Beyond β, σ and clubs: A unified approach to measure economic convergence

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Abstract

The study of convergence examines whether lagging economies are catching up with high-income economies. However, as the growth literature has recently pointed out the approach must be done by considering multi-country models. There are theoretical and empirical models that have attempted to overcome the weakness of a uni-country model. However, no one has been able to reconcile this new theoretical approach with the empirical methodology to measure cross-country convergence. In this paper, drawing on complexity and graph theory, I propose a novel method that bridges the gap between convergence theory and its empirical measurement within a unified framework. This approach enables a more comprehensive assessment of both country-level convergence and the convergence of the global economic system. When estimating a convergence model with this methodology we observe: a) there is convergence between countries b) there is convergence in the global system in the period (1960-2019) but with a decreasing trend c) the graph allows us to identify between two and five groups of countries. This reflects the trend towards multipolarity in the globalized world.

Keywords: growth, network, complexity, convergence, ERGM

1 Introduction

Understanding economic convergence remains a fundamental basis of the economic growth literature. At its core, convergence seeks to determine whether poorer

economies tend to catch up with wealthier ones in terms of income levels over time. Based on the Solow (1956) growth model, this framework suggest an inverse relationship between a country's initial capital (or income) per capita and its growth rate. This issue was first empirically checked by Barro and Sala-i Martin (1992) and Sala-i Martin (1996), making convergence a fundamental topic in macroeconomic research. Yet, despite decades of inquiry, critical questions remain unresolved: How robust is the convergence hypothesis across different contexts? What factors drive or impede convergence? And how do these dynamics evolve over time? More importantly, Is convergence well measured?

The empirical convergence literature has evolved along several dimensions. Traditional approaches, such as β and σ-convergence, focus on income growth and dispersion across countries, respectively. Seminal works by Quah (1993a, 1993b, 1996a, 1996b) introduced distributional perspectives, while cluster regression models (e.g., Canova (2004)) emphasized heterogeneity among groups of countries. However, these methods rely heavily on simplifying assumptions, often neglecting the interdependence between countries. Convergence, in reality, is not an isolated process; it emerges from complex, interconnected relationships among economies. A fundamental gap in the literature is the lack of frameworks that capture these networked interactions.

Recent advances in complexity economics offer a promising lens to address this gap. Complexity theory reconceptualizes the economy as a system of interconnected actors, where interactions shape macroeconomic outcomes (Foster, 2005). While macroeconomics has increasingly embraced this perspective (Fagiolo, 2016), its application to the study of convergence remains underexplored. For example, the ongoing debates between Acemoglu and Molina (2021) and Kremer, Willis, and You (2021) on the existence or not of convergence, ignore its inherently networked nature. These works focus in the discussion of convergence without taking into account the methodological weaknesses of the estimation methods. This illustrate the need for more general frameworks. To date, no studies in the convergence literature explicitly incorporate network theory.

This paper seeks to fill this gap by proposing a novel framework for measuring economic convergence, based on the principles of complexity and network theory. We conceptualize convergence as a system of interdependent relationships, represented through a convergence matrix that captures the likelihood of convergence between pairs of countries over time. Using data from the Penn World Table (1950–2020), we construct a graph-based representation of these relationships and employ the Exponential Random Graph Model (ERGM) to estimate both pairwise and global convergence probabilities. This approach moves beyond traditional methods, offering a unified framework that integrates individual, bilateral, and systemic convergence dynamics.

Our contributions are threefold. First, a theoretical advancement. We align the measurement of convergence with the complexity perspective, challenging the traditional focus on country-specific steady states Mankiw, Romer, and Weil (1992). By modeling convergence as a networked process, we provide a more comprehensive understanding of its dynamics. Second, a methodological innovation. We introduce ERGM to the convergence literature, bridging the gap between applied graph statistics and

macroeconomics. This enables us to capture both, the structure and drivers of convergence networks, overcoming the limitations of limited number of countries (e.g., Jang and Yang (2022)) or purely descriptive analyses levels without including inferential methods. Finally, we obtain empirical insights. Our results identify 2–5 convergence clubs among countries, reveal cyclical patterns in convergence (e.g., increased likelihood during crises), and highlight the opposing effects of external sector gaps (positive) and investment and complexity gaps (negative) on convergence probabilities. Importantly, our analysis shows that global convergence exists but has declined over time. This work not only advances the convergence literature but also contributes to broader debates in economic growth by integrating tools from network science. By offering a more robust framework to analyze convergence, it provides insights into the structural factors influencing long-term income equality and macroeconomic stability.

The remainder of the paper is structured as follows. Section 2 reviews the theoretical and empirical literature on economic convergence. Section 3 introduces the data and methodology, including the construction of the convergence matrix and application of ERGM. Section 4 presents the results, and Section 5 discusses their implications in the context of economic growth. Finally, Section 6 concludes with key findings and further research.

2 Theoretical Framework and literature background

The global economic system can be understood as a complex system (Fagerberg & Srholec, 2017; Fagiolo, 2016). This implies a system in which its components have intricate or interrelated relationships, giving rise to emergent properties – global properties of the system that cannot be reduced to the sum of its individual parts. Therefore, such a system would be impossible to comprehend if we only studied its components separately.

These systems rely on all interactions between agents. Therefore, there are no atomized agents, but the global economy should be considered as a whole, in which interactions constantly occur between different countries linked through the flow of people, goods, and other intangible products. In this way, it has been shown that growth in one country is determined by complex networks of production, both at the local (Carvalho and Tahbaz, 2019) and international levels (Bernard and Moxnes, 2018). Within the global system, interactions between countries involve direct interrelations that affect the growth of each other, as well as indirect interrelations in related countries and hardly imperceptible movements in more distant countries (understanding "distance" in this work as income disparity).

Direct or indirect interrelations are intrinsic to the economic problem; therefore, economic trajectories between regions/countries should not be analyzed separately. It is considered appropriate to review the methods used in the discipline of growth from the perspective of complexity to better understand macroeconomic dynamics (Fagiolo, 2016). In this article a link or interdependence between two countries represent if this two countries have converge in a period.

2.1 Convergence

There is agreement among the major theories that institutions, population, international trade, innovation, and structural change are fundamental channels for explaining long-term growth and, consequently, their effects on convergence (Aghion & Howitt, 1992; Galor & Weil, 2000; Kongsamut, Rebelo, & Xie, 2001; Romer, 1990; Solow, 1956). In parallel, there are factors related to domestic demand within each country, but primarily on a global scale, where trade factors are essential for exchanging goods and services, expanding a country's consumption possibilities, and positioning local goods in the global market, leading to increased economic growth (Frankel & Romer, 1999). However, there is still much to understand in growth theory to explain why economies exhibit such heterogeneous income levels. Bruns and Ioannidis (2020) confirm this presumption by identifying 145 explanatory variables for economic growth and their subsequent impact on convergence.

In particular, studies on convergence emerged in the 1980s with single-country models (without interactions), driven by renewed interest in long-term economic growth theory. The early works showed the existence of convergence (Barro, 2015) but faced methodological criticisms. These criticisms primarily refer to sample biases in the selected countries and low statistical power (Bernard & Durlauf, 1996; De Long, 1988). This led to a discussion of the validity of the theory and the differentiation between absolute (or unconditional) and conditional convergence. Unconditional convergence postulates that all countries converge in the long run. However, in conditional convergence, there is convergence (toward the same steady state) if and only if the characteristics of countries are similar.

Empirical evidence on this phenomenon has been generated using various methodologies. Initially, linear models were employed to investigate the existence of convergence between countries. By specifying the equation 1, the test examined whether the convergence rate (β) was negative; in that case, convergence was present (Baumol, 1986; Barro, 1991; Dowrick, 1992):

$$\log\left(y_{it}/y_{it-\tau}\right) = \alpha - \beta \log\left(y_{it-\tau}\right) + \vec{\gamma}.\vec{X}_{it-\tau} + \epsilon_{it} \tag{1}$$

Where $y_{it-\tau}$ is the total economic income of a country at a $t-\tau$ moment in time, ϵ_{it} is the model error, $\vec{X}$ is a vector of relevant variables in the model that explain income and $\vec{\gamma}$ is the vector of associated coefficients.

Several studies document a natural convergence rate (β) of around 2%, meaning they find convergence, but at a rate lower than what the Solow model theoretically predicts. However, Abreu, De Groot, and Florax (2005b) in their extensive review of over 600 articles, show that the results on convergence are far from conclusive, with both positive values (convergence) and negative values (divergence) being reported.

However, within this methodological approach, many other authors have found conditional convergence at the country level (Abreu, De Groot, & Florax, 2005a). Similar estimates have been made using nonlinear models. Nonparametric methods have also been used to estimate the effect of variables on growth (and its corollary, convergence), yielding heterogeneous results depending on the samples used (OECD countries vs. other countries) (Maasoumi, Racine, & Stengos, 2007). Some studies have also explored convergence using time series analysis, they examine whether there

are stationary income series between countries, which would validate the presence of convergence (Matheson & Oxley, 2007; Phillips & Sul, 2007). Regardless of the models used, the growth literature has not yet settled this debate.

However, it's worth noting that the study of convergence has been primarily conducted from a fragmented perspective among countries. This implies that the results of a single-country model may be replicated as-is for the global economy as a whole.

At this point, it's important to note a key aspect for the conceptual development of the rest of the work: β strictly does not measure convergence between countries but rather measures the convergence of a country relative to its own long-term growth trajectory. This is where the need to incorporate a systemic approach arises, moving from the country-level analysis to a dyadic level, meaning pairs of countries.

2.2 Alternative convergence tests

The main limitation of β convergence is due to the fact that convergence occurs between a country and its own stationary state (Mankiw et al., 1992). This limit the correct interpretability of β in this sense, it became necessary to explore other approaches. Leaving this weakness aside, there are other arguments to seek new tests of the phenomenon. On the one hand, the identification of β through appropriate econometric methods. But more important is the Galton fallacy described by Quah (1993a) where a β that implies convergence is feasible with an unchanged income distribution of the countries. Therefore, better measurement needs to test beyond averages and an individual country type against itself (Quah, 1997).

In this regard, different approaches to measure convergence are discussed. The first approach is the best known, the *sigma* convergence. It focuses on the reduction of income dispersion. It implies convergence when the variance of income ($\sigma_{\log y}$) decreases from one period (t) to another (t+T)(Durlauf, Kourtellos, & Tan, 2008):

$$\sigma^2_{\log y,t} - \sigma^2_{\log_y,t+T} > 0 \tag{2}$$

A posterior and complementary approach is based on income polarization and the emergence of convergence clubs (Anderson, Linton, & Leo, 2012; M. Bianchi, 1997; Van Dijk & Paap, 2008), where the results show multimodality with at least two modes of countries. This concept of multiple equilibriums can be related to both complexity theory and neoclassical approaches with poverty traps (Cayssials & Picasso, 2020).

The closest approach to the work presented in the passage is associated with studying movements within the distribution of per capita incomes (Quah, 1996a, 1996b). These works model transitions using a Markov process. However, these approaches do not provide insights into the key factors driving convergence or whether it occurs at the individual or group level. The passage suggests that this challenge could be addressed by combining graph theory and econometrics within a complex systems framework.

2.3 Convergence, a Complex Measurement

While there is an extensive literature on the issue of convergence, as far as is known, it has not been directly addressed from the perspective of complexity theory (Acemoglu & Molina, 2021; Durlauf et al., 2008; Kremer et al., 2021). It is understandable that,

given the course of research on the subject, this view was not taken into account in the early experiences of the 1980s and 1990s. However, in light of the availability of new theory and computational methods for validation, it seems necessary to consider the limitations of the assumption of independence and incorporate the complexity approach as a way to improve the validity of measurement.

Measure convergence in a more appropriate manner implies a model with a structure like the one in equation 3.

$$\log(\frac{gdp_{it}}{gdp_{it-1}}) = \alpha + \sum_{j \neq i} \beta_j . gdp_{jt-1} + \vec{\beta}_{iz} \vec{X}_{it-z} + \epsilon_{it} \tag{3}$$

Where the coefficient β_j represents the convergence of country j with country i. $\vec{X}_{iz}$ is a vector of variables associated with country i correlated with gdp_i, β_{iz} represents the coefficient associated with this variable and ϵ_{it} the typical error term of a regression model.

However, it would be impractical to econometrically estimate it due to the high number of degrees of freedom needed to test the null hypotheses of convergence. Perhaps partly due to this, a reductionist approach has predominated.

