Collateral Requirements and Delinquency Rates in a Two-Agent New Keynesian Model*

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Abstract

We study the effects of monetary policy shocks in a two-agent New Keynesian model with a bank-based prudential measure capturing the cyclicality of credit and defaults in the data. In our theory, prudential collateral requirements counter-cyclically respond to defaults and credit growth. They are implemented to tackle distortions arising from the peculiar behavior of defaults in the private sector. An active collateral requirement amplifies the impact of a monetary policy shock on output and labor relative to an economy with a constant collateral requirement. The extent to which monetary policy affects output and employment depends on bank-based prudential practices and whether the defaults fluctuate due to some exogenous or cyclical deviation. Monetary policy cannot prevent significant employment and output losses when the default probability constant. When the defaults are endogenous, monetary policy can create long-run distortions in employment while amplifying output loss. A countercyclical response to defaults and credit supply is optimal for the constrained and unconstrained households when both have access to the housing markets. Nevertheless, when only borrowers consume housing goods, a constant collateral requirement rule induces significant welfare gain. Although the monetary policy and the collateral requirement measures are costly, this policy mix emerges as a potential tool for preventing the risk of delinquency in the short run. Specifically, we find that monetary policy in a model that features an anti-default collateral requirement rule that leans against the credit cycle leads to a temporary yet significant decline in defaults in the private sector and induces moderate volatility in house prices and consumption.

Keywords: bank-based prudential practice, delinquency rate, monetary policy
JEL Codes: E32, E44, G21.

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1. Introduction

Borrowers' financial conditions matter for financial institutions' lending policies, and their capacity to pay back their debt is a crucial consideration for banks when approving loans. A theory of credit policies by Rajan (1994) suggests that regulatory interventions and demand-side conditions can have unforeseen consequences on banks' credit policies. During economic downturn, banks often experience a high level of delinquent debt and tighten the loan-to-value (LTV) limits to mitigate this risk.[1] Bank-based prudential measures like maximum LTV requirements have recently gained popularity in many countries (Aikman et al. 2021); however, the adoption of these tools depends on whether policy-makers find it desirable. In practice, these measures do not vary with the cycle, though there is a large class of optimal LTV limit literature that considers time-varying LTV rules (Studies of this type include, e.g., (Angelini, Neri, and Panetta 2012; Gelain, Lansing, and Mendicino 2013; Lambertini, Mendicino, and Punzi 2013; Rubio and Carrasco-Gallego 2014; Aikman et al. 2019; Ferrero, Harrison, and Nelson 2024)). These and other studies state that aggregate loan-to-value requirements could be used as a corrective action to financial vulnerability in real-time, reducing large debt built-up during expansion and consequently limiting debt deleveraging during a credit bust. The bank-based prudential measure proposed in this article is an anti-default time-varying measure that counter-cyclically responds to default and credit growth. Such a bank-based prudential instrument that creates a more stable and resilient financial system could be useful in addressing financial distortions and disrupting the feedback loop during economic downturns.

In this paper, we assess the effectiveness of bank-based prudential practices for sustaining aggregate credit to the private sector and containing systemic risk in financial markets. In particular, we focus on loan-to-value limits or, equivalently, collateral requirements. We assume that in response to high delinquency rates, banks adopt tight collateral conditions and decrease the maximum LTV requirements.[2] Our goal is to understand how prudential practices, e.g., collateral limits, affect macroeconomic outcomes and evaluate the effectiveness of joint monetary and bank-based prudential policies in preventing default risk. The idea that collateral limit may be an important source of macroeconomic fluctuations is widely discussed in the literature, including by Justiniano, Primiceri, and Tambalotti (2015), Iacoviello (2015), Guerrieri and Iacoviello (2017), and Mendicino et al. (2020).

We present a two-agent New Keynesian framework where borrowers can default on their loans and face a collateral constraint. The model has three main ingredients. First, there are two types of households: borrowers and savers. The borrowers are assumed to be collateral-constrained. Second, firms

[1] Throughout the paper, we use the terms collateral requirement limit, bank-based prudential practice, and loan-to-value limit interchangeably.

[2] The terms delinquency rate and charge-off rate are used interchangeably to refer to the percentage of loans that banks have classified as a loss relative to total lending.

are financially constrained and face collateral constraints when borrowing from banks. An interpretation of the debt contract between borrowers and lenders holds that the bank lends money, and the borrowers pledge collateral that serves as protection for the lender should the borrowers fail to make their loan payments. The specifics of the collateral constraint in our framework follow those in Kiyotaki and Moore (1997); however, we assume that financial intermediaries set the loan-to-value limit, which reacts inversely to the level of delinquency in the borrowing sector and credit supply. Third, the financial intermediaries in this economy observe the delinquency rates on business and mortgage loans and adjust the collateral requirement limit in each period according to the observed deterioration of borrowers' financial situation. In other words, it is optimal to change the LTV limit whenever they observe the impending negative performance of borrowers. This feature distinguishes our analysis from previous studies that assumed the loan-to-value limit does not vary with the cycle or is given exogenously. We also hypothesize that firm debt is mainly asset-based. We are aware that decomposing firm debt into cash flow-based debt and asset-based debt would make our analysis consistent with the empirical evidence in Lian and Ma (2021). Nevertheless, we believe our formulation of the profit-maximizing firm that faces a collateral constraint is a fair approximation. For instance, if firms that pledged future cash flows against debt cannot meet their interest payments, they may reschedule the debt, issue new equity or debt, or sell their assets. The latter case relies on the same logic as asset-based debt.

We estimate the model using US aggregate data and financial data over the period 1984Q1–2021Q4. We investigate the impact of an increase in the loan-to-value limit for business and mortgage loans on output and labor. We compare the impact of a contractionary monetary policy on output and labor under different scenarios: (i) in the baseline model where lenders adjust their collateral requirements limit based on the borrowers' financial conditions, and credit supply, (ii) in a model with constant LTV, and (iii) in a model where lenders are not concerned by the boom and bust in credit supply and by the level of delinquency when setting the loan-to-value limit. We also compare the macroeconomic outcome of a monetary policy shock in several additional counterfactuals, including under deterministic and endogenous default probability. Finally, we examine the effects of a contractionary monetary policy shock that coexists with the bank-based prudential measure on the delinquency rate and assess its welfare implications.

Our results lead to four main conclusions. First, a prudential policy that attempts to raise the collateral limits and expand credit to businesses and households boosts aggregate demand, and increases output and labor. These results, while obvious, are important as they indicate that the model can reproduce the key features of a credit boom episode. Second, through the lens of our model, we show that an active prudential policy creates a strong link between central bank policy rates and economic activity. It amplifies the responses of output and labor to a monetary policy shock. These findings are then

contrasted with two counterfactual exercises: where the LTV rule is less active and takes the form of an autoregressive process of order one, and where the LTV is kept constant over time. The analysis indicates that implementing a contractionary monetary policy can still trigger a slump in output, but the effect is less pronounced than in the baseline case. In addition, we find that the declines in output and employment due to a contractionary monetary policy are significantly severe in the deterministic default probability model relative to the baseline scenario. Notably, the bank-based prudential instrument cannot reduce the delinquency rates (since it is fixed in every period) or mitigate the associated debt loss, which generates greater volatility in output and employment loss. On the other hand, the monetary policy amplifies the fall in output and labor in the endogenous default probability model. The amplification of this shock can be attributed to the additional endogenous adjustment of monitoring costs, which causes a meaningful loss in output and employment.

Third, and most importantly, we test the optimality of the bank-based prudential measures that explicitly consider default and lean against the credit cycle. We show that the suboptimal scenario for unconstrained households (savers) is when the economy features an active LTV requirement under the assumption that both households have access to the housing market. When we impose the assumption that only borrowers have access to the housing market and are financially constrained, an economy with a constant LTV is socially optimal, with large gains for borrowers at the expense of savers. In general, we find that the welfare gains are sensitive to the model assumptions and the form of the LTV rule. Furthermore, the results regarding the default in the private sector reveal that the bank-based prudential practice in our baseline model can limit movement in default probability. A contractionary monetary policy can be effective in reducing the risk of delinquency in the short run as it lessens the charge-off rate on business and mortgage loans. In terms of consumption and house prices, we find that the baseline model features low volatility relative to the scenario where the LTV is kept constant over the cycle.

Besides the welfare gain from implementing countercyclical bank-based measures, there are several other possible justifications for using such tools. First, the financial instability following the 2008 global financial crisis provided a credible justification for a monetary authority that intervenes to stabilize the financial market or an independent financial authority that is solely concerned with addressing distortion in the financial market. However, a monetary authority that leans against the wind (increases interest rates during booms) can arguably prolong the liquidity trap when monetary policy is supplemented with a macro-prudential policy; see Bianchi and Coulibaly (2022) and Farhi and Werning (2016). Nevertheless, for the macro-prudential policy to work reasonably well, it must be inseparable from the conduct of monetary policy and its target. Second, higher monitoring costs associated with household and business debt raise the burden on loan contractors as risk increases. The costly state verification thus leads to inefficient fluctuations in credit spread, as argued by Carrillo et al. (2021). Finally, the fluctuations in credit

supply drive abnormal movement in delinquency rates. The boom and bust in delinquency rates represent a serious concern for financial stability and accentuate the vulnerability of lenders and borrowers in credit markets. This issue is explored by Bianchi and Mendoza (2018), who emphasize that the frequency of crises can be reduced substantially when a social planner employs an optimal macro-prudential policy without committing to future policies. However, in our context, financial intermediaries set a prudential LTV instrument designed to counter defaults and moderate credit supply in every period.

A substantial body of literature incorporates financial intermediation into mainstream macroeconomic models to improve the policy analysis and predictions of dynamic general equilibrium models, including classic models such as those by Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2014). There has been renewed interest in banking intermediation and financial frictions, particularly in the effect of various policies and financial regulations on financial stability and banking system efficiency (see Brunnermeier and Sannikov (2014), Curdia and Woodford (2016), Jermann and Quadrini (2012), Jakab and Kumhof (2015), He and Krishnamurthy (2012), Fuentes-Albero (2019), Bianchi and Saki (2022), and Akinci and Queralto (2022)). However, most of these papers only consider the impact of monetary policy and banking regulation on the economy. In contrast, we focus on the effects of bank-based prudential measure where both households and firms are financially constrained. This prudential rule ensures that the loan-to-value limit reacts to the borrowers' delinquency rate and credit supply.

Influential contributions to the literature on New Keynesian models with financial frictions include Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Iacoviello (2015), Justiniano, Primiceri, and Tambalotti (2015), Guerrieri and Iacoviello (2017), and Gertler, Kiyotaki, and Prestipino (2017). These studies emphasized the importance of modeling financial frictions in macroeconomic models. We contribute to this body of literature by accounting for financial policy, which sets a countercyclical loan-to-value limit according to the severity of the borrowers' financial condition and credit supply. We demonstrate that a mix of bank-based prudential and monetary policies can diminish the risk of delinquency in business and mortgage debt in the short run. Recent contributions such as Cerutti, Claessens, and Laeven (2017), Kim and Mehrotra (2018), Franz (2020), Martin, Mendicino, and Van der Ghote (2021), Van der Ghote (2021), and Ottonello, Perez, and Varraso (2022) analyze the effects of macro-prudential policies. Our quantitative findings are related to this literature and highlight the importance of switching between tighter and normal collateral terms rather than a loan-to-value limit that does not vary with the cycle. See Gatt (2024) for a detailed discussion on the loan-to-value limit as a policy tool in theory and practice.

This study also contributes to the literature on collateral and its macroeconomic impact. Several influential studies, including Williamson (2016) and Geanakoplos and Zame (2007), highlight the role of collateral. Iacoviello (2005) study the propagation mechanism of a negative monetary shock when the collateral channel is operational, focusing on how the collateral generates amplification effects on

demand shocks, and Mendicino et al. (2020) and Ottonello and Song (2022) investigate optimal capital requirements. Guerrieri and Lorenzoni (2017) identify the channels by which the shock to borrowing limits propagates. Becard and Gauthier (2022) develop a macroeconomic model with heterogeneous agents in which both households and firms face collateral constraints. Other related contributions include Christensen et al. (2009), Alpandaa and Zubairy (2017), Lambertini, Mendicino, and Punzi (2013), Rubio and Carrasco-Gallego (2014), Bianchi and Mendoza (2018), and Drechsel and Kim (2022). Our paper differs from previous studies as we consider a bank-based prudential measure that depends both on the demand and supply sides. In our model, financial intermediaries monitor the changes in the severity of borrowers' conditions and adjust the collateral requirement limit. Our contribution includes the development of a tractable two-agent New Keynesian model with a bank-based prudential measure and use it to understand the effectiveness of these measures and its interaction with monetary policy.

The remainder of the paper is structured as follows. Section 2 provides key insights on bank-based prudential measures. Section 3 describes the theoretical model. Section 4 presents the estimation results. Section 5 discusses the effects of collateral requirements on macroeconomic aggregates, examines the effects of a monetary policy supplemented with a bank-based prudential measure on output, employment, and delinquency rates, and analyzes the welfare implications of the bank-based prudential practice. Section 6 concludes the paper.

2. Understanding Bank-based Prudential Practices

In this section, we discuss bank-based prudential practice and then examine the effects of tighter collateral terms on credit supply using local projection.

2.1. Empirical Measure of Bank-based Prudential Practices

To identify changes in bank-based prudential measures, we rely on the Senior Loan Officer Opinion Survey on Bank Lending Practices, which provides a quarterly measure of the changes in collateral terms on business loans (Figure 1). Our data sample covers 1999-2022. This measure reveals that more than 50% of banks reported tight collateral requirement limits in the middle of the COVID-19 pandemic, which temporarily increased the strain on borrowers before decreasing at the end of 2021. Large variations also occurred after 1999 and during the 2008 crisis. These observations suggest that banks pursue a tight collateral limit policy during uncertain times and can reflect changes in the loan-to-value limit. An alternative interpretation of changes in this measure is that it can be thought of as changes in the LTV limit because of some macro-level regulation (maximum LTV limits) imposed by a financial stability authority. Of course, a clear distinction between bank practice and prudential regulation matters in practice;

nevertheless, this survey data is the best available information on changes in collateral requirement limits.

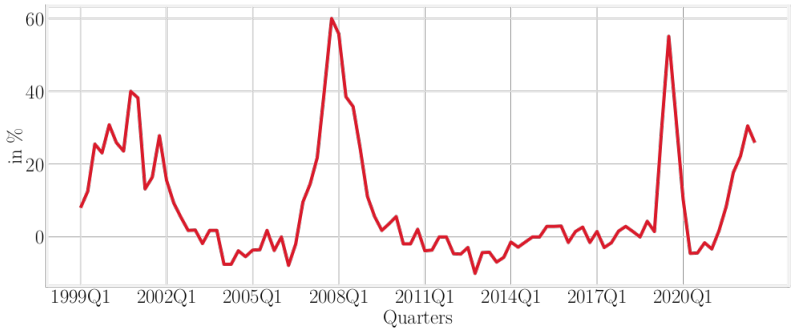


FIGURE 1. Percentage of Banks Reporting a Change in Collateral Requirements for Small Firms

Notes: This figure shows the measure of tightness in collateral conditions (SUBLPDCISTQNQ). Source: Board of Governors of the Federal Reserve System (US), Senior Loan Officer Opinion Survey on Bank Lending Practices. Quarterly data from 1999-2022.

We now present the bank-based prudential instrument in our model, where changes in collateral conditions are correlated with changes in demand-side conditions. Our view is that the changes in the delinquency rate conveyed signals about the state of the borrowers' financial conditions that affected the credit policies of banks through changes in the loan-to-value limit. We use the delinquency rate as a proxy for the deterioration of borrowers' conditions. Thus, we adopt the following specification for the LTV limit ϕ established by financial intermediaries on real estate and business loans:

$$\phi_t^i = \rho_{\phi i}\phi_{t-1}^i - \alpha^{\phi i}(\chi_{t-1}^i - \chi_{ss}^i) - \alpha^{mi}(m_t^i - m_{t-1}^i) + \epsilon_t^{\phi i}.$$

The borrowers are indexed by $\{i \in e, h\}$, while h and e denote households and firms, respectively. ϕ_t^i represents the collateral requirement limit imposed on either households' or firms' loans, $\rho_{\phi i}$ is the collateral limit smoothing parameter, α^{mi} represents the response of LTV limit to the credit supply, and $\alpha^{\phi i}$ denotes the collateral limit weight on the loan delinquency rate. Moreover, χ_{t-1}^i represents the delinquency rate, m_t^i is the credit supply, while $\epsilon_t^{\phi i}$ is the innovation to the collateral requirement limit, which is an i.i.d. with a mean of zero and variance $\sigma_{\phi i}$. Under this specification, the collateral requirement limit imposed by financial intermediaries on borrowers is mechanically linked to the percentage of total loans that have been charged off and to the changes in credit level, and has some degree of history dependence. The assumption that the collateral requirement limit reacts to the deterioration of borrowers' financial conditions distinguishes our analysis from Lambertini, Mendicino, and Punzi (2013) and Rubio and Carrasco-Gallego (2014), who defined a regulatory loan-to-value ratio that responds counter-cyclically to credit growth. Our rule also differs from that proposed by Alpandaa and Zubairy (2017), who introduced

an optimal loan-to-value limit to address households' indebtedness.

2.2. Collateral Conditions and Credit Markets: An Intuitive Channel

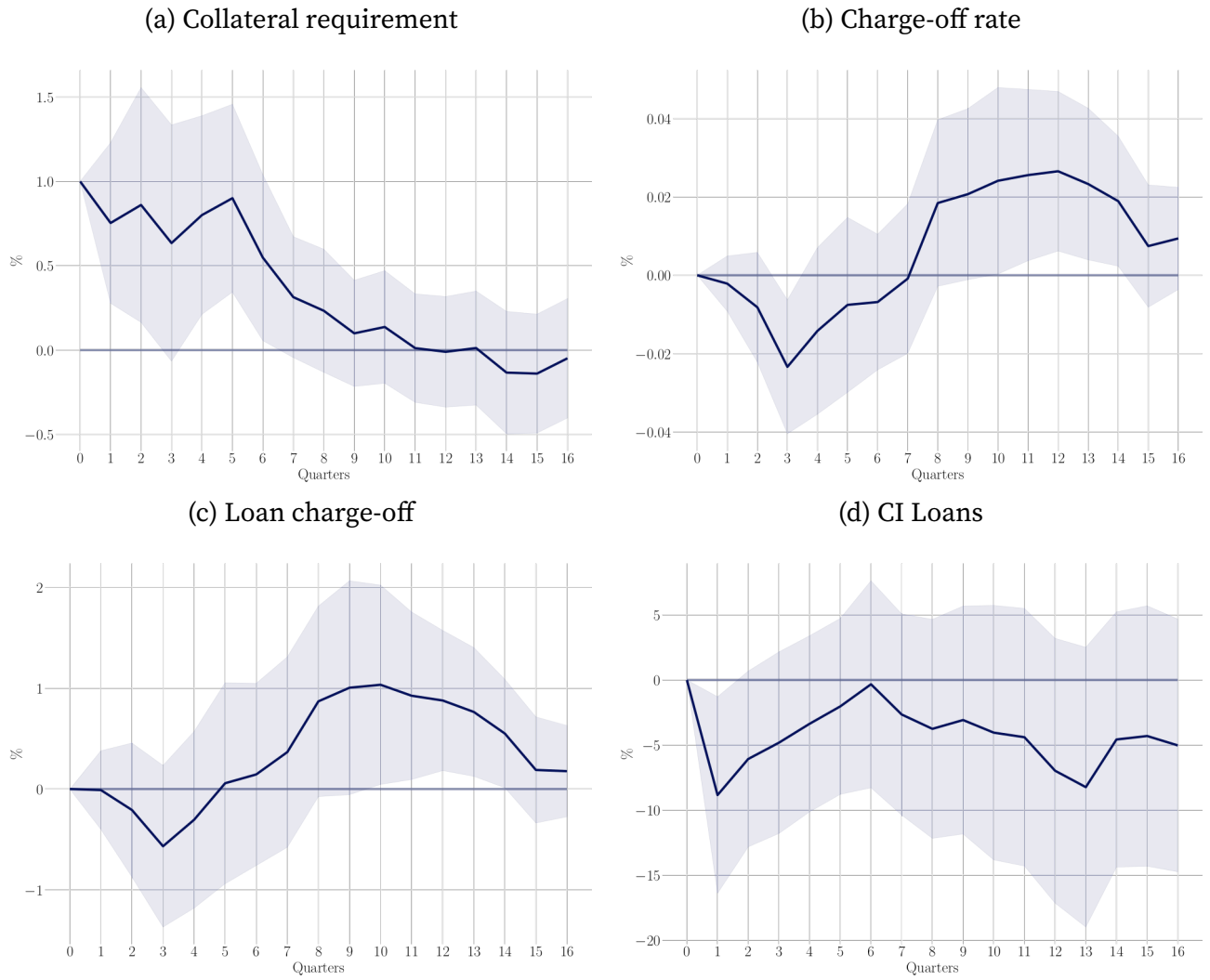


FIGURE 2. IRF to collateral requirement shocks

Notes: Local projection results. Sample: 1999Q1-2022Q3. We rescaled the IR in Panel (c) by 100, and in Panel (d) by 1000.

