Monetary-Fiscal-Capital Account Interactions in a Small Open Economy*

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Abstract

I add capital account policy to a small open economy (SOE) model where monetary and fiscal policy interact. I find three policy regimes where the price level and exchange rate are uniquely determined. Two regimes are open economy generalizations of well-known 'monetary dominance' and 'fiscal dominance' regimes. They are generalized to a SOE by adding the condition that capital account policy must ensure the SOE's external solvency. The third regime—a 'capital account dominance' regime—is one where capital account policy uniquely determines the price level and exchange rates by targeting the real exchange rate. Monetary and fiscal policies in this regime accommodate to capital account policy's objective by ensuring government and external solvency. Notably, monetary contractions fail to reduce inflation in this new regime due to a lack of 'capital account backing'. These results have important theoretical implications for policymaking in SOEs: i) inflation-targeting central banks require the support of both fiscal and capital account policies to be able to control inflation and ii) a whole class of equilibria exists where monetary (and fiscal) policies cannot be used to stabilize inflation or exchange rates.

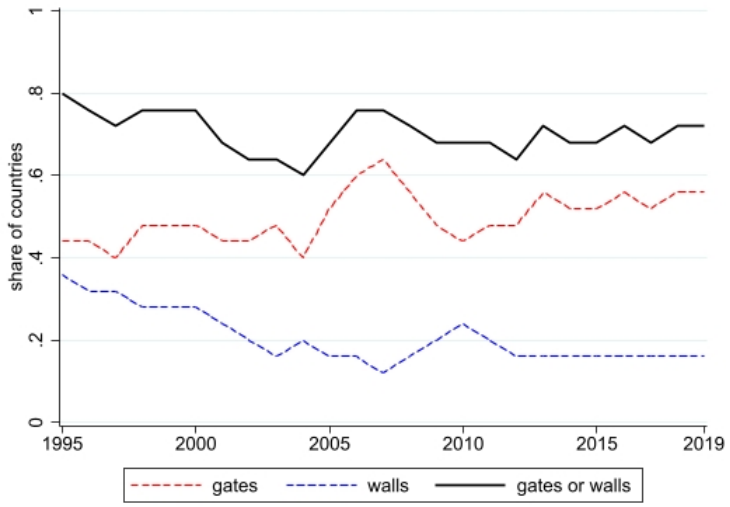
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1 Introduction

A subject that has long interested economists and policymakers is the effect of policy interventions on macroeconomic aggregates. In small open economies (SOEs), the question of how policy decisions can impact inflation and exchange rates has been one of particular interest due to a long history of policy-induced crises in these countries. A large body of literature has studied these episodes of price instability through the lens of their monetary and/or fiscal roots.[1] Less attention has been devoted to how capital account policies can also influence inflation and exchange rates movements. The pervasiveness of such policies in SOEs, however, makes a strong case for including them in the policy mix when studying the effects of policy on price stability. As Figure 1 shows, a

Figure 1: Share of 'Gates' and 'Walls' (1995-2019)



Notes: Figure shows the share of selected SOEs classified as 'Gates' and 'Walls' using the capital account openness index in Fernández et al., 2015. 'Wall' countries are defined as having capital controls on more than 70 percent of their transactions subcategories on a given year, 'Gate' countries are defined as having controls on less than 70 but more than 30 percent of their transaction subcategories on a given year (Klein, 2012). See Fernández et al., 2015 for more details.

significant share of the selected SOEs has consistently imposed moderate (classified as 'gates') to heavy (classified as 'walls') capital account controls since 1995.[2] Although the share of countries imposing heavy controls has decreased considerably since then, most SOEs still did not have what is considered to be an 'open' capital account in 2019. This is an interesting fact since the traditional

[1] A well-studied example is the case of Brazil. The hyperinflation of the late 1980s (Loyo, 1999) and the severe crisis of 2003 (Blanchard, 2004; Favero and Giavazzi, 2004) have been largely attributed to inconsistent monetary and fiscal policies. See also Kehoe and Nicolini, 2022 for examples in other Latin American countries such as Argentina in 2001 or Mexico in 1982.

[2] The sample includes 25 selected SOEs: Argentina, Australia, Brazil, Canada, Chile, Colombia, Czech Republic, Ecuador, Hungary, Indonesia, Israel, Korea, Rep., Malaysia, Mexico, New Zealand, Paraguay, Peru, Philippines, Poland, Romania, Singapore, South Africa, Sweden, Thailand and Uruguay.

'trilemma' of international macroeconomics would predict these countries should have had fully open capital accounts.[3] A now vast literature has studied the role of capital account policies as macroprudential tools to promote financial stability (e.g. Bianchi, 2011; Farhi and Werning, 2014; Jeanne and Korinek, 2019; Korinek, 2011; Schmitt-Grohé and Uribe, 2016) which could account for the observed prevalence of capital account controls. Moreover, and in line with this view, the IMF recently updated its institutional view in favor of countries using capital controls to promote financial stability (see Adrian et al., 2022). A more favorable view toward the use of capital account policies suggests that SOEs will likely continue to make extensive use of these policies. This underscores the importance of developing a theory of price level and exchange rate determination for SOEs that includes monetary, fiscal *and* capital account policies.

In this paper I ask two simple questions: How are the price level and exchange rate determined in a SOE? What is the role that policy plays in determining these prices? To answer these questions, I extend a monetary-fiscal interactions framework of price determination of Leeper, 1991; Sims, 1994; Woodford, 1994 to the case of a SOE. In particular, I add capital account policy to a stylized SOE model where monetary and fiscal policies interact through a government budget constraint. Following Leeper, 1991—and the literature of monetary-fiscal interactions more broadly—I explicitly model the dynamics of government (and external) debt. I find that, as a result of the three-way interaction between monetary, fiscal, and capital account policies, there exist three policy regimes where the price level and the exchange rate are uniquely determined. Identifying these three regimes extends the determinacy result of Leeper, 1991 to a SOE model by showing that equilibrium determinacy in these models requires one 'active' and *two* 'passive' policies as opposed to one 'active' and one passive' as the author derived for a closed economy.[4]

Two of these three regimes are open-economy generalizations of the closed-economy 'monetary dominance' (Regime M) and 'fiscal dominance' (Regime F) regimes of Leeper, 1991 and extend much of the same intuition of their closed economy counterparts to a SOE. However, they are generalized to a SOE by adding the condition that capital account policy must be 'passive'. The third regime—Regime K—is one where capital account policy is 'active' while monetary *and* fiscal policy accommodate to this target (i.e. are 'passive'). Since the labels 'active' and 'passive' have not been used to describe capital account policies before, I specifically define their meaning and

[3]Over the same time period, most of these countries also maintained flexible exchange rates and independent monetary policies.

