Testing Euler Equation with Stock Market Data: A Heterogeneous Story

Penglu Fu[a], Zongxin Qian[a,b], Wendun Wang[c]

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a. School of Finance, Renmin University of China
b. China Financial Research Center, Renmin University of China
c. Econometric Institute, Erasmas University Rotterdam

Abstract

Testing the household Euler equation with consumption data faces econometric challenges caused by large measurement errors in the data and a short time span. We adopt a framework to test the Euler equation with stock market data to alleviate the measurement error and short time span issues. Utilizing a data-driven group panel data method, we identify a heterogeneous pattern of Euler equation failure among different groups of listed firms. The identified degree of Euler equation failure is significantly related to firm characteristics that are associated with famous stock anomalies. We show that the correlations between the degree of Euler equation failure and firm characteristics provide a new set of stylized facts that can help us distinguish between different economic theories on Euler equation failures and asset pricing anomalies, and identify challenges facing current theories.

Keywords: Euler equation; Stock anomalies; Rationality; Financial constraint

JEL codes: E21; E71; G12; G40

Declaration of interests:

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1 Introduction

Households' Euler equation is one of the key components of modern dynamic macroeconomics. It describes how households optimally plan their lifetime consumption path to maximize their lifetime utility. A representative rational household trades off between current and future consumption according to their preferences and expected investment returns. With a perfect financial market, households can easily borrow and save, and hence, consume according to their expected lifetime income, also called permanent income, rather than current income. We typically call this prediction the permanent income hypothesis (PIH). Whether the permanent income hypothesis fails and how it fails matter for the transmission of various macroeconomic policies and optimal policy design. For example, if the PIH holds, any policy that just changes the time path of households' income should have no impact on consumption demand if the net present value of households' income stays unchanged. Typical fiscal policy, such as a debt-financed tax cut, falls into this category. Violating the rationality assumption underlying the Euler equation, waves of over-optimism and over-pessimism could generate excessive volatility in individual decisions, which translates to excessive aggregate fluctuations (Bordalo *et al.*, 2022; Bordalo *et al.*, 2024b).

A huge literature has tested the households' Euler equation using consumption data. However, directly testing the Euler equation with consumption data could be controversial because of large measurement errors in the consumption data. Runkle (1991) finds that around 76 percent of the variance in the growth rate of consumption in the Panel Study of Income Dynamics unexplained by interest rates is due to measurement error. Alan and Browning (2010) estimate that 86% of the variance in observed consumption growth is due to measurement error. Micro-econometric evidence is further challenged by econometric issues associated with the short time span of consumption surveys. Attanasio and Low (2004) and Alan *et al.* (2019) suggest that the estimation bias of log-linear Euler equation is small if the time span of the estimation is long enough, but typical survey consumption data are not long enough. This paper tests the Euler equation with data from the American stock market. Since the households' choice of future consumption path is realized through their investment, the consumption Euler equation could also be taken as the households' investment Euler equation. It implies that the households' marginal utility cost of giving up one dollar of current consumption to buy an asset, such as a stock, must equal the

expected marginal consumption utility gain brought by the future investment return. The comparative advantage of the stock market data is that financial data are more precisely measured and have a very long span.

The testing framework we adopt avoids estimation uncertainty caused by using consumption data. More specifically, our test is based on the Campbell and Shiller (1988) decomposition of stock return (henceforth, we shall call this the CS decomposition). Campbell and Shiller (1988) decompose unexpected, or surprise, stock return into two components. One is the cash flow news, which summarizes investors' updates of expectations on current and all future-period cash flows upon receipt of new information. The other is the expected return news, which summarizes investors' updates of expectations on all future-period expected returns upon receipt of new information. The decomposition is derived from the definition of stock return and holds as an identity in a first-order approximation. Imposing the Euler equation not only gives the identity an economic interpretation but also creates a framework to test the Euler equation.

More specifically, in a first-order approximation, the Euler equation implies that minus expected returns of all stocks equal households' discount rates. Intuitively, fixing the investment cost at one dollar, changes in expected discount rates must offset changes in expected returns to balance the marginal cost and benefit of stock investing. Therefore, assuming that the Euler equation holds, we could interpret the expected return news as discount rate news. Since households discount all stocks with the same discount factor, the difference in the changes of discount factor for different stocks is zero. This means that the discount rate news of any particular stock equals that of the market index. In other words, the market-adjusted discount rate news should be zero for all stocks. Combining this result with the CS decomposition of the market-adjusted surprise stock return, we obtain a one-to-one correspondence between the market-adjusted surprise stock return and the market-adjusted cash flow news of the stock. The failure of this one-to-one correspondence leads to the rejection of the log-linear Euler equation.

We find that about 3 quarters of the firms' market-adjusted surprise stock returns do not have a one-to-one correspondence to the market-adjusted cash flow news, thus rejecting the log-linear Euler equation. Using household survey data and account-level household payment data, two recent studies (Crump *et al.*, 2022; Baugh *et al.*, 2021) also reject the log-linear Euler equation.

Compared with those papers, the more interesting part of this paper is that our econometric framework generates an indicator of theheterogeneous degree of (log-linear) Euler equation failure at the cross-section of different group of firms. By linking the heterogeneous degree of Euler equation failure to different firm characteristics, we are able to distinguish between various theoretical explanations for the identified Euler equation failure. We believe this provides important empirical facts for future theoretical development.

Actually, our paper is not the first paper which tests the households' Euler equation with stock market data. There is a large literature testing the Euler equation by asking the question whether the observed American equity premium could be reconciled with a reasonable elasticity of inter-temporal substitution. A well-known puzzle, called the equity premium puzzle, is that the American equity premium seems to be too high to be justified by a reasonable elasticity of inter-temporal substitution (Mehra and Prescott, 1985). Intuitively, as the households tradeoff between one dollar and its future investment return, what matters is how the households, or investors, discount the future investment return. Suppose the households have a utility function with constant relative risk aversion (CRRA), the coefficient of relative risk aversion (the inverse of the elasticity of inter-temporal substitution) has to be unreasonably high to match the American equity premium. Major rational explanations of the equity premium puzzle modify the utility function to allow for either consumption habit (Campbell and Cochrane, 1999) or preference for early resolution of uncertainty (Bansal and Yaron, 2004). A complication of this literature is that the elasticity of inter-temporal substitution is usually estimated from consumption data. Harvanek (2015) reviews 2735 estimates of the elasticity of inter-temporal substitution and finds that the estimates range from -5 to 5, suggesting large uncertainty in the estimation. Moreover, there is also estimation uncertainty about the degree of consumption habit. Harvanek *et al.* (2017) survey 597 estimates on habit formation and find that the estimated strength of habit formation ranges from -1 to 1.5. Our approach leaves aside the explanation of the equity premium puzzle in the aggregate time series dimension, but explores the heterogeneity in the cross-sectional dimension to facilitate theoretical discussions.

Cohen *et al.* (2002) and Vuolteenaho (2002) use a similar framework to this paper but interpret the failure of the Euler equation as the failure of the rationality assumption. Intuitively, if the Euler equation holds, the market-adjusted discount rate is zero; all changes in market-adjusted

stock returns have to come from changes in market-adjusted expected cash flows. In other words, the "reaction coefficient" of market-adjusted stock return to cash flow news should be one. If the reaction coefficient is larger (or smaller) than one, Cohen *et al.* (2002) and Vuolteenaho (2002) interpret that as investors irrationally over-reacting (or under-reacting) to cash flow news. Using pooled panel data models, they find that the reaction coefficient is much smaller than one and arrive at the conclusion that American investors under-react to cash flow news.

In this paper, we show that the pooling test of Cohen *et al.* (2002) and Vuolteenaho (2002) disguises a rich heterogeneous pattern in the reaction coefficients of different stocks. First, we show that the serious under-reaction finding of Cohen *et al.* (2002) and Vuolteenaho (2002) comes from distorted information. There are firms of which the underlying financial data are non-stationary and have no well-defined cash flow news indicator. Excluding these firms from the sample, the pooled test obtains a reaction coefficient that is much closer to one. Second, the pooled model is not the best econometric model. Our data-driven grouping methods put firms into different groups of which the reaction coefficients differ. There are some groups of firms of which the reaction coefficients are, on average, statistically indifferent from zero while both under-reaction and over-reaction exist in some groups. More importantly, the heterogeneity among different group of firms is not random but closely related to different firm characteristics. Interestingly, these firm characteristics are linked to various anomalies (Table 1) identified in the asset pricing literature. While some of the anomalies are usually explained by mis-valuation, which might be related to irrationality, others are linked to market frictions such as borrowing constraints. Therefore, the term "over (under)-reaction" used in this paper and Cohen *et al.* (2002) and Vuolteenaho (2002) is just a concise way of saying that the estimated reaction coefficient of market-adjusted stock return to cash flow news violates the prediction of the (log-linear) Euler equation. The mis-reaction could be caused by not only irrationality but also market frictions.

There are at least two contributions of this paper to the literature. First, we provide a new framework to test the households' Euler equation using stock market data. While making use of the advantage of small measurement errors and the long span of financial data, our framework generates an indicator of heterogeneous degrees of model failure, which helps us link the degree of model failure to various firm characteristics. This gives us more scope for in-depth research on tracing the source of the failure. Particularly, the possibility of linking the firm characteristics

related to model failure to various asset pricing anomalies helps further bridge the macroeconomic literature and the asset pricing literature. We believe this opens a new direction for future theoretical research. A theoretical model that could remedy the Euler equation failure found in our test should also be able to explain the asset pricing anomaly associated with that failure, and vice versa. Notably, our heterogeneous approach is founded on solid econometric ground. Intuitively, in a panel data model, if the regression coefficients are heterogeneous, pooling introduces biases into the estimation.[1] However, pooling does improve efficiency if some individual stock information is noisy. Although the measurement error issue in stock data is believed to be smaller than in consumption data, there could still be scope for efficiency gain from pooling. Following Bonhomme and Manresa (2015), we automatically group individual stocks into different groups according to the Bayesian information criterion, and pool at the group level. This approach is a better balance between consistency and efficiency. On the one hand, the optimal group of our baseline model is 40, far smaller than the total number of stocks 2,739 in the sample, indicating the efficiency gain from pooling. On the other hand, the single group model is also not the optimal model. More importantly, the neglect of the heterogeneity in the degree of Euler equation failure misses useful information which can help us trace the sources of the model failure. Our heterogeneous approach is closely linked to the huge literature on cross-sectional anomalies in stock pricing. Like our paper, the stock market anomaly literature, with Fama and French (1993, 2015) as famous examples, also explores the cross-sectional heterogeneity to test various theories. Moreover, this literature also assigns firms into different groups and tests different theories with the return spread of those different groups. But there is a vital difference. Our heterogeneous degree of Euler equation failure is generated by a data-driven grouping method which optimizes the balance of estimation consistency and efficiency of the econometric model. By contrast, the stock market anomaly literature groups the firms according to some prior theoretical considerations. The disadvantage of the latter approach is the risk of publication bias is high. Harvey *et al.* (2016) test 316 stock pricing factors in a multiple testing framework and find that many published factors are false discoveries. Hou *et al.* (2020) test 452 anomalies and find that 65% to 82% of the anomalies are not robust while replicated anomalies have much smaller economic magnitude than

[1] In our Euler equation testing context, using popular heterogeneous panel data models such as the random coefficient model of Swamy (1970) and the group mean estimator of Pesaran and Smith (1995) to estimate the average regression effect does not solve the problem. See Chen *et al.* (2023) for a formal proof.

originally reported. The correlation between our data-driven indicator of model failure and the firm characteristics associated with famous stock market failures is important supporting evidence for both the validity of our empirical methodology and the anomalies. Moreover, the specific correlation pattern between our indicator of model failure and the firm characteristics imposes a restriction on plausible theoretical explanations for the anomalies. Any theory with contradictory predictions on the cross-sectional correlation pattern to our finding is not consistent with the data.

Second, our test rejects the Euler equation, which confirms evidence from the consumption data. Given the data quality and econometric issues related to consumption data, this additional piece of evidence for model failure is important. In contrast to the prediction of the permanent income hypothesis, studies using consumption data find that consumption over-reacts to income changes that do not alter expected lifetime income (Crump *et al.*, 2022; Baugh *et al.*, 2021; Kubota and Fukushige, 2016; Li, 2009; Agarwal *et al.*, 2007; Jappelli *et al.*, 1998). Consumption co-moves with the business cycle. The correlation between current income and current consumption is high over the working life. Students and retired workers seem to consume too little compared to their lifetime income (Browning and Crossley, 2001). Typical explanations for these model failures are non-separable utility functions, liquidity constraints, and irrationality. Two recent papers by Crump *et al.* (2022) and Baugh *et al.* (2021) reject the first two explanations and refer to irrational behaviors as the main explanations. Our paper partially echoes their findings. More specifically, just changing the utility function cannot explain the heterogeneous model failure we found in this paper. For example, a specific utility functional form could introduce a time-varying degree of risk aversion and hence a time-varying price of risk. In this case, aggregate shocks could simultaneously change investors' cash flow and future return forecasts, leading to a correlation between cash flow news and expected return news, and hence, the failure of the log-linear Euler equation. However, just changing the utility function is not adequate to explain why the cash flow news and expected return news are positively correlated in some firms while negatively correlated in others. Therefore, consistent with Crump *et al.* (2022) and Baugh *et al.* (2021), our finding suggests that the utility function alone is not enough to explain the (log-linear) Euler equation failure. Our identified model failure is associated with various stock market anomalies which could be explained by some behavioral theories, supporting the view of Crump *et al.* (2022) and Baugh *et al.* (2021) that irrational behaviors might be major sources of model failure. Our finding is also

in line with the findings of Kubota and Fukushige (2016). With direct survey data information on households' rationality and self-control in Japan, Kubota and Fukushige (2016) find that the permanent income hypothesis fails only in the sub-sample of households with irrationality and self-control problems.

However, our testing results from stock market data also differ from results from those consumption-based tests in important aspects. First, we find that irrationality alone is not adequate to explain the heterogenous pattern of Euler equation failure. Particularly, our findings are also consistent with models with imperfect financial markets. Frazzini and Pedersen (2014) suggest that leverage constraint generates a preference for high-beta stocks, which increases systematic risk-taking. We find that investors appear to over-react to cash flow news of high-beta stocks, suggesting that financial constraints might be an important source of the Euler equation failure. This result is different from the finding of Baugh *et al.* (2021) that liquidity constraints do not help explain consumption anomalies. As pointed out by Baugh *et al.* (2021), their sample covers less constrained households, which might have reduced the explanatory power of liquidity constraints for consumer behavior. Our finding suggests that liquidity constraints as a potential source of Euler equation failure still deserve further research. Similarly, existing studies (Drake *et al.*, 2011; Johnson and So, 2012) find that short sale constraints lead to under-representation of pessimistic investors in the formation of stock prices, particularly when investors disagree on their earnings forecasts of firms. Moreover, short sale costs drive investors with negative news to the options market. We find that investors react less to cash flow news of firms with more dispersed analyst earnings forecasts and higher option trading volume, suggesting that short sale constraints are a potential explanation for the Euler equation failure.

Second, the heterogeneous pattern of reaction coefficient and its correlation with different firm characteristics provide empirical facts which help distinguish different behavioral explanations for cross-sectional stock pricing anomalies. More specifically, as we show in the paper, the magnitude of the estimated reaction coefficient relative to one is a reverse indicator of the sign of correlation between the firms' cash flow news and expected return news. Under-reaction suggests a positive correlation while over-reaction suggests a negative correlation. We find that value firms (with high book-to-market values), high-profit firms, conservatively investing firms tend to over-react, suggesting that their cash flow news is negatively correlated to expected return news. The reverse is

true for growth firms (with low book-to-market values), low-profit firms, and aggressively investing firms. Behavioral models which predict that value, high-profit, conservatively investing firms have a smaller forecasting error than growth, low-profit, aggressively investing firms can explain the value effect, profitability effect and investment effect.[2] Examples include the expectation extrapolation theory of La Porta *et al.* (1997), the sticky information model of Bouchaud *et al.* (2019) and the belief over-reaction theory of Bordalo *et al.* (2024a). According to La Porta *et al.* (1997), investors under-predict the cash flow of value firms because they extrapolate the poor performance of those firms into the future. Bouchaud *et al.* (2019) suggest that due to investors' slow update of information, currently high-profit firms' earnings are under-predicted. Bordalo *et al.* (2024a) argue that all firms' future earnings are over-predicted by investors due to representative bias, but earnings of value, high-profit, conservatively investing firms are less over-predicted. The prices of the firms of which the earnings are more under-predicted (less over-predicted) tend to increase more (decrease less) than other firms. Therefore, buying these firms while selling other firms generates positive returns. Consistent with these theories, we find that analyst earnings forecast errors on average decreases with the BM ratio and profitability of our data-driven groups while increases with the asset growth rate of our data-driven groups. However, deviations from rational expectations alone are not adequate to explain the coexistence of positive and negative correlations between cash flow news and expected return news. Actually, as we demonstrate in the paper, biased earnings forecasts alone always predict negative correlations between cash flow news and expected return news. We illustrate that a combination of biased earnings forecasts and short sale constraints could potentially simultaneously explain the stock anomalies and the heterogeneous pattern of reaction coefficients identified in this paper. We believe this is an interesting future direction of theoretical development both in macroeconomics and asset pricing.

The remaining of this paper proceeds as follows: section 2 reviews the literature; section 3 introduces our methodology; section 4 describes the data and variables; section 5 presents the Euler equation testing results; section 6 links the testing results to well-known anomalies in the asset pricing literature and uses the empirical results to distinguish between various asset pricing

[2]In this paper, the forecasting error is defined as the forecasted value minus the realized value. Hence, a smaller error means a lower forecasted value relative to the realized value.

theories; section 7 concludes.

[Insert Table 1 here]

2 Literature review

With CRRA utility function and no financial frictions, typical households' optimal consumption decision in a dynamic stochastic general equilibrium (DSGE) model takes the following form:

$$\begin{aligned} E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{t+1}\right] &= 1, \\ \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{t+1} = \varepsilon_{t+1};\ E_t\left(\varepsilon_{t+1}\right) &= 1, \end{aligned} \tag{1}$$

where C_t is households' consumption at time t, R_{t+1} is the investment return at time $t+1$, β is the subjective discount factor, γ is the coefficient of relative risk aversion or the inverse of the elasticity of inter-temporal substitution, ε_{t+1} is the expectation error, $E_t(\cdot)$ is the expectation operator.

This optimal condition is also called the consumption Euler equation. The equation can be estimated by general method of moments (GMM) using past interest rates, consumption, and income as instrument variables if there is no measurement error.

However, C_t is measured with errors. Denote observed consumption by C_t^o, let $C_t^o = C_t\eta_t$, where η_t is the measurement error in period t, then,

$$\beta\left(\frac{C_{t+1}^o}{C_t^o}\right)^{-\gamma} R_{t+1} = \left(\frac{\eta_{t+1}}{\eta_t}\right)^{-\gamma} \varepsilon_{t+1}. \tag{2}$$

The existence of measurement error leads to problems for GMM estimation.

The log-linear version of (2) takes the form:

$$\Delta \ln\left(C_{t+1}^o\right) = \alpha + \frac{1}{\gamma}\ln R_{t+1} + z_{t+1}, \tag{3}$$

where z_{t+1} includes expectation error, measurement error and approximation error. Assuming that the approximation error is independent of the interest rate, estimating (3) generates consistent results. This estimator is called the approximation GMM or AGMM estimator.