A simple experiment we consider in this section is how tightness in collateral conditions affects credit markets. We use the percentage of banks reporting tighter collateral terms from the *Senior Loan Officer Opinion Survey on Bank Lending Practices*, which is reported in Figure 1, as a proxy of changes in the bank-based prudential practice, such as the loan-to-value limit. We focus on the period between 1999Q1

and 2022Q3. The empirical specification of our model is as follows:

$$y_{t+h} = \delta_t + \beta_h s_t + \nu_{t+h}; \quad t = 1, \ldots, T, \tag{2.1}$$

where δ_t denotes the time fixed effects, h represent the horizon for the impulse response functions, y_{t+h} is the vector of the endogenous variable, and s_t represents the vector of the predetermined variables and controls. The variables used in our analysis are the collateral limit measure, CI loans, loan charge-offs, and the charge-off rate. The coefficient β_h reflects the response of y, and ν_{t+h} is the error term. As in Jordà (2005), we use Newey-West heteroscedasticity and autocorrelation consistent estimator.

Figure 2 shows the impulse response function to a collateral condition shock. A tighter collateral condition initially reduces the net charge-off rate on business loans, although the effect is significant only in the third quarter. The effect becomes positive and significant ten quarters after the shock. The response of the total loans, which is classified as a loss to the shock, is nearly insignificant but becomes positive and significant ten quarters after the shock. The business loan supply declines more gradually, with a significant impact observed one quarter after the shock. It is worth noting that the decline in the business loan delinquency rate is plausibly driven by the large decrease in the loan supply, which outpaces the decline in the loan charge-off. Moreover, stricter conditions on the pledged collateral may result in a boom and bust in the delinquency rate and total loan charge-offs. Thus, tighter bank-based prudential instruments do not have a significant persistent impact on business credit supply, however, they can induce a substantial shrink in credit supply in the short run and result in lower default levels.

3. A Model with Collateral Requirements

The model developed in this paper shares the key features of New Keynesian models in the financial sector, including forward-looking agents who live infinitely and maximize their profits in a discrete-time economy. These agents maximize their utility and profits with the presence of nominal rigidities in prices and wages. The main agents in this framework are two types of households (constrained and unconstrained), entrepreneurs, a financial intermediary, and a central bank. In equilibrium, all households and entrepreneurs behave optimally, and all markets clear. To conserve space, we show the full derivation of the model in Appendix A. In this setting, the key ingredients are (i) collateral-constrained households, (ii) collateral-constrained entrepreneurs, and (iii) a financial intermediary that can conduct a liberal or tight lending policy through an internal LTV limit that inversely respond to default and credit growth. Below, we illustrate how this feature is important for the central bank's monetary policy and how it mitigates the default risk.

Unconstrained Households - Savers. Consider unconstrained households that maximize their lifetime utility by providing labor services, consuming the final goods, and making deposits. The utility function takes the form

$$E_0 \Sigma_{t=0}^{\infty} \beta_u^t \left\{ \zeta_{c,t} (\log(c_{u,t} - b_u c_{u,t-1}) - \psi_l \frac{(l_{u,t})^{1+\sigma_l}}{1+\sigma_l} \right\}, \tag{3.1}$$

subject to $c_{u,t} + d_t \leq w_t l_{u,t} + (1 + r_t) d_{t-1}/\pi_t$. Here, $0 < \beta_u < 1$ is the discount factor, $c_{u,t}$ is the per capita consumption, d_t is the deposit, w_t is the hourly earnings, $l_{u,t}$ represents the hours worked, b_u is the internal habit in consumption, π_t is the gross inflation rate, and r_t is the nominal interest rate. The parameter σ_l is the curvature on the disutility of labor, and ψ_l is the disutility weight on labor. Consumers face consumption preference shocks $\zeta_{c,t}$, which is assumed to evolve as follows: $\zeta_{c,t} = \rho_{\zeta_c} \zeta_{c,t-1} + \epsilon_t^{\zeta_c}$. The consumer decides the level of consumption, deposit, and labor with the marginal utility $\lambda_{u,t}$.

Collateral-Constrained Households - Borrowers. Financially constrained households choose the consumption $c_{c,t}$, hours worked $l_{c,t}$, debt m_t^h to maximize their expected lifetime utility:

$$E_0 \Sigma_{t=0}^{\infty} \beta_c^t \left\{ (\log(c_{c,t} - b_c c_{c,t-1}) - \psi_l \frac{(l_{c,t})^{1+\sigma_l}}{1+\sigma_l} + \chi \log(h_t) \right\}, \tag{3.2}$$

subject to the flow budget constraint $c_{c,t} + q_t^h h_t + m_{t-1}^h (1 + r_t^e)/\pi_t \leq w_t l_{c,t} + q_t^h h_{t-1} + m_t^h$ and collateral constraint $\phi_t^h q_t^h h_t \geq (1 + r_t^e) m_{t-1}^h / \pi_t$. Here, b_u denotes the internal habit in consumption, ψ is the weight of labor in the utility function and σ_l is the inverse of the Frisch elasticity of labor supply. The parameter β_c denotes the discount factor. We assume that constrained households discount the future more than unconstrained households, that is, $\beta_c < \beta_u$. χ is the weight of assets in the utility of households, and ϕ_t^h is the LTV requirement imposed on constrained households. Total housing stock h is exogenous and follows the autoregressive process $h_t = \rho_q h_{t-1} + \epsilon_{h,t}$. We assume that collateral-constrained households are aware of the gross inflation rate π_t, the nominal interest rate r_t^e, and the wage rate w_t. Financially constrained households decide the consumption c, hours l, and debt m^h with marginal utility $\lambda_{c,t}$.

We define the probability that a debt is charged-off as $\chi_t^h = \left(1/(1 + e^{\sigma_t^h}) \right)$ for all $\sigma_t^h \in (0, +\infty)$, where σ_t^h reflects the volatility that can lead to a high or low probability of debt being uncollectible with no chance of being repaid. This shock follows a standard autoregressive process of order 1: $\sigma_t^h = \rho_h \sigma_{t-1}^h + \epsilon_t^h$. Moreover, we assume that financial intermediaries establish an LTV requirement on real estate loans. The simplest specification of the LTV requirement rule adopted here is

$$\phi_t^h = \rho_{\phi h} \phi_{t-1}^h - \alpha^{\phi h} (\chi_{t-1}^h - \chi_{ss}^h) - \alpha^{mh} (m_t^h - m_{t-1}^h) + \epsilon_t^{\phi h},$$

with the innovation $\epsilon_t^{\phi h} \sim N(0, \sigma_{\phi h}^2)$. A rule of this type is primarily concerned with moderating the credit cycle and reducing the default rate. In this economy, the collateral requirements at time t adjust to

its previous level ϕ^h_{t-1}, and respond to the debt m^h and the loan charge-off rate χ^h_{t-1} in a counter-cyclical manner. It is worth mentioning that the collateral constraint does not guarantee risk-free debt. Indeed, constrained households perceive the debt as risky in the sense that an exogenous shifter ϕ^h_t can either relax or tighten the borrowing constraint, and the collateral price is affected by the household's marginal utility of consumption. Thus, by borrowing more, households increase current consumption and raise equilibrium collateral prices.

Furthermore, we assume that unconstrained households make loans to collateral-constrained households at the nominal interest rate r^e_t, via a financial intermediary that has the participation constraint $(1-\chi^h_t)(1+r^e_t)m^h_{t-1}/\pi_t + (1-\mu^h)\chi^h_t\phi^h_t q^h_t h_{t-1} \geq (1+r_t)m^h_{t-1}/\pi_t$. This condition gives an equilibrium in which banks incur no loss when providing loans to collateral-constrained households. With probability $(1-\chi^h_t)$, the borrowers can pay back the debt $(1+r^e_t)m^h_{t-1}/\pi_t$; with probability χ^h_t, the bank cannot collect debt and classifies it as delinquent. In the case of delinquency, this condition implies a cost $(1-\mu^h)$ associated with the repossession of the collateral $\phi^h_t q^h_t h_{t-1}$. Regardless of whether the borrower defaults on their loans, the banks always receive total earnings from lending equal to $(1+r_t)m^h_{t-1}/\pi_t$. Finally, aggregating the labor and consumption of the two household types yields $l_{i,t} = [l_{u,t} + l_{c,t}]$ and $c_{i,t} = [c_{u,t} + c_{c,t}]$.

Labor Market and Wage Stickiness. Labor contractors hire households $i \in \{u, c\}$ and sell homogeneous labor services to intermediate good producers. Labor services take the form $l_t = [\int_0^1 (l_{i,t})^{1/\nu_l} di]^{\nu_l}$ subject to $\int_0^1 w_{i,t} l_{i,t} di = w_t l_t$, where ν_l denotes the fixed wage markup and $w_{i,t}$ is the wage rate. There is a monopoly union that represents all workers, sets wages, and faces Calvo-style frictions: $w_t = [(1-\zeta_l)(w_{t-1})^{1/(1-\nu_l)} + \zeta_l (w^*_t)^{1/(1-\nu_l)}]^{1-\nu_l}$. Here, $1-\zeta_l$ is the probability that the monopoly reoptimizes the wages, while with probability ζ_l, the monopoly cannot reoptimize.

Goods Production and Price Stickiness. The economy is populated by a continuum of firms and operates under monopolistic competition. Each firm has the final goods stock $y_t = [\int_0^1 y_{j,t}^{1/(1+\nu_p)} d_j]^{1+\nu_p}$. The final good is indexed by $j \in [0, 1]$, and $1 < \nu_p < \infty$. A higher price markup ν_p implies that firms have market power and that the good is less substitutable for other goods. Using capital and labor inputs, the monopolist firm produces the intermediate good y_t, and the intermediate good production function takes the form

$$y_t = k^{\alpha}_{t-1}(z_t l_t)^{1-\alpha} \tag{3.3}$$

where k_t is the capital input, l_t is the labor input and z_t is an exogenous productivity shock $z_t = \rho_z z_{t-1} + \epsilon^z_t$. The parameter $\alpha \in (0, 1)$ measures the responsiveness of output to changes in capital. We assume that

prices are sticky by using a variant of Calvo-type frictions, as in Christiano, Motto, and Rostagno (2014). The monopolist can set the price p_t of the good by reoptimizing the price with probability $1-\zeta_p$, or without reoptimizing the price with probability ζ_p, and where p_t^* is the price level that maximizes the expected discounted value of future profits. The price level is defined by $p_t = [(1-\zeta_p)(p_{t-1})^{\nu_p/(1-\nu_p)} + \zeta_p(p_t^*)^{\nu_p/(1-\nu_p)}]^{(1-\nu_p)/\nu_p}$. After setting prices and quantities, the monopolist minimizes the production cost $w_t l_t + r_t^k k_t$ subject to the production function 3.3. The producer chooses the level of labor and capital allocation with a marginal cost μ_t.

Capital Markets. In capital markets, the previous capital is combined with investment goods i_t to produce new capital k_t with the following technology:

$$k_t = (1-\delta)k_{t-1} + [1 - S/2(i_t/i_{t-1} - 1)^2]i_t \tag{3.4}$$

where capital decays at the fixed rate $0 < \delta \leq 1$. The quantity of investment at period t is proportional to the adjustment cost function S. Capital producers choose investments to maximize the present value and future operating profits minus the total investment cost: $q_t^k i_t - [1 + S/2(i_t/i_{t-1} - 1)^2]i_t$. Based on the law of motion for capital stock k_t and investment expenditures, capital producers sell capital to entrepreneurs at the price q^k. Capital producers' optimal investment equalizes the marginal revenue product of capital to the marginal cost of investment goods.[3]

Entrepreneurs and Collateral Constraints. Each entrepreneur purchases the capital goods k_t at the price q_t^k using loans m_t obtained from the bank. Their net worth net is $k_t q_t^k = m_t^e + n_t^e$ for all t. Entrepreneurs are hit by a shock $\mathcal{X}_t^e = \left(1/(1+e^{\sigma_t^e})\right)$, which describes the probability that the entrepreneurs are unable to pay their debt. Letting σ_t^e obey the AR(1) process $\sigma_t^e = \rho_e \sigma_{t-1}^e + \epsilon_t^e$, the default probability increases when σ_t^e goes up. The collateral constraint for entrepreneurs is defined as $\phi_t^e(1+r_t^k)q_t^k k_t \geq (1+r_t^e)m_t^e/\pi_t$. The entrepreneur's debt is charged-off with probability $\mathcal{X}_t^e$. As the entrepreneur is unable to pay the interest and principal, the pledged collateral is seized by the bank. Financial intermediaries adopt a collateral requirements limit ϕ_t^e which evolve over time according to the following rule:

$$\phi_t^e = \rho_{\phi e}\phi_{t-1}^e - \alpha^{\phi e}(\mathcal{X}_{t-1}^e - \mathcal{X}_{ss}^e) - \alpha^{me}(m_t^e - m_{t-1}^e) + \epsilon_t^{\phi e}.$$

This rule indicates that lenders adjust the level of collateral requirements to the previous level at $t-1$, where $\rho_{\phi e}$ determines the LTV weight on previous collateral requirements. α^{me} is the response of the

[3]The capital producer's profit is given by $q_t^k k_t - [1 + S/2(i_t/i_{t-1} - 1)^2]i_t$, where the law of motion for capital is $k = k_0 + [1 + S/2(i_t/i_{t-1} - 1)^2]i_t$ given that, in a steady state, $S(.) = 0$ becomes $k - k_0 = i_t$. Then, the capital producer's problem can be written as $q_t^k i_t - [1 + S/2(i_t/i_{t-1} - 1)^2]i_t$.

LTV limit to business credit, and $\alpha^{\phi e}$ is interpreted as the LTV weight on the charge-off rate of business loans. The innovation $\epsilon_t^{\phi e}$ is assumed to be an i.i.d. with a mean zero and standard deviation $\sigma^{\phi e}$.

The entrepreneur's expected earnings are given by

$$E_t\{[(1+r_t^k)q_t^k k_t - (1+r_t^e)m_t^e/\pi_t](1-\mathcal{X}_{t-1}^e)\}, \tag{3.5}$$

where r_t^k is the average real rate of return on capital and r_t^e is the nominal interest rate paid by the entrepreneurs on their debt m_t^e. For the sake of simplicity, we define the probability that entrepreneurs will default on their loans as $\mathcal{X}_{t-1}^e$ and the probability that they meet their scheduled debt as $1-\mathcal{X}_{t-1}^e$. We assume that the leverage level of entrepreneurs is $Lev_t = m_t^e/n_t^e$. Thus, the entrepreneurs' problem is to maximize their entrepreneurial earnings[4] subject to the participation constraint:

$$(1-\mathcal{X}_t^e)(1+r_t^e)m_t^e/\pi_t + (1-\mu^e)\mathcal{X}_t^e(1+r_t^k)\phi_t^e q_t^k k_t \geq (1+r_t)m_t^e/\pi_t. \tag{3.6}$$

Note that the first term on the left-hand side corresponds to returns from non-defaulting entrepreneurs; the second term corresponds to the returns from defaulting entrepreneurs whose collateral is seized by banks, net of the monitoring cost μ^e. The right-hand side describes the return on loans given the interest rate demanded by banks r_t. We define the entrepreneur's expected net worth as $n_{t+1}^e = \gamma^E((1-\mathcal{X}_t^e - (1-\mathcal{X}_t^e)\phi_t^e)(1+R_t^k)k_t q_{t-1}^k + k_t(1-\delta)q_{t-1}^k$.

Monetary Policy Rule. The linearized for of the monetary policy rule is:

$$\tilde{r}_t = \rho_p \tilde{r}_{t-1} + (1-\rho_p)[a_\pi(\tilde{p}_t - p_{ss}) + a_{\Delta y}(\tilde{y}_t)] + \epsilon_t. \tag{3.7}$$

Here, the nominal interest rate r_t depends on price gap and output. ϵ_t is the monetary policy shock with $\epsilon_t \in N(0, \sigma_r^2)$.

Resource Constraints and Equilibrium We complete the model setup by specifying the main aggregate resource constraints

$$y_t = c_t + f_t^e + f_t^h + i_t + + g_t \tag{3.8}$$

Here, c_t is the aggregate consumption, f_t^e represents the banks spending on monitoring entrepreneurs with $f_t^e = \phi_t^e \mu^e \mathcal{X}_t^e (1+r_t^k) q_t^k k_t$, and the term $f_t^h = \phi_t^h \mu^h \mathcal{X}_t^h q_t^h h_t$ represents the costs associated with

[4] See the Technical Appendix for detailed computations.

monitoring households. g_t denotes government consumption and is given by $gt = \eta_{g,t} y_t$. The share of government consumption in the total output follows an AR(1) process with $\eta_{g,t} = \rho_g \eta_{g,t} + \epsilon_t^g$, and i_t is the aggregate investment. A competitive equilibrium consists of stochastic processes $\{p_t, p_t^*, F_{p,t}, K_{p,t}, w_t, w_t^*, F_{w,t}, k_t, i_t, \mu_t, q_t^k, r_t^k, \bar{r}_t^k, n_t^e, y_t, l_t, l_{u,t}, l_{c,t}, \lambda_t, \lambda_{u,t}, \lambda c, t, c_t, c_{u,t}, c_{c,t}, d_t, h_t, m_t^h, q_t^h, r_t^e, f_t^e, f_t^h, m_t^e\}$ that evolve according to a system of equations. This model is driven by eight exogenous shocks: a government spending shock $\eta_{g,t}$, a technology shock z_t, a consumption preference shock ζ_t^c, a monetary policy shock r_t, a delinquency rate shock on firm loans χ_t^e, a delinquency rate shock on mortgage loans χ_t^h, a business loan collateral requirement shock ϕ_t^e, and a mortgage loan collateral requirement shock ϕ_t^h.

4. Estimation

In this section, we first present the solution method and the calibrated parameters of the model. Then, we estimate the model parameters and shocks using Bayesian methods.

Methods and Data. We approximate the equilibrium system by log-linearizing around the steady state. Hereafter, all variables in log deviations from the steady-state values are identified with a tilde (the model derivations and log-linearized model equations are reported in the Technical Appendix). We estimate the model by proceeding in three stages: (i) we construct the empirical data to be combined with the model's equations, (ii) we calibrate key economic parameters, and (iii) we estimate the remaining economic parameters and shock parameters using Bayesian methods.

