. Their price dynamics are given by:

$$
\begin{aligned}
dS_t &= (\beta_t S_t - \delta_t) dt + \sigma_t S_t dZ_t, && (9)\\
dB_t &= r_t B_t dt, && (10)
\end{aligned}
$$

where β_t is the expected rate of return of the stock, r_t is the instantaneous interest rate, and σ_t is the stock's return volatility. Assuming the aggregate dividend D_t equals the aggregate endowment Y_t and that only one share of the stock exists, the dividend per share per unit of time is given by $\delta_t = Y_t$.

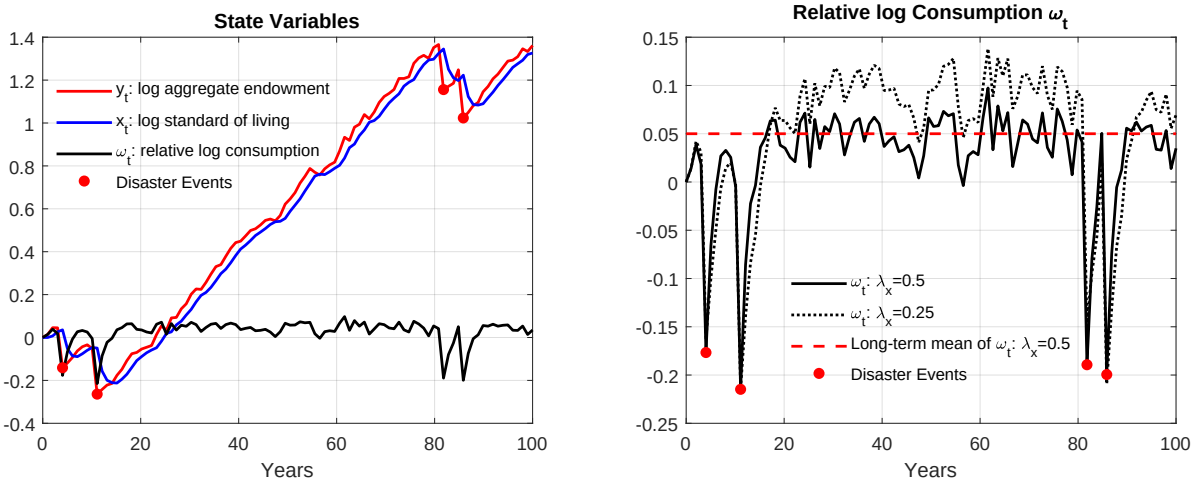


Figure 3: **Aggregate State Processes.** The left panel presents the log aggregate endowment, y_t, modeled as a jump-diffusion process with parameters $\mu = 0.0252$, $\sigma = 0.020$, and $B = -0.2357$. It also displays the log standard of living, x_t, and the relative (log) consumption, ω_t, which follows a mean-reverting jump-diffusion process with a long-run mean $\bar{\omega} = 0.0059$ and a mean reversion rate $\lambda_x = 0.5$. The disaster probability, λ_t, evolves according to a CIR process with parameters $\kappa = 0.080$, $\bar{\lambda} = 0.0355$, and $\sigma_\lambda = 0.067$. The right panel illustrates ω_t for two values of λ_x: 0.5 (continuous line) and 0.25 (dotted line), highlighting its sensitivity to disaster events.

2.4 Agents

The economy consists of two agents who differ in risk aversion and beliefs about the probability of disaster events. The more risk-averse agent is also pessimistic, referred to as the Averse-Pessimistic (AP) agent, while the less risk-averse agent is optimistic, referred to as the Tolerant-Optimistic (TO) agent. Their preferences follow the *catching up with the Joneses* framework (e.g., Abel, 1990; Chan and Kogan, 2002), where an external standard of living influences their utility function.

Preferences and Heterogeneity in Risk Aversion. The investor's utility function, representing this preferences type, follows a constant relative risk aversion (CRRA) specification with a relative risk aversion coefficient (RRA) γ_k for agent $k \in \{AP, TO\}$, given by

$$u(c_{kt}, X_t) = \frac{(c_{kt}/X_t)^{1-\gamma_k}}{1-\gamma_k}, \quad \gamma_{AP} > \gamma_{TP}, \tag{11}$$

where X_t represents the standard of living in the economy, which agents seek to adjust to. Thus, as the standard of living increases, agents adjust their consumption accordingly. As shown by

Chan and Kogan (2002), this complementary effect of X_t on current consumption requires the following condition holds:

$$\frac{\partial^2 u(c_{kt}, X_t)}{\partial X_t \partial c_{kt}} \equiv \frac{\partial u_c(c_{kt}, X_t)}{\partial X_t} = (1-\gamma_k) c_{kt}^{-\gamma_k} X_t^{\gamma_k - 2} \geq 0, \quad k \in \{AP, TO\}$$

This condition holds when $\gamma_k \geq 1$; thus, we restrict our analysis to relative risk aversion values greater than one. Additionally, as mentioned in the previous section, X_t is modeled as a geometric average of past realizations of the aggregate endowment (Eq. 3).

Since the relative (log) consumption ω_t summarizes the state of the economy, we rewrite the agent's utility function (11) in terms of ω_t as follows:

$$u(c_{kt}, X_t) = \frac{(c_{kt}/X_t)^{1-\gamma_k}}{1-\gamma_k} \equiv \frac{1}{1-\gamma_k} \left(\tilde{c}_{kt} e^{\omega_t}\right)^{1-\gamma_k} \equiv u(\tilde{c}_{kt}, \omega_t), \tag{12}$$

where $\tilde{c}_{kt} = c_{kt}/Y_t$ represents the agent k's share of aggregate consumption and serves as the new control variable.

Heterogeneity in Beliefs. Estimating the probability of a disaster poses significant challenges for investors due to the scarcity of historical data, the infrequency of disaster events, and the time-varying nature of disaster risk. Consequently, agents are likely to hold heterogeneous beliefs regarding its dynamics. Specifically, we model agents as differing in their assessments of the long-term mean disaster probability, $\bar{\lambda}$: while the averse-pessimistic (AP) agent believes that the long-term mean of disaster probability is high, the tolerant-optimistic (TO) agent believes the opposite, implying that $\bar{\lambda}^{AP} > \bar{\lambda}^{TO}$. Furthermore, agents have equivalent probability measures $\mathcal{P}^k$, where $k \in \{AP, TO\}$, which are also equivalent to the physical measure $\mathcal{P}$. Consequently, there exists a Radon-Nikodym derivative $\eta_k = d\mathcal{P}^k/d\mathcal{P}$ such that the expectation of any stochastic variable H_t under the probability measure $\mathcal{P}^k$ can be equivalently written in the probability measure $\mathcal{P}$ as follows:

$$\mathbb{E}_{\mathcal{P}^k}\left[H_t\right] = \mathbb{E}_{\mathcal{P}}\left[H_t \times \frac{d\mathcal{P}^k}{d\mathcal{P}}\right] = \mathbb{E}_{\mathcal{P}}\left[H_t \times \eta_k\right], \quad k = \{AP, TO\} \tag{13}$$

We use the property (13) in the social planner's problem, as will be discussed later. For each agent, given their perceived long-run mean of disaster probability $\bar{\lambda}^k$, the perceived disaster probability follows:

$$d\lambda_t^k = \kappa\left(\bar{\lambda}^k - \lambda_t^k\right) dt + \sigma_\lambda \sqrt{\lambda_t^k}\, dZ_{\lambda,t}^k, \quad k \in \{AP, TO\}, \tag{14}$$

where $dZ_{\lambda,t}^k$ is a standard Brownian motion defined in the probability space of agent k with probability measure $\mathcal{P}^k$.

Next, we define the relationship between agent k's subjective beliefs and the true physical probability through the variable $\eta_{k,t}$, as stated in Lemma 2.2:

Lemma 2.2. *Utilizing the Girsanov theorem and Itô's lemma for jump-diffusion processes, along with the dynamics of λ_t (equation 2), the Radon-Nikodym derivative for each agent k is given by:*

$$\eta_{k,t} = \frac{d\mathcal{P}^k}{d\mathcal{P}} \equiv \exp\left(-\int_0^t \lambda_s \varphi^k ds + \left[\ln\left(1+\varphi^k\right)\right] N_t\right), \quad k \in \{AP, TO\}, \tag{15}$$

where $\varphi^k = \bar{\lambda}^k/\bar{\lambda} - 1$ represents the deviation of agent k's beliefs about the long-run average disaster probability from the true value $\bar{\lambda}$.

