Structural Reforms in Granular Economies*

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Abstract

This paper investigates how granularity affects the outcome of market reforms intended to reduce concentration and enhance competition. We use a baseline model of endogenous entry and exit by heterogeneous producers to show that the shape of the distribution of firm size matters for both the short- and long-term effects of product market reform. In turn, these affect the extent to which the economy is granular by altering the equilibrium number and size distribution of operating firms. When the economy is granular, reforms yield larger long-run gains, but at the cost of more short-run pain, than in a non-granular environment. By facilitating competitor entry that puts upward pressure on labor costs, reforms initially cause the cutoff productivity for producer activity to rise, and the number of operating firms to fall. As a result, the economy endogenously becomes more granular (market concentration rises) in the immediate aftermath of reforms. However, the number of producers gradually recovers, making the economy less granular over time. In the presence of idiosyncratic shocks, aggregate volatility generated by shocks to the largest producers temporarily rises before eventually decreasing in the long run. We conclude by providing suggestive empirical evidence in support of our theoretical results in cross-section regressions and a panel VAR for OECD countries.

JEL Codes: E32; L16; L51.

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1 Introduction

Structural reforms designed to increase market flexibility and enhance competition have become a standard ingredient of the diet recommended to ailing economies. Existing literature has emphasized the tradeoff between long-term gains and short-run pain from reforms and the importance of conditions at the time of their implementation in shaping the resulting dynamics. For instance, Cacciatore et al. (2015, 2016a, 2016b and 2017) show product and labor market reforms in different scenarios for business cycles and their interaction with macro policy.

This paper studies how the effects of product market reforms (reductions in barriers to producer entry) are affected by granularity (a fat-tailed distribution of firm size) as another important initial condition. Our interest in this issue has roots in the experiences of countries that implemented significant reforms: During the Asian financial crisis of 1997-98, the IMF provided bailout funds for crisis-hit countries, tying the funds to economic reforms (the "Structural Adjustment Package"). As shown in Figure 1, post-reform transition dynamics were costly, especially so in Korea, a prime example of granular economy. This motivates us to investigate the relation between the shape of the distribution of firm size and the effects of reforms.

Our results suggest that reforms in more granular economies are likely to be associated with larger long-run gain, but more short-run pain than those in less granular ones. In a granular economy, the average size of entrants is bigger given its fat-tailed distribution of firm size, and so is the marginal impact of new entry on firms' survival. Therefore, the product market reform leads to a larger reduction of the number of operating firms[1] as well as consumption at the immediate aftermath of reforms in the granular economy. However, since this relatively tougher survival condition enhances the average productivity of firms more in the economy, the productivity gain brings larger benefits over time as the total number of firms increases. An empirical study by Marrazzo and Terzi (2017), which suggests that emerging economies experience bigger short-term pain from their reforms but reap more benefits than advanced countries in the long-run, is very relevant to our result considering advanced countries are likely to be less granular than emerging ones.

More importantly, we shed light on the effect of the product market reform on the economy's granularity and further on aggregate volatility in response to idiosyncratic shocks. As Gabaix (2011) suggests, granularity requires fat-tailed distribution of firm size, and in this environment, idiosyncratic shocks to large firms have aggregate effects. Therefore, policy actions that affect the number and the size distribution of operating firms matter for the extent to which the economy is granular. Product

[1] We intentionally distinguish between operating firms and total firms in an economy. In our setup, only some fraction of firms operates at any period due to the existence of cutoff productivity under fixed operating costs.

market reform lowers entry barriers and induces an influx of new entrants, causing the economy less granular. On the other hand, it also creates fierce competition and makes less productive firms fall behind, leading to more concentration on larger and more productive firms. Depending on the magnitudes of these conflicting effects, the degree of granularity varies over time. We theoretically and empirically find out that the product market reform makes an economy become more vulnerable to idiosyncratic shocks in the short-run but more resistant to them in the long-run, and that the marginal impact of the reform on aggregate volatility is larger in a (more) granular economy regardless of time horizon. To show and support the results, we derive analytical formulas of the Herfindahl index, a typical measure of the degree of granularity, and the aggregate volatility and we also estimate a panel VAR for OECD countries. This is the first study which investigates the interrelationship between the product market reform and the granularity based on a full dynamic general equilibrium model with a continuum of heterogeneous firms.

The rest of the paper is organized as follows. Section 2 develops a simple theoretical framework as a baseline and Section 3 provides analytical solutions of the model as well as transition dynamics based on a calibrated model. Section 4 extends the baseline model with idiosyncratic shocks and Section 5 analyzes the impact of the product market reform on the aggregate volatility. Section 6 presents suggestive empirical findings, which support our theoretical study. Section 7 concludes.

2 The Basic Model

Our baseline model is set up by modifying the Ghironi and Melitz (2005) model. We simplify it as a closed economy version in order to focus on the pure effects of product market reforms and incorporate fixed production costs which enable endogenous changes in the firm-size distribution. In the absence of fixed production costs, every monopolistically competitive firm in a closed economy earns positive profits regardless of its own productivity, implying that all firms continue production unless they are hit by exogenous exit shocks. So, the degree of granularity of the model economy is exogenously pinned down by the shape parameter of the Pareto distribution, because the size distribution of firms is time-invariant. On the other hand, once we include fixed production costs, there exist some firms that are unable to cover their fixed costs and only firms productive enough operate. The cutoff productivity changes depending on overall economic conditions as well as policy actions, leading to endogenous changes in the size distribution of operating firms. Thus, the degree of granularity in our model can endogenously vary over time. This major difference from the Ghironi and Melitz (2005) plays a key role in analyzing the impact of product market reforms on the granularity. We basically follow the

setup in the Ghironi-Melitz (2005) except the above departures. Hence, this section focuses on the description of new features hereafter, and we refer readers to the article for details.

Household Preferences

The economy is populated by a unit mass of atomistic households. The representative household supplies one unit of labor inelastically in each period at the nominal wage rate W_t and maximizes the expected intertemporal utility from consumption:

$$E_t\left[\sum_{s=t}^{\infty}\frac{\beta^{s-t}C_s^{1-\gamma}}{1-\gamma}\right] \tag{1}$$

where $\beta \in (0,1)$ is the discount factor and $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution. The household consumes the basket of goods C_t defined over a continuum of goods Ω: $C_t = \left(\int_{\omega\in\Omega} c_t(\omega)^{(\theta-1)/\theta}d\omega\right)^{\theta/(\theta-1)}$, while only a subset of goods $\Omega_t \subset \Omega$ is available at any given time t, and $\theta > 1$ is the elasticity of substitution across goods. Let $p_t(\omega)$ denote the nominal price of each good $\omega \in \Omega_t$, then the consumption-based price index for the economy is $P_t = \left(\int_{\omega\in\Omega} p_t(\omega)^{1-\theta}d\omega\right)^{1/(1-\theta)}$, and the household's demand for a good ω becomes $c_t(\omega) = (p_t(\omega)/P_t)^{-\theta}C_t$.

Production

There is a continuum of monopolistically competitive and heterogeneous firms, and each firm produces a differentiated good $\omega \in \Omega_t$ with different technologies indexed by firm-specific productivity z. The firm-specific productivity z is determined upon entry and remains fixed thereafter. After paying a sunk entry cost[2] of $f_{E,t}$ effective labor units, firms draw their productivity level z from a time-invariant Pareto distribution G(z) with support on $[z_{min}, \infty)$ and shape parameter $\kappa > \theta - 1$. Then, the size distribution of firms also becomes Pareto, for the size of a firm depends on its productivity z. So, the firm sizes are increasingly concentrated toward their lower bound z_{min} as the exogenous shape parameter κ increases.

With labor as the only factor of production, a firm with productivity z produces $Z_t z$ units of output per worker, where Z_t denotes aggregate labor productivity. As stated in the Ghironi and Melitz (2005), heterogeneity in productivities across firms translate into heterogeneity in the unit labor costs of production. In addition to the labor cost, production incurs a fixed cost of $f_{X,t}$ units of consumption[3].

[2]We interpret changes in $f_{E,t}$ as changes in product market regulations facing firms in the economy.

[3]The fixed cost $f_{X,t}$ is set as units of consumption instead of effective labor units to match an important empirical

Endogenous changes in the size distribution of operating firms In the presence of fixed production costs, a firm with productivity z produces if and only if its profit $d_t(z)$ is non-negative. Specifically, only firms with productivity higher than a cutoff level, $z_t^c = \inf\{z : d_t(z) > 0\}$, operate and produce goods at any given time t. Otherwise, the firm is assumed to be switched off without losing its sunk investment. So, firms make decisions on whether to produce or not in every period and permanently exit only when they are hit by a "death" shock, that takes place with probability $\delta \in (0, 1)$. As a reminder, the productivity distribution of all firms is represented by G(z) with lower bound z_{min} in the Ghironi and Melitz (2005), implying the distribution is time-invariant and set exogenously by its parameters. However, in our model, the productivity distribution of operating firms endogenously varies with the time-varying lower bound z_t^c depending on economic conditions at every period as shown in Figure 6. This plays a crucial role in generating endogenous changes in the degree of granularity, which is a major departure from the Ghironi and Melitz (2005). In this setup, the number of operating firms at time t is given by a time-varying portion of all firms in the economy $N_{o,t} = N_t[1 - G(z_t^c)] = N_t(z_{min}/z_t^c)^\kappa$.

As for the entry, there exists an unbounded mass of prospective entrants who can anticipate the stream of expected future profits correctly. So, prospective entrants compare their expected post-entry value and the entry cost and enter the market until the value and the cost are equalized. This leads to the free entry condition $\tilde{e}_t = w_t f_{E,t}/Z_t$, where the expected post-entry value $\tilde{e}_t$ is defined by the present discounted value of the expected profit stream $\{\tilde{d}_s\}_{s=t+1}^{\infty}$:

$$\tilde{e}_t \equiv E_t \sum_{s=t}^{\infty} \beta_{t,s}(1-\delta)^{s-t}(1 - G(z_s^c))\tilde{d}_s \tag{2}$$

We assume that there is a one-period time-to-build lag for the entrants and that the exit shock occurs after entry. Thus, the number of total firms at time t obeys the following law of motion $N_t = (1-\delta)(N_{t-1} + N_{E,t-1})$.

Firms set their prices as the same proportional markup $\theta/(\theta-1)$ over marginal cost under constant elasticity θ. So, the real price of a good produced by a firm with productivity z is set as:

$$\rho_t(z) \equiv \frac{p_t(z)}{P_t} = \frac{\theta}{\theta - 1}\frac{w_t}{Z_t z} \tag{3}$$

regularity. If it is set as effective labor units like the sunk entry costs, the number of operating firms becomes less than the initial number after the product market reform even in the long-run, which is not consistent with data.

and its profit is given by:

$$d_t(z) = \frac{1}{\theta}\rho_t(z)^{1-\theta}Y_t^c - f_{X,t} \tag{4}$$

where aggregate demand for the consumption bundle $Y_t^c \equiv C_t + N_{o,t}f_{X,t}$ as the sum of household consumption and the use of the bundle by operating firms to cover fixed costs.

Firm averages are defined by following Melitz (2003). Since the productivity levels of operating firms at time t are distributed over $[z_t^c, \infty)$ given by G(z), the average productivity is defined as

$$\tilde{z}_t \equiv \left[\frac{1}{1-G(z_t^c)}\int_{z_t^c}^{\infty} z^{\theta-1}dG(z)\right]^{\frac{1}{\theta-1}} \tag{5}$$

and by letting $\nu \equiv \left[\kappa/(\kappa-\theta+1)\right]^{1/(\theta-1)}$, the average productivity is given by $\tilde{z}_t = \nu z_t^c$, which implies the average is proportional to the cutoff productivity. Then, the model is isomorphic to one where $N_{o,t}$ firms with the same productivity level $\tilde{z}$ produce, for the average summarizes all the relevance of the distribution of productivity levels for aggregate variables as shown in Melitz (2003). Next, the price index of the economy can be written as

$$1 = \tilde{\rho}_t^{1-\theta}N_{o,t} \tag{6}$$

where the average real price of products in the market is given by $\tilde{\rho}_t \equiv \rho_t(\tilde{z})$. Also, the average profit of operating firms is obtained with the average productivity as $\tilde{d}_t \equiv d_t(\tilde{z}_t)$.

