A Game Theoretic Model of Tax Structure Choice

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Author Note

I want to thank my parents for allowing me to aim high. Without them, I would not have been able to live and submit this research paper.

Abstract

This paper integrates a Bayesian game of incomplete information between governments and households with a Laffer curve-based market dynamics framework to investigate the factors influencing business tax structures across countries. The paper explores how tax auditing costs, consumer behavior, government auditing strategies, and household evasion strategies interact, shaping both the design of tax systems and the distributional effects of business taxation. As governments worldwide consider tax reforms understanding the drivers of variation in tax structures is critical to informing policy decisions. While it is known that developed countries tend to rely more on direct taxes and developing countries on indirect taxes, the underlying institutional and economic drivers of these patterns are less understood. This paper fills that gap by offering a theoretical explanation for the observed variation, showing that the optimal mix of direct and indirect taxes is deeply influenced by a country's tax auditing capacity and institutional frameworks. Our findings provide novel insights into how these factors affect business investment, tax evasion, and government revenue, offering a comprehensive view of how tax policy design and auditing capabilities shape the economic and distributional outcomes of business taxation.

Keywords: Bayesian game, Laffer curve, tax policy design, tax evasion, incomplete information.

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A fundamental question in public finance is why countries differ in their reliance on direct versus indirect taxes to raise government revenue. Empirical evidence has consistently shown that developed countries tend to rely more heavily on direct taxes while developing countries derive a larger share of their revenue from indirect taxes. These disparities have far-reaching economic consequences for economic growth, equity, and government revenue stability. Understanding underlying factors that cause these differences in tax structures is crucial for designing effective tax policies that align with the country's economic contexts and government administrative capabilities.

This paper provides a theoretical framework to explain these observed patterns, focusing on the roles of household tax evasion strategies, government enforcement strategies, consumer behavior, and market dynamics. By integrating a Bayesian game of incomplete information with a Laffer curve-based market dynamics framework, this paper explores how a country's institutional capabilities and governance constraints shape its tax structure. Our models demonstrate that higher tax auditing costs, often seen in underdeveloped countries, lead the government to rely more heavily on indirect taxes to raise government revenue. In contrast, developed nations, with lower auditing costs due to their stronger enforcement mechanisms, are better equipped to raise revenue through the collection of direct taxes.

The importance of this topic lies in its potential to inform tax policy design. Understating the factors that influence tax structure choices can provide policymakers with actionable insights to design systems to balance administrative feasibility, fairness, and economic growth. While the existing literature highlights the empirical patterns of tax structure differences, there is a significant gap in the theoretical framework that attempts to link these patterns to the underlying factors in a country. By addressing this gap in the literature, this paper provides valuable insights into the factors that influence tax structure choices across countries and their implications for market dynamics, tax compliance behavior, and revenue generation.

2. Literature Review

The literature on taxation explores the optimal design of tax systems, focusing on the choice of direct and indirect forms of taxation. Additionally, game theory has emerged as an adept mathematical tool in economics, specifically in taxation research. In particular, game theory has been employed in understanding the interactions between fiscal authorities and citizens of a country. This literature review synthesizes key contributions of optimal tax policy design, the impacts of differing tax systems on countries, and the role of game-theoretic frameworks used in taxation research.

Recent literature has demonstrated the varied impact of direct and indirect taxes on economic growth. According to Hakim (2020), a study examining the effects of direct and indirect taxes on economic growth across 51 countries, found that direct taxes have a negative impact on economic growth, while taxes, though more efficient at generating government revenue, have a positive but insignificant effect. In contrast, Nguyen (2019) emphasized the positive impact of indirect taxes rather than direct taxes on economic growth in Vietnam, implying that indirect taxes may be more growth-promoting due to effective government investment or a structure that preserves consumption incentives. Furthermore, in the case of developing and developed countries, Hakim et al. (2022) found that the economic impact of direct and indirect taxes varies depending on the country's level of development. These results may be attributed to the mechanisms of differing economic structures, administrative capacities, and spending efficiencies. Developing countries often rely more on indirect taxes due to limited administrative resources and a larger informal sector, while developed countries can implement and enforce direct taxes more effectively. These studies indicate that the effects of direct and indirect taxes on economic growth are complex and may differ based on a country's economic context.

Additionally, a key starting point in optimal tax policy design is the Atkinson-Stigliz (1976) theorem, which asserts that when governments can impose a general income tax, assuming individuals vary only in their wage-earning abilities and the utility functions are separable between labor and

commodities, indirect taxes are unnecessary. However, this theorem acknowledges that the use of indirect taxes may depend on more complex consumer preferences and constraints on income taxation. Additionally, the theorem does not account for the cost of tax administration, which can significantly influence the design of an optimal tax structure. When considering the administration of different types of taxation, enforcement and evasion issues have been shown to critically affect the optimal ratio of direct and indirect taxation. Boadway et. al (1994) proved that if different taxes have varying levels of evasion risk, an optimal tax structure including indirect taxes emerges naturally. In their analysis, they assume that income taxes are more prone to evasion. Research by Farhi and Gabaix (2015) further expanded the field of optimal taxation theory by incorporating behavioral biases into traditional frameworks, finding that foundational results in optimal taxation theory fail with behavioral agents.

Game theory plays a crucial role in understanding the interactions between taxpayers and the government, specifically on taxpayer compliance. Bardsley (1994) introduced a Bayesian Nash equilibrium model with imperfect auditing, where both taxpayers and auditors observe noisy signals of true tax liability. Alm et al. (1993) empirically supported the game-theoretic approach, finding nonrandom audit strategies and small underreporting elasticities related to tax rates and income. In contexts such as Brazil, game theory analysis demonstrated that tax avoidance emerges as the optimal strategy for companies due to tax complexities and special installments (Gomes et al., 2023). Moreover, Alm and McKee (2003) explored tax compliance as a coordination game, demonstrating that while pre-game communication facilitates coordination on low tax compliance, fiscal authorities can counteract this through strategic audit rule modifications. These studies collectively highlight the complex interplay between taxpayer behavior, audit strategies, and policy design in tax compliance games, emphasizing the strength of game theory in analyzing the interactions between taxpayers and the government.

In conclusion, this literature review highlights key contributions of optimal tax policy design, the impacts of differing tax systems on countries, and the role of game-theoretic frameworks used in taxation

research. Existing literature indicates that tax policies have complex and varied impacts on economic growth depending on the economic contexts of different countries. Through optimal taxation theory, policymakers can identify strategies to align revenue goals with social welfare objectives. Additionally, game-theoretic approaches are adept in analyzing taxpayer behavior and compliance strategies, revealing the interactions between taxpayers, businesses, and governments.

3. Key Contributions

The incorporation of the Bayesian game and the Laffer curve-based framework represents the groundbreaking contribution of our paper. By integrating Bayesian game theory with the Laffer curve market dynamics framework, this paper demonstrates a previously underexplored relationship between the strategic interactions of households and the government and the broader macroeconomic dynamics of tax revenue generation. Specifically, this novel framework connects tax auditing costs, consumer behavior in the broader economy, government auditing strategies, and household evasion strategies to optimize the design of tax systems. Through this paper, our models offer valuable insights for policymakers in both developed and developing countries on how different factors influence various economic outcomes and direct and indirect tax revenue generation. Finally, our paper fills the gap in the literature by illustrating novel insights into how differing tax structures emerge due to varying administrative capacities and economic contexts in different countries. These contributions enhance the understanding of tax system design by integrating game theoretic concepts with economic theory, providing both theoretical explanations and practical insights into tax policy.

4. Theoretical Framework

In this paper, we explore the interactions between a fiscal authority, a household, and a firm. Each of these players plays a distinct role in the functioning of the economy. By examining the interactions between these players, our models aim to analyze the mechanisms driving the effectiveness of direct and

indirect taxation and the specific parameters influencing the choice and efficacy of different tax methodologies.

This paper presents two interconnected models to analyze the interactions within a taxation framework. The first model employs Bayesian games to explore the strategic interactions between the government and households in the context of direct taxes, focusing on tax compliance and auditing strategies. The second model examines the impact of indirect taxes on firms and markets, analyzing their revenue, profit behaviors, and market dynamics while incorporating the derivation of the Laffer curve, with profits, wages, and elasticity as key variables. By linking household compliance, government auditing strategies, and market dynamics, these models together provide a holistic perspective on the trade-offs and dynamics of taxation policies, highlighting how direct and indirect tax mechanisms influence one another within the broader economic system.