However, there is a heavy debate on whether the potential correlation between the approximation error and interest rate in the AGMM estimation leads to large biases (Carroll, 2001; Attanasio and Low, 2004; Alan *et al.*, 2019). Simulation results of both Attanasio and Low (2004) and Alan *et al.* (2019) suggest that the estimation bias caused by the approximation error is small if the time span of the estimation is long enough. However, typical survey consumption data have very short time spans. Cross-sectional aggregation helps average out measurement error and increase sample length. However, Toda and Walsh (2015) show that higher-order cross-sectional moments of consumption might not exist, leading to inconsistent GMM estimation using cross-sectional moments.

In summary, due to the problem of large measurement errors and short time spans, Euler equation estimation and testing are subject to econometric problems. As a result, the literature generates a wide range of estimates for the elasticity of inter-temporal substitution. Harvanek (2015) finds that estimates for the elasticity of inter-temporal substitution vary with researcher's choice of utility function, data characteristics, control variables, variable definition, etc.

Besides directly estimating the Euler equation, the literature also tests the predictions of the theory, particularly the permanent income hypothesis (Browning and Crossley, 2001). A typical failure of the PIH identified is households' over-reaction to anticipated income changes (Crump *et al.*, 2022; Baugh *et al.* 2021; Kubota and Fukushige, 2016; Li, 2009; Agarwal *et al.*, 2007; Jappelli *et al.*, 1998). Agarwal *et al.* (2007) find that consumers save more when received anticipated tax rebates by repaying credit cards. However, they soon increase their spending, violating the PIH. This effect is stronger for more liquidity-constrained people, suggesting that the failure of the PIH might come from liquidity constraints. Iacoviello (2004) formally shows that housing price and stock enter the aggregate consumption Euler equation through their impacts on the tightness of borrowing constraint. Such liquidity constraint assumption has strong implications for consumption volatility (Iacoviello and Neri, 2010), the business cycles (Liu *et al.*, 2013), and transmission of monetary (Iacoviello, 2005) and fiscal policy (Heathcote, 2005).

Despite the analytical appeal, the liquidity constraint explanation for the failure of the Euler equation is not without challenge. Baugh *et al.* (2021) find consumption increases when households receive expected tax refunds, however, consumers smooth consumption when paying taxes by

transferring funds among liquid accounts. This asymmetric phenomenon is inconsistent with the liquidity constraint explanation. Moreover, the phenomenon also exists for households with a wide spectrum of liquidity conditions. Baugh *et al.* (2021) suggest that their finding is more consistent with mental accounting. Instead of estimating (3) where the left-hand side is realized consumption, Crump *et al.* (2022) use survey expectations of consumption growth on the left-hand side, which alleviates the concern about correlations between the expectation error and right-hand side variables. They control demographic and labor supply variables to alleviate the impact of non-separability in utility. Non-separability indicates that the marginal utility of consumption is influenced by the level of labor or other inputs in the utility function. They find that even for the households not facing substantial liquidity constraints, the planned consumption exhibits excess sensitivity to expected income changes, and thus challenges liquidity constraint as an explanation for the excessive sensitivity. Similar to Baugh *et al.* (2021), Crump *et al.* (2022) concludes that irrational behavior might explain the failure of the Euler equation. There is also survey evidence supporting the Euler equation. Using a consumption survey of German consumers, Drager and Nghiem (2021) find that consumers increase current spending with planned spending and higher expected inflation, supporting the Euler equation. Interestingly, they also find that expectational effects are stronger when there is news about monetary policy or financial markets. Li (2009) explains the Euler equation failure by transaction costs. He argues that transferring cash from asset accounts is costly. Therefore, when the income change is small, agents may choose not to smooth consumption, while they smooth consumption when the news of income changes is large.

The equity premium puzzle is another famous failure of the Euler equation. Mehra and Prescott (1985) found that the equity premium in the United States could not be rationalized by standard neoclassical financial economic models. There are four major explanations for the equity premium puzzle. Campbell and Cochrane (1999) suggest that consumers have consumption habit, and when their decline in wealth threats the consumption habit, they turn much more risk-averse than usual, justifying the high-risk premium observed in the American stock market. Bansal and Yaron (2004) build a model with long run risks, that is, shocks to long-horizons expected growth. Agents that prefer early resolution of uncertainty fear shocks to long-run growth and volatility and require a high positive risk-premium for holding risky assets. Barro (2006) suggests that there are rare disaster risks faced by investors which justify the high-risk premium. There are

also behavioral or belief-based explanations, such as learning and parameter uncertainty (Andrei *et al.*, 2019), ambiguity (Collard *et al.*, 2018; Ju and Miao, 2012), model uncertainty (Bidder and Dew-Becker, 2016), prospect theory (Barberis *et al.*, 2001), and extrapolative expectations (Barberis *et al.*, 2015). The use of consumption data also complicates the testing and explanation for the equity premium puzzle. Using cross-sectional moments of consumption, Balduzzi and Yao (2007) show that their heterogeneous-agent model fits the US equity premium with reasonable values of relative risk aversion. They interpret the results as financial market incompleteness and limited stock market participation explain the equity premium puzzle. Brav *et al.* (2002), Cogley (2002), Kocherlakota and Pistaferri (2009) use similar cross-sectional averaging technique for the Euler equation estimation. However, Toda and Walsh (2015) argue that these papers rely on the existence of higher cross-sectional moments of consumption to work. They show that the higher cross-sectional moments of consumption might not exist.

In the cross-section dimension, the prominent asset pricing puzzle is the existence of significant cross-sectional stock return heterogeneity which cannot be explained by the systematic risk measure, beta coefficient, of the capital asset pricing model (CAPM). The unexplained heterogeneity by CAPM is usually linked to various firm characteristics and is called stock pricing anomalies. While the literature has found hundreds of anomalies, the most influential ones are those summarized in Fama and French (1993, 2015). More specifically, conditional on the differences in beta, stocks of smaller firms (Banz,1981), value firms (Stattman,1980), high-profit firms (Novy-Marx, 2013), low-asset growth firms (Cooper *et al.*, 2008) have higher returns than large firms, growth firms, low-profit firms, and high-asset growth firms. There are at least three alternative explanations for the anomalies. Along the line of rational expectations framework, a strand of literature explains the anomalies by systematic risk exposures of firms which are associated with the firm characteristics but not with the beta coefficients (Berk *et al.*, 1999; Zhang, 2005; Kogan and Papanikolaou, 2014; Gu *et al.*, 2018; Gomez and Schmid, 2021). Another strand of the literature explains the anomalies by financial frictions (Drake *et al.*,2011; Johnson and So, 2012; Frazzini and Pedersen, 2014; Detzel *et al.*, 2023). The third line of research suggests that investors' forecasts on firm earnings, or firm fundamentals, deviate from rational forecasts with perfect information due to non-Bayesian expectations formation (La Porta *et al.*, 1997; Barberis *et al.*, 1998; Daniels *et al.*, 1998; Bordalo *et al.*, 2019; Bordalo *et al.*, 2024a) or sluggish dissemination of information

(Bouchaud *et al.*, 2019). Those theories generate different predictions on the correlation between cash flow news and expected return news. Therefore, our empirical findings on the heterogeneous correlation between cash flow news and expected return news can help distinguish between those theories.

3 Methodology

3.1 Return decomposition

Previous studies (Campbell, 1991; Vuolteenaho, 2002; Cohen *et al.*, 2002; Lochstoer and Tetlock, 2020) show that the log return of firm i at time t can be decomposed as follows[3]:

$$r_{i,t} - E_{t-1}\left(r_{i,t}\right) \approx N_{i,t}^{\mathrm{cf}} - N_{i,t}^{r}, \tag{4}$$

$$N_{i,t}^{\mathrm{cf}} \equiv \Delta E_t \left[\sum_{j=0}^{\infty} \rho^j \left(\mathrm{roe}_{i,t+j}\right)\right], \tag{5}$$

$$N_{i,t}^{r} \equiv \Delta E_t \left[\sum_{j=1}^{\infty} \rho^j r_{i,t+j}\right], \tag{6}$$

where $r_{i,t} = \mathrm{lnR}_{i,t+1}$ is the log return of firm i at time t, $\mathrm{roe}_{i,t} \equiv \log\left(1 + \mathrm{ROE}_{i,t}\right)$, ROE is the firm's return on equity which is a proxy for the firm's ability to generate cash flows for its investors, cash-flow news $N_{i,t}^{\mathrm{cf}}$ is the sum of discounted changes in expectations concerning current and future cash flows, whereas expected return news $N_{i,t}^{r}$ is the sum of discounted changes in expectations concerning future stock returns, ρ is a constant in (0,1), E_t indicates investors' expectations at time t, and $\Delta E_t\left(\cdot\right)$ represents the change in expectations from time *t-1* to time t, defined as $\Delta E_t\left(\cdot\right) = E_t\left(\cdot\right) - E_{t-1}\left(\cdot\right)$.

The return decomposition says that unexpected, or surprise, changes in a firm's returns could come from two sources: revisions in investors' forecasts of the future cash flows of the firms, and

[3]See appendix A for detailed derivations.

revisions in investors' forecasts of the future returns of the firms. The decomposition is derived from the definition of the firm's log return and return on equity and holds as an identity up to a first-order approximation. Particularly, no households' optimization behavior has to be imposed to obtain this decomposition. However, to arrive at the first-order approximation, a technical assumption is made, that is,

$$\lim_{T\to\infty} \rho^{T+1} E_t \left(\mathrm{bm}_{i,t+T}\right) = 0, \tag{7}$$

$$\Delta E_t \left[\sum_{j=0}^{\infty} \rho^j \left(\mathrm{roe}_{i,t+j}\right)\right] \text{ and } \Delta E_t \left[\sum_{j=1}^{\infty} \rho^j r_{i,t+j}\right] \text{ are finite.} \tag{8}$$

This assumption is often neglected in previous studies. However, we shall show later that neglecting this technical assumption generates misleading results.

3.2 Testing the Euler equation

The Euler equation (1) can be rewritten as:

$$E_t \left[M_{t+1} R_{t+1}\right] = 1, \tag{9}$$

where $M_{t+1}=\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$ is the the stochastic discount factor. Up to a first order approximation, we have

$$E_t m_{t+1} = -E_t r_{t+1}, \tag{10}$$

where $m_{t+1} = log(\mathrm{M}_{t+1})$. Equation (10) intuitively suggests that if the expected return from a one-dollar investment increases by one percent, the corresponding expected discount factor has to decrease by one percent to keep the marginal benefit of the investment equal to the marginal cost to satisfy the households' utility maximization condition. Imposing the log-linear Euler equation, we can translate the expected return news in equation (6) to discount rate news. Now the return decomposition has an intuitive economic meaning: unexpected changes in stock returns of firm i are caused by either investors' revisions on the firm's current and future cash flow forecasts or their revisions on the firm's future discount rate forecasts.

In the representative household framework widely used in macroeconomic models, the discount factor is determined by the same household's preference and consumption plan. Therefore, it is

the same across all stocks. As a result, equation (10) implies that expected return changes are the same across all stocks.

In a heterogeneous households framework with an incomplete financial market, the discount factor can be constructed as a weighted sum of all households' inter-temporal marginal rates of substitution (Brav *et al.*, 2002; Cogley, 2002), which is again the same for all stocks. Balduzzi and Yao (2007) and Kocherlakota and Pistaferri (2009) use the ratio of cross-sectional average of consumption moments to construct the discount factor for heterogeneous households. The discount factor is also the same for all stocks. Therefore, equation (10) holds not only in the representative household model with complete financial markets but also in a set of heterogeneous households models with incomplete financial markets.

However, in the presence of liquidity constraint, equation (10) fails. Iacoviello (2004, 2005) shows that in a model with households which are heterogeneous in patience and borrowing constraint faced by impatient borrowers, the aggregate Euler equation takes the following form:

$$c_t = -\gamma (1-\lambda)(r_t + l_t) + \omega\lambda (q_t + r_t - E_t(q_{t+1}) + \lambda q_t + \lambda\theta h_t), \tag{11}$$

where q_t, h_t are respectively housing price and stock, c_t and l_t are respectively the log total consumption and the long term real interest rate, r_t is the log real interest rate, and λ, θ, and ω are structural parameters. Housing price and stock enter the equation because houses are used as collateral for borrowers. Despite this modification, the modified Euler equation still holds for all stocks.

Since equation (4) holds for both an individual firm i and the market portfolio (denoted by the subscript mkt), Cohen *et al.* (2002) derive a decomposition for market-adjusted return $\widetilde{r}_{i,t}$ (see equation 12),

$$\widetilde{r}_{i,t} - E_{t-1}(\widetilde{r}_{i,t}) \approx \widetilde{N}^{\text{cf}}_{i,t} - \widetilde{N}^{r}_{i,t}, \tag{12}$$

where $\widetilde{r}_{i,t} \equiv r_{i,t} - r_{\text{mkt},t}$, $\widetilde{N}^{\text{cf}}_{i,t} \equiv N^{\text{cf}}_{i,t} - N^{\text{cf}}_{\text{mkt},t}$, and $\widetilde{N}^{r}_{i,t} \equiv N^{r}_{i,t} - N^{r}_{\text{mkt},t}$ are market-adjusted log return, market-adjusted cash flow news, and market adjusted discount rate news, respectively. The log-linear Euler equation (10) implies that the market-adjusted expected return news is zero. Note that the existence of heterogeneous liquidity constraints faced by households does not change this result, given that all stocks are bought by the same cohorts of households.

Therefore, we have

$$\widetilde{r}_{i,t} = a + b\widetilde{N}^{\text{cf}}_{i,t} + u_{\text{it}}, \tag{13}$$

where a=0, b=1, u_{it}=$-\widetilde{N}^{r}_{i,t}$ is the mean-zero discount rate news. Intuitively, since the discount rate is the same for all stocks, the market-adjusted discount rate news is zero. As a result, all unexpected changes in a firm's returns have to come from its cash flow news. Therefore, if the Euler equation (10) were to hold, the regression coefficient of equation (13) must be 1. Cohen *et al.* (2002) and Vuolteenaho (2002) call b the reaction coefficient. An estimated b<1 is termed under-reaction while b>1 is termed over-reaction. Cohen *et al.* (2002) and Vuolteenaho (2002) interpret $b \neq 1$ as an indicator of investor irrationality. As we shall see later, while irrationality is a plausible explanation for $b \neq 1$, there could other explanations. Therefore, we shall follow the tradition of Cohen *et al.* (2002) and Vuolteenaho (2002) to use the terms reaction coefficient, under-reaction, and over-reaction, but keep in mind that it is an indicator of the model failure rather than an indicator of individual rationality without further theoretical and empirical justifications.

3.3 The Forecasting Model and Implementation of the Euler Equation Test

To estimate equation (13), we need a forecasting model to calculate the cash flow news. From the definition of return on equity and log return, we have (see Equation A5 in appendix A), up to a first-order approximation,

$$r_{i,t} = \text{roe}_{i,t} - \rho \text{bm}_{i,t} + \text{bm}_{i,t-1}, \tag{14}$$

where $\text{bm}_{i,t}$ is the log book-to-value ratio. This result suggests that current return on equity, current and one-period lag of the book-to-market ratio contain all information on current stock return. Based on this observation, Vuolteenaho (2002) proposed to use a vector autoregression (VAR) model of log return, ROE, and book-to-value ratio as the forecasting model for stock returns. Cohen *et al.* (2002) adds institutional ownership as an additional endogenous variable. We use the four-variate VAR model of Cohen *et al.* (2002) as the baseline forecasting model.

3.3.1 The Pooled VAR Model

Cohen *et al.* (2002) assume that the market-adjusted log returns of stocks can be forecasted using a VAR(1) model (see equation 15):

$$\begin{aligned} z_{i,t} &= \Gamma z_{i,t-1} + u_{i,t}, \\ z_{i,t} &= \begin{pmatrix} \widetilde{r}_{i,t} & \widetilde{\text{bm}}_{i,t} & \widetilde{\text{roe}}_{i,t} & \widetilde{\text{if}}_{i,t} \end{pmatrix}' \end{aligned} \tag{15}$$

The elements in the K*1 vector $z_{i,t}$ are market-adjusted firm-specific endogenous variables describing firm i at time t, where $i = 1, \ldots, N$, and N is the number of firms in the sample. Specifically, We set K=4 in the baseline model. The first element of $z_{i,t}$ is the market-adjusted log stock return $\widetilde{r}_{i,t}$. The other elements in $z_{i,t}$ are the market-adjusted log book-to-market ratio $\widetilde{\text{bm}}_{i,t}$, the market-adjusted log accounting return on equity $\widetilde{\text{roe}}_{i,t}$, and the market-adjusted fraction of institutional ownership $\widetilde{\text{if}}_{i,t}$. Γ is the matrix of parameters to be estimated. $u_{i,t}$ is the vector of error terms in the VAR process, which is independent of any information available at time *t-1*.

Given that the maximum of the absolute eigenvalues of $\rho\Gamma$ is less than 1, the market-adjusted discount rate news $\widetilde{N}^r_{i,t}$ can be calculated as:

$$\widetilde{N}^r_{i,t} = e' \rho\Gamma \left(I - \rho\Gamma\right)^{-1} u_{i,t}, \tag{16}$$

and the market-adjusted cash flow news can be calculated as:

$$\widetilde{N}^{\text{cf}}_{i,t} = e' \left[I + \rho\Gamma \left(I - \rho\Gamma\right)^{-1}\right] u_{i,t}. \tag{17}$$

In these equations, e is a K*1 vector whose first element is 1 while all other elements are zeros. I is a K*K identity matrix. Replacing Γ with the VAR estimates generates a panel data set of firm-specific cash flow news, which allows the implementation of the regression test (13) to be possible.

3.3.2 The Group VAR model

While intuitive, the pooled VAR model implicitly imposed a parameter homogeneity assumption on the VAR coefficient matrix. This assumption is not innocuous. It requires the dynamic interactions between the endogenous variables of the VAR to be homogeneous. As we shall show

in the empirical section, this assumption is far from the reality. Violation of this assumption leads to biased pooled VAR estimates of firm-specific cash flow news. At the other side of the extreme, one can assume the VAR coefficient matrix to be firm-specific. Estimation with this assumption brings consistency but may be inefficient when endogenous variables of different firms follow similar dynamic processes.

To achieve a better balance between consistency and efficiency, we utilize and extend the recently developed econometric model of Bonhomme and Manresa (2015) that introduces unobserved grouped patterns of heterogeneity into linear panel data models. More specifically, we incorporate unobserved grouped patterns of heterogeneity into the VAR(1) process as follows:

$$\begin{aligned} z_{i,t} &= \alpha_{g_i} + \Gamma_{g_i} z_{i,t-1} + u_{i,t}, \\ u_{i,t} &\sim i.i.d.N(0,\ \Omega_{g_i}). \end{aligned} \tag{18}$$

The model assumes that the pattern of heterogeneity is group-specific. The number of groups is G, and $g_i \in \{1,\ \ldots,\ G\}$ indicates the group membership of firm i. The group membership g_i is unobserved and can be estimated from the data. The vector of error terms $u_{i,t}$ is assumed to follow an i.i.d. normal distribution. The parameters α_{g_i}, Γ_{g_i}, and the covariance matrix Ω_{g_i} are group-specific parameters to be estimated.