Household Budget Constraint and Intertemporal Choices

The representative household enters period t with x_t shares in a mutual fund of firms. The mutual fund pays total profits from all the operating firms in each period, $N_{o,t}\tilde{d}_t$. During the period, the household purchases x_{t+1} mutual fund shares of all the incumbents and the new entrants, $N_t + N_{E,t}$. Even though only $N_{o,t+1}$ firms will operate and be able to pay dividends at time $t+1$, the household, who does not know which firms will survive from the exit shock and have higher productivity than the cutoff, must finance all the potential firms for next period's production. And the price of a claim to the future dividend payments of the mutual fund at time t is equal to the average price of claims

to the future stream of profits, $\tilde{e}_t$.

Then, the household's period budget constraint is given by

$$C_t + \tilde{e}_t(N_t + N_{E,t})x_{t+1} = \Big[\big(1 - G(z_t^c)\big)\tilde{d}_t + \tilde{e}_t\Big]N_t x_t + w_t \tag{7}$$

which implies the household's income consists of the mutual fund dividends from operating firms, the revenue from selling its initial share holdings, and the wage, while the household allocates the income between current and future consumption by investing in mutual fund shares carried into next period. The household maximizes its expected intertemporal utility (1) subject to the budget constraint (7). From the maximization problem, the Euler equation for shareholdings is given by

$$\tilde{e}_t = \beta(1-\delta)E_t\left[\Big(\frac{C_{t+1}}{C_t}\Big)^{-\gamma}(\tilde{e}_{t+1} + \Big(\frac{z_{min}}{z_{t+1}^c}\Big)^{\kappa}\tilde{d}_{t+1})\right] \tag{8}$$

of which forward iteration together with the assumption of the absence of speculative bubbles yields the equation (2).

Aggregate Accounting

Aggregating the budget constraint (7) across households and imposing equity market equilibrium ($x_{t+1} = x_t = 1$) yields [4] the following aggregate accounting equation as

$$Y_t^c + N_{E,t}\tilde{e}_t = w_t + N_{o,t}\tilde{d}_t \tag{9}$$

implying that aggregate consumption and investment must equal aggregate income from labor and profit in each period.

Summary

The main equilibrium conditions of the baseline model are summarized in Table 2. The system consists of 10 equations in 10 endogenous variables: w_t, $\tilde{e}_t$, $\tilde{d}_t$, N_t, $N_{E,t}$, $\tilde{z}_t$, z_t^c, Y_t^c, C_t, $\tilde{\rho}_t$. Also, the model features 3 exogenous variables: Z_t, $f_{E,t}$, $f_{X,t}$.

[4] We omit the transversality condition for mutual fund shares which is needed to ensure optimality.

The Herfindahl Index

To track changes in the degree of granularity after the reform in a rigorous way, we derive the Herfindahl index[5], the standard measure of market concentration and granularity. The Herfindahl index at any given time t obtained from the model is given by

$$HHI_t = \frac{\theta(\kappa - \theta + 1)}{\kappa - 2(\theta - 1)} \frac{f_{X,t}}{Y_t^c} \tag{10}$$

if $\kappa > 2(\theta - 1)$ holds. The above equation of the Herfindahl index implies that higher aggregate consumption demand is associated with lower Herfindahl index and that higher fixed costs are related to higher concentration. The former is in line with di Giovanni and Levchenko (2012), who find the inverse relationship between the country size and the degree of granularity. The latter is intuitive, for heavy fixed costs are likely to cause only relatively larger and more productive firms to survive in the market. Also, we can reconfirm that the degree of granularity is affected not only by exogenous setting (represented by the Pareto shape parameter κ which determines the distribution of possible productivity draws) but also by endogenous changes in the economy (represented by the aggregate consumption demand at the period).

Elasticities

By log-linearizing the model equations around the steady-state, we get the equations in Table 2. Then, the 10 log-linearized equations can be boiled down to the following 2 equations:

$$\hat{N}_{t+1} = -\delta(\psi-1)\left(\frac{\theta-2}{\theta-1}+\frac{1}{\kappa}\right)\hat{C}_t+\left(1-\delta+\frac{\delta(\psi-1)}{\kappa}\right)\hat{N}_t-\delta(\psi-1)\left(\frac{1}{\theta-1}-\frac{1}{\kappa}\right)\hat{f}_{X,t}+\delta\psi\hat{Z}_t-\delta\hat{f}_{E,t} \tag{11}$$

$$\begin{aligned}\left[\gamma - \frac{\kappa-(\theta-1)}{\kappa(\theta-1)}\right]\hat{C}_t = \left[\gamma - 1 + \beta(1-\delta)\frac{(\theta-1)(\kappa+1)-\kappa}{\kappa(\theta-1)}\right]E_t\hat{C}_{t+1} + \left[1-\beta(1-\delta)\frac{\kappa+1}{\kappa}\right]\hat{N}_{t+1} \\ +\frac{1}{\kappa}\hat{N}_t - \frac{\kappa-(\theta-1)}{\kappa(\theta-1)}\left(\hat{f}_{X,t} - \beta(1-\delta)\hat{f}_{X,t+1}\right) + \hat{f}_{E,t} - \beta(1-\delta)\hat{f}_{E,t+1}\end{aligned} \tag{12}$$

where ψ is defined by $\psi = \frac{1-\delta}{\alpha\delta} > 0$.

[5]The details of its derivation are provided in the Appendix A.

Under the following assumptions: 1)$Z_t = \phi_z Z_{t-1} + \epsilon_{Z,t}$, 2)$f_{E,t} = \phi_{f_E} f_{E,t-1}$, and 3)$f_{X,t} = \phi_{f_X} f_{X,t-1}$ where $\epsilon_{Z,t}$ is i.i.d., normal innovation with zero mean and variance $\sigma^2_{\epsilon_Z}$, the unique solution to the system (15)-(16) takes the form:

$$N_{t+1} = \eta_{NN} N_t + \eta_{NZ} Z_t + \eta_{Nf_E} f_{E,t} + \eta_{Nf_X} f_{X,t} \tag{13}$$

$$C_t = \eta_{CN} N_t + \eta_{CZ} Z_t + \eta_{Cf_E} f_{E,t} + \eta_{Cf_X} f_{X,t} \tag{14}$$

where each η_{xy} represents the elasticity of variable x with respect to variable y.

3 Analysis under the Baseline Model

Steady States

First, we consider the long-run effects of product market reforms from the new steady-state. The key variables in which we are interested are the total number of firms N, the number of operating firms N_o, the cutoff productivity z^c, the household consumption C, and the average profit of operating firms $\tilde{d}$. The steady-state levels of the variables can be derived analytically as displayed in Table 5. The signs of partial derivatives of them with respect to the policy variable, entry costs f_E, are as follows

$$\frac{\partial N}{\partial f_E} < 0, \quad \frac{\partial N_o}{\partial f_E} < 0, \quad \frac{\partial z^c}{\partial f_E} < 0, \quad \frac{\partial C}{\partial f_E} < 0, \quad \frac{\partial \tilde{d}}{\partial f_E} = 0 \tag{15}$$

which imply that lowering entry costs eventually leads to higher cutoff, a larger number of firms and more consumption.

The intuition is straightforward. The number of total firms N in the economy increases as new firms N_E enter following the product market deregulation (reduction in sunk entry costs f_E). Increased competition raises the cutoff z^c as well as the average productivity $\tilde{z}$. Despite the higher cutoff, the expansion of total firms N prevails in determining the number of operating firms N_o. Thus, more operating firms with higher productivities lead to increase in production, wage, and consumption C. Meanwhile, the average profit $\tilde{d}$ stands still because bigger aggregate consumption demands Y^c and smaller market share per firm offset each other. We can also find that the reform reduces the Herfindahl index in the long run, for the larger number of operating firms N_o prevails on higher cutoff z^c in inducing lower concentration.

Transition dynamics

Now, we analyze short- to medium-term dynamics triggered by the product market reform and compare the dynamics in a granular scenario to those in a non-granular one. As shown in Table 4, we log-linearize the system of equilibrium conditions around the unique steady-state. With properly calibrated parameters, we solve for the dynamic responses to an exogenous policy shock.

Calibration We interpret periods as quarters and calibrate parameters. Following the standard choices in the business cycle literature, we set the discount factor $\beta = .99$ and the risk aversion coefficient $\gamma = 2$. The exogenous exit shock δ is equal to .025 as set in Ghironi and Melitz (2005) to match the yearly 10 percent job destruction rate from the U.S. empirical data. We also use the elasticity of substitution $\theta = 3.8$ following Ghironi and Melitz (2005). As for the fixed operation costs $f_{X,t}$ and the sunk entry costs $f_{E,t}$, we follow Collard-Wexler (2013) which reports the ratio of entry costs to fixed production costs as around 4.5. Then, we normalize the Pareto support z_{min} and the aggregate productivity Z to 1. The calibration results are summarized in Table 6.

Figure 7 shows transitional changes in response to 1% permanent reduction in entry costs as percent deviations from steady-state. The changes in the cutoff productivity z^c and the number of firms N are the key to understand the mechanism. Lower entry barriers f_E induce new firms N_E to enter the market, and households have to cut their consumption spending for financing the new entrants. Together with the influx of new firms, the sudden decrease in the consumption demand makes firms' survival tougher, leading to the immediate jump of the cutoff z^c. Since entry costs and the time-to-build lag transform N into a state variable that behaves like a capital stock, the number of operating firms N_o decreases in the short-run because the increase in z^c is immediate whereas the increase in N is gradual. Hence, the economy experiences contraction as well as more concentration of economic activity (higher degree of granularity measured by the Herfindahl index) in the immediate aftermath of reform.

However, as N piles up over time, the previously contracted production and real wages begin to rebound. This raises aggregate consumption demand and slows down the increase in z^c. So, the number of operating firms N_o also rebounds and eventually exceeds the initial steady-state level. Finally, with more operating firms N_o with higher average productivity $\tilde{z}$, the economy reaps the gains of reforms in terms of more variety, higher GDP, and lower degree of granularity.

Next, Figure 8 compares the transitional changes in response to the same permanent deregulation under different degrees of granularity set by different values of the Pareto shape parameter κ. Since

an economy is granular if $\frac{\kappa}{\theta-1} < 2$ according to Gabaix (2011), we compare impulse responses under $\kappa = 2.9$ ($\frac{\kappa}{\theta-1} = 1.035$) and $\kappa = 11.2$ ($\frac{\kappa}{\theta-1} = 4$) as granular (red, solid line) and non-granular (blue, dotted line) cases respectively. Overall, the transition patterns are similar in both cases. But there are differences in the magnitude of the responses. What is notable here is that the different magnitudes generate crossings of the transition paths of the key variables in both cases: the number of operating firms, the consumption, and the Herfindahl index[6]. The crossings which occur around 4 years after the reform suggest that reforms in granular economies are likely to be associated with larger long-term gain, but also more acute short-term pain. The economic intuition is as follows. Given its fat-tailed distribution of firm size, the average size of entrants is bigger in a granular economy, and so is the marginal impact of new entry on firms' survival. It implies the initial jump of the cutoff productivity z_t^c is larger in the granular economy. Hence, the number of operating firms N_o declines more, which leads to more concentration (higher Herfindahl index) as well as larger reduction of consumption at the immediate aftermath of reforms compared to the non-granular economy.

However, as this relatively tougher survival condition enhances the average productivity $\tilde{z}$ of operating firms more in the granular economy, the productivity gain brings larger benefits over time as the total number of firms rises. Therefore, this bigger N_o and higher average productivity $\tilde{z}$ bring more expansionary effect, and market concentration falls more in the granular economy.