Furthermore, the dynamics of $\eta_{k,t}$ evolves as:

$$d\log\eta_{k,t} = -\varphi^k \lambda_t dt + \ln\left(1+\varphi^k\right) dN_t. \tag{16}$$

Expression (15) aligns with the findings of Chen et al. (2012), who examine the role of belief heterogeneity in the equity premium. Intuitively, an agent's disagreement regarding the true value of $\bar{\lambda}$, as reflected in $\eta_{k,t}$, accounts for both normal and disaster events. For instance, if $\bar{\lambda}^k < \bar{\lambda}$ (i.e., agents underestimate the long-term average disaster probability), then in normal times without disasters, the likelihood ratio $\eta_{k,t}$ increases at a rate of $(\bar{\lambda} - \bar{\lambda}^k)/\bar{\lambda}$. Conversely, during disaster events, $\eta_{k,t}$ jumps by a factor of $\ln\left(\bar{\lambda}^k/\bar{\lambda}\right)$. This follows directly from Eq. (15) when incorporating the definition of φ^k:

$$\eta_{k,t} = \exp\left(\int_0^t \lambda_s \left(\frac{\bar{\lambda} - \bar{\lambda}^k}{\bar{\lambda}}\right) ds + \left[\ln\left(\frac{\bar{\lambda}^k}{\bar{\lambda}}\right)\right] N_t\right).$$

Furthermore, under the assumption $\bar{\lambda}^k < \bar{\lambda}$, the pure jump process N_t has a negative jump size, since $\left(\ln\left(\bar{\lambda}^k/\bar{\lambda}\right) < 0\right)$, implying that disaster events reduce the likelihood ratio $\eta_{k,t}$.

Following Chen et al. (2010), we next examine the Radon-Nikodym derivatives *between* agents' probability measures, denoted as $\eta_t = d\mathcal{P}^{TO}/d\mathcal{P}^{AP}$, which characterizes belief heterogeneity between the two agents. Using Eq. (15), we define η_t as:

$$\eta_t = \frac{\eta_{TO,t}}{\eta_{AP,t}} = \frac{d\mathcal{P}^{TO}}{d\mathcal{P}^{AP}} \equiv \exp\left(-\int_0^t \lambda_s \left(\frac{\bar{\lambda}^{TO} - \bar{\lambda}^{AP}}{\bar{\lambda}}\right) ds + \ln\left(\frac{\bar{\lambda}^{TO}}{\bar{\lambda}^{AP}}\right) N_t\right), \tag{17}$$

where AP denotes the Averse-Pessimist agent ($k = 1$) and TO denotes the Tolerant-Optimistic agent ($k = 2$). The Radon-Nikodym derivative $\eta_t \equiv \eta_{TO,t}/\eta_{AP,t}$ quantifies the extent of belief divergence, with $\eta_t = 1$ in the absence of heterogeneity. The impact of belief heterogeneity is further revealed through the dynamics of η_t:

$$d\log\eta_t = -\tilde{\varphi}_0 \lambda_t dt + \tilde{\varphi}_1 dN_t, \tag{18}$$

where $\tilde{\varphi}_0 = \left(\bar{\lambda}^{TO} - \bar{\lambda}^{AP}\right)/\bar{\lambda}$ governs the impact of belief heterogeneity in normal times, and $\tilde{\varphi}_1 = \log\left(\bar{\lambda}^{TO}/\bar{\lambda}^{AP}\right)$ determines the magnitude of jumps in η_t during disaster events.

Figure 4 illustrates the Radon-Nikodym derivatives $\eta_{k,t}$, capturing the relationship between $\mathcal{P}^k$ and $\mathcal{P}$, as well as η_t, which quantifies belief heterogeneity between the agents by comparing $\mathcal{P}^{AP}$ and $\mathcal{P}^{TO}$. The left and middle panels show that $\eta_{TO,t}$ is more sensitive to rare disasters than $\eta_{AP,t}$ since the jump magnitude, given by $\ln\left(\bar{\lambda}^k/\bar{\lambda}\right)$, is larger for the TO agent.

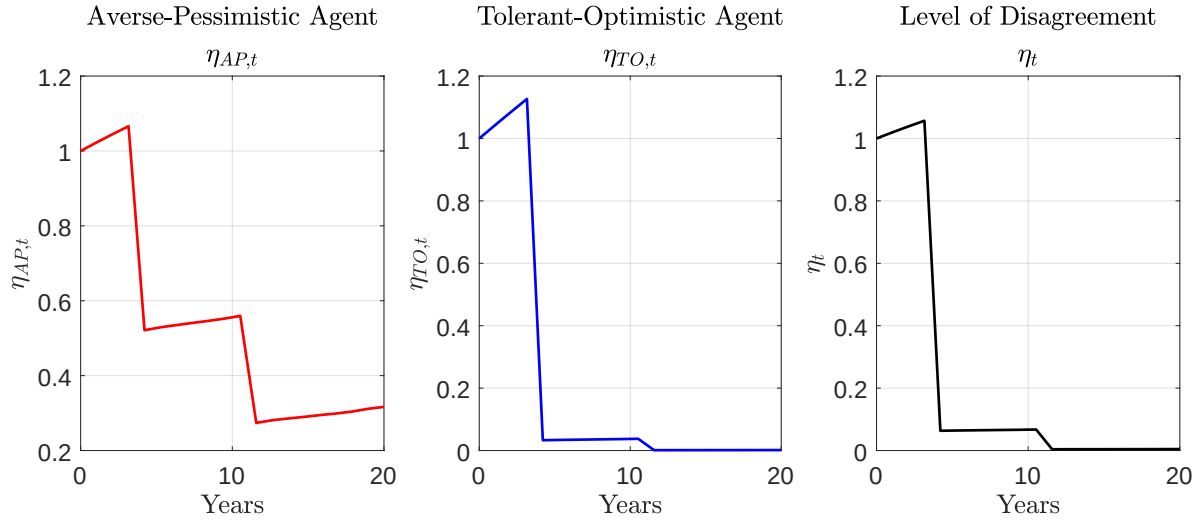


Figure 4: **Radon-Nikodym Derivative.** The left and middle panels depict the Radon-Nikodym derivative $\eta_{k,t}$ for each agent, assuming that the disaster probability λ_t follows a CIR process with parameters $\kappa = 0.080$, $\bar{\lambda} = 3.55\%$, and $\sigma_\lambda = 0.067$. Agents' beliefs are specified as $\bar{\lambda}^{AP} = 1.7\%$ and $\bar{\lambda}^{TO} = 0.1\%$ (Chen et al., 2012). The right panel illustrates $\eta_t = \eta_{TO,t}/\eta_{AP,t}$, highlighting the role of the pure jump process N_t as a key driver of belief disagreement.

2.5 The Equilibrium

Given that markets are complete, the equilibrium allocations are characterized by the solution to the social planner problem. Utilizing the property of the equivalent probability measure, as stated in equation (13), we reformulate the objective function of each agent under the objective probability measure $\mathcal{P}$ as follows:

$$\mathbb{E}_{\mathcal{P}^k}\left[\int_0^\infty e^{-\rho t}u(\tilde{c}_{kt},\omega_t)dt\right] = \mathbb{E}_{\mathcal{P}}\left[\int_0^\infty \frac{d\mathcal{P}^k}{d\mathcal{P}}e^{-\rho t}u(\tilde{c}_{kt},\omega_t)dt\right] = \mathbb{E}_{\mathcal{P}}\left[\int_0^\infty \eta_{k,t}e^{-\rho t}u(\tilde{c}_{kt},\omega_t)dt\right], \tag{19}$$

where $\eta_{k,t}$ denotes the Radon-Nikodym derivative of agent k. Under this transformation, the social planner problem can be specified as:

$$\max_{\{\tilde{c}_{1t},\tilde{c}_{2t}\}} \mathbb{E}_{\mathcal{P},0}\left[\int_0^\infty e^{-\rho t}\left(\alpha_{1,0}\eta_{1,t}u(\tilde{c}_{1t},\omega_t)+\alpha_{2,0}\eta_{2,t}u(\tilde{c}_{2t},\omega_t)\right)dt\right], \tag{20}$$

subject to the resource constraint:

$$\tilde{c}_{1t}+\tilde{c}_{2t}=1, \tag{21}$$

Since the total endowment is consumed, and no resources are transferred over time in the Social Planner problem, the problem becomes static. Consequently, Eq. (20) can be reformulated as:

$$\alpha_{1,0}\eta_{1,t}\times\max_{\{\tilde{c}_{1t},\tilde{c}_{2t}\}}\left\{e^{-\rho t}\left[u(\tilde{c}_{1t},\omega_t)+\alpha\eta_t u(\tilde{c}_{2t},\omega_t)\right]\right\}, \tag{22}$$

subject to Eq. (21), where $\alpha=\alpha_{2,0}/\alpha_{1,0}$ and $\eta_t=\eta_{2,t}/\eta_{1,t}$. This formulation follows Basak (2005), who demonstrates that under heterogeneity in beliefs, the weights used to construct the social planner's utility function are stochastic processes, represented by $\alpha_{k,0}\eta_{k,t}$, where $\alpha_{k,0}$ is a constant. For notational simplicity, we denote the Averse-Pessimist (AP) agent as $k=1$ and the Tolerant-Optimist (TO) agent as $k=2$, implying $\eta_t=\eta_{2,t}/\eta_{1,t}=\eta_{TO,t}/\eta_{AP,t}$.

