Household Consumption, Informality, and the Transmission of Monetary Policy*

Mohamed Safouane Ben Aïssa[a,1], Firas Nfikha[b,1,*]

[a] *FSEGT, University of Tunis ElManar, LaReQuad, Tunis, Tunisia*

[b] *FSEGT, University of Tunis ElManar,LaReQuad, Tunis, Tunisia*

Abstract

We measure the decrease in household spending on informal goods as total expenditure increases by estimating the slope of the Informality Engel Curve from survey data. We reproduce the results in a Duel-sector Heterogeneous Agent New Keynesian Model and we evaluate the transmission of monetary policy in an economy characterized by a sizable informal sector. Our calibration and estimation of the model using the Bayesian method affirms the existence of the buffering effect of the informal sector but reveals that unemployed and informal workers are more vulnerable to the redistributive effect of monetary policy shocks than their formal counterparts in general, especially at lower wealth levels.

Keywords: Monetary policy, Heterogeneous agent, Informal sector, Informality Engel Curve, Bayesian estimation

JEL: E12, E52, E26, E31

*This is a draft submitted for participation in the NBER conference: "Monetary Policy in Emerging Markets" May 7-8, 2025 – Istanbul, Türkiye

**This document is part of the third chapter in the thesis: Informality, Unemployment, and Monetary Policy: Further Dynamics and Effect.

The authors acknowledge the valuable guidance from Nooman Rebei and Hafedh Bouakez.

*Corresponding author

Contents

1 Introduction **4**

2 Informality from the Prospective of Household Spending **6**

2.1 Heterogeneity in household informal spending. 6

2.2 Identification of the informal budget share 7

2.3 Estimation of the IEC 10

3 A Heterogeneous Agent New Keynesian Model With informality **12**

3.0.1 Households 12

3.0.2 Goods Producers 14

3.0.3 Capital Producer 15

3.0.4 Monetary Policy 15

3.0.5 Government 15

3.0.6 Equilibrium 16

4 Transmission of Monetary Policy: Single Vs Duel Sector Models **17**

4.1 Calibration 17

4.2 Model comparison 19

5 The Transmission of Monetary Policy from an Estimated Model **20**

5.1 Numerical Solution and Estimation Algorithm 20

5.2 Data & Priors 21

5.3 The Transmission of Monetary Policy In Tunisia 22

6 Conclusion **24**

1. Introduction

In recent years, central banks' monetary policy quickly went from accommodating economic recovery following the COVID-19 pandemic to a restrictive stance in response to a worldwide surge in inflation. The effects of this transition were not felt equally among households. Indeed, the monetary policy presents itself as a systematic action that is often based on the evolution of macroeconomic aggregates but affects a wide selection of heterogeneous economic agents and in the process generates asymmetric outcomes, i.e. winners and losers. In the literature, this is often called the "redistributive effect" and recently, a growing number of papers have provided theoretical and empirical evidence for its importance for the transmission of monetary policy. However, these papers only focus on the case of developing economies and, as a result, some key features of emerging market and developing economies are omitted. Of these, a sizable informal sector might be one of the most prominent.

The informal economy, defined in this paper as economic activities that are hidden from official authorities mainly to avoid taxation, can have major implications on the transmission of monetary policy and its redistributive effects in more than one way. On the one hand, the prominence of informal labor is an additional source of employment insecurity and, by extension, increases the probability of uninsurable income shocks, especially in lower wealth quantities. These shocks are at the core of the redistributive effect of monetary policy (Kaplan et al. (2018)). On the other hand, the informal sector can dampen the effects of shocks especially when it comes to household consumption (Castillo and Montoro (2012), Alberola and Urrutia (2020)), which can lead to a lower marginal propensity to consumption (MPC), another important factor in the redistributive effect of monetary policy. The prevalence of informality can also manifest as a large share of household budget used for informal purchases. This share is not constant among households, and in fact, the existence of an Informality Engel Curve (IEC) has been documented for a large set of countries (Bachas et al. (2023)). Since formal and informal sectors react differently to monetary shocks, they generally it is less affected, the existence of the IEC can create an additional buffer effect for lower-income households and add additional distortions to policy transmission. In some way, this effect could hint at a "progressive" [1] nature of monetary policy in the presence of a sizable informal sector.

In the first step, we follow the methodology used in Bachas et al. (2023) and estimate

[1] As in progressive tax.

the IEC using data from the 2021 Tunisian National Survey on Budget, Consumption, and Living Standers (ENBCNV). The identification is based on the classification of the place of purchase of consumption goods to modern and traditional stores. Since modern stores are more likely to comply with tax regulations, we can use this classification to approximate the share of informal spending per household. We propose a baseline identification and 2 calibrated scenarios based on available data. We use the processed data to estimate the slope of the IEC. We found a statistically significant negative ICE slope for the 3 cases supporting the case for a decreasing share of informal depending as income increases and thus a reduced exposure to informal market distortions. We then develop an one asset medium size duel sector Heterogeneous Agent new Keynesian (HANK) model. One sector in the model assumes the role of the informal sector and includes features like lower productivity, tax evasion, and lower friction. We set up the model to recreate the Informality Engel Curve (IEC) we estimated. Using a calibration version, we compare monetary policy transmission with that of a baseline single-sector model. We still confirm the presence of the buffering effect that reduces the impact on Consumption, Investment, and GDP as reported in the literature (Castillo and Montoro (2010), Castillo and Montoro (2012), Alberola and Urrutia (2020)). However, we find a stronger transmission to inflation when compared with the baseline model i.e. a restrictive monetary shock results in a greater reduction in inflation with a smaller loss in GDP and consumption. Finally, we then estimate a set of key parameters for friction and shocks using Bayesian inference. We use the use the toolbox developed in Liu and Plagborg-Møller (2023). Since we use continuous-time HANK models for their advantage (Ahn et al. (2017), Achdou et al. (2021)), we adopt the using use the method proposed in Christensen et al. (2024) based on an exact discrete-time representation for estimating continuous models using discrete data. From the generated results, we found typical reaction functions from output and inflation, showcasing the buffering effect, but an ambiguous reaction from consumption characterized by a long brief increase followed by a prolonged downward deviation. We also found that unemployed and informal workers seem to be most affected by the monetary shock, especially with lower wealth.

The paper will proceed as follows, In section 2, we identify the informal budget share from household consumption data and estimate the slope of the Informality Engel curve (IEC). In section 3, We develop a duel-sector HANK model with heterogeneity that recreates the IEC and we compare the transmission with that of a single-sector HANK model. In section 4, we present our empirical strategy for estimating the model using the Bayesian methods we then report the main results on the transmission for monetary policy and associated

redistributive effect. Finally, we conclude.

2. Informality from the Prospective of Household Spending

Informality is a complex and multifaceted phenomenon especially affecting emerging market economies. Despite its prevalence, it is not well explored in the macroeconomic literature mainly due to the lack of reliable data. Indeed, indicators of informal activity can only be inferred using a variety of methods. For estimates on the size of informal economic activity, methods such as national account discrepancies, general equilibrium simulations, and, the Multiple-Indicators and Multiple-Causes (MIMIC) have been used. Notable works attempting to estimate the size of informal activity include Medina et al. (2018),Medina and Schneider (2019), Elgin et al. (2021), and more recently Asllani et al. (2024). Even then, these estimates present a fuzzy picture of the prevalence of informality. Taking Tunisia as an example, figure 1 presents the evolution of multiple informal size estimates for Tunisia across the years. With the exception of the national account estimate, the three other methods suggest the presence of a shadow economy representing more than 30% of GDP. For its part, the Tunisian National Institute of Statistics (INS) includes an estimate of informal sector activity in their official GDP figures putting it at 27,4% of GDP in 2015. Similarly, informal labor estimates are hard to infer since they rely mostly on direct methods like labor surveys. ILO (2018) provide an estimate of 58.8 % while The Tunisian National Institute for Statistics estimates of the size of informal laborers from 2019 puts it at 44.8% of the working force.