4 Model Extension

So far, we have studied how reforms are propagated and affect the distribution of firm size in absence of idiosyncratic volatility. It is necessary to understand how reforms can affect the environment in which idiosyncratic shocks may have aggregate effects. By affecting the number of operating firms N_o, cutoff productivity z^c, and the market concentration, reforms cause the extent to which the economy is granular to vary over time. Since the Herfindahl index initially rises before its eventual decrease, the impact of idiosyncratic shocks on the aggregate economy in a granular environment may initially be magnified before eventually becoming smaller. Next, we explore this intuition by extending the model to account for idiosyncratic volatility.

[6] As stated in the section 2, the Herfindahl index in our model can be defined only under non-granular ranges of κ (when $\frac{\kappa}{\theta-1} > 2$). Thus, we draw an alternative case (green, solid line) that is non-granular but still very close to granular instead of the granular case.

Idiosyncratic shock

To the baseline model, we incorporate an idiosyncratic shock $a_t(\omega)$, which is drawn from a time-invariant distribution at every period as in di Giovanni and Levchenko (2012). From now on, the productivity of a firm ω is time-varying, for it is determined not only by the firm-specific productivity $z(\omega)$ that is unchanging after drawn upon entry but also by the idiosyncratic shock $a_t(\omega)$ at time t. Then, the firm's profit-maximizing price is given by

$$\rho_t(\omega) = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t z(\omega) a_t(\omega)} \tag{16}$$

Also, we take the following assumptions in di Giovanni and Levchenko (2012):

Assumption 1: The marginal firm is small enough that it does not take the impact of its own realization of $a_t(\omega)$ on the total expenditure (X_t) and the price level (P_t) of the economy into consideration.

Assumption 2: The marginal firm treats X_t and P_t as fixed.

As stated in di Giovanni and Levchenko (2012), the second assumption, which is needed to take X_t and P_t out of the expectation operator, leads to simplicity in analyses as well as computation without affecting main results. It is particularly true in the model economy, where aggregate outcomes are driven not by marginal firms but by large ones.

Discretization

In extending the baseline model to incorporate the idiosyncratic shock, there is an important problem with the assumption of a continuum of firms. Under the assumption, the model economy has mathematically an infinite number of firms, and thus the effect of any idiosyncratic shocks on aggregate volatility will average out.

To avoid this problem while maintaining the advantages of a continuous setup, we discretize the model inspired by Al-Najjar (1995). First, we modify the productivity distribution as a truncated Pareto distribution with lower bound z_{min} and upper bound z_{max}. Then, the productivity of operating firms at time t is distributed over $[z_t^c,\ z_{max}]$ given by the distribution G(z). By partitioning the firms according to their firm-specific productivities with an interval defined by $i_t = (z_{max} - z_t^c)/\bar{N}$, the

mathematically infinite number of operating firms can be discretized into a very large, finite number $\bar{N}$ as shown in Figure (11). Each set of firms can be interpreted as a firm in which there is a set of product lines. Here, shocks are assumed to be idiosyncratic to each set. Then, the system is isomorphic to a model with a finite number $\bar{N}$ of firms.

Of course, one may ask why we do not model with a finite number of firms from the beginning. Our setup with a continuum of firms has some advantages in terms of model tractability and the derivations of the Herfindahl index and aggregate volatility as shown in Appendix A and B. For example, di Giovanni and Levchenko (2012) have to compute the Herfindahl index by repeating simulation exercises over 1,000 times and taking the median value of it. However, we can obtain the values of the Herfindahl index and the aggregate volatility from the theoretically derived solutions. So, our work provides a new, tractable way of granularity analysis, showing that one can analyze the effect of fat-tail distribution using a continuous model and that one can still study the relationship between idiosyncratic shocks and aggregate volatility by discretizing the framework properly.

Aggregate Volatility

Since the extended model becomes isomorphic to a model with $\bar{N}$ firms operating at any given time t under the above discretization, idiosyncratic shocks no longer average out but can generate aggregate fluctuations. Following Gabaix (2011) and di Giovanni and Levchenko (2012), the aggregate volatility is given by

$$\sigma^2_{GDP,t} = \sigma^2 \sum_{j=1}^{\bar{N}} \left(\frac{x_{j,t}}{X_t} \right)^2 \tag{17}$$

where σ is the standard deviation of the growth rate of individual firm sales, $x_{j,t}$ is the sales of the j-th firm, in other words, the j-th set of products, and $X_t \equiv P_t Y_t^c$ denotes the total sales of the economy at time t. From the equation of the demand for an individual product, the sale of a product ω within a set is given by

$$x_t(\omega) \equiv p_t(\omega) c_t(\omega) = \rho_t(\omega)^{1-\theta} X_t \tag{18}$$

and by applying the equation (16) of profit-maximizing price in the presence of idiosyncratic shocks, we obtain the market share of the product ω as

$$\frac{x_t(\omega)}{X_t} = \left(\frac{\theta}{\theta - 1}\frac{w_t}{Z_t z(\omega) a_t(\omega)}\right)^{1-\theta} \tag{19}$$

And under normalization of the idiosyncratic shocks such that $E_a[a^{\theta-1}] = 1$, the expected market share of the product ω is given by

$$E_a\left[\frac{x_t(\omega)}{X_t}\right] = \left(\frac{\theta}{\theta - 1}\frac{w_t}{Z_t z(\omega)}\right)^{1-\theta} \tag{20}$$

Then, we can calculate the market share of the j-th set of products by using the equation (20) and applying Jensen's inequality. So, the upper bound of expected aggregate volatility is given by

$$\sigma^2_{GDP,t} \;\leq\; \sigma^2\left(\frac{\theta f_{X,t}}{Y^c_t}\right)\left[\frac{\theta - \kappa - 1}{2(\theta - 1) - \kappa}\right]\left[\frac{(z_{max}/z^c_t)^{2(\theta-1)-\kappa} - 1}{(z_{max}/z^c_t)^{\theta-1-\kappa} - 1}\right] \tag{21}$$

as shown in Appendix B. The aggregate volatility is affected by exogenous parameters such as the Pareto shape parameter κ, and by endogenous changes in the cutoff productivity z^c_t and the real wage w_t. The derivatives with respect to κ, z^c_t, and w_t are

$$\frac{\partial \sigma_{GDP}}{\partial \kappa} < 0, \quad \frac{\partial \sigma_{GDP}}{\partial z^c_t} > 0 \quad \text{and} \quad \frac{\partial \sigma_{GDP}}{\partial w_t} < 0$$

The first derivative confirms the result of Gabaix (2011) that *ceteris paribus*, more granular economy (firm-size distribution is fatter-tailed) is more volatile. Figure (9) displays aggregate volatility rises exponentially as $\kappa/(\theta - 1)$ goes beyond the threshold of granularity. The second one with respect to the cutoff is also intuitive. As z^c_t increases, fewer firms can survive in the market, making the economy more granular and ultimately more volatile. As for the third derivative, a wage increase implies there are more firms operating and/or producing larger quantity in the economy. So, the wage increase is related to the decrease in granularity as well as volatility.

5 The Impact on Aggregate Volatility

Based on the extended model[7], we analyze the impact of the product market reform on the aggregate volatility of the economy. Figure 10 displays the impulse response of aggregate volatility (precisely, its upper bound) to the product market reform. The blue, dotted line is the case of $\kappa/(\theta-1) = 2.0035$, which is very close to the granular threshold[8], whereas the green, solid line is the case of $\kappa/(\theta-1) = 4$, which is far enough from the threshold. The former and the latter can be interpreted as relatively more and less concentrated economies, respectively.

As shown in the Figure, the product market reform causes the extent to which the economy is exposed to idiosyncratic shocks to vary over time. The impulse responses of aggregate volatility share the two main characteristics observed in those of the Herfindahl index under the baseline model: 1) an initial jump followed by a gradual decrease, and 2) larger short-term pain and long-term gain in the relatively more granular case, represented by the crossing of the impulse responses under different granularities. This confirms our intuition about the effect of the product market reform on the aggregate volatility in the baseline model.

The intuition behind the first characteristic is as follows. Aggregate consumption demand Y^c initially falls due to financing burden for new entrants, causing the cutoff productivity z_t^c to jump and the market share of large firms to rise. Thus, the economy becomes more volatile in the immediate aftermath of the reform. But over time, aggregate demand Y^c recovers as the economy grows thanks to gains of the reforms in the form of higher productivity and more production. So, the increase in the cutoff z_t^c is gradually subdued, whereas the real wage w_t rises under tighter labor market conditions. Therefore, the economy becomes more resistant to idiosyncratic shocks. The second characteristic, the marginal impacts of reform on the aggregate volatility are larger in a (more) granular economy regardless of time horizon, can be understood by the same intuition in the section 3 because the magnitudes of impulse responses of aggregate volatility depend on those of the cutoff z_t^c as well as the real wage w_t, which are already explained in the section.

[7]The main equilibrium conditions of the extended model are summarized in Table 3.

[8]There is a caveat here. If we compare the impulse responses of strictly granular and non-granular cases, the results do not show a crossing as in Figure 10. Even though the long-run characteristic, the bigger fall of aggregate volatility in a granular economy, is still preserved, the short-run one, the bigger rise of aggregate volatility in the granular country, no longer exists. This may stem from the fact that the signs of the powers of z_{max} and z_t^c in the equation (21) flip as κ moves across the granular threshold, $\kappa < 2(\theta-1)$. We believe that the comparison between the values that do not cause the sign of the power to flip is fair, and this is why we choose the value of κ in Figure 10.

6 Suggestive Empirical Evidences

Data Description

We use the OECD indicators of Product Market Regulation (PMR) from OECD Statistics database as our main indicator for product market regulation. The indicators are a comprehensive and internationally-comparable, and measure the regulatory and market environments in 34 OECD countries and in 13 non-OECD countries[9] including BRICs. Next, we calculate the Herfindahl-Hirschman index, another main indicator in this chapter as a measure of the degree of granularity, with the operating revenues of the top 25 firms in each country from the Mint Global database[10]. Other data: yearly GDP, GDP per capita and trade-to-GDP ratio, come from the World Development Indicators database, while quarterly GDP is from the OECD Statistics database.

An important caveat is that the available years of the two main indicators are limited. Since it is impossible to obtain credible time-series data of the Herfindahl index, we only have the index calculated with data at the latest available year of each firm, which is mostly 2017. Thus, the Herfindahl index can be regarded as the degree of market concentration as of 2017. Also, the OECD indicators of PMR are provided for the following 4 years only: 1998, 2003, 2008 and 2013, and available years vary across countries. To utilize all the available data, we take the average of each variable (except the Herfindahl index) over 1998-2013 and match the availability of the OECD indicators of PMR.

The Results of Empirical Analysis

First, we investigate the relationship between product market regulations, particularly legal barriers to entry, and the degree of granularity. As presented in Table 1, we regress the latest Herfindahl index against the legal barrier to entry which is a component of the OECD PMR indicators. Other variables such as trade-to-GDP ratio, GDP share in world GDP, and GDP per capita are also used following di Giovanni and Levchenko (2012). The coefficients of legal barriers to entry are significant under 5% or 10%, and R-squared of the regressions are over 0.34 in most cases, implying that if a country's legal barrier to entry is high, market concentration is likely to be high as well. Also, we find that the trade-to-GDP ratio is very significant mostly under 1% in line with di Giovanni and Levchenko (2012). Figure 2 is a partial regression plot from the regression in Table 1, displaying the relationship between the Herfindahl index and the average of the legal barriers to entry for a sample of 44 countries over the

[9]The countries are: Brazil, Bulgaria, China, Croatia, Cyprus, India, Indonesia, Latvia, Lithuania, Malta, Romania, Russia, and South Africa.

[10]The Mint Global is a streamlined version of the Orbis database and contains comprehensive information on 275 million companies worldwide, mainly medium and large-sized firms. Though the Orbis covers more small-sized firms, the Mint Global is good enough for obtaining data of top 25 firms and much cheaper.

period 1998-2013, after netting out the impact of trade-to-GDP ratio during the period. Intuitively, countries with higher entry barriers tend to be more concentrated.