As noted by Cincotti, Raberto, and Teglio (2022), reductionism postulates that we should stop studying nature as an organic whole and should reduce the problem to simpler, quantifiable phenomena by breaking down the complex structure into smaller parts that can be mathematically represented. In contrast, there is a holistic stance that posits that the whole is greater than the sum of its parts. Systems with low coupling (independence between parts) work well with the reductionist approach, but when this is not the case and there are strong interactions with the external world, a holistic approach is required. For example, traditional macroeconomic models (Dynamic Stochastic General Equilibrium models) do not capture the fact that complex systems do not behave like a Gaussian distribution but rather like a power law. To incorporate the power law, found in many social systems, one or more assumptions of the central limit theorem (CLT) must be violated (Cincotti et al., 2022).

Analyzing these types of problems requires aligning the theoretical framework of convergence and complexity, incorporating a method that allows working with the issue of non-independence and achieving more appropriate measures of convergence. A technique that meets these requirements is ERGM. This approach to measuring convergence allows for considering as necessary conditions that random variables are dependent and that heterogeneous agents exist.

2.4 Worlwide interactions

Ertur and Koch (2011) ask whether the growth process is better explained "once worldwide interdependence and interactions between countries are fully taken into account?". The authors argue that this question has not yet found a clear, direct, and convincing answer. Their work continues the line of previous studies that consider it relevant to incorporate interactions between countries (Ertur & Koch, 2011; Howitt & Mayer-Foulkes, 2002; Keller, 2004). The underlying argument is that countries cannot be considered as independent islands (Quah, 1996b).

Keller (2004) presents a solid argument for considering that interdependencies are important and must be taken into account. His argument stems from technological diffusion, where productivity differences were attributed to this phenomenon. The incorporation of external technology explained more than 90% of productivity growth. By deriving a model from Eaton and Kortum (1999); Kortum (1997), Keller (2004) integrates technology and its diffusion, finding that the speed of technological diffusion is crucial for understanding convergence.

In this context, several studies incorporate the conceptual framework of interdependence. (Aghion, Howitt, & Mayer-Foulkes, 2005) develop a multi-country model to explain convergence, where they derive an econometric equation approximation as follows:

$$g_i - g_1 = \beta_0 + \beta_f F_i + \beta_y \cdot (y_i - y_1) + \beta_{fy} \cdot F_i \cdot (y_i - y_1) + \beta_x X_i + \varepsilon_i, \tag{4}$$

Where g_i is the average growth of country i and g_1 is the growth of the frontier economy, F_i is the country's average level of financial intermediation, y_i is the log of per-capita GDP in the country at the start of the period, y_1 is the log of per capita GDP in the United States at the start of the period, β_0, β_f and β_y are constant coefficients and ϵ_i a classical disturbance term. This specification integrates the concept of convergence between countries and closely aligns with the core idea of this paper. However, it relies on comparisons with a leading country—in this case, the United States. While this represents an advancement over traditional growth equations, it still presents a key limitation: the necessity of benchmarking against a single reference country. Given the evolving nature of economic dynamics, the technological frontier—traditionally represented by the United States—has become less distinct. Therefore, a comprehensive approach that accounts for the entire global system is essential for a more accurate and robust estimation. Where as we saw with the equation (3) is no longer possible.

In light of this discussion, this paper aims: i) demonstrate that the issue of convergence should be approached from a new perspective that incorporates an appropriate method for the problem; ii) propose a method that contributes to addressing the methodological issues present in previous works, and; iii) answer the question of whether income convergence between countries has occurred in the period 1960-2019 and what macro, meso, and micro-level factors may be influencing it.

3 Methodology

3.1 A Distance Measure to Identify Convergence

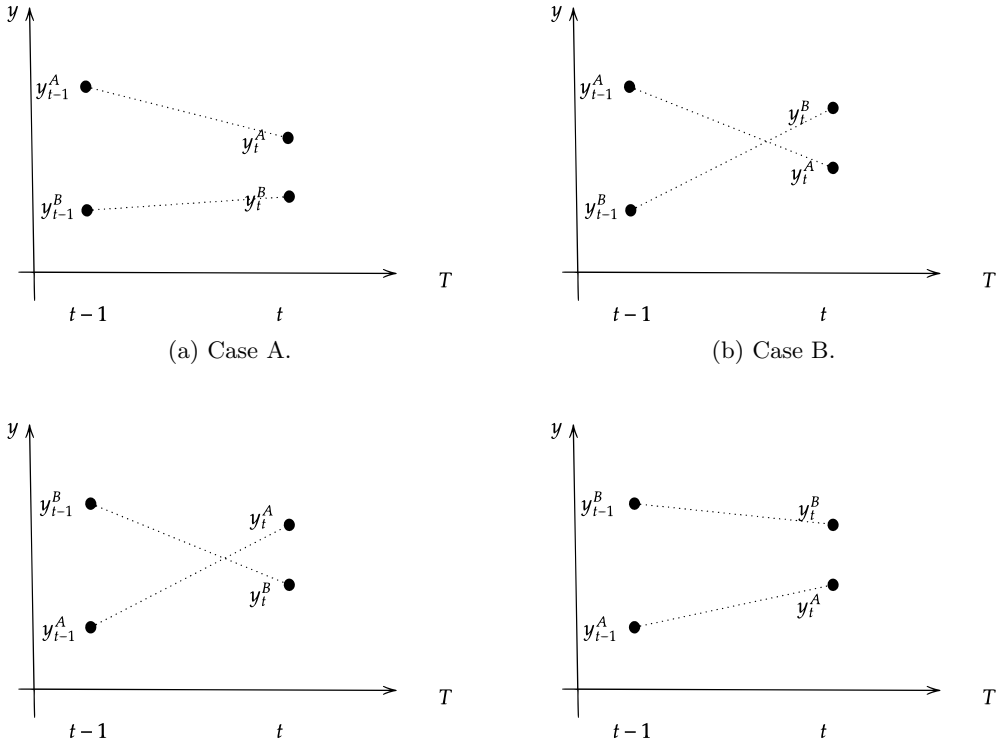
The empirical strategy for identifying convergence relies on a distance measure between economies. In algebraic terms, a novel distance measure is defined, as unlike other measures used (García-Algarra, Gómez-Bengoechea, & Mouronte-López, 2023; Ye & Robertson, 2016), in this case, the distance between each pair of countries A and B is used. Thus, the distance is defined as the difference between the GDPs of both economies at a given point in time:

$$\left| y_j^A - y_j^B \right| \tag{5}$$

Where y_j^i is the GDP of country i in year j. Under this distance measure, the definition of convergence is postulated through inequality 6. Under this definition, the concept being sought to capture is the convergence but is operationalized in a better way. It is considered an appropriate measure to capture this phenomenon:

$$\left|y_t^A - y_t^B\right| < \left|y_{t-1}^A - y_{t-1}^B\right| \tag{6}$$

Therefore, if the distance between two countries A and B decreases from one period to another, there is convergence. The possible cases for this to occur are four and are described in the figure 1.



(a) Case A.

(b) Case B.

(c) Case C.

(d) Case D.

Fig. 1: Convergence between A and B

3.2 Constructing the Convergence Graph: The Absolute Value Method

The unit of analysis is not the country but the pair A and B of countries; let's call this unit of analysis a "dyad" to maintain the language of networks.

To estimate convergence, a specification such as equation (3) should be employed. The degrees of freedom issue associated with this equation can be resolved by

reconceptualizing convergence (or divergence) as a systemic process arising from the interconnections between countries. By reformulating the approach to represent each pairwise interaction as an element within a matrix, the challenge of measuring convergence can be effectively addressed using statistical methods from graph theory literature.

Therefore, the first step in studying the mechanisms that explain convergence is to construct the network (graph) that determines which pairs of countries have converged at a given time, as shown in Figure (2). Here, a graph is simply a matrix composed of zeros and ones, where ones indicate income convergence in a given period between two countries. To illustrate this, we provide an example of a graph $y = y(nodes, edges)$. In this context, nodes represent countries, and edges represent the dichotomous values indicating convergence. In Figure (2), we can observe that countries A and B converge with each other, as do countries A and C. However, country D does not converge with any other country.



Fig. 2: Example of a Convergence Graph.

For each moment t, a matrix (y) of size NxN is obtained, with N being the number of countries, where it is one if there is convergence between i and j, and zero otherwise:

$$y_{ij} \longmapsto \begin{cases} 1 \text{ if } \left|y_t^i - y_t^j\right| < \left|y_{t-1}^i - y_{t-1}^j\right| \\ 0 \text{ in other case} \end{cases} \forall\, i, j \in \text{ set of countries under analysis.}$$

The graph G is constructed based on the absolute value difference at two points in time. For this reason, it will be referred to as the Absolute Value Method (AVM) from this point on. The network offers a range of mathematical and statistical tools for analyzing this information, which would be hardly impossible with the usual cross-sectional longitudinal analysis. It information will be disaggregated in the following sections to provide more insights into convergence.

3.3 Exponential Random Graph Model

From the convergence graph (y), an adaptation of the Exponential Random Graph Models (ERGMs) methodology will be applied with the aim of answering which characteristics are relevant to explain the position of each of the nodes (countries) in the network (Robins, Pattison, Kalish, & Lusher, 2007). ERGMs are designed to explain the overall structure of the network, and they also allow for explaining ties through

interactions between actors. These interactions are captured by the distance that exists in the measurements of any variable between two actors. This analysis corresponds to the meso level and will be referred to as dyadic analysis.

ERGM models are modified logistic regressions that allow estimating the probability of a link to depend on the presence of other links within the network, among other variables. ERGMs can account for directed interactions as well as weighted interactions between nodes (Van der Pol, 2019).

Mathematically, an ERGM is defined as (Robins et al., 2007):

$$\mathrm{p}(y \mid \theta) = \left(\frac{1}{\kappa}\right) \exp\left\{\theta^T g(y)\right\} \tag{7}$$

Where $y = [y_{i,j}]$ is the realization of the random adjacency matrix $Y = [Y_{i,j}]$ associated with a graph $y = (N, v)$. $g\,(y) = [g_1(y), g_2(y), .., g_Z(y)]^T$ is a Z dimensional vector composed of statistics (and/or known functions) of y and attributes of the nodes (E, in this case, countries) in the network. $\kappa \overset{\text{def}}{=} \kappa(\theta)$ is the normalization constant. $\kappa = \sum_{y'} \exp\left\{\theta^T g(y')\right\}$, where $g(y')$ is the vector of network statistics for each possible network and $\mathcal{Y}$ is the set of all possible networks with the same number of nodes as Y. θ represents the magnitude and direction of the effects of $g\,(y)$ on the overall probability of the network.

The probability of the complete graph can be re-expressed on a logit scale, in terms of the conditional probability of observing an edge (V, in this case, convergence) while keeping the rest of the network fixed. It can be expressed:

$$\text{logit}\ \Pr\left(y_{i,j} = 1 \mid y_{-(i,j)}\right) = \theta^T \delta_{i,j}(y) \tag{8}$$

Where $y_{-(i,j)}$ corresponds to the vector y, excluding the observation $y_{i,j}$. $\delta_{i,j}(y)$ is the estimate of change and is calculated as $g(y \mid y_{i,j} = 1) - g(y \mid y_{i,j} = 0)$, while keeping the values of the rest of the edges or dyads (convergence between country i and j) in the network y constant. The θ^T can be interpreted as the probability of observing a particular edge, conditioned on the rest of the dyads remaining unchanged.

Using these models, it is possible to estimate the probability that two countries are connected due to their attributes, i.e., their observable characteristics. The specification to be estimated will be as follows, with minimal variations depending on the specific estimation of the model:

$$\text{logit}\ y_{t,ij} = \sum_{z=1}^{Z} \theta_z X_{t,i}^z + \sum_{z=1}^{Z} \theta_z X_{t,j}^z + \theta_{d_{i,j}} |gdppc_{t-1,i} - gdppc_{t-1,j}| + \sum_{z=1}^{Z} \theta_z X_{t,ij}^z + \vec{\theta}_s \vec{S}_{t-z} - \ln(\kappa) \tag{9}$$

For the estimations, taking advantage of the network's evolution over time, the dynamic version of the ERGM model, called Temporal Exponential Random Graph Models, will be used. This is relevant because it allows capturing the general evolution of the system. In other words, it can reveal whether countries tend to cluster together or disperse over time, providing a measure of structural convergence or systemic convergence.

