Household Preferences, Security Design, and Volatility Prices

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Abstract

We examine how households' non-standard preferences may dampen volatility (and dividend) prices by fueling demand for Short Put Products (SPP) – a globally popular type of retail structured product that offers high headline rates in exchange for selling a put option. By studying SPP issuances since market inception, we empirically document that these products offer higher headline rates and sell in higher volumes when the realized volatility of the underlying asset is high. However, the volatility premium decreases as SPP issuance volume increases. To uncover the underlying mechanism and quantify equilibrium effects of the development of these innovative products, we develop a structural model in which households underweight left-tail risk, leading to a robust demand for SPPs. Risk-averse financial intermediaries optimize the headline rate offered on these products while imperfectly hedging their exposure. As volatility rises, the stronger retail demand for SPPs resulting from higher headline rates exert a counteracting downward pressure on option prices, particularly at strikes below 100%. Our findings highlight the role of household preferences in shaping volatility price dynamics through demand for securities with non-linear payoffs.

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Keywords: Security Design, Volatility, Dividend, Options, Structured Products, Market Segmentation

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I. Introduction

Across the world, financial institutions have been designing and marketing retail products with non-linear payoffs to cater to households with "non-standard" preferences. These products include retail structured products (Celerier and Vallee (2017), in Europe; Vokata (2023), in the U.S.), structured funds (Gao, Hu, Kelly, Peng and Zhu (2024), in China), or structured annuities (Koijen and Yogo (2022), in the U.S.). Collectively, these financial products represent several trillion dollars of assets under management as of 2023. While their penetration rate has historically been high in Europe and Asia, this class of products has experienced rapid growth in the U.S. in recent years.[1]

Despite the economic significance of this phenomenon, its asset pricing implications are not well understood. The financial press and regulators have suggested that issuance of certain structured products may affect volatility levels and prices, while recognizing the challenge of measuring these effects.[2] Such an hypothesis is motivated by a growing literature, beginning with Gârleanu, Pedersen and Poteshman (2009), that explores demand pressure effects on option prices. If instruments with non-linear payoffs allow households to express a specific tolerance for tail-risk, their development should influence the equilibrium price of insurance against such risk. In turn, this phenomenon could have important implications for financial stability. This study thus seeks to address the following questions: Do household preferences or beliefs affect volatility prices through their demand for securities with nonlinear payoffs? What are the underlying mechanism and equilibrium effects? How does the magnitude of these pricing effects vary with market conditions?

To address these questions, we focus on the development of Short Put Products (SPP) – globally popular structured products that offer high headline rates in exchange for implicitly selling put options. More specifically, the payoff formula depends on the performance of an underlying asset, typically an equity index or stock, and is determined by two parameters:

[1] In Europe, approximately 15% of households hold retail products with non-linear payoffs.

[2] See for instance *Are structured products to blame for suppressed volatility?*, Financial Times, March 8, 2024.

the *headline rate*, a fixed coupon paid periodically until maturity, and the *barrier*, usually set between 50% and 80% of the underlying price at issuance. At maturity, the investor receives their capital back if and only if the underlying asset is above the barrier. If the final value of the underlying falls below the barrier, the investor's final payment is adjusted for the negative performance of the underlying asset. Hence, SPPs offer a fixed coupon in most scenarios, while fully exposing retail investors to left tail risk. Figure 1 represents the payoff diagram at maturity for a typical SPP.

Our empirical analysis, encompassing both reduced-form and structural approaches, leverages a dataset that provides comprehensive coverage of all SPPs issued globally since the market's inception, which we merge with data on option prices for most underlying equity indices. Our dataset is similar to the one used in Celerier and Vallee (2017), with a sample that covers a longer time series and a broader set of countries but only focuses on SPP within retail structured products. For each SPP issuance, we observe the underlying asset, the maturity, the detailed payoff design, and the issuance volume and date. We measure the SPP headline rate, a central outcome of interest, through a text analysis algorithm of the payoff description, which provides a better precision than the one reported by the data provider. For each underlying equity index, we collect data on the implied volatility of option contracts of different strikes and maturities. Our final dataset covers over $1.3 trillion of cumulative issuance of more than 360,000 SPPs and option prices for over 20 underlying equity indices, spanning 17 years and across three continents: North America, Europe, and Asia. The median maturity, headline rate, and barrier are 3 years, 7.5%, and 60%, respectively. Figure 2 illustrates the market growth for these products over recent years in the U.S., Europe, and Asia.

INSERT FIGURE 1 AND 2

In the first part of our study, we document three key empirical facts. First, the level of realized volatility, and to a lesser extent the level of interest rates, predict SPP headline rates. The elasticity of headline rate to the 90 day-volatility amounts to around 0.4. Second,

higher headline rates are associated with higher SPP issuance volume, with an elasticity of issuance volume to headline rates again around 0.4. Together, these results mean that households buy more SPPs when the left tail risk they embed is more likely to be realized, and therefore provide a form of countercyclical insurance for the equity markets.

Third, we document that SPP issuance volume is predictive of lower implied volatility, and a lower volatility premium –defined as the spread between implied and realized volatility– . This relationship is the most pronounced around the 60% moneyness, which corresponds to the most popular level for the SPP barrier, but exists for all moneyness below 100%. This empirical fact suggests that SPPs may depress volatility prices, in a way that diffuses along the volatility smile. These patterns are persistent at a relatively long horizon, and therefore do not correspond to the micro-structure concept of price impact. This persistence suggest that the asset pricing phenomenon we are exploring results from long run demand and supply equilibrium effects.

We next investigate the economic mechanism that can rationalize these empirical findings. Financial intermediaries, who issue SPPs to households and trade options and the underlying asset to hedge SPP contracts, are a natural focus of our analysis. When structuring short put products, intermediaries hedge the associated market risks. The typical hedge for a SPP requires the intermediaries to short a portfolio of puts on the underlying asset and short the dividend exposure through dividend swaps. If competitive intermediaries could hedge their risks perfectly, as would be the case in a Black-Scholes-Merton economy, options would be redundant assets and hedging would have no effect on option prices. In reality, however, even intermediaries cannot hedge options perfectly because trading cannot be implemented in continuous time due to transactions costs and because the volatility of the underlying evolves stochastically over time (Figlewski, 1989). In addition, intermediaries are sensitive to risk, for instance due to capital constraints and agency costs.

We accordingly develop a structural model with these features, which we use to estimate the demand for structured products and trace its impact on option prices. The economy

is populated by a representative household, a representative dealer, and outside investors with exogenous demand for options, in the style of Gârleanu et al. (2009), who all trade in discrete time. The investment set includes a riskless asset, an equity index, an SPP, and a set of plain-vanilla European puts on the equity index. Financial markets are driven by an exogenous state vector consisting of the prevailing interest rate and the stochastic volatility of the stock index. The dealer sets the headline rate and put prices, matches the demand for the SPP from households and puts from institutional investors, and partially hedges by trading the stock index. In line with the intermediary asset pricing literature, the dealer has CARA utility. The household can invest in the risk-free asset, the equity index, and the SPP, but cannot trade options. Each period, household demand for the SPP is affected by their time-invariant preferences, the endogenous SPP design (that is, the headline rate that it offers given a fixed maturity and barrier), and time-varying market conditions. This setting allows for substantial flexibility in modeling household preferences and beliefs.

We estimate the model by likelihood-based inference, as follows. For each level of the risk-free rate and the equity index volatility, the model provides the household demand for the SPP, the SPP headline rate set by the dealer, and the implied volatility of each put. We compare these values with monthly data on the swap rate, equity index return, SPP headline rate, and implied volatility of puts, which allows us to estimate the structural parameters of the model by maximum likelihood. We apply this method to products tied to the Eurostoxx 50.

The model predicts that household demand for SPPs and headline rates increase when volatility increases and interest rates decrease, which is consistent with the data. The demand for the SPP is null when the representative household has Epstein-Zin Utility. However, we do obtain significant household demand for SPPs when households underweight the left-tail risk, for instance when the household neglects to some extent the probability that the SPP may not pay the headline rate.

We conduct a counterfactual analysis showing that the implied volatility of a put contract

on the equity index with a 60% strike is ca. 2 percentage points higher in the absence of the SPP compared to a market in which the SPP is traded. Furthermore, the equilibrium impact of SPPs increases with both the volume of issuance of SPPs, and the volatility of the equity index.

We conclude our analysis by first noting that a similar phenomenon is at play for dividend futures, and by developing a trading strategy based on our empirical findings. When the outstanding volume of SPPs goes up, the term structure of dividends flattens. We show that a long-short trading strategy in volatility markets that sort stocks by their associated SPP issuance volumes yields attractive Sharpe ratios, which further speaks to the significance and persistence of the equilibrium effects we document.

Our results suggest that the development of markets for innovative securities can affect the supply and demand equilibrium for volatility prices by incorporating non-standard household preferences into prices through intermediaries hedging strategies. As option markets prices are key inputs for financial institutions risk management models, the effects we document have the potential to affect their risk-taking, and in turn financial stability. More broadly, our results speak to the equilibrium effects of a change in the set of participants for a given financial market.

This paper contributes to several strands of the literature. First, our findings are consistent with models that emphasize supply and demand factors, such as funding constraints of financial intermediaries or limits to arbitrage, as key drivers of asset prices. Gârleanu et al. (2009) develop a demand-based option pricing model and show that demand pressures from the put-call imbalance explain cross-sectional variations in volatility skewness across U.S. equity options. Vayanos and Vila (2009), Greenwood and Vayanos (2014) and Greenwood and Vissing-Jørgensen (2018) explain the term structure of riskless returns in a segmented supply and demand framework. Risk-averse intermediaries trade with end clients with strong preferences for specific-maturity bonds, hence driving price and return variations across different maturities. Brown, Davies and Ringgenberg (2018) study non-fundamental demand

in ETF market. Our contribution is to provide a micro-foundation for demand pressures by pinning down household as a source of demand for left-tail risk and quantify its asset pricing implications.

The paper also adds to the literature on the volatility risk premium (Bakshi, Carr and Wu, 2008; Bollerslev, Tauchen and Zhou, 2009; Todorov, 2010; Han and Zhou, 2012; Cao and Han, 2013) and on spillovers between distinct but related capital markets, such as derivative markets and their underlying asset markets (Calvet, Gonzalez-Eiras and Sodini, 2004; Henderson, Pearson and Wang, 2015).

Finally, this study speaks to the general equilibrium effects of the class of financial products studied in Celerier and Vallee (2017), Vokata (2021), Calvet, Célérier, Sodini and Vallee (2023), retail structured products, which have been shown to be effective at catering to, or mitigating, household behavioral biases.

II. Background and Data

A. Short Put Products

Short Put Products (SPPs), also often labeled Yied Enhancement Products (YEPs), are the most popular category of retail structured products. Retail structured products include any investment products marketed to retail investors that offer a payoff varying automatically and non-linearly with the performance of an underlying asset.[3] Typically designed with embedded options, these products leave no room for discretionary investment decisions during the life of the investment. The underlying assets are mainly equity indices and individual stocks but may also include commodities, fixed income assets, or alternative indices.

The retail market for structured products emerged in Europe at the beginning of the

[3]Exchange traded funds, which have payoffs that are a linear function of a given underlying financial index, are not retail structured products.

2000s and has subsequently experienced steady growth worldwide, particularly so in Asia. As of 2023, global outstanding is estimated at more than $2 trillion, with Asia representing half of that amount.[4] While synthetic Capital Guarantee Products (CGPs) were initially the most popular design, SPPs are now dominating the asset class in most markets.[5] A distinct feature of SPPs is that the payoff at maturity embeds the sale of a put option, hence exposing retail investors to tail risk. More precisely, the capital is protected on the downside as long as the underlying asset stays beyond a barrier. If the underlying goes below the barrier, the investor participates in the performance of the underlying. The product pays a fixed coupon every period until maturity. Hence, the payoff formula of a standard SPP includes two parameters: the headline rate, the fixed coupon paid every period until maturity, and the barrier, most often set between 50% and 80% of the underlying price at issuance. Figure 1 shows the payoff diagram at maturity of a typical SPP.

B. Hedging SPP Issuances

When issuing structured products, the issuing bank typically reduces the risk of paying large payoffs to the investors by buying or selling options that offset these payoffs. More specifically for SPPs, the bank sells put options that mirror the exposure of the end user, or roll similar positions with a shorter maturity. Banks often rely on a mix of hedge at inception with options and dynamic gamma-hedging with the underlying asset, for instance because the maturity of these products is longer than the maturity of the options that can be traded efficiently to build the hedge. When investing in SPPs, retail investors get a return that does not take into account dividends. Therefore, as banks usually hedge in a way that accounts for dividends – either through options or stocks–, they are long in dividend. Banks can offset this long exposure to dividend risk by entering into a short position in dividend futures.

[4]See SRP Global Market Review: H1 2024.

[5]See Calvet et al. (2023) for more details on CGPs.

C. Data

C.1. Short Put Products

We exploit a dataset from a specialized private data provider that collects all information on retailed structured products issued globally since market inception, including SPPs. For each product, we observe the underlying asset, the maturity, the exact payoff design, the volumes and issuance date. While most papers on retail structured products focus on specific geographic areas, this study leverages the global coverage of the dataset, exploiting data from products issued in North America, Europe and Asia. This comprehensive coverage offers a large cross-section to explore. It is also crucial for our analysis, as a large share of products are structured on foreign underlying assets. Therefore, we need to observe both domestic and foreign issuance to comprehensively capture the demand pressure on hedging markets.

Our dataset includes all SPPs issued globally since market inception and that have matured before 2020. In total, the dataset covers 362,000 products issued across 52 countries, for a total of $1.3 trillion of cumulated issuance in current dollars. We improve the reliability of the information on each product's headline rate and barrier using a textual analysis algorithm analyzing the textual description of the payoff formula, as in Celerier and Vallee (2017). The textual analysis algorithm also allows us to identify the key payoff features of each product, which affect the headline rates.[6]

C.2. Market Data

We obtain from a major investment bank a panel dataset of implied volatility surfaces for the stocks and indices most frequently used as underlying asset, and complement it with Bloomberg and Optionmetrics data. The volatility data represents a monthly panel from 2002 to 2019 at the underlying asset, moneyness, and maturity level.

For any given index at a time t, we observe implied volatility for moneyness of 50 to 150 % by 10% increments, over 3, 12 and 24 months maturity as well. Our current data covers

[6]The code is available in the Internet Appendix.

the Euro Stoxx 50, the Nikkei and the S&P 500. We are in the process of expanding the sample of underlying indices by incorporating data from OptionMetrics.

We also obtain a panel data of dividend term structures for the stocks used as underlying assets to retail structured products. The term structure spans 1 to 5 years.

C.3. Matched Panel

To perform our reduced form analysis, we construct a monthly panel of underlying assets with information on corresponding SPP volumes and volatility prices. To do so, we merge the data on SPPs with the volatility and dividend data for the three main indices: the Eurostoxx 50, S&P 500, Nikkei 225.[7]

For each index, we calculate the issuance volumes of SPPs that use them as underlying asset, as well as the outstanding volumes, taking into account the early redemptions. We build a panel at the month and underlying level. For each month-underlying observation, the panel includes the corresponding outstanding volumes of SPPs, volatility data at three maturity and different moneyness, as well as dividend data.

D. Summary Statistics

Figure 2 plots the yearly volumes of SPP issuance across the U.S., Europe and Asia. This figure evidences the fast penetration of such products in Europe relative to other geographic areas, and the more recent rise in Europe and the U.S.

Turning to the design of SPPs, Figure 3 shows that most issuance volumes occur with a barrier level of 50, 60 or 70%.

INSERT FIGURE 3

Tables I provides summary statistics on the main variables of our analysis. The sample is restricted to SPPs with the Eurostox50, Nikkei225 and S&P500 as underlying.[8] The strike

[7] We are in the process of expanding this sample.

[8] We will expand this sample to all underlying assets in future versions of the paper.

of the embedded put option ranges from 58% of the initial value of the underlying (10^{th} percentile) to 70% (90^{th} percentile). The median maturity is 3 years. On average, SPPs offer a headline rate and excess ex-post return of 9.7% and 5.7%, respectively.

INSERT TABLE I

Panel B of Figure 3 reports the distribution of ex post excess returns across SPPs. We observe that the bulk of SPPs earned positive excess returns, while a small minority of contracts earned negative excess returns.

III. Stylized Facts

A. Security Design, Issuance Volumes and Market Conditions

We first document that the headline rate of SPPs significantly varies with market conditions. In Figure 4, we plot the average headline rates of SPP linked to the EuroStoxx 50 and the 90-day historical volatility for that inde between 2005 and 2020. We observe a clear positive correlation between these two quantities.