As a logical consequence of the above result, if rigid product market regulation is likely to raise the degree of granularity, it may lead to higher volatility as well. Figure 3 depicts another partial regression plot showing the relationship between aggregate volatility and product market regulation after netting out the impact of country size in the sample of 42 OECD and non-OECD countries over the period 1998-2013. The volatility is defined by the standard deviation of quarterly GDP growth rate over the period, and the country size is the portion of the country in world GDP. The country size and the PMR indicator are averages over the same period and logged. As expected, this figure suggests that more product market regulations are likely to be associated with bigger aggregate fluctuation of the country. The result is significant with a t-statistic of the product market regulation variable as 2.19 and R-squared over 0.43.

Last, since our research questions are whether short- to long-term results of structural reforms are different according to the degrees of granularity of countries, and if so, how different the results will be, we empirically estimate the impacts of the product market reform using a panel VAR following Cacciatore and Fiori (2016). They estimate a panel VAR with harmonized annual data for the sample of 21 OECD countries[11] over the period 1982-2005. We extend their data set[12] by adding the standard deviation of quarterly GDP each year over the same period and divide the set into two according to the countries' Herfindahl indexes. So, we consider four macroeconomic variables - the unemployment rate, real GDP, real investment and GDP volatility - and OECD indicator of regulatory impediments, which reflects barriers to entry. Figure 4 and 5 present impulse responses following a one-standard deviation reduction in product market regulation with continuous lines representing median responses and dashed lines representing 68 percent confidence bands. The impulse responses of both groups display so-called "Short-term pain and Long-term gain" of structural reforms in the current literature. The product market reform leads to a decrease in output as well as an increase in volatility in the immediate aftermath of the reform, but eventually brings an upturn in output and a decline of volatility. The notable difference between the groups is that relatively more granular group is likely to experience a bigger negative impact in the short-run in terms of larger GDP drop and volatility rise, followed by bigger positive effects in the medium-term as shown in Figure 4 and 5. This finding sheds

[11] The countries are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

[12] Refer to the Appendix of Cacciatore and Fiori (2016) for data description except for the GDP volatility that is added in this paper.

light on the importance of granularity for short- to medium-term performances of structural reforms, which is the main idea of this chapter.

7 Conclusions

We studied the effect of the product market reform in relation to the granularity based on a full general equilibrium setup. First, the short- to long-term impacts of the reform on productivity, consumption, and output are affected by the initial degree of granularity. In line with the existing literature of structural reforms, the product market reform is beneficial in the long-run, but the benefits are accompanied by transition costs in the short-run. We further found out that the degree of granularity affects the magnitude of short-term pain as well as long-term gain, specifically, the marginal impact of the reform is larger in more granular economies than in less granular ones. On the other hand, the degree of granularity is also affected by the product market reform. The economy becomes more concentrated in the immediate aftermath of the reform before getting less concentrated eventually. So, the changes in the degree of granularity after the reform suggests that the impact of idiosyncratic shocks on the aggregate economy may initially be magnified before eventually becoming smaller.

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Table 1: OLS Regression Results

	(1)	(2)	(3)	(4)
Dep. Var: Log(Herfindahl index)				
Log(Legal barriers to entry)	0.536	0.866**	0.825*	0.802*
	(1.56)	(2.89)	(2.67)	(2.68)
Log(Trade-to-GDP ratio)		1.529***	1.715***	1.434**
		(4.26)	(3.67)	(3.98)
Log(GDP share)			0.0853	
			(0.63)	
Log(GDP per capita)				0.232
				(1.44)
Constant	5.451***	-1.104	-1.859	-2.952
	(29.05)	(-0.71)	(-0.94)	(-1.48)
Observations	44	44	44	44
R^2	0.0548	0.3450	0.3514	0.3774

Note: t statistics in parentheses. * significant at 10%, ** at 5%, *** at 1%

Table 2: Summary of the Basic Model

$$\tilde{e}_t = \beta(1-\delta)E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(\tilde{e}_{t+1} + \left(\frac{z_{min}}{z^c_{t+1}}\right)^{\kappa}\tilde{d}_{t+1})\right] \quad (1)$$

$$\tilde{e}_t = w_t f_{E,t}/Z_t \quad (2)$$

$$N_t = (1-\delta)(N_{t-1} + N_{E,t-1}) \quad (3)$$

$$\tilde{\rho}_t = \frac{\theta}{\theta-1}\frac{w_t}{Z_t\tilde{z}_t} \quad (4)$$

$$C_t + N_t\left(\frac{z_{min}}{z^c_t}\right)^{\kappa} f_{X,t} = w_t + N_t\left(\frac{z_{min}}{z^c_t}\right)^{\kappa}\tilde{d}_t - N_{E,t}\tilde{e}_t \quad (5)$$

$$\tilde{d}_t = \frac{1}{\theta}\tilde{\rho}_t^{1-\theta}Y^c_t - f_{X,t} \quad (6)$$

$$1 = \tilde{\rho}_t^{1-\theta}N_t\left(\frac{z_{min}}{z^c_t}\right)^{\kappa} \quad (7)$$

$$\tilde{z}_t = \nu z^c_t \quad (8)$$

$$z^c_t = \frac{\theta^{\frac{\theta}{\theta-1}}}{\theta-1}\frac{w_t}{Z_t}\left(\frac{f_{X,t}}{Y^c_t}\right)^{\frac{1}{\theta-1}} \quad (9)$$

$$Y^c_t = C_t + N_t\left(\frac{z_{min}}{z^c_t}\right)^{\kappa} f_{X,t} \quad (10)$$

Note: $C, Y^c, \tilde{\rho}, N, N_E, w, \tilde{d}, \tilde{e}, z^c, \tilde{z}$ are the 10 endogenous variables determined by these equations. Other variables in the table are determined as described in the text.

Table 3: Summary of the Extended Model

$$\tilde{e}_t = \beta(1-\delta)E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(\tilde{e}_{t+1} + \left\{\frac{\left(\frac{z_{min}}{z^c_{t+1}}\right)^{\kappa} - \left(\frac{z_{min}}{z_{max}}\right)^{\kappa}}{1-\left(\frac{z_{min}}{z_{max}}\right)^{\kappa}}\right\}\tilde{d}_{t+1})\right] \tag{1}$$

$$\tilde{e}_t = w_t f_{E,t}/Z_t \tag{2}$$

$$N_t = (1-\delta)(N_{t-1} + N_{E,t-1}) \tag{3}$$

$$\tilde{\rho}_t = \frac{\theta}{\theta-1}\frac{w_t}{Z_t\tilde{z}_t} \tag{4}$$

$$C_t + N_t\left[\frac{\left(\frac{z_{min}}{z^c_t}\right)^{\kappa} - \left(\frac{z_{min}}{z_{max}}\right)^{\kappa}}{1-\left(\frac{z_{min}}{z_{max}}\right)^{\kappa}}\right]f_{X,t} = w_t + N_t\left[\frac{\left(\frac{z_{min}}{z^c_t}\right)^{\kappa} - \left(\frac{z_{min}}{z_{max}}\right)^{\kappa}}{1-\left(\frac{z_{min}}{z_{max}}\right)^{\kappa}}\right]\tilde{d}_t - N_{E,t}\tilde{e}_t \tag{5}$$

$$\tilde{d}_t = \frac{1}{\theta}\tilde{\rho}_t^{1-\theta}Y_t^c - f_{X,t} \tag{6}$$

$$1 = \tilde{\rho}_t^{1-\theta}N_t\left[\frac{\left(\frac{z_{min}}{z^c_t}\right)^{\kappa} - \left(\frac{z_{min}}{z_{max}}\right)^{\kappa}}{1-\left(\frac{z_{min}}{z_{max}}\right)^{\kappa}}\right] \tag{7}$$

$$\tilde{z}_t = \left[\left(\frac{\kappa}{\theta-\kappa-1}\right)\left(\frac{(z_{max})^{\theta-\kappa-1} - (z^c)^{\theta-\kappa-1}}{(z^c)^{-\kappa} - (z_{max})^{-\kappa}}\right)\right]^{\frac{1}{\theta-1}} \tag{8}$$

$$z_t^c = \frac{\theta^{\frac{\theta}{\theta-1}}}{\theta-1}\frac{w_t}{Z_t}\left(\frac{f_{X,t}}{Y_t^c}\right)^{\frac{1}{\theta-1}} \tag{9}$$

$$Y_t^c = C_t + N_t\left[\frac{\left(\frac{z_{min}}{z^c_t}\right)^{\kappa} - \left(\frac{z_{min}}{z_{max}}\right)^{\kappa}}{1-\left(\frac{z_{min}}{z_{max}}\right)^{\kappa}}\right]f_{X,t} \tag{10}$$

Note: $C, Y^c, \tilde{\rho}, N, N_E, w, \tilde{d}, \tilde{e}, z^c, \tilde{z}$ are the 10 endogenous variables determined by these equations. Other variables in the table are determined as described in the text.

Table 4: Log-Linearized System

$$\hat{e}_t = E_t\Big[-\gamma(\hat{C}_{t+1} - \hat{C}_t) + \beta(1-\delta)\hat{e}_{t+1} + (1-\beta(1-\delta))\hat{d}_{t+1} - \kappa(1-\beta(1-\delta))\hat{z}^c_{t+1}\Big] \quad (1)$$

$$\hat{e}_t = \hat{w}_t + \hat{f}_{E,t} - \hat{Z}_t \quad (2)$$

$$\hat{N}_t = (1-\delta)\hat{N}_{t-1} + \delta\hat{N}_{E,t-1} \quad (3)$$

$$\hat{\rho}_t = \hat{w}_t - \hat{z}_t - \hat{Z}_t \quad (4)$$

$$\hat{Y}^c_t = \frac{\mathsf{w}}{\mathsf{Y^c}}\hat{w}_t + \frac{\mathsf{N}\tilde{\mathsf{d}}}{\mathsf{Y^c}}\Big(\frac{z_{min}}{\mathsf{z^c}}\Big)^{\kappa}(\hat{N}_t + \hat{d}_t - \kappa\hat{z}^c_t) - \frac{\mathsf{N_E}\tilde{\mathsf{e}}}{\mathsf{Y^c}}(\hat{N}_{E,t} + \hat{e}_t) \quad (5)$$

$$\hat{d}_t = \frac{\tilde{\mathsf{p}}^{1-\theta}\mathsf{Y^c}}{\theta\tilde{\mathsf{d}}}((1-\theta)\hat{\rho}_t + \hat{Y}^c_t) - \frac{f_X}{\tilde{\mathsf{d}}}\hat{f}_{X,t} \quad (6)$$

$$(1-\theta)\hat{\rho}_t + \hat{N}_t = \kappa\hat{z}^c_t \quad (7)$$

$$\hat{z}_t = \hat{z}^c_t \quad (8)$$

$$\hat{z}^c_t = \hat{w}_t - \hat{Z}_t + \frac{1}{\theta-1}(\hat{f}_{X,t} - \hat{C}_t) \quad (9)$$

$$\hat{Y}^c_t = \frac{\mathsf{C}}{\mathsf{Y^c}}\hat{C}_t + \frac{\mathsf{N}f_X}{\mathsf{Y^c}}\Big(\frac{z_{min}}{\mathsf{z^c}}\Big)^{\kappa}(\hat{N}_t + \hat{f}_{X,t} - \kappa\hat{z^c_t}) \quad (10)$$

Note: Sans-serif fonts represent steady-states while "hat" does log deviations from them.