3 Solving for the Equilibrium

3.1 Calibration

Our calibration, presented in Table 1, is based on parameter values widely used in the disaster risk literature (Barro, 2006; Barro and Ursúa, 2012), models of time-varying disaster intensity (Wachter, 2013), and studies on habit consumption (Xiouros and Zapatero, 2010). Specifically, the disaster size follows the estimates of Barro and Ursúa (2012), who uses GDP and consumption data from 40 and 20 countries, respectively, covering a non-continuous sample period from 1870 to 2009.

Regarding beliefs about disaster probabilities, we assume that the averse-pessimistic agent perceives a long-run average disaster probability of 1.7%, following Barro (2006). In contrast, the optimistic agent believes disasters occur once every 100 years, implying a subjective disaster probability of 1% ($\bar{\lambda}^{TO}=1\%$).

The parameter values governing the dynamics of aggregate endowment and time-varying disaster intensity are drawn from Wachter (2013), who calibrates these values to match postwar aggregate consumption data for G7 countries and the disaster distribution documented by Barro and Ursúa (2008).

Additionally, we set the speed of mean reversion for relative (log) consumption, λ_x, to 9.2% and the subjective discount rate to 4.19%, based on Xiouros and Zapatero (2010), a study that

examines habit consumption and risk-aversion heterogeneity in asset pricing.

Finally, we select the relative risk aversion parameters for the more and less risk-averse agents, γ_1 and γ_2, within the standard macro-finance range of 1 to 10. Specifically, we assume $\gamma_1 = 2$ and $\gamma_2 = 1.2$. Overall, our calibration aligns with key empirical asset pricing patterns, including the procyclical price-dividend ratio and the countercyclical nature of both the price of risk and stock return volatility.

Table 1: Calibration

Parameters	Symbol	Value	Source
(A) Aggregate consumption and habits			
Subjective discount rate (%)	ρ	4.19	Xiouros and Zapatero (2010)
Mean endowment growth (%)	μ	2.52	Wachter (2013)
S.D. endowment growth (%)	σ	2.0	Wachter (2013)
Speed of mean reversion of ω_t (%)	λ_x	9.2	Xiouros and Zapatero (2010)
(B) Disaster size and intensity			
Disaster size (%)	B	-23.57	Barro and Ursúa (2012)
Speed of mean reversion of λ_t (%)	κ	8.0	Wachter (2013)
Long-run mean of λ_t (%)	$\bar{\lambda}$	3.55	Wachter (2013)
S.D. coefficient of λ_t (%)	σ_λ	6.7	Wachter (2013)
(C) Heterogeneity in RRA and Beliefs			
Beliefs of averse-pessimistic agent (%)	$\bar{\lambda}^{AP}$	1.7	Barro (2006)
Beliefs of tolerant-optimistic agent (%)	$\bar{\lambda}^{TO}$	1	Disasters occur once every 100 years
RRA of more risk-averse agent	γ_1	2	Conventional
RRA of less risk-averse agent	γ_2	1.2	Conventional

Parameter values are in annual terms.

3.2 Risk-sharing and Optimal Consumption

We next derive expressions for the risk-sharing rule and the optimal consumption for each agent by solving the social planner's problem (Eq. 22 subject to 21), as summarized in the following lemma.

Lemma 3.1. *Under complete financial markets and the absence of arbitrage opportunities, the social planner equilibrium is characterized by the following results:*

- ***Risk-Sharing Rule:*** *The optimal risk-sharing condition is given by*

$$u_{\tilde{c}_{1t}}(\tilde{c}_{1t}, \omega_t) = \alpha \eta_t \, u_{\tilde{c}_{2t}}(\tilde{c}_{2t}, \omega_t), \tag{23}$$

  where $u_{\tilde{c}_{kt}}(\tilde{c}_{kt}, \omega_t) = \frac{\partial u(\tilde{c}_{kt}, \omega_t)}{\partial \tilde{c}_{kt}}$ *denotes the marginal utility function of agent* k*, for* $k \in \{1, 2\}$ *or equivalently* $k \in \{AP, TO\}$*.*

- ***Optimal Consumption:*** *The consumption allocations for the two agents are given by*

$$\tilde{c}_{1t} = \tilde{\alpha}^{-1/\gamma_1} \eta_t^{-1/\gamma_1} e^{\omega_t(\gamma_2 - \gamma_1)/\gamma_1} (1 - \tilde{c}_{1t})^{\gamma_2/\gamma_1}, \tag{24}$$

$$\tilde{c}_{2t} = 1 - \tilde{c}_{1t}, \tag{25}$$

  where $\tilde{\alpha} = \alpha_{2,0}/\alpha_{1,0}$ *and* $\eta_t = \eta_{2,t}/\eta_{1,t}$*. Furthermore, the optimal consumption of the first agent can be expressed as* $\tilde{c}_{1t} = \tilde{c}_{1t}(\omega_t, \eta_t)$*.*

To gain insight into the role of risk aversion, belief heterogeneity, and rare disasters, Figure 5 plots the optimal consumption of the averse-pessimistic agent, $\tilde{c}_{1t}$, across four different model specifications.

The upper panels present $\tilde{c}_{1t}$ for the baseline model, which features consumption habit and risk-aversion heterogeneity, both without disaster events (*Model 1*) and with disaster events (*Model 2*). The upper-left graph illustrates the negative relationship between optimal consumption $\tilde{c}_{1t}$ and relative (log) consumption, ω_t, in both models. This indicates that during bad times (low levels of ω_t), $\tilde{c}_{1t}$ is higher than in normal times (high levels of ω_t). This behavior is primarily driven by the averse agent's reduced exposure to aggregate risk. Moreover, in the presence of disaster events, aggregate consumption drops significantly—more than the standard of living—resulting in a large but negative ω_t. Consequently, the relative consumption of the tolerant agent ($k = 2$) declines substantially, leading to an increase in $\tilde{c}_{1t}$, as evidenced by the higher levels of $\tilde{c}_{1t}$ under Model 2 compared to Model 1.

Since Models 1 and 2 do not incorporate belief heterogeneity, the level of disagreement, η_t, remains equal to one, as shown in the upper-middle graph. The upper-right graph simulates

the consumption process $\tilde{c}_{1t}$, illustrating that $\tilde{c}_{1t}$ under Model 2 exhibits greater volatility than in Model 1 due to the presence of disaster events.

The bottom panels depict $\tilde{c}_{1t}$ in a more comprehensive model that incorporates consumption habit, disaster events, and heterogeneity in both risk aversion and beliefs regarding the long-run mean of disaster probability (*Model 3*). Additionally, the panels include results for a variation of this model in which risk-aversion heterogeneity is removed (*Model 4*).

First, under belief heterogeneity, the relationship between $\tilde{c}_{1t}$ and ω_t becomes non-linear and lacks a clear pattern, as shown in the bottom-left graph. Second, $\tilde{c}_{1t}$ is a decreasing and non-linear function of the level of disagreement between agents, η_t, as illustrated in the bottom-middle graph. Since the tolerant-optimistic (TO) agent perceives a lower disaster frequency than the averse-pessimistic (AP) agent, η_t is generally below one. When disagreement diminishes, meaning the TO agent's beliefs converge toward those of the AP agent, the TO agent becomes more pessimistic, reducing market participation and increasing current consumption. This negatively affects the consumption share of the AP agent, $\tilde{c}_{1t}$. Consequently, a negative relationship between $\tilde{c}_{1t}$ and η_t emerges endogenously. Third, under belief heterogeneity, the AP agent dominates the economy in the long run since their beliefs are closer to the true disaster probability, a result consistent with Chen et al. (2012).

When risk-aversion heterogeneity is removed (*Model 4*), the relationships between $\tilde{c}_{1t}$, ω_t, and η_t persist but at a different scale: the consumption level $\tilde{c}_{1t}$ is lower when only belief heterogeneity is present (*Model 4*). This underscores the importance of risk-aversion heterogeneity in the economy: the more risk-averse agent allocates a greater share of resources to the risk-free asset, reducing exposure to disaster events and increasing the consumption share $\tilde{c}_{1t}$ during economic downturns and disaster events. Thus, pessimistic beliefs are reinforced by heightened risk aversion, further elevating $\tilde{c}_{1t}$.

3.3 Asset Prices

Equipped with the risk-sharing rule, the agents' optimal consumption, and the fact that aggregate uncertainty in this economy is driven by two independent standard Brownian motions (Z_t and $Z_{\lambda,t}$) and a jump (Poisson) process N_t, we now derive expressions for the stochastic discount factor (SDF), the risk-free interest rate, and the price of risk.

