The Divergence Between the Return of Capital and the Risk-Free Rate: A Micro-Anatomy*

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Preliminary and Incomplete, Please Do Not Circulate

The persistent decline in the risk-free rate since the 1980s, coupled with the stable return on capital, presents a challenge for the standard Neoclassical growth model. In this paper, we employ a general equilibrium firm-level framework to decompose in closed form the contributions of key explanations proposed in the literature—risk premium, profits, and capital-related frictions. While confirming the significant role of profits, we highlight a novel finding: contrary to previous literature emphasizing the role of risk, rising frictions associated with intangible capital have been the other key factor preventing the gap between the return on capital and the risk-free rate from closing. This rise is primarily driven by increasing intangible capital intensity among large firms in newer cohorts. Our results highlight the growing importance of intangible capital and differences across cohorts in shaping firm-level and macroeconomic dynamics.

Keywords: Return on capital, risk-free rate, intangible capital, capital frictions, profits, micro-to-macro

JEL Codes: D24, D25, E22, E23, E43, G12, G31

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1 Introduction

It is well established that the risk-free rate in the U.S. has experienced a secular decline since the 1980s, indicating low returns on savings. However, over the same period, the return on capital has remained stable or even slightly increased. This divergence challenges the standard Neoclassical growth model, which predicts that these two rates should equalize as investors are expected to be indifferent between investment opportunities, and it has profound policy implications.

High returns on capital, especially during periods of low economic growth like post-1980s U.S., imply that a larger share of national income is going to capital owners, potentially contributing to the sharp rise in wealth inequality (Piketty, 2014; Piketty and Zucman, 2014). Moreover, the secular decline in the risk-free rate may signal economic stagnation (Summers, 2014). However, the consistently high returns on capital challenge this view (Gomme, Ravikumar and Rupert, 2015) and complicates central banks' efforts to determine which long-run rate should guide their policy actions (Reis, 2022).

The existing literature highlights several complementary explanations for this divergence, including the rise in profits (Atkeson, Heathcote and Perri, 2024; Eggertsson, Robbins and Wold, 2021), risk premia (Farhi and Gourio, 2018), tangible and intangible capital frictions (Asriyan, Laeven, Martin, Van der Ghote and Vanasco, 2024). While these explanations have similar positive implications, their normative implications differ significantly, with some calling for policy intervention and others not. Therefore, any policy conclusions drawn from the divergence between the return on capital and the risk-free rate require careful evaluation of which mechanism is most relevant in practice. However, much of the literature examines these mechanisms in isolation, which can overstate their individual explanatory power.[1]

The objective of this paper is to examine *all* these explanations *jointly* and provide a quantitative estimate of each one's role in the divergence between the return on capital and the risk-free rate. To achieve this, we extend a standard Neoclassical model by introducing firm heterogeneity, allowing for differences in risk premia, market power, fixed costs, and frictions related to both tangible and intangible capital.

Using this framework we make three contributions. First, we demonstrate that the *aggregate* divergence between the return on capital and the risk-free rate can be decomposed in

[1] A problem akin to that observed in the labor share literature (Grossman and Oberfield, 2022).

closed form into a capital-weighted average of the *firm-level* counterparts of all the hypothesis in the literature, creating a direct micro-to-macro link. The framework highlights how neglecting one or multiple complementary explanations can lead to an overestimation of the others. Additionally, using standard firm-level data, we demonstrate how the identification of each of these complementary channels naturally follows from the structure of our framework.

Second, our decomposition shows that the rise in profit rates—returns from firm ownership—accounts for 83 percent of the divergence between the *measured* return on capital and the risk-free rate, while changes in the *true* return on capital explain the remaining 17 percent. This suggests that although the true return on capital has declined significantly over time, it has not been sufficient to fully close the gap with the risk-free rate. Contrary to previous literature, we find that this remaining gap has been driven primarily by the rising contribution of frictions related to intangible capital, rather than by the risk premium, which has actually declined since the 1980s.

Third, leveraging the direct micro-to-macro link provided by our framework, we investigate the role of sectors and firms in driving the rise in frictions associated with intangible capital. Our analysis shows that changes in the sectoral composition of the U.S. economy have not contributed to the increase in these frictions. Instead, the growing role of intangible capital frictions is entirely driven by the increasingly higher use of intangible capital—with its associated high frictions—relative to other inputs cross newer cohorts of large firms.

Model. We extend a standard Neoclassical growth model by introducing heterogeneous firms that face standard downward-sloping demand curves, risk premia, and frictions related to both tangible and intangible capital. These frictions are modeled as reduced-form wedges that do not allow inputs' marginal revenue products to equalize across firms as in Hsieh and Klenow (2009). The model also nests the firm problem posed in De Loecker and Warzynski (2012) that allows heterogeneous markups across firms and generalizes the firm-level CAPM model. The equilibrium conditions reveal how analyzing different narratives in isolation inevitably leads to biased results. Additionally, the model enables us to use standard firm-level data sources and apply state-of-the-art IO and asset pricing techniques to estimate markups and risk premia. The structure allows us to explicitly account for mismeasurement following Bils, Klenow and Ruane (2021), and identify frictions as residuals once all other factors are measured.

The model also allows for a closed-form decomposition of the divergence between the return on capital and the risk-free rate into various firm-level components tied to each narrative, weighted by the appropriate capital shares. This enables us to quantitatively separate the contribution of each narrative to the aggregate divergence. Moreover, the model distinguishes between the accounting measure of return on capital—calculated as the return from non-labor income, which includes economic profits—and the economic return on capital, which correctly attributes profits to a separate source of income: firm ownership. Finally, it allows us to infer the role of changing sectoral composition, as well as the impact of incumbent firms, capital reallocation across firms, and changes in firm composition due to entry and exit dynamics.

Firm-level measurement. We demonstrate how to apply our model to Compustat data, which offers several advantages for this study. It covers the largest firms in the economy—those that account for the majority of aggregate capital (Crouzet and Mehrotra, 2020)—and spans a wide range of sectors. Unsurprisingly, the aggregate return on capital from Compustat closely tracks that of the national accounts. Moreover, the dataset allows us to observe sales, variable and fixed costs, and tangible capital, while also enabling the measurement of intangible capital using state-of-the-art approaches from the corporate finance literature (Peters and Taylor, 2017; Ewens, Peters and Wang, 2024).

Using this standard dataset, we identify firm-level markups, risk premia, measurement errors, and capital-specific frictions. Specifically, following the literature, we estimate firm-level production function elasticities following Foster, Haltiwanger and Syverson (2008), markups as in De Loecker and Warzynski (2012), and risk premia by expanding the CAPM model from Hou, Xue and Zhang (2015) to account for markups, capital-specific frictions, and measurement errors. We assess mismeasurement following Bils, Klenow and Ruane (2021) and measure capital-specific frictions as residuals in line with Hsieh and Klenow (2009). Our findings are robust to various alternative production function estimators, markup measures not based on the De Loecker and Warzynski (2012) approach, and CAPM models commonly used in the literature, as discussed in detail in the main body of the paper.

Quantitative results. Our firm-level decomposition of the divergence between the measured return on capital and the risk-free rate reveals four main facts:

> **Fact 1:** *The gap between measured return on capital and the risk-free rate increased by 8.33 percentage points between 1982 and 2019. Rising Profits account for 83 percent of this*

gap, while movements in the true return on capital account for the remaining 17 percent.

Fact 2: *Contrary to previous findings, risk premium declined in the economy and led to convergence between the two returns. Meanwhile, rising frictions associated with intangible capital have been the main force behind the divergence between the true return on capital and the risk-free rate.*

Fact 3: *Changes in the sectoral composition of the U.S. economy played no role in the rise in frictions associated to intangible capital.*

Fact 4: *The gap between the true return on capital and the risk-free rate is entirely driven by an increase in the aggregate intangible capital wedge. This rise is due solely to changes in the composition of firms over time, with newer cohorts becoming more intangible-intensive, rather than simply exhibiting higher wedges.*

These four facts highlight a novel perspective on the divergence between the measured return on capital and the risk-free rate. A dominant portion of this divergence is driven by the rising profit rates, with the rise in markups documented by De Loecker et al. (2020) and the review by Syverson (2024), which have been improperly attributed to the return on capital. Once this is correctly accounted for, we observe that the true return on capital has indeed declined, though not enough to fully close the gap with the risk-free rate. The dominant quantitative force preventing the true return on capital from converging to the level of the risk-free rate is the rise in frictions associated with intangible capital.

The rise in frictions associated with intangible capital is not due to changes in sectoral composition but rather changes in the composition oof large firms in Compustat within sectors over time. Specifically, overall frictions tied to intangible capital have risen in the U.S. economy because intangible capital plays a larger role in newer cohorts. Although these new cohorts differ in their intangible capital intensity, i.e., the ratio of intangible to tangible capital, they closely resemble older cohorts in other life-cycle patterns, such as levels and growth rates of sales and variable costs, consistent with findings by Karahan, Pugsley and Şahin (2024) and Hopenhayn, Neira and Singhania (2022). This indicates that while firms remain similar in many aspects, their production processes have increasingly shifted toward greater reliance on intangible capital over time, a trend documented by Chiavari and Goraya (2024) in the U.S. and more recently by Aum and Shin (2024) in Korea. Our findings underscore the importance

of understanding the differences across firm cohorts and their role in shaping macroeconomic outcomes.

Related literature. This paper relates to several strands of literature. First, it contributes to the literature on the divergence between the return on capital and the risk-free rate by presenting a framework that decomposes in closed form the *joint* role of the most prominent explanations—risk premium, profits, and frictions associated with both tangible and intangible capital—at the *firm level* using standard data sources. In contrast, the existing literature has typically examined only a subset of these explanations at a time, with their conclusions being limited by the ability of aggregate data to distinguish between them. For instance, Marx, Mojon and Velde (2021) highlights the role of risk, while Eggertsson, Robbins and Wold (2021) emphasizes the impact of rising profits. Farhi and Gourio (2018) find that both risk and profits contribute significantly to the divergence. On the contrary, Reis (2022) emphasized the role of friction to tangible capital. While confirming the significant role of profits, we emphasize the novel finding that rising frictions associated with intangible capital—driven by increasing intangible capital intensity across cohorts of large firms over time—have been the key factor preventing the gap between the return on capital and the risk-free rate from closing.

Second, a rapidly growing body of literature highlights the significant macroeconomic implications of cohort differences. Our contribution to this literature is the novel evidence that newer cohorts of large firms are becoming increasingly intangible-intensive, with important aggregate implications for the divergence between the return on capital and the risk-free rate. Previously, Sterk, Sedláček and Pugsley (2021) estimated that most of these differences are driven by ex-ante heterogeneity rather than persistent ex-post shocks. Business cycle frequency analyses by Moreira (2016) and Sedláček and Sterk (2017) showed that firms born during recessions remain smaller than those born during booms. Longer-term studies by Bowen III, Frésard and Hoberg (2023) and Ma, Pugsley and Zimmermann (2024) demonstrate that cohorts starting during periods of rapid technological and organizational change experience lasting benefits, as postulated by Arrow (1974), Jovanovic (2001), and Jovanovic and Rousseau (2005).

Third, this paper contributes to the literature on the implications of rising intangible capital, particularly those focusing on its firm-level properties, by highlighting how its increased usage among recent cohorts of large firms and the associated frictions have impeded the con-

vergence between the return on capital and the risk-free rate. The observation that recent cohorts of large firms are using increasingly more intangible capital is supported by the works of Chiavari and Goraya (2024) and Aum and Shin (2024), which demonstrate that firm-level production processes have become more intangible-intensive. The role of frictions associated with intangible capital has been extensively studied: Peters and Taylor (2017), Belo, Gala, Salomao and Vitorino (2022), Cloyne, Martinez, Mumtaz and Surico (2022), Crouzet and Eberly (2023), and Chiavari and Goraya (2024) provide evidence of significant adjustment costs tied to intangible capital, while Caggese and Pérez-Orive (2022), Falato, Kadyrzhanova, Sim and Steri (2022), Bøler, Moxnes and Ulltveit-Moe (2023), and Zhang (2024) highlight the high financial frictions linked to this type of capital. Our findings confirm the results of this literature and show that the riser of this capital with its specific frictions is crucial to understanding the recent dynamics of the return on capital.

Finally, by providing a general equilibrium micro-to-macro framework that explicitly accounts for the aggregate effects of firm-level factors, we contribute to the vast literature on factors misallocation. Since the seminal work of Hsieh and Klenow (2009) demonstrated how to use standard data sources to measure the aggregate impact of firm-level frictions, several papers have extended this approach to distinguish capital frictions from other confounding factors. David and Venkateswaran (2019), building on De Loecker and Warzynski (2012), separate capital frictions from markups explicitly, while David, Schmid and Zeke (2022) highlight how to differentiate risk premia from capital frictions. Additionally, Bils, Klenow and Ruane (2021) addresses the evolution of measurement error in the wedges identified by Hsieh and Klenow (2009). We merge insights from these studies into a unified framework, enabling the closed-form decomposition of the divergence between the return on capital and the risk-free rate into distinct components: risk premia, markups, and frictions associated with both tangible and intangible capital.

Outline. Section 2 reviews the baseline divergence between the return on capital, as measured in national accounts, and the risk-free rate, along with the main narratives for this divergence outlined in the literature. Section 3 presents our framework, which leads to the decomposition of this divergence into its components at the firm level. Section 4 discusses the data and details the firm-level measurement of markups, risk premia, and capital-specific frictions. Section 5 presents the main results. Finally, Section 6 concludes.

2 Return on Capital and Risk-Free Rate Divergence

This section reviews the basic facts regarding the evolution of the return on capital and the risk-free rate over the last decade and presents the leading explanations in the literature for their behavior.

2.1 Basic Stylized Facts

Here, we review the basic facts about the evolution of the return on capital and the risk-free rate since the 1980s, the period most often studied in the literature (e.g., Farhi and Gourio, 2018). While the risk-free rate is typically observed directly in the data, the return on capital is a model-implied concept, which we discuss in detail below.

We measure the aggregate return on capital in line with established literature. As noted by Caselli and Feyrer (2007), under the standard assumptions of constant returns to scale, perfect competition, no risk, and no capital mismeasurement, the aggregate net output can be expressed as follows:

$$Y_t - \delta K_t = (R_t - \delta) K_t + W_t L_t, \tag{1}$$

where Y represents value added, δ is the depreciation rate, K is total capital, R is the rental price of capital, and WL is total labor. Equation (1) indicates that net output, $Y - \delta K$, is allocated either to payments to labor services, WL, or capital services, $(R - \delta) K$. Hence, while W denotes the return on labor, $R^k \equiv R - \delta$ represents the return on capital, which can be measured as follows:

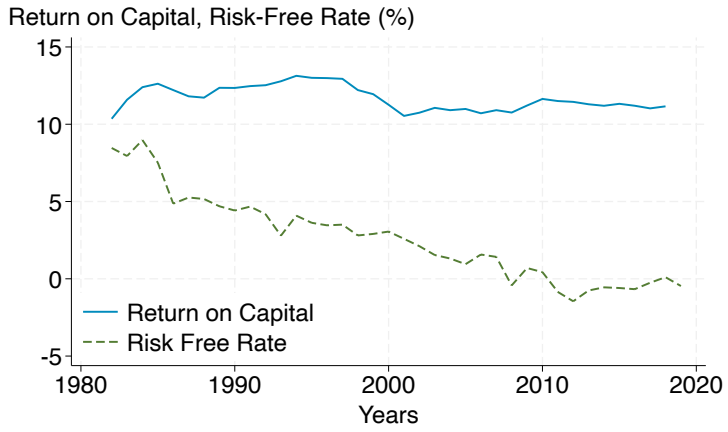
$$R_t^k = \frac{Y_t - \delta K_t - W_t L_t}{K_t}. \tag{2}$$

Note that, since investors should be indifferent between investing one unit of output in capital or an investment yielding a risk-free return r, we would expect R^k to equal r.

Figure 1 illustrates the evolution of the return on capital and the risk-free rate. The return on capital is calculated using equation (2) with data from the BEA, while the risk-free rate is represented by the real market yield on U.S. Treasury securities with a 10-year constant maturity. Additional details on these calculations are provided in Appendix A.

Since the 1980s, the risk-free rate has steadily declined, falling from just below 10 percent to near 0 percent. In contrast, the return on capital has remained relatively stable, consistently

Figure 1: Evolution of Return on Capital and Risk-Free Rate



Note. Figure 1 shows the evolution of the return on capital and of the risk-free rate, measured in percent, since 1982. The return on capital is measured using equation (2) with NIPA data. The risk-free rate is the market yield on U.S. Treasury securities with a 10-year constant maturity net of expected inflation from Michigan.

hovering just above 10 percent. This divergence, following decades of convergence between the two rates (see Appendix A for the historical evolution of these measures), has puzzled researchers and spurred various potential explanations, which we review in the following section.

Appendix A demonstrates that alternative measures of the risk-free rate, as outlined in Rachel and Summers (2019), including the real Aaa corporate bond yield, the real Baa corporate bond yield, and the real S&P 500 earnings yield, exhibit a similar downward trend.

Appendix A further shows that alternative measures of the return on capital based on Gomme, Ravikumar and Rupert (2011) yield similar patterns, with no evidence of a decline over time. They report measures based solely on the business sector, excluding the impact of housing, which produces results closely aligned with ours. Additionally, they present measures both with and without capital gains and before and after taxes, demonstrating that neither adjustment accounts for the missing decline in the return on capital.

2.2 Explanations in Literature

The literature has proposed several complementary explanations for this divergence, all linked to potential departures from the assumptions used in calculating the return on capital. Since neither tax nor capital gains adjustments account for the missing decline, most of the focus has shifted to the role of profits, capital-related frictions or mismeasurement, and risk premia.

