Competitive Dynamics in Organizations

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Abstract

While economic research has extensively examined dynamic market competition, the study of competitive dynamics within organizations remains underexplored. Our study contributes to the literature in organizational economics by analyzing the dynamic of internal competition and offering new insights into productivity divergence and institutional reform.

Existing literature often conceptualizes internal competition as a static, high-powered incentive mechanism designed by superiors. We challenge this view by demonstrating that top performers are not always selected as winners, as superiors must balance productivity goals with the basic need to maintain authority and personal ties with subordinates. This paper develops a principal-agent model within a three-layer hierarchical structure to examine how superiors balance productivity goals with the socio-political imperative to maintain authority and subordinate networks.

Our findings demonstrate that superiors strategically adjust the level of subordinate competition to retain discretionary control over promotions and rewards, reconciling economic efficiency with sociopolitical objectives. The model shows larger networks lead to higher competition intensity and greater productivity, while stronger networks reduce competition intensity, resulting in lower productivity. The interplay of the two network structures impacts the optimal level of internal competition and the organizational productivity in a counterintuitive way. Within this dynamic framework, fostering subordinate networks emerges as a rational choice for superiors, challenging the high-powered incentive assumption that dominates traditional theories of internal competition.

Keywords: informal networks; competitive dynamics; productivity; inequality

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Introduction

Competitive dynamics play a crucial role in driving fluctuations in economic efficiency. For centuries, economists have primarily focused on competition in markets. These studies have evolved from the initial framework of pure competition to the polar opposite of perfect monopoly, where competition is absent, and finally to the recognition that real-world markets operate under imperfect competition—situated between free competition and perfect monopoly due to exogenous factors such as barriers to entry and exit, product differentiation, information asymmetry, and other market conditions beyond the control of individual firms (Smith 1776; Chamberlin 1951; Robinson 1969; Lamadon et al. 2022).

In contrast, less attention has been paid to systematically studying competitive dynamics within organizations, where allocation rules are deliberately designed by superiors rather than emerging from the unconscious workings of the ”invisible hand” in markets (Coase 1937; Arrow 1974). A key consequence of this gap is the limited understanding of how superiors' dynamic preferences for different levels of competitive intensity within organizations shape economic outcomes. This lack of insight hinders our ability to fully explain variations in organizational productivity from an endogenous perspective. To address this gap, our study develops a novel competition theory that incorporates human nature, advancing the understanding of internal competitive dynamics and the sources of productivity divergence in organizations.

Studies on internal competition in organizations can be traced back to the early 1980s. Influenced by Lazear and Rosen's (1981) seminal work on rank-order tournaments, the current mainstream literature in organizational economics often constructs internal competition as an ideal incentive scheme. In this ideal competition, each subordinate in the lower hierarchy is always granted equal opportunity to secure resources or positions in the upper hierarchy, and only the top-performing subordinates are rewarded or promoted.

The importance of their work cannot be emphasized enough. First, their studies have greatly enhanced our understanding of competition's incentive mechanism. By mitigating external shocks through relative performance rankings, subordinates receive stronger motivation to exert effort due to the reduction of noise in their performance evaluation (e.g. Lazear and Rosen 1981; Holmstrom 1982; Nalebuff and Stiglitz 1983; Green and Stokey 1983; Vickers 1995).

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Second, they are the first to differentiate competition within organizations from that in the market. They define internal competition within organizations as an incentive scheme designed by powerful people in the organization. Following this logic, it is understandable that while pure competition is seldom achieved in markets, it is attainable within organizations. This distinction highlights the fundamental difference between the two contexts. In markets, when imperfect competition occurs, it cannot be deliberately adjusted by the ”invisible hand” to reach an ideal state, nor do monopolies have the incentive to enhance market competition intensity. Consequently, the emergence of pure competition in markets is largely a random phenomenon which can hardly be observed. In contrast, within organizations, superiors possess both the authority and motivation to implement an effective incentive mechanism because their organizations must produce to survive in the marketplace. Thus, an ideal competition, such as the standard rank-order tournament, can be the dominant incentive option. In their view, the typical hierarchical structure embedded within organizations further justifies the reliance on an ideal internal competition. Hierarchies naturally shape a perfect pyramid structure of resource distribution, which facilitates employees at lower levels to compete for scarce resources arranged at upper levels, such as high salary and great power.

While it is reasonable to believe that ideal competition is more likely to occur in organizations than in markets, real-world organizations rarely adopt it as the dominant mode. In family firms, for example, subordinates who are family members are more likely to be promoted to key positions than the highest-performing employees who lack family ties. Similarly, law firms promote associates who are deemed most suitable for partnership, rather than simply those with the highest performance. The once widely accepted argument that personnel promotions in Chinese local governments are purely performance-based has also been increasingly questioned by scholars (Shih, Adolph, and Liu Shih et al., Landry 2008). Some researchers have even found evidence suggesting that the level of competition within local governments fluctuates depending on changes in political leadership (Wang, Yang and Yang 2025). All of this evidence suggests that subordinates are rarely treated equally, and that ideal competition within organizations is the exception rather than the norm.

One essential reason pure competition is less likely to be adopted as a common incentive scheme in real-world organizations is that it threatens superiors' authority, thus undermining organizational stability. Van den Steen (2010) and Adams and Ferreira (2007) point out that high-powered incentive measures can lead employees to focus all their efforts on improving their own performance, making them less willing to comply with their superiors' directives. Ideal competition involves high-intensity incentives, where the decision to reward or promote a subordinate completely depends on whose performance ranks top. Therefore, in organizations where internal resource allocations

heavily depend on power/authority, a persistent implementation of ideal competition that leads to subordinates' disobedience in lower hierarchies erodes organizational cohesion and hinders effective leadership. For firms, disobedience could lead to internal management chaos and reduce production efficiency. For governments and nations, it might trigger more severe political conflicts, thereby fundamentally undermining the stability of social and political systems. Consequently, to avoid organizational instability, subordinates with the best performance are not always promoted to higher ranks. Instead, those who demonstrate loyalty to decision-makers are more likely to be rewarded, even if their performance rankings are not high (Tullock 1987, Mesquita and Smith 2011, Svolic 2012).

An additional reason ideal competition cannot be frequently observed in real-world organizations is the broader informal network economy that permeates organizations. We argue that the hierarchical structure, where most of the power and resources are disproportionately allocated in the higher hierarchy, is designed not only to facilitate competition but also to enhance the dependent relationship between superiors and their subordinates. This makes real-world organizations not merely a production machine but also a breeding ground for fostering broader informal networks across the vertical hierarchy (Greve and Mitsuhashi 1982).

Informal networks are widely regarded by social scientists as matrices of social links or sets of multiple dyadic alliances involving the exchange of favors (Mayer 1966, Landé 1977). By favoring some subordinates in internal competitions in exchange for their loyalty and compliance, superiors essentially consolidate and expand the informal networks with their subordinates. In many situations, those dyadic alliances are institutionalized as formal organizational structures.[1] The benefits gained from cultivating such networks, however, often exceed the amount derived from straightforward authority consolidation. First, people exchange favors to reduce uncertainty. Favors exchanged in informal networks often involve power, capacity, and willingness, which cannot be acquired through markets and formal contracts. Unlike transactions involving risks and specific products/services through markets or formal contracts, favors exchanged in informal networks address uncertainty. The value of such favors lies not only in their market value but also in their timely provision: they are given when they are "really needed." This intangible weight can greatly reduce uncertainty from unforeseen future events.

Second, favors flowing through informal networks involve asymmetrical utility exchanges, which is the fundamental reason why informal networks can occur automat-

[1]For example, by institutionalizing succession planning—such as establishing clear succession policies and procedures, including criteria for evaluating successors, training programs, and timelines for succession—these informal relationships can be formalized, thereby ensuring a smooth transition in key positions and increasing confidence among employees and external stakeholders in the company's stability.

ically. Exchange automation is also understood in economics as trust, loyalty, reputation, etc. Asymmetrical utility exchange can be self-enforcing because the beneficiary gains high utility from a resource or service provided by the benefactor, while the utility loss the benefactor suffers is low. The asymmetrical nature of utility exchange in informal networks encourages the superiors and subordinates who are able to provide the needed resources or services to enhance their personal relationship by sustaining asymmetrical exchanges with each other. For example, in his famous theory of structural holes, Burt (2018) indicates subordinates who occupy more social capital and an information advantage are indeed more likely to gain the favor of their superiors and, consequently, tend to receive higher salaries and faster promotions. This ongoing asymmetrical exchange catalyzes the formation of informal networks between the superior and some of his subordinates. The favoritism toward network subordinates may ultimately hinder the superior from implementing an ideal internal competition purely based on relative performance rankings.

Of course, the strength of internal informal networks depends on the expected amount of network benefits they can receive.[2] When both parties experience minimal costs while gaining substantial benefits, they are strongly motivated to enhance the relationship, as it leads to mutual gains and reinforces their desire to continue the connection. On the contrary, when the net gain derived from a network decreases, networks may still function, but individuals may rely less on them to facilitate exchanges. When the superior's reliance on networks changes, the degree of favoritism toward his network subordinates varies. As a result, the relative importance of economic incentives versus network maintenance fluctuates.

We believe that the shift between economic incentives and the need to maintain informal networks for obtaining either authority or broader social network benefits can influence the superior's preference for different levels of internal competition, ultimately resulting in competitive dynamics within organizations and productivity divergence.

By building a principal-agent model, we investigate how informal networks between the superior and his subordinates shape internal competitive dynamics and their impact on organizational productivity. In the context of a hierarchical structure, Our model shows the mechanism is intriguingly interactive. The superior's preference for the optimal level of competition is mutually shaped by the interplay of two dimensions of the network structure: the strength of the subordinate network (the amount of network gain the superior can obtain) and the size of the subordinate network (the number of subordinates who are network members out of the total number of subordinates at

[2]Two social psychologists, Thibaut and Kelley (1959), note in their classic study of dyadic relations that such personal connections tend to thrive when each individual can offer highly valuable rewards to the other at little personal cost.

the same lower hierarchy). When the superior has stronger (or weaker) ties with subordinates, he tends to set a lower (or higher) level of internal competition, which in turn leads to lower (or higher) organizational productivity. On the other hand, when the superior manages a larger (or smaller) subordinate network, he is inclined to set a higher (or lower) level of internal competition, resulting in increased (or decreased) organizational productivity.

The interaction between these two dimensions of network structure reveals an interesting pattern in internal competitive dynamics and the formation of productivity differences. We show that the ideal competition assumed by the current literature occurs only when the network ties between the superior and the subordinates are extremely weak, which is accompanied by the highest organizational productivity. The level of internal competition and the corresponding production level are moderated and swing in most cases with different combinations of the two network structures. As a result, organizational productivity decreases and fluctuates. When network ties are strengthened, organizations with larger subordinate networks implement more intense competition than those with smaller subordinate networks. The former demonstrates significantly greater resilience than the latter to the downward trend in overall productivity. In a few extreme cases where the network size is very small, organizational productivity is very fragile: it rapidly drops to the lowest level shortly after the network strength is enhanced.