INSERT FIGURE 4

To more precisely measure the relationship between headline rates and market conditions, we run the following regressions at the SPP issuance level:

$$\log(HeadlineRate_i) = \alpha + \beta_v HistVol_{a,t} + \beta_r IR_t \gamma_i + \epsilon_i, \tag{1}$$

where $HeadlineRate_i$ is the headline rate of issuance i, $HistVol_t$ is the 90-day historical volatility in month t of the underlying asset a, and IR_t is the 3-year CMS swap rate in the country of issuance in month t, and γ_i are a set of fixed effects, namely month, underlying assets, product design and country fixed effects. Regression coefficients are presented in

Table II. We observe that even in this saturated specification the level of historical volatility is highly predictive of the headline rate offered by a SPP, with an elasticity around 0.4. Interest rates also predict headline rate, but the economic magnitude of this relationship is less pronounced, which is consistent with banks aiming to preserve the headline rate when interests come down, as documented in Celerier and Vallee (2017).

INSERT TABLE II

We also investigate whether headline rates, and relatedly historical volatility, are predictive of SPP issuance volumes. We regress the log of the monthly volume of issuance for SPP linked to a given index on the log of the average headline rate for these issuances in column 4 of Table II,, and on historical volatility in column 5. Consistent with household preferences for high headline rates, we observe that SPP issuance volume is significantly higher when headline rates offered are high, and relatedly when historical volatility is high. Together, these findings highlight the endogenous nature of SPP design and volume.

B. SPP Issuances and Volatility Prices

We now turn to studying whether SPP issuances predicts the level of implied volatility. In Panel A of Figure 5, we plot the average yearly issuance volume of SPP linked to a given index, scaled by the index market capitalization. In Panel B, we plot the volatility risk premium for the three indices, i.e. the difference between the 1 year implied volatility and the 90-day realized volatility, at the 60 to 100% moneyness, with 10% increments. We observe that the volatility risk premium is negatively correlated with the amount of SPP issuance, and that this relationship is more pronounced for moneyness around 60%.

INSERT FIGURE 5

To identify the existence of this relationship in the time-series, we now run the following specification:

$$IV^u_{K=k,t} = \alpha + \beta SPPVolumes + \beta_v HistVol_{a,t} + \beta_r IR_t + \gamma_t + \gamma^u + \varepsilon^u_t, \quad (2)$$

where $IV^u_{K=k,t}$ is the 1-year implied volatility of underlying asset u and moneyness k, in month t, $SPPVolume^u_t$ is the SPP volume with underlying asset u in month t. γ_t and γ^u are month and underlying fixed effects. Standard errors are clustered at the month level. We run this regression in levels in Panel A of Table III, and in logs in Panel B of this same table. We use absolute SPP issuance volumes in columns 1 to 6, and scale SPP issuance volumes by indices market capitalization in columns 7 to 12. We span 60% moneyness (columns 1, 2, 7 and 8), 80% (columns 3,4, 9 and 10), and 100% (columns 5, 6, 11 and 12). Overall, we observe that the amount of SPP issuance is predictive of lower implied volatility. This result is robust when running regressions in levels or in logs, and when scaling volumes by indices market capitalization. When looking at the cross-section of moneyness, we observe as in the cross-sectional figure that the relationship is more pronounced at 60%, and decreases as moneyness rises.

INSERT TABLE III

To better visualize these relationships, we plot in Figure 6 the coefficient estimates for dummies for quintiles of issuance volumes in Panel A, and strikes, in Panel B in a regression where the dependent variable is the 1 year implied volatility (in level). These figures suggest a concave relationship between SPP volumes and implied volatility, and confirm the monotonic relationship relative to moneyness.

INSERT FIGURE 6

IV. Structural Model

Having established that SPP volumes are endogenous to the security design, and that SPP volumes are predictive of implied volatility levels, we develop a quantitative model that

captures the equilibrium that jointly determines these three outcome variables. This theoretical exercise allows us to quantify these equilibrium effects by implementing counterfactual exercises, and to investigate how household preferences affect volatility prices through the development of innovative security design. In this framework, we consider market conditions, namely the level of historical volatilty and interest rate, as state variables, which also allows to condition our findings for these dimensions. Motivated by our empirical analysis, we focus on the headline rate as the main parameter of the SPP design, and consider the barrier and maturity as fixed.

A. Assets

We consider a discrete-time financial market defined at dates $t = 0, \ldots, \infty$. In the empirical implementation of the model, the time index t will refer to a month. At each date t, financial market participants can trade four securities: (i) a riskless zero-coupon bond, (ii) an equity index fund, (iii) a short put product (SPP) written on the equity index, and (iv) a set of plain-vanilla European puts on the equity index. The zero-coupon bond, the SPP, and the European puts all reach maturity at date $t+T$. In the empirical application, we set T equal to 36 months, which is the median maturity of an SPP on the Eurostoxx 50.

Zero-coupon bond and equity index. Let $R_{f,t:t+T}$ denote the net arithmetic return on the zero-coupon bond between t and $t+T$, and let $R_{\text{M},t:t+T}$ denote the net arithmetic return on the equity index over the same period. The zero-coupon bond and the equity index have infinitely elastic supplies, so that their returns are taken as exogenous to the model.

The exogenous state vector:

$$z_t = (z_{1,t}, z_{2,t})'. \tag{3}$$

captures time-varying conditions in the bond and equity markets. The state variable $z_{1,t}$ drives relevant interest rates and the state variable $z_{2,t}$ the volatility of the equity index return, as we now explain.

We define the state variable $z_{1,t}$ as the yield on the riskless zero-coupon bond issued at t and maturing at date $t+T$, expressed in annual units. We assume that the yield satisfies a first-order autoregression:

$$z_{1,t} - \mu_1 = \phi_1(z_{1,t-1} - \mu_1) + \sigma_1 \varepsilon_{1,t}$$

for every t. The return on the zero-coupon bond over its life, $R_{f,t:t+T}$, is linked to the annualized yield, $z_{1,t}$, via the usual relationship: $R_{f,t:t+T} = (1 + z_{1,t})^{T/12} - 1$.

For simplicity, we assume that the spread between the T-period zero yield and the 1-period zero yield is time-invariant, and we denote by π denote this constant spread. The net arithmetic interest rate per period is therefore $R_{f,t:t+1} = (1 + z_{1,t} - \pi)^{1/12} - 1$ for every t.

We model the equity index by assuming that its log excess return follows a process with constant mean and stochastic volatility. Specifically, let $r^e_{\text{M},t:t+1} = \ln(1 + R_{\text{M},t:t+1}) - \ln(1 + R_{f,t:t+1})$ denote the log excess return on equity between t and $t+1$. We posit that:

$$r^e_{\text{M},t:t+1} = \mu_r + e^{z_{2,t+1}/2}\varepsilon_{\text{M},t+1}, \tag{4}$$

where μ_r is a constant, $z_{2,t+1}$ is the state variable controlling stochastic volatility, and $\varepsilon_{\text{M},t+1}$ is a stochastic shock. The state variable $z_{2,t}$ satisfies the first-order autoregression:

$$z_{2,t} - \mu_2 = \phi_2(z_{2,t-1} - \mu_2) + \sigma_2 \varepsilon_{2,t} \tag{5}$$

in every period t. The dynamics of the equity index return between t and $t+T$ follows from the identity $R_{\text{M},\,t:t+T} = (1 + R_{\text{M},t:t+1}) \ldots (1 + R_{\text{M},T-1:T}) - 1$.

We close the specification of bond and equity returns by assuming that the vector of stochastic shocks, $\varepsilon_{t+1} = (\varepsilon_{1,t+1}, \varepsilon_{2,t+1}, \varepsilon_{\text{M},t+1})'$, is independent through time. Furthermore,

it is Gaussian with zero mean and variance-covariance matrix:

$$\Sigma = \begin{pmatrix} 1 & \rho_{1,2} & \rho_{1,\mathrm{M}} \\ \rho_{1,2} & 1 & \rho_{2,\mathrm{M}} \\ \rho_{1,\mathrm{M}} & \rho_{2,\mathrm{M}} & 1 \end{pmatrix} \tag{6}$$

in every period t.

To sum up, equations (3)-(6) fully specify the dynamics of bond and equity returns. Throughout the paper, we consider the state vector, z_t, the vector of shocks, ε_t, and the parameters μ_1, ϕ_1, σ_1, μ_r, μ_2, ϕ_2, σ_2, $\rho_{1,2}$, $\rho_{1,\mathrm{M}}$, and $\rho_{2,\mathrm{M}}$ as exogenous inputs of the general equilibrium model.

Short put product. The SPP is issued at t and is illiquid until it reaches maturity at date $t+T$. Its payoff at date T depends on a barrier, B, defined at the contract's origination, and is contingent on the performance of the equity index over the period, $R_{\mathrm{M},t:t+T}$. If the return is above the barrier, the SPP pays off a coupon rate y_t. Otherwise, the SPP has the same return as the equity index. More compactly, the net payoff of the SPP per unit of capital invested is given by:

$$R_{\mathrm{SPP},\,t:t+T} = y_t \, \mathbb{1}_{\left\{R_{\mathrm{M},\,t:t+T} > B\right\}} + R_{\mathrm{M},\,t:t+T} \, \mathbb{1}_{\left\{R_{\mathrm{M},\,t:t+T} \leq B\right\}},$$

where y_t is the coupon rate per period and B is the barrier. For SPPs written on the Eurostoxx 50, the barrier B has a median value of -40%. To avoid arbitrage opportunities, we we assume that $B < 0$ and $R_{f,t:t+T} > B/2$. in every state of the world.

In the general equilibrium model defined in Section IV.C, the barrier is taken as exogenous, but the coupon rate per period is endogenously determined by supply and demand forces. The coupon yield will then be a function of the state:

$$y_t = Y(z_t), \tag{7}$$

where the mapping $Y(\cdot)$ is determined in general equilibrium.

Puts. Investors can trade a set of plain-vanilla puts written on the equity index, which are issued at t and reach maturity at date $t+T$. Their strikes are exogenous and denoted by $K_1, \ldots, K_N$. A European put with strike K_n pays off $(K_n - 1 - R_{\mathrm{M},\,t:t+T})_+$ at date $t+T$, where x_+ denotes the maximum of x and 0 for every real number x. The prices of the European puts, $P_{\mathrm{PUT},n,t}$, and their net returns, $R_{\mathrm{PUT},n,t} = (K_n - 1 - R_{\mathrm{M},\,t:t+T})_+ / P_{\mathrm{PUT},n,t} - 1$, are endogenously determined in general equilibrium.

An important property of our setup is that the SPP can be hedged partially, but not fully, by trading the zero-coupon bond, the equity index, and European puts at date t. The explanation is that the payoff of the SPP is discontinuous at $R_{\mathrm{PUT},n,t} = B$, while the assets used for hedging purposes (bond, equity, puts) have continuous payoffs. Put slighlty differently, an SPP contract delivers the same payoff at maturity as a portfolio containing (i) a short position in 1 put option on the equity index with strike $1+B$, (ii) B units of the zero-coupon bond, and (iii) $(y_t - B)$ units of a digital call options paying \$1 if the equity index return exceeds the barrier. We do not assume that a digital call option of maturity T is traded in our setting, a reflection of the fact that digital call options are thinly traded and difficult to hedge in practice (Gallus, 1999). Hence, the SPP is not a redundant asset in our setting.

B. Agents

Every period t, financial markets are populated by three types of investors: a long-lived representative household, a representative dealer, and a group of investors with an inelastic demand for puts.

Households. The representative household at t trades and consumes at dates t, $t+T$, $t+2\,T, \ldots, \infty$. It can trade the riskless asset, the equity index fund, and the SPP. Importantly,

it does not have access to put markets.

The household's consumption-portfolio problem is defined as follows. The household starts period t with wealth W_t^H. It allocates C_t to consumption and invests the remaining wealth, $FW_t^H = W_t^H - C_t^H$, in financial assets through the dealer. The dealer receives a fraction ϕ of gross invested wealth, FW_t^H, as compensation for his services. The household's net invested wealth is therefore

$$X_t^H = (1-\phi)\, FW_t^H$$

at the end of period t. The household allocates a fraction $\alpha_{\text{M},t}^H$ of net invested wealth to the index fund, a fraction $\alpha_{\text{SPP},t}^H$ to the SPP, and the remaining fraction, $1 - \alpha_{\text{M},t}^H - \alpha_{\text{SPP},t}^H$, to the zero-coupon bond. The financial wealth available to the household at the beginning of period $t+T$ is therefore

$$W_{t+T}^H = X_t^H \left(1 + R_{f,t:t+T} + \alpha_{\text{M},t}^H R_{\text{M},t:t+T}^e + \alpha_{\text{SPP},t}^H R_{\text{SPP},t:t+T}^e\right),$$

where $R_{\text{M},t:t+T}^e = R_{\text{M},t:t+T} - R_{f,t:t+T}$ and $R_{\text{SPP},t:t+T}^e = R_{\text{SPP},t:t+T} - R_{f,t:t+T}$ denote, respectively, the excess return on the index fund and the SPP over the period.

The household selects consumption and portfolio weights by maximizing the Epstein-Zin recursive utility:

$$V^H(W_t^H; z_t, Y) = \max_{\{C_t^H, \alpha_{\text{M},t}^H, \alpha_{\text{SPP},t}^H\}} \left[(1-\delta^T)\,(C_t^H)^{1-1/\psi} + \delta^T\,(\mu_{t+T}^H)^{1-1/\psi}\right]^{\frac{1}{1-1/\psi}}, \tag{8}$$

where Y is the state-contingent SPP coupon yield defined in (7) and

$$\mu_t^H = \left\{\mathbb{E}_t^{\mathbb{P}}[V^H (W_{t+T}^H)^{1-\gamma}]\right\}^{\frac{1}{1-\gamma}}$$

is the certainty equivalent of future consumption. The household is a price-taker viewing

the state-contingent SPP yield and other asset prices as exogenous. We denote by

$$\alpha^H_{\text{SPP},t} = \alpha^H_{\text{SPP}}(z_t, Y)$$

the optimal SPP weight in period t.

Dealers. The dealer available in period t has no initial wealth but receives from the household the management fee $X^D_t = \phi F W^H_t = \phi X^H_t/(1-\phi)$, or equivalently

$$X^D_t = X^H_t/\lambda^H,$$

where $\lambda^H = (1-\phi)/\phi$. The dealer allocates a fraction $\alpha^D_{\text{M},t}$ of his wealth X^D_t to the index fund, a fraction $\alpha^D_{\text{SPP},t}$ to the SPP, a fraction $\alpha^D_{\text{PUT},n,t}$ to each put n, and the remaining fraction to the zero-coupon bond.

The dealer's wealth in period $t+T$ is

$$W^D_{t+T} = X^D_t \left(1 + R^D_{t:t+T}\right),$$

where

$$R^D_{t:t+T} = R_{f,\,t:t+T} + \alpha^D_{\text{M},t}\, R^e_{\text{M},\,t:t+T} + \alpha^D_{\text{SPP},t}\, R^e_{\text{SPP},\,t:t+T} + \sum_{n=1}^{N} \alpha^D_{\text{PUT},n,t}\, R^e_{\text{PUT},n,\,t:t+T} \tag{9}$$

denotes the dealer's portfolio return, and $R^e_{\text{PUT},n,\,t:t+T} = R_{\text{PUT},n,\,t:t+T} - R_{f,t:t+T}$ is the excess return of each put n over the period.

The dealer has constant absolute risk aversion (CARA) utility, $U_t = -\mathbb{E}^{\mathbb{P}}_t(e^{-A_t W^D_{t+T}})$. The CARA specification is commonly used in intermediary asset pricing (see, e.g., He and Krishnamurthy (2018)) because it permits the dealer to absorb the losses than can be caused by imperfect hedging or mismatches between supply and demand. We assume that the CARA coefficient in period t satisfies $A_t = \gamma^D/X^D_t$, which allows us to obtain stationary portfolio

weights, as we now explain.

The dealer solves the optimization problem

$$\max_{\alpha_t^D} \mathbb{E}_t \left[-e^{-\gamma^D \left(W_{t+T}^D / X_t^D \right)} \right],$$

where $\alpha_t^D = (\alpha_{\mathrm{M},t}^D, \alpha_{\mathrm{SPP},t}^D, \alpha_{\mathrm{PUT},1,t}^D, \ldots, \alpha_{\mathrm{PUT},N,t}^D)$. Since $W_{t+T}^D / X_t^D = 1 + R_{t:t+T}^D$, the solution to this problem is independent of the wealth level X_t^D. We denote by

$$\alpha^D[z_t; Y(z_t)]$$

the optimal portfolio of the dealer.

We observe that since the dealer is short-lived, his portfolio depends on the current state of the bond and equity markets, z_t, and on the SPP coupon yield in the current state, $Y(z_t)$. By contrast, the household's optimal portfolio depends on the coupon yield in *all* states, as the household is long-lived and the mapping Y characterizes all current and future investment opportunities.