Under the condition that the maximum absolute eigenvalues of $\rho\Gamma_g$ are less than 1 for all $g \in \{1,\ \ldots,\ G\}$, the market-adjusted discount rate news $\widetilde{N}^r_{i,t}$ can be calculated as equation (19), and the market adjusted cash flow news $\widetilde{N}^{\text{cf}}_{i,t}$ can be calculated as equation (20):

$$\widetilde{N}^r_{i,t} = e^{'} \rho\Gamma_{g_i} \left(I - \rho\Gamma_{g_i}\right)^{-1} u_{i,t}, \tag{19}$$

$$\widetilde{N}^{\text{cf}}_{i,t} = e^{'} \left[I + \rho\Gamma_{g_i} \left(I - \rho\Gamma_{g_i}\right)^{-1}\right] u_{i,t}. \tag{20}$$

Note that the model composed of equations (18) to (20) is a general setup that encompasses the pooling model of equations (15), (16), (17) as a special case for $G = 1$. Another special case is the firm-specific model where G=N.

We use the Iterative algorithm in Bonhomme and Manresa (2015) to estimate the model (18) and use BIC to determine the number of groups. The algorithm and the equation of BIC are represented in appendix B. The standard errors and confidence intervals are estimated based on bootstrapping with 5000 replications. For each replication, we sample firms with replacements.

3.4 Heterogeneity and Determinants of the failure of Euler function

Another advantage of accounting for heterogeneity is that the estimation of optimal group membership enables us to investigate the degree of Euler equation failure in different groups. Specifically, given the group membership estimated in the model (18), we calculate the group-specific degree of Euler equation failure by running regression (21) in each group.

$$\widetilde{r}_{i,t} = a_g + b_g \widetilde{N}^{\text{cf}}_{i,t} + \varepsilon_{i,t},\ i \in \{i | g_i = g\}\,. \tag{21}$$

We also run a regression of b_g on various firm characteristics as in equation (22), where x_g indicates group-mean firm characteristics, to investigate the relationship between b_g and firm characteristics.

$$b_g = \mu + \theta x_g + \epsilon_g. \tag{22}$$

4 Data and Variables

This paper performs the Euler function test using yearly data of publicly traded common stocks in the United States, spanning the sample period from 1980 to 2020. Stock returns and data used in calculating equity capital are sourced from the Center for Research in Security Prices (CRSP). Firms' book-to-market ratios (BM), return on equity (ROE), operating profitability, and investment are calculated using data from Compustat. We also use book equity data from Davis, Fama, and French (2000) to supplement missing data, following Lochstoer and Tetlock (2020). Institutional ownership data is obtained from Refinitiv, while all other firm characteristic data is sourced from the database constructed by Chen and Zimmermann (2021)[4].

We apply the same filters as Cohen *et al.* (2002). The filters from Cohen *et al.* (2002) that we follow are as follows: (1) the stocks are common stocks listed on NYSE, AMEX, or NASDAQ; (2) the firms' fiscal year-end at time *t-1* should be December; (3) firms must have book equity at fiscal year-ends for years *t-1*, *t-2*, and *t-3*; (4) market value must be available for time *t-1*, *t-2*, and *t-3*; (5) firms with a market value less than \$10 million, a book-to-market value greater than 100, or less than 0.01 at time *t-1* are excluded; (6) there must be a valid trade in the month preceding

[4]The definitions of those variables are listed in Appendix C.

the yearly returns period; (7) there must be at least one monthly return recorded in each yearly returns period, from year *t-5* to year *t-1*.

For variable construction, log returns at time t are defined as $r_{i,t} \equiv log(R_{i,t})$, where $R_{i,t}$ is one plus the yearly simple return for firm i from the beginning of July of year t to the end of June of year *t+1*. This timing ensures that investors have access to December accounting data prior to the return period. For firms with multiple classes of common stock, we calculate the value-weighted return. Returns are adjusted for delisting following Cohen *et al.* (2002): if delisting is performance-related and the delisting return is missing, we set the delisting return to -30%. When compounding monthly returns, missing monthly returns are assumed to be zero.

In calculating the book-to-market ratio ($\text{BM}_{i,t}$), the market value of firms at time t ($\text{ME}_{i,t}$) is the market value in June of year *t+1*, and the book value ($\text{BE}_{i,t}$) is stockholders' equity plus deferred taxes minus preferred equity at the fiscal year-end of year t. If $\text{ME}_{i,t}$ is missing, we assume no change in the stock price or the number of shares outstanding and use the lagged market value to impute $\text{ME}_{i,t}$. If $\text{BE}_{i,t}$ is missing or non-positive, we impute the book value by multiplying the lagged book-to-market ratio by the market value at time t. For firms with multiple share classes, we sum $\text{ME}_{i,t}$ and $\text{BE}_{i,t}$ across all classes before calculating $\text{BM}_{i,t}$. The log book-to-market ratio is defined as $\text{bm}_{i,t} \equiv log(\text{BM}_{i,t})$.

The log return on equity $\text{roe}_{i,t} \equiv log(1+\text{ROE}_{i,t})$, where $\text{ROE}_{i,t}$ is the return on equity of stock i.[5] $\text{ROE}_{i,t}$ is calculated as the firm's earnings at the fiscal year-end of December year t divided by lagged book equity. We use net income ("ni") in Compustat to proxy firms' earnings; if net income is missing, we use the change in book value plus dividends to calculate earnings. If dividends are missing, we assume they are zero. Again, for firms with multiple share classes, we sum earnings and $\text{BE}_{i,t}$ across all classes before calculating $\text{ROE}_{i,t}$.

Institutional ownership at time t ($\text{if}_{i,t}$) is the institutional ownership in June of the year t, as defined in Cohen *et al.* (2002).

In cases where firms go bankrupt within the sample period, 1+ROE and BM may approach zero, making log transformation ill-defined. For these firms, we follow Cohen *et al.* (2002) and

[5] We do not use log $\text{ROE}_{i,t}$ because net income could be negative.

substitute them with a portfolio comprising 90% common stocks of the firm and 10% Treasury bills based on market values. Operating profitability and investment are calculated following Fama and French (2015).

We proxy the characteristics of the market portfolio ($r_{mkt,t}$, $\text{bm}_{mkt,t}$, $\text{roe}_{mkt,t}$, and $\text{if}_{mkt,t}$) as the cross-sectional sample mean for each year. For a variable $x_{i,t}$, the market-adjustment is defined as $\widetilde{x}_{i,t} \equiv x_{i,t} - x_{mkt,t}$.

Finally, since stocks of which return and other firm characteristics follow non-stationary processes have very different stochastic property from stationary processes. Our subsequent empirical study will distinguish between stationary and non-stationary stocks. Table 2 shows the summary statistics of the variables for both the stationary sample and the full sample.

[Insert Table 2 here]

5 Testing the Euler equation

5.1 Pooled testing results

Table 3 summarizes the estimation results of equation (13), where the cash flow news of the firms is calculated using the pooled VAR model. Panel A uses the full sample for the pooled VAR and reaction coefficient estimation. The column labeled "b_news_cf" reports the estimated reaction coefficient (row "b") and bootstrapping standard error (row "se"). The estimated reaction coefficient of equation (13) is 0.7243, with a standard error of 0.0153. Similar to the finding of Cohen *et al.* (2002), investors appear to under-react to cash flow news. More specifically, on average, investors under-react to a 1-dollar cash flow news by 28 cents. The failure of the Euler equation appears to be economically important. The degree of under-reaction is also statistically significant. The estimated reaction coefficient is significantly smaller than 1 at the 1% level (bootstrapping 1 percent upper bound of the confidence interval is 0.7608). The column labeled "b_cons" reports the estimated intercept of equation (13) (row "b") and its bootstrapping standard error (row "se"). The intercept appears to be statistically indifferent from zero. Columns

labeled "var_cf", "var_r", and "cov" report the variance decomposition results of equation (12) using the pooled VAR coefficients. Row "b" reports the variance of cash flow news (column "var_cf"), the variance of expected return news (column "var_r"), and the covariance between cash flow news and expected return news (column "cov"). The corresponding row "se" reports the bootstrapping standard errors. The last column "corr" reports the correlation coefficient of cash flow news and expected return news (row "b") and its bootstrapping standard error (row "se"). Consistent with Cohen *et al.* (2002), panel A of Table 3 suggests that cash flow news is the main driver of firm-level stock returns. It also suggests that the cash flow news and expected return news are highly positively correlated (the estimated correlation coefficient is 0.7041 with a bootstrapping standard error of 0.0233). This is just another manifestation of the Euler equation failure. Technically, equation (12) suggests that

$$\text{Cov}\left(\widetilde{r}_{i,t}, \widetilde{N}^{\text{cf}}_{i,t}\right) = bVar\left(\widetilde{N}^{\text{cf}}_{i,t}\right) + Cov\left(\widetilde{N}^{\text{cf}}_{i,t}, u_{\text{it}}\right) \tag{23}$$

Note that u_{it}=$-\widetilde{N}^{r}_{i,t}$, so

$$\text{Cov}\left(\widetilde{r}_{i,t}, \widetilde{N}^{\text{cf}}_{i,t}\right) = bVar\left(\widetilde{N}^{\text{cf}}_{i,t}\right) - Cov\left(\widetilde{N}^{\text{cf}}_{i,t}, \widetilde{N}^{r}_{i,t}\right) \tag{24}$$

Diving both sides by $Var(\widetilde{N}^{\text{cf}}_{i,t})$, we obtain

$$\widehat{b} = b - \frac{cov(\widetilde{N}^{\text{cf}}_{i,t}, \widetilde{N}^{r}_{i,t})}{var(\widetilde{N}^{\text{cf}}_{i,t})} \tag{25}$$

where $\widehat{b}$ is the OLS estimate of b. Obviously, a higher correlation between market-adjusted cash flow news and expected return news leads to a lower value of the estimated reaction coefficient.

Suppose the log-linear Euler equation is a good approximation; the expected return news is the negative of the discount rate news. If the fair discount factor is the same across stocks, as predicted by the Euler equation, the market-adjusted discount rate news should be zero and uncorrelated with the market-adjusted cash flow news. The significant correlation between market-adjusted cash flow news and discount rate news rejects the log-linear Euler equation. As we discussed before, this also means it is better not to interpret the deviation of the estimated reaction coefficient from one as evidence of irrational mis-reaction too quickly. Before any theoretical discussions, it simply says that the log-linear Euler equation (10) fails. If the approximation error from the log-linear approximation is large, we should not assume that expected return news is the negative of discount

rate news. More importantly, the market adjustment can no longer render the expected return news to (almost) zero. In this case, the correlation between market-adjusted cash flow news and expected return news deviates from zero. As we discussed in the literature review section, such a non-zero correlation could be generated by various sources including not only irrationality but also time-varying risk premia or financial constraints. We shall discuss this in more detail later.

[Insert Table 3 here]

The pooled VAR estimation, however, imposes a strong assumption on the data-generating process (DGP). More specifically, it assumes that the dynamic interactions between stock return, return on equity, book-to-market ratio, and institutional investor ownership are the same across all firms. We call this assumption the "homogeneous interaction" assumption. On the other side of the extreme, one could assume that the dynamic interactions between stock return, return on equity, book-to-market ratio, and institutional investor ownership are firm-specific. We call this assumption the "firm-specific interaction" assumption. Pooled VAR estimation generates more efficient results if the homogeneous interaction assumption holds. However, if the "firm-specific interaction" assumption holds, each firm's dynamic interaction effects could be very different from the average effects, even though we could obtain consistent estimates of the average effects using methods such as the random coefficient model (Swamy, 1970) and mean group estimator (Pesaran and Smith, 1995). In this case, cash flow news generated with the firm-specific VAR coefficients in the true DGP will be very different from the cash flow news generated with the pooled VAR coefficients.

To see this, we tabulate the distribution of firm-level cash flow news generated using pooled VAR coefficients and firm-specific VAR coefficients (see equation 26) in Table 5. The percentiles of the VAR coefficient matrix are reported in Table 4.

$$z_{i,t} = \alpha_i + \Gamma_i z_{i,t-1} + u_{i,t}. \tag{26}$$

[Insert Table 4 here]

[Insert Table 5 here]

Notably, the tails of the cash flow news generated using firm-specific VAR include very large absolute numbers. The largest five numbers in the right tail are 280.92123, 285.64767, 300.17044, 302.04498, and 398.66345. The smallest five numbers in the left tail are -451.42145, -314.50665, -285.81119, -247.53609, and -219.40817. This actually indicates the first important violation of the "homogeneous interaction" assumption. More specifically, about 24.23% firm-specific VAR models are not stationary (it accounts for 22.62% firm-year observations). Non-stationary VARs are obviously different DGPs from stationary VARs. Moreover, the infinite sums in the definition of cash flow news and expected return news in equation (16) and (17) are not necessarily well defined when the VAR model is non-stationary[6], according to condition (8). Therefore, we re-estimate the pooled VAR model with only stationary data and test the Euler equation with cash flow news generated by the new pooled VAR model. About 38.20% firms do not have enough data to estimate firm-specific VAR models (it accounts for 8.74% firm-year observations). Data of these firms are also quite noisy and it is difficult to judge whether these firms have the same VAR DGP as other firms. We also drop those firms in the new estimation. The results are summarized in panel B of Table 3.

Surprisingly, just accounting for the obvious heterogeneity caused by non-stationarity leads to quite different qualitative results. Now the failure of the Euler equation is economically and statistically less serious. The estimated average reaction coefficient is 0.9643, with a bootstrapping standard error of 0.0175. On average, investors under-react to a 1-dollar cash flow news by less than 4 cents. The reaction coefficient is statistically smaller than 1 at the 5 percent level but not at the 1 percent level. In other words, the Euler equation nearly holds. Not surprisingly, by excluding firms with non-stationary data, we obtain a much smaller estimated variance of both cash flow and expected return news. The finding that the main driver of firm-level stock return is cash flow news remains. However, the estimated correlation between cash flow and expected return news is much lower (the correlation coefficient is 0.1087 with a standard error of 0.0529).

[6]In our empirical analysis, we call a stock with "non-stationary" firm-specific VAR, if the maximum absolute eigenvalues of estimated Γ_i in equation (26) is no less than 1.

5.2 Group testing results

Section 5.1 shows that considering the heterogeneity in the data-generating process might lead to very different testing results. However, the heterogeneity considered is confined to the stationarity of the DGP. In this section, we follow Bonhomme and Manresa (2015) to classify firms into different groups according to the Bayesian information criterion. The dynamic interaction effects between stock return, return on equity, book-to-market ratio, and institutional investor ownership are the same across firms in the same group but different among different groups. The "The homogeneous interaction" assumption and "firm-specific interaction" assumption are just two special cases of the grouping method. The former is the case in which the number of groups is 1. The latter is the case in which the number of groups equals the number of firms in the sample (N). However, *ar priori*, there is no guarantee that 1 or N is the optimal number of groups. Intuitively, the optimal choice of number of groups searches for the best balance between consistency and efficiency. The homogeneous interaction assumption leads to inconsistent estimates for firm-specific cash flow news if the interaction effects are heterogeneous among firms. The firm-specific interaction assumption generates consistent estimates but may be inefficient when there are commonalities in the dynamic interactions among firms. Table 6 reports the BIC values of different number of groups of our baseline VAR model. The BIC is minimized when the number of groups is set to 40. According to this optimal choice of grouping, we estimate pooled VAR for each group. Table 7 reports the percentiles of the group VAR coefficient matrices. According to the table, the percentiles are very different from each other. For example, the 1st percentile of the coefficient of $\widetilde{r}_{i,t-1}$ in the equation whose dependent variable is $\widetilde{r}_{i,t}$ is -0.2534, while the 99th percentile is 0.1336. Notice that the coefficient in the pooled VAR model is 0.1292, very different from the group VAR estimation for many firms.

[Insert Table 6 here]

[Insert Table 7 here]

[Insert Table 8 here]

Table 8 reports the pooled Euler equation testing results using cash flow news generated from the group VARs. The estimated reaction coefficient is 0.9875, with a standard error of 0.0372. The coefficient is statistically indifferent from 1, suggesting that on average the Euler equation does

not fail. The variance decomposition results still suggest that cash flow news is the main driver of firm-level stock returns while the correlation between cash flow and expected return news is small and statistically insignificant (the correlation coefficient is 0.0336 with a standard error of 0.0831).

This is the end of the story if the estimated average reaction coefficient is representative for all firms. However, the group heterogeneity in the dynamic interaction effects suggests that there might also be heterogeneity in the reaction coefficient. To raise an extreme example: suppose half of the sample firms have reaction coefficients significantly larger than 1 while the reaction coefficients are significantly smaller than 1 for the other half. The average reaction coefficient may still be 1 if the deviations from 1 in both halves cancel out. In this case, the Euler equation fails for all firms but the average reaction coefficient leads us to a misleading conclusion that the Euler equation holds. Table 9 suggests that the pooling estimation of the reaction coefficient does hide important heterogeneity. Table 9 classifies the group estimates of reaction coefficients into four quartiles. Rows with label "b" report the average reaction coefficient, intercept in equation (13), variance of cash flow news, variance of expected return news, and the covariance of cash flow and expected return news in each quartile. Rows with the label "se" report the standard errors. Firms in the first quartile have the most volatile cash flow and expected return news and the highest covariance (0.0738 with a standard error of 0.0344) between cash flow and expected return news. Investors, on average, under-react to cash flow news of firms in this quartile (the average reaction coefficient is 0.7997 with a standard error of 0.0356). On average, the Euler equation is not rejected in the second quartile and the correlation between cash flow and expected return news is insignificant. Investors appear to, on average, over-react to cash flow news in the third and fourth quartile (the average reaction coefficient is respectively 1.1956 and 1.4139). Firms in these two quartiles have the least volatile cash flow and expected return news and negative covariance (-0.0102 and -0.0226) between cash flow and expected return news. In contrast to the pooling test, our grouped test suggests that on average, three quarters of the sample firms fail the Euler equation test.

[Insert Table 9 here]

5.3 Robustness to alternative VAR specifications

As indicated in the methodology section, the return decomposition and hence the calculation of cash flow news depend on the specification of the forecasting model of stock returns. To check the robustness of our Euler equation testing results we re-do our tests with three different VAR specifications in this section.

(1) The first alternative VAR is used by Vuolteenaho (2002). Compared to our baseline model, the VAR model of Vuolteenaho (2002) omits institutional ownership as an endogenous variable and further drops the observations if *t-1* or *t-2* net income or long-term debt is missing. At the cost of losing the information about other endogenous variables contained in the variation of institutional ownership, the model extends our sample length by starting the sample from 1952.

(2) The second alternative augments the VAR model of Cohen *et al.* (2002) by equity market factors extracted using the variable set from Chen and Zhao (2009).[7] Chen and Zhao (2009) argue that the decomposition of the market return might be distorted by model uncertainty in the predictive VAR and suggest augmenting the predictive VAR model by factors extracted from a set of additional potential predictors of market return. We use these factors to supplement the baseline VAR model with information on changes in the discount factor and common changes in firm cash flows.

(3) The third alternative augments the VAR model of Cohen *et al.* (2002) by factors extracted using a large set of macroeconomic variables used in the monetary policy analysis of McCracken and Ng (2016).[8] These factors might affect the risk-free rate and aggregate risk premium and consequently indirectly affect the discount factor. They might also affect firm cash flows through their impacts on business cycles.