4.1 Game Theoretic Foundation

Game theory is utilized immensely in the study of government taxation, as it provides a rigorous framework for analyzing strategic interactions between governments and taxpayers. In this paper, we employ game theory to formulate a Bayesian game under conditions of incomplete information. Taxation systems around the world inherently involve information asymmetry, where governments do not have full knowledge of taxpayers' incomes, assets, or behaviors, while taxpayers lack full clarity about government enforcement strategies or auditing practices.

In this Bayesian game model, taxpayers are categorized into different types based on their decision to evade or comply with taxes, while the government adopts a probabilistic approach to its auditing strategy informed by its belief about the evasion behavior of the household. This interaction between households and governments creates a dynamic of decision-making, where the households aim to

minimize their tax burden and penalties, while the government aims to maximize tax revenue and minimize auditing costs.

This Bayesian framework is adept in capturing this dynamic due to its incorporation of asymmetric information and the strategic interactions among players based on their beliefs, which are accurate and relevant in the real world. Through the analysis of this game, this paper sheds light on the factors that influence compliance or evasion decisions and provides insights into how various factors may influence enforcement policies and tax rates around the world.

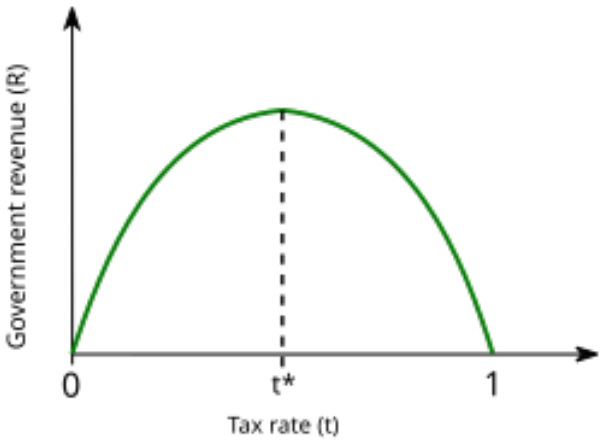
4.2 Economic and Market Dynamics of Indirect Taxation

4.2.1 Theoretical background of the Laffer curve

The Laffer curve illustrates a theoretical framework for the relationship between the tax rate and the total government revenue. It states that tax revenue is zero at both extremes, at 0% and 100%, because at 0% there is no taxation and at 100% there is no incentive to work or produce. Since at both extremes, the revenue generated is zero, from the intermediate value theorem, there must exist some a maximum between these two rates, on the condition that the Laffer curve is continuous which it is.

Figure 1

Illustration of the Laffer curve



The Laffer curve framework is crucial in taxation policy as it highlights the trade-offs between higher tax rates and economic behavior. At low tax rates, increasing taxes generally leads to higher

revenue. However, as tax rates rise beyond a certain point, the incentive to comply decreases due to higher tax burdens, leading to reduced economic activity, increased tax evasion, or both. This interplay between compliance and tax burden underscores the need for policymakers to identify optimal rates that balance revenue maximization with minimal distortions in economic behavior.

4.2.2 Market Impacts of Indirect Taxes

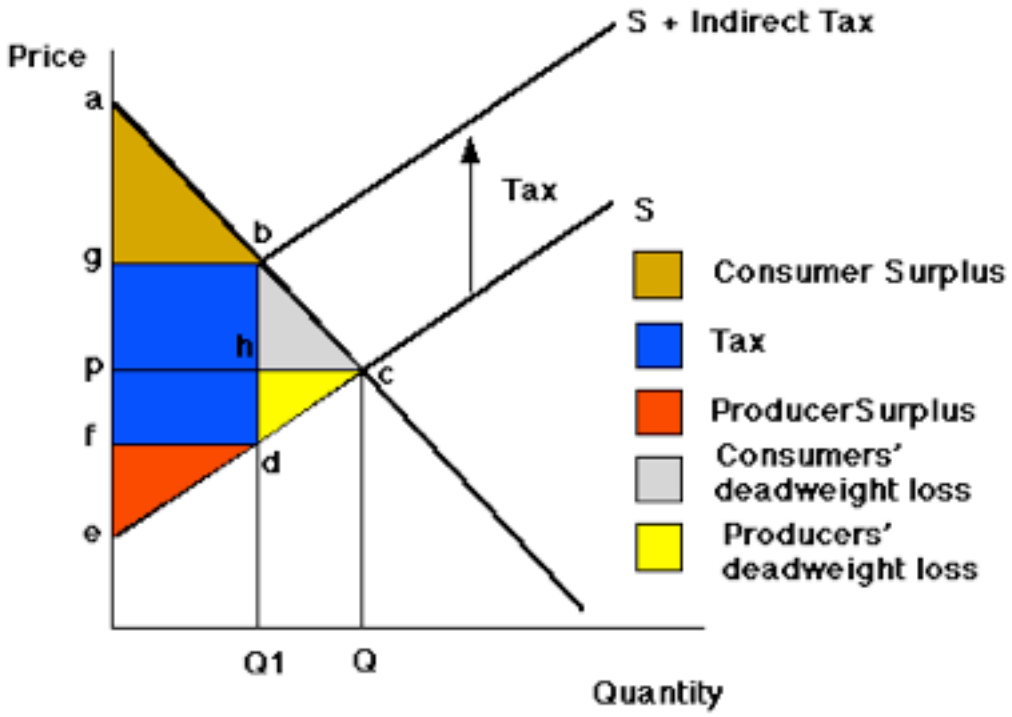
Indirect taxes of the ad valorem type (a tax based on the value of the goods) such as value-added taxes (VAT) and excise duties, have a direct influence on firm behavior, pricing strategies, and consumer demand. When an indirect tax is imposed, consumers face a higher cost to purchase a good or service, and firms face higher production costs. This may lead to a reduction in demand depending on the elasticity of the good or service.

Goods with inelastic demand (necessities) tend to see smaller reductions in quantity demanded when taxes increase, leading to more stable tax revenues. Conversely, for goods with elastic demand, even a small increase in price due to taxes can lead to a significant drop in demand, reducing both firm revenue and government tax collections.

In addition to elasticity, wages play a significant role in determining how indirect taxes affect the market equilibrium. A higher direct tax may lower disposable income, affecting consumer purchasing power. For example, substitute goods may have an increased demand, while the demand for normal goods may decrease.

Figure 2

Supply and demand graph showing the impact of indirect taxes on consumers and producers



Note: The demand curve (sloping down) can also shift to the left due to the increase in price caused by the indirect tax.

By altering the cost of a product and increasing the price of goods, indirect taxes can shift the equilibrium conditions of markets, causing changes in a firm's profitability and the welfare of consumers.

4.2.3 Indirect Taxes and Economic Inefficiency

Indirect taxes can have far-reaching consequences in markets, economies, and resource allocation. On one hand, a well-structured indirect tax can generate substantial revenue with minimal administrative costs while promoting fiscal stability. However, a poorly structured indirect tax can lead to decreased economic growth, market inefficiencies, and inefficient resource allocation.

As seen in Figure 2, in the gray and yellow colors showing consumers' and producers' deadweight loss, indirect taxes can reduce economic welfare and lower productivity by shifting consumption away from taxed goods to untaxed substitutes. If the indirect tax were to be increased, the producers of the good in Figure 2 would experience higher costs leading to the reduction in the supply of

the good. This reduction in supply decreases both consumer and producer surplus but increases the overall tax revenue generated and the deadweight loss of the market, leading to economic inefficiency and reduced welfare.

Therefore, the design and implementation of indirect taxes must balance revenue generation while attempting to minimize distortion in the market and the economy.

5. Model 1: Direct Tax Evasion Bayesian Game Under Asymmetric Information

In this section, we build a Bayesian game to model the strategic interactions between a fiscal authority (the government) and the household (the taxpayer), which incorporates incomplete information. In this Bayesian game, the government faces uncertainty about the household's strategy and type, specifically the household's decision to evade or comply with the direct tax. The taxpayer's action is based on their own private information about their willingness to comply, which is unknown to the government. This situation creates an incomplete information scenario, which is imperative to understanding the dynamics of tax compliance and auditing in the real world.

We aim to link this game theoretic model of direct taxes with a Laffer curve market dynamics framework to model how different countries utilize direct and indirect taxes differently based on their economic and institutional factors. This connection is a novel aspect of this paper, as it unveils a previously underexplored interplay between individual tax compliance decisions and the broader economic dynamics encapsulated by both the Bayesian game and the Laffer curve.

In Section 5.1, the foundation elements of our direct tax evasion Bayesian game are defined, outlining the players, types, strategies, and payoff structures. In Section 5.2, the mixed strategy Bayesian Nash equilibrium is derived. In Section 5.3, the equilibrium is analyzed, with propositions introduced and proven to formalize assumptions and explore the variables that influence equilibrium outcomes.