We use quarterly US data from 1984Q1 to 2021Q4 (see Figure A1 in the Appendix). The time series are normalized to have a mean zero. For example, data on the gross domestic product, consumption, investment, commercial industrial average outstanding, commercial industrial total charge-offs, real estate loans average outstanding, real estate loans total charge-offs, and government spending are deflated using their specific implicit price deflator. Then, we express these aggregate variables in per capita terms, divide each variable by the civilian non-institutionalized population over 16, and take the logarithmic first difference. The data on hours worked are normalized by population only, and then transformed by taking the first difference of its logarithm. We also take the logarithmic first difference of the data on the delinquency rate of commercial industrial loans and the delinquency rate of real estate loans. Finally, we end up with eleven observables:

$$[\Delta \log y_t^{obs}, \Delta \log c_t^{obs}, \Delta \log m_t^{e,obs}, \Delta \log d_t^{h,obs}, \Delta \log i_t^{obs},$$
$$\Delta \log l_t^{obs}, \Delta \log f_t^{e,obs}, \Delta \log f_t^{h,obs}, \Delta \log g_t^{obs}, \Delta \log \chi_t^{h,obs}, \Delta \log \chi_t^{e,obs}]$$

These observables are growth in output $\Delta \log y_t^{obs}$, growth in consumption $\Delta \log c_t^{obs}$, growth in commer-

cial industrial $\Delta \log m_t^{e,obs}$, real estate loans $\Delta \log d_t^{h,obs}$, growth in investment $\Delta \log i_t^{obs}$, growth in hours worked $\Delta \log l_t^{obs}$, growth in commercial industrial loans total charge-offs $\Delta \log f_t^{e,obs}$, growth in real estate loans total charge-offs $\Delta \log f_t^{h,obs}$, growth in government spending $\Delta \log g_t^{obs}$, growth in the delinquency rate of real estate loans $\Delta \log \chi_t^{h,obs}$, and growth in the delinquency rate of firm loans $\Delta \log \chi_t^{e,obs}$. The Technical Appendix describes the system of measurement equations in detail.

TABLE 1. Calibrated Parameters

Parameter	Description	Value
Household		
β	Discount rate - unconstrained households	0.990
β_c	Discount rate - constrained households	0.950
σ^l	Curvature on disutility of labor	1.000
b_u	Habit persistence parameter - unconstrained households	0.630
b_c	Habit persistence parameter - constrained households	0.630
ν^l	Steady-state markup suppliers of labor	1.050
μ^h	Fraction of realized profits lost in bankruptcy	0.940
χ	Utility weight on housing	1.500
ϕ^h	Steady-state collateral requirements on mortgage loan	0.980
Production		
δ	Depreciation rate on capital	0.025
S	Adjustment cost function	5.000
α	Power on capital in production function	0.400
ν^p	Steady-state markup, intermediate good firms	0.200
Entrepreneurs		
γ^e	Percentage of entrepreneurs who survive	0.976
μ^E	Fraction of realized profits lost in bankruptcy	0.940
ϕ^e	Steady-state collateral requirements on business loan	0.980
Policy and shocks		
$\eta_{g,ss}$	Share of government consumption	0.200

Notes: The time frequency is quarterly.

Parameter Estimates. The model is parameterized such that the key variables are matched to the US data over the sample period 1984–2021. The parameterization is described in Table 1. As a standard practice in

the business cycle literature, such as in Iacoviello (2005), we set the discount factor for unconstrained households β_u to 0.99 and the discount factor for constrained household β_c to 0.95. This reflects the assumption that collateral-constrained households are more impatient than unconstrained households. We set the elasticity of labor supply σ^L to 1 and the consumption habits of constrained and unconstrained households, b^c and b^u respectively, to 0.63. We also set the wage markup ν^l to 1.05, which is close to the value of labor supply elasticity in Justiniano, Primiceri, and Tambalotti (2011). We assume the steady-state price of housing stock q^h is equal to 1. The price markup of intermediate good firms $\nu^p = 0.2$ and the power on capital in the production function $\alpha = 0.4$ are in line with the values used in the literature. We set the utility weight on housing χ to 1.5, which is similar to the value used by Justiniano, Primiceri, and Tambalotti (2014). Concerning investment, we set the depreciation rate on capital δ to 0.025, a standard value in the business cycle literature. To target the delinquency rate on mortgage loans $\chi^h = 8\%$, we set σ^h to 2.5, such that $\chi_t^h = \left(1/(1+e^{\sigma^h})\right) = 0.0759$. Similarly, we set σ^e to 2.5 to target a value of 8% for the default rate on business loans. The monitoring cost of business loans μ^e is set to 0.94, as in Christiano, Motto, and Rostagno (2014). We also set the monitoring cost of real estate loans μ^h to 0.94. The collateral requirement limits ϕ^h and ϕ^e are equal to 0.98, as in Iacoviello and Pavan (2013) and Justiniano, Primiceri, and Tambalotti (2014). We set the percentage of entrepreneurs who survive γ^e to 0.68. Moreover, we set standard values for the Calvo parameters of sticky prices ζ^p and sticky wages ζ^l to 0.702 and 0.771, respectively. Finally, we set the value of the steady-state government spending to output ratio η_g to 0.18, which is consistent with the US data.

We now describe the procedure for estimating the unknown parameters. Following a Bayesian approach, we choose prior distributions of selected parameters for estimation. Researchers typically rely on existing empirical literature or beliefs derived from macro and micro-level evidence. The posterior distribution was constructed over the period 1984Q1-2021Q4 using the Metropolis-Hastings algorithm, with 10,000 draws per chain needed to achieve convergence. The estimation results are reported in Table 2.

For habit parameters b_u and b_c, we assume the prior obeys the Beta distribution with a mean of 0.5 and a standard deviation of 0.1, as in Iacoviello and Pavan (2013). Including habit formation in the model allows for hump-shaped responses of macroeconomic variables to economic shocks. The Calvo price stickiness parameters ζ_l and ζ_p are assumed to follow the Beta distribution, with a mean of 0.75 and a standard deviation of 0.1. According to our estimates, the model exhibits a high degree of price and wage rigidity, and the values of these two estimates are consistent with Guerrieri and Iacoviello (2017). We also assume that the prior of adjustment cost function S follows a Normal distribution with a mean of 5 and a standard deviation of 0.1, such as in Christiano, Motto, and Rostagno (2014) and Guerrieri and Iacoviello (2017).

TABLE 2. Prior and Posterior Distribution of Estimated Parameters

		Prior Distribution			Posterior Distribution	
		Density	Mean	Std. Dev.	Mean	Std. Dev.
Macroeconomic Parameters						
b_u	Habit parameter - unconstrained households	Beta	0.500	0.100	0.552	0.001
b_c	Habit parameter - constrained households	Beta	0.500	0.100	0.692	0.001
ζ_l	Calvo wage stickiness	Beta	0.750	0.100	0.797	0.002
ζ_p	Calvo price stickiness	Beta	0.750	0.100	0.660	0.004
S	Investment adjustment cost curvature	Normal	5.000	0.100	4.992	0.001
Shock Parameters: Persistence						
α^{pi}	Policy weight on inflation	Normal	1.500	0.250	1.697	0.001
α^{Δ_y}	Policy weight on output growth	Normal	0.250	0.100	0.214	0.001
ρ_p	Policy smoothing parameter	Beta	0.500	0.100	0.839	0.002
ρ_g	Autoc. gov. Spend. shock	Beta	0.700	0.100	0.692	0.001
ρ_z	Autoc. tech. shock	Beta	0.700	0.100	0.938	0.001
ρ_{ζ_c}	Autoc. cons. pref. shock	Beta	0.700	0.100	0.852	0.001
ρ_{σ_e}	Autoc. b. loan delinq. rate shock	Beta	0.700	0.100	0.463	0.001
ρ_{σ_h}	Autoc. r. r. loan delinq. rate shock	Beta	0.700	0.100	0.500	0.003
ρ_{ϕ_e}	Collateral policy smoothing parameter - B. Loan	Beta	0.700	0.100	0.527	0.000
ρ_{ϕ_h}	Collateral policy smoothing parameter - R. E. Loan	Beta	0.700	0.100	0.523	0.001
$\alpha^{\phi e}$	Collateral policy weight on b. loan delinq. rate	Beta	0.500	0.100	0.508	0.000
$\alpha^{\phi h}$	Collateral policy weight on r. e. loan delinq. rate	Beta	0.500	0.100	0.582	0.001
α^{me}	Collateral policy weight on b. loan	Beta	0.500	0.100	0.673	0.003
α^{mh}	Collateral policy weight on r. e. loan	Beta	0.500	0.100	0.532	0.002
Shock Parameters: Standard Deviation						
ϵ_g	St. dev. gov. spend. shock	Inv. G.	0.001	Inf	0.021	0.001
ϵ_z	St. dev. tech. shock	Inv. G.	0.001	Inf	1.015	0.002
ϵ_{ζ_c}	St. dev. cons. pref. shock	Inv. G.	0.001	Inf	0.048	0.005
ϵ_{σ_e}	St. dev. b. loan delinq. rate. shock	Inv. G.	0.001	Inf	1.040	0.065
ϵ_{σ_h}	St. dev. r. e. loan delinq. Rate. Shock	Inv. G.	0.001	Inf	4.281	0.057
ϵ_{ϕ_e}	St. dev. b. loan coll. requir. shock	Inv. G.	0.001	Inf	0.282	0.017
ϵ_{ϕ_h}	St. dev. r. e. loan coll. requir. shock	Inv. G.	0.001	Inf	0.001	0.001
ϵ_h	St. dev. r. e. volume shock	Inv. G.	0.001	Inf	1.046	0.004
ϵ_ρ	St. dev. m. p. shock	Inv. G.	0.001	Inf	0.210	0.014
Measurement Errors on The Observables						
$\rho_{y^{obs}}$	Measurement error on y_t	Inv. G.	0.001	Inf	1.615	0.027
$\rho_{i^{obs}}$	Measurement error on i_t	Inv. G.	0.001	Inf	0.937	0.016
$\rho_{l^{obs}}$	Measurement error on l_t	Inv. G.	0.001	Inf	0.017	0.002
$\rho_{f^{e,obs}}$	Measurement error on f_t^e	Inv. G.	0.001	Inf	0.968	0.007
$\rho_{f^{h,obs}}$	Measurement error on f_t^h	Inv. G.	0.001	Inf	0.032	0.053

Notes: The table reports the results of Bayesian estimation, posterior statistics are constructed using 10,000 draws per chain.

We assume the priors are centered at 0.7 for the persistence of shock processes (government spending, technology, consumption preference, real estate delinquency rate, business loan delinquency rate, collateral requirement on business loans, and collateral requirement on real estate loans) with a Beta distribution that ensures the estimates are bounded between 0 and 1 (the standard deviation of the prior is 0.1). We also assume the prior of standard deviation of structural shocks and the measurement errors on the observables follow an Inverse Gamma distribution with a mean of 0.001 and a standard deviation of ∞.

With nine orthogonal shocks and five measurement errors, we use the eleven observable variables described above to estimate the model. Regarding the identification of the two main financial shocks in the model, collateral requirements and delinquency rate shocks, we use aggregated mortgage and firm debt data. We also add the delinquency rates on firm and mortgage loans, and the total charge-offs on firm and real estate debt data from the FDIC to identify these two shocks.

We interpret large positive swings in the delinquency rates, which is treated as an exogenous object in our model, as a higher probability of a crisis occurring. Several papers treat default as either exogenous or endogenous. For example, Gunn and Johri (2013) and Ajello (2016) assume that intermediation costs evolve exogenously over time and interpret these disturbances as financial shocks, and Gunn, Johri, and Letendre (2022) show how endogenous default and news shocks to the default costs help in matching the model with the observed variance in the data. Further,Candian and Dmitriev (2020) provide a micro-foundation for fluctuations in bankruptcy costs, highlighting how endogenous liquidation costs amplify the response of macroeconomic aggregates to financial shocks. In Section 5, we demonstrate how our economy reacts counterfactually to a contractionary monetary policy under deterministic and endogenous delinquency rates.

Recall that the corresponding equations for the collateral requirement limits on real estate loans and firm loans are

$$\varphi_t^h = 0.523\phi_{t-1}^h - 0.582(\mathcal{X}_{t-1}^h - \mathcal{X}_{ss}^h) - 0.532(m_t^h - m_{t-1}^h) + \epsilon_t^{\Phi h}, \tag{4.1}$$

$$\varphi_t^e = 0.527\phi_{t-1}^h - 0.508(\mathcal{X}_{t-1}^e - \mathcal{X}_{ss}^e) - 0.673(m_t^e - m_{t-1}^e) + \epsilon_t^{\Phi e} \tag{4.2}$$

Based on these two regressions, the feedback coefficient of collateral requirements to the mortgage delinquency rate is relatively high (0.582); similarly, the collateral requirement limit on firm loans appears to react very strongly to the firm loan delinquency rate (0.508). The coefficients of the lagged collateral requirement are 0.523 for mortgage loans and 0.527 for business loans. The feedback coefficient of the LTV requirement to firm credit supply is 0.532, and to mortgage supply is 0.673. According to these collateral limit rules, financial intermediaries will adjust their collateral requirement limits as they observe the variations in the default rates. Higher delinquency rates and credit boom conveyed negative signals about

the borrower's financial conditions; thus, the tightening of collateral terms is the natural outcome of peculiar behaviors in default among borrowers.

Before presenting the results, it is useful to discuss the identification of the collateral requirement (Equations 4.1 and 4.2). This specification captures the idea that lenders require a tight loan-to-value ratio as long as the probability of default is high, and they set loose collateral conditions when the default probability is low. After observing the percentage of total loans that have been charged-off—an indicator of the deterioration of the financial situation of borrowers— financial intermediaries set the collateral requirement limit. The design of the prudential limit proposed in this article echoes the ideas in Korinek and Mendoza (2014), who explain that a social planner intervenes whenever a positive probability of sudden stop events is high. In this case, the planner reduces borrowing in the present to relax future collateral constraints and mitigate the severity of sudden stops. In many existing collateral constraint models, bank-based prudential measures counter-cyclically react to credit supply or the level of indebtedness, whereas we assume it reacts to the credit supply and default probability. It is worth emphasizing that our specification is subject to the Lucas critique since the parameters of the equation are unlikely to remain constant. One way to test the stability of the parameters $\rho_{\phi i}$, $\alpha^{\phi i}$ and α^{mi} is to control for confounding variables with a similar empirical specification using microdata. However, even with this approach, we cannot completely rule out endogeneity bias. A full analysis of the identification of the loan-to-value limit, while important, is not the primary objective of this article.

5. Monetary Policy and Bank-based Prudential Measure

In this section, we first look at the response of output and the shifts in labor to changes in the LTV limits. We examine the propagation of a monetary policy shock, in particular when the central bank conducts a contractionary policy by raising the interest rate, and assess any potential conflicts between the bank-based prudential and monetary policies. The goal here is not to test whether our model matches the facts quantitatively. Instead, we focus on emphasizing the impact of a monetary policy shock that coexists with a bank-based prudential practice in multiple versions of the model, with exogenous, endogenous, and deterministic default probability, and an additional model extension that allows for a constant LTV limit. Finally, we examine the welfare implication of the LTV limits under different model assumptions.

How Does the Collateral Limit Affect Output and Labor? We consider the effects of collateral limit friction on output and labor. The responses to a positive loan-to-value limit shock displayed in Panel (a) and (c) of Figure 3 reflect the case where the economy is hit by a shock that facilitates access to business credit. This shock leads to a sharp rise in the total output followed by a gradual recovery toward the steady-state level up to the 10th quarter. This increase in business activity translates then into higher labor demand,

and as collateral limit pressure subsides, labor adjusts gradually to reach the steady-state level. Panels (b) and (d) of Figure 3 display the response of output and labor to an increase in the collateral limit on mortgage loans. The increase in the loan-to-value limit creates a long-run positive impact on output and entails a large increase in aggregate employment, which is interpreted here as the total hours worked.

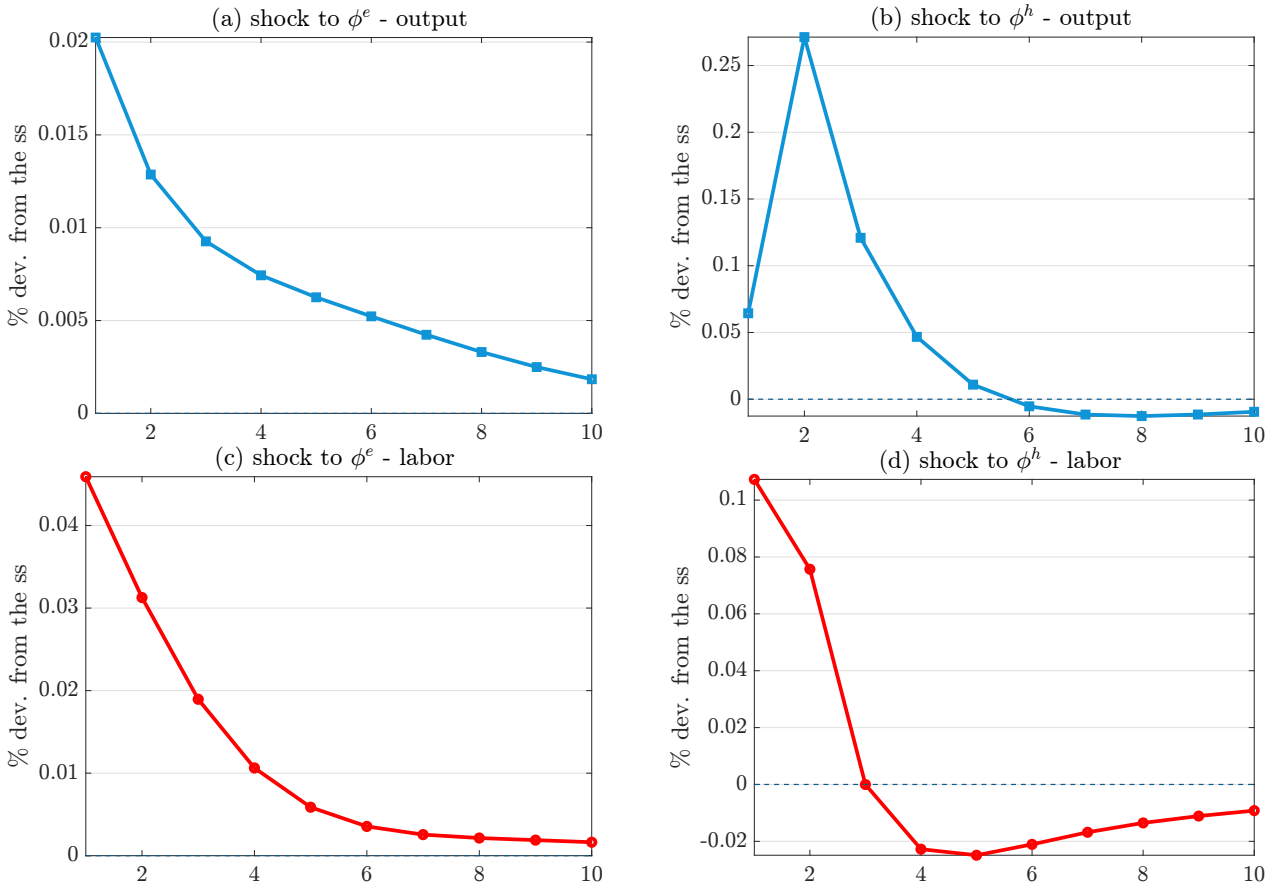


FIGURE 3. Positive Collateral Requirement Shocks: ϕ^e and ϕ^h

Notes: Panels (a) and (c) report the response of labor and output after an increase in the LTV limit ϕ^e. Panels (b) and (d) show the response of labor and output after an increase in the LTV limit ϕ^h. The time frequency is quarterly.