Table 5: Steady States

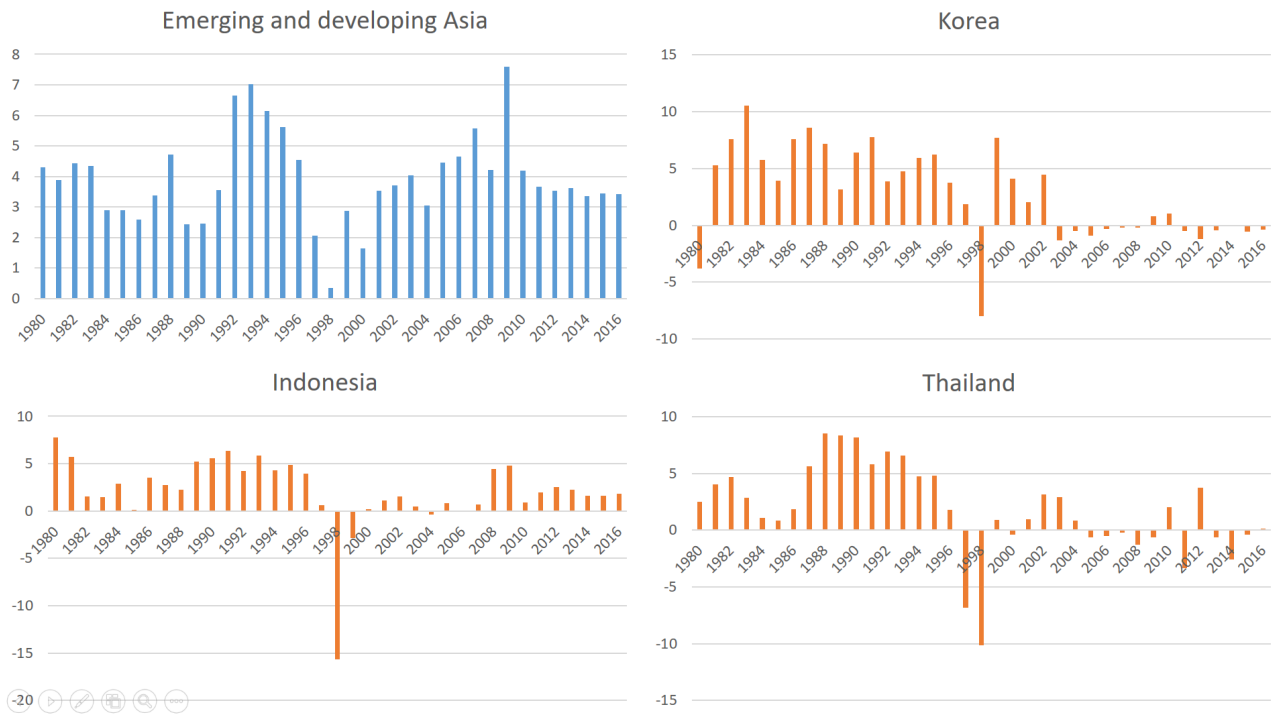
Variables	Steady states	Partial derivatives
$\tilde{d}$	$(\nu^{\theta-1}-1)f_X$	$\frac{\partial \tilde{d}}{\partial f_E}=0$
N	$\frac{\alpha Z}{f_E}$	$\frac{\partial N}{\partial f_E}<0$
N_o	$\left[(\alpha Z)^{\kappa+1}\frac{\zeta^{\kappa}}{f_X^{\kappa}f_E}\right]^{\frac{\theta-1}{\kappa(\theta-2)+(\theta-1)}}$	$\frac{\partial N_o}{\partial f_E}<0$
z^c	$\left[\frac{f_X^{\theta-1}}{\alpha\zeta^{\theta-1}f_E^{\theta-2}Z}\right]^{\frac{1}{\kappa(\theta-2)+(\theta-1)}}$	$\frac{\partial z^c}{\partial f_E}<0$
C	$(\theta\nu^{\theta-1}-1)\left[\frac{\zeta^{\kappa(\theta-1)}(\alpha Z)^{(\kappa+1)(\theta-1)}}{f_E^{\theta-1}f_X^{\kappa-\theta+1}}\right]^{\frac{1}{\kappa(\theta-2)+(\theta-1)}}$	$\frac{\partial C}{\partial f_E}<0$

Note: $\alpha=\frac{(1-\nu^{1-\theta})\beta(1-\delta)}{\theta(1-\beta(1-\delta))-(1-\beta)(1-\nu^{1-\theta})}>0$, $\zeta=\left(\frac{\theta-1}{\theta}\right)\left(\frac{1-\beta(1-\delta)}{\beta(1-\delta)}\right)\left(\frac{\nu}{\nu^{\theta-1}-1}\right)>0$

Table 6: Calibration

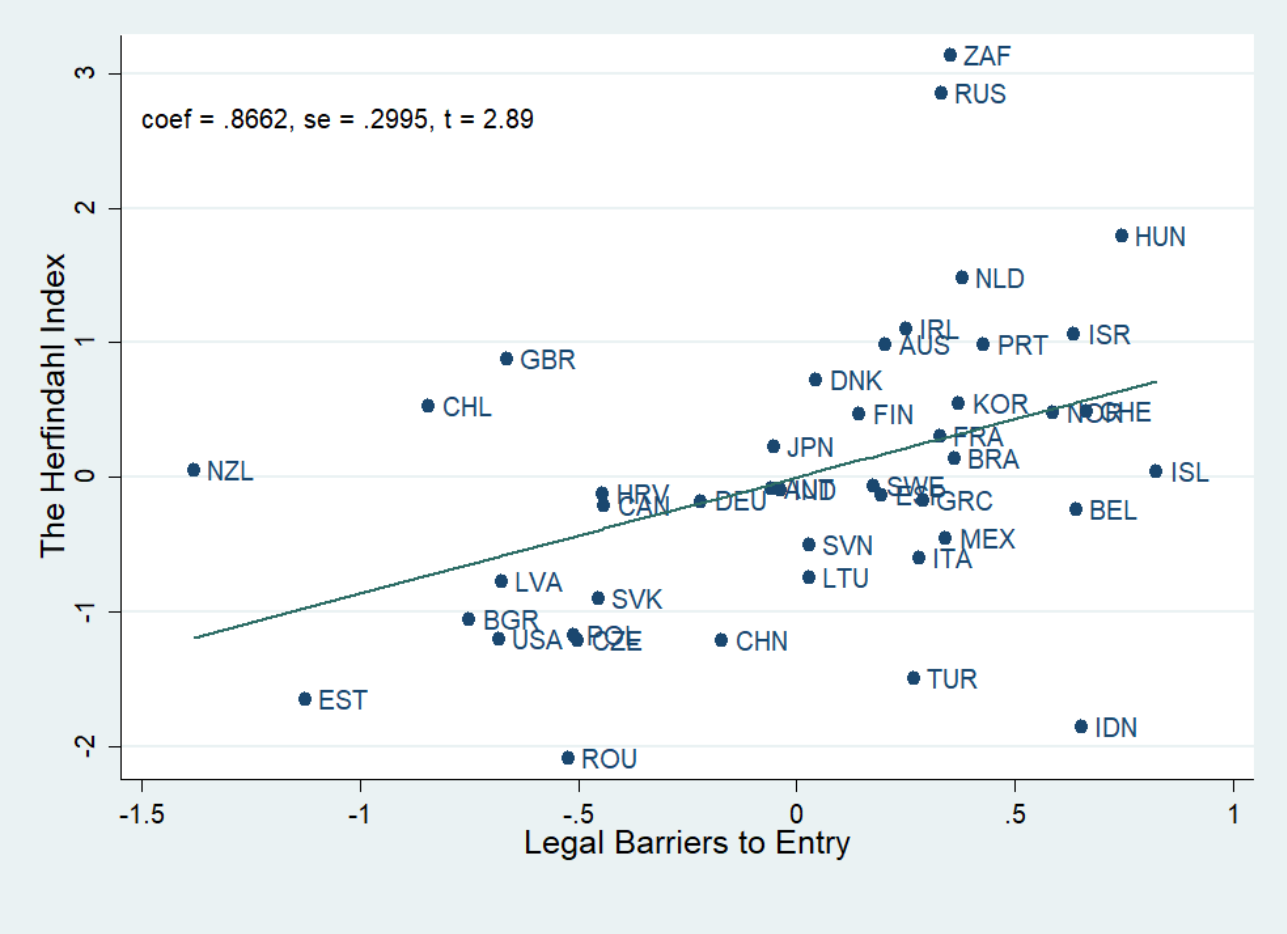
Parameter	Symbol	Value	Source
Elasticity of substitution	θ	3.8	Ghironi and Melitz (2005)
Risk aversion	γ	2	Standard in the literature
Discount factor	β	0.99	”
Exogenous producer exit probability	δ	0.025	”
Std. deviation of firm sales' growth rate	σ	0.1	di Giovanni and Levchenko (2012)
Fixed production cost	f_X	0.27	Collard-Wexler (2013)
Producer entry cost	f_E	1	Normalization
Pareto support	z_{min}	1	”
Aggregate productivity	Z	1	”

Figure 1: Growth Differentials before and after the Asian Crisis



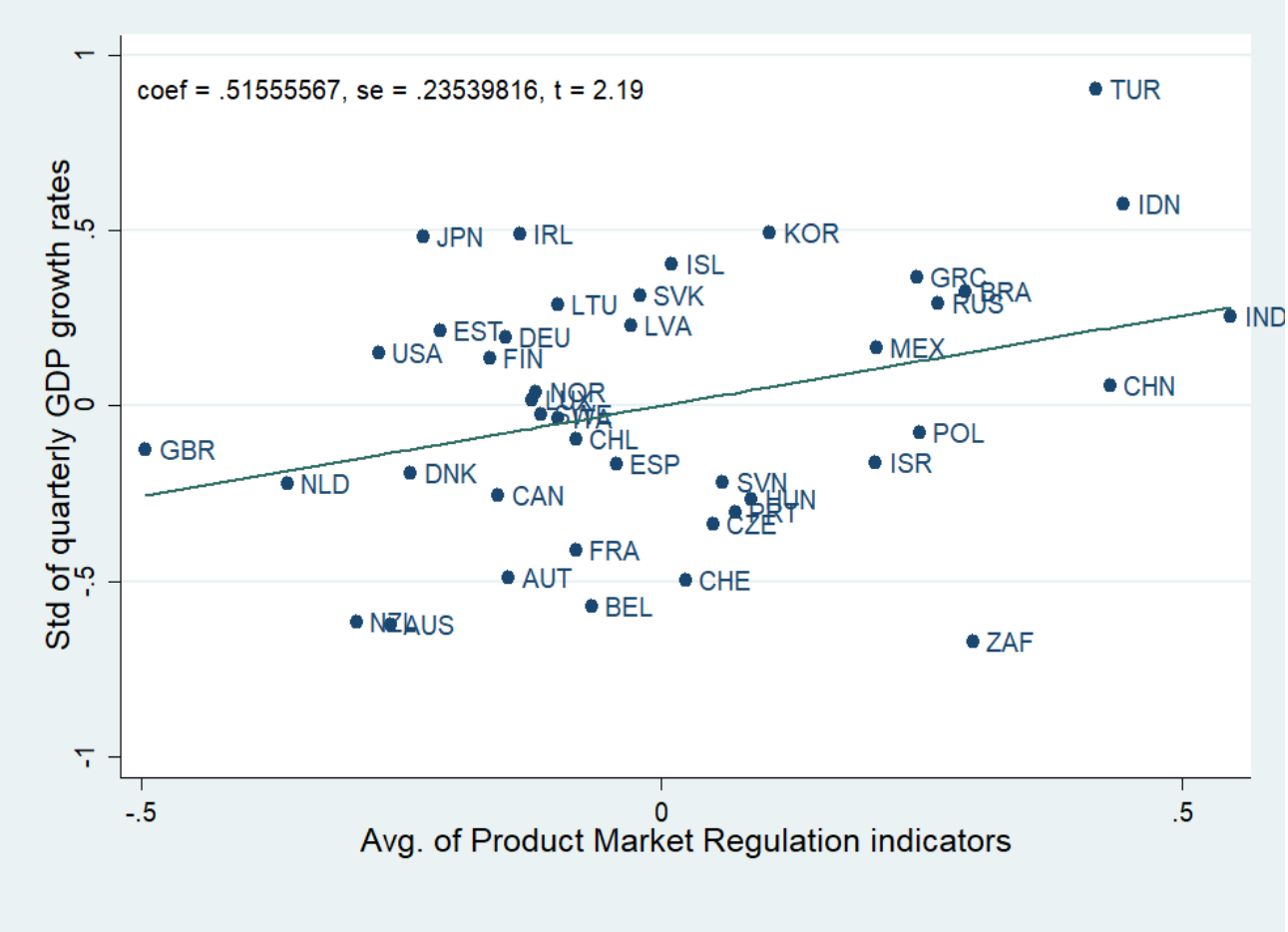
Notes: This figure shows the differentials of real GDP growth rates between the country and the world. (ex: differential (%) = "given year's GDP growth rate of Korea" − "that of the World"). A positive value of the differential implies that the country or region outperforms the world average in the year. Data source: The IMF World Economic Outlook database.