[4]Leeper, 1991 defines active policy as when policy behaves unconstrained by the economy's budgetary conditions and passive policy as when it behaves consistent with budgetary conditions and agents' optimal behavior.

argue that they can be mapped into two well-studied types of capital account policies: prudential and mercantilist. In doing so I show that, by allowing for both active and passive capital account policies, this framework nests a large class of SOE models found in the literature, including SOE models where capital controls are absent.

I show that active capital account policy is equivalent to targeting the real exchange rate and that, by pursuing this target, capital account policy is able to uniquely determine inflation and exchange rates. Active capital account can therefore be linked to real-world policies whereby capital account interventions—such as capital controls or foreign reserves accumulation—are used to undervalue or otherwise manipulate a country's currency (see e.g. Costinot et al., 2014; Jeanne, 2013; Korinek and Serven, 2016). Notably, because equilibrium inflation and real exchange rate in this regime depend only on capital account policy surprises, this shows that a whole class of equilibria exists in SOE models where monetary and fiscal policies cannot be used to control inflation or stabilize exchanges rates. This result contributes to a literature on policy interactions that has long argued how equilibrium responses of inflation and exchange rates to shocks critically depend on the prevailing policy environment.

Early theoretical contributions to the monetary-fiscal interactions literature (Cochrane, 1998; Leeper, 1991; Sims, 1994; Woodford, 1994) argued that policy behavior plays a key role in determining the dynamics of equilibrium prices in a closed economy. Extending this framework to open economy models and studying the implications for price determinacy was explored early on by Dupor, 2000; Loyo, 1997. Dupor, 2000 found that the price level and exchange rate are indeterminate in a two-country, cash-in-advance model (with complete markets) if the nominal interest rate is pegged and fiscal surpluses are constant over time.[5] Witheridge, 2024 recently extended a monetary-fiscal interactions framework to a New-Keynesian SOE model à la Gali and Monacelli, 2005 (i.e. with complete markets and a continuum of measure-zero SOEs) and found two uniquely determined policy regimes ('monetary-led' and 'fiscal-led') which also echoed the two closed economy regimes of Leeper, 1991. In contrast to these papers, I use a flexible-price, two-country SOE model (De Paoli, 2009; Sutherland, 2005) with incomplete markets (Ghironi, 2006, 2008) where households exhibit an exogenous home bias in consumption and where the government can impose capital controls (Bianchi and Mendoza, 2010; Devereux et al., 2015) to show that prices are uniquely determined under multiple policy specifications. Related to my third regime, Ding and Jiang, 2024; Leith and Wren-Lewis, 2008 extend a framework of monetary-fiscal interactions to a two-country

[5]This result echoed the findings of Sargent and Wallace, 1975 and Kareken and Wallace, 1981.

model and also identify three policy regimes where prices are uniquely determined. Their third regime, however, differs from mine in that uniqueness obtains from active monetary-fiscal policies in the foreign country whereas, in my framework, uniqueness is ensured by an active capital account policy in the domestic country.

In a closed economy model like Leeper's, 1991, government solvency is a sufficient condition for the existence of an equilibrium. Uniqueness of an equilibrium therefore requires monetary and fiscal policies to coordinate so that one ensures government solvency while the other uniquely determines the price level. This yields the two policy regimes described in Leeper, 1991. The first corresponds to a conventional 'monetary dominance' equilibrium characteristic of New-Keynesian models where monetary policy targets inflation and fiscal policy adjusts as needed to satisfy the government budget constraint given the monetary authority's inflation target. The second corresponds to a 'fiscal dominance' equilibrium in which fiscal policy determines inflation by setting taxes independently from the government's finances and monetary policy adjusts to ensure the necessary financing for this fiscal objective.

In an open economy model, however, cross-country ownership of debt implies external solvency of the economy as a whole—reflected by the current account condition—is also a necessary condition for existence. Policy must therefore coordinate to ensure two independent constraints (the government budget constraint and the current account condition) are satisfied and the price level and exchange rate are uniquely determined. It follows that two policy authorities must accommodate (i.e. be passive) while a third follows an independent policy objective (i.e. is active) for a unique equilibrium to exist in a SOE model. In Regime K, where capital account policy is active, this implies monetary *and* fiscal policy must accommodate to ensure that government and external solvency are satisfied given capital account policy's real exchange rate target. I refer to this policy behavior as monetary and fiscal policy providing the necessary 'joint backing' for capital account policy.[6]

As argued above, a joint backing is necessary for *any* type of policy objective to be consistent with an equilibrium. In the last section of the paper, I turn to the implications of this for the theoretical conditions under which a central bank can target inflation in a SOE. Following the approach of Ljungqvist and Sargent, 2018, I specialize the model to illustrate in a simple setting how 'capital account backing' is just as critical as fiscal backing for the existence of an equilibrium

[6]See Aiyagari and Gertler, 1985; Cochrane, 2011; Sims, 2005 for discussions on the necessary 'fiscal backing' for monetary policy in a closed economy. A necessary 'joint backing' is a natural extension of this concept in the context of a SOE where monetary, fiscal, and capital account policies interact.

where the central bank can target inflation. In particular, I show that the conventional monetarist result whereby higher interest rates reduce inflation on impact critically depends on the behavior of both fiscal *and* capital account policy in a SOE. A monetary tightening, if not supported by both fiscal and capital account policy, will increase inflation in the future instead of reducing it today. This result echoes Sargent and Wallace's, 1981 'unpleasant monetarist arithmetic' and extends it to the case of a SOE. Despite the theoretical nature of the result, this has significant implications for SOE policymaking: a successful implementation of an inflation-targeting framework critically hinges on consistent fiscal *and* capital account policies being in place.