θ_s correspond with the effects os the structural variables $\vec{S}$. Where $\vec{S}$ are the structural variables: *edges*, *transivities*, *triangle*, *memory*, *time* and *gwsd*.

Geometrically Weighted Shared Partners (*GWSD*) measures the tendency of pairs of actors to be connected through shared partners, with the shared connections weighted by the strength of dyadic relationships (Geometrically Weighted Dyadwise Shared Partner distribution). This captures the likelihood that two actors with a common connection will also form a direct tie with each other (McMillan, Felmlee, & Ashford, 2022). For example, in the context of international relations, it reflects the tendency for two countries converging with the same third country to eventually converge with each other

memory is a form of intertemporal dependence designed to capture the temporal process of the complete network. In these specifications, a "memory" term is controlled for, which takes into account whether both the presence and absence of convergence are maintained over time. A positive coefficient would confirm the stability of the network. This indicates that the effect of diminishing marginal returns seems to operate in the system, or the existence of certain stationarity in the system is confirmed. In other words, there are divergent patterns but they are dampened by the dynamics of a closed system, making the overall structure of the system convergent.

time is a variable collected of the temporal ERGM model. It is a fundamental variable because capture the temporal dimension of the process. It measure the probability of convergence in terms of the complete network over time. So, we call to this variable also as *systemic convergence*. In these sense this variable allows to infer if the system tend to converge or not over time. For this reason, this variable is named as σ_t. In a clear analogy to the concept of conditional convergence, which takes up a similar idea. For the rest of the variables of the S vector see the table (1).

Thus, the probability of dyadic convergence is defined as the income gap between country i and j through the following distance measure ($\theta_{d_{i,j}}$), which quantitatively differs from the usual β (see Equation 1):

$$\theta_{d_{i,j}} = \frac{\partial \text{ logit } y_{t,ij}}{\partial |gdp_{pc_{t-1,i}} - gdp_{pc_{t-1,j}}|} \neq \beta \tag{10}$$

Through equation **??**, it is possible to identify not only the effects typically used as controls, i.e., *individual level* or microscopic effects (by country in this case), *edge level* (pairs of countries or mesoscopic effects), and *structural level* or macroscopic effects. In addition to the distance between the GDP of two countries that identifies the concept of convergence, it measures the distance between relevant variables at the macroeconomic level. Finally, it controls for the structural endogeneity of the convergence network. This control aims to corroborate whether the hypotheses that consider the importance of dyads and triangles are supported by identifying a positive and significant probability of convergence.

Finally, for the estimations, the pseudolikelihood method using bootstrapping will be employed, implemented through the BTERGM library in the R software. This library enables the efficient temporal estimation of ERGM models (Leifeld, Cranmer, & Desmarais, 2018).

3.4 Variables and data sources

Taking into account the diversity of factors influencing economic growth, some typical variables are used to explain growth from the supply side. These variables include economic complexity captured through the Economic Complexity Index (ECI) (Hausmann & Hidalgo, 2011) and the lagged investment rate (inv) relative to the national economy's output. According to the economic complexity approach, the production of goods and services requires typical production factors, as well as tacit and codified knowledge (know-how), the incorporation of which is crucial for achieving diversified structural transformation that drives sustained economic growth with high value-added (Hausmann et al., 2011) . The ECI measures the complexity of a nation's production based on the diversity of its production and the ubiquity of the products it produces. This measure is strongly correlated with the economic growth by the channel of the structural change achieved thanks to a more complex economy (Hidalgo & Hausmann, 2009).

If we consider a typical production function for capital per unit of output ($\frac{investment}{GDP}$), which determines the level of aggregate production in an economy, *eci* and *inv* are crucial in identifying the factors explaining GDP growth. Moreover, the ECI is an increasingly used measure, and while it doesn't represent innovation, it allows distinguishing between differentiated intensive uses of capital. Thus, it enables distinguishing between less complex production functions given a level of production factors, approximating an economy's ability to combine resources in a complex manner.

On the other hand, as mentioned, the external factor is essential. In this regard, we chose to use the *export margin* variable as a proxy for the influence of the external sector on growth and convergence. This variable is a refinement of classic measures like the terms of trade and the real exchange rate. It doesn't consider all world prices but instead focuses on prices affecting the country's basket of exportable goods (**?**).

The primary data source used is the Penn World Table (PWT) version 10.0. This database is commonly employed for comparative analysis of real GDP between countries (Feenstra, Inklaar, & Timmer, 2015). It allows us to estimate whether there is convergence at time t between country i and country j (v_{ij}). Another advantage of this database is its extensive coverage, both in terms of time (1950-2020) and geography (182 countries). It's worth noting that despite the initially large number of countries in the database, it is significantly reduced due to missing data on both GDP and the variables describing the explanatory factors of convergence. For this reason, the database is refined to obtain a balanced panel used for regression estimations. We aimed to have the most extensive panel possible in terms of time and countries. Therefore, depending on the type of network analyzed for the ERGM model estimations, the number of countries will vary between 83 and 106.

Finally, in Table 1, some key concepts from the network literature are defined. These concepts will be used in the following section and are necessary for interpreting the results of the analysis.

Table 1: Networks indicators

Indicator	Definition	Interpretation
Number of nodes (N)	Objects that makeup the network	N represents the countries
Edges (E)	An edge, link, or dyad is the presence of a connection between two countries	It implies convergence between two countries
Average links or degree (deg)	$deg = \frac{N}{E}$	Indicates the average number of countries with which the type node converges.
Component	Sub-graph in which all nodes are directly or indirectly connected to each other	Countries that directly or indirectly converge with each other.
Edge density	The ratio between the existing links and the possible links	The ratio of the number of countries that converge to all possible cases of convergence.
Isolates	These are nodes that do not have connections in the network	Countries that do not converge with other countries
Triads	Given three nodes, at least two of them converge with each other	This is the case where one country (i) converges with two other countries (j and z), regardless of whether j and z converge with each other
Triangles	The number of close triads (all three nodes have pairwise links)	Countries taken in groups of three, in which all of them converge with each other
Transitivity	It measures the proportion of triangles in the network relative to the existing triads (whether closed or not)	It measures the triads of countries where all of them converge with each other, relative to the triads of countries where at least one of them converges with the rest
Centrality	Level of concentration in the degree distribution among the nodes in the network	High levels of concentration imply that there are only a few countries that converge with many countries
Cluster	Clustering algorithms that select similar groups within the network, creating disjoint groups	The number of clusters indicates the number of disjoint groups that contain nearby/similar countries.

4 Hypotheses

The general assumption of the work posits that to adequately measure complex phenomena like convergence, it is necessary to analyze the structure of an interdependent graph. As seen in equation (3) and also in (4), it is essential to move from the country unit of analysis to a conception of pairwise interactions, where the unit of analysis

is the dyadic relationship. This enables obtaining measurements consistent with the research question.

The main postulate of this work posits that the traditional measurement of convergence yields results that are, if not erroneous, at least limited due to the failure to consider the systemic effect. This effect cannot be measured with traditional methodology for two reasons: i) lack of degrees of freedom to validate hypotheses and ii) lack of independence.

In this sense, once the need for operationalization through a dyadic approach has been justified, the hypotheses about independence are tested. The logical-deductive reasoning is as follows: if there is a complex structure among the actors in the economic system, a high proportion of connections that form different subgraphs or substructures within the network should be observed, which will be manifested through dyads, triangles, and clusters (see the definition in Table 1).

4.1 Absence of Independence

4.1.1 Dyads

To justify the approach to convergence as a set of interdependencies, there must indeed be cases of convergence. This means that the structure of the convergence network should have dyadic relationships that explain the convergence graph (referred to as y). Therefore, a trivial relationship will be tested by the ERGM model, specifically, if the variable measuring the prominence of dyads in the network (referred to as an edge in the model's jargon) is significant. This is essential to ensure that the model makes sense, as it expresses the relationships of dependence as probable for the given network structure.

This hypotheses might suggest a tautological relationship, but the existence of convergence from the AVM definition is not assured (see proposition (1)). It depends on the starting point and the growth trajectory of each country. Assuming convergence is based on previous theoretical models and not on the construction of the graph. Therefore, at this point, the aim is to verify whether the convergence between pairs of countries indeed contributes to the probability of forming the graph or system.

H1.1: Dyadic relationships (edges) will be a significant explanatory variable for explaining the network's structure: $\theta_{edges} \neq 0$.

Although the hypotheses is a necessary condition for estimating such models, it doesn't guarantee that dyadic relationships exist beyond random associations. It is crucial to understand that the definition of convergence does not guarantee actual convergence between countries.

4.1.2 Transitivity

Until now, we have primarily considered dyadic relationships (two-way interactions) in the network. However, in network studies, nodes often have more complex relationships, including triangles, stars, trees, forests, cliques, and other configurations (Desmarais & Cranmer, 2012). These structures reflect the underlying interactions between nodes and are crucial for capturing the endogeneity of the data in estimations.

Given that we have argued for the need to approach the problem as a complex one and have emphasized the importance of dyads in defining the concept of convergence, the next logical step is to examine triadic structures in the network (see Figure 3). Triads are configurations of three nodes that determine more complex structures. Their existence raises the question of how likely it is to observe them and whether they imply the convergence of groups of three countries, meaning triads in which countries are converging in groups of three.

Thus, following the line of network studies (Goodreau, Kitts, & Morris, 2009), it is likely that triangular structures or transitivity relationships between actors will emerge. This means recognizing, for this study, that if there is convergence between two economies, under certain conditions, these will converge to a third one. In terms of graph mathematics, this postulates that a triad (A converges with A and C) is statistically plausible to eventually close (B will converge with C, see Figure 3).

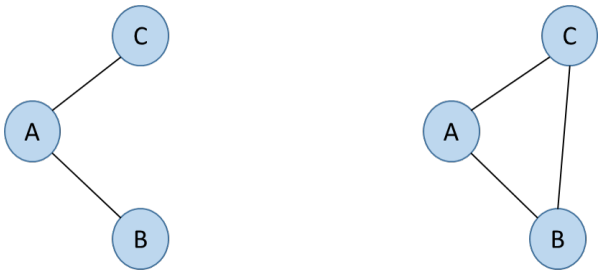


Fig. 3: Example of a triad (left) and a triangle (right).

The idea is simple. For example, in the European Union countries, even if not all of them converge with each other, leading countries like Germany and France ended up converging with others like Spain. This integration of these countries led to connected triangular structures (Matesanz & Ortega, 2016). But is this always feasible? Under what circumstances does this happen?

The interconnected system of countries would suggest that there's a nonzero probability of convergence. The theoretical framework seems to support this idea. But is there a theoretical possibility of convergence in groups of three countries? If the answer is yes, how likely is this convergence to occur? When there's convergence between B and C, given the convergence of A with B and C, theoretically, according to the cases in 1, there are 10 possible combinations of A's convergence with B and C, where, according to mathematical Proposition 1, 6 of them imply convergence.

Proposition 1. *If any country A converges with two arbitrary countries B and C, there are 6 out of the 10 possible cases that imply convergence.*

Proof. see B □

This suggests that there is an expectation of a non-zero probability of triangles explaining the network. This result is expected, as convergence does not occur in a sterile and random context but depends on various factors.

While, in theoretical terms, nothing ensures that there is convergence among triads of countries, the proposition exposes that this phenomenon is plausible and has a high probability. This empirical task is difficult to test, but graph theory provides tools to approach this phenomenon. The proposition also highlights the importance of analyzing the economic system as interdependent. Moreover, countries, just by being in a system (the world), can only push the system to form subgroups with a non-negligible probability.

In operational terms, network literature offers a test that validates a significant relationship of transitivity. In this case, it implies obtaining triadic clubs to explain convergence. This implies testing the following null hypotheses:

H1.2 The network structure will have a non-zero probability of convergence of three countries:

$$\theta_{transitivity} \neq 0$$

.

4.1.3 Clusters

Following the idea of convergence conditioned on groups of countries, several studies (Corrado, Martin, & Weeks, 2005; Corrado & Weeks, 2011; Hobijn & Franses, 2000) provide evidence of clubs of converging countries (Papadimitriou, Gogas, & Sarantitis, 2016).

As also seen in Quah's works (Quah, 1993a, 1993b, 1996b, 1997), bimodal distributions of countries are observed, which provides previous evidence of possible clustered dynamics among countries.

Specifically, through the study of the network's topology, we expect to find several clubs of convergent countries.

H1.3 The network structure will appear fragmented, but K disjoint groups of countries will be identified, with K being greater than 1.