Lemma 3.2. *Under the assumptions of complete markets and no-arbitrage opportunities, the dynamics of the SDF (m_t) are given by*

$$\frac{dm_t}{m_t} = -\tilde{r}_t dt - \tilde{\psi}_t dZ_t - \tilde{\psi}_{\lambda,t} dZ_{\lambda,t} - \tilde{\theta}_t dN_t, \tag{26}$$

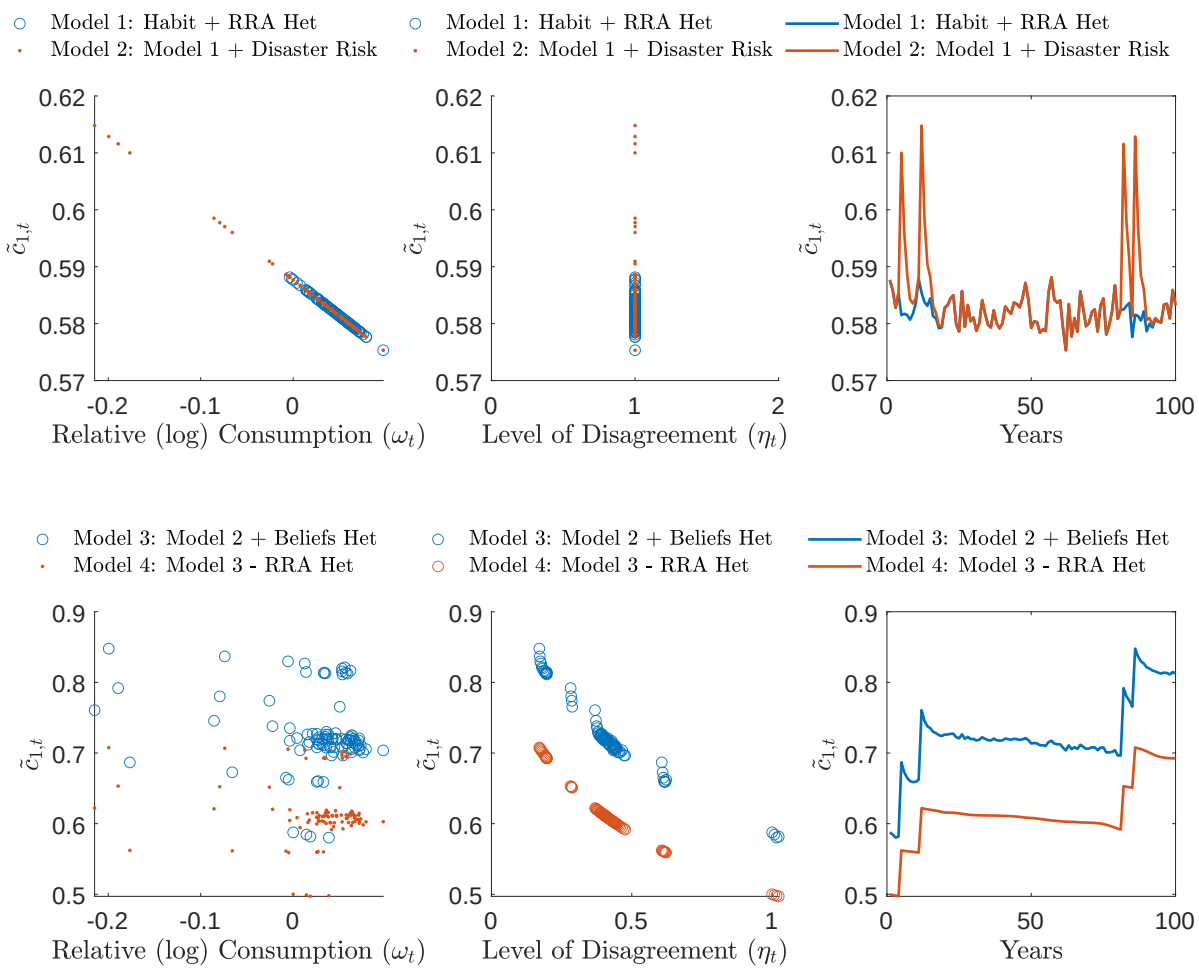


Figure 5: **Optimal Consumption of the Averse-Pessimistic Agent ($\tilde{c}_{1t}$).** This figure displays the optimal consumption share $\tilde{c}_{1t}$ over a 100-period simulation for four models. The upper panel (without belief heterogeneity) compares *Model 1*, the baseline model featuring consumption habit and risk-aversion heterogeneity, with *Model 2*, which extends Model 1 by incorporating disaster risk. The lower panel (with belief heterogeneity) examines *Model 3*, which builds on Model 2 by introducing belief heterogeneity, and *Model 4*, which removes risk-aversion heterogeneity from Model 3.

where $\tilde{r}_t$ denotes the risk-free interest rate without the effect of disaster events, $\tilde{\psi}_t$ denotes the price of risk associated with the endowment level shock, $\tilde{\psi}_{\lambda,t}$ denotes the price of risk associated with the disaster probability shock, and $\tilde{\theta}_t$ is the price of risk associated with the disaster event shock. The endowment level shock is represented by dZ_t, the disaster probability shock is represented by $Z_{\lambda,t}$, and the disaster event shock is represented by dN_t. Furthermore, m_t can be expressed as:

$$m_t = e^{-\rho t} e^{-\gamma_1 \omega_t - x_t} \tilde{c}_{1t}^{-\gamma_1} \equiv f\left(t, \omega_t, x_t, \eta_t; \Theta\right), \tag{27}$$

where $\tilde{c}_{1t}(\eta_t, \omega_t)$ denotes the optimal consumption share of the more risk-averse agent ($\tilde{c}_{1t} = c_{1t}/Y_t$), and Θ represents the set of parameters of the model. The stochastic discount factor can also be expressed as:

$$m_t = \hat{m}_{1,t}\tilde{c}_{1,t}^{-\gamma_1} \equiv \hat{m}_{2,t}\tilde{c}_{2,t}^{-\gamma_2}, \tag{28}$$

with

$$\hat{m}_{k,t} = e^{-\rho t}\alpha_{k,0}\eta_{k,t}e^{-x_t(1-\gamma_k)-\gamma_k y_t}, \quad k = 1, 2.$$

Using the definition of the stochastic discount factor, we proceed to calculate the equilibrium interest rate and price of each risk, shown in lemma 3.3.

Lemma 3.3. *A Social Planner equilibrium can be constructed because financial markets are complete, and there are no arbitrage opportunities. Based on that, the following holds*

- ***The risk-free interest rate:*** *The risk-free interest rate in equilibrium, in an economy with disaster risk and heterogeneous beliefs on disaster probability, is given by*

$$r_t = -\mathbf{E}_t\left[\frac{dm_t}{m_t}\right] \equiv \tilde{r}_t + \tilde{\theta}_t\lambda_t, \tag{29}$$

where the interest rate in normal times (without jumps), $\tilde{r}_t$, is given by:

$$\tilde{r}_t \quad = \quad \rho + \mathbf{R}_t\mu - \frac{\sigma^2}{2}\mathbf{R}_t\mathbf{P}_t - \lambda_x(\mathbf{R}_t - 1)\omega_t + \lambda_t JRP_t.$$

where $\mathbf{R}_t$ is the aggregate risk aversion and $\mathbf{P}_t$ is the aggregate prudence, both defined as:

$$\mathbf{R}_t = \left(\frac{\tilde{c}_{1,t}}{\gamma_1} + \frac{\tilde{c}_{2,t}}{\gamma_2}\right)^{-1}, \quad \mathbf{P}_t = (1+\gamma_1)\left(\frac{\mathbf{R}_t}{\gamma_1}\right)^2\tilde{c}_{1,t} + (1+\gamma_2)\left(\frac{\mathbf{R}_t}{\gamma_2}\right)^2\tilde{c}_{2,t},$$

and JRP_t is the aggregate jump-risk premium defined as the weighted arithmetic mean of the jump-risk premium of individual agents.

$$JRP_t = w_{1,t}\frac{\bar{\lambda}^{AP}}{\bar{\lambda}} + w_{2,t}\frac{\bar{\lambda}^{TO}}{\bar{\lambda}}, \quad JRP^k = \frac{\bar{\lambda}^k}{\bar{\lambda}}, \quad w_{k,t} = \frac{\tilde{c}_{k,t}\mathbf{R}_t}{\gamma_2}.$$

- ***The price of risk:*** *The price of risk of the three shocks are given by:*

$$\text{endowment level shock} \quad : \quad \tilde{\psi}_t = \mathbf{R}_t\sigma \tag{30}$$

$$\text{disaster probability shock} \quad : \quad \tilde{\psi}_{\lambda,t} = 0 \tag{31}$$

$$\text{disaster event shock} \quad : \quad \tilde{\theta}_t = -\widehat{JRP}_t + B\mathbf{R}_t, \tag{32}$$

where $\widehat{JRP}_t$ is the aggregate (log) jump-risk premium defined as the weighted arithmetic mean of the (log) jump-risk premium of individual agents:

$$\widehat{JRP}_t = \left[w_{1,t}\ln\left(\frac{\bar{\lambda}^{AP}}{\bar{\lambda}}\right) + w_{2,t}\ln\left(\frac{\bar{\lambda}^{TO}}{\bar{\lambda}}\right)\right],$$

and B denotes a constant disaster size.