2 A Small Open Economy Model

The model is a simplified version of a two-country SOE model (e.g. De Paoli, 2009; Sutherland, 2005) rather than a model of a continuum of SOEs (e.g. Gali and Monacelli, 2005). The home country is a SOE and the foreign (large) country can be interpreted as the rest of the world (foreign variables are identified with a '*'). The SOE is populated by a continuum of identical and infinitely-lived households and a government split into three policy authorities: monetary, fiscal, and capital account. There are only two tradable goods (home and foreign). Households in the SOE receive a constant endowment of y units of the domestic good each period. There are only two assets: domestic and foreign one-period nominal government bonds. Domestic households are assumed to only trade in domestic government bonds while foreign households are able to trade in both domestic and foreign bonds. Markets are therefore incomplete as households will not be able to fully share risk across borders. The only sources of uncertainty in the model are monetary, fiscal, and capital account policy shocks.

2.1 Households

2.1.1 Preferences

The representative SOE household maximizes expected lifetime utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log C_t$$

where aggregate consumption C_t is defined as a Cobb-Douglas composite of two tradable goods $C_t = C_{Ht}^{1-\gamma} C_{Ft}^{\gamma}$ where C_{Ht}, C_{Ft} is domestic consumption of the home and foreign good. The

parameter γ reflects the degree of preference for the foreign good in the domestic consumption basket ($1-\gamma$ is the degree of home bias). The price of aggregate consumption P_t (i.e. the price level) takes the form $P_t = P_{Ht}^{1-\gamma} P_{Ft}^{\gamma}$ where P_{Ht}, P_{Ft} are the local currency prices of the domestic and foreign good. Households in the SOE face the flow budget constraint:

$$C_t + \frac{B_{Ht}}{P_t R_t} = \frac{P_{Ht}}{P_t} y - \tau_t + \frac{B_{Ht-1}}{P_t} \tag{1}$$

where B_{Ht} are domestic holdings of the domestic bond. I assume the law of one price holds for each individual good i.e. $\mathcal{E}_t P_{Ht}^* = P_{Ht}$ and $\mathcal{E}_t P_{Ft}^* = P_{Ft}$ where $\mathcal{E}_t$ is the nominal exchange rate defined as units of home currency per unit of the foreign currency.[7] In the spirit of Bianchi and Mendoza, 2010; Devereux et al., 2015, I assume the government taxes foreign purchases of domestic bonds at a rate of τ_t^c. This means $\tau_t^c > 0$ is equivalent to a tax on capital inflows and $\tau_t^c < 0$ is a subsidy.

2.1.2 Optimality

Households in each country face a consumption-savings decision. Domestic household optimality implies the following bond pricing equation:

$$\frac{1}{R_t} = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-1} \frac{P_t}{P_{t+1}} \right] \tag{2}$$

Since there are no foreign shocks in this model and shocks in the SOE have no effect on foreign variables, foreign inflation and consumption will be constant ($C_t^* = C^*$ and $\pi_t^* = \pi^*$ for all t). The foreign stochastic discount factor will therefore be constant and equal to one. The foreign bond pricing equations are:

$$\frac{1}{R_t^*} = \frac{\beta}{\pi^*} \tag{3}$$

$$\frac{1+\tau_t^c}{R_t} = \frac{\beta}{\pi^*} \mathbb{E}_t \left[\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right] \tag{4}$$

where the capital controls tax τ_t^c shows up as a wedge between the price and expected return of domestic bonds. Equations (3) and (4) imply the no-arbitrage condition:

$$0 = \mathbb{E}_t \left[R_t^* - \frac{R_t}{1+\tau_t^c} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right] \tag{5}$$

[7] If $\mathcal{E}_t$ goes up the home currency depreciates and/or the foreign currency appreciates.

which states that the expected foreign-currency return of domestic bonds adjusted for capital controls $\frac{R_t}{1+\tau_t^c}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$ must be equal to the return of foreign bonds R_t^*. Equation (5) implies the uncovered interest rate parity (UIP) will not hold when $\tau_t^c \neq 0$ and deviations from UIP will be driven by capital account policy.

Households also face an expenditure minimization problem given a level of aggregate consumption C_t, C_t^*. Domestic households minimizing their consumption expenditure implies demand for each good is:

$$C_{Ht} = (1-\gamma)\left(\frac{P_{Ht}}{P_t}\right)^{-1} C_t \quad ; \quad C_{Ft} = \gamma\left(\frac{P_{Ft}}{P_t}\right)^{-1} C_t \tag{6}$$

while foreign demand for the domestic good (demand for exports) is:

$$C_{Ht}^* = \gamma^*\left(\frac{P_{Ht}^*}{P_t^*}\right)^{-1} C_t^* \tag{7}$$

where $P_t^* = {P_{Ht}^*}^{\gamma^*} {P_{Ft}^*}^{1-\gamma^*}$ is the foreign price level and γ^* is foreign trade openness. Following De Paoli, 2009; Sutherland, 2005, I will consider the case of $\gamma^* \to 0$ as a way to reflect the fact that the home country is a SOE.[8] Lastly, an optimal consumption-savings plan must satisfy the transversality condition (TVC):

$$\lim_{T\to\infty} \mathbb{E}_t \Theta_{t+T} \frac{B_{Ht+T}}{P_{t+T+1}} = 0 \tag{8}$$

where $\Theta_{t+T} = \beta^T \left(\frac{C_{t+T}}{C_t}\right)^{-1}$ is the household's stochastic discount factor T periods ahead. The TVC rules out equilibria where households run a Ponzi scheme on the government by imposing that household assets cannot be expected to grow faster than the domestic real interest rate (i.e. exhibit explosive dynamics).

[8] The SOE assumption can be expressed as $\frac{y}{y^*} \to 0$. As in De Paoli, 2009; Sutherland, 2005 I relate the openness parameter γ^* to the SOE's relative size: $\gamma^* = \frac{y}{y^*}\gamma z$. Therefore, $\gamma^* \to 0$ follows directly from assuming the home country is a SOE. I add the shifting parameter z to allow for a non-zero trade balance in steady state. Importantly, this assumption and the foreign resource constraint $C_t^* = y^*$ imply $\gamma^* C_t^* \to \gamma z y \neq 0$. That is, SOE exports are not zero in steady state despite the fact that the foreign country is virtually a closed economy. See Appendix A for more details.