Table 10 summarizes the BIC values of various group VAR specifications. The panels "V02", "FAVAR_CZ", "FAVAR_Macro" respectively report BIC values of the VAR model of Vuolteenaho

[7]See appendix D for the list of variables used to extract the equity market factors.
[8]See appendix E for the list of variables used to extract the macro factors.

(2002), the VAR model augmented by equity market factors and the VAR model augmented by macroeconomic factors, corresponding to the number of groups indicated in the column labeled "number of groups". In all specifications, the optimal number of groups is not one, suggesting some heterogeneity in the dynamic interaction effects among stock return, return on equity, and book-to-market ratio.

Tables 11 to 13 classify the group estimates of reaction coefficients based on cash flow news generated from different VAR models into four quartiles. Rows with label "b" report the average reaction coefficient, intercept in equation (13), variance of cash flow news, variance of expected return news, and the covariance of cash flow and expected return news in each quartile. Rows with the label "se" report the standard errors. The following results are robust across all VAR specifications. Firms in the first quartile have a positive correlation between cash flow and expected return news. Investors, on average, under-react to cash flow news of firms in this quartile. On average, the Euler equation is not rejected in the second quartile. Investors, on average, over-react to cash flow news in the third and fourth quartile. Firms in these two quartiles have less volatile cash flow and expected return news and negative correlation coefficients between cash flow and expected return news. Therefore, the Euler equation fails in general and the degree of failure appears to be heterogeneous.

[Insert Table 10 here]

[Insert Table 11 here]

[Insert Table 12 here]

[Insert Table 13 here]

5.4 Robustness to alternative construction method of the cash flow news

So far, we follow the literature to calculate the cash flow news as a residual. More specifically, we use a VAR model to forecast the returns and construct the expected return news as forecast revisions on returns. The cash flow news is then backed out using equation (4). In this subsection, we follow Vuolteenaho (2002) and Chen and Zhao (2009) to conduct a robustness test by directly

constructing the cash flow news with VAR forecasts on the return on equity. Table 14 presents the results. The key qualitative results remain.

[Insert Table 14 here]

6 Potential sources of the heterogeneous model failure

This section explores the potential sources of the heterogeneous model failure. To facilitate discussions, we shall continue to call the estimated coefficient of cash flow news in the Euler equation test the "reaction coefficient." Note that, *ar priori*, a reaction coefficient which is different from 1 does not necessarily indicate that investors are irrational. Rather, it might suggest other sources of Euler equation failure. That being said, we also do not exclude irrational behaviors as sources of model failure. Also recall from equation (25), a low reaction coefficient suggests a high correlation between cash flow news and expected return news. Therefore, the relationship between the reaction coefficient and the firm characteristics found in this section indicates a reverse relationship between the correlation coefficient and the firm characteristics.

6.1 Earnings volatility

Recall that the last section finds that firms with high volatility in cash flow news appear to have a smaller than 1 reaction coefficient while firms with low cash flow news volatility have a larger than 1 reaction coefficient. Is this pattern related to some well-known anomalies in the stock market which are associated with earnings volatility?

Panel A of Table 15 reports regression results of our reaction coefficient on earnings volatility. Increasing earnings stability (decreasing earnings volatility) leads to a higher reaction coefficient.

There are two competing behavioral hypotheses that link firms' earnings volatility to investor reaction to their cash flow news. In both cases, earnings volatility could be taken as a reverse indicator of earnings consistency.

The gambler's fallacy (Rabin, 2002) predicts that investors tend to believe that trends in

earnings require immediate balancing by the opposite outcome. Therefore, investors irrationally under-react to trends. Loh and Warachka (2012) finds that buying stocks with positive earnings streaks and selling with negative streaks generates a positive abnormal portfolio return, supporting the gambler's fallacy.

On the contrary, the representative bias (Tversky and Kahneman, 1974) predicts that investors over-react to trends in earnings. Based on this behavioral assumption, the model of Barberis *et al.* (1998) suggests that a string of consecutive earnings movements in the same direction causes over-reaction. Daniel *et al.* (1998) built a behavioral model of investors' over-confidence caused by biased self-attrition. Investors over-estimate their ability to generate private information. They might act according to their private information. Attribution bias leads them to attribute results confirming their initial actions to high ability while attributing results disconfirming their initial actions to external noises. Therefore, consistency of firm financial performance could deepen market over-reaction. Alwathainani (2009) finds that investors over-react to the consistency of earnings growth.

The positive (negative) relationship between the "reaction coefficient" and earnings stability (volatility) in Panel A of Table 15 seems to support the representative bias hypothesis. Loh and Warachka (2012) measure earnings consistency with the length of earnings streak. The second row in Panel A of Table 15 reports the estimation results of the reaction coefficient on the average length of earnings streak in the group of stocks. The results continue to suggest a positive relationship between earnings consistency and the reaction coefficient. As we have discussed before, the mis-reaction from our OLS estimation is just an indicator of bias caused by the correlation between cash flow news and expected return news. Therefore, the positive relationship between earnings consistency and the reaction coefficient are better interpreted as a negative relationship between earnings consistency and the correlation coefficient. More specifically, Panel A of Table 15 suggests that stronger earnings consistency leads to a lower correlation between cash flow news and expected return news. Representative bias can indeed serve as a potential explanation for this pattern in the data. Upon observing positive cash flow news, investors with representative bias might extrapolate the good news, which leads to low future returns since the investors will eventually realize that they were overly optimistic. Therefore, our expected return news calculated under the rationality assumption is negative. This generates a negative correlation between our cash flow news and

expected return news. Our finding suggests that firms with more consistent earnings tend to have a stronger negative correlation between cash flow news and expected return news, supporting the idea that earnings consistency leads to investor over-reaction.

[Insert Table 15 here]

6.2 The Fama-French anomalies

Fama and French (2015) summarize famous failures of the capital asset pricing model (CAPM) in their five-factor model. Four additional factors, the size, value, profit, and investment factors, are used to augment the market factor suggested by the CAPM. These factors are motivated by famous anomalies in the asset pricing literature. The first is the size effect: conditional on the differences in beta, stocks of smaller firms have higher returns than larger firms (Banz,1981). The second is the value effect: value stocks that have a higher book-to-market ratio generate higher returns than growth stocks that have a lower book-to-market ratio (Stattman,1980). The third is the profitability anomaly: stocks of firms with higher profitability have higher returns (Novy-Marx, 2013). The fourth is the investment anomaly: stocks of firms with higher asset growth rates have lower returns (Cooper *et al.*, 2008). Given the explanatory power of the Fama-French five-factor model for cross-sectional stock return heterogeneity, it is interesting to see if they are associated with the heterogeneous Euler equation failure. Panel B of Table 15 reports regression results of our reaction coefficient on average log market equity (a measure of firm size), book-to-market ratio, operating profitability and asset growth rate.

Firm size does not appear to explain the heterogeneous Euler equation failure. Except when we construct the group-specific cash flow news using the VAR model of Vuolteenaho (2002), the group heterogeneity in estimated reaction coefficients is not significantly explained by firm size differences. When we construct the group-specific cash flow news using the model of Vuolteenaho (2002), the reaction coefficient decreases in firm size. The effect of firm size is only weakly significant at the 10 percent level. This is in sharp contrast with the finding of Vuolteenaho (2002) which suggests that the reaction coefficient is increasing in firm size. The vital difference between Vuolteenaho (2002) and this paper is that the grouping of firms in this paper is data-driven to

achieve an optimal balance between estimation consistency and efficiency. In Vuolteenaho (2002), the reaction coefficients are separately estimated in difference size groups which means that size is designated as a grouping criterion without any statistical support.

The estimated reaction coefficient is increasing in the group average book-to-market ratio and operating profitability while decreasing in the group average asset growth rate, suggesting that the correlation between cash flow news and expected return news is decreasing in book-to-market ratio and operating profitability while increasing in asset growth rate. In other words, value firms and high-profit firms tend to have a negative correlation between cash flow news and expected return news while firms that are aggressive in investment tend to have a positive correlation between cash flow news and expected return news.

Previous literature finds systematic under-prediction of earnings of value firms and high-profit firms. La Porta *et al.* (1997) finds that earnings surprises are systematically more positive for value firms, suggesting that value firms are underpriced before earnings announcements. Bouchaud *et al.* (2019) finds that analysts systematically underestimate future profits when current profits are high. La Porta *et al.* (1997) explain the value effect with behavioral biases while Bouchaud *et al.* (2019) explains the profitability effect with investors' infrequent updating of information. Based on their theoretical and empirical studies, we could provide a uniform explanation for the value effect, profitability effect, and the negative correlation between cash flow news and expected return news in value and high-profit firms:

Behavioral biases or informational frictions lead some investors to under-predict the earnings of value firms and high-profit firms. In other words, earnings forecasts of those firms are too low compared to rational full-information forecasts. The under-prediction leads to stronger forecast revision compared to rational full-information forecasts upon the arrival of good news on earnings. Therefore, those investors appear to over-react to cash flow news of rational full-information forecasters. Their buying actions following the forecast revision push up the stock price, which explains the value effect and profitability effect. Rational full-information investors should understand the under-prediction of other investors and predict the return increase following forecast revision and buying actions of those investors. Therefore, the return forecast for the stocks of rational full-information investors was high before the release of new earnings information. The

price increase after the information release at least partially realizes the forecasted return, leading to a downward revision in future return forecasts. This generates a negative correlation between cash flow news and expected return news. When the firm is hit by negative cash flow news, the forecast revision of under-predicting investors could be negative or positive, depending on the degree of under-prediction compared to the size of the shock, therefore, the sign of the correlation between cash flow news and expected return news is undetermined. However, the negative correlation generated by positive cash flow news always dominates because investors on average under-predict earnings. We formalize the intuition in a three-period model in appendix F.

There is also evidence on the over-prediction of high asset growth firms' future earnings. Cooper *et al.* (2008) find that the asset growth effect on stock returns is weaker in times of increased corporate oversight, suggesting that the asset growth effect arises in part from managerial over-investment. However, investors under-appreciate the negative impact of managerial empire building on future earnings, leading to over-prediction of firm earnings. Lyandres *et al.* (2008) suggest that asset expansion is usually accompanied by external financing, and there is a negative correlation between net external financing and future firm profitability. However, investors under-appreciate this phenomenon when forecasting firm earnings, which leads to over-prediction and negative future returns. Based on the prior literature, a potential explanation for the investment effect is as follows:

Irrational investors or investors facing information frictions over-predict the earnings of firms that are aggressive in investment. When negative cash flow news hits, the over-prediction leads to downward forecast revision and selling of stocks. The resulting price decrease explains the investment effect. However, the resulting correlation between cash flow news and expected return news is negative, contradicting our finding that aggressively investing firms tend to have a positive correlation between cash flow news and expected return news. Intuitively, the price decrease after the negative cash flow news at least partially realizes the forecasted price depreciation by rational forecasters, leading to an upward revision in future return forecasts. This creates a negative correlation between cash flow news and expected return news. When the firm is hit by positive cash flow news, the forecast revision of over-predicting investors could be negative or positive, depending on the degree of over-prediction compared to the size of the shock. Therefore, the sign of the correlation between cash flow news and expected return news is undetermined. However,

the negative correlation generated by negative cash flow news always dominates because investors on average over-predict earnings. We formalize the intuition in a three-period model in appendix F.

Table 16 presents the regression coefficients of earnings forecast errors[9] on the average BM ratio, operating profit, and asset growth rate of groups of firms identified with our group heterogeneous VAR models. Consistent with the behavioral finance models and the sticky information model, the forecasting error decreases with the BM ratio and operating profit, while increases with the asset growth rate[10]. Therefore, biased earnings forecasts serve as important explanations for the Fama-French anomalies. As discussed above, the prices of the firms whose earnings are more under-predicted (less over-predicted) tend to increase more (decrease less) than those firms whose earnings are less under-predicted (more over-predicted). Therefore, buying the former firms while selling the latter ones generates positive returns. Bordalo *et al.* (2024a) assume that investors over-react to earnings shocks of all firms.[11] However, they over-react less to earnings shocks of value, high-profit, and aggressive-investing firms than to growth, low-profit, and conservatively-investing firms. The cross-sectional difference in belief over-reaction explains the anomalies associated with value, profitability, and investment.

[Insert Table 16 here]

As introduced above, the literature uses both behavioral biases and informational frictions to explain the biased prediction of firm earnings. La Porta *et al.* (1997) suggests that investors have

[9]The earnings forecast errors with a prediction interval of h years in these regressions are the group mean of $\text{Deps}_{i,t,t+h}$. $\text{Deps}_{i,t,t+h}$ represents the EPS forecast error for stock i with a prediction interval of h years, divided by the stock's price. It is defined as $\text{Deps}_{i,t,t+h} \equiv (\text{Feps}_{i,t,t+h} - \text{eps}_{i,t+h})/P_{i,t}$, where $\text{Feps}_{i,t,t+h}$ is the median of analysts' forecasts for $\text{eps}_{i,t+h}$ at year t, $\text{eps}_{i,t+h}$ is the EPS in year $t+h$, and $P_{i,t}$ is the stock price at time t. We follow Da *et al.* (2014) to calculate $\text{Feps}_{i,t,t+h}$.

[10]Our results show that the coefficient corresponding to value is negative but not significant. This could be because the value effect is driven by profitability (Fama and French, 2015; Novy-Marx, 2013), or it could be due to the influence of outliers.

[11]Bordalo *et al.* (2019) show that the belief overreaction might arise from the representative bias. More specifically, earnings development is driven by the firm's unobservable fundamentals and transitory shocks. Analysts infer the fundamental using a Kalman filter from recent earnings news. The representative bias exaggerates the signal-to-noise ratio relative to the standard Kalman filter, leading to overreaction.

extrapolative expectations. Investors tend to extrapolate the poor earnings growth of value firms in the past to the future, leading to under-prediction of value firm earnings. Barberis and Shleifer (2003) suggest that the extrapolative expectations might come from representative bias and the existence of rational investors does not correct the mis-pricing (see also Barberis *et al.*, 2015). Cooper *et al.* (2008) find that earnings announcements for low asset growth firms are associated with positive abnormal returns and earnings announcements for high growth firms are associated with negative abnormal returns, supporting the extrapolative expectations theory of La Porta *et al.* (1997).

Piotroski (2000) suggests that the success of the value strategy relies on the strong performance of a few firms. He finds that less than 44% of high BM firms earn positive market-adjusted returns in the two years following portfolio formation. Therefore, investors could benefit by discriminating between value firms with strong and weak fundamentals. Piotroski (2000) constructs a summary indicator of firm profitability, financing risk, and operating efficiency, the F-score. A higher F-score suggests better firm fundamentals. Selecting firms with high F-scores from the value portfolio generates better returns. Piotroski (2000) argues that this is because value firms (high BM firms) attract less investors' attention and the superior return comes from the slow dissemination of information. Applying the sticky information model of Mankiw and Reis (2002) to the asset pricing context, Bouchaud *et al.* (2019) explain the profitability anomaly by analysts' sluggish updating of earnings forecasts. Given the positive autocorrelation of profits, investors' delay in updating expectations leads to underestimation of future profits. Recent empirical evidence seems to support the sticky information story of Bouchaud *et al.* (2019). Using media coverage as a proxy for investor attention, Chen *et al.* (2023) find that the profitability effect is more pronounced in firms with less investor attention. Using high-frequency data on stock returns and news, Jiang *et al.* (2021) also find supporting evidence for investor inattention. Famer *et al.* (2023) find time-varying stock return predictability in the U.S. stock market. More specifically, stock returns appear to be predictable within short periods, but unpredictable outside of those short periods. They find that models with time-varying price of risk cannot explain their findings, while the sticky information model fits the data well. Guo *et al.* (2020) finds that analysts do not fully use the information in anomaly signals when making recommendations, impeding the efficient correction of mispricing.

Despite the supporting evidence from Table 16 and all the external evidence discussed above, the biased earnings forecast story alone is not adequate to explain our heterogeneous pattern of Euler equation failure. As mentioned above, the main difficulty lies in the over-predicting part of the story. The predictions of behavioral theory alone indicate that stocks with low BM ratio, low profit, and high asset growth rate that tend to be over-priced, will have a negative correlation between cash flow news and expected return news. On the contrary, our empirical results show that firms with low BM ratio, low profit, and high asset growth rate tend to have over-optimistic analyst forecasts , but at the same time, there is a positive correlation between cash flow news and expected return news (i.e., a smaller than 1 estimated reaction coefficient). While the over-optimism in analyst forecasts can help explain the value, profitability, and investment effect, the positive correlations appear to contradict the theory.

Combining biased earnings forecast with short sale constraints might be a solution to the puzzle. Based on the evidence from D'Avolio (2002), Nutz and Scheinkman (2020) argue that selling stocks is asymmetrically more costly than buying stocks. Moreover, the cost of selling stocks is increasing with the short position. In appendix F, we discuss in more detail on how a short sale constraint that is increasing with the short position could generate a positive correlation between cash flow news and expected return news when the earnings of firms are initially over-predicted. Intuitively, as we discussed above, a negative cash flow shock leads to downward earnings forecast revisions by initially over-predicting investors. These investors would like to sell the stock, but the existence of a selling cost limits the selling orders and the resulting price decrease. In another word, the correction of the over-valuation of the stock takes more time; hence, the expected return news will be less positive than if there were no selling costs. This reduces the absolute value of the negative correlation between cash flow news and expected return news generated by the negative cash flow shock. A positive cash flow shock could still lead to downward revisions of earnings forecasts by irrational investors if the earnings forecast was very optimistic before the release of cash flow news. Absent any selling cost, those investors will sell some of the stocks, which leads to a price decrease of the stock. Since the price decrease is anticipated by rational investors, their expected return forecast is partially realized, and the return forecast will be revised upward. Such a scenario generates a positive correlation between cash flow news and expected return news. The existence of selling costs will prevent the price decrease and expected

return forecast revision, which weakens the positive correlation. While both positive and negative correlations between cash flow news and expected return news could be weakened by short sale constraints, the limit on the negative correlation is stronger since the selling cost is increasing in the short position, and the downward revision of stock valuation is stronger when the cash flow shock is negative.

There are also risk-based explanations for the stock anomalies. Berk *et al.* (1999) explain the value premium by the relatively high average systematic risk of projects held by the value firms. Zhang (2005) endogenizes higher systematic risk of value firms relative to growth firms proposed by Berk *et al.* (1999) using asymmetric capital adjustment costs. Gu *et al.* (2018) extend the costly capital adjustment model of Zhang (2005) to study the interaction effect between operating leverage and costly asset scale adjustment on stock returns. Kogan and Papanikolaou (2014) use investment-specific technology (IST) shock to explain the value premium. A common feature of those models is that they assume or predict a negative correlation between cash flow news and discount rate news for all firms. Whether those models could generate the coexistence of positive and negative correlations between cash flow news and expected return news is unclear when the log-linear Euler equation fails. In other words, expected return news is no longer equal to the negative of discount rate news, and market adjustment could not turn expected return news to zero. Quantitative studies of those risk-based models to see whether they could generate the coexistence of positive and negative correlations between cash flow news and expected return news of firms with different characteristics could be an interesting future research direction.