Under the three aforementioned hypotheses, this implies identifying a fragmented network where some countries are closely connected while dissociated from other groups of countries. There exists a statistical dependency among countries, which is manifested through the interconnected structure of the network, forming interconnected clusters. If this occurs, we are facing a lack of independence. Not rejecting the set of hypotheses regarding the network's topology provides evidence, as Quah (1996b) argues, for the necessity of assuming non-independence among countries, violating the central limit theorem. In other words, it suggests a violation of the assumptions of traditional linear models. Confirming the importance of the incorporation of recent models with worldwide interactions (Eaton & Kortum, 1999; Ertur & Koch, 2011; Keller, 2004; Kortum, 1997). Which justifies the use of graph methods for obtaining consistent estimates.

4.2 Convergences hypothesis

Once the method's relevance has been demonstrated, we proceed with testing convergence in a different way than previous studies (Acemoglu & Molina, 2021; Barro & Sala-i Martin, 1992; Kremer et al., 2021). We propose to measure the probability of convergence from the perspective of complexity theory and graphs, and in operational terms, we will use Exponential Random Graph Models (ERGMs). We will specify an econometric model that measures the probability of convergence $\theta_{d_{i,j}}$, where $d_{i,j}$ is a measure of the distance between the incomes of two countries at a given point in time.

It's important to note that $\theta_{d_{i,j}}$ does not refer to the speed of convergence, and quantitatively, it has nothing to do with the β convergence discussed in Barro (1991), Acemoglu and Molina (2021), and Kremer et al. (2021). That being said, the conceptual approach is the same: the greater the income gap between countries at time t-z periods, the higher the probability of convergence. This is what canonical convergence literature postulates (Kremer et al., 2021). The higher the accumulation of capital and other inputs associated with income generation in a country, the lower the potential future growth. In systemic terms, dyadic in this case, it is recognizing that the greater the income distance between two countries in previous periods to the current one, the relatively poorer country will have to grow faster. Therefore, there will be a higher probability of convergence.

This will contrast the convergence hypotheses in light of the new method:

H2 - The probability of convergence will increase as the distance, $d_{i,j}$, between incomes from the previous period between two countries i and j becomes greater.

$$\theta_{d_{i,j}} > 0$$

. So, if the $\theta_{d_{i,j}}$ is positive there is convergence between countries.

Although there is evidence of some convergence between countries (Kremer et al., 2021), the evidence is not conclusive. Evidence in previous work (Papadimitriou et al., 2016) also shows the tendency to form clusters of countries. In this sense, the methodology in this paper has an advantage. It allows to see the convergence of the system, where both options are perfectly possible: countries converging with other countries, but only or in their majority proportion, with some specific countries in a cluster. Thus, the hypothesis is as follows:

H3 - There will be a tendency of the system to be more unequal in the period.

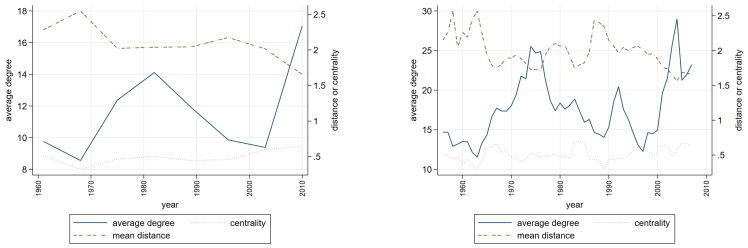
$$\sigma_t < 0$$

5 Results

The temporal dimension of the analysis spans almost 60 years (1960-2020). This implies making methodological decisions regarding the time unit. Typically, in the growth literature, data are averaged over a set of years to smooth out variations caused by shocks or measurement errors that could distort the variable values. The network is observed according to the time unit used, with the "short network" being the one

constructed using seven-year GDP averages, while the "long network" is the network that uses annual GDP values with moving averages.

The results show a cyclical pattern with waves of convergence between countries in the early 1980s, around 2000, and around 2008 (see Figure 4a and 4b). This is strongly linked to global crises such as the Latin American debt crisis of the 1980s, the Asian financial crisis at the end of the 20th century, and the U.S. mortgage crisis of 2008. The result is suggestive and deserves further exploration to understand why this phenomenon of increased convergence occurred during times of crisis. The annual network shows too much variability when using an annual time unit, so it is advisable to use the short network from now on, which uses five-year averages.



(a) "Short network" (b) "Long network"

Fig. 4: Dynamic of convergence graphs

The Figures 4a and 4b show the evolution of network centrality, which is the number of countries on average that the typical country in the network converges with. It can be observed that on average, convergence starts with a minimum of 9 countries at the beginning of the period and grows to around 17 countries in the later years (see graph 4a). It follows a slightly increasing trend. In terms of extreme values, this is also evident in the annual network, as shown in graph 4b, although with much greater instability. This shows how, on average, countries converge with between 10% and 20% of the others.

The results of the short network (which will be used from now on) allow for a first conclusion. Under the assumption that connections are established through a random process (such as an Erdos-Renyi network), the probability of connection per node (according to Barabási, 2013) would be $2 \cdot \frac{L}{N-1}$. Here, L represents the number of links (convergences), and N is the number of countries; in this case, with 108 countries, the probability of convergence between two countries would range between 0.26 and 0.43.

However, as demonstrated in the proposition 1 and as will be described below, connections are far from independently. The network's topology and the countries' characteristics, linked to their history, mean that countries do not converge independently of these characteristics. The rest of the article delves into this point: convergence occurs between countries, which tend to converge in triads and form clusters associated with various types of distances.

5.1 Tryads

Another distinctive characteristic of complexity is confirmed: the strong presence of triangles in the network. On the vertical axis to the left, one can observe the numerical values corresponding to the dashed line depicted in Figure 5. This line represents the progression of triangles in relation to the total number of potential triangles within the network.Typically, a useful measure to assess the tendency to form convergence clubs is to look at the network's global transitivity. This measure is reflected in the solid line whose values are shown on the right axis, and it measures the occurrence of triangles given the number of existing triads. Note that this number starts around 0.5, reaching its maximum in the mid-1980s, with a probability of 0.6. However, from that point on, the probability systematically decreases.

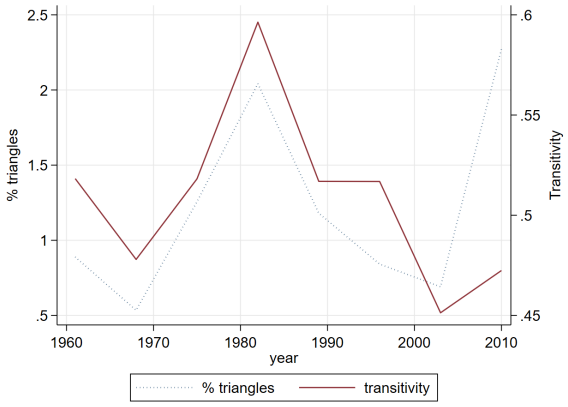


Fig. 5: % of triangles and transitivity of short network.

The previous result contrasts with Proposition 1. If the cases were equiprobable, the probability of forming groupings should be 0.6. This does not happen, which might suggest that not all cases are empirically plausible. However, it's not possible to make this claim based on this indicator. Another less linear interpretation is that it's unlikely that very dissimilar countries have synchronous (similar) movements that lead them to converge with a third country. What is evident is that the confirmatory theoretical proposition of network literature is confirmed in the data, with a non-negligible and high probability, close to 0.5, of the existence of triads, i.e., inter-dependencies.

5.2 Clusters

One way to summarize the information in the graph is to observe groups of countries that are closer to each other compared to another group. In this case, closeness implies groups of countries that tend to converge to a greater extent among themselves compared to another group of countries. Technically, the mathematical tools that handle this are community clustering algorithms, in this case, the Louvain and Fastgreedy

community clustering algorithms. For example, the Louvain algorithm seeks to maximize the modularity of the graph, which means finding partitions of the graph in which connectivity within the community is much greater than between communities.

Applying these classic clustering algorithms to the graph confirms similar results to other studies (Papadimitriou et al., 2016). Cluster structures in the network emerge, typically ranging from 2 to 5 groups depending on the period and the algorithm used (see Figure 6). This provides evidence to support the hypotheses in Section 4.1.3. This results confirm the twin-peaks finding by Quah (1997). But also find a new evidence, the stratification structure (several groups of convergence) instead of polarization (Anderson et al., 2012) in almost all periods.

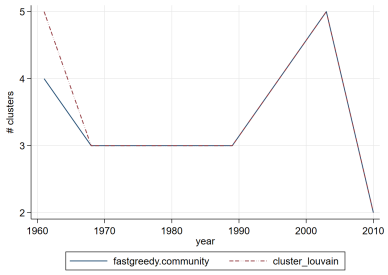
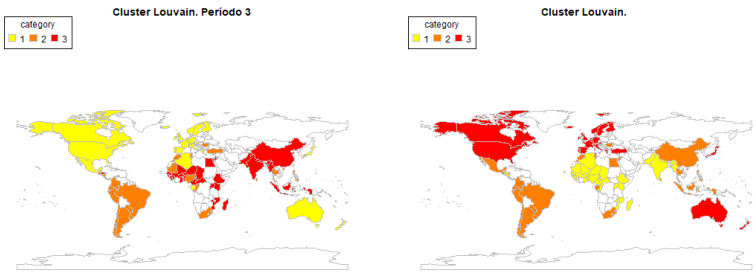


Fig. 6: Number of clusters.

There is evidence of clustering into three groups (see figure 7). Primarily, convergence is observed among Europe, the United States, Oceania, and Argentina in South America. Subsequently, convergence can be seen in African countries associated with a global economic center like India. Finally, an interesting aspect is the income convergence between South America and China, which may indicate shifts of the economic center of gravity from North America to Asia by these countries (Quah, 2011). This phenomenon has also been documented in other works (Zhu et al., 2014), possibly reflecting a convergent process among countries that have had political interactions, such as some BRICS members. However, other studies show long-term divergence processes, so the interpretation of this second network should be approached with caution. Additionally, when examining the regularity patterns period by period (see figures in the appendix D), the pattern is not clear-cut. There are times when China approaches South America, but there are periods when they diverge. The only consistently clear pattern is the convergence of the central northern group consisting of Europe, North America, and other countries, like Australia.

This result highlights the emergence of structures that go beyond the individual, dyads, and triads. It provides information that is impossible to discern through individual analysis. Observing the number of countries with which a country converges does not add substantial information. When ranking countries by convergence, there are numerous countries with many (or few) links on all continents and with varying income levels (see tables C2, C3, and C4 in annex). This does not allow for a clear



(a) Network "1971-1977". (b) Network "total-1960-2019".

Fig. 7: Georeferenced clusters.

pattern of convergence to be observed. Therefore, it is more suitable to move towards more structural levels of analysis to discern patterns more clearly.

For any of the proposed convergence graphs, the hypotheses from section 4.1.1 is confirmed, which is a necessary condition for developing the argument of this article. Without a high number of dyads, approaching the topic from a complexity perspective would neither be necessary nor viable.

The transitivity coefficient measures the proportion of triangles in the network, showing a probability of occurrence greater than 50% in all types of graphs. This provides evidence to support the hypotheses presented in section 4.1.2. This result will be further validated through specification of the ERGM model.

The above results show a high number of links in the AVM network and the clustering of countries into clusters of at least three groups, depending on the method and network type. This confirms the hypotheses discussed in section 4.1.3.

5.3 Estimation of the probability of convergence

Below, we estimate ERGM models, exploiting the presence of the convergence network over time. This allows for time-varying estimates, enabling the control of shocks and testing the network's stability.

The results show a positive and significant effect at the dyadic level, where $\theta_{d_{i,j}}$ is either 0.69 or 0.60, depending on the number of lags chosen. This implies that the probability of convergence increases by an average of 65% to 67%, conditional on the rest of the network. Furthermore, the results confirm the importance of investment and economic complexity in increasing the probability of convergence.

There is a negative effect of individual parameter of complexity (eci) affecting the network. Increasing levels of complexity correspond to a reduced probability of convergence. This can be explained by the fact that countries that are more successful in produce more complex goods and services tend to break away from the system.

An important result is that a higher GDP increases the probability of convergence. It confirms the convergence hypothesis that the higher the GDP the slower the growth

rate, so the greater the probability that other countries will grow faster and catch up with the richer countries.