There are several takeaways from these results. First, when the credit supply is expanded to households and businesses, the aggregate output and employment will be at their highest level. These over-time gains in employment and output induced from relaxing the LTV limits are relatively large and perhaps justify their use as a corrective action. Second, under such a bank-based rule, the business and household borrowing channels appear to transmit this shock disproportionately, and exhibit distinct dynamic interactions. For example, the output and employment responses to changes in the collateral limit on mortgage loans up-weights their responses to changes in collateral limit on business loans.

Effects of a Contractionary Monetary Policy Shock. This next section considers three alternative definitions of a bank-based prudential measure. As described in the baseline setup, entrepreneurs and collateral-constrained households face a collateral requirement limit in the form of:

$$\phi_t^i = \rho_{\phi i}\phi_{t-1}^i - \alpha^{\phi i}(\mathcal{X}_t^i - \mathcal{X}_{ss}^i) - \alpha^{mi}(m_t^i - m_{t-1}^i) + \epsilon_t^{\phi i},$$

where $i \in e, h$ refers to the relevant household or entrepreneur group, $\mathcal{X}_t^i$ represents the delinquency rate, and m^i can be either mortgage or business loan. For the second counterfactual scenario, we consider the variant collateral requirement limit by shutting down its response to the charge-off rates and credit supply. In this case, the collateral requirements for entrepreneurs and households are defined as an autoregressive process $\phi_t^i = \rho_{\phi i}\phi_{t-1}^i + \epsilon_t^{\phi i}$. We refer to this specification as the less active prudential policy, which has some degree of history dependence. We also modify the model and assume that the maximum LTV requirements do not vary with the cycle. We refer to this counterfactual as the constant LTV model.

Figure 4 reports the model impulse response to a monetary policy shock under three scenarios: baseline, less active LTV measure, and constant LTV. This is useful to reveal how the economy reacts to a monetary policy shock when the LTV rule responds inversely to credit growth and default. Panel (a) of Figure 4 indicates that a contractionary monetary policy shock limits borrowers' leverage and drives down aggregate output. This shock reduces the level of external finance for businesses and subsequently propagates through the labor demand channel by decreasing aggregate employment (see Panel (b) of Figure 4). Through the investment channel, firms are forced to reduce their aggregate production after a monetary policy hike as their capacity to finance investment is restricted. Regarding the inflation and output gap responses displayed in Panels (c) and (d), we show that a positive monetary policy shock leads to a decrease in the output gap, and a short-lived decline in inflation followed by a reverse response up to the 5th quarter. An economy with an active LTV rule (the baseline scenario) is greatly affected by the shock: it experiences a drop in total output by 0.16% and can have a relatively large negative effect on labor with a 0.35% decline. On the other hand, an economy with a less active LTV rule implies a limited decline in total output and a reduced effect on employment after an increase in policy interest rates. The absence of the charge-off rate and credit supply in the collateral requirement rule makes output and employment less sensitive to the monetary policy shock. Thus, this shock is followed by an immediate decline in output by 0.13%, and causes a decrease in aggregate labor by 0.27%. Under a constant LTV measure, the output and employment responses are slightly lower relative to the response in the baseline model; this is also true for the responses of inflation and output gap to monetary policy shock. Collectively, these results suggest that an LTV measure that moderates both the credit cycle and stabilizes the delinquency rate in real-time, tends to amplify the effects of monetary policy on output and employment along with

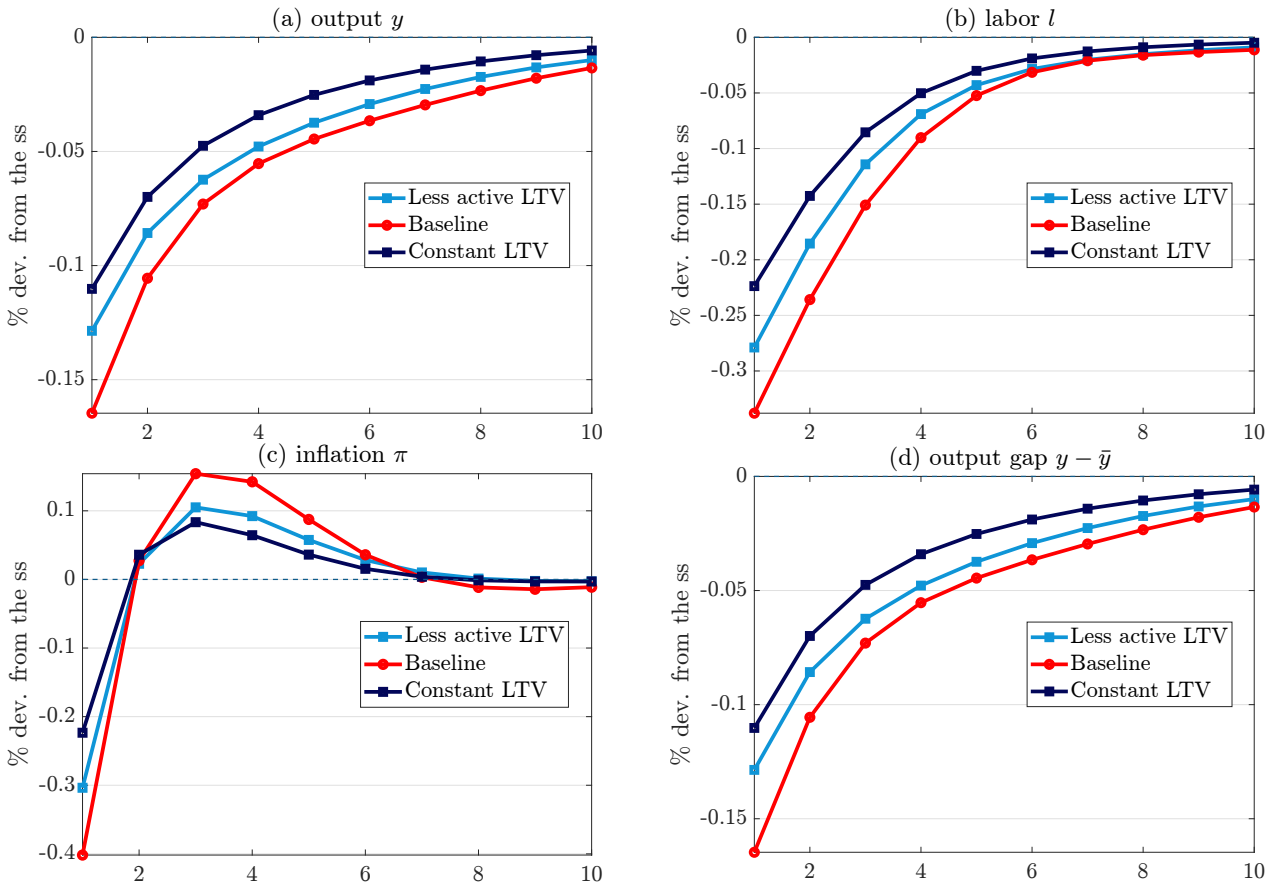


FIGURE 4. Impulse Responses to a Monetary Policy Shock: Baseline Estimates

Notes: Panel (a) reports the response of output to a positive monetary policy shock under three scenarios; similarly, Panel (b) shows the response of labor; Panel (c) shows the response of inflation and Panel (d) presents the response of output gap to a positive monetary policy shock. The time frequency is quarterly.

inflation and the output gap. Different from the baseline scenario, the constant LTV does not amplify the responses of these aggregates.

Deterministic Probability of Default. We extend the analysis by assuming the default probability is deterministic, which implies that the probability of default remains constant over time and does not fluctuate with the cycle. This means that the financial intermediary updates the loan-to-value ratio constantly but considers explicitly a constant probability of default. To this end, we set: $\tilde{\sigma}_t^e = \tilde{\sigma}_{ss}^e$ and $\tilde{\sigma}_t^h = \tilde{\sigma}_{ss}^h$.

Figure 5 contrasts the benchmark model's response with the counterfactual results. Panel a shows the response of output, and Panel b shows the response of labor to a positive monetary policy shock. In general, holding the probability of default constant produces a large decline in output following an increase in the policy interest rate. This result underscores that as borrowers receive limited financial

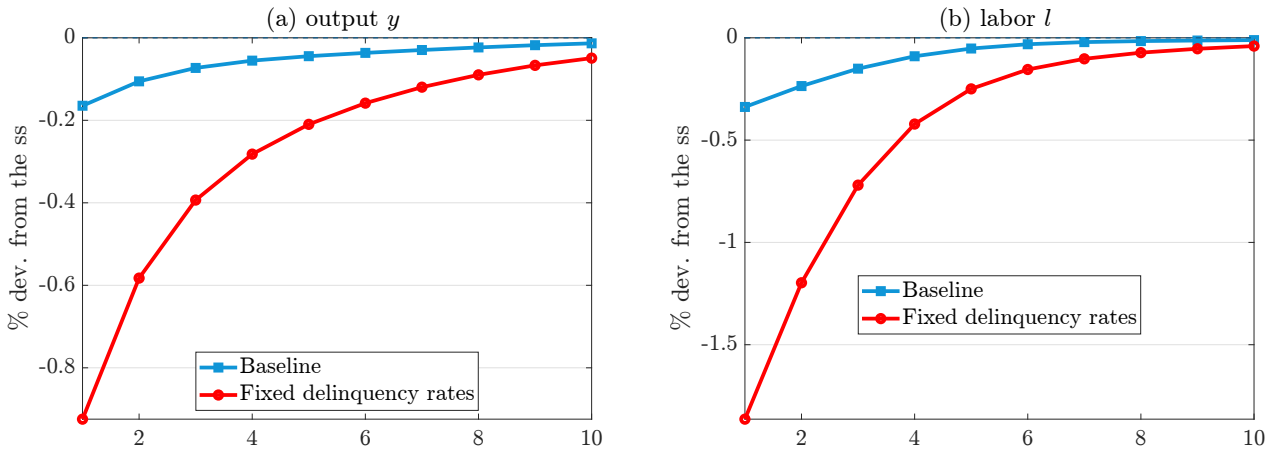


FIGURE 5. Impulse responses to a monetary policy shock: time-variant versus fixed delinquency rates

Notes: Panel a reports the response of output to a positive monetary policy shock and Panel b shows the response of labor to a positive monetary policy shock. The time frequency is quarterly.

resources, their default probability cannot be reduced but remains at its initial level, which will eventually amplify the downturn and create large output losses. It is useful to note that when the default is constant, the effect of monetary policy on employment is much greater compared to the baseline results, thereby strengthening employment feedback to changes in interest rates.

Several observations can be made from this analysis. First, a deterministic delinquency rate leads to worse labor outcomes in the short term and slightly prolongs the recovery following the contractionary shock. Revealing that an LTV tool that moderates the credit supply but does not reduce the default probability may harm the economy more since a rise in interest rates generates large negative effects. The effects of monetary policy on output depend on the form of the bank-based measure and whether the default fluctuates due to exogenous or cyclical deviations. Therefore, the combination of constant default and changes in the LTV limits could act as friction, negatively influencing firm production decisions and distorting the consumption smoothing of constrained households. Accounting for uncertainty in the default probability, instead of a simple deterministic default probability, can be important in reducing output volatility. For instance, a bank-based prudential rule that reacts to an exogenous default probability can minimize output volatility and ease the downturn more effectively than in the case of a deterministic default probability. Jones, Midrigan, and Philippon (2022) evaluate the extent to which monetary policy and credit shocks affect employment. They emphasize that the effects depend on the degree of idiosyncratic risk volatility and show that precautionary motives dominate as uncertainty increases.

Introducing Endogenous Default Probability. In the literature on credit cycles and prudential policies, the probability of a financial crisis occurring is mostly treated as an equilibrium outcome. We now consider another model extension by assuming the default probability of both entrepreneurs and households is endogenous. In this context, we assume that monitoring costs μ_t^e and μ_t^h are decision variables characterized by the following optimality conditions:

$$\chi_t^e m_t^e (1 - r_t^e)/\pi_t - (1 + r_t^k)\phi_t^e q_t^k k_t = 0, \qquad \chi_t^h m_t^h (1 - r_t^e)/\pi_t - \phi_t^h q_t^h h_t = 0 \tag{5.1}$$

Figure 6 compares the impulse responses of employment and output to a positive monetary policy shock in the baseline model and the endogenous delinquency rate model. At higher interest rates, borrowers deleverage, which affects consumer expenditure and business investment and causes a decline in aggregate output and labor. The decline in output and employment is considerably larger in the endogenous delinquency rate model compared to the model with an exogenous probability of default. The amplification of the effects of monetary policy shock could be driven by the additional adjustment of monitoring costs that put further downward pressure on aggregate output and potentially explain the large employment loss observed in the model with endogenous delinquency rates.

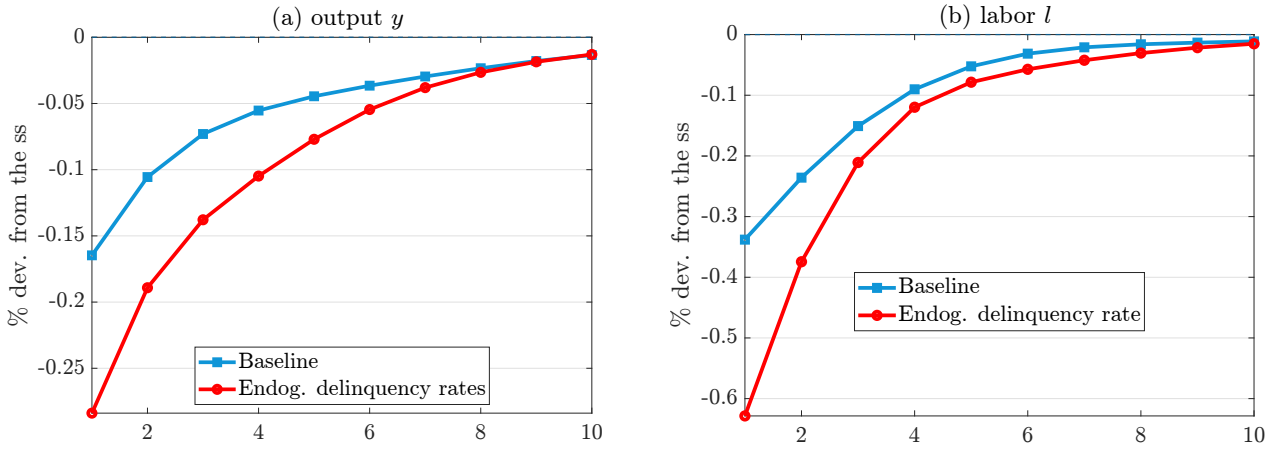


FIGURE 6. Impulse Responses to a Monetary Policy Shock: Exogenous Versus Endogenous Delinquency Rates

Notes: Panel (a) reports the response of output to a positive monetary policy shock, and Panel (b) shows the response of labor to a positive monetary policy shock. The time frequency is quarterly.

Overall, a contractionary monetary policy shock can depress economic activity; however, the impact differs when the policy interacts with various types of bank-based prudential instruments. The form of the bank-based rule seems important in the transmission of the monetary policy. Our results are relevant in light of previous studies providing evidence of strong complementarity between monetary

and bank-based prudential ruleS. These include Altavilla, Laeven, and Peydro (2020), who show that prudential policies matter for the conduct of monetary policy, and Carrillo et al. (2021), who highlight that the transmission of monetary policy becomes stronger.

Welfare Implications of the Bank-based Prudential Rule To make a proper normative statement on the effectiveness of the time-varying LTV rule proposed in our model, we compute the welfare maximizing function for both the constrained and unconstrained households and the social welfare gain from introducing the LTV rule as follows:

$$W_t = (1-\beta_c)W_{c,t} + (1-\beta_u)W_{u,t}, \tag{5.2}$$

where the welfare function for the constrained households is $W_{c,t} = \beta_c\{(\log(c_{c,t}-b_c c_{c,t-1})-\psi_l((l_{c,t})^{1+\sigma_l})/(1+\sigma_l)+\chi\log(h_t)\}$, and constrained households is $W_{u,t} = \beta_u\{\zeta_{c,t}(\log(c_{u,t}-b_u c_{u,t-1})-\psi_l((l_{u,t})^{1+\sigma_l})/(1+\sigma_l)\}$. The welfare weighted average function was previously used by Lambertini, Mendicino, and Punzi (2013) and Rubio and Carrasco-Gallego (2014).

Panel (a) of Table 3 provides detailed individual and social welfare under different model assumptions. An anti-default LTV rule that leans against the credit cycle induces large welfare for constrained households (borrowers in the baseline). Their welfare gains were slightly diminished in the scenario with less active LTV. Note that if we assume that both households consume housing, the implied individual and social welfare are the highest. The suboptimal scenario for unconstrained households (savers) is when they participate in the housing market while featuring an LTV rule, as in the baseline model. Comparing the results in columns (1) and (4), constant LTV improves social welfare, it increases the welfare gain of borrowers but induces large welfare loss for savers. Savers may indeed prefer the baseline LTV relative to a constant LTV. However, under less active LTV, the implied social welfare gain is lower than the induced gains under the constant LTV model and under the baseline assumptions. In terms of optimal policy, one can conclude that the counter-cyclical response to defaults and credit supply is optimal for the two agents, as shown in column (5). However, having a constant LTV rule induces significant welfare gain when only borrowers consume housing goods.

We now examine how the delinquency rate responds to a monetary policy that coexists with an active prudential policy. Panel (b) of Table 3 reports a scenario in which a contractionary monetary policy shock hits the economy. Maintaining the baseline model assumption of the loan-to-value limit rule, the economy experiences a significant drop in the business loan delinquency rate following a monetary policy shock in the short run (0.38% in the sixth quarter). The contraction in the business debt delinquency rate reverts to a modest rise in the twentieth quarter. The prudential policy, which induces oscillations from boom to bust in business credit and default probability, may not be effective in

TABLE 3. Welfare Evaluation: Model Comparison

	LTV-baseline	Less active LTV	LTV-baseline: Fixed delinquency rates	Constant LTV	LTV-baseline: both HHs consume housing	LTV-baseline: Endog. delinquency rates
	(1)	(2)	(3)	(4)	(5)	(6)
Panel (a): Welfare						
W	0.16	0.13	0.82	0.96	2.82	0.26
W_c	3.49	2.81	17.91	20.90	37.77	5.79
W_u	-1.46	-1.17	-8.01	-8.73	92.82	-2.59
Panel (b): Effects of monetary policy on business loan delinquency rate $\mathcal{X}^e$ in %						
Short run (6th Quarter)	-0.38	0.40	-	-0.48	1.53	0.15
Long run (20th Quarter)	0.02	0.01	-	-0.01	0.00	0.00
std (q^h)	0.20	0.11	0.09	0.91	0.31	0.39
std (c)	0.41	0.33	0.22	2.63	0.62	0.71
Panel (c): Effects of monetary policy on mortgage loan delinquency rate $\mathcal{X}^h$ in %						
Short run (6th Quarter)	-0.08	0.43	-	-1.28	-0.28	1.46
Long run (20th Quarter)	0.00	-0.02	-	-0.02	-0.01	1.27

Notes: Panel (a) shows the welfare implication of a contractionary monetary policy, Panel (b) shows the response of the delinquency rate on business loans $\mathcal{X}^e$ to a contractionary monetary policy, percent deviation from the steady-state and reports the standard deviation of house prices q^h and consumption c. Panel (c) shows the response of the delinquency rate on mortgage loans $\mathcal{X}^h$ to a contractionary monetary policy, percent deviation from the steady-state. For the results shown on Column 5, we allow for the unconstrained households to access to the housing market and rewrite their utility as $E_0\Sigma_{t=0}^{\infty}\beta^t\{\zeta_{c,t}(\log(c_{u,t}-b_u c_{u,t-1}))-\psi_l\frac{(l_{u,t})^{1+\sigma_l}}{1+\sigma_l}+\chi\log(h_{u,t})\}$, subject to budget constraint $c_{u,t}+d_t+q_t^h h_{u,t}\leq w_t l_{u,t}+(1+r_t)d_{t-1}/\pi_t+q_t^h h_{u,t-1}$, with an aggregate housing stock clearing condition: $h_{c,t}+h_{u,t}=h_t$. The utility function of constrained households is $E_0\Sigma_{t=0}^{\infty}\beta^t\{\zeta_{c,t}(\log(c_{c,t}-b_c c_{c,t-1}))-\psi_l\frac{(l_{c,t})^{1+\sigma_l}}{1+\sigma_l}+\chi\log(h_{c,t})\}$, subject to budget constraint $c_{c,t}+m_{t-1}^h(1+r_t^e)/\pi_t+q_t^h h_{c,t}\leq w_t l_{c,t}+m_t^h+q_t^h h_{c,t-1}$,

preventing the increase in delinquency rate when the LTV disturbance pressure subsides. This situation is not too different from the results in Bianchi and Mendoza (2018), showing that when the economy enters a stable credit region, and debt falls, crises are unlikely in the short run but more likely in the long run. Under a constant LTV scenario, the decline in the delinquency rate is more pronounced in the short-run (-0.48%). Interestingly, under the less active LTV rule, the business loan delinquency rate responds positively to a monetary policy shock (0.40%) in the short-run. Furthermore, the nature of

the LTV rule is also relevant for the volatility of consumption and house prices. Indeed, the constant LTV model is characterized by high volatility in consumption relative to the baseline model. In terms of house price volatility, the baseline model features low volatility relative to the scenario where the LTV is kept constant over the cycle. Comparing the results in columns (1) and (5), one can see how allowing unconstrained households to consume housing raises the volatility of house prices and consumption. Panel (c) of Table 3 reports the short-run and long-run responses of the net charge-off rate on mortgage debt when a contractionary monetary policy shock hits the economy. Under the baseline scenario, the economy experiences a decline in the mortgage loan delinquency rate in the short-run, with an aggregate effect of −0.08%; however, after the twentieth quarter, the impact becomes null. Under the constant LTV rule, the effects is negative with a decrease of 1.28% in the sixth quarter and gradually less strong in the long-run.

