Business Closures and the Deepening of Labor Market Inequalities *

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Abstract

We study how business closure orders in 2020 affected labor demand and wages of workers of different age, skill and nationality in the US, and highlight key empirical facts between 2018 and 2021. The composition of working age population in the CPS data shows that only 16% of individuals are immigrants, 23% are young and slightly less than 40% are high-skilled (i.e., with at least a bachelor's degree). From this data, we observe that hourly earning inequalities were evident over the entire period from 2018 to 2021. Middle-aged foreign workers earned lower wages than native workers with similar skills and age. The average hourly earnings of young, low-skilled foreign workers are less than those of native workers with the same characteristics. Remarkably, young, high-skilled foreign workers earn more than their native counterparts. We develop a macroeconomic model with heterogeneous labor to illustrate the distributional general equilibrium effects of business restrictions on workers of different types. We estimate the average of the elasticity of substitution between different categories of workers over time and across industries using CPS data on wages and hours worked. Business closure order is modelled as a negative productivity shock, and estimated using industry-level GDP, hours worked and the share of workers affected by the business restrictions. Qualitatively, our model results indicate that the business-closure orders disproportionately affected the most vulnerable workers. Young, low-skilled foreign and middle-aged native workers experience the largest reduction in total hours worked. In terms of wages, middle-aged, low-skilled native workers face the most significant decrease.

Keywords: productivity shock, labor inequalities, heterogeneous labor, business-closure orders.
JEL Codes: E20, E24, J01.

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1 Introduction

The COVID-19 outbreak significantly impacted the U.S. labor market. Total hours worked in the nonfarm business sector dropped sharply in 2020, after several states ordered citizens to stay home and imposed a shutdown of nonessential businesses. The enormous decline in hours and the curtailment of economic activity during the COVID-19 lockdown might have exacerbated the pre-existing labor inequalities, which appeared as one of the most worrying trends, particularly among the most vulnerable workers (Yasenov, 2020).[1] Adams-Prassl et al. (2020) reached a similar conclusion, showing that the pandemic hit the fragile segments of the labor force harder. Given the scale of the crisis, certain groups of workers, such as young, low-skilled, and non-native workers, would seem likely to lag far behind other segments in how fully they would recover from the crisis. Quantifying the labor-market implications of business-closure orders requires a framework that captures the distributional effect of this shock on hours worked and wages, as well as its repercussions for different types of workers.

In this paper, we study the effects of the business-closure orders shock on the U.S. labor markets. In particular, we look at the responses in terms of labor demand and hourly earnings. To illustrate the distributional general equilibrium effects of business restrictions, we develop a model of labor demand with nested CES structure to capture different substitutability between native and foreign, low and high skilled, and young and middle-aged workers.[2]

We start the analysis by documenting some key stylized facts about hourly earnings and hours worked during 2018-2021 using the Current Population Survey (CPS). There is a solid evidence about labor inequalities: as an example, a young, low-skilled foreign worker has an average hourly earning of 14.45$ (in 2021) lower than a native worker with the same characteristics (a native with the same characteristic earns on average 15.89$ per hour in 2021). Remarkably, in 2021, a young, high-skilled foreign worker earns on average 31.76$ per

[1] Data from the US Bureau of Labor Statistics confirms this. The U.S. second and third quarters' total hours (year-over-year) fell by 13.7% and 7%, respectively. The U.S. economy experienced a huge decrease in the gross value added of nonfarm businesses (11.86% lower in the second quarter of 2020 as compared to 2019).

[2] One can ask why we needed a general equilibrium model and did not simply try to causally identify the effect of business closures on different categories of workers, using standard reduced-form methods. One reason for conducting this study was to capture the distributional general equilibrium effects of productivity shocks and business-closure orders on US labor markets. The impact of the pandemic on workers was unequally distributed, and many vulnerable segments of the labor force were more exposed than others to the business-closure order. We attempt to propose a model that could generate distributional effects consistent with this reality. The use of the model with heterogeneous labor, in which household decisions are age-dependent, is convenient for our analysis since the age category of the worker is an important feature of heterogeneity in the model.

hour more than her native counterpart. Throughout this same period, middle-aged foreign workers earned lower wages than native workers of similar skills and age. The crisis that entailed the interruption or closure of nonessential businesses reduced the average weekly hours worked substantially across all worker groups. In line with this evidence, many workers during the pandemic outbreak reported greater inability to work. This was specifically the case for middle-aged, low-skilled foreign workers. Further, the post-pandemic episode was characterized by a downward pressure on hours worked, declining by 4 hours per week, and a strong recovery of hourly wages, increasing by 12%.

The specifics of the theoretical model are as follows. Representative firm within sector i is a price-taker, use capital and labor to produce output which is determined according to a CES production function. The CES production formulation captures the notion of imperfect substitutability between different labor inputs. Labor is disaggregated according to several criteria, including age, skills, and nationality. These key elements enable the model to generate a differential effect of the productivity shock across workers. In terms of group partition, we differentiate between young (15-30) and middle-aged (30-64) workers. We define foreign workers on the basis of the nationality and identify highly skilled workers as those who held at least a bachelor's degree. Each household consumes, saves and allocates units of time on labor conditional on being employed. Business closures is specified as an exogenous negative productivity shock, so that productivity responds proactively to business closure orders. This is estimated using industry-level GDP, hours worked, and share of workers affected by the restrictions at the industry level. For this we use the Current Population Survey (CPS), the Occupational Information Network (ONET) and the Bureau of Economic Analysis (BEA) data over the period 2018-2021 (pre- and post-COVID-19 crisis). Similarly, we use the CPS data on wages and hours worked to estimate the elasticity of substitution between types of labor. We use both time and industry variation to identify the elasticity parameters and also focus on the period 2018-2021. We use a simple labor demand function as in (Ottaviano and Peri, 2012) to estimate the elasticity of substitution between workers. In our model, the degree of substitutability or complementarity between workers shape wages and labor demand. Specifically, the demand for labor is sensitive to workers characteristics (age, education, nationality): firms draw on these elements when hiring workers and allocating labor for production. Our own elasticity of substitution estimates range between 0.05 and 0.19. After providing new estimates of labor-labor elasticities, we use the model to undertake

a counterfactual where firms face a negative productivity shock. Through the lens of our model, a fall in productivity during the pandemic crisis led to a heterogeneous impact on hours worked and hourly earnings.

Our model distinguishes between hours worked and the unemployment rate. It predicts that a negative productivity shock (a 1 percent decrease) leads to a dramatic decline in labor demand which varies between 2.45 and 0.67 percent. Most importantly, we observe greater responsiveness of hours worked to a negative productivity shock among middle-aged workers, who experienced the largest decline in hours worked (a decrease ranging from 2.45 to 1.11 percent), as well as among foreign workers (a decrease ranging from 2.45 to 1.34 percent). We find that among native workers, middle-aged, low-skilled workers were the most disadvantaged, experiencing the largest decrease in total hours worked (a decrease by 2.12 percent). Furthermore, we show that within the foreign workers population the response of hours worked by middle-aged, high-skilled workers to a decline in productivity was quantitatively the largest (a decrease by 2.45). It is relevant to mention that using the elasticity of substitution estimates from (Ottaviano and Peri, 2012), the response of hours worked is consistent with the results in the baseline model albeit the effect is amplified. In general, it appears that this shock would continue to affect middle-aged workers most (a decrease ranging from 4.70 to 4.10 percent), and still leads to a substantial reduction in foreign workers' hours worked (a decrease ranging from 4.70 to 1.56 percent). In any case, this exercise shows that age, nationality, and education are key in generating differences in hours-worked responses.

Considering the wage responses to the productivity shock, we show that all types of workers experienced a drop in wage rates in response to the negative productivity shock, and the effects range between 3.49 and 4.82 percent. Age makes a difference in terms of hourly earnings, with middle-aged workers appearing to be the most affected (a decrease ranging from 4.39 to 4.82 percent). Further, middle-aged low-skilled native workers experienced the most significant reduction in nominal wages (a decrease by 4.82 percent). When we use the elasticity of substitution estimates from (Ottaviano and Peri, 2012), the results underestimate the impact on young workers but remain robust for middled-aged workers (the negative impact ranging from 2.62 to 5.25). We find that middle-aged workers' wages fall substantially relative to young workers (a decrease ranging from 4.35 to 5.25 percent).

Finally, we test the validity of our results under different model specifications: in partic-

ular, with inelastic labor supply, using a production function without physical capital, and through a model with a fiscal side. In terms of welfare evaluation, we analyze the implied difference in welfare for different groups of workers and under government programs such as unemployment benefits. Following a negative shock to productivity and the expansion of unemployment benefits, we show that all workers are better off when they receive unemployment benefits. In particular, among young workers, high-skilled workers gain the most from the extension of unemployment benefits. Within the native population, middle-aged, low-skilled workers gained the most. That said, unemployment insurance generosity mitigates significant welfare loss during economic recessions.

This paper relates, first, to the literature on the effects of COVID-19 on the labor markets, showing that the disparity between workers' reduced hours of work and employment was mostly due to the lower flexibility of jobs (Borjas and Cassidy, 2020; Yasenov, 2020) and the higher spread of the virus among less "remotable" jobs (Basso et al., 2022).[3] Recent important contributions to this literature include Cajner et al. (2020), Cortes and Forsythe (2023), Cortes and Forsythe (2020), Adams-Prassl et al. (2020), and Leyva and Urrutia (2022). While the literature on the distributional effects of COVID-19 on labor markets mainly focuses on some segments of the labor force, we propose instead a model with heterogeneous labor, to estimate and understand the impact of the productivity shock on labor markets.

Our study follows the literature that adopted a structural approach to modeling heterogeneous labor. Our main reference was a recent paper by Busch et al. (2020), who employ an overlapping generations model to estimate the impact of refugees' flow to Germany between 2015 and 2016 on different categories of workers. In particular, the authors assess the gross wage responses to the immigration inflow.[4] Instead, we develop a model with heterogeneous workers to assess the effects of the business-closure orders on different types of workers, assuming heterogeneity by age, origin, and skill. We also augment the model with a business-closure shock to understand how the adoption of these orders in the US during the crisis

[3] Fasani and Mazza (2023) used European data and found that extra-EU migrants were more likely than natives to be exposed to a high risk of losing their job, but only within industry and occupation. Work identified as *essential occupations* employed many foreign workers.

[4] See also Imrohoğlu et al. (2017), who focuses on how immigration policy based on attracting highly skilled workers can mitigate Japan's fiscal-imbalances problem. Storesletten (2000) employed an overlapping generations model to examine whether a selective immigration policy, based on skills and age, can have strong quantitative implications for U.S. public finance. He showed that an increased inflow of middle-aged and high- and medium-skilled immigrants can sustain U.S. fiscal policy. Other papers adopting a structural approach to model immigration are Chassamboulli and Palivos (2014), Moreno-Galbis and Tritah (2016), and Ortega (2000), which focus on the effects of immigration on destination countries' labor markets.

contributed to the curtailment of economic activity and substantially affected the dynamics in wages and hours worked. Other contemporaneous papers that explain the post-pandemic dynamics in the labor market are Faccini et al. (2022), who attribute the dynamics in the labor market to the high propensity of workers searching for a new job; Peri and Zaiour (2023), who emphasize the fall in the number of foreign workers as the main driver of US labor market dynamics; Domash and Summers (2022), who attempt to explain the labor market tightness with the increase in several vacant jobs in the US; and Tuzemen (2022), who documents a decline in the participation rate, due to a large number of new retirees.[5] Our paper differs from the aforementioned studies, as our model can generate differential effects of business-closure orders on workers and suggests that business closures add to the existing disparities between workers.

Finally, we contribute to the literature that analyzes labor inequality,[6] and estimate the elasticity of substitution between types of labor. This growing literature includes Berger et al. (2022), who focus on the differential effects of minimum wage, as well as Engbom and Moser (2022) and Hurst et al. (2022). The active literature on the estimation of elasticity of substitution includes León-Ledesma et al. (2011), who employ a normalized multi-level production function; Borjas (2003), who uses a simple supply-demand framework; Alvarez-Cuadrado et al. (2018), who account for sectoral differences; and Gechert et al. (2022), who revisit the existing approach in the estimation of the elasticity of substitution. We add to this literature by providing estimates of labor-labor elasticity, crucial to understanding the distributional effects of business-closure orders on the labor market. Furthermore, the pandemic productivity shock mainly hit non-essential and less remotable industries that were more likely to have a high share of low-skilled, young, and non-native workers. The model we present in this paper takes a stance by including these key features. Thus, we view our work as partly related to past papers that analyzed the concentration of immigrants in specific sectors. Among these, Foged and Peri (2015) showed the concentration of migrants in some low-skill sectors with high-level manual and low-level communication content. Their entry into the labor market led low-skilled natives to shift toward nonmanual (i.e., communication-intensive) jobs that allowed for native upgrading and an increase in natives'

[5] See also Muehlemann and Leiser (2018), who show a positive association between labor market tightness and hiring costs using Swiss data.

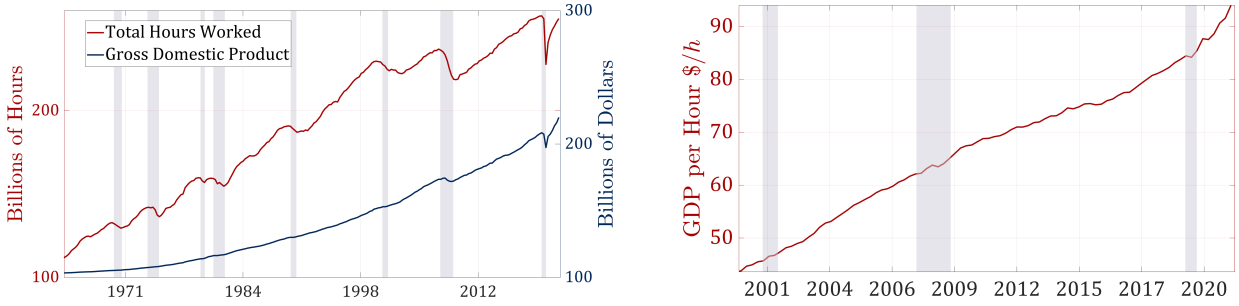
[6] See for example Card et al. (2018), who interrogate whether firm-specific productivity differentials drive differences in wages.

wages. In addition, Burstein et al. (2020) explored the labor impacts of immigration with worker heterogeneity in occupational productivity. They found that the effect varies across occupations, due to the concentration of high-skilled immigrant workers in computer-related jobs, and low-skilled immigrants in agriculture and manufacturing sectors.

Layout. The rest of the paper proceeds as follows. Section 2 presents US trends in productivity and labor inequalities and empirical evidence on the effects of the pandemics on labor market outcomes. Section 3 lays out the theoretical model. Section 4 describes the data and the estimation of model parameters. Section 5 discusses the effects of productivity shocks on hours worked and wages. Section 6 performs sensitivity tests and presents model extensions. Section 7 concludes the paper.

2 Labor Inequality and Business Closure: What Do the Data Tell Us?

We start our analysis by documenting the recent trends in labor inequality. Next, we examine how business-closure orders made labor inequality worse, with implications for production and the labor market that we explain later.



A. Hours and Ouput B. Labor Productivity

Figure 1: Labor Productivity, Hours Worked and Output Dynamics

Notes: Panel A depicts the dynamics in gross domestic product and hours worked between 1964 and 2021. Panel B displays the dynamics of labor productivity, measured as the ratio of gross domestic product to hours worked between 2000 and 2021. The shaded red areas are NBER defined recessions. Source: Federal Reserve Economic Data.

Productivity and Hours Worked. As the US economy emerged from the recent pandemic, the phases of this unprecedented crisis became worth examining. Figure 1 plots the gross domestic product and total hours worked in the USA from Q1-1964 to Q4-2021. Though the 2020 recession was the shortest, it drove down total hours. Indeed, total hours worked fell dramatically in the second quarter of 2020, after most states across the US shut down nonessential business to curb the spread of the virus. During this period, gross domestic product reached the bottom in the second quarter. Many states relaxed their restrictions on economic activities starting in the third quarter of that year. Then, the total hours worked increased, and following this trend, the gross domestic product rose rapidly, signaling economic recovery.

Panel B of Figure 1 showcases U.S. labor productivity amid the COVID-19 crisis. During the second quarter of 2020, labor productivity decreased slightly but recovered rapidly in the subsequent quarter. The very nature of the business-closure orders we considered was that they involved reduced working hours and lay-offs. The time series observations in Panel B suggest that US labor productivity did not experience a significant decline during the crisis. The decline in hours worked by less productive workers who may have dropped out of the labor force could have been the primary driver of the adjustment we observe. On the other hand, workers who kept their jobs and effectively participated in the labor force during the pandemic were likely the most productive, and their performance was highest during this period. Studies in line with this interpretation are Tuzemen (2022), who argues that the decline in the participation rate is attributable to older adults exiting the labor force, and Hobijn and Şahin (2022), who document a downward long-run trend in participation rate apparent before the COVID crisis.

This crisis draws attention to the impact of the reduction in productivity on the labor market. As Figure 2 shows, the impact of the crisis varied by sector, and this effect was likely due to the possibility of teleworking by sector and its *emergency* nature. For example, such sectors as Construction, Merchandise Stores, and Manufacturing, which require the physical presence of a high share of workers, were substantially affected by the crisis. Conversely, it had little or even no impact on production in the Data Processing sector, with its high share of workers who can work remotely.

Distribution of the Average Hourly Wage Before examining the distribution of wages across different types of workers, it is important to clarify that this analysis is not intended to assess

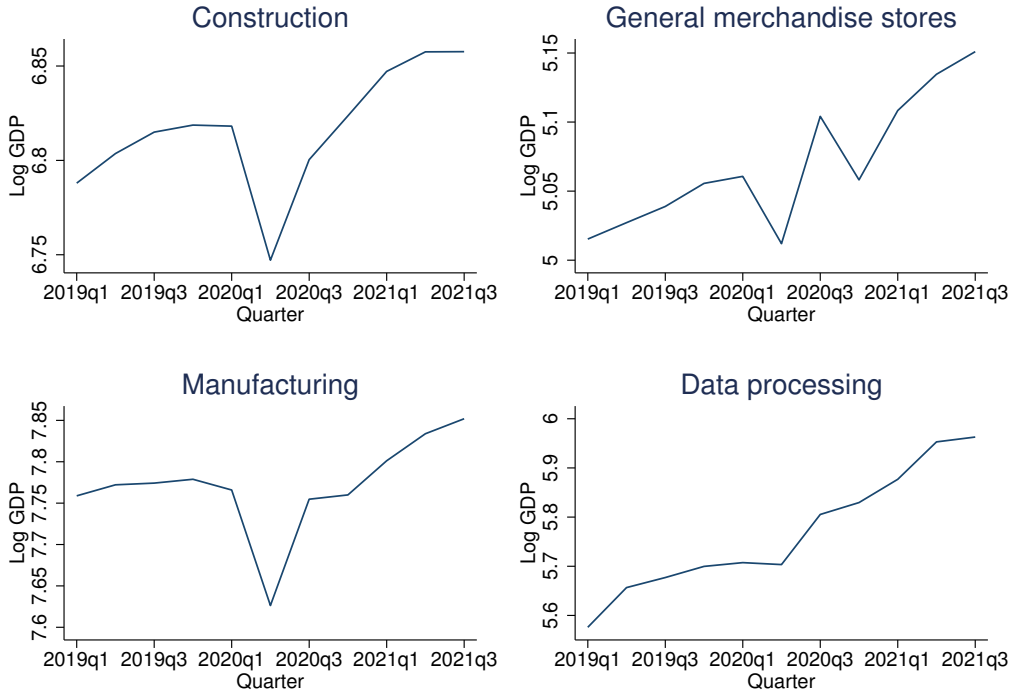


Figure 2: Log-GDP by Sector: Some Examples

Notes: The left-hand side panel describes the dynamics of log GDP in the construction sector (top) and the manufacturing sector (bottom). The right-hand side panel describes the dynamics of log GDP in the general merchandise stores sector (top) and the data processing sector (bottom). Time: 2019-2021. Source: Bureau of Economic Analysis.

the impact of the pandemic on wage inequality. Rather, we mean to underline the degree to which hourly earnings were unequally distributed. Specifically, we present the unequal distribution of hourly earnings over the whole period of 2018-2021, and the conclusions that Figure 3 suggests may not necessarily reflect the pandemic's effects.

Exploring CPS data from period 2018-2021 revealed the potential difference in the hourly earning distribution for different types of workers. The panels in Figure 3 show the average wage distribution of natives and foreign workers in four broad classes.[7] Two important facts emerge from these figures. Between 2018 and 2021, wages were much more unequally distributed between native and foreign workers in the middle-aged, low-skilled category (Figure 3, panel A). However, this divide between these two types of workers is less evident

[7] Note that observations in Figure 3 are a pure approximation of hourly earnings, in terms of labor income/hours worked. With such an approximation, we end up with workers who report hourly earnings below the minimum wage. We recognize the measurement errors in the CPS; fortunately, the fraction of observations that report an hourly wage under the minimum wage is very low.

in the other classes. We doubt that this difference (see Figure 3 panel A) is because foreign workers newly arriving in the labor force in the US may have faced difficulties integrating into the labor market. They may have seen their job experience unrecognized or evaluated differently and continued to earn less than their counterparts.

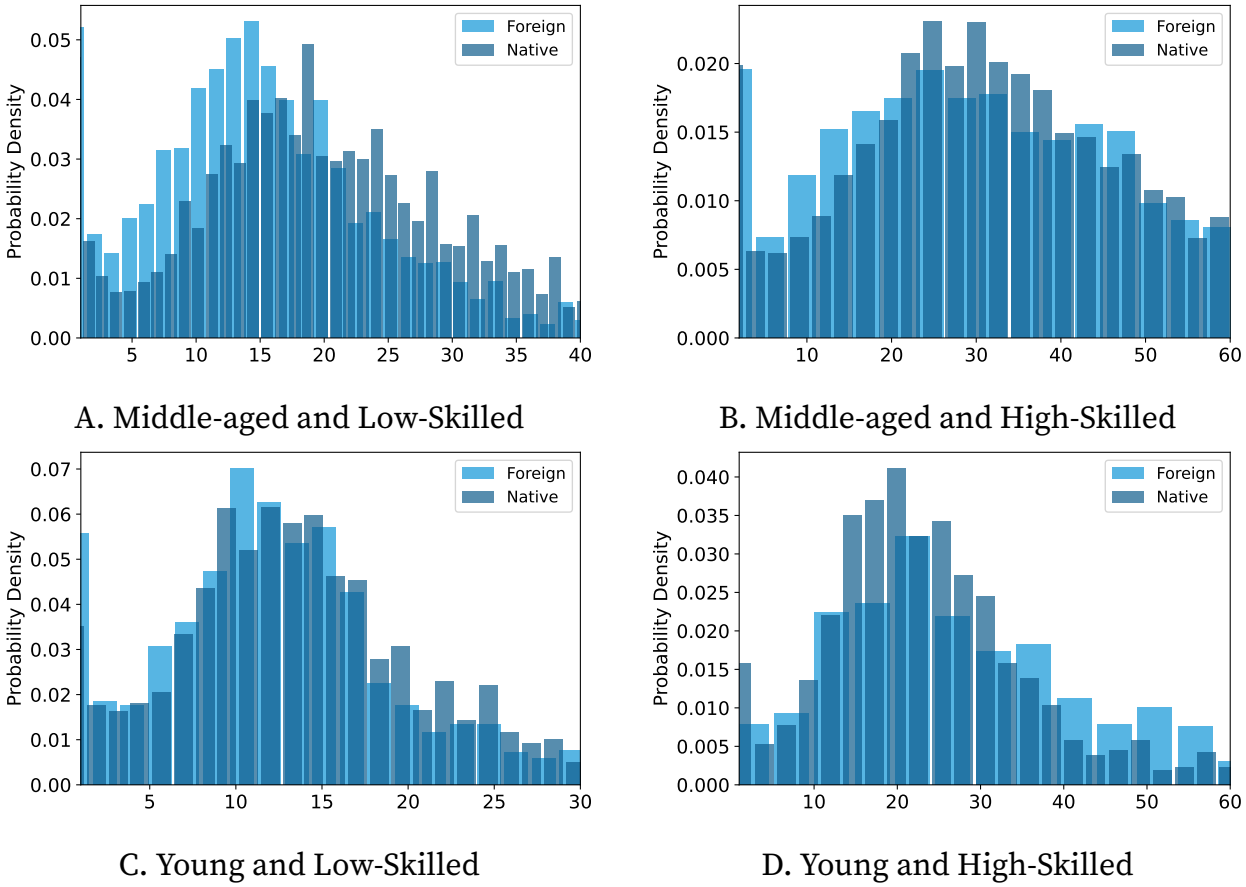


Figure 3: Distribution of Hourly Wages of Native and Foreign Workers across Age and Skill Levels

Notes: Average hourly earnings in USD. The top panels describe the distribution of hourly wages across native and foreign workers when they are middle-aged and low-skilled (left), and the same demographic high-skilled (right). The bottom panels present the distribution of hourly wages across native and foreign workers when they are young and low-skilled (left) and the same demographic whne workers are young and high-skilled (right). Time: 2018-2021. Source: Current Population Survey

Figure 3, Panel B reports a small difference in the distribution of average wage rates between natives and foreigners in the middle-aged, high-skilled category. A small shift into the upper echelons of the hourly earnings distribution appears among native workers. Figure 3, Panel C illustrates that the average wage rate of workers with less education lags further behind workers with more education. However, among young workers with lower educational

levels, there is no clear difference between native and foreign workers's hourly wage rate (Figure 3, Panel C).

Throughout the same period, Figure 3 Panel D indicates that the wage distribution of young, high-skilled workers skews to the right, reflecting that, on average, young workers were paid less than middle-aged workers with similar skills. Contrary to our expectations, we observe that the difference in earnings distribution between natives and foreigners when young and highly skilled is very small, and the hourly earnings of foreign workers only becomes dominant at the top of the distribution (Figure 3, Panel D).[8]

We rely on COVID-19-related questions in the CPS to understand how the pandemic outbreak affected workers' ability to work. We focus on the responses to the question: *Were you unable to work due to COVID-19?* recorded as *Yes* or *No*, from May 2020 to December 2021. Figure 4 reports differences in the answers to the question by respondent's age, skills, and origin. Interestingly, we find that the pandemic outbreak affected foreign workers much more than natives. We also show that middle-aged workers were more exposed to the pandemic than young workers, while low-skilled workers' ability to work was more significantly affected than that of high-skilled workers. These observations suggest that the pandemic more significantly impacted the most fragile segments of the US labor market, particularly, low-skilled, middle-aged, and foreign workers, than other worker categories. This evidence is in line with our thesis that COVID-19 hit workers differently, by age, skills, and origin.