Exogenous Demand for Puts. We assume that an unmodelled group of investors has an inelastic demand for puts, as in Gârleanu et al. (2009). Specifically, this group invests

$$d_n^E \, X_t^D$$

units of each put n, where $d_n^E \in \mathbb{R}$. We take the number of puts demanded per unit of wealth, d_n^E, as exogenous. Importantly, the exogenous demand, $d_n^E \, X_t^D$, grows with the size of the economy, so that the pricing impact of the demand pressure remains sizable over time.

C. *Stationary General Equilibrium*

We focus on a stationary general equilibria, in which the joint distribution of asset returns conditional on the state, z_t, remains the same over time.

Definition 1 (Stationary General Equilibrium): *A stationary general equilibrium consists of a state-contingent SPP yield, $Y(z_t)$, and state-contingent put prices, $P_{\text{PUT},n}(z_t)$, such that markets clear:*

$$\begin{aligned} \alpha^D_{\text{SPP},t}[z_t; Y(z_t)]\, X^D_t + \alpha^H_{\text{SPP},t}(z_t, Y)\, X^H_t &= 0 \\ \alpha^D_{\text{PUT},n}[z_t; Y(z_t)]\, X^D_t + d^E_n\, X^D_t\, P_{\text{PUT},n}(z_t) &= 0 \end{aligned}$$

in every period t and state z_t.

We divide the equilibrium equations by the dealer's wealth and obtain:

$$\alpha^D_{\text{SPP},t}[z_t; Y(z_t)] = -\lambda^H\, \alpha^H_{\text{SPP},t}(z_t, Y) \tag{10}$$

$$\alpha^D_{\text{PUT},n}[z_t; Y(z_t)] = -d^E_n\, P_{\text{PUT},n}(z_t). \tag{11}$$

These equations show that the dealer offsets the position of the household sector in the SPP market, and the exogenous demand in the market for each put. In general equilibrium the dealer finds it optimal to offset the positions of other agents.

PROPOSITION 1: *In a stationary general equilibrium, the SPP yield, $Y(z_t)$, and the dealer's hedge, $\alpha^D_{\text{M}}[z_t, Y(z_t)]$, satisfy:*

$$\mathbb{E}\left(R^e_{\text{M},t:t+T}\, M_{t+T} \middle|\, z_t\right) = 0, \tag{12}$$

$$\mathbb{E}\left(R^e_{\text{SPP},t:t+T}\, M_{t+T} \middle|\, z_t\right) = 0, \tag{13}$$

where the excess return on the SPP satisfies

$$R^e_{\text{SPP},t+1} = Y(z_t)\,\mathbb{1}_{\left\{R_{\text{M},t:t+T} > B\right\}} + R_{\text{M},t:t+T}\,\mathbb{1}_{\left\{R_{\text{M},t:t+T} \leq B\right\}} - R_{f,t:t+T},$$

and the stochastic discount factor is given by:

$$M_{t+T} = e^{-\gamma^D \left\{\alpha^D_{\text{M}}[z_t, Y(z_t)]\, R^e_{\text{M},t:t+T} - \lambda^H\, \alpha^H_{\text{SPP}}(z_t, Y)\, R^e_{\text{SPP},t:t+T} - \sum_{n=1}^N d^E_n\, (K_n - 1 - R_{\text{M},t:t+T})_+ \right\}}.$$

The price of each put n is given by:

$$P_{\text{PUT},n}(z_t) = \frac{\mathbb{E}_t\left[(K_n - 1 - R_{\text{M},t:t+T})_+\, M_{t+T} |\, z_t\right]}{(1 + R_{f,t:t+T})\mathbb{E}\left(M_{t+T} |\, z_t\right)}, \tag{14}$$

for every t and z_t.

We henceforth denote by $\mathcal{V}_n(z_t)$ the implied volatility of put n resulting from $P_{\text{PUT},n}(z_t)$.

PROPOSITION 2: *There exists a unique stationary equilibrium.*

D. Estimation Strategy

The estimation is based on the following structural equations:

$$\begin{aligned}
z_{1,t} &= \mu_1 + \phi_1(z_{1,t-1} - \mu_1) + \sigma_1\, \varepsilon_{1,t}, && (15)\\
z_{2,t} &= \mu_2 + \phi_2(z_{2,t-1} - \mu_2) + \sigma_2\, \varepsilon_{2,t}, && (16)\\
r^e_{\text{M},t} &= \mu_r + e^{z_{2,t}/2}\varepsilon_{\text{M},t} && (17)\\
y_{SPP,t} &= Y(z_t) + \sigma_{spp} u_t && (18)\\
IV_{n,t} &= \mathcal{V}_n(z_t) + \sigma_{IV}\eta_{n,t} && (19)
\end{aligned}$$

The model is specified by the following parameters. The interest rate and equity index returns are parameterized by $\mu_1, \phi_1, \sigma_1, \mu_r, \mu_2, \phi_2$, and σ_2. In simulations, we have verified

that the demand for the SPP, α exhibits limited variation with respect to the patience parameter, δ, and the elasticity of intertemporal subsitution, ψ. For this reason, we calibrate and we estimate the model parameters

$$\theta = (\underbrace{\mu_1, \phi_1, \sigma_1, \mu_r, \mu_2, \phi_2, \sigma_2, \rho_{1,2}, \rho_{1,\mathrm{M}}, \rho_{2,\mathrm{M}}}_{\text{assets}}, \underbrace{\gamma, \lambda^H}_{\text{household}}, \underbrace{\gamma^D}_{\text{dealer}}, \underbrace{d^E}_{\text{demand pressure}}, \underbrace{\sigma_{spp}, \sigma_{IV}}_{\text{measurement}}).$$

The vector θ has $K + 15$ components, where K is the number of puts.

The estimation of θ requires the following. For a given θ, we compute the household's demand schedule for the SPP, $\alpha^H_{\mathrm{SPP}}[z_t; Y(z_t)]$, by solving the Bellman equation (1). We then obtain the dealer's demand for the equity index and the SPP yield spread in each state z_t by solving the set of first-order conditions (10) and (11).

We estimate the parameter vector θ by likelihood-based inference. The observation vector contains the interest rate, the equity excess return, and the endogenous prices:

$$y_t = (z_{1,t}, r^e_{\mathrm{M},t}, X_t), \tag{20}$$

where

$$X_t = (y_{SPP,t}, IV_{1,t}, \ldots, IV_{n,t}) \tag{21}$$

contains equilibrium quantities;

The one-step likelihood of y_{t+1} is given by

$$f(y_{t+1}|y_{1:t}) \;=\; \iint h(y_{t+1}|y_{1:t}, z_{2,t}, z_{2,t+1}) f(z_{2,t}, z_{2,t+1}|y_{1:t}) dz_{2,t} dz_{2,t+1}.$$

We note that $h(y_{t+1}|y_{1:t}, z_{2,t}, z_{2,t+1}) = h(y_{t+1}|z_{1,t}, z_{2,t}, z_{2,t+1})$ and therefore

$$\begin{aligned} h(y_{t+1}|y_{1:t}, z_{2,t}, z_{2,t+1}) \;=\; & f(X_{t+1}|z_{1,t+1}, z_{2,t+1}) \;\; f(r^e_{M,t+1}|z_{1,t}, z_{2,t}, z_{1,t+1}, z_{2,t+1}) \\ & \times f(z_{1,t+1}|z_{1,t}, z_{2,t}, z_{2,t+1}), \end{aligned}$$

where $f(X_{t+1}|z_{1,t+1}, z_{2,t+1})$, $f(R^e_{M,t+1}|z_{1,t+1}, z_{2,t+1}, z_{1,t}, z_{2,t})$ and $f(z_{1,t+1}|z_{2,t+1}, z_{1,t}, z_{2,t})$ have closed-form expressions provided in the Appendix.

Since the volatility state is not observed directly, we sequentially approximate its distribution by a particle filter $\{z^{(b)}_{2,t}\}^B_{b=1}$.

1. We simulate forward each particle and obtain $\{(z^{(b)}_{2,t}, \tilde{z}^{(b)}_{2,t+1})\}^B_{b=1}$, which targets the distribution of $(z_{2,t}, z_{2,t+1})$ conditional on $y_{1:t}$.

2. We approximate the one-step ahead likelihood by:

$$\hat{f}(y_{t+1}|y_{1:t}) = \frac{1}{B}\sum_{b=1}^{B} h(y_{t+1}|y_t, z^{(b)}_{2,t}, \tilde{z}^{(b)}_{2,t+1})$$

3. We assign to each particle the nonormalized importance weight:

$$p^{(b)}_{t+1} = \frac{h(y_{t+1}|y_t, z^{(b)}_{2,t}, z^{(b)}_{2,t+1})}{\sum_{b'=1}^{B} h(y_{t+1}|y_t, z^{(b)}_{2,t}, z^{(b)}_{2,t+1})}$$

4. We sample the particle $z^{(b)}_{2,t+1}$ from the candidate particles $\tilde{z}^{(1)}_{2,t+1}, \ldots, \tilde{z}^{(B)}_{2,t+1}$ with probabilities $p^{(1)}_{t+1}, \ldots, p^{(B)}_{t+1}$. We repeat the same procedure to obtain $z^{(2)}_{2,t+1}, \ldots, z^{(B)}_{2,t+1}$.

E. Estimation Results

Preliminary results from the model estimation are provided in Table IV. We consider a version of the model under which investors neglect the probability that the equity index may generate a return lower than the barrier and the SPP may therefore pay less than the headline rate. The demand for the SPP is then substantial. The unrestricted estimate in column 1 corresponds to our baseline model. The restricted estimate in column2 corresponds to model where λ_h is set to zero, i.e. where household demand for SPP cannot play a role in the stochastic discount factor. The comparison between these two estimations allows us

to reject the null hypothesis that household demand for SPPs do not play a role in the stochastic discount factor.

The SPP yield spread increase with the interest rate and equity volatility, which is consistent with the empirical evidence in Figure 4.

INSERT TABLE IV

In Figure 7, we compare the implied volatility curve produced by the model estimation with the average one observed in the data, along SPP volumes and across moneyness. The model outputs closely match their empirical counterparts, which provides a validation for the framework.

INSERT FIGURE 7

V. Additional Results

This section presents additional empirical results. We first document that a similar phenomenon is at play in the dividend future market. We also describe a trading strategy that allows to monetize the effect on implied volatility we document in the paper.

A. On Dividend Term Structure

In addition to affecting implied volatility, issuances of retail structured products might affect the dividend term structure as banks typically hedge their long dividend exposure by taking a short position in dividend swaps.

We test this hypothesis by regressing the level of dividend swaps on the volume of outstanding retail structured products.

Table V indicates the results. We find a significant negative relationship between the outstanding volume of short put products and the overall level of the dividend term structure. When looking at the impact along the term structure, we find a magnitude increasing with

maturity, i.e., a flattening of the term structure. Such pattern is consistent with the rigidity of the short end of the dividend term structure.

INSERT TABLE V

B. Trading Strategy

To further quantify the impact of structured retail products on market prices, we consider a long-short trading strategy that exploits the supply and demand imbalances in the volatility market due to structured product issuance-related hedging. As previously documented, structured retail products dampen options implied volatility, particularly for moneyness that correspond to the strikes of short-put products. The following trading strategy exploits these price distortions by purchasing options (long on volatility or variance swaps) on stocks that have a relatively large amount of short-put product outstanding volumes and selling options (short on volatility or variance swaps) on stocks that have little or no associated short-put product issuance.

At each month, we form five volatility portfolios by sorting the underlying stocks based on the total outstanding volumes of short put products scaled by the market capitalization of each stock. We calculate the hold-to-maturity returns on one-year volatility ($r^{volswap}$) or variance ($r^{varswap}$) swaps for each individual stock by subtracting the implied fair strikes (K_{it}^{vol} and K_{it}^{var}) from the ex-post one-year realized volatility ($\sigma_{R,it}$) or variance ($\sigma_{R,it}^2$). The return on the volatility swap in units of dollar vega notional ($1 per volatility point) is

$$r_{it}^{volswap} = \sigma_{R,it} - K_{it}^{vol}, \tag{22}$$

and the returns on variance swap is

$$r_{it}^{varswap} = \frac{1}{2K_{it}^{var}} \left(\sigma_{R,it}^2 - K_{it}^{var} \right). \tag{23}$$

The term $\frac{1}{2K_{it}^{var}}$ is the variance notional or variance units that scales the variance swap similarly to volatility swap. If realized volatility is 1 point above the strike at maturity, the payoff will approximately be equal to this notional amount.

The variance swap fair strike, K_{it}^{var}, is calculated using the 80% moneyness and at-the-money implied volatility following the approximation from Demeterfi, Derman, Kamal and Zou (1999) assuming that the skew is linear in strike:

$$K_{it}^{var} \approx \sigma_{ATM,it}\sqrt{1 + 3T \times skew_{it}^2}, \tag{24}$$

where $skew_{it}$ is the slope of the implied volatility curve that we calculate to be $\sigma_{m=80,it} - \sigma_{ATM,it}$. Assuming a log-linear skew-to-strike relationship does not materially alter our results. Calculating *skew* using the difference between 90% moneyness and ATM implied volatility also does not materially affect our results. The volatility swap fair strike is approximated as $K_{it}^{vol} \approx \frac{1}{2}\left(\sigma_{ATM,it} + K_{it}^{var}\right)$.[9]

Table VI reports the summary statistics associated with the returns on volatility and variance swap portfolios sorted on the total outstanding volume of short put products, scaled by the market cap for each stock. The table shows that the portfolio returns increase monotonically with short put product exposure. While the portfolio with the lowest quintile exposure to short put products outstanding volume has negative return, the returns increases to positive for highly exposed portfolios. The high-minus-low long-short portfolio (last row) is constructed by taking a long position on volatility swaps/variance swaps on the stocks with the most short-put products outstanding and taking a short position on those with the least short-put products exposures.

INSERT TABLE VI

Figure 8 shows the time series of the cumulative P&L for the long-short volatility and

[9]Volatility swap payoffs cannot be statically replicated as in the case of variance swaps. The approximation above is used commonly by market practitioners as the boundary for volatility swap fair strikes are determined by the ATM implied volatility and variance swap fair strikes, $\sigma_{ATM,it} < K_{it}^{vol} < K_{it}^{var}$.

variance swap portfolios constructed as the highest-quintile minus the lowest-quintile portfolio based on sorting by short put product exposures. This cumulative P&L assumes that the investor forms a long-short portfolio each month by sorting stocks into those most exposed to short-put products exposure and those with the least exposure, taking long (short) volatility or variance swap positions on the most (least) exposed stocks. Examining the time series, it appears that this particular strategy did not perform prior to 2008 and had the strongest performance from around 2008 to 2016. This pattern generally aligns with the pick up in the issuance of short-put products after the GFC and the eventual saturation of arbitrage capital in this market.

INSERT FIGURE 8

Table VII reports the sharpe ratios associated with long-short portfolios formed based on sorting by alternative variables. These variables reflect either the stock or the flow of short put products. We find that the stock effect (outstanding volume) appears to be stronger than the flow effect (Net Issuance), particularly when scaled by the market capitalization of the underlying stock. Additionally, sorting on volume of issuance has the opposite effect than sorting on maturing volumes. This further support our general finding that structured retail products have a noticeable impact on the options volatility market.

INSERT TABLE VII

VI. Conclusion

The paper documents the price effects of the demand for securities with innovative payoff designs.

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Proof of Proposition 1

The dealer selects the portfolio weight of equity, $\alpha^D_{\mathrm{M},t}$, the weight of the SPP, $\alpha^D_{\mathrm{SPP},t}$ and weight of each put, $\alpha^D_{\mathrm{PUT},n,t}$, that maximizes expected utility. The corresponding first-order conditions are:

$$\begin{aligned}
\mathbb{E}_t\left[R^e_{\mathrm{M},\,t:t+T}\, e^{-\gamma^D\left(\alpha^D_{\mathrm{M},t}\, R^e_{\mathrm{M},\,t:t+T}+\alpha^D_{\mathrm{SPP},t}\, R^e_{\mathrm{SPP},\,t:t+T}+\sum_{n=1}^N \alpha^D_{\mathrm{PUT},n,t}\, R^e_{\mathrm{PUT},n,\,t:t+T}\right)}\right] &= 0,\\
\mathbb{E}_t\left[R^e_{\mathrm{SPP},t+1} e^{-\gamma^D\left(\alpha^D_{\mathrm{M},t}\, R^e_{\mathrm{M},t+1}+\alpha^D_{\mathrm{SPP},t}\, R^e_{\mathrm{SPP},t+1}+\sum_{n=1}^N \alpha^D_{\mathrm{PUT},n,t}\, R^e_{\mathrm{PUT},n,t+1}\right)}\right] &= 0,\\
\mathbb{E}_t\left[R^e_{\mathrm{PUT},n,t+1} e^{-\gamma^D\left(\alpha^D_{\mathrm{M},t}\, R^e_{\mathrm{M},t+1}+\alpha^D_{\mathrm{SPP},t}\, R^e_{\mathrm{SPP},t+1}+\sum_{n=1}^N \alpha^D_{\mathrm{PUT},n,t}\, R^e_{\mathrm{PUT},n,t+1}\right)}\right] &= 0,
\end{aligned}$$

for every $n \in \{1,\ldots,N\}$. Let

$$M^0_{t+T} = e^{-\gamma^D\left(\alpha^D_{\mathrm{M},t}\, R^e_{\mathrm{M},\,t:t+T}+\alpha^D_{\mathrm{SPP},t}\, R^e_{\mathrm{SPP},\,t:t+T}+\sum_{n=1}^N \alpha^D_{\mathrm{PUT},n,t}\, R^e_{\mathrm{PUT},n,\,t:t+T}\right)}. \tag{1}$$

We infer from (4) and (5) that

$$M^0_{t+T} = e^{-\gamma^D\left[\alpha^D_{\mathrm{M},t}\, R^e_{\mathrm{M},\,t:t+T}-\lambda^H\alpha^H_{\mathrm{SPP},t}\, R^e_{\mathrm{SPP},\,t:t+T}-\sum_{n=1}^N d^E_n\,(K_n-1-R_{\mathrm{M},t:t+T})_+ +(1+R_{f,t:t+T})\sum_{n=1}^N d^E_n P_n\right]}. \tag{2}$$

Since $(1+R_{f,t:t+T})\sum_{n=1}^N d^E_n P_n$ is known conditional on z_t, we infer that is also a stochastic discount factor.