Profits. In the presence of profits, either due to deviations from perfect competition or constant returns to scale, the net output can be expressed as $Y_t - \delta K_t = R_t^k K_t + W_t L_t + \Pi_t$. This leads to the following measured return on capital: $\widetilde{R}^k = R_t^k + \frac{\Pi_t}{K_t}$.

Therefore, if profits have been rising over time and are incorrectly attributed to capital, this could explain the observed divergence. The literature has identified two potential drivers for changes in profits over time: shifts in markups (Gutiérrez and Philippon, 2017; Barkai, 2020; De Loecker, Eeckhout and Unger, 2020) or changes in fixed costs (Hsieh and Rossi-Hansberg, 2023; De Ridder, 2024).

Capital-related frictions and mismeasurement. Frictions related to capital investment, such as adjustment costs and financial frictions, raise the average rental price of capital. Additionally, mismeasurement of capital would result in a higher measured return on capital due to an artificially low denominator in (2). In both cases, the measured return on capital would be higher than the risk-free rate.

The literature has highlighted several reasons to believe that increasing frictions or mismeasurement of aggregate capital could contribute to the documented divergence. Regarding *tangible capital*, Döttling, Gutierrez and Philippon (2017), Reis (2022), and Asriyan, Laeven, Martin, Van der Ghote and Vanasco (2024) argue that rising financial frictions, which constrain investment in tangible capital, may help explain the growing excess return over the risk-free rate.

A substantial body of literature points to several reasons why the rise of *intangible capital* may be driving the increasing wedge between the return on capital and the risk-free rate. Corrado, Hulten and Sichel (2009), Lev (2016), Haskel and Westlake (2017), Crouzet and Eberly (2018), Crouzet and Eberly (2023) demonstrate that intangible capital is significantly undermeasured and could lead to mismeasurement in the firm-level and aggregate stock of capital. Furthermore, Peters and Taylor (2017), Belo, Gala, Salomao and Vitorino (2022), Cloyne, Martinez, Mumtaz and Surico (2022), Crouzet and Eberly (2023), and Chiavari and Goraya (2024) provide evidence of large adjustment costs associated with intangible capital, while Caggese and Pérez-Orive (2022), Falato, Kadyrzhanova, Sim and Steri (2022), Bøler, Moxnes and Ulltveit-Moe (2023) and Zhang (2024) show high financial frictions linked to this type of capital.

Risk premia. In the presence of risk, firms are expected to pay a premium above the

risk-free rate to rent capital. Therefore, if risk has increased over the past few decades, as hypothesized by Caballero, Farhi and Gourinchas (2017), Farhi and Gourio (2018), and Marx, Mojon and Velde (2021), this could explain why the return on capital has not declined in line with the drop in the risk-free rate.

3 A Microfounded Structural Decomposition

This section introduces a structural framework for identifying, using firm-level data, the various explanations proposed in the literature. It then outlines how to decompose the divergence documented in Section 2, attributing it to the contribution of each explanation.

3.1 The Firms' Side

Time is discrete and indexed by t. Firms are static, indexed by i, and produce using a production function that includes variable inputs, tangible capital, and intangible capital, expressed as follows:

$$q_{it} = q(z_{it}, \ell_{it}, k^T_{it}, k^I_{it}), \tag{3}$$

where output quantity is denoted by q, z represents Hicks-neutral productivity, ℓ is the variable input, k^T is tangible capital, and k^I is intangible capital. We define the output-elasticity to each input as follows:

$$\mathcal{E}_\ell \equiv \frac{\partial q_{it}}{\partial \ell_{it}} \frac{\ell_{it}}{q_{it}}, \quad \mathcal{E}_{k^T} \equiv \frac{\partial q_{it}}{\partial k^T_{it}} \frac{k^T_{it}}{q_{it}}, \quad \text{and} \quad \mathcal{E}_{k^I} \equiv \frac{\partial q_{it}}{\partial k^I_{it}} \frac{k^I_{it}}{q_{it}}. \tag{4}$$

Throughout the remainder of the paper, we will use bold notation for the firm-level variables that we consider as potential drivers of the aggregate divergence documented in Section 2. These include (i) firm-level risk premia, (ii) capital-specific firm-level mismeasurement or frictions, (iii) firm-level markups, and (iv) firm-level fixed costs.

Firms borrow both types of capital from investors (more on them later), paying a rental rate comprising a risk-free rate r, a firm-level risk premium $\boldsymbol{\zeta}$, and a capital-specific depreciation rate δ. Additionally, we account for the possibility of capital-specific mismeasurement or frictions, which we represent in reduced form as a wedge $\boldsymbol{\tau}^j$, where $j \in \{T, I\}$. Appendix B.1 provides various alternative microfoundations to mismeasurement for this term, relating it

explicitly to either adjustment costs or financial frictions. Finally, firms face an overhead cost $\boldsymbol{c}$ to operate their production technology. Thus, the Lagrangian objective function associated with the firm's cost minimization problem can be expressed as:

$$\mathcal{L}(\ell_{it}, k_{it}^T, k_{it}^T, \xi_{it}) = p_{it}^{\ell}\ell_{it} + \sum_{j\in\{T,I\}} \left(r_t + \boldsymbol{\zeta}_{it} + \delta^j + \boldsymbol{\tau}_{it}^j\right) p_t^j k_{it}^j + \boldsymbol{c}_{it} - \xi_{it}\left(q(\cdot) - q_{it}\right), \tag{5}$$

where p^{ℓ} is the price of the variable input, p^T is the price of tangible capital, p^I is the price of intangible capital, ξ is the Lagrange multiplier, $q(\cdot)$ represents the technology as specified in equation (3), and q_{it} is a is a scalar. We assume that variable input prices are exogenous to the firm. Taking the first-order conditions of the minimization problem expressed in equation (5) yields the following set of equations:

$$\boldsymbol{\mu}_{it} = \mathcal{E}_{\ell}\frac{p_{it}q_{it}}{p_{it}^{\ell}\ell_{it}}, \tag{6}$$

$$r_t + \boldsymbol{\zeta}_{it} + \delta^T = \mathcal{E}_{k^T}\frac{1}{\boldsymbol{\mu}_{it}}\frac{p_{it}q_{it}}{p_t^T k_{it}^T} - \boldsymbol{\tau}_{it}^T, \tag{7}$$

$$r_t + \boldsymbol{\zeta}_{it} + \delta^I = \mathcal{E}_{k^I}\frac{1}{\boldsymbol{\mu}_{it}}\frac{p_{it}q_{it}}{p_t^I k_{it}^I} - \boldsymbol{\tau}_{it}^I; \tag{8}$$

where $\boldsymbol{\mu}$ represents markups of price over marginal costs and p_{it} is the firm-level output price.

Note that equation (6) determines the markups as a function of observables, akin to Hall (1988) and De Loecker and Warzynski (2012).[2] Similarly to Hsieh and Klenow (2009), equations (7) and (8) determine the revenue-based marginal products of tangible capital ($MRPK^T$) and intangible capital ($MRPK^I$), as defined by the right-hand side of each equation. However, unlike their approach, where the capital first-order conditions allowed for a single unobservable in the wedge $\boldsymbol{\tau}^j$, we also consider that heterogeneity in firm-level risk premia may contribute significantly to the variation in the revenue-based marginal product of capital, as emphasized by David, Schmid and Zeke (2022).

Note that because the revenue-based marginal products depend on both observable variables and the wedge $\boldsymbol{\tau}^j$, it is useful for the remainder of the paper to define the observable

[2]This approach has recently garnered significant attention, as reviewed in De Loecker and Syverson (2021). Bond et al. (2021) notes that, without price data, this method cannot estimate the aggregate markup level. However, as will become clear later, our focus is exclusively on changes over time—specifically, changes relative to the aggregate markup level—which, as De Ridder, Grassi and Morzenti (2022) demonstrates, the method accurately captures.

revenue-based marginal product, $\widetilde{MRPK}$, as follows:

$$\widetilde{MRPK}^T_{it} = MRPK^T_{it} + \boldsymbol{\tau}^T_{it}, \quad \text{and} \quad \widetilde{MRPK}^I_{it} = MRPK^I_{it} + \boldsymbol{\tau}^I_{it}, \tag{9}$$

which implicitly states that the true capital-specific, revenue-based marginal product of capital is equal to the risk-free rate r, the risk premium $\boldsymbol{\zeta}^j$, and the capital-specific depreciation rate δ^j.

Equations (6)-(8) define a system of three equations with four unknowns: markups ($\boldsymbol{\mu}$), risk premia ($\boldsymbol{\zeta}$), and tangible and intangible capital-specific wedges ($\boldsymbol{\tau}^T$, $\boldsymbol{\tau}^I$). They also demonstrate the importance of considering the various narratives proposed by the literature together. Examining them in isolation would lead to improper attribution of the missing factors to the one under consideration. For example, an increase in the gap between the observed capital-specific revenue-based marginal product and the risk-free rate could be mistakenly attributed to a rise in risk premia if capital-specific wedges are not accounted for, or conversely, to capital-specific wedges if risk premia are ignored. The same holds true if the role of markups is overlooked.

Nonetheless, equations (6)-(8) form the core of our approach. In particular, our identification strategy relies on the fact that, given firm-level measures of risk premia ($\boldsymbol{\zeta}$), these equations allow us to derive markups and firm-level capital-specific wedges ($\boldsymbol{\mu}$, $\boldsymbol{\tau}^T$, $\boldsymbol{\tau}^I$) as functions of observables. Next, we demonstrate how to derive firm-level measures of risk premia from the problem of investors lending capital to firms.

3.2 The Investors' Side

The investor side represents the asset supply in the model. Appendix B.2 provides a micro-foundation for this, starting from a standard consumption-saving problem, and can be summarized by the following set of first-order conditions:

$$1 = \mathbb{E}_{t-1}\left[M_t\left(MRPK_{it} + \sum_{j\in\{T,I\}} \kappa^j_{it-1}(1-\delta^j)\frac{p^j_t}{p^j_{t-1}}\right)\right], \tag{10}$$

where M represents the stochastic discount factor and $MRPK \equiv \sum_{j\in\{T,I\}} \kappa^j_{-1} MRPK^j$, and $\kappa^j \equiv \frac{p^j k^j}{p^T k^T + p^I k^I}$. By substituting the observed revenue-based marginal product of capital

into equation (10), realizing that by definition $\boldsymbol{\tau}$ is uncorrelated with M, incorporating the fact that $\mathbb{E}_{t-1}M_t = (1+r_{t-1})^{-1}$, and performing some straightforward algebra, we can derive the following equation for the capital-specific risk premium:

$$\widetilde{MRPK}_{it} - r_{t-1} - \delta_{it} = \boldsymbol{\tau}_{it} - g_{it} + \overbrace{\underbrace{\frac{\mathbb{C}_{t-1}\left[M_t, \widetilde{MRPK}_{it}\right]}{\mathbb{V}_{t-1}\left[M_t\right]}}_{\equiv \boldsymbol{\beta}_{it}} \underbrace{\left(-\frac{\mathbb{V}_{t-1}\left[M_t\right]}{\mathbb{E}_{t-1}\left[M_t\right]}\right)}_{\equiv \boldsymbol{\lambda}_t}}^{\equiv \boldsymbol{\zeta}_{it}} + \varepsilon_{it}, \tag{11}$$

where $\widetilde{MRPK} \equiv \sum_{j\in\{T,I\}} \kappa^j_{-1} \widetilde{MRPK}^j$, $\delta \equiv \sum_{j\in\{T,I\}} \kappa^j_{-1}\delta^j$, $\boldsymbol{\tau} \equiv \sum_{j\in\{T,I\}} \kappa^j_{-1}\tau^j$, and $g \equiv \sum_{j\in\{T,I\}} \kappa^j_{-1}(1-\delta^j)\left(\frac{p^j}{p^j_{-1}} - 1\right)$. Thus, the firm-level excess return can be expressed as a function of the wedge $\boldsymbol{\tau}_{it}$, capital gains g_{it}, a risk premium $\boldsymbol{\zeta}_{it} \equiv \boldsymbol{\beta}_{it}\boldsymbol{\lambda}_t$, and an expectational error ε_{it}.

Equation (11) expresses the risk premium as the product of two components: the exposure of the return to movements in the stochastic discount factor—i.e., the firm's riskiness—summarized by $\boldsymbol{\beta}_{it}$, and the aggregate price of risk, summarized by $\boldsymbol{\lambda}_t$. Thus, it generalizes standard asset pricing models by explicitly accounting for wedges $\boldsymbol{\tau}_{it}$ and the presence of markups $\boldsymbol{\mu}_{it}$ through the observed revenue-based marginal product of capital $\widetilde{MRPK}_{it}$.[3]

Similarly to equations (6)-(8), equation (11) also highlights the importance of considering all narratives together. Ignoring the role of markups and capital-specific wedges would lead to incorrectly attributing changes in the gap between the capital-weighted, capital-specific revenue-based marginal product and the risk-free rate to shifts in the risk premium, thus overemphasizing its role in this divergence.

Thus, equation (11), together with equations (6), (7), and (8), define a system of four equations in four unknowns $(\boldsymbol{\mu}, \boldsymbol{\tau}^T, \boldsymbol{\tau}^I, \boldsymbol{\zeta})$. These equations characterize all the firm-level unobservables of interest, except for the fixed cost $\boldsymbol{c}$, which we measure directly from the data as explained later. Next, we demonstrate how these firm-level variables account for the aggregate divergence documented in Section 2.

[3]It is important to note that ignoring the presence of markups and using the usual measure of the revenue-based marginal product of capital unadjusted for firm-level markups, $(p_{it}q_{it})/(p^j_t k^j_{it})$, as dependent variable would lead to a bias in the regression given by the following $\widetilde{MRPK}_{it}(\boldsymbol{\mu}_{it} - 1)/\boldsymbol{\mu}_{it}$, which vanishes only in the presence of perfect competition.

3.3 Micro-to-Macro Link

Here, we demonstrate how the divergence of the return on capital, R^k, from the risk-free rate, r, can be explained by (i) firm-level risk premia, (ii) capital-specific firm-level mismeasurement or frictions, (iii) firm-level markups, and (iv) firm-level fixed costs.

To accomplish this, we begin by defining the *measured* firm-level returns on capital, R^k_{it}, as the firm-level counterpart to the aggregate return on capital introduced in Section 2, defined as follows:

$$R^k_{it} \equiv \frac{p_{it}q_{it} - \boldsymbol{c}_{it} - \delta^T p^T_t k^T_{it} - \delta^I p^I_t k^I_{it} - p^\ell_{it}\ell_{it}}{p^T_t k^T_{it} + p^I_t k^I_{it}}, \tag{12}$$

This measured firm-level return on capital can be further decomposed into the sum of the *true* return on capital, $\mathcal{R}^k_{it}$, and the profit rate, π_{it}, as follows:

$$R^k_{it} = \mathcal{R}^k_{it} + \pi_{it}, \tag{13}$$

where true return on capital is defined as follows:

$$\mathcal{R}^k_{it} \equiv r_t + \boldsymbol{\zeta}_{it} + \kappa^T_{it}\boldsymbol{\tau}^T_{it} + \kappa^I_{it}\boldsymbol{\tau}^I_{it}, \tag{14}$$

which is simply the sum of the risk-free rate, the risk premium, and the weighted average of the two capital-specific wedges. In contrast, the profit rate is defined as follows:

$$\pi_{it} \equiv \frac{p_{it}q_{it}}{p^T_t k^T_{it} + p^I_t k^I_{it}}\left(1 - \frac{1}{\boldsymbol{\mu}_{it}}\right) - \frac{\boldsymbol{c}_{it}}{p^T_t k^T_{it} + p^I_t k^I_{it}}, \tag{15}$$

which represents the return from charging markups, net of the share attributed to fixed costs. The fact that the measured return on capital equals the true one plus the profit rates stems from the fact that standard measurement assumes zero profits. Therefore, our methodology enables the distinction between movements in the true return on capital and movements in the return to firm ownership, as captured by profits.

To link these firm-level objects to the aggregate divergence of interest we start by the definition of measured aggregate capital return R^k presented in Section 2 that can expressed

as follows:

$$R_t^k = \frac{Q_t - \delta K_t - W_t L_t}{K_t}, \tag{16}$$

$$= \sum_i \omega_{it} \left(\frac{p_{it} q_{it} - \boldsymbol{c}_{it} - \delta^T p_t^T k_{it}^T - \delta^I p_t^I k_{it}^I - p_{it}^\ell \ell_{it}}{p_t^T k_{it}^T + p_t^I k_{it}^I} \right), \tag{17}$$

$$= \sum_i \omega_{it} R_{it}^k; \tag{18}$$

with $K_t^T \equiv \sum_i p_t^T k_{it}^T$ is aggregate tangible capital, $K_t^I \equiv \sum_i p_t^I k_{it}^I$ is aggregate intangible capital, $Q_t - W_t L_t \equiv \sum_i (p_{it} q_{it} - p_{it}^\ell \ell_{it} - \boldsymbol{c}_{it})$ is total GDP net of the aggregate wage bill, $K_t \equiv K_t^T + K_t^I$ is aggregate total capital, $\delta K_t = \delta^T K_t^T + \delta^I K_t^I$ is total aggregate depreciation, and $\omega_{it} = (p_t^T k_{it}^T + p_t^I k_{it}^I)/K_t$ is the share of firm-level share of total capital into aggregate total capital. Equations (16)-(18) demonstrate that the measured aggregate return on capital can be precisely represented as the capital-weighted average of the measured firm-level returns on capital.