To summarize, a monetary policy that coexists with a bank-based prudential practice leads to a temporary yet significant decline of default in the private sector as it reduces both default rates on real estate and business loans in the short run. However, it can be counterproductive in the long run as it leads to an increase in delinquency rates, although this increase is not particularly large. This analysis differs from Van der Ghote (2021) since, in our model, the monetary and bank-based prudential policies are not coordinated, but they still influence optimal decisions of each other. Lenders in our model are concerned with the loan-to-value limit that stabilizes the delinquency rate, whereas the central bank sets the policy rate according to the Taylor rule to stabilize inflation and the output gap. Therefore, a monetary policy alone will fail to tackle distortions arising from credit cycle inefficiencies, as noted in Carrillo et al. (2021). This view also aligns with the findings of Akinci and Queralto (2022), who argued that macro-prudential policies are useful in reducing the risk of crisis, and Altavilla, Laeven, and Peydro (2020), who stressed the strong complementarity of the two policies as the effects of monetary policy on bank lending are large.

6. Conclusion

Bank-based prudential instruments were designed to control and reduce the high risk of delinquency in the private sector. We asked the following question: What is the impact of these measures on the economy, and how do they interact with the monetary policy in a two-agent New Keynesian model? Our study emphasized the role of collateral limits in the context of contractionary monetary policy: output becomes less sensitive to changes in monetary policy shocks when banks conduct a less active collateral policy. However, output responds considerably to a monetary policy shock under an active bank-based prudential measure. Employment analysis produces similar findings. A more active collateral policy leads to a large response of employment to a monetary policy shock. Moreover, we considered the response of

the delinquency rate to the monetary policy shock under the active bank-based prudential measure. The overall conclusion from our experiments is that the transmission of a contractionary monetary policy can reduce the charge-off rates in the short-run, and the largest impact of monetary policy is on the path of the delinquency rate in the first quarter. Our model could be augmented with heterogeneous agents to test whether it reaches similar conclusions and provides new insights into the heterogeneous effects of bank-based prudential measures; we intend to pursue this topic in future research.

The crux of the matter is that the severity of borrowers' financial conditions affects the way banks assess pledged collateral. Lenders restore financial stability through a bank-based prudential measure that reacts actively to an uncertain default probability. It is evident from our model that the effects of the monetary policy are amplified when financial intermediaries adopt an active bank-based prudential measure, which makes banks less vulnerable to economic shocks.

In summary, bank-based prudential policies have gained growing importance and recognition as effective tools that financial intermediates can use to mitigate the severity of economic downturns. These measures serve to reduce bank risk exposure, help in dealing with delinquent debt, and guarantee that the liquidation of collateral is sufficient to cover losses. The implementation of prudential financial measures and how they relate to credit policies is increasingly crucial and merit further assessment

References

Aikman, David, Jonathan Bridges, Anil Kashyap, and Caspar Siegert. 2019. "Would macroprudential regulation have prevented the last crisis?" *Journal of Economic Perspectives* 33 (1): 107–130.

Aikman, David, Robert Kelly, Fergal McCann, and Fang Yao. 2021. "The Macroeconomic Channels of Macroprudential Mortgage Policies." *Financial Stability Notes, Central Bank of Ireland.* 11.

Ajello, Andrea. 2016. "Financial Intermediation, Investment Dynamics, and Business Cycle Fluctuations." *American Economic Review* 106 (8): 2256–2303.

Akinci, Ozge, and Albert Queralto. 2022. "Credit Spreads, Financial Crises, and Macroprudential Policy." *American Economic Journal: Macroeconomics*.

Alpandaa, Sami, and Sarah Zubairy. 2017. "Addressing Household Indebtedness: Monetary, Fiscal or Macroprudential Policy?." *European Economic Review* 92: 47–73.

Altavilla, Carlo, Luc Laeven, and Jose Luis Peydro. 2020. "Monetary and Macroprudential Policy Complementarities: Evidence from European Credit Registers."

Angelini, Paolo, Stefano Neri, and Fabio Panetta. 2012. "Monetary and Macroprudential Policies." ECB Working Paper Series 1449, European Central Bank.

Becard, Yvan, and David Gauthier. 2022. "Collateral Shocks." *American Economic Journal: Macroeconomics* 14 (1): 83–103.

Bernanke, Ben, Mark Gertler, and Simon Gilchrist. 1999. "The Financial Acceleratorin a Quantitative Business Cycle Framework." *Handbook of Macroeconomics*.

Bianchi, Javier, and Louphou Coulibaly. 2022. "Liquidity traps, prudential policies, and international spillovers."

Bianchi, Javier, and Enrique G Mendoza. 2018. "Optimal Time-consistent Macroprudential Policy." *Journal of Political Economy* 126 (2): 588–634.

Bianchi, Javier, and Bigio Saki. 2022. "Banks Liquidity Management and Monetary Policy." *Econometrica* 90 (1): 391–454.

Brunnermeier, Markus K., and Yuliy Sannikov. 2014. "A Macroeconomic Model with a Financial Sector." *American Economic Review*.

Candian, Giacomo, and Mikhail Dmitriev. 2020. "Default Recovery Rates and Aggregate Fluctuations." *Journal of Economic Dynamics and Control* 121: 104011.

Carrillo, Julio A, Enrique G Mendoza, Victoria Nuguer, and Jessica Roldán-Peña. 2021. "Tight money-tight credit: coordination failure in the conduct of monetary and financial policies." *American Economic Journal: Macroeconomics* 13 (3): 37–73.

Cerutti, Eugenio, Stijn Claessens, and Luc Laeven. 2017. "The Use and Effectiveness of Macroprudential Policies: New Evidence." *Journal of financial stability* 28: 203–224.

Christensen, Ian, Paul Corrigan, Caterina Mendicino, and Shin-Ichi Nishiyama. 2009. "Consumption, Housing Collateral, and the Canadian Business Cycle." *Bank of Canada Working Paper* 26.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno. 2014. "Risk Shocks." *American Economic Review*.

Curdia, Vasco, and Michael Woodford. 2016. "Credit Frictions and Optimal Monetary Policy." *Journal of Monetary Economics*.

Drechsel, Thomas, and Seho Kim. 2022. "Macroprudential Policy with Earnings-based Borrowing Constraints."

Farhi, Emmanuel, and Iván Werning. 2016. "A theory of macroprudential policies in the presence of nominal rigidities." *Econometrica* 84 (5): 1645–1704.

Ferrero, Andrea, Richard Harrison, and Benjamin Nelson. 2024. "House Price Dynamics, Optimal LTV Limits and the Liquidity Trap." *The Review of Economic Studies* 91 (2): 940–971.

Franz, Thorsten. 2020. "The Effects of Borrower-Based Macroprudential Policy: An Empirical Application." *International Journal of Central Banking* 64.

Fuentes-Albero, Cristina. 2019. "Financial Frictions, Financial Shocks, and Aggregate Volatility." *Journal of Money, Credit and Banking* 51 (6): 1581–1621.

Gatt, William. 2024. "Loan-to-value limits as a macroprudential policy tool: Developments in theory and practice." *Journal of Economic Surveys* 38 (1): 232–267.

Geanakoplos, John, and William R. Zame. 2007. "Collateralized Asset Markets." *Discussion Paper, Yale University*.

Gelain, Paolo, Kevin Lansing, and Caterina Mendicino. 2013. "House Prices, Credit Growth, and Excess Volatility: Implications for Monetary and Macroprudential Policy." *International Journal of Central Banking* 9: 219–276.

Gertler, Mark, and Peter Karadi. 2011. "A Model of Unconventional Monetary Policy." *Journal of Monetary Economics*.

Gertler, Mark, and Nobuhiro Kiyotaki. 2010. "Financial Intermediation and Credit Policy in Business Cycle Analysis." *Handbook of Monetary Economics, Ed. by Benjamin M. Friedman and MichaelWoodford*.

Gertler, Mark, Nobuhiro Kiyotaki, and Andrea Prestipino. 2017. "A Macroeconomic Model with Financial Panics." *NBER Working Paper*.

Van der Ghote, Alejandro. 2021. "Interactions and Coordination between Monetary and Macroprudential Policies."

American Economic Journal: Macroeconomics 13 (1): 1–34.

Guerrieri, Luca, and Matteo Iacoviello. 2017. "Collateral Constraints and Macroeconomic Asymmetries." *Journal of Monetary Economics*.

Guerrieri, Veronica, and Guido Lorenzoni. 2017. "Credit Crises, Precautionary Savings and the Liquidity Trap." *Quarterly Journal of Economics*.

Gunn, Christopher M, and Alok Johri. 2013. "An Expectations-driven Interpretation of the "Great Recession"." *Journal of Monetary Economics* 60 (4): 391–407.

Gunn, Christopher M, Alok Johri, and Marc-André Letendre. 2022. "Charge-offs, Defaults and the Financial Accelerator." *The BE Journal of Macroeconomics* 23 (1): 427–471.

He, Zhiguo, and Arvind Krishnamurthy. 2012. "A Macroeconomic Framework for Quantifying Systemic Risk." *NBB Working Paper*.

Iacoviello, Matteo. 2005. "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle." *American Economic Review* 95 (3): 739–764.

Iacoviello, Matteo. 2015. "Financial Business Cycles." *Review of Economic Dynamics*.

Iacoviello, Matteo, and Marina Pavan. 2013. "Housing and Debt Over the Life Cycle and Over the Business Cycle." *Journal of Monetary Economics* 60 (2): 221–238.

Jakab, Zoltan, and Michael Kumhof. 2015. "Banks Are Not Intermediaries of Loanable funds – and Why This Matters." *Bank of England Working Paper*.

Jermann, Urban, and Vincenzo Quadrini. 2012. "Macroeconomic Effects of Financial Shocks." *American Economic Review*.

Jones, Callum, Virgiliu Midrigan, and Thomas Philippon. 2022. "Household leverage and the recession." *Econometrica* 90 (5): 2471–2505.

Jordà, Òscar. 2005. "Estimation and inference of impulse responses by local projections." *American economic review* 95 (1): 161–182.

Justiniano, Alejandro, Giorgio E Primiceri, and Andrea Tambalotti. 2014. "The Effects of the Saving and Banking Glut on the US Economy." *Journal of International Economics* 92: S52–S67.

Justiniano, Alejandro, Giorgio E Primiceri, and Andrea Tambalotti. 2015. "Household Leveraging and Deleveraging." *Review of Economic Dynamics*.

Justiniano, Alejandro, Giorgio Primiceri, and Andrea Tambalotti. 2011. "Investment Shocks and the Relative Price of Investment." *Review of Economic Dynamics*.

Kim, Soyoung, and Aaron Mehrotra. 2018. "Effects of Monetary and Macroprudential Policies—Evidence from Four Inflation Targeting Economies." *Journal of Money, Credit, and Banking* 50 (5): 967–992.

Kiyotaki, Nobuhiro, and John Moore. 1997. "Credit Cycles." *Journal of political economy* 105 (2): 211–248.

Korinek, Anton, and Enrique G Mendoza. 2014. "From Sudden Stops to Fisherian Deflation: Quantitative Theory and Policy." *Annual Review of Economics* 6 (1): 299–332.

Lambertini, Luisa, Caterina Mendicino, and Maria Teresa Punzi. 2013. "Leaning Against Boom–bust Cycles in Credit and Housing Prices." *Journal of Economic Dynamics & Control* 37.

Lian, Chen, and Yueran Ma. 2021. "Anatomy of Corporate Borrowing Constraints." *The Quarterly Journal of Economics* 136 (1): 229–291.

Martin, Alberto, Caterina Mendicino, and Alejandro Van der Ghote. 2021. "On the Interaction between Monetary and Macroprudential Policies." *ECB Working Paper Series* 2527.

Mendicino, Caterina, Kalin Nikolov, Juan Rubio-Ramirez, Javier Suarez, and Dominik Supera. 2020. "Twin Defaults and Bank Capital Requirements." *Working Paper*.

Ottonello, Pablo, Diego J Perez, and Paolo Varraso. 2022. "Are Collateral-constraint Models Ready for Macroprudential Policy Design?." *Journal of International Economics*: 103650.

Ottonello, Pablo, and Wenting Song. 2022. "Financial Intermediaries and the Macroeconomy: Evidence from a High-Frequency Identification." *Working Paper*.

Rajan, Raghuram G. 1994. "Why Bank Credit Policies Fluctuate: A Theory and Some Evidence." *The Quarterly Journal of Economics* 109 (2): 399–441.

Rubio, Margarita, and José A. Carrasco-Gallego. 2014. "Macroprudential and Monetary Policies: Implications for Financial Stability and Welfare." *Journal of Banking & Finance* 49.

Williamson, Stephen. 2016. "Scarce Collateral, the Term Premium, and Quantitative Easing." *Journal of Economic Theory*.

Appendix A. Model Derivations

Households optimality conditions: We start with the households utility maximization.

Unconstrained Households. The objective function is give by

$$\begin{aligned} &\text{maximize} \quad E_0 \Sigma_{t=0}^{\infty} \beta^t \left\{ \zeta_{c,t} (\log(c_{u,t} - b_u c_{u,t-1})) - \psi_l \frac{(l_{u,t})^{1+\sigma_l}}{1+\sigma_l} \right\} \\ &\text{subject to} \quad c_{u,t} + d_t \leq w_t l_{u,t} + (1+r_t) d_{t-1}/\pi_t \end{aligned} \tag{A1}$$

The maximization problem yields the following first-order conditions with respect to consumption, deposit and labor:

$$\frac{\partial \mathcal{L}_t}{\partial c_{u,t}} : \quad \lambda_{u,t} - \frac{\zeta_{c,t}}{c_{u,t} - b c_{u,t-1}} + b\beta E_t \frac{\zeta_{c,t+1}}{c_{u,t+1} - b c_{u,t}} = 0 \tag{A2}$$

$$\frac{\partial \mathcal{L}_t}{\partial d_t} : \quad \lambda_{u,t} - \beta E_t \lambda_{u,t+1} (1 + r_{t+1})/\pi_{t+1} = 0 \tag{A3}$$

$$\frac{\partial \mathcal{L}_t}{\partial l_{u,t}} : \quad -\psi_l (l_{u,t})^{\sigma_l} + w_t \lambda_{u,t} = 0 \tag{A4}$$

where $\lambda_{u,t}$ is the marginal utility.

Collateral Constrained Households. Financially constrained households maximize their expected lifetime utility

$$\begin{aligned} &\text{maximize} \quad E_0 \Sigma_{t=0}^{\infty} \beta_c^t \left\{ \zeta_{c,t} (\log(c_{c,t} - b_c c_{c,t-1})) - \psi_l \frac{(l_{c,t})^{1+\sigma_l}}{1+\sigma_l} + \chi \log(h_t) \right\} \\ &\text{subject to} \quad c_{c,t} + q_t^h h_t + m_{t-1}^h (1 + r_t^e)/\pi_t \leq w_t l_{c,t} + q_t^h h_{t-1} + m_t^h \\ &\qquad\qquad \phi_t^h q_t^h h_t \geq (1 + r_t^e) m_{t-1}^h / \pi_t \end{aligned} \tag{A5}$$

The first order condition with respect to consumption c, hours l, and debt m^h

$$\frac{\partial \mathcal{L}_t}{\partial c_t} : \quad \lambda_{c,t} - \frac{\zeta_{c,t}}{c_{c,t} - b c_{c,t-1}} + b\beta_c E_t \frac{\zeta_{c,t+1}}{c_{c,t+1} - b c_{c,t}} = 0 \tag{A6}$$

$$\frac{\partial \mathcal{L}_t}{\partial m_t^h} : \quad \beta_c E_t \lambda_{c,t+1} (1 + r_{t+1}^e)/\pi_{t+1} - \lambda_{c,t} - \lambda_{c,t+1} (1 + r_{t+1}^e)/\pi_{t+1} = 0 \tag{A7}$$

$$\frac{\partial \mathcal{L}_t}{\partial l_{c,t}} : \quad -\psi_l (l_{c,t})^{\sigma_l} + w_t \lambda_{c,t} = 0 \tag{A8}$$

Unconstrained households make loans to collateral constrained households at nominal interest rate r^e, via a financial intermediary that has a participation constraint

$$(1 - \chi_t^h)(1 + r_t^e) m_{t-1}^h/\pi_t + (1 - \mu^h) \chi_t^h \phi_t^h q_t^h h_t \geq (1 + r_t) m_{t-1}^h/\pi_t$$

which can be simplified to: $m_{t-1}^h/\pi_t = \frac{\phi_t^h \left(1 - \chi_t^h\right) q_t^h h_t + h_t q_t^h \phi_t^h \chi_t^h \left(1 - \mu^h\right)}{1 + r_t}$. We also define households deposit condition $d_{t-1} = m_t^h$. Aggregating labor and consumption of the two type of households yields $l_{i,t} = [l_{u,t} + l_{c,t}]$, and $c_{i,t} = [c_{u,t} + c_{c,t}]$.

Labor Markets Labor contractors hire households $i \in \{u, c\}$ and sell homogeneous labor services to the intermediate good producers. Labor services takes the form

$$l_t = \left[\int_0^1 (l_{i,t})^{\frac{1}{\nu_l}} di \right]^{\nu_l}$$

subject to

$$\int_0^1 w_{i,t} l_{i,t} di = w_t l_t$$

where ν_l denote the fixed wage markup and $w_{i,t}$ is the wage rate.
There is a monopoly union that represent all workers and set wage and faces Calvo style frictions,

$$w_t = \left[(1 - \zeta_l)(w_{t-1})^{\frac{1}{1-\nu_l}} + \zeta_l (w_t^*)^{\frac{1}{1-\nu_l}} \right]^{1-\nu_l} \tag{A9}$$

Where $1 - \zeta_l$ is the probability that the monopoly reoptimizes the wage, while with probability ζ_l the monopoly cannot reoptimize.