The first-order condition for holdings of put n can be rewritten as:

$$\mathbb{E}_t\left[(K_n-1-R_{\mathrm{M}})_+\, M_{t+T}\right] = (1+R_{f,t})P_n\mathbb{E}_t\left[(K_n-1-R_{\mathrm{M}})_+\, M_{t+T}\right],$$

and we conclude that the proposition holds.

Equity Premium

We assume the following:

$$r^e_{\mathrm{M},t+1} = \mu_r(z_t) + e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1} \tag{1}$$

$$\mathbb{E}_t(r^e_{\mathrm{M},t+1}) = \lambda\, Var_t(r^e_{\mathrm{M},t+1}) \tag{2}$$

$$\mathbb{E}_t(R_{\mathrm{M},t:t+T} - R_{f,t:t+T}) = RP, \tag{3}$$

where RP is an exogenous constant. The values of $\mu_r(z_t)$ and λ are unique and can be computed as follows.

LEMMA 2: *The drift in each state is given by*

$$\mu_r(z_t,\lambda) = \lambda\,\mathbb{E}_t\left\{\left[e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1} - \mathbb{E}_t(e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1})\right]^2\right\} - \mathbb{E}_t(e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1}). \tag{4}$$

The parameter λ is the unique solution to the equation:

$$\rho(\lambda) = RP, \tag{5}$$

where

$$\rho(\lambda) = \mathbb{E}\left\{\prod_{i=1}^{T}(1+z_{1,t+i-1})^{\frac{1}{12}}\exp\left[\mu_r(z_{t+i-1}) + e^{z_{2,t+i}/2}\varepsilon_{\mathrm{M},t+i}\right] - (1+z_{1;t})^{T/12}\right\} \tag{6}$$

for every $\lambda \in \mathbb{R}$.

Proof. The system has the following properties. Since

$$\mathbb{E}_t(r^e_{\mathrm{M},t+1}) = \mu_r(z_t) + \mathbb{E}_t(e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1}),$$

we obtain that

$$Var_t(r^e_{\mathrm{M},t+1}) = \mathbb{E}_t\left\{\left[e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1} - \mathbb{E}_t(e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1})\right]^2\right\},$$

and therefore

$$\mu_r(z_t) = \lambda\,\mathbb{E}_t\left\{\left[e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1} - \mathbb{E}_t(e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1})\right]^2\right\} - \mathbb{E}_t(e^{z_{2,t+1}/2}\varepsilon_{\mathrm{M},t+1}). \tag{7}$$

Under the assumptions of the model, we obtain:

$$\begin{aligned}
1 + R_{\mathrm{M},t:t+T} &= (1+R_{f,t:t+1})\ldots(1+R_{f,t+T-1:t+T})\exp\left(r^e_{M,t:t+1} + \cdots + r^e_{\mathrm{M},t+T-1:t+T}\right) \\
&= \prod_{i=1}^{T}(1+R_{f,t+i-1:t+i})\exp\left[\mu_r(z_{t+i-1}) + e^{z_{2,t+i}/2}\varepsilon_{\mathrm{M},t+i}\right] \\
&= \prod_{i=1}^{T}(1+z_{1,t+i-1})^{\frac{1}{12}}\exp\left[\mu_r(z_{t+i-1}) + e^{z_{2,t+i}/2}\varepsilon_{\mathrm{M},t+i}\right].
\end{aligned}$$

and

$$1 + R_{f,t:t+T} = (1+z_{1,t})^{T/12}, \tag{8}$$

Let

$$\rho(\lambda) = \mathbb{E}\left\{\prod_{i=1}^{T}(1+z_{1,t+i-1})^{\frac{1}{12}}\exp\left[\mu_r(z_{t+i-1}) + e^{z_{2,t+i}/2}\varepsilon_{\mathrm{M},t+i}\right] - (1+z_{1;t})^{T/12}\right\} \tag{9}$$

We obtain λ by solving □

Existence and Uniqueness of One-Step Equilibrium

The consumption-portfolio problem of the household at date t depends both on (i) the value of the SPP yield spread on contracts issued at date t and (ii) the value of the SPP yield

spread in future states, which determines the continuation value of future wealth. To prove the existence and uniqueness of a stationary general equilibrium, it is useful to distinguish between current and future spreads.

We accordingly rewrite the household's decision problem as follows. Let $\mathcal{Z}$ denote the state space and let $Y_0 : \mathcal{Z} \to \mathbb{R}_+$ denote a continuous mapping providing the state-contingent SPP yield spread at dates $t+T, t+2T, \ldots, \infty$. The household selects consumption and portfolio weights at date t by maximizing the Epstein-Zin recursive utility:

$$V^H[W_t^H; y(z_t), Y_0, z_t] = \max_{\{C_t^H, \alpha_{\text{M},t}^H, \alpha_{\text{SPP},t}^H\}} \left[(1-\delta^T)\,(C_t^H)^{1-1/\psi} + \delta^T\,(\mu_{t+T}^H)^{1-1/\psi}\right]^{\frac{1}{1-1/\psi}}, \tag{1}$$

where the excess return on the SPP is given by:

$$R^e_{\text{SPP},\,t:t+T} = y_(z_t)\,\mathbb{1}_{\left\{R_{\text{M},\,t:t+T} > B\right\}} + R^e_{\text{M},\,t:t+T}\,\mathbb{1}_{\left\{R_{\text{M},\,t:t+T} \leq B\right\}}, \tag{2}$$

the wealth available at the beginning of date $t+T$ by:

$$W_{t+T}^H = X_t^H\,\left(1 + R_{f,t:t+T} + \alpha_{\text{M},t}^H R^e_{\text{M},t:t+T} + \alpha_{\text{SPP},t}^H\,R^e_{\text{SPP},t:t+T}\right),$$

and the certainty equivalent of future wealth by:

$$\mu_{t+T}^H = \left(\mathbb{E}_t^{\mathbb{P}}\left\{V^H\left[W_{t+T}^H; Y_0(z_{t+T}), Y_0, z_{t+T}\right]^{1-\gamma}\right\}\right)^{\frac{1}{1-\gamma}}.$$

We denote by

$$\alpha_{\text{SPP},t}^H = \alpha_{\text{SPP}}^H[y(z_t), Y_0, z_t]$$

the household's optimal SPP weight in period t. We know that $\alpha_{\text{SPP}}^H(y_t, Y_0, z_t)$ is contained in $[0,1]$ and is a continuous function of y_t, Y_0, and z_t. We do not need to reformulate the dealer's problem since her decisions do not depend on future investments opportunities.

More generally, we establish the existence of a one-step equilibrium under the following set of sufficient conditions.

CONDITION 1 (Sufficient conditions for existence of one-step equilibrium): *The state vector z_t is first-order Markov and contained in a compact subset, $\mathcal{Z}$, of a Euclidean space. The household share of the SPP,*

$$\alpha_{\text{SPP}}^H(y_t, Y_0, z_t), \tag{3}$$

is contained in $[0,1]$, is continuous and increasing in y_t, and satisfies $\alpha_{\text{SPP}}^H(0, Y_0, z_t) = 0$ for every z_t.

The condition $\alpha_{\text{SPP}}^H(0, Y_0, z_t) = 0$ states that if the SPP yield spread is zero, the SPP is dominated by the bond and the household allocates no funds to the SPP. Let $\mathcal{C}_+$ denote the set of continuous mappings define on $\mathcal{Z}$ and taking values in $\mathbb{R}_+$. In the rest of the section, we derive the following result.

PROPOSITION 3 (Existence and uniqueness of one-step equilibrium): *Assume that Condition 2 holds. For every $Y_0 \in \mathcal{C}_+$, there exists a unique $Y_1 \in \mathcal{C}_+$ such that for every state*

$z_t \in \mathcal{Z}$, *the dealer maximizes her utility and the SPP and put markets clear:*

$$\alpha^D_{\text{SPP},t}[z_t, Y_1(z_t)] = -\lambda^H \, \alpha^H_{\text{SPP},t}[Y_1(z_t), Y_0, z_t], \tag{4}$$

$$\alpha^D_{\text{PUT},n}[z_t; Y_1(z_t)] = -d^E_n \, P_{\text{PUT},n}(z_t). \tag{5}$$

We denote by $\mathcal{T}$ *the transformation on* $\mathcal{C}_+$ *that maps the future yield spread,* Y_0*, to the current equilibrium yield spread:* $Y_1 = \mathcal{T}(Y_0)$.

The proof of the proposition proceeds as follows. Consider the stochastic discount factor

$$M_{t+T}(a^D_{\text{M}}, y_t, a^H_{\text{SPP}}) = e^{-\gamma^D \left[a^D_{\text{M}} \, R^e_{\text{M},\, t:t+T} - \lambda^H \, a^H_{\text{SPP}} \, R^e_{\text{SPP},\, t:t+T}(y_t) - \sum_{n=1}^N d^E_n \, (K_n - 1 - R_{\text{M},t:t+T})_+\right]}$$

where $R^e_{\text{SPP},\, t:t+T}(y_t) = y_t \, \mathbb{1}_{\left\{R_{\text{M},\, t:t+T} > B\right\}} + R^e_{\text{M},\, t:t+T} \, \mathbb{1}_{\left\{R_{\text{M},\, t:t+T} \leq B\right\}}$ expresses the SPP's excess return as a function of the the current yield.

LEMMA 3 (Auxiliary function G): *For a given state,* z_t*, SPP yield spread,* y_t*, and household demand for the SPP,* $\alpha^H_{\text{SPP},t}$*, the dealer's first-order condition in* z_t*:*

$$\mathbb{E}_t \left[R^e_{\text{M},\, t:t+T} \, M_{t+T}(\alpha^D_{\text{M},t}, y_t, \alpha^H_{\text{SPP},t})\right] = 0. \tag{6}$$

has a unique solution, $\alpha^D_{\text{M},t}$*, which we denote by* $G(y_t, \alpha^H_{\text{SPP},t}, z_t)$*. Furthermore, the dealer's equity share,* $\alpha^D_{\text{M},t} = G(y_t, \alpha^H_{\text{SPP},t}, z_t)$*, is strictly increasing in the SPP yield spread,* y_t*, and in the household's demand for the SPP,* $\alpha^H_{\text{SPP},t}$.

Proof. Consider the function $f(\alpha^D_{\text{M},t}, y_t, \alpha^H_{\text{SPP},t}) = \mathbb{E}_t \left[R^e_{\text{M},\, t:t+T} \, M_{t+T}(\alpha^D_{\text{M},t}, y_t, \alpha^H_{\text{SPP},t})\right]$, or equivalently

$$f(\alpha^D_{\text{M},t}, y_t, \alpha^H_{\text{SPP},t}) = \mathbb{E}_t \left\{ R^e_{\text{M},\, t:t+T} \, e^{-\gamma^D \left[\alpha^D_{\text{M},t} \, R^e_{\text{M},\, t:t+T} - \lambda^H \alpha^H_{\text{SPP},t} \, R^e_{\text{SPP},\, t:t+T}(y_t) - \sum_{n=1}^N d^E_n \, (K_n - 1 - R_{\text{M},t:t+T})_+\right]} \right\}.$$

We note that

$$\frac{\partial f}{\partial \alpha^D_{\text{M},t}} = -\gamma^D \, \mathbb{E}_t \left[(R^e_{\text{M},\, t:t+T})^2 \, M_{t:t+T}\right] < 0,$$

$$\frac{\partial f}{\partial y_t} = \gamma^D \, \lambda^H \, \alpha^H_{\text{SPP},t} \, \mathbb{E}_t \left(R^e_{\text{M},\, t:t+T} \, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}} \, M_{t:t+T}\right),$$

$$\frac{\partial f}{\partial \alpha^H_{\text{SPP},t}} = \gamma^D \, \lambda^H \, \mathbb{E}_t \left(R^e_{\text{M},\, t:t+T} \, R^e_{\text{SPP},\, t:t+T} \, M_{t+T}\right).$$

By the Implicit Function Theorem, the partial derivatives of G are given by:

$$\frac{\partial G}{\partial y_t} = -\frac{\partial f}{\partial y_t} \times \left(\frac{\partial f}{\partial \alpha^D_{\text{M},t}}\right)^{-1} = \lambda^H \, \alpha^H_{\text{SPP},t} \, \frac{\mathbb{E}_t \left(R^e_{\text{M},\, t:t+T} \, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}} \, M_{t:t+T}\right)}{\mathbb{E}_t \left[(R^e_{\text{M},\, t:t+T})^2 \, M_{t+T}\right]}, \tag{7}$$

$$\frac{\partial G}{\partial \alpha^H_{\text{SPP},t}} = -\frac{\partial f}{\partial \alpha^H_{\text{SPP},t}} \times \left(\frac{\partial f}{\partial \alpha^D_{\text{M},t}}\right)^{-1} = \lambda^H \, \frac{\mathbb{E}_t \left(R^e_{\text{M},\, t:t+T} \, R^e_{\text{SPP},\, t:t+T} \, M_{t+T}\right)}{\mathbb{E}_t \left[(R^e_{\text{M},\, t:t+T})^2 \, M_{t+T}\right]}. \tag{8}$$

Since $\mathbb{E}_t\left(R^e_{\mathrm{M},\,t:t+T}\,M_{t+T}\right)=0$, we know that

$$\mathbb{E}_t\left(R^e_{\mathrm{M},\,t:t+T}\,\mathbb{1}_{\{R_{\mathrm{M},\,t:t+T}>B\}}M_{t+T}\right)=-\mathbb{E}_t\left[R^e_{\mathrm{M},\,t:t+T}\,\mathbb{1}_{\{R_{\mathrm{M},\,t+T}\le B\}}M_{t:t+T}\right]>0. \tag{9}$$

Hence the function G strictly increases in y_t.