Building on these insights, substituting equations (14)-(16) into equation (18) results in the following expression:

$$R_t^k = \mathcal{R}_t^k + \Pi_t, \tag{19}$$

$$= \sum_i \omega_{it} \mathcal{R}_{it}^k + \sum_i \omega_{it} \pi_{it}, \tag{20}$$

$$= \sum_i \omega_{it} \left(r_t + \boldsymbol{\zeta}_{it} + \kappa_{it}^T \boldsymbol{\tau}_{it}^T + \kappa_{it}^I \boldsymbol{\tau}_{it}^I + \pi_{it} \right); \tag{21}$$

where $\mathcal{R}_t^k \equiv \sum_i \omega_{it} \mathcal{R}_{it}^k$ is the *true* aggregate return on capital and $\Pi_t \equiv \sum_i \omega_{it} \pi_{it}$ is the aggregate profit rate. Equation (21) demonstrates how the measured aggregate return on capital can be represented as a capital-weighted average of the various firm-level narratives identified in the literature.

Rearranging equation (21), we write the difference between the return on capital and the

risk-free rate as follows:

$$R_t^k - r_t = \underbrace{\sum_i \omega_{it} \left(\mathcal{R}_{it}^k - r_t\right)}_{\text{True return on capital}} + \underbrace{\sum_i \omega_{it} \pi_{it}}_{\text{Profits}}, \tag{22}$$

$$= \underbrace{\sum_i \omega_{it} \boldsymbol{\zeta}_{it}}_{\text{Risk}} + \underbrace{\sum_i \omega_{it} \kappa_{it}^T \boldsymbol{\tau}_{it}^T}_{\text{Tangible}} + \underbrace{\sum_i \omega_{it} \kappa_{it}^I \boldsymbol{\tau}_{it}^I}_{\text{Intangible}} + \underbrace{\sum_i \omega_{it} \pi_{it}}_{\text{Profits}}. \tag{23}$$

Equation (22) demonstrates that the divergence between the measured return on capital, R^k, and the risk-free rate, r, can be decomposed into two components: the divergence between the capital-weighted average of firm-level true returns on capital, $\mathcal{R}_{it}^k$, and the risk-free rate, r, as well as the capital-weighted average profit rate.

Instead, Equation (23) shows that the divergence between R^k and r can be decomposed into four aggregate components, each representing a capital-weighted average of the corresponding firm-level factors. Risk premia reflect a reduced willingness of investors to supply capital to firms, which forces firms to operate with less capital, resulting in higher returns on capital. Instead, capital-specific wedges appear in equation (23) as adjustments to the observable capital-specific revenue-based marginal product, allowing us to recover the true capital-specific revenue-based marginal product, as defined in equation (9).

Shifting to profit-related explanations, markups and fixed costs, as defined by the profit share in equation (15), influence the wedge between the measured aggregate return on capital and the risk-free rate. This occurs because they represent deviations from the perfect competition assumption, which standard measurements in the literature rely on, as explained in Section 2. When this assumption does not hold, the rise in returns to firm ownership is incorrectly attributed to capital ownership. Therefore, while an increase in markups, by boosting profits, can contribute to the widening gap between R^k and r, a rise in fixed costs would have the opposite effect, as it implies a reduction in profits.

Overall, the firm-level equation (10), along with equations (6), (7), and (8), which are linked to the macro level through equation (23), provide a micro-to-macro methodology applicable to standard firm-level data source for decomposing exactly the divergence between R^k and r into various potential competing explanations debated in the literature. The following sections detail how to implement this methodology and the results it yields.

4 Data, Variable Construction, and Measurement

4.1 Data Sources

Firm-level data. The primary data source is Compustat, a firm-level database that provides balance sheet information for all U.S. publicly traded firms covering the entire period of interest. The key advantage of using Compustat is its comprehensive coverage of the relevant time frame and a wide range of sectors. While publicly traded firms represent only a small fraction of the total number of firms, they are often among the largest in the economy, accounting for approximately 30% of U.S. employment Davis, Haltiwanger, Jarmin, Miranda, Foote and Nagypal (2006). More relevant to our focus on capital, the particularly right-skewed distribution of capital across firms makes these large firms highly informative for understanding aggregate capital, as noted by Crouzet and Mehrotra (2020). According to our calculations, they represent approximately 60 percent of total non-residential capital.

Aggregate risk factors data. We obtain data on aggregate risk factors from various sources. For the standard CAPM, the five-factors data from Hou, Xue and Zhang (2015) can be accessed at http://global-q.org/factors.html. Additionally, the five-factors, three-factors, and one-factor data from Fama and French (2023) are available on Kenneth French's website at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/. Finally, when using the consumption CAPM, we employ a single-factor model that incorporates the consumption growth rate from the NIPA tables.

4.2 Variable Definitions

In this section, we provide a brief overview of how we compute the main variables of interest used in the analysis. See Appendix C.1 for further details on the cleaning process and for summary statistics of our main variables.

Tangible and intangible capital. Tangible capital is constructed using the perpetual inventory method:

$$k_{it}^T = (1 - 0.07)k_{it-1}^T + x_{it}^T, \tag{24}$$

where $x_{it}^T - 0.07k_{it-1}^T \equiv \texttt{ppent}_{it} - \texttt{ppent}_{it-1}$, and values are deflated. The initial capital stock is set as $k_{i0}^T = \texttt{ppegt}_{it}$.

We measure intangible capital stock as the sum of three components, following best prac-

tices in corporate finance (Peters and Taylor, 2017; Ewens, Peters and Wang, 2024). The first component, knowledge capital, is capitalized R&D, defined as:

$$k_{it}^{KNWL} = (1 - \delta_{s(i)}^{KNWL})k_{it-1}^{KNWL} + \texttt{xrd}_{it}, \tag{25}$$

where sector-level depreciation rates, $\delta_{s(i)}^{KNWL}$, are from Ewens, Peters and Wang (2024), and $k_{i0}^{KNWL} = 0$. The second component, organizational capital, is constructed by capitalizing a portion of selling, general, and administrative expenses as follows:

$$k_{it}^{ORG} = (1 - 0.20)k_{it-1}^{ORG} + \gamma_{s(i)}\texttt{xsga}_{it}, \tag{26}$$

where $\gamma_{s(i)}^{ORG}$ is from Ewens, Peters and Wang (2024), and $k_{i0}^{ORG} = 0$. The third component, balance sheet intangible capital net of goodwill, is defined as: $k_{it}^{BS} = \texttt{intano}_{it}$.[4] Total intangible capital is calculated as: $k_{it}^{I} = k_{it}^{KNWL} + k_{it}^{ORG} + k_{it}^{BS}$, and values are deflated.

Output, and variable and fixed costs. The Compustat data include detailed firm-level financial statements, such as sales, input expenditures, capital stock, and industry classifications. We use sales, $\texttt{sale}_{it}$, to measure firm output, cost of goods sold, $\texttt{cogs}_{it}$, to capture variable input, and the non-capital fraction of selling, general, and administrative expenses, $(1 - \gamma_{s(i)}^{ORG})\texttt{xsga}_{it}$, to measure fixed costs. We deflate all variables to obtain their real values.

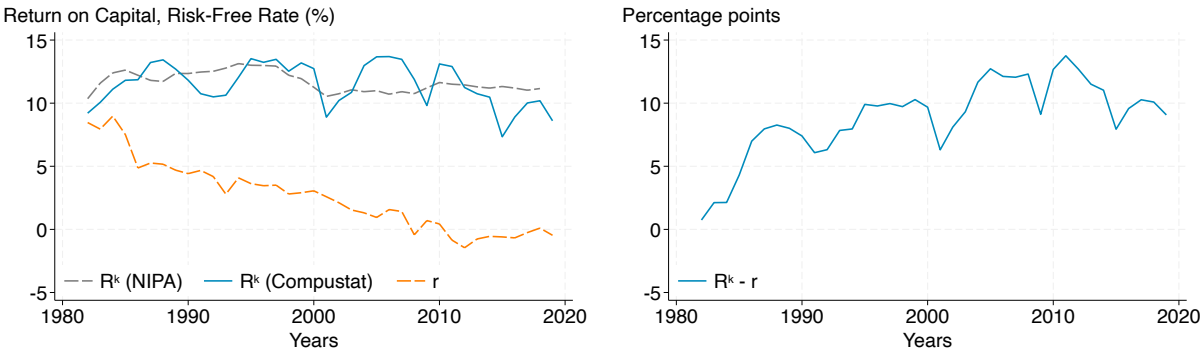
Relative prices. We measure the relative price of tangible capital, p_t^T, as the ratio of the tangible capital investment deflator to the GDP deflator, and the relative price of intangible capital, p_t^I, as the ratio of the intangible capital deflator to the GDP deflator.

Firm-level and aggregate return on capital. We calculate the firm-level return on capital, R_{it}^k, using equation (12). To assess how closely the aggregate return on capital from Compustat aligns with national accounts, we aggregate the firm-level returns, weighting them by total capital: $R_t^k = \sum_i \omega_{it} R_{it}^k$, in accordance with equation (18).

Figure 2a displays the evolution of the return on capital from national accounts (NIPA) and firm-level data (Compustat), showing a strong correlation between the two. This is unsurprising, given the well-established fact that the distribution of capital is heavily skewed toward larger firms (Crouzet and Mehrotra, 2020), which are highly representative of aggre-

[4]This component is crucial to include, as it typically accounts for most software on the balance sheet, which is the fastest-growing segment of aggregate intangible capital according to BEA data. For a detailed discussion on software accounting standards, see Chiavari and Goraya (2024) and Aum and Shin (2024).

Figure 2: Comparison Between National Accounts and Compustat



(a) Return on Capital and Risk-Free Rate **(b) Return on Capital and Risk-Free Rate Gap**

Note. Figure 2a illustrates the evolution of the aggregate return on capital from national accounts and the risk-free rate, as previously presented in Figure 1, alongside the aggregate return on capital from Compustat. Figure 2b highlights the aggregate divergence between the return on capital from Compustat and the risk-free rate.

gate movements; as mentioned before, based on our calculations, they account for approximately 60 percent of total non-residential capital. This confirms that Compustat is a reliable dataset for studying the divergence of the return on capital from the risk-free rate. Figure 2b quantifies this divergence, which grew from nearly zero in 1982 to almost 10 percentage points by 2019. Next, we examine the firm-level components from equation (23) that may have contributed to this divergence, assessing their quantitative explanatory power.

4.3 Firm-Level Measurements

4.3.1 Production Function Elasticities and Markups

This section outlines our baseline production function and markup measurement, describes the main findings, and presents a series of additional exercises we conduct to evaluate their robustness.

Production function and markup measurement. In our baseline calculations, equation (3) is represented by a constant return to scale Cobb-Douglas production function. We measure its elasticities at the sector-time level using firm-level cost share averages (Foster, Haltiwanger and Syverson, 2008), approximating the user cost of capital with the risk-free rate and the depreciation rate. This approach is both flexible and transparent, avoiding the need for firm-level output price deflators, which can pose challenges for other production function estimation methods commonly used in the literature (Bond, Hashemi, Kaplan and

Zoch, 2021).

While estimating these elasticities is necessary for our analysis, it is important to emphasize that we are primarily concerned with their trends rather than their levels, as our analysis focuses on changes over time. Furthermore, any mismeasurement of these elasticities will be accounted for through the explicit inclusion of measurement error in our methodology.

Then, we use equation (6) to compute firm-level markups, μ_{it}. This ratio estimator has gained popularity in the literature due to its modest data requirements and its demonstrated ability to accurately capture markup dispersion and growth, as shown by De Ridder, Grassi and Morzenti (2022).[5]

Baseline estimates. Appendix C.2 presents the evolution of the estimated elasticities of the production function. In line with Chiavari and Goraya (2024), we observe a steady increase in the elasticity of intangible capital over time, while the elasticities of variable costs and tangible capital have modestly declined. Additionally, we find a consistent increase in average markups, which aligns with the extensive literature documenting this trend (see Syverson, 2024 for a recent review).

Robustness exercises. Given the range of preferred methods for estimating production function elasticities and markups, we demonstrate in Appendix C.2 that our results are robust across a variety of methodologies. As an alternative estimation approach, we employ a translog production function that allows for non-unitary elasticity of substitution between inputs, an aspect recently emphasized by Oberfield and Raval (2021) and Aum and Shin (2024) in the presence of both tangible and intangible capital. To ensure identification in estimating gross output production functions and account for potential mismeasurment in capital inputs, we impose constant returns to scale, following Flynn, Traina and Gandhi (2019), and incorporate capital-specific measurement error as in Collard-Wexler and De Loecker (2021). Additionally, to measure markups, we apply the accounting profit approach, following Baqaee and Farhi (2020), where markups are calculated as the ratio of sales to total costs, encompassing both variable and fixed overhead costs in our model.

[5]While the literature acknowledges that measuring the aggregate markup level presents additional challenges (Bond, Hashemi, Kaplan and Zoch, 2021), this issue is not central to our analysis, which focuses on changes over time.

4.3.2 Risk Premium

Equation (11) naturally leads to a two-stage procedure for estimating risk premia, following the standard practice in the empirical asset pricing literature.

First stage. The first step involves estimating $\boldsymbol{\beta}_{it}$, which represents the firm-level exposure to movements in the stochastic discount factor. To achieve this, we perform the following regression for each firm:

$$\widetilde{MRPK}_{it} = \alpha_{i\tau} + \boldsymbol{\beta}_{i\tau} M_t + \varepsilon_{it}, \tag{27}$$

We estimate these regressions using backward-looking rolling windows of N_τ years. Specifically, for each year τ, we use the data from the period $t \in [\tau - N_\tau, \tau]$.

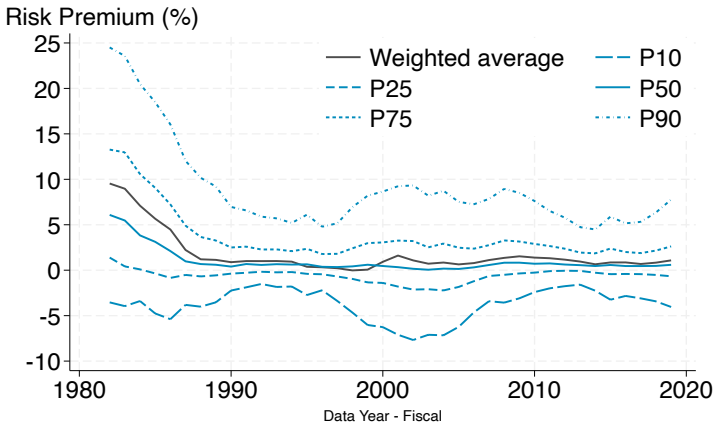
Second stage. Given a time-varying firm-level estimate $\boldsymbol{\beta}_{it}^{j}$, we can estimate the aggregate price of risk $\boldsymbol{\lambda}_t$ using equation (11). In practice, this implies to estimate the following cross-sectional regression in each year:

$$\widetilde{MRPK}_{it} - r_{t-1} - \delta_{it} = \alpha_{s(i)t} + p(age_{it}; \phi_t) + \boldsymbol{\beta}_{it}\boldsymbol{\lambda}_t + \varepsilon_{it}, \tag{28}$$

We use a combination of sector-time fixed effects and a third-degree polynomial in age to control for firm-level measurement errors or frictions, as well as time-varying capital gains present in equation (11). Although this step is not necessary for identification—since by construction, measurement errors and capital gains are uncorrelated with the risk premium—it helps to improve precision in small samples. Thus, the time-varying capital-specific firm-level premium is measured as $\boldsymbol{\zeta}_{it} = \boldsymbol{\beta}_{it}\boldsymbol{\lambda}_t$.

Empirical implementation and results. We implement this two-stage procedure by approximating the stochastic discount factor, M_t, using the five-factor model from Hou, Xue and Zhang (2015), which has been shown to track cross-sectional firm-level variation well and was recently applied in a similar context by David, Schmid and Zeke (2022). These factors include (i) the market return, (ii) the return on a portfolio that is long in small firms and short in large firms, (iii) the return on a portfolio that is long in low-investment firms and short in high-investment firms, (iv) the return on a portfolio that is long in high-profitability (return on equity) firms and short in low-profitability firms, and (v) the return on a portfolio that is long in firms with high expected 1-year-ahead investment-to-assets changes and short in firms with low ones.

Figure 3: Evolution of the Risk Premium Distribution



Note. Figure 3 shows the evolution of various moments in the distribution of firm-level risk premia from 1982 to 2019, specifically reporting the 10th, 25th, 50th, 75th, and 90th percentiles, as well as the capital-weighted average.

Figure 3 illustrates the evolution of estimated firm-level risk premia.[6] It highlights the 10th, 25th, 50th, 75th, and 90th percentiles, along with the capital-weighted average, which serves as the input for equation (23). We find that the distribution of risk premia exhibits substantial right-skewness, with most firms displaying relatively small risk premia, while a few experience significantly larger ones, as reflected by the pronounced right tail represented by the P90 in Figure 3.

Moreover, we find that the distribution of risk premia has shifted significantly over time. Most firms experienced a decline in risk premia between 1982 and 1990. Safer firms—those with negative or near-zero risk premia, represented by the P10 and P25—saw little to no change. In contrast, riskier firms, which make up the majority and are represented by the P50 to P90 in Figure 3, experienced a sharp reduction in risk premia. This has resulted in the overall distribution becoming more compressed around approximately 2 percent over time. These shifts in the distribution of firm-level risk premia have resulted in a decline in the capital-weighted risk premium during the early part of the sample, followed by a period of stabilization, with the premium remaining relatively flat in the later years.