5.1 Foundational Elements of the Bayesian Game

There are two players in our game: The government (player 1) and the household (player 2). The government is responsible for implementing tax policies and auditing taxpayers, and the household is

responsible for deciding whether to comply fully, evade taxes partially, or evade taxes completely. Thus let $\mathrm{N} = \{1,2\}$ represent the set of players.

To introduce asymmetric information, we assume that player 2's, the household's, information about their compliance is private. Therefore, let $T_2 = \{\mathrm{E}, \mathrm{C}\}$ represent the set of taxpayer types:

$$T_2 = \begin{cases} E, & with\ probability\ p \\ C, & with\ probability\ 1-p \end{cases}$$

Where $t_2 = E$ is an evasive household, and $t_2 = C$ is a fully compliant household. The type $t_2 \in T_2$ is the type of the household. Let $\pi_2 = \{p, 1-p\}$ denote the probability distribution of the household's types E and C, respectively. The government's type, $T_1 = \{G\}$, is a singleton, which occurs with a probability of 1.

Defining the strategy sets for both players, let $S_1 = \{A, \neg A\}$ represent the set of strategies for the government, where A denotes the strategy to audit the household, and $\neg A$ denotes the strategy of not auditing the household. On the other hand, the strategy set for the household depends on its type. If the household is of type $t_2 = E$, their strategy is $S_2(E) = s$, where $s \in [0,1)$ represents their level of compliance. Specifically, $s = 0$ means the taxpayer is fully avoiding taxes and $s = 1$ represents the taxpayer is fully compliant to the tax rate. If household is of type $t_2 = C$ their strategy is simply $S_2(C) = s = 1$, meaning they are fully compliant with the direct tax.

Since the household's types are private information, their compliance rate, s, is also private. This creates a scenario of incomplete information within the Bayesian framework, where the government does not have direct access to the household's compliance behavior. Therefore, the government forms beliefs about the taxpayer's compliance. In the real world, these beliefs may come from previous interactions between the taxpayer and the government, such as past audit results, compliance history, or patterns in declared income. Therefore, the government may leverage historical or sector-specific data to refine their beliefs based on the taxpayer by using the theoretical framework of Bayesian updating. By incorporating observed data into their prior beliefs, the government can adjust their compliance estimates to better

optimize their auditing strategies. This leads to an iterative process that enhances the precision of enforcement efforts. Mathematically this belief updating process can be represented as follows:

$$P(\theta|X) = \frac{P(X|\theta)P(\theta)}{P(X)}$$

Here, $P(\theta|X)$ is the posterior probability of taxpayer compliance, θ, given the observed data, X. $P(X|\theta)$ represents the likelihood of observing the data given a specific compliance behavior. $P(\theta)$ is the government's prior belief about compliance levels before any data is observed, while $P(X)$ is the marginal likelihood that ensures the distribution is properly normalized. By using historical or sector-specific data as the observed evidence X, the government can iteratively refine its prior beliefs $P(\theta)$ and develop a more accurate auditing strategy tailored to specific taxpayer profiles.

In this paper, we shall assume that the government's belief about the taxpayer's compliance rate, s, is modeled by a truncated normal distribution. The use of a truncated normal distribution is motivated by practical and theoretical considerations. The normal distribution is widely used in mathematics, economics, and behavioral sciences for modeling uncertain quantities, as it provides a reasonable approximation for many real-world phenomena. The truncated normal distribution accounts for the fact that the compliance rate, s, is constrained between the interval [0,1].

The probability density function (PDF) of this normal distribution is given by:

$$f(s) = \frac{\phi(s; \mu, \sigma^2)}{\Phi(1; \mu, \sigma^2) - \Phi(0; \mu, \sigma^2)}, \quad s \in [0,1]$$

Where $\phi(s; \mu, \sigma^2)$ is the PDF of the normal distribution with mean μ and variance σ^2, defined as:

$$\phi(s; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(s-\mu)^2}{2\sigma^2}\right)$$

and $\Phi(s; \mu, \sigma^2)$ is the cumulative distribution function (CDF) of the normal distribution, given by:

$$\Phi(s; \mu, \sigma^2) = \int_{-\infty}^{s} \phi(x; \mu, \sigma^2) dx$$

This truncated normal distribution reflects the belief about s, where the value of s lies between 0 and 1, capturing both partial and full compliance levels.

Intuitively speaking, based on the idea of Bayesian updating of belief structures, if the government assumes a higher mean compliance rate, μ, the government may audit less frequently, as the government assumes that taxpayers are mostly compliant. Additionally, if the government assumes a high variance across taxpayers, σ^2, the government may adopt a more targeted auditing strategy based on highly evasive groups.

Below, in Table 1, illustrate the payoff matrix between households and the government based on their strategies and types.

Table 1

Payoff matrix

Household	Government	
	Audit (A)	No Audit ($\neg A$)
Evade	$(1-s\tau_d)W-(1-s)PW, s\tau_d W+(1-s)PW-C$	$(1-s\tau_d)W, s\tau_d W$
Comply	$(1-\tau_d)W, \tau_d W-C$	$(1-\tau_d)W, \tau_d W$

From Table 1, each cell shows the payoff of the government and the household for each action that they choose. The household's payoff is on the left and the government payoff is on the right. The direct tax rate is denoted by τ_d, where $\tau_d \in [0,1]$. The penalty rate for tax evasion is denoted by P, $P > \tau_d$ to disincentivize tax evasion. The wage of the household is denoted as W, and the auditing cost for the government is C, where $C > 0$.

If the taxpayer is of the type to evade and chooses a compliance rate of $s \in [0,1]$ and the government decides to audit, the taxpayer's payoff comes from his post-tax wages after noncompliance, $(1 - s\tau_d)W$, subtracted by their penalty rate incurred due to non-compliance, (1 − s)PW, that is $(1 - s\tau_d)W - (1 - s)PW$. On the other hand, the government's payoff comes from the compliance to the tax rate, $s\tau_d W$, added by the penalty received due to catching households evading, $(1 - s)PW$, subtracted by the cost of auditing, C. Therefore, the taxpayer's and government's payoff for evading and auditing is $(1 - \mathrm{s}\tau_\mathrm{d})\mathrm{W} - (1 - \mathrm{s})\mathrm{PW}$ and $\mathrm{s}\tau_\mathrm{d}\mathrm{W} + (1 - \mathrm{s})\mathrm{PW} - \mathrm{C}$, respectively.

Table 1 describes the normal form of our Bayesian game with incomplete information. The payoffs, U_1 and U_2 for the government and the household, are given in the table above. In each of the cells the payoff U_1 is to the right and the payoff U_2 is to the left. For example, given that the household is of the complying type and the government decides to audit, we have $\mathrm{U}_1(\mathrm{S}_1(\mathrm{C}), \mathrm{A}) = \tau_\mathrm{d}\mathrm{W} - \mathrm{C}$ and $\mathrm{U}_2(\mathrm{S}_1(\mathrm{C}), \mathrm{A}) = (1 - \tau_\mathrm{d})\mathrm{W}$.

To conclude, our direct tax Bayesian game is the collection $J = \{N, T_1, T_2, \pi, S_1, S_2, U_1, U_2\}$. $N = \{1,2\}$ denotes the set of players, where player 1 represents the government, and player 2 represents the household/taxpayer. T_i is the set of types of players $i \in N$, where $T_2 = \{E, C\}$, where $t_2 = E$ if the taxpayer is an evader and $t_2 = C$ if the taxpayer is fully compliant. $T_1 = \{G\}$ is the set of types of government which is a singleton. π_i is the probability distribution over the types of players $i \in N$, where $\pi_1 = 1$ and $\pi_2(t_2 = E) = p,\ \pi_2(t_2 = C) = 1 - p$. S_i is the set of actions in $i \in N$. For player 1, the government's set of actions, $S_1 = \{A, \neg A\}$, represent whether to audit or not. For player 2, the household, the set of actions depends on their type:

$$S_2(t_2) = \begin{cases} s, & if\ t_2 = E, where\ s \in [0,1); \\ 1, & if\ t_2 = C. \end{cases}$$

Finally, U_i is the payoff functions for player $i \in N$.

5.2 Mixed Strategy Bayesian Nash Equilibrium

In a game of incomplete information, players account for their private types and the strategies of other players to maximize their own payoff. A Bayesian Nash equilibrium (BNE) in a mixed strategy situation specifies a strategy for each type of player such that no player has an incentive to deviate unilaterally, given their beliefs and the strategy of other players. The BNE of this game requires that the government choose a strategy that maximizes its expected payoff given their beliefs about the household's strategies and types.