The above pattern validates our inference regarding the dynamics of internal competition within organizations. Since the dynamic nature of internal competition stems from the socioeconomic foundation of organizations, it provides a systematic perspective for understanding internal competitive dynamics. As the superior derives greater benefits from his subordinate network (stronger network ties), he balances the tradeoff between network gains and productivity by implementing a less competitive incentive scheme. Reduced competitive intensity increases his discretion over rewarding or promoting subordinates, enabling him to favor network members with a higher probability of winning without entirely undermining productivity. At the same time, the level of favoritism in internal competition is influenced by the size of the subordinate network within the hierarchical structure. When the network is large (e.g., 4 out of 5 subordinates in the lower hierarchy are network members), the superior does not need to sacrifice competitive intensity for a high likelihood of rewarding or promoting a network subordinate. Consequently, his optimal strategy is to maintain a relatively high level of competition. However, when the network size shrinks significantly (e.g., only 1 out of 5 subordinates is a network member), his dominant strategy is to substantially lower competition intensity to ensure a relatively higher probability of rewarding or promoting his network subordinate. The strategy of setting an extremely low level of competition is rational when network gains completely outweigh organizational pro-

ductivity.

This interactive mechanism contributes new insight to understanding differences in organizational productivity. Productivity differences are a fundamental research topic in economics, with many classical economic theories offering explanations for them. As a supplement to these classical studies, we capture the human factor that shapes the internal competitive dynamics and the variations in productivity. Our study indicates that even if we controlled for all exogenous factors that impact productivity, such as property rights, technology, etc., productivity divergence can still exist because it originates from the dynamic socioeconomic nature of organizations.

Our model also demonstrates that the production gaps between subordinates who are network members and those who are not are endogenously determined by the superior's preference for different competition levels under different combinations of informal network structures. These production gaps occur in internal organizations as long as the competitive levels are moderated. Additionally, we show that the production gaps between subordinates with different network identities change regardless of the level of organizational productivity. If we apply this mechanism to explain economic inequality, it may imply that any degree of unbalanced economic development can happen in any country regardless of its economic performance. This is because the dynamic internal competition implemented in the central-state-local government hierarchies shapes unbalanced economic incentives toward subordinate entities with different network identities. A consequence is that government officers holding positions in different regions may have different levels of incentives to develop the local economy.

In our model extension, we explore the superior's preferred structure of his subordinate network and its impact on internal competitive dynamics. We reveal that the superior is always motivated to enlarge the subordinate network. This finding provides theoretical support for the pervasiveness of informal networks in organizations. We also discover some tipping points that shift the superior's preferred network strength when the size of his network is small. These properties reinforce our main argument that the optimal level of competition can hardly remain at the ideal level.

This paper is structured as follows: Section 1 reviews the related literature. Section 2 details our model. Section 3 visualizes our results. Section 4 discusses the implications of our theoretical findings. Section 5 extends our model. Section 6 summarizes the implications for institutional reforms. Finally, Section 7 concludes the paper and highlights areas for future analysis.

Literature Review

Literature on internal competition

Our study relates to several literatures. We fill the gap in the literature of internal competition by pointing out the socioeconomic nature of competitive dynamics in organizations. The current literature on internal competition in organizations assumes organizations are pure production machines and that superiors maximize organizational productivity. Thus, this branch of literature mainly focuses on strategies that can maintain competition's incentive efficiency at the highest level (see the literature review by Connelly et al. 2014 on rank-order tournaments). For example, some economists have noted that the incentive cost of competition may increase as the number of competitors surges. To avoid inefficient competition, they point out that the principal can limit the number of competitors or set a multiple-prize contest (Nalebuff and Stiglitz 1983; O'keeffe et al. 1984; Berry 1993, Eriksson 1999). Sabotage is another competitor behavior that weakens competition's incentive efficiency. To limit the negative impact of sabotage, the principal can decrease the prize difference, increase the number of competitors, or increase the punishment/monitoring of sabotage behavior (Lazear Lazear; Konrad 2000; Curry and Mongrain Curry and Mongrain). Economists also recognize that competitors' heterogeneity can be an alternative factor that undermines internal competition's incentive level (Cai and Treisman 2005; Lazear and Rosen 1981). Thus, to minimize the efficiency loss of the competitive mechanism caused by heterogeneity, they argue that the superior can group competitors with similar individual endowments in advance or implement a multi-round elimination process to achieve de-heterogenization (Lazear Lazear; Rosen 1986).

This literature explores many exogenous factors that may cause the ideal competition to be less efficient, as well as ideas on how the superior can maintain the ideal competition by avoiding these factors. However, few of them ask whether the ideal internal competition is always the dominant strategy for the superior. Some of them are aware that the heterogeneity of subordinates may weaken the incentive function of the ideal competition, but none of them consider the heterogeneity of subordinates as a property of organizations that can hardly be removed. Different from these studies, we examine the competitive dynamics within organizations. Rather than taking the level of internal competition as exogenous, we explore how the competitive intensity is endogenized by the socioeconomic nature of organizations.

Literature on economic effects of informal networks

Our theory also builds on the literature on the economic effect of informal networks in organizations. The fundamental statement that economic action is embedded in

ongoing social ties was proposed by Granovetter (1985). His argument of social embeddedness bridges sociological and economic perspectives in organizational studies. However, it lacks its own concrete account of how personal relations impact economic outcomes. A few economists later explore the explicit mechanism of personal ties and their economic impact with formal models. In general, economic theorists treat personal ties between superiors and subordinates as a source of economic inefficiency. For example, Tirole (1992) considers the exchange between superiors and their subordinates as bribery to obtain desirable outcomes. Prendergast (1993); Prendergast and Topel (1996) capture the broader social nature of the exchange between superiors and subordinates. They point out that the social incentive for workers to conform to the opinions of their supervisors and the superior's motivation to favor subordinates they personally prefer can distort economic incentives within organizations.

While our work shares some similarities with their research, it introduces two key extensions. First, we treat informal networks as part of the inherent organizational structure rather than as an exogenous phenomenon. We expand the exchange between one superior and one subordinate in their models to exchanges in the informal networks between one superior and multiple subordinates. This makes our model closer to the real-world hierarchical structure of organizations. In their model, the pyramid structure of the organization is simplified to a one-superior–one-subordinate framework. Thus, their findings only shed partial light and cannot capture the nonlinear effect of informal networks on economic outcomes. Second, in contrast to prior studies focusing on a social tie's influence on a single agent's economic decision-making, our model examines how informal networks dynamically shape both (1) the competitive behaviors of multiple subordinates within organizations and (2) their collective impact on broader organizational productivity.

Literature on loyalty-competence tradeoff

In the field of political science, some studies discuss similar topics to ours. These studies use formal models to investigate the loyalty-competence tradeoff faced by dictators when promoting a candidate. Egorov and Sonin (2011) conclude that rulers, especially those who are weak and vulnerable, sacrifice the competence of their agents by hiring mediocre but loyal subordinates, which leads to economic inefficiency in the long run. Zakharov (2016) finds that dictators who prioritize economic performance tend to appoint more competent subordinates, while incompetent subordinates are more likely to be hired when dictators place greater weight on future payoffs or when the benefits of competence are relatively lower. These findings, derived from the assumption that more competent subordinates are less loyal to the ruler, literally echo the special case of a strong but small subordinate network in our model.

In our model, we show that a superior who has a small network size (referring to a weak and vulnerable ruler with a scarce number of political supporters) but heavily relies on the subordinate network (as in a dictatorship with weak institutions) tends to implement a low level of competition (resulting in a decline in productivity) among all subordinates in exchange for a higher likelihood of selecting his network subordinate. In this regard, their result is compatible with ours. However, the fundamental difference between our assumption and theirs is that rather than assuming a fixed negative relationship between subordinates' capacity and their loyalty, we assume a flexible relation between capacity and loyalty. We endogenize subordinates' capacity to the economic incentives of internal competition and their loyalty to the strength of informal networks between subordinates and the ruler in a hierarchical framework.

An advantage of our perspective is that it enables our model to explain broader phenomena. For example, why do some autocrats still prefer to implement a high-powered economic incentive scheme even if their political survival heavily relies on subordinates' loyalty (e.g., China since 1978, South Korea during Park Chung-Hee's military dictatorship, Chile under Augusto Pinochet's regime)? Why do some democratic regimes suffer economic stagnation and societal instability while others obtain economic prosperity?

The Model

Background

By building a principle-agent model in a three-layer organizational hierarchy, we analyze a socially embedded multi-player competition scenario in which n subordinates with heterogeneous network identities in the bottom hierarchy respectively exert efforts to win a resource $V > 0$ located in the middle hierarchy after observing the competition rules set by the Superior D in the top hierarchy.[3]

We assume an informal network preexists between the superior D and some of the subordinates. Thus, subordinates in the bottom hierarchy are identically heterogeneous. Among them, $m \in \{0, 1, 2, \ldots, n\}$ subordinates are D's network members, while the remaining $n-m$ subordinates are not network members. For conciseness, we name network subordinates as favorites and nonnetwork subordinates as underdogs. Given the total number of subordinates (n), the number of favorites (m) represents the relative size of the subordinate networks. The more favorites compared to underdogs, the the larger size of the surbordinate network in an organization.

In practice, many complex factors cause changes in network size. At the organiza-

[3]The type of resource V could be a job position, money, political power, or any type of reward that is valuable for those competitors.

tional level, the degree of power centralization, types of leadership, the leader's (real) authority level, personal reputation, economic incentives, and corporate culture can promote or prevent the formation of an informal network. At the societal level, the spread of kinship, the mix of culture, norms, and formal institutions may also shape informal networks in varying sizes. Our model does not focus on the mechanisms causing changes in network density. We take the number of favorites m as a given variable and focus on how superiors' private preference and economic outcome change when m changes.

Besides network size m, we also incorporate the superior's network gain $\alpha > 0$, which is a private benefit the superior D derives from his favorites.

In our model, favorites differ from underdogs in that a favorite reciprocates D with $\alpha > 0$ after D allocates V to that favorite, while an underdog who is not socially tied to D causes D to incur a loss of network gain $-\alpha$ if V is rewarded to that underdog.[4] The amount of D's gains α derived from the network reflects the degree of heterogeneity between favorites and underdogs.[5] For D, higher α indicates a greater difference that favorites can distinguish from underdogs, while lower α means favorites are less distinguishable from underdogs. The m favorites are identical because each favorite reciprocates the same amount of α if she wins V. Similarly, the $n-m$ underdogs are also identical because each underdog incurs the same amount of loss of $-\alpha$ if she wins V. As long as $\alpha > 0$, D has the motivation to maintain the network by preferring favorites over underdogs.[6]

D's preference for favorites over underdogs is reflected in the competition rules he sets. Before the n competitors choose their effort levels to compete for the resource V, D announces that the potential winner is selected from competitors whose performances rank in the top $k \in \{1, 2, \ldots, n\}$, where k reflects the competition intensity chosen by D. Another common knowledge is that D always selects the potential winner from the favorite(s) who rank(s) in the top k if at least one favorite's performance ranks in the top k; otherwise, D selects the winner from underdogs who rank in top k.

This competition rule has two implications. First, favorites are conditionally preferred by D. Favorites can obtain priority only if their performances meet the basic qualification (ranking to the top k). Second, the only case in which an underdog wins the resource V is when no favorites' performances are qualified (no favorites rank to

[4]Favorites' precommitment, the potential loss caused by underdogs, and the superior's commitment to allocate the resource V to any subordinate are considered credible when all these actions are at low to zero costs. This judgment is based on the two characteristics of informal networks: reduce uncertainty and asymetrical exchange. Even if there is a positive cost with favorites and underdogs to provide $\pm\alpha$, it can be transmitted to a lower level of α or a smaller V. Thus, for simplicity, we assume allocating V or providing $\pm\alpha$ is at zero costs.