Labor Inequalities: Suggestive Evidence. If we think of labor inequalities, we might consider the average wage rate and hours worked by type of worker. Table 1 provides an example that illustrates substantial differences between comparable workers' wages and hours worked.

Productivity shocks may contribute to labor inequalities between workers of different origins, skills, and ages. For instance, low-skilled workers performing nonremoteable jobs can be more vulnerable to government-mandated business closure. Temporarily closing nonessential businesses can lead to worsening the pre-existing disparities between workers with similar characteristics. Yet, in Table 1, we find that the average hours worked per week fell for all age and skill groups in 2020. A conceivable explanation for this variation may be

[8] Interested readers can consult the Online Appendix H, which contains the distribution of hourly earnings before and after the pandemic.

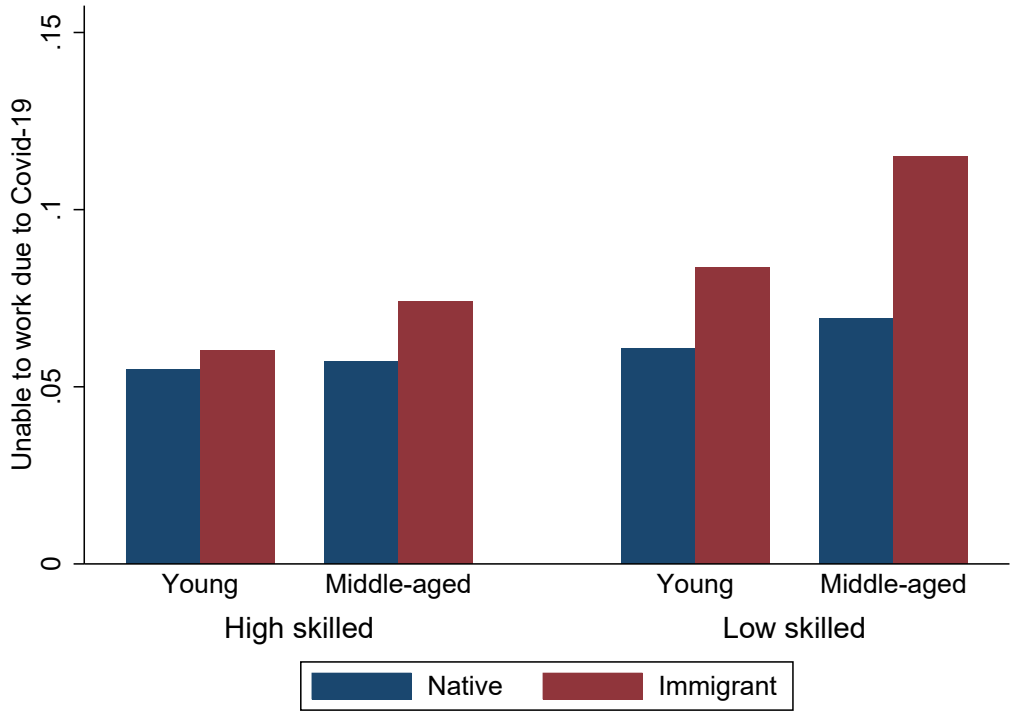


Figure 4: The effect of Covid-19 on the ability to work

Notes: The Figure shows the answers to the question "Were you unable to work due to COVID-19?"from the CPS between May 2020 and December 2021.

the restrictions on economic activities during the pandemic outbreak. We also see a large difference between average hours worked per week by type of worker, differences among workers that are also intriguing in how they exacerbate labor disparities. As the table shows, we could attribute the decline in average hours worked to the reduced working hours of workers who remained employed, as well as those who lost their jobs during the pandemic.[9] In Table 1, we report the average weekly wage rate from the CPS during 2018-2021. This table shows reliable evidence of an increase in hourly earnings inequality between native and immigrant workers within age and skill categories. Throughout the period, middle-aged foreign workers earned lower wages than native workers of similar skills and age. Also, a young, low-skilled foreign worker earned less than a comparable native worker. Perhaps surprisingly, the young immigrant with high-level skills earned more than the native.

We provide an intuition of the patterns of wages and hours worked during the years 2018-2021 in Table 1. Many plausible factors explain the discrepancy between native and foreign

[9] It is worth mentioning that the CPS has surveyed both employed and unemployed individuals.

Table 1: Weekly Hourly Wage and Hours Worked 2018-2021

	Hourly Wage				*Hours*			
	Low skill		High skill		Low skill		High skill	
	Immigrant	Native	Immigrant	Native	Immigrant	Native	Immigrant	Native
Young workers								
2018	14.47	15.65	31.65	24.76	35.37	34.23	38.58	38.54
	(0.04)	(0.03)	(0.02)	(0.02)	(0.02)	(0.03)	(0.02)	(0.02)
2019	14.43	15.74	31.78	24.79	35.42	34.20	38.57	38.56
	(0.02)	(0.04)	(0.04)	(0.03)	(0.02)	(0.03)	(0.02)	(0.01)
2020	14.45	15.78	**31.74**	24.90	**35.31**	**34.18**	**38.53**	**38.53**
	(0.02)	(0.04)	(0.03)	(0.02)	(0.02)	(0.03)	(0.02)	(0.02)
2021	14.45	15.89	31.76	24.93	35.33	34.23	38.57	38.54
	(0.02)	(0.05)	(0.03)	(0.04)	(0.03)	(0.04)	(0.01)	(0.02)
Middle aged workers								
2018	17.29	22.02	36.89	38.03	38.60	40.02	39.85	40.78
	(0.06)	(0.10)	(0.08)	(0.19)	(0.03)	(0.09)	(0.03)	(0.05)
2019	17.53	22.62	36.79	38.46	38.61	40.05	39.95	40.83
	(0.08)	(0.12)	(0.07)	(0.13)	(0.03)	(0.09)	(0.03)	(0.05)
2020	**17.42**	23.89	36.97	38.70	**38.36**	**39.31**	**39.77**	**40.53**
	(0.04)	(0.20)	(0.14)	(0.14)	(0.04)	(0.10)	(0.03)	(0.05)
2021	17.57	23.95	37.16	39.47	38.48	39.80	39.87	40.59
	(0.07)	(0.14)	(0.23)	(0.26)	(0.04)	(0.10)	(0.04)	(0.05)

Notes: The table describes the weekly hourly wages and hours worked across worker categories between 2018 and 2021. We report the mean for each year and group with the corresponding standard errors in parentheses. Source: Current Population Survey.

workers. One explanation is the concentration of immigrants in low-paying occupations or sectors. Also, there may have been barriers to getting into occupations that required professional licensing. Not only the concentration of immigrants in specific sectors may have been important in explaining low wages among these workers; those foreign workers may have been less informed about the market wages and, thus, could have accepted lower wages.

We are not claiming that all variations in hours worked stem from business-closure orders only. Numerous factors, including the collapse in international trade, the decline in participation rate due to workers leaving the labor force, the increase in the retired share of the US population, the decrease in immigration inflow, and the expansion of unemployment insurance claims, among others, may potentially explain labor market dynamics. Furthermore, a focus on the effects of business-closure orders is crucial in providing insight into post-pandemic dynamics in the US labor market. In terms of the quantitative relevance of this channel, several studies investigated business closures during the pandemic. For instance,

Bartik et al. (2020) decomposed the percentage change in hours, using high-frequency data, and showed that the decline in hours was mainly due to business shutdowns and lay-offs. These factors appear to have the same quantitative effect on total hours, whereas cuts in hours at firm level explained a minor share of the variation. In a different study, Chetty et al. (2020) found that re-openings had little impact on employment. Kong and Prinz (2020) indicated that nonessential business-closures explained only 6.4 % of unemployment insurance claims, using high-frequency Google search data.

Based on the empirical facts on hourly earnings and hours worked, we propose a model that not only accounts for labor heterogeneity but also considers the feedback from business closures to labor markets, through the productivity channel. Our macroeconomic model allows us to generate the differential impact of productivity shocks and business closures across workers with different education levels, age, and origins.

Empirical evidence from reduced form regressions We, first, examine the pre- and post-pandemics empirical evidence from least squares regressions of the outcomes (i.e., wages and hours worked) on workers' characteristics (i.e., age, immigration status, and skill level). Table 2 reports the results from these regressions.

Column 1 shows the associations between worker characteristics and yearly wage income, using individual-level data from the CPS (for years 2018-2021). Our estimates reveal that on average, workers enjoyed a 6.7 % nominal wage growth during the post-COVID period of 2021, in comparison to the pre-COVID period. While "postCovid-19" period is associated with an increase in yearly wages, the increase is smaller (though not significantly) for middle-aged and low-skilled workers and larger for middle-aged low-skilled immigrant workers by 14.7 %. This may potentially be interpreted as a strong recovery of labor income after most COVID-19 restrictions on businesses had been lifted.

Column 2 reports the associations between worker characteristics and hourly wages. We find that "postCovid-19" episode is associated with an increase in hourly wages by 12 % on average. While middle-aged workers report a smaller increase in wages of 11.94 %, middle-aged, low-skilled workers enjoyed an increase (though insignificant) of 12.06 %. Those workers who exited the labor market, who might have had lower wages, may have driven these associations. As a result, only those with higher wages remained in the labor markets. Moreover, since the data was registered yearly, our findings should be considered as an

Table 2: Least Squares Regression: Wages and Hours Worked

VARIABLES	(1) Log Wage	(2) Log Hourly Wage	(3) Hours usually worked per week at all jobs
Post Covid	0.067***	0.120***	-4.734**
	(0.020)	(0.024)	(2.062)
Middle-aged · Post Covid	-0.008	-0.051**	1.679
	(0.020)	(0.024)	(1.835)
Low skilled · Post Covid	0.007	-0.035	3.668*
	(0.022)	(0.029)	(2.106)
Low-skilled · Middle-aged · Post Covid	0.001	0.060**	-4.034*
	(0.023)	(0.030)	(2.173)
Immigrant · Post Covid	0.068	-0.061	6.043
	(0.068)	(0.060)	(5.196)
Middle-aged · Immigrant · Post Covid	-0.082	0.061	-3.728
	(0.065)	(0.058)	(5.981)
Low-skilled · Immigrant · Post Covid	-0.126	0.061	-10.836
	(0.087)	(0.078)	(7.823)
Low-skilled · Middle-aged · Immigrant · Post Covid	0.147*	-0.049	9.485
	(0.083)	(0.076)	(8.657)
Observations	189,941	189,927	2,695,063
Adjusted R-squared	0.395	0.241	0.384
Industry FE	Yes	Yes	Yes
County FE	Yes	Yes	Yes
Individual FE	Yes	Yes	Yes
Number of counties	281	281	281

Notes Standard error clustered at county level. Standard error in parentheses *p<.05; **p<.01; ***p<.001

average by year.

Finally, Column 3 shows the associations between worker characteristics and hours worked per week[10]. We find that hours worked decreased by 4 hours per week after COVID-19, a decrease less pronounced among low-skilled workers. This reduction seems more evident among immigrant workers and, especially, young, low-skilled immigrant workers. This suggestive evidence stresses the downward pressure on hours worked, which did not fully recover during the post-COVID period.

These post-COVID dynamics in wages and hours worked may reflect the decline in labor supply and the labor shortage, as Faberman et al. (2022) show. The US labor market has suffered from significant tightness, apparent in both an increase in labor demand and a shortfall in labor supply (Lee et al., 2023). As Hobijn and Şahin (2022) highlight, a decline in

[10] While the CPS only reports wages once per year in the ASEC supplement, it reports monthly data on hours worked. Therefore, the measurement of hours worked is more reliable than wages.

the population growth rate since the pandemic further restrained the labor supply. Hobijn and Şahin (2022) clarifies that the fall in the participation rate was evident even before the pandemic and argues that this decline in labor supply, coupled with a scarcity of workers, dampened job creation. An acute labor shortage may induce firms' eagerness to hire and increase wages, to attract prospective workers. Domash and Summers (2022) also discuss the issue of labor market tightness and skill shortage in the US, providing evidence that the US labor market was extremely tight, and its inflationary pressure was persistent. This mounting inflationary pressure on firms push them to adjust their price and margin and, eventually, pass through the wage cost to inflated prices.

3 A Model with Heterogeneous Labor

We start this section by presenting a simple model with heterogeneous labor supply, in which we adopt a nested constant elasticity of substitutions (CES) production function. The worker categorization in different types enables us to consider the differential effect of business-closure orders, that are treated as a negative shock on productivity. We allow for imperfect substitutability (i.e., elasticity of substitution different from one) among workers of a given age (young vs. middle-aged) and, later, between workers of different origin (native vs. foreign), and skills (high-skilled vs. low-skilled), to understand how pandemic-induced recession increased inequalities in the labor market. By construction the model does allow for distinguishing between employment responses, hours worked adjustments, and wage effects of the pandemic-induced economic recession.

Employment To explicitly consider extensive and intensive margins of labor supply responses, we assume that workers can be either employed E or unemployed U, and N represents the labor force, with $N = E+U$. The labor force is normalized to 1, and the employment rate is given by $e_t = 1 - u_t$. Further, the specification of total employment E_t in this economy is measured as man-hours quantity supplied and takes the form:

$$E_t = (1 - \mathcal{Q}) \sum_{s \in h,l} \sum_{k \in f,n} \sum_{j \in y,m,o} l^j_{s,k,t} + sU_t$$

where $l^j_{s,k,t}$ denotes the hours supplied by different types of workers and U_t denotes total unemployment. The term $1 - \mathcal{Q}$ governs how hours supplied are connected to total employment, representing the rate at which employed workers remain employed. We assume that unemployed workers become employed with probability s and employed workers become unemployed with probability $\mathcal{Q}$. From Equation 3, we have $E_t/N = e_t = (1 - \mathcal{Q}) \sum_{s \in h,l} \sum_{k \in f,n} \sum_{j \in y,m,o} l^j_{s,k,t}/N + s u_t$. Written this way, we treat both employment rate and unemployment rate as endogenous objects.

Households We consider three type of agents where $j \in \{y, m, o\}$ identifies age groups: young, middle, and old, each alive at any point in time and maximizing lifetime utility.

$$U(c^j_{s,k,t}, l^j_{s,k,t})$$

Let the instantaneous utility function $U(c)$ be a logarithmic function and $U(l)$ be a constant relative risk aversion $(l^j_t)^{1+\eta}/1 + \eta$, where η is the curvature on the disutility of labor. We assume that the parameter η is the same for all types of workers. We let s denote the skill of the agent with $s \in \{h, l\}$ indexes skill categories: high-skilled and low-skilled, and let k represent the origin of the agent with nationality $k \in \{f, n\}$ (foreign and native, respectively). Note that each agent faces a specific budget constraint, but the marginal utilities are the same across worker groups. Therefore, the budget constraint of each agent is:

$$c^j_{s,k,t} + a^j_{s,k,t} = e_t l^j_{s,k,t} \frac{\omega^j_{s,k,t}}{\pi_t} + a^j_{s,k,t-1}(1 + r_{t-1})$$

A household has units of time to spend on labor, conditional on being employed with probability e_t. Let $l^j_{s,k}$ represent the labor supply, $\omega^j_{s,k}$ denote the average hourly earnings of a worker, and $c^j_{s,k}$ denote the consumption. We further assume that each agent saves $a^j_{s,k}$ and receives the return at a risk-free interest rate r. The risk-free interest rate is determined by the demand and supply of assets a. If assets are perfectly elastically supplied then a no-arbitrage condition should determine interest rate r. All agents consume and receive gross return on assets $a^j_{s,k,t}(1 + r)$, but only young and middle-aged workers participate in the labor market in each period (specifically, we impose $l^o_{s,k,t} = 0$ and $\omega^o_{s,k,t} = 0$). There is a capital market, where each worker chooses to save by renting capital to firms for a risk-free rate, therefore we have $k_t = a_t = \sum_{s \in h,l} \sum_{k \in f,n} \sum_{j \in y,m,o} a^j_{s,k,t}$. Finally, we define real wages

as follows: $\omega^j_{r,s,k,t} = \omega^j_{s,k,t}/\pi_t$, where π_t corresponds to the gross inflation. Wage inflation is defined as $\pi_{w,t} = \omega^j_{s,k,t+1}/\omega^j_{s,k,t}$.

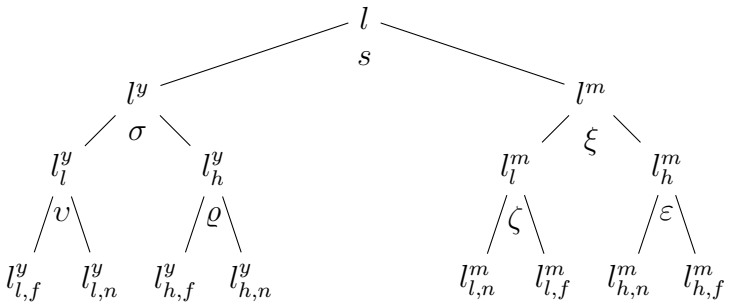


Figure 5: Schemes of the nested CES production function

Labor-Labor Substitution. Now, we present the aggregate labor equations. Labor is disaggregated according to several criteria, adding such features as workers' age, skills, and origin. Figure 5 illustrates the nesting order in the CES production function. We distinguish labor by age (young and middle-aged), education (high- and low-skilled) and national origin (natives and foreigners). Let l^y_t denote the hours worked by younger individuals:

$$l^y_t = [(\phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}} + \phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}} + (\phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}} + \phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}}]^{\sigma},$$

This equation emphasizes that hours worked by younger individuals comprise various categories of labor: hours worked by high-skilled foreign workers $l^y_{h,f,t}$; by high-skilled native workers $l^y_{h,n,t}$; by low-skilled foreign workers $l^y_{l,f,t}$; and by low-skilled native workers $l^y_{l,n,t}$. Let ϱ (υ) denote the parameter of substitution between high- (low-) skilled native and foreign workers of young age. The parameter σ denotes the parameter of substitution between high- and low-skilled workers when young.

Similarly, the hours worked by middle-aged individuals are:

$$l^m_t = [(\theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\varepsilon}} + \theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\xi}} + (\theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}} + \theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}}]^{\xi},$$

The term l^m_t represents the total hours worked by middle-aged individuals and encompasses four labor groups: hours worked by high-skilled foreign workers $l^m_{h,f,t}$; by high-skilled native workers $l^m_{h,n,t}$; by low-skilled foreign workers $l^m_{l,f,t}$; and by low-skilled native workers $l^m_{l,n,t}$. Within the middle-aged population, we also distinguish the elasticity of substitution

between native and foreign workers, with ε (ζ) representing the parameter of substitution between high- (low-) skilled native and foreign workers. We also assume that the parameter of substitution between high- and low-skill workers is ξ.

A notable study on the substitution between workers by Ottaviano and Peri (2008) adopts a nested constant elasticity of substitution production function. The key assumption in their nested CES framework is that for labor inputs, workers of different education levels are split into specific education subgroups, which are then nested into groups with different experience levels. Within the same education and experience group, they identify U.S.-born and foreign-born workers. Instead, our framework considers age categories (young vs middle-aged workers) and assumes a split between high-skilled native workers, high-skilled foreign workers, low-skilled native workers, and low-skilled foreign workers. Moreover, the relative demand elasticities between different worker categories are not identical. We do not restrict the elasticity of substitution to being the same between high- and low-skilled workers of different ages (young and middle-aged). We also do not restrict the elasticity of substitution to being the same between foreign and native workers of different ages (young and middle-aged) and skills (high-skill and low-skill).

Technology Representative firms within industry i choose capital and labor to maximize profits, subject to the production function. The production function that relates outputs to inputs takes the constant elasticity of substitution (CES) form:

$$y_{i,t} = (k_{i,t})^{1-\gamma}(z_{i,t}(\alpha(l^y_{i,t})^{\frac{1}{s}} + (1-\alpha)(l^m_{i,t})^{\frac{1}{s}})^s)^{\gamma} \tag{3.1}$$

where α is a distribution parameter that determines how important the two labor factors are in aggregate production, $k_{i,t}$ is the capital input and $\gamma \in (0,1)$ is the labor share parameter. The aggregate capital stock $k_t = \int k_{i,t}di$ is then expressed by the capital accumulation function: $k_{t+1} = (1-\delta^k)k_t + i_t$ with i_t the total investment and δ^k is the capital depreciation rate. $l^y_{i,t}$ and $l^m_{i,t}$ refer to the firm's labor input, with aggregate functions of hours worked by young workers $l^y_t = \int l^y_{i,t}di$ and middle-aged workers $l^m_t = \int l^m_{i,t}di$. $z_{i,t}$ is the aggregate productivity shock, and s denotes the elasticity of substitution between hours worked by young ($l^y_{i,t}$) and middle-aged workers ($l^m_{i,t}$). For each age group, we distinguish between natives and foreigners; and between high- and low-skilled workers. When $s > 1$, the young and middle-aged workers

are gross substitutes; when $s < 1$, they are complements.[11]

We model business closures as a negative aggregate productivity shock, following Gourio (2013), and assume that aggregate productivity shock $z_{i,t}$ takes the form:

$$\begin{aligned}
\ln(z_{i,t}) &= \rho^z \ln(z_{i,t-1}) + g_{i,t};\\
g_{i,t} &= \delta d_{i,t} + \epsilon^z_{i,t};\\
d_{i,t} &= \begin{cases} 0 < d_{i,t} \leq 1, & \text{if business restriction were imposed} \\ 0, & \text{otherwise} \end{cases}
\end{aligned}$$

where ρ^z is the productivity smoothing parameter, δ is the productivity response to the business closure orders. $d_{i,t}$ is our measure of business closure order variable, and $\epsilon^z_{i,t}$ is the productivity shock. We assume that $\epsilon^z_{i,t}$ are i.i.d. N(0, 1). According to this specification, agents do not anticipate a negative or a positive productivity shock ($\epsilon^z_{i,t}$) or a possibility of business-closure orders occurring ($d_{i,t}$). However, agents do observe changes in productivity ($z_{i,t}$) that these two innovations drive. Note that the business-closure order shock is not persistent, but agents can anticipate a decline or an increase of $z_{i,t}$ in the next period.

Following Koren and Pető (2020), we construct a measure that reflects the share of workers the business-closure restrictions affect. We can interpret this measure as the occurrence of the business-closure orders and how they affect economic activity. For example, prior to and after the pandemic crisis, this share will equal zero, but when the COVID shock hits, the share will have a value between 0 and 1 and will vary across industries, given that some industries may have high- or low-level shares of affected workers.

Pricing decision. We assume that firms operate in a monopolistic competitive market and adjust their price by adopting a variant of Calvo price friction $p_t = \theta p_{t-1} + (1-\theta)p^*_t$, firms can reoptimze the price with probability $1-\theta$; otherwise, it cannot reoptimize with probability θ. We can derive the inflation equation which is fairly standard $\pi_t = \beta\pi_{t+1} + \kappa\mu_t$, where μ_t is the marginal cost and $\kappa = (1-\theta)(1-\beta\theta)/\theta$ depends on the the discount factor and frequency of price adjustment.

[11] One could also consider industries with different production functions and assume that each industry has a specific return to scale parameter. This is an interesting approach to model industry heterogeneity that we could explore.

Market Clearing Condition. The market clearing condition is:

$$k_{t+1} - (1-\delta^k)k_t = y_t - \Sigma_{s\in h,l}\Sigma_{k\in f,n}\Sigma_{j\in y,m,o}c^j_{s,k,t}.$$

This equation reveals that total investment in this economy equals the difference between total output and the consumption.

The Definition of Equilibrium. We define equilibrium as a collection of quantities and prices, such that (i) Households choose $\{c^y_{h,f,t}, c^m_{h,f,t}, c^o_{h,f,t}, c^y_{l,f,t}, c^m_{l,f,t}, c^o_{l,f,t}, c^y_{h,n,t}, c^m_{h,n,t}, c^o_{h,n,t}, c^y_{l,n,t}, c^m_{l,n,t}, c^o_{l,n,t}, l^y_{h,f,t}, l^m_{h,f,t}, l^y_{l,f,t}, l^m_{l,f,t}, l^y_{h,n,t}, l^m_{h,n,t}, l^y_{l,n,t}, l^m_{l,n,t}, a^y_{h,f,t}, a^m_{h,f,t}, a^o_{h,f,t}, a^y_{l,f,t}, a^m_{l,f,t}, a^o_{l,f,t}, a^y_{h,n,t}, a^m_{h,n,t}, a^o_{h,n,t}, a^y_{l,n,t}, a^m_{l,n,t}, a^o_{l,n,t},\}$ to maximize their utility subject to the budget constraints; (ii) Producers choose how much of each input to employ $\{l^y_{h,f,t}, l^m_{h,f,t}, l^y_{l,f,t}, l^m_{l,f,t}, l^y_{h,n,t}, l^m_{h,n,t}, l^y_{l,n,t}, l^m_{l,n,t}, k_t\}$ to minimize their production cost. The first order conditions yield the market prices at equilibrium $\{\omega^y_{h,f,t}, \omega^m_{h,f,t}, \omega^y_{l,f,t}, \omega^m_{l,f,t}, \omega^y_{h,n,t}, \omega^m_{h,n,t}, \omega^y_{l,n,t}, \omega^m_{l,n,t}, r^k_t\}$. (iii) Productivity z_t is exogenous, and the conditions that determine employment rate e_t, unemployment rate u_t are satisfied. (iii) Equilibrium requires the market clearing condition $k_{t+1} - (1-\delta^k)k_t = y_t - c_t$ to hold. [12]

4 Data and Estimation

In this section, we set the value of the main parameters of the model such that it matches the key facts occurred on the US labor market. We measure the remaining parameters directly from US data. The model is applied to data at quarterly frequency.

4.1 Data, Summary Statistics and Construction of Variables.

Data source. For this analysis we focus on the period 2019-2021. Several states have enforced business-closure orders starting from the second quarter of 2020 to curb the spread of the virus. These restrictions were largely lifted in the third quarter of 2020. This is why we use a sample that covers 2019-2021, the period before and after the public health restrictions on businesses.

[12] Detailed computations appear in the Online appendix D. The model is solved using first-order perturbation method.