2.1.3 International Risk-Sharing and Net Foreign Assets

The risk-sharing condition for the SOE can be obtained by combining equations (2) and (4) and using the definition of the real exchange rate $Q_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}$:

$$0 = \mathbb{E}_t \left\{ \left[\frac{Q_t}{Q_{t+1}} \frac{1}{1+\tau_t^c} - \frac{C_t}{C_{t+1}} \right] \frac{1}{\pi_{t+1}} \right\} \tag{9}$$

The assumption that markets are incomplete implies that consumption growth is related to the real depreciation rate only in expectation.[9] That is, since households are only able to trade in non-contingent assets, they are unable to perfectly smooth their consumption across states of the economy. While the complete markets assumption is fairly common in the SOE literature (e.g. Benigno and Benigno, 2008; Gali and Monacelli, 2005), I deviate from this practice as market completeness effectively shut downs the current account channel in open economy models and therefore precludes the net foreign asset position from playing any role in the propagation of shocks in these models (see Ghironi, 2006, 2008; Leith and Wren-Lewis, 2008 for a discussion of this point). As I will show below, net foreign assets turn out to be a key state variable for the transmission of policy shocks to inflation and exchange rates.

2.2 Government

2.2.1 Budget Constraint and Limiting Conditions

The SOE government is composed of three policy authorities: monetary, fiscal, and capital account. Monetary policy is conducted by setting the gross nominal interest rate R_t while capital account policy consists of setting the capital controls tax rate τ_t^c. Fiscal policy consists of issuing one-period nominal debt B_t and levying lump-sum taxes net of transfers τ_t. Government revenues therefore consist of i) net tax revenues, ii) issuance of one-period nominal bonds sold at a price of $1/R_t$, and iii) capital controls revenues. The consolidated government budget constraint is:

$$\tau_t + \frac{B_t}{P_t R_t} + \frac{B_{Ft}}{P_t R_t} \tau_t^c = \frac{B_{t-1}}{P_t} \tag{10}$$

[9] In the absence of capital controls ($\phi_k = 0$), the incomplete markets assumption implies consumption, the real exchange rate, and the trade balance all follow a random walk in equilibrium. As discussed in Schmitt-Grohé and Uribe, 2003 there are several computationally similar methods to induce stationary equilibrium dynamics so that a log-linear approximation to the model is accurate. To keep the main theoretical point simple, I abstract from the case whe $\phi_k = 0$. However, any of these stationarity-inducing features can be introduced to this model to study equilibria where $\phi_k = 0$ with no effect on my main results.

where $\frac{B_{Ft}}{P_t R_t}\tau_t^c$ are capital control revenues. In the spirit of Bergin, 2000; Sims, 1997, I assume that the following limiting condition for total government debt holds in equilibrium:

$$\lim_{T\to\infty} \mathbb{E}_t \Theta_{t+T} \left(\frac{B_{Ht+T}}{P_{t+T+1}} + \frac{B_{Ft+T}}{P_{t+T+1}} \right) = 0 \tag{11}$$

Similarly to before, this condition states that, in equilibrium, total government debt cannot grow faster than the real interest rate.[10] Combining (8) and (11) yields a similar limiting condition for the country's net foreign liabilities (NFLs):

$$\lim_{T\to\infty} \mathbb{E}_t \Theta_{t+T} \frac{B_{Ft+T}}{P_{T+t+1}} = 0 \tag{12}$$

Equations (8),(11),(12) will be critical for assessing the existence of an equilibrium under different policy mixes.

2.2.2 Policy Rules

I assume the central bank and fiscal authority set their policy instruments R_t, τ_t, τ_t^c following policy rules. In specifying these rules, I use variables without subscript to denote steady state values. The central bank conducts monetary policy following a Taylor-type interest rate rule:

$$\frac{R_t}{R} = \alpha^M + \phi_\pi \frac{\pi_t}{\pi} + \varepsilon_t^M \tag{13}$$

where ε_t^M is a monetary policy shock and ϕ_π is a policy parameter governing the sensitivity of the nominal interest rate to deviations of inflation from steady state. The fiscal authority sets lump-sum taxes τ_t following a tax rule similar to Leeper, 1991:

$$\frac{\tau_t}{\tau} = \alpha^F + \phi_b \frac{b_t}{b} + \varepsilon_t^F \tag{14}$$

where $b_t \equiv \frac{B_{t-1}}{P_t}$ is the real value of total outstanding government debt at time t, ε_t^F is a fiscal policy shock, and ϕ_b is a policy parameter governing the sensitivity of taxes to deviations of government debt from steady state. The capital account authority conducts capital account policy by setting

[10] This limiting condition rules out equilibria where one government issues infinite debt and agents in the other country accumulate infinite assets. In this scenario, the purchase of these assets is financed by an infinite lending from the domestic government. While consistent with both domestic and foreign households' transversality conditions, this type of equilibrium implies infinite transfers of wealth from one country to the other. Bergin, 2000; Sims, 1997 have argued that political economy and political stability considerations make ruling out such equilibria a reasonable assumption.

the capital inflow tax on foreign bonds according to:

$$\frac{1+\tau_t^c}{1+\tau^c} = \alpha^K + \phi_k \frac{b_{Ft}}{b_F} + \varepsilon_t^K \tag{15}$$

where $b_{Ft} \equiv \frac{B_{Ft-1}}{P_t}$ is the real value of outstanding NFLs at time t, ε_t^K is a capital account policy shock, and ϕ_k is a policy parameter governing the sensitivity of capital controls to deviations of NFLs from steady state. I motivate the general form of policy rule (15) in two ways. First, there is some empirical evidence (see Aizenman and Pasricha, 2013; Fratzscher, 2012) showing that changes in capital controls in emerging economies have been associated to changes in capital flows and exchange rates.[11] Second, the IMF has recently issued explicit recommendations to use capital controls as tools to stabilize capital flows and promote financial stability (see Adrian et al., 2022). Since capital flows are simply the change in a country's NFLs, this policy rule captures these facts.