Appendix C.3 validates our firm-level estimates of risk premia. Specifically, we show that firms with higher risk premia tend to exhibit higher returns on equity, lower levels of both tangible and intangible capital, and a lower capital-to-variable-costs ratio for both types

[6]Given our longer-term focus, we apply a 5-year moving average to firm-level risk premia to smooth out excess volatility.

of capital, as expected. Moreover, consistent with the findings of David, Schmid and Zeke (2022), we observe that sectors with more dispersed risk premia also display greater dispersion in revenue-based marginal products of tangible capital, intangible capital, and revenue productivity.

Moreover, in Appendix C.3, we demonstrate that alternative factor models commonly used in the literature, such as the Fama and French (2023) 5-factor, 3-factor, and 1-factor models, as well as the consumption CAPM, produce a quantitatively similar evolution of the average capital-weighted risk premium over time when compared to the Hou, Xue and Zhang (2015) model. With these validations in place, we proceed to discuss how these estimates align with the existing evidence on risk premia in the literature.

Relation with existing evidence. Our estimated risk premium is consistent with a broad body of evidence and the asset pricing literature. Common risk indicators, such as the VIX Index, the SKEW Index, the spread between the Fed Funds Rate and the Three-Month Treasury Bill Rate (e.g., Drechsler, Savov and Schnabl, 2018), the spread between risky and safe assets in Jordà, Knoll, Kuvshinov, Schularick and Taylor (2019), the Excess Bond Premium from Gilchrist and Zakrajšek (2012), the Economic Policy Uncertainty Index by Baker, Bloom and Davis (2016), Robert Shiller's CAPE Ratio, the Chicago Fed's NFCI risk subindex, the financial uncertainty index of Jurado, Ludvigson and Ng (2015), the risk appetite index of Bauer, Bernanke and Milstein (2023), the risk aversion index of Bekaert, Engstrom and Xu (2022), and the variance risk premium from Bekaert and Hoerova (2014), show no upward trend over time. In fact, many of these indicators have shown a decline since the 1980s, while others have remained constant over time, aligning with our findings.

Moreover, several papers in asset pricing have estimated the risk premium over time.[7] Campbell and Thompson (2008) find that the risk premium has either remained constant or declined since the 1980s, while Lettau, Ludvigson and Wachter (2008) and Avdis and Wachter (2017) estimate a significant decline in the risk premium since the early 1980s. Using a Gordon-like stock valuation model, Jagannathan, McGrattan and Scherbina (2001) document a decline in the risk premium from the 1980s to the 1990s. Martin (2017) reports a risk premium that has been constant since the mid-1990s. Similarly, Gagliardini, Ossola and Scaillet (2016), using

[7]Most of the existing asset pricing literature has concentrated on the equity risk premium. However, an important contribution by David, Schmid and Zeke (2022) demonstrates that equity risk premia and capital risk premia are closely related and proportional, meaning that movements in one directly inform the other.

a time-varying cross-sectional risk premium estimator, finds a stable risk premium over time. Gormsen and Huber (2023) estimate a constant perceived risk premium since the early 2000s, and Duarte and Rosa (2015), in their review of the asset pricing literature, show that risk premium estimates are generally either constant or declining. Overall, we conclude that our estimates are broadly consistent with those reported in the asset pricing literature.

An exception in the literature is Farhi and Gourio (2018), who argue that while the risk premium declined between the 1980s and 1990s, it has been rising since the 2000s, particularly around the Great Recession. This is not inconsistent with our findings, which confirm the historical decline in the risk premium while also noting a rise around the Great Recession, though small compared to its historical levels in the early part of our sample.

4.3.3 Tangible and Intangible Capital Mismeasurement and Frictions

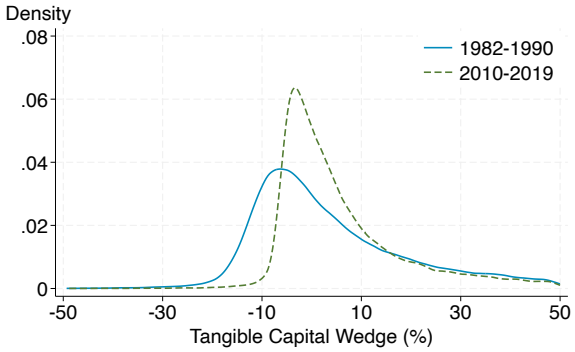
With measures of firm-level markups and risk premia in hand, we can now calculate the tangible capital wedge $\boldsymbol{\tau}^T$ and the intangible capital wedge $\boldsymbol{\tau}^I$ using equations (7) and (8).

Evolution over time. Figures 4a and 4b summarize the distribution of these wedges across different periods. Two key observations emerge: First, tangible capital wedges show less dispersion compared to intangible capital wedges in both periods, consistent with findings from Chiavari and Goraya (2024). Second, while the dispersion of tangible capital wedges has significantly decreased over time, a similar reduction is not observed for intangible capital wedges.

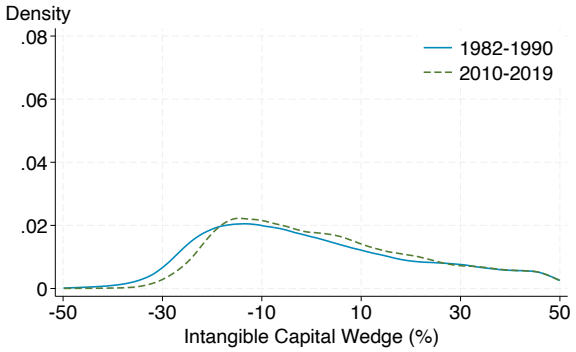
Figure 4c illustrates the evolution of the adjusted weighted average of each wedge over time, represented by $\sum_i \omega_i \kappa_i^T \boldsymbol{\tau}_i^T$ and $\sum_i \omega_i \kappa_i^I \boldsymbol{\tau}_i^I$ in equation (23). The findings are clear: both measures exhibit an upward trend, but their magnitudes differ significantly. Specifically, the average increase in the tangible capital wedge is approximately 3.85 percentage points, while the intangible capital wedge shows a larger rise of 6.12 percentage points.

Mismeasurment or frictions? Here, we investigate the nature of the estimated $\boldsymbol{\tau}^T$ and $\boldsymbol{\tau}^I$. Specifically, we assess the role of mismeasurement and the impact of frictions, such as financial constraints and adjustment costs, in the rise of capital-specific wedges over time. Since wedges either reflect frictions or mismeasurement, our approach is to assess the extent of mismeasurement in the observed $\boldsymbol{\tau}^T$ and $\boldsymbol{\tau}^I$, as well as its evolution over time. Based on these mismeasurement trends, we infer the role of frictions in the increase of capital-specific

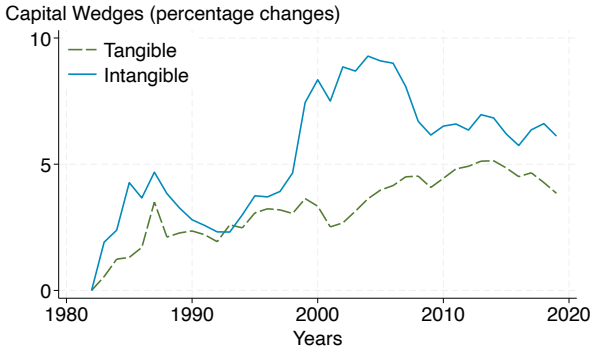
Figure 4: Tangible and Intangible Capital Wedges



(a) Tangible Capital Wedge Distribution



(b) Intangible Capital Wedge Distribution



(c) Tangible and Intangible Wedges Evolution

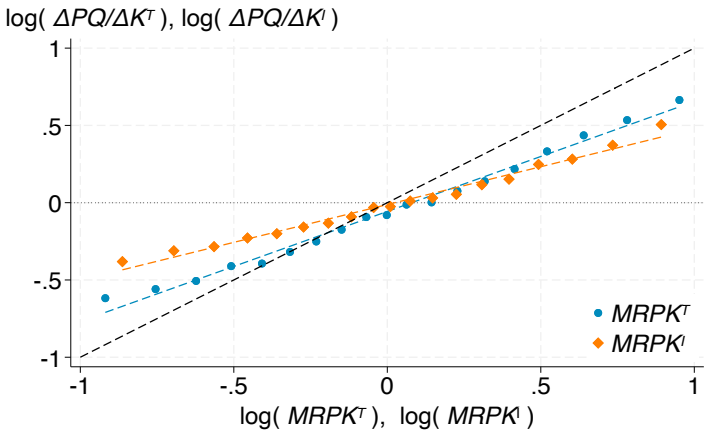
Note. Figures 4a and 4b display the distribution of firm-level $\boldsymbol{\tau}_i^T$ and $\boldsymbol{\tau}_i^I$ for the periods 1982-1990 and 2010-2019. Figure 4c illustrates the evolution of the cumulative change in the adjusted weighted average of each wedge. This average is calculated as the capital-weighted average of firm-level $\boldsymbol{\tau}_i^j$ multiplied by the firm-level j-specific capital share κ_i^j, as specified in equation (23).

wedges and validate our findings against standard proxies for frictions commonly used in the empirical literature.

We begin by investigating the potential extent of measurement error in our estimated tangible and intangible capital wedges using a simple approach that leverages the definition of the revenue-based marginal product of an input, calculated as the change in output relative to the change in the corresponding input. In our context, this approach implies that the observable revenue-based marginal product of each type of capital is given by $\widetilde{MRPK^j} \approx \Delta pq/\Delta k^j$, where $j \in \{T, I\}$. This differencing not only provides a complementary measure of revenue-based marginal product but also helps eliminate most persistent measurement error, as suggested by Bai, Jin and Lu (2024). Therefore, comparing our baseline measure with this alternative measure yields a simple diagnostic indicating the potential extent of measurement error in each component. Specifically, in the absence of measurement error, the two mea-

sures should be perfectly correlated. Conversely, if all of our original variation arises from measurement error, the two measures should be uncorrelated.

Figure 5: Alternative Measures of Revenue-Based Marginal Product of Both Capitals



Note. Figure 5 illustrates the relationship between our baseline measure of observed revenue-based marginal product for each type of capital and the measure constructed using first differences of sales over capital. Both variables are deviations from sector time averages. The blue dots represent tangible capital, while the orange diamonds denote intangible capital, with each point corresponding to one of the 25 percentiles. The dotted line indicates the best fit line.

Figure 5 displays the correlation between our baseline measure and the one constructed using first differences. Overall, we find that both measures are highly correlated, suggesting that measurement error accounts for only part of the observed variation in our $\boldsymbol{\tau}^T$ and $\boldsymbol{\tau}^I$. Specifically, the correlation for tangible capital is 0.68, while for intangible capital it is 0.59. The weaker correlation for intangible capital—indicated by a flatter line in Figure 5—suggests that this type of capital is affected by a larger amount of measurement error, consistent with existing literature (e.g., Lev, 2016).

To further corroborate our results, we employ a more structured test for measurement error based on the seminal work of Bils, Klenow and Ruane (2021), as applied in David and Venkateswaran (2019) and Bai, Jin and Lu (2024). This method allows us to estimate the extent of additive measurement error. It essentially involves estimating the following regression:

$$\Delta \log p_{it} q_{it} = \alpha_\kappa^j + \beta_\kappa^j \log \Delta k_{it}^j + \varepsilon_{it}^j, \tag{29}$$

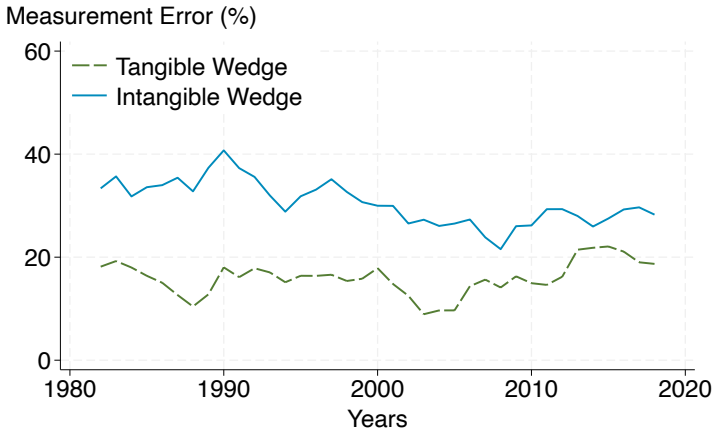
where $\Delta \log pq$ and $\Delta \log k^j$ represent the log-changes in sales and capital, κ denotes the decile of $\widetilde{MPRK^j}$ and $j \in \{T, I\}$. The key focus is the coefficient β_k^j. Intuitively, if the

measured deviations in $\widetilde{MPRK^j}$ are primarily due to additive measurement error, then firms with high observed $\widetilde{MPRK^j}$ will exhibit a lower elasticity of sales with respect to capital j. Bils, Klenow and Ruane (2021) demonstrate that, under certain assumptions, this coefficient captures the extent of measurement error, leading to the following corrected measure:

$$\log \widehat{MRPK^j} = \log \widetilde{MPRK^j} + \log \beta_\kappa^j. \tag{30}$$

Hence, the ratio of the variance of $\log \beta_x^j$ to the variance of $\log \widetilde{MPRK^j}$ informs the extent of measurement error in capital-specific wedges. Figure 6 presents our results. Several key observations arise. First, intangible capital wedges exhibit substantially more measurement error than tangible capital wedges. On average, intangible capital has a 30.5 percent measurement error, while tangible capital has 15.9 percent. Second, measurement error in tangible capital has remained relatively stable over time, while measurement error in intangible capital has decreased.

Figure 6: Evolution of Mismeasurement



Note. Figure 6 presents the evolution of measurment error in the tantgible capital wedge and the intangible capital wedge, measured ad the variance of $\log \beta_x^j$ to the variance of $\log \widetilde{MPRK^j}$ as in Bils, Klenow and Ruane (2021).

This pattern aligns with findings in the literature. It is well-known that purchased intangible assets, such as software, have become a larger share of total intangible capital over time, both in national accounts and firm-level data (e.g., Chiavari and Goraya, 2024; Aum and Shin, 2024). Since externally produced intangible assets are easier to measure as they often have market prices, the shift towards purchased intangibles likely contributed to improvements in the quality of intangible capital measurement. Additionally, these measurements align

with the findings of Bils, Klenow and Ruane (2021), which show that measurement error in revenue-based total factor productivity has increased over time. This increase is explained by the growing importance of intangible capital, which tends to have higher measurement errors, thus raising the overall measurement error in total factor productivity revenue.

Overall, we conclude that mismeasurement in capital-specific wedges is substantial, particularly for intangible capital. However, since measurement error is either stable or declining, it is unlikely to be driving the rise in capital-specific wedges documented in Figure 4c. Therefore, we conclude that frictions are a far more likely candidate for the rise documented in Figure 4c. To support this conclusion, Appendix C.4 shows that $\boldsymbol{\tau}^T$ and $\boldsymbol{\tau}^I$ are highly correlated with sectoral proxies of capital adjustment frictions from Cooper and Haltiwanger (2006) and firm-level proxies of financial frictions from Hoberg and Maksimovic (2015). Furthermore, the fact that frictions have been rising over time aligns with a substantial body of evidence, as discussed in Section 2.2.

5 Micro-Anatomy of Return to Capital and Risk-Free Rate Divergence

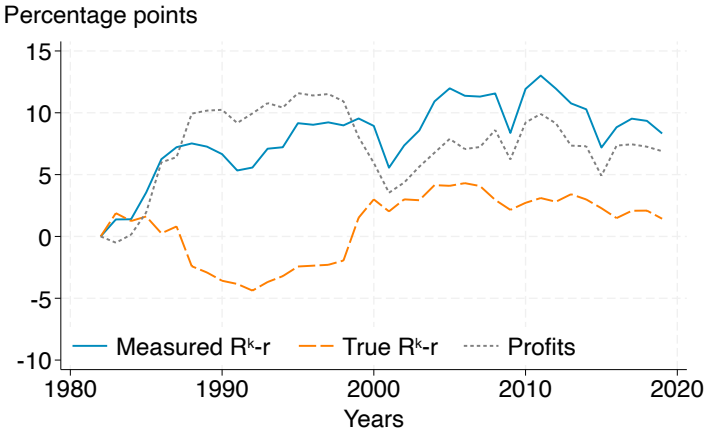
This section presents the main empirical results of the paper. First, it provides the results from the decomposition of the divergence between the return on capital and the risk-free rate, as outlined in Section 3. Second, it documents the contribution of changes in sectoral composition to this divergence, as well as the role of firm-level dynamics and within-sector reallocation in driving the divergence.

5.1 The Macroeconomic Drivers

Here, after measuring all the necessary firm-level variables required by our theory, we analyze the joint contribution of each narrative associated with the divergence between the measured aggregate return on capital and the risk-free rate. To achieve this, we first isolate the contribution of profits from the underlying dynamics of the true return on capital, using equation (22).

Figure 7 illustrates the evolution of the divergence between the measured return on capital and the risk-free rate, along with the evolution of each component as calculated in equation

Figure 7: The Role of Profits and True Return on Capital



Note. Figure 7 presents the decomposition based on equation (22) for the period 1982 to 2019. It illustrates the evolution of the gap between the return on capital and the risk-free rate (solid blue line), alongside the contributions of profits (dash-dotted green line) and the true return on capital (long-dashed orange line).