The best response condition for a BNE ensures that each player's strategy maximizes their expected utility, given their own type, the strategy of other players, and the beliefs about the other player types. For a player i with type t_i, their strategy $s_i^*(t_i)$ satisfies the condition:

$$\mathrm{s_i^*(t_i)} \in \arg\max_{\mathrm{s_i \in S_i}} \mathrm{E_{t_{-i} \sim \pi_{-i}}}\left[\mathrm{U_i(s_i, s_{-i}^*(t_{-i}); t_i, t_{-i})}\right]$$

This expression states that for any player i with type t_i, their equilibrium strategy, $s_i^*(t_i)$, is chosen from the set of all possible strategies, S_i, such that it maximizes its expected payoff based on the strategies of other players and their types, $U_i(s_i, s_{-i}^*(t_{-i}); t_i, t_{-i})$. The expected payoff, denoted as $E_{t_{-i} \sim \pi_{-i}}[\cdot]$, is determined by the types of the other players t_{-i} based on player i's belief of the distribution of those types, π_{-i}.

To determine the government's Bayesian Nash equilibrium in a mixed strategy setting, we first aim to solve for the value of p, which represents the probability that the household is of the type to evade, then formalize a relationship between the value of p and the optimal mixed strategy. Thus, our goal is to analyze the variables that influence the optimal mixed strategy of the government, which determines the government's indifference between its available strategies of A and $\neg A$. This condition ensures the government's payoff is equal for both of its actions, thereby yielding a mixed strategy Bayesian Nash equilibrium.

Let the government choose a mixed strategy μ_1, which is a probability distribution over its actions to audit or to not audit. This probability distribution is denoted as $\mu_1(A)$ and $\mu_1(\neg A)$, where $\mu_1(A)$ is the probability that the government chooses to audit, $\mu_1(\neg A) = 1 - \mu_1(A)$.

When the government decides to audit or not its payoff is determined by how compliant the taxpayer is with the tax code. As mentioned above in section 5.1 the government forms a belief about the taxpayer compliance rate, s, to be a truncated normal distribution between 0 and 1. Given this belief, the general form for the expected payoff for the government auditing, denoted as $E_{t_2 \sim \pi_2}[U_1(\mathrm{A})]$, would be:

$$E_{t_2 \sim \pi_2}[U_1(A)] = \sum_{t_2} \pi_2(t_2) \cdot U_1(A \mid t_2)$$

Here, the expected payoff for the government when choosing to audit is the weighted sum of the payoffs associated with each possible taxpayer type t_2, where each payoff is weighted by the probability $\pi_2(t_2)$ of observing that specific taxpayer type. We can write this sum in expanded form as:

$$E_{t_2 \sim \pi_2}[U_1(A)] = pU_1(A \mid t_2 = E) + (1-p)U_1(A \mid t_2 = C)$$

The payoffs are, $U_1(A \mid t_2 = E) = s\tau_d W + (1-s)PW - C$ and $U_1(A \mid t_2 = C) = s\tau_d W$, from Table 1. Since s is a random variable and its distribution is described by the PDF $f(s)$, from Section 5.1, the expected payoff for the government decision to audit must account for the distribution of s. This is done by integrating over the possible values of s from 0 to 1, weighted by $f(s)$. Thus, the expected payoff for $E_{t_{-2} \sim \pi_{-2}}[U_1(A)]$ is:

$$E_{t_{-2} \sim \pi_{-2}}[U_1(A)] = p \int_0^1 [(1-s)PW + s\tau_d W - C] f(s)\, ds + (1-p)[\tau_d W - C]$$

Using the same logic the expected payoff for not auditing, $E_{t_{-2} \sim \pi_{-2}}[U_1(\neg A)]$, is:

$$E_{t_{-2} \sim \pi_{-2}}[U_1(\neg A)] = p \int_0^1 [s\tau_d W] f(s)\ ds + (1-p)[\tau_d W]$$

In order to solve for the mixed strategy Bayesian Nash equilibrium, we first solve for the value of p by setting the two expected payoffs equal. To simply computation we let $\mathrm{E(s)} = \int_0^1 \mathrm{s\,f(s)\,ds}$.

$$E_{t_{-2} \sim \pi_{-2}}[U_1(A)] = E_{t_{-2} \sim \pi_{-2}}[U_1(\neg A)]$$

$$p\big[\big(1-E(s)\big)PW+E(s)\tau_d W-C\big]+(1-p)[\tau_d W-C]=p[E(s)\tau_d \mathrm{W}]+(1-p)[\tau_d W]$$

Solving for p we get:

$$p=\frac{C}{\big(1-E(s)\big)PW}$$

This equation represents the threshold probability p at which the government is indifferent between auditing and not auditing. If the perceived belief that a household is of type E exceeds the value of p the government will prefer to audit as the expected payoff of auditing is greater than not auditing, and vice versa. In other words, for the government to choose auditing, the perceived likelihood of a household being of the type to evade must be greater than p.

Assuming this likelihood is chosen randomly, the probability of auditing therefore be $1-p$. Given this assumption, the mixed strategy probability of the government choosing to audit is:

$$\mu_1(A)=1-p=1-\frac{C}{\big(1-E(s)\big)PW}$$

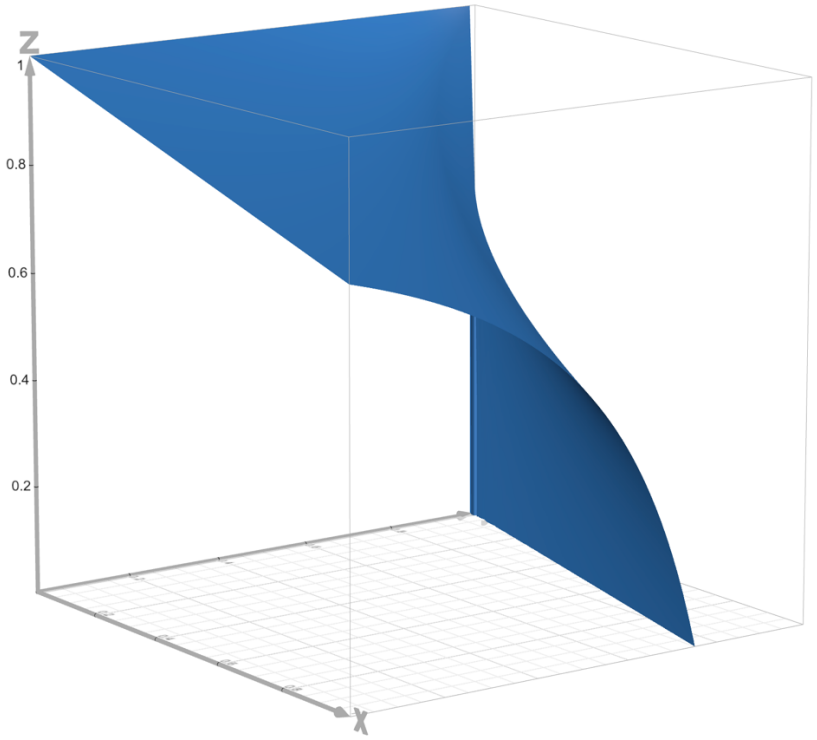
Thus, the best response for the government in a mixed strategy Bayesian Nash equilibrium is to audit with probability $\mu_1(A)$, where $\mu_1(A)$ is determined from the equation above. In the optimal mixed strategy, if the cost of auditing increases, C, the auditing probability will decrease, and if the penalty rate, P, increases the auditing probability will increase. The variation of the auditing probability aligns with our basic intuition, as if the cost of auditing increases the government will be less inclined to audit frequently as to save on costs, additionally, the increase in the penalty rate will cause taxpayers to be more compliant leading to a decrease in auditing probability.

In the context of the Bayesian Nash equilibrium, the government's mixed strategy ensures that there is no incentive for unilateral deviation from the chosen strategy. Specifically, by choosing $\mu_1(A)$ for auditing the government is indifferent between his actions of auditing and not auditing, since the payoff of these are equal.

5.3 Equilibrium Analysis

Figure 3

Graph illustrating the mixed strategy equilibrium.



Note: Showing the relationship between the probability of auditing $\mu_1(A)$, z-axis; the cost of auditing (C), x-axis; and expected value of s (E(s)), y-axis.

Lemma 1. *An increase in the cost of auditing, C, will decrease the probability of auditing,* $\mu_1(A)$.