Our main source for workers' data is the Current Population Survey conducted by the Census Bureau (Flood et al., 2021). This survey collects information on each individual every month over the age of 15 in households surveyed. Relevant information includes the average hours worked per week, labor earnings, education level, age, nationality, industry, and geographic information (county and state). The summary statistics is reported in Table 3. Our initial dataset contains 4,274,781 individuals. We drop observations with missing or zero usual hours worked and missing wages and, then, winsorize wages at 1 and 99 percentiles. This leaves us with 1,910,679 (monthly) observations. The hours worked of individuals surveyed show substantial variation, with a weekly average hours worked of 38, and a standard deviation of 13 hours. Sixteen percent of individuals are immigrants, 23% are young and slightly less than 40% are high-skilled (i.e., with at least a bachelor's degree). Around 18% of middle-aged are immigrant workers, and only 11.2% of young population are immigrant workers. The yearly wage exhibits substantial volatility, with the mean equal to $53,322 and the standard deviation being $52,007.

The basis of our definition of native workers and foreign workers in the US is the nationality indicated in the survey. Of course, we did not capture how long an immigrant stayed in the US, whether a native worker had recently acquired US citizenship, or the worker's place of birth. Regarding the broad skill group partition, we define high-skilled workers as those who attained high educational levels, disregarding skill acquisition and the years spent in the US labor market.

To calculate total factor productivity time series, we use the gross domestic product and labor input at the industry level (3-digit NAICS code level). Specifically, we use quarterly data on gross domestic product by industry level, released by the Bureau of Economic Analysis (BEA) of the US Department of Commerce.

Data for the estimation of the elasticities. As the CPS includes questions on wages only in the ASEC module (the March module) and wages are not reported monthly, we construct a measure of the hourly wage rate (measured in $) as follows:

$$\text{Hourly Wage Rate} = \frac{\text{Wage Income Annual}}{\text{Weeks} \times \text{Hours Worked per Week}}$$

Table 3: Summary statistics

Variable	Mean	Std. Dev.	N
Immigrant	0.158	0.365	1,910,679
Young (15-30)	0.232	0.422	1,910,679
Middle-aged (30-64)	0.692	0.462	1,910,679
Old (>64)	0.076	0.265	1,910,679
High skilled (Degree or higher)	0.395	0.489	1,910,679
Low skilled (Less than degree)	0.605	0.489	1,910,679
Hours worked last week	38.653	13.034	1,910,679
Wage and salary income (yearly data)	53,322.105	52,007.357	222,240
Low-skilled native (young)	0.638	0.481	646,178
Low-skilled native (middle-aged)	0.470	0.499	1921792
High-skilled native (young)	0.250	0.433	646178
High-skilled native (middle-aged)	0.350	0.477	1921792
Low-skilled immigrant (young)	0.075	0.263	646,178
Low-skilled immigrant (middle-aged)	0.109	0.311	1,921,792
High-skilled immigrant (young)	0.037	0.189	646,178
High-skilled immigrant (middle-aged)	0.071	0.256	1,921,792

where *Hours Worked per Week* is computed from the monthly CPS.
Using the CPS data, we computed weighted averages of hourly wage rate and hours worked by skill level (considering those who earned a degree as high-skilled and others as low-skilled), age category (young: 15-30, middle-aged: 30-64, old: $>$64), industry, nationality (immigrant or native), and county between 2019 and 2021. This collapse leaves us with a sample of 449,556 observations.

To estimate elasticities, we reshape the data from long to wide format, such that each row contains columns for hours worked and hourly wages across eight worker types. Each row is uniquely identified by a combination of year, quarter, industry code, state, and county.

There are also empty cells. For instance, it may happen that a specific industry does not exist in a specific county. To overcome this issue, we replace missing values with the mean of each column. Lastly, our final sample contained 45,125 observations.

Data for estimation of the productivity shock. To construct a measure for the productivity shock $z_{i,t}$, we use the CPS data on hours worked (initial dataset with 1,910,679 observations). We assign the specific industry (3-digit NAICS) code for each sub-sector to standardize it to the format of the BEA dataset.[13] We then aggregate the hours worked by quarter and industry, and end up with 693 observations.

[13] For a complete description of the industry codes, see Appendix B.

Concerning the business-closure order shock, we follow Koren and Pető (2020) in using the Occupational Information Network dataset. This dataset includes the share of workers affected by COVID-19 restrictions in each industry at the 3-digit NAICS code level. The business-closure orders $d_{i,t}$ at the industry level is defined as follows:

$$d_{i,t} = \text{Affected Share}_{i,t}$$

The variable $d_{i,t}$ is proxied by the affected share in industry i at time t from the ONET dataset. Some industries may have been more severely affected than others. One can see that the affected share is a measure of the percentage of workers hit by the restrictions, in this case an appropriate proxy for business-closure orders.[14]

Data on total output at the quarter and sector level (with 671 observations) from the BEA is then merged with the CPS data and the ONET data. Our final sample covers the period Q1-2019 and Q3-2021 and contains 671 observations that we use to construct $z_{i,t}$ and estimate the parameter δ. We approximate the value of productivity $z_{i,t}$ using the following equation:

$$\underbrace{\ln z_{i,t}}_{\text{productivity}} = \underbrace{\ln y_{i,t}}_{\text{GDP}} - \underbrace{\ln l_{i,t}}_{\text{hours worked}}$$

Here again i indexes an industry and t a quarter.[15] The estimation is applied to data at quarterly frequency, the relevant frequency for labor market dynamics. Notice also that capital is missing from this equation because data on capital inputs is not available at industry and quarterly levels. Even when we consider the BEA-BLS data on industry-level total factor productivity, it is available only on an annual basis.

[14] Koren and Pető (2020) used the Occupational Information Network (ONET) dataset to measure the share of workers the restrictions affected, by industry. To compute this share, the authors constructed three indices (Teamwork, Customer, and Presence) that account for face-to-face interactions along these dimensions, in more than 1,000 occupations that the ONET dataset included.

[15] One could also use total factor productivity (TFP) from the KLEMS data to estimate the parameters of the productivity shock. This dataset provides TFP at the industry level with only two-digit NAICS codes. However, for our analysis, we use the Bureau of Economic Analysis data on gross domestic product and labor supply at industry and quarter level, which is available in three-digit NAICS codes. This is convenient since the business-closure orders data from the Occupational Information Network (ONET) are also available at three-digit NAICS level.

4.2 Estimation

Preset parameters. We first set the parameters that the literature commonly uses. We set the value of the discount factor $\beta = 0.97$ to target an interest rate of 3%. We set the value of the curvature on the disutility of labor $\eta = 1.5$ based on a standard value in the literature (see, e.g., Chetty et al. (2011)). We set the value of capital depreciation rate δ^k to 0.025. The labor share γ is set to 0.3, consistent with (Harrison, 2024). We fix the value of the probability that employed workers become unemployed $\mathcal{Q}$ to 0.55, and the probability that unemployed workers become employed s to 0.45, to target an unemployment rate of 5%. We set the slop coefficient on marginal cost to $kappa = (1-\theta)(1-\beta\times\theta)/\theta = (1-0.75)(1-0.95\times 0.75)/0.75 = 0.0958$.

Direct computation of the labor distribution parameters. We first calculate the share of young and middle-aged workers. To do so, we use the following equation:

$$\alpha = \text{young population}/(\text{young population} + \text{middle-aged population})$$

to pin down the youth labor share parameter α, implying a value of 0.25 (see Panel B in Table 5).

In the second step, we estimate the distribution parameters measuring the specific labor intensity of given labor category within younger workers population:

$$\begin{aligned}
\phi_{h,n} &= \text{high-skilled native/young population} = 0.25,\\
\phi_{h,f} &= \text{high-skilled foreign/young population} = 0.04,\\
\phi_{l,n} &= \text{low-skilled native/young population} = 0.64,\\
\phi_{l,f} &= \text{low-skilled foreign/young population} = 0.07.
\end{aligned}$$

Similarly, for middle-aged workers:

$$\begin{aligned}
\theta_{h,n} &= \text{high-skilled native/middle-aged population} = 0.35,\\
\theta_{h,f} &= \text{high-skilled foreign/middle-aged population} = 0.07,\\
\theta_{l,n} &= \text{low-skilled native/middle-aged population} = 0.47,\\
\theta_{l,f} &= \text{low-skilled foreign/middle-aged population} = 0.11.
\end{aligned}$$

Note that Panel C in Table 5 presents the values assigned to each parameter.

Estimated parameters for labor-labor substitution. We use the generic variant of the linearized labor demand function, while maintaining the assumption of stable labor intensity, which is crucial for identifying the elasticity parameters.[16] We use both time and industry variation to identify the elasticity parameters. Similar empirical approach to estimate the corresponding elasticity of substitution can be found in influential work by Ottaviano and Peri (2012). In this specification, let i denote industry, c denote county, and t denote year. We add the county α_c and year α_t fixed effects to absorb time trends and county characteristics that may affect the wage differentials. We augmented this specification with a constant and an error term $\epsilon_{i,t}$. Hence, we write our specifications as follows:

Foreign vs native (highly skilled, young)

$$\ln \omega^{y}_{i,h,f,t,c} - \ln \omega^{y}_{i,h,n,t,c} = \alpha + 0.118(\ln l^{y}_{i,h,f,t,c} - \ln l^{y}_{i,h,n,t,c}) + \alpha_t + \alpha_c + \epsilon_{i,t}.$$

Foreign vs native (low skilled, young)

$$\ln \omega^{y}_{i,l,f,t,c} - \ln \omega^{y}_{i,l,n,t,c} = \alpha + 0.167(\ln l^{y}_{i,l,f,t,c} - \ln l^{y}_{i,l,n,t,c}) + \alpha_t + \alpha_c + \epsilon_{i,t}.,$$

Foreign vs native (highly skilled, middle-aged)

$$\ln \omega^{m}_{i,h,f,t,c} - \ln \omega^{m}_{i,h,n,t,c} = \alpha + 0.126(\ln l^{m}_{i,h,f,t,c} - \ln l^{m}_{i,h,n,t,c}) + \alpha_t + \alpha_c + \epsilon_{i,t}.$$

Foreign vs native (low skilled, middle-aged)

$$\ln \omega^{m}_{i,l,f,t,c} - \ln \omega^{m}_{i,l,n,t,c} = \alpha + 0.187(\ln l^{m}_{i,l,f,t,c} - \ln l^{m}_{i,l,n,t,c}) + \alpha_t + \alpha_c + \epsilon_{i,t}.$$

Middle-aged vs young

$$\ln \omega^{m}_{i,t,c} - \ln \omega^{y}_{i,t,c} = \alpha + 0.119(\ln l^{m}_{i,t,c} - \ln l^{y}_{i,t,c}) + \alpha_t + \alpha_c + \epsilon_{i,t}.$$

High skilled vs low skilled (middle-aged)

$$\ln \omega^{m}_{i,h,t,c} - \ln \omega^{m}_{i,l,t,c} = \alpha + 0.133(\ln l^{m}_{i,h,t,c} - \ln l^{m}_{i,l,t,c}) + \alpha_t + \alpha_c + \epsilon_{i,t}.$$

High skilled vs low skilled (young)

$$\ln \omega^{y}_{i,h,t,c} - \ln \omega^{y}_{i,l,t,c} = \alpha + 0.047(\ln l^{y}_{i,h,t,c} - \ln l^{y}_{i,l,t,c}) + \alpha_t + \alpha_c + \epsilon_{i,t}.$$

Overall, our estimates suggest some degree of substitutability between foreign and native

[16] The optimality conditions that characterize the elasticity of substitution parameters appear in the Online Appendix D.

workers. An increase in the supply of labor by foreign workers relative to natives, whether they are both young, middle-aged, highly skilled or with lower educational levels, leads to an increase in wages of foreign workers relative to natives with the same characteristics. The change in labor supply by foreign workers turns out to raise the wage differentials (though modestly). The corresponding elasticity of substitution parameters $\{\varrho, \upsilon, \varepsilon, \zeta\}$ are listed in Table 4.

Explaining how our estimates relate to the emerging literature trying to estimate labor demand elasticity is important. In particular, early on, Ottaviano and Peri (2008) and D'Amuri et al. (2009) raised the question of imperfect substitution between native and immigrant workers and found imperfect substitutability between the two. Our estimates differ substantially, primarily because we have a different specification of the estimated optimality equation and employ a different CES nested order. Additionally, our estimation relates to the pre- and post-pandemic context, restricting the estimation sample to the years between 2019 and 2021. All these elements are certainly important, with consequences on our estimated elasticities.

Moving to the estimation of the elasticity between young and middle-aged workers, our estimates favor some degree of substitution between these two groups. We find that an increase of 1 % in the labor supply by middle-aged workers relative to young workers is associated with an increase in wages of middle-aged workers relative to young workers by 0.119 % ($1/s - 1 = 0.119$).

Finally, we find that the elasticity of substitution between high- and low-skilled workers is $1/\xi - 1 = 0.133$ among middle-aged workers, and $1/\sigma - 1 = 0.047$ among young workers. These results imply that high-skilled workers and low-skilled workers are perfect substitutes. These estimates substantially differ from the estimates of McAdam et al. (2011), who find a high elasticity of substitution between skilled and unskilled labor. We report all the estimates of the elasticity of substitutions in Table 4, while detailed estimation results under various specifications appear in Appendix G.

One could reasonably question the model identification for the estimation of the labor-labor elasticities. Our model assumes an upward-sloping labor supply curve and a downward-sloping labor demand curve. The optimality conditions for firms in our model determine the optimal demand for labor and thus wages. These conditions are both necessary and convenient for our study, since its focus is to account for key ingredients that identify the equilibrium relationships between wages and hours worked, and to inform us of the degree

Table 4: Elasticity of substitution parameters

Elasticity of substitution						
$1/\varrho - 1$	$1/\upsilon - 1$	$1/\varepsilon - 1$	$1/\zeta - 1$	$1/s - 1$	$1/\sigma - 1$	$1/\xi - 1$
0.118*** (0.011)	0.167*** (0.014)	0.126*** (0.014)	0.187*** (0.017)	0.119*** (0.013)	0.047*** (0.011)	0.133*** (0.014)
Elasticity of substitution parameters						
ϱ	υ	ε	ζ	s	σ	ξ
0.8945	0.8569	0.8881	0.8425	0.8937	0.9551	0.8826

Note: The estimates in this table are obtained using a simple OLS with county and year fixed effects, the number of observation is equal to 24186. Homoscedastic standard errors in parentheses. $*p < .1, **p < .05, ***p < .01$

of substitutability between workers. Nevertheless, we should mention that estimating the elasticities of substitution is difficult, and it is challenging to come up with a clean identification for these elasticities. Our specification could potentially suffer from major omitted variable biases that we cannot completely rule out. Therefore, we include various checks of our estimates with time and country fixed effects. We also run additional regressions to check the robustness of our estimates and find that the estimates are not sensitive to the exclusion of the pandemic sample period and are largely robust to specifications that allow for county and year fixed effects (see Appendix G and Online Appendix G.2 for more details).

Business-closure orders. Turning to business-closure orders effects on productivity, we estimate a regression of the following form

$$\ln(z_{i,t}) = \rho^z \ln(z_{i,t-1}) + g_{i,t}$$
$$g_{i,t} = \delta d_{i,t} + \epsilon^z_{i,t};$$

where $\ln(z_{i,t}) = \ln(y_{i,t}) - \ln(l_{i,t})$, is constructed using data on gross domestic product and labor supply at industry level. The parameter ρ^z captures the persistence of productivity shocks, $g_{i,t}$ represents the random component, and $\epsilon^z_{i,t}$ is an i.i.d. normal random variable with mean zero and variance σ^z. We aim to capture the idea that business closures can affect the productivity and, thus, affect aggregate output. To do so, we add a variable $d_{i,t}$ to this specification, which reflects the affected share in industry i at time t. The term d measures the share of economic activities that have been disrupted by COVID-19. Last, we insert the

Table 5: Summary of Parameters

Param.	Description	Value	Source
Panel A: Household - preset parameters			
β	Discount factor	0.9500	Standard
η	The curvature on the disutility of labor	1.5000	Chetty et al. (2011)
Panel B: Producer - preset parameters			
γ	Labor share	0.3000	Harrison (2024)
δ^k	Capital depreciation rate	0.0250	Standard
κ	Slop coefficient on marginal cost	0.0958	$\theta = 0.75$ Galı and Gertler (1999)
Panel D: Productivity shock - estimated			
ρ^z	Autocorrelation parameter of technology shock	0.9832	BEA
δ	Responses of productivity to the business closure orders	-0.0017	BEA/ ONET
Panel E: Labor - Distribution parameters - direct computation			
α	Youth labor share parameter	0.2500	CPS
$\theta_{h,n}$	High-skilled middle-aged native labor share parameter	0.3500	CPS
$\theta_{h,f}$	High-skilled middle-aged foreign labor share parameter	0.0700	CPS
$\theta_{l,n}$	Low-skilled middle-aged native labor share parameter	0.4700	CPS
$\theta_{l,f}$	Low-skilled middle-aged foreign labor share parameter	0.1100	CPS
$\phi_{h,n}$	High-skilled young native labor share parameter	0.2500	CPS
$\phi_{h,f}$	High-skilled young foreign labor share parameter	0.0400	CPS
$\phi_{l,n}$	Low-skilled young native labor share parameter	0.6400	CPS
$\phi_{l,f}$	Low-skilled young foreign labor share parameter	0.0700	CPS
Panel F: Labor - Elasticity of subst. parameters (young vs middle-aged) - estimated			
s	Elasticity of subst. between labor factor l^y and l^m	0.9300	CPS
Panel G: Labor - Elasticity of subst. parameters (high vs low-skilled) - estimated			
σ	Elasticity of subst. between labor factor l^y_h and l^y_l	0.9746	CPS
ξ	Elasticity of subst. between labor factor l^m_h and l^m_l	0.9116	CPS
Panel H: Labor - Elasticity of subst. parameters (natives vs foreigners) - estimated			
ϱ	Elasticity of subst. between labor factor $l^y_{h,f}$ and $l^y_{h,n}$	0.8969	CPS
υ	Elasticity of subst. between labor factor $l^y_{l,f}$ and $l^y_{l,n}$	0.8818	CPS
ε	Elasticity of subst. between labor factor $l^m_{h,f}$ and $l^m_{h,n}$	0.9017	CPS
ζ	Elasticity of subst. between labor factor $l^m_{l,f}$ and $l^m_{l,n}$	0.8569	CPS
Var.	**Description**	**Model**	**Data**
Panel I: Targeted moments			
u_t	Unemployment rate	0.0500	0.0500 - FRED
Panel J: Non-targeted moments			
$l^y_{h,n}/l^y$	High-skilled native labor share (young)	0.2600	0.2500 - CPS
$l^y_{h,f}/l^y$	High-skilled foreign labor share (young)	0.1600	0.0400 - CPS
$l^y_{l,n}/l^y$	Low-skilled native labor share (young)	0.4100	0.6400 - CPS
$l^y_{l,f}/l^y$	Low-skilled foreign labor share (young)	0.1700	0.0700 - CPS
$l^m_{h,n}/l^m$	High-skilled native labor share (middle-aged)	0.3100	0.3500 - CPS
$l^m_{h,f}/l^m$	High-skilled foreign labor share (middle-aged)	0.1300	0.0700 - CPS
$l^m_{l,n}/l^m$	Low-skilled native labor share (middle-aged)	0.3800	0.4700 - CPS
$l^m_{l,f}/l^m$	Low-skilled foreign labor share (middle-aged)	0.1700	0.1100 - CPS

estimates for $\rho^z = 0.98$ and $\delta = -0.0017$ into our model. For more details, refer to Appendix C.

Discussion of model consistency with observed facts. In Panels I and J of Table 5, we report model implied moments and their consistency with observed facts. In general, we show that the model fairly matches empirical moments. Although targeted, the unemployment rate predicted by the model corresponds to the value observed before the pandemic crisis in the US with a value of around 4%. Turning to untargeted moments, we find that the model matches closely the age, skill, and origin structure of the U.S. labor market the CPS data indicates.

5 Results

We divide our analysis of the impact of the productivity shock on the labor markets into three parts. First, we study the responses of total hours worked to a negative productivity shock. Second, we investigate the implications of this shock on the wage rate. Finally, we provide empirical evidence on the dynamics of wages and hours worked after the pandemic crisis of 2019.

5.1 Labor Response to Productivity and Business-Closure Shocks

In Figure 6, we document the considerable decline in aggregate hours worked during the pandemic for all types of workers. The primary economic explanation for the drop in hours after a productivity shock is that strict restrictions on businesses partially suspend or entirely cease business activity. This unprecedented supply shock across US industries curtailed economic activity and caused a significant decrease in labor demand. As Panels A and B of Figure 6 illustrate, a business-closure order affecting workers disproportionately generates a fall in total hours worked of 2.45 % for middle-aged, high-skilled foreign workers, and 1.74 % for young, high-skilled foreign workers. The same economic forces lead to a significant decline in hours worked by highly educated native workers, a decrease of 2.11 % when middle-aged and 1.11 % when young (plotted in Panels C and D).

We now turn to the impact of the productivity shock on low-skilled workers. Panels E, F, G, and H of Figure 6 depict the responses of hours worked in this category. One consequence of a negative productivity disturbance is a considerable decline in hours worked by middle-aged, low-skilled foreign workers of 2.44 % (Panel E). Within the foreign population, young workers

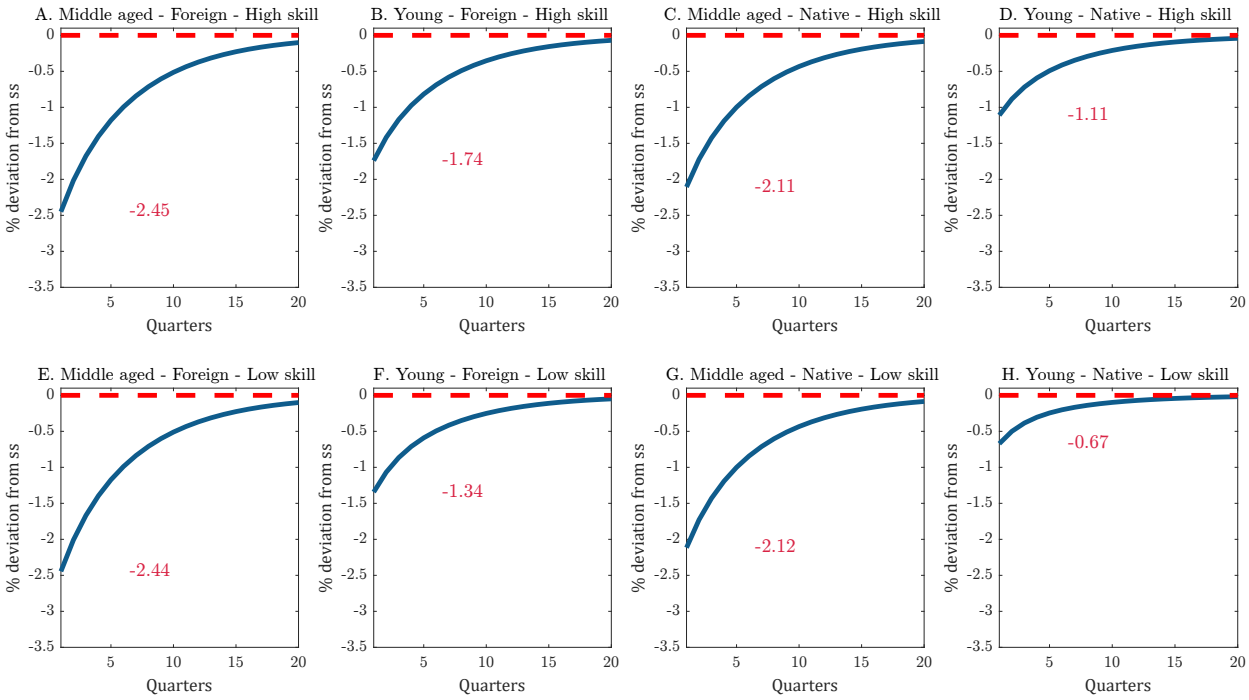


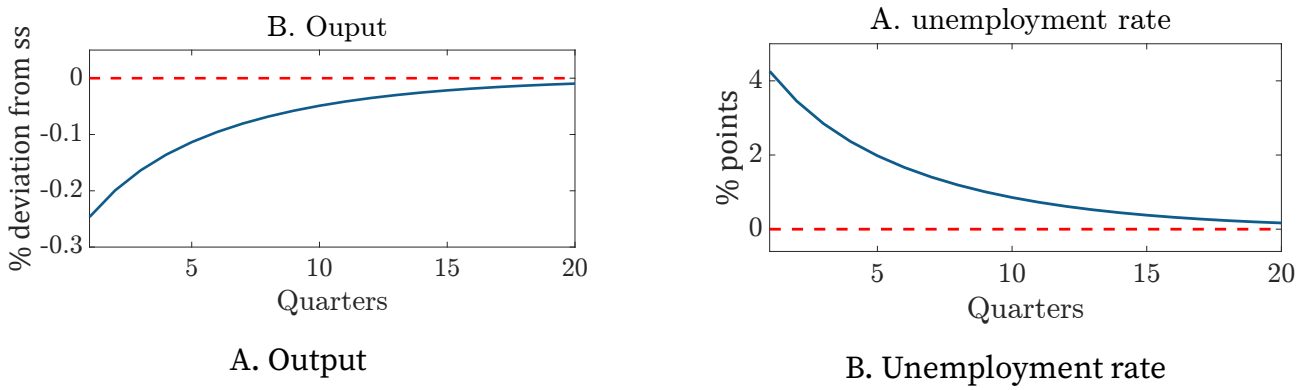
Figure 6: Hours responses to a negative productivity shock

Notes: Panels A, B, E and F show impulse responses to a negative productivity shock of hours worked by foreign workers of different skills and ages. Panels C, D, G, and H show impulse responses to a negative productivity shock of hours worked by native workers of different skills and ages.

experience a fall in hours of 1.34 % (Panel F). Furthermore, the decline in productivity affects low-skilled native workers substantially, as it depresses the total hours worked by middle-aged workers by 2.12 % and 0.67 % for young workers (see Panels G and H).

The thought experiment documented here emphasizes that the fall in productivity during the pandemic crisis led to heterogeneous responses of hours worked. Age, nationality and education are key in generating differences between workers' responses to a negative productivity shock. Generally speaking, middle-aged workers suffered more from business closure than young workers, whereas low-skilled workers experienced a larger reduction in hours worked than highly skilled workers. We observe higher levels of exposure to productivity shock among less-educated young workers, who experienced the largest decline in hours worked. Furthermore, we find that, among natives, the largest fall in total hours worked disadvantaged the middle-aged workers the most. Such disturbance most obviously affects hours worked negatively and adds to the existing disparities between workers.