Similarly, since $R^e_{\mathrm{M},\,t:t+T}\,R^e_{\mathrm{SPP},\,t:t+T}=(R^e_{\mathrm{M},\,t:t+T})^2\mathbb{1}_{\{R_{\mathrm{M},\,t:t+T\le B\}}}+y_t\,R^e_{\mathrm{M},\,t:t+T}\,\mathbb{1}_{\{R_{\mathrm{M},\,t:t+T>B\}}}$, we infer that

$$\begin{aligned}\mathbb{E}_t\left(R^e_{\mathrm{M},\,t:t+T}\,R^e_{\mathrm{SPP},\,t:t+T}\,M_{t+T}\right) &= \mathbb{E}_t\left[(R^e_{\mathrm{M},\,t:t+T})^2\mathbb{1}_{\{R_{\mathrm{M},\,t:t+T}\le B\}}M_{t+T}\right]\\ &\quad +y_t\,\mathbb{E}_t\left[R^e_{\mathrm{M},\,t:t+T}\,\mathbb{1}_{\{R_{\mathrm{M},\,t:t+T>B\}}}M_{t+T}\right]>0.\end{aligned}$$

Hence the function G strictly increases in $\alpha^H_{\mathrm{SPP},t}$. □

In a one-step equilibrium, the following first-order conditions hold:

$$\mathbb{E}_t\left(R^e_{\mathrm{M},\,t:t+T}\,M_{t+T}\right) = 0, \tag{10}$$

$$\mathbb{E}_t\left(R^e_{\mathrm{SPP},t+1}\,M_{t+T}\right) = 0. \tag{11}$$

By equation (2), the first-order condition (11) can be rewritten as:

$$y_t\,\mathbb{E}_t\left(\mathbb{1}_{\left\{R_{\mathrm{M},\,t:t+T}>B\right\}}M_{t+T}\right)+\mathbb{E}_t\left(R^e_{\mathrm{M},\,t:t+T}\,\mathbb{1}_{\left\{R_{\mathrm{M},\,t:t+T}\le B\right\}}\,M_{t+T}\right)=0, \tag{12}$$

or

$$y_t=-\frac{\mathbb{E}_t\left(R^e_{\mathrm{M},\,t:t+T}\,\mathbb{1}_{\left\{R_{\mathrm{M},\,t:t+T}\le B\right\}}\,M_{t+T}\right)}{\mathbb{E}_t\left(\mathbb{1}_{\left\{R_{\mathrm{M},\,t:t+T}>B\right\}}M_{t+T}\right)}>0. \tag{13}$$

We infer from equation (10) that

$$y_t=\frac{\mathbb{E}_t\left(R^e_{\mathrm{M},\,t:t+T}\,\mathbb{1}_{\{R_{\mathrm{M},\,t:t+T}>B\}}\,M_{t+T}\right)}{\mathbb{E}_t\left(\mathbb{1}_{\{R_{\mathrm{M},\,t:t+T}>B\}}\,M_{t+T}\right)}. \tag{14}$$

Since the return on the SPP is deterministic for $R_{\mathrm{M},\,t:t+T}>B$, this equation reduces to:

$$y_t=\frac{\mathbb{E}_t\left[R^e_{\mathrm{M},\,t:t+T}\,\mathbb{1}_{\{R_{\mathrm{M},\,t:t+T}>B\}}\,M^*_{t+T}(\alpha^D_{\mathrm{M},t})\right]}{\mathbb{E}_t\left[\mathbb{1}_{\{R_{\mathrm{M},\,t:t+T}>B\}}\,M^*_{t+T}(\alpha^D_{\mathrm{M},t})\right]}, \tag{15}$$

where

$$M^*_{t+T}(\alpha^D_{\mathrm{M},t})=e^{-\gamma^D\left[\alpha^D_{\mathrm{M},t}\,R_{\mathrm{M},t:t+T}-\sum_{n=1}^N d^E_n\,(K_n-1-R_{\mathrm{M},t:t+T})_+\right]}. \tag{16}$$

We now study the properties of the function on the right-hand side of equation (15).

LEMMA 4 (Auxiliary function H): *The function $H(\alpha^D_{\mathrm{M},t},z_t)$ strictly decreases in $\alpha^D_{\mathrm{M},t}$ for every state z_t.*

Proof. We let

$$H(a, z_t) = \frac{u(a, z_t)}{v(a, z_t)} - R_{f,t,t:T}, \tag{17}$$

where

$$\begin{aligned} u(a, z_t) &= \mathbb{E}_t \left[R_{\text{M},\, t:t+T}\, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}}\, M^*_{t+T}(a) \right], \\ v(a, z_t) &= \mathbb{E}_t \left[\mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}}\, M^*_{t+T}(a) \right]. \end{aligned}$$

We note that

$$\frac{\partial u}{\partial a} = -\gamma^D\, \mathbb{E}_t \left[(R_{\text{M},\, t:t+T})^2\, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}}\, M^*_{t+T} \right]$$

$$\frac{\partial v}{\partial a} = -\gamma^D\, \mathbb{E}_t \left[R_{\text{M},\, t:t+T}\, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}}\, M^*_{t+T} \right] = -\gamma^D\, u_t(a).$$

We infer that

$$\frac{\partial H}{\partial a} = \gamma^D\, \frac{\begin{array}{c} \left\{ \mathbb{E}_t \left[R_{\text{M},\, t:t+T}\, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}}\, M^*_{t+T} \right] \right\}^2 - \\ \left\{ \mathbb{E}_t \left[(R_{\text{M},\, t:t+T})^2\, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}}\, M^*_{t+T} \right] \right\} \left\{ \mathbb{E}_t \left[\mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}}\, M^*_{t+T} \right] \right\} \end{array}}{\left\{ \mathbb{E}_t [\mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}}\, M^*_{t+T}] \right\}^2}.$$

By the Cauchy Schwarz inequality, the numerator is negative.[10] Hence, H is strictly decreasing in a. We also note that

$$\left| \frac{\partial H}{\partial a}(a, z_t) \right| \leq \gamma^D\, \frac{\mathbb{E}_t \left[(R_{\text{M},\, t:t+T})^2\, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}}\, M^*_{t+T} \right]}{\mathbb{E}_t [\mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}}\, M^*_{t+T}(a)]}.$$

for every a and z_t. □

We consider the function

$$\Phi(y_t, Y_0, z_t) = y_t - H \left\{ G[y_t, \alpha^H_{\text{SPP}}(y_t, Y_0, z_t), z_t], z_t \right\}.$$

Under Condition 2, the function α^H_{SPP} increases in y_t. By Lemmas 3 and 4, the function Φ therefore increases in y_t.

When $y_t = 0$, the household has zero demand for the SPP; $\alpha^H_{\text{SPP}}(0, Y_0, z_t) = 0$, as per Condition 2. Hence,

$$\Phi(0, Y_0, z_t) = -H \left[\hat{\alpha}(z_t), z_t \right],$$

where $\hat{\alpha}(z_t) = G(0, 0, z_t)$ denotes the dealer's optimal equity share in the absence of the SPP contract. The equity share, $\hat{\alpha}(z_t)$, satisfies the first-order condition:

$$\mathbb{E}_t \left\{ R^e_{\text{M},\, t:t+T}\, e^{-\gamma^D \left[\alpha^D_{\text{M},t}\, R^e_{\text{M},\, t:t+T} - \sum_{n=1}^N d^E_n\, (K_n - 1 - R_{\text{M},t:t+T})_+ \right]} \right\} = 0, \tag{18}$$

[10] Let $X = R_{\text{M},\, t:t+T}\, \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}}\, [M^*_{t+T}(a)]^{1/2}$ and $Y = \mathbb{1}_{\{R_{\text{M},\, t:t+T} > B\}}\, [M^*_{t+T}(a)]^{1/2}$. By the Cauchy Schwarz inequality, $[E(XY)]^2 < E(X^2)E(Y^2)$. The inequality is strict because X and Y are not perfectly correlated.

or, more compactly, $\mathbb{E}_t\left\{R^e_{\text{M},\,t:t+T}\,M^*_{t+1}[\hat{\alpha}(z_t)]\right\}=0$. Since

$$\mathbb{E}_t\left\{R^e_{\text{M},\,t:t+T}\,\mathbb{1}_{\{R_{\text{M},\,t:t+T}>B\}}\,M^*_{t+1}[\hat{\alpha}(z_t)]\right\}=-\mathbb{E}_t\left\{R^e_{\text{M},\,t:t+T}\,\mathbb{1}_{\{R_{\text{M},\,t:t+T}\leq B\}}\,M^*_{t+1}(\hat{\alpha}(z_t))\right\}>0,$$

we infer that $H\left[\hat{\alpha}(z_t),z_t\right]>0$ and therefore $\Phi(0,Y_0,z_t)<0$.

We also note that

$$\frac{\partial\Phi(y_t,Y_0,z_t)}{\partial y_t}\geq 1. \tag{19}$$

Hence $\Phi(y_t,Y_0,z_t)$ is positive for y_t sufficiently large. We infer that the equation $\Phi(y_t,Y_0,z_t)=0$ has a unique solution, so that there exists a unique one-step equilibrium for every z_t.

We further note that

$$Y_1(z_t,Y_0)\leq\left|\,H\left[G(0,0,z_t),z_t\right]\,\right| \tag{20}$$

for every $z_t\in\mathcal{Z}$ and $Y_0\in\mathcal{C}_+$.

Proof of Proposition 2

Let $\mathcal{C}$ denote the set of continuous functions defined on $\mathcal{Z}$ and taking values on the real line. Let $\|f\|_\infty=\sup_{z\in\mathcal{Z}}|f(z)|$ denote the sup norm. We know that $(\mathcal{C},\|f\|_\infty)$ is a Banach space.

Consider

$$\bar{y}=\max_{z_t\in\mathcal{Z}}\left|H\left[G(0,0,z_t),z_t\right]\right|<\infty. \tag{1}$$

We know that for every function $Y_0\in\mathcal{C}$, the transform $Y_1=\mathcal{T}\,Y_0$ takes values between 0 and $\bar{y}$, as equation (20) implies. For this reason, we henceforth focus on the subset of functions:

$$\mathcal{K}=\{f\in\mathcal{C}:0\leq f(z)\leq\bar{y}\text{ for every }z\in\mathcal{Z}\}.$$

The set $\mathcal{K}$ is convex and closed.

Consider the function $f(y_t,a_{\text{SPP}},z_t)=H\left[G(y_t,a_{\text{SPP}},z_t),z_t\right]$. By the chain rule,

$$\frac{\partial f}{\partial a_{\text{SPP}}}(y_t,a_{\text{SPP}},z_t)=\frac{\partial H}{\partial a_M}\left[G(y_t,a_{\text{SPP}},z_t),z_t\right]\,\frac{\partial G}{\partial a_{\text{SPP}}}(y_t,a_{\text{SPP}},z_t). \tag{2}$$

The functions on the right-hand side are bounded and we let

$$K_1=\sup\left\{\left|\frac{\partial f}{\partial a_{\text{SPP}}}(y_t,a_{\text{SPP}},z_t)\right|;\ (y_t,a_{\text{SPP}},z_t)\in[0,\bar{y}]\times[0,1]\times\mathcal{Z}\right\}.$$

Importantly, we note that the upper bounds $\bar{y}$ and K_1 do not depend on the household demand for the SPP.[11]

[11]Furthermore, we infer from and that

$$K_1\leq\lambda^H\,\gamma^D\sup_{(y_t,a_{\text{SPP}},z_t)\in[0,\bar{y}]\times[0,1]\times\mathcal{Z}}\frac{\mathbb{E}_t\left\{R^e_{\text{M},\,t:t+T}\,R^e_{\text{SPP},\,t:t+T}\,M_{t+T}[G(a_{\text{SPP}},z_t),y_t,z_t]\right\}}{\mathbb{E}_t\left\{\mathbb{1}_{\{R_{\text{M},\,t:t+T}>B\}}\,M_{t+T}[G(a_{\text{SPP}},z_t),y_t,z_t]\right\}}. \tag{3}$$

CONDITION 2 (Sufficient conditions for existence of stationary equilibrium): *There exists* $k_{\text{SPP}} > 0$ *such that the household share of the SPP,* $\alpha^H_{\text{SPP}}(y_t, Y_0, z_t)$*, satisfies*

$$|\alpha^H_{\text{SPP}}(y_t, Y_0', z_t) - \alpha^H_{\text{SPP}}(y_t, Y_0, z_t)| \leq k_{\text{SPP}} \, \|Y_0' - Y_0\|_\infty$$

for every $Y_0, Y_0' \in \mathcal{K}$*,* $y_t \in [0, \bar{y}]$*, and* $z_t \in \mathcal{Z}$*. Furthermore, the constant* k_{SPP} *satisfies* $k_{\text{SPP}} < 1/K_1$*.*

We consider $Y_0, Y_0' \in \mathcal{K}$. Let $Y_1 = \mathcal{T}Y_0$ and $Y_1' = \mathcal{T}Y_0'$. For a given state z_t, we let $y_t = Y_1(z_t)$ and $y_t' = Y_1'(z_t')$. Since $\Phi(y_t', Y_0', z_t) = \Phi(y_t, Y_0, z_t) = 0$, we infer that

$$\Phi(y_t', Y_0', z_t) - \Phi(y_t', Y_0, z_t) = \Phi(y_t, Y_0, z_t) - \Phi(y_t', Y_0, z_t). \tag{4}$$

We infer from (19)

$$|y_t' - y_t| \leq |\Phi(y_t, Y_0, z_t) - \Phi(y_t', Y_0, z_t)|. \tag{5}$$

Sufficient Condition 2 implies

$$|\Phi(y_t', Y_0', z_t) - \Phi(y_t', Y_0, z_t)| \leq k_{\text{SPP}} \, K_1 \, \|Y_0' - Y_0\|_\infty. \tag{6}$$

We infer from (4), (5) and (6) that

$$|y_t' - y_t| \leq k_{\text{SPP}} \, K_1 \, \|Y_0' - Y_0\|_\infty.$$

Since this inequality holds for every state z_t, the transformation $\mathcal{T}$ is a contraction:

$$\|\mathcal{T}Y_0' - \mathcal{T}Y_0\|_\infty \leq k_{\text{SPP}} \, K_1 \, \|Y_0' - Y_0\|_\infty.$$

Hence, the contraction mapping $\mathcal{T}$ has a unique fixed point.

Alternative specification of the dealer

The wealth of the dealer in period t is

$$X_t^D = \phi F W_t^H = \phi X_t^H / (1 - \phi).$$

We write

$$X_t^H = \lambda^H X_t^D,$$

where $\lambda^H = (1 - \phi)/\phi$.

The dealer's wealth in period $t + T$ is

$$W_{t+T}^D = X_t^D \left(1 + R_{t:t+T}^D\right).$$

where

$$R^D_{t:t+T} = R_{f,\,t:t+T} + \alpha^D_{\mathrm{M},t}\, R^e_{\mathrm{M},\,t:t+T} + \alpha^D_{\mathrm{SPP},t}\, R^e_{\mathrm{SPP},\,t:t+T} + \sum_{n=1}^{N} \alpha^D_{\mathrm{PUT},n,t}\, R^e_{\mathrm{PUT},n,\,t:t+T} \tag{1}$$

denotes the dealer's return over the period.

The dealer has constant absolute risk aversion (CARA) utility:

$$\mathbb{E}^{\mathbb{P}}_0 \sum_{t=1}^{\infty} \delta^t u_t(W^D_t)$$

where $u_{t+T}(W^D_{t+T}) = -e^{-A_t W^D_{t+T}}/A_t$ and $A_t = \gamma^D/X^D_t$.

Let $\alpha^D_t = (\alpha^D_{\mathrm{M},t}, \alpha^D_{\mathrm{SPP},t}, \alpha^D_{\mathrm{PUT},1,t}, \ldots, \alpha^D_{\mathrm{PUT},K,t})$. Assume that to minimize attention costs etc., the dealer chooses the portfolio α^D_t and is then precluded from modifying it thereafter.

The dealer solves the optimization problem

$$\max_{\alpha^D_t} \mathbb{E}_t \left[-e^{-\gamma^D \left(W^D_{t+T}/X^D_t\right)} \right].$$

Since $W^D_{t+T}/X^D_t = 1 + R^D_{t:t+T}$, the solution to this problem is independent of the wealth level X^D_t. We denote by

$$\alpha^D[z_t; Y(z_t)]$$

the optimal portfolio of the dealer.

We can also assume that the dealer consumes X^D_t at date t and keeps the residual risk:

$$X^D_t \left(\alpha^D_{\mathrm{M},t}\, R^e_{\mathrm{M},\,t:t+T} + \alpha^D_{\mathrm{SPP},t}\, R^e_{\mathrm{SPP},\,t:t+T} + \sum_{n=1}^{N} \alpha^D_{\mathrm{PUT},n,t}\, R^e_{\mathrm{PUT},n,\,t:t+T} \right). \tag{2}$$

Likelihood

As we explain in the main text, the one-step likelihood of y_{t+1} is

$$f(y_{t+1}|y_{1:t}) \;=\; \iint h(y_{t+1}|y_{1:t}, z_{2,t}, z_{2,t+1}) f(z_{2,t}, z_{2,t+1}|y_{1:t})\, dz_{2,t}\, dz_{2,t+1},$$

where

$$h(y_{t+1}|y_{1:t}, z_{2,t}, z_{2,t+1}) \;=\; f(z_{1,t+1}|z_t, z_{2,t+1}) \;\; f(r^e_{M,t+1}|z_t, z_{t+1}) \;\; f(X_{t+1}|z_{t+1}). \tag{1}$$

We explain below how to compute the conditional densities on the right-hand side of (1).

First, conditional on z_t, the state vector z_{t+1} is Gaussian with mean $(\mu_1 + \phi_1(z_{1,t} - \mu_1), \mu_2 + \phi_2(z_{2,t} - \mu_2))'$ and variance-covariance matrix

$$\begin{pmatrix} \sigma_1^2 & \rho_{12}\,\sigma_1\,\sigma_2 \\ \rho_{12}\,\sigma_1\,\sigma_2 & \sigma_2^2 \end{pmatrix}$$

Hence

$$f(z_{1,t+1}|z_t, z_{2,t+1}) = n\left[z_{1,t+1}; \mu_1 + \phi_1(z_{1,t} - \mu_1) + \sigma_1\rho_{12}\varepsilon_{2,t+1}; \sigma_1^2(1-\rho_{12}^2)\right],$$

where

$$\varepsilon_{2,t+1} = \frac{1}{\sigma_2}[z_{2,t+1} - \mu_2 - \phi_2(z_{2,t} - \mu_2)]$$

denotes the normalized innovation on state 2.