Proof: From above the expression for $\mu_1(A)$ is:

$$\mu_1(A) = 1 - \frac{C}{\big(1 - E(s)\big)PW}$$

Differentiating $\mu_1(A)$ with respect to C:

$$\frac{\partial \mu_1(\mathrm{A})}{\partial \mathrm{C}} = -\frac{\partial}{\partial \mathrm{C}}\left(\frac{\mathrm{C}}{\big(1 - \mathrm{E(s)}\big)\mathrm{PW}}\right)$$

$$\frac{\partial \mu_1(A)}{\partial C} = -\frac{1}{\big(1 - E(s)\big)PW}$$

Since $\big(1 - E(s)\big)PW > 0, \frac{\partial \mu_1(A)}{\partial C} < 0$. Thus, we have proven that an increase in the cost of auditing leads to a decrease in the probability of auditing. This result aligns with economic intuition. ■

Proposition 1. *Higher auditing costs lead to a decrease in the direct tax rate.*

Proof: To begin we will find the mixed strategy Bayesian Nash equilibrium for the household. The household's action revolves around their decision to comply with taxes ($t_2 = C$) or to evade taxes ($t_2 = E$). In a mixed strategy, the household will randomize its strategy based on the expected payoffs for evading and complying.

The expected payoff for compliance is:

$$E[U_2(C)] = (1 - \tau_d)W$$

The expected payoff for evasion depends on whether the household is audited or not, which is the percent $\mu_1(A)$. Thus the expected payoff for evasion is:

$$E[U_2(E)] = \mu_1(A)[(1 - s\tau_d)W - (1 - s)PW] + \big(1 - \mu_1(A)\big)[(1 - s\tau_d)W]$$

Setting both expected payoffs equal we get:

$$E[U_2(C)] = E[U_2(E)]$$

$$(1 - \tau_d)W = \mu_1(A)[(1 - s\tau_d)W - (1 - s)PW] + \big(1 - \mu_1(A)\big)[(1 - s\tau_d)W]$$

Solving for $\mu_1(A)$, we get:

$$\mu_1(A) = \frac{\tau_d}{P}$$

From above $\mu_1(A) = 1 - \frac{C}{(1-E(s))PW}$. Setting both of these expressions equal and differentiating τ_d with respect to C, we get:

$$\frac{\tau_d}{P} = 1 - \frac{C}{\big(1 - E(s)\big)PW}$$

$$\frac{1}{P}\left(\frac{\partial \tau_d}{\partial \mathrm{C}}\right) = -\frac{\partial}{\partial C}\left(\frac{C}{\big(1 - E(s)\big)PW}\right)$$

$$\frac{\partial \tau_\mathrm{d}}{\partial \mathrm{C}} = -\frac{1}{\big(1 - \mathrm{E(s)}\big)\mathrm{W}}$$

Thus, since $\frac{\partial \tau_d}{\partial C} < 0$ we show that as the cost of auditing increases the direct tax rate decreases to maintain equilibrium. ■

Proposition 2. *Higher disposable income is realized by households when the government has a high cost of auditing.*

Proof: Building on Lemma 1, where we established that an increase in auditing costs reduces the probability of auditing, and Proposition 1, which demonstrates that higher auditing costs lead to a decrease in the tax rate, we now proceed to analyze how auditing costs influence the expected value of the household.

From the mixed strategy Bayesian Nash equilibrium of the household:

$$E[U_2(E)] + E[U_2(C)] = \mu_1(A)[(1 - s\tau_d)W - (1 - s)PW] + \big(1 - \mu_1(A)\big)[(1 - s\tau_d)W] + (1 - t_d)W$$

For ease of notation, we set $E[U_2] = E[U_2(E)] + E[U_2(C)]$. Using the chain rule and differentiating $E[U_2]$ in terms of the auditing cost C we get:

$$\frac{\partial E[U_2]}{\partial C} = \frac{\partial \mu_1(A)}{\partial C}[(1 - s\tau_d)W - (1 - s)PW] + \mu_1(A)\frac{\partial \tau_d}{\partial C}(-sW) - \frac{\partial \mu_1(A)}{\partial C}[(1 - s\tau_d)W] + \big(1 - \mu_1(A)\big)\frac{\partial \tau_d}{\partial C}(-sW) - \frac{\partial \tau_d}{\partial C}(W)$$

Recalling that $\frac{\partial \tau_d}{\partial C} = -\frac{1}{(1-E(s))W}$ and $\frac{\partial \mu_1(A)}{\partial C} = -\frac{1}{(1-E(s))PW}$, we plug in these functions and simplifying:

$$\frac{\partial E[U_2]}{\partial C} = -\frac{(1 - s\tau_d)}{(1 - E(s))P} + \frac{(1 - s)}{(1 - E(s))} + \frac{\mu_1(A)s}{(1 - E(s))} + \frac{(1 - s\tau_d)}{(1 - E(s))P} + \frac{s}{(1 - E(s))} - \frac{\mu_1(A)s}{(1 - E(s))} + \frac{1}{(1 - E(s))}$$

Canceling like terms we arrive at our result:

$$\frac{\partial E[U_2]}{\partial C} = \frac{2}{1 - E(s)}$$

Since $1 - E(s) > 0$, as $E(s)$ ranges from 0 and 1, we conclude that $\frac{\partial E[U_2]}{\partial C} > 0$. This result implies that as the cost of auditing increases, the expected utility of the household increases. In other words, higher auditing costs lead to higher expected payoffs, which in turn leads to a higher disposable income for households. ■

6. Model 2: Indirect Taxes, Market Dynamics, and Laffer-Type Model

In this section, we introduce a Laffer curve-based market dynamics framework that focuses on the interactions between indirect taxes, market dynamics, and the implications for government revenue. Specifically, we examine how ad valorem taxes (a type of indirect tax levied as a percentage of the price of a good or service) affect the behavior of the government, firms, and consumers.

A central element of this model is household disposable income, which plays a pivotal role in connecting this framework to the Bayesian game from Section 5**.** In Section 5.3 proposition 2 we analyzed how the cost of auditing influenced the disposable income of households. In this Laffer-type model, we illustrate how the effects of the Bayesian game can carry over into the market, ultimately affecting market dynamics and the implementation of indirect taxes. This link between direct and indirect taxation allows us to answer the question of why countries differ in their reliance on these taxes.

Section 6.1 introduces the key assumptions and definitions underlying our Laffer-type model; Section 6.2 formulates the Laffer-type model; and, Section 6.3 analyzes the results and establishes propositions.

6.1 Assumptions and definitions of Laffer-type model

In this section, we lay the foundations of the model by establishing key assumptions and definitions that guide its structure.

Firstly, we consider a competitive market where firms produce goods and services subject to an ad valorem tax. The participants in this market include households, who gain utility from using the goods and services; Firms, which aim to maximize profits by producing goods and setting prices; the government, which seeks to optimize revenue and balance economic efficiency through indirect taxes.

In many different countries, governments use many different types of indirect taxes to generate revenue, such as value added tax (VAT), sales tax, excise tax, etc. In this paper, we primarily focus on VAT and sales tax, as both are ad valorem taxes that play a significant role in everyday transactions, particularly in global markets. Ad valorem taxes are defined as a percentage of the price of goods and services. Mathematically the price paid by a consumer of a good subject to an ad valorem tax can be

represented as $p_c = p(1 + \tau_i)$, where p_c represents the price paid by consumers, p denotes the pre-tax price, and τ_i denotes the indirect ad valorem tax rate. For firms $p_f = p$, in other words, the pre-tax price is the same price firms receive from selling a good subjected to an ad valorem tax. It is important to note that we assume the tax burden primarily falls on the consumer, meaning that the consumer effectively bears most of the cost of the tax through higher prices, even though the firm collects the tax on behalf of the government.

For the government, it is important to recognize that its primary goal is to maximize the revenue generated by indirect taxes while balancing economic efficiency and welfare. In a market where ad valorem taxes are imposed, a general mathematical representation of government revenue is:

$$R = \tau_i \cdot p \cdot q(p_c)$$

Where τ_i is the ad valorem tax rate, p is the price of the good, and $q(p_c)$ is the quantity demanded as a function of the consumer's cost. This representation of revenue generated by indirect taxes is distinct in comparison to the case in Figure 2, as the indirect tax in that market is a per unit tax which has a revenue function of: $R = \tau_i * q$, where τ_i is a fixed tax per unit of the good and q is just the quantity sold.