Equation (29) can be expressed in a more comprehensive form as follows:

$$r_t = \underbrace{\rho + \mathbf{R}_t\mu - \frac{\sigma^2}{2}\mathbf{R}_t\mathbf{P}_t - \lambda_x(\mathbf{R}_t - 1)\omega_t}_{\text{Interest rate with habit and RRA heterogeneity}} + \underbrace{B\mathbf{R}_t\lambda_t}_{\text{Disaster risk}} + \underbrace{\left(\text{JRP}_t - \widehat{\text{JRP}}_t\right)\lambda_t}_{\text{Belief heterogeneity}}. \tag{33}$$

Equation (33) shows that the interest rate can be decomposed into three components: the baseline interest rate in an economy with habit consumption and heterogeneous risk aversion, a component related to disaster risk, and a term capturing belief heterogeneity regarding disaster probability.

A disaster risk event, characterized by a negative shock of magnitude B, reduces the interest rate, as shown in Figure 6 when comparing Models 1 and 2. This occurs because, in the presence of disaster risk, agents shift resources toward the risk-free asset, increasing its price and lowering the interest rate. This effect is amplified when the economy becomes more risk-averse, as reflected in a higher aggregate risk aversion $\mathbf{R}_t$.

When agents also differ in their beliefs about disaster probability, the impact on the interest rate can strengthen or weaken the effect of disaster risk. If both agents are optimistic but hold different beliefs, they reduce their allocation to the risk-free asset, lowering its price and thereby increasing the interest rate, as shown in Figure 6 when comparing Models 2 and 3. However, if both agents are pessimistic—believing that the long-run average disaster probability is higher than its true value—they rebalance their portfolios toward the risk-free asset, further reducing the interest rate.

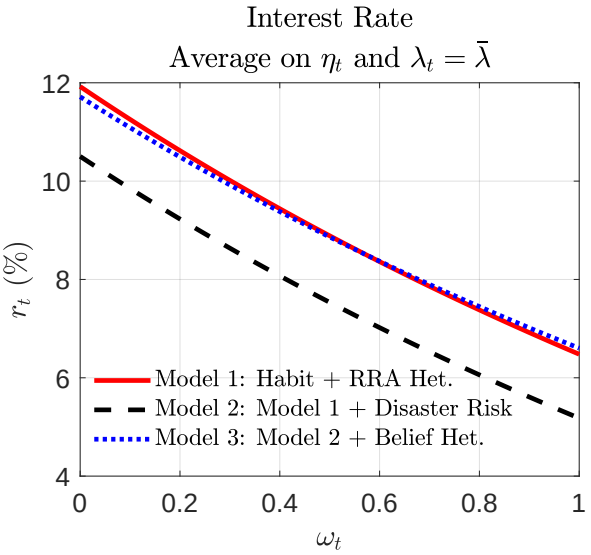


Figure 6: **Instantaneous Interest Rate r_t.** This figure illustrates the interest rate across three models: (i) Model 1, incorporating consumption habit and RRA heterogeneity; (ii) Model 2, extending Model 1 with rare disasters; and (iii) Model 3, further accounting for belief heterogeneity. The current disaster probability is set to its long-term mean $\bar{\lambda}$. The analysis is based on a grid of 100 values for ω_t within $[0, 1]$ and 50 values for η_t within $[0.4, 1.4]$, ensuring positive consumption across all grid points.

4 The Equity Term Structure

In this section, we study the model's implications for the term structure of equity yields. Our goal in this section is to compare the equity term structure generated by our model with the data.

4.1 The Price of Dividend Strips

We define a *dividend strip* as an asset that pays $Y_{t+\tau}$ units of consumption good in τ periods from now with current price, $h_t^{(\tau)}$ defined as follows:

$$h_t^{(\tau)} = \mathbb{E}_{\mathcal{P},t}\left[\frac{m_{t+\tau}}{m_t} Y_{t+\tau}\right], \quad \tau \in \{1, 2, \ldots\}, \tag{34}$$

where τ represents the maturity. The stochastic discount factor $m_{t+\tau}/m_t$, using Eq. (27), is expressed as:

$$\frac{m_{t+\tau}}{m_t} = e^{-\rho\tau} e^{-\gamma_1(\omega_{t+\tau}-\omega_t)-(x_{t+\tau}-x_t)} \left(\frac{\tilde{c}_{1,t+\tau}}{\tilde{c}_{1,t}}\right)^{-\gamma_1}. \tag{35}$$

Equipped with an expression for the SDF, the dividend-strip price equation (34) is expressed as:

$$h_t^{(\tau)} = e^{-\rho\tau} \frac{e^{x_t+\gamma_1\omega_t}}{\tilde{c}_{1t}^{-\gamma_1}} \mathbb{E}_{\mathcal{P},t}\left[e^{-(\gamma_1-1)\omega_{t+\tau}} \tilde{c}_{1,t+\tau}^{-\gamma_1}\right] \equiv h_t^{(\tau)}(x_t, \omega_t, \eta_t; \tau, \gamma_1), \tag{36}$$

where η_t is a function of the disaster probability as $\eta_t = \eta(\lambda_t)$.

4.2 The Term Structure of Equity Yields

Equity yields are defined as:

$$r_t^\tau = \frac{1}{\tau} \log \frac{Y_t}{h_t^{(\tau)}}, \tag{37}$$

where Y_t is the dividend at period t and τ represents maturity. Considering $h_t^{(\tau)}$ from Eq. (36), the equity yield (Eq. 37) becomes:

$$\begin{aligned}
r_t^\tau &= \frac{1}{\tau} \log \left[\frac{e^{\omega_t+x_t}}{e^{-\rho\tau+x_t+\gamma_1\omega_t} \tilde{c}_{1t}^{\gamma_1} \mathbb{E}_{\mathcal{P},t}\left[e^{-(\gamma_1-1)\omega_{t+\tau}} \tilde{c}_{1,t+\tau}^{-\gamma_1}\right]}\right], &(38)\\
&= \frac{1}{\tau} \log \left[\frac{e^{\rho\tau-(\gamma_1-1)\omega_t} \tilde{c}_{1t}^{-\gamma_1}}{\mathbb{E}_{\mathcal{P},t}\left[e^{-(\gamma_1-1)\omega_{t+\tau}} \tilde{c}_{1,t+\tau}^{-\gamma_1}\right]}\right], \\
&= \rho - \frac{1}{\tau} \underbrace{\left[(\gamma_1-1)\omega_t + \gamma_1 \log \tilde{c}_{1t}\right]}_{z(\omega_t,\eta_t)} - \frac{1}{\tau}\left(\log \mathbb{E}_{\mathcal{P},t} \underbrace{\left[e^{-(\gamma_1-1)\omega_{t+\tau}} \tilde{c}_{1,t+\tau}^{-\gamma_1}\right]}_{g(\omega_{t+\tau},\eta_{t+\tau})}\right), &(39)\\
r_t^\tau &= \rho - \frac{1}{\tau} z(\omega_t, \eta_t) - \frac{1}{\tau} \log \mathbb{E}_{\mathcal{P},t}\left[g(\omega_{t+\tau}, \eta_{t+\tau})\right]. &(40)
\end{aligned}$$

Equation (40) shows that equity yields depend on the current and future distribution of relative (log) consumption, ω, and the degree of belief disagreement about disaster probability, η.

Deriving an analytical expression for r_t^τ is challenging due to the nonlinear nature of the risk-sharing rule, which arises from heterogeneity in agents' preferences and beliefs. These heterogeneities complicate the computation of the expectation term, $\mathbb{E}_{\mathcal{P},t}\left[g(\cdot,\cdot)\right]$.

To better understand the effects of belief heterogeneity on equity yields, we approximate the risk-sharing rule through linearization. This approach allows us to express the expectation $\mathbb{E}_{\mathcal{P},t}\left[g(\cdot,\cdot)\right]$ as a function of affine jump-diffusion state variables, making it suitable to the transform analysis developed by Duffie et al. (2000). Consequently, the expectation takes the form of an exponential function involving a linear combination of state variables, yielding an analytical expression for equity yields. The approximated risk-sharing rule and the resulting equity yield expression are presented in the following lemma.