However, this effect is individual, it is a marginal effect from a ceteris paribus point of view. For even if these are types of countries with an increasing rate of GDP or complexity, they could never do so indefinitely due to the ergodicity of the system itself, or in other words, to diminishing marginal returns. Moreover, the literature shows (Keller, 2004) that network effects of knowledge spillovers exist and have been very important for growth and convergence.

However, the effects are not all in the same direction. The margin in the external sector determines an opposite effect, resulting in a higher probability of convergence. This is observed in countries dependent on commodities, where reliance on international prices is crucial in the margin as well as in growth (C. Bianchi, Isabella, & Picasso, 2023). The margin acts as a convergence factor, likely in subgroups of countries that depend on this factor for growth.

Finally, the individual effect of investment is ambiguous depending on the specification and is therefore inconclusive in this paper.

In addition to the standard controls for individuals and the previously explained $\theta_{d_{i,j}}$ (equation 10), it is worth noting that the variables $|eci_i(t-1) - eci_j(t-1)|$, $|margin_i(t-1) - margin_j(t-1)|$, and $|inv_i(t-1) - inv_j(t-1)|$ reflect how the distance network among individuals regarding ECI, margin, and investment has implications for explaining the convergence network. This is derived from the ERGM model itself, which explains the dependent variable "convergence network" with individual covariates and other explanatory networks.

When the parameters associated with the variables mentioned above are positive, it indicates that a greater distance between the values of these variables in two countries i and j increases the probability of convergence. Strikingly, a negative effect of the investment gap, eci gap and the ECI gap on the probability of convergence is observed, but an effect in the opposite direction of the income gap. The latter means that a greater distance between incomes in earlier periods means an increase in the probability of convergence, validating the idea that there are spillovers that allow this gap to be narrowed, and thus convergence to occur.

This could be due to the following. If the gap between, for example eci, in $t-1$ between countries widens, it increases the distance between incomes in t. Note that the direct effect is that it reduces the probability of convergence. This implies an increase in the income gap between country i and j, so that, under classic terms, if there are diminishing marginal returns in capital accumulation, the probability of convergence in $t+1$ should increase. This underscores the importance of networks and the temporal dimension of the system. There is a constant feedback loop between countries and time, determining divergences and convergences at the pairwise level.

The temporal ERGM model allows us to examine other variables of a structural or systemic dimension (the complete network) not yet mentioned in this section: $gwsdp$, $memory$, and σ_t. These variables are conceptually important because they capture the systemic dimension of the phenomenon. The reasoning in the previous paragraph highlights this dimension, as time and inter dependencies beyond dyads appear to be crucial.

The relationship between investment gaps, income gaps, and convergence, as mentioned, is interesting to cross-reference with the *memory* variable. This is a form of intertemporal dependence designed to capture the temporal process of the complete network. In these specifications, a "memory" term is controlled for, which takes into account whether both the presence and absence of convergence are maintained over time. A positive coefficient would confirm the stability of the network. This indicates that the effect of diminishing marginal returns seems to operate in the system, or the existence of certain stationarity in the system is confirmed. In other words, there are divergent patterns but they are dampened by the dynamics of a closed system, making the overall structure of the system convergent.

However, regarding the temporal dimension of the evolution of the network (σ_t), the trend is significant and negative. It shows a tendency to decrease the probability of convergence in terms of the complete network over time. This reflects a consistent result with the divergence literature, which shows a tendency to increase the gap in recent decades. In other words, the probability of convergence tends to decrease over time. However, another interpretation is that the temporal dimension is no longer as relevant for convergence processes, and structural changes that alter accumulation processes or the path dependence of the system should be considered.

Table 2: Effects θ by specification

Effects θ by level	(1)	(2)	(3)
country			
eci (t-1)	-0.08*	-0.08*	-0.07*
	[-0.07; -0.17]	[-0.06; -0.18]	[-0.05; -0.16]
margin (t-1)	0.05*	0.06*	0.04*
	[0.04; -0.15]	[0.07; -0.13]	[0.04; -0.15]
% investement(t-1)	-0.08*	-0.04*	0.01*
	[0.40; -0.46]	[0.36; -0.52]	[0.37; -0.48]
rgdpo_percap(t-1)	0.40*	0.41*	0.39*
	[0.39; 0.23]	[0.40; 0.23]	[0.39; 0.21]
dyads			
$d_{i,j}$(t-2)	0.68*	0.67*	
	[0.81; 0.47]	[0.78; 0.50]	
$d_{i,j}$(t-1)			0.62*
			[0.72; 0.44]
$\lvert eci_i(t-1) - eci_j(t-1)\rvert$	-0.25*	-0.25*	-0.21*
	[-0.24; -0.44]	[-0.24; -0.48]	[-0.21; -0.45]
$\lvert margen_i(t-1) - margen_j(t-1)\rvert$	-0.07*	-0.07*	-0.04*
	[-0.03; -0.20]	[-0.05; -0.25]	[-0.02; -0.18]
$\lvert inv_i(t-1) - inv_j(t-1)\rvert$	-0.09*	-0.12*	-0.22*
	[-0.23; -1.17]	[-0.16; -1.21]	[-0.21; -1.24]
structurals			
edges	-9.16*	-8.15*	-8.99*
	[-10.22; -25.67]	[-8.61; -11.06]	[-10.00; -18.89]
transitiveties	1.07*		1.08*
	[1.68; 0.76]		[1.59; 0.67]
triangle	0.14*	0.13*	0.13*
	[0.16; 0.12]	[0.15; 0.12]	[0.15; 0.12]
gwdsp	-0.11*	-0.12*	-0.11*
	[-0.13; -0.17]	[-0.13; -0.17]	[-0.12; -0.17]
memory	0.16*	0.17*	0.27*
	[0.12; -0.06]	[0.15; 0.01]	[0.27; 0.09]
σ_t	-0.26*	-0.26*	-0.24*
	[-0.20; -0.56]	[-0.22; -0.70]	[-0.19; -0.48]

$*$ Null hypotheses value outside the confidence interval.

Finally, robustness tests were conducted comparing the AVM with other method for construct the graph (see A for more details). In summary, of all the estimations made for the BBK and AVM methods (see Appendix E) , which are significant in the variable of interest, all of them have a positive and significant contribution to the probability of convergence in terms of distance, also confirming hypotheses 4.2.

6 Conclusions and agenda for future research

This paper advances the study of economic convergence by introducing a novel framework that integrates complexity theory and network analysis. Departing from traditional approaches that analyze convergence in isolation, it conceptualizes the global economy as a complex system of interdependent relationships. Using graph theory and Exponential Random Graph Models (ERGM), this methodology reconciles theoretical insights from complexity science with robust empirical techniques, offering a more comprehensive understanding of convergence dynamics.

The findings reveal that convergence is not a universal process but occurs within specific clusters of countries, shaped by historical, geographical and economical factors. Contrary to the traditional center-periphery view, convergence clusters exhibit a stratified structure, with intermediate groups emerging. Structural factors, such as past convergence relationships and macroeconomic disparities in investment, complexity, and external sector margins, significantly influence the probability of convergence. Importantly, the results demonstrate that while localized convergence occurs, global convergence has declined over time, underscoring the persistence of income inequalities.

This work highlights the limitations of traditional convergence metrics, such as β convergence, which focus on individual countries' steady states. The proposed framework generalizes these methods by capturing individual, dyadic and systemic effects, emphasizing the role of inter-country relationships in shaping convergence patterns. The cyclical nature of convergence—marked by phases of impulses and brakes—suggests that both structural and policy-driven factors perpetuate global income disparities. The analysis identifies three primary convergence clusters: Northern countries and Oceania, African nations and India, and Latin America with China, reflecting diverse development trajectories.

The study opens new avenues for research by advocating for the incorporation of higher-order relationships in convergence analysis. Future work could explore generalizing ERGM models to continuous variables, refining methods for identifying convergence networks, and further integrating complexity science into growth economics. Addressing these challenges would provide deeper insights into the mechanisms driving or hindering global income convergence.

In conclusion, while globalization and technological progress have reduced certain barriers between nations, they have not ensured global income convergence. Persistent inequalities reflect differences in countries' starting conditions, institutional capacities, and abilities to leverage internal and external knowledge. By bridging the conceptual gap between complexity theory and empirical growth analysis, this paper contributes to a more nuanced understanding of economic convergence, providing valuable insights for future research on this topic.

Appendix A A second graph construction option: the Backbone-Kalofolias method (BBK).

In addition to constructing the graph using the distance measure, a novel methodology will be used, which will be referred to as the *Back-Bone-Kalofolias* method. This methodology combines two recently developed techniques. Kalofolias (2016) provides an optimization method to create a graph based on the attributes (p) or features of each node (countries in this case, is called X to continue with the author's nomenclature) complemented by the Back-Bone (BB) extraction method (Neal, 2022). In this case, each X vector of p dimension corresponds to the per capita incomes of countries. After obtaining the graph using the Kalofolias method, the network is subjected to the BB method to identify the relevant structure within the network.

The initial problem sought to be solved by methods such as Kalofolias (2016) is: given a set of observations $\mathcal{X} := \{\mathbf{x}_p\}_{p=1}^{P}$ we seek to identify a graph G such that the attributes (or signals) $\mathcal{X}$ are smooth in G.

Smooth refers to signals having similarity in attributes and are formed from criteria such as homophily or proximity in latent space. The criterion behind the elicitation (or learning of the graph from the signals) is the smoothness of the Dirichlet energy in the graph G with Laplacian L measured by the total variance (TV):

$$\mathrm{TV}(\mathbf{x}) = \mathbf{x}^\top \mathbf{L}\mathbf{x}$$

Kalofolias (2016) proposes noting that the total variation can be rewritten as follows:

$$\sum_{p=1}^{P} \mathrm{TV}\left(\mathbf{x}_p\right) = \mathrm{trace}\left(\mathbf{X}^\top \mathbf{L}\mathbf{X}\right) = \frac{1}{2}\|\mathbf{A} \circ \mathbf{Z}\|_1 \tag{A1}$$

Where Z is the Euclidean matrix: $\mathbf{Z} \in \mathbb{R}_+^{N\times N}$, where $Z_{ij} := \|\overline{\mathbf{x}}_i - \overline{\mathbf{x}}_j\|^2$. Where N is the number of nodes (countries). Where X: $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_P] \in \mathbb{R}^{N\times P}$, and $\overline{\mathbf{x}}_i^\top \in \mathbb{R}^{1\times P}$. What this equation does is to promote sparsity by favouring edges (i,j) associated with small values of Z_{ij}

The Kalofolias method allows the construction of a graph based on each country's GDP series. To achieve this, a connection matrix is first created based on the similarity of the GDP series, assuming that greater similarity between the attributes (also known as signals) of two nodes implies a higher probability of a link between them. Thus, a best-fit GDP series matrix optimisation is performed using the model of Kalofolias (2016):

$$\min_{\mathbf{A}} \left\{ \|\mathbf{A} \circ \mathbf{Z}\|_1 - \alpha \mathbf{1}^\top \log(\mathbf{A}\mathbf{1}) + \frac{\beta}{2}\|\mathbf{A}\|_F^2 \right\}$$
$$\text{s. a} \quad \mathrm{diag}(\mathbf{A}) = \mathbf{0}, A_{ij} = A_{ji} \geq 0, i \neq j$$

The first minimization term, which has already been discussed, regulates smoothness and sparsity. The second term avoids isolated nodes since adding rows of the

matrix A (adjacency) results in the degree of the nodes, which when applying the logarithm avoids isolated nodes. The third term regulates the sparsity of the graph, since as β grows, more edges are promoted in the graph.

The BB method identifies and preserves only the most important links in the network, its backbone, generating a core network for better comprehension. It involves comparing the weight of a link in the network (similarity of GDP vector) with the weight of the link under a random model. If the difference is significant, the link is retained.

Furthermore, the BB method is necessary because the pure Kalofolias method does not allow for the use of the proposed ERGM estimation technique, as the values of the matrix obtained in Kalofolias take continuous values. Therefore, by applying the Back-Bone extraction method, a transformation from a weighted network to a dichotomous network (presence of a link vs. absence) is performed. This results in an unweighted graph that will serve as the network (dependent variable) for convergence estimations.

It's worth noting that this second methodology, by its construction, incorporates per capita GDP over time, but it loses the temporal dimension of the analysis. For this reason, in order to compare both methods, the AVM methodology without a temporal dimension is used, meaning a network that observes convergence within a time window between 1960 and 2019. This allows for comparability between the two methods.