(22), and summarizes our first finding:

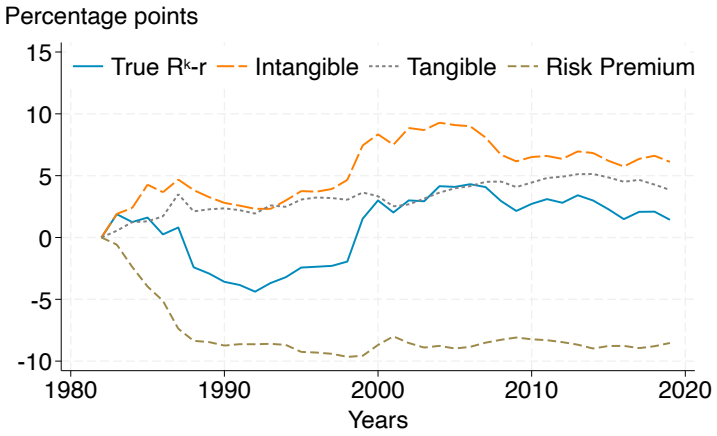
Fact 1: *Profits account for 83 percent of the divergence between the measured return on capital and the risk-free rate, while movements in the true return on capital account for the remaining 17 percent.*

Figure 7 shows that over time, the gap between the return on capital and the risk-free rate has widened, reaching approximately 8.33 percentage points in 2019. It also presents the counterfactual evolution of this gap, illustrating scenarios in which only profits or only the true return on capital have changed over time. We find that if profits were the sole driver of the divergence, the gap would have increased by 6.89 percentage points, accounting for 83 percent of the divergence. In contrast, if only the true return on capital were responsible, the divergence would have reached just 1.44 percentage points, or 17 percent. Therefore, we conclude that if profits were correctly accounted for, the measured gap between the return on capital and the risk-free rate would have risen significantly less over time.

We emphasize that the finding that a significant rise in profits explains a substantial portion of the divergence between the aggregate measured return on capital and the risk-free rate aligns with the seminal work of De Loecker, Eeckhout and Unger (2020), which documents a notable increase in market power. The extensive literature that followed has confirmed the broad notion of rising markups, though the magnitude of this increase varies depending on

how markups are measured and aggregated. This body of work has been recently reviewed and summarized by Syverson (2024). Appendix D.1 demonstrates that decomposing the role of profits into markups and fixed costs reveals that both factors contributed roughly equally to the widening gap between the return on capital and the risk-free rate.

Figure 8: The Drivers of The True Return on Capital and Risk-Free Rate Divergence



Note. Figure 8 presents the decomposition based on equation (23) for the period 1982 to 2019. It illustrates the evolution of the gap between the true return on capital and the risk-free rate (solid blue line), alongside the contributions of frictions associated to intangible capital (dash-dotted green line), frictions associated to tangible capital (long-dashed orange line), and the risk premium (dashed brown line).

Next, we examine why the true return on capital has diverged from the risk-free rate, albeit to a much lesser extent than if profits were not properly accounted for. We leverage the decomposition in equation (23) to shed light on which of the possible explanations—risk premium or frictions related to intangible or tangible capital—best explains this divergence. Figure 8 presents the results from this decomposition. Particularly, it plots the divergence between the true return on capital and the risk-free rate, along with the evolution of each component as calculated in equation (23), and summarizes our second finding:

Fact 2: *Contrary to previous findings, the rising frictions associated with intangible capital—rather than the risk premium, which has actually driven convergence between the two returns—have been the main force behind the divergence between the true return on capital and the risk-free rate.*

Figure 8 illustrates that frictions associated with both tangible and intangible capital have been rising, contributing to the divergence between the true return on capital and the risk-

free rate. However, the rise in frictions related to intangible capital is quantitatively much larger, predicting a divergence of 6.12 percentage points, whereas frictions tied to tangible capital would have predicted a smaller increase of 3.85 percentage points. In contrast to the previous consensus (e.g., Farhi and Gourio, 2018), we find that the decline in the risk premium over time, consistent with extensive asset pricing literature and evident in many risk proxies reviewed in Section 4.3.2, would have predicted a substantial convergence of 8.53 percentage points between the true return on capital and the risk-free rate. Therefore, to understand why the gap between the true return on capital and the risk-free rate has persisted over time, it is essential to focus on the significant rise in frictions related to intangible capital, which we will explore in greater detail in the following sections. Appendix D.2 shows that the results presented in Figure 7 are robust to the use of different production functions, measures of markups, and risk premium, as discussed in Sections 4.3.1 and 4.3.2.

Before proceeding, we want to highlight one of the key advantages of our framework. In particular, the findings in this section highlight that if we were to examine the factors in isolation or in groups, rather than considering all the causes identified in the literature jointly, we might conclude, for example, that profits and intangible capital together account for more than 150 percent of the total divergence.[8] This mirrors the point raised by Grossman and Oberfield (2022) about how the literature attempting to explain the decline in labor share of national income ended up collectively over-explaining the phenomenon. Instead, our methodology, which allows for a joint analysis of these contributing factors, provides a unified picture of the divergence.

Another advantage of our methodology is that it ties the various forces driving the divergence between the return on capital and the risk-free rate to their micro-level sources. This allows us to investigate the roles that sectors, firms, reallocation, and entry of new cohorts have played in shaping this divergence. We explore these factors in the following sections.

5.2 The Role of Sectors

This section explores the role of sectors and their changing importance over time in driving the gap between the true return on capital and the risk-free rate. We focus specifically on the

[8] As already mentioned, profits alone would result in a counterfactual gap of 6.89 percentage points, while frictions associated with intangible capital predict a gap of 6.12 percentage points. The actual total gap stands at 8.33 percentage points.

frictions associated with intangible capital, as shown in the previous section to be the primary quantitative driver. Appendix D.3 provides a similar analysis for the other components outlined in equation (23).

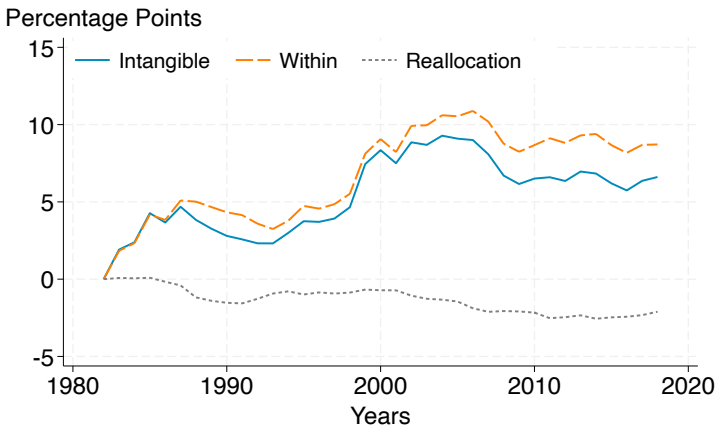
To achieve this, we implement the following sectoral decomposition:

$$\Delta y_t = \underbrace{\sum_s \omega_{st-1}\Delta y_{st}}_{\Delta\text{within}} + \underbrace{\underbrace{\sum_s \Delta\omega_{st} y_{st-1}}_{\Delta\text{between}} + \underbrace{\sum_s \Delta\omega_{st}\Delta y_{st}}_{\Delta\text{cross term}}}_{\Delta\text{reallocation}}, \tag{31}$$

where y_t is the intangible-related component in equation (23) and equals $\sum_i \omega_{it}\kappa^I_{it}\boldsymbol{\tau}^I_{it}$, y_{st} equals $\sum_{i\in s} \omega_{it}\kappa^I_{it}\boldsymbol{\tau}^I_{it}$, ω_{st} equals $\frac{\sum_{i\in s}\omega_{it}}{\sum_i \omega_{it}}$, and s represents a 2-digit NAICS sector.

Equation (31) breaks down the change in the gap between the return on capital and the risk-free rate into three components: Δwithin, Δbetween, and Δcross terms. The Δwithin component captures the portion of the change due to shifts in the sector-level wedges while holding sector-level weights constant. The Δbetween component captures the change due to shifts in sector-level weights, keeping the sector-level wedges constant. The Δcross term reflects the covariance between changes in weights and sector-level wedges. We combine the Δbetween and Δcross terms into a single Δreallocation component.

Figure 9: Sectoral Decomposition of the Return on Capital and Risk-Free Rate Gap



Note. Figure 9 illustrates the results of the sectoral decomposition in equation (31) for the period from 1982 to 2019. The solid blue line represents the evolution of the frictions associated to intangible capital, while the long dashed orange line depicts the evolution of the Δwithin component. The short dashed grey line shows the evolution of the Δreallocation component.

Figure 9 presents the results of the sectoral decomposition described in equation (31) for the period from 1982 to 2019 and summarizes our third finding:

Fact 3: *Changes in the sectoral composition of the U.S. economy played no role in the rise in frictions associated to intangible capital.*

This finding is striking: the Δwithin component accounts for the entire divergence between the return on capital and the risk-free rate, while the Δreallocation component contributes nothing to this divergence, and if anything played a dampening role. This holds true even when examining each part of the Δreallocation component separately, namely the Δbetween and Δcross term components, both of which remain very small throughout the analysis period. Appendix D.3 confirms that this is also the case when considering the other individual component contributing to the divergence in the true return on capital, specifically tangible capital and the risk premium.

Overall, our findings suggest that while the U.S. economy has experienced substantial deindustrialization over the analysis period, characterized by a continuous shrinkage of its manufacturing sector and a rise in services, the sources of the widening gap between the true return on capital and the risk-free rate, and in particular of the rise in frictions associated to intangible capital must be sought elsewhere. In fact, what Figure 9 indicates is that, to uncover the root causes of this divergence, we must focus on within-sector forces. This sets the stage for our next section, which investigates the roles of firms, reallocation among them, and the dynamics of firm entry and exit.

5.3 The Role of Firms and Reallocation

This section delves into the role of firm dynamics, reallocation among firms, and the impact of firm entry and exit on the rise in frictions related to intangible capital. Similar to the previous section, Appendix D.4 offers a comparable analysis for the other components outlined in equation (23). To accomplish this, we employ the firm-level decomposition from Haltiwanger

(1997):

$$\Delta y_t = \underbrace{\sum_{i \in \text{Continue}} \omega_{it-1} \Delta y_{it}}_{\Delta\text{within}} + \underbrace{\underbrace{\sum_{i \in \text{Continue}} \Delta\omega_{it}(y_{it-1} - y_{t-1})}_{\Delta\text{between}} + \underbrace{\sum_{i \in \text{Continue}} \Delta\omega_{it}\Delta y_{it}}_{\Delta\text{cross term}}}_{\Delta\text{reallocation}} + \underbrace{\sum_{i \in \text{Entry}} \omega_{it}(y_{it} - y_{t-1}) - \sum_{i \in \text{Exit}} \omega_{it-1}(y_{it-1} - y_{t-1})}_{\Delta\text{net entry}}, \tag{32}$$

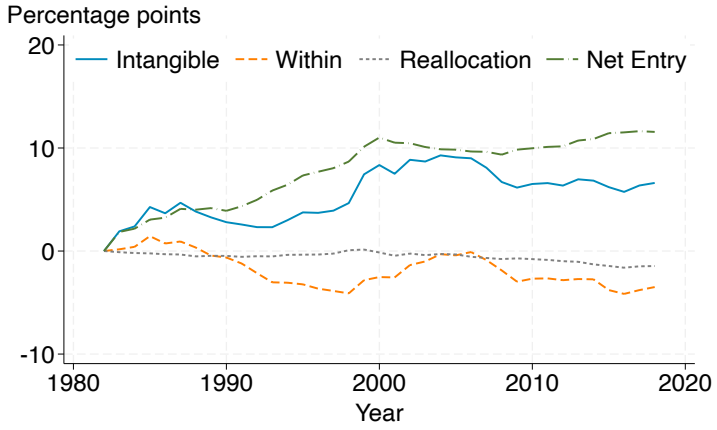
where ω_{it} represents firm-level capital weights, and y_{it} equals the firm-level adjusted intangible capital wedge, $\kappa^I_{it}\tau^I_{it}$.

The Δwithin term measures the average change resulting solely from changes in the firm-level adjusted intangible capital wedge, while keeping the weights constant from the previous period. In contrast, the Δbetween term captures the change due to shifts in weights, while holding the firm-level adjusted intangible capital wedge fixed. The Δcross term reflects the joint change in both the firm-level adjusted intangible capital wedge and the weights. Since both the Δbetween and Δcross term capture the effect of changing weights over time, we group them under Δreallocation.[9] Finally, the Δnet entry term measures the effect of entry and exit on the adjusted intangible capital wedge. This captures the change in the composition of firms in the market. If the entering firms have higher adjusted intangible capital wedge than the exiting firms, for example, this term will be positive.

Figure 10 presents the results of the firm-level decomposition from equation (32) for the period from 1982 to 2019. It shows that the rise in frictions associated with intangible capital is entirely driven by the Δnet entry component, reflecting firm entry and exit dynamics, while the Δwithin component, capturing changes within firms, and the Δreallocation component, reflecting reallocation among firms, had little quantitative impact and, if anything, dampened the overall increase. This is a significant finding, as it underscores that the rise in these frictions—and consequently, the divergence between the true return on capital and the risk-free rate—is not due to incumbent firm dynamics but rather to shifts in the composition of the firm pool over time. Note that "entry" here refers to entry into Compustat, which is important to keep in mind when interpreting the results. We will expand on this point at the

[9]The Δcross term is quantitatively very small in our counterfactual analysis.

Figure 10: Firm-Level Decomposition of the Adjusted Intangible Capital Wedge



Note. Figure 10 illustrates the results of the firm-level decomposition in equation (32) for the period from 1982 to 2019. The solid blue line represents the evolution of the adjusted intangible capital wedge, while the long dashed orange line depicts the evolution of the within component. The short dashed grey line shows the evolution of the reallocation component. The dash-dotted green line shows the net entry component

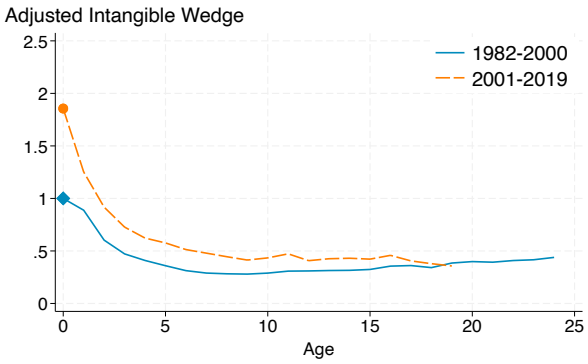
end of this section, when commenting our overall findings.

Figure 11a further illustrates this point by showing the life-cycle evolution of the adjusted intangible capital wedge, $\kappa^I \boldsymbol{\tau^I}$, for cohorts in the first and second halves of our sample period. Different cohorts exhibit markedly distinct life-cycle trajectories for their average adjusted intangible capital wedge. Notably, later cohorts start with significantly higher adjusted intangible capital wedges at entry, a difference that persists for more than fifteen years. Since the adjusted intangible capital wedge is the product of two components—the intangible capital wedge, $\boldsymbol{\tau^I}$, and the intangible capital intensity, κ^I—we analyze their respective contributions to its evolution by presenting their life-cycle dynamics across different cohorts. Figures 11b and 11c present their evolution, highlighting our fourth result:

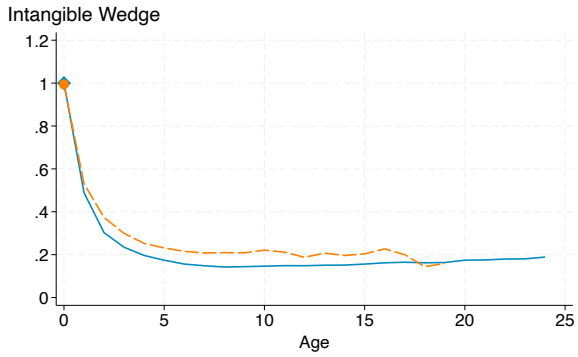
Fact 4: *The increase in the adjusted intangible capital wedge is entirely driven by changes in the composition of firms over time, with later cohorts being more intangible-intensive rather than simply exhibiting higher wedges.*

Figures 11b and 11c present the life-cycle evolution of the intangible capital wedge, $\boldsymbol{\tau^I}$, and intangible capital intensity, κ^I, for cohorts from the first and second halves of our sample period. A few key observations stand out. First, Figures 11b and 11c shows that intangible capital wedges decline significantly over the firm life cycle as intangible capital intensity

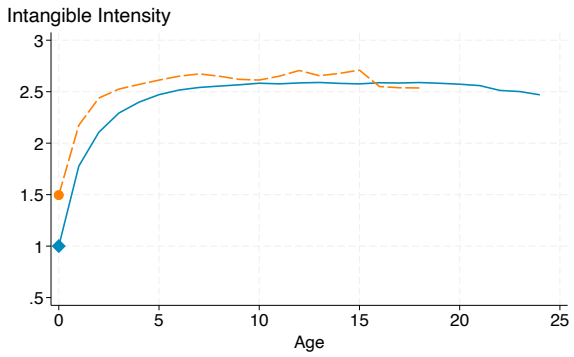
Figure 11: Unpacking the Adjusted Intangible Capital Wedge



(a) Adjusted Intangible Capital Wedge



(b) Intangible Capital Wedge



(c) Intangible Intensity

Note. Figures 11a, 11b, and 11c illustrate for the first and second halves of our sample period the life-cycle evolution of the adjusted intangible capital wedge, $\kappa^I \boldsymbol{\tau^I}$, and its components: the intangible capital wedge, $\boldsymbol{\tau^I}$ and the intangible capital intensity, κ^I. The solid blue line shows the life-cycle evolution of each component before 2000, while the long-dashed orange line reflects their evolution from 2000 onward. Each line represents the average across firms, with all values normalized to the first cohort's initial entry observation.

increases, which is consistent with standard life cycle models where financial frictions and adjustment costs gradually diminish as firms grow.[10] The fact that intangible capital grows more than tangible capital over the life cycle reinforces the view that intangible capital faces more binding frictions compared to tangible capital. Extensive literature highlights substantial frictions associated with intangible capital, showing they are significantly higher than those for tangible capital, both in terms of adjustment costs (e.g., Peters and Taylor, 2017; Belo, Gala, Salomao and Vitorino, 2022; Cloyne, Martinez, Mumtaz and Surico, 2022; Crouzet and Eberly, 2023; Chiavari and Goraya, 2024) and financial frictions (e.g., Caggese and Pérez-

[10]This pattern further supports the findings in Section 4.3.3, suggesting that mismeasurement alone cannot fully account for these frictions, as there is no reason to expect it to fade over a firm's life cycle, whereas there are strong economic reasons for the decline.