Figure 7 depicts the impact of the pandemic-induced business-closure orders on total output and inflation. Our framework predicts that a negative shock leads to a decline in



A. Output

B. Unemployment rate

Figure 7: Aggregates response to a negative productivity shock

Notes: Panel A presents the responses of output. Panel B displays the response of unemployment rate.

nominal wages and a decrease in labor demand. This creates a downward pressure on households' earnings that, in turn, pushes them to cut their spending. Consequently, output falls and unemployment rate rises (see Panels A and B). The key result that stands out from this analysis is that a negative shock to productivity driven by business-closure orders leads to a contraction in total hours worked. This is consistent with several studies, including (Cajner et al., 2020), who find that business closures explains 18 % of the decline in aggregate employment. Baek et al. (2021) use state-level data and conclude that one-week stay-at-home orders raised unemployment claims by 1.9 %.

However, there are many reasons to believe that workers throughout the pandemic period became less willing to work, for several reasons unrelated to business restrictions. For instance, unemployment benefit extensions, fear of the virus, and lack of childcare, among other factors. From Brinca et al. (2021) and other studies, we recognize that labor supply shocks are more significant than labor demand shocks in explaining the decline in sectoral hours worked during the pandemic. Forsythe et al. (2020) reveal that vacancy postings declined by 44 % and employment collapsed by 13 % between February and April 2020. They argue that stay-at-home orders were not the sole cause of the decline in employment. In a landmark study, Aum et al. (2021) explore an interesting case that pertained to the South Korean context, where such measures as lockdowns or business closures were not implemented. Most governmental policies centered on social distancing and school closures. They show that the contraction in economic activity and international factors, particularly China's economic downturn, were the primary drivers of employment decline.

5.2 Hourly Earnings and Productivity Shocks

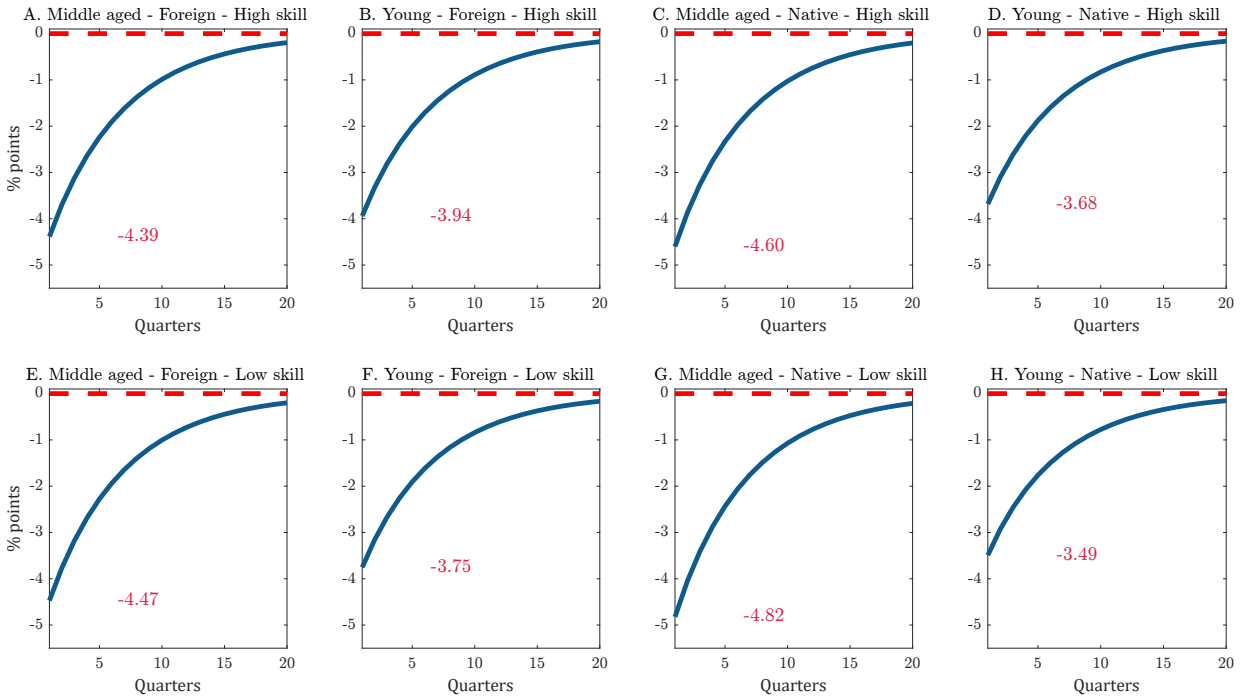


Figure 8: Wages responses to a negative productivity shock

Notes: Panels A, B, E, and F show impulse responses of wages, among foreign workers of different skills and ages to a negative productivity shock. The left panels C, D, G and H show impulse responses of wages, among native workers of different skills and ages, to a negative productivity shock.

Figure 8 shows the response of hourly earnings after a negative productivity shock. Panel A presents the response of middle-aged, high-skilled foreign workers to the productivity shock. A 1 percent decrease in productivity reduces the wage rate of this category by 4.39 %. On the other hand, this shock depresses the hourly wage rate of the young, high-skilled foreign workers by 3.94 % (see Panel B). We observe a similar effect for the high-skilled native population (See Panels C and D). A one percent decrease in productivity reduces the hourly earnings of middle-aged workers by 4.60 % and young workers by 3.68 %.

Moving to the case of low-skilled workers, we find that middle-aged foreign workers experienced a wage decline in response to a fall in productivity (Panel E), a result not very different from the case of high-skilled workers (Panel A). Quantitatively, the decrease in the hourly wage rate is larger for foreign middle-aged workers (4.47 %, see Panel E) than for foreign young workers (3.75 %, see Panel F). Similarly, native workers faced a drop in wage rate after the negative productivity shock, with a decrease of 4.82 % for middle-aged workers

(Panel G), and a decrease of 3.49 % for young workers (Panel H).

Again, age is an important factor, shaping the wage effects of productivity shock. After the chronic shock to productivity, the decline in hourly earnings is especially apparent among middle-aged workers. At the margin, middle-aged workers have much more exposure to productivity shock. The hourly wage rate of middle-aged workers normally exceeds that of young workers, given the level of experience. Highly educated workers appear to be less affected by this shock and the decline in wages for this category is less pronounced. A recent study by Cortes and Forsythe (2021) documents heterogeneous employment losses across types of jobs and groups of workers, finding that the pandemic disproportionately affected low-wage jobs.

6 Model Extensions

In this section, we test the sensitivity of our model results. First, we perform a simple sensitivity analysis using the elasticity of substitution estimates of Ottaviano and Peri (2012), testing whether the findings remain consistent with ours. Second, we present the model results of an additional extension that accounts for inelastic labor supply and abstracts from physical capital in the production function. Finally, we discuss the extensions of the model that consider government programs, such as unemployment benefits (UB), and perform welfare comparisons for different worker groups.

Sensitivity analysis with respect to the estimated elasticities of substitution To perform this exercise, we use the estimates from Ottaviano and Peri (2012) and test whether our findings remain consistent. Their estimates of substitutability between natives and foreigners are in the neighborhood of -0.05. However, they report estimates of -0.32 for the elasticity of substitution between high- and low-education groups, and -0.28 between young and old workers (See Table 7 in Appendix A). Regarding a negative productivity shock, we show that hours worked decrease for all workers categories, though the response of hours is amplified in the counterfactual exercise than in our baseline model. Both models predict that the effect on young workers is less than that on middle-aged workers. The impulse response functions appear in Figure 9 in Appendix A. The implied relative wage effect of the pandemic-induced business disruption in our model when we consider the Ottaviano and Peri (2012) estimates

indicates that middle-aged workers experienced a greater decline in hourly earnings than young workers. Interestingly, foreign workers' wages appear to be much more affected than those of native workers. The results are provided in Figure 10 in Appendix A.

Model without physical capital In Figures 13 and 14 in Appendix A, we report the response of hourly earnings and hours worked. In this counterfactual model, we abstract from physical capital in the production function. This dampens substantially the response of wages and hours to the shock. Initially, wages decline, then increase again and return to the steady state level. Disregarding capital adjustment in this framework, our model leads to a modest hours worked response to the shock.

Model with inelastic labor supply curve In Figures 15 and 16 in Appendix A, we report the consequence of a negative productivity shock on hours and hourly earnings when we assume an inelastic labor supply curve. This model shows the mechanical effects of the pandemic when wage rates set by workers are fixed. Thus, the labor supply is inelastic to changes in wages. This model version predicts declining patterns in nominal wages and, surprisingly, the underlying response of hours worked by all types of workers turns positive. Both hours worked and wages feature persistent responses. The results are reported in the Appendix A.

Model with unemployment benefits We now modify our baseline model to account for a fiscal side to examine the endogenous response in hours worked and relative wage effects of COVID-induced business shutdowns. This version is identical to the baseline framework, with the only difference that workers receive unemployment benefits and the government collects taxes. The new budget constraint is given by

$$c^j_{s,k,t} + a^j_{s,k,t} = (1-\tau)e_t \frac{\omega^j_{s,k,t}}{\pi_t} l^j_{s,k,t} + \chi^j_{s,k,t} + a^j_{s,k,t-1}(1+r_{t-1})$$

where τ is the labor income tax, and $\chi^j_{s,k,t}$ represent the unemployment benefit. The optimal choice of hours worked $l^j_{s,k,t}$ is derived as follows $\frac{\partial \mathcal{L}_t}{\partial l^j_{s,k,t}} : \beta(l^j_{s,k,t})^\eta = (1-\tau^c)\lambda_t e_t \omega^j_{s,k,t}/\pi_t$. We follow Cai and Heathcote (2023) and assume that unemployment benefit $\chi^j_{s,k,t}$ received by unemployed workers depends on a fixed floor $\delta^u = 0.05$ and replace up to 50% of worker's earnings. We, then, assume that unemployment benefit is defined as: $\chi^j_{s,k,t} = u_t(\delta^u + \kappa l^j_{s,k,t}\omega^j_{s,k,t})$,

with $\kappa = 0.5$. The government finances social transfers $\chi^j_{s,k,t}$ by collecting labor income taxes, therefore

$$\tau e_t \frac{\omega^j_{s,k,t}}{\pi_t} l^j_{s,k,t} = \chi^j_{s,k,t}.$$

Figure 9 in Appendix A displays the hours worked in response to a negative productivity shock in the benchmark model and the counterfactual scenario which accounts for public spending programs, particularly unemployment benefits. It is worth mentioning that in the benchmark model, workers are not self-insured against unemployment risk and pay no taxes on labor income. Quantitatively, in the version with unemployment benefits, the impact of this shock is negative in the short run but positive in the long run. These results suggest a stabilizing effect of this policy as it mitigates the effects of the business-induced recession on hours worked and generates a persistent increase in hours worked in the long run.

Unemployment benefits are determined according to the government budget constraint, keeping the replacement rate constant, thus this entails procyclical unemployment benefits. Mitman and Rabinovich (2015) argue that the optimal unemployment insurance policy should be procyclical, higher during normal periods and low during recessions. Our findings differ from their results primarily because their analysis regards the optimal unemployment benefits and their framework assumes sluggish wages and exogenous unemployment benefits rule that reacts proactively to productivity.

Figure 12 in Appendix A illustrates the response of nominal wages to a negative productivity shock. The unemployment benefit mitigates the transmission of a negative productivity shock on nominal wages in the short run. When workers receive unemployment benefits, the impact turns out to be less pronounced than in the baseline model. This negative shock primarily affects firm labor demand and the endogenous unemployment benefits, creating a feedback mechanism that reduces wages initially and generates a persistent increase in wages in the long run.

Welfare comparison between counterfactual scenarios Given the significant magnitude and wide scope of policy response during the COVID-19 crisis, measuring the impact of the expansion of unemployment benefits and implied welfare for different groups of workers is imperative for understanding labor inequalities during this episode. Technically, we

use the utility function to compute the welfare gain of each worker. Thus, we compute $U(c^j_{s,k,t}, l^j_{s,k,t}) = \beta\{\ln((1-\tau^c)e_t\frac{\omega^j_{s,k,t}}{\pi_t}l^j_{s,k,t} + \chi^j_{s,k,t} + a^j_{s,k,t-1}(1+r_{t-1}) - a^j_{s,k,t}) - \frac{(l^j_{s,k,t})^{1+\eta}}{1+\eta}\}$, and report $U = \sum_{t=0}^{20} U_t$ the sum of utility responses to z_t shock over 20 quarters.

Table 6: Welfare Comparison between Counterfactual Scenarios

	Welfare gain or loss (Worker)		
	U Without UB	U With UB	Difference in U
Panel A. Young workers			
$U(c^y_{h,f}, l^y_{h,f})$ High-skilled foreign	-25.05	-17.72	7.33
$U(c^y_{h,n}, l^y_{h,n})$ High-skilled native	-24.73	-18.57	6.15
$U(c^y_{l,f}, l^y_{l,f})$ Low-skilled foreign	-24.84	-21.49	3.35
$U(c^y_{l,n}, l^y_{l,n})$ Low-skilled native	-25.74	-24.68	1.06
Panel B. Middle-aged workers			
$U(c^m_{h,f}, l^m_{h,f})$ High-skilled foreign	-24.94	-16.72	8.21
$U(c^m_{h,n}, l^m_{h,n})$ High-skilled native	-20.08	-10.33	9.75
$U(c^m_{l,f}, l^m_{l,f})$ Low-skilled foreign	-24.25	-15.87	8.38
$U(c^m_{l,n}, l^m_{l,n})$ Low-skilled native	-17.00	-6.06	10.94

Notes: "U Without UB" represents the welfare gain or loss in the baseline model. "U With UB" is the welfare gain or loss in the model with unemployment benefits . "Difference in U" is the welfare difference between the two scenarios. Negative values represent welfare loss.

Table 6 presents the model-implied welfare following a negative productivity shock under two scenarios: the baseline model and the model with unemployment benefits. In the latter, we allow for each type of worker to receive transfers if unemployed. We observe that the pandemic-induced economic disruption affected the economic well-being of young workers more than that of middle-aged workers. Looking at welfare differences, reported in column "Difference in U", we see that all workers are better off when they receive unemployment benefits, as it mitigates the welfare loss. Within the native population, middle-aged, low-skilled workers gained the most. Among young workers, high-skilled workers had the largest welfare gain from unemployment benefits. Throughout this analysis, we show that government programs, such as unemployment insurance generosity, may imply significant welfare gains across different types of workers. Therefore, extending generous unemployment insurance to workers during bad times can raise welfare for all categories of workers. Consistent with our mechanism, (Landais et al., 2018) argue that the optimal unemployment insurance policy should be countercyclical. Providing a more generous unemployment benefit during a recession achieves sizeable welfare gains to workers. Our finding in table 6 is more about the consequences of unemployment insurance on welfare, and we do not attempt to explore

the implication of variations in unemployment insurance (different replacement rate) or the optimal unemployment benefit formula on welfare.

It was not only through unemployment insurance generosity that the government could maximize the welfare of workers. Direct transfers to households and firms also played a role in mitigating the impact of the crisis on workers. A recent study by Gorn and Trigari (2024) argues that unemployment benefits can be an important instrument to stabilize the aggregate demand channel. When temporary closing of nonessential businesses was the proper action to curb the spread of the virus, the result was a major shift in US government policies towards redistributive policies. There are several studies that made concrete use of quantitative experiments to understand the consequences of such policies. For example, Mitman and Rabinovich (2015) show that unemployment insurance policy amplifies the response of unemployment and delivers substantial welfare gains. They suggested that the optimal unemployment schedule is procyclical, exhibiting a positive correlation with labor productivity and a negative correlation with unemployment, whereas Kekre (2023) find that this policy can be contractionary under flexible prices.

7 Conclusion

In this paper, we study the effects on the US labor market of business closures during the pandemic crisis. The inclusion of restrictions on US business to the specification in the aggregate productivity shock generates a reduction in labor market activity and translates into a severe episode of economic downturn.

We emphasize the estimation of labor-labor elasticity, deemed crucial to map the wage differentials and differences in hours worked across types of workers. Our model makes clear that a negative shock to aggregate productivity leads to a contraction in total hours worked and wages. Finally, the magnitude of the shock responses differs by worker age, skill, and origin, suggesting that the productivity shock was unequally distributed during the pandemic.

The intuition for this result is that the business-closure orders deliver a fall in total hours worked. Recent work by Peri and Zaiour (2023) points out a presumable fall in the number of foreign workers in 2021. Tuzemen (2022) emphasizes a similar conclusion, documenting a decline in the labor force and the participation rate in the US at the onset of the pandemic.

Coibion et al. (2020) study the implication of the pandemic for the labor force participation rate and unemployment, using large-scale household survey data. The nature of the business-closure orders shock and the dramatic increase in the unemployment rate during the crisis episode affected labor demand and dampened nominal wages. However, considering another channel through which the pandemic-induced economic disruption affected labor markets, such as the decline in labor supply, the direct health shock to workers, and the induced skill shortage, would be fruitful for understanding post-pandemic labor market dynamics. We believe our paper provides many directions for future investigation.

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APPENDIX:

Appendix A Model extensions: results

In this section, we present the results summarized in Section 6. First, we describe the baseline framework results using estimates from (Ottaviano and Peri, 2012). Second, the impulse response function results of the model with capital, a fiscal side, and households face a risk of unemployment. Third, the model with inelastic labor supply: We assume that both young and middle wages ω^y and ω^m are constant implying an inelastic labor supply curve.

In Table 7 we report the elasticity of substitution parameters from (Ottaviano and Peri, 2012), which are used in the sensitivity analysis section.

Table 7: Sensitivity analysis: elasticity of substitution parameters

	Elasticity of substitution						
	$1/\varrho - 1$	$1/\upsilon - 1$	$1/\varepsilon - 1$	$1/\zeta - 1$	$1/s$	$1/\sigma - 1$	$1/\xi - 1$
Values	-0.05	-0.05	-0.05	-0.05	-0.28	-0.32	-0.32
	Elasticity of substitution parameters						
	ϱ	υ	ε	ζ	s	σ	ξ
Values	1.0526	1.0526	1.0526	1.0526	1.3888	1.4705	1.4705

Note: The implied elasticity of substitution parameters using the estimates from Ottaviano and Peri (2012).

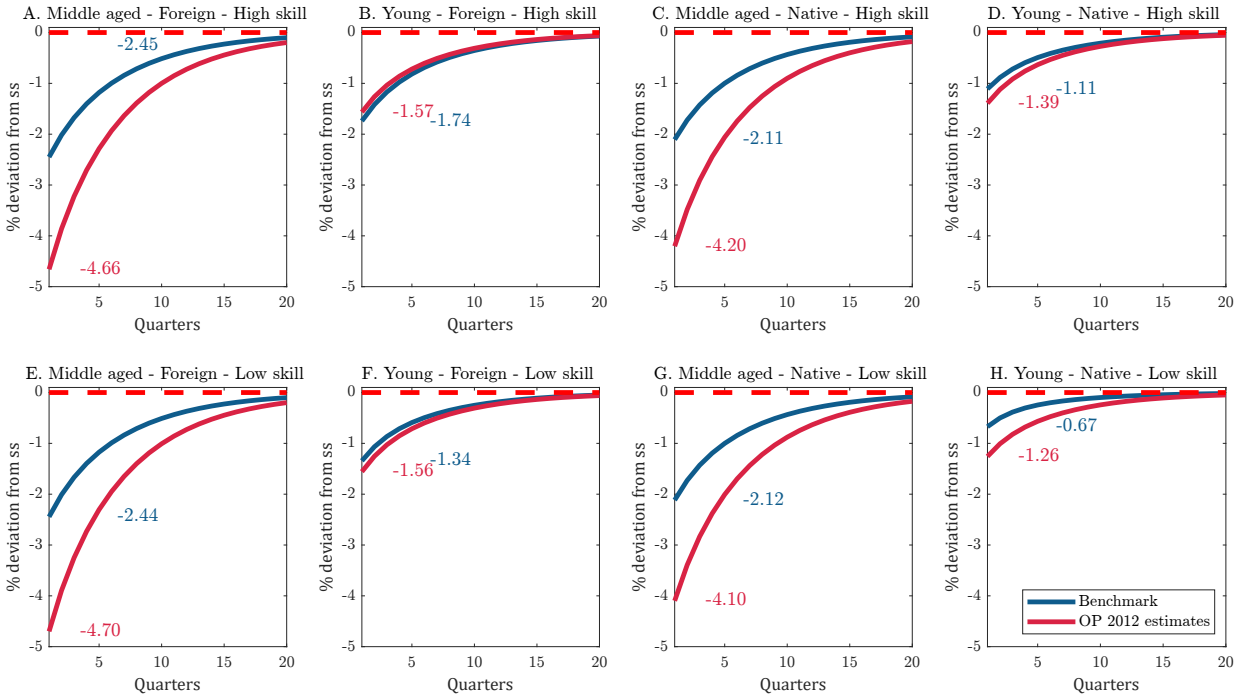


Figure 9: The effect of negative productivity shock on hours worked

Notes: The Figure shows hours worked impulse response functions to a negative shock to productivity of the benchmark model and the counterfactual model using (Ottaviano and Peri, 2012) estimates.

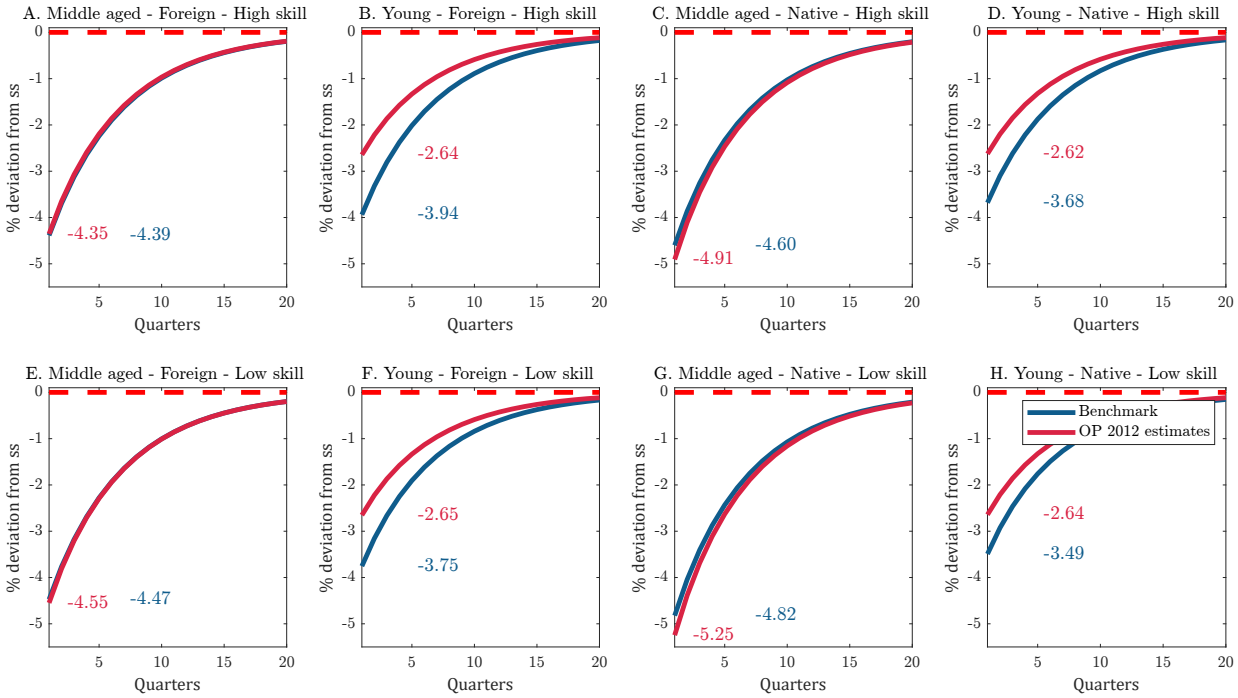


Figure 10: The effect of negative productivity shock on wages

Notes: The Figure shows wage impulse response functions to a negative shock to productivity of the benchmark model and the counterfactual model using (Ottaviano and Peri, 2012) estimates.

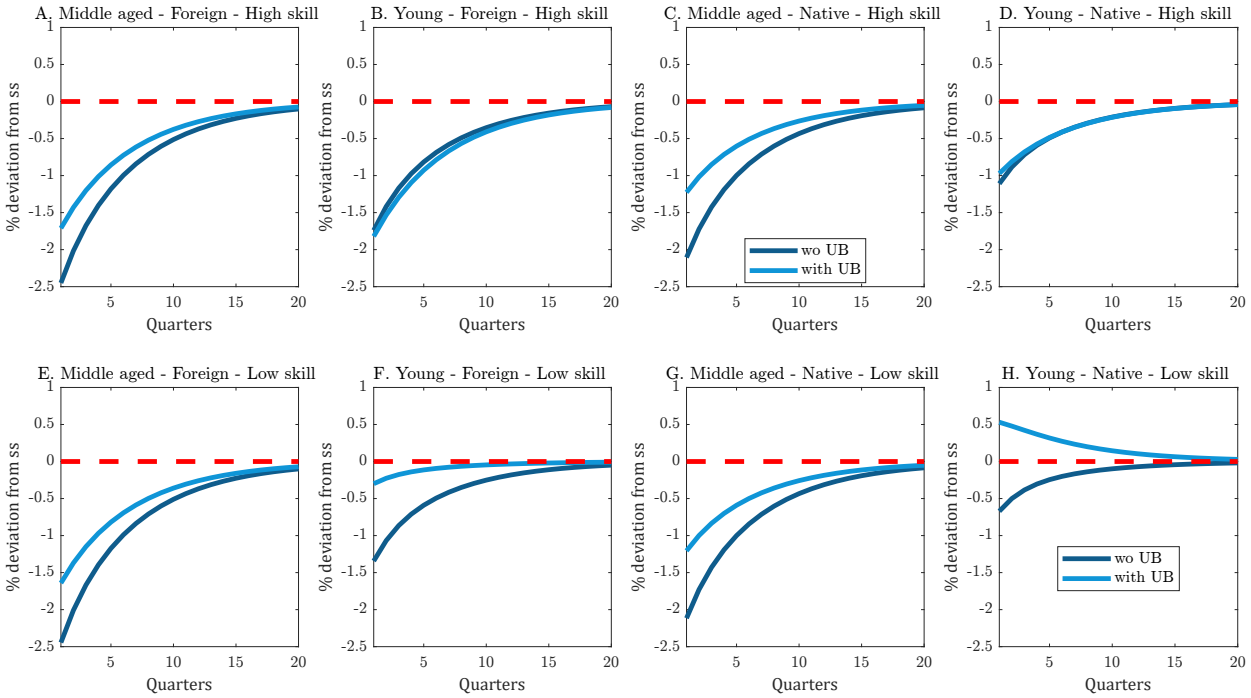


Figure 11: The effect of negative productivity shock on hours worked

Notes: The Figure shows hours worked impulse response functions to a negative shock to productivity of the benchmark model and the counterfactual model with extension of unemployment benefits.