[5]In reality, the amount of network gain derived from informal networks is determined by complex factors.

[6]Different levels of α may be caused by different social norms, cultures, or the strengths of formal institutions, such as power checks and balances and the monitoring capacity of a system.

the top k). In other words, even if an underdog's performance ranks in the top k, her winning likelihood is zero as long as at least one favorite simultaneously ranks in the top k. When no favorites rank to the top k, an underdog's winning likelihood after her performance ranks in the top k is $\frac{1}{k}$. Each favorite who ranks in the top k has an equal conditional winning likelihood $\frac{1}{i} \geq \frac{1}{k}$, where $i \leq k$ is the number of favorites whose performance successfully enters the top k.

In our model, the competition intensity k set by D is not assumed as given; rather, it is adjustable. This differs from traditional literature that either holds productivity maximization (merely focus cases with $k = 1$, and only considers the economic attribute). The adjustable competition intensity $k \in \{1, 2, \ldots, n\}$ in our model reflects the dynamic process in which the superior makes tradeoffs between his social and economic natures to maximize his utility function. When D sets $k = 1$ (the winner is the best-performing competitor), he implements the most intense and unbiased competition. The competition is less intense when he sets $1 < k < n$ (refers to a medium level of minimum performance standard), creating D with greater discretionary room to prioritize his favorites. When D sets $k = n$, competition intensity reduces to zero, and each favorite enjoys absolute priority over underdogs regardless of their performance rankings.

Competitors' Strategies

The n competitors' strategy is choosing their effort level respectively to maximize their own utilities after observing the competition rule k set by D.

To be specific, favorite $s(s = 1, ..., m)$ exerts effort level x_{f_s} to maximize her utility:

$$U_{f_s} = P_{f_s} V - \frac{x_{f_s}^2}{2}, (s = 1, ..., m; m \leq n) \tag{1}$$

Where $P_{f_s}$is favorite s' s likelihood to win V, while $\frac{x_{f_s}^2}{2}$ is favorite s' s effort cost when she chooses effort level x_{f_s}.

While underdog $t(t = 1, ..., n - m)$ exerts effort level x_{u_t} to maximize her utility:

$$U_{u_t} = P_{u_t} V - \frac{x_{u_t}^2}{2}, (t = 1, ..., n - m; m \leq n) \tag{2}$$

Where $P_{u_t}$is underdog t' s likelihood to win V while $\frac{x_{u_t}^2}{2}$ is underdog t' s effort cost when she exerts effort level x_{u_t}.

The mathematical expression of P_{f_s} and P_{u_t} are developed from Berry's (1993) Contest Success Function that describes an agent's probability of being rewarded in a competition with multiple winners. To incorporate D's preference toward favorites over underdogs whose performances rank in the top k, we make an adjustment to Berry's (1993) Contest Success Function by including the winning likelihood conditioning on

the performance have ranked in the top k.[7]

Specifically, for underdog t, she is rewarded V only if no favorites rank in the top k. This implies the fact that underdog t' s winning likelihood is positive (P_{u_t}>0) can happen only when the number of underdogs is no less than the competition intensity k set by D ($n-m \geq k$). Otherwise, D always selects the winner from favorites ranking in the top k, and $P_{u_t} = 0$. The expression of underdog t' s winning likelihood is as follows:

$$P_{u_t} = \begin{cases} \frac{1}{k} \frac{\binom{n-m-1}{k-1} x_{u_t} + (n-m-1)\binom{n-m-2}{k-2} x_{u_{\bar{t}}}}{\binom{n-1}{k-1}[x_{u_t} + (n-m-1)x_{u_{\bar{t}}} + m x_{f_s}]} & \text{if } n-m \geq k \\ 0, & \text{if } n-m < k \end{cases} \tag{3}$$

Where x_{u_t} is underdog t' s $(t = 1, ..., n-m)$ effort level, $x_{u_{\bar{t}}}$ is the effort level exerted by any of the rest of underdogs $\bar{t}(t = 1, ..., t-1, t+1, ...n-m)$, and x_{f_s} is the effort level exerted by favorite $s(s = 1, 2, ..., m)$.

When $n-m \geq k$, P_{u_t} is decoded as two parts. The first part $\frac{1}{k}$ refers to the conditional winning probability for underdog $t(t = 1, ..., n-m)$ if her performance ranked in the top k. The second part is the probability that underdog t ranks in the top k and the rest of the top k competitors are all underdogs. Obviously, underdog t' s positive winning likelihood depends not only on her own effort level but also on the rest of the underdogs' effort levels, favorites' effort levels, the value of k, m, and n.

For favorite s, her winning likelihood $P_{f_s}(s = 1, ..., m)$ not only depends on the probability that her performance ranks in the top k but also relies on the total number of favorites whose performances rank in the top k. Based on the two cases of P_{u_t}, P_{f_s} is a piecewise function accordingly. In particular, when $n-m \geq k$, the calculation of P_{f_s} is:

$$P_{f_s} = \frac{\begin{array}{l} x_{f_s} \sum_{i=1}^{\min[m,k]} \frac{1}{i}\binom{m-1}{i-1}\binom{n-m}{k-i} + (m-1)x_{f_{\bar{s}}} \sum_{i=2}^{\min[m,k]} \frac{1}{i}\binom{m-2}{i-2}\binom{n-m}{k-i} \\ +(n-m)x_{u_t} \sum_{i=1}^{\min[m,k]} \frac{1}{i}\binom{m-1}{i-1}\binom{n-m-1}{k-i-1} \end{array}}{\binom{n-1}{k-1}[x_{f_s} + (m-1)x_{f_{\bar{s}}} + (n-m)x_{u_t}]} \tag{4}$$

Where i ($1 \leq i \leq \min(k, m)$) is the total number of favorites whose performances successfully rank in the top k. $\frac{1}{i}$ is favorite s' s $(s = 1, ..., m)$ conditional probability if her performance ranks in the top k. $x_{f_{\bar{s}}}$ is the effort level exerted by any one of the rest of favorites ($\bar{s} = 1, ..., s-1, s+1, ..., m$) except favorite s.

When $n-m < k$, where $P_{u_t} = 0$ and underdog t' s equilibrium effort level $x^*_{u_t} = 0$),

[7]Please see Appendix for more details about Berry's (1993) Contest Success Function and the adjustment we made to adapt to our model.

favorite s' s $(s = 1, ..., m)$ winning likelihood P_{f_s} is calculated as

$$P_{f_s} = \frac{\sum_{j=0}^{n-m} \frac{1}{k-j}\binom{n-m}{j}\binom{m-1}{k-j-1}x_{f_s} + (m-1)\sum_{j=0}^{n-m} \frac{1}{k-j}\binom{n-m}{j}\binom{m-2}{k-j-2}x_{f_{\bar{s}}}}{\binom{n-1}{k-1}[x_{f_s} + (m-1)x_{f_{\bar{s}}}]} \tag{5}$$

Where j is the number of underdogs whose performances rank in the top k, and $\frac{1}{k-j}$ is the conditional winning likelihood of favorite s if her performance ranks in the top k.

In summary, the piecewise expression of favorite s's winning likelihood $P_{f_s}(s = 1, ..., m)$ is

$$P_{f_s} = \begin{cases} \frac{x_{f_s}\sum_{i=1}^{\min[m,k]} \frac{1}{i}\binom{m-1}{i-1}\binom{n-m}{k-i} + (m-1)x_{f_{\bar{s}}}\sum_{i=2}^{\min[m,k]} \frac{1}{i}\binom{m-2}{i-2}\binom{n-m}{k-i} + (n-m)x_{u_t}\sum_{i=1}^{\min[m,k]} \frac{1}{i}\binom{m-1}{i-1}\binom{n-m-1}{k-i-1}}{\binom{n-1}{k-1}[x_{f_s}+(m-1)x_{f_{\bar{s}}}+(n-m)x_{u_t}]} & \text{if } n-m \geq k \\ \frac{\sum_{j=0}^{n-m} \frac{1}{k-j}\binom{n-m}{j}\binom{m-1}{k-j-1}x_{f_s}+(m-1)\sum_{j=0}^{n-m} \frac{1}{k-j}\binom{n-m}{j}\binom{m-2}{k-j-2}x_{f_{\bar{s}}}}{\binom{n-1}{k-1}[x_{f_s}+(m-1)x_{f_{\bar{s}}}]}, & \text{if } n-m < k \end{cases} \tag{6}$$

Obviously, favorite s' s winning likelihood depends not only on her own effort level, but also on the rest of the favorites' effort levels, underdogs' effort levels, and the value of k, m, i, and n.

The Superior's Strategy

The superior D chooses competition intensity k to maximize his socioeconomic utility function W:

$$W = (\sum_{s=1}^{m} x_{f_s} + \sum_{t=1}^{n-m} x_{u_t}) + \alpha(P_f - P_u) \tag{7}$$

Where $\sum_{s=1}^{m} x_{f_s} + \sum_{t=1}^{n-m} x_{u_t}$ is the overall productivity function, which is the sum of effort levels of all favorites and underdogs. P_f is the likelihood that any favorite wins the reward V while P_u is the likelihood that any underdog wins the reward V. $\pm\alpha$ is the amount of network benefit that D gains/ loses if a/an favorite/underdog wins the resource V. Thus, $\alpha(P_f - P_u)$ is D' s expected network gains from rewarding a favorite.

Since $P_u = 1 - P_f$, Equation (7) can be rewritten as:

$$W = (\sum_{s=1}^{m} x_{f_s} + \sum_{t=1}^{n-m} x_{u_t}) + \alpha(2P_f - 1) \tag{8}$$

Where the math expression of P_f is:

$$P_f = \begin{cases} 1 - \frac{(n-m)\binom{n-m-1}{k-1}x_{u_t}}{\binom{n-1}{k-1}[(n-m)x_{u_t}+mx_{f_s}]} & \text{if } n-m \geq k \\ 1, & \text{if } n-m < k \end{cases} \tag{9}$$

Where $\frac{(n-m)\binom{n-m-1}{k-1}x_{u_t}}{\binom{n-1}{k-1}[(n-m)x_{u_t}+mx_{f_s}]}$ is the likelihood P_u that D rewards an underdog when $n-m \geq k$. When $n-m < k$, at least one favorite's performance ranks in the top k. In this case, D rewards a favorite with certainty ($P_f = 1$).

Given the network size m and D's network gain α, the following sections show how the superior D selects the optimal competition intensity k^* to maximize his socioeconomic utility, and how competition intensity k^* impacts subordinates' effort levels and the overall productivity in equilibriums.

Visual Application When $n = 5$

In this section, we visualize our analysis with specific applications to present the equilibrium solutions evidently. Our main purpose is to untangle the relationships between network size m, the competition intensity k, D's network gain α, favorites' and underdogs' effort levels $x_{f_s}(s = 1, ..., m)$ and $x_{u_t}(1, ..., n-m)$, and the overall productivity $mx_{f_s} + (n-m)x_{u_t}$.

To analyze the competitive dynamics ($k \geq 1$), we should consider cases where $n > 2$. To maintain simplicity while preserving generality, we explore the relationship between m, k, α, $x_{f_s}(s = 1, ..., m)$, $x_{u_t}(1, ..., n-m)$, and $mx_{f_s} + (n-m)x_{u_t}$ in the context of five competitors ($n = 5$).[8]

For simplicity, we set $V = 1$ for visualization.