6.3 Financial constraint

Beta measures firms' systematic risk exposure. Investors can increase their risk-taking by either leveraging a low-beta investment or buying a high-beta stock. Frazzini and Pedersen (2014) suggest that investors are constrained in the leverage that they can take. As a result, investors overweigh risky securities instead of using leverage. This behavior of tilting towards high-beta assets suggests that high-beta assets require lower risk-adjusted returns than low-beta assets that require leverage. Therefore, the demand for high-beta stocks is not only affected by their expected future cash flows and price changes, but also by variations in leverage constraints faced by investors. The

standard Euler equation (1) obviously fails to account for the heterogeneous effects of changing variations in leverage constraints on stock pricing for different betas. Panel C of Table 15 reports regression results of our reaction coefficient on the beta coefficients calculated following Frazzini and Pedersen (2014). There is a robust significantly negative correlation between the beta value of firms and the reaction coefficient, supporting the theory of Frazzini and Pedersen (2014).

Note that heterogeneous households' leverage constraints alone do not necessarily lead to the heterogeneous Euler equation failures found in this paper. Iacoviello (2004, 2005) introduces heterogeneous liquidity constraints of different households into a general equilibrium model. The introduction of heterogeneous liquidity constraints adds aggregate housing price and stock into the aggregate Euler equation. However, in our market-adjusted Euler equation testing framework, anything that is common across all stocks is eliminated during the market adjustment and cannot generate the heterogeneous Euler equation failure pattern identified in the last section. It is the combination of heterogeneous households' leverage constraints and heterogeneous leveraging effects of different stocks that provides a potential explanation for the heterogeneous Euler equation failure.

There are short sale constraints in the stock market. Drake *et al.* (2011) find that short sellers are underrepresented in price formation whenever they disagree with analysts. Johnson and So (2012) suggest that short sale costs result in a negative relation between relative option volume and future firm value. More specifically, short sale costs drive investors with negative news to option markets. Given this theory, we expect the reaction coefficient to be negatively related to the relative option volume. Panel C of Table 15 suggests this is indeed the case.

7 Conclusion

We test the households Euler equation using stock market data. Based on the Campbell and Shiller (1988) stock return decomposition, we decompose American firm-level stock returns into a cash flow news component and an expected return news component. The standard households Euler equation with rational expectations implies that the market-adjusted expected return news should be zero. Therefore, all surprise market-adjusted firm returns should react to market-

adjusted cash flow news with a coefficient one. Using a heterogeneous panel data approach, we show that this implication of the Euler equation is clearly rejected. This result is consistent with the recent Euler equation testing results using consumption data (Crump *et al.*, 2022; Baugh *et al.* 2021).

Our novel data-driven approach of Euler equation testing creates a heterogeneous cross-sectional pattern in the Euler equation failure. We show that the model failure indicates the correlations between cash flow news and expected returns news of different firms. More specifically, overreaction indicates a negative correlation while underreaction indicates a positive correlation. Interestingly, the signs of the correlations and their relationship with firm characteristics provide empirical facts which help us distinguish between various asset pricing theories. Key empirical facts are as follows:

(1) The majority of the firms fail the Euler equation test and exhibit significant positive or negative correlations between cash flow news and expected return news.

(2) Those correlations are associated with firm characteristics which are linked to famous asset pricing anomalies in the cross-section of listed firms. Firms with more stable earnings, value firms, high-profit firms, and conservatively investing firms tend to over-react to cash flow news, suggesting that their cash flow news are negatively correlated with expected return news. Firms with opposite characteristics tend to under-react to cash flow news, suggesting positive correlations between cash flow news and expected return news.

(3) Firm characteristics which are linked to leveraging and short sale constraints could help explain the cross-sectional heterogeneity of Euler equation failure.

Those empirical facts help us distinguish between various asset pricing theories which could potentially explain the Euler equation failures. Our finding is consistent with the conclusion of Crump *et al.* (2022) and Baugh *et al.* (2021) that utility function alone cannot explain the Euler equation failure. Also consistent with Crump *et al.* (2022) and Baugh *et al.* (2021), we find that a few behavioral models in which households produce biased forecasts of earnings could help explain the negative correlation between cash flow news and expected return news. These models feature either extrapolative expectations or sluggish updating of information. However, behavioral models alone could not explain the positive correlation between cash flow news and expected return news.

Combining biased earnings forecasts with short sale constraints could help explain the positive correlation. While Crump *et al.* (2022) and Baugh *et al.* (2021) suggest that liquidity constraint cannot explain the Euler equation failure, we find that when combined with firm heterogeneity, financial constraint still has the potential to be a powerful explanation for Euler equation failures.

Our framework provides a unified data-driven framework to test households Euler equation and various asset pricing theories. It complements the literature on the consumption-based tests of households Euler equation by combining the tests to cross-sectional firm heterogeneity and utilizing the relatively longer span and better data quality of stocks. It contributes to the asset pricing literature in two aspects. First, compared with the traditional approach of grouping stocks into various characteristic groups with theoretical priors, our data-driven approach of grouping is based on estimation consistency and efficiency concerns, which is not only econometrically more sensible but also less subject to the risk of data mining. Therefore, our grouping method is a natural robustness test of various group criteria used in anomaly studies. Second, the rich pattern of cross-section heterogeneity in the reaction coefficient provides stylized facts to be explained by asset pricing theories, which should help future theoretical developments.

Our testing framework is both intuitive and flexible. Intuitive in the sense that it is derived from simple definitions such as log return and ROE. To implement the test, we only have to choose a stock forecasting model to mimic rational investors' expectations. The popular GMM approach of consumption Euler equation test relies on some degree of predictability of endogenous variables using lags of the endogenous variables. In this sense, the GMM approach is similar to our approach of forecasting endogenous variables using their lags. However, there is a subtle difference between the underlying assumptions for consistent tests. The GMM approach requires strong predictability to avoid weak instrument problems while our approach requires optimal use of forecasting information which mimics the forecasting behavior of rational investors. The latter feature actually suggests the flexibility of our framework, any advances in econometric modelling which could improve the forecasting of endogenous variables in our test could be integrated into the test. The difference in assumptions also makes our approach a useful robustness test for the GMM testing results.

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A Derivation of Return Decomposition

Starting from the definition of stock returns and returns of equity (see equation A1 and equation A2), where $r_{i,t}$ represents the log return of firm i's stock at time t, $R_{i,t}$ denotes one plus the simple return of firm i, $\text{ME}_{i,t}$ is the market value of firm i, $\text{BE}_{i,t}$ is the book value, $D_{i,t}$ is the dividend of the firm i at time t, and Δ denotes the first-order difference operator.

$$r_{i,t} \equiv \log\left(R_{i,t}\right) = log\left(1 + \frac{\Delta \text{ME}_{i,t} + D_{i,t}}{\text{ME}_{i,t-1}}\right) \tag{A1}$$

$$\text{roe}_{i,t} \equiv \log\left(1 + \text{ROE}_{i,t}\right) = log\left(1 + \frac{\Delta \text{BE}_{i,t} + D_{i,t}}{\text{BE}_{i,t-1}}\right) \tag{A2}$$

We further define firm i's log dividend growth as $\Delta d_{i,t} \equiv \log\left(\frac{D_{i,t}}{D_{i,t-1}}\right)$, log dividend-to-price ratio as $\text{dp}_{i,t} \equiv \log\left(\frac{D_{i,t}}{\text{ME}_{i,t}}\right)$, and log dividend-to-book value ratio as $\Delta\text{db}_{i,t} \equiv \log\left(\frac{D_{i,t}}{\text{BE}_{i,t}}\right)$, equations (A1) and (A2) can be rewritten as follows:

$$\begin{aligned} r_{i,t} &= log\left(1 + \frac{\Delta \text{ME}_{i,t} + D_{i,t}}{\text{ME}_{i,t-1}}\right) \\ &= \log\left(\frac{\text{ME}_{i,t} + D_{i,t}}{D_{i,t}}\right) + \log\left(\frac{D_{i,t}}{D_{i,t-1}}\right) + \log\left(\frac{D_{i,t-1}}{\text{ME}_{i,t-1}}\right) \\ &= log\left(1 + exp\left(-\text{dp}_{i,t}\right)\right) + \Delta d_{i,t} + \text{dp}_{i,t-1} \end{aligned} \tag{A3}$$

$$\begin{aligned} \text{roe}_{i,t} &= log\left(1 + \frac{\Delta \text{BE}_{i,t} + D_{i,t}}{\text{BE}_{i,t-1}}\right) \\ &= \log\left(\frac{\text{BE}_{i,t} + D_{i,t}}{D_{i,t}}\right) + \log\left(\frac{D_{i,t}}{D_{i,t-1}}\right) + \log\left(\frac{D_{i,t-1}}{\text{BE}_{i,t-1}}\right) \\ &= log\left(1 + exp\left(-\text{db}_{i,t}\right)\right) + \Delta d_{i,t} + \text{db}_{i,t-1} \end{aligned} \tag{A4}$$

Subtracting equation (A3) from equation (A4) and using a first-order approximation, we obtain:

$$\begin{aligned} \text{roe}_{i,t} - r_{i,t} &= \log\left(1 + exp\left(-\text{db}_{i,t}\right)\right) - log\left(1 + exp\left(-\text{dp}_{i,t}\right)\right) + \text{db}_{i,t-1} - \text{dp}_{i,t-1} \\ &\approx \rho \text{bm}_{i,t} - \text{bm}_{i,t-1} \end{aligned} \tag{A5}$$

Here, ρ is a constant close to but less than 1, and $\text{bm}_{i,t} \equiv log\left(\text{BM}_{i,t}\right) = log\left(\frac{\text{BE}_{i,t}}{\text{ME}_{i,t}}\right)$ is the log book-to-market ratio. By iterating equation (A5) forward, we can obtain a decomposition of $\text{bm}_{i,t-1}$

(see equation A6).

$$\mathrm{bm}_{i,t-1} \approx \sum_{j=0}^{T} \rho^j r_{i,t+j} - \sum_{j=0}^{T} \rho^j \mathrm{roe}_{i,t+j} + \rho^{T+1} \mathrm{bm}_{i,t+T} \tag{A6}$$

As $T \to \infty$, the last term in equation (A6) converges to zero. Thus, subtracting the expectation $E_{t-1}(\mathrm{bm}_{i,t-1})$ from $E_t(\mathrm{bm}_{i,t-1})$, we obtain:

$$0 = E_t(\mathrm{bm}_{i,t-1}) - E_{t-1}(\mathrm{bm}_{i,t-1}) \approx \Delta E_t \left[\sum_{j=0}^{\infty} \rho^j r_{i,t+j} \right] - \Delta E_t \left[\sum_{j=0}^{\infty} \rho^j (\mathrm{roe}_{i,t+j}) \right] \tag{A7}$$

where E_t indicates investors' expectations at time t, and $\Delta E_t(\cdot)$ represents the change in expectations from time t-1 to time t, defined as $\Delta E_t(\cdot) = E_t(\cdot) - E_{t-1}(\cdot)$.

Rearranging the terms, the unexpected stock returns can be decomposed into a component of cashflow news ($N^{\mathrm{cf}}_{i,t}$) and a component of discount rate news ($N^{r}_{i,t}$) at the first-order approximation, as outlined in equations (A8)-(A10) (Campbell, 1991; Vuolteenaho, 2002).

$$r_{i,t} - E_{t-1}(r_{i,t}) \approx N^{\mathrm{cf}}_{i,t} - N^{r}_{i,t} \tag{A8}$$

$$N^{\mathrm{cf}}_{i,t} \equiv \Delta E_t \left[\sum_{j=0}^{\infty} \rho^j (\mathrm{roe}_{i,t+j}) \right] \tag{A9}$$

$$N^{r}_{i,t} \equiv \Delta E_t \left[\sum_{j=1}^{\infty} \rho^j r_{i,t+j} \right] \tag{A10}$$

B Model Estimation

To estimate the model (18) with unobserved group patterns, we use the Iterative algorithm in Bonhomme and Manresa (2015) to search for parameters that maximize the log-likelihood lnfy_G (see equation B1). The subscript G in lnfy_G indicates that the number of groups should be determined before estimation.

$$\text{lnfy}_G = -\frac{K * Obs}{2}\log(2\pi) + \sum_{g=1}^{G}\left\{\frac{\text{Obs}_g}{2} * log\left(\left|\Omega_g^{-1}\right|\right) - \frac{1}{2}\sum_{i\in\{i|g_i=g\}}\sum_{t=1}^{T} u_{i,t}^{'}\Omega_g^{-1}u_{i,t}\right\} \quad \text{(B1)}$$

Here, K is the number of rows of $z_{i,t}$ (in the case of our main text, K=4). We define obs_i as the number of observations for firm i for the vector $\begin{pmatrix} z_{i,t}^{'} & z_{i,t-1}^{'}\end{pmatrix}$. $\text{Obs} \equiv \sum_{i=1}^{N}\text{obs}_i$ is the total number of observations in the sample, and $\text{Obs}_g \equiv \sum_{i\in\{i|g_i=g\}}\text{obs}_i$ is the number of observations in group g. T is the length of the sample period. To address the issue of an unbalanced panel, if the observation of the vector $\begin{pmatrix} z_{i,t}^{'} & z_{i,t-1}^{'}\end{pmatrix}$ for firm i is missing at some time t, we set $u_{i,t} = 0$. Ω_g is the covariance matrix of $u_{i,t}$ in group g.

The computational algorithm is as follows, and we run 10,000 iterations in our estimation (Nit=10,000):

ALGORITHM —Iterative:

1. Set nit=1, Nit=10000, and go to step 2.

2. **Initial Value:** Generate random initial values for $(\alpha_g,\ \Gamma_g,\ \Omega_g)$ as $\left(\alpha_g^{(nit,0)},\ \Gamma_g^{(nit,0)},\ \Omega_g^{(nit,0)}\right)$, for all $g \in \{1,\ \ldots,\ G\}$. Set s=0, $\text{Obj}^{(nit,0)} = 10^5$ (or any other large value). For time t, if firm i has observations for $\begin{pmatrix} z_{i,t}^{'} & z_{i,t-1}^{'}\end{pmatrix}$, define $u_{i,t}^{(nit,0)} = z_{i,t} - \alpha_{g_i}^{(nit,0)} - \Gamma_{g_i}^{(nit,0)} z_{i,t-1}$; otherwise, set $u_{i,t}^{(nit,0)} = 0$. go to step 3.

3. **Assignment:** Compute the optimal group membership for all $i \in \{1,\ \ldots,\ N\}$ using the following equation, given the values of $\left(\alpha_g^{(nit,\ s)},\ \Gamma_g^{(nit,s)},\ \Omega_g^{(nit,s)}\right)$:

$$g_i^{(nit,\ s+1)} = \underset{g_i\in\{1,\ ...,\ G\}}{\arg\min}\left\{(\text{obs}_i) * log(|\Omega_{g_i}^{(nit,s)}|) + \sum_{t=1}^{T}\left(u_{i,t}^{(nit,s)}\right)^{'}\left(\Omega_{g_i}^{(nit,s)}\right)^{-1}\left(u_{i,t}^{(nit,s)}\right)\right\}$$

Set $\text{Obs}_g^{(nit,s+1)} = \sum_{i\in\left\{i|g_i^{(nit,s+1)}=g\right\}}\text{obs}_i$, for all $g \in \{1,\ \ldots,\ G\}$. Go to step 4.

4. **Update:** Estimate parameters $\left(\alpha_g^{(nit,s+1)},\ \Gamma_g^{(nit,s+1)},\ \Omega_g^{(nit,s+1)}\right)$ for all $g \in \{1,\ \ldots,\ G\}$, given the group membership $g_i^{(nit,s+1)}$ for $i \in \{1,\ \ldots,\ N\}$:

$$\left(\alpha_g^{(nit,s+1)},\ \Gamma_g^{(nit,s+1)},\ \Omega_g^{(nit,s+1)}\right) = \underset{(\alpha_g,\ \Gamma_g,\ \Omega_g)}{\arg\min} \left\{ \text{Obs}_g^{(nit,s+1)} * log(|\Omega_g|) + \sum_{i\in\{i|g_i^{(nit,s+1)}=g\}} \sum_{t=1}^{T} \left(u_{i,t}\right)' \left(\Omega_g\right)^{-1} \left(u_{i,t}\right) \right\}$$

where $u_{i,t} = z_{i,t} - \alpha_g - \Gamma_g z_{i,t-1}$ if firm i has observations for $\begin{pmatrix} z_{i,t}' & z_{i,t-1}' \end{pmatrix}$; otherwise, $u_{i,t} = 0$. For time t, define $u_{i,t}^{(nit,s+1)} = z_{i,t} - \alpha_{g_i}^{(nit,s+1)} - \Gamma_{g_i}^{(nit,s+1)} z_{i,t-1}$ if firm i has observations for $\begin{pmatrix} z_{i,t}' & z_{i,t-1}' \end{pmatrix}$; otherwise, set $u_{i,t}^{(nit,s+1)} = 0$. Set the optimal value of the objective function using the following equation:

$$\text{Obj}^{(nit,s+1)} = \sum_{g=1}^{G} \left\{ (\text{Obs}_g^{(nit,s+1)}) * log(|\Omega_g^{(nit,s+1)}|) + \sum_{i\in\{i|g_i^{(nit,s+1)}=g\}} \sum_{t=1}^{T} \left(u_{i,t}^{(nit,s+1)}\right)' \left(\Omega_{g_i}^{(nit,s+1)}\right)^{-1} \left(u_{i,t}^{(nit,s+1)}\right) \right\}$$

Go to step 5.

5. If $\text{Obj}^{(nit,s+1)} = \text{Obj}^{(nit,s)}$, go to step 6 and set $S^{\text{nit}} = s$; otherwise, set $s = s + 1$ and go to step 3.