Importantly, these policy rules aim at modeling government behavior in a tractable, yet empirically relevant way. They are not meant to capture optimal policy.

2.3 Equilibrium

2.3.1 Market Clearing

There are three domestic markets: the goods market, the bond market, and the foreign currency (FX) market. The domestic goods market clearing condition equates the supply (endowment) of the domestic good y to total demand (domestic and foreign):

$$\begin{aligned} y &= C_{Ht} + C_{Ht}^* \\ &= (1-\gamma)\left(\frac{P_{Ht}}{P_t}\right)^{-1} C_t + \gamma^* \left(\frac{P_{Ht}^*}{P_t^*}\right)^{-1} C_t^* \\ &= Q_t^{\frac{\gamma}{1-\gamma}}(1-\gamma)C_t + Q_t^{\frac{1}{1-\gamma}}\gamma^* C_t^* \end{aligned} \tag{16}$$

where I use individual demands for C_{Ht}^* and C_{Ft} and the fact that, from the definitions of P_t, P_t^*, we have $\frac{P_{Ht}}{P_t} = Q_t^{-\frac{\gamma}{1-\gamma}}$. Using equation (16) we can write aggregate consumption as a function of

[11] Modifying the policy rule so that capital inflow taxes respond to the real exchange rate (another way of capturing changes in capital flows), e.g. $\frac{1+\tau_t^c}{1+\tau^c} = \alpha^K + \phi_k \frac{Q_t}{Q} + \varepsilon_t^K$, produces isomorphic equilibrium dynamics.

only the real exchange rate and exogenous variables:

$$C_t = \frac{Q_t^{-\frac{\gamma}{1-\gamma}} y - Q_t \gamma^* C_t^*}{1-\gamma} \tag{17}$$

This expression shows that domestic consumption will be inversely related to the real exchange rate. The real exchange rate in this model can be interpreted as the relative price of the foreign good in terms of the domestic good. In particular, we can use the Law of One Price for each good and the definitions of P_t, P_t^*, to write:

$$Q_t = \left(\frac{P_{Ft}}{P_{Ht}}\right)^{1-\gamma}$$

where I also make use of the fact that $\gamma^* \to 0$. Hence, a higher real exchange rate implies the domestic good becomes relatively cheaper. This induces higher demand for exports and also lowers the value of the domestic endowment. Therefore domestic consumption falls as domestic households feel poorer and external demand for goods increases.

The condition that ensures the FX market clears is the familiar current account condition that equates the change of a country's net foreign asset position to its trade balance:

$$-\left(\frac{B_{Ft}}{P_t R_t}(1+\tau_t^{c*}) - \frac{B_{Ft-1}}{P_t}\right) = \frac{\mathcal{E}_t P_{Ht}^*}{P_t} C_{Ht}^* - \frac{P_{Ft}}{P_t} C_{Ft} \tag{18}$$

Combining the household budget constraint (1) and the government budget constraint (10), and imposing market clearing, we can obtain this economy's aggregate resource constraint:

$$C_t + tb_t = \frac{P_{Ht}}{P_t} y \tag{19}$$

where I define $tb_t \equiv \frac{\mathcal{E}_t P_{Ht}^*}{P_t} C_{Ht}^* - \frac{P_{Ft}}{P_t} C_{Ft}$ as the SOE's trade balance. Combining the resource constraint (19) with the expression for consumption (17), we can also write the trade balance as a function of only the real exchange rate and exogenous variables:

$$tb_t = \frac{Q_t \gamma^* C_t^* - Q_t^{-\frac{\gamma}{1-\gamma}} \gamma y}{1-\gamma} \tag{20}$$

Equations (17) and (20) will be useful below to show that the present value of trade balances can be expressed in terms of only current and future real exchange rates.

By Walras' Law, the bond market will always clear when conditions (16) and (18) hold. It is nonetheless useful to state the domestic-issued bond market clearing condition:

$$B_t = B_{Ht} + B_{Ft} \tag{21}$$

which simply states that the quantity supplied by the government B_t must equal the sum of domestic and foreign demands. Importantly, when invoking Walras' Law, I follow the rest of the SOE literature in assuming that all foreign markets clear independent of SOE outcomes. As mentioned in footnote 8 above, this follows from the SOE assumption $\frac{y}{y^*} \to 0$ which implies that the foreign country is virtually a closed economy (i.e. $\gamma^* \to 0$). See Appendix A for details.

2.3.2 Intertemporal Conditions

Solving the government's budget constraint (10) forward, imposing limiting condition (11), we can obtain the government's intertemporal solvency condition:

$$\frac{B_{t-1}}{P_t} = \tau_t + \sum_{j=1}^{\infty} \mathbb{E}_t \Theta_{t+j} (\tau_{t+j} + \tau^c_{t+j-1} b_{Ft+j}) \tag{22}$$

This condition states that, in equilibrium, the real value of total outstanding government liabilities must equal the present value of current and future government primary surpluses. In contrast to closed-economy versions of (22), government liabilities now have an external debt component (since $B_{t-1} = B_{Ht-1} + B_{Ft-1}$) and primary surpluses are composed of net taxes τ_{t+j} and net capital controls revenue $\tau^c_{t+j-1} b_{Ft+j}$.

Solving the current account condition (18) forward, imposing limiting condition (12), we obtain the intertemporal current account condition:

$$\frac{B_{Ft-1}}{P_t} = tb_t + \sum_{j=1}^{\infty} \mathbb{E}_t \Theta_{t+j} \left(tb_{t+j} + \tau^c_{t+j-1} b_{Ft+j} \right) \tag{23}$$

This condition similarly states that, in equilibrium, the real value of the country's outstanding net foreign liabilities (NFLs) must equal the present value of current and future trade balances tb_{t+j} and capital controls revenue $\tau^c_{t+j-1} b_{Ft+j}$.