2.1. Heterogeneity in household informal spending.

The prevalence of informality can manifest as a sizable share of household spending used for the consumption of informal products. This manifestation isn't observed to affect households equally. Bachas et al. (2023) document the existence and significance of this type of heterogeneity for a set of countries to assess the redistributive effect of taxes on consumption in developing countries. The authors showcase the existence of a downward-sloping informality Engel curve (IEC): the informal budget share steeply declines with household income and by proxy consumption. The identification of the informal budget share is based on the probability of the place of purchase to comply with taxation. The authors classified data on the purchase place of products mainly to traditional and modern type stores, Transaction

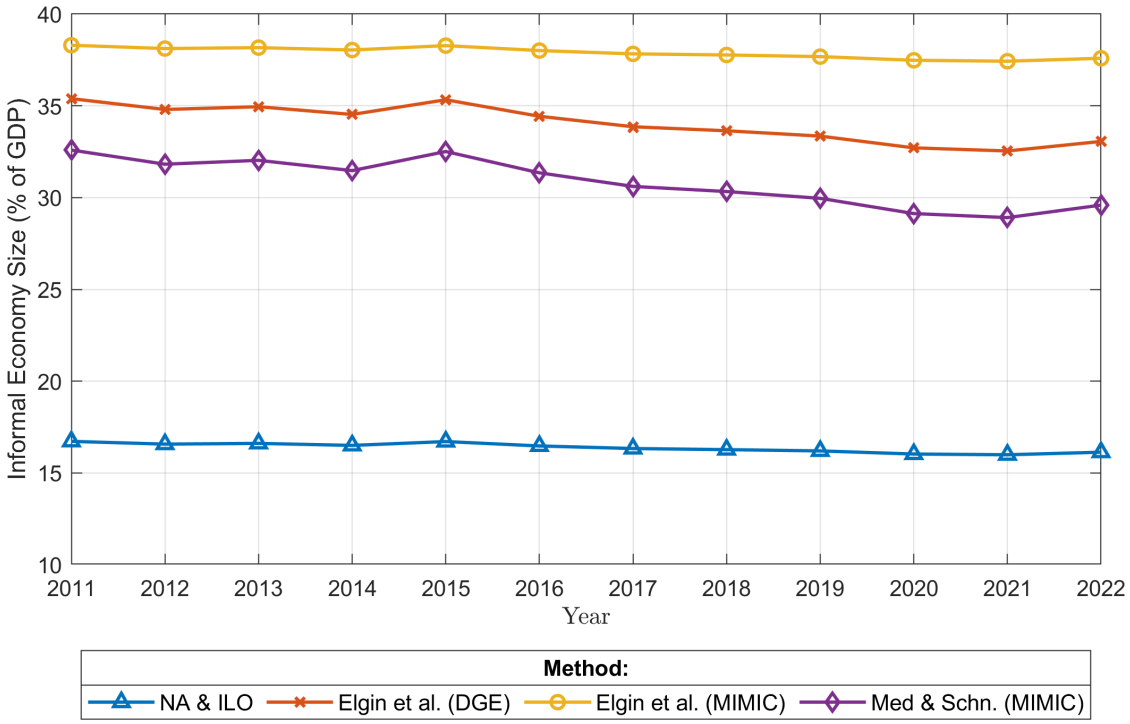


Figure 1: Asllani et al. (2024): The Size of Informal Economy In Tunisia

from each category is later used to identify the per household share of informal spending. Since modern stores are in general larger, employ more people, and are better equipped to hold inventory and accounting records. Also, fiscal authorities are more likely to inspect larger stores making them more likely to comply with taxes. Since factors like household characteristics, store accessibility, and non-homothetic preferences can affect where people shop, they can help explain the emergence of the ICE in empirical data. Using household data from the most recent consumption survey data from Tunisia, we validate the existence of the IEC and estimate its slope. [2] [3]

2.2. Identification of the informal budget share

We use the 2021 Tunisian National Survey on Budget, Consumption, and Living Standards (ENBCNV) to construct a consumption-based measure of informality at the household level.

[2] In Bachas et al. (2023), the authors use the 2010 Tunisian National Survey on Budget, Consumption, and Living Standards to calculate the slope of the IEC in Tunisia.

[3] It should be stressed here that we are not attempting to provide a measure of the aggregate size of the informal sector rather we estimate the potential share of spending allocated to informal products in the household budget.

We follow the methodology used in Bachas et al. (2023) and identify the share of informal budget based on the place of purchase of each product[4]. In this survey, the place of purchase variable takes on modalities: (1) "Private shop", (2) "Supermarket", (3) "Fixed market", (4) "Weekly market", (5) "Exhibition", (6) "Online", (7), "Other acquisition place", (8) "Not declared", and (9) "Missing". Inspecting the volume of transactions by store type (Table 1), we find that "Privet shop" and "Missing" categories account for roughly 90% of transaction volume (61.58% and 28.53% receptively). Classifying transaction to goods and services[5], we find that the majority of good purchase are from privet shops while the majority of "service purchases" are in the "Missing" category.

Lieu acquisition prdt	Goods (%)	Services (%)	Total (%)
Private shop	46.254%	15.332%	61.586%
Supermarket	1.842%	0.223%	2.065%
Fixed market	1.474%	0.007%	1.481%
Weekly market	4.008%	0.026%	4.033%
Exhibition	0.017%	0.001%	0.017%
Online	0.062%	0.013%	0.076%
Other acqu.	1.566%	0.114%	1.679%
Not declared	0.182%	0.343%	0.525%
Missing	3.841%	24.697%	28.538%
Total	59.244%	40.756%	100.000%

Table 1: Share of expenditure by type of store and nature of good

We mapped the type of expenditure by store type across expenditure levels by household (2), revealing that, ceteris paribus, across seven out of the nine categories, there exists a statistically significant[6] relation between the share of purchases from the store type and the log total household expenditure. We find positive slope coefficients for categories the private shop, supermarket, and online categories and negative slope coefficients for the rest. In particular, we find the steepest slope coefficients for the "Privet shop" and the "Missing" categories (5.74 and -4.37 respectively).

Baseline identification: // The simplest method to identify the share of the informal budget is to follow strict traditional/modern stop-type categories. We assign categories 1,2,

[4]The 2021 ENBCNV survey follows the 3 conditions mentioned in Bachas et al. (2023): (1) National representative, (2) Open diary consumption dairy, (3) Reporting the place of purchase of each product.

[5]We classify transaction to "good purchase" or "service purchase" based on the COICOP code of the product in question.

[6]p-valoue $(P > |t|) \leq 0.05$

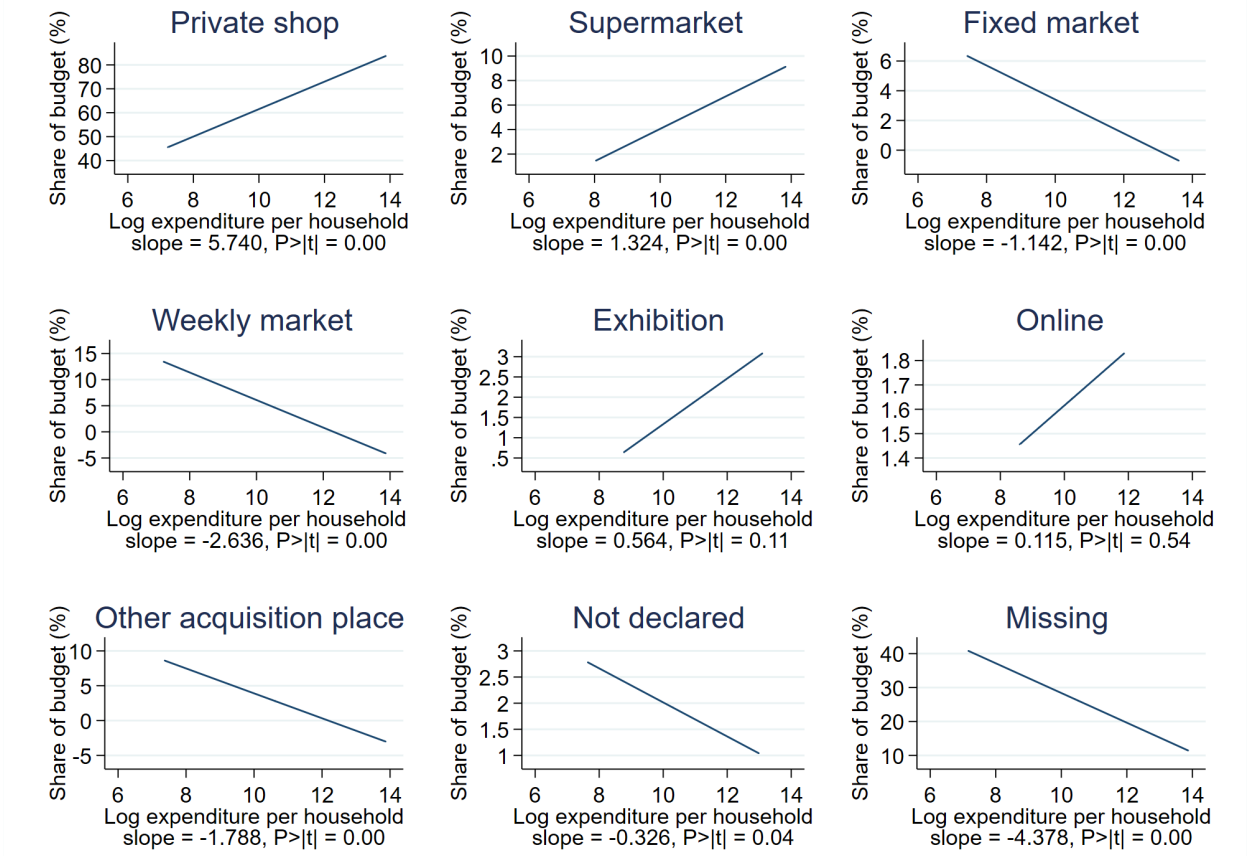


Figure 2: The relation between the share of total consumption and household expenditure

and 6 to the modern store type and categories 3, 4, and 5 to the traditional shop. We classify 7 as informal since it is mostly goods and 8 and 9 as formal since they consist of services.