The AVM method is simple in its construction, making it easy to interpret, but it has the limitation that it doesn't use inference measures to identify the graphs. In contrast, the Backbone-Kalofolias method has the advantage of constructing the graph using a statistical method to identify the links, which provides statistical robustness. However, it has the limitation that it can identify as many graphs as the chosen parameter values for identifying the optimal graph, making it difficult to determine which graph is the "right" one. For this reason, both approaches are used to provide a sense of robustness or sensitivity of the results based on the constructed graph.

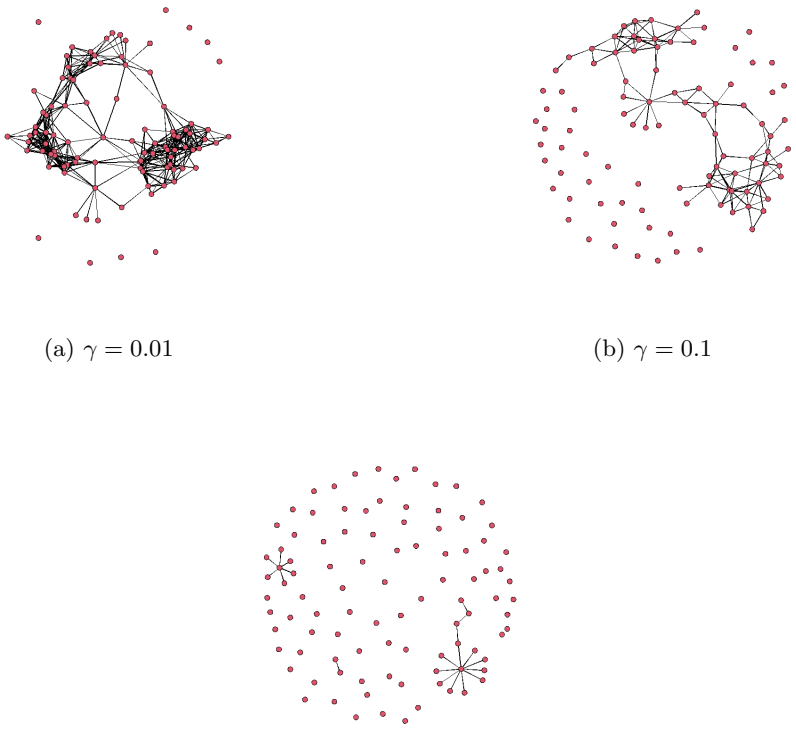
A.1 AVM vs BBK

Another way to observe convergence is through a single network. This approach sacrifices variability in cases and the temporal evolution of the graph but gains in conceptual clarity by understanding that convergence is typically seen as a long-term concept. Thus, a single graph is constructed using the AVM method. The graph G is built using the same inequality (5) but considering only the initial per capita GDP of each country (average from 1950-1970) and the final per capita GDP (average from 2000-2019). To provide comparability to the results obtained by the AVM method, a complementary graph is generated using the BBK method.

For the BBK method, the GDP series of each country in the sample is used. This results in a single graph with a specific structure (see Figure A1). The parameter γ penalizes larger weights to control the graph's spread. A higher value of the parameter γ leads to a sparser graph. Since there are no benchmarks to assign a particular value of γ due to the novelty of the Kalofolias method in general and its application to economics, the procedure consists of iterating over different values of γ to obtain several possible convergence graphs. This is why multiple representations of the convergence

graph are selected to later subject it to the ERGM model and the probability test of convergence.

Once the convergence matrix is obtained using the Kalofolias method, a probabilistic model was applied to detect the links with the highest probability of occurrence. This method, in addition to presenting a good fit for link occurrence, allowed the generation of a cleaned matrix of zeros and ones, a necessary condition for applying the ERGM method.



(a) $\gamma = 0.01$

(b) $\gamma = 0.1$

(c) $\gamma = 0.5$

Fig. A1: Graph of convergence by the BBK method according to the parameter γ of the Kalofolias model.

The new single graph, using the Absolute Value Method (AVM), serves a conceptual purpose, as it allows observing average long-term convergence over the entire period. It also serves a methodological purpose, enabling comparison with the network structure using the BBK method and inferring potential differences based on the graph construction method used.

The construction using the AVM method generates a fully connected network, as indicated by the presence of a single component in the network where 100% of the nodes are part of the main component. This implies that all countries converged with at least one other country, with an average of 6.28 countries with which each country converges. These indicators contrast with the BBK method, where there are cases of countries that do not converge with any other country, meaning that there are more than one component in the network and a non-zero percentage of isolated nodes.

The AVM method results in a denser graph (edge density) with a value of 0.13, compared to values between 0.04 and 0.096 for BBK. Despite the heterogeneities in the constructed convergence graphs, certain regularities are observed, including the presence of several larger clusters, a high number of connections (dyads), and a high level of transitive relationships (percentage of triangles in relation to the existing triads), with 49% in AVM compared to 57% and 45% in the denser BBK networks.

	BBK			AVM
Indicator	0.01	0.1	0.5	
countries	97	97	97	100
mean links	4.588	1.402	0.206	6.28
% countries in major component	91%	63%	14%	100%
% countries in second major component	1%	1%	7%	NA
edge density	0.096	0.029	0.004	0.126
% isolates	9%	37%	76%	0
triangles	2970	321	0	4632
% triangles	0%	0%	0%	31.2%
transitivity	57%	45%	0%	49%
centrality	0.12	0.09	0.10	0.29
eigen centrality	13.5	6.8	3.2	17.6
mean distance	3.2	5.2	2.3	3.4
cluster edge betweenness	15	42	78	3
fastgreedy.community	12	41	78	3
cluster louvain	14	41	78	4

Table A1: Structure of unique network by method.

Appendix B Proposition 1

Proposition 1. *If any country A converges with two arbitrary countries B and C, there are 6 out of the 10 possible cases that imply convergence.*

Proof. To demonstrate the existence of convergence, it is necessary to test all possible combinations of convergence cases derived from 1. This includes C_2^4 plus the repeated cases.

1. **A converge with B and with C, by case A (see fig 1b)**
   Case 1:

$$\left.\begin{array}{l} 0 < y^A - y^B < y^A_{t-1} - y^B_{t-1} \\ 0 < y^A - y^C < y^A_{t-1} - y^C_{t-1} \end{array}\right\} \Rightarrow 2y^A_t - y^B_t - y_C < 2.y^A_{t-1} - y^B_{t-1} - y^C_{t-1} \quad \text{(B2)}$$

Adding y^B_t:

$$y^B - y^C < +y^B_t + y^B_t - 2y^A_t + 2y^A_{t-1} - y^B_{t-1} + -y^C_{t-1} \quad \text{(B3)}$$

Adding y^B_{t-1}:

$$y^B_t - y^C_t + y^B_{t-1} < +y^B_{t-1} + y^B_t - 2y^A_t + 2y^A_{t-1} - y^B_{t-1} + -y^C_{t-1} \quad \text{(B4)}$$

Then:

$$y^B_t - y^C_t + y^B_{t-1} + y^B_{t-1} < +y^B_{t-1} + 2y^B_t - 2y^A_t + 2y^A_{t-1} - y^B_{t-1} + -y^C_{t-1} + y^B_{t-1} \quad \text{(B5)}$$

Grouping:

$$y^B_{t-1} + y^B_{t-1} < +y^B_{t-1} + 2y^B_t - 2y^A_t + 2y^A_{t-1} - y^B_{t-1} + -y^C_{t-1} + y^B_{t-1} - (y^B_t - y^C_t) \quad \text{(B6)}$$

$$y^B_{t-1} + y^B_{t-1} < y^B_t - 2y^A_t + 2y^A_{t-1} - y^B_{t-1} + (y^B_{t-1} - y^C_{t-1}) + y^B_{t-1} - (y^B_t - y^C_t) \quad \text{(B7)}$$

$$2y^B_{t-1} - (2y^B_t - 2y^A_t + 2y^A_{t-1}) < (y^B_{t-1} - y^C_{t-1}) - (y^B_t - y^C_t) \quad \text{(B8)}$$

Rearrange terms:

$$2\left[(y^B_{t-1} - y^B_t + y^A_t - y^A_{t-1})\right] < (y^B_{t-1} - y^C_{t-1}) - (y^B_t - y^C_t) \quad \text{(B9)}$$

Rearrange:

$$2\left[\underbrace{(-y^B_t + y^A_t)}_{>0} + \underbrace{(y^B_{t-1} - y^A_{t-1})}_{>0}\right] < (y^B_{t-1} - y^C_{t-1}) - (y^B_t - y^C_t) \quad \text{(B10)}$$

This demonstrates that under case 1, there is always convergence.

2. **A converges with B according to case A (see fig 1a). A converges with C according to case B. (ver fig 1b)**
   Case 2:

$$\left.\begin{array}{l} 0 < y^A - y^B_t < y^A_{t-1} - y^B_{t-1} \\ 0 < -y^A + y^C < y^A_{t-1} - y^C_{t-1} \end{array}\right\} \Rightarrow y^C_t - y^B_t < 2.y^A_{t-1} - y^B_{t-1} - y^C_{t-1} \quad \text{(B11)}$$

Adding and subtracting y^B_{t-1}:

$$\Rightarrow y^C_t - y^B_t < 2.y^A_{t-1} - y^B_{t-1} - y^C_{t-1} + y^B_{t-1} - y^B_{t-1} \tag{B12}$$

It is rearranged in such a way that the definition of convergence between B and C appears, and it is observed if one of the convergence conditions is met:

$$\Rightarrow (y^C_t - y^B_t) - (-y^C_{t-1} + y^B_{t-1}) < 2.\underbrace{(y^A_{t-1} - y^B_{t-1})}_{>0\ by\ definition} \tag{B13}$$

$$-2.\underbrace{(y^A_{t-1} - y^B_{t-1})}_{>0\ by\ definition} < (-y^C_{t-1} + y^B_{t-1}) - (y^C_t - y^B_t) \tag{B14}$$

In this case, convergence may or may not exist. The general postulate of this case can be summarized as:

$$\left.\begin{array}{l} y^A_{t-1} > y^B_{t-1} \\ If\ \ y^A_{t-1} > y^C_{t-1} \\ y^B_t < y^A_t < y^C_t \end{array}\right\} \Rightarrow \text{There is not necessarily convergence.} \tag{B15}$$

Therefore, it can be concluded that

$$\left.\begin{array}{l} y^A_{t-1} > y^B_{t-1} \\ Si\ \ y^A_{t-1} > y^C_{t-1} \\ y^B_t < y^A_t < y^C_t \\ y^C_{t-1} > y^B_{t-1} \end{array}\right\} \Rightarrow \text{there is no convergence.} \tag{B16}$$

3. **A converges with B according to case A (see fig 1a). A converges with C according to case C. (see fig 1c)**
Case 3:

$$\left.\begin{array}{l} 0 < y^A_t - y^B_t < y^A_{t-1} - y^B_{t-1} \\ 0 < y^A - y^C < -y^A_{t-1} + y^C_{t-1} \end{array}\right\} \Rightarrow 2.y^A_t - y^B_t - y^C_t < -y^B_{t-1} + y^C_{t-1} \tag{B17}$$

Adding and subtracting y^C_t

$$\Rightarrow 2.y^A_t - y^B_t - y^C_t < -y^B_{t-1} + y^C_{t-1} + y^C_t - y^C_t \tag{B18}$$

So, by rearranging terms:

$$\Rightarrow 2.y^A_t - y^B_t - y^C_t < -y^B_{t-1} + y^C_{t-1} + y^C_t - y^C_t \tag{B19}$$

$$\Rightarrow 2.\underbrace{(y^A_t - y^C_t)}_{>0\ by\ definition} < (y^C_{t-1} - y^B_{t-1}) - (y^C_t - y^B_t) \tag{B20}$$

This ensures that under case 3, there is convergence.