If the monopoly cannot reoptimize, then she sets the wage w_t^* according to

$$w_t^* = \left(\frac{E\Sigma_{s=0}^{\infty} (\beta \zeta_l)^s \psi_l \nu_l \left(l_{t,s}\right)^{(1+\sigma_l)} \left(w_{t+s}^{\frac{\nu_l}{1-\nu_l}} \right)^{-(1+\sigma_l)}}{E\Sigma_{s=0}^{\infty} (\beta \zeta_l)^s \lambda_{t+s} w_{t+s}^{-\frac{\nu_l}{1-\nu_l}} l_{t+s}} \right)^{\frac{\nu_l - 1}{\sigma_l \nu_l + \nu_l - 1}}$$

Labor union optimality conditions: The objective of households is to choose the wage level that maximizes the utility s periods into the future

$$\begin{aligned} \underset{w_{i,t}}{\text{maximize}} \quad & E\Sigma_{s=0}^{\infty} (\beta \zeta_l)^s \left[\lambda_{t+s} w_{i,t} l_{i,t+s} - \psi_l \frac{l_{i,t+s}^{1+\sigma_l}}{1+\sigma_l} \right] \\ \text{subject to} \quad & l_{i,t+s} = \left(\frac{w_{i,t}}{w_{t+s}} \right)^{\frac{\nu_l}{1-\nu_l}} l_{t+s}, \end{aligned}$$

We first simplify the expression

$$E\Sigma_{s=0}^{\infty} (\beta \zeta_l)^s \left[\lambda_{t+s} w_{i,t} l_{i,t+s} - \psi_l \frac{l_{i,t+s}^{1+\sigma_l}}{1+\sigma_l} \right]$$

$$E\Sigma_{s=0}^{\infty} (\beta \zeta_l)^s \lambda_{t+s} w_{i,t} l_{i,t+s} - E\Sigma_{s=0}^{\infty} (\beta \zeta_l)^s \psi_l \frac{l_{i,t+s}^{1+\sigma_l}}{1+\sigma_l}$$

$$E\Sigma_{s=0}^{\infty} (\beta \zeta_l)^s \lambda_{t+s} w_{i,t} \left(\left(\frac{w_{i,t}}{w_{t+s}} \right)^{\frac{\nu_l}{1-\nu_l}} l_{t+s} \right) - E\Sigma_{s=0}^{\infty} (\beta \zeta_l)^s \psi_l \frac{\left(\left(\frac{w_{i,t}}{w_{t+s}} \right)^{\frac{\nu_l}{1-\nu_l}} l_{t+s} \right)^{1+\sigma_l}}{1+\sigma_l}$$

Then we derive the first order condition with respect to $w_{i,t}$:

$$\frac{\partial \mathcal{L}}{\partial w_{i,t}}: \quad E\Sigma_{s=0}^{\infty}(\beta\zeta_l)^s\lambda_{t+s}\frac{1}{1-\nu_l}w_{i,t}^{\frac{1}{1-\nu_l}-1}w_{t+s}^{-\frac{\nu_l}{1-\nu_l}}l_{t+s}-E\Sigma_{s=0}^{\infty}(\beta\zeta_l)^s\psi_l\frac{\nu_l\left(l_{t,s}w_{i,t}^{\frac{\nu_l}{1-\nu_l}}\right)^{(1+\sigma_l)}\left(w_{t+s}^{\frac{\nu_l}{1-\nu_l}}\right)^{-(1+\sigma_l)}}{(1-\nu_l)w_{i,t}}$$

$$w_{i,t}^{\frac{1}{1-\nu_l}-1}=E\Sigma_{s=0}^{\infty}(\beta\zeta_l)^s\psi_l\frac{\nu_l\left(l_{t,s}w_{i,t}^{\frac{\nu_l}{1-\nu_l}}\right)^{(1+\sigma_l)}\left(w_{t+s}^{\frac{\nu_l}{1-\nu_l}}\right)^{-(1+\sigma_l)}}{(1-\nu_l)w_{i,t}}\frac{(1-\nu_l)}{E\Sigma_{s=0}^{\infty}(\beta\zeta_l)^s\lambda_{t+s}w_{t+s}^{-\frac{\nu_l}{1-\nu_l}}l_{t+s}}$$

$$w_{i,t}w_{i,t}^{\frac{1}{1-\nu_l}-1}=\frac{E\Sigma_{s=0}^{\infty}(\beta\zeta_l)^s\psi_l\nu_l(l_{t,s}w_{i,t}^{\frac{\nu_l}{1-\nu_l}})^{(1+\sigma_l)}\left(w_{t+s}^{\frac{\nu_l}{1-\nu_l}}\right)^{-(1+\sigma_l)}}{E\Sigma_{s=0}^{\infty}(\beta\zeta_l)^s\lambda_{t+s}w_{t+s}^{-\frac{\nu_l}{1-\nu_l}}l_{t+s}}$$

$$\frac{w_{i,t}w_{i,t}^{\frac{1}{1-\nu_l}-1}}{(w_{i,t}^{\frac{\nu_l}{1-\nu_l}})^{(1+\sigma_l)}}=\frac{E\Sigma_{s=0}^{\infty}(\beta\zeta_l)^s\psi_l\nu_l(l_{t,s})^{(1+\sigma_l)}\left(w_{t+s}^{\frac{\nu_l}{1-\nu_l}}\right)^{-(1+\sigma_l)}}{E\Sigma_{s=0}^{\infty}(\beta\zeta_l)^s\lambda_{t+s}w_{t+s}^{-\frac{\nu_l}{1-\nu_l}}l_{t+s}}$$

$$w_{i,t}^{\frac{\sigma_l\nu_l+\nu_l-1}{\nu_l-1}}=\frac{E\Sigma_{s=0}^{\infty}(\beta\zeta_l)^s\psi_l\nu_l(l_{t,s})^{(1+\sigma_l)}\left(w_{t+s}^{\frac{\nu_l}{1-\nu_l}}\right)^{-(1+\sigma_l)}}{E\Sigma_{s=0}^{\infty}(\beta\zeta_l)^s\lambda_{t+s}w_{t+s}^{-\frac{\nu_l}{1-\nu_l}}l_{t+s}}$$

$$w_{i,t}=\left(\frac{E\Sigma_{s=0}^{\infty}(\beta\zeta_l)^s\psi_l\nu_l\left(l_{t,s}\right)^{(1+\sigma_l)}\left(w_{t+s}^{\frac{\nu_l}{1-\nu_l}}*\right)^{-(1+\sigma_l)}}{E\Sigma_{s=0}^{\infty}(\beta\zeta_l)^s\lambda_{t+s}w_{t+s}^{-\frac{\nu_l}{1-\nu_l}}l_{t+s}}\right)^{\frac{\nu_l-1}{\sigma_l\nu_l+\nu_l-1}}$$

Now assume that $w_t^*=w_{i,t}$. The wage level according the Calvo type frictions is given by

$$w_t=\left[(1-\zeta_l)(w_{t-1})^{\frac{1}{1-\nu_l}}+\zeta_l(w_t^*)^{\frac{1}{1-\nu_l}}\right]^{1-\nu_l} \tag{A10}$$

Goods Production The economy is populated by a continuum of firms and operates under a monopolistic competition. Each firm has the final good stock, which writes $y_t=\left[\int_0^1 y_{j,t}^{\frac{1}{1+\nu_p}}dj\right]^{1+\nu_p}$. The final good is indexed by $j\in[0,1]$, and $1\le\nu_p<\infty$. A higher price markup ν_p implies that firms have market power and that the good is less substitutable for other goods.

With capital and labor inputs, the monopolist produce the intermediate good y_t, the intermediate good production function takes the following form

$$y_t=(k_t)^{\alpha}(z_tl_t)^{1-\alpha} \tag{A11}$$

Where l_t is the capital input, l_t is the labor input and z_t is an exogenous productivity shock $z_t=\rho_z z_{t-1}+\epsilon_t^z$. The parameter $\alpha\in(0,1)$ measures the responsiveness of output to changes in capital.

We assume that prices are sticky, by using a variant of Calvo type frictions. The monopolist can set the price p_t of the good by reoptimizing the price with probability $1-\zeta_p$, or without reoptimzing the price with probability ζ_p, and p_t^* is the price level that maximizes the expected discounted value of future profits into the future. The price level is defined by

$$p_t = \left[(1-\zeta_p)(p_{t-1})^{\frac{\nu_p}{1-\nu_p}} + \zeta_p (p_t^*)^{\frac{\nu_p}{1-\nu_p}}\right]^{\frac{1-\nu_p}{\nu_p}} \tag{A12}$$

After setting the prices and the quantities the monopolist minimize the production cost

$$w_t l_t + r_t^k k_t$$

subject to production function (A11). The first-order conditions, with respect to labor and capital, yield the following:

$$\mu_t = \frac{w_t}{(1-\alpha)(k_t)^{\alpha}(z_t l_t)^{-\alpha}}, \tag{A13}$$

$$\mu_t = \frac{r_t^k}{\alpha(z_t l_t)^{1-\alpha}(k_t)^{\alpha-1}}. \tag{A14}$$

We can also characterize an alternative expression for the marginal cost μ_t

$$\mu_t = \left(\frac{1}{1-\alpha}\right)^{(1-\alpha)}\left(\frac{1}{\alpha}\right)^{\alpha}(r_t^k)^{\alpha} w_t^{(1-\alpha)} \tag{A15}$$

Monopolistic competition - optimality condition The economy is populated by a continuum of firms and operates under monopolistic competition. Each firm has the final good stock, which writes:

$$y_t = \left[\int_0^1 y_{j,t}^{\frac{1}{1+\nu_p}} dj\right]^{1+\nu_p}$$

Final good producers purchase the good and resell it to consumers. Their objective is to maximize their profits.

$$p_t y_t - \int_0^1 p_{j,t} y_{j,t} dj$$

$$p_t\left[\int_0^1 y_{j,t}^{\frac{1}{1+\nu_p}} dj\right]^{1+\nu_p} - \int_0^1 p_{j,t} y_{j,t} dj.$$

The first-order condition with respect to $y_{j,t}$ is given by

$$(\partial y_{j,t}): \quad p_t(1+\nu_p)\left[\int_0^1 y_{j,t}^{\frac{1}{1+\nu_p}} dj\right]^{1+\nu_p-1} \frac{1}{1+\nu_p} y_{j,t}^{(\frac{1}{1+\nu_p})-1} - p_{j,t} = 0$$

$$\left[\int_0^1 y_{j,t}^{\frac{1}{1+\nu_p}} dj\right]^{\nu_p} y_{j,t}^{(\frac{-\nu_p}{1+\nu_p})} = \frac{p_{j,t}}{p_t}$$

$$\left[\int_0^1 y_{j,t}^{\frac{1}{1+\nu_p}} dj\right]^{-(1+\nu_p)} y_{j,t} = \left(\frac{p_{j,t}}{p_t}\right)^{\frac{1+\nu_p}{\nu_p}}$$

$$y_t^{-1} y_{j,t} = \left(\frac{p_{j,t}}{p_t}\right)^{-\frac{1+\nu_p}{\nu_p}}.$$

From the first-order condition, $y_{j,t}$ is given by

$$y_{j,t} = \left(\frac{p_{j,t}}{p_t}\right)^{-\frac{1+\nu_p}{\nu_p}} y_t.$$

To define the aggregate price of the final good p_t, I use an expression for the output, which is equal to price times quantities $p_t y_t = \int_0^1 p_{j,t} y_{j,t} dj$; then, I simplify to obtain the expression for the aggregate price of the final good:

$$p_t y_t = \int_0^1 p_{j,t} \left(\frac{p_{j,t}}{p_t}\right)^{-\frac{1+\nu_p}{\nu_p}} y_t dj$$

$$p_t = \left[\int_0^1 p_{j,t}^{-\frac{1+\nu_p}{\nu_p}} dj\right]^{-\frac{\nu_p}{1+\nu_p}}.$$

Calvo Pricing- optimality condition: We assume that prices are sticky, in this environment firms choose the price level that maximizes their profits:

$$\underset{p_{j,t}}{\text{maximize}} \quad E\Sigma_{s=0}^{\infty} \beta^s (1-\zeta^p)[y_{j,t}(p_{j,t}^* - \mu_{j,t+s} z_t p_{j,t+s})]$$

$$\text{subject to} \quad y_{j,t} = y_t \left(\frac{p_{j,t}}{p_t}\right)^{\frac{\nu_p}{1-\nu_p}},$$

We solve the firm problem by substituting $y_{j,t}$ into the firm's profits function:

$$(\partial p_{j,t}^*): \qquad p_{j,t}^* = \nu_p \frac{E_t \Sigma_{s=0}^{\infty} \beta^s (1-\zeta^p) \left(\frac{p_{j,t}}{p_{t+s}}\right)^{\frac{\nu_p}{1-\nu_p}} y_{j,t+s} z_t \mu_{j,t+s} p_{j,t+s}}{E_t \Sigma_{s=0}^{\infty} \beta^s \zeta_s^p \left(\frac{p_{j,t}}{p_{t+s}}\right)^{\frac{\nu_p}{1-\nu_p}} y_{j,t+s}}$$

The recursive form is written as:

(A16) $$K_{p,t} = z_t \mu_t y_t + \beta(1-\zeta_p) E_t K_{p,t+1}$$

(A17) $$F_{p,t} = \mu_t y_t + \beta(1-\zeta_p) E_t \frac{p_{t-1}}{p_t} F_{p,t+1},$$

Capital Market In capital market, the previous capital is combined with investment goods to produce a new capital, which is supplied to entrepreneurs with the following technology:

(A18) $$k_t = (1-\delta) k_{t-1} + \left(1 - \frac{S}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right) i_t,$$

where capital decays at the fixed rate $0 < \delta \leq 1$. According to this equation, the new capital depends on the existing capital and investment good i_t. The quantity of investment at period t is proportional to the adjustment cost function S.

Capital producers choose investments to maximize the present value and future operating profits, less the total investment cost, as shown below:[5]

$$q_t^k i_t - [1 + \frac{S}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2] i_t$$

Given by the law of motion for capital stock k_t and investment expenditures, entrepreneurs sell capital to firms at price q^k. Capital producers' optimal investment policy would equalize the marginal revenue product of capital q^k to the marginal cost of investment goods, as shown below:

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial i_t}: \qquad & (-\mu_t)\, q_t^k + \mu_t \left(1 + \frac{S}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 + \left(\frac{i_t}{i_{t-1}} - 1\right) S \frac{i_t}{i_{t-1}}\right) \\ & - S\beta\mu_{t+1}\left(\frac{i_{t+1}}{i_t}\right)^2 \left(\frac{i_{t+1}}{i_t} - 1\right) = 0 \end{aligned} \tag{A19}$$

We assume that capital producers choose the optimal the quantity of capital k_t which equivalent to choosing r_{t-1}^k, using the following function:

$$\Theta_t^k = k_t - r_{t-1}^k k_{t-1} - (1-\delta)k_{t-1} \tag{A20}$$

where k_t represents the provision of capital to firms at time t, $r_{t-1}^k k_{t-1}$ is the return on renting capital services, and $(1-\delta)k_{t-1}$ is the previous capital k_{t-1} which they receive back and decays at fixed rate δ. The first order condition with respect to capital gives

$$\mu_t = E_t \beta \mu_{t+1} r_t^k + E_t \beta \mu_{t+1}(1-\delta) \tag{A21}$$

the entrepreneur enjoys the average gross nominal rate of return on capital $(1 + r_t^k)$.

Entrepreneurs and Introducing Collateral Requirement Shock. Each entrepreneur purchase capital good k_t at price q_t^k using loans m_t^e obtained from bank and net worth n_t^e. Then,

$$k_t q_t^k = m_t^e + n_t^e \tag{A22}$$

Entrepreneurs are hit by a shock $\mathcal{X}_{e,t} = 1/(1 + e^{\sigma_{e,t}})$ which describe the probability that entrepreneurs are unable to pay their debt. Letting $\sigma_{e,t}$ obey to an AR(1) process $\sigma_{e,t} = \rho_{\sigma_e}\sigma_{e,t-1} + \epsilon_t^{\sigma_e}$, thus, the default probability increases when $\sigma_{e,t}$ goes up. The collateral constraint for entrepreneurs can be defined as

$$\phi_t^e (1 + r_t^k) q_t^k k_t \geq (1 + r_t^e) m_t^e / \pi_t.$$

[5]The capital producer profit is given by $q_t^k k_t - [1 + S/2\,(i_t/i_{t-1} - 1)^2] i_t$, where the law of motion for capital $k = k' + [1 + S/2\,(i_t/i_{t-1} - 1)^2] i_t$, given that in a steady state, $S(.) = 0$ becomes $k - k' = I_t$. Then the capital producer problem can be written as $q_t^k i_t - [1 + S/2\,(i_t/i_{t-1} - 1)^2] i_t$.

Entrepreneur's debt is charged-off with probability $\mathcal{X}^e_{t-1}$, as the entrepreneur is unable to pay the interest and principle, as a result the pledged collateral is sized by the bank. we assume that collateral requirements $\phi_{e,t}$ evolve over time according to the following law of motion $\phi^e_t = \rho_{\phi_e}\phi^e_{t-1} - \alpha^{\phi e}(\mathcal{X}^e_{t-1} - \mathcal{X}^e_{ss}) - \alpha^{me}(m^e_t - m^e_{t-1}) + \epsilon^{\phi e}_t$. This rule reveals that lenders adjust the level of collateral requirement to the previous level at $t-1$ where ρ_{ϕ_e} determine the LTV policy weight on its previous level. Additionally, financial intermediaries can observe the level of charge-offs rate of business loans and adjust collateral requirement accordingly, with $\alpha^{\phi e}$ interpreted as the LTV weight on the level of charge-offs rate. The innovation $\epsilon^{\phi e}_t$ is assumed to be an i.i.d. with mean zero and standard deviation σ^{ϕ_e}.

The entrepreneur expected earnings is given by

$$E_t\left\{\left[(1+r^k_t)q^k_t k_t - (1+r^e_t)m^e_t/\pi_t\right](1-\mathcal{X}^e_t)\right\}$$

where r^k_t is the rate of return on capital and r^e_t is the nominal interest rate paid by entrepreneurs on their debt m^e_t. For the sake of simplicity, we define the probability that entrepreneurs will default on their loans $\mathcal{X}^e_t$, and the probability that they can make meet scheduled repayment on their debt $1-\mathcal{X}^e_t$. We assume that the leverage level of entrepreneurs is defined as $Lev_t = m^e_t/n^e_t$. Thus, the problem of the entrepreneur is to maximize the entrepreneurial earnings subject to participation constraint

$$E_t\left\{\left(1-\mathcal{X}^e_t\right)(1+r^e_t)m^e_t/\pi_t + (1-\mu^e)\mathcal{X}^e_t(1+r^k_t)\phi^e_t q^k_t k_t \geq (1+r_t)m^e_t/\pi_t\right\} \tag{A23}$$

Note that the equation of entrepreneur bank participation (A23) must hold with strict equality in every state of nature. The first term on the left-hand side of the equation corresponds to returns from non-defaulting entrepreneurs, and the second term corresponds to the returns from defaulting entrepreneurs whose collateral is sized by banks and net of monitoring cost μ^e. The right-hand side describes the return on loans given the interest rate demanded by banks r_{t+1}. We define the entrepreneur expected net worth as $n^e_{t+1} = \gamma^E\left(\left(1-\mathcal{X}^e_t - \left(1-\mathcal{X}^e_t\right)\phi^e_t\right)\left(1+R^k_t\right)k_t q^k_{t-1} + k_t\left(1-\delta\right)q^k_{t-1}\right)$.