Second, we turn to the conditional distribution of

$$r^e_{M,t+1} = \mu_r + e^{z_{2,t+1}/2}\varepsilon_{M,t+1} \tag{2}$$

conditional on density z_t and z_{t+1}). The variance-covariance matrix of the Gaussian vector $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{m,t})'$:

$$V_\varepsilon = \begin{pmatrix} 1 & \rho_{12} & \rho_{1M} \\ \rho_{12} & 1 & \rho_{2M} \\ \rho_{1M} & \rho_{2M} & 1 \end{pmatrix}$$

The Cholesky decomposition of V_ε can be written as

$$V_\varepsilon = L\,L^\top$$

where $L = (L_{ij})$ is a lower triangular matrix. We consider the random vector $u_t = L^{-1}u_t$, or equivalently

$$\varepsilon_{t+1} = L\,u_{t+1}, \tag{3}$$

The vectors are u_t is iid $\mathcal{N}(0, I)$ and satisfy:

$$\begin{aligned} \varepsilon_{1,t+1} &= L_{1,1}\,u_{1,t+1} \\ \varepsilon_{2,t+1} &= L_{2,1}\,u_{1,t+1} + L_{2,2}\,u_{2,t+1} \\ \varepsilon_{\mathrm{M},t+1} &= L_{3,1}\,u_{1,t+1} + L_{3,2}\,u_{2,t+1} + L_{3,3}\,u_{3,t+1}. \end{aligned}$$

If the vectors z_t and z_{t+1} are known, then $\varepsilon_{1,t+1}$, $\varepsilon_{2,t+1}$ $u_{1,t+1}$ and $u_{2,t+1}$ are also known.

We note that $r^e_{M,t+1} = \mu_r + e^{z_{2,t+1}/2}\varepsilon_{M,t+1}$ satisfies

$$r^e_{M,t+1} = \mu_r + e^{z_{2,t+1}/2}(L_{3,1}u_{1,t+1} + L_{3,2}u_{2,t+1} + L_{3,3}u_{3,t+1}) \tag{4}$$

Hence

$$f(r^e_{\mathrm{M},t+1}|z_t, z_{t+1}) = n\left[r^e_{\mathrm{M},t+1}; \mu_r + e^{z_{2,t+1}}(L_{3,1}u_{1,t+1} + L_{3,2}u_{2,t+1}); e^{z_{2,t+1}}L_{33}^2)\right], \tag{5}$$

where

$$\begin{aligned} u_{1,t+1} &= \varepsilon_{1,t+1}/L_{1,1}, & (6) \\ u_{2,t+1} &= (\varepsilon_{2,t+1} - L_{2,1}\,u_{1,t+1})/L_{2,2}. & (7) \end{aligned}$$

Price Elasticity

Let v_t^{SPP} denote the volume of SPP. The stochastic discount factor can be written as $M_{t+T} = e^{-m_{t+T}(z_t, v_t^{\text{SPP}})}$, where

$$m_{t+T}(z_t, v_t^{\text{SPP}}) = \gamma^D \left[\alpha_{\text{M}}^D(z_t, v_t^{\text{SPP}})\, R^e_{\text{M},t:t+T} - \lambda^H\, v_t^{\text{SPP}}\, R^e_{\text{SPP},t:t+T} - \sum_{n=1}^{N} d_n^E\, (K_n - 1 - R_{\text{M},t:t+T})_+ \right].$$

We know that

$$\begin{aligned} \mathbb{E}[\, R^e_{\text{M},t:t+T} M_{t+T}(z_t, v_t^{\text{SPP}}) \big|\, z_t] &= 0 \\ \mathbb{E}[\, R^e_{\text{SPP},t:t+T} M_{t+T}(z_t, v_t^{\text{SPP}}) \big|\, Z_t] &= 0, \end{aligned}$$

for every v_t^{SPP} and z_t.

By (14), the price of a put satisfies:

$$\begin{aligned} \ln[P_{\text{PUT},n}(z_t, v_t^{\text{SPP}})] &= \ln\{\mathbb{E}\,[(K_n - 1 - R_{\text{M},t:t+T})_+\, M_{t+T}(z_t, v_t^{\text{SPP}}) |\, z_t]\} \\ &\quad - \ln\{\mathbb{E}\,[M_{t+T}(z_t, v_t^{\text{SPP}}) |\, z_t]\} - \ln(1 + R_{f,t:t+T}). \end{aligned}$$

We observe that

$$\frac{\partial \mathbb{E}\,(M_{t+T} |\, z_t)}{\partial v_t^{\text{SPP}}} = \gamma^D\, \mathbb{E}\left(\left[\lambda^H\, R^e_{\text{SPP},t:t+T} - \frac{\partial \alpha_{\text{M}}^D(z_t, v_t^{\text{SPP}})}{\partial v_t^{\text{SPP}}}\, R^e_{\text{M},t:t+T} \right] M_{t+T} \,\middle|\, z_t \right) = 0.$$

Hence,

$$\begin{aligned} \frac{\partial \ln[P_{\text{PUT},n}(z_t)]}{\partial v_t^{\text{SPP}}} &= \frac{\partial \ln\{\mathbb{E}\,[(K_n - 1 - R_{\text{M},t:t+T})_+\, M_{t+T} |\, z_t]\}}{\partial v_t^{\text{SPP}}} \\ &= \frac{1}{\mathbb{E}\,[(K_n - 1 - R_{\text{M},t:t+T})_+\, M_{t+T} |\, z_t]} \frac{\partial \{\mathbb{E}\,[(K_n - 1 - R_{\text{M},t:t+T})_+\, M_{t+T} |\, z_t]\}}{\partial v_t^{\text{SPP}}} \\ &= \frac{1}{P_{\text{PUT},n}(1 + R_{f,t:t+T})\mathbb{E}\,(M_{t+T} |\, z_t)} \frac{\partial \{\mathbb{E}\,[(K_n - 1 - R_{\text{M},t:t+T})_+\, M_{t+T} |\, z_t]\}}{\partial v_t^{\text{SPP}}} \\ &= \frac{\gamma^D}{P_{\text{PUT},n}(1 + R_{f,t:t+T})\mathbb{E}\,(M_{t+T} |\, z_t)} \times \\ &\quad \mathbb{E}\left\{ (K_n - 1 - R_{\text{M},t:t+T})_+ \left[\lambda^H\, R^e_{\text{SPP},t:t+T} - \frac{\partial \alpha_{\text{M}}^D(z_t, v_t^{\text{SPP}})}{\partial v_t^{\text{SPP}}}\, R^e_{\text{M},t:t+T} \right] M_{t+T} \,\middle|\, z_t \right\}. \end{aligned}$$

By the chain rule,

$$\begin{aligned} \frac{\partial \ln P_{\text{PUT},n}}{\partial v_t^{\text{SPP}}} &= \frac{1}{P_{\text{PUT},n}} Vega_{n,t} \frac{\partial IV_{n,t}}{\partial v_t^{\text{SPP}}} \\ &= \frac{Vega_{n,t}}{P_{\text{PUT},n}} \frac{IV_{n,t}}{v_t^{\text{SPP}}} \frac{\partial \ln IV_{n,t}}{\partial \ln v_t^{\text{SPP}}} \end{aligned}$$

Hence

$$\frac{\partial \ln IV_{n,t}}{\partial \ln v_t^{\text{SPP}}} = \frac{\gamma^D \, v_t^{\text{SPP}}}{Vega_{n,t} IV_{n,t}(z_t)(1+R_{f,t:t+T})\mathbb{E}\left(M_{t+T}|\, z_t\right)} \times \mathbb{E}\left\{ (K_n - 1 - R_{\text{M},t:t+T})_+ \left[\lambda^H \, R^e_{\text{SPP},t:t+T} - \frac{\partial \alpha^D_{\text{M}}(z_t, v_t^{\text{SPP}})}{\partial v_t^{\text{SPP}}} \, R^e_{\text{M},t:t+T}\right] M_{t+T} \middle| \, z_t \right\}.$$

LEMMA 5: *The sensitivity of the dealer's optimal risky share with respect to the SPP volume is given by*

$$\frac{\partial \alpha^D_{\text{M}}}{\partial v_t^{\text{SPP}}} = \lambda^H \frac{\kappa_0\kappa_2 + \kappa_1^2}{\kappa_0(\kappa_2+\kappa_3) + \lambda^H \gamma^D \, v_t \kappa_1 (\kappa_3\kappa_0 - \kappa_1^2)/\kappa_0}, \tag{1}$$

where

$$\begin{aligned}
\kappa_0 &= \mathbb{E}\left(1_{\{R_M > B\}} M_{t+T}\right) > 0 \\
\kappa_1 &= -\mathbb{E}\left(R^e_{\text{M}} \, 1_{\{R_M < B\}} M_{t+T}\right) > 0 \\
\kappa_2 &= \mathbb{E}\left[(R^e_{\text{M}})^2 \, 1_{\{R_M < B\}} M_{t+T}\right] > 0 \\
\kappa_3 &= \mathbb{E}\left[(R^e_{\text{M}})^2 \, 1_{\{R_M > B\}} M_{t+T}\right] > 0
\end{aligned}$$

Proof. The dealer's optimal risky share and the SPP yield solve:

$$f(\alpha^D_{\text{M}}, y_t; v_t^{\text{SPP}}) = E(R^e_{\text{M}} \, M) = 0 \tag{2}$$

$$g(\alpha^D_{\text{M}}, y_t; v_t^{\text{SPP}}) = E(R^e_{\text{SPP}} \, M) = 0 \tag{3}$$

By the Implicit Function Theorem,

$$\frac{\partial \alpha^D_{\text{M}}}{\partial v_t^{\text{SPP}}} = \frac{\frac{\partial f}{\partial v}\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}\frac{\partial g}{\partial v}}{\frac{\partial f}{\partial y}\frac{\partial g}{\partial \alpha^D_{\text{M}}} - \frac{\partial f}{\partial \alpha^D_{\text{M}}}\frac{\partial g}{\partial y}}, \tag{4}$$

We note that

$$\begin{aligned}
\frac{\partial f}{\partial \alpha^D_{\text{M}}} &= -\gamma^D(\kappa_2+\kappa_3) \\
\frac{\partial f}{\partial y} &= \lambda^H \gamma^D \, v_t^{\text{SPP}} \, \kappa_1 \\
\frac{\partial f}{\partial v} &= \lambda^H \gamma^D \, (\kappa_2 + y_t \, \kappa_1) \\
\frac{\partial g}{\partial \alpha^D_{\text{M}}} &= -\gamma^D \, (\kappa_2 + y_t \, \kappa_1) \\
\frac{\partial g}{\partial y} &= (1 + \lambda^H \gamma^D \, v_t \, y_t)\kappa_0 \\
\frac{\partial g}{\partial v} &= \lambda^H \gamma^D \, (\kappa_2 + y_t^2 \, \kappa_0),
\end{aligned}$$

where $y_t = \kappa_1/\kappa_0$. We verify that

$$\frac{\partial f}{\partial v}\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}\frac{\partial g}{\partial v} = \lambda^H \gamma^D \left(\kappa_0\kappa_2 + \kappa_1^2\right)$$

$$\frac{\partial f}{\partial y}\frac{\partial g}{\partial \alpha_{\text{M}}^D} - \frac{\partial f}{\partial \alpha_{\text{M}}^D}\frac{\partial g}{\partial y} = \gamma^D \left[\kappa_0(\kappa_2+\kappa_3) + \lambda^H \gamma^D v_t \kappa_1(\kappa_3\kappa_0 - \kappa_1^2)/\kappa_0\right]$$

Hence,

$$\frac{\partial \alpha_{\text{M}}^D}{\partial v_t^{\text{SPP}}} = \lambda^H \frac{\kappa_0\kappa_2 + \kappa_1^2}{\kappa_0(\kappa_2+\kappa_3) + \lambda^H \gamma^D v_t \kappa_1(\kappa_3\kappa_0 - \kappa_1^2)/\kappa_0}. \tag{5}$$

□

The sensitivity is thus

$$\begin{aligned}\frac{\partial \ln IV_{n,t}}{\partial \ln v_t^{\text{SPP}}} &= \gamma^D \lambda^H v_t^{\text{SPP}} \times \\ &\quad \frac{\mathbb{E}\left\{(K_n - 1 - R_{\text{M},t:t+T})_+ \left[R^e_{\text{SPP},t:t+T} - \frac{\kappa_0\kappa_2+\kappa_1^2}{\kappa_0(\kappa_2+\kappa_3)+\lambda^H\gamma^D v_t\kappa_1(\kappa_3\kappa_0-\kappa_1^2)/\kappa_0} R^e_{\text{M},t:t+T}\right] M_{t+T}\middle| z_t\right\}}{Vega_{n,t} IV_{n,t}(z_t)(1+R_{f,t:t+T})\mathbb{E}\left(M_{t+T}\middle| z_t\right)}.\end{aligned}$$

If $K_n \leq 1 + B$,

$$\begin{aligned}\frac{\partial \ln IV_{n,t}}{\partial \ln v_t^{\text{SPP}}} &= \lambda^H \gamma^D v_t^{\text{SPP}} \frac{(\kappa_0\kappa_3 - \kappa_1^2)(1 + \lambda^H \gamma^D v_t \kappa_1/\kappa_0)}{\kappa_0(\kappa_2+\kappa_3) + \lambda^H \gamma^D v_t \kappa_1(\kappa_3\kappa_0 - \kappa_1^2)/\kappa_0} \\ &\quad \times \frac{\mathbb{E}\left\{(K_n - 1 - R_{\text{M},t:t+T})_+ R^e_{\text{M},t:t+T} M_{t+T}\middle| z_t\right\}}{Vega_{n,t} IV_{n,t}(z_t)(1+R_{f,t:t+T})\mathbb{E}\left(M_{t+T}\middle| z_t\right)},\end{aligned}$$

Finally, if $K_n = 1 + B$,

$$\begin{aligned}\frac{\partial \ln IV_{n,t}}{\partial \ln v_t^{\text{SPP}}} &= -\lambda^H \gamma^D v_t^{\text{SPP}} \frac{(\kappa_0\kappa_3 - \kappa_1^2)(1 + \lambda^H \gamma^D v_t \kappa_1/\kappa_0)}{\kappa_0(\kappa_2+\kappa_3) + \lambda^H \gamma^D v_t \kappa_1(\kappa_3\kappa_0 - \kappa_1^2)/\kappa_0} \\ &\quad \times \frac{\kappa_2 - (R_{f,t} - B)\kappa_1}{Vega_{n,t} IV_{n,t}(z_t)(1+R_{f,t:t+T})\mathbb{E}\left(M_{t+T}\middle| z_t\right)},\end{aligned}$$

which is approximately equal to

$$\frac{\partial \ln IV_{n,t}}{\partial \ln v_t^{\text{SPP}}} = -\lambda^H \gamma^D v_t^{\text{SPP}} \frac{\kappa_0\kappa_3 - \kappa_1^2}{\kappa_0(\kappa_2+\kappa_3)} \frac{\kappa_2 - (R_{f,t} - B)\kappa_1}{Vega_{n,t} IV_{n,t}(z_t)(1+R_{f,t:t+T})\mathbb{E}\left(M_{t+T}\middle| z_t\right)}.$$

Appendix A. Tables

Table I
Summary Statistics on Short Put Products (2004-2018)

	Mean	p10	p50	p90
Product Characteristics, N =182,000				
Maturity, in years	3.1	1.5	3	5.0
Headline Rate (annualized), in %	9.7	3	7.5	19.1
Strike, in % of the initial value	58	49	60	70
Yearly Excess Ex-post Return, in %	5.7	.7	6.3	9.7
Issuance Volumes, in million $	6.4	0.2	2.3	15.5
Market Parameters, across 220 Months and 22 Indices				
1-year Risk Free Rate	1.1	-0.25	0.28	3.878
Historical Volatility, in %	20.1	11.2	18.3	29.0
Implied Volatility, strike 60%	31.7	25.2	30.8	39.2
Implied Volatility, strike 80%	31.7	25.2	30.8	39.2
Implied Volatility, strike 100%	31.7	25.2	30.8	39.2
SPP Issuance Volumes, in billion $	1.1	0.1	0.8	2.5
SPP Scaled Issuance Volumes, in %				

This table reports summary statistics on the volume and design of SPPs with indices as underlying assets, as well as corresponding market data.