Lemma 4.1. *Given the solution of the theoretical model, the following holds:*

1. ***Optimal Consumption:*** *From risk-sharing rule (24), there exists an approximate linear relationship between* $cc_{1,t+\tau} \equiv \log(\tilde{c}_{1,t+\tau})$ *and both* $\omega_{t+\tau}$ *and* $\eta_{t+\tau}$*, given by:*

$$cc_{1,t+\tau} \approx -\frac{\log\tilde{\alpha}+\gamma_2}{\gamma_1+\gamma_2} - \frac{\gamma_1-\gamma_2}{\gamma_1+\gamma_2}\omega_{t+\tau} - \frac{1}{\gamma_1+\gamma_2}\log\eta_{t+\tau}, \quad \tilde{\alpha}=\alpha_{2,0}/\alpha_{1,0}, \quad \forall\tau\geq 0. \tag{41}$$

   This approximation follows from the first-order expansion $e^{cc_{1,t+\tau}} \approx 1 + cc_{1,t+\tau}$.

2. ***The Nonlinear Expectation:*** *Given the approximated risk-sharing rule (41), the dependence of* $\log\eta_t$ *on* λ_t *(Eq. 18), and Proposition 1 of Duffie et al. (2000), the component* $\log\mathbb{E}_{\mathcal{P},t}\left[g(\omega_{t+\tau},\eta_{t+\tau})\right]$ *of equity yields* r_t^τ *from Eq. (40) can be expressed as a linear function of* ω_t, $\log\eta_t$, *and* λ_t*, given by:*

$$\log\mathbb{E}_{\mathcal{P},t}\left[g(\omega_{t+\tau},\eta_{t+\tau})\right] = a(\log\tilde{\alpha}+\gamma_2) + \alpha_t + a\log\eta_t + be^{-\lambda_x\tau}\omega_t + \beta_{3,t}\lambda_t, \tag{42}$$

   where the parameter a *and* b *are defined as:*

$$a=\gamma_1/(\gamma_1+\gamma_2), \quad b=(\gamma_1+\gamma_2-2\gamma_1\gamma_2)/(\gamma_1+\gamma_2)<0,$$

   and the functions α_t *and* $\beta_{3,t}$ *satisfy the differential equations:*

$$\dot{\alpha}_t = -\lambda_x\bar{\omega}\beta_{2,t} - \frac{1}{2}\sigma^2\beta_{2,t}^2 - \kappa\bar{\lambda}\beta_{3,t}, \quad \textit{with} \quad \alpha_{t+\tau}=0, \tag{43}$$

$$\dot{\beta}_{3,t} = \kappa\beta_{3,t} - \frac{1}{2}\sigma_\lambda^2\beta_{3,t}^2 + \tilde{k}_{0,\tau}, \quad \textit{with} \quad \beta_{3,t+\tau}=0, \tag{44}$$

   where

$$\tilde{k}_{0,\tau} = \tilde{\varphi}_0\beta_{1,t} - e^{\beta_{1,t}\tilde{\varphi}_1+\beta_{2,t}B} + 1, \quad \beta_{1,t}=a, \quad \textit{and} \quad \beta_{2,t}=be^{-\lambda_x\tau}.$$

Furthermore, using expression (41) evaluated at $\tau = 0$, $z(\omega_t, \eta_t)$ can be written as:

$$z(\omega_t, \eta_t) \approx -a(\log \tilde{\alpha} + \gamma_2) - a \log \eta_t - b\omega_t. \tag{45}$$

3. ***Equity Yields:*** *Using expressions (42) and (45), the equity yields (Eq. 40) can be expressed as follows:*

$$r_t^\tau \approx \rho + \frac{1}{\tau}\left[\left(1 - e^{-\lambda_x \tau}\right) b\omega_t - \alpha_t - \beta_{3,t}\lambda_t\right]. \tag{46}$$

After solving the system of ordinary differential equations (43) and (44),the coefficients α_t and $\beta_{3,t}$ become functions of the maturity τ, denoted as $\alpha_t = \alpha(\tau)$ and $\beta_{3,t} = \beta_3(\tau)$.

Incorporating this result into Equation (46), we observe that equity yields depend on relative (log) consumption and disaster probability, as belief disagreement η is itself a function of disaster probability. However, heterogeneity in beliefs about disaster probability is specifically captured by $\beta_{3,t}$, as will be shown later.

Furthermore, substituting α_t and $\beta_{3,t}$ into Equation (46), we find that equity yields are determined by the mean-reversion behavior of relative (log) consumption and a premium for disaster probability (if $\beta_{3,t} < 0$), as expressed in Equation (47).

$$r_t^\tau = \rho + \frac{1}{\tau}\Bigg\{\underbrace{(1 - e^{-\lambda_x \tau})b\omega_t - \tau\left(\lambda_x \bar{\omega} b e^{-\lambda_x \tau} + \frac{1}{2}\sigma^2 b^2 e^{-2\lambda_x \tau}\right)}_{f(\omega_t - \bar{\omega})}\Bigg\} + \underbrace{-\frac{\beta_{3,t}}{\tau}\left(\tau\kappa\bar{\lambda} + \lambda_t\right)}_{\text{Disaster Risk Premium}} \tag{47}$$

This expression indicates that, in equilibrium, the risk of a disaster event is priced into equity yields for all maturities, regardless of whether the economy is in a recession or normal times.

Next, we plot equity yields for three maturities: very short-term ($\tau = 1$), medium-term ($\tau = 10$), and long-term ($\tau = 20$) in Figure 7. The first key observation is that equity yields are higher during recessions (low ω) than during expansions (high ω). Second, a higher disaster probability leads to higher equity yields due to the disaster risk premium. However, this effect diminishes as maturity increases.

4.2.1 Disaster Risk Premium and Beliefs Heterogeneity

We then inspect the disaster risk premium after considering the factor $1/\tau$ in the calculation of equity yields. This premium has two components: the first one is independent of the current level of disaster probability ($-\beta_{3,t}\kappa\bar{\lambda}$) and the second one is disaster-probability dependent ($-\frac{1}{\tau}\beta_{3,t}\lambda_t$):

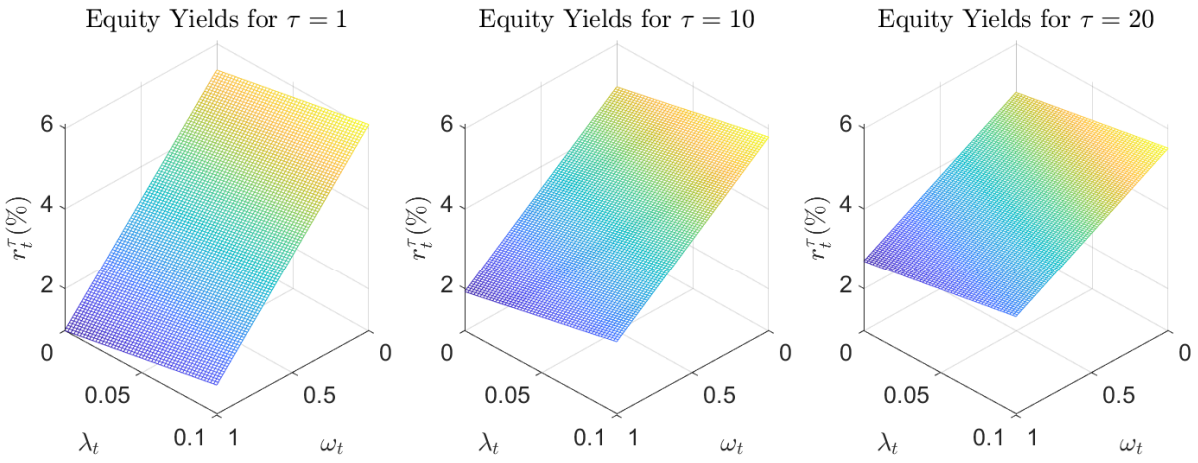


Figure 7: **Equity Yields.** This figure presents equity yields as a function of the two state variables, ω_t and λ_t, for three maturities ($\tau = 1, 10, 20$). We construct a grid of 100 values for ω_t in the range $[0, 1]$ and a grid of 50 values for λ_t in the range $[0, 0.1]$.

$$-\frac{1}{\tau}\beta_{3,t}\left(\tau\kappa\bar{\lambda} + \lambda_t\right) \equiv -\beta_{3,t}\left(\kappa\bar{\lambda} + \frac{1}{\tau}\lambda_t\right) \tag{48}$$

Equation (48) indicates that the current level of disaster probability, λ_t, has diminishing effects on equity yields across maturities. This implies that λ_t exerts a stronger influence on short-term dividend strips than on long-term ones. A crucial component of the disaster risk premium is the coefficient $\beta_{3,t}$, which is highly nonlinear in the model's parameters.