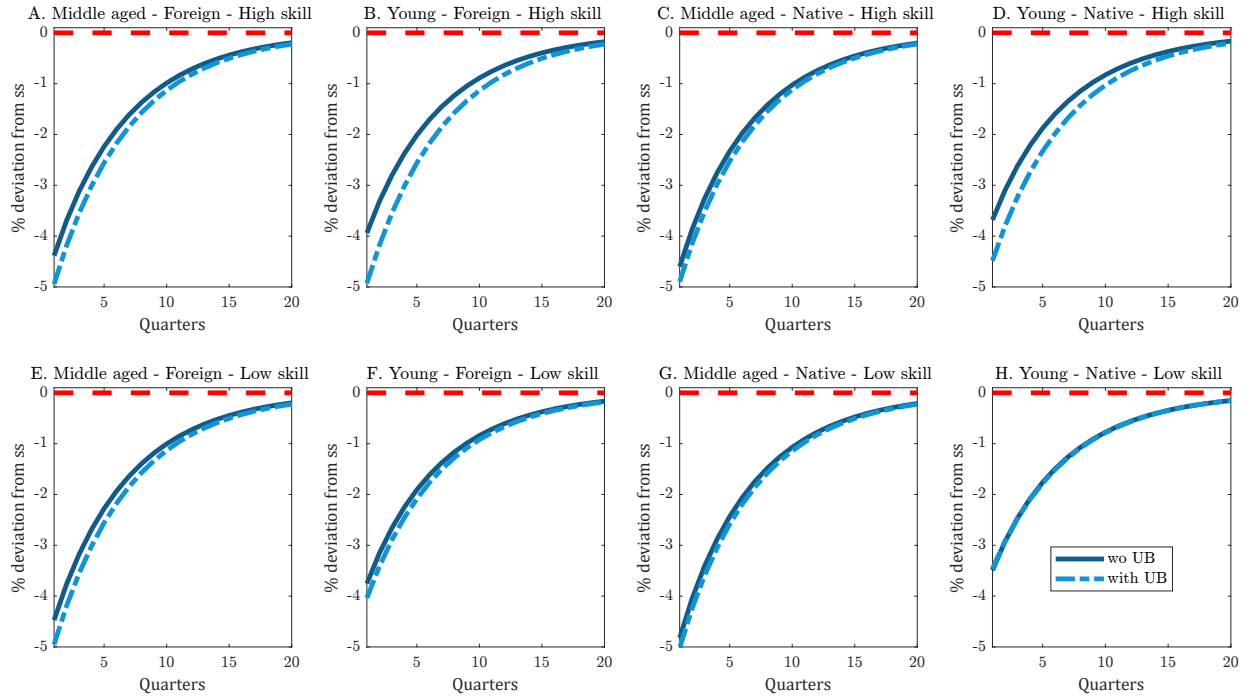


Figure 12: The effect of negative productivity shock on wages

Notes: The Figure shows wage impulse response functions to a negative shock to productivity of the benchmark model and the counterfactual model with extension of unemployment benefits.

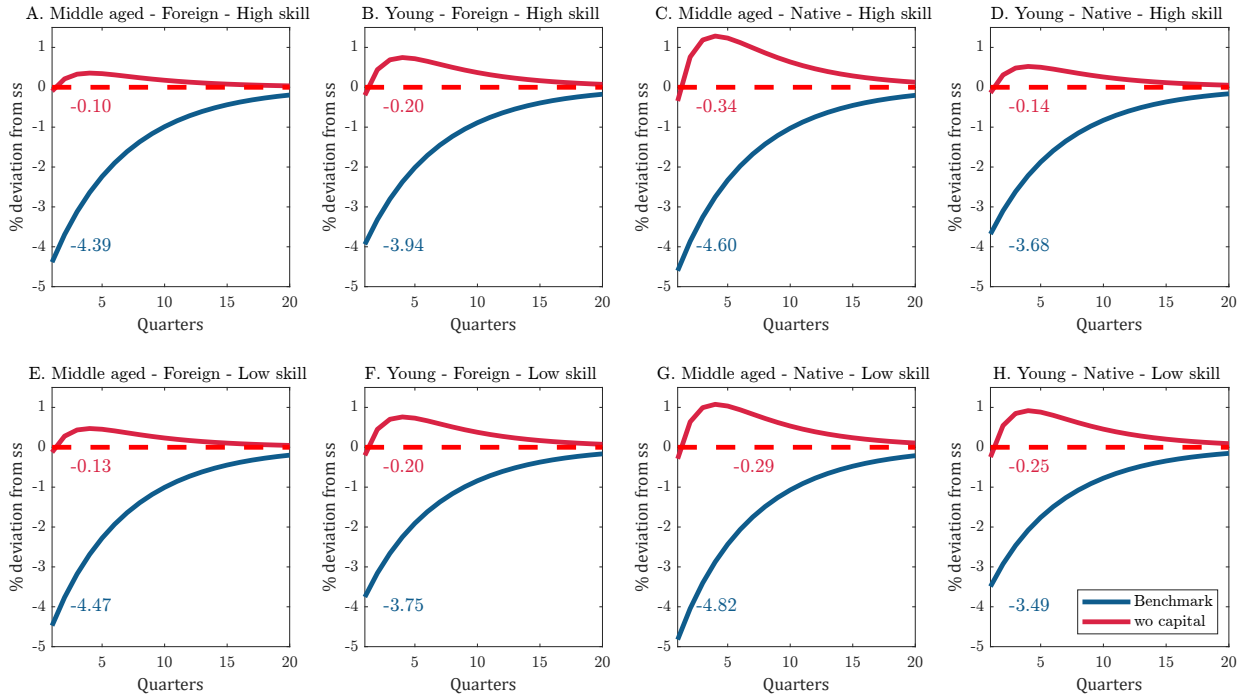


Figure 13: Wage response to a negative productivity shock

Notes: The Figure presents wage impulse response functions to a negative shock to productivity of the benchmark model and the counterfactual model without capital.

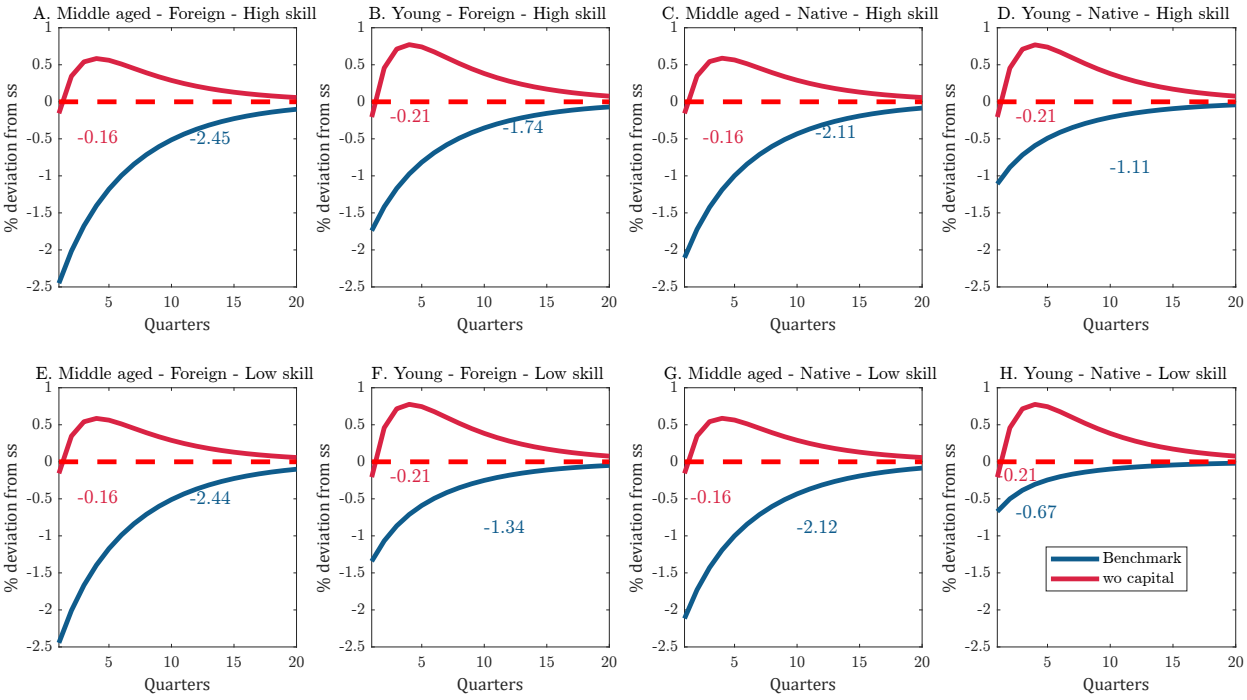


Figure 14: Labor response to a negative productivity shock

Notes: The Figure shows hours worked impulse response functions to a negative shock to productivity of the benchmark model and the counterfactual model without capital.

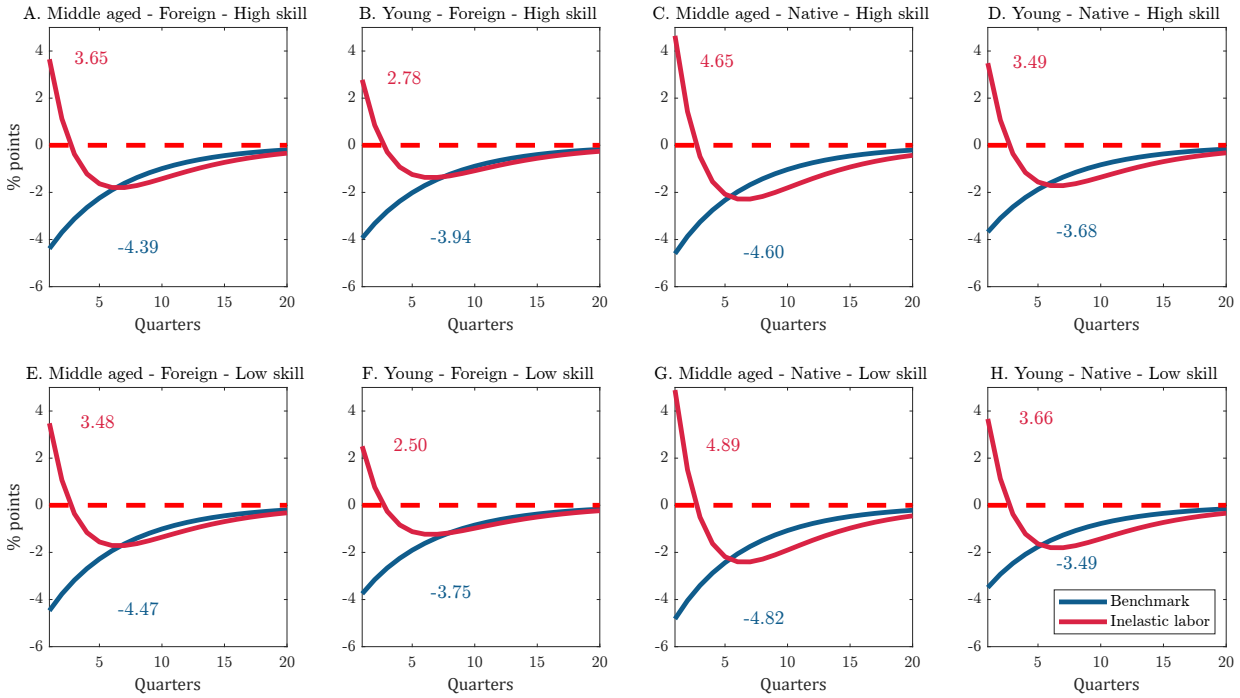


Figure 15: Wage response to a negative productivity shock

Notes: The Figure presents wage impulse response functions to a negative shock to productivity of the benchmark model and the counterfactual model with inelastic labor supply.

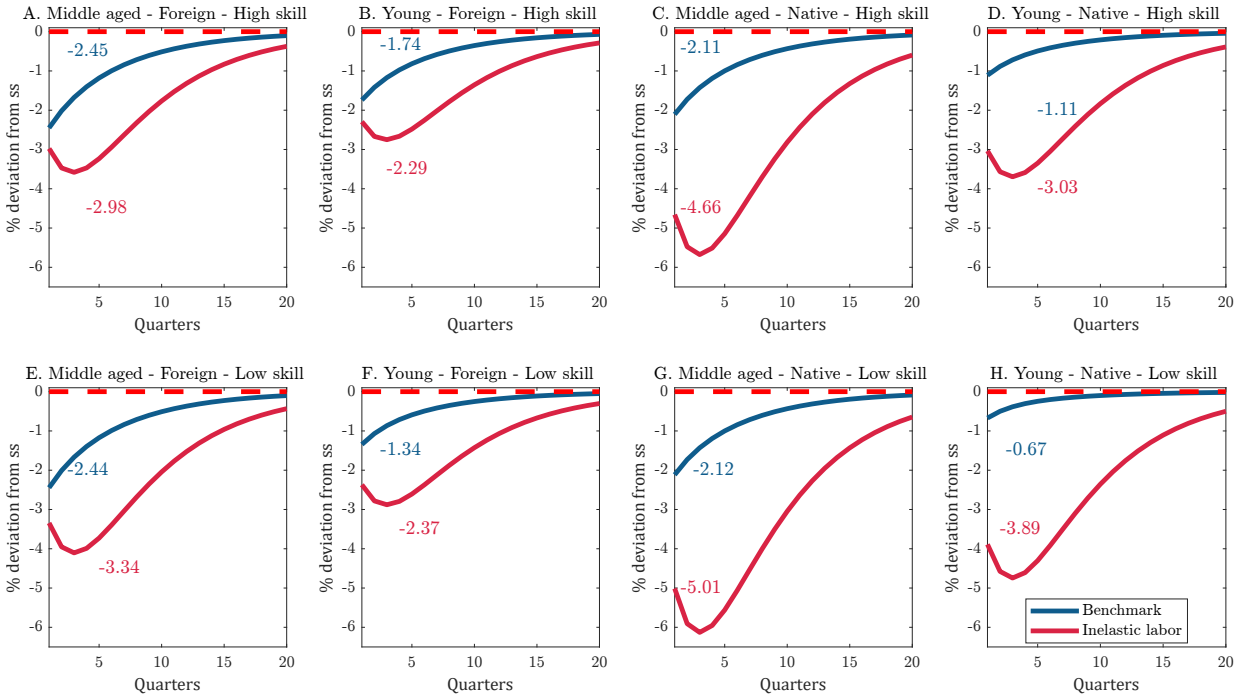


Figure 16: Labor response to a negative productivity shock

Notes: The Figure shows hours worked impulse response functions to a negative shock to productivity of the benchmark model and the counterfactual model with inelatic labor supply.

Appendix B Sample

We describe the 3 digits NIACS code for each sector of our sample.

Table 8: Industry code

Industry description	Code	Industry description	Code
Accommodation	721	Wood products	321
Administrative and support services	561	Air transportation	481
Ambulatory health care services	621	Broadcasting and telecommunications	515-517
Amusements, gambling, and recreation industries	713	Data processing, internet publishing, and other information services	518-519
Apparel and leather and allied products	315-316	Educational services	611
Chemical products	325	Farms	111-112
Computer and electronic products	334	Food and beverage stores	445
Electrical equipment, appliances, and components	335	Forestry, fishing, and related activities	113-114-115
Fabricated metal products	332	General merchandise stores	451-452-453-454
Federal Reserve banks, credit intermediation, and related activities	521-522	Management of companies and enterprises	551
Food and beverage and tobacco products	311-312	Mining, except oil and gas	212
Food services and drinking places	722	Motion picture and sound recording industries	512
Furniture and related products	337	Motor vehicle and parts dealers	441
Hospitals	622	Oil and gas extraction	211
Insurance carriers and related activities	524	Other retail	442-443-444-446-447-448
Machinery	333	Other transportation and support activities	487-488-491-492
Miscellaneous manufacturing	339	Pipeline transportation	486
Nonmetallic mineral products	327	Professional, scientific, and technical services	541
Nursing and residential care facilities	623	Publishing industries, except internet (includes software)	511
Paper products	322	Rail transportation	482
Performing arts, spectator sports, museums, and related activities	711-712	Support activities for mining	213
Petroleum and coal products	324	Transit and ground passenger transportation	485
Plastics and rubber products	326	Truck transportation	484
Primary metals	331	Warehousing and storage	493
Printing and related support activities	323	Water transportation	483
Real estate	531	Construction	23
Rental and leasing services and lessors of intangible assets	532-533	Utilities	221
Social assistance	624	Wholesale trade	423-424-425
Textile mills and textile product mills	313-314	Motor vehicles, bodies and trailers, and parts and Other transportation equipment	336
Waste management and remediation services	562	Funds, trusts, and other financial vehicles and Securities, commodity contracts, and investments	523-525
		Other services, except government	811-812-813

[a] Note: We assign "Not specified utilities" to the 221 NAICS code, "Not specified food industries" to 312 NAICS code. We assign "Knitting mills, and apparel knitting mills" to 315 NAICS code, and 315M to 315. We assign "Not specified metal industries" to 332 NAICS code, and 333MS to 333. We assign "Not specified wholesale" trade to 424. We assign 44511 to 445, 45121 to 452, 454110 to 454. We assign "Not specified retail trade" to 454. We assign "Banking and related activities" to 522, "Securities, commodities, funds, trusts, and other financial investments" to 523, and 532M2 to 532. We assign "Commercial, industrial, and other intangible assets rental and leasing" to 533, we also assign 55 to 551.

[b] In the Bureau of Economic Analysis data, the "Construction" industry is aggregated by sector, whereas we have affected share observations by three sub sectors of the "Construction" industry. To overcome this limitation, we assign the value of affected share of the sub-sector "Construction of buildings" to the overall sector "construction". Data sources: The Bureau of Economic Analysis.

Appendix C Productivity shock parameters

upadated:

Table 9: Estimation of the productivity shock parameters

Dependent variable	$\ln(z_{i,t})$	$\ln(y_{i,t})$
$d_{i,t}$	-0.002***	-0.003***
	(0.001)	(0.001)
$\ln(l_{i,t})$		0.477***
		(0.004)
$\ln(z_{i,t-1})$	0.983***	
	(0.003)	
Observations	610	671
R^2	0.997	0.970
Adjusted R^2	0.997	0.970
Residual Std. Error	0.303	0.903
	(df=608)	(df=669)
F Statistic	108899.366***	14389.943***
	(df=2; 608)	(df=2; 669)

Note: Standard error in parentheses are clustered at quarter level $*p < .1, **p < .05, ***p < .01$

Appendix D Model Computations

The model derivations of four model versions are pesented in this Appendix.

- Benchmark framework
- Model without capital
- Model with a fiscal side and unemployment insurance benefits
- Model with an inelastic labor supply

To simplify notation, we suppress industry indexes i.

D.1 Benchmark Model

In this Section, we present the benchmark model computation. This model accounts for labor-intensive margin with a risk of unemployment, government collects no taxes and make no transfers, and a Nested CES production function with capital input.

D.1.1 Employment and Labor Market Matching

Households produce labor services, the employment rate e_t is the share of workers in the labor firce $\mathcal{N}$, that is

$$e_t = (1 - u_t)$$

We assume that workers can be either employed E or unemployed U, and N represents the labor force, with $N = E + U$. The labor force is normalized to 1, and the employment rate is given by $e_t = 1 - u_t$. Total employment E_t in this economy is measured as man-hours quantity supplied and takes the form:

$$E_t = (1 - \mathcal{Q}) \sum_{s \in h,l} \sum_{k \in f,n} \sum_{j \in y,m,o} l^j_{s,k,t} + sU_t$$

where $l^j_{s,k,t}$ represents the hours supplied by different types of workers and U_t denotes total unemployment. The term $1 - \mathcal{Q}$ governs how hours supplied are connected to total employment, representing the rate at which employed workers remain employed. We assume that unemployed workers become employed with probability s and employed workers become unemployed with probability $\mathcal{Q}$. This equation can be expresses in terms of employment rate by normalizing by labor force object N, that is we obtain

$$E_t/N = e_t = (1 - \mathcal{Q}) \sum_{s \in h,l} \sum_{k \in f,n} \sum_{j \in y,m,o} l^j_{s,k,t}/N + su_t.$$

D.1.2 Households

The lifetime utility is given by

$$maximize \quad U(c^j_{s,k,t}, l^j_{s,k,t}) = \beta\Bigg\{ \ln(c^j_{s,k,t}) - \frac{(l^j_{s,k,t})^{1+\eta}}{1+\eta} \Bigg\}$$

where $0 \leq \beta \leq 1$ is the discount factor and η is the curvature on the disutility of labor. All agents maximize the utility subject to the budget constraint and capital accumulation.

$$c^j_{s,k,t} + a^j_{s,k,t} = e_t \frac{\omega^j_{s,k,t}}{\pi_t} l^j_{s,k,t} + a^j_{s,k,t-1}(1 + r_{t-1})$$

We let $c^j_{s,k}$ denote the per capita consumption across age groups j at time t where $j \in \{y, m, o\}$ identifies age groups: young, middle-aged, old age. l^y and $l^m_{s,k}$ denote labor supply by young and middle-aged individuals at wage rate $\omega^j_{s,k}$. The young and middle-aged agents consume and work in each period, in the first and second period of life, while old agent earn no labor income in the third period of life. Both young and middle-aged agent save, but only the middle-aged and old age agents receive asset earnings at risk-free interest rate r.

Using the substitution method we can then write the Lagrangian as follows

$$\begin{aligned}\mathcal{L}_t = \Bigg[&\beta\Bigg\{ \ln(c^m_{s,k,t}) - \frac{(l^m_{s,k,t})^{1+\eta}}{1+\eta} \Bigg\} \\ &+ \lambda_t \Bigg(e_t \frac{\omega^j_{s,k,t}}{\pi_t} l^j_{s,k,t} + a^j_{s,k,t-1}(1 + r_{t-1}) - c^j_{s,k,t} - a^j_{s,k,t} \Bigg) \Bigg]\end{aligned}$$

The household optimality conditions with respect to consumption: $c^m_{s,k,t}$, $c^y_{s,k,t-1}$, $c^o_{s,k,t+1}$, and labor: $l^m_{s,k,t}$, and $l^y_{s,k,t-1}$ are derived as follow

$$\frac{\partial \mathcal{L}_t}{\partial c^j_{s,k,t}} : \qquad \frac{\beta}{c^j_{s,k,t}} = \lambda_t \tag{D.1}$$

$$\frac{\partial \mathcal{L}_t}{\partial l^j_{s,k,t}} : \qquad \beta (l^j_{s,k,t})^\eta = \lambda_t e_t \frac{\omega^j_{s,k,t}}{\pi_t} \tag{D.2}$$

$$\frac{\partial \mathcal{L}_t}{\partial a^j_{s,k,t}} : \qquad \lambda_t = \lambda_{t+1}(1 + r_t) \tag{D.3}$$

from D.1 and D.2 we obtain

$$\frac{1}{c^j_{s,k,t}} = \frac{(l^j_{s,k,t})^\eta \pi_t}{e_t \omega^j_{s,k,t}}$$

D.1.3 Capital accumulation and market clearing condition

Capital is a predetermined variable that evolves according to the following function:

$$k_{t+1} = (1 - \delta_k)k_t + i_t \tag{D.4}$$

Market clearing condition

$$k_{t+1} - (1 - \delta^k)k_t = y_t - \Sigma_{s\in h,l}\Sigma_{k\in f,n}\Sigma_{j\in y,m,o}c^j_{s,k,t}. \tag{D.5}$$

D.1.4 Nested production function

Technology We introduce a standard CES production function

$$y_t = (k_t)^{1-\gamma}(z_t(\alpha(l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^s)^\gamma \tag{D.6}$$

where z_t is the productivity shock, α is the share parameter of labor input. For simplicity, the producer uses labor factor only as input. l^y_t and l^m_t denote the hours worked by young and middle-aged workers, respectively. The parameter s represents the elasticity of substitution. The hours worked by young is given by

$$l^y_t = [(\phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}} + \phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}} + (\phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}} + \phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}}]^\sigma$$

where l^y_t denotes the total hours worked by young individuals. This equation emphasizes that hours worked by younger individuals young is broken down into various origin (native or foreign) and skill (high or low) groups. We assume that labor inputs l^y_t is a vector of four types of labor: hours worked by foreign worker in high skilled sector $l^y_{h,f,t}$; hours worked by native worker in high skilled sector $l^y_{h,n,t}$; hours worked by foreign worker in low skilled sector $l^y_{l,f,t}$; and hours worked by native worker in low skilled sector $l^y_{l,n,t}$.
Similarly, the hours worked by middle-aged individuals are given by:

$$l^m_t = [(\theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\varepsilon}} + \theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\xi}} + (\theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}} + \theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}}]^\xi$$

where the term l^m_t represents the total hours worked by young individuals. The hours worked by middle-aged individuals include hours worked by natives and foreigners and by high and low skilled. The labor inputs l^m_t is then a vector of four types of labor: hours worked by foreign worker in high skilled sector $l^m_{h,f,t}$; hours worked by native worker in high skilled sector $l^m_{h,n,t}$; hours worked by foreign worker in low skilled sector $l^m_{l,f,t}$; and hours worked by native worker in low skilled sector $l^m_{l,n,t}$.