The economic efficiency impacted by the tax rate can be expressed as the deadweight loss caused by the indirect tax. This is because the addition of an indirect tax on the market drives a wedge between the price paid by consumers p_c and the price received by firms, p_f. This results in economic inefficiency, referred to as deadweight loss, which represents the value of transactions that no longer occur due to the imposition of the tax. The deadweight loss can be represented as the area of the triangle between the supply and demand curves at the post-tax equilibrium quantity. In Figure 2 the deadweight loss is represented as triangle bcd.

$$DWL = \frac{1}{2}(\tau_i p)(q_e - q_i)$$

In this expression, $\tau_i p$ represents the tax wedge which can be rewritten as $p_c - p_f$, the price different between consumers and producers. $q_e - q_i$ represents the reduction in quantity from the equilibrium quantity (without tax), q_e, to the quantity traded when the indirect tax is imposed, q_i.

Additionally, we assume that the indirect taxes will primarily affect the demand curve in the market. As the tax increases it raises the price consumers must pay for the good, which in turn, leads to a reduction in quantity demanded, by the law of demand. For simplicity, we model the quantity demanded function as:

$$q(p, \tau_i, W) = a + \beta W - \varepsilon p(1 + \tau_i)$$

Where a is the base demand constant, β is the wage multiplier (which indicates the extent to which an increase in wage influences consumer demand), $\beta > 0$ as we assume the good is a normal good, W denotes the disposable income of consumers, ε denotes the elasticity of demand (showing how sensitive demand is to price changes), p is the price, and τ_i is the indirect tax rate. Note that the equilibrium quantity without tax is $q(p, 0, W)$.

The goal for firms is to maximize their profits, considering their production cost, the effect of indirect taxes on the demand of the good, and the price of the good. We denote the profit function for the firms as:

$$\pi(p) = (p - c)\big(q(p, \tau_i, W)\big)$$

Where p is the price set by the firm, c is the constant cost per unit of production, $q(p, \tau_i, W)$ is the quantity demanded function for consumers. For simplicity, we use a constant per-unit cost of production, however, for more complex models a concave down cost function could be employed to reflect economies of scale.

6.2 Laffer-type model

Firstly, in order to generate a Laffer-type model we must find the market equilibrium price and quantity after the indirect tax has been imposed.

Lemma 2. *The profit maximizing price of the firm is the market equilibrium price of a perfectly competitive market.*

Proof: At market equilibrium conditions in a perfectly competitive market, the marginal revenue is equal to the marginal cost, $MR = MC$, as the producer has no incentive to produce more nor gain any benefit by producing less.

A general function for a firm's total revenue is given by: $R(q) = q * P(q)$, where $P(q)$ is the price, the firm can charge at quantity q. Therefore, the profit function of a general firm is: $\pi(q) = R(q) - C(q)$, where $C(q)$ is the cost of producing q units.

To maximize profits, we take the first derivative of $\pi(q)$ with respect to q and set it equal to zero:

$$\frac{d\pi(q)}{dq} = \frac{dR(q)}{dq} - \frac{dC(q)}{dq} = 0$$

$$\frac{dR(q)}{dq} = \frac{dC(q)}{dq}$$

$$MR = MC$$

Thus, for profits to be maximized, $MR = MC$. Notice that if we substituted in $R(q) = q * P(q)$ we would obtain $MR = P(q) + q\frac{dP(q)}{dq} = \frac{dC(q)}{dq}$.

In a perfectly competitive market, the firm operates as a price taker meaning $P(q)$ is a constant, thus $\frac{dP(q)}{dq} = 0$, simplifying the condition to $P(q) = MR = MC$. ■

Utilizing the framework outlined in Lemma 2, we find the price of the market equilibrium by first finding the profit maximizing price for the firm. Recall the profit function of the firm is $\pi(p) = (p - c)\big(q(p, \tau_i, W)\big)$ and the quantity demanded function $q(p, \tau_i, W) = a + \beta W - \varepsilon p(1 + \tau_i)$. Substituting the functions together:

$$\pi(p) = (p - c)\big(a + \beta W - \varepsilon p(1 + \tau_i)\big)$$

To find the profit maximizing price, p, we take the FOC of $\pi(p)$ with respect to p, set it equal to 0, and solve for p:

$$\frac{d\pi(p)}{dp} = \big(a + \beta W - \varepsilon p(1 + \tau_i)\big) + (p - c)\big(-\varepsilon(1 + \tau_i)\big) = 0$$

$$0 = a + \beta W - \varepsilon p(1 + \tau_i) - \varepsilon p(1 + \tau_i) + c\varepsilon(1 + \tau_i)$$

$$p^* = \frac{a + \beta W + c\varepsilon(1 + \tau_i)}{2\varepsilon(1 + \tau_i)}$$

From Lemma 2, p^* is the profit maximizing and equilibrium price of the perfect competitive market in our model.

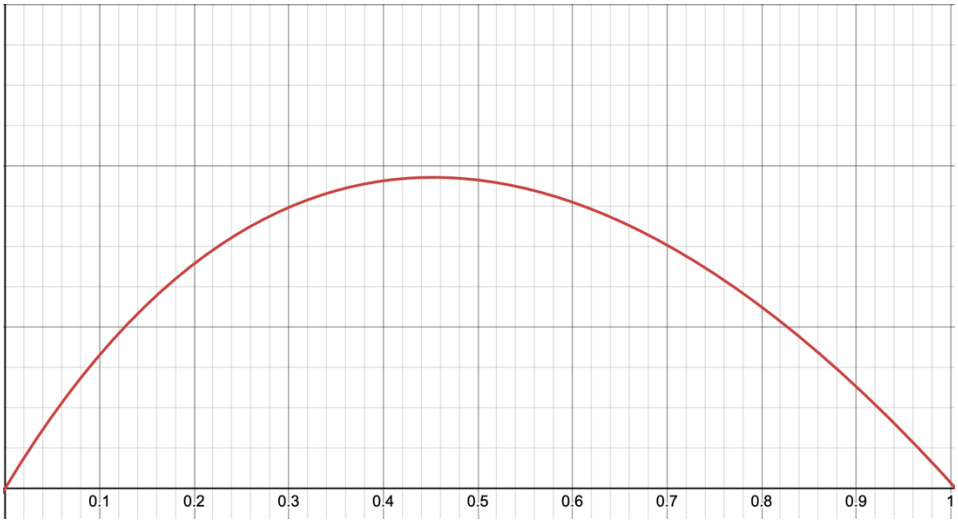
Recall that government revenue generated from indirect tax is modeled by: $R = \tau_i \cdot \mathrm{p} \cdot q(p, \tau_i, W)$. Therefore, at the market equilibrium with the indirect tax imposed, the government revenue is:

$$R = \tau_i \cdot \mathrm{p}^* \cdot q(p^*, \tau_i, W)$$

$$\mathrm{R} = \tau_i p^* \big(a + \beta W - \varepsilon p^* (1 + \tau_i)\big)$$

Figure 4

Graph of government revenue function at market equilibrium under the imposition of an indirect tax



Note: indirect tax rate, τ_i, is the x-axis, government revenue, R, is the y-axis

Figure 4 illustrates the government revenue function, which closely resembles the Laffer curve from Figure 1. It is evident that there exists an optimal tax rate that maximizes government revenue. Below this optimal rate, the revenue function slopes upward, while it slopes downward for tax rates above the optimal level.

To find the optimal indirect tax rate τ_i^*, we take the FOC of R with respect to τ_i set it equal to zero and solve for τ_i^*:

$$R = \tau_i p^*\left(a + \beta W - \varepsilon p^*(1+\tau_i)\right)$$

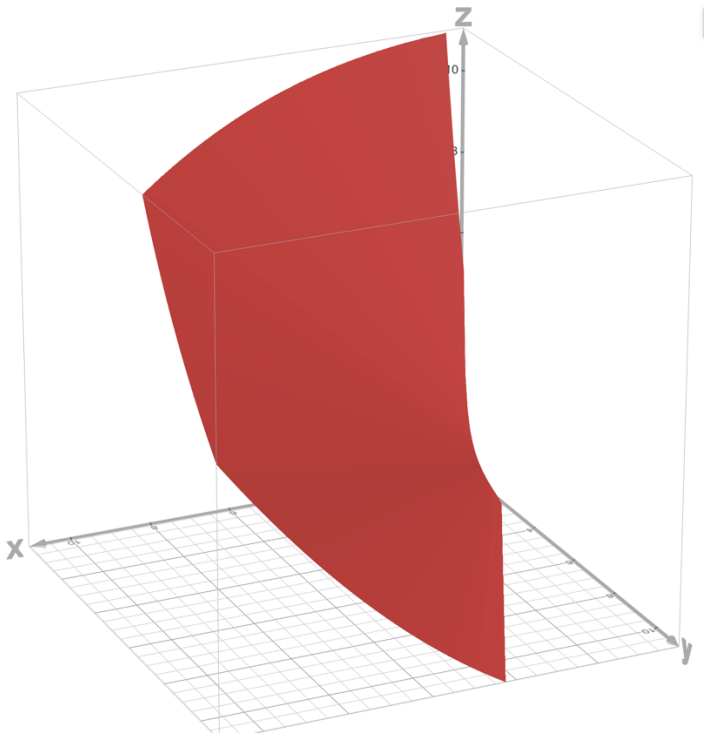
After plugging $p^* = \frac{a+\beta W+c\varepsilon(1+\tau_i)}{2\varepsilon(1+\tau_i)}$ into the expression above and doing the chain rule and a lot of algebra, τ_i^* turns out to be:

$$\tau_i^* = \frac{(W\beta + a)^2}{4\varepsilon(x+1)^2} - \frac{c^2\varepsilon(2x+1)}{4}$$

6.3 Analysis of Laffer-type model

Figure 5

Graph showing τ_i^* in terms of wage, W (x-axis), and elasticity, ε (y-axis).