Table II
Market Conditions and Product Design

	Headline Rate, in %			Headline Rate, in Log		
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	
Historic Volatility, in %	0.117***	0.110***	0.0415***			
	(0.0256)	(0.0256)	(0.00747)			
Historical Volatility, in Log				0.307***	0.290***	0.117***
				(0.0599)	(0.0612)	(0.0221)
Fixed Effects						
Underlying	Yes	Yes	Yes	Yes	Yes	Yes
Design	-	Yes	Yes	-	Yes	Yes
Market Country	-	Yes	Yes	-	Yes	Yes
Year	-	-	Yes	-	-	Yes
Observations	180,585	180,582	180,582	180,585	180,582	180,582
R^2	0.156	0.298	0.516	0.156	0.321	0.508

This table displays the coefficients from regressing SPP headline rates on historical volatility. The sample includes all SPP products issued over the 2004-2018 period with one or several indices as underlyings. Design fixed effects include fixed effects for maturity, strike, number of payoff features, number of scenari. Standard errors are clustered at the month and index level. *, **, and *** represent statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

Table III
The Impact of SPP Volumes on Implied Volatility

	Dependent Variable : Implied Volatility											
	Issuance Volumes						Scaled Issuance Volumes					
Strike	60%		80%		100%		60%		80%		100%	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A. In Levels												
SPP Volumes	-0.64**	-0.59***	-0.35*	-0.43***	-0.33	-0.59***	-45.66***	-44.78***	-25.59**	-33.23***	-6.14	-21.29***
	(0.26)	(0.21)	(0.21)	(0.15)	(0.22)	(0.14)	(14.00)	(11.56)	(10.01)	(7.63)	(10.52)	(7.75)
Historical Volatility		0.22***		0.24***		0.30***		0.27***		0.29***		0.33***
		(0.02)		(0.02)		(0.02)		(0.04)		(0.03)		(0.02)
Risk-free Rate		0.16		-0.03		-0.27**		0.44***		0.12		-0.11
		(0.13)		(0.09)		(0.11)		(0.16)		(0.09)		(0.09)
Panel B. In Logs												
Log(SPP Volumes)	-0.03***	-0.03***	-0.02**	-0.02**	-0.02	-0.04***	-0.04***	-0.05***	-0.02**	-0.02***	0.00	-0.01*
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
(mean) hvol90d		0.01***		0.01***		0.01***		0.01***		0.01***		0.01***
		(0.00)		(0.00)		(0.00)		(0.00)		(0.00)		(0.00)
(mean) rf_1y		0.00		-0.00		-0.02***		0.01		0.00		-0.01*
		(0.00)		(0.00)		(0.01)		(0.01)		(0.00)		(0.01)
Fixed Effects												
Month	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Index	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	465	465	465	465	465	465	545	545	545	545	545	545
R^2	0.97	0.98	0.96	0.98	0.94	0.97	0.93	0.95	0.94	0.97	0.93	0.97

This table displays the coefficients from regressing monthly index implied volatility at 60, 80 and 100% on SPP volumes with corresponding underlying. In Columns 1 to 6, SPP volumes are issuance volumes, in Columns 7 and 12, SPP issuance volumes are scaled by market cap. In Panel A, implied volatility measures and volumes are in levels, while they are in logs in Panel B. The model includes month and index fixed effects. The sample includes monthly implied volatility data at 1 year for the three main indices of the US, Europe and Asia: S&P500, Nikkei225, and Eurostoxx50. Standard errors are clustered at the month and at the index-year levels. *, **, and *** represent statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

Table IV
Maximum Likelihod Estimation
of Structural Model

	Unrestricted Estimate (1)	Restricted Estimate (2)
Interest Rate		
Mean μ_1	0.01928	0.01924
Persistence parameter ϕ_1	0.99699	0.99676
Volatility σ_1	0.00212	0.00211
Equity Index		
Mean log volatility μ_2	-5.49322	-5.50605
Persistence parameter ϕ_2	0.93439	0.93319
Volatility of log volatility σ_2	0.33565	0.33668
Correlations		
ρ_{12}	0.10121	0.10291
ρ_{1M}	0.25972	0.28945
ρ_{2M}	-0.13300	-0.14236
Stochastic discount factor		
Sensitivity of SDF to SPP volume, λ_H	0.38810	0
Payoff of Exogenous Put Portfolio:		
$S_T = 0.0$	0.63072	0.53537
$S_T = 0.5$	-1.11587	-1.14221
$S_T = 0.6$	-0.82323	-0.82332
$S_T = 0.7$	-0.66744	-0.65700
$S_T = 0.8$	-0.88680	-0.86121
$S_T = 0.9$	-0.41482	-0.33398
Measurement errors		
Yield spread	0.01556	0.01615
Put prices	0.00629	0.00633
Test of the restriction $\lambda_H = 0$		
Log likelihood	6286.82283	6283.32066
LR statistic	7.00325	
p value	0.00814	

This table displays the ML estimates of the structural model outlined in Section IV.

Table V
Dividend Term Structure and Outstanding Volume of Short Put Products: Equity Indices

	Implied Dividend				
	(1)	(2)	(3)	(4)	(5)
Outstanding Volume Scaled by Market Cap	-0.524**	-0.545***	-0.524***	-0.417***	
	(0.195)	(0.122)	(0.126)	(0.097)	
Term=2y × Outstanding Volume Scaled by Market Cap				0.016	-0.097***
				(0.066)	(0.018)
Term=3y × Outstanding Volume Scaled by Market Cap				-0.111	-0.270***
				(0.092)	(0.038)
Term=4y × Outstanding Volume Scaled by Market Cap				-0.190	-0.408***
				(0.125)	(0.056)
Term=5y × Outstanding Volume Scaled by Market Cap				-0.273	-0.536***
				(0.163)	(0.066)
Historical Dividend		0.005***	0.005***	0.005***	
		(0.001)	(0.001)	(0.001)	
Fixed Effects					
Month	Yes	Yes			
Security × Maturity	Yes	Yes	Yes	Yes	
Month × Maturity			Yes	Yes	
Maturity					Yes
Security × Month					Y
Observations	5,339	5,339	5,339	5,339	5,327
R^2	0.821	0.893	0.915	0.916	0.958

This table displays the coefficients from regressing future dividend yield on outstanding short put product volumes scaled by the stock market capitalization. Standard errors are double-clustered at the stock and month levels. *, **, and *** represent statistical significance at the 10%, 5%, and 1% confidence levels, respectively.

Table VI
Volatility and Variance Swap Portfolios
Sorted on Outstanding Short Put Products

		2004-2018		2007-2018		2009-2018	
Portfolio		Var Swap	Vol Swap	Var Swap	Vol Swap	Var Swap	Vol Swap
Low	Mean	-14.79	-18.46	-13.19	-17.71	-47.91	-47.53
	Sd	31.36	28.98	33.78	31.21	18.60	20.03
	Sharpe	-0.47	-0.64	-0.39	-0.57	-2.58	-2.37
2	Mean	-11.43	-17.10	-6.82	-13.58	-39.93	-41.43
	Sd	31.10	28.25	33.32	30.28	18.64	19.94
	Sharpe	-0.37	-0.61	-0.20	-0.45	-2.14	-2.08
3	Mean	-4.49	-11.26	0.83	-7.36	-29.64	-32.94
	Sd	30.85	27.94	33.01	29.96	21.08	21.61
	Sharpe	-0.15	-0.40	0.03	-0.25	-1.41	-1.52
4	Mean	3.28	-5.66	8.61	-2.31	-24.21	-28.44
	Sd	34.58	29.32	37.07	31.49	21.08	21.20
	Sharpe	0.09	-0.19	0.23	-0.07	-1.15	-1.34
High	Mean	22.93	10.06	28.41	13.42	-15.24	-19.36
	Sd	42.11	33.26	45.07	35.53	23.95	22.77
	Sharpe	0.54	0.30	0.63	0.38	-0.64	-0.85
High-Low	Mean	37.72	28.52	41.60	31.12	32.67	28.17
	Sd	16.80	10.99	17.51	11.08	10.27	8.57
	Sharpe	2.25	2.59	2.38	2.81	3.18	3.29

This table presents return statistics associated with variance and volatility swap portfolios formed by sorting on the total outstanding of short put products scaled by the underlying stocks' market capitalization. At each month, stocks are sorted into five portfolios based on the total outstanding of structured retail products associated with each stock scaled by the market cap. Variance and volatility swaps with one year of maturity are formed for each stock and grouped into the sorted portfolios. The ex-post hold-to-maturity returns in units of vega notionals are reported for each portfolio group. The returns for each monthly portfolio groups are annualized (hold-to-maturity return means are multiplied by 12 and standard deviations are multiplied by square root of 12).

Table VII
Sharpe Ratios for Long Short Volatility and Variance Swap Portfolios with Alternative Sorting Variables

Type	2004-2018		2007-2018		2009-2018	
Sort Variable	VarSwap	VolSwap	VarSwap	VolSwap	VarSwap	VolSwap
Outstanding_MarketCapScaled	2.25	2.59	2.38	2.81	3.18	3.29
Outstanding	1.78	1.92	1.91	2.05	3.20	3.13
NetIssuance_MarketCapScaled	1.87	2.11	1.89	2.15	2.31	2.27
NetIssuance	1.28	1.28	1.40	1.45	1.92	1.73
Maturity	-1.24	-1.28	-1.30	-1.38	-1.85	-1.72
Issuance	0.86	0.74	0.96	0.84	0.83	0.65

This table presents the long-short portfolio sharpe ratio associated with variance and volatility swap portfolios formed by sorting on variables capturing the stock and flow of short put products. At each month, stocks are sorted into five portfolios based on the sorting variable. Variance and volatility swaps with one year of maturity are formed for each stock and grouped into the sorted portfolios. The ex-post hold-to-maturity returns are reported in units of vega notionals for each portfolio group. The sharpe ratio is based on annualized returns (each porfolio's hold-to-maturity mean return is multiplied by 12, and standard deviations are multiplied by square root of 12).

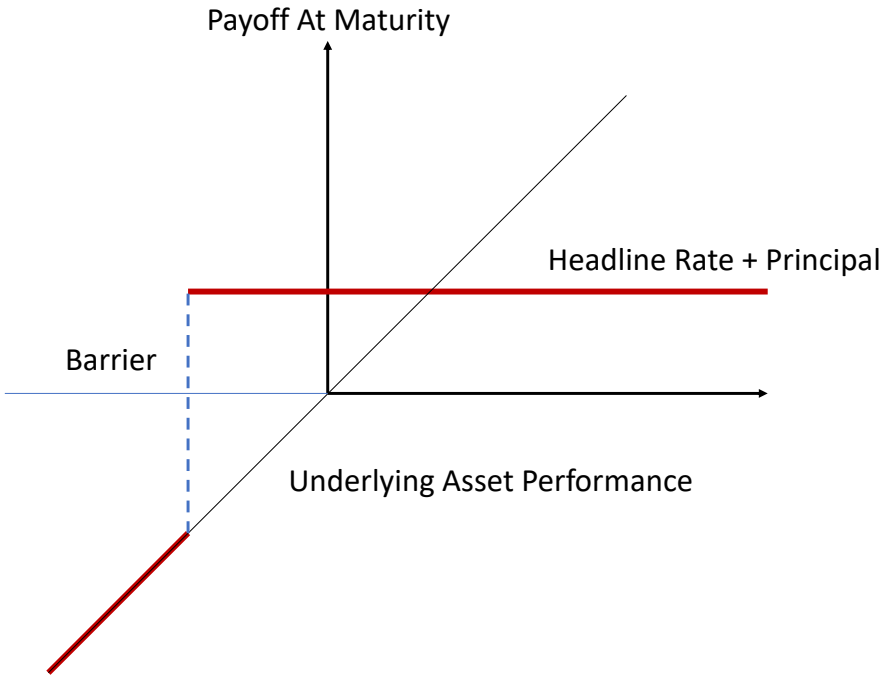


Figure 1. Design of a Simple Short Put Product

This figure shows the pay-off diagram of a typical Short Put Product.

Appendix B. Figures

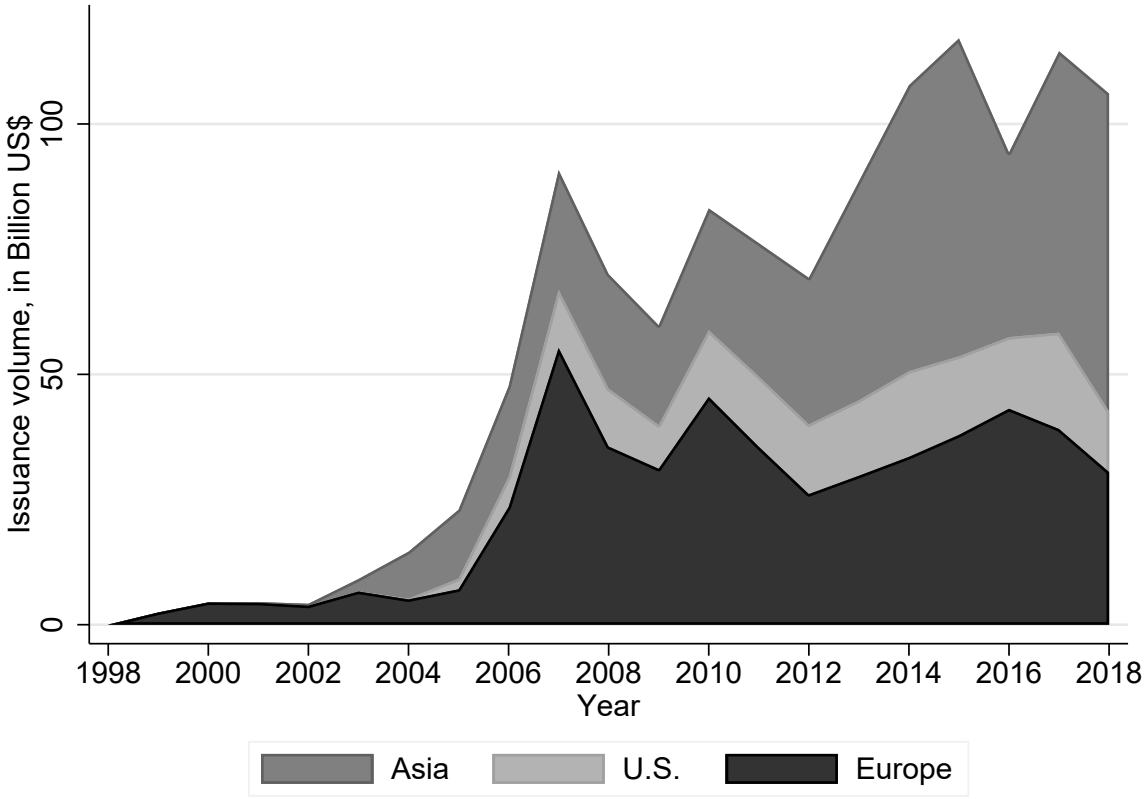
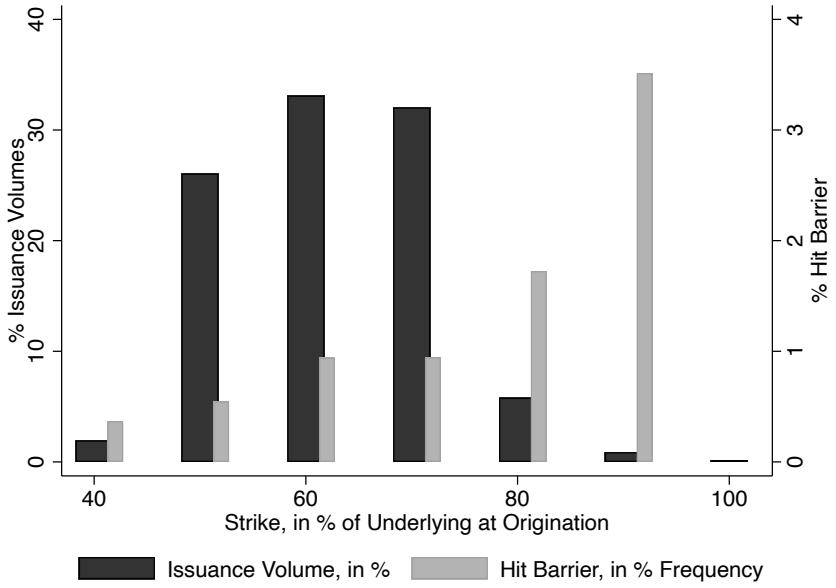


Figure 2. SPP Yearly Issuance Volumes Across Regions

The figure plots the yearly issuance volumes of Short Put Products (SPP) across Europe, Asia and the U.S. in billion dollars from 1998 to 2018.