6. if nit=Nit, output the estimation of $\left(\hat{\alpha}_g,\ \hat{\Gamma}_g,\ \hat{\Omega}_g\right) = \left(\alpha_g^{(\text{nit}_{\text{opt}},\ \text{S}^{\text{nit}_{\text{opt}}})},\ \Gamma_g^{(\text{nit}_{\text{opt}},\ \text{S}^{\text{nit}_{\text{opt}}})},\ \Omega_g^{(\text{nit}_{\text{opt}},\ \text{S}^{\text{nit}_{\text{opt}}})}\right)$, and $\hat{g}_i = g_i^{(\text{nit}_{\text{opt}},\ \text{S}^{\text{nit}_{\text{opt}}})}$ for all $i \in \{1,\ \ldots,\ N\}$ and $g \in \{1,\ \ldots,\ G\}$, where $(\text{nit}_{\text{opt}}) = \underset{\text{nit}\in\{1,\ \ldots,Nit\}}{\arg\min}\ \text{Obj}^{(nit,S^{\text{nit}})}$; otherwise, set nit=nit+1 and go to step 2;

We also follow Bonhomme and Manresa (2015) in using the Bayesian Information Criterion (BIC) to determine G. Specifically, when G>1, the BIC is calculated as in equation (B2). When G=1, the BIC is calculated as in equation (B3). Compared to equation (B2), the BIC in equation (B3) is reduced by N, because when G=1, the group membership is pre-determined and the number of parameters is reduced by N.

$$BIC = Obj + \left[N + G * K + K^2 * G + G * K * (K+1)/2)\right] * log\left(\text{Obs}\right),\ when\ G > 1 \qquad \text{(B2)}$$

$$BIC = Obj + \left[G * K + K^2 * G + G * K * (K+1)/2)\right] * log\left(\text{Obs}\right),\ when\ G = 1 \qquad \text{(B3)}$$

C Variables Definition

[Insert Table C1 here]

D Variables used in extracting equity market factors

[Insert Table D1 here]

E Variables used in extracting macro factors

[Insert Table E1 here]

F A three-period model of stock pricing

F.1 The model

The model has 3 periods and two types of agents. The 3 period are time 0, time 1 and time 2. In each period, λ rational agents with rational expectations and $1-\lambda$ irrational agents [12] who have biased earnings forecasts buy and price the stocks in the model. Stocks are assumed to have i.i.d cash flow (earnings) news at time 1 and time 2 (denoted as c_1 and c_2). c_1 and c_2 equal c^+ with probability p, and equal $c^- < c^+$ with probability $1-p$. (see equation F2). The mean of the distribution of cash flow news is μ^c, and the variance is σ^{c2}.

$$c \sim i.i.d(\mu^c, \sigma^{c2}) \tag{F1}$$

$$c_i = \begin{cases} c^+ & \text{with probability } p \\ c^- & \text{with probability } 1\text{-}p \end{cases} \tag{F2}$$

[12] The biased expectations could come from either irrationality or information frictions. For Brevity, we call investors with biased expectations "irrational investors" regardless of the underlying reasons for the biases.

$$\mu^c \equiv pc^+ + (1-p)c^- \tag{F3}$$

Accordingly, the valuation of stocks are as follows:

$$P_1 = \lambda v_1^s + (1-\lambda)v_1^{ir}, \tag{F4}$$

$$v_1^s = c_1 + E_1^s(P_2), \tag{F5}$$

$$v_1^{ir} = c_1 + E_1^{ir}(P_2), \tag{F6}$$

$$P_2 = c_2, \tag{F7}$$

where $E_t^s(\cdot)$ ($E_t^{ir}(\cdot)$) is the expectation of rational (irrational) agents at time t. And v_t^s (v_t^{ir}) denotes the valuation of stocks by rational (irrational) agents at time t. It is easy to see that

$$E_0^s(c_1) = E_1^s(c_2) = \mu^c; \quad v_1^s = c_1 + \mu^c; \quad v_1^{ir} = c_1 + E_1^{ir}(c_2). \tag{F8}$$

Therefore,

$$P_1 = c_1 + \lambda\mu^c + (1-\lambda)E_1^{ir}(c_2), \tag{F9}$$

$$E_1^s(R_2) = \frac{E_1^s(c_2)}{c_1 + \lambda\mu^c + (1-\lambda)E_1^{ir}(c_2)} = \frac{\mu^c}{c_1 + \lambda\mu^c + (1-\lambda)E_1^{ir}(c_2)}, \tag{F10}$$

$$\begin{aligned} E_0^s(R_2) &= \frac{E_0^s(c_2)}{[\lambda E_0^s(c_1) + (1-\lambda)E_0^{ir}(c_1)] + \lambda E_0^s(c_2) + (1-\lambda)E_0^{ir}(c_2)} \\ &= \frac{\mu^c}{2\lambda\mu^c + (1-\lambda)E_0^{ir}(c_1) + (1-\lambda)E_0^{ir}(c_2)}. \end{aligned} \tag{F11}$$

F.2 The correlation between cash flow news and expected return news

The cash flow news and expected return news are respectively

$$N_1^{cf} = \Delta E_1^s(c_1) = c_1 - \mu_1^c, \tag{F12}$$

$$N_1^r = \Delta E_1^s(R_2) = E_1^s(R_2) - E_0^s(R_2). \tag{F13}$$

Hence,

$$corr(N_1^{cf}, N_1^r) = corr((c_1 - \mu_1^c), (E_1^s(R_2) - E_0^s(R_2))). \tag{F14}$$

The sign of the correlation coefficient is the same as the sign of the covariance, so we focus the discussion on the sign of the covariance.

$$
\begin{aligned}
cov(N_1^{cf}, N_1^r) &= cov((c_1 - \mu_1^c), (E_1^s(R_2) - E_0^s(R_2))) \\
&= E(c_1 - \mu_1^c)(E_1^s(R_2) - E_0^s(R_2)) - E(c_1 - \mu_1^c)E(E_1^s(R_2) - E_0^s(R_2)) \\
&= E(c_1 - \mu_1^c)(E_1^s(R_2) - E_0^s(R_2)) \\
&= p(c_1^+ - \mu_1^c)(E_1^s(R_2|c_1 = c_1^+) - E_0^s(R_2)) + (1-p)(c_1^- - \mu_1^c)(E_1^s(R_2|c_1 = c_1^-) - E_0^s(R_2)) \\
&= p(1-p)(c_1^+ - c_1^-)(E_1^s(R_2|c_1 = c_1^+) - E_1^s(R_2|c_1 = c_1^-)) \qquad \text{(F15)}
\end{aligned}
$$

Obviously, the sign of the covariance is the same as the magnitude of the return forecast revision upon a positive cash flow news relative to a negative cash flow news, $(E_1^s(R_2|c_1 = c_1^+) - E_1^s(R_2|c_1 = c_1^-))$. Assuming that prices are always positive, equation (F10) implies that the sign of $(E_1^s(R_2|c_1 = c_1^+) - E_1^s(R_2|c_1 = c_1^-))$ is opposite to the sign of $B^+ - B^-$, where $B^+ = c_1^+ + \lambda\mu^c + (1-\lambda)E_1^{ir}(c_2|c_1 = c_1^+)$ and $B^- = c_1^- + \lambda\mu^c + (1-\lambda)E_1^{ir}(c_2|c_1 = c_1^-)$.

Since

$$B^+ - B^- = (c_1^+ - c_1^-) + (1-\lambda)[E_1^{ir}(c_2|c_1 = c_1^+) - E_1^{ir}(c_2|c_1 = c_1^-)], \qquad \text{(F16)}$$

$B^+ - B^- > 0$ if

$$E_1^{ir}(c_2|c_1 = c_1^+) - E_1^{ir}(c_2|c_1 = c_1^-) > 0. \qquad \text{(F17)}$$

Equation (F17) holds if the irrational agents have more optimistic expectations on period 2 earnings when good news hits in period 1. This is typically assumed in behavioral finance models, such as La Porta et al.(1997), Barberis et al.(1998), and Daniels et al.(2001). Therefore, we follow this assumption here. In this case, we obtain $cov(N_1^{cf}, N_1^r) < 0$.

F.3 Under-prediction, over-prediction, and $cov(N_1^{cf}, N_1^r) < 0$

Below, we show the co-movements of N_1^{cf} and N_1^r in cases where the irrational agents under-valuate and over-valuate stocks, respectively. We use $E_t^u(\cdot)$ $(E_t^o(\cdot))$ to denote the expectation of irrational agents who under-valuate (over-valuate) stocks. Similarly, v_t^u (v_t^o) represents valuation of stocks by irrational agents who under-valuate (over-valuate) stocks.

F.3.1 Under-prediction

From equation (F10) and (F11), we have

$$\frac{E_1^u(R_2)}{E_0^u(R_2)} = \frac{E_0(P_1)}{P_1} = \frac{2\lambda\mu^c + (1-\lambda)(E_0^u(c_1) + E_0^u(c_2))}{c_1 + \lambda\mu^c + (1-\lambda)E_1^u(c_2)} \tag{F18}$$

we further define $A_u \equiv 2\lambda\mu^c + (1-\lambda)(E_0^u(c_1) + E_0^u(c_2))$ and $B_u \equiv c_1 + \lambda\mu^c + (1-\lambda)E_1^u(c_2)$. We also use B_u^+ (B_u^-) to denote the value of B_u when the cash flow shock at time 1 is positive (negative).

$$\begin{aligned} B_u - A_u &= (1-\lambda)\left[c_1 - E_0^u(c_1)\right] + (1-\lambda)\left[E_1^u(c_2) - E_0^u(c_2)\right] + \lambda(c_1 - \mu^c) \\ &= (1-\lambda)\left\{c_1 - E_0^u(c_1) + E_1^u(c_2) - E_0^u(c_2)\right\} + \lambda(c_1 - \mu^c) \end{aligned} \tag{F19}$$

Given that price is positive, $B_u - A_u > 0$ is equivalent to $E_1^s(R_2) < E_0^s(R_2)$. The term in $\{\quad\}$ is the sum of two periods' forecast revision by the underpricing agents, on average, it is positive by assumption of "underprice", that is, on average, $\frac{E_1^s(R_2)}{E_0^s(R_2)} < 1$ and the expected return news is negative.

We present different scenarios in which the correlation between cash flow news and expected return news differs below.

Case 1: Positive Cash Flow Shock When Under-prediction

Upon a positive shock at time 1, we have

$$B_u^+ - A_u = (1-\lambda)\left\{c^+ - E_0^u(c_1) + E_1^u(c_2|c_1 = c^+) - E_0^u(c_2)\right\} + \lambda(1-p)(c^+ - c^-) > 0 \tag{F20}$$

Because $c^+ > E_0^u(c_1)$, $E_1^u(c_2|c_1 > c^+) - E_0^u(c_2)$, and $c^+ > c^-$, $E_1^s(R_2)$ is smaller than $E_0^s(R_2)$, expected return news is negative. **The signs of cash flow news and expected return news at time 1 are opposite.**

Intuitively, rational investors know that irrational agents under-predict the earnings, anticipating the follow-up buying orders of irrational investors and the price increase. The realization of a positive shock partially realizes their expectations, leading to a downward revision of rational return forecast.

Case 2: Negative Cash Flow Shock When Under-prediction

When $c_1 = c^-$:

$$B_u^- - A_u = (1-\lambda)\left\{c^- - E_0^u(c_1) + E_1^u(c_2|c_1 = c^-) - E_0^u(c_2)\right\} + \lambda p(c^- - c^+) \tag{F21}$$

The sign of $B_u^- - A_u$ depends on the extent to which irrational investors underestimate the value of stocks. Define the following terms:

$$\Delta_1 \equiv c^- - E_0^u(c_1); \quad \Delta_2 \equiv E_1^u(c_2|c_1 = c^-) - E_0^u(c_2); \quad \Delta_3 \equiv \lambda p(c^- - c^+) < 0 \tag{F22}$$

- **Case 2.1:** If $c_1 = c^- \geq E_0^u(c_1) \Rightarrow E_1^u(c_2|c_1 = c^-) \geq E_0^u(c_1) \Rightarrow \Delta_1 \geq 0, \Delta_2 \geq 0$
  - **Case 2.1.1:** If $(1-\lambda)|\Delta_1 + \Delta_2| > |\Delta_3|$, then $B_u^- > A_u$, $\dfrac{E_1^s(R_2)}{E_0^s(R_2)} = \dfrac{A_u}{B_u^-} < 1$, $N_1^r < 0$ and $N_1^{cf} < 0$. **The signs of cash flow news and expected return news are the same.**
  - **Case 2.1.2:** If $(1-\lambda)|\Delta_1 + \Delta_2| < |\Delta_3|$, and $N_1^r > 0$ and $N_1^{cf} < 0$. **The signs of cash flow news and expected return news are opposite.**
- **Case 2.2:** if $c_1 = c^- < E_0^u(c_1) \Rightarrow E_1^u(c_2|c_1 = c^-) < E_0^u(c_1) \Rightarrow \Delta_1 < 0, \Delta_2 < 0 \Rightarrow N_1^r > 0$ and $N_1^{cf} < 0$. **The signs of cash flow news and expected return news are opposite.**

Intuitively, when a negative cash flow shock hits and the shock is so large that the irrational investors become even more pessimistic than before, their selling order will cut the stock price and the rational agents will revise their return forecast upward rather than downward. Conversely, when the negative cash flow shock is small, the realized earning might still be higher than the irrational investors' forecast, leading to their buying orders and a price increase. The subsequent return forecast revision of the rational investors could be negative.

F.3.2 Over-prediction

Similarly, from equation (F10) and (F11), we have

$$\frac{E_1^o(R_2)}{E_0^o(R_2)} = \frac{2\lambda\mu^c + (1-\lambda)(E_0^o(c_1) + E_0^o(c_2))}{c_1 + \lambda\mu^c + (1-\lambda)E_1^o(c_2)} \tag{F23}$$

We define $A_o \equiv 2\lambda\mu^c + (1-\lambda)(E_0^o(c_1) + E_0^o(c_2))$, $B_o \equiv c_1 + \lambda\mu^c + (1-\lambda)E_1^o(c_2)$, and use B_o^+ (B_o^-) to denote the value of B_o when the cash flow shock at time 1 is positive (negative).

$$\begin{aligned} B_o - A_o &= (1-\lambda)\left[c_1 - E_0^o(c_1)\right] + (1-\lambda)\left[E_1^o(c_2) - E_0^o(c_2)\right] + \lambda(c_1 - \mu^c) \\ &= \lambda(c_1 - \mu^c) + (1-\lambda)\left[E_1^o(c_2) - E_0^o(c_2)\right] + (1-\lambda)\left[c_1 - E_0^o(c_1)\right] \end{aligned} \tag{F24}$$

On average, the first term is zero, the second and third term is negative, therefore, on average, $B_o - A_o < 0$, $E_1^s(R_2) > E_0^s(R_2)$, expected return news is positive.

Case 3: Positive Cash Flow Shock When Over-prediction

Upon a positive shock at time 1 ($c_1 = c^+$), we have

$$B_o^+ - A_o = \lambda(1-p)(c^+ - c^-) + (1-\lambda)\left\{E_1^o(c_2|c_1 = c^+) - E_0^o(c_2)\right\} + (1-\lambda)\left\{c^+ - E_0^o(c_1)\right\} \quad \text{(F25)}$$

In equation (F25), the first term is positive, whereas the signs of the second and third term depend on the conditions. Define the following terms:

$$\Delta_1 \equiv c^+ - E_0^o(c_1); \quad \Delta_2 \equiv E_1^o(c_2|c_1 = c^+) - E_0^o(c_2); \quad \Delta_3 \equiv \lambda(1-p)(c^+ - c^-) > 0 \quad \text{(F26)}$$

- **Case 3.1:** If $c_1 = c^+ > E_0^o(c_1)$, then $E_1^o(c_2|c_1 = c^+) > E_0^o(c_2)$, $B_o^+ - A_o > 0$, and The expected return news is negative. **The signs of cash flow news and expected return news are opposite.**
- **Case 3.2:** If $c_1 = c^+ < E_0^o(c_1)$, then $E_1^o(c_2|c_1 = c^+) < E_0^o(c_2)$, $\Delta_1 < 0$, and $\Delta_2 < 0$.
  - **Case 3.2.1:** If $(1-\lambda)|\Delta_1 + \Delta_2| > |\Delta_3|$, then $B_o^+ < A_o$, $\dfrac{E_1^s(R_2)}{E_0^s(R_2)} = \dfrac{A_o}{B_o^+} > 1$, $N_1^r > 0$ and $N_1^{cf} > 0$. **The signs of cash flow news and expected return news are the same.**
  - **Case 3.2.2:** If $(1-\lambda)|\Delta_1 + \Delta_2| < |\Delta_3|$, and $N_1^r < 0$ and $N_1^{cf} > 0$. **The signs of cash flow news and expected return news are opposite.**

Case 4: Negative Cash Flow Shock When Over-prediction

When $c_1 = c^-$, we have

$$B_o^- - A_o = (1-\lambda)\left\{c^- - E_0^o(c_1) + E_1^o(c_2|c_1 = c^-) - E_0^o(c_2)\right\} + \lambda p(c^- - c^+) < 0 \quad \text{(F27)}$$

Hence $E_1^s(R_2)$ is larger than $E_0^s(R_2)$, the expected return news is positive. **The signs of cash flow news and expected return news at time 1 are opposite.**

Intuitively, when irrational investors over-predict the cash flow of the firm and a negative cash flow shock leads them to revise their earnings forecast downward and sell the stock. The resulting price decease partially realizes the rational investors' anticipation, leading to upward revision of their return forecast.

When the cash flow shock is positive and large, the shock confirms the optimistic forecast by irrational agents, leading to buying orders and downward return forecast by rational agents. However, when the initial forecasts of irrational agents are very optimistic, which usually happens in a mania, even a positive cash flow shock will fail their expectations and lead to a price fall. In this case, the return forecast of rational investors could be positive.

F.4 Short-selling constraints and $cov(N_1^{cf}, N_1^r)$, when over-prediction

We assume that short-selling constraints imply that the cost of selling increases with the amount sold. Next, we present a special case where a positive correlation between cash flow news and expected return news can arise when $c_1 = c^+ < E_0^o(c_1)$ and $(1-\lambda)|\Delta_1 + \Delta_2| > |\Delta_3|$, i.e., when irrational agents are extremely optimistic.

As mentioned earlier, when there are no short-selling costs, **Case 4** dominates the sign of $cov(N_1^{cf}, N_1^r)$ because, in this case, a unit cash flow shock leads investors to sell more stocks, resulting in a greater impact on expected returns. However, when short-selling constraints are present, the effect of a negative cash flow shock on expected return news is limited. Under such a shock, investors may want to sell the stock, but the existence of selling costs restricts the volume of sales and limits the resulting price decline. In other words, the correction of overvaluation takes longer time, and thus the expected return news is less positive than it would be without these costs. This reduces the absolute value of the negative correlation between cash flow news and expected return news caused by a negative cash flow shock. In an extreme case, the expected return news could even be zero.

Meanwhile, **Case 3.2.1** is less constrained by short-selling limits compared to **Case 4**. This is because, in **Case 3.2.1**, irrational agents lower their expectations for stocks less drastically, resulting in fewer stock sales than in **Case 4**. In a particular instance, the positive co-movements

in **Case 3.2.1** could dominate the correlation between cash flow news and expected return news more than in **Case 4**, and the correlation could be positive. More specifically, in **Case 3.2.1**, a positive cash flow shock prompts irrational investors to revise their earnings forecasts downward if their prior expectations were overly optimistic. Without selling costs, these investors would sell some of their stocks, causing a price drop. Since rational investors anticipate this price decline, part of their expected return forecast is realized, leading to an upward revision in the return forecast. This scenario generates a positive correlation between cash flow news and expected return news. The presence of selling costs, however, prevents the price drop and limits the revision in the expected return forecast, weakening the positive correlation. While both **Case 4** and **Case 3.2.1** are affected by short-selling constraints, the limitation on the negative correlation is more pronounced in **Case 4**. This is because selling costs increase with the size of the short position, and downward revisions in stock valuation are more severe in **Case 4**.

Table 1: Anomalies linked to the cross-section pattern of Euler equation failure

Anomalies	
Panel A: The Fama-French (2015) anomalies	
Value anomaly	High book-to-market (BM) ratio firms (value firms) have higher returns than low BM ratio firms (growth firms).
Profitability anomaly	High-profit firms have higher returns.
Investment anomaly	Firms with high asset growth rate earn less returns than firms with low asset growth rate. than low-profit firms.
Panel B: Earnings consistency	
Earnings consistency effect	Investors over-react to a consistently positive or negative string of earnings.
Panel C: Financial constraint related anomalies	
Betting against beta (BAB) effect	Return spread between high and low beta stocks is narrower than predicted by the capital asset pricing model.
Earnings forecasts dispersion effect	Stocks with more dispersed earnings forecasts earn lower returns.
The option volume effect	Stocks with higher relative option volume earn lower returns.