4. **A converges with B according to case A. (see fig 1a). A converges with B according to case D. (see fig 1d)**
Case 4:

$$\left.\begin{array}{l} 0 < y_t^A - y_t^B < y_{t-1}^A - y_{t-1}^B \\ 0 < -y^A + y^C < -y_{t-1}^A + y_{t-1}^C \end{array}\right\} \Rightarrow y_t^C - y_t^B < y_{t-1}^C - y_{t-1}^B \tag{B21}$$

By definition, there is convergence. In other words, if

$$\left.\begin{array}{l} y_t^B < y_t^A < y_t^C \\ y_{t-1}^B < y_{t-1}^A < y_{t-1}^C \end{array}\right\} \Rightarrow Convergence \tag{B22}$$

5. **A converges with B according to case B (see fig 1b). A converges with B according to case B (see fig 1b)**
Case 5:

$$\left.\begin{array}{l} 0 < -y_t^A + y_t^B < y_{t-1}^A - y_{t-1}^B \\ 0 < -y^A + y^C < y_{t-1}^A - y_{t-1}^C \end{array}\right\} \Rightarrow -2.y_t^A + y_t^B + y_t^C < 2y_{t-1}^A - y_{t-1}^B - y_{t-1}^C \tag{B23}$$

Adding and subtracting y_t^B y $+y_{t-1}^C$

$$\Rightarrow -2.y_t^A + y_t^B + y_t^C < 2y_{t-1}^A - y_{t-1}^B - y_{t-1}^C + y_t^B - y_t^B + y_{t-1}^C - y_{t-1}^C \tag{B24}$$

$$\Rightarrow -2.y_t^A + y_t^B < 2y_{t-1}^A - y_{t-1}^B - y_{t-1}^C - (y_t^C - y_t^B) - y_t^B + y_{t-1}^B - y_{t-1}^B \tag{B25}$$

$$\Rightarrow -2.y_t^A + y_t^B < 2y_{t-1}^A - y_{t-1}^C - (y_t^C - y_t^B) - y_t^B + (y_{t-1}^C - y_{t-1}^B) - y_{t-1}^C \tag{B26}$$

$$\Rightarrow -2.y_t^A + 2y_t^B < 2y_{t-1}^A - y_{t-1}^C - (y_t^C - y_t^B) + (y_{t-1}^C - y_{t-1}^B) - y_{t-1}^C \tag{B27}$$

$$\Rightarrow -2.y_t^A + 2y_t^B < 2y_{t-1}^A - 2y_{t-1}^C - (y_t^C - y_t^B) + (y_{t-1}^C - y_{t-1}^B) \tag{B28}$$

$$\Rightarrow -2.y_t^A + 2y_t^B - 2y_{t-1}^A + 2y_{t-1}^C < -(y_t^C - y_t^B) + (y_{t-1}^C - y_{t-1}^B) \tag{B29}$$

$$\Rightarrow 2 \underbrace{(-y_t^A + y_t^B)}_{>0 \text{ by definition}} + 2 \underbrace{(y_{t-1}^C - y_{t-1}^A)}_{>0 \text{ by definition}} < -(y_t^C - y_t^B) + (y_{t-1}^C - y_{t-1}^B) \tag{B30}$$

Then, as $(y_t^C - y_t^B) < (y_{t-1}^C - y_{t-1}^B)$ there is convergence between country C and B.

6. **A converges with B according to case B (see fig 1b). A converges with C according to case C (see fig 1c)**
   Case 6:

$$\left.\begin{array}{l} 0 < -y_t^A + y_t^B < y_{t-1}^A - y_{t-1}^B \\ 0 < y_t^A - y_t^C < -y_{t-1}^A + y_{t-1}^C \end{array}\right\} \Rightarrow y_t^B - y_t^C < -y_{t-1}^B + y_{t-1}^C \tag{B31}$$

7. **A converges with B according to case B (see fig 1b). A converges with C according to case D (see fig 1d)**
   Case 7:

$$\left.\begin{array}{l} 0 < -y_t^A + y_t^B < y_{t-1}^A - y_{t-1}^B \\ 0 < -y_t^A + y_t^C < -y_{t-1}^A + y_{t-1}^C \end{array}\right\} \Rightarrow -2.y_t^A + y_t^B + y_t^C < y_{t-1}^C - y_{t-1}^B \tag{B32}$$

$$\Rightarrow -2.y_t^A + y_t^B + y_t^C - y_t^B + y_t^B < y_{t-1}^C - y_{t-1}^B \tag{B33}$$

$$\Rightarrow -2.y_t^A + y_t^B + y_t^C - y_t^B + y_t^B < y_{t-1}^C - y_{t-1}^B \tag{B34}$$

$$\Rightarrow -2.y_t^A + 2y_t^B < y_{t-1}^C - y_{t-1}^B - (y_t^C - y_t^B) \tag{B35}$$

$$\Rightarrow 2.(-y_t^A + y_t^B) < y_{t-1}^C - y_{t-1}^B - (y_t^C - y_t^B) \tag{B36}$$

   Since $(-y_t^A + y_t^B)$ is positive, then there is convergence.

8. **A converges with B according to case C (see fig 1c). A converges with C according to case C (see fig 1c)**
   Case 8:

$$\left.\begin{array}{l} 0 < y_t^A - y_t^B < -y_{t-1}^A + y_{t-1}^B \\ 0 < y^A - y^C < -y_{t-1}^A + y_{t-1}^C \end{array}\right\} \tag{B37}$$

$$\Rightarrow 2.y_t^A - y_t^B - y_t^C < -2y_{t-1}^A + y_{t-1}^B + y_{t-1}^C \tag{B38}$$

   Adding and subtracting y_t^B:

$$\Rightarrow 2.y_t^A - y_t^B - y_t^C + y_t^B - y_t^B < -2y_{t-1}^A + y_{t-1}^B + y_{t-1}^C \tag{B39}$$

   Adding and subtracting y_{t-1}^B:

$$\Rightarrow 2.y_t^A - y_t^B - y_t^C + y_t^B - y_t^B < -2y_{t-1}^A + y_{t-1}^B + y_{t-1}^C + y_{t-1}^B - y_{t-1}^B \tag{B40}$$

$$\Rightarrow 2.y_t^A - y_t^B - y_t^C + y_t^B - y_t^B < -2y_{t-1}^A + y_{t-1}^B + y_{t-1}^B + (y_{t-1}^C - y_{t-1}^B) \tag{B41}$$

$$\Rightarrow 2.y_t^A - y_t^B - y_t^B < -2y_{t-1}^A + y_{t-1}^B + y_{t-1}^B + (y_{t-1}^C - y_{t-1}^B) - (-y_t^C + y_t^B) \tag{B42}$$

$$\Rightarrow 2.y_t^A - 2.y_t^B + 2y_{t-1}^A - 2y_{t-1}^B < +(y_{t-1}^C - y_{t-1}^B) - (-y_t^C + y_t^B) \tag{B43}$$

$$\Rightarrow 2.(\underbrace{y_t^A - .y_t^B + y_{t-1}^A - y_{t-1}^B}_{<0\ by\ definition}) < (y_{t-1}^C - y_{t-1}^B) - (-y_t^C + y_t^B) \tag{B44}$$

By the equation B52, since $y_t^A - .y_t^B + y_{t-1}^A - y_{t-1}^B$ is negative , nothing ensures that $(y_{t-1}^C - y_{t-1}^B) - (-y_t^C + y_t^B)$ is positive. Therefore, nothing ensures that there is convergence.

9. **A converges with B according to case C (see fig 1c). A converges with C according to case D (see fig 1d)**
Case 9:

$$\left.\begin{array}{c} 0 < y_t^A - y_t^B < -y_{t-1}^A + y_{t-1}^B \\ 0 < -y^A + y^C < -y_{t-1}^A + y_{t-1}^C \end{array}\right\} \tag{B45}$$

$$\Rightarrow y_t^C - y_t^B < -2y_{t-1}^A + y_{t-1}^B + y_{t-1}^C - y_{t-1}^C + y_{t-1}^C \tag{B46}$$

$$\Rightarrow y_t^C - y_t^B < -2y_{t-1}^A + 2.y_{t-1}^C + (-y_{t-1}^C + y_{t-1}^B) \tag{B47}$$

$$\Rightarrow 0 < -2y_{t-1}^A + 2.y_{t-1}^C + (-y_{t-1}^C + y_{t-1}^B) - (y_t^C - y_t^B) \tag{B48}$$

$$\Rightarrow -2.(-y_{t-1}^A + .y_{t-1}^C) < (-y_{t-1}^C + y_{t-1}^B) - (y_t^C - y_t^B) \tag{B49}$$

In this case there may or may not be convergence. The general postulate of this case is summarized as:

$$Si\ \left.\begin{array}{c} y_{t-1}^A < y_{t-1}^B \\ y_{t-1}^A < y_{t-1}^C \\ y_t^B < y_t^A < y_t^C \end{array}\right\} \Rightarrow \text{there is no convergence necessarily} \tag{B50}$$

Furthermore, since $y_t^B < y_t^C$ it can be stated that:

$$Si\ \left.\begin{array}{c} y_{t-1}^A < y_{t-1}^B \\ y_{t-1}^A < y_{t-1}^C \\ y_t^B < y_t^A < y_t^C \\ y_{t-1}^B < y_{t-1}^C \end{array}\right\} \Rightarrow \text{there is no convergence} \tag{B51}$$

10. **A converges with B according to case D (see fig 1d). A converges with C according to case D (ver fig 1d)**
Case 10:

$$\left.\begin{array}{c} 0 < -y_t^A + y_t^B < -y_{t-1}^A + y_{t-1}^B \\ 0 < -y^A + y^C < -y_{t-1}^A + y_{t-1}^C \end{array}\right\} \tag{B52}$$

$$\Rightarrow -2y_t^A + y_t^C + y_t^B < -2y_{t-1}^A + y_{t-1}^B + y_{t-1}^C \tag{B53}$$

Adding and subtracting y_{t-1}^B y y_t^B

$$\Rightarrow -2y_t^A + y_t^C + y_t^B < -2y_{t-1}^A + y_{t-1}^B + y_{t-1}^C + y_{t-1}^B - y_{t-1}^B + y_t^B - y_t^B \tag{B54}$$

$$\Rightarrow -2y_t^A + (y_t^C - y_t^B) + y_t^B < -2y_{t-1}^A + y_{t-1}^B + (y_{t-1}^C - y_{t-1}^B) + y_{t-1}^B - y_t^B \tag{B55}$$

Reordering

$$\Rightarrow -2y_t^A + (y_t^C - y_t^B) + 2.y_t^B < -2y_{t-1}^A + 2.y_{t-1}^B + (y_{t-1}^C - y_{t-1}^B) \tag{B56}$$

$$\Rightarrow 2(-y_t^A + y_t^B) + (y_t^C - y_t^B) < 2.(-y_{t-1}^A + .y_{t-1}^B) + (y_{t-1}^C - y_{t-1}^B) \tag{B57}$$

$$\Rightarrow 2\left[-y_t^A + y_t^B - (-y_{t-1}^A + y_{t-1}^B)\right] < (y_t^C - y_t^B) - (y_{t-1}^C - y_{t-1}^B) \tag{B58}$$

In this case there may or may not be convergence. The general postulate of this case is summarized as:

$$\left.\begin{array}{l} Si \\ \\ \\ \end{array}\begin{array}{l} y_{t-1}^A < y_{t-1}^B \\ y_{t-1}^A < y_{t-1}^C \\ y_t^A < y_t^B \\ y_t^A < y_t^C \end{array}\right\} \Rightarrow \text{There is not necessarily convergence.} \tag{B59}$$

Out of the ten possible configurations, six of them result in assured convergence, while in others, there may be convergence or divergence. Therefore, it can be concluded that in at least $\frac{6}{10}$ of the cases, there is convergence. □

Appendix C Centrality

Table C2: Top and bottom 10 centrality

Period 2		Period 3		Period 4	
Country	cent	Country	cent	Country	cent
Malta	73	Republic of Korea	65	Algeria	37
Thailand	67	Nigeria	61	Argentina	33
Egypt	62	Switzerland	59	Australia	45
Algeria	61	Tunisia	59	Austria	39
Republic of Korea	60	New Zealand	58	Barbados	53
Romania	56	Thailand	58	Belgium	42
Morocco	54	Australia	57	Bolivia	25
Uruguay	53	Malta	57	Brazil	37
Pakistan	51	Romania	57	Burkina Faso	37
Taiwan	51	United Kingdom	54	Cameroon	65
Madagascar	20	Morocco	20	Taiwan	35
Cameroon	19	Mozambique	20	Thailand	52
Mozambique	19	Kenya	19	Trinidad and Tobago	32
Burkina Faso	15	Madagascar	17	Tunisia	48
Uganda	15	Cameroon	16	Turkey	36
Chad	14	Chad	15	Uganda	11
Indonesia	14	Burkina Faso	14	United Kingdom	36
Rwanda	10	Ethiopia	14	United States	42
Myanmar	7	Myanmar	10	Uruguay	37
Mali	2	Mali	9	Venezuela	45