Producer Problem The problem of the intermediate good producer is to minimize the cost

$$\omega^y_{h,f,t}l^y_{h,f,t} + \omega^y_{h,n,t}l^y_{h,n,t} + \omega^y_{l,f,t}l^y_{l,f,t} + \omega^y_{l,n,t}l^y_{l,n,t} + \omega^m_{h,f,t}l^m_{h,f,t} + \omega^m_{h,n,t}l^m_{h,n,t} + \omega^m_{l,f,t}l^m_{l,f,t} + \omega^m_{l,n,t}l^m_{l,n,t}$$

subject to the production function that relates outputs to inputs. The first order conditions with respect to $l^y_{h,f,t}$ and $l^y_{h,n,t}$ are given by:

$$\frac{\partial \mathcal{L}_t}{\partial l^y_{h,f,t}}: \quad \omega^y_{h,f,t} - \mu_t \gamma k_t^{1-\gamma} \alpha z_t \phi_{h,f} (l^y_{h,f,t})^{\frac{1}{\varrho}-1} (\phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}} + \phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}-1}$$
$$((\phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}} + \phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}} + (\phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}} + \phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}})^{\sigma-1}$$
$$(l^y_t)^{\frac{1}{s}-1} (\alpha (l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1}$$
$$(z_t(\alpha(l^y_t)^{\frac{1}{s}} + (l^m_t)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0$$

and

$$\frac{\partial \mathcal{L}_t}{\partial l^y_{h,n,t}}: \quad \omega^y_{h,n,t} - \mu_t \gamma k_t^{1-\gamma} \alpha z_t \phi_{h,n} (l^y_{h,n,t})^{\frac{1}{\varrho}-1} (\phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}} + \phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}-1}$$
$$((\phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}} + \phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}} + (\phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}} + \phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}})^{\sigma-1}$$
$$(l^y_t)^{\frac{1}{s}-1} (\alpha (l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1}$$
$$(z_t(\alpha(l^y_t)^{\frac{1}{s}} + (l^m_t)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0$$

The optimality equation is

$$\frac{\omega^y_{h,f,t}}{\omega^y_{h,n,t}} = \frac{\phi_{h,f}}{\phi_{h,n}} \left(\frac{l^y_{h,f,t}}{l^y_{h,n,t}} \right)^{\frac{1}{\varrho}-1}$$

The first order conditions with respect to $l^y_{l,f,t}$ and $l^m_{l,n,t}$ are given by:

$$\frac{\partial \mathcal{L}_t}{\partial l^y_{l,f,t}}: \quad \omega^y_{l,f,t} - \mu_t \gamma k_t^{1-\gamma} \alpha z_t \phi_{l,f} (l^y_{l,f,t})^{\frac{1}{\upsilon}-1} (\phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}} + \phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}-1}$$
$$((\phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}} + \phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}} + (\phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}} + \phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}})^{\sigma-1}$$
$$(l^y_t)^{\frac{1}{s}-1} (\alpha (l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1}$$
$$(z_t(\alpha(l^y_t)^{\frac{1}{s}} + (l^m_t)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0$$

and

$$\frac{\partial \mathcal{L}_t}{\partial l^y_{l,n,t}}: \quad \omega^y_{l,n,t} - \mu_t \gamma k_t^{1-\gamma} \alpha z_t \phi_{l,n} (l^y_{l,n,t})^{\frac{1}{\upsilon}-1} (\phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}} + \phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}-1}$$
$$((\phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}} + \phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}} + (\phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}} + \phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}})^{\sigma-1}$$
$$(l^y_t)^{\frac{1}{s}-1} (\alpha (l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1}$$
$$(z_t(\alpha(l^y_t)^{\frac{1}{s}} + (l^m_t)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0$$

The optimality equation is given by

$$\frac{\omega^y_{l,f,t}}{\omega^y_{l,n,t}} = \frac{\phi_{l,f}}{\phi_{l,n}} \left(\frac{l^y_{l,f,t}}{l^y_{l,n,t}} \right)^{\frac{1}{\upsilon}-1}$$

The first order conditions with respect to $l^m_{h,f,t}$ and $l^m_{h,n,t}$ are given by:

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial l^m_{h,f,t}}: \quad & \omega^m_{h,f,t} - \mu_t \gamma k_t^{1-\gamma}(1-\alpha) z_t \theta_{h,f} (l^m_{h,f,t})^{\frac{1}{\epsilon}-1} (\theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}} + \theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}-1} \\ & ((\theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}} + \theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}} + (\theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}} + \theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}})^{\xi-1} \\ & (l^m_t)^{\frac{1}{s}-1} (\alpha (l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} \\ & (z_t(\alpha (l^y_t)^{\frac{1}{s}} + (l^m_t)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial l^m_{h,n,t}}: \quad & \omega^m_{h,n,t} - \mu_t \gamma k_t^{1-\gamma}(1-\alpha) z_t \theta_{h,n} (l^m_{h,n,t})^{\frac{1}{\epsilon}-1} (\theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}} + \theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}-1} \\ & ((\theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}} + \theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}} + (\theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}} + \theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}})^{\xi-1} \\ & (l^m_t)^{\frac{1}{s}-1} (\alpha (l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} \\ & (z_t(\alpha (l^y_t)^{\frac{1}{s}} + (l^m_t)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0\end{aligned}$$

The optimality equation is given by

$$\frac{\omega^m_{h,f,t}}{\omega^m_{h,n,t}} = \frac{\theta_{h,f}}{\theta_{h,n}} \left(\frac{l^m_{h,f,t}}{l^m_{h,n,t}} \right)^{\frac{1}{\epsilon}-1}$$

The first order conditions with respect to $l^m_{l,f,t}$ and $l^m_{l,n,t}$ are given by:

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial l^m_{l,f,t}}: \quad & \omega^m_{l,f,t} - \mu_t \gamma k_t^{1-\gamma}(1-\alpha) z_t \theta_{l,f} (l^m_{l,f,t})^{\frac{1}{\zeta}-1} (\theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}} + \theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}-1} \\ & ((\theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}} + \theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}} + (\theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}} + \theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}})^{\xi-1} \\ & (l^m_t)^{\frac{1}{s}-1} (\alpha (l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} \\ & (z_t(\alpha (l^y_t)^{\frac{1}{s}} + (l^m_t)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial l^m_{l,n,t}}: \quad & \omega^m_{l,n,t} - \mu_t \gamma k_t^{1-\gamma}(1-\alpha) z_t \theta_{l,n} (l^m_{l,n,t})^{\frac{1}{\zeta}-1} (\theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}} + \theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}-1} \\ & ((\theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}} + \theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}} + (\theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}} + \theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}})^{\xi-1} \\ & (l^m_t)^{\frac{1}{s}-1} (\alpha (l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} \\ & (z_t(\alpha (l^y_t)^{\frac{1}{s}} + (l^m_t)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0\end{aligned}$$

The optimality equation

$$\frac{\omega^m_{l,f,t}}{\omega^m_{l,n,t}} = \frac{\theta_{l,f}}{\theta_{l,n}} \left(\frac{l^m_{l,f,t}}{l^m_{l,n,t}} \right)^{\frac{1}{\zeta}-1}$$

Further optimality conditions

$$\begin{aligned}
&\mathcal{L}_t = \omega^y_{h,t} l^y_{h,t} + \omega^y_{l,t} l^y_{l,t} + \omega^m_{h,t} l^m_{h,t} + \omega^m_{l,t} l^m_{l,t} \\
&- \mu_t k_t^{(1-\gamma)} (z_t(\alpha(((\phi_h(l^y_{h,t}))^{\frac{1}{\sigma}} + (\phi_l(l^y_{l,t}))^{\frac{1}{\sigma}})^{\sigma})^{\frac{1}{s}} + (1-\alpha)(((\theta_h(l^m_{h,t}))^{\frac{1}{\xi}} + (\theta_l(l^m_{l,t}))^{\frac{1}{\xi}})^{\xi})^{\frac{1}{s}})^{s})^{\gamma}
\end{aligned}$$

The first order conditions with respect to $l^y_{h,t}$ and $l^y_{l,t}$ are given by:

$$\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial l^y_{h,t}} : \quad & \omega^y_{h,t} = \alpha\gamma k_t^{1-\gamma} \mu t \phi_h z_t (l^y_{h,t}\phi_h)^{\frac{1}{\sigma}-1} \\
& ((l^y_{h,t}\phi_h)^{1/\sigma} + (l^y_{l,t}\phi_l)^{1/\sigma})^{\sigma-1} (l^y_t)^{\frac{1}{s}-1} \\
& (\alpha (l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} \\
& (z_t(\alpha(l^y_t)^{\frac{1}{s}} + (l^m_t)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial l^y_{l,t}} : \quad & \omega^y_{l,t} = \alpha\gamma k_t^{1-\gamma} \mu t \phi_l z_t (l^y_{l,t}\phi_l)^{\frac{1}{\sigma}-1} \\
& ((l^y_{h,t}\phi_h)^{1/\sigma} + (l^y_{l,t}\phi_l)^{1/\sigma})^{\sigma-1} (l^y_t)^{\frac{1}{s}-1} \\
& (\alpha (l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} \\
& (z_t(\alpha(l^y_t)^{\frac{1}{s}} + (l^m_t)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0
\end{aligned}$$

The optimality equation is given by

$$\frac{\omega^y_{h,t}}{\omega^y_{l,t}} = \frac{\phi_h}{\phi_l} \frac{\left(\phi_h l^y_{h,t}\right)^{\frac{1}{\sigma}-1}}{\left(\phi_l l^y_{l,t}\right)^{\frac{1}{\sigma}-1}}$$

The first order conditions with respect to $l^m_{h,t}$ and $l^m_{l,t}$ are given by

$$\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial l^m_{h,t}} : \quad & \omega^m_{h,t} = (1-\alpha)\gamma k_t^{1-\gamma} \mu t \theta_h z_t (l^m_{h,t}\theta_h)^{\frac{1}{\xi}-1} \\
& ((l^m_{h,t}\theta_h)^{1/\xi} + (l^m_{l,t}\theta_l)^{1/\xi})^{\xi-1} (l^m_t)^{\frac{1}{s}-1} \\
& (\alpha (l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} \\
& (z_t(\alpha(l^y_t)^{\frac{1}{s}} + (l^m_t)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0
\end{aligned}$$

and

$$\frac{\partial \mathcal{L}_t}{\partial l_{l,t}^m}: \quad \omega_{l,t}^m = (1-\alpha)\gamma k_t^{1-\gamma}\mu t\theta_l z_t (l_{l,t}^m\theta_l)^{\frac{1}{\xi}-1} ((l_{h,t}^m\theta_h)^{1/\xi} + (l_{l,t}^m\theta_l)^{1/\xi})^{\xi-1}(l_t^m)^{\frac{1}{s}-1} (\alpha(l_t^y)^{\frac{1}{s}} + (1-\alpha)(l_t^m)^{\frac{1}{s}})^{s-1} (z_t(\alpha(l_t^y)^{\frac{1}{s}} + (l_t^m)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0$$

The optimality equation is given by

$$\cdot\frac{\omega_{h,t}^m}{\omega_{l,t}^m} = \frac{\theta_h}{\theta_l}\frac{\left(\theta_h l_{h,t}^m\right)^{\frac{1}{\xi}-1}}{\left(\theta_l l_{l,t}^m\right)^{\frac{1}{\xi}-1}}$$

$$\mathcal{L}_t = \omega_t^y l_t^y + \omega_t^m l_t^m - \mu_t k_t^{(1-\gamma)}(z_t(\alpha(\phi(l_t^y))^{\frac{1}{s}} + (1-\alpha)(\theta(l_t^m))^{\frac{1}{s}})^s)^\gamma$$

The first order conditions with respect to l_t^y and l_t^m are given by:

$$\frac{\partial \mathcal{L}_t}{\partial l_t^y}: \quad \omega_t^y = \alpha\gamma k_t^{1-\gamma}\mu_t\phi z_t (l_t^y\phi)^{\frac{1}{s}-1} \left(\alpha(l_t^y\phi)^{\frac{1}{s}} + (l_t^m\theta)^{\frac{1}{s}}(1-\alpha)\right)^{s-1} \left(z_t(\alpha(l_t^y\phi)^{\frac{1}{s}} + (l_t^m\theta)^{\frac{1}{s}}(1-\alpha))^s\right)^{\gamma-1}$$

and

$$\frac{\partial \mathcal{L}_t}{\partial l_t^m}: \quad \omega_t^m = (1-\alpha)\gamma k_t^{1-\gamma}\mu_t\theta z_t (l_t^m\theta)^{\frac{1}{s}-1} \left(\alpha(l_t^y\phi)^{\frac{1}{s}} + (l_t^m\theta)^{\frac{1}{s}}(1-\alpha)\right)^{s-1} \left(z_t(\alpha(l_t^y\phi)^{\frac{1}{s}} + (l_t^m\theta)^{\frac{1}{s}}(1-\alpha))^s\right)^{\gamma-1}$$

The optimality equation is given by

$$\frac{\omega_t^y}{\omega_t^m} = \frac{\alpha\phi\left(\phi l_t^y\right)^{\frac{1}{s}-1}}{(1-\alpha)\,\theta\left(\theta l_t^m\right)^{\frac{1}{s}-1}}$$

D.2 Model without capital

We present the model computations assuming that firm use only labor as input. We further assume that household face a risk of unemployment, and the economy has no fiscal side. The household optimality condition remain the same as in the baseline model.

D.2.1 Nested production function

Technology We introduce a standard CES production function

$$y_t = z_t(\alpha(l_t^y)^{\frac{1}{s}} + (1-\alpha)(l_t^m)^{\frac{1}{s}})^s \tag{D.7}$$

where z_t is the productivity shock, α is the share parameter of labor input. For simplicity, the producer use labor factor only as in input. l_t^y and l_t^m denote the hours worked by young and middle-aged workers, respectively. The parameter s represent the elasticity of substitution. The hours worked by young is given by

$$l_t^y = [(\phi_{h,f}(l_{h,f,t}^y)^{\frac{1}{\varrho}} + \phi_{h,n}(l_{h,n,t}^y)^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}} + (\phi_{l,f}(l_{l,f,t}^y)^{\frac{1}{\upsilon}} + \phi_{l,n}(l_{l,n,t}^y)^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}}]^{\sigma}$$

where l_t^y denotes the total hours worked by young individuals. This equation emphasizes that hours worked by young workers are broken down into various origin (native or foreign) and skill (high or low) groups. We assume that labor input l_t^y is a vector of four types of labor: hours worked by foreign worker in high skilled sector $l_{h,f,t}^y$; hours worked by native worker in high skilled sector $l_{h,n,t}^y$; hours worked by foreign worker in low skilled sector $l_{l,f,t}^y$; and hours worked by native worker in low skilled sector $l_{l,n,t}^y$.
Similarly, the hours worked by middle-aged individuals are given by:

$$l_t^m = [(\theta_{h,f}(l_{h,f,t}^m)^{\frac{1}{\varepsilon}} + \theta_{h,n}(l_{h,n,t}^m)^{\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\xi}} + (\theta_{l,f}(l_{l,f,t}^m)^{\frac{1}{\zeta}} + \theta_{l,n}(l_{l,n,t}^m)^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}}]^{\xi}$$

the term l_t^m represents the total hours worked by young individuals. The hours worked by middle-aged individuals include hours worked by natives and foreigners and by high and low skilled. The labor input l_t^m is then a vector of four types of labor: hours worked by foreign worker in high skilled sector $l_{h,f,t}^m$; hours worked by native worker in high skilled sector $l_{h,n,t}^m$; hours worked by foreign worker in low skilled sector $l_{l,f,t}^m$; and hours worked by native worker in low skilled sector $l_{l,n,t}^m$.

Producer Problem The problem of the intermediate good producer is to minimize the cost

$$\omega_{h,f,t}^y l_{h,f,t}^y + \omega_{h,n,t}^y l_{h,n,t}^y + \omega_{l,f,t}^y l_{l,f,t}^y + \omega_{l,n,t}^y l_{l,n,t}^y + \omega_{h,f,t}^m l_{h,f,t}^m + \omega_{h,n,t}^m l_{h,n,t}^m + \omega_{l,f,t}^m l_{l,f,t}^m + \omega_{l,n,t}^m l_{l,n,t}^m$$

subject to the production function that relates outputs to inputs. The first order conditions with respect to $l_{h,f,t}^y$ and $l_{h,n,t}^y$ are given by:

$$\frac{\partial \mathcal{L}_t}{\partial l_{h,f,t}^y}: \qquad \omega_{h,f,t}^y - \mu_t \alpha z_t \phi_{h,f}(l_{h,f,t}^y)^{\frac{1}{\varrho}-1}(\phi_{h,f}(l_{h,f,t}^y)^{\frac{1}{\varrho}} + \phi_{h,n}(l_{h,n,t}^y)^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}-1}$$

$$((\phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}} + \phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}} + (\phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}} + \phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}})^{\sigma-1}$$
$$(l^y_t)^{\frac{1}{s}-1}(\alpha(l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} = 0$$
$$\omega^y_{h,f,t} - \mu_t \alpha z_t \phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}-1}(\phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}} + \phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}-1}$$
$$(l^y_t)^{\frac{\sigma-1}{\sigma}}$$
$$(l^y_t)^{\frac{1}{s}-1}(\alpha(l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} = 0$$

and

$$\frac{\partial \mathcal{L}_t}{\partial l^y_{h,n,t}}: \quad \omega^y_{h,n,t} - \mu_t \alpha z_t \phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}-1}(\phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}} + \phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}-1}$$
$$((\phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}} + \phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}} + (\phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}} + \phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}})^{\sigma-1}$$
$$(l^y_t)^{\frac{1}{s}-1}(\alpha(l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} = 0$$

The optimality equation is

$$\frac{\omega^y_{h,f,t}}{\omega^y_{h,n,t}} = \frac{\phi_{h,f}}{\phi_{h,n}}\left(\frac{l^y_{h,f,t}}{l^y_{h,n,t}}\right)^{\frac{1}{\varrho}-1}$$

The first order conditions with respect to $l^y_{l,f,t}$ and $l^m_{l,n,t}$ are given by:

$$\frac{\partial \mathcal{L}_t}{\partial l^y_{l,f,t}}: \quad \omega^y_{l,f,t} - \mu_t \alpha z_t \phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}-1}(\phi_{lf}(l^y_{l,f,t})^{\frac{1}{\upsilon}} + \phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}-1}$$
$$((\phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}} + \phi_{ln}(l^y_{l,n,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}} + (\phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}} + \phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}})^{\sigma-1}$$
$$(l^y_t)^{\frac{1}{s}-1}(\alpha(l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} = 0$$

and

$$\frac{\partial \mathcal{L}_t}{\partial l^y_{l,n,t}}: \quad \omega^y_{l,n,t} - \mu_t \alpha z_t \phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}-1}(\phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}} + \phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}-1}$$
$$((\phi_{ln}(l^y_{l,n,t})^{\frac{1}{\upsilon}} + \phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}} + (\phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}} + \phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}})^{\sigma-1}$$
$$(l^y_t)^{\sigma})^{\frac{1}{s}-1}(\alpha(l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} = 0$$

The optimality equation is given by

$$\frac{\omega^y_{l,f,t}}{\omega^y_{l,n,t}} = \frac{\phi_{l,f}}{\phi_{l,n}}\left(\frac{l^y_{l,f,t}}{l^y_{l,n,t}}\right)^{\frac{1}{\upsilon}-1}$$

The first order conditions with respect to $l^m_{h,f,t}$ and $l^m_{h,n,t}$ are given by:

$$\frac{\partial \mathcal{L}_t}{\partial l^m_{h,f,t}}: \quad \omega^m_{h,f,t} - \mu_t (1-\alpha) z_t \theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}-1}(\theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}} + \theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}-1}$$

$$((\theta_{hf}(l^m_{h,f,t})^{\frac{1}{\epsilon}} + \theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}} + (\theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}} + \theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}})^{\xi-1}$$
$$(l^m_t)^{\frac{1}{s}-1}(\alpha(l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} = 0$$

and

$$\frac{\partial \mathcal{L}_t}{\partial l^m_{h,n,t}}: \quad \omega^m_{h,n,t} - \mu_t(1-\alpha)z_t\theta_{hn}(l^m_{h,n,t})^{\frac{1}{\epsilon}-1}(\theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}} + \theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}-1}$$
$$((\theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}} + \theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}} + (\theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}} + \theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}})^{\xi-1}$$
$$(l^m_t)^{\frac{1}{s}-1}(\alpha(l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} = 0$$

The optimality equation is given by

$$\frac{\omega^m_{h,f,t}}{\omega^m_{h,n,t}} = \frac{\theta_{h,f}}{\theta_{h,n}}\left(\frac{l^m_{h,f,t}}{l^m_{h,n,t}}\right)^{\frac{1}{\epsilon}-1}$$

The first order conditions with respect to $l^m_{l,f,t}$ and $l^m_{l,n,t}$ are given by:

$$\frac{\partial \mathcal{L}_t}{\partial l^m_{l,f,t}}: \quad \omega^m_{l,f,t} - \mu_t(1-\alpha)z_t\theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}-1}(\theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}} + \theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}-1}$$
$$((\theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}} + \theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}} + (\theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}} + \theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}})^{\xi-1}$$
$$(l^m_t)^{\frac{1}{s}-1}(\alpha(l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} = 0$$

and

$$\frac{\partial \mathcal{L}_t}{\partial l^m_{l,n,t}}: \quad \omega^m_{l,n,t} - \mu_t(1-\alpha)z_t\theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}-1}(\theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}} + \theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}-1}$$
$$((\theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}} + \theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}} + (\theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}} + \theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}})^{\xi-1}$$
$$(l^m_t)^{\frac{1}{s}-1}(\alpha(l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} = 0$$

The optimality equation

$$\frac{\omega^m_{l,f,t}}{\omega^m_{l,n,t}} = \frac{\theta_{l,f}}{\theta_{l,n}}\left(\frac{l^m_{l,f,t}}{l^m_{l,n,t}}\right)^{\frac{1}{\zeta}-1}$$

Further optimality conditions The first order conditions with respect to $l^y_{h,t}$ and $l^y_{l,t}$ are given by:

$$\frac{\partial \mathcal{L}_t}{\partial l^y_{h,t}}: \quad \omega^y_{h,t} = \alpha\mu t\phi_h z_t(l^y_{h,t}\phi_h)^{\frac{1}{\sigma}-1}$$
$$((l^y_{h,t}\phi_h)^{1/\sigma} + (l^y_{l,t}\phi_l)^{1/\sigma})^{\sigma-1}(l^y_t)^{\frac{1}{s}-1}$$
$$(\alpha(l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} = 0$$

and

$$\frac{\partial \mathcal{L}_t}{\partial l^y_{l,t}}: \quad \omega^y_{l,t} = \alpha\mu t\phi_l z_t (l^y_{l,t}\phi_l)^{\frac{1}{\sigma}-1} \left((l^y_{h,t}\phi_h)^{1/\sigma} + (l^y_{l,t}\phi_l)^{1/\sigma}\right)^{\sigma-1} (l^y_t)^{\frac{1}{s}-1} (\alpha(l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} = 0$$

The optimality equation is given by

$$\frac{\omega^y_{h,t}}{\omega^y_{l,t}} = \frac{\phi_h}{\phi_l} \frac{\left(l^y_{h,t}\right)^{\frac{1}{\sigma}-1}}{\left(l^y_{l,t}\right)^{\frac{1}{\sigma}-1}} \tag{D.8}$$

The first order conditions with respect to $l^m_{h,t}$ and $l^m_{l,t}$ are given by:

$$\frac{\partial \mathcal{L}_t}{\partial l^m_{h,t}}: \quad \omega^m_{h,t} = (1-\alpha)\mu t\theta_h z_t (l^m_{h,t}\theta_h)^{\frac{1}{\xi}-1} \left((l^m_{h,t}\theta_h)^{1/\xi} + (l^m_{l,t}\theta_l)^{1/\xi}\right)^{\xi-1} (l^m_t)^{\frac{1}{s}-1} (\alpha(l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} = 0$$

and

$$\frac{\partial \mathcal{L}_t}{\partial l^m_{l,t}}: \quad \omega^m_{l,t} = (1-\alpha)\mu t\theta_l z_t (l^m_{l,t}\theta_l)^{\frac{1}{\xi}-1} \left((l^m_{h,t}\theta_h)^{1/\xi} + (l^m_{l,t}\theta_l)^{1/\xi}\right)^{\xi-1} (l^m_t)^{\frac{1}{s}-1} (\alpha(l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1} = 0$$

The optimality equation is give by

$$\frac{\omega^m_{h,t}}{\omega^m_{l,t}} = \frac{\theta_h}{\theta_l} \frac{\left(l^m_{h,t}\right)^{\frac{1}{\xi}-1}}{\left(l^m_{l,t}\right)^{\frac{1}{\xi}-1}} \tag{D.9}$$

The first order condition with respect to l^y_t and l^m_t are given by

$$\frac{\partial \mathcal{L}_t}{\partial l^y_t}: \quad \omega^y_t = \alpha\mu_t\phi z_t (l^y_t\phi)^{\frac{1}{s}-1} \left(\alpha(l^y_t\phi)^{\frac{1}{s}} + (l^m_t\theta)^{\frac{1}{s}}(1-\alpha)\right)^{s-1}$$

and

$$\frac{\partial \mathcal{L}_t}{\partial l_t^m}: \qquad \omega_t^m = (1-\alpha)\mu_t \theta z_t (l_t^m \theta)^{\frac{1}{s}-1} \left(\alpha (l_t^y \phi)^{\frac{1}{s}} + (l_t^m \theta)^{\frac{1}{s}} (1-\alpha)\right)^{s-1}$$

The optimality equation is given by

$$\frac{\omega_t^y}{\omega_t^m} = \frac{\alpha \phi \left(l_t^y\right)^{\frac{1}{s}-1}}{(1-\alpha)\,\theta \left(l_t^m\right)^{\frac{1}{s}-1}} \tag{D.10}$$

D.3 Model with a fiscal side and unemployment insurance benefits

This version is identical to the baseline framework, the only difference is that workers receive unemployment benefits and the government collects taxes. Only household labor decision will change. The household optimality condition with respect to labor: $l^j_{s,k,t}$ is derived as follows

$$\frac{\partial \mathcal{L}_t}{\partial c^j_{s,k,t}}: \qquad \frac{\beta}{c^j_{s,k,t}} = \lambda_t \tag{D.11}$$

$$\frac{\partial \mathcal{L}_t}{\partial l^j_{s,k,t}}: \qquad \beta(l^j_{s,k,t})^\eta = (1-\tau^c)\lambda_t e_t \frac{\omega^j_{s,k,t}}{\pi_t} \tag{D.12}$$

$$\frac{\partial \mathcal{L}_t}{\partial a^j_{s,k,t}}: \qquad \lambda_t = \lambda_{t+1}(1+r_t) \tag{D.13}$$

from these conditions we obtain

$$\frac{1}{c^j_{s,k,t}} = \frac{(l^j_{s,k,t})^\eta \pi_t}{(1-\tau^c)e_t \omega^j_{s,k,t}}$$

The government finance social transfers by collecting labor taxes, which are given by

$$\tau e_t \frac{\omega^j_{s,k,t}}{\pi_t} l^j_{s,k,t} = \chi^j_{s,k,t}$$

D.4 Model with an inelastic labor supply

In this model version, labor supply is inelastic by assuming that wages in the household maximisation problem are fixed.