Orive, 2022; Falato, Kadyrzhanova, Sim and Steri, 2022; Bøler, Moxnes and Ulltveit-Moe, 2023; Zhang, 2024).

Second, examining the evolution of wedges and intangible capital intensities across cohorts from the first and second halves of our sample, Figure 11b shows that the magnitude of firms' frictions has not significantly increased in more recent cohorts. In contrast, Figure 11c highlights that intangible capital intensities—representing the share of intangible capital in total capital—have increased substantially across cohorts, particularly at the point of entry, and this difference remains notable for at least ten to fifteen years.

This point is crucial for understanding the drivers behind the rise in the adjusted intangible capital wedge. It suggests that increasingly binding frictions over time are unlikely to explain this rise, whereas the growing intangible capital intensities are. This aligns with the findings of Chiavari and Goraya (2024) and Aum and Shin (2024), who show that firm-level production processes have become more intensive in intangible capital over time, often at the expense of labor. This shift implies a greater reliance on intangible capital as an input, rather than labor or even tangible capital. Thus, the rise in aggregate frictions linked to intangible capital is not the result of firms encountering more frictions or a reallocation of economic activity toward more friction-prone firms. Instead, it is driven by newer cohorts of firms increasingly relying on a more frictional input—intangible capital—in their production processes.

We emphasize that while different cohorts vary in their intangible intensity, particularly at younger ages, in Appendix D.4 we show that they are remarkably similar in terms of sales levels, variable costs, and growth rates. These findings align with Karahan, Pugsley and Şahin (2024) and Hopenhayn, Neira and Singhania (2022), who demonstrate that the life cycle of firms has not changed significantly across cohorts. In fact, our results suggest that the key difference over time lies not in the average life cycle, which has remained surprisingly consistent across cohorts, but in the composition of inputs within firms, with more recent cohorts relying substantially more on intangible capital than tangible capital at younger ages.

We conclude by emphasizing, as mentioned earlier, that since age in Compustat represents years since incorporation in the dataset, young firms should not be mistaken for new firms. Instead, they should more accurately be understood as newer large firms. As discussed, due to the highly skewed distribution of capital toward these firms, they are highly informative

of aggregate capital movements. Our findings highlight that these markedly different new cohorts of large firms, with their initially higher intangible capital intensity, are central to the aggregate rise in frictions associated with intangible capital, and consequently, to the divergence between the true return on capital and the risk-free rate.

6 Conclusion

This paper presents a closed-form general equilibrium framework that decomposes the divergence between the return on capital and the risk-free rate into the joint effects of the most prominent explanations—risk premium, profits, and frictions associated with both tangible and intangible capital—at the firm level, drawing on standard data sources. While confirming the significant role of profits, we highlight a novel finding: contrary to previous literature that emphasizes the role of risk, rising frictions associated with intangible capital—driven by the increasing intangible capital intensity across cohorts of large firms over time—have been the key factor preventing the gap between the return on capital and the risk-free rate from closing. These findings underscore the importance of understanding the macroeconomic impact of differences across cohorts of firms and the growing role of intangible capital in production, raising several questions for future research.

First, our work suggests that cohorts of large firms may differ in intangible capital intensity due to variations in production processes over time, particularly in their early stages. We believe that further analysis of the interaction between new cohorts and technological change is crucial, as these new cohorts may engage with emerging technologies differently than incumbent firms, which could have significant implications for business dynamism and aggregate growth.

Second, while our framework suggests that intangible capital is subject to large frictions, even compared to tangible capital, it does not specify their origins. Similar to the existing literature, we find that these frictions may be linked to both adjustment costs and financial frictions. However, we believe that studying both types of frictions simultaneously within a structural framework is a promising avenue for future research, as they may lead to different normative implications.

Finally, our model adopts a neoclassical approach to the interaction between intangible capital and profits. However, a broader perspective could account for economic interactions;

for instance, investment in intangible assets such as product innovation or brand capital may directly contribute to a firm's profit generation. Incorporating these interactions would extend the neoclassical framework presented here and could potentially strengthen the links between intangible capital and profits, further increasing the share of the aggregate divergence attributed to intangible capital if it is a driver of rising profits. This, in turn, could blur the distinction between the return on capital and the return on firm ownership. We leave this exploration for future research.

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The Divergence Between the Return of Capital and the Risk-Free Rate: A Micro-Anatomy

Andrea Chiavari, Julia Faltermeier, and Sampreet S. Goraya

Online Appendix

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A Additional Evidence on the Return on Capital and Risk-Free Rate Divergence

Here, we explain how the return on capital is calculated in the main text, present its historical evolution in comparison to the risk-free rate, and demonstrate that their divergence remains robust across various measures of both return on capital and the risk-free rate.

Measurement of return of capital. To measure the aggregate return on capital in the national accounts, we follow the principles outlined in Section 2.1, particularly equation (1). For aggregate measures we follow Koh et al. (2020). In practice, our baseline measure of the return on capital is calculated as follows:

$$R^k = \frac{NOS - CE - PI - DEP}{K}, \tag{A.1}$$

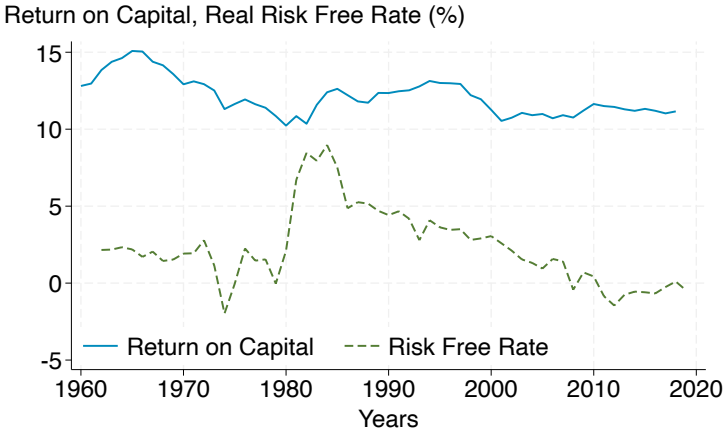
where NOS is the net operating surplus, CE represents compensation for employees (from NIPA Table 1.12), PI refers to proprietors' income (also from NIPA Table 1.12), DEP stands for depreciation (from the Fixed Assets Accounts Tables), and K represents the fixed assets as defined by the BEA. This includes all non-residential structures, equipment, and intellectual property products (IPP).

Historical evolution of return of capital versus the risk-free rate. Figure A.1 presents the evolution of the return on capital and the risk-free rate since the 1960s. We find that the return on capital remains relatively flat over these six decades, consistent with the findings of Gomme, Ravikumar and Rupert (2011) and Reis (2022). In contrast, the risk-free rate follows an inverted U-shaped pattern, in line with Rachel and Summers (2019).

Additional measures of the return on capital and the risk-free rate. Figures A.2a and A.2b show the evolution of alternative measures of the return on capital and the risk-free rate since the 1980s.

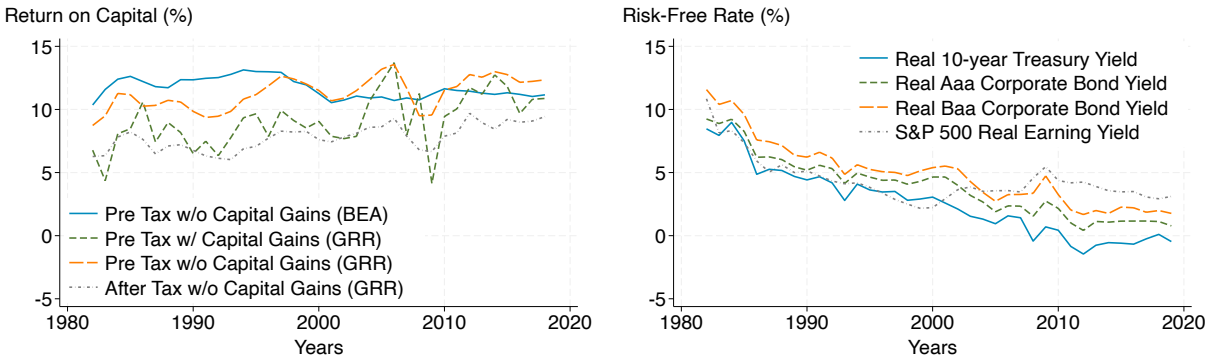
Figures A.2a illustrates the evolution of several alternative measures of the return on capital from Gomme, Ravikumar and Rupert (2011). These measures focus on the business sector, thus excluding housing, and explicitly account for taxation and capital gains. While the inclusion of taxation and capital gains has only a modest impact on the level of the return on capital, it does not affect its remarkable stability over time. This robustness to various adjustments is also documented in Reis (2022).

Figure A.1: Historical Evolution of the Return on Capital and Risk-Free Rate



Note. Figure A.1 shows the evolution of the return on capital and the risk-free rate, measured in percent, since 1960. The return on capital is measured using equation (2) with BEA data. The risk-free rate is the market yield on U.S. Treasury securities with a 10-year constant maturity net of expected inflation from Michigan.

Figure A.2: Alternative Measures of Return on Capital and Risk-Free Rate Over Time



(a) Return on Capital **(b) Risk-Free Rate**

Note. Figure A.2a presents the evolution of alternative measures of the return on business capital from Gomme, Ravikumar and Rupert (2011), both before and after tax, and with and without capital gains. Figure A.2b displays alternative measures of the risk-free rate, as reported in Rachel and Summers (2019).

Figure A.2b presents the evolution of several alternative measures of the risk-free rate, similar to those used in Rachel and Summers (2019). Specifically, it shows the real Aaa Corporate Bond yield, the real Bbb Corporate Bond yield, and the S&P 500 Real Earnings yield, all of which exhibit a notably similar decline over time.

B Further Details on the Microfounded Structural Decomposition

B.1 Additional Microfoundations for the Firms' Problem

In this section, we present alternative microfoundations for the wedges $\boldsymbol{\tau^j}$, where $j \in \{T, I\}$, as used in the main text. Specifically, we demonstrate that these wedges can be interpreted as financial frictions, adjustment costs, or measurement error.

Financial frictions. Assuming the presence of time varying firm i specific capital j specific capital constraints κ_{it}^j, capturing flexible the presence of financial friction contraining capital expansions of the firm, then the Lagrangian objective function associated with the firm's cost minimization problem can be expressed as:

$$\begin{aligned}\mathcal{L}(\ell_{it}, k_{it}^T, k_{it}^T, \boldsymbol{\tau}_{it}^j, \xi_{it}) = p_{it}^{\ell}\ell_{it} + \sum_{j\in\{T,I\}} \left(r_t + \boldsymbol{\zeta}_{it} + \delta^j\right) p_t^j k_{it}^j + \boldsymbol{c}_{it} \\ - \sum_{j\in\{T,I\}} \boldsymbol{\tau}_{it}^j \left(\kappa_{it}^j - p_t^j k_{it}^j\right) - \xi_{it}\left(q(\cdot) - q_{it}\right),\end{aligned} \tag{B.2}$$

where all variables are defined as in the main text, except that $\boldsymbol{\tau}_{it}^j$ now represents the Lagrange multiplier associated with the constraint.

The first-order conditions for both types of capital, along with the complementary slackness condition, are given by:

$$r_t + \boldsymbol{\zeta}_{it} + \delta^j + \boldsymbol{\tau}_{it}^j = \frac{\xi_{it}}{p_t^j}\frac{\partial q(\cdot)}{\partial k_{it}^j} \tag{B.3}$$

$$\left(\kappa_{it}^j - p_t^j k_{it}^j\right) \geq 0 \tag{B.4}$$

$$\boldsymbol{\tau}_{it}^j \left(p_t^j k_{it}^j - \kappa_{it}^j\right) = 0. \tag{B.5}$$

Note that equation (B.3) is isomorphic to equations (7) and (8) in the main text, as the right-hand side is exactly equal to $\widetilde{MRPK}_{it}^j$.

Adjustment costs. Assuming that firms face capital-specific adjustment frictions, $\boldsymbol{\tau^j}(k_{it}^j)$, the Lagrangian objective function for the firm's cost minimization problem can be expressed

as:

$$\mathcal{L}(\ell_{it}, k_{it}^T, k_{it}^T, \xi_{it}) = p_{it}^{\ell}\ell_{it} + \sum_{j\in\{T,I\}} \left(r_t + \boldsymbol{\zeta}_{it} + \delta^j\right) p_t^j k_{it}^j - \sum_{j\in\{T,I\}} p_t^j \boldsymbol{\tau}^{\boldsymbol{j}}(k_{it}^j) + \boldsymbol{c}_{it} - \xi_{it}\left(q(\cdot) - q_{it}\right). \tag{B.6}$$

The first-order conditions for both types of capital are given by:

$$r_t + \boldsymbol{\zeta}_{it} + \delta^j + \boldsymbol{\tau}_{it}^j = \frac{\xi_{it}}{p_t^j}\frac{\partial q(\cdot)}{\partial k_{it}^j}, \tag{B.7}$$

where $\boldsymbol{\tau}_{it}^j \equiv \partial\boldsymbol{\tau}^{\boldsymbol{j}}(k_{it}^j)/\partial k_{it}^j$. Again, note that equation (B.7) is isomorphic to equations (7) and (8) in the main text, as the right-hand side is exactly equal to $\widetilde{MRPK}_{it}^j$.

B.2 Additional Microfoundations for the Investors' Problem

Here, we provide the microfoundations for equation (10). Assuming a risk-neutral investor who discounts the future using a stochastic discount factor M_t and has total per-period income I_t, who invests in each type of capital j and firm i, the optimization problem is formulated as follows:[11]

$$\begin{aligned} &\max_{\{k_{it+1}^T, k_{it+1}^I\}_{\forall i}} \mathbb{E}_t \sum_{\tau}^{\infty} M_{t+\tau}^{\tau} \sum_i d_{it+\tau} \\ &\text{s.t.} \quad d_{it} = I_t - \sum_{j\in\{T,I\}} p_t^j \left(k_{it+1}^j - (1-\delta^j)k_t^j\right) + \sum_{j\in\{T,I\}} \left(r_t + \boldsymbol{\zeta}_{it}^j + \delta^j\right) p_{t-1}^j k_{it}^j. \end{aligned} \tag{B.8}$$

The first-order conditions of this problem are given by the following set of equations:

$$1 = \mathbb{E}_{t-1}\left[M_t\left(MRPK_{it}^j + (1-\delta^j)\frac{p_t^j}{p_{t-1}^j}\right)\right], \quad \forall i, \forall j \in \{T, I\}. \tag{B.9}$$

For each firm i, multiplying its first-order condition for tangible and intangible capital by $\frac{p_{t-1}^T k_{it-1}^T}{p_{t-1}^T k_{it-1}^T + p_{t-1}^I k_{it-1}^I}$ and $\frac{p_{t-1}^I k_{it-1}^I}{p_{t-1}^T k_{it-1}^T + p_{t-1}^I k_{it-1}^I}$ respectively, and then summing them up, yields

[11] Note that we leave the source of I_t unspecified. It could originate from labor income, profits, or both, and can be easily microfounded through a representative household or a mutual fund investing on its behalf.

exactly equation (10). From equation (10) it is easy to get:

$$1 + r_{t-1} = \mathbb{E}_{t-1}\left[MRPK_{it} + \sum_{j\in\{T,I\}} \kappa^j_{it-1}(1-\delta^j)\frac{p^j_t}{p^j_{t-1}}\right] + \frac{\mathbb{C}_{t-1}\left(M_t, MRPK_{it}\right)}{\mathbb{E}_{t-1}\left[M_t\right]}, \quad \text{(B.10)}$$

since $\mathbb{E}_{t-1}\left[M_t\right] = (1+r_{t-1})^{-1}$ and $\mathbb{C}_{t-1}\left(M_t, \sum_{j\in\{T,I\}} \kappa^j_{it-1}(1-\delta^j)\frac{p^j_t}{p^j_{t-1}}\right) = 0$. Substituting into equation (B.10) equation (9), replacing expectations with an expectational error ε_{it}, and rearranging the terms yields the following:

$$\widetilde{MRPK}_{it} - r_{t-1} = \boldsymbol{\tau}_{it} + 1 - \sum_{j\in\{T,I\}} \kappa^j_{it-1}(1-\delta^j)\frac{p^j_t}{p^j_{t-1}} - \frac{\mathbb{C}_{t-1}\left(M_t, MRPK_{it}\right)}{\mathbb{E}_{t-1}\left[M_t\right]} + \varepsilon_{it} \quad \text{(B.11)}$$

Subtracting the average depreciation rate, $\delta_{it} \equiv \sum_{j\in\{T,I\}} k^j_{it-1}\delta^j$, from both sides of equation (B.11) results in equation (11) in the main text.

C Further Details on Data, Variable Construction, and Measurement

C.1 Data Cleaning and Summary Statistics

Here, we explain the data cleaning process. For data quality purposes, we interpret values for `sale`, k^T_{it}, k^I_{it}, `cogs`, or `xsga` as errors if they are zero, negative, or missing, and we exclude those observations. If `xrd`, `intano`, or `am` are negative or missing, we treat them as zeros. To make all variables real, we deflate them using the GDP deflator. Finally, we winsorize `sale`, k^T_{it}, k^I_{it}, `cogs`, and `xsga` at the 0.5 percent level. Table C.1 provides summary statistics for these variables.