To see how these conditions relate monetary policy to fiscal and capital account policies, we

can substitute (22) back into (10) and (23) into (18) to obtain:

$$\frac{B_t}{R_t P_t} = \sum_{j=1}^{\infty} \mathbb{E}_t[\Theta_{t+j}\tau_{t+j}] + \sum_{j=2}^{\infty} \mathbb{E}_t[\Theta_{t+j}\tau^c_{t+j-1}b_{Ft+j}] \tag{24}$$

$$\frac{B_{Ft}}{R_t P_t} = \sum_{j=1}^{\infty} \mathbb{E}_t[\Theta_{t+j}tb_{t+j}] + \sum_{j=2}^{\infty} \mathbb{E}_t[\Theta_{t+j}\tau^c_{t+j-1}b_{Ft+j}] \tag{25}$$

Equations (24) and (25) relate the market value of government and external debt to the present value of the future expected payoffs associated with holding each type of debt. Government debt B_t is valued as a claim on future government surpluses since it is by raising taxes (lump-sum or capital controls) that the government will be able to pay off its liabilities. Similarly, domestic-issued debt B_{Ft} is valued as a claim on the future surpluses that the domestic economy as a whole will be able to generate.[12] In this case, the domestic economy's surpluses are equal to trade surpluses and the resources the government extracts from foreigners via capital controls.

Equations (24) and (25) show how monetary (R_t), fiscal (τ_t), and capital account (τ^c_t) policies are all linked by the fact that the government and the SOE as a whole must be intertemporally solvent in equilibrium. As I will show below, this implies that different monetary-fiscal-capital account policy mixes can deliver different equilibrium inflation and exchange rate dynamics.

3 Equilibrium Determinacy in a Small Open Economy

3.1 Log-Linearized Model

I log-linearize the model around a symmetric zero-inflation deterministic steady state where purchase power parity (PPP) holds (i.e. $Q = 1$ and $\pi = \pi^* = 1$) and where capital controls are zero (i.e. $\tau^c = 0$).[13]. To keep the main theoretical point simple, I concentrate in a steady state where the SOE is a net borrower (i.e. $b_F > 0$).[14] Below, I use the notation $\hat{x}_t$ to denote log (or percent) deviations from steady state i.e. $\hat{x}_t = \log(\frac{x_t}{x}) \approx \frac{x_t - x}{x}$. Equilibrium dynamics can be reduced to a system of four difference equations in the real exchange rate Q_t, CPI inflation π_t, outstanding real

[12] I use the term 'domestic-issued debt' to highlight the fact that trade surpluses back the portfolio of all domestic-issued and externally-held debt (call it D_t). Because in this model we only allow for government-issued debt in the domestic economy, we have $D_t = B_{Ft}$. This important distinction is the reason why lump-sum taxes τ_t do not affect the market value of external debt. Higher taxes, while increasing the government's repayment capacity, also reduce domestic households' capacity to repay their external liabilities. Foreign investors only care about the capacity of the domestic economy as a whole to repay its foreign liabilities and so transfers between government and households will have no effect on their valuation of domestic debt.

[13] See Appendix B for details about the deterministic steady state.

[14] For a discussion of the case when $b_F \leq 0$ see Ferrer (2025).

government debt b_t, and outstanding real NFLs b_{Ft}:[15]

$$\mathbb{E}_t[\hat{Q}_{t+1}] = \hat{Q}_t - \frac{1-\gamma}{1+\kappa}\left(\phi_k \hat{b}_{Ft} + \varepsilon_t^K\right) \tag{26}$$

$$\mathbb{E}_t[\hat{\pi}_{t+1}] = \phi_\pi \hat{\pi}_t + \varepsilon_t^M - \frac{\gamma+\kappa}{1+\kappa}(\phi_k \hat{b}_{Ft} + \varepsilon_t^K) \tag{27}$$

$$\mathbb{E}_t[\hat{b}_{t+1}] = \frac{1-(1-\beta)\phi_b}{\beta}\hat{b}_t - \frac{b_F}{b}\phi_k \hat{b}_{Ft} + \phi_\pi \hat{\pi}_t - \mathbb{E}_t[\hat{\pi}_{t+1}] + \varepsilon_t^M - \frac{1-\beta}{\beta}\varepsilon_t^F - \frac{b_F}{b}\varepsilon_t^K \tag{28}$$

$$\mathbb{E}_t[\hat{b}_{Ft+1}] = \left(\frac{1}{\beta} - \phi_k\right)\hat{b}_{Ft} + \phi_\pi \hat{\pi}_t - \mathbb{E}_t[\hat{\pi}_{t+1}] - \frac{\omega}{\beta b_F}\hat{Q}_t + \varepsilon_t^M - \varepsilon_t^K \tag{29}$$

where $\omega = \gamma^* C^* + \gamma C \frac{\gamma+\kappa}{1-\gamma}$ and $\kappa = \frac{\gamma^* C^*}{(1-\gamma)C}$ are constants related to steady state consumption. Equations (26) and (27) are the incomplete markets risk-sharing condition and Fisher equation governing the dynamics of the real exchange rate and inflation while equations (28) and (29) are the government budget constraint and current account condition governing the dynamics of government and external debt.

3.2 Determinate Equilibria

The dynamic system formed by equations (26)-(29) consists of two forward-looking (or 'jump') variables π_t, Q_t and two backward-looking (or state) variables b_t, b_{Ft}. A sufficient condition for the existence of a unique saddle-path equilibrium is that two of the system's eigenvalues are greater than one and two are less than one in absolute value (Blanchard and Kahn, 1980). This implies the existence of three types of determinate equilibria (or regimes) defined by the values of the policy parameters ϕ_π, ϕ_b, ϕ_k.

Proposition 1. *Assuming the SOE is a net borrower ($b_F > 0$), the system of equations* (26)-(29) *has a (locally) unique and bounded equilibrium if and only if policy parameters ϕ_π, ϕ_b, ϕ_k satisfy one of the following mutually exclusive conditions:*

1. $|\phi_\pi|, |\phi_b| > 1$ *and* $\phi_k \in [0, \overline{\phi_k}]$ *(Regime M*)*
2. $|\phi_\pi|, |\phi_b| < 1$ *and* $\phi_k \in [0, \overline{\phi_k}]$ *(Regime F*)*
3. $|\phi_\pi| < 1$, $|\phi_b| > 1$ *and* $\phi_k \notin [0, \overline{\phi_k}]$ *(Regime K)*

where $\overline{\phi_k} = \frac{2(1+\beta^{-1})}{\frac{1-\gamma}{1+\kappa}(\frac{\omega}{\beta b_F}+2)} > 0$.