D.4.1 Households

The lifetime utility is given by

$$maximize \quad U(c^j_{s,k,t}, l^j_{s,k,t}) = \beta\Bigg\{ \ln(c^j_{s,k,t}) - \frac{(l^j_{s,k,t})^{1+\eta}}{1+\eta} \Bigg\}$$

where $0 \leq \beta \leq 1$ is the discount factor and η is the curvature on the disutility of labor. All agents maximize the utility subject to the budget constraint and capital accumulation.

$$c^j_{s,k,t} + a^j_{s,k,t} = e_t \frac{\omega^j_{s,k}}{\pi_t} l^j_{s,k,t} + a^j_{s,k,t-1}(1 + r_{t-1})$$

We let $c^j_{s,k}$ denote the per capita consumption across age groups j at time t where $j \in \{y, m, o\}$ identifies age group: young, middle-aged, old age. l^y and $l^m_{s,k}$ denote labor supply by young and middle-aged individuals at wage rate $\omega^j_{s,k}$. The young and middle-aged agents consume and work in each period, in the first and second period of life, while old agents earn no labor income in the third period of life. Both young and middle-aged agents save, but only the middle-aged and old age agents receive asset earnings at risk-free interest rate r.

Using the substitution method we can then write the Lagrangian as follows

$$\mathcal{L}_t = \Bigg[\beta\Bigg\{ \ln(c^m_{s,k,t}) - \frac{(l^m_{s,k,t})^{1+\eta}}{1+\eta} \Bigg\} + \lambda_t \Bigg(e_t \frac{\omega^j_{s,k}}{\pi_t} l^j_{s,k,t} + a^j_{s,k,t-1}(1 + r_{t-1}) - c^j_{s,k,t} - a^j_{s,k,t} \Bigg)\Bigg]$$

The household optimality conditions with respect to consumption: $c^m_{s,k,t}$, $c^y_{s,k,t-1}$, $c^o_{s,k,t+1}$, and labor: $l^m_{s,k,t}$, and $l^y_{s,k,t-1}$ are derived as follows

$$\frac{\partial \mathcal{L}_t}{\partial c^j_{s,k,t}} : \qquad \frac{\beta}{c^j_{s,k,t}} = \lambda_t \tag{D.14}$$

$$\frac{\partial \mathcal{L}_t}{\partial l^j_{s,k,t}} : \qquad \beta (l^j_{s,k,t})^{\eta} = \lambda_t e_t \frac{\omega^j_{s,k}}{\pi_t} \tag{D.15}$$

$$\frac{\partial \mathcal{L}_t}{\partial a^j_{s,k,t}} : \qquad \lambda_t = \lambda_{t+1}(1 + r_t) \tag{D.16}$$

from D.14 and D.15 we obtain

$$\frac{1}{c^j_{s,k,t}} = \frac{(l^j_{s,k,t})^{\eta} \pi_t}{e_t \omega^j_{s,k}}$$

Appendix E List of equilibrium conditions

Productivity shock
$$\frac{1}{z_{ss}}\tilde{z}_t = \frac{1}{z_{ss}}\rho^z \tilde{z}_{t-1} + \delta d_t - \epsilon_t^z \tag{E.1}$$

Inflation
$$0 = -\pi_t + \beta\pi_{t+1} - \kappa\mu_t \tag{E.2}$$

Saving decision
$$\lambda_t = \lambda_{t+1}\beta(1+r_t) \tag{E.3}$$

Employment rate
$$e_t = (1-\mathcal{Q})(l^y_{h,f,t} + l^y_{h,n,t} + l^y_{l,f,t} + l^y_{l,n,t} + l^m_{h,f,t} + l^m_{h,n,t} + l^m_{l,f,t} + l^m_{l,n,t})/N + su_t \tag{E.4}$$

Unemployment rate
$$u_t = (1-e_t) \tag{E.5}$$

Consumption (y)
$$\frac{\beta}{c^y_t} = \lambda_t \tag{E.6}$$

Consumption (m)
$$\frac{\beta}{c^m_t} = \lambda_t \tag{E.7}$$

Consumption (o)
$$\frac{\beta}{c^o_t} = \lambda_t \tag{E.8}$$

Labor supply (m)
$$l^m_t = (\lambda_t e_t \frac{\omega^m_t}{\beta\pi_t})^{1/\eta} \tag{E.9}$$

Labor supply (m,h,f)
$$\beta(l^m_{h,f,t})^\eta = \lambda_t e_t \frac{\omega^m_{h,f,t}}{\pi_t} \tag{E.10}$$

Labor supply (m,h,n)
$$\beta(l^m_{h,n,t})^\eta = \lambda_t e_t \frac{\omega^m_{h,n,t}}{\pi_t} \tag{E.11}$$

Labor supply (m,l,f)
$$\beta(l^m_{l,f,t})^\eta = \lambda_t e_t \frac{\omega^m_{l,f,t}}{\pi_t} \tag{E.12}$$

Labor supply (m,l,n)
$$\beta(l^m_{l,n,t})^\eta = \lambda_t e_t \frac{\omega^m_{l,n,t}}{\pi_t} \tag{E.13}$$

Labor supply (y)
$$l^y_t = (\lambda_t e_t \frac{\omega^y_t}{\beta\pi_t})^{1/\eta} \tag{E.14}$$

Labor supply (y,h,f)
$$\beta(l^y_{h,f,t})^\eta = \lambda_t e_t \frac{\omega^y_{h,f,t}}{\pi_t} \tag{E.15}$$

Labor supply (y,h,n)
$$\beta(l^y_{h,n,t})^\eta = \lambda_t e_t \frac{\omega^y_{h,n,t}}{\pi_t} \tag{E.16}$$

Labor supply (y,l,f)
$$\beta(l^y_{l,f,t})^\eta = \lambda_t e_t \frac{\omega^y_{l,f,t}}{\pi_t} \tag{E.17}$$

Labor supply (y,l,n)
$$\beta(l^y_{l,n,t})^\eta = \lambda_t e_t \frac{\omega^y_{l,n,t}}{\pi_t} \tag{E.18}$$

Youth labor supply
$$\alpha = l^m_t/(l^m_t + l^y_t) \tag{E.19}$$

Labor demand (y,h,f)
$$\omega^y_{h,f,t} - \mu_t\gamma k_t^{1-\gamma}\alpha z_t \phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}-1}(\phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}} + \phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}-1}$$
$$((\phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}} + \phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}} + (\phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}} + \phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}})^{\sigma-1}$$
$$(l^y_t)^{\frac{1}{s}-1}(\alpha(l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^{s-1}$$

$$(z_t(\alpha(l_t^y)^{\frac{1}{s}} + (l_t^m)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0 \tag{E.20}$$

Labor demand (y,h,n)
$$\omega^y_{h,n,t} - \mu_t\gamma k_t^{1-\gamma}\alpha z_t\phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}-1}(\phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}} + \phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}-1}$$
$$((\phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}} + \phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}} + (\phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}} + \phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}})^{\sigma-1}$$
$$(l_t^y)^{\frac{1}{s}-1}(\alpha(l_t^y)^{\frac{1}{s}} + (1-\alpha)(l_t^m)^{\frac{1}{s}})^{s-1}$$
$$(z_t(\alpha(l_t^y)^{\frac{1}{s}} + (l_t^m)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0 \tag{E.21}$$

Labor demand (y,l,f)
$$\omega^y_{l,f,t} - \mu_t\gamma k_t^{1-\gamma}\alpha z_t\phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}-1}(\phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}} + \phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}-1}$$
$$((\phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}} + \phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}} + (\phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}} + \phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}})^{\sigma-1}$$
$$(l_t^y)^{\frac{1}{s}-1}(\alpha(l_t^y)^{\frac{1}{s}} + (1-\alpha)(l_t^m)^{\frac{1}{s}})^{s-1}$$
$$(z_t(\alpha(l_t^y)^{\frac{1}{s}} + (l_t^m)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0 \tag{E.22}$$

Labor demand (y,l,n)
$$\omega^y_{l,n,t} - \mu_t\gamma k_t^{1-\gamma}\alpha z_t\phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}-1}(\phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}} + \phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}-1}$$
$$((\phi_{l,n}(l^y_{l,n,t})^{\frac{1}{\upsilon}} + \phi_{l,f}(l^y_{l,f,t})^{\frac{1}{\upsilon}})^{\frac{\upsilon}{\sigma}} + (\phi_{h,n}(l^y_{h,n,t})^{\frac{1}{\varrho}} + \phi_{h,f}(l^y_{h,f,t})^{\frac{1}{\varrho}})^{\frac{\varrho}{\sigma}})^{\sigma-1}$$
$$(l_t^y)^{\frac{1}{s}-1}(\alpha(l_t^y)^{\frac{1}{s}} + (1-\alpha)(l_t^m)^{\frac{1}{s}})^{s-1}$$
$$(z_t(\alpha(l_t^y)^{\frac{1}{s}} + (l_t^m)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0 \tag{E.23}$$

Labor demand (m,h,f)
$$\omega^m_{h,f,t} - \mu_t\gamma k_t^{1-\gamma}(1-\alpha)z_t\theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}-1}(\theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}} + \theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}-1}$$
$$((\theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}} + \theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}} + (\theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}} + \theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}})^{\xi-1}$$
$$(l_t^m)^{\frac{1}{s}-1}(\alpha(l_t^y)^{\frac{1}{s}} + (1-\alpha)(l_t^m)^{\frac{1}{s}})^{s-1}$$
$$(z_t(\alpha(l_t^y)^{\frac{1}{s}} + (l_t^m)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0 \tag{E.24}$$

Labor demand (m,h,n)
$$\omega^m_{h,n,t} - \mu_t\gamma k_t^{1-\gamma}(1-\alpha)z_t\theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}-1}(\theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}} + \theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}-1}$$
$$((\theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}} + \theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}} + (\theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}} + \theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}})^{\xi-1}$$
$$(l_t^m)^{\frac{1}{s}-1}(\alpha(l_t^y)^{\frac{1}{s}} + (1-\alpha)(l_t^m)^{\frac{1}{s}})^{s-1}$$
$$(z_t(\alpha(l_t^y)^{\frac{1}{s}} + (l_t^m)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0 \tag{E.25}$$

Labor demand (m,l,f)
$$\omega^m_{l,f,t} - \mu_t\gamma k_t^{1-\gamma}(1-\alpha)z_t\theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}-1}(\theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}} + \theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}-1}$$
$$((\theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}} + \theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}} + (\theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}} + \theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}})^{\xi-1}$$
$$(l_t^m)^{\frac{1}{s}-1}(\alpha(l_t^y)^{\frac{1}{s}} + (1-\alpha)(l_t^m)^{\frac{1}{s}})^{s-1}$$
$$(z_t(\alpha(l_t^y)^{\frac{1}{s}} + (l_t^m)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0 \tag{E.26}$$

Labor demand (m,l,n)
$$\omega^m_{l,n,t} - \mu_t\gamma k_t^{1-\gamma}(1-\alpha)z_t\theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}-1}(\theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}} + \theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}-1}$$
$$((\theta_{l,n}(l^m_{l,n,t})^{\frac{1}{\zeta}} + \theta_{l,f}(l^m_{l,f,t})^{\frac{1}{\zeta}})^{\frac{\zeta}{\xi}} + (\theta_{h,n}(l^m_{h,n,t})^{\frac{1}{\epsilon}} + \theta_{h,f}(l^m_{h,f,t})^{\frac{1}{\epsilon}})^{\frac{\epsilon}{\xi}})^{\xi-1}$$
$$(l_t^m)^{\frac{1}{s}-1}(\alpha(l_t^y)^{\frac{1}{s}} + (1-\alpha)(l_t^m)^{\frac{1}{s}})^{s-1}$$
$$(z_t(\alpha(l_t^y)^{\frac{1}{s}} + (l_t^m)^{\frac{1}{s}}(1-\alpha))^s)^{\gamma-1} = 0 \tag{E.27}$$

Output
$$y_t = (k_t)^{1-\gamma}(z_t(\alpha(l_t^y)^{\frac{1}{s}} + (1-\alpha)(l_t^m)^{\frac{1}{s}})^s)^{\gamma} \tag{E.28}$$

Capital choice $\mu_t = r_t^k \frac{k_t}{(1-\gamma)y_t}$ (E.29)

Capital rental rate $r_t^k = r_t$ (E.30)

Labor demand (y)
$$\omega_t^y = \alpha\gamma k_t^{1-\gamma}\mu_t\phi z_t(l_t^y\phi)^{\frac{1}{s}-1} \left(\alpha(l_t^y\phi)^{\frac{1}{s}} + (l_t^m\theta)^{\frac{1}{s}}(1-\alpha)\right)^{s-1} \left(z_t(\alpha(l_t^y\phi)^{\frac{1}{s}} + (l_t^m\theta)^{\frac{1}{s}}(1-\alpha))^s\right)^{\gamma-1}$$ (E.31)

Labor demand (m)
$$\omega_t^m = (1-\alpha)\gamma k_t^{1-\gamma}\mu_t\theta z_t(l_t^m\theta)^{\frac{1}{s}-1} \left(\alpha(l_t^y\phi)^{\frac{1}{s}} + (l_t^m\theta)^{\frac{1}{s}}(1-\alpha)\right)^{s-1} \left(z_t(\alpha(l_t^y\phi)^{\frac{1}{s}} + (l_t^m\theta)^{\frac{1}{s}}(1-\alpha))^s\right)^{\gamma-1}$$ (E.32)

Real wage (y,h,n) $\omega_{r,h,n,t}^y = \omega_{h,n,t}^y/\pi_t$ (E.33)

Wage inflation (y,h,n) $\pi_{w,t} = \omega_{h,n,t+1}^y/\omega_{h,n,t}^y$ (E.34)

Real wage (y,h,f) $\omega_{r,h,f,t}^y = \omega_{h,f,t}^y/\pi_t$ (E.35)

Wage inflation (y,h,f) $\pi_{w,t} = \omega_{h,f,t+1}^y/\omega_{h,f,t}^y$ (E.36)

Real wage (y,l,n) $\omega_{r,l,n,t}^y = \omega_{l,n,t}^y/\pi_t$ (E.37)

Wage inflation (y,l,n) $\pi_{w,t} = \omega_{l,n,t+1}^y/\omega_{l,n,t}^y$ (E.38)

Real wage (y,l,f) $\omega_{r,l,f,t}^y = \omega_{l,f,t}^y/\pi_t$ (E.39)

Wage inflation (y,l,f) $\pi_{w,t} = \omega_{l,f,t+1}^y/\omega_{l,f,t}^y$ (E.40)

Real wage (m,h,n) $\omega_{r,h,n,t}^m = \omega_{h,n,t}^m/\pi_t$ (E.41)

Wage inflation (m,h,n) $\pi_{w,t} = \omega_{h,n,t+1}^m/\omega_{h,n,t}^m$ (E.42)

Real wage (m,h,f) $\omega_{r,h,f,t}^m = \omega_{h,f,t}^m/\pi_t$ (E.43)

Wage inflation (m,h,f) $\pi_{w,t} = \omega_{h,f,t+1}^m/\omega_{h,f,t}^m$ (E.44)

Real wage (m,l,n) $\omega_{r,l,n,t}^m = \omega_{l,n,t}^m/\pi_t$ (E.45)

Wage inflation (m,l,n) $\pi_{w,t} = \omega_{l,n,t+1}^m/\omega_{l,n,t}^m$ (E.46)

Real wage (m,l,f) $\omega_{r,l,f,t}^m = \omega_{l,f,t}^m/\pi_t$ (E.47)

Wage inflation (m,l,f) $\pi_{w,t} = \omega_{l,f,t+1}^m/\omega_{l,f,t}^m$ (E.48)

Real wage (y) $\omega_{r,t}^y = \omega_t^y/\pi_t$ (E.49)

Wage inflation $\pi_{w,t} = \omega_{t+1}^y/\omega_t^y$ (E.50)

Real wage (m) $\omega_{r,t}^m = \omega_t^m/\pi_t$ (E.51)

Wage inflation (m) $\pi_{w,t} = \omega_{t+1}^m/\omega_t^m$ (E.52)

Market clearing condition $-(k_{t+1} - (1-\delta^K) * k_t) + y_t - c_t^y - c_t^m - c_t^o$ (E.53)

Appendix F Alternative nesting order in the CES production function

Suppose now that producers have access to the following production function

$$y_t = z_t(\alpha(l_t^y)^{\frac{1}{s}} + (1-\alpha)(l_t^m)^{\frac{1}{s}})^s \tag{F.1}$$

We assume that firms combine heterogeneous labor to produce good y. The hours worked by younger individuals are given by:

$$l_t^y = \phi_{h,f} l_{h,f,t}^y + \phi_{h,n} l_{h,n,t}^y + \phi_{l,f} l_{l,f,t}^y + \phi_{l,n} l_{l,n,t}^y$$

This equation emphasizes that hours worked by young workers are broken down into various origin (native or foreign) and skill (high or low) groups. Similarly, the hours worked by middle-aged individuals are given by:

$$l_t^m = \theta_{h,f} l_{h,f,t}^m + \theta_{h,n} l_{h,n,t}^m + \theta_{l,f} l_{l,f,t}^m + \theta_{l,n} l_{l,n,t}^m$$

the hours worked by middle-aged individuals include hours worked by natives and foreigners and by high- and low-skilled.

Producer Problem The problem of the intermediate good producer is to minimize the cost

$$\omega_{h,f,t}^y l_{h,f,t}^y + \omega_{h,n,t}^y l_{h,n,t}^y + \omega_{l,f,t}^y l_{l,f,t}^y + \omega_{l,n,t}^y l_{l,n,t}^y + \omega_{h,f,t}^m l_{h,f,t}^m + \omega_{h,n,t}^m l_{h,n,t}^m + \omega_{l,f,t}^m l_{l,f,t}^m + \omega_{l,n,t}^m l_{l,n,t}^m$$

subject to the production function that relates outputs to inputs. The first order conditions with respect to $l_{h,f,t}^y$ and $l_{h,f,t}^m$ are given by:

$$\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial l_{h,f,t}^y} : \quad & \omega_{h,f,t}^y - \mu_t \alpha z_t \phi_{h,f} (l_t^y)^{\frac{1}{s}-1} ((l_t^y)^{\frac{1}{s}} + (l_t^m)^{\frac{1}{s}})^{s-1} = 0, \\
\frac{\partial \mathcal{L}_t}{\partial l_{h,f,t}^m} : \quad & \omega_{h,f,t}^m - \mu_t (1-\alpha) z_t \theta_{h,f} (l_t^m)^{\frac{1}{s}-1} ((l_t^y)^{\frac{1}{s}} + (l_t^m)^{\frac{1}{s}})^{s-1} = 0.
\end{aligned}$$

We derive the optimality equation

$$\frac{\omega_{h,f,t}^y}{\omega_{h,f,t}^m} = \frac{\alpha}{(1-\alpha)} \frac{\phi_{h,f}}{\theta_{h,f}} \left(\frac{l_t^y}{l_t^m} \right)^{-\frac{1}{s}},$$

The first order conditions wrt $l_{h,n,t}^y$ and $l_{h,n,t}^m$ are:

$$\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial l_{h,n,t}^y} : \quad & \omega_{h,n,t}^y - \mu_t \alpha z_t \phi_{h,n} (l_t^y)^{\frac{1}{s}-1} ((l_t^y)^{\frac{1}{s}} + (l_t^m)^{\frac{1}{s}})^{s-1} = 0, \\
\frac{\partial \mathcal{L}_t}{\partial l_{h,n,t}^m} : \quad & \omega_{h,n,t}^m - \mu_t (1-\alpha) z_t \theta_{h,n} (l_t^m)^{\frac{1}{s}-1} ((l_t^y)^{\frac{1}{s}} + (l_t^m)^{\frac{1}{s}})^{s-1} = 0.
\end{aligned}$$

Combining these two equations gives the following optimality equation

$$\frac{\omega^y_{h,n,t}}{\omega^m_{h,n,t}} = \frac{\alpha}{1-\alpha}\frac{\phi_{h,n}}{\theta_{h,n}}\left(\frac{l^y_t}{l^m_t}\right)^{-\frac{1}{s}},$$

first order conditions with respect to $l^y_{l,f,t}$ and $l^m_{l,f,t}$

$$\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial l^y_{l,f,t}}: &\qquad \omega^y_{l,f,t} - \mu_t \alpha z_t \phi_{l,f} (l^y_t)^{\frac{1}{s}-1}((l^y_t)^{\frac{1}{s}} + (l^m_t)^{\frac{1}{s}})^{s-1} = 0,\\
\frac{\partial \mathcal{L}_t}{\partial l^m_{l,f,t}}: &\qquad \omega^m_{l,f,t} - \mu_t (1-\alpha) z_t \theta_{l,f} (l^m_t)^{\frac{1}{s}-1}((l^y_t)^{\frac{1}{s}} + (l^m_t)^{\frac{1}{s}})^{s-1} = 0,
\end{aligned}$$

The optimality equation

$$\frac{\omega^y_{l,f,t}}{\omega^m_{l,f,t}} = \frac{\alpha}{1-\alpha}\frac{\phi_{l,f}}{\theta_{l,f}}\left(\frac{l^y_t}{l^m_t}\right)^{-\frac{1}{s}},$$

first order conditiosn with respect to $l^y_{l,n,t}$ and $l^m_{l,n,t}$

$$\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial l^y_{l,n,t}}: &\qquad \omega^y_{l,n,t} - \mu_t \alpha z_t \phi_{l,n} (l^y_t)^{\frac{1}{s}-1}((l^y_t)^{\frac{1}{s}} + (l^m_t)^{\frac{1}{s}})^{s-1} = 0,\\
\frac{\partial \mathcal{L}_t}{\partial l^m_{l,n,t}}: &\qquad \omega^m_{l,n,t} - \mu_t (1-\alpha) z_t \theta_{l,n} (l^m_t)^{\frac{1}{s}-1}((l^y_t)^{\frac{1}{s}} + (l^m_t)^{\frac{1}{s}})^{s-1} = 0,
\end{aligned}$$

The optimality equation

$$\frac{\omega^y_{l,n,t}}{\omega^m_{l,n,t}} = \frac{\alpha}{1-\alpha}\frac{\phi_{l,n}}{\theta_{l,n}}\left(\frac{l^y_t}{l^m_t}\right)^{-\frac{1}{s}},$$

The producer marginal cost is

$$mc = \mu_t = \frac{\omega_t(l^y_t + l^m_t)}{z_t(\alpha(l^y_t)^{\frac{1}{s}} + (1-\alpha)(l^m_t)^{\frac{1}{s}})^s},$$

Appendix G Labor elasticity of substitution

If we assume constant labor intensity parameters $\{\theta, \phi\}$ over time, we can easily estimate the elasticity parameters. The estimation of the elasticity of substitution $\{\varrho, \upsilon, \varepsilon, \zeta\}$ between types of labor can be identified using the optimality equations from the producer optimization. We use the firm optimality equations to estimate the elasticity of substitution parameters. More formally, we take the logarithm of the optimality equations: $\ln(\omega^y_{h,f,t}) - \ln(\omega^y_{h,n,t}) = (1/\varrho - 1)\left(\ln(l^y_{h,f,t}) - \ln(l^y_{h,n,t})\right)$. We introduce in the elasticity of substitution specification the county α_c and year α_t fixed effects to absorb time trends and county characteristics that may affect the wage differentials, such that:

$$\ln \omega^y_{i,h,f,t,c} - \ln \omega^y_{i,h,n,t,c} = \alpha + 0.118(\ln l^y_{i,h,f,t,c} - \ln l^y_{i,h,n,t,c}) + \alpha_t + \alpha_c + \epsilon_{i,t}.$$

The estimation of the elasticity of substitution between foreign and native workers of equal skills and age shows that young high-skilled foreign and native workers are substitutes (when the elasticity is positive, $1/\varrho - 1 = 0.118$). This differs from the estimates of Ottaviano and Peri (2008). Our estimates suggest some degree of substitutability between foreign and native workers. An increase in the supply of labor by foreign workers relative to natives, specifically when they are both young and highly skilled, leads to an increase in wages of foreign workers relative to natives with the same characteristics.

In a simple form, we also estimate the elasticity of substitution between young low-skilled foreign and native workers: $\ln(\omega^y_{l,f,t}) - \ln(\omega^y_{l,n,t}) = (1/\upsilon - 1)\left(\ln(l^y_{l,f,t}) - \ln(l^y_{l,n,t})\right)$, We augment the equation with a constant α and error term $\epsilon_{i,t}$. Our empirical specification includes the time fixed effects as well as the county fixed effects.

$$\ln \omega^y_{i,l,f,t,c} - \ln \omega^y_{i,l,n,t,c} = \alpha + 0.167(\ln l^y_{i,l,f,t,c} - \ln l^y_{i,l,n,t,c}) + \alpha_t + \alpha_c + \epsilon_{i,t}.$$

We also find a positive elasticity ($1/\upsilon - 1 = 0.167$) between young low-skilled foreign and native workers. An increase by 1 % of labor supply from foreign workers relative to natives leads to an increase in wages of foreign workers, relative to native workers by 0.167 %.

In addition, the elasticity between middle-aged foreign and native workers with high-level skills is $\ln(\omega^m_{h,f,t}) - \ln(\omega^m_{h,n,t}) = (1/\epsilon - 1)\left(\ln(l^m_{h,f,t}) - \ln(l^m_{h,n,t})\right)$. We augment this specification with a constant term, time fixed effects, county fixed effects, and an error term, and rewrite it as:

$$\ln(\omega^m_{i,h,f,t,c}) - \ln(\omega^m_{i,h,n,t,c} = \alpha + 0.126(\ln l^m_{i,h,f,t,c} - \ln l^m_{i,h,n,t,c}) + \alpha_t + \alpha_c + \epsilon_{i,t}.$$

Our estimate favors some degree of substitution between the two categories. A positive elasticity $1/\varepsilon - 1 = 0.126$ implies that an increase of 1% in labor supply by foreign workers relative to native workers leads to an increase of 0.126 % in wages of foreign workers relative to native workers.