An inspection of the expression for $\beta_{3,t}$ (Eq. 44) reveals its nonlinear dependence on parameters related to heterogeneous beliefs, heterogeneous risk aversion, disaster size, and the dynamics of the two state variables in the economy:

$$\beta_{3,t} = \beta_3(\tau;\ \underbrace{\tilde{\varphi}_0, \tilde{\varphi}_1}_{\substack{\text{Heterogeneous}\\ \text{Beliefs}}},\ \underbrace{a, b}_{\substack{\text{Heterogeneous}\\ \text{Risk Aversion}}},\ \underbrace{B}_{\text{Disaster Size}},\ \underbrace{\lambda_x, \kappa, \sigma_\lambda}_{\substack{\text{Dynamics of}\\ \omega_t \text{ and } \lambda_t}}). \tag{49}$$

Heterogeneous beliefs on disaster probability is captured by the the coefficient $\tilde{\varphi}_0$ and $\tilde{\varphi}_1$, which can be expressed in terms of jump-risk premium (JRP) of agent $k \in \{TO, AP\}$, defined as:

$$\text{JRP}^k = \frac{\lambda_t^k}{\lambda_t} = 1 + \varphi^k = \frac{\bar{\lambda}^k}{\bar{\lambda}}, \quad k \in \{TO, AP\}.$$

If JRP^k is less than one, disaster events are *less* frequent under $\mathcal{P}^k$ than under $\mathcal{P}$. Our analysis assumes that $\text{JRP}^k < 1$ since both agents beliefs that the long-term average of disaster probability (λ_t^k) is lower than the true value (λ_t). However, the optimistic agent beliefs that its

value is even more lower than the pessimistic one. Equipped with this definition, $\tilde{\varphi}_0$ and $\tilde{\varphi}_1$ are defined as:

$$\tilde{\varphi}_0 = \frac{\bar{\lambda}^{TO} - \bar{\lambda}^{AP}}{\bar{\lambda}} \equiv \text{JRP}^{TO} - \text{JRP}^{AP}, \quad \tilde{\varphi}_1 = \log\left(\frac{\bar{\lambda}^{TO}}{\bar{\lambda}^{AP}}\right) = \log \text{JRP}^{TO} - \log \text{JRP}^{AP}.$$

We next examine the role of heterogeneous beliefs in shaping $\beta_{3,t}$ and, consequently, the disaster risk premium. Panel A of Figure 8 illustrates $\beta_{3,t}$, the key driver of the disaster risk premium, across three cases: (i) no heterogeneous beliefs in an optimistic economy ($\bar{\lambda}^{TO} = \bar{\lambda}^{AP} = 1.7\% < \bar{\lambda}$), (ii) moderate heterogeneous beliefs, where the Tolerant-Optimistic (TO) agent perceives a lower long-term average disaster probability than the Averse-Pessimistic (AP) agent ($\bar{\lambda}^{TO} = 1.35\%$, $\bar{\lambda}^{AP} = 1.7\%$), and (iii) substantial heterogeneous beliefs ($\bar{\lambda}^{TO} = 1\%$, $\bar{\lambda}^{AP} = 1.7\%$).

When beliefs are homogeneous, $\beta_{3,t}$ is positive, implying a negative disaster risk premium, as shown in Panel B. This, in turn, depresses equity yields, with stronger effects on short-term dividend strips that gradually weaken over longer maturities. In this economy, agents shift toward short-term assets, increasing their prices and reducing their yields.

However, when beliefs heterogeneity is present, $\beta_{3,t}$ turns negative, with its magnitude increasing as belief heterogeneity grows. Consequently, the disaster risk premium becomes positive, raising equity yields, particularly at longer maturities. This occurs because disaster risk accumulates over time, leading to a decline in the price of long-term dividend strips and an increase in their equity yields. Furthermore, this effect is amplified by the current level of disaster probability: when the disaster probability is high, the impact of belief heterogeneity intensifies, leading to a significant increase in equity yields.

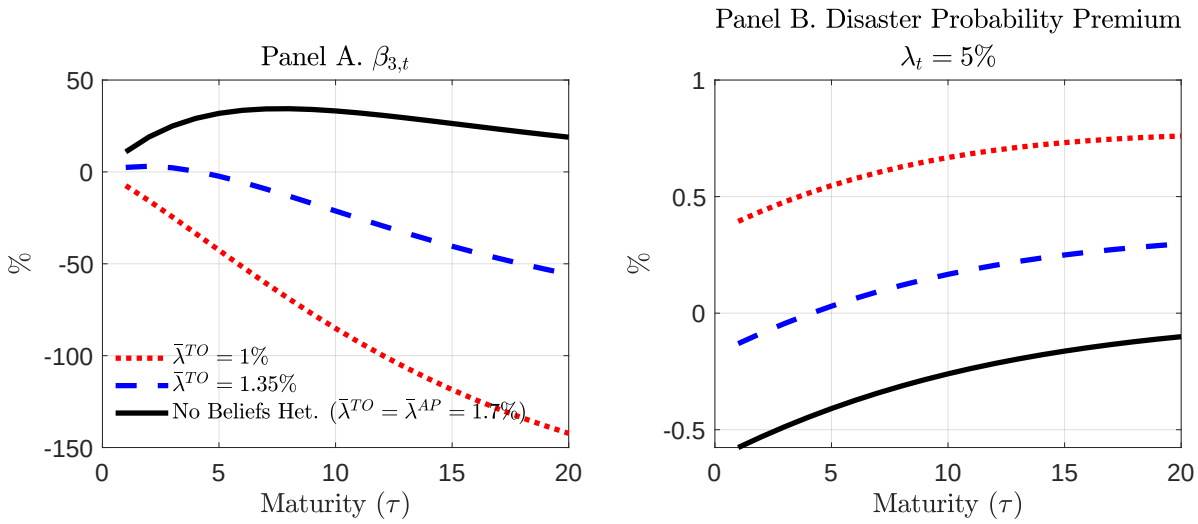


Figure 8: **Disaster Risk Premium.** This figure presents the disaster risk premium (Eq. 48) and its main determinant $\beta_{3,t}$ for a fixed level of current disaster probability $\lambda_t = 5\%$.

4.2.2 Inspecting the Slope of Equity Yields

In this section, we compute the equity term structure by simulating equity yields from Eq. (47) across a grid of 100 values for the relative (log) consumption, $\omega_t \in [0, 1]$, and 50 values for the disaster intensity, $\lambda_t \in [0, 0.1]$. This simulation is conducted for each maturity $\tau \in [1, 20]$, with the results displayed in Figure 9.

Panel A illustrates the equity term structure of equity yields. For each ω_t, we compute the average equity yield for each maturity. We further report average equity yields under three conditions: recessions (first decile of ω_t), normal times (last decile of ω_t), and the unconditional case (entire sample of ω_t). Our model generates a procyclical slope of the equity yield term structure, consistent with empirical findings (e.g., Bansal et al., 2021; Giglio et al., 2024). This pattern is primarily driven by the mean-reverting nature of relative (log) consumption: in recessions ($\omega_t < \bar{\omega}$), investors demand higher equity yields for short-term claims than for long-term ones. Since their marginal utility is heightened and highly sensitive to consumption fluctuations, they require greater compensation to defer consumption in favor of financial investments. Furthermore, because the economy is expected to recover due to the mean-reverting behavior of ω_t, investors anticipate lower long-term risk, leading to lower required returns on long-term assets.

Conversely, during expansions, agents' marginal utility is low and less sensitive to consumption changes. In the short term, they are more willing to reallocate resources toward financial assets, exerting downward pressure on short-term equity yields. However, because the economy is expected to revert to its long-term mean—implying lower future consumption—investors demand higher returns for long-term assets to hedge against potential consumption declines. As a result, when the economy is in an expansionary phase, short-term equity yields are lower than long-term yields.

The fact that relative (log) consumption fluctuates around its long-term mean provides a mechanism for generating a procyclical slope of the equity yield term structure. This aligns with the idea that business cycles are stationary and mean-reverting, linking them to financial markets.

Heterogeneity in risk aversion and beliefs about disaster probability primarily influences the level of the equity term structure, as we will show in the next section.

Panel B explores the role of disaster probability in the equity term structure. For each value of λ_t, we compute the average equity yield for each maturity and report average equity yields when the current disaster probability is low (first decile of λ_t), high (last decile of λ_t), and unconditional (entire sample of λ_t). Panel B shows that the term structure of equity yields is

always positively sloped, implying that investors require higher yields for long-term claims than for short-term ones. This is primarily because investors anticipate an accumulation of disaster events over time, increasing disaster risk for long-term assets. Additionally, the level of equity yields is higher when λ_t is high than when it is low across all maturities. A higher current disaster probability signals greater disaster risk, thereby raising equity yields.