A.1. Model with Endogenous Default Probability

To endogenize the probability of default of entrepreneurs, and households, we assume that the monitoring cost μ^e_t and μ^h_t are decision variables. That is, the objective function is given by $\Xi^e_t = \mu^e_t\mathcal{X}^e_t(m^e_t - r^e_t/\pi_t m^e_t)$ subject to $\mu^e_t(1+r^k_t)\phi^e_t q^k_t k_t \geq \mathcal{X}^e_t(1+r_t)m^e_t/\pi_t$. The first order condition with respect to μ^e_t is given by

$$\mathcal{X}^e_t m^e_t(1-r^e_t)/\pi_t - (1+r^k_t)\phi^e_t q^k_t k_t = 0 \tag{A24}$$

The log-linearized equation:

$$\tilde{\mathcal{X}}^e_t = \tilde{\phi}^e_t + \tilde{r}^k_t + \tilde{q}^k_t + \tilde{k}_t - \tilde{m}^e_t + \tilde{r}^e_t - \tilde{\pi}_t \tag{A25}$$

The objective function is $\Xi_t^h = \mu_t^h \chi_t^h (m_t^h - m_t^h r_t^e/\pi_t)$, subject to $\mu_t^h \phi_t^h q_t^h h_t \geq \chi_t^h (1+r_t) m_t^h/\pi_t$. The first order condition with respect to μ_t^h is

$$\chi_t^h m_t^h (1 - r_t^e) - \phi_t^h q_t^h h_t = 0 \tag{A26}$$

The log-linearized equation:

$$\tilde{\chi}_t^h = \tilde{\phi}_t^h + \tilde{q}_t^h + \tilde{h}_t - \tilde{m}_t^h + \tilde{r}_t^e - \tilde{\pi}_t \tag{A27}$$

A.2. Alternative Model assumptions

Unconstrained Households. The objective function is give by

$$\begin{aligned} &\text{maximize} \quad E_0 \Sigma_{t=0}^{\infty} \beta^t \left\{ \zeta_{c,t} (\log(c_{u,t} - b_u c_{u,t-1})) - \psi_l \frac{(l_{u,t})^{1+\sigma_l}}{1+\sigma_l} + \chi log(h_{u,t}) \right\} \\ &\text{subject to} \quad c_{u,t} + d_t + q_t^h h_{u,t} \leq w_t l_{u,t} + (1+r_t) d_{t-1}/\pi_t + q_t^h h_{u,t-1} \end{aligned} \tag{A28}$$

The maximization problem yields the following first-order conditions with respect to consumption, deposit and labor:

$$\frac{\partial \mathcal{L}_t}{\partial c_{u,t}} : \quad \lambda_{u,t} p_t - \frac{\zeta_{c,t}}{c_{u,t} - b c_{u,t-1}} + b\beta E_t \frac{\zeta_{c,t+1}}{c_{u,t+1} - b c_{u,t}} = 0 \tag{A29}$$

$$\frac{\partial \mathcal{L}_t}{\partial d_t} : \quad \lambda_{u,t} - \beta E_t \lambda_{u,t+1} (1 + r_{t+1})/\pi_t = 0 \tag{A30}$$

$$\frac{\partial \mathcal{L}_t}{\partial l_{u,t}} : \quad -\psi_l (l_{u,t})^{\sigma_l} + \chi_t \lambda_{u,t} = 0 \tag{A31}$$

where $\lambda_{u,t}$ is the marginal utility. Aggregate housing stock clearing condition $h_{c,t} + h_{u,t} = h_t$.

Appendix B. Steady states.

After calibrating model parameters, we obtain the steady state of the model. First, we set $\pi_{ss} = 1$ we solve for r^k_{ss}

$$\begin{aligned}\lambda_t &= E_t\beta\lambda_{t+1}r^k_t + E_t\beta\lambda_{t+1}(1-\delta)\\ \lambda_{ss} &= \beta\lambda_{ss}r^k_{ss} + \beta\lambda_{ss}(1-\delta)\\ r^k_{ss} &= \frac{1}{\beta} - (1-\delta).\end{aligned}$$

We pin down the value of r_{ss} in the steady state

$$\begin{aligned}\lambda_t - \beta E_t\lambda_{t+1}(1+r_{t+1}) &= 0\\ \lambda_{ss} - \beta\lambda_{ss}(1+r_{ss}) &= 0\\ r_{ss} &= \frac{1}{\beta} - 1,\end{aligned}$$

we also compute the value of q^k_{ss}

$$\begin{aligned}&q^k_t\lambda_t - \lambda_t[1 + S\left(\zeta_{i,t}\frac{i_t}{i_{t-1}}\right) + \zeta_{i,t}\frac{i_t}{i_{t-1}}S'\left(\zeta_{i,t}\frac{i_t}{i_{t-1}}\right)] + \beta\lambda_{t+1}\zeta_{i,t+1}\left(\frac{i_{t+1}}{i_t}\right)^2 S''\left(\zeta_{i,t+1}\frac{i_{t+1}}{i_t}\right) = 0\\ &q^k_{ss} = 1,\end{aligned}$$

and p^*_{ss} is

$$\begin{aligned}p_t &= \left[(1-\zeta_p)(p_{t-1})^{\frac{\nu_{p,t}}{1-\nu_{p,t}}} + \zeta_p(p^*_t)^{\frac{\nu_{p,t}}{1-\nu_{p,t}}}\right]^{\frac{1-\nu_{p,t}}{\nu_{p,t}}}\\ p_{ss} &= \left[(1-\zeta_p)(p_{ss})^{\frac{\nu_{p,ss}}{1-\nu_{p,ss}}} + \zeta_p(p^*_{ss})^{\frac{\nu_{p,ss}}{1-\nu_{p,ss}}}\right]^{\frac{1-\nu_{p,ss}}{\nu_{p,ss}}}\\ p^*_{ss} &= \left(\frac{(p_{ss})^{\frac{\nu_{p,ss}}{1-\nu_{p,ss}}} - (1-\zeta_p)(p_{ss})^{\frac{\nu_{p,ss}}{1-\nu_{p,ss}}}}{\zeta_p}\right)^{\frac{1-\nu_{p,ss}}{\nu_{p,ss}}}.\end{aligned}$$

The steady state value of w^*_{ss} is given by

$$\begin{aligned}w_t &= \left[(1-\zeta_l)(w_{t-1})^{\frac{1}{1-\nu_l}} + \zeta_l(w^*_t)^{\frac{1}{1-\nu_l}}\right]^{1-\nu_l}\\ w_{ss} &= \left[(1-\zeta_l)(w_{ss})^{\frac{1}{1-\nu_l}} + \zeta_l(w^*_{ss})^{\frac{1}{1-\nu_l}}\right]^{1-\nu_l}\\ w^*_{ss} &= \left(\frac{(w_{ss})^{\frac{1}{1-\nu_l}} - (1-\zeta_l)(w_{ss})^{\frac{1}{1-\nu_l}}}{\zeta_l}\right)^{1-\nu_l},\end{aligned}$$

and the marginal cost of production μ_{ss} is

$$\begin{aligned}p_t &= (1+\nu_{p,t})\mu_t\\ \mu_{ss} &= \frac{1}{(1+\nu_{p,ss})}p_{ss}.\end{aligned}$$

Given that $q^k_{ss}k_{ss} = m^e_{ss} + n^e_{ss}$ and $Lev_{ss} = m^e_{ss}/n^e_{ss} = (q^k_{ss}k_{ss} - n^e_{ss})/n^e_{ss}$. We use the participation constraint and the expected net worth to solve simultaneously for k_{ss} and n^e_{ss}

$$\left((1-\mathfrak{X}^e_t) + (1-\mu^e)\mathfrak{X}^e_t\right)\phi^e_t(1+r^k_t)q^k_t k_t \geq (1+r_t)(q^k_t k_t - n^e_t)$$
$$\left((1-\mathfrak{X}^e_{ss}) + (1-\mu^e)\mathfrak{X}^e_{ss}\right)\phi^e_{ss}(1+r^k_{ss})q^k_{ss}k_{ss} \geq (1+r_{ss})(q^k_{ss}k_{ss} - n^e_{ss}),$$

$$n^e_t = \left(1-\mathfrak{X}^e_t\right) - \phi^e_t(1-\mathfrak{X}^e_t))(1+r^k_t)(Lev_{ss}+1)n^e_t\left(\frac{1+r_t}{1+r^k_t}\right) + (1-\delta)q^k_t k_t$$
$$n^e_{ss} = \left(1-\mathfrak{X}^e_{ss}\right) - \phi^e_{ss}(1-\mathfrak{X}^e_{ss}))(1+r^k_{ss})(Lev_{ss}+1)n^e_{ss}\left(\frac{1+r_{ss}}{1+r^k_{ss}}\right) + (1-\delta)q^k_{ss}k_{ss}.$$

We then solve for investment i_{ss}

$$k_t = (1-\delta)k_{t-1} + \left(1 - S\left(\zeta_{i,t}, \frac{i_t}{i_{t-1}}\right)\right)i_t$$
$$k_{ss} = (1-\delta)k_{ss} + \left(1 - S\left(\zeta_{ss}, \frac{i_{ss}}{i_{ss}}\right)\right)i_{ss}$$
$$k_{ss} = (1-\delta)k_{ss} + i_{ss}$$
$$i_{ss} = k_{ss} - (1-\delta)k_{ss}$$
$$i_{ss} = \delta k_{ss}.$$

We also calculate m^e_{ss}

$$m^e_t = q_t k_t - n^e_t$$
$$m^e_{ss} = q_{ss}k_{ss} - n^e_{ss}.$$

Next we find, the value of labor in steady state l_{ss}

$$\mu_t = \frac{r^k_t}{\alpha\gamma_t(z_t l_t)^{1-\alpha}(k_t)^{\alpha-1}}$$
$$l_t = \left(\frac{r^k_t}{\alpha\gamma_t\mu_t(k_t)^{\alpha-1}}\right)^{\frac{1}{1-\alpha}}\frac{1}{z_t}$$
$$l_{ss} = \left(\frac{r^k_{ss}}{\alpha\gamma_{ss}\mu_t(k_{ss})^{\alpha-1}}\right)^{\frac{1}{1-\alpha}}\frac{1}{z_{ss}},$$

and subsequently solve for output y_{ss}

$$y_t = \gamma_t(k_t)^{\alpha}(z_t l_t)^{1-\alpha}$$
$$y_{ss} = \gamma_{ss}(k_{ss})^{\alpha}(z_{ss}l_{ss})^{1-\alpha}.$$

We obtain the steady state value of the government spending g_{ss}

$$g_t = \eta_g y_t$$
$$g_{ss} = \eta_g y_{ss}.$$

We compute r^e_{ss}

$$\begin{aligned}
\beta E_t \lambda_{t+1}(1 + r^e_t) - \lambda_t &= 0 \\
\beta \lambda_{ss}(1 + r^e_{ss}) - \lambda_{ss} &= 0 \\
\beta \lambda_{ss}(1 + r^e_{ss}) &= \lambda_{ss} \\
\beta(1 + r^e_{ss}) &= 1 \\
(1 + r^e_{ss}) &= \frac{1}{\beta} \\
r^e_{ss} &= \frac{1}{\beta} - 1,
\end{aligned}$$

we also obtain the steady state value of d^h_s

$$\begin{aligned}
(1 - \chi^h_{ss})\phi_{ss} q^h_{ss} h_{ss} + (1 - \mu^h)\chi^h_{ss}\phi_{ss} q^h_{ss} h_{ss} &= (1 + r_{ss}) d^h_s \\
d^h_s &= \frac{(1 - \chi^h_{ss})\phi_{ss} q^h_{ss} h_{ss} + (1 - \mu^h)\chi^h_{ss}\phi_{ss} q^h_{ss} h_{ss}}{(1 + r_{ss})},
\end{aligned}$$

we compute the value of f^e_{ss}

$$\begin{aligned}
f^e_t &= \phi_t \mu^e \chi^e_t (1 + r^k_t) q^k_t k_t \\
f^e_{ss} &= \phi_{ss} \mu^e \chi^e_{ss} (1 + r^k_{ss}) q^k_{ss} k_{ss}.
\end{aligned}$$

Finally, we solve for f^h_{ss}

$$\begin{aligned}
f^h_t &= \phi_t \mu^h \chi^h_t q^h_t h_t \\
f^h_{ss} &= \phi_{ss} \mu^h \chi^h_{ss} q^h_{ss} h_{ss}.
\end{aligned}$$

Appendix C. Log-Linearized Model: Baseline

Business Loan Delinq. Rate Shock $\tilde{\sigma}_t^e = \epsilon_{\sigma_e,t} + \rho_e \tilde{\sigma}_{t-1}^e$

Real Estate Loan Delinq. Rate Shock $\tilde{\sigma}_t^h = \epsilon_{\sigma_h,t} + \rho_h \tilde{\sigma}_{t-1}^h$

Business Loan Delinq. Rate $\frac{1}{\chi^e}\tilde{\chi}_t^e - \tilde{\sigma}_t^e \sigma_{ss}^e = 0$

Real Estate Loan Delinq. Rate $\frac{1}{\chi^h}\tilde{\chi}_t^h - \tilde{\sigma}_t^h \sigma_{ss}^h = 0$

Business Loan Coll. Req. Shock $\tilde{\phi}_t^e = \epsilon_{\phi_e,t} + \rho_e \tilde{\phi}_{t-1}^e - \alpha^{\phi e}(\tilde{\chi}_t^e - \tilde{\chi}_{ss}^e) - \alpha^{me}(\tilde{m}_t^e - \tilde{m}_{t-1}^e)$

Real Estate Loan Coll. Req. Shock $\tilde{\phi}_t^{\ h} = \epsilon_{\phi_h,t} + \rho_h \tilde{\phi}_{t-1}^h - \alpha^{\phi h}(\tilde{\chi}_t^h - \tilde{\chi}_{ss}^h) - \alpha^{mh}(\tilde{m}_t^h - \tilde{m}_{t-1}^h)$

Gov. Spend. Shock $\tilde{\eta}_{g,t} = \epsilon_{g,t} + \rho_g \tilde{\eta}_{g,t-1}$

Tech. Shock $\tilde{z}_t = \epsilon_{z,t} + \rho_z \tilde{z}_{t-1}$

Cons. Pref. Shock $\tilde{\zeta}_t^c = \epsilon_{\zeta_c,t} + \rho_{\zeta c} \tilde{\zeta}_{t-1}^c$

Price according to Calvo $\zeta_p \tilde{p}_t^* = \tilde{p}_t - (1-\zeta_p)\tilde{p}_{t-1}$

Price Recursive $\tilde{F}_{p,t} = \tilde{\mu}_t + \tilde{y}_t + (1-\zeta_p)\,\beta\,\tilde{p}_{t-1} - (1-\zeta_p)\,\beta\,\tilde{p}_t + (1-\zeta_p)\,\beta \tilde{F}_{p,t+1}$

Price Recursive $\tilde{K}_{p,t} = \tilde{\mu}_t + \tilde{y}_t + \tilde{z}_t + (1-\zeta_p)\,\beta \tilde{K}_{p,t+1}$

F.O.C. w.r.t. Price $-(1+\nu_p)((1-\alpha)\tilde{z}_{t+1} + (1-\alpha)\tilde{l}_{t+1} - (1-\alpha)\tilde{k}_{t+1}) + \tilde{p}_t^* - \tilde{p}_{t+1} = 0$

Wage according to Calvo $\zeta_l \tilde{w}_t^* = \tilde{w}_t - (1-\zeta_l)\tilde{w}_{t-1}$

Wage Recursive $\tilde{\mu}_t \frac{1}{\nu_l} - \tilde{w}_t^* + \frac{1}{\nu_l}\tilde{l}_t + \beta\,(1-\zeta_l)\,\tilde{F}_{w,t+1} - \tilde{F}_{w,t} = 0$

Wage Recursive $\tilde{F}_{w,t}\frac{1-\zeta_l}{1+\frac{1+\nu_l}{\nu_l}+\sigma^l} + \tilde{w}_t^* \frac{1-\zeta_l}{1+\frac{1+\nu_l}{\nu_l}+\sigma^l} + \zeta_l \tilde{w}_{t-1} - \tilde{w}_t = 0$

Capital Law of Motion $\tilde{k}_t = (1-\delta)\tilde{k}_{t-1} + \delta\tilde{i}_t - \tilde{i}_t S + S\tilde{i}_{t-1}$

F.O.C. w.r.t. Investment $-\tilde{q}_t^k + S\tilde{i}_t - S\tilde{i}_{t-1} + S\tilde{i}_t - S\tilde{i}_{t-1}$

$S + \tilde{i}_t - S\tilde{i}_{t-1} - S\beta\tilde{i}_{t+1} + S\beta\tilde{i}_t + S\beta\tilde{i}_{t+1} - S\beta\tilde{i}_t$

Producer Euler Equation $\tilde{\mu}_{t+1} = \alpha\tilde{r}_t^k + (1-\alpha)\tilde{w}_t$

F.O.C. w.r.t Capital-Capital Producer $-\tilde{R}_{t+1}^k + \beta\tilde{r}_t^k + (1-\delta)\tilde{q}_{t+1}^k - \tilde{q}_t^k = 0$

F.O.C. w.r.t Capital-Firm $-\tilde{r}_t^k + (1-\alpha)\tilde{z}_t + \tilde{l}_t(1-\alpha) - (1-\alpha)\tilde{k}_t = 0$

F.O.C. w.r.t Labor-Firm $-\tilde{l}_t - (1/-\alpha)\tilde{z}_t + (1/-\alpha)\tilde{w}_t - \alpha\tilde{k}_t = 0$;

Entrepreneur Optimality Condition $\tilde{R}_{t+1}^K - \tilde{r}_t - \tilde{n}_t^e + \tilde{\chi}_t^e + \tilde{\phi}_t^e = 0$

Entrepreneur Net Worth Definition $-\tilde{n}_t^e - \tilde{\phi}_t^e + \tilde{R}_t^k - \tilde{r}_{t-1} + \tilde{k}_{t-1} + \tilde{q}_{t-1}^k + \tilde{n}_{t-1}^e = 0$

Production Function $(1-\alpha)\tilde{l}_t + (1-\alpha)\tilde{z}_t + \alpha\tilde{k}_t - \tilde{y}_t = 0$

Monetary Policy Shock $\tilde{r}_t = \rho_p \tilde{r}_{t-1} + (1-\rho_p)[a_\pi(\tilde{p}_t - p_{ss}) + a_{\Delta y}(\tilde{y}_t)] + \epsilon_t$.