Table 2: Summary Statistics

	Nobs	mean	sd	p5	p25	p50	p75	p95
Panel A: Summary statistics in the sample of stationary stocks								
$r_{i,t}$	49,467	0.0690	0.4130	-0.6190	-0.1154	0.0953	0.2884	0.6533
$\text{bm}_{i,t}$	49,467	-0.5846	0.7848	-1.9697	-1.0254	-0.5084	-0.0736	0.5383
$\text{roe}_{i,t}$	49,467	0.0592	0.3270	-0.3514	0.0354	0.0982	0.1570	0.3106
$\text{if}_{i,t}$	49,467	0.4799	0.2783	0.0381	0.2427	0.4841	0.7144	0.9167
Operating profitability	47,814	0.2116	0.5926	-0.2151	0.1260	0.2125	0.3125	0.5242
Asset growth	47,854	0.1171	0.4670	-0.1821	-0.0087	0.0581	0.1492	0.5282
Log market equity	48,839	13.369	2.0621	10.147	11.839	13.310	14.775	16.915
Frazzini-Pedersen Beta	48,872	0.9376	0.2069	0.6110	0.7595	0.9735	1.0745	1.3099
EPS forecast dispersion	48,872	0.1741	0.0627	0.1066	0.1303	0.1564	0.1942	0.3207
Earnings streak length	48,872	1.5228	0.2540	1.1563	1.3293	1.5622	1.6561	1.8333
Option to stock volume	34,267	719.24	139.20	494.81	615.16	742.32	836.01	957.53
ROA volatility	48,872	0.0185	0.0052	0.0088	0.0161	0.0189	0.0219	0.0268
Panel B: Summary statistics in the full sample								
$r_{i,t}$	79,944	0.0360	0.5191	-0.8109	-0.1552	0.0862	0.3011	0.7094
$\text{bm}_{i,t}$	79,944	-0.5930	0.8754	-2.1135	-1.0562	-0.5118	-0.0537	0.6436
$\text{roe}_{i,t}$	79,944	0.0224	0.4665	-0.6007	0.0120	0.0927	0.1564	0.3474
$\text{if}_{i,t}$	79,944	0.4401	0.2939	0.0168	0.1748	0.4204	0.6896	0.9247

Note: This table displays the summary statistics of the variables that we used in our study. In the column labeled "Nobs", we report the number of firm-year observations. Columns labeled "mean" and "sd" report the mean and standard deviation, respectively. The columns labeled "p5", "p25", "p50", "p75", and "p95" show the 5th percentile, 25th percentile, 50th percentile, 75th percentile, and 95th percentile of variables, respectively.

Table 3: Pooled testing results using pooled VAR model

	b_news_cf	b_cons	var_cf	var_r	cov	corr
Panel A: Full sample						
b	0.7243###	0.0000	0.3688	0.0565	0.1017 ***	0.7041 ***
se	(0.0153)	(0.0006)	(0.0165)	(0.0064)	(0.0098)	(0.0233)
Panel B: Sample consisting of stocks with stationary individual VAR model						
b	0.9643##	-0.0015 **	0.1392	0.0141	0.0048 **	0.1087 **
se	(0.0175)	(0.0006)	(0.0056)	(0.0019)	(0.0026)	(0.0529)

Note: This table presents the results of the rationality tests from equation (13) and the return decomposition results from equation (12) using a pooled VAR model to forecast returns. Panel A reports the results for the full sample, where the data have been filtered according to the methodology of Cohen et al. (2002). Panel B presents results for a subsample that includes only stocks with stationary individual VAR models. In the rows labeled "b," the point estimates are reported, while the rows labeled "se" provide the bootstrap standard errors. The column labeled "b_news_cf" ("b_cons") reports the estimated reaction coefficient (constant) from equation (13). The columns "var_cf" and "var_r" display the variances of cash flow news and discount rate news, respectively. The columns labeled "cov" and "corr" report the covariance and correlation between cash flow news and discount rate news, respectively. "#", "##", and "###" in the column labeled "b_news_cf" indicate the reaction coefficient is different from 1 at 0.1, 0.05, and 0.01 significance levels, respectively. "*", "**", and "***" denote that the number is different from zero at 0.1, 0.05, and 0.01 significance levels, respectively. The significance levels are calculated using bootstrap methods.

Table 4: Coefficient Matrices of Pooled VAR model and Firm-Specific VAR model

variables			$\widetilde{r}_{i,t-1}$	$\widetilde{\text{bm}}_{i,t-1}$	$\widetilde{\text{roe}}_{i,t-1}$	$\widetilde{\text{if}}_{i,t-1}$
$\widetilde{r}_{i,t}$	Pooled var		0.1292	0.0450	0.0836	0.1291
	Firm-Specific VAR	p1	-5.1724	-5.0952	-20.583	-39.994
		p25	-0.2948	-0.05763	-1.0410	-1.4765
		p50	0.02788	0.2226	-0.1417	-0.1772
		p75	0.3698	0.6534	0.5824	0.9319
		p99	5.0902	6.8862	18.945	37.206
$\widetilde{\text{bm}}_{i,t}$	Pooled VAR		0.1362	0.8716	0.0414	-0.0667
	Firm-Specific VAR	p1	-5.1887	-5.2318	-17.244	-41.801
		p25	-0.4314	-0.01080	-0.6000	-1.0828
		p50	-0.05595	0.4699	0.1933	0.1818
		p75	0.2837	0.7903	1.0061	1.5889
		p99	5.4160	5.3616	19.907	37.907
$\widetilde{\text{roe}}_{i,t}$	Pooled var		0.2317	-0.0458	0.2075	0.1236
	Firm-Specific VAR	p1	-3.4916	-4.4726	-6.6598	-23.778
		p25	-0.09838	-0.3088	-0.2849	-0.4051
		p50	0.03541	-0.1162	0.03211	0.01040
		p75	0.1623	-0.002029	0.3412	0.4542
		p99	3.3909	3.4079	8.1991	22.384
$\widetilde{\text{if}}_{i,t}$	Pooled var		0.0483	-0.0070	0.0111	0.9232
	Firm-Specific VAR	p1	-1.3553	-1.3296	-3.5679	-3.2807
		p25	-0.04655	-0.08435	-0.1131	0.2243
		p50	0.01744	-0.01611	0.01768	0.6012
		p75	0.08534	0.04261	0.1852	0.8432
		p99	0.9829	1.2169	3.9919	4.2006

Note: This table presents the estimated coefficient matrix Γ from the pooled VAR model (15) in the rows labeled "pooled VAR" and the percentiles of estimated coefficient matrix Γ_i from the Firm-Specific VAR models (26) in the rows labeled "Firm-Specific VAR". The first column of the table indicates the dependent variables in the VAR system, while the first row lists the lagged dependent variables. In the third column, rows labeled "p1", "p25", "p50", "p75", and "p99" indicate the 1st percentile, 25th percentile, 50th percentile, 75th percentile, and 99th percentile of the coefficients of individual VAR models, respectively.

Table 5: Summary Statistics of Cash Flow News using Pooled VAR model and Firm-Specific VAR model

	Nobs	**Nfirms**	**mean**	**min**	**p25**	**p50**	**p75**	**max**
Pooled VAR	71201	7290	0.0000	-10.5830	-0.2044	0.0472	0.2865	5.2958
Firm-Specific VAR	64979	4505	0.0000	-451.4200	-0.1098	0.0000	0.1125	398.6600

Note: This table displays the summary statistics of cash flow news calculated using the pooled (firm-specific) VAR model to forecast returns in the row labeled "pooled VAR" ("firm-specific VAR"). In the columns labeled "Nobs" and "Nfirms", we report the number of firm-year observations and the number of firms, respectively. The number of firms (observations) is less when using firm-specific VAR models than when using pooled VAR models because we drop firms with observations less than 5 when estimating firm-specific VAR models. Columns labeled "mean", "min", and "max" report the mean, minimum, and maximum of cash flow news, respectively. The columns labeled "p25", "p50", and "p75" show the 25th percentile, 50th percentile, and 75th percentile of cash flow news, respectively.

Table 6: BIC of Different Groups: Baseline

num of groups	BIC
1	-389876.5159
10	-534917.6903
20	-545011.8200
30	-548173.6821
40	**-549011.7344**#
50	-548917.7924
60	-547461.2583
70	-546129.4898
80	-544417.6831

Note: This table reports the BICs defined in the equation (B2) and equation (B3) in the appendix B. The first column reports the number of groups, while the second column of the table represents the BICs. The hash sign in the column labeled "BIC" indicates when BIC is the lowest. The optimal number of groups is 40.

Table 7: Coefficient Matrices of Pooled VAR model and Group VAR model

variables			$\widetilde{r}_{i,t-1}$	$\widetilde{\text{bm}}_{i,t-1}$	$\widetilde{\text{roe}}_{i,t-1}$	$\widetilde{\text{if}}_{i,t-1}$
$\widetilde{r}_{i,t}$	pooled VAR		0.1292	0.0450	0.0836	0.1291
	group VAR	p1	-0.2534	-0.0976	-0.0243	-0.5114
		p25	-0.0241	-0.0408	0.0065	-0.0913
		p50	0.0385	0.0065	0.0330	-0.0039
		p75	0.0900	0.0377	0.0617	0.0475
		p99	0.1336	0.1224	0.1608	0.3793
$\widetilde{\text{bm}}_{i,t}$	pooled VAR		0.1362	0.8716	0.0414	-0.0667
	group VAR	p1	-0.2672	-0.1511	0.5754	-0.0852
		p25	-0.0387	0.0513	0.7872	0.0855
		p50	-0.0166	0.1186	0.8600	0.1594
		p75	0.0214	0.1625	0.9156	0.3217
		p99	0.0999	0.2381	0.9923	0.5957
$\widetilde{\text{roe}}_{i,t}$	pooled VAR		0.2317	-0.0458	0.2075	0.1236
	group VAR	p1	-0.6251	-0.0419	-0.3127	-0.0329
		p25	0.0148	0.0543	-0.1205	0.1339
		p50	0.0471	0.0779	-0.0611	0.3115
		p75	0.0628	0.1185	-0.0353	0.3638
		p99	0.1094	0.2843	0.0350	0.5981
$\widetilde{\text{if}}_{i,t}$	pooled VAR		0.0483	-0.0070	0.0111	0.9232
	group VAR	p1	-0.0190	-0.0100	-0.0330	-0.0552
		p25	-0.0005	0.0257	-0.0145	-0.0086
		p50	0.0061	0.0372	-0.0056	0.0084
		p75	0.0117	0.0572	-0.0026	0.0280
		p99	0.0473	0.1162	0.0074	0.1616

Note: This table presents the estimated coefficient matrix Γ from the pooled VAR model (15) in the rows labeled "pooled VAR" and the percentiles of estimated coefficient matrix Γ_g from the group VAR models (18) in the rows labeled "group VAR". The first column of the table indicates the dependent variables in the VAR system, while the first row lists the lagged dependent variables. In the third column, rows labeled "p1", "p25", "p50", "p75", and "p99" indicate the 1st percentile, 25th percentile, 50th percentile, 75th percentile, and 99th percentile of the coefficients of group VAR models, respectively.

Table 8: Group testing results using group VAR model

	b_news_cf	b_cons	var_cf	var_r	cov	corr
b	**0.9875**	**0.0413 *****	**0.1245**	**0.0172**	**0.0016**	**0.0336**
se	(0.0372)	(0.0016)	(0.0071)	(0.0041)	(0.0053)	(0.0831)

Note: This table presents the results of the rationality tests from equation (13) and the return decomposition from equation (12) using a group VAR model to forecast returns. In the rows labeled "b," the point estimates are reported, while the rows labeled "se" provide the bootstrapping standard errors. The column labeled "b_news_cf" ("b_cons") reports the estimated reaction coefficient (constant) from equation (13). The columns "var_cf" and "var_r" display the variances of cash flow news and discount rate news, respectively. The columns labeled "cov" and "corr" report the covariance and correlation between cash flow news and discount rate news, respectively. "#", "##", and "###" in the column labeled "b_news_cf" indicate the reaction coefficient is different from 1 at 0.1, 0.05, and 0.01 significance levels, respectively. "*", "**", and "***" denote that the number is different from zero at 0.1, 0.05, and 0.01 significance levels, respectively. The significance level is calculated with bootstrapping.

Table 9: Heterogeneity in the Reaction Coefficient and (Co-)Variances of news

Quartile		b_news_cf	var_cf	var_r	cov_cf_r
1	b	**0.7997$^{\#\#\#}$**	**0.3345**	**0.0480**	**0.0738 *****
	se	(0.0356)	(0.0544)	(0.0274)	(0.0344)
2	b	**1.0413**	**0.1730**	**0.0109**	**-0.0064**
	se	(0.0211)	(0.0298)	(0.0056)	(0.0045)
3	b	**1.1956$^{\#\#\#}$**	**0.0538**	**0.0101**	**-0.0102 *****
	se	(0.0218)	(0.0163)	(0.0028)	(0.0026)
4	b	**1.4139$^{\#\#\#}$**	**0.0603**	**0.0240**	**-0.0226 *****
	se	(0.0492)	(0.0080)	(0.0034)	(0.0026)

Note: This table classifies the group estimates of reaction coefficients into four quartiles and reports the mean of the estimated group-specific reaction coefficient ("b_news_cf"), the mean of group-specific variance of cash flow news ("var_cf"), the mean of group-specific variance of expected return news ("var_r"), and the mean of covariance of cash flow news and expected return news ("cov_cf_r") in each quartile. The first column indicates the quartile. the rows labeled "b" report the point estimation, while the rows labeled "se" show the bootstrapping standard errors. "#", "##", and "###" in the column labeled "b_news_cf" indicate the reaction coefficient is different from 1 at 0.1, 0.05, and 0.01 significance levels, respectively. "*", "**", and "***" denote that the number is different from zero 0.1, 0.05, and 0.01 significance levels, respectively. The significance level is calculated with bootstrapping.

Table 10: BICs for Alternative Model Specifications

Panel A: V02		Panel B: FAVAR_CZ		Panel C: FAVAR_Macro	
G	**BIC**	**G**	**BIC**	**G**	**BIC**
1	-318315.9882	1	-341204.2850	1	-437312.8029
10	-509940.3317	10	-474583.8583	10	-566913.5333
30	-528277.3175	20	-482458.4957	20	-574986.7163
50	-532547.7331	**30**	**-483858.0379#**	**30**	**-575763.3365#**
60	-533121.8195	40	-483017.8748	40	-575158.2539
70	**-533393.7643#**	50	-480558.9644	50	-572777.8923
80	-533037.5482	60	-477955.6327	60	-569893.7078
90	-532638.7376				
100	-532063.7372				

Note: This table reports BICs for alternative model specifications. The panels "V02", "FAVAR_CZ", and "FAVAR_Macro" respectively report BIC values of the VAR model of Vuolteenaho (2002), the VAR model augmented by equity market factors, and the VAR model augmented by macroeconomic factors. The columns labeled "G" report the number of groups, while the columns labeled "BIC" of the table represent the BIC values. The hash sign in the column labeled "BIC" indicates when BIC is the lowest.

Table 11: Heterogeneity in the Reaction Coefficient and (Co-)Variances of news using the grouped version of VAR model of Vuolteenaho (2002)

group		b_news_cf	b_cons	var_cf	var_r	cov_cf_r
1	b	**0.7503$^{\#\#\#}$**	**0.0011**	**0.2179**	**0.0363**	**0.0608 *****
	se	(0.0250)	(0.0135)	(0.1655)	(0.1580)	(0.1598)
2	b	**0.9793**	**-0.0275**	**0.2435**	**0.0102**	**0.0051 ***
	se	(0.0145)	(0.0151)	(0.0290)	(0.0035)	(0.0039)
3	b	**1.1569$^{\#\#\#}$**	**0.0248**	**0.1230**	**0.0155**	**-0.0175 *****
	se	(0.0166)	(0.0131)	(0.0197)	(0.0032)	(0.0035)
4	b	**1.4578$^{\#\#\#}$**	**0.0490 *****	**0.0556**	**0.0206**	**-0.0203 *****
	se	(0.0197)	(0.0076)	(0.0080)	(0.0024)	(0.0028)

Note: This table shows the heterogeneity in the reaction coefficient and (Co-)Variances of news using the grouped version of VAR model of V (2002) to forecast returns. The table classifies the group estimates of reaction coefficients into four quartiles and reports the mean of the estimated group-specific reaction coefficient ("b_news_cf"), the mean of group-specific variance of cash flow news ("var_cf"), the mean of group-specific variance of expected return news ("var_r"), and the mean of covariance of cash flow news and expected return news ("cov_cf_r") in each quartile. The first column indicates the quartile. the rows labeled "b" report the point estimation, while the rows labeled "se" show the bootstrapped standard errors. "#", "##", and "###" in the column labeled "b_news_cf" indicate the reaction coefficient is different from 1 at 0.1, 0.05, and 0.01 significance levels, respectively. "*", "**", and "***" denote that the number is different from zero 0.1, 0.05, and 0.01 significance levels, respectively. The significant level is calculated with bootstrapping.

Table 12: Heterogeneity in the Reaction Coefficient and (Co-)Variances of news using the group VAR model augmented by equity market factor

group		b_news_cf	b_cons	var_cf	var_r	cov_cf_r
1	b	**0.8303###**	**-0.0327**	**0.4084**	**0.0569**	**0.0905 *****
	se	(0.0317)	(0.0133)	(0.0605)	(0.0298)	(0.0397)
2	b	**1.0364**	**0.0322 *****	**0.1099**	**0.0062**	**-0.0059**
	se	(0.0217)	(0.0129)	(0.0259)	(0.0021)	(0.0029)
3	b	**1.1937###**	**0.0650 *****	**0.0504**	**0.0054**	**-0.0099 *****
	se	(0.0248)	(0.0113)	(0.0166)	(0.0027)	(0.0027)
4	b	**1.4099###**	**0.0683 *****	**0.0484**	**0.0214**	**-0.0189 *****
	se	(0.0311)	(0.0055)	(0.0063)	(0.0026)	(0.0020)

Note: This table shows the heterogeneity in the reaction coefficient and (Co-)Variances of news using the group VAR model augmented by equity market factor to forecast returns. The table classifies the group estimates of reaction coefficients into four quartiles and reports the mean of the estimated group-specific reaction coefficient ("b_news_cf"), the mean of group-specific variance of cash flow news ("var_cf"), the mean of group-specific variance of expected return news ("var_r"), and the mean of covariance of cash flow news and expected return news ("cov_cf_r") in each quartile. The first column indicates the quartile. the rows labeled "b" report the point estimation, while the rows labeled "se" show the bootstrapped standard errors. "#", "##", and "###" in the column labeled "b_news_cf" indicate the reaction coefficient is different from 1 at 0.1, 0.05, and 0.01 significance levels, respectively. "*", "**", and "***" denote that the number is different from zero 0.1, 0.05, and 0.01 significance levels, respectively. The significance level is calculated with bootstrapping.