Calibrated identification: // Generally, informality has spillover effects into the modern stores and the ability of the state to levy taxes is not perfect. Using data from retail stores, Bachas et al. (2023) estimated that, on average 90% 80% of modern stores escape taxation while around 10% of traditional stores are included. They also found that these results are fairly consistent between countries at different levels of development. Following this reasoning we propose a calibrated identification process based on the additional data on informal activity in Tunisia. The INS estimates that, in 2015, 40% of the service sector is conducted in the informal sector [7]. **Case one:** In our first case scenario, we set the share of informal activity in modern stores and the share of formal activity to 0.05%. We classify

[7]Source (in French): Les Comptes Nationaux changent de base

'Other acqui' as being from traditional stores. "Not declared" transactions, we apply the 40% rate. For missing values, we choose an informality rate of 40%. **Case two:** In our first case scenario, we set the share of informal activity in modern stores and the share of formal activity to 0.1%. We classify 'Other acqu' as being from traditional stores. Not declared transactions, we apply the 40% rate. For missing values, we choose an informality rate of 65%. We especially this decision based on the inspection of the composition of this category which reveals that most of the expenses are rent while the rest are utility and telecommunication spending.

For these three cases, we calculate the share of informal spending in each household and report the summary statistics in 2. The baseline identification we get an average share of 9.423%. It is easy to see how these results do not reflect the true scale of informal consumption in Tunisia. The calibrated identification yields more possible results. On the lower end, we get an average share of 22.686% in the first case and 33.467% in the second. In what follows, we focus on the case 2 while reporting results for all 3 cases.

Variable	Mean	Std. Dev.	Min	Max
Log Consumption per capita	9.678	0.599	7.168	13.866
Share of informal budget: Baseline	9.423	9.210	0	97.679
Share of informal budget: Case 1	22.686	7.316	5.1271	93.292
Share of informal budget: Case 2	33.467	8.569	10.194	88.742

Table 2: Summary Statistics

2.3. Estimation of the IEC

Using the generated informal sector, we estimate the slope of the IEC for each of our cases using the following regression:

$$Informalshare_i = \beta \ln(consumption_i) + \Gamma X_i + \epsilon_i \tag{1}$$

X_i is a vector of control variables related to the household; size, head's sex, head's age, head's education level, and the urban state. We report the results in Table 3. Given household characteristics, we find a statistically significant slope coefficient and constant for all cases. In case 2 we get a β equals -5.423 implying, a redaction of informal budget share by one as expenditure increases by roughly 20%. Figure 3 showcases the downward trend of the IEC in Tunisia.

	Baseline	Case 1	Case 2
Log household expenditure	-2.311***	-4.239***	-5.423***
	(0.121)	(0.100)	(0.119)
Household size	0.726***	0.273***	-0.0366
	(0.0465)	(0.0385)	(0.0458)
Household head size	0.00586	0.0126**	0.0157**
	(0.00509)	(0.00421)	(0.00501)
1.Household head sex	-	-	-
	(.)	(.)	(.)
2.Household head sex	-0.914***	-0.173	0.105
	(0.177)	(0.146)	(0.174)
1.Household head eduction	-	-	-
	(.)	(.)	(.)
2.Household head eduction	-0.997***	-0.235	0.0335
	(0.187)	(0.155)	(0.184)
3.Household head eduction	-1.723***	0.0287	0.656**
	(0.214)	(0.177)	(0.210)
4.Household head eduction	-2.451***	1.169***	2.778***
	(0.273)	(0.226)	(0.269)
1.Urban	0	0	0
	(.)	(.)	(.)
2.Rural	5.172***	1.216***	-0.187
	(0.144)	(0.119)	(0.142)
_cons	27.61***	61.31***	84.65***
	(1.145)	(0.947)	(1.127)
N	17118	17118	17118

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3: Slope of the IEC

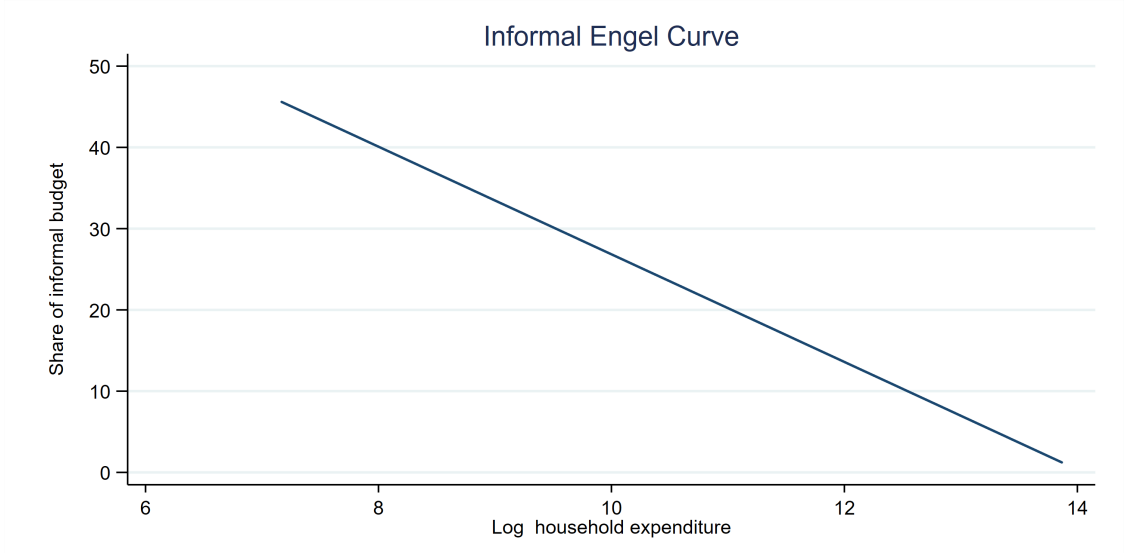


Figure 3: Informality Engel Curve (IEC) for Tunisia

3. A Heterogeneous Agent New Keynesian Model With informality

We develop a medium-scale model with heterogynous agent. The model is a one-type-asset HANK model, in the spirit of Aiyagari (1994) and Krusell and Smith (1998), augmented with a dual-sector structure.

3.0.1. Households

The economy is populated by a continuum of infinitely lived heterogeneous households. Heterogeneity between households can be summarized by three idiosyncratic states (a, z, s). Here, $a \in \mathbb{R}^+$ is the wealth of the household. $z \in \mathbb{R}^+$ is the productivity of the household. We assume that z follows an exogenous Markov process. $s \in \{0, s^i, s^f\}$ is the employment state of the household and reflects state-specific productivity. Here, $s = 0$ indicates the unemployment state, while s^i and s^f indicate employment in the informal and formal sectors respectively. Since informal firms are generally smaller and less productive than their formal counterparts (Porta and Shleifer (2008). La Porta and Shleifer (2014)), we can impose that $z^f > z^i$.

For simplicity, we assume that the labor, sector, and productivity are exogenous and follow a Poisson process with an arrival probability $\lambda^j; \quad j \in \{0, i, f\}$. We note by $\mu(da, dz)$ the joint distribution for the state of the economy. μ has a support $\mathcal{M} := \mathbb{R}^+ \times \times \{z^0, z^i, z^f\}$.