Government Spending Share $\tilde{y}_t - \tilde{g}_t + \tilde{\eta}_{g,t} = 0$

Labor Supply-unconstrained Households $\tilde{l}_{u,t} = \tilde{l}_t$

Labor Supply-constrained Households $\tilde{l}_{c,t} = \tilde{l}_t$

F.O.C. w.r.t. Labor-unconstrained Households $\sigma^l \tilde{l}_{u,t} = \tilde{w}_t + \tilde{\lambda}_{u,t}$

F.O.C. w.r.t. Labor-constrained Households $\sigma^l \tilde{l}_{c,t} = \tilde{w}_t + \tilde{\lambda}_{c,t}$

F.O.C. w.r.t. Consumption-unconstrained Households $\tilde{\lambda}_{u,t} - \tilde{\zeta}_{c,t} + \tilde{c}_{u,t} - b_u \tilde{c}_{u,t-1} + \beta b_u \tilde{\zeta}_{c,t+1} = 0$

F.O.C. w.r.t. Consumption-constrained Households $\beta_c b_c \tilde{\zeta}_{c,t+1} + \tilde{\lambda}_{c,t} - \tilde{\zeta}_{c,t} + \tilde{c}_{c,t} - b_c \tilde{c}_{c,t-1} = 0$

Reals Estate Volume Shock $\tilde{h}_t = \epsilon_{h,t} + \rho_q \tilde{h}_{t-1}$

Budget constraint-unconstrained Households $\tilde{c}_{u,t} = \tilde{r}_{t-1} + \tilde{w}_t + \tilde{l}_{u,t} - \tilde{d}_t - \pi_t$

Households Deposit Defintion-unconstrained Households $-\tilde{d}_{t-1} + \tilde{m}_t^h = 0$

Participtaion Constraint-unconstrained Households $-\tilde{m}_{t-1}^h + \tilde{\chi}_t^h + \tilde{\phi}_t^h + \tilde{q}_t^h + \tilde{h}_{t-1} + \tilde{\chi}_t^h + \tilde{\phi}_t^h + \tilde{q}_t^h + \tilde{h}_{t-1} - \tilde{r}_t = 0$

Housholds Budget Constraint-unconstrained Households $\tilde{c}_{c,t} = \tilde{m}_t^h + \tilde{h}_{t-1} + \tilde{q}_t^h + \tilde{w}_t + \tilde{l}_{c,t} - \tilde{q}_t^h + \tilde{h}_t - \tilde{m}_{t-1}^h + \tilde{r}_t^e - \pi_t$

Business Loan Total Charge-offs $\tilde{q}_{t-1}^k + \tilde{k}_{t-1} - \tilde{f}_t^e + \tilde{\chi}_{t-1}^e + \tilde{\phi}_{t-1}^e + \tilde{R}_{t-1}^K = 0$

Real Estate Loan Total Charge-offs $\tilde{h}_{t-1} - \tilde{f}_t^h + \tilde{\chi}_{t-1}^h + \tilde{\phi}_{t-1}^h + \tilde{q}_{t-1}^h = 0$

Business Loan Total Definition $\tilde{m}_t = \tilde{k}_t + \tilde{q}_t^k - \tilde{n}_{t+1}^e$

Total Consumption $\tilde{c}_t = \tilde{c}_{u,t} + \tilde{c}_{c,t}$

Firm Value $\tilde{v}_t = -\tilde{\chi}_{t+1}^h - \tilde{\phi}_{t+1}^e + \tilde{\chi}_{t+1}^h + \tilde{R}_{t+1}^K + \tilde{q}_t^k + \tilde{k}_t + \tilde{n}_t^e + \tilde{R}_{t+1} - \tilde{R}_{t+1} + \tilde{q}_t^k + \tilde{k}_t$

Fundamnetal Value and Average Capital Price $\tilde{s}_t = \tilde{k}_t + \tilde{q}_t^k - \tilde{v}_t$

Housing Stock Fundamnetal Value $\tilde{q}_t^f = -\chi^h \tilde{h}_t - (\chi^h + \beta_c)\tilde{\lambda}_{c,t} + \tilde{\phi}_t^h + \tilde{q}_t^h - \tilde{r}_t^e + \beta_c \tilde{\lambda}_{c,t+1} - \beta_c \tilde{q}_{t+1}^h$

Fundamnetal Value and Average Housing Stock Price $\tilde{resi}_t = \tilde{q}_t^f - \tilde{q}_t^h$

Measurement Equations:

Gross Domestic Product $\text{gdpobs}_t = \tilde{y}_t + \rho_{y^{obs},t}$

Consumption $\text{consumptionobs}_t = \tilde{c}_t$

CI Loans $\text{CIloanobs}_t = \tilde{m}_t$

Real Estate Loans $\text{Hloanobs}_t = \tilde{m}_t^h$

Investment $\text{investobs}_t = \tilde{i}_t + \rho_{i^{obs},t}$

Total Hours $\text{hoursobs}_t = \tilde{l}_t + \rho_{l^{obs},t}$

CI Charge-offs $\text{CIchargoffobs}_t = \tilde{f}_t^e + \rho_{f^{e,obs},t}$

Real estate Charge-offs $\text{Hcharfoffobs}_t = \tilde{f}_t^h + \rho_{f^{h,obs},t}$

Gov. Spending $\text{govobs}_t = \tilde{g}_t$

Delinquency Rate-Real Estate Loans $\text{defaulthobs}_t = \tilde{\chi}_t^h$

Delinquency Rate-Business Loans $\text{defaulteobs}_t = \tilde{\chi}_t^e$

Appendix D. Log-Linearized Model: Extensions

Next, we present the log-linearized form of deterministic and endogenous default probability models. In principle, all the optimality condition of the benchmark model remains unchanged except the equations that characterize the default probability.

D.1. Log-Linearized Model: Endogenous Default Probability

$$\text{Business Loan Delinq. Rate} \quad \tilde{\chi}_t^e = \tilde{\phi}_t^e + \tilde{r}_t^k + \tilde{q}_t^k + \tilde{k}_t - \tilde{m}_t^e + \tilde{r}_t^e - \tilde{\pi}_t$$

$$\text{Real Estate Loan Delinq. Rate} \quad \tilde{\chi}_t^h = \tilde{\phi}_t^h + \tilde{q}_t^h + \tilde{h}_t - \tilde{m}_t^h + \tilde{r}_t^e - \tilde{\pi}_t$$

D.2. Log-Linearized Model: Deterministic Endogenous Default Probability

$$\text{Business Loan Delinq. Rate Shock} \quad \tilde{\sigma}_t^e = \tilde{\sigma}_{ss}^e$$

$$\text{Real Estate Loan Delinq. Rate Shock} \quad \tilde{\sigma}_t^h = \tilde{\sigma}_{ss}^h$$

$$\text{Business Loan Delinq. Rate} \quad \frac{1}{\chi^e}\tilde{\chi}_t^e - \tilde{\sigma}_t^e \sigma_{ss}^e = 0$$

$$\text{Real Estate Loan Delinq. Rate} \quad \frac{1}{\chi^h}\tilde{\chi}_t^h - \tilde{\sigma}_t^h \sigma_{ss}^h = 0$$

Appendix E. Equilibrium Definition

A competitive equilibrium consists of stochastic processes $\{p_t, p_t^*, F_{p,t}, K_{p,t}, w_t, w_t^*, F_{w,t}, k_t, i_t, \mu_t, q_t^k, r_t^k, \bar{r}_t^k, n_t^e, y_t, l_t, l_{u,t}, l_{c,t}, \lambda_t, \lambda_{u,t}, \lambda_{c,t}, c_t, c_{u,t}, c_{c,t}, d_t, h_t, m_t^h, q_t^h, r_t^e, f_t^e, f_t^h, m_t^e\}$ that evolve according to a system of equations. This model is driven by 8 exogenous shocks: government spending shock $\eta_{g,t}$, technology shock z_t, consumption preference shock ζ_t^c, monetary policy shock r_t, delinquency rate shock on firm loan χ_t^e, delinquency rate shock on mortgage loan χ_t^h, business loan collateral requirement shock ϕ_t^e, and mortgage loan collateral requirement shock ϕ_t^h.

- *In each period t, firms maximize their profits by converting intermediate goods to final goods, setting prices and wages under nominal inertia.*
- *Entrepreneurs maximize profits by providing capital services and borrowing from banks. Entrepreneurs are collateral constrained and can default on their loans.*
- *Unconstrained households maximize their lifetime utility by providing labor services, consuming the final good, and making a deposit.*
- *Constrained households maximize their lifetime utility by providing labor services, consuming the final good, purchasing real estate assets, and obtaining a loan to finance their purchase. Collateral constrained households face collateral constraints and can default on their loans.*
- *Banks intermediate the flow of deposits and loans.*
- *A prudential policy is conducted by a lenders who adjusts the collateral requirement to the level of the charge-off rate.*
- *A central bank set the policy rate depending on expected inflation and the deviation of output from its steady state.*

Appendix F. Data.

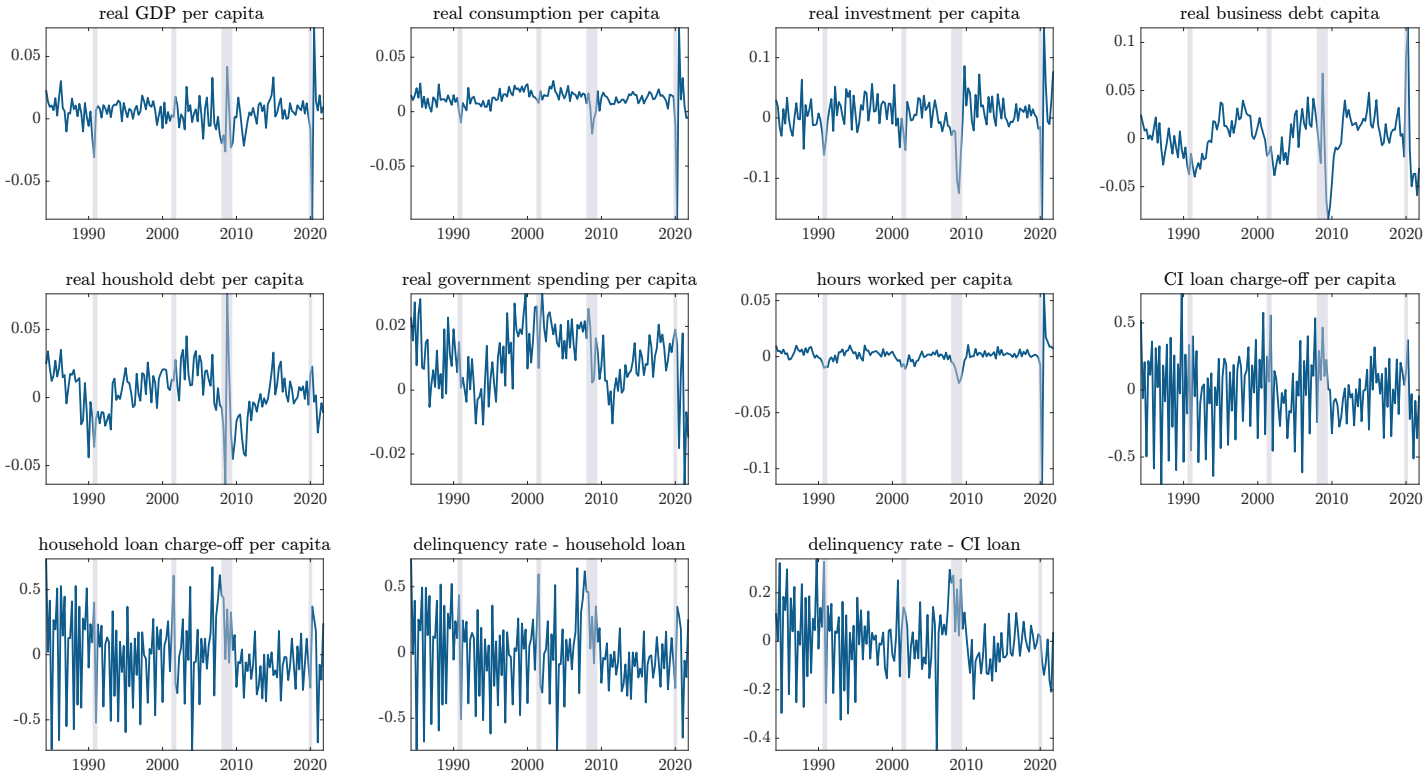


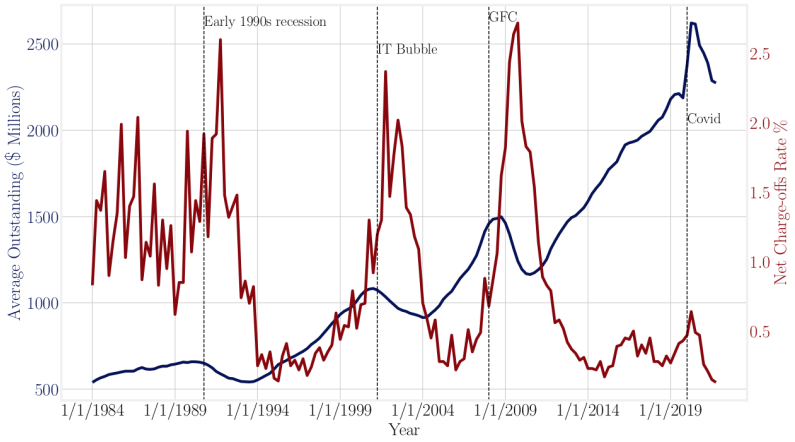
FIGURE A1. US Data

Note: Quarterly data retrieved from Federal Reserve Bank of St. Louis Database (FRED) and Federal Deposit Insurance Corporation (FDIC)

Appendix G. Loan Supply during the IT Bubble in the 1990s and the Great Recession

Next, we provide information on loan charge-off rates and collateral liquidation in the US, and discuss the dynamics of credit supply between 1984 and 2021.

(a) CI loans vs charge-off rate



(b) Mortgage loans vs charge-off rate

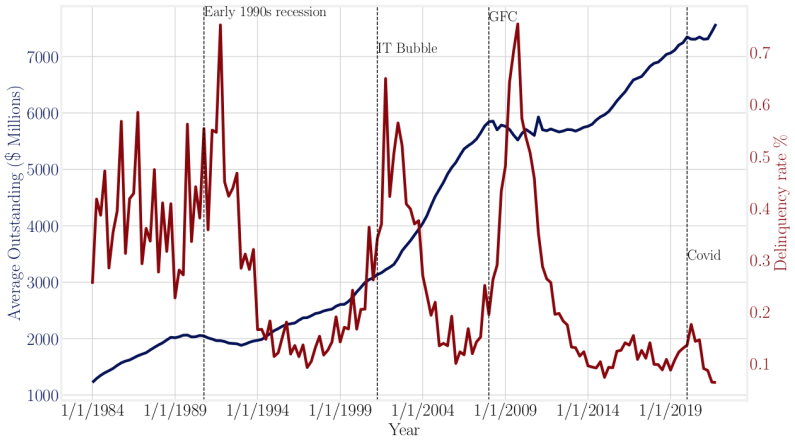


FIGURE A2. Business and Mortgage Loans vs charge-off rate

Notes: Panel (a) shows the dynamics of commercial industrial loans and the charge off rate. Panel (b) presents the evolution of mortgage loans and charge-off rate. Source: FDIC. Sample: 1984Q1-2021Q4

Panel (a) of Figure A2 presents the dynamics of firm debt and the net charge-off rate in the US between 1984 and 2021. It offers a clear summary of the antecedents to the 2001 and 2008 crises and suggests that these crises were preceded by high credit growth and a relaxation of lending conditions. By focusing on these two recessions, starting from 1992, we can see that firm debt rose considerably and the boom ended approximately in 1998. Following the peak, business loan growth went down, entering the bust

phase and reaching the bottom in 2002. Banks that overextended credit prior to the IT bubble burst and the financial crisis of 2008 faced higher level of charge-off rates. This faster credit growth, with poor economic performance, exerted a downward impact on the willingness of banks to lend. Interestingly, an abnormal increase in the net charge-off rate occurs when the economy enters a crisis or periods of high uncertainty. Following an increase in the delinquency rate, business lending experiences downward pressure until the charge-off rate starts to decrease.

The dynamics of mortgage loans are shown in panel (b) of Figure A2. The credit cycle phases of firm debt are broadly similar to the patterns observed in mortgage loans. Loans to households exhibit steady movement during crises while the defaults on household loans rise. Before the 2008 crisis, a period characterized by relaxed credit standards and thus collateral terms, banks conducted an expansionary credit policy. When the crisis hit the economy, mortgage loans declined slightly, whereas the delinquency rate rapidly increased. This echoes the empirical evidence in Section 3 (Figure 1) and reflects the idea that banks relax their collateral terms during good times and become more prudent during economic downturns.

Appendix H. Uncollectible Debt and Collateral Liquidation

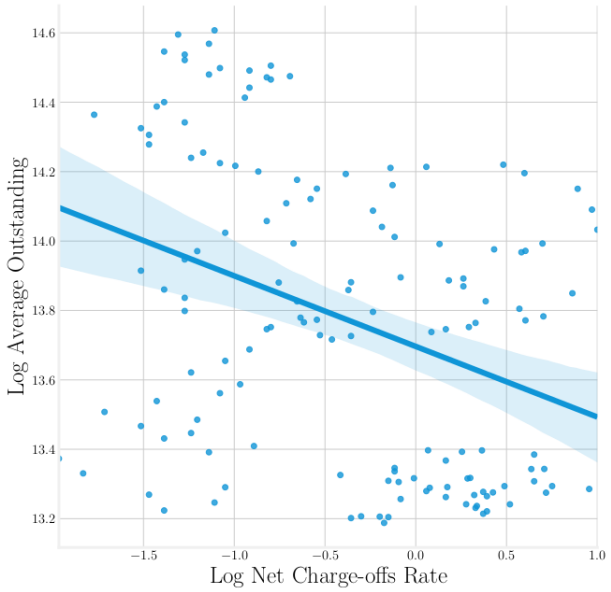


FIGURE A3. Correlation between Average Outstanding Debt and Net charge-off rate

Notes: Quarterly data retrieved from the Federal Deposit Insurance Corporation (FDIC); Sample: 1984Q1-2021Q4

The collateral channel proposed in this study posits that finical intermediaries may restore financial stability through an active internal prudential policy rule that reacts to the delinquency rates and credit

supply. Figure A3 displays the inverse relationship between total debt and the net charge-off rate. The vertical axis measures the log of the total debt. The total debt comprises commercial and industrial loans and real estate loans. The horizontal axis represents the log of the net charge-off rate. The observations are quarterly and represent the aggregated data at the national level from the Federal Deposit Insurance Corporation (FDIC). Given the high level of default, the lender can anticipate loan losses and change the loan-to-value limit, which will cause a decline in the credit supply to firms and individuals. Furthermore, it is likely that when the charge-off rate is high, banks experience a decline in liquidity that forces them to adopt a tight collateral policy, thus reducing the loan supply.

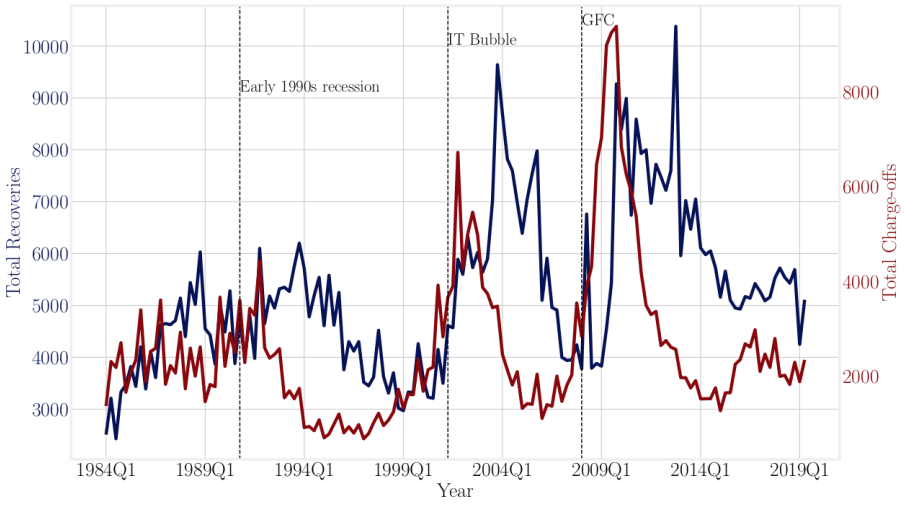


FIGURE A4. US Banks' Total Recoveries on CI and HH Loans

Notes: This figure plots the banks total recoveries on commercial, indutrial and household loans. Quarterly data retrieved from Federal Deposit Insurance Corporation (FDIC); Sample: 1984Q1-2021Q4

While we focus on the changes in bank-based prudential measures, it is useful to consider the evolution of bank recoveries. Banks facing a high level of loan charge-off are also subject to decreasing liquidity; in this situation, banks can repossess and liquidate the collateral and recover loan losses. The dynamics of collateral liquidation, shown in Figure A4, indicate that collateral liquidation lags the delinquent business and mortgage debt. This is evident during the economic downturn, with collateral liquidation peaking two years after a higher level of delinquent debt. The total recoveries peaked in 2004 and 2013 and decreased in subsequent years. These significant lagged bank recoveries reveal that the collateral repossessed by banks becomes illiquid and difficult to sell when the crisis hits the economy.