We also used the optimality equation to estimate the elasticity of substitution between middle-aged foreign and native workers with low-level skills $\ln(\omega^m_{l,f,t}) - \ln(\omega^m_{l,n,t}) = (1/\zeta - 1)\left(\ln(l^m_{l,f,t}) - \ln(l^m_{l,n,t})\right)$. We augment the specification with a constant term, time fixed effects, county fixed

effects, and an error term. Hence, we rewrite the specification as follows:

$$\ln(\omega^m_{i,l,f,t,c}) - \ln(\omega^m_{i,l,n,t,c}) = \alpha + 0.187(\ln l^m_{i,l,f,t,c} - \ln l^m_{i,l,n,t,c}) + \alpha_t + \alpha_c + \epsilon_{i,t}.$$

We also find evidence of substitutability between middle-aged foreign and native workers with low-level skills, with positive elasticity of substitution: $1/\zeta - 1 = 0.187$. An increase of 1 % in labor supply by foreign workers relative to native workers, especially when both are low-skilled and middle-aged workers, leads to an increase in wages of foreign workers relative to native workers with the same characteristics by 0.187 %.
Before moving to the estimation of the elasticity between young and middle-aged workers, explaining how our estimates relate to the emerging literature trying to estimate labor demand elasticity is important. In particular, early on, Ottaviano and Peri (2008) and D'Amuri et al. (2009) raised the question of imperfect substitution between native and immigrant workers and found imperfect substitutability between the two. Our estimates differ substantially because we have a different specification of the estimated optimality equations.

An additional parameter that turns out to be important for our empirical analysis is the elasticity of substitution between middle-aged and young workers s. Note that the producer optimization implies an optimality equation $\omega^m_t/\omega^y_t = (1-\alpha/\alpha)(l^m_t/l^y_t)^{-1/s}$. From the first-order condition for producer's cost minimization, one can write in logarithmic form and augment the specification with a constant term, county fixed effects, time fixed effects, and an error term, as follows:

$$\ln(\omega^m_{i,t,c}) - \ln(\omega^y_{i,t,c}) = \alpha + 0.119(\ln l^m_{i,t,c} - \ln l^y_{i,t,c}) + \alpha_t + \alpha_c + \epsilon_{i,t}.$$

We include both county and year fixed effects in the regression. Here, we note that the estimated $\frac{1}{s}$ is positive and statistically significant. An increase by 1 % of labor supply by middle-aged workers relative to young workers leads to an increase in wages of middle-aged workers relative to young workers by 0.119 %.
We estimate the elasticity of substitution between high- and low-skilled middle-aged workers, $(1/\xi - 1)$ using the expression: $\ln(\omega^m_{h,t}) - \ln(\omega^m_{l,t}) = (1/\xi - 1)\,(\ln(l^m_{h,t}) - \ln(l^m_{l,t}))$. We also augment the specification with a constant term, country fixed effects, time fixed effects, and an error term. Hence, we rewrite the model to be estimated as follows:

$$\ln(\omega^m_{i,h,t,c}) - \ln(\omega^m_{i,l,t,c}) = \alpha + 0.133(\ln l^m_{i,h,t,c} - \ln l^m_{i,l,t,c}) + \alpha_t + \alpha_c + \epsilon_{i,t}.$$

We find that the elasticity of substitution between high- and low-skilled workers is $1/\xi - 1 = 0.133$ among middle-aged workers. This suggests that high- and low-skill middle-aged workers are substitutes. An increase of 1 % in labor supply by high-skilled middle-aged workers relative to low-skilled middle-aged workers leads to an increase in wages of high-skilled workers relative to low-skilled workers of 0.133 %.
Finally, we run the regression for the optimality equation that captures the elasticity of substitution between high-skilled and low-skilled young workers $\ln(\omega^y_{h,t}) - \ln(\omega^y_{l,t}) = (1/\sigma - 1)\,(\ln(l^y_{h,t}) - \ln(l^y_{l,t}))$. The specification accounts for a constant term, time fixed effects, and county fixed

effects, and an error term, which we rewrite as follows:

$$\ln(\omega^y_{i,h,t,c}) - \ln(\omega^y_{i,l,t,c}) = \alpha + 0.047(\ln l^y_{i,h,t,c} - \ln l^y_{i,l,t,c}) + \alpha_t + \alpha_c + \epsilon_{i,t}.$$

This specification includes county and year fixed effects to remove time trends and specific county factors that can explain wage differentials. Ignoring the origin of the worker, once again the elasticity of substitution between high- and low-skilled young workers $1/\sigma - 1$ is positive and equals 0.047. An increase of 1 % in labor supply by high-skilled workers relative to low-skilled workers when both young leads to an increase in wages of high-skilled workers relative to low-skilled workers by 0.047 percent. These results imply that the two groups are perfect substitutes. These estimates substantially differ from the estimates of McAdam et al. (2011), who find high elasticity of substitution between skilled and unskilled labor.

G.1 Estimation

Table 10: Labor elasticity of substitution σ

	Dependent variable: $\ln(\omega^y_{h,t,c}) - \ln(\omega^y_{l,t,c})$			
	(1)	(2)	(3)	(4)
const	0.600***	0.600***	0.600***	0.600***
	(0.001)	(0.001)	(0.001)	(0.001)
$\ln(l^y_{h,t,c}) - \ln(l^y_{l,t,c})$	0.024***	0.025***	0.024***	0.025***
	(0.008)	(0.008)	(0.008)	(0.008)
Observations	45125	45125	45125	45125
N. of groups	30	30	30	30
R^2	0.000	0.000	0.000	0.000
Residual Std. Error	0.002	0.002	0.002	0.002
F Statistic	9.160***	9.276***	9.101***	9.331***
Year FE	Yes	Yes	No	No
County FE	Yes	No	Yes	No
Note:				*p<0.1; **p<0.05; ***p<0.01

G.2 Robustness checks: dropping the pandemic period

Appendix H Distribution of hourly earnings

Table 11: Labor elasticity of substitution s

	Dependent variable: $\ln(\omega^m_{t,c}) - \ln(\omega^y_{t,c})$			
	(1)	(2)	(3)	(4)
const	-0.257***	-0.257***	-0.257***	-0.257***
	(0.001)	(0.001)	(0.001)	(0.001)
$\ln(l^m_{t,c}) - \ln(l^y_{t,c})$	0.111***	0.109***	0.109***	0.112***
	(0.009)	(0.010)	(0.009)	(0.010)
Observations	45125	45125	45125	45125
N. of groups	30	30	30	30
R^2	0.003	0.003	0.003	0.003
Residual Std. Error	0.010	0.010	0.009	0.010
F Statistic	137.723***	131.892***	131.063***	138.450***
Year FE	Yes	Yes	No	No
County FE	Yes	No	Yes	No
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 12: Labor elasticity of substitution ζ

	Dependent variable: $\ln(\omega^m_{l,f,t,c}) - \ln(\omega^m_{l,n,t,c})$			
	(1)	(2)	(3)	(4)
Constant	-0.009***	-0.009***	-0.009***	-0.009***
	(0.002)	(0.002)	(0.002)	(0.002)
$\ln(l^m_{l,f,t,c}) - \ln(l^m_{l,n,t,c})$	0.188***	0.192***	0.187***	0.193***
	(0.012)	(0.012)	(0.012)	(0.012)
Observations	45125	45125	45125	45125
N. of groups	30	30	30	30
R^2	0.006	0.006	0.006	0.006
Residual Std. Error	0.040	0.041	0.040	0.041
F Statistic	261.867***	271.995***	260.242***	273.623***
Year FE	Yes	Yes	No	No
County FE	Yes	No	Yes	No
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 13: Labor elasticity of substitution ε

	Dependent variable: $\ln(\omega^m_{h,f,t,c}) - \ln(\omega^m_{h,n,t,c})$			
	(1)	(2)	(3)	(4)
Constant	-0.146***	-0.146***	-0.146***	-0.146***
	(0.003)	(0.003)	(0.003)	(0.003)
$\ln(l^m_{h,f,t,c}) - \ln(l^m_{h,n,t,c})$	0.127***	0.126***	0.126***	0.127***
	(0.010)	(0.010)	(0.010)	(0.010)
Observations	45125	45125	45125	45125
N. of groups	30	30	30	30
R^2	0.004	0.004	0.004	0.004
Residual Std. Error	0.044	0.043	0.044	0.044
F Statistic	170.189***	165.252***	166.888***	168.449***
Year FE	Yes	Yes	No	No
County FE	Yes	No	Yes	No
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 14: Labor elasticity of substitution ξ

	Dependent variable: $\ln(\omega^m_{h,t,c}) - \ln(\omega^m_{l,t,c})$			
	(1)	(2)	(3)	(4)
Constant	0.625***	0.625***	0.625***	0.625***
	(0.002)	(0.002)	(0.002)	(0.002)
$\ln(l^m_{h,t,c}) - \ln(l^m_{l,t,c})$	0.118***	0.118***	0.117***	0.120***
	(0.010)	(0.010)	(0.010)	(0.010)
Observations	45125	45125	45125	45125
N. of groups	30	30	30	30
R^2	0.003	0.003	0.003	0.003
Residual Std. Error	0.019	0.019	0.019	0.019
F Statistic	132.822***	132.468***	129.410***	135.908***
Year FE	Yes	Yes	No	No
County FE	Yes	No	Yes	No
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 15: Labor elasticity of substitution υ

	Dependent variable: $\ln(\omega^y_{l,f,t,c}) - \ln(\omega^y_{l,n,t,c})$			
	(1)	(2)	(3)	(4)
Constant	0.238***	0.238***	0.238***	0.238***
	(0.001)	(0.001)	(0.001)	(0.001)
$\ln(l^y_{l,f,t,c}) - \ln(l^y_{l,n,t,c})$	0.120***	0.121***	0.120***	0.121***
	(0.010)	(0.010)	(0.010)	(0.010)
Observations	45125	45125	45125	45125
N. of groups	30	30	30	30
R^2	0.003	0.003	0.003	0.003
Residual Std. Error	0.010	0.010	0.010	0.010
F Statistic	155.547***	157.969***	155.613***	157.866***
Year FE	Yes	Yes	No	No
County FE	Yes	No	Yes	No
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 16: Labor elasticity of substitution ϱ

	Dependent variable: $\ln(\omega^y_{h,f,t,c}) - \ln(\omega^y_{h,n,t,c})$			
	(1)	(2)	(3)	(4)
Constant	-0.076***	-0.076***	-0.076***	-0.076***
	(0.001)	(0.001)	(0.001)	(0.001)
$\ln(l^y_{h,f,t,c}) - \ln(l^y_{h,n,t,c})$	0.091***	0.091***	0.091***	0.091***
	(0.008)	(0.008)	(0.008)	(0.008)
Observations	45125	45125	45125	45125
N. of groups	30	30	30	30
R^2	0.003	0.003	0.003	0.003
Residual Std. Error	0.015	0.015	0.015	0.015
F Statistic	141.362***	141.197***	141.778***	140.780***
Year FE	Yes	Yes	No	No
County FE	Yes	No	Yes	No
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 17: Labor elasticity of substitution

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	-0.076***	0.238***	-0.146***	-0.009***	-0.257***	0.600***	0.625***
	(0.001)	(0.001)	(0.003)	(0.002)	(0.001)	(0.001)	(0.002)
$\ln(l^y_{h,f,t,c}) - \ln(l^y_{h,n,t,c})$	0.091***						
	(0.008)						
$\ln(l^y_{l,f,t,c}) - \ln(l^y_{l,n,t,c})$		0.120***					
		(0.010)					
$\ln(l^m_{h,f,t,c}) - \ln(l^m_{h,n,t,c})$			0.127***				
			(0.010)				
$\ln(l^m_{l,f,t,c}) - \ln(l^m_{l,n,t,c})$				0.188***			
				(0.012)			
$\ln(l^m_{t,c}) - \ln(l^y_{t,c})$					0.111***		
					(0.009)		
$\ln(l^y_{h,t,c}) - \ln(l^y_{l,t,c})$						0.024***	
						(0.008)	
$\ln(l^m_{h,t,c}) - \ln(l^m_{l,t,c})$							0.118***
							(0.010)
Observations	45125	45125	45125	45125	45125	45125	45125
N. of groups	30	30	30	30	30	30	30
R^2	0.003	0.003	0.004	0.006	0.003	0.000	0.003
Residual Std. Error	0.015	0.010	0.044	0.040	0.010	0.002	0.019
F Statistic	141.362***	155.547***	170.189***	261.867***	137.723***	9.160***	132.822***
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
County FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Note: *p<0.1; **p<0.05; ***p<0.01

Table 18: Labor elasticity of substitution σ

	Dependent variable: $\ln(\omega^y_{h,t,c}) - \ln(\omega^y_{l,t,c})$			
	(1)	(2)	(3)	(4)
const	0.599***	0.599***	0.599***	0.599***
	(0.001)	(0.001)	(0.001)	(0.001)
$\ln(l^y_{h,t,c}) - \ln(l^y_{l,t,c})$	0.047***	0.047***	0.047***	0.047***
	(0.011)	(0.011)	(0.011)	(0.011)
Observations	24186	24186	24186	24186
N. of groups	30	30	30	30
R^2	0.001	0.001	0.001	0.001
Residual Std. Error	0.003	0.003	0.003	0.003
F Statistic	17.430***	17.455***	17.419***	17.469***
Year FE	Yes	No	No	Yes
County FE	Yes	No	Yes	No

Note: *p<0.1; **p<0.05; ***p<0.01

Table 19: Labor elasticity of substitution s

	Dependent variable: $\ln(\omega^m_{t,c}) - \ln(\omega^y_{t,c})$			
	(1)	(2)	(3)	(4)
const	-0.248***	-0.248***	-0.248***	-0.248***
	(0.002)	(0.002)	(0.002)	(0.002)
$\ln(l^m_{t,c}) - \ln(l^y_{t,c})$	0.119***	0.120***	0.119***	0.120***
	(0.013)	(0.013)	(0.013)	(0.013)
Observations	24186	24186	24186	24186
N. of groups	30	30	30	30
R^2	0.003	0.003	0.003	0.003
Residual Std. Error	0.010	0.010	0.010	0.010
F Statistic	82.529***	84.345***	82.772***	84.074***
Year FE	Yes	No	No	Yes
County FE	Yes	No	Yes	No
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 20: Labor elasticity of substitution ζ

	Dependent variable: $\ln(\omega^m_{l,f,t,c}) - \ln(\omega^m_{l,n,t,c})$			
	(1)	(2)	(3)	(4)
Constant	-0.002	-0.002	-0.002	-0.002
	(0.003)	(0.003)	(0.003)	(0.003)
$\ln(l^m_{l,f,t,c}) - \ln(l^m_{l,n,t,c})$	0.187***	0.192***	0.187***	0.193***
	(0.017)	(0.017)	(0.017)	(0.017)
Observations	24186	24186	24186	24186
N. of groups	30	30	30	30
R^2	0.005	0.006	0.005	0.006
Residual Std. Error	0.039	0.040	0.039	0.040
F Statistic	128.265***	135.239***	127.890***	135.608***
Year FE	Yes	Yes	No	No
County FE	Yes	No	Yes	No
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 21: Labor elasticity of substitution ε

	Dependent variable: $\ln(\omega^m_{h,f,t,c}) - \ln(\omega^m_{h,n,t,c})$			
	(1)	(2)	(3)	(4)
Constant	-0.124***	-0.124***	-0.124***	-0.124***
	(0.005)	(0.005)	(0.005)	(0.005)
$\ln(l^m_{h,f,t,c}) - \ln(l^m_{h,n,t,c})$	0.126***	0.126***	0.126***	0.126***
	(0.014)	(0.014)	(0.014)	(0.014)
Observations	24186	24186	24186	24186
N. of groups	30	30	30	30
R^2	0.003	0.003	0.003	0.003
Residual Std. Error	0.041	0.041	0.041	0.041
F Statistic	83.203***	82.763***	83.186***	82.775***
Year FE	Yes	Yes	No	No
County FE	Yes	No	Yes	No
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 22: Labor elasticity of substitution υ

	Dependent variable: $\ln(\omega^y_{l,f,t,c}) - \ln(\omega^y_{l,n,t,c})$			
	(1)	(2)	(3)	(4)
const	0.240***	0.240***	0.240***	0.240***
	(0.001)	(0.001)	(0.001)	(0.001)
$\ln(l^y_{l,f,t,c}) - \ln(l^y_{l,n,t,c})$	0.167***	0.166***	0.167***	0.166***
	(0.014)	(0.014)	(0.014)	(0.014)
Observations	24186	24186	24186	24186
N. of groups	30	30	30	30
R^2	0.006	0.006	0.006	0.006
Residual Std. Error	0.013	0.013	0.013	0.013
F Statistic	136.940***	134.604***	136.803***	134.738***
Year FE	Yes	Yes	No	No
County FE	Yes	No	Yes	No
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 23: Labor elasticity of substitution ϱ

	Dependent variable: $\ln(\omega^y_{h,f,t,c}) - \ln(\omega^y_{h,n,t,c})$			
	(1)	(2)	(3)	(4)
Constant	-0.074***	-0.074***	-0.074***	-0.074***
	(0.002)	(0.002)	(0.002)	(0.002)
$\ln(l^y_{h,f,t,c}) - \ln(l^y_{h,n,t,c})$	0.118***	0.119***	0.118***	0.119***
	(0.011)	(0.011)	(0.011)	(0.011)
Observations	24186	24186	24186	24186
N. of groups	30	30	30	30
R^2	0.005	0.005	0.005	0.005
Residual Std. Error	0.019	0.019	0.019	0.019
F Statistic	121.355***	122.301***	121.434***	122.209***
Year FE	Yes	Yes	No	No
County FE	Yes	No	Yes	No
Note:				*p<0.1; **p<0.05; ***p<0.01

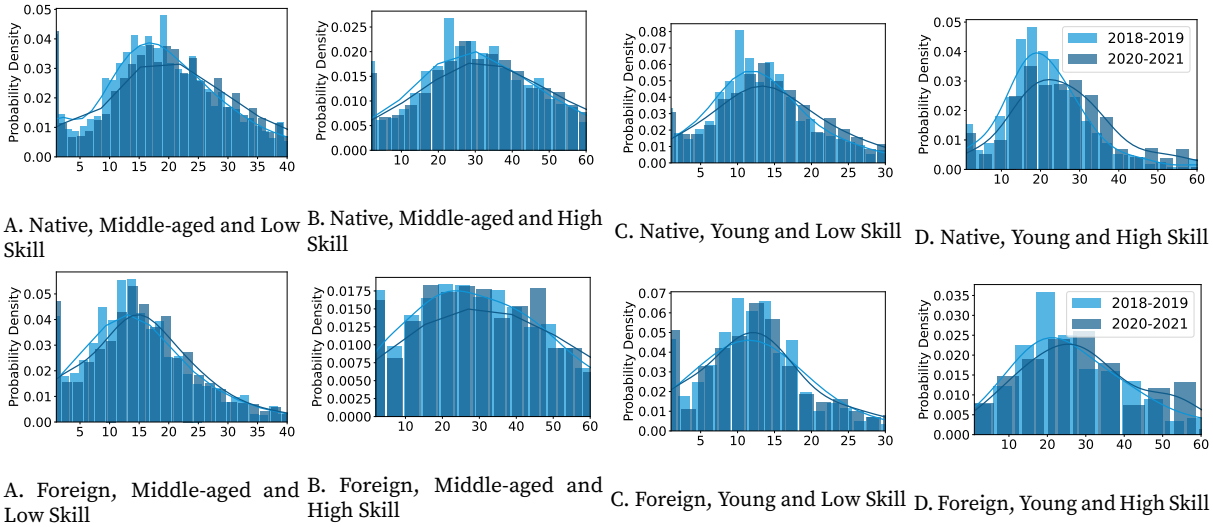
Table 24: Labor elasticity of substitution ξ

	Dependent variable: $\ln(\omega^m_{h,t,c}) - \ln(\omega^m_{l,t,c})$			
	(1)	(2)	(3)	(4)
const	0.636***	0.636***	0.636***	0.636***
	(0.002)	(0.002)	(0.002)	(0.002)
$\ln(l^m_{h,t,c}) - \ln(l^m_{l,t,c})$	0.133***	0.136***	0.133***	0.136***
	(0.014)	(0.014)	(0.014)	(0.014)
Observations	24186	24186	24186	24186
N. of groups	30	30	30	30
R^2	0.004	0.004	0.004	0.004
Residual Std. Error	0.021	0.021	0.021	0.021
F Statistic	85.329***	88.547***	85.177***	88.699***
Year FE	Yes	No	No	Yes
County FE	Yes	No	Yes	No
Note:				*p<0.1; **p<0.05; ***p<0.01

Table 25: Labor elasticity of substitution

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
const	-0.074***	0.240***	-0.124***	-0.002	-0.248***	0.599***	0.636***
	(0.002)	(0.001)	(0.005)	(0.003)	(0.002)	(0.001)	(0.002)
$\ln(l^y_{h,f,t,c}) - \ln(l^y_{h,n,t,c})$	0.118***						
	(0.011)						
$\ln(l^y_{l,f,t,c}) - \ln(l^y_{l,n,t,c})$		0.167***					
		(0.014)					
$\ln(l^m_{h,f,t,c}) - \ln(l^m_{h,n,t,c})$			0.126***				
			(0.014)				
$\ln(l^m_{l,f,t,c}) - \ln(l^m_{l,n,t,c})$				0.187***			
				(0.017)			
$\ln(l^m_{t,c}) - \ln(l^y_{t,c})$					0.119***		
					(0.013)		
$\ln(l^y_{h,t,c}) - \ln(l^y_{l,t,c})$						0.047***	
						(0.011)	
$\ln(l^m_{h,t,c}) - \ln(l^m_{l,t,c})$							0.133***
							(0.014)
Observations	24186	24186	24186	24186	24186	24186	24186
N. of groups	30	30	30	30	30	30	30
R^2	0.005	0.006	0.003	0.005	0.003	0.001	0.004
Residual Std. Error	0.019	0.013	0.041	0.039	0.010	0.003	0.021
F Statistic	121.355***	136.940***	83.203***	128.265***	82.529***	17.430***	85.329***
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
County FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Note: *p<0.1; **p<0.05; ***p<0.01



A. Native, Middle-aged and Low Skill

B. Native, Middle-aged and High Skill

C. Native, Young and Low Skill

D. Native, Young and High Skill

A. Foreign, Middle-aged and Low Skill

B. Foreign, Middle-aged and High Skill

C. Foreign, Young and Low Skill

D. Foreign, Young and High Skill

Figure 17: Distribution of Hourly Wage by Middle-aged and Young, Native and Foreign Workers of Different Skill before and after the Pandemic Crisis

Notes: Average hourly earnings in USD. The top panels describe the distribution of hourly wages across native and foreign workers when they are middle-aged and low-skilled (left), and the same demographic high-skilled (right). The bottom panels present the distribution of hourly wages across native and foreign workers when they are young and low-skilled (left) and the same demographic when workers are young and high-skilled (right). Time: 2018-2021. Source: Current Population Survey

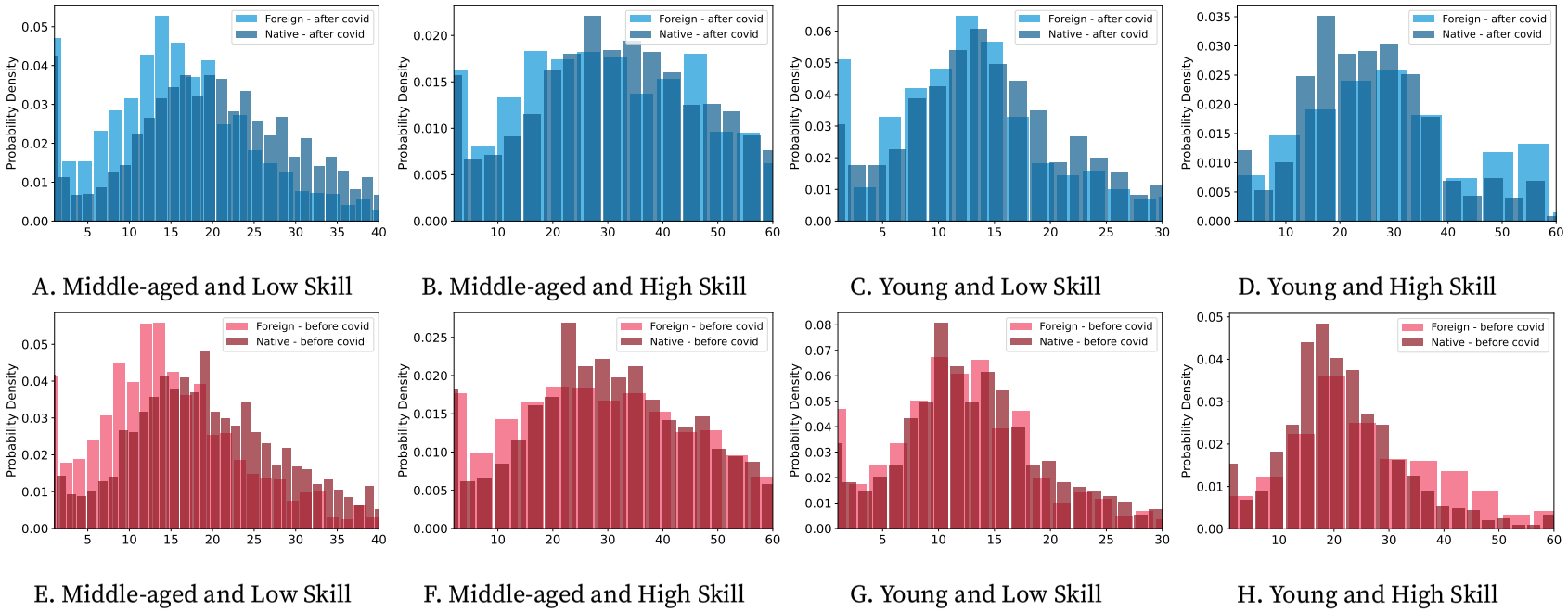


Figure 18: Distribution of hourly wage by native and foreign workers of different skills and age before and after the pandemic

Notes: The panels A and B describe the distribution of hourly wages across middle-aged native and foreign workers with low and high-skills. The panel C and D describe the distribution of hourly wages across young native and foreign workers with low and high-skills. Panels E and F present the distribution of hourly wages of middle-aged native and foreign workers with low and high skills. The panels G and H describe the distribution of hourly wages across young native and foreign workers with low and high skills. Time: 2018-2021. Source: Current Population Survey