Households receive utility u from consumption c and disutility from labor l. Preferences are conditional on savings and the future discount rate $\rho \leq 0$. The household maximization problem differs between labor and sector states.

$$\mathbb{E}_0 \int_0^\infty e^{-(\rho+\zeta)t} u(c_t^f, c_t^i, l_t) dt. \tag{2}$$

To embed the heterogeneous informal budget share in our model, we propose to separate the consumption utility of the two categories of goods [8]. We assume that household utility follows:

$$u(c_t^f, c^i, l_t) = \frac{(c_t^f)^{1-\sigma_f}}{1-\sigma_f} + \Phi_i \frac{(c_t^i)^{1-\sigma_i}}{1-\sigma_i} + \Phi_l \frac{(l_t)^{1+\phi_l}}{1+\phi_l} \tag{3}$$

with σ_f and σ_i represent the elasticities of formal and informal goods respectively, ϕ_l is the inverse Frish elasticity of labor. Φ_i and Φ_l are scaling parameters for informal consumption and labor respectively.

Household's asset holdings evolve according to:

$$\dot{a} = w_t(z_t) z_t l_t + r_t a_t + T_t + \Pi - (1+\tau_c) c_t^f - p_t^i c_t^i \tag{4}$$

Where $w_t(z_t)$ is the wage level associated the state z_t i.e. formal sector wage $(1-\tau_w) w_t^f$ for z_f, w_t^i for z_i and 0 for unemployed workers and l_t is the labor supply. r_t is the real return on assets and T_t is a universal government transfer to all households. Combined profits from formal and informal firms are distributed to households as a dividend, Π_t, based on their productivity level [9].

Assets:

Unlike the model in Kaplan et al. (2018) we do not distinguish between illiquid and illiquid wealth in the economy. The wealth a is composed of bond holdings, b, and physical capital,k. Household weal can be expressed as:

$$a_t = b_t + q_t k_t \tag{5}$$

Where q_t is the capital price. We assume that households can shift between the two types of assets without cost. As such we impose the non-arbitrage condition:

$$\frac{r_t^k - \delta q_t - \dot{q}_t}{q_t} = r_t \tag{6}$$

[8] This is in contrast to the constant share across households implied by a CES function where formal and informal goods are combined to produce a single final consumption good like in model presented in the previous section.

[9] As explained in Kaplan et al. (2018), the distribution process of assets plays a critical role in shaping the resulting distribution of wealth.

where r_t^k is the rental rate of capital to formal firms and *delta* is the depreciation rate of capital.

3.0.2. Goods Producers

In the economy, there are two types of firms; Formal, indexed by f, and informal, indexed by i. **Intermediate Goods Producers:**
There is a continuum of intermediate-good producers, indexed by j_s, operating in each sector $s; s \in \{f, i\}$ and using the sector specific production function:

$$\begin{aligned} y^f_{j_f,t} &= Z^f_t k^\alpha_t (n^f_{j_s,t})^{(1-\alpha)} \\ y^i_{j_i,t} &= Z^i_t n^s_{j_s,t} \end{aligned} \tag{7}$$

Here, Z^f_t and Z^i_t are two aggregate productivity shocks associated with the formal and informal sectors respectively. Both shocks follow the Ornstein-Uhlenbeck process:

$$dZ^s_t = -\theta_{z_s} Z^s_t dt + \sigma_{z_s} dW^{z_s}_t; \quad s \in \{f, i\} \tag{8}$$

We assume that the marginal cost of firms in both sectors is subject to a cost-push shock. We have:

$$\begin{aligned} mc^f_t &= \frac{\Xi^f_t}{w^f_t} \left(\frac{w^f_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{r^k_t}{\alpha} \right)^{\alpha} \\ mc^i_t &= \frac{\Xi^i_t}{Z^i_t} w^i_t \end{aligned} \tag{9}$$

The two cost-push shocks, Ξ^f_t and Ξ^f_t, follow the Ornstein-Uhlenbeck process:

$$d\Xi^s_t = -\theta_{\Xi_s} \Xi^s_t dt + \sigma_{\Xi_s} dW^{\Xi_s}_t; \quad s \in \{f, i\} \tag{10}$$

Firms are subject to sector-specific quadratic price adjustment costs following Rotemberg (1982):

$$\Theta^s_t \left(\frac{\dot{p^s_t}}{p^s_t} \right) = \frac{\Omega^s}{2} \left(\frac{\dot{p^s_t}}{p^s_t} \right)^2 y^s_t \tag{11}$$

From the firm's optimization problem, we can determine the following New Keynesian Phillips curves:

$$\begin{aligned} \left(r_t - \frac{\dot{y^f_t}}{y^f_t} \right) \pi^f_t &= \frac{\xi_f}{\Omega^f} \left(m^f_t - p^f_t \frac{\xi_f - 1}{\xi_f} \right) + \dot{\pi^f_t} \\ \left(r_t - \frac{\dot{y^i_t}}{y^i_t} \right) \pi^i_t &= \frac{\xi_i}{\Omega^i} \left(m^i_t - p^i_t \frac{\xi_i - 1}{\xi_i} \right) + \dot{\pi^i_t} \end{aligned} \tag{12}$$

where π_t^f is the inflation rate of formal goods and π_t^i is the inflation rate of informal goods.

Final-Goods Producers:
Final goods produced by formal firms and final goods produced by informal firms are produced using intermediate inputs, y_t^f and y_t^i, from formal and informal intermediate goods producers respectively.

$$y_t^s = \left[\int_0^1 \left(y_{j_s,t}^s\right)^{\frac{1}{\xi_s}} dj_s\right]^{\xi_s} \quad s \in \{f, i\} \tag{13}$$

3.0.3. Capital Producer

A competitive capital producer transforms formal goods into capital goods bought by households at the price q_t. We assume that the production process is subject to capital adjustment cost and that the cost function of the capital producer is $\Upsilon(\iota_t + \frac{\Omega}{2}(\iota - \delta)^2)K_t$ where ι_t is the investment rate and Υ_t is a marginal efficiency of investment shock. The producer maximizes the expected profit stream, discounted at the stochastic discount factor of the household:

$$\begin{aligned} W_t = &\max_{\iota_t, K_t} \int_0^\infty \Lambda_{0,t} \left(q_t \iota - \Upsilon(\iota_t + \frac{\Omega}{2}(\iota - \delta)^2)\right) K_t dt \\ &s.t. \quad \dot{K}_t = (\iota_t - \delta) K_t \end{aligned} \tag{14}$$

3.0.4. Monetary Policy

Monetary policy is set using the following Taylor rule:

$$i_t = \bar{r} + \phi_\pi \pi_t + \phi_y (y_t - \bar{y}) + \varepsilon_t^{MP} \tag{15}$$

ε_t^{MP} follows the Ornstein-Uhlenbeck process:

$$d\varepsilon_t^{mp} = -\theta_{mp} \varepsilon_t^{mp} + \sigma_{mp} . dW_t^{mp} \tag{16}$$

where dW_t is the innovation to a standard Brownian motion, θ_{MP} is the rate of mean reversion, and σ captures the size of innovations.

3.0.5. Government

The government budget satisfies the constraint given by:

$$\dot{B}_t^g + G_t + T_t = \tau_w \int w_t^f z^f l^f d\mu_t(a, z, s) + \tau_c \int c_t^f d\mu_t(a, z, s) + \tau_f \tilde{\Pi}_t^f + r_t B_t^g \tag{17}$$

Where the government adjusts lump transfers T_t to satisfy the budget constraints. We assume that government expenditure follows the Ornstein-Uhlenbeck process:

$$dG_t = -\theta_g G_t dt + \sigma_G dW_t^G \tag{18}$$

Finally, to maintain fiscal the budget balance, we assume that the government uses transfers, T_t, as its policy instrument[10].

3.0.6. Equilibrium

The equilibrium in this economy is defined as the path of household and firm decisions $\{a_t, c_t^f, c_t^i, l_t, n_t^f, n_t^i, k_t, \iota_t\}_{t\geq 0}$, prices $\{r_t, p_t^i, w_t^f, w_t^i, q_t\}_{t\geq 0}$, inflation rates $\{\pi_t, \pi_t^i\}_{t\geq 0}$, fiscal variables $\{\tau_t^w, \tau_t^f, \tau_t^c, T_t, G_t, B_t\}_{t\geq 0}$, measures $\{g_t\}_{t\geq 0}$

The model has 6 markets, the bonds market, the capital market, the formal and informal labor markets, and the formal and informal goods markets. The bond market clearing:

$$B_t^g = \int b_t d\mu_t(a, z, s) \tag{19}$$

The capital market clearing:

$$K_t = \int k_t d\mu_t(a, z, s) \tag{20}$$

Clearing labor markets for the formal and informal sectors:

$$\begin{aligned} L^f &= \int s^f z l_t \mu_t(a, z, s^f) da \\ L^i &= \int s^i z l_t \mu_t(a, z, s^i) da \end{aligned} \tag{21}$$

The formal goods market clearing condition:

$$Y_t^f = C_t^f + I_t + G_t + \Theta_t^f \tag{22}$$

And finally clearing the informal goods market

$$Y_t^i = C_t^i + \Theta_t^i \tag{23}$$

We define GDP as:

$$GDP_t = p_t^f Y_t^f + p_t^i Y_t^i \tag{24}$$

[10]The choice of fiscal instrument has an important role in the transmission of shocks as showcased in Kaplan et al. (2018). Other specifications could utilize taxes or government expenditures. Exploring these alternative cases is left out of our analysis

4. Transmission of Monetary Policy: Single Vs Duel Sector Models

In this section we compare the the monetary policy transmission from our model (Full model)and that from a single-sector model (Baseline).