Proposition 3. As the consumer's disposable income, W, increases, the optimal indirect tax rate, τ_i^*, increases.

Proof: Taking a partial derivative of τ_i^* with respect to W, we get:

$$\frac{\partial \tau_i^*}{\partial W} = \frac{\beta(W\beta + a)}{2\varepsilon(x+1)^2}$$

Since $\frac{\partial \tau_i^*}{\partial W} > 0$, τ_i^* will increase as the consumer's disposable income increases. ■

Proposition 4. As the elasticity of demand, ε, increases, the optimal tax rate, τ_i^*, decreases.

Proof: Taking a partial derivative of τ_i^* with respect to ε, we get:

$$\frac{\partial \tau_i^*}{\partial \varepsilon} = \frac{\beta(W\beta + a)}{4\varepsilon^2(x+1)^2} - \frac{c^2(2x+1)}{4}$$

Since $\frac{\partial \tau_i^*}{\partial \varepsilon} < 0$, τ_i^* will decrease as the elasticity of demand increases. ■

Proposition 4 is very intuitive, as if consumers are more sensitive to price changes in markets, where demand is more elastic, a higher tax rate would lead to a significant reduction in quantity demanded, reducing government revenue. To avoid this, the government would aim for a lower indirect tax rate to balance the trade-off between tax revenue and the loss in quantity demanded, which is caused by consumers' sensitivity to the price increases driven by the indirect tax.

7. Linking the Bayesian Game Theoretic Model with the Laffer-type Model

From Section 5, we developed a Bayesian Game of incomplete information to model the interactions between the government and the household. In this game, we focused on the compliance behavior and optimal auditing strategy by the government to maximize taxation revenue. In Proposition 2 we proved that with an increase in the cost of auditing the household will realize an increase in disposable income.

In Section 6, we modeled the quantity demanded function, $q(p, \tau_i, W)$, as a function of the price of the good, the tax rate, and the disposable income of the consumer. This section will integrate the Bayesian game model with the Laffer-type framework to analyze how the government's tax structure choices are influenced by various variables. We will explore how changes in key factors, such as auditing costs, consumer behavior, and government strategies, impact the optimal tax rates for direct and indirect taxes and overall tax structure employed by the government.

In Proposition 2, we found the result that an increase in the auditing costs for the government will increase the expected utility of households:

$$\frac{\partial E[U_2]}{\partial C} = \frac{2}{1 - E(s)}$$

Given this relationship, we can assume that as compliance becomes more costly for the government, the auditing probability and enforcement effort for governments becomes weaker, leading to higher levels of income remaining in households. Thus, resulting in more disposable income. Moreover, it is logical to assume that as households' disposable income increases, they will allocate a larger share of this income to spend in the economic markets, thereby stimulating more demand in the economy. This increased demand in the economy is captured by:

$$\frac{\partial q(p, \tau_i, W)}{\partial W} = \frac{\partial}{\partial W}\left(a + \beta W - \varepsilon p(1 + \tau_i)\right)$$

$$\frac{\partial q(p, \tau_i, W)}{\partial W} = \beta > 0 \ (since\ \beta \text{ reflects wage sensitivity } for\ normal\ goods)$$

Thus, an increase in government auditing costs will lead to higher disposable income in households, which in turn will increase demand in the market.

Next, we establish a broader connection between the government's auditing strategies and their economic implications, focusing on how the Bayesian game influences market dynamics and the indirect tax rate. Firstly, we define the disposable wage, W, as equal to the expected utility of the household, $E[U_2]$, as the post tax-income translates directly into disposable income spent in the economy.

$$\frac{\partial q(p, \tau_i, W)}{\partial C} = \frac{\partial q(p, \tau_i, E[U_2])}{\partial E[U_2]} \cdot \frac{\partial E[U_2]}{\partial C}$$

$$\frac{\partial q(p, \tau_i, W)}{\partial C} = \frac{2\beta}{1 - E(s)}$$

Next, in Proposition 3 we proved that an increase in the disposable income of consumers increases the indirect tax rate.

$$\frac{\partial \tau_i^*}{\partial W} = \frac{\beta(W\beta + a)}{2\varepsilon(x + 1)^2}$$

As stated above, since disposable income is equal to the expected utility of household, the disposable income is correlated to the auditing costs of the government. Thus, the impact of auditing cost on indirect taxes is:

$$\frac{\partial \tau_i^*}{\partial C} = \frac{\partial \tau_i^*}{\partial E[U_2]} \cdot \frac{\partial E[U_2]}{\partial C} = \frac{\beta(W\beta + a)}{\varepsilon(x+1)^2\big(1 - E(s)\big)} > 0$$

Thus, the tradeoff between indirect taxes and direct taxes depends on the auditing costs of the government. In proposition 1 and the expression above:

$$\frac{\partial \tau_d}{\partial C} = -\frac{1}{\big(1 - E(s)\big)W} < 0$$

$$\frac{\partial \tau_i^*}{\partial C} = \frac{\beta(W\beta + a)}{\varepsilon(x+1)^2\big(1 - E(s)\big)} > 0$$

In conclusion, we demonstrate a clear relationship between the government's auditing costs, household disposable income, market demand, and indirect and direct tax rates. Higher auditing costs weaken enforcement efforts, lowering the direct tax rate, and increasing household disposable income, which drives greater market demand and necessitates adjustments in the indirect tax rate to optimize government revenue. This integrated approach provides a framework for understanding why different countries adopt varying tax structures, as differences in auditing costs, consumer behavior, and enforcement capabilities shape the balance between direct and indirect taxation in distinct economic contexts.

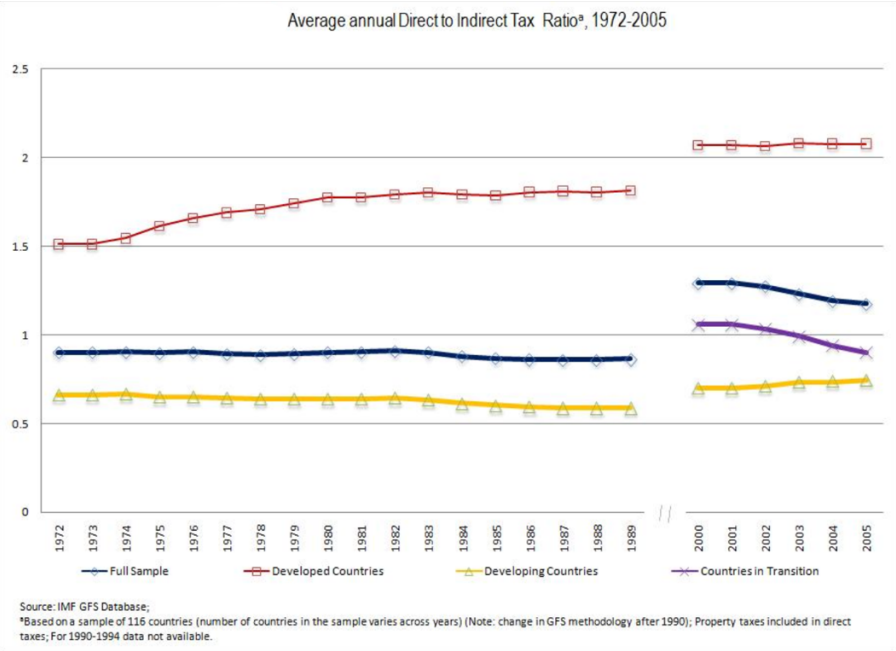
8. Supporting Evidence: Cross-Country Variations in Auditing Costs and Tax Structures

In this section, we utilize empirical evidence to support our theoretical models, focusing on the relationships between tax auditing costs and the structure of indirect and direct tax rates. By examining cross country variations in tax administration and tax structure, we investigate how factors such as administrative capacity, informal economies, and technological capabilities influence the government's tax auditing costs. Using our theoretical framework in both the Bayesian game and the market dynamics Laffer model, we show that countries with a higher auditing cost are more likely to adopt tax structure that rely on indirect taxes, while those with lower auditing costs are better positioned to leverage direct tax systems.