Partial Equilibrium Visualization

Figure 1 depicts the relationship between network size m, competition intensity k that are possibly selected by the superior D, and the overall productivity $mx_{f_s} + (n-m)x_{u_t}$ in the partial equilibrium.

Figure 1 reveals several interesting properties.

Theorem 1 *Overall productivity is determined by competition intensity. More intense competition leads to higher overall productivity, while less intense competition results in lower overall productivity.*

[8]We did the same analysis in cases where $n = 3$, $n = 6$, and $n = 7$, the conclusions from the case where $n = 5$ remains valid.

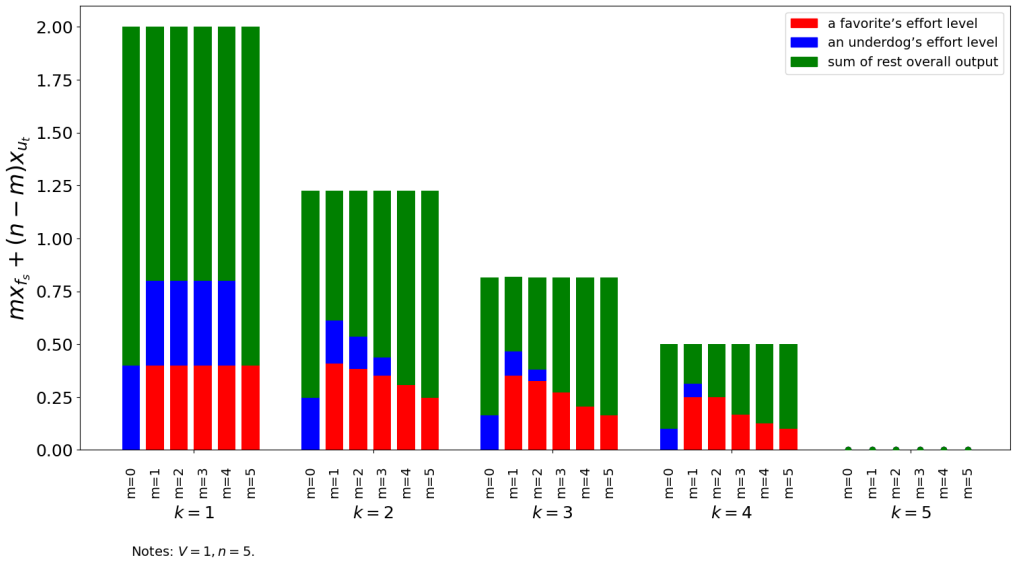


Figure 1 COMPETITION INTENSITY, NETWORK SIZE AND OUTPUT IN PARTIAL EQUILIBRIUM

The first discovery in Figure 1 is that the overall output $mx_{f_s} + (n-m)x_{u_t}$ solely depends on the competition intensity k. Specifically, regardless of the network size m, the overall productivity increases when the competition intensity increases (smaller k). Conversely, the overall productivity decreases when the competition intensity decreases (larger k). When the competition is most intense ($k = 1$), the overall productivity reaches its highest level. In this scenario, both favorites and underdogs have equally strong incentives to exert effort because the superior D rewards any of them who performs best, regardless of their identities.

On the other extreme, when the competition intensity completely disappears ($k = 5$), overall productivity drops to zero. In cases where subordinates' identities are heterogeneous ($0 < m < 5$), the economic incentive drops to zero because D only rewards one of the subordinates who is a favorite. In other words, the reward rule is entirely based on subordinates' network identities, regardless of performance ranking. Therefore, no favorite or underdog is motivated to exert effort. When $k = 5$ and subordinates have homogeneous social identities (all underdogs ($m = 0$) or all favorites ($m = 5$)), D rewards one of them randomly, regardless of their performance. In summary, when $k = 5$, competitors lack the motivation to produce, even though favoritism is absent.

The second property of Figure 1 lies in the relationship between favorite and underdog's effort levels x_{f_s} and x_{u_t}, the network size m, and the possible competition intensity k that could be selected by D.

Theorem 2 *Both m and k have a partial non-positive impact on x_{f_s} and x_{u_t}.*[9].

When m is constant, the reason for the decrease in the effort levels of a signle fa-

[9]There is a special case in which favorites' effort level x_{f_s} increases slightly from 0.400 to 0.408 when $m = 1$ and k increases from 1 to 2.

vorite and underdog is that less intensified competition (larger k) increases the likelihood that favorites' performance rank in the top k while needlessly exerting larger effort x_{f_s}. This makes underdogs less likely to get the reward even if their performances are top k ranked. Therefore, underdogs are demotivated to exert effort. According to Equation (16) in the Appendix, favorites' effort levels are positively associated with underdogs' effort levels in a competitive relationship. Therefore, favorites exert less effort x_{f_s} when they expect underdogs, their rivals, to decrease their effort x_{u_t}.

When k is given, as the number of favorites m increases, both favorites and underdogs do not increase their effort levels x_{f_s} and x_{u_t}. Specifically, when $k = 1$ or $k = 5$, favorites and underdogs maintain the same effort levels ($x_{f_s} = 4$ or $x_{u_t} = 0$) regardless of variations in m. This is because when $k = 1$, favoritism does not work because favorites and underdogs have equal chances of being rewarded if they exert the same effort level. In the case of $k = 5$, superior D's decision to reward is independent of competitors' performance ranking; thus, all competitors, regardless of their identities, exert zero effort. When $1 < k < 5$, an increase in m not only demotivates underdogs to exert effort but also directly decreases a favorite's winning likelihood by reducing her priority of being rewarded if her performance ranks in the top k.

Note that although the effort levels of both favorites and underdogs decrease as k or m increases, the degree of decline in their efforts varies.

Theorem 3 *Favorites show much stronger resilience than underdogs to the downward pressure on their effort levels.*

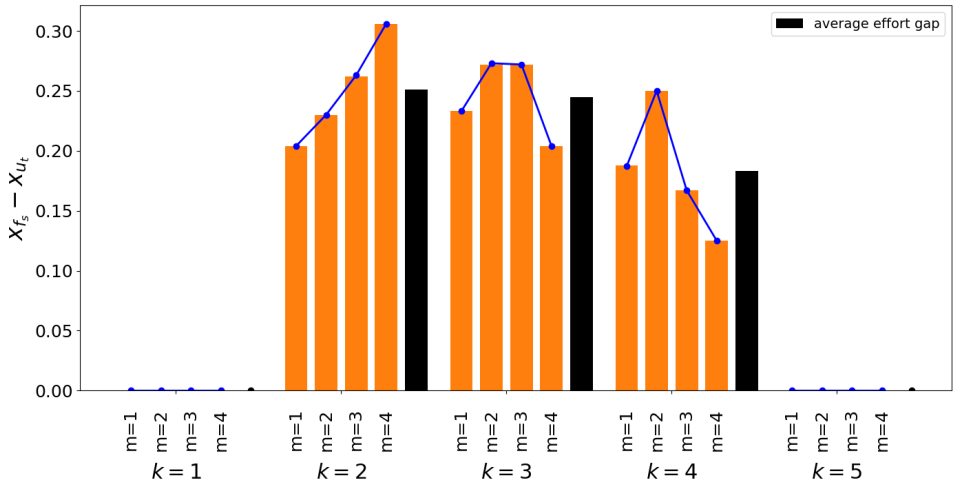
This is because, compared to underdogs, favorites have a higher winning likelihood derived from some degree of priority once their performance ranks in the top k. Favorites' priority reaches the highest level when $n - m < k$, where underdogs completely slack off ($x_{u_t} = 0$) because their winning chances are zero regardless of their effort levels.

Because of the differing resilience of favorites and underdogs to the downward trend in effort levels as m and k increase, effort gaps between favorites and underdogs emerge. In addition to overall productivity, the effort gap is another variable of interest because it has important practical implications for unbalanced development and inequality.

Figure 2 illustrates how effort gaps between favorites and underdogs vary with m and k.[10]

Theorem 4 *There are no gaps between favorites' and underdogs' effort levels when* $k = 1$ *or* $k = 5$. *Positive gaps between favorites' and underdogs' effort levels (*$x_{f_s} - x_{u_t} > 0$*) occur only when* $1 < k < 5$.

[10] In Figure 2, we exclude cases of $m = 0$ and $m = 5$ because competitors' identities are homogeneous (all underdogs or all favorites) in these cases. This means each competitor exerts the same level of effort regardless of k, making it meaningless to discuss effort gaps in these scenarios.



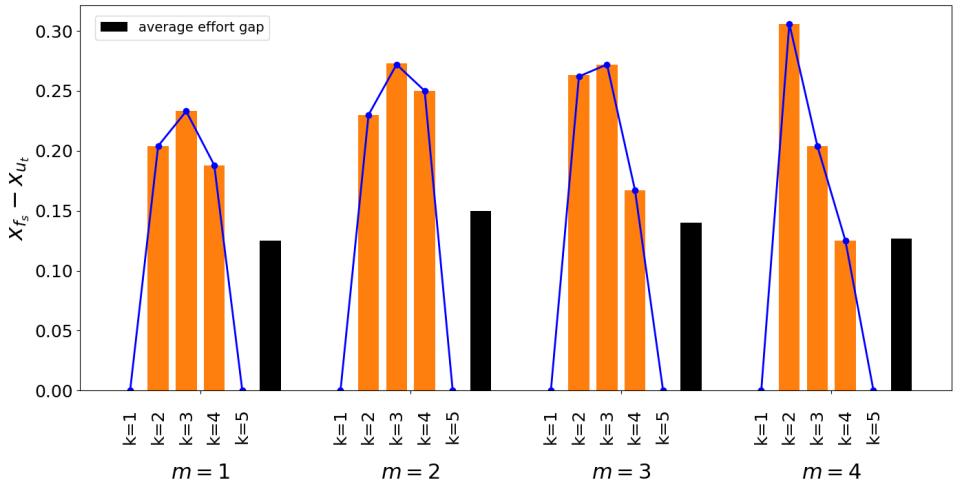


Figure 2 EFFORT GAPS BETWEEN FAVORITES AND UNDERDOGS

When $k = 1$, regardless of the network size m, favorites and underdogs have equal chances of being rewarded, so they exert the same effort level ($x_{f_s} = x_{u_t}$). When $k = 5$, the superior D's decision to reward is independent of competitors' performance ranking, so all subordinates, regardless of their identities, exert zero effort. Thus, effort gaps between favorites and underdogs are zero.

In Figure 2, when $1 < k < 5$, the relationship between network size m, competition intensity k, and effort gaps is not linear. Specifically, when k is given, effort gaps generally rise first and then fall as the number of favorites m increases, and the average effort gaps for a given k decrease as the competition becomes less intense (k increases). When m is given, effort gaps generally rise first and then fall as the competition intensity decreases (k increases), and the average effort gaps for a given m decrease first and then increase as m increases.

General Equilibrium Visualization

The above partial equilibrium visualization offers a basic decoding of the relationship between overall output, effort gaps, network size, and the given competition intensity that may be selected by the superior D. However, this is far from achieving a comprehensive understanding of the dynamic interaction between these social and economic factors in reality. In practice, the superior D adjusts the competition intensity k to the optimal level k^* so as to maximize his utility when the network size m and the social network strength $\alpha > 0$ change. To capture how the optimal competition intensity k^* are affected by m and α, and to present a comprehensive pattern of how these social and economic factors interactively impact economic outcomes, this part visualizes general equilibrium.