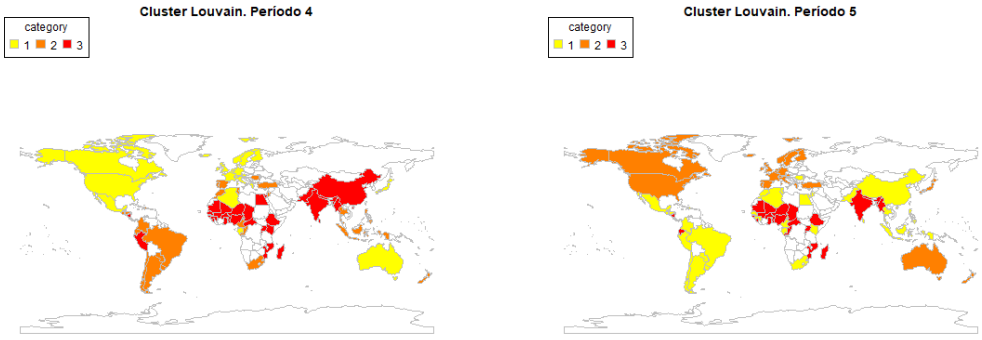
Table C3: Top and bottom 10 centrality

Period 5		Period 6		Period 7	
Country	cent	Country	cent	Country	cent
Trinidad and Tobago	71	Egypt	67	Nigeria	82
China	65	Iceland	58	Syrian Arab Republic	73
Egypt	64	Iran	57	Mozambique	72
Morocco	55	Canada	54	Mali	71
Gabon	54	Myanmar	53	Myanmar	70
Indonesia	54	India	52	Congo	67
Thailand	54	Switzerland	52	India	66
Chad	53	China	51	Uganda	58
Algeria	50	Indonesia	51	Burkina Faso	57
Venezuela	48	Sweden	51	China	57
China, Hong Kong SAR	21	Senegal	19	Netherlands	21
D.R. Congo	20	Cameroon	18	Senegal	21
Sweden	20	China, Hong Kong SAR	18	Pakistan	20
Burkina Faso	19	Niger	15	CAte d'Ivoire	16
Rwanda	18	Syrian AR	13	Kenya	16
Syrian Arab Republic	18	D.R. of the Congo	12	Chad	14
Uganda	17	Madagascar	12	Niger	14
Myanmar	16	Nigeria	10	Ethiopia	11
Niger	15	Rwanda	9	Norway	9
Mozambique	11	Ethiopia	7	D.R. Congo	3

Table C4: Top and bottom 10 centrality

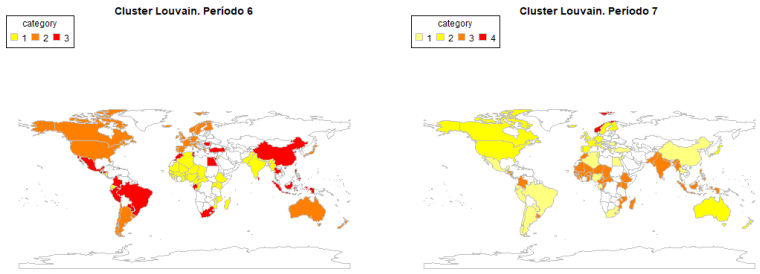
Period 8		Period 9		sum networks 1 a 9	
Country	cent	Country	cent	Country	cent
United States	77	Ethiopia	81	Algeria	423
Myanmar	73	Norway	77	Venezuela	408
Nigeria	71	CÃte d'Ivoire	72	Trinidad and Tobago	397
Canada	67	D.R. Congo	72	Equatorial Guinea	372
Chad	67	Myanmar	71	Iran	368
Ethiopia	66	China, Hong Kong SAR	70	South Africa	364
Iceland	66	Kenya	70	Ghana	362
Japan	65	Rwanda	68	Gabon	360
China, Hong Kong SAR	64	Bolivia	66	Guinea	355
Syrian Arab Republic	64	India	65	Barbados	354
Switzerland	35	Switzerland	38	Singapore	218
Trinidad and Tobago	35	Singapore	37	Netherlands	217
CÃte d'Ivoire	32	Uganda	33	Australia	214
Mozambique	30	Mozambique	30	Denmark	213
Norway	23	Niger	28	Sweden	212
Singapore	22	Senegal	28	Germany	203
Cameroon	20	Congo	26	Switzerland	198
Senegal	19	Madagascar	22	United States	194
Burkina Faso	14	Chad	16	Austria	192
Niger	5	Ireland	11	Ireland	184

Appendix D Clusters



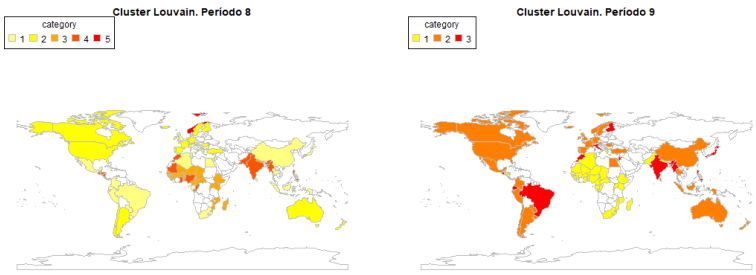
(a) Network "1978-1984". (b) Network "1985-1991".

Fig. D2: Georeferenced clusters.



(a) Network "1992-1998". (b) Network "1999-2005".

Fig. D3: Georeferenced clusters.



(a) Network "2006-2011". (b) Network "2012-2019".

Fig. D4: Georeferenced clusters.

Appendix E Estimation using a convergence graph: AVM vs. BBK

Using the AVM matrix has a higher conceptual intuition compared to the BBK method, in the sense that it is constructed with a simple mathematical inequality, and it is solidly complemented by the theoretical proposition that underlies the use of estimation methods through graphs.

BBK, to be precise, is sensitive to the use of the parameters used, namely α and γ in the development of the graph identification model (Kalofolias, 2016). This gives the method versatility and flexibility, but it also reveals the degrees of freedom that the technique has as a way of identifying the graph. In the extreme, there are infinite possibilities. However, this statement must be qualified because the γ used to maximize the probability of correctly identifying the graph should be low to identify more edges. This is why we started with a γ close to one and descended to identify edges. Thus, estimates are offered for γ values close to zero. Then, due to the absence of graph dispersion (many ones), which does not provide relevant information, it was purified using a probabilistic method to identify the relevant edges. Thus, through the technique of backbone extraction, a graph that is neither full of ones nor full of zeros, a necessary requirement for identifying the effects when estimating, was identified.

From the above discussions, it is clear that an effort has been made to conduct rigorous work to identify "the graph," based on statistical methods from previous work. This serves as a way to provide robustness to the analysis, although statistically speaking, it is observed in the following table that the effects of most variables are not significant. In addition, our variable $d_{i,j}$, relevant when discussing convergence, is shown with opposite signs, although in no case can it be said to be different from zero. When observing the goodness of fit of the models, poor fits can also be seen, indicating that nothing suggests that the results found with the AVM method should be discarded.

	(0.01) z=5	(0.01)z=4	(0.1) z=5	(0.1)z=4	(0.5)z=5	(0.5)z=4
countries						
gdp pc(t-z)	-0.06	0.02	-0.06	-0.13	-0.32	-0.18
	-0.05	-0.05	-0.1	-0.09	-0.24	-0.26
eci(t-z)	0.04	-0.01	0.07	0.15	0.3	0.42
	-0.05	-0.05	-0.09	-0.09	-0.24	-0.28
margin	0.11	0.21*	0.21	0.09	0.5	0.81*
	-0.07	-0.08	-0.15	-0.13	-0.34	-0.33
inv	0.26	0.5	1.4	0.52	-0.7	-1.05
	-0.4	-0.49	-0.82	-0.71	-2.15	-2.51
dyads						
$d_{i,j}$(t-z)	-0.03	-0.03	-0.1	-0.14	0.47	0.69*
	-0.07	-0.07	-0.12	-0.12	-0.3	-0.31
$\lvert eci_i(t-z) - eci_j(t-z)\rvert$	-0.02	-0.04	0	0.01	-0.31	-0.48
	-0.07	-0.07	-0.12	-0.12	-0.35	-0.37
$\lvert margin_i(t-z) - margin_j(t-z)\rvert$	-0.08	0.15	0.18	-0.07	-1.02	-0.91
	-0.08	-0.11	-0.19	-0.14	-0.58	-0.52
$\lvert inv_i(t-z) - inv_j(t-z)\rvert$	1.27*	1.2	-0.89	-0.04	-1.44	-0.96
	-0.53	-0.65	-1.16	-0.97	-3.03	-3.5
structurals						
edges(t-z)	-1.82*	-3.63***	-3.47	-1.63	-0.98	-4.23
	-0.86	-1.04	-1.81	-1.49	-4.05	-4.77
AIC	2938.11	2932.46	1237	1241.4	262.79	258.59
BIC	2996.12	2990.47	1295.01	1299.42	320.81	316.6
Log Likelihood	-1460.06	-1457.23	-609.5	-611.7	-122.4	-120.29

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table D5: convergence networks ERGM (single network). BBK method.

Despite this poor fit, the only case in which $\theta_{d_{i,j}}$ is significant remains positive, as in the estimates in Table 2. As confirmation of these results, the unique graph using the AVM technique also has a positive and significant $\theta_{d_{i,j}}$ (see E7). In summary, of all the estimates made for the BBK and AVM methods that are significant in the variable of interest, all have a significant probability in the distance, confirming hypotheses 4.2.

	z=6	z=5	z=4
country			
lnrgdpo pc(t-z)	-0.03	-0.04	-0.05
	-0.03	-0.03	-0.03
eci(t-z)	0.19***	0.15***	0.26***
	-0.03	-0.03	-0.03
margin(t-z)	0.10*	0.18***	0.15**
	-0.04	-0.05	-0.05
inv(t-z)	0.22	0.53*	0.4
	-0.23	-0.26	-0.3
dyads			
$d_{i,j}$ (t-z)	0.38***	0.30***	0
	-0.04	-0.04	-0.04
$\lvert eci_i(t-z) - eci_j(t-z)\rvert$	-0.90***	-0.95***	-0.81***
	-0.05	-0.05	-0.05
$\lvert margin_i(t-z) - margin_j(t-z)\rvert$	-0.01	0.01	0.21**
	-0.04	-0.05	-0.07
$\lvert inv_i(t-z) - inv_j(t-z)\rvert$	-0.65*	-0.78*	-2.02***
	-0.32	-0.35	-0.41
structurals			
edges(t-z)	0.48	0.63	1.21*
	-0.49	-0.53	-0.61
AIC	6355.15	6202.61	6250.9
BIC	6413.72	6261.18	6309.47
Log Likelihood	-3168.58	-3092.31	-3116.45

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table D6: convergence networks ERGM (single network). AVM method.

Supplementary information.

Acknowledgements.

Declarations

Some journals require declarations to be submitted in a standardised format. Please check the Instructions for Authors of the journal to which you are submitting to see if you need to complete this section. If yes, your manuscript must contain the following sections under the heading 'Declarations':

- Funding
- Conflict of interest/Competing interests (check journal-specific guidelines for which heading to use)
- Ethics approval and consent to participate

	z=6(AVM)	z=5 (AVM)	(0.5) z=4 (BBK)
country			
lnrgdpo pc(t-z)	-0.03	-0.04	-0.18
eci(t-z)	0.19***	0.15***	0.42
margin(t-z)	0.10*	0.18***	0.81
inv(t-z)	0.22	0.53*	-1.05
díadas			
$d_{i,j}$ (t-z)	0.38***	0.30***	0.69*
$\|eci_i(t-z) - eci_j(t-z)\|$	-0.90***	-0.95***	-0.48
$\|margen_i(t-z) - margen_j(t-z)\|$	-0.01	0.01	-0.91
$\|inv_i(t-z) - inv_j(t-z)\|$	-0.65*	-0.78*	-0.96
structurals			
edges(t-z)	0.48	0.63	
AIC	6355.15	6202.61	258.89
BIC	6413.72	6261.18	316.6
Log Likelihood	-3168.58	-3092.31	-120.29

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table E7: Estimations using the AVM and BBK methods for which d_{ij} was significant.

- Consent for publication
- Data availability
- Materials availability
- Code availability
- Author contribution

If any of the sections are not relevant to your manuscript, please include the heading and write 'Not applicable' for that section.

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