Firstly, there are numerous factors that contribute to higher tax auditing costs in underdeveloped countries. Systemic corruption in underdeveloped countries often compromises information technology systems meant to improve tax administration. Additionally, the prevalence of informal economies and lack of IT infrastructure among small and medium-sized enterprises in underdeveloped countries hinder effective e-tax administration (Umar & Mas'ud, 2020). In developing countries, tax audits are a three-dimensional process involving audit probability, detection, and sanction, all of which must work together for effectiveness. However, corruption can compromise each stage, rendering the entire process ineffective leading to a higher cost to ensure compliance to the tax code (Umar et al., 2019). Additionally, inefficient tax administration, large informal sectors, and ineffectiveness of governments lead to higher auditing costs in developing countries (Muzurura et al., 2021). These factors collectively contribute to disproportionately higher tax auditing costs in underdeveloped countries compared to developed economies. Connecting these findings to our model, we can expect the auditing cost, C, for the governments in underdeveloped countries to be higher than in those of developed countries, leading to an emphasis in indirect tax collection.

Figure 6

Average Annual Direct to Indirect Tax Ratio. Adapted from Martinez-Vazquez, Vulovic, and Liu (2009)

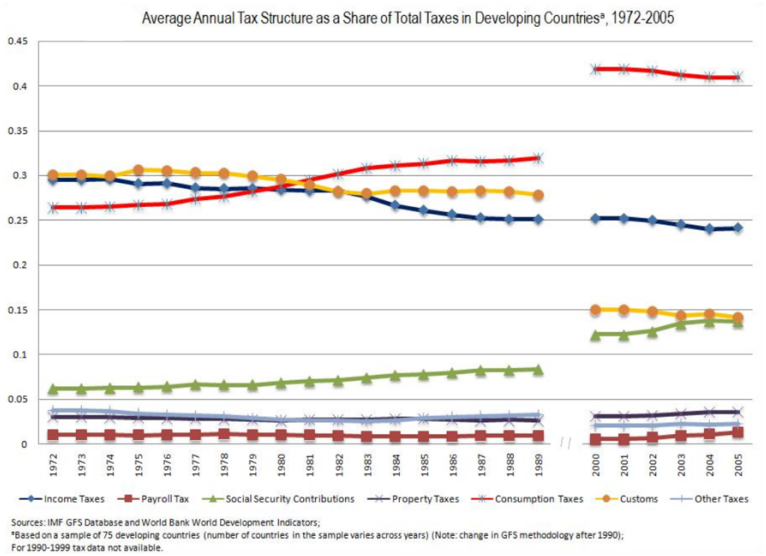


Note: Data unavailable for 1990-1994 due to change in GFS methodology.

In Figure 6 we can clearly see a difference in the ratio of direct taxes and indirect taxes between developed countries and developing/underdeveloped countries. To better understand what's influencing the ratio of direct to indirect tax, figures 7 and 8 show the components of the direct and indirect tax for developed and underdeveloped countries.

Figure 7

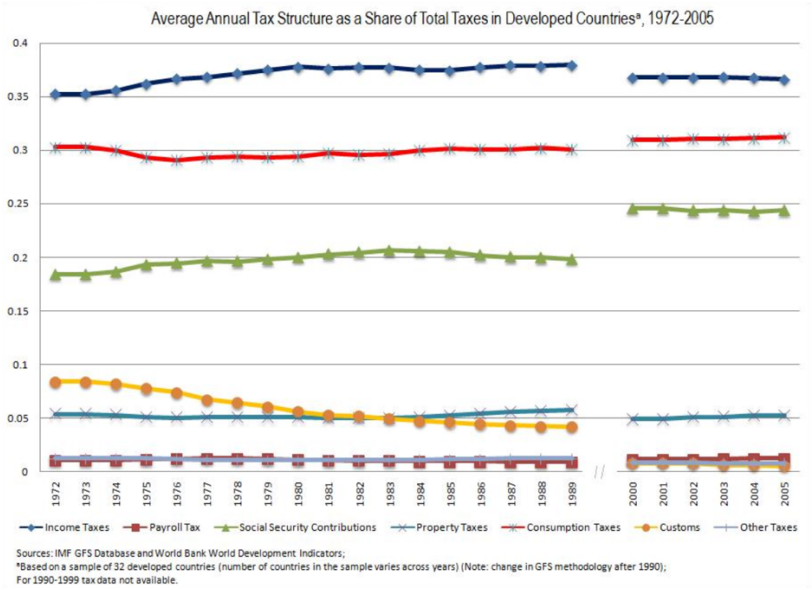
Average annual tax structure as a share of total taxes in developing countries. Adapted from Adapted from Martinez-Vazquez, Vulovic, and Liu (2009)



Note: Data unavailable for 1990-1994 due to change in GFS methodology.

Figure 8

Average Annual Tax structure as a share of total taxes in developed countries. Adapted from Adapted from Martinez-Vazquez, Vulovic, and Liu (2009)



Note: Data unavailable for 1990-1994 due to change in GFS methodology.

In Figure 6, the direct tax to indirect tax ratio for developed countries approaches 2, while the ratio for underdeveloped countries hovers around 0.7. Looking closely at the makeup of the total tax revenue generated by developed and underdeveloped countries in Figures 7 and 8, we can see that from

the period 2000 onwards developing countries generated 43% of their total tax revenue based on consumption taxes, which constitute indirect taxes, while income taxes generated about 25% of the total tax revenue. On the other hand, developed countries generate about 38% of their total tax revenue from income taxes while consumption taxes generate 31% of the total tax revenue.

The evidence presented in Figures 6,7, and 8 provide strong empirical support for our theoretical models, which demonstrated that the optimal mix of direct and indirect taxes is driven by the governments cost of tax auditing.

In developing countries, where auditing costs are higher due to weaker administrative capacities, large informal sector, corruption, and more, reliance of indirect taxes becomes more prominent. Indirect taxes, such as consumption taxes, are easier to enforce and collect since they are embedded in the transactions of purchasing goods. Additionally, with the decrease in the direct tax rate, more consumers have increased disposable income, further generating more revenue from indirect taxes. These ideas align with the observation in Figure 7 outlining the data that developing countries derive 43% of their tax revenue from consumption taxes while generating only 25% from income taxes.

Conversely, in developed countries, lower auditing costs from a robust administrative capacity enable governments to collect a significant share of revenue through direct taxes. This idea is evident in Figure 8 where developed countries have a nearly balanced contribution of direct and indirect taxes sitting at 38% and 31% respectively.

This evidence validates our theoretical framework by implying that auditing costs play a critical role in determining a country's tax structure. By establishing these observations in our models, we demonstrate that the difference in tax structures across countries is influenced by economic and institutional constraints.

9. Conclusion

In conclusion, this paper offers a novel theoretical framework that examines the factors influencing the design of optimal tax systems, particularly the balance between direct and indirect taxes. By integrating a Bayesian game of incomplete information with a Laffer curve-based market dynamics model, we model the complex relationships between tax auditing costs, consumer behavior, and tax evasion strategies. Our findings demonstrate that the optimal combination of direct and indirect taxes is significantly shaped by a country's tax auditing costs.

Through this framework, we provide novel insights into the variations in tax structures across countries, emphasizing the role of tax enforcement and the institutional context in shaping these differences. The paper also extends the existing literature by incorporating enforcement costs and tax evasion dynamics into the traditional Laffer curve framework, offering a comprehensive understanding of the intricacies involved in tax policy design. Our research has important policy implications, particularly for countries at different stages of development. It highlights the challenges faced by developing countries in implementing and enforcing direct taxes due to limited administrative resources and large informal sectors, which often lead to a reliance on indirect taxes. In contrast, developed countries, with stronger administrative capacities, are better positioned to implement and enforce direct taxes.

Overall, this paper contributes to the ongoing discourse on tax policy design by offering a theoretical explanation for the observed differences in tax structures across nations. Future research could further refine these models by exploring more complex dynamics with monopolistic markets, nonlinear supply and demand curves, Stackelberg game framework, and more.

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