Panel A. Histogram of SPP Strikes



Panel B. Histogram of SPP Ex Post Excess Returns

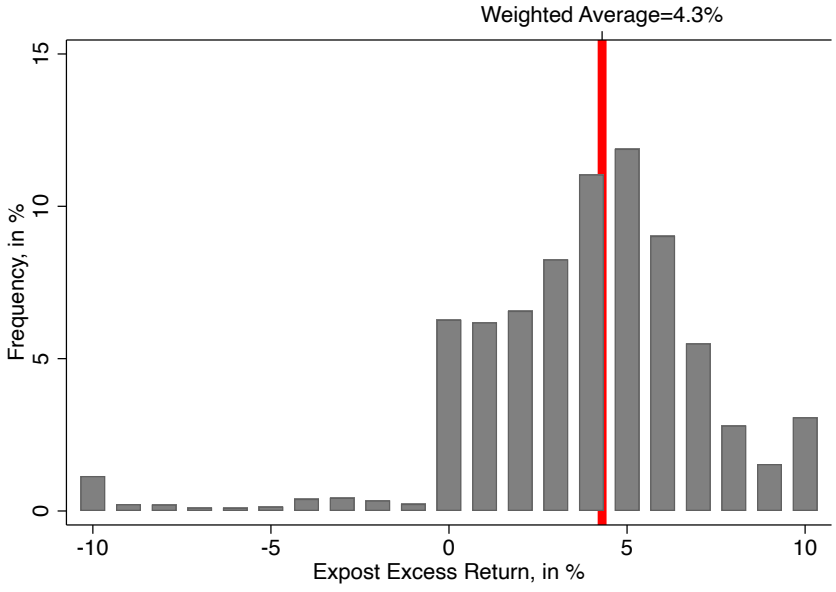


Figure 3. SPP Issuance volumes across Strikes and Ex-Post Excess Returns

Panel A displays the distribution of SPP issuance volumes by strikes over the 2002-2015 period as well as the percentage of products that hit the barrier. Panel B displays the histogram of SPP ex post excess returns over the 2002-2015 period. The sample is restricted to SPPs with S&P500, Nikkei225, and Eurostoxx50 as underlying assets.

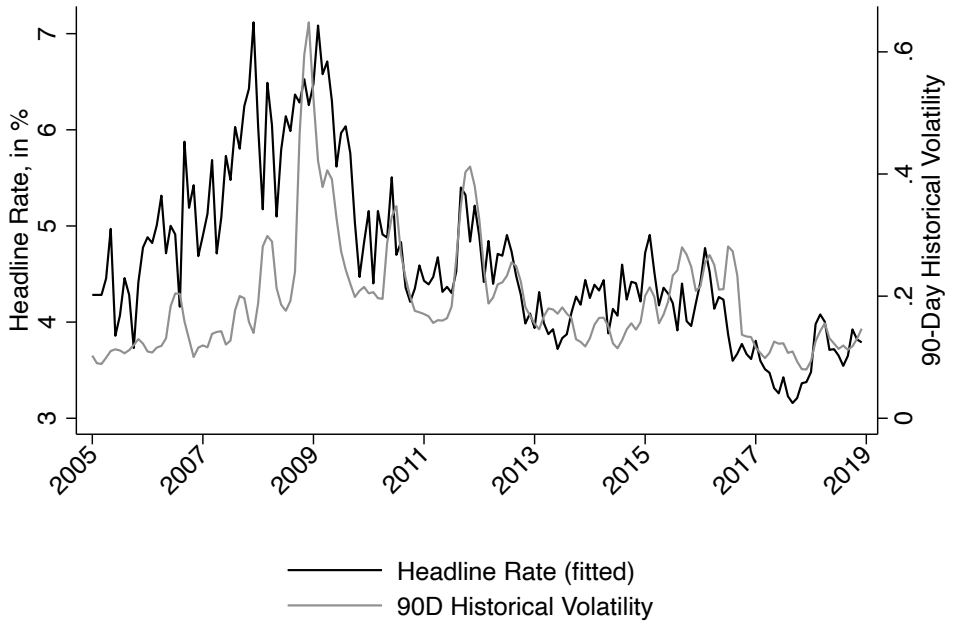


Figure 4. SPP Headline Rates and Historical Volatility

This figure displays the fitted headline rates of SPPs issued globally from 2005 to 2018, with the Eurostoxx 50 as the underlying asset, alongside the 90-day historical volatility of the Eurostoxx 50. The sample is structured as a monthly panel, where historical volatility represents the average 90-day historical volatility for each month, and the fitted headline rate corresponds to the predicted headline rate for a simple SPP with a 3-year maturity and a 60% strike. We obtain the predicted headline rate by regressing the headline rates of 50,000 products in our sample on month, maturity, strike, the number of payoffs, the number of underlying assets (fixed effects), and dummies for key payoff features, such as worst-off, protection tracker, and knock-in.

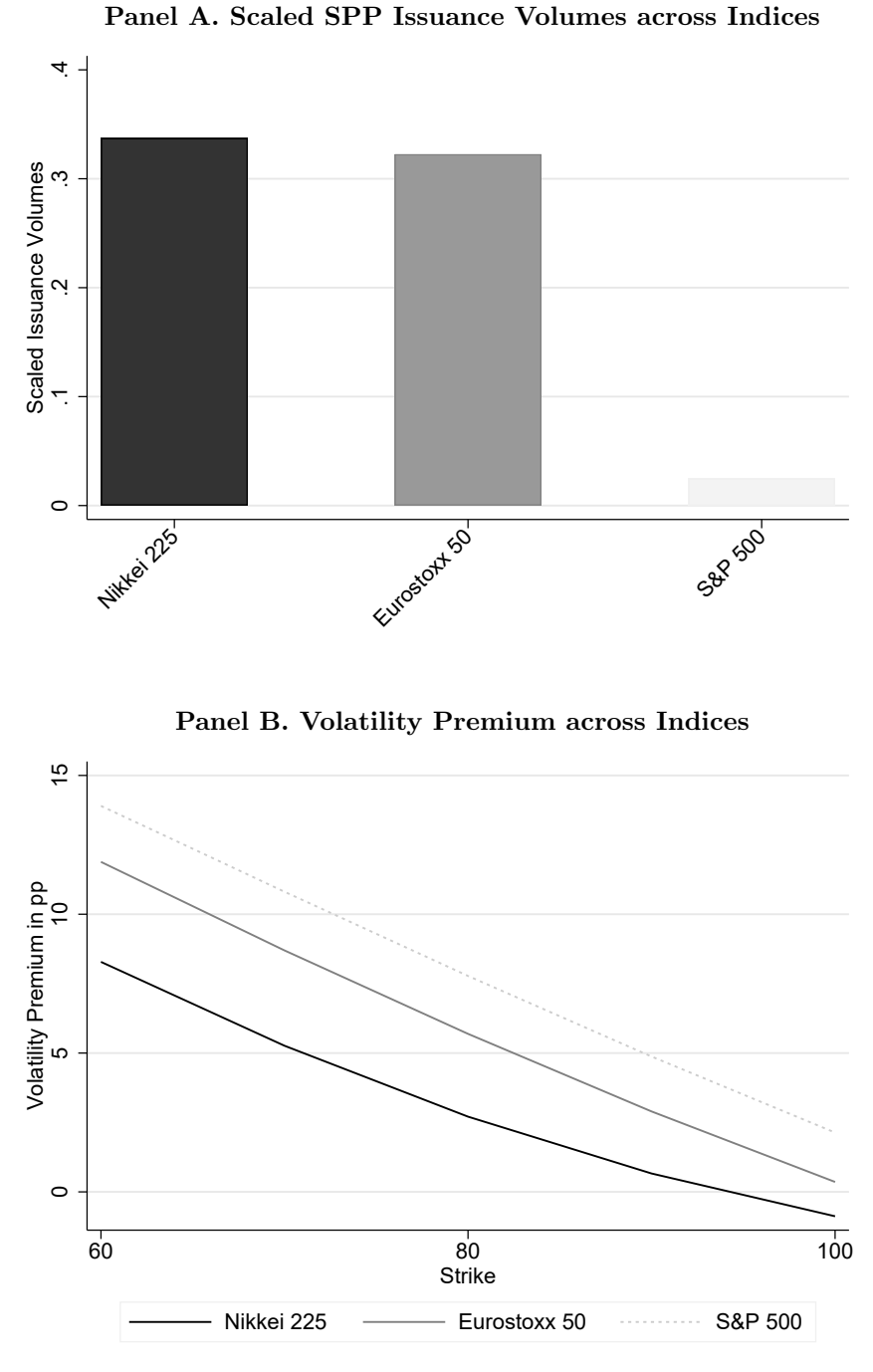
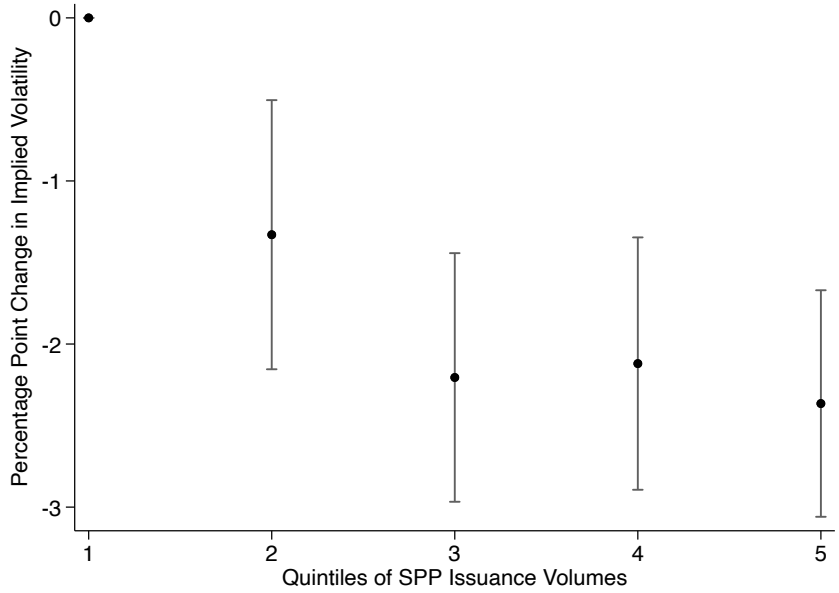


Figure 5. Average Scaled Volumes and Volatility Premium across Indices

Panel A plots the average yearly issuance volumes of SPP scaled by market capitalization across the three indices, S&P500, Nikkei225, and Eurostoxx50. Panel B plots the volatility premium, i.e. the difference between 1 year implied volatility and 90Day-historical volatility, at the 60, 70, 80, 90, and 100% strikes for the three indices, S&P 500, Nikkei 225, and Eurostoxx 50.

Panel A. Across SPP Issuance Volumes



Panel B. Across Strikes

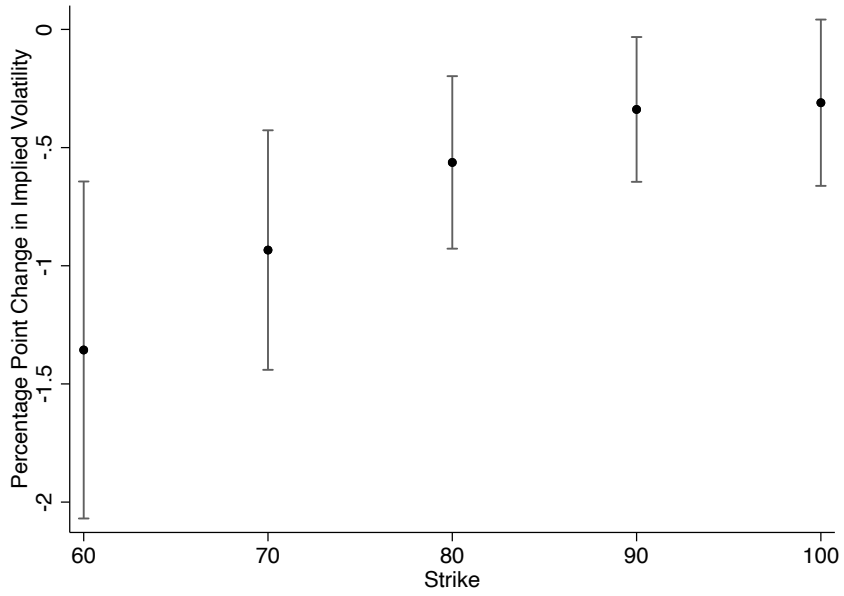
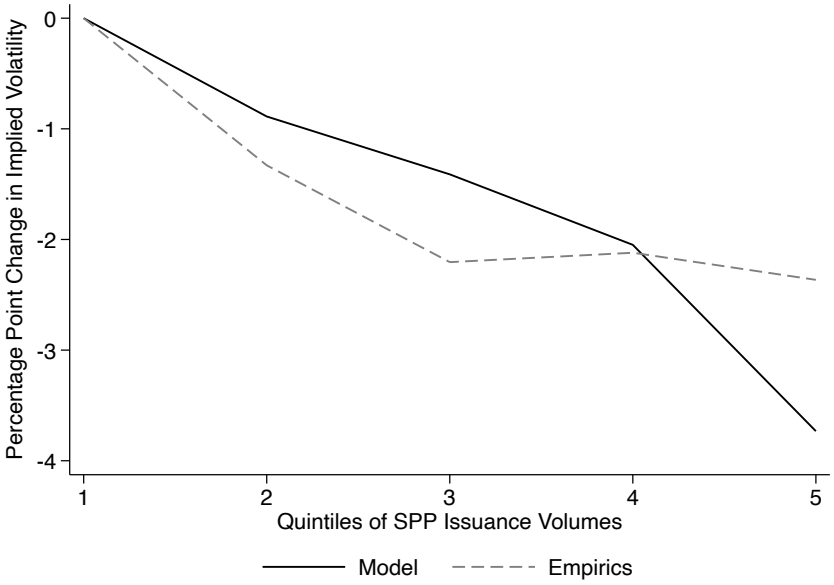


Figure 6. Impact of SPP Issuance Volumes on Implied Volatility: Empirical Results

This figure plots the coefficient estimates of dummies for quintiles of issuance volumes in Panel A, and strikes, in Panel B in a regression where the dependent variable is the 1 year implied volatility. The sample is a monthly panel of the three indices, S&P500, Nikkei225, and Eurostoxx50. The regression includes month and underlying fixed effects.

Panel A. Across SPP Issuance Volumes



Panel B. Across Strikes

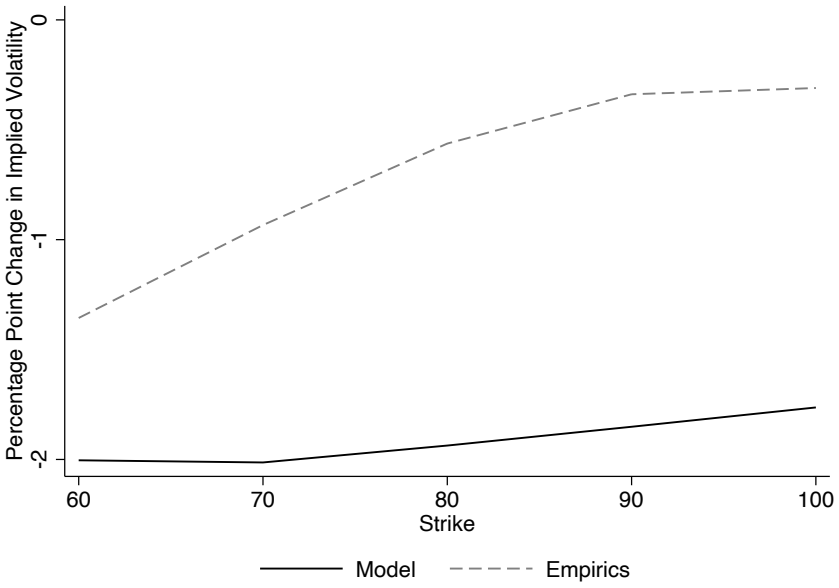


Figure 7. Impact of SPP Issuance Volumes on Implied Volatility: Model versus Empirics

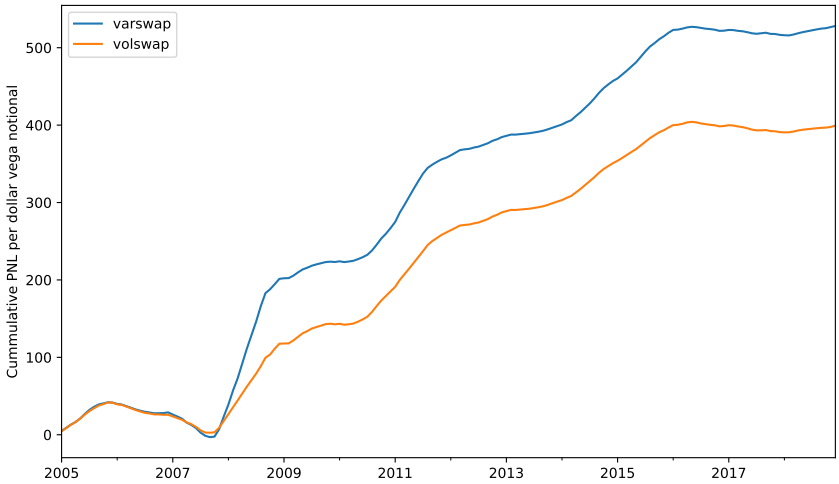


Figure 8. Cumulative P&L on long-short volatility and variance swaps portfolio

This figure plots the cumulative gains from implementing a monthly long-short trading strategy held to its 1 year maturity, based on sorting stocks on the amount of outstanding volumes of short put products.