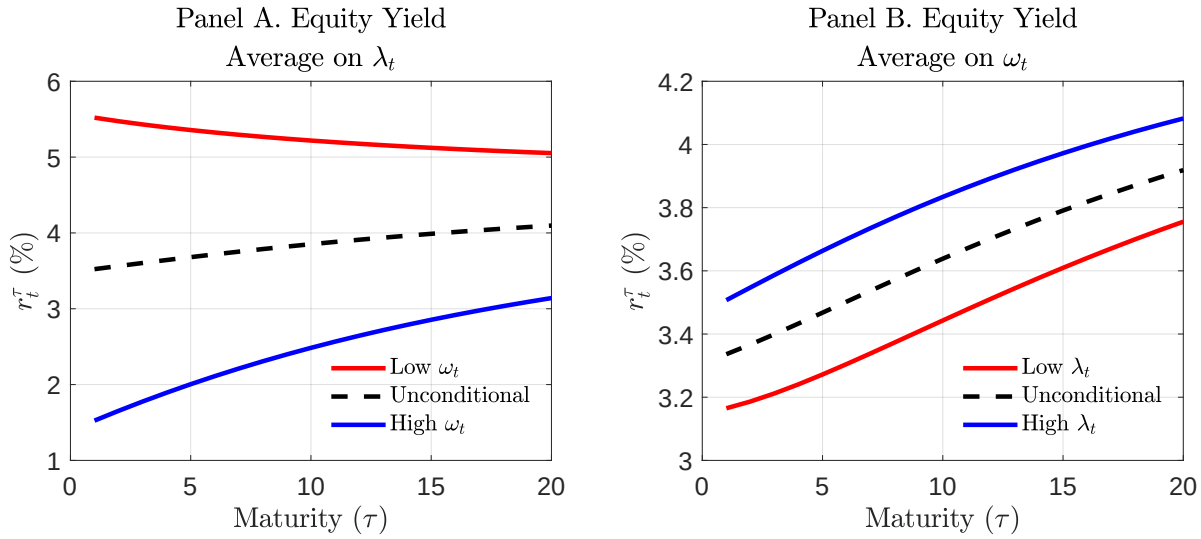


Figure 9: **The Term Structure of Equity Yields.** Panel A presents the average equity yield curve over maturities for three levels of relative (log) consumption (ω_t): recessions (first decile of ω_t), expansions (highest decile of ω_t), and unconditional (entire sample of ω_t). Panel B displays equity yields for three levels of disaster probability (λ_t): low (first decile of λ_t), high (last decile of λ_t), and unconditional (entire sample of λ_t). We construct a grid of 100 values for ω_t in the range $[0, 1]$ and 50 values for λ_t in the range $[0, 0.1]$.

4.2.3 Heterogeneity in Preferences and Beliefs

In this section, we examine the impact of heterogeneity in both preferences and beliefs on the term structure of equity yields. To do so, we compare the equity term structures generated by three models. The first model, our benchmark, features habit preferences and disaster risk, with a representative agent whose relative risk aversion parameter is $\gamma = 2$. The second model extends the benchmark by introducing heterogeneity in risk aversion, where agents have $\gamma_1 = 2$ and $\gamma_2 = 1.2$. Finally, the third model incorporates belief heterogeneity regarding disaster probability: the less risk-averse agent (TO agent) is optimistic about the long-run average disaster probability, while the more risk-averse agent (AP agent) is pessimistic, reflected in $\bar{\lambda}^{TO} < \bar{\lambda}^{AP} < \bar{\lambda}$.

Figure 10 presents the equity term structures generated by these three models. The key findings are as follows.

First, introducing risk-aversion heterogeneity (Model 2) affects both the level and slope of the equity term structure. In recessions (Panel A), short-term equity yields decline, whereas long-term yields increase relative to the benchmark model. This occurs because the risk-sharing mechanism allocates more risk to the less risk-averse agent, who, in turn, reallocates his portfolio toward short-term assets. This increased demand pushes up their prices, lowering their yields. Conversely, during expansions (Panel C), investors anticipate a reversion to the long-term mean alongside a potential rare disaster, which further suppresses expected future consumption. As a result, agents demand an additional risk premium for holding dividend strips, increasing their yields. Furthermore, disaster events introduce an additional channel through which risk-aversion heterogeneity influences equity yields, as formalized in Eq. (49).

Second, incorporating belief heterogeneity regarding disaster probability (Model 3) leads to jumps in equity yields during both recessions and expansions (Panel A and C), elevating the overall level of equity yields. This effect arises because the difference in agents' jump-risk premia ($-\tilde{\varphi}_0$) increases with belief heterogeneity, amplifying the disaster risk premium and, consequently, raising equity yields.

Third, on average (Panel B), the combination of risk-aversion and belief heterogeneity reinforces each other, further increasing equity yields. These factors influence the risk-sharing rule between agents, thereby affecting the stochastic discount factor and, ultimately, the pricing and yields of dividend strips.

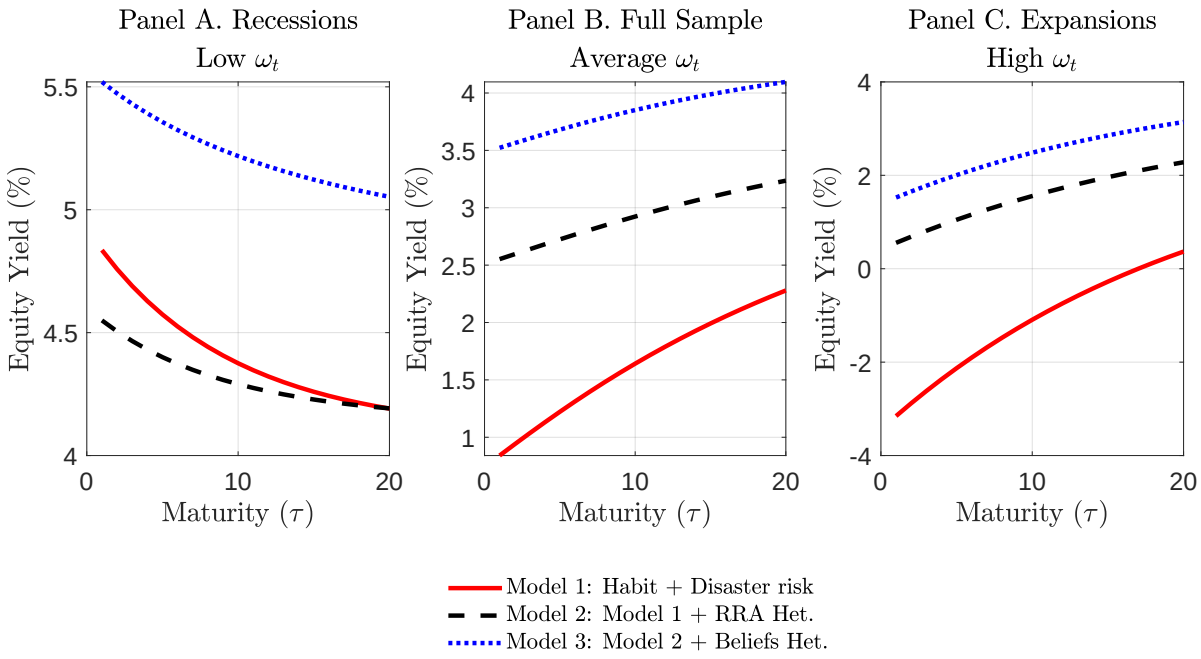


Figure 10: **The Role of Preferences and Belief Heterogeneity.** This figure presents the equity term structure across three models: (i) Model 1, the benchmark model incorporating habit formation and disaster risk; (ii) Model 2, which extends Model 1 by introducing heterogeneity in risk aversion; and (iii) Model 3, which further incorporates heterogeneous beliefs into Model 2. Panel A displays the equity term structure in recessions (low ω_t), Panel B presents results for the full sample, and Panel C focuses on expansionary periods (high ω_t). Parameter values are calibrated as reported in Table 1.

5 Conclusions

This paper examines how belief heterogeneity in disaster probability affects the equity term structure. We develop a two-agent model featuring heterogeneity in risk aversion and beliefs, along with catching up with the Joneses preferences, to analyze asset prices across different maturities.

We show that belief disagreement increases equity yields across maturities, with this effect being amplified when the economy's speed of reversion to its long-run mean is low. Additionally, risk-aversion heterogeneity reinforces this impact when the optimistic agent is also the less risk-averse agent. Furthermore, we demonstrate that the disaster risk premium is a nonlinear function of the parameters governing investor behavior, unveiling a rich interaction between equity yields, disaster risk, and investor characteristics.

This article also opens a promising research agenda. Our model can be extended in at least two directions. First, since disaster probability is plausibly countercyclical—rising in recessions—incorporating this feature into our model would allow us to study how the disaster risk premium evolves and its subsequent effects on equity yields. Second, we currently assume that the persistence of relative (log) consumption is fixed and independent of economic conditions. However, empirical evidence suggests that policymakers implement measures to accelerate economic recovery, influencing the speed at which the economy returns to its long-run mean. Accounting for this variation in persistence across recessions and expansions could have important implications for the equity term structure, particularly in the behavior of equity yields and one-period returns.

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