In Figure 3, we show how the optimal competition intensity k^* as well as the overall productivity $mx_{f_s} + (n - m)x_{u_t}$ vary with the network size m and network strength α in the general equilibrium. Figure 4 presents the pattern of effort gaps between favorites and underdogs $x_{f_s} - x_{u_t}$ in different social structures (m and α) in the general equilibrium.

Figure 3 includes several key conclusions of our model, highlighting a clear socioeconomic mechanism helps us to understand the variation in productivity across social backgrounds with different network sizes and the strength of network.

Theorem 5 *As network size m increases, the superior D monotonically increases the optimal competition intensity k^*, resulting in higher overall productivity, and vice versa. Conversely, as network strength α increases, D monotonically decreases competition intensity k^*, resulting in lower overall productivity, and vice versa.*

From the vertical perspective in Figure 3, for any given $\alpha > 0$, the superior D sets a monotonically increasing competition intensity (smaller k) as network size m

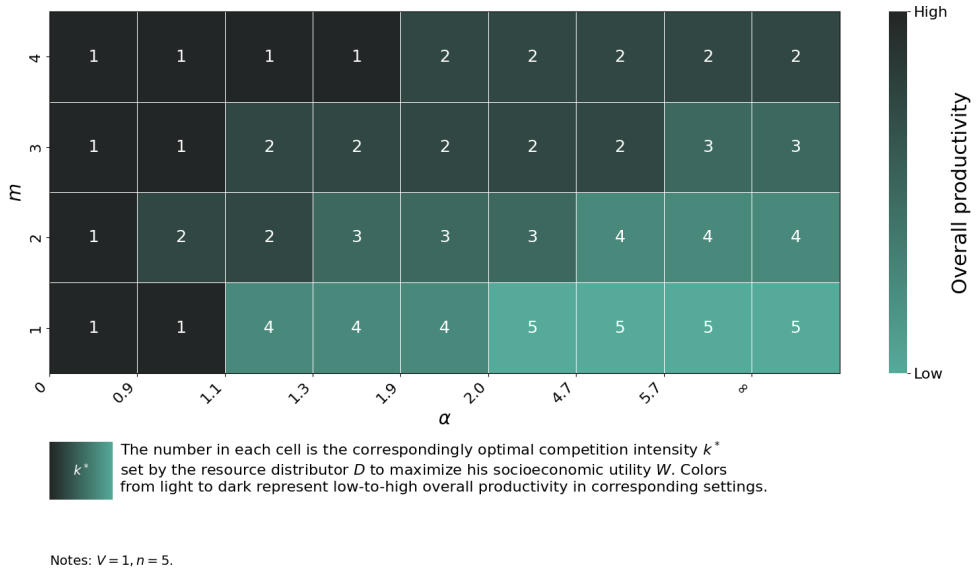


Figure 3 COMPETITION INTENSITY, NETWORK SIZE AND OUTPUTS

increases.[11] In Figure 1, network size m does not directly impact overall productivity in partial equilibrium. However, Figure 3 indicates that an increase in network size indirectly leads to equal or higher overall productivity (as the green color darkens) because D intensifies the optimal competition intensity k^* when network size increases, while network strength α remains constant. On the other hand, when α is fixed, a decrease in network size indirectly results in equal or lower overall productivity as D reduces the optimal competition intensity k^*.

This is straightforward because having more favorites guarantees D a higher likelihood of rewarding a favorite and a lower likelihood of rewarding an underdog without reducing competition intensity. Therefore, D does not need to decrease the competition intensity (increase k) at the cost of overall productivity to achieve a higher expected network gain.

From the horizontal perspective in Figure 3, for any given m, the superior D prefers a monotonically decreasing competition intensity (greater k^*) as α increases, which indirectly reduces overall productivity (as the color green lightens).

This pattern is intuitive because it becomes more appealing for D to reward favorites rather than underdogs as network strength enhances. Although less intensified competition demotivates both groups, the expected benefit from increasing the likelihood of rewarding a favorite (by increasing k^*) compensates for the productivity loss. Thus, decreasing competition intensity can still be optimal for D to maximize socioeconomic utility within a given range of increased network strength.

Theorem 6 *As the network strength decreases, overall productivity in cases with*

[11]This tendency is briefly interrupted in a small area when $m = 1$ and $0.9 < \alpha \leq 1.1$, where D sets a more intensified competition ($k^{=}1$) compared to the area where $m = 2$ ($k^* = 2$) while maintaining the same network strength $0.9 < \alpha \leq 1.1$.

larger network size shows greater resistance to downward trends compared to cases with smaller network size.

THEOREM 7 demonstrates the positive impact of network size on overall productivity.

In summary, Figure 3 provides insights into complexity regarding the impact of informal networks on economic outcomes. The interplay of the effect of network size and strength on the optimal competition intensity causes the influence of informal networks on economic outcomes to vary. For instance, while D prefers favorites to maintain his informal network, the level of favoritism—reflected through optimal competition intensity (k^*)—decreases as network size (m) increases and vice versa. The level of favoritism can also diminish as network strength (α) shrinks, or strengthens as it increases. Furthermore, the effects of network strength on overall productivity through optimal competition intensity may change as the network size varies.

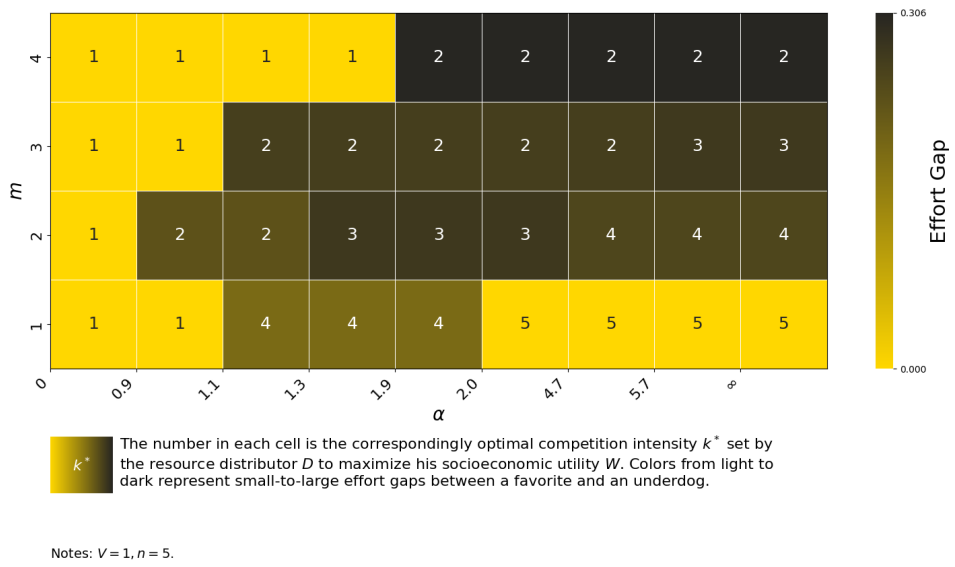


Figure 4 COMPETITION INTENSITY, NETWORK SIZE AND OUTPUTS

Figure 4 depicts the relationship between m, k^*, α, and the gaps between favorites' and underdogs' effort levels $x_{f_s} - x_{u_t}$. As shown in Figure 4, Effort gaps emerge in cases where $1 < k^* < 5$. Greater effort gaps (darker colors) are more likely to appear in cases where both network size m and network strenght α are larger. In particular, when $m \geq 3$ and $\alpha \geq 1.9$, the effort gaps on average are the greatest, compared with cases when $m \leq 2$ or $\alpha < 1.9$. Effort gaps decrease to zero when the superior sets the most intensified competition ($k^* = 1$), or the least intensified competition ($k^* = 5$). This is because in the cases of $k^* = 1$, favorites and underdogs have equal winning chances. In the cases of $k^* = 5$, both favorites are completely demotivated because the reward rule only relies on their identity rather than their performance rankings.

Theorem 7 *No causal relationship exists between the optimal competition intensity k^* and the effort gaps between favorites and underdogs. This indicates that any level of economic inequality can coexist with any level of overall productivity.*

An interesting observation in Figure 4 is that there is no causal relationship between the optimal competition intensity k^* and effort gaps. For instance, greater overall productivity can coexist with both the smallest effort gaps ($k^* = 1$) and the largest effort gaps ($k^* = 2$, $m = 4$). Conversely, lower overall productivity can be associated with the smallest effort gaps ($k^* = 5$) and larger effort gaps ($k^* = 3$, $m = 2$). So long as the optimal competition intensity k^* is not equal to 1 or 5, the interplay of network size and strength determines the combination of effort gaps and overall productivity in an unpredictable manner.

Practical Implications

According to our analysis, different levels of network size and strength derived from informal networks may lead superiors to select different levels of competition intensity to serve their private interests. This can result in divergence in productivity across countries with different social backgrounds.

This socioeconomic mechanism may offer insights to some long-term perplexing phenomena.

For more than half a century, scholars have investigated the associations between political regimes and countries' economic performance, yet they continue to struggle to reach a consensus. Some scholars assert that democracy fosters economic growth (Lipset 1960; Persson and Tabellini 2006; Acemoglu et al. 2019). Others argue that the overall effect of democracy on economic growth is negative (Huntington 1973; Tavares and Wacziarg 2001; Mobarak 2005). Other researchers even indicate that the effect of democracy on economic performance is neutral (Przeworski 2000; Gerring et al. 2005).

We attribute these conflicting conclusions to the inherent complexities and varying social contexts within the political perspective itself. Rather than any specific type of political regime, we argue the varying competition intensity may determined by particular social factors (network size and strength) of a country, that causes divergence in overall productivity at the country level. Specifically, our model indicates that any degree of competition intensity and corresponding different productivity levels can occur in any type of political regime, as each regime type can be located at different unit cells in Figure 3.

For instance, non-democratic countries, characterized by relatively weak formal institutions where superiors may heavily rely on their personal ties or political loyalists

(relatively greater network strength α), can experience less intensified competition intensity and lower economic performance when the number of political loyalists or individuals tied to them is small (at the bottom right of Figure 3). On the other hand, more intensified competition rule and higher economic performance can occur in non-democratic countries if the network size in their society is greater (located at the top right of Figure 3). This mechanism echoes Blanchard and Shleifer's (2001) observation between Russia and China. In their research, they find that China favors growth much more than Russia because China has a high level of political centralization while Russia does not.

Similarly, democratic countries may experience one of two outcomes based on the strength of their formal institutions and social structures. If their institutions are strong, limiting superiors' reliance on personal ties (resulting in smaller α), they can achieve highly intensified competition and high economic performance. However, if their institution weakens (greater network strength) and network size shrinks, they may encounter low level of competition and low economic performance.

This socioeconomic perspective also applies to productivity differences at the firm level. It indicates that a firm's productivity is determined by the competition intensity, which is determined by the leader's network structure (network size and strength) within the firm. To some extent, our results are reflected in Bloom and Van Reene's (2007, 2010) empirical findings. In their research, they find that companies that select the CEO from all family members perform better than those that select the CEO based on primogeniture. They also notice that European product market competition levels are lower than those in the United States because of the fact that primogeniture is much more common in Europe.