C.2 Additional Production Functions and Markups Measurements

In our baseline approach, we estimate the production function using the cost share method, as in Foster, Haltiwanger and Syverson (2008), and markups using the ratio estimator from De Loecker and Warzynski (2012). Here, we demonstrate that our results remain robust across a variety of alternative methodologies.

Table C.1: Summary Statistics

	Sale	Cost of Goods Sold	Tangible Capital Stock	Intangible Capital Stock	Selling General & Administrative
Mean	1088,446	716,663	681,002	344,926	128,177
p25	24,905	13,199	6,812	7,985	4,291
p50	123,044	69,643	35,971	33,523	16,037
p75	586,797	350,030	218,133	152,703	68,144
No. Obs.	161,317	161,317	161,317	161,317	161,317

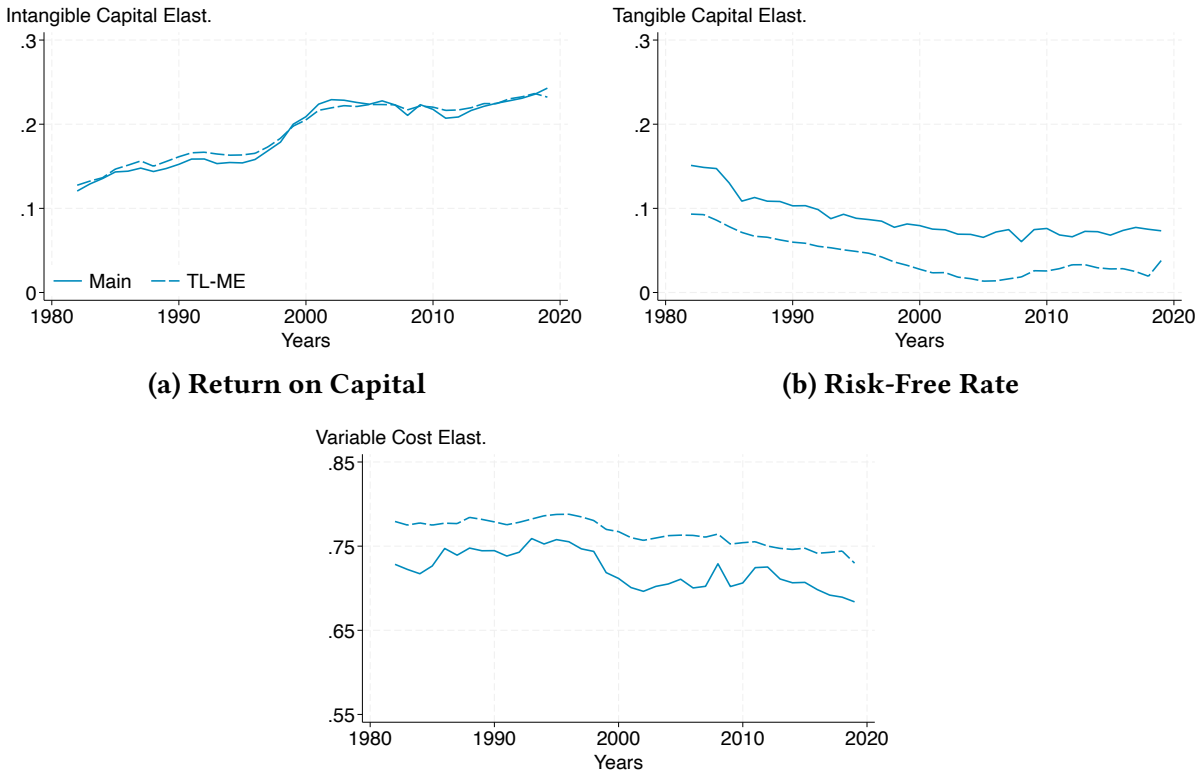
Note. Summary statistics of cleaned Compustat dataset between 1982 and 2019. All variables are in thousands of U.S.$.

For alternative production function estimation techniques, we employ a translog production function that allows for non-unitary elasticity of substitution between inputs, an aspect recently emphasized by Oberfield and Raval (2021) and Aum and Shin (2024) in the presence of both tangible and intangible capital. To ensure identification in estimating gross output production functions and account for potential mismeasurment in capital inputs, we impose constant returns to scale, following Flynn, Traina and Gandhi (2019), and incorporate capital-specific measurement error as in Collard-Wexler and De Loecker (2021). For alternative measures of markups, we apply the accounting profit approach, following Baqaee and Farhi (2020), where markups are calculated as the ratio of sales to total costs, encompassing both variable and fixed overhead costs in our model.

Figures C.3a, C.3b, and C.3c illustrate the evolution of elasticities for intangible capital, tangible capital, and variable costs across different production function estimation approaches. Figure C.4a presents the evolution of the various markup measures. Overall, we find that the elasticities of alternative production functions across inputs are broadly consistent, exhibiting similar trends despite some level differences. A similar conclusion holds for the different markup estimates, which display comparable trends over time, albeit with some differences in levels, as already noted in the literature (e.g., De Ridder et al., 2022).

To further assess the similarity across different markup measures and move beyond simple mean comparisons, Tables C.2 and C.2 present the distribution of markups for the various alternative measures, along with their correlations. We find that the different markup measures exhibit surprisingly similar distributions across several moments and are highly and positively correlated. This is not entirely unexpected, as while there is some disagreement

Figure C.3: Alternative Production Functions

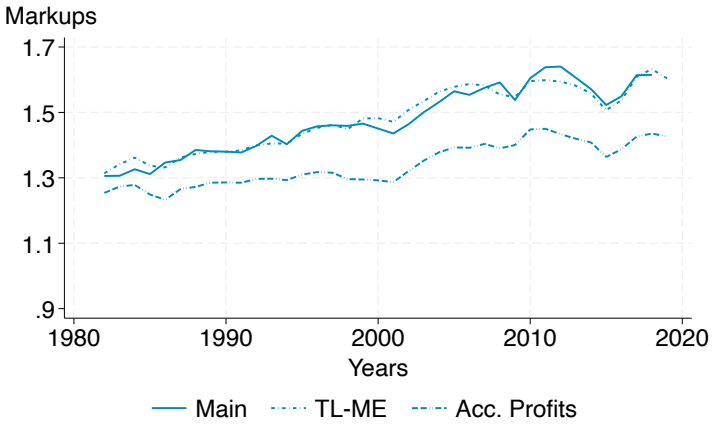


(a) Return on Capital **(b) Risk-Free Rate**

(c) Risk-Free Rate

Note. Figures C.3a, C.3b, and C.3c show the evolution of elasticities for intangible capital, tangible capital, and variable costs across different production function estimation approaches.

Figure C.4: Alternative Markups Measures



(a) Risk-Free Rate

Note. Figure C.4a presents the evolution of the various alternative markup measures. Markups are presented as simple averages.

regarding the preferred method for estimating markups, there is broad consensus that most alternative measures display a positive trend and share comparable properties (e.g., Syverson, 2024). In conclusion, since the different measures exhibit very similar trends and only slight variations in their initial levels—differences that are not central in our methodology that focuses on trends relative to initial year—we conclude that neither the choice of markup measures nor the production function estimation methods significantly impact our results.

Table C.2: Summary Statistics and Correlations for Alternative Markup Estimates

	Mean	p10	p50	p90	SD
Main	1.48	0.85	1.19	2.23	1.03
TL	1.34	0.90	1.18	1.90	0.62
A. Profits	1.33	0.98	1.22	1.87	0.49

	Main	TL	A. Profits
Main	1.00		
TL	0.95	1.00	
A. Profits	0.67	0.74	1.00

C.3 Additional Validations and Measurements of Risk Premium

Here, we present additional validations of our firm-level risk premium measure, demonstrating that it aligns well with standard theoretical predictions. We then show that alternative measures of firm-level risk premia exhibit a similar decline over time, supporting the robustness of our conclusions.

Validations. To validate our estimates of firm-level risk premia, we examine its correlation with firm-level equity returns, tangible and intangible capital used in production, and the ratio of tangible and intangible capital to variable costs. Theory predicts that firms with higher risk premia on capital should exhibit higher equity returns (David, Schmid and Zeke, 2022), lower levels of tangible and intangible capital due to a higher user cost of capital, and a lower ratio of tangible and intangible capital to variable costs, as risk premia affect the user cost of capital inputs but not of variable inputs.

Table C.3 presents the correlations between our firm-level risk premium on capital and firm-level equity returns, tangible and intangible capital used in production, as well as the ratio of tangible and intangible capital to variable costs. We find that all correlations, except one, are statistically significant, and all align with standard theoretical predictions.

To further validate our measure, we test an additional prediction highlighted by David, Schmid and Zeke (2022). Sectors with more dispersed risk premia are expected to exhibit higher proxies of misallocation, given by greater dispersion in revenue-based marginal prod-

Table C.3: Correlations Between Risk Premia and Other Firm-Level Variables

Dependent Variable	r^e (1)	k^T (2)	k^I (3)	k^T/ℓ (4)	k^I/ℓ (5)
Risk Premium, ζ	0.003***	-0.002***	-0.001	-0.011***	-0.012***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Fixed Effect					
Firm	✓	✓	✓	✓	✓
Sector × Year	✓	✓	✓	✓	✓
Controls					
Age^2	✓	✓	✓	✓	✓
Observations	87,749	87,749	87,749	87,506	87,402

Note. All dependent variables but the equity return are in logs. Standard errors are clustered at the industry-year level and reported in parentheses. *, **, and *** denote 10, 5, and 1% statistical significance respectively.

ucts of both types of capital and in revenue productivity, which is simply the geometric average of the two. This is the case because dispersion in risk premia implies dispersion in the user cost of both types of capital. Table C.4 tests and confirms these predictions in the data, showing that dispersion in risk premia is positively and statistically significantly associated with the dispersion of proxies for misallocation used in the literature. Overall, this evidence, along with the findings presented above, suggests that our measure captures many desirable properties of a good risk premium measure.

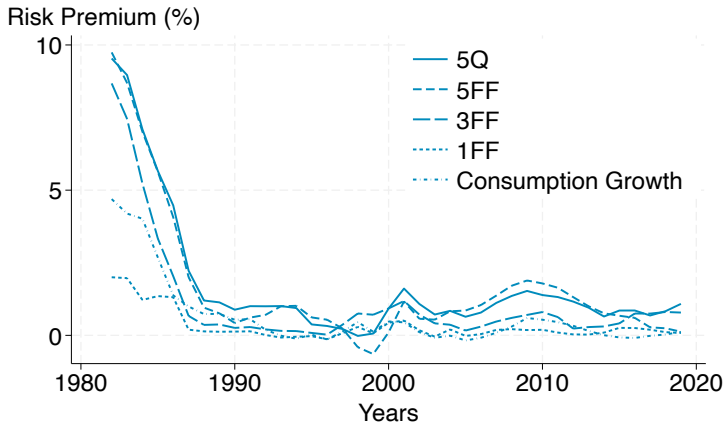
Table C.4: Risk Premia Dispersion and (Mis)Allocation

Dependent Variable	$\sigma(MRPK^T)$ (1)	$\sigma(MRPK^I)$ (2)	$\sigma(TFPR)$ (3)
Std. Risk Premium, $\sigma(\zeta)$	0.020***	0.013***	0.003**
	(0.004)	(0.003)	(0.001)
Fixed Effect			
Sector	✓	✓	✓
Year	✓	✓	✓
Observations	3,003	3,003	3,003

Note. Standard errors are clustered at the industry-year level and reported in parentheses. *, **, and *** denote 10, 5, and 1% statistical significance respectively.

Robustness. Here, we present the evolution of the risk premium obtained using alternative factor models commonly found in the literature, including the 5-factor, 3-factor, and

Figure C.5: Alternative Risk Premium Measures



Note. Figure C.5 illustrates the evolution of the average capital-weighted risk premium across various factor models, including the baseline model, the Fama-French 5-factor model, the Fama-French 3-factor model, the Fama-French 1-factor model, and the consumption CAPM.

1-factor models from Fama and French (2023), as well as the consumption CAPM. Figure C.5 illustrates the evolution of the risk premium derived from these different factor models. Overall, they exhibit a quantitatively similar trend in the average capital-weighted risk premium over time compared to the model proposed by Hou, Xue and Zhang (2015), suggesting that the specific asset pricing model employed does not significantly influence the trend of the risk premium.

Additionally, to further validate the robustness of our results, we demonstrate in Table C.5 that the various firm-level risk premia derived from different asset pricing models are positively and highly correlated. This suggests that, despite some differences, they all capture a similar source of underlying firm risk, beyond just their average evolutionary trends.

Table C.5: Correlation Matrix for Risk Premiums

	MAIN	5FF	3FF	1FF	CCAPM
MAIN	1.00				
5FF	0.83	1.00			
2FF	0.70	0.69	1.00		
1FF	0.46	0.39	0.51	1.00	
CCAPM	0.52	0.51	0.57	0.50	1.00

C.4 Additional Validations of Capital-Specific Wedges

Here, we demonstrate that our estimated capital-specific wedges correlate with proxies for adjustment costs and financial frictions. While measuring these directly in the data is known to be challenging, we adopt popular proxies from the literature. For adjustment costs, we adopt some of the identifying moments emphasized by Cooper and Haltiwanger (2006). For financial frictions, we use the popular proxies introduced by Hoberg and Maksimovic (2015).

Figure C.6 shows the correlation between capital-specific wedges (in absolute terms) and sector-level proxies for adjustment costs. We measure capital-specific wedges in absolute terms since adjustment costs impact the deviation of firm-level wedges from zero. Sectors are classified as having high adjustment costs if they exhibit above-median "spike rates"—investment rates exceeding 20 percent in absolute term—or a high standard deviation of investment, as these are key moments related to adjustment costs, as highlighted by Cooper and Haltiwanger (2006). We find that sectors with higher proxy-adjustment costs show greater deviations in capital-specific wedges from zero, as predicted by theory, and all relationships are statistically significant.

Table C.6: Adjustment Costs and Capital Wedges

Dependent Variable	$\lvert\tau^T\rvert$ (1)	$\lvert\tau^T\rvert$ (2)	$\lvert\tau^T\rvert$ (3)	$\lvert\tau^T\rvert$ (4)	$\lvert\tau^I\rvert$ (5)	$\lvert\tau^I\rvert$ (6)	$\lvert\tau^I\rvert$ (7)	$\lvert\tau^I\rvert$ (8)
High spike, k^T	0.167***		0.169***					
	(0.003)		(0.003)					
High std, k^T		0.045***		0.047***				
		(0.003)		(0.003)				
High spike, k^I					0.046***		0.042***	
					(0.006)		(0.006)	
High std, k^I						0.117***		0.121***
						(0.006)		(0.006)
Fixed Effects								
Year			✓	✓			✓	✓
Observations	88,142	88,142	88,142	88,142	88,143	88,143	88,143	88,143

Note. Standard errors are reported in parentheses. *, **, and *** denote 10, 5, and 1% statistical significance respectively.

Table C.7 presents the correlation between proxies of financial frictions and capital-specific wedges. These financial friction proxies, introduced by Hoberg and Maksimovic (2015), can be summarized as follows: i) Delay: Higher values indicate firms similar to those likely to delay investments due to liquidity issues; 2) Equity Delay: Higher values indicate firms likely

to delay investments due to liquidity concerns and are planning to issue equity, presumably to alleviate these issues; 3) Debt Delay: Similar to Equity Delay, but the firm indicates plans to issue debt to address liquidity problems; and 4) Private Delay: Analogous to the above, but the firm indicates plans to issue private placements to resolve liquidity challenges. Overall, we find that either these proxies show no significant correlation with capital-specific wedges or that they are positively correlated and highly statistically significant, suggesting that some form of financing friction may be influencing capital-specific wedges in the data.

Table C.7: Correlation of Capital-Specific Wedges with Financing Constraints

Dependent Variable	τ^I	τ^T
	(1)	(2)
Delay	0.069	0.035
	(0.105)	(0.044)
Equity Delay	-0.052	-0.046
	(0.133)	(0.055)
Debt Delay	0.241***	0.178***
	(0.084)	(0.032)
Private Delay	-0.016	-0.054
	(0.091)	(0.037)
Constant	0.504***	0.190***
	(0.003)	(0.001)
Observations	46,374	46,275
Fixed Effects		
Firm	✓	✓
Year	✓	✓

Note. Standard errors are reported in parentheses. *, **, and *** denote 10, 5, and 1% statistical significance respectively.

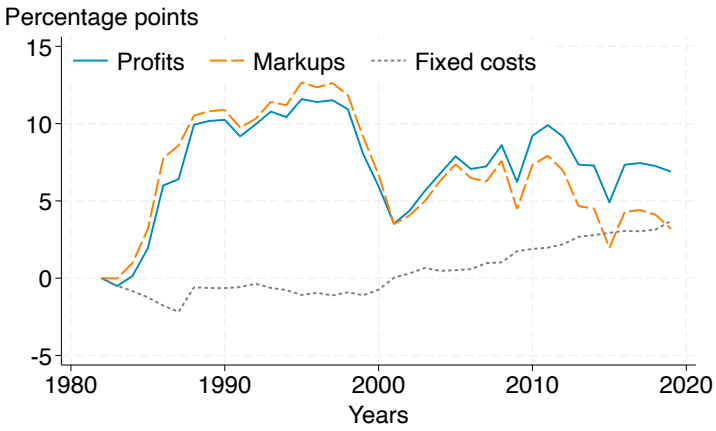
In conclusion, our findings show that the estimated capital-specific wedges are associated with well-known proxies for adjustment costs and financial frictions. This suggests that these wedges largely capture capital frictions, beyond the measurement error addressed in the main analysis.