4.1. Calibration

We present our calibration choice in Table 4. We calibrate the model using standard values from the literature on New Keynesian models, and to replicate key features from the Tunisian economy. Since we will be evaluating the model at zero steady-state inflation, we set the discount factor to 0.015 and the capital depreciation rate to 0.015. σ_f and σ_i are calibrated to 1.45 and 1.85 to control the slope of the implied IEC generated by the model. ξ_f and ξ_i are chosen to allow a mark-up rate of 20% and 10% in the formal and informal markets respectively. $\bar{s}$ is chosen to match the average household consumption from the model with the one observed in the survey data. The normalized productivity vector $[s_u, s_i, s_f]$ is set to [0, 0.8, 1] implying that an informal worker, is on average 80% as productive as its formal counterpart. For the size of the informal sector, we choose a steady value of 25%. We also calibrate the employment state jump matrix to obtain an unemployment rate as close to the data.

Symbol	Parameter	Full model	Baseline
ρ	Discount factor	1.5%	1.5%
α	Share of capital	0.3	0.3
δ	Capital depreciation rate	1.5%	1.5%
σ_f	Risk aversion for formal consumption	1.45	1.45
σ_i	Risk aversion for informal consumption	1.85	-
ϕ	Inverse Frish elasticity	2	2
Φ	Labor disutility	25	25
ξ_f	Formal goods elast.	6	6
ξ_i	Informal goods elast.	11	-
Ω_f	Formal adj. cost	100	100
Ω_i	Informal adj. cost	75	-
Ω_k	Capital adj. cost	1	1
τ_w	Labor tax	25%	25%

τ_f	Corporate profits tax	20%	25%
τ_c	Consumption tax	18%	18%
ϕ_π	Taylor inflation	1.25	1.25
ϕ_y	Taylor output gap	0.3	0.3
$\bar{s}$	Average productivity	4.75	6
s	Normalized productivity vector	[0, 0.8, 1]	[0, 1]
$\frac{B}{GDP}$	Government Debt to GDP	0.6%	0.6%
$\frac{T}{GDP}$	Government transfers to GDP	0.06%	0.06%
π	Steady state inflation	0	0
θ_{mp}	Reversion rate, mon. pol.	0.75	0.75
σ^2_{mp}	Innovation size, mon. pol.	0.05	0.05

Table 4: Calibrated Values

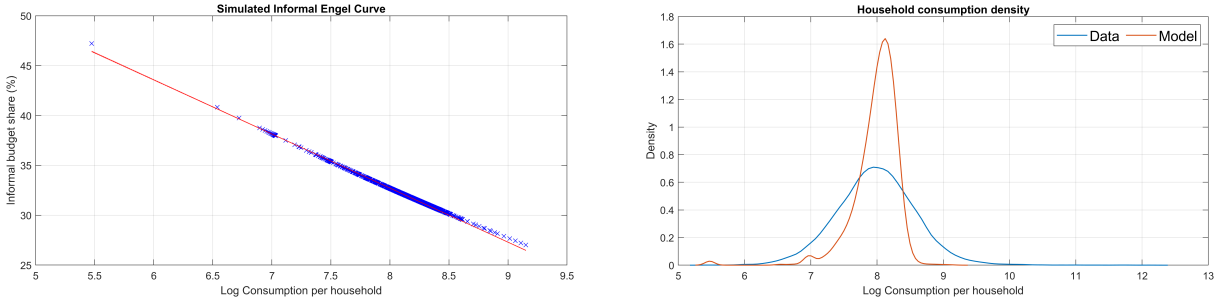
We report some of the steady state ratios results in Table 5. The main difference between the two models lay in the real interest rate steady state 0.358% vs 0.998% and in the share of consumption and government spending to GDP since we try to keep our parameters constant.

Symbol	Name	Full model	Baseline	Data
Φ_i	Informal utility scaling parameter	0.2684	-	-
r	Interest rate	0.358%	0.998%	-
$\frac{c}{GDP}$	Consumption to GDP	69.39%	62.45%	70.67%
$\frac{i}{GDP}$	Investment to GDP	15.39%	15.01%	18.48%
$\frac{g}{GDP}$	Government spending to GDP	15.21%	22.56%	20.26%
U	Unemployment	15.75%	15.75 %	15.78%
$\frac{Y^i}{GDP}$	Informal output share	25%	-	27.4%
$\frac{N^i}{N}$	Informal employment share	44.71%	-	44.8%
$\ln(\bar{c})$	Log quarterly expenditure per Household	7.988	7.970	7.984
$\frac{\bar{c^i}}{\bar{c}}$	Average share of informal consumption	32.74%	-	33.467%

Table 5: Steady state results

In Figure 4, we present results from our measurement of the implied IEC generated by the model and the kernel density estimates of the log per household quarterly consumption

distribution. We compare these with the impractical findings obtained from the household consumption survey. We were able to recreate the IEC with a slope of -5.4262 vs -5.423 that we found earlier. On the other hand, we were not able to recreate the consumption distribution. We can mainly attribute this to the inability of our model to create fat tail distributions.



(a) Informality Engel Curve (b) Consumption distributions

Figure 4: Steady State Informality Engel Curve and Consumption Distribution

4.2. Model comparison

With our objective in mind, we simulate the response of the economy to a non-anticipated monetary policy innovation. We report the Impulse Response Functions (IRFs) generated by a positive (restrictive) monetary policy shock in Figure 5. The IRFs measure the percent deviation of key macroeconomic variables from their respective steady-state values following the realization of the shock. We mainly find that results from the HANK model with informality deviate substantially from the others. This effect can be seen in lower losses in consumption, investment, and GDP, in line with the 'buffering effect' characteristic of informality. On the other hand, for inflation, we register a stronger reaction translated as a greater downward deviation of inflation. This comes in contrast to the RANK model where the response to inflation is lower than the models with only the formal part which is the case often reported in the literature (Castillo and Montoro (2010), Castillo and Montoro (2012), Alberola and Urrutia (2020)).

In Figure 6, we present the transmission of monetary policy to aggregates and their components. Here we can see how the informal sector can mitigate shocks, especially for GDP. For inflation, we find similar results for the two sectors. We can also see that as a

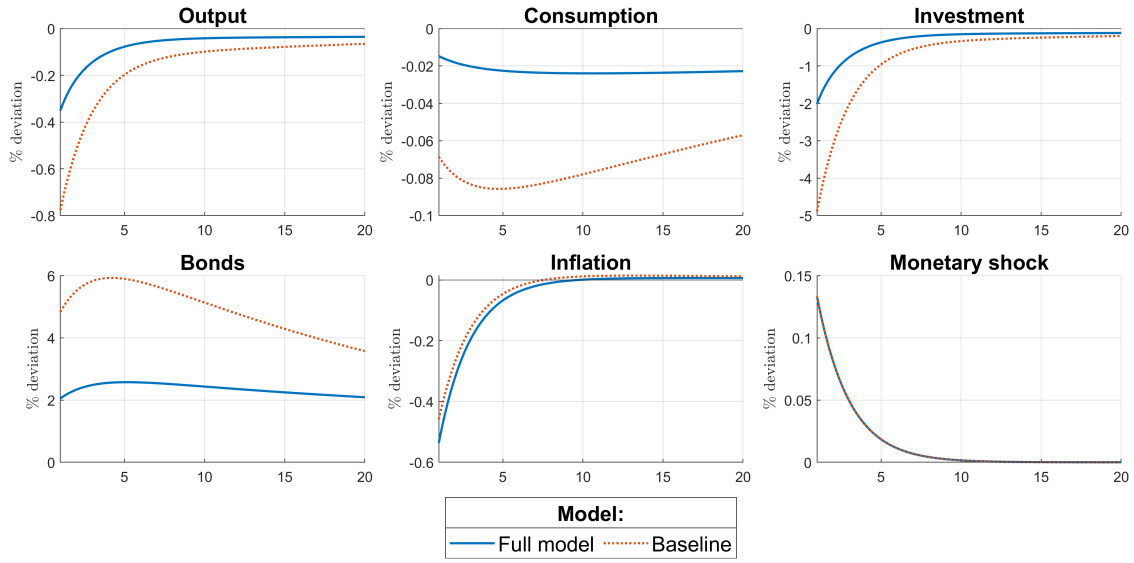


Figure 5: Transmission of monetary policy in different models

result of monetary policy shock, the share of the informal sector in the economy increases for the short term by up to 0.35% before returning to its steady state.