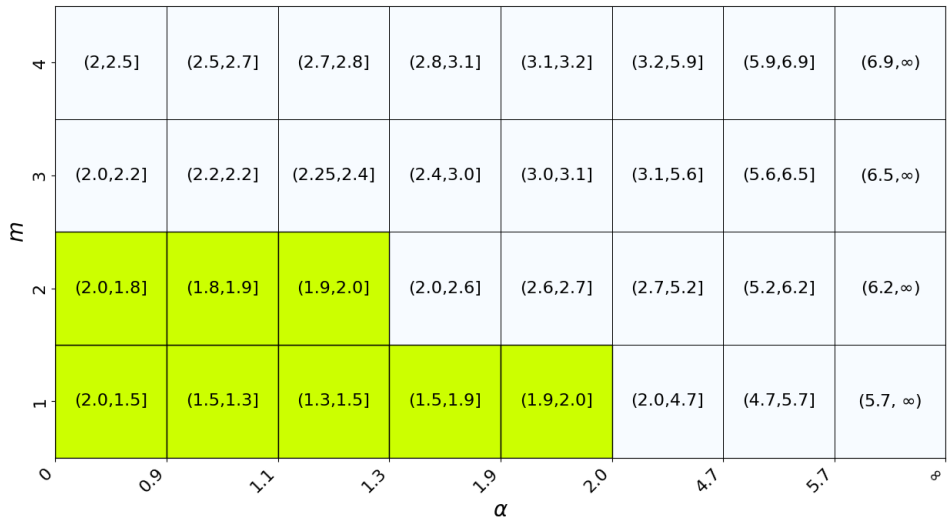
Second, we also provide a new perspective on understanding economic inequality. Unlike those studies trying to demonstrate correlations between economic growth and inequality, our model indicates no direct link between the two. According to our model shown in Figure 4, economic inequality (gaps between effort levels of favorites and underdogs) and overall productivity are independent products of competition intensity. It is possible that extremely low inequality accompanies both the highest and lowest levels of productivity (when $k^* = 1$ or $k^* = 5$). Conversely, the severe inequality can coexist with high productivity ($m = 4, k^* = 2$). In general, inequality is a relatively less severe issue when network size is smaller ($m \leq 3$), but it has some exceptions ($m = 2$, $k^* = 3$).

To summarize, in practice, a high level of overall productivity ($k^* = 1$) can be realized without economic inequality by enlarging network size and weakening network strength. Otherwise, it may be unavoidable to experience either both productivity inefficiency or economic inequality.

Extention: the Optimal Network Size and the Tipping Point of Network Strength

The analysis above assumes that selecting the optimal competition intensity (k^*) is the only strategy available to the superior D. This assumption holds in scenarios where network size (m) and the strength of network (α) are determined exogenously. However, in real-world organizations, the superior is capable of deciding both (m) and (α). For example, through institutional reform or political struggle, the superior may exercise discretionary power over network size by fostering more or fewer competitors into his network, or indirectly influence the network strength by enhancing or weakening the irreplaceability of the resource (V).

We now relax this assumption by allowing the superior D not only to choose (k^*), but also to adjust both the network size (m) and strength (α). We then investigate the optimal values of (m) and (α) that maximize D's socioeconomic utility (W).



Notes: The interval in each cell represents the range for *max*(*W*), the maximized socioeconomic utility of the resource distributor *D* within the specified range of *α* and the value of *m*.
Notes: $V = 1, n = 5$.

Figure 5 Superior D's Maximized Utility Ranges

Figure 5 presents the range of the superior D's maximized utility W^* across corresponding values of α and m and k^*. We first analyze the optimal m^* given α, then followed by an analysis of the optimal α^* given m.

In Figure 5, we find the range of utility W^* in each column (given α) increases as network size m increases. It indicates largest m is the optimal m^* for the superior.

Theorem 8 *Regardless of the strength of network, increasing network size is always a dominant strategy for the superior. Because W^* increases as m increases.*

The proof is straightforward. Developing a new relationship with an additional competitor ($m + 1$) increases the likelihood of rewarding favorites. As a result, the superior's expected network benefit ($P_f\alpha - P_u\alpha$) increases, which subsequently enhances the superior's utility. This finding justifies the prevalence of informal networks in practice.

The relationship between network strength α and superior D's maximized utility W^* is more complex than the correlation between the network size m and W^*. Therefore, to analyze the optimal α^*, we need to consider the situation on a case-by-case basis.

Figure 5 divides the pattern into two cases: some unit cells are highlighted in neon green, while others remain uncolored. Holding m constant, areas in neon green represent cases where the superior is demotivated to increase α if the initial level of the network strength is extremely low. We name the largest value of α in the colored row as tipping point α_{tp}.[12] Above the tipping point in the uncolored areas, the superior D is motivated to increase the network strength α.

Let's examine the ranges of utility W^* in each row from left to right in the uncolored area. Holding m constant, there is a clear positive relationship between the network strength α and the superior D's maximized utility W^* in the uncolored areas. This implies that holding network size constant, enhancing the strength of the network is a dominant strategy for the superior if he is able to increase network strength above the tipping point α_{tp}. For instance, when $m = 1$, the optimal value of α is the largest possible $\alpha \to \infty$, if the superior can implement an α greater than 2.0, approaching infinity.

However, if the superior D can only achieve a relatively smaller range of α in the neon green areas (e.g., $\alpha \in [0, 2.0]$ when $m = 1$), the superior D can be demotivated to increase α. Specifically, in the colored areas, the superior D's maximized utility W^* reaches its highest level in the leftmost unit cell of each row, meaning that given m constant, any strategy to increase α within the colored areas is either dominated or indifferent for the superior D. In other words, holding m constant, an organization initially with an extremely weak network tie (e.g., $0 < \alpha \leq 0.9$) can remain its status quo because the superior is demotivated to strengthen his reliance on the surbordinate network if he is constrained to increase network strength α below the tipping point α_{tp}.

Theorem 9 *There exist tipping points α_{tp} that can shift the superior D's motivation to increase α. The optimal $\alpha^* \to 0$ when the initial network strength lies within the range of (0, 0.9] and D is constrained to keeping α below α_{tp}. If D is capable of increasing α above the tipping point α_{tp} ($\alpha_{tp} = 2.0$ when $m = 1$; $\alpha_{tp} = 1.3$ when $m = 2$), he will prefer the largest α available.*

[12] For example, $\alpha_{tp} = 2.0$ when $m = 1$; and $\alpha_{tp} = 1.3$ when $m = 2$.

The formation of tipping points α_{tp} occurs in the combination of small network size and weak network strength. A smaller network size makes it much harder to reward a favorite due to the smaller number of favorites. The reduced likelihood of rewarding a favorite directly leads to a negative expected network gain in D's utility W^*. Therefore, when both the network size m and strength α are at extremely low levels, slightly increasing network strength decreases D's maximized utility W^*. This negative relationship between α and W^* persists until D is able to select an α that exceeds a specific point, known as the tipping point α_{tp}. This is because once the superior selects an optimal α^* above the tipping point α_{tp}, it becomes lucrative enough for him to adjust the likelihood of rewarding a favorite to be higher than that of rewarding an underdog by loosening the optimal competition intensity k^*. This increase in k^* ultimately shifts the negative expected network gain $\alpha(P_f - P_u) < 0$ to positive ($\alpha(P_f - P_u) > 0$) in his utility function. Therefore, once the superior can select $\alpha > \alpha_{tp}$, D is motivated to choose the largest α available.

Theorem 10 *When network size m increases, the value of tipping point α_{tp} gets smaller (when $m = 2$), or disappears (when $m > 2$). It indicates the superior D's preference for weaker network and highly intensified competition ($k^* = 1$) is less sustainable as the network size increases.*

This is easy to understand. As m increases, the likelihood of rewarding a favorite rises. This makes a small increase in α sufficient to turn negative network gain into positive by reducing the optimal competition intensity k^* (P_f increases). Thus, as m grows larger, α_{tp} becomes smaller. When m is sufficiently large, the expected network gain in D's maximized utility W^* is consistently positive ($\alpha(P_f - P_u) > 0$), and the increase in expected network gain from raising α compensates for the corresponding loss in overall productivity caused by the reduced optimal competition intensity k^*. Consequently, the tipping point disappears when m becomes sufficiently large ($m = 3$ and $m = 4$)[13].

Implications for the Stability of Economic Development

The analysis in Section IV tells us that an organization or society can drive economic growth through highly intensified competition by institutional designs that increase network size m or decrease network strength α. To further achieve high economic growth without inequality, the analysis suggests that institutional reform should gener-

[13] See the superior D's maximized utility expression in Appendix

ally guarantee a very weak network or prudently balance m and α to ensure the optimal competition intensity $k^* = 1$.

These findings provide valuable guidance for institutional design aimed at improving productivity and reducing economic inequality. However, they do not address the stability of these conditions over time. In other words, our basic model does not provide insights into whether such an ideal economic state—characterized by high productivity and low inequality—can be sustained in the long term.

To address this gap, Section V extends the basic model and offers additional insights regarding the stability of economic development.

In summary, the analysis in Section V suggests that maintaining the stability of an ideal economic state is challenging. This is because, regardless of the combination of network size and strength in an organiztion or society, the superior always prefers to increase network size. This finding implies that when the superior's discretion over network size is unrestricted, even if an organization or society is initially at the optimal state depicted in the bottom-left cell of Figure 5 ($m = 1$, $\alpha \leq 0.9$), it can eventually shift to the top-right cell ($m = 4$, $\alpha > 6.9$), where economic inequality is most severe. Although the superior may be motivated to weaken network strength within the green area of Figure 5, a further increase in network size will ultimately lead to the displacement outside the green area. Once the organization or society moves outside the green area, the superior's preference shifts towards strengthening his reliance on the network. As a consequence, the displacement speeds up.

In the real world, the superior's discretion on the level of network size and strength may be restricted into any combination because of intricate and complex factors that are beyond our theory. This may lead to different organization and societies locate in various unit cells, presenting volatile states of economic development. And the status of their economic development can change to any unit cell depending on the power restriction to the superior.

To sustain a high level of overall productivity without inequality, the key technique is to implement institutions that demotivate the superior to increase the number of his network members and his reliance on his network up to the tipping point (α_{tp}). If the institution fails to constrain the superior's discretion over network size (m) or strength (α) below the tipping point, the organization or society may face risks that the superior sets policies to increase either or both. Changes in network size (m) or strength (α) can indirectly cause fluctuations in overall productivity and economic inequality by altering the superior's optimal competition intensity (k^*). This may potentially lead the organization or society into economic and social instability. For example, if the superior increases α from an initial state of $m = 1$ and $\alpha = 2$, it leads to economic collapse (with $k^* = 5$ and zero overall productivity ($mx_{f_s} + (n - m)x_{u_t} = 0$). Similarly, if the superior increases the number of his favorites from $m = 1$ to $m = 4$, he will be further moti-

vated to choose the largest α available, which ultimately leads to the greatest economic inequality, even if it results in the second-highest overall productivity ($k^* = 2$).

Conclusion

This paper propose a new competition theory to explain internal competitive dynamics within organizations and productivity divergence across organizations and societies. We challenge the traditional assumptions of productivity maximization and the exogeneity of competition intensity in classic competition theories. Using a multifaceted principal-agent model, we explore how social structures dynamically shape competition intensity in the hierarchical structure of organizations and drive variations in productivity. Our theory contributes to a broader understanding of competition and productivity in social science, with implications across multiple fields.