***Proof**: See Appendix D.*

[15] See Appendix C for a derivation of the linearized model.

The first two regimes in Proposition 1 are open-economy generalizations of the active monetary, passive fiscal regime (Regime M) and active fiscal, passive monetary (Regime F) identified by Leeper, 1991 in a closed economy. I label these open economy regimes Regimes M* and F*. Note these regimes differ from their closed economy counterparts only in that the condition for passive capital account policy ($\phi_k \in [0, \overline{\phi_k}]$) is added to the usual closed economy conditions for active/passive monetary policy. The third regime, the 'capital account dominance' regime or Regime K, corresponds to a policy regime where capital account policy is active ($\phi_k \notin [0, \overline{\phi_k}]$) and monetary and fiscal policy are passive ($|\phi_\pi| < 1$, $|\phi_b| > 1$).

Since the terms 'active' and 'passive' capital account policy have not been used before in the literature, I now will precisely define their meaning and argue that they are simply a relabeling of two well-known types of capital controls.

3.3 Defining 'Active' and 'Passive' Capital Account Policy

Starting from Leeper's (1991) definition of active/passive policy, active capital account policy is defined as when policy sets capital controls as if unconstrained by current budgetary conditions. In this model, the economy's budgetary conditions can be summarized by the government budget constraint (10) and the current account condition (18). On the other hand, passive capital account policy is defined as when capital controls are set to satisfy (10) and (18) given the actions of any active policy authority and agents' optimal behavior.

Using these definitions, we can now relate these labels to the regions for ϕ_k shown in Proposition 1. Consider the example of an unanticipated monetary policy tightening that increases domestic interest rates ($\varepsilon_t^M > 0$). Higher nominal rates, which lower bond prices, will raise foreign demand for domestic bonds B_{Ft} and, as inflation tends to fall due to the monetary tightening, also raise outstanding NFLs $b_{Ft} = \frac{B_{Ft-1}}{P_t}$.

If $\phi_k \in [0, \overline{\phi_k}]$, policy rule (15) implies that higher outstanding NFLs, b_{Ft}, will lead policy to *increase* the capital inflow tax τ_t^c. Foreign investors, facing a higher tax on their purchases of domestic bonds, demand less of these bonds which offsets the effect of lower bond prices. As implied by policy rule (15), the capital account authority will keep τ_t^c above steady state until the initial increase in demand for domestic bonds is completely offset by higher bond taxes and b_{Ft} returns to steady state. Hence, when $\phi_k \in [0, \overline{\phi_k}]$, capital account policy behaves consistent with the stabilization of NFLs and therefore ensures the current account condition holds at every period. By the above definition, I associate this behavior with passive capital account policy. This case

illustrate how a capital account policy stabilizes NFLs by responding in the 'right direction' (i.e. increase τ_t^c when NFLs increase).

If $\phi_k < 0$, higher outstanding NFLs, b_{Ft}, now would imply a *lower* capital inflow tax. Since foreign investors now face lower taxes on their purchases of domestic bonds, they will demand more of these bonds. This increase in demand piles on the initial increase caused by higher domestic rates. Next period's outstanding NFLs, $b_{Ft+1} = \frac{B_{Ft}}{P_{t+1}}$, increase due to both effects. Since $\phi_k < 0$, capital account policy will lower next period's bond tax τ_{t+1}^c even more which, in turn, will induce an even higher demand for NFLs in $t+2$. It is easy to see how, in the absence of monetary or fiscal responses, these dynamics cause NFLs to diverge and violate limiting condition (12). Thus, when $\phi_k < 0$, capital account policy behaves as if unconstrained by the current account condition. By the above definition, I associate this behavior with active capital account policy. This case illustrates how capital account policy responding in the 'wrong direction', drives agents (acting optimally) to explode the SOE's NFL position.

If $\phi_k > \overline{\phi_k}$, even though policy responds in the 'right direction' since $\overline{\phi_k} > 0$, this response is strong enough to destabilize NFLs like in the case when $\phi_k < 0$. When lower inflation raises b_{Ft}, the capital account authority raises τ_t^c by 'too much', that is, the increase in the bond tax τ_t^c is enough to fully offset the initial increase due to higher rates and also bring NFLs down to below their steady state level. Since $\phi_k > \overline{\phi_k} > 0$, this then prompts the capital account authority to strongly *lower* the tax τ_t^c causing NFLs to rise above their steady state again. We can see how, in the absence of monetary-fiscal responses, $\phi_k > \overline{\phi_k}$ creates unstable and oscillating NFL dynamics. As in the case when $\phi_k < 0$, NFLs diverge and thus violate limiting condition (12). Therefore, when $\phi_k > \overline{\phi_k}$, capital account policy again behaves as if unconstrained by the current account condition and therefore I also associate with active capital account policy.

Finally, consider the case of $\phi_k = 0$. This case can be classified as a special case of passive capital account policy. We defined passive policy behavior as being consistent with current budgetary conditions and, therefore, with NFL stability. When capital account policy keeps the bond tax constant, it allows for a fully market-determined—as opposed to policy-determined—equilibrium real exchange rate. As any market-determined price, a market-determined real exchange rate will adjust to ensure markets clear. In this case, it will ensure the FX market clears in every period. Therefore, capital account policy, by not systematically adjusting τ_t^c, ensures current account solvency and NFL stability by allowing the real exchange rate to float freely. Therefore, this behavior satisfies the definition of passive policy.

3.4 Active/Passive Capital Account Policy in the Literature

A natural question that arises from the above discussion is: How is the behavior implied by active/passive capital account policies related to how they have been modeled in the vast literature on capital controls? In this section I argue that active/passive capital account policy can be thought of as relabeling two well-known motives for imposing capital controls. In doing so, I show that this framework can nest a large class of SOE models in the literature.