5. The Transmission of Monetary Policy from an Estimated Model

In this section, we lay down our approach to empirically quantify the Transmission of monetary policy in the case of Tunisia.

5.1. Numerical Solution and Estimation Algorithm

Nowadays, solving the HANK model can be considered a feasible task thanks to toolboxes kindly provided online for the scientific community. For continuous-time models, Ahn et al. (2017) provides an easy-to-use toolbox, for solving and simulating HANK models with aggregate shocks. The method is based on the works of Achdou et al. (2021) to find the steady state equilibrium using finite difference methods and of Reiter (2009), among others, to solve the linearized model. On the other hand, estimating the HANK model is still a new area of research, and unlike model solutions, it is a more challenging task. Most of the available literature uses calibrated models made to replicate key empirical moments for the heterogeneous states (from income or consumption surveys). Only a minority of works attempt to estimate HANK models including Bayer et al. (2024),Auclert et al. (2021), Fernández-Villaverde et al. (2023), and Acharya et al. (2023) to name a few. For our case, we use the

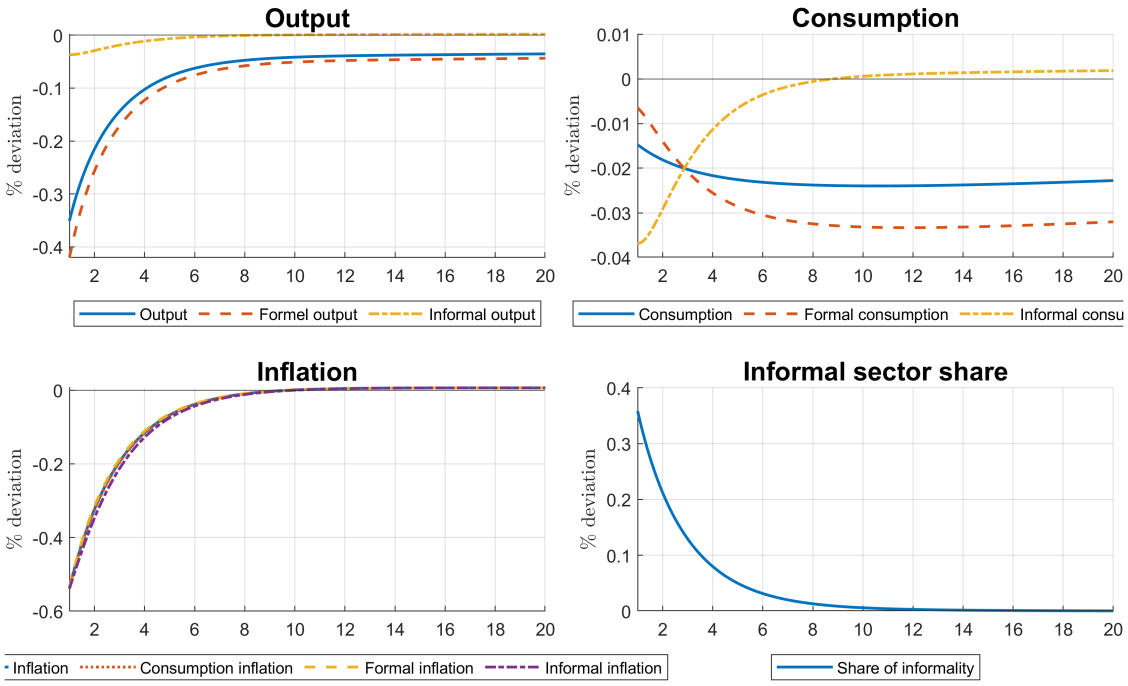


Figure 6: Transmission of monetary policy: Detailed

estimation method proposed in Liu and Plagborg-Møller (2023). While we limit ourselves to a macro data-only estimation, the proposed method in this article allows for the incorporation of microdata for a full-information estimation, a task we are willing to tackle in future works. However, the inference method presented in Liu and Plagborg-Møller (2023) is developed for discrete-time models. Applying it to our continuous time model requires additional tinkering. To solve this problem, we use the method proposed in Christensen et al. (2024) for estimating continuous models using discrete data. The approach is based on an exact discrete-time representation.

5.2. Data & Priors

For estimation, we use an observations sample of 7 macroeconomic variables, at a quarterly frequency, from the Tunisian economy from 2015$Q2$ to 2022$Q4$. We include in our sample the policy rate, GDP inflation, CPI inflation, GDP, household final consumption, investment, and government spending. All variables are filtered using the one-way hp-filter. Real quantities are expressed in real values and per capita terms. For household final consumption, investment, and government spending, data is only available in annual frequencies. We opt to apply a quadratic low to high-frequency transformation filter to obtain data at

quarterly frequency[11].

For our estimation, we use a random walk Metropolis-Hasting chain with 10000 draws after a burn-in of 2000 draws. We estimate in total 16 parameters, focusing on frictions and shock processes. We report the prior used and the resulting posterior in Table 6.

	Parameter description	Prior			Postorior	
		Distr	Mean	st.d	Mean	st.d
Ω_f	Formal adj. cost	Gamma	100	25	75.233	0.0530
Ω_i	Informal adj. cost	Gamma	75	20	75.106	0.0740
Ω_k	Capital adj. cost	Gamma	0.3	0.2	0.644	0.1138
ϕ_π	Taylor rule, inflation	Normal	2	1	2.082	0.0361
θ_{mp}	Reversion rate, mon. pol.	Beta	0.75	0.2	0.3626	0.0523
θ_{z_f}	Reversion rate, formal prod.	Beta	0.75	0.2	0.2758	0.0740
θ_{z_i}	Reversion rate, informal prod.	Beta	0.75	0.2	0.2408	0.1003
θ_g	Reversion rate, gov. exp.	Beta	0.75	0.2	0.3007	0.0863
θ_Υ	Reversion rate, MEI	Beta	0.75	0.2	0.4052	0.0628
σ_{mp}	Innovation size, mon. pol.	Inv-Gamma	0.25	Inf	0.3578	0.0655
σ_{z_f}	Innovation size, formal prod.	Inv-Gamma	0.25	Inf	0.4780	0.0676
σ_{z_i}	Innovation size, informal prod.	Inv-Gamma	0.25	Inf	0.7036	0.1436
σ_g	Innovation size, gov. exp.	Inv-Gamma	0.25	Inf	0.4361	0.0912
σ_{Ξ_f}	Innovation size, formal cost	Inv-Gamma	0.25	Inf	0.4033	0.0407
σ_{Ξ_i}	Innovation size, informal cost	Inv-Gamma	0.25	Inf	0.5585	0.1179
σ_Υ	Innovation size, MEI	Inv-Gamma	0.25	Inf	0.6328	0.1215

Table 6: Estimation priors and results

5.3. The Transmission of Monetary Policy In Tunisia

To evaluate the transmission of monetary policy in Tunisia, we study the generated Impulse Response Functions (IRFs) generated by a positive 1% standard deviation monetary policy shock at t0. We report the IRFs generated at the mean posterior distribution[12].

[11] We can instead use mixed frequency estimation facilitated by the algorithm provided in Christensen et al. (2024)

[12] Not to be confused with the mean IRFS generated which is often reported as Bayesian IRFs.

In general, results are qualitatively in line with expectations based on existing literature. For output, we find that formal output is more affected than informal output but recovers slightly faster. However, we find that consumption showcases an opposite pattern. Formal consumption deviates upward at first before diving below its steady-state level. We should note that consumption movement is less important exhibiting only one-fifth that of output deviation. For Inflation, we find that formal and informal price inflation exhibit similar path trajectories although informal inflation deviates slightly lower leading to a slightly lower consumption price inflation. These results are in line with those estimated in End et al. (2020) using a var model and other estimates for the Tunisian economy using DSGE models. As a result, the size of the informal sector to GDP goes up at first by up to 0.3% before reversing its trajectory by the fourth period and maintaining a downward deviation.