Table 13: Heterogeneity in the Reaction Coefficient and (Co-)Variances of news using the group VAR model augmented by macroeconomic factor

group		b_news_cf	b_cons	var_cf	var_r	cov_cf_r
1	b	**0.8854###**	**-0.0434**	**0.3578**	**0.0198**	**0.0417 *****
	se	(0.0306)	(0.0197)	(0.0536)	(0.0115)	(0.0185)
2	b	**1.0295**	**0.0661 ****	**0.0782**	**0.0046**	**-0.0018**
	se	(0.0211)	(0.0166)	(0.0330)	(0.0024)	(0.0032)
3	b	**1.2095###**	**0.0536 *****	**0.0598**	**0.0100**	**-0.0128 *****
	se	(0.0236)	(0.0099)	(0.0117)	(0.0027)	(0.0027)
4	b	**1.4053###**	**0.0643 *****	**0.0527**	**0.0197**	**-0.0204 *****
	se	(0.0372)	(0.0074)	(0.0078)	(0.0029)	(0.0024)

Note: This table shows the heterogeneity in the reaction coefficient and (Co-)Variances of news using the group VAR model augmented by macroeconomic factor to forecast returns. The table classifies the group estimates of reaction coefficients into four quartiles and reports the mean of the estimated group-specific reaction coefficient ("b_news_cf"), the mean of group-specific variance of cash flow news ("var_cf"), the mean of group-specific variance of expected return news ("var_r"), and the mean of covariance of cash flow news and expected return news ("cov_cf_r") in each quartile. The first column indicates the quartile. the rows labeled "b" report the point estimation, while the rows labeled "se" show the bootstrapped standard errors. "#", "##", and "###" in the column labeled "b_news_cf" indicate the reaction coefficient is different from 1 at 0.1, 0.05, and 0.01 significance levels, respectively. "*", "**", and "***" denote that the number is different from zero 0.1, 0.05, and 0.01 significance levels, respectively. The significance level is calculated with bootstrapping.

Table 14: Heterogeneity in the Reaction Coefficient and (Co-)Variances of news using the group VAR model to directly forecast return on equity

group		b_news_cf	b_cons	var_cf	var_r	cov_cf_r
1	b	**0.3985**$^{\#\#\#}$	-0.0285	0.4876	0.0445	0.0333 ***
	se	(0.0244)	(0.0108)	(0.0440)	(0.0269)	(0.0210)
2	b	**0.9141**$^{\#\#\#}$	0.0353	0.1330	0.0146	0.0076
	se	(0.0315)	(0.0104)	(0.0262)	(0.0045)	(0.0066)
3	b	**1.2212**$^{\#\#\#}$	0.0498	0.0631	0.0105	-0.0096 ***
	se	(0.0295)	(0.0085)	(0.0105)	(0.0030)	(0.0022)
4	b	**1.5324**$^{\#\#\#}$	0.0504	0.0417	0.0236	-0.0159 ***
	se	(0.0472)	(0.0059)	(0.0052)	(0.0032)	(0.0017)

Note: This table shows the heterogeneity in the reaction coefficient and (Co-)Variances of news using the group VAR model of Cohen et al (2002) to directly forecast returns on equity. The table classifies the group estimates of reaction coefficients into four quartiles and reports the mean of the estimated group-specific reaction coefficient ("b_news_cf"), the mean of group-specific variance of cash flow news ("var_cf"), the mean of group-specific variance of expected return news ("var_r"), and the mean of covariance of cash flow news and expected return news ("cov_cf_r") in each quartile. The first column indicates the quartile. the rows labeled "b" report the point estimation, while the rows labeled "se" show the bootstrapped standard errors. "#", "##", and "###" in the column labeled "b_news_cf" indicate the reaction coefficient is different from 1 at 0.1, 0.05, and 0.01 significance levels, respectively. "*", "**", and "***" denote that the number is different from zero 0.1, 0.05, and 0.01 significance levels, respectively. The significance level is calculated with bootstrapping.

Table 15: Regressions Results of Group-Specific Reaction Coefficients on Firm Characteristics

	CPV	V02	FAVAR_CZ	FAVAR_Macro	Direct
Panel A: Earnings volatility					
RoA Volatility	-4.0011 ***	-2.3632 ***	-6.2345 ***	-4.8707 ***	-8.7543 ***
	(0.9110)	(0.4731)	(1.0078)	(0.9522)	(0.9889)
Earnings Streak Length	0.7374 ***	0.5067 ***	0.8629 ***	0.5950 ***	1.6451 ***
	(0.1532)	(0.1341)	(0.2084)	(0.1631)	(0.2036)
Panel B: The Fama-French anomalies					
Log Market Equity	-0.0117	-0.0292 *	-0.0250	-0.0151	-0.0511 **
	(0.0191)	(0.0163)	(0.0181)	(0.0202)	(0.0219)
Book-to-Market Ratio	0.7441 ***	0.1710 ***	0.7047 ***	0.5856 ***	1.9253 ***
	(0.1936)	(0.0610)	(0.1525)	(0.1401)	(0.2385)
Operating Profitability	0.3269 ***	-0.0640	0.4660 ***	0.2247 **	0.5763 ***
	(0.1053)	(0.0666)	(0.1230)	(0.1093)	(0.1153)
Asset Growth	-1.9244 ***	-0.8373 ***	-2.4708 ***	-2.3800 ***	-3.8071 ***
	(0.4605)	(0.2194)	(0.4183)	(0.4316)	(0.5744)
Panel C: Financial constraint					
Frazzini-Pedersen Beta	-0.4194 ***	-0.3328 ***	-0.6103 ***	-0.5156 ***	-0.8038 ***
	(0.1784)	(0.0839)	(0.0952)	(0.1165)	(0.1752)
EPS Forecast Dispersion	-0.5601 ***	-0.3630 ***	-0.7564 ***	-0.6595 ***	-0.9018 ***
	(0.1595)	(0.0745)	(0.1255)	(0.1360)	(0.1840)
Option to Stock Volume	-0.0004 ***	-0.0002 *	-0.0005 ***	-0.0001 *	-0.0009 ***
	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0002)

Note: This table shows the estimated θ of the regressions (22) that regress group-specific reaction coefficients on group-mean firm characteristics that are related to cross-section stock anomalies and indicate the source of the failure of the Euler equation. The first column of the table denotes the independent variables in regression (22). In Panel A, Panel B, Panel C, we report the results concerning the proxies of earnings volatility, the Fama-French anomalies, and financial constraint, respectively. The panels "CPV", "V02", "FAVAR_CZ", and "FAVAR_Macro" respectively report results when the cash flow news is constrcuted from the VAR model of the baseline model, Vuolteenaho (2002), the VAR model augmented by equity market factors, and the VAR model augmented by macroeconomic factors. The panel "Direct" report the results when we directly forecast return on equity with the baseline VAR model. The bootstrapping standard errors are reported in parentheses. "*", "**", and "***" denote that the coefficients are different from zero 0.1, 0.05, and 0.01 significance levels, respectively. The significant level is calculated with bootstrapping.

Table 16: Regression Results of Forecasting Errors on Firm Characteristics

Forecast horizon	BM		Operating profit		Asset growth rate	
	b	se	b	se	b	se
Panel A: CPV						
1 year	-0.0369	(0.0339)	-0.1500 ***	(0.0167)	0.2867 ***	(0.0676)
2 year	-0.055	(0.0420)	-0.1927 ***	(0.0195)	0.3575 ***	(0.0846)
3 year	-0.0369	(0.0386)	-0.1688 ***	(0.0193)	0.2571 ***	(0.0834)
4 year	-0.0348	(0.0466)	-0.2009 ***	(0.0236)	0.2841 ***	(0.1020)
5 year	-0.047	(0.0528)	-0.2304 ***	(0.0263)	0.3475 ***	(0.1140)
Panel B: V02						
1 year	0.0089	(0.0193)	-0.0774 ***	(0.0189)	0.1717 ***	(0.0451)
2 year	-0.0104	(0.0240)	-0.1119 ***	(0.0225)	0.1449 **	(0.0593)
3 year	-0.0084	(0.0207)	-0.1122 ***	(0.0181)	0.0328	(0.0531)
4 year	-0.014	(0.0243)	-0.1371 ***	(0.0208)	0.0126	(0.0627)
5 year	-0.0249	(0.0282)	-0.1491 ***	(0.0251)	0.021	(0.0727)
Panel C: FAVAR_CZ						
1 year	-0.0356	(0.0367)	-0.1512 ***	(0.0212)	0.3504 ***	(0.0801)
2 year	-0.0743	(0.0460)	-0.2031 ***	(0.0250)	0.4366 ***	(0.1054)
3 year	-0.0522	(0.0391)	-0.1676 ***	(0.0218)	0.3268 ***	(0.0938)
4 year	-0.027	(0.0488)	-0.2023 ***	(0.0269)	0.3530 ***	(0.1190)
5 year	-0.0304	(0.0566)	-0.2334 ***	(0.0314)	0.3826 **	(0.1407)
Panel D: FAVAR_Macro						
1 year	-0.0006	(0.0356)	-0.1307 ***	(0.0217)	0.2699 ***	(0.0924)
2 year	-0.012	(0.0453)	-0.1794 ***	(0.0246)	0.2925 **	(0.1225)
3 year	-0.0019	(0.0457)	-0.1767 ***	(0.0258)	0.2324 *	(0.1281)
4 year	-0.0097	(0.0550)	-0.2178 ***	(0.0298)	0.2774 *	(0.1544)
5 year	-0.0096	(0.0651)	-0.2538 ***	(0.0363)	0.2732	(0.1859)

Note: This table shows the estimated coefficient of the regressions of earnings forecasting errors on group-mean firm characteristics that are related to cross-section stock anomalies. The first column of the table denotes the horizon of the forecast. "BM", "Operating profit", "Asset growth rate" indicate the regressor is respectively the average BM ratio, operating profit and asset growth rate. The panels "CPV", "V02", "FAVAR_CZ", and "FAVAR_Macro" respectively report results when the cash flow news is constructed from the VAR model of the baseline model, Vuolteenaho (2002), the VAR model augmented by equity market factors, and the VAR model augmented by macroeconomic factors. The bootstrapping standard errors are reported in parentheses. "*", "**", and "***" denote that the coefficients are different from zero 0.1, 0.05, and 0.01 significance levels, respectively. The significant level is calculated with bootstrapping.

Table C1: Variables Definition

variables	**definition**
RoA Volatility	Rolling standard deviation of quarterly return on assets (roaq) over 4 years (minimum 2 years).
Earnings Streak Length	Number if consecutive 4-quarter increases in Income Before Extraordinary Items (the variable "ibq" in the Compustat database), up to 8.
Frazzini-Pedersen Beta	Using daily data, call tempRi the sum of today's return and the past 2 days, call tempRm the sum of today's mkt return and the past 2 days mkt return. Regress return on tempRi and tempRm using past 5 years of data, with a minimum of 3 years. BetaFP is the square root of the r square from this regression times the ratio of the idiosyncratic stock vol and the market vol.
EPS Forecast Dispersion	Standard deviation of earnings estimates scaled by mean earnings estimate.
Option to Stock Volume	Total monthly option volume (volume) over all puts and calls, divided by monthly stock trading volume (vol). Exclude if price less than 1 or share code (in CRSP database) greater than 11 or option volume or stock volume data are missing for the previous month.

Table D1: Variables used in extracting equity market factors

Variables	Description
bm	The ratio of book value to market value for the Dow Jones Industrial Average.
tbl	Treasury-bill rates.
ntis	Net Equity Expansion: The ratio of 12-month moving sums of net issues by NYSE listed stocks divided by the total end-of-year market capitalization of NYSE stocks.
infl	Inflation
eqis	Percent Equity Issuing: The ratio of equity issuing activity as a fraction of total issuing activity.
svar	Stock Variance: Computed as the sum of squared daily returns on the S&P 500.
excess_ret	Excess S&P 500 index returns, with dividends.
ep	Earnings Price Ratio: Earnings are 12-month moving sums of earnings on the S&P 500 index
tms	The Term Spread: The difference between the long-term yield on government bonds and the Treasury bill.
dfy	The Default Yield Spread: The difference between BAA and AAA-rated corporate bond yields
dfr	The Default Return Spread: The difference between long-term corporate bond and long-term government bond returns.
value_spread	Value Spread: The difference of equal-weighted returns between the decile with the highest BM and the decile with the lowest BM

Table E1: Variables used in extracting macro factors

Variables	Description
Panel A: Output and Income	
RPI	Real Personal Income
W875RX1	Real personal income ex transfer receipts
INDPRO	IP Index
IPFPNSS	IP: Final Products and Nonindustrial Supplies
IPFINAL	IP: Final Products (Market Group)
IPCONGD	IP: Consumer Goods
IPDCONGD	IP: Durable Consumer Goods
IPNCONGD	IP: Nondurable Consumer Goods
IPBUSEQ	IP: Business Equipment
IPMAT	IP: Materials
IPDMAT	IP: Durable Materials
IPNMAT	IP: Nondurable Materials
IPMANSICS	IP: Manufacturing (SIC)
IPB51222S	IP: Residential Utilities
IPFUELS	IP: Fuels
CUMFNS	Capacity Utilization: Manufacturing
Panel B: Consumption and Orders	
HOUST	Housing Starts: Total New Privately Owned
HOUSTNE	Housing Starts, Northeast
HOUSTMW	Housing Starts, Midwest
HOUSTS	Housing Starts, South
HOUSTW	Housing Starts, West
PERMIT	New Private Housing Permits (SAAR)
PERMITNE	New Private Housing Permits, Northeast (SAAR)
PERMITMW	New Private Housing Permits, Midwest (SAAR)
PERMITS	New Private Housing Permits, South (SAAR)
PERMITW	New Private Housing Permits, West (SAAR)
Panel C: Money and Credit	
M1SL	M1 Money Stock
M2SL	M2 Money Stock
M2REAL	Real M2 Money Stock
BOGMBASE	Monetary Base; Total
TOTRESNS	Total Reserves of Depository Institutions
NONBORRES	Reserves Of Depository Institutions
BUSLOANS	Commercial and Industrial Loans
REALLN	Real Estate Loans at All Commercial Banks
NONREVSL	Total Nonrevolving Credit
CONSPI	Nonrevolving consumer credit to Personal Income
DTCOLNVHFNM	Consumer Motor Vehicle Loans Outstanding
DTCTHFNM	Total Consumer Loans and Leases Outstanding
INVEST	Securities in Bank Credit at All Commercial Banks

Table E1: Variables used in extracting macro factors (Continue)

Panel D: Labor Market	
HWI	Help-Wanted Index for United States
HWIURATIO	Ratio of Help Wanted/No. Unemployed
CLF16OV	Civilian Labor Force
CE16OV	Civilian Employment
UNRATE	Civilian Unemployment Rate
UEMPMEAN	Average Duration of Unemployment (Weeks)
UEMPLT5	Civilians Unemployed - Less Than 5 Weeks
UEMP5TO14	Civilians Unemployed for 5-14 Weeks
UEMP15OV	Civilians Unemployed - 15 Weeks & Over
UEMP15T26	Civilians Unemployed for 15-26 Weeks
UEMP27OV	Civilians Unemployed for 27 Weeks and Over
CLAIMSx	Initial Claims
PAYEMS	All Employees: Total nonfarm
USGOOD	All Employees: Goods-Producing Industries
CES1021000001	All Employees: Mining and Logging: Mining
USCONS	All Employees: Construction
MANEMP	All Employees: Manufacturing
DMANEMP	All Employees: Durable goods
NDMANEMP	All Employees: Nondurable goods
SRVPRD	All Employees: Service-Providing Industries
USTPU	All Employees: Trade, Transportation & Utilities
USWTRADE	All Employees: Wholesale Trade
USTRADE	All Employees: Retail Trade
USFIRE	All Employees: Financial Activities
USGOVT	All Employees: Government
CES0600000007	Avg Weekly Hours : Goods-Producing
AWOTMAN	Avg Weekly Overtime Hours : Manufacturing
AWHMAN	Avg Weekly Hours : Manufacturing
CES0600000008	Avg Hourly Earnings : Goods-Producing
CES2000000008	Avg Hourly Earnings : Construction
CES3000000008	Avg Hourly Earnings : Manufacturing
Panel E: Orders and Inventories	
DPCERA3M086SBEA	Real personal consumption expenditures
CMRMTSPLx	Real Manu. and Trade Industries Sales
RETAILx	Retail and Food Services Sales
ACOGNO	New Orders for Consumer Goods
AMDMNOx	New Orders for Durable Goods
ANDENOx	New Orders for Nondefense Capital Goods
AMDMUOx	Unfilled Orders for Durable Goods
BUSINVx	Total Business Inventories
ISRATIOx	Total Business: Inventories to Sales Ratio
UMCSENTx	Consumer Sentiment Index

Table E1: Variables used in extracting macro factors (Continue)

Panel F: Interest rate and Exchange Rates	
FEDFUNDS	Effective Federal Funds Rate
CP3Mx	3-Month AA Financial Commercial Paper Rate
TB3MS	3-Month Treasury Bill
TB6MS	6-Month Treasury Bill
GS1	1-Year Treasury Rate
GS5	5-Year Treasury Rate
GS10	10-Year Treasury Rate
AAA	Moody's Seasoned Aaa Corporate Bond Yield
BAA	Moody's Seasoned Baa Corporate Bond Yield
COMPAPFFx	3-Month Commercial Paper Minus FEDFUNDS
TB3SMFFM	3-Month Treasury C Minus FEDFUNDS
TB6SMFFM	6-Month Treasury C Minus FEDFUNDS
T1YFFM	1-Year Treasury C Minus FEDFUNDS
T5YFFM	5-Year Treasury C Minus FEDFUNDS
T10YFFM	10-Year Treasury C Minus FEDFUNDS
AAAFFM	Moody's Aaa Corporate Bond Minus FEDFUNDS
BAAFFM	Moody's Baa Corporate Bond Minus FEDFUNDS
TWEXAFEGSMTHx	Trade Weighted U.S. Dollar Index: Major Currencies
EXSZUSx	Switzerland / U.S. Foreign Exchange Rate
EXJPUSx	Japan / U.S. Foreign Exchange Rate
EXUSUKx	U.S. / U.K. Foreign Exchange Rate
EXCAUSx	Canada / U.S. Foreign Exchange Rate
Panel G: Stock Market	
S&P 500	S&P's Common Stock Price Index: Composite
S&P: indust	S&P's Common Stock Price Index: Industrials
S&P div yield	S&P's Composite Common Stock: Dividend Yield
S&P PE ratio	S&P's Composite Common Stock: Price-Earnings Ratio
VIXCLSx	CBOE Volatility Index

Table E1: Variables used in extracting macro factors (Continue)

Panel H: Prices	
WPSFD49207	PPI: Finished Goods
WPSFD49502	PPI: Finished Consumer Goods
WPSID61	PPI: Intermediate Demand by Commodity Type: Processed Goods for Intermediate Demand
WPSID62	PPI: Intermediate Demand by Commodity Type: Unprocessed Goods for Intermediate Demand
OILPRICEx	Crude Oil, spliced WTI and Cushing
PPICMM	PPI: Metals and metal products
CPIAUCSL	CPI : All Items
CPIAPPSL	CPI : Apparel
CPITRNSL	CPI : Transportation
CPIMEDSL	CPI : Medical Care
CUSR0000SAC	CPI : Commodities
CUSR0000SAD	CPI : Durables
CUSR0000SAS	CPI : Services
CPIULFSL	CPI : All Items Less Food
CUSR0000SA0L2	CPI : All items less shelter
CUSR0000SA0L5	CPI : All items less medical care
PCEPI	Personal Cons. Expend.: Chain Index
DDURRG3M086SBEA	Personal Cons. Exp: Durable goods
DNDGRG3M086SBEA	Personal Cons. Exp: Nondurable goods
DSERRG3M086SBEA	Personal Cons. Exp: Services