Figure 2: Legal Barriers to Entry and Market Concentration



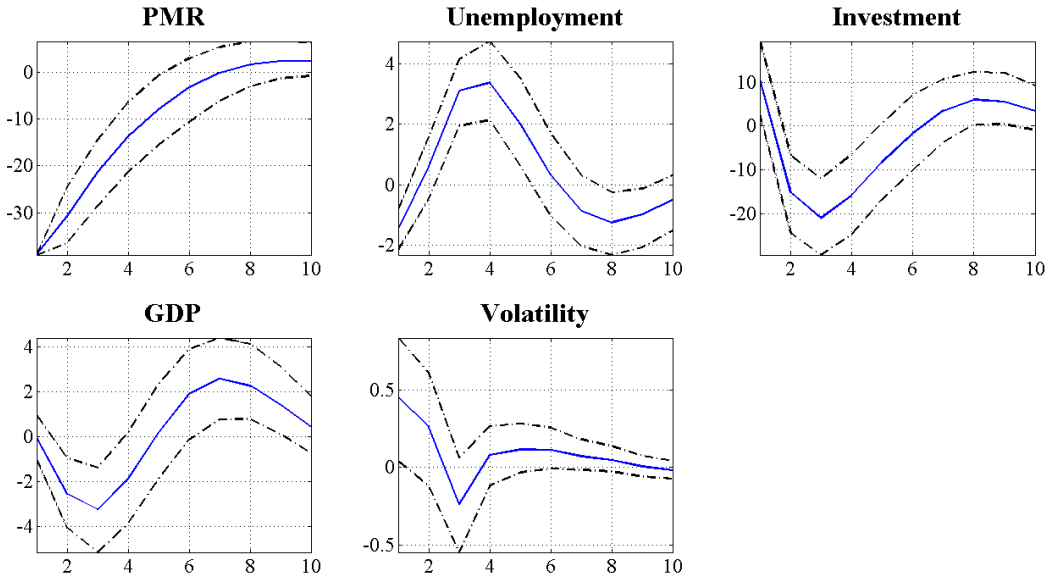
Notes: This is a partial regression plot showing the relationship between legal barriers to entry and the Herfindahl market concentration index in 44 countries. Both axes are in log scale. Sources: the OECD, the World Bank and the Mint Global.

Figure 3: Product Market Regulation and Volatility



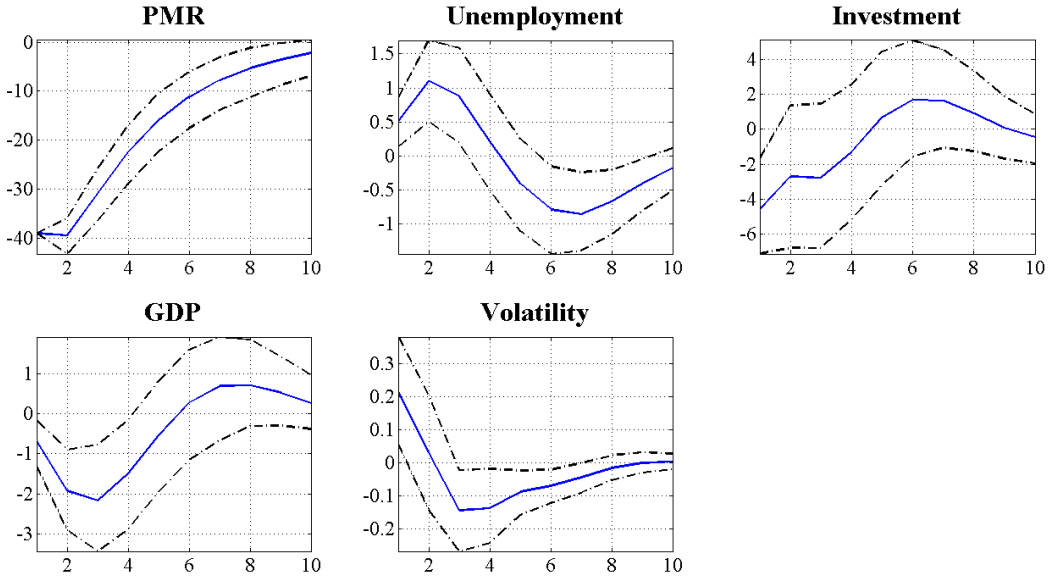
Notes: This figure reports the partial regression plot of aggregate volatility, measured as the standard deviation of the quarterly growth rate of GDP over 1998-2013, on the y-axis against the average of product market regulation index over the same period on the x-axis, after netting out the impact of country size (GDP share in world GDP). Data source: The OECD Statistics database.

Figure 4: Impulse Responses in Relatively More Granular Countries



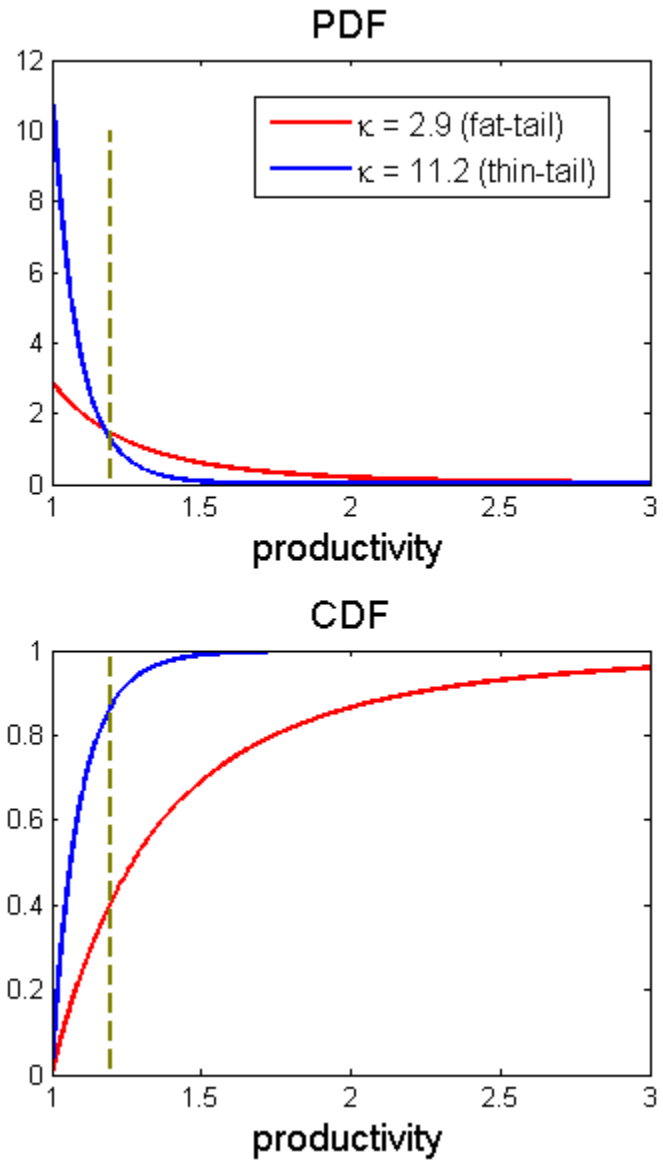
Notes: Panel VAR, impulse responses to Product Market Regulation (PMR) shocks. GDP and Investment are in percent from baseline; Unemployment rate and volatility are in deviations from baseline. Data source: Cacciatore and Fiori (2016) and the OECD Statistics database

Figure 5: Impulse Responses in Relatively Less Granular Countries



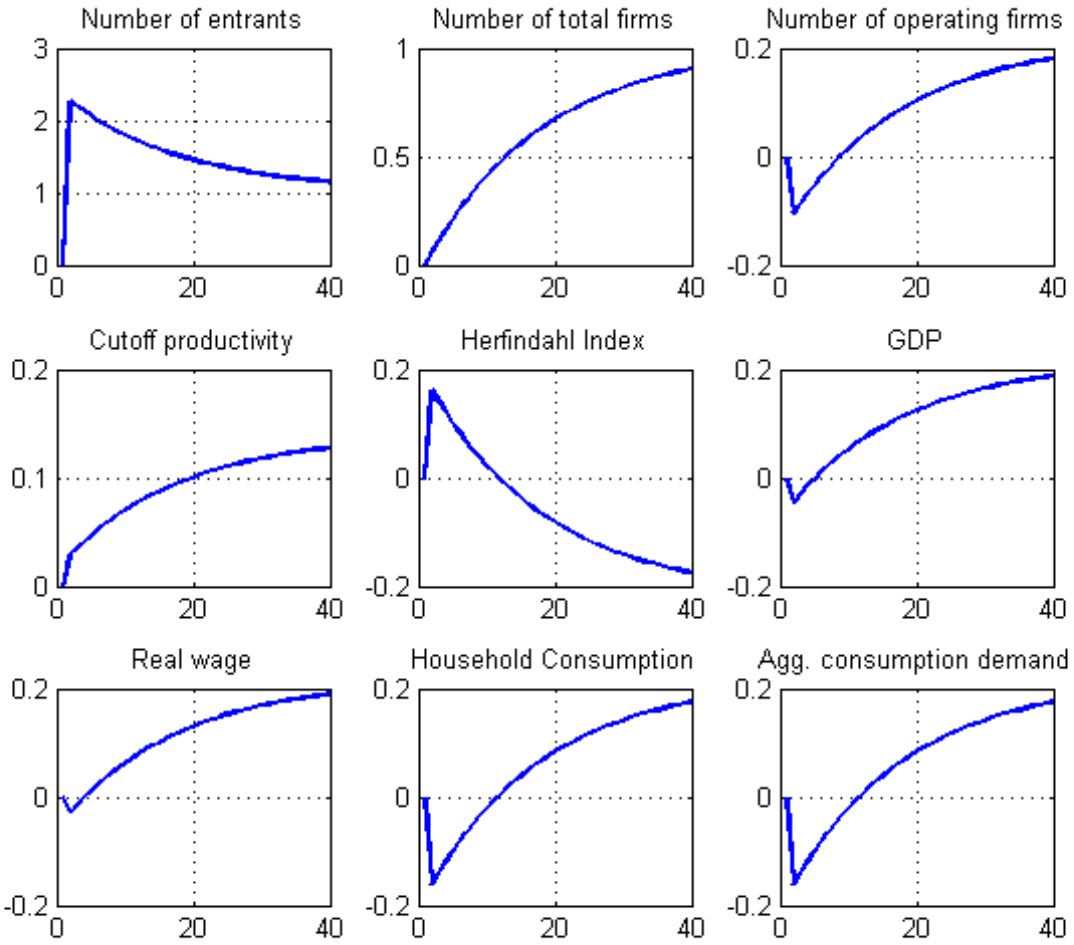
Notes: Panel VAR, impulse responses to Product Market Regulation (PMR) shocks. GDP and Investment are in percent from baseline; Unemployment rate and volatility are in deviations from baseline. Data source: Cacciatore and Fiori (2016) and the OECD Statistics database