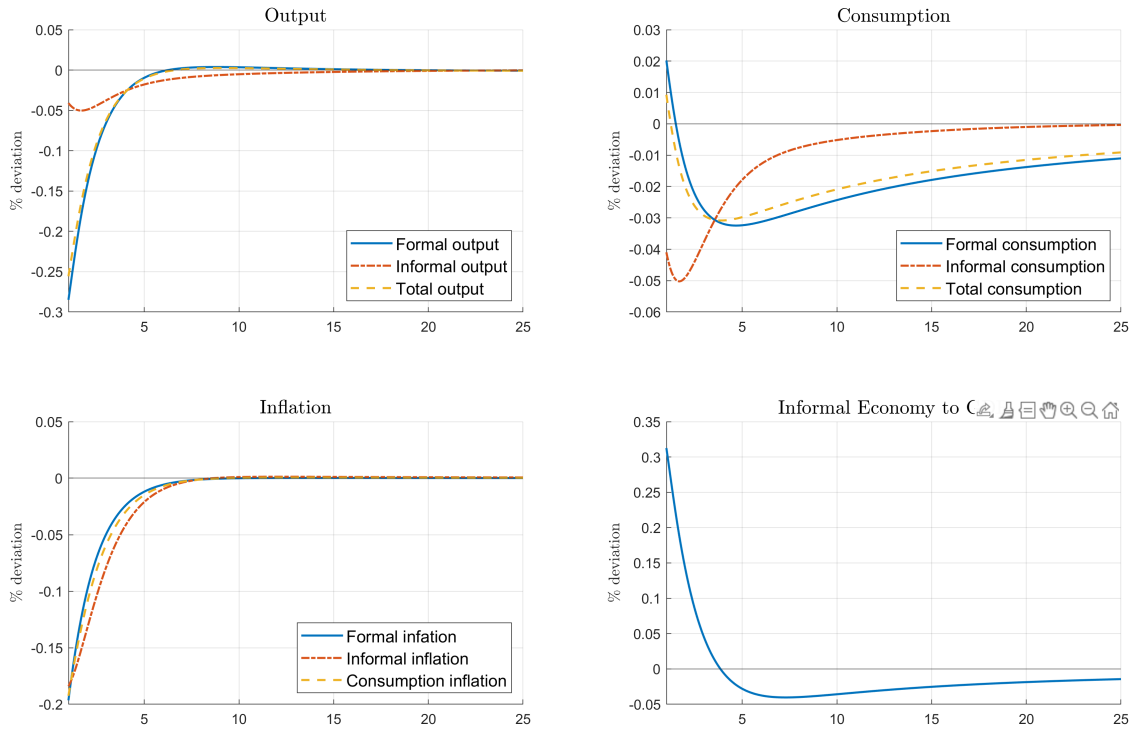


Figure 7: Impulse Response Function at Posterior Mean

One advantage of heterogeneous agent models is that they can be used to generate specific IRFs for each type of household. If we take consumption, we generate the formal consumption reaction of each worker type at the extremities of our grid and a midpoint. Results are presented in Figure 8. Household formal consumption is not homogeneous across the two states. First, size and diversion between reactions seem to decrease with wealth. Second, all results seem to exhibit qualitatively similar trajectories to aggregate consumption i.e.

positive deviation at first followed by a prolonged downward deviation. We attribute this reaction to three factors. First, the indirect effects due to general equilibrium response to changes in household disposable income. Second, the increase of government transfers as a result of increased demand for bonds exceeding revenue losses from taxation. Finally, in this model specification distributed profits increase with restrictive monetary policy as these are counter-cyclical, in the presence of only price rigidities[13].

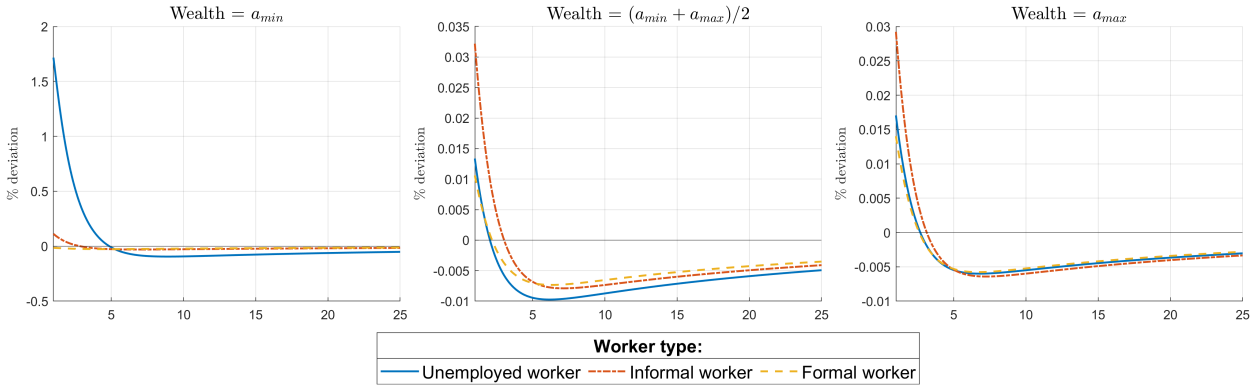


Figure 8: Impulse Response Function by Wealth and Worker Type

Another advantage of the HANK model is that it provides the ability to map the reaction of household distributions to aggregate shocks. In our case, we report the reaction of wealth distribution to a monetary shock in Figure 9. For the clarity of the illustration, we increased the size of the initial shock. We can note that the wealth distribution shifts to the left. This effect is especially noticeable for unemployed and informal workers and at lower wealth levels. This result suggests that unemployed and informal workers are more vulnerable to the redistributive effect of monetary policy in comparison with their formal counterparts. By t = 4 (one year after the shock), the distribution returns to its steady state.

6. Conclusion

In this paper, we examine the transmission of monetary policy in an economy characterized by heterogeneous households and a sizable informal sector. We focus on the heterogeneous spending of households on informal goods. We start by identifying the share of this spending using data from the 2021 Tunisian National Survey on Budget, Consumption, and Living

[13]In Kaplan et al. (2018) a similar problem arises and is dressed by controlling the share of profits distributed as liquid and illiquid assets.