D Further Results and Robustness on the Micro-Anatomy of Return on Capital and Risk-Free Rate Divergence

D.1 Further Results on Main Decomposition

Here, we break down the evolution of profits to highlight the roles of markups and fixed costs. Figure D.6 illustrates this relationship, showing that profits' upward trend is both driven by markups and fixed costs.

Figure D.6: Profits, Markups, and Fixed Costs



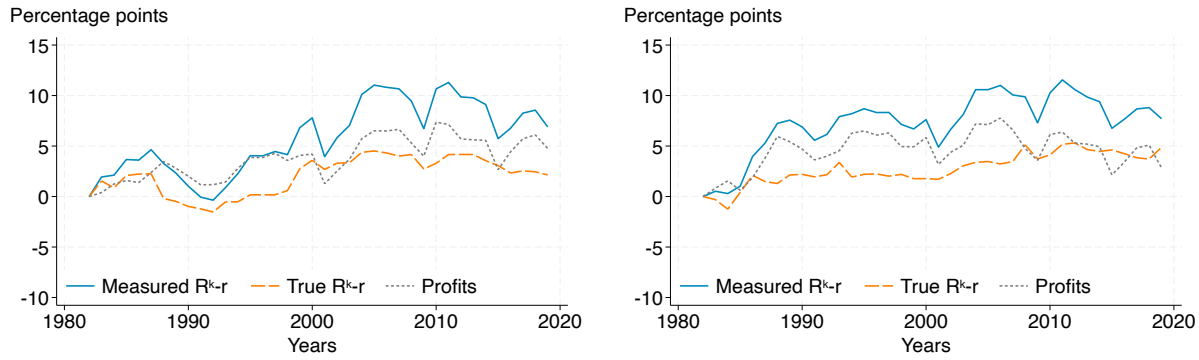
Note. Figure D.6 displays the progression of profits over time, breaking down the overall profit evolution into contributions from markups and fixed costs.

D.2 Robustness Exercises of Section 5.1

Here we show that using alterative markups measures and production function estimation techniques as outlines in Appendix C.2, yields very similar insights to those outlined in the main text.

Figures D.7a and D.7b show the decomposition of the measured return on capital into the profit rate and the true return on capital for different robustness exercises. In Figures D.7a, we retain the cost-share approach to estimate the production function while measuring markups using the accounting approach. In D.7b, we estimate a translog production function with measurement error. Both figures show that profit rates played a quantitatively important role in the divergence between the measured return on capital and the risk-free rate, although not enough to close the the gap between the true return on capital and the risk free rate, as in the

Figure D.7: Robustness Figure 7

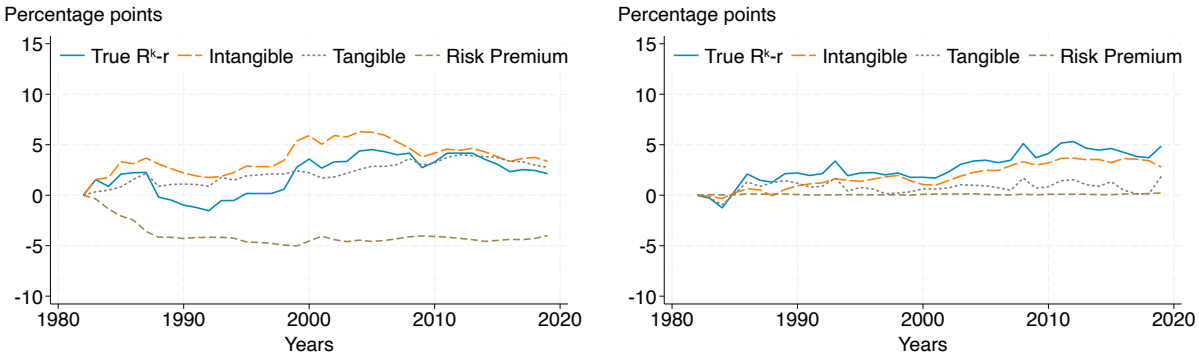


(a) Cost Shares and Accounting Markups **(b) Translog Production Function**

Note. Figures D.7a and D.7b show the decomposition of the measured return on capital into the profit rate and the true return on capital for different robustness exercises. In Figures D.7a, we retain the cost-share approach to estimate the production function while measuring markups using the accounting approach. In D.7b, we estimate a translog production function with measurement error, as outlined in Appendix C.2.

the main text.

Figure D.8: Robustness Figure 8



(a) Cost Shares and Accounting Markups **(b) Translog Production Function**

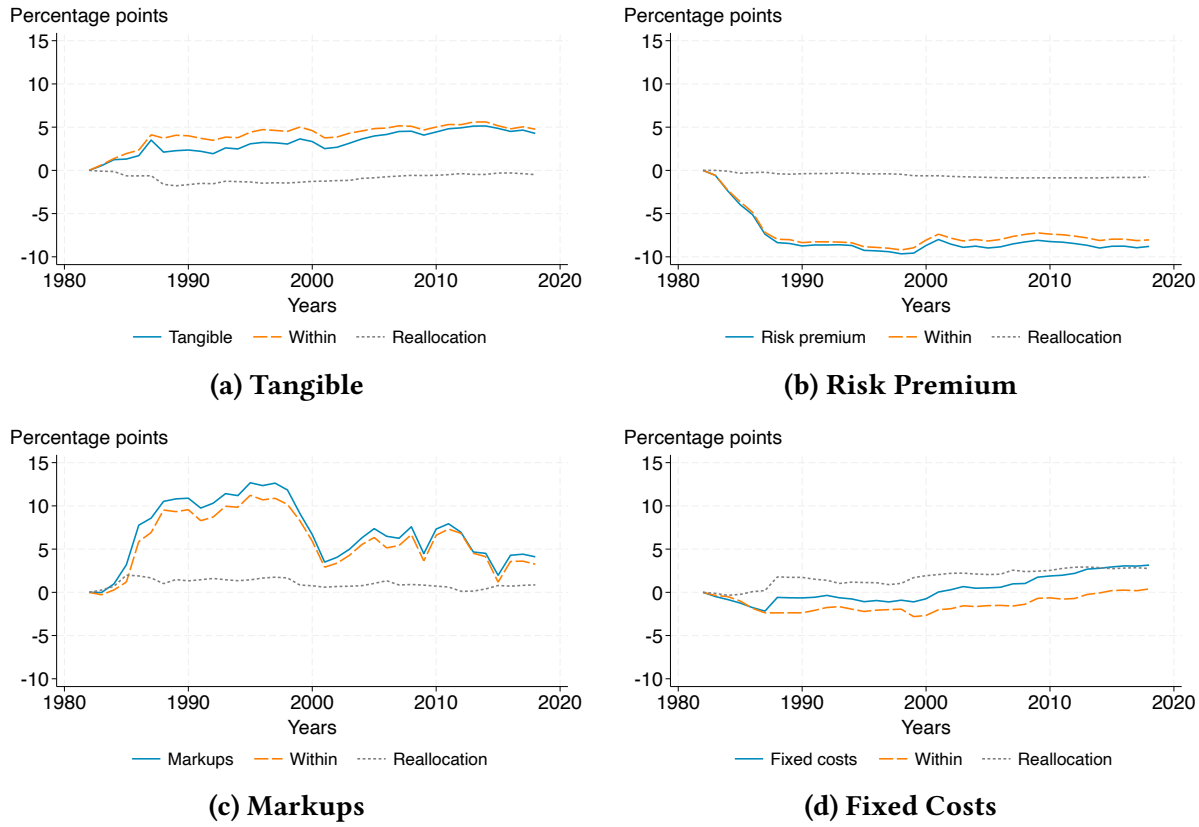
Note. Figures D.8a and D.8b show the decomposition of the true return on capital into the risk premium and the capital-specific wedges for different robustness exercises. In Figures D.8a, we retain the cost-share approach to estimate the production function while measuring markups using the accounting approach. In D.8b, we estimate a translog production function with measurement error, as outlined in Appendix C.2.

Figures D.8a and D.8b show the decomposition of the true return on capital into the risk premium and the capital-specific wedges for different robustness exercises. In Figures D.8a, we retain the cost-share approach to estimate the production function while measuring markups using the accounting approach. In D.8b, we estimate a translog production function with measurement error. Both figures show that intangible capital wedges played a quantitatively

dominant role in the divergence between the true return on capital and the risk-free rate, as in the the main text. Finally, for brevity, we do not report additional robustness tests for the baseline approach with alternative risk premium estimates. Figure C.5 in Appendix C.3 illustrates the evolution of the capital-weighted risk premium across the various asset pricing models used in our robustness analysis. All models display either a declining or stable trajectory over time. Since these risk premiums serve as shifting factors for both capital wedges in the decomposition, they do not alter the main finding: intangible capital wedges remain the primary quantitative driver.

D.3 Additional Findings of Sectoral Decomposition

Figure D.9: Tangible Wedge, Risk Premium, Markups, and Fixed Costs Sectoral Decomposition



(a) Tangible **(b) Risk Premium**

(c) Markups **(d) Fixed Costs**

Note. Figures D.9a, D.9b, D.9c, and D.9d show the sectoral decomposition of the tangible capital wedge, risk premium, markups, and fixed costs.

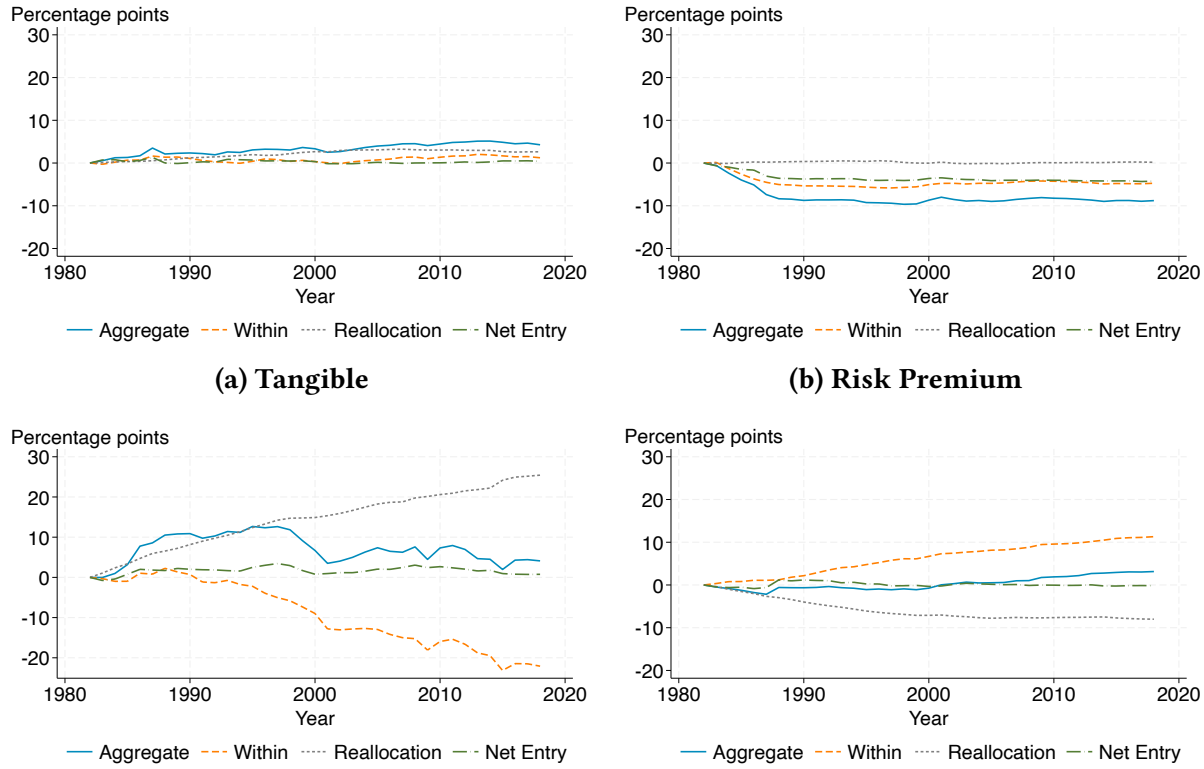
Figures D.9a, D.9b, D.9c, and D.9d illustrate the sectoral decomposition of the tangible capital wedge, risk premium, markups, and fixed costs. Overall, we find that the Within component

of the sector-level decomposition is the primary driver of the evolution of these factors over time. This is consistent with the main text findings for the intangible capital wedge, which also highlights the main quantitative role of the Within component in driving overall changes.

D.4 Additional Findings of Firm-Level Decomposition

Firm-level decomposition of other variables. In the main text, we presented the firm-level decomposition of the intangible capital wedge. Here, we extend this analysis to provide a firm-level decomposition for other variables in our model: the tangible capital wedge, the risk premium, markups, and fixed costs.

Figure D.10: Tangible Wedge, Risk Premium, Markups, and Fixed Costs Firm-Level Decomposition



(a) Tangible **(b) Risk Premium**

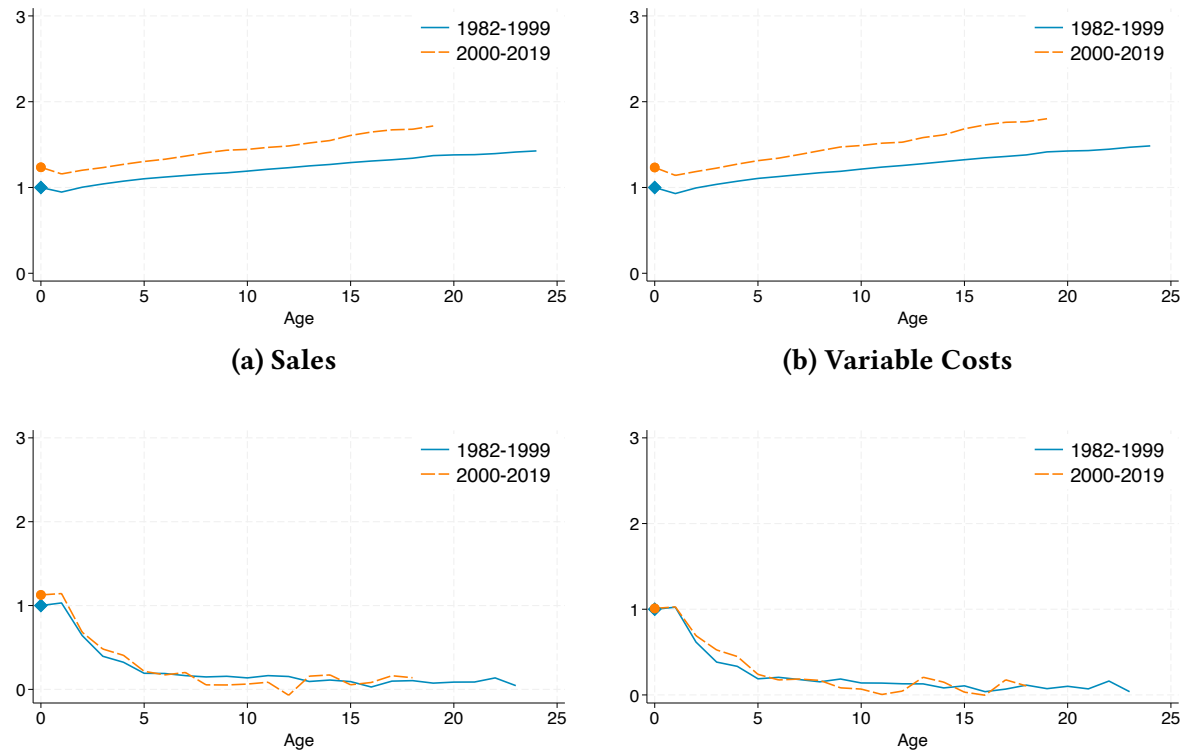
(c) Markups **(d) Fixed Costs**

Note. Figures D.10a, D.10b, D.10c, and D.10d show the firm-level decomposition of the tangible capital wedge, risk premium, markups, and fixed costs.

Figures D.10a, D.10b, D.10c, and D.10d present the firm-level decomposition of the tangible capital wedge, risk premium, markups, and fixed costs, respectively. Several insights emerge. First, the increase in the tangible capital wedge is primarily driven by the Realloca-

tion component. Second, the decline in the risk premium results from a mix of the Within and Net Entry components, suggesting that while large U.S. firms are generally becoming safer, cohort-based changes are also favoring safer firms overall. Third, the rise in markups is entirely attributable to the Reallocation component, aligning with existing literature. Finally, the increase in fixed costs is driven by the Within component.

Figure D.11: Changes in Cohort Dynamics Over Time



(a) Sales **(b) Variable Costs**

(c) Sales **(d) Variable Costs**

Note. Figures D.11a, D.11b, D.11c, and D.11d illustrate for the first and second halves of our sample period the life-cycle evolution of sales and variable costs in logs. The solid blue line shows the life-cycle evolution of each component before 2000, while the long-dashed orange line reflects their evolution from 2000 onward. Each line represents the average across firms, with all values normalized to the first cohort's initial entry observation.

Comparison with the literature. In the main text, we show that different cohorts exhibit varying levels of intangible intensity, especially at younger ages. Here, we further demonstrate that while these cohorts differ in intangible intensity early on, they are remarkably consistent in terms of sales levels, variable costs, and growth rates. Figures D.11a, D.11b, D.11c, and D.11d reveal that both sales and variable costs follow closely aligned trajectories across cohorts, with only minor deviations, as discussed in prior literature. Additionally, these

figures indicate that the growth trajectories across cohorts are nearly identical for both variables. These results align with findings from Karahan, Pugsley and Şahin (2024) and Hopenhayn, Neira and Singhania (2022), which indicate that the firm life cycle has remained relatively stable across cohorts.