Figure 6: PDF and CDF of Pareto Distribution



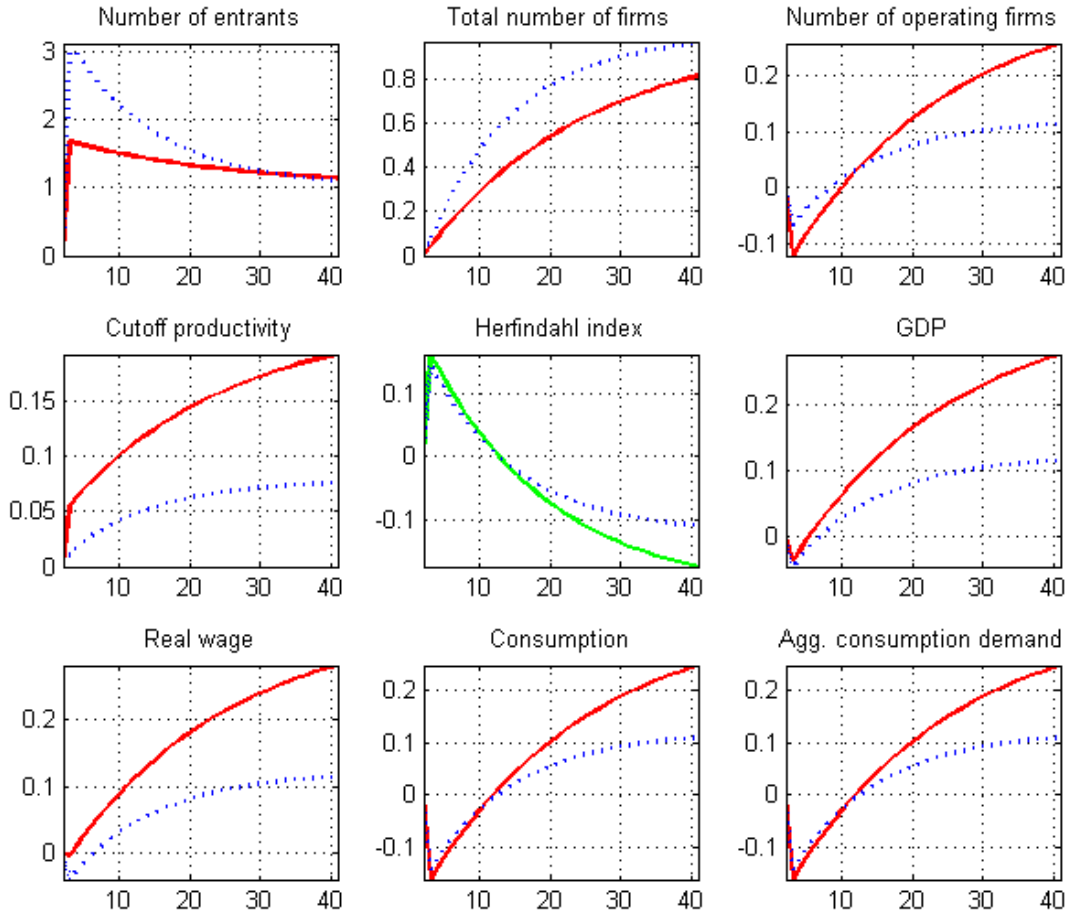
Notes: Blue(non-granular), Red(granular), and Yellow(cutoff productivity) under the value of $\theta = 3.8$.

Figure 7: Impulse Response to the Permanent Reduction in Entry costs



Notes: This IRF is under the value of $\kappa = 5.63$, and variables are in percentage deviations from the steady-state.

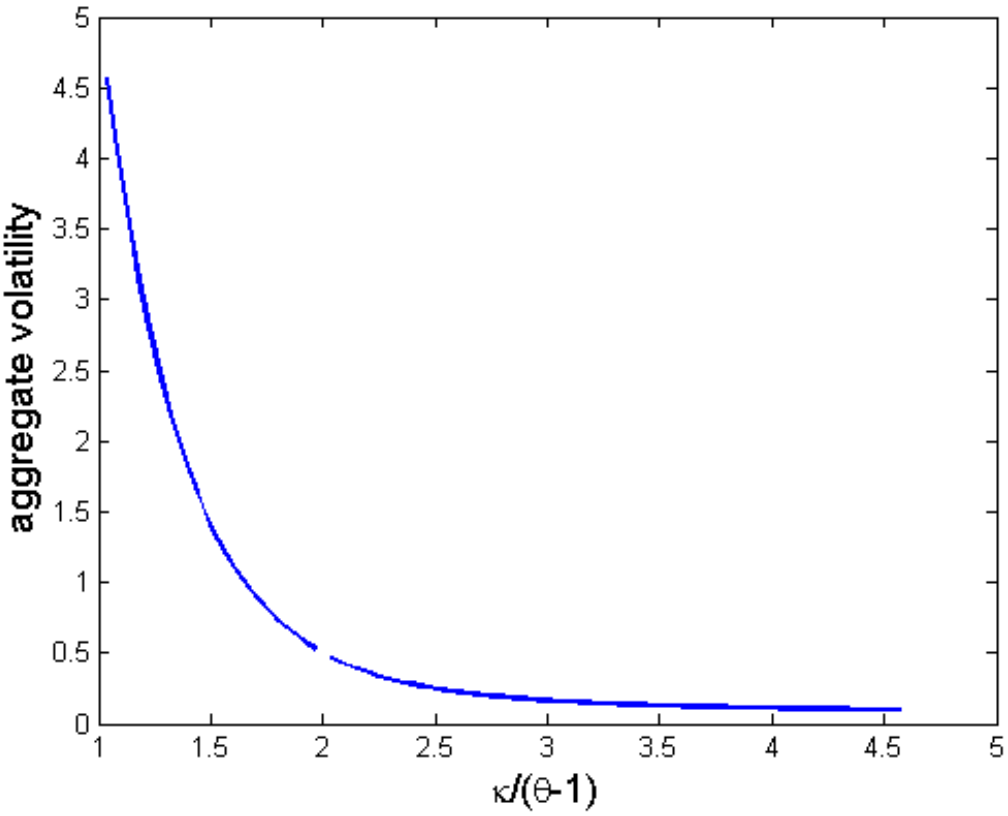
Figure 8: Impulse Response under Different Degrees of Granularity



Notes: Red(granular: $\kappa = 2.9$), Blue(non-granular: $\kappa = 11.2$), and Green(non-granular but very close to granular threshold: $\kappa = 5.63^a$). Variables are in percentage deviations from the steady-state.

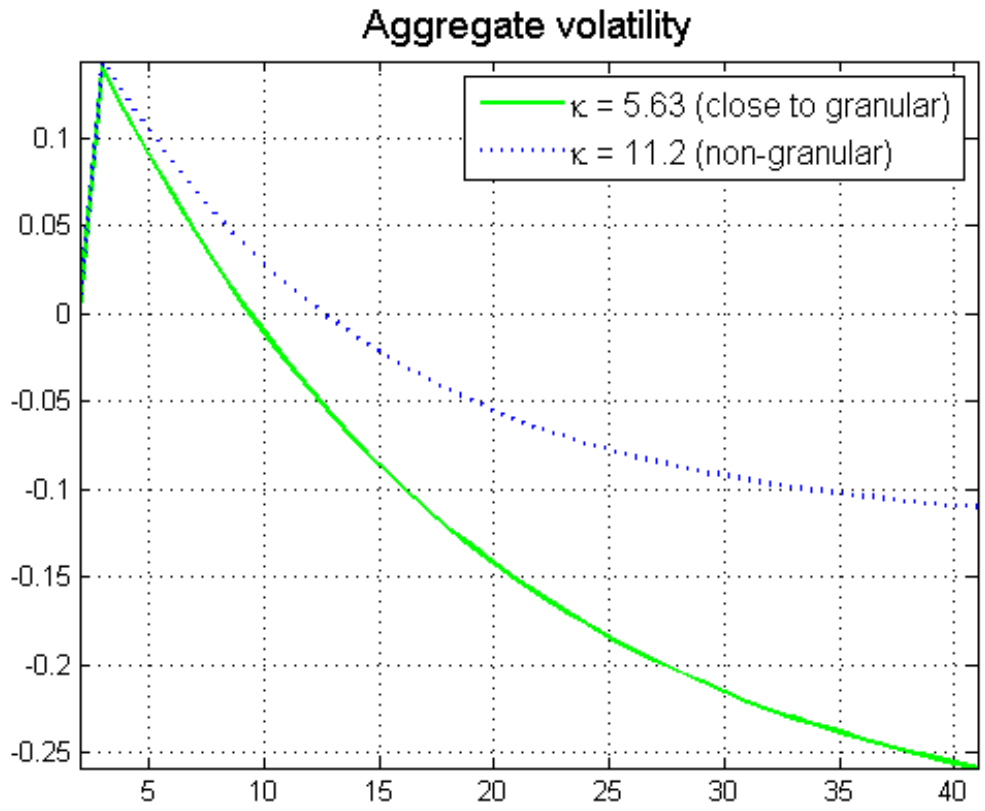
[a]Since the Herfindahl index in the model is not defined if the value of κ is within the granular range, we use a value beyond the granular threshold.

Figure 9: Aggregate Volatility under Different Values of Shape Parameter κ



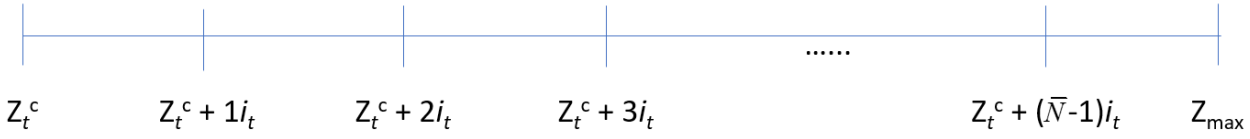
Notes: This figure displays volatilities calculated according to the equation (21) under different values of Pareto shape parameter κ. The granular threshold is $\kappa/(\theta-1)=2$.

Figure 10: Impulse Response of Aggregate Volatility to the Product Market Reform



Notes: Blue(non-granular) and Green(non-granular but very close to granular threshold). Variable on the y-axis is percentage deviation from the steady-state.

Figure 11: Concept of Discretization



Structural Reforms in Granular Economies: Appendix

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A. Derivation of the Herfindahl index

The Herfindahl index (HHI) is the sum of the squared market share of all firms in an economy. Under the assumption of a continuum of firms, the HHI of the baseline model economy at any given time t is defined by

$$HHI_t \equiv \int_\omega \left(\frac{x_t(\omega)}{X_t}\right)^2 d\omega \tag{1}$$

where $x_t(\omega)$ and X_t are the sale of an individual firm ω and the total sales of the economy, respectively. Since the demand for a firm ω's good is given by $y_t(\omega) = (p_t(\omega)/P_t)^{-\theta} Y_t^c$, the sale of the firm ω is

$$x_t(\omega) = \left(\frac{p_t(\omega)}{P_t}\right)^{1-\theta} X_t \tag{2}$$

where $x_t(\omega) \equiv p_t(\omega) y_t(\omega)$ and $X_t \equiv P_t Y_t^c$. The, the market share of a firm ω is simply $x_t(\omega)/X_t = \rho_t(\omega)^{1-\theta}$. By using the equation of profit-maximizing price (**??**), the market share can be rewritten as

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$$\frac{x_t(\omega)}{X_t} = \left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t z(\omega)}\right)^{1-\theta} \tag{3}$$

and the HHI is given by

$$HHI_t = \int_\omega \left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t z(\omega)}\right)^{2(1-\theta)} d\omega \tag{4}$$

Then, by using the fact that the sum is equal to the product of the average and the total number, the HHI can be calculated by

$$HHI_t = N_{o,t}\left[\frac{1}{1-G(z_t^c)}\int_{z_t^c}^{\infty}\left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t z}\right)^{2(1-\theta)} dG(z)\right] \tag{5}$$

and it can be rearranged as

$$HHI_t = N_{o,t}\left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t}\right)^{2(1-\theta)}\left[\frac{1}{1-G(z_t^c)}\int_{z_t^c}^{\infty} z^{2(\theta-1)} dG(z)\right] \tag{6}$$