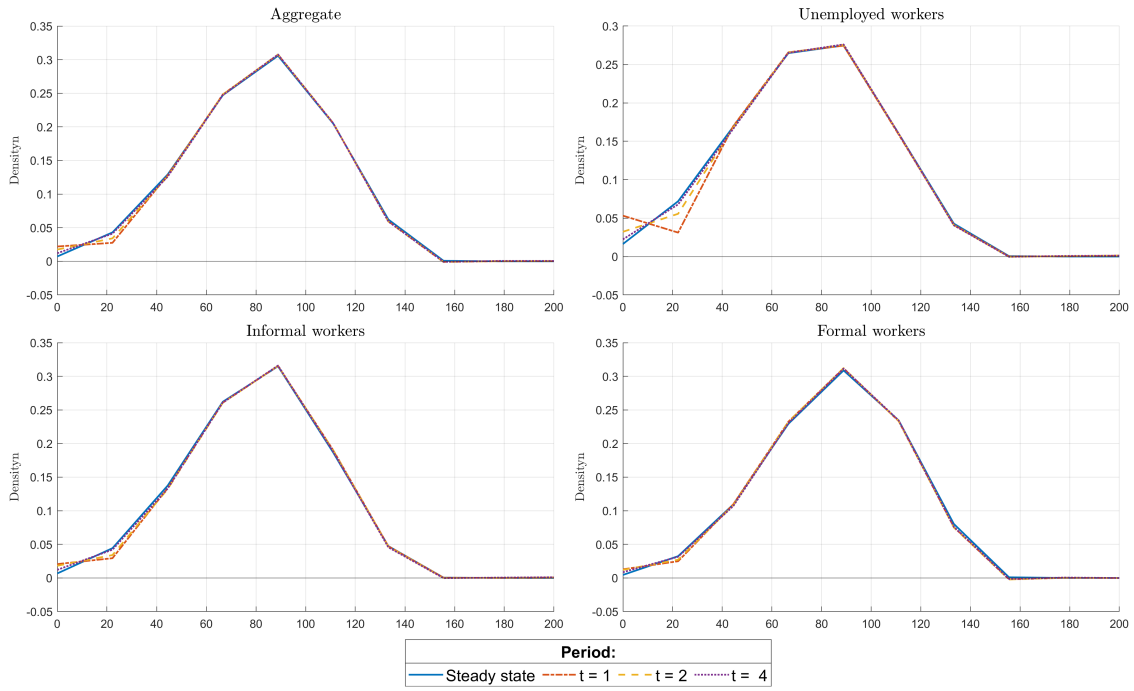


Figure 9: Impulse response function

Standers (ENBCNV). The identification process relies on the classification of the place of purchase of each product into one from a traditional or a modern store. We provide 3 cases: one baseline and 2 calibrated scenarios. Using the processed data we then estimate the slope of the Informality Engel Curve (IEC). We find a statistically significant negative slope with respect to log total household expenditure i.e. as the consumption spending increases, the share of spending on informal goods decreases. Based on these findings, we develop a one-asset duel sector medium-sized HANK model. We assume that one sector represents the informal side of the economy. The model is able to re-recreate the IEC slope after careful calibration. Using this version, we compare monetary policy transmission with that of a baseline single-sector model. We still confirm the presence of the buffering effect that reduces the impact on Consumption, Investment, and GDP as reported in the literature. In addition, we find a stronger transmission to inflation when compared with the baseline model i.e. a restrictive monetary shock results in a greater reduction in inflation with a smaller loss in GDP and consumption. In the final step, we estimate key parameters for the model dynamics using the Bayesian method. To achieve this step we combine different methods to obtain a discrete state space representation of the continuous time model using the exact discrete-time representation which will allow us to evaluate the likelihood function using macroeconomic data. From the results generated by estimating our model, we found

that unemployed and informal workers seem to be most affected by the monetary shock, especially those with lower wealth. These results showcase the importance of informality and heterogeneity in monetary policy analysis. Monetary authorities in emerging economies like Tunisia can use these insights to improve the efficacy of their policy at lower social costs. This can be especially important for achieving mandates related to price stability, fostering economic growth, improving public trust, and the accountability of these institutions. Also, coordination with fiscal authorities can prove useful to mitigate harmful repercussions of monetary tightening on vulnerable segments of the population without sacrificing policy objectives.

References

S. Acharya, W. Chen, M. D. Negro, K. Dogra, A. Gleich, S. Goyal, D. Lee, E. Matlin, R. Sarfati, and S. Sengupta. Estimating HANK for Central Banks. Staff Reports 1071, Federal Reserve Bank of New York, Aug. 2023. URL https://ideas.repec.org/p/fip/fednsr/96600.html.

Y. Achdou, J. Han, J.-M. Lasry, P.-L. Lions, and B. Moll. Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach. *The Review of Economic Studies*, 89(1):45–86, 04 2021. ISSN 0034-6527. doi: 10.1093/restud/rdab002. URL https://doi.org/10.1093/restud/rdab002.

S. Ahn, G. Kaplan, B. Moll, T. Winberry, and C. Wolf. When Inequality Matters for Macro and Macro Matters for Inequality. In *NBER Macroeconomics Annual 2017, volume 32*, NBER Chapters, pages 1–75. National Bureau of Economic Research, Inc, 12 2017.

S. R. Aiyagari. Uninsured Idiosyncratic Risk and Aggregate Saving. *The Quarterly Journal of Economics*, 109(3):659–684, 1994. URL https://ideas.repec.org/a/oup/qjecon/v109y1994i3p659-684..html.

E. Alberola and C. Urrutia. Does informality facilitate inflation stability? *Journal of Development Economics*, 146:102505, 2020. ISSN 0304-3878. doi: https://doi.org/10.1016/j.jdeveco.2020.102505. URL https://www.sciencedirect.com/science/article/pii/S0304387820300808.

A. Asllani, R. Dell'Anno, and F. Schneider. Mapping the Informal Economy Around the World with an Enhanced MIMIC Approach: New Estimates for 110 Countries from 1997-2022. CESifo Working Paper Series 11416, CESifo, 12 2024.

A. Auclert, B. Bardóczy, M. Rognlie, and L. Straub. Using the sequence-space jacobian to solve and estimate heterogeneous-agent models. *Econometrica*, 89(5):2375–2408, 2021. doi: https://doi.org/10.3982/ECTA17434. URL https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA17434.

P. Bachas, L. Gadenne, and A. Jensen. Informality, consumption taxes, and redistribution. *The Review of Economic Studies*, 91(5):2604–2634, 09 2023. ISSN 0034-6527. doi: 10.1093/restud/rdad095. URL https://doi.org/10.1093/restud/rdad095.

C. Bayer, B. Born, and R. Luetticke. Shocks, frictions, and inequality in us business cycles. *American Economic Review*, 114(5):1211–47, May 2024. doi: 10.1257/aer.20201875. URL https://www.aeaweb.org/articles?id=10.1257/aer.20201875.

P. Castillo and C. Montoro. Monetary policy in the presence of informal labour markets. Working Papers 2010-009, Banco Central de Reserva del Perú, 2010.

P. Castillo and C. Montoro. Inflation Dynamics in the Presence of Informal Labour Markets. BIS Working Papers 372, Bank for International Settlements, Feb. 2012.

B. J. Christensen, L. Neri, and J. C. Parra-Alvarez. Estimation of continuous-time linear dsge models from discrete-time measurements. *Journal of Econometrics*, 244(2):105871, 2024. ISSN 0304-4076. doi: https://doi.org/10.1016/j.jeconom.2024.105871. URL https://www.sciencedirect.com/science/article/pii/S0304407624002161.

C. Elgin, F. Ohnsorge, and S. Yu. Understanding Informality. CEPR Discussion Papers 16497, C.E.P.R. Discussion Papers, Aug. 2021. URL https://ideas.repec.org/p/cpr/ceprdp/16497.html.

N. End, M. E. H. Khatat, and R. Kolsi. Tunisia monetary policy since the arab spring: The fall of the exchange rate anchor and rise of inflation targeting. *IMF Working Papers*, 2020(167):A001, 2020. doi: 10.5089/9781513555027.001.A001. URL https://www.elibrary.imf.org/view/journals/001/2020/167/article-A001-en.xml.

J. Fernández-Villaverde, S. Hurtado, and G. Nuño. Financial frictions and the wealth distribution. *Econometrica*, 91(3):869–901, 2023. doi: https://doi.org/10.3982/ECTA18180. URL https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA18180.

ILO. Women and men in the informal economy: A statistical picture. third edition, Apr. 2018. URL https://www.ilo.org/wcmsp5/groups/public/---dgreports/---dcomm/documents/publication/wcms_626831.pdf.

G. Kaplan, B. Moll, and G. L. Violante. Monetary policy according to hank. *American Economic Review*, 108(3):697–743, 03 2018. doi: 10.1257/aer.20160042. URL https://www.aeaweb.org/articles?id=10.1257/aer.20160042.

P. Krusell and A. A. Smith, Jr. Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5):867–896, 1998. doi: 10.1086/250034. URL https://doi.org/10.1086/250034.

R. La Porta and A. Shleifer. Informality and development. *Journal of Economic Perspectives*, 28(3):109–26, 09 2014. doi: 10.1257/jep.28.3.109. URL https://www.aeaweb.org/articles?id=10.1257/jep.28.3.109.

L. Liu and M. Plagborg-Møller. Full-information estimation of heterogeneous agent models using macro and micro data. *Quantitative Economics*, 14(1):1–35, 2023. doi: https://doi.org/10.3982/QE1810. URL https://onlinelibrary.wiley.com/doi/abs/10.3982/QE1810.

L. Medina and F. Schneider. Shedding Light on the Shadow Economy: A Global Database and the Interaction with the Official One. CESifo Working Paper Series 7981, CESifo, Dec. 2019.

L. Medina, F. Schneider, and A. Fedelino. Shadow economies around the world: What did we learn over the last 20 years? *IMF Working Papers*, 2018(017):A001, 2018. doi: 10.5089/9781484338636.001.A001. URL https://www.elibrary.imf.org/view/journals/001/2018/017/article-A001-en.xml.

R. L. Porta and A. Shleifer. The unofficial economy and economic development. *Brookings Papers on Economic Activity*, 2008:275–352, 2008. ISSN 00072303, 15334465. URL http://www.jstor.org/stable/27720402.

M. Reiter. Solving heterogeneous-agent models by projection and perturbation. *Journal of Economic Dynamics and Control*, 33(3):649–665, 2009. ISSN 0165-1889. doi: https://doi.org/10.1016/j.jedc.2008.08.010. URL https://www.sciencedirect.com/science/article/pii/S0165188908001528.

J. J. Rotemberg. Monopolistic price adjustment and aggregate output. *The Review of Economic Studies*, 49(4):517–531, 1982.