There remain several open questions that future research could address. We highlight two in particular. First, the generalization of the model presents a valuable direction for exploration. In this study, we focused specifically on the case of $n = 5$ due to mathematical limitations. However, extending the model could provide insights into how the number of competitors influences competition intensity, and how these factors interact with network size and strength. Second, while our findings offer a theoretical foundation for designing institutions that enhance overall productivity and reduce economic inequality, and while we discuss the challenges of sustaining ideal economic development in organizations and societies, practical strategies for shaping an optimal social structure—one that promotes higher productivity alongside reduced inequality—lie beyond the scope of our current theory and warrant further investigation.

Appendix

Theories of Winning Likelihood

The mathematical expression of the winning likelihood applied in this paper is based on Berry's (1993) Multi-winner Success Function. To understand Berry's (1993) Multi-winner Success Function and the extension of the winning likelihood function applied in our paper, it is necessary to introduce the Contest Success Function (CSF) and the Tullock Success Function (TSF) as the baseline.

The CSF is a core component of a contest situation, which depicts each competitor's winning likelihood given others' level of effort. Let $x = (x_1, x_2, ..., x_n)$ denote a vector of efforts for n players. Each player i's winning probability is denoted by $Prob^i(x)$ ($Prob^i(x) : [0, X]^n \to \mathbb{R}$ where $X > x_i$ for all $i \in N$). For a contest that only has one

winner, each player's winning probability should meet the following three properties (Skaperdas 1996):

(1A)$\sum_{i\in N} Prob^i(x) = 1$ and $Prob^i(x) \geq 0$ for all $i \in N$ and all x; if $x_i > 0$ then $Prob^i(x) > 0$.

(2A) For all $i \in N$, $Prob^i(x)$ is increasing in x_i and decreasing in x_j for all $j \neq i$.

(3A) For any permutation π of N (i.e., a bijection $\pi : N \to N$), it has

$$Prob^{\pi(i)}(x) = Prob(x_{\pi_1}, x_{\pi_2}, ..., x_{\pi_n}) \forall i \in N.$$

Tullock Success Function (TSF) is expressed below,

$$Prob^i(x) = \frac{x_i^r}{x_1^r + x_2^r + ... + x_n^r} \tag{10}$$

TSF is a special type of CSF that meets the above three properties. It depicts the winning likelihood for an individual competitor in a single-winner contest. In the mathematical expression of TSF, the nominator only includes information relating to competitor i's effort level x_i because competitor i is the only winner in the contest. The denominator $\sum_{i=1}^{n} x_i^r$ is the sum of all information about each competitor's effort levels, and r refers to an index of external uncertainty/ the marginal cost of influencing the probability of winning (Rai and Sarin 2009). In a single-winner contest setting, the ratio of nominator and denominator of TSF is the winning likelihood for competitor i, given other competitors' effort level $x_{\bar{i}}$ and the uncertainty level r.

TSF functions as the baseline for Barry's Multi-winner Success Function. In a contest with k winners ($1 < k \leq n$), each player's winning probability would be more significant than when there is one winner. Hence, to describe the winning probability for each competitor in a multi-winner contest, Berry (1993) made some adjustments to TSF while keeping the three properties of CFS.

Let $Prob_k^i(x)(i = 1, 2, ...n)$ denote competitor i's probability to get into the top k. In other words, $Prob_k^i(x)(i = 1, 2, ...n)$ depicts competitor i's winning likelihood in a k-winner contest. We have the expression of $Prob_k^i(x)$ below.

The numerator displays all possible combinations of winner i and the other $k-1$ winners, which are denoted as $j \neq i$. The expression in each bracket $(x_{j_q^1}^r + x_{j_q^2}^r + ... + x_{j_q^{k-1}}^r)$, where $q = 1, ..., \binom{n-1}{k-1}$ represents the qth set of different combinations of the other winner j 's efforts.

In a k-winner contest of n players, there are $\binom{n-1}{k-1}$ possibilities for player i ($i = 1, ..., n$) to get into top k qualified, where $\binom{n-1}{k-1}$ represents the combination of $n-1$ objects taking $k-1$ at a time. $(x_{z_p^1}^r + x_{z_p^2}^r + ... + x_{z_p^k}^r)$, $p = 1, ..., \binom{n}{k}$ in the denominator denotes the pth combination of any players that is possible to win the contest. Similarly, there are $\binom{n-1}{k-1}$ winning combinations in total. The ratio between the numerator and

the denominator is calculated as the winning probability for competitor i in a multi (k)-winner competition.

$$
\begin{aligned}
Prob_k^i(x) = & \frac{\begin{array}{l} x_i^r + (x_{j_1^1}^r + x_{j_1^2}^r + ... + x_{j_1^{k-1}}^r) + x_i^r + (x_{j_2^1}^r + x_{j_2^2}^r + ... + x_{j_2^{k-1}}^r) \\ +... + x_i^r + \left(\begin{matrix} x_{j^1}^r \\ \binom{n-1}{k-1} \end{matrix} + \begin{matrix} x_{j^2}^r \\ \binom{n-1}{k-1} \end{matrix} + ... + \begin{matrix} x_{j^{k-1}}^r \\ \binom{n-1}{k-1} \end{matrix} \right) \end{array}}{\begin{array}{l} (x_{z_1^1}^r + x_{z_1^2}^r + ... + x_{z_1^k}^r) + (x_{z_2^1}^r + x_{z_2^2}^r + ... + x_{z_2^k}^r) \\ +... + \left(\begin{matrix} x_{z^1}^r \\ \binom{n}{k} \end{matrix} + \begin{matrix} x_{z^2}^r \\ \binom{n}{k} \end{matrix} + ... + \begin{matrix} x_{z^k}^r \\ \binom{n}{k} \end{matrix} \right) \end{array}} \\
= & \frac{\binom{n-1}{k-1} x_i^r + \binom{n-2}{k-2} \sum_{j \neq i} x_j^r}{\binom{n-1}{k-1} \sum_{z=1}^{n} x_z^r}
\end{aligned}
$$

Where $j \neq i$, $j, i \in N$.

Calculation Details for Section II

When $n - m \geq k$,

$$
\begin{aligned}
\frac{\partial P_{f_s}}{\partial x_{f_s}} = & \frac{\begin{array}{l} \sum_{i=1}^{\min(m,k)} \frac{1}{i} \binom{m-1}{i-1} \binom{n-m}{k-i} [x_{f_s} + (m-1)x_{f_s} + (n-m)x_{u_t}] \\ - \Big[x_{f_s} \sum_{i=1}^{\min(m,k)} \frac{1}{i} \binom{m-1}{i-1} \binom{n-m}{k-i} + (m-1)x_{f_s} \sum_{i=2}^{\min(m,k)} \frac{1}{i} \binom{m-2}{i-2} \binom{n-m}{k-i} \\ +(n-m)x_{u_t} \sum_{i=1}^{\min(m,k)} \frac{1}{i} \binom{m-1}{i-1} \binom{n-m-1}{k-i-1} \Big] \end{array}}{\binom{n-1}{k-1} [x_{f_s} + (m-1)x_{f_s} + (n-m)x_{u_t}]^2} \\
= & \frac{(\binom{n-m}{k-1} + \sum_{i=2}^{\min(m,k)} \frac{1}{i} \binom{m-2}{i-1} \binom{n-m}{k-i})(m-1)x_{f_s} + (n-m)x_{u_s} \sum_{i=1}^{\min(m,k)} \binom{m-1}{i-1} \binom{n-m-1}{k-i}}{\binom{n-1}{k-1} [x_{f_s} + (m-1)x_{f_s} + (n-m)x_{u_t}]^2} \\
= & \frac{x_{f_s} \sum_{i=1}^{min(m,k)} \binom{m-1}{i} \binom{n-m}{k-i} + \frac{n-m}{m} x_{u_t} \sum_{i=1}^{min(m,k)} \binom{m}{i} \binom{n-m-1}{k-i}}{\binom{n-1}{k-1} [m x_{f_s + (n-m)x_{u_t}}]^2}
\end{aligned}
\tag{12}
$$

When $n-m<k$, we get:

$$\begin{aligned}\frac{\partial P_{f_s}}{\partial x_{f_s}} &= \frac{(m-1)x_{f_s}(\sum_{i=0}^{n-m}\frac{1}{k-i}\binom{n-m}{i}\binom{m-1}{k-i-1}-\sum_{i=0}^{n-m}\frac{1}{k-i}\binom{n-m}{i}\binom{m-2}{k-i-2})}{\binom{n-1}{k-1}[x_{f_s}+(m-1)x_{f_s}]^2} \\ &= \frac{(m-1)x_{f_s}\sum_{i=0}^{n-m}\frac{1}{k-i}\binom{n-m}{i}\binom{m-2}{k-i-1}}{\binom{n-1}{k-1}[x_{f_s}+(m-1)x_{f_s}]^2} \\ &= \frac{\sum_{i=0}^{n-m}\binom{n-m}{i}\binom{m-1}{k-i}}{\binom{n-1}{k-1}m^2x_{f_s}}\end{aligned} \tag{13}$$

When $n-m\geq k$, the calculation of $\frac{\partial P_{u_t}}{\partial x_{u_t}}$ is:

$$\begin{aligned}\frac{\partial P_{u_t}}{\partial x_{u_t}} &= \frac{\binom{n-m-1}{k-1}[x_{u_t}+(n-m-1)x_{u_t}+mx_{f_s}]-[\binom{n-m-1}{k-1}x_{u_t}+\binom{n-m-2}{k-2}(n-m-1)x_{u_t}]}{k\binom{n-1}{k-1}[x_{u_t}+(n-m-1)x_{u_t}+mx_{f_s}]^2} \\ &= \frac{\binom{n-m-1}{k-1}[mx_{f_s}+(n-m)x_{u_t}]-[\binom{n-m-1}{k-1}x_{u_t}]+\binom{n-m-2}{k-2}(n-m-1)x_{u_t}}{k\binom{n-1}{k-1}[mx_{f_s}+(n-m)x_{u_t}]^2} \\ &= \frac{\binom{n-m-1}{k-1}mx_{f_s}+(n-m-1)\binom{n-m-2}{k-1}x_{u_t}}{k\binom{n-1}{k-1}[mx_{f_s}+(n-m)x_{u_t}]^2} \\ &= \frac{k\frac{m}{n-m}\binom{n-m}{k}x_{f_s}+k\binom{n-m-1}{k}x_{u_t}}{k\binom{n-1}{k-1}[mx_{f_s}+(n-m)x_{u_t}]^2} \\ &= \frac{\frac{m}{n-m}\binom{n-m}{k}x_{f_s}+\binom{n-m-1}{k}x_{u_t}}{\binom{n-1}{k-1}[mx_{f_s}+(n-m)x_{u_t}]^2}\end{aligned} \tag{14}$$

By combining equation (14) and equation (15), we get the expression of x_{f_s} as a function of x_{u_t} when $n-m\geq k$ as follows:

$$\frac{x_{f_s}\sum_{i=1}^{\min[m,k]}\binom{m-1}{i}\binom{n-m}{k-i}+\frac{n-m}{m}x_{u_t}\sum_{i=1}^{\min[m,k]}\binom{m}{i}\binom{n-m-1}{k-i}}{x_{f_s}}=\frac{\binom{n-1}{k-1}[mx_{f_s}+(n-m)x_{u_t}]^2}{V}$$

$$\Rightarrow \frac{mx_{f_s}\sum_{i=1}^{\min[m,k]}\binom{m-1}{i}\binom{n-m}{k-i}+(n-m)x_{u_t}\sum_{i=1}^{\min[m,k]}\binom{m}{i}\binom{n-m-1}{k-i}}{mx_{f_s}}$$

$$=\frac{m\binom{n-m}{k}x_{f_s}+(n-m)\binom{n-m-1}{k}x_{u_t}}{(n-m)x_{u_t}} \quad (15)$$

$$\Rightarrow x_{f_s}=\frac{\sum_{i=1}^{\min[m,k]}\binom{m-1}{i}\binom{n-m}{k-i}-\binom{n-m-1}{k}+\left[(\sum_{i=1}^{\min[m,k]}\binom{m-1}{i}\binom{n-m}{k-i}-\binom{n-m-1}{k})^2+4\binom{n-m}{k}\sum_{i=1}^{\min[m,k]}\binom{m}{i}\binom{n-m-1}{k-i}\right]^{\frac{1}{2}}}{\frac{2m}{n-m}\binom{n-m}{k}}x_{u_t}$$

Solving for Equilibrium

In this subsection, we use backward induction to analyze the optimal behaviors of the m favorites, the $n-m$ underdogs, and superior D given V, m, n, and k.