First, the square bracket on the right-hand side of (6) can be solved

$$\begin{aligned}
\frac{1}{1-G(z_t^c)}\int_{z_t^c}^{\infty} z^{2(\theta-1)} dG(z) &= \left(\frac{z_t^c}{z_{min}}\right)^{\kappa}\int_{z_t^c}^{\infty} z^{2(\theta-1)}\frac{\kappa z_{min}^{\kappa}}{z^{\kappa+1}} dz \\
&= \kappa (z_t^c)^{\kappa}\int_{z_t^c}^{\infty}\frac{z^{2(\theta-1)}}{z^{\kappa+1}} dz \\
&= \kappa (z_t^c)^{\kappa}\int_{z_t^c}^{\infty} z^{2(\theta-1)-\kappa-1} dz \\
&= \frac{\kappa (z_t^c)^{\kappa}}{2(\theta-1)-\kappa} z^{2(\theta-1)-\kappa}\Big|_{z_t^c}^{\infty}
\end{aligned}$$

and it boils down to the following equation under an additional assumption on the range of $\kappa > 2(\theta-1)$[1]

[1] If the shape parameter of Pareto distribution falls in the granular range, $\kappa < 2(\theta-1)$, it goes to infinity. So, this calculation of the Herfindahl index can only be applied to non-granular cases. This caveat stems from the assumption of the infinite upper bound of the Pareto distribution used in the baseline model, which is impossible in real world.

$$\frac{1}{1-G(z_t^c)}\int_{z_t^c}^{\infty} z^{2(\theta-1)}dG(z) = \frac{\kappa}{\kappa-2(\theta-1)}(z_t^c)^{2(\theta-1)} \tag{7}$$

Second, by using the equation (3) and (6) in the baseline model, the rest of the equation (6) can be rearranged

$$\begin{aligned} N_{o,t}\left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t}\right)^{2(1-\theta)} &= N_{o,t}\left[\tilde{\rho}^{1-\theta}\tilde{z}_t^{1-\theta}\left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t}\right)^{1-\theta}\right] \\ &= \tilde{z}_t^{1-\theta}\left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t}\right)^{1-\theta} \\ &= (\nu z_t^c)^{1-\theta}\left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t}\right)^{1-\theta} \end{aligned}$$

and from the definition of ν, we get the following

$$N_{o,t}\left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t}\right)^{2(1-\theta)} = \frac{\kappa-(\theta-1)}{\kappa}(z_t^c)^{1-\theta}\left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t}\right)^{1-\theta} \tag{8}$$

Then, by substituting (7) and (8) into (6), the HHI can be rewritten as

$$\begin{aligned} HHI_t &= \frac{\kappa-(\theta-1)}{\kappa-2(\theta-1)}\left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t}\right)^{1-\theta}(z_t^c)^{\theta-1} \\ &= \frac{\kappa-(\theta-1)}{\kappa-2(\theta-1)}\left(\frac{\theta}{\theta-1}\frac{1}{Z_t}\right)^{1-\theta}\left(\frac{z_t^c}{w_t}\right)^{\theta-1} \end{aligned}$$

and since we know the following from the equation (9) in the Table 2,

$$\frac{\theta f_{X,t}}{Y_t^c} = \left(\frac{\theta}{\theta-1}\frac{1}{Z_t}\right)^{1-\theta}\left(\frac{z_t^c}{w_t}\right)^{\theta-1} \tag{9}$$

the Herfindahl index at time t in the baseline model is given by

$$HHI_t = \frac{\theta(\kappa - \theta + 1)}{\kappa - 2(\theta - 1)} \frac{f_{X,t}}{Y_t^c} \tag{10}$$

if $\kappa > 2(\theta - 1)$ holds.

B. Derivation of the Aggregate Volatility

The aggregate volatility of an economy in the extended model with the idiosyncratic shocks and the discretization is defined by

$$\sigma_{GDP}^2 = \sigma^2 \sum_{j=1}^{\bar{N}} \left(\frac{x_j}{X} \right)^2 \tag{11}$$

where x_j/X denotes the market share of a firm j which is simply the sum of market shares of each product ω in the firm

$$\frac{x_j}{X} = \int_{\omega \in \Omega_j} \frac{x_j(\omega)}{X} d\omega \tag{12}$$

With the realization of idiosyncratic shock $a(\omega)$, the market share of a product ω in the firm j is defined by

$$\frac{x_j(\omega)}{X} = \left(\frac{\theta}{\theta - 1} \frac{w}{Zz(\omega)a(\omega)} \right)^{1-\theta} \tag{13}$$

and under normalization of the idiosyncratic shocks such that $E_a[a^{\theta-1}] = 1$, the expected market share of the product ω is given by

$$E_a \left[\frac{x_j(\omega)}{X} \right] = \left(\frac{\theta}{\theta - 1} \frac{w}{Zz(\omega)} \right)^{1-\theta} \tag{14}$$

Now, using the equation (12) and (14), the expected market share of the firm j is

$$E_a \left[\frac{x_j}{X} \right] = \int_{\omega \in \Omega_j} E_a \left[\frac{x_j(\omega)}{X} \right] d\omega = \int_{\omega \in \Omega_j} \left(\frac{\theta}{\theta - 1} \frac{w}{Zz(\omega)} \right)^{1-\theta} d\omega \tag{15}$$

Here, recall that the expected market share of the firm j, which is the sum of the expected market shares of each product in the firm, can be calculated by multiplying the average of the market shares of the products in the firm and the mass of them:

$$E_a\left[\frac{x_j}{X}\right] = N_j\left[\frac{1}{G(z^c+ji)-G(z^c+(j-1)i)}\int_{z^c+(j-1)i}^{z^c+ji}\left(\frac{\theta}{\theta-1}\frac{w}{Zz}\right)^{1-\theta}dG(z)\right] \tag{16}$$

where N_j denotes the mass of products in the firm j and is defined by $N_{j,t} = N_t\big[G(z_t^c+ji_t) - G(z_t^c+(j-1)i_t)\big]$, which is the portion of the firm j in the total mass of products. Then, the squared market share of the firm j is

$$\left(E_a\left[\frac{x_j}{X}\right]\right)^2 = N_j^2\left[\frac{1}{G(z^c+ji)-G(z^c+(j-1)i)}\int_{z^c+(j-1)i}^{z^c+ji}\left(\frac{\theta}{\theta-1}\frac{w}{Zz}\right)^{1-\theta}dG(z)\right]^2 \tag{17}$$

Since what we ultimately need is the sum of squared market shares of each firm in order to obtain the aggregate volatility, we have to solve the above equation and sum up over all the firms. But due to cross terms in the process, it is impossible to get a closed-form expression. So, we avoid the problem by relying on Jensen's inequality $\varphi(E[Y]) \leq E[\varphi(Y)]$ where Y is a random variable and φ is a convex function:

$$E_a\left[\left(\frac{x_j}{X}\right)^2\right] = N_j\left[\frac{1}{G(z^c+ji)-G(z^c+(j-1)i)}\int_{z^c+(j-1)i}^{z^c+ji}\left(\frac{\theta}{\theta-1}\frac{w}{Zz}\right)^{2(1-\theta)}dG(z)\right] \tag{18}$$

Hence, the result can be interpreted as the upper bound of the squared market share of the firm. By solving the equation (18), we obtain the following

$$E_a\left[\left(\frac{x_j}{X}\right)^2\right] = N\int_{z^c+(j-1)i}^{z^c+ji}\left(\frac{\theta}{\theta-1}\frac{w}{Zz}\right)^{2(1-\theta)}dG(z)$$

$$= N\left(\frac{\theta}{\theta-1}\frac{w}{Z}\right)^{2(1-\theta)}\left[\frac{1}{2(\theta-1)-\kappa}\right]\left[\frac{\kappa(z_{min})^\kappa}{1-(z_{min}/z_{max})^\kappa}\right]$$

$$\left\{\left[z^c+ji\right]^{2(\theta-1)-\kappa}-\left[z^c+(j-1)i\right]^{2(\theta-1)-\kappa}\right\}$$

and summing up all the squared market shares of $\bar{N}$ firms gives us the following:

$$\sum_{j=1}^{\bar{N}} E_a\left[\left(\frac{x_j}{X}\right)^2\right] = N\left(\frac{\theta}{\theta-1}\frac{w}{Z}\right)^{2(1-\theta)}\left[\frac{1}{2(\theta-1)-\kappa}\right]\left[\frac{\kappa(z_{min})^\kappa}{1-(z_{min}/z_{max})^\kappa}\right]$$

$$\sum_{j=1}^{\bar{N}}\left\{\left[z^c+ji\right]^{2(\theta-1)-\kappa}-\left[z^c+(j-1)i\right]^{2(\theta-1)-\kappa}\right\}$$

$$= N\left(\frac{\theta}{\theta-1}\frac{w}{Z}\right)^{2(1-\theta)}\left[\frac{1}{2(\theta-1)-\kappa}\right]\left[\frac{\kappa(z_{min})^\kappa}{1-(z_{min}/z_{max})^\kappa}\right]$$

$$\left[\left(z_{max}\right)^{2(\theta-1)-\kappa}-\left(z^c\right)^{2(\theta-1)-\kappa}\right]$$

From the cutoff productivity equation (9) in the Table 3 in the main text, we know the following

$$\left(\frac{\theta}{\theta-1}\frac{w}{Z}\right)^{2(1-\theta)} = \left(\frac{\theta f_X}{Y^c}\right)^2\left(z^c\right)^{2(1-\theta)} \tag{19}$$

And by combining the equation (4) and (7) in the Table 3 in the main text and the equation (19), the following is obtained:

$$N\left(\frac{\theta f_X}{Y^c}\right) = \left(\frac{\tilde{z}}{z^c}\right)^{1-\theta}\left[\frac{1-(z_{min}/z_{max})^\kappa}{(z_{min}/z^c)^\kappa-(z_{min}/z_{max})^\kappa}\right] \tag{20}$$

where the average productivity $\tilde{z}$ is

$$\tilde{z} \equiv \left[\frac{1}{1-G(z^c)}\int_{z^c}^{z_{max}} z^{\theta-1}dG(z)\right]^{\frac{1}{\theta-1}} = \left[\left(\frac{\kappa}{\theta-\kappa-1}\right)\left(\frac{(z_{max})^{\theta-\kappa-1}-(z^c)^{\theta-\kappa-1}}{(z^c)^{-\kappa}-(z_{max})^{-\kappa}}\right)\right]^{\frac{1}{\theta-1}} \tag{21}$$

Then, by normalizing $z_{min}=1$ and by using the above equations (19), (20), and (21), the sum of the squared market shares of $\bar{N}$ firms is given by

$$\sum_{j=1}^{\bar{N}} E_a\left[\left(\frac{x_j}{X}\right)^2\right] = \left(\frac{\theta f_X}{Y^c}\right)\left[\frac{\theta-\kappa-1}{2(\theta-1)-\kappa}\right]\left[\frac{(z_{max}/z^c)^{2(\theta-1)-\kappa}-1}{(z_{max}/z^c)^{\theta-1-\kappa}-1}\right]$$

Therefore, the upper bound of expected aggregate volatility[2] of the economy is given by

$$\sigma^2_{GDP} \;\leq\; \sigma^2 \sum_{j=1}^{\bar{N}} E_a\left[\left(\frac{x_j}{X}\right)^2\right] = \sigma^2\left(\frac{\theta f_X}{Y^c}\right)\left[\frac{\theta-\kappa-1}{2(\theta-1)-\kappa}\right]\left[\frac{(z_{max}/z^c)^{2(\theta-1)-\kappa}-1}{(z_{max}/z^c)^{\theta-1-\kappa}-1}\right]$$

[2]Unlike the analytical solution of the Herfindahl index in Appendix , an additional assumption on the range of κ is not needed to derive the result here. But the signs of the powers of z_{max} and z_t^c flip according to the value of κ, which requires caution in using it.