After observing k set by the superior D, favorite s and underdog t maximize their own expected utilities by choosing their effort levels x_{f_s} and x_{u_t} respectively.

For favorite $s(s=1,2,...,m)$, given k, $x_{f_{\bar{s}}}(\bar{s}=1,...,s-1,s+1,...,m-1)$, and $x_{u_t}(t=1,...,n-m)$, she chooses her effort level x_{f_s} to maximize her expected function in Equation (1). Thus, the first-order condition of favorite s' s utility function is:

$$\frac{\partial P_{f_s}}{\partial x_{f_s}}V-x_{f_s}=0 \quad (16)$$

where

$$\frac{\partial P_{f_s}}{\partial x_{f_s}}=\begin{cases}\dfrac{x_{f_s}\sum_{i=1}^{min[m,k]}\binom{m-1}{i}\binom{n-m}{k-i}+\frac{n-m}{m}x_{u_t}\sum_{i=1}^{min[m,k]}\binom{m}{i}\binom{n-m-1}{k-i}}{\binom{n-1}{k-1}[mx_{f_s+(n-m)x_{u_t}}]^2} & \text{if } n-m\geq k\\ \dfrac{\sum_{i=0}^{n-m}\binom{n-m}{i}\binom{m-1}{k-i}}{\binom{n-1}{k-1}m^2x_{f_s}}, & \text{if } n-m<k\end{cases} \quad (17)$$

For underdog $t(t=1,2,...,n-m)$, given k, $x_{u_{\bar{t}}}(\bar{t}=1,...,t-1,t+1,...,n-m-1)$ and $x_{f_s}(s=1,...,m)$, she chooses her effort level x_{u_t} to maximize her expected function in Equation (2). The first-order condition of underdog t is:

$$\frac{\partial P_{u_t}}{\partial x_{u_t}}V-x_{u_t}=0 \quad (18)$$

$$\frac{\partial P_{u_t}}{\partial x_{u_t}} = \begin{cases} \frac{\frac{m}{n-m}\binom{n-m}{k}x_{f_s}+\binom{n-m-1}{k}x_{u_t}}{\binom{n-1}{k-1}[mx_{f_s}+(n-m)x_{u_t}]^2} & \text{if } n-m \geq k \\ 0, & \text{if } n-m < k \end{cases} \tag{19}$$

Bring Equation (11) to (10). When $n-m \geq k$, we get:

$$A = \frac{x_{f_s} \sum_{i=1}^{\min[m,k]} \binom{m-1}{i}\binom{5-m}{k-i} + \frac{5-m}{m}x_{u_t} \sum_{i=1}^{\min[m,k]} \binom{m}{i}\binom{5-m-1}{k-i}}{x_{f_s}} \tag{20}$$

where $A = \frac{\binom{n-1}{k-1}[mx_{f_s}+(n-m)x_{u_t}]^2}{V}$.

Bring Equation (13) to (12). When $n-m \geq k$, we get:

$$A = \frac{\frac{m}{n-m}\binom{n-m}{k}x_{f_s} + \binom{n-m-1}{k}x_{u_t}}{x_{u_t}} \tag{21}$$

By calculating the combination of (14) and (15), we get the positive relationship between favorite's effort level $x^*_{f_s}$ and underdog's effort level $x^*_{u_t}$ when $n-m \geq k$ in the partial equilibrium:

$$x^*_{f_s} = Bx^*_{u_t} \ (B > 0) \text{ if } n-m \geq k \tag{22}$$

where $B = \dfrac{\sum_{i=1}^{\min[m,k]} \binom{m-1}{i}\binom{n-m}{k-i} - \binom{n-m-1}{k} + \left[\left(\sum_{i=1}^{\min[m,k]} \binom{m-1}{i}\binom{n-m}{k-i} - \binom{n-m-1}{k}\right)^2 + 4\binom{n-m}{k}\sum_{i=1}^{\min[m,k]} \binom{m}{i}\binom{n-m-1}{k-i}\right]^{\frac{1}{2}}}{\frac{2m}{n-m}\binom{n-m}{k}}$.

Theorem 11 *When $n-m \geq k$, the positive relationship between $x^*_{f_s}$ and $x^*_{u_t}$ in the partial equilibrium ($x^*_{f_s} = Bx^*_{u_t}$) describes the competition relationship between favorites and underdogs. The strength of the competition relationship ($B > 0$) depends on competition intensity k, the number of favorites m, and the total number of competitors n.*

Bring (16) back to (10) and (12), we get the expression of $x^*_{f_s}$ and $x^*_{u_t}$ when $n-m \geq k$:

$$x^*_{f_s} = B\left[\frac{V\binom{n-m}{k}}{\binom{n-1}{k-1}(n-m)(n-m+Bm)}\right]^{\frac{1}{2}} \text{ if } n-m \geq k \tag{23}$$

$$x_{u_t}^* = \left[\frac{V\binom{n-m}{k}}{\binom{n-1}{k-1}(n-m)(n-m+Bm)} \right]^{\frac{1}{2}} \text{ if } n-m \geq k \tag{24}$$

When $n-m<k$, the winning likelihood for any underdog ($P_{u_t}(t=1,...,n-m)$) is zero. Thus underdog t exerts zero effort $x_{u_t}^* = 0$ in the equilibrium. In this situation, favorite s's $(s=1,...,m)$ partial equilibrium effort level x_{f_s} is:

$$x_{f_s}^* = \frac{1}{m} \left[\frac{\sum_{i=0}^{n-m} \binom{n-m}{i}\binom{m-1}{k-i}}{\binom{n-1}{k-1}} V \right]^{\frac{1}{2}} \text{ if } n-m<k \tag{25}$$

In summary, favorite s and underdog t's partial equilibrium effort levels $x_{f_s}^*$ and $x_{u_t}^*$ are:

$$x_{f_s}^* = \begin{cases} B\left[\frac{V\binom{n-m}{k}}{\binom{n-1}{k-1}(n-m)(n-m+Bm)} \right]^{\frac{1}{2}} & \text{if } n-m \geq k \\ \frac{1}{m}\left[\frac{\sum_{i=0}^{n-m} \binom{n-m}{i}\binom{m-1}{k-i}}{\binom{n-1}{k-1}} V \right]^{\frac{1}{2}}, & \text{if } n-m<k \end{cases} \tag{26}$$

$$x_{u_t}^* = \begin{cases} \left[\frac{V\binom{n-m}{k}}{\binom{n-1}{k-1}(n-m)(n-m+Bm)} \right]^{\frac{1}{2}} & \text{if } n-m \geq k \\ 0, & \text{if } n-m<k \end{cases} \tag{27}$$

Superior D chooses k^* to maximize his socioeconomic utility function in Equation (8) based on $x_{f_s}^*$ and $x_{u_t}^*$:

$$k^* = \arg\max_k \left[W = \begin{cases} \left[\frac{V\binom{n-m}{k}(n-m+Bm)}{\binom{n-1}{k-1}(n-m)} \right]^{\frac{1}{2}} + \alpha \left[1 - \frac{2(n-m)\binom{n-m-1}{k-1}}{\binom{n-1}{k-1}(n-m+Bm)} \right] & \text{if } n-m \geq k^* \\ \left[V \frac{\sum_{i=0}^{n-m} \binom{n-m}{i}\binom{m-1}{k-i}}{\binom{n-1}{k-1}} \right]^{\frac{1}{2}} + \alpha, & \text{if } n-m<k^* \end{cases} \right] \tag{28}$$

Table 1 PARTIAL EQUILIBRIUM EXAMPLE WITH $n = 5$

G / x_{f_s} / x_{u_t}		k = 1	k = 2	k = 3	k = 4	k = 5
m	0	2 N/A 0.400	1.225 N/A 0.245	0.816 N/A 0.163	0.500 N/A 0.100	0 N/A 0
	1	2 0.400 0.400	1.225 0.408 0.204	0.816 0.350 0.117	0.500 0.250 0.063	0 0 0
	2	2 0.400 0.400	1.225 0.383 0.153	0.816 0.327 0.054	0.500 0.250 0	0 0 0
	3	2 0.400 0.400	1.225 0.350 0.087	0.816 0.272 0	0.500 0.167 0	0 0 0
	4	2 0.400 0.400	1.225 0.306 0	0.816 0.204 0	0.500 0.125 0	0 0 0
	5	2 0.400 N/A	1.225 0.245 N/A	0.816 0.163 N/A	0.500 0.100 N/A	0 0 N/A

G is the total output ($G = mx_{f_s} + (n - m)x_{u_t}$). x_{f_s} is favorite s's partial equilibrium effort level. x_{u_t} is underdog u's partial equilibrium effort level. k is competition intensity. m is the number of favorites.

Charts and Figures

Table 2 THE SUPERIOR'S UTILITY

W		k = 1	k = 2	k = 3	k = 4	k = 5
m	0	2	$\frac{\sqrt{6}}{2}$	$\frac{\sqrt{6}}{3}$	$\frac{1}{2}$	0
	1	$2 - \frac{3}{5}\alpha$	$\frac{\sqrt{6}}{2}$	$\frac{\sqrt{6}}{3} + \frac{3\alpha}{7}$	$\frac{1}{2} + \frac{3\alpha}{4}$	α
	2	$2 - \frac{\alpha}{5}$	$\frac{\sqrt{6}}{2} + \frac{5\alpha}{8}$	$\frac{\sqrt{6}}{3} + \frac{14\alpha}{15}$	$\frac{1}{2} + \alpha$	α
	3	$2 + \frac{\alpha}{5}$	$\frac{\sqrt{6}}{2} + \frac{13\alpha}{14}$	$\frac{\sqrt{6}}{3} + \alpha$	$\frac{1}{2} + \alpha$	α
	4	$2 + \frac{3\alpha}{5}$	$\frac{\sqrt{6}}{2} + \alpha$	$\frac{\sqrt{6}}{3} + \alpha$	$\frac{1}{2} + \alpha$	α
	5	2α	$\frac{\sqrt{6}}{2}\alpha$	$\frac{\sqrt{6}}{3}\alpha$	$\frac{1}{2}\alpha$	α

*

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