Two main motives for imposing capital controls have been identified in the capital controls literature: a prudential and a competitiveness motive. The prudential motive can be described as imposing capital controls to promote financial and macroeconomic stability often by correcting type of externality that leads agents to over-borrow from abroad (see e.g. Korinek, 2011). This type of policy has often been described as 'counter-cyclical' capital controls or as 'leaning against the wind' (see Farhi and Werning, 2014; Fernández et al., 2015) since policy tightens controls during booms and loosens during busts. As argued above, passive capital account policy stabilizes NFLs. This can be readily interpreted as having a prudential motive since it aims at undoing deviations of NFLs from some target level i.e. prevent over-borrowing. Moreover, it is easy to show in this model that, when capital account policy is passive, capital controls behave counter-cyclically.[16] Prudential capital controls can therefore be interpreted as passive capital account policy in this framework.

The competitiveness motive consists of using capital controls with the objective to promote export-led growth by managing or manipulating the country's terms of trade (see Pasricha, 2022). This type of capital controls have also been referred to as 'mercantilist' capital controls. As I show below, when capital account policy is active it effectively targets the real exchange rate and therefore the country's terms of trade since $\mathcal{T}_t \equiv \frac{P_{Ft}}{P_{Ht}} = Q_t^{\frac{1}{1-\gamma}}$. Mercantilist capital controls can therefore be interpreted as active capital account policy in this framework.

A more general case is the class of SOE models with no capital controls. This can be captured by setting $\phi_k = 0$ and $\tau^c = 0$. Since, as argued above, $\phi_k = 0$ can be seen as a special case of passive capital account policy, it follows that specifying a SOE model without capital account policy is implicitly assuming that capital account policy is passive and is therefore also nested by this framework.

[16] When $\phi_k \in [0, \overline{\phi_k}]$, higher NFLs, which is equivalent to higher capital inflows, leads to higher controls. Hence policy raises controls when capital inflows increase i.e. controls are counter-cyclical.

4 The Capital Account Dominance Regime

As Proposition 1 shows, bringing capital account policy into a SOE model with monetary-fiscal interactions has non-trivial implications for the regions of the parameter space where a unique equilibrium exists. Adding capital controls as a third policy instrument creates the possibility of a third class of equilibria where the price level and exchange rate are uniquely determined: a 'capital account dominance' regime—or Regime K for short. Below, I describe the main properties of this new regime emphasizing how the price level and exchange rate are determined and the implications this has for policy.

4.1 Active Capital Account Policy as Real Exchange Rate Targeting

Proposition 2. *Under active capital account policy and passive monetary-fiscal policies, capital account policy can target the real exchange rate. In particular, the equilibrium real exchange rate is:*

$$\hat{Q}_t = \sum_{k=0}^{\infty} \lambda_k \mathbb{E}_t[\varepsilon_{t+k}^K] \tag{30}$$

where the λ_k depend on model parameters and the policy parameter ϕ_k.
***Proof**: See Appendix E*

Proposition 2 formalizes the idea that, when capital account policy responds to changes in NFLs in a way that makes them diverge (i.e. is active), it effectively controls the real exchange rate. Active capital account policy can therefore be interpreted as policy having a specific policy objective in mind: targeting the real exchange rate. The reason that a real exchange rate target is associated with diverging NFLs is the existence of a policy trade-off between stabilization of both variables.

4.1.1 Trade-Off Between Real Exchange Rate and NFL Stability

Of central importance to understand the result stated in Proposition 2 is the fact that a policy trade-off generally exists between real exchange rate and NFL stabilization. This is not a specific feature the class of equilibria where capital account policy is active (i.e. Regime K) but of household optimizing behavior. To see this, we can solve the household's risk-sharing condition (26) forward

to obtain:

$$\hat{Q}_t = \frac{1-\gamma}{1+\kappa}\sum_{j=0}^{\infty}\mathbb{E}_t\widehat{1+\tau^c_{t+j}}$$

Then, the current account condition (29) can be written as:

$$\mathbb{E}_t[\hat{b}_{Ft+1}] = \beta^{-1}\hat{b}_{Ft} + i_t - \mathbb{E}_t[\hat{\pi}_{t+1}] - \frac{\omega}{\beta b_F}\hat{Q}_t - \widehat{1+\tau^c_t}$$

From these two equilibrium conditions we see that changes in τ^c_t will have opposite effects on the real exchange rate Q_t and NFLs b_{Ft+1}. If, for instance, policy increases τ^c_t, this will increase Q_t as a higher tax on foreign bond purchases causes foreign households to demand less domestic bonds and capital inflows into the SOE fall. The current account condition (18) implies that as capital inflows fall (i.e. the change in NFLs falls), the trade balance must increase to clear the FX market. The real exchange rate must therefore rise to induce higher demand for domestic exports. On the other hand, it is straightforward to see that higher τ^c_t causes NFLs to fall as a higher tax on foreign bond purchases reduces foreign demand for these bonds. Notice that, in both cases, the effect does not depend on the values of policy parameters, implying that this trade-off is not specific to any policy regime.

The opposite signs of these effects imply that capital account policy must choose between stabilizing the real exchange rate and NFLs (capital inflows). This is why targeting the real exchange rate results in policy destabilizing NFLs. Capital account policy preventing real exchange rate volatility will inevitably create NFL volatility as each attempt to offset changes in Q_t will create an opposite-signed change in b_{Ft+1}. As mentioned above, identifying this trade-off allows us to rationalize active capital account policy not as creating unstable NFL dynamics for no reason, but as a specific policy objective to stabilize the real exchange rate. This interpretation of active capital account serves to bring Regime K closer to the reality of policy-making as SOEs have resorted to real exchange rate (or terms of trade) stabilization policies in times of crisis in the past.[17]

[17]Examples include Chile in the late 1980s after a severe banking crisis (see Caputo and Saravia, 2018) or Brazil in the early 2010s after the financial crisis (see Benelli et al., 2014)

4.2 Inflation Dynamics under Capital Account Dominance

4.2.1 Price Level Determination

By targeting the real exchange rate, capital account policy also pins down a unique price level and therefore can freely determine inflation under Regime K. To see why, notice the intertemporal current account condition (23) can be written as:

$$\frac{B_{Ft-1}}{P_t} = tb_t + \sum_{j=1}^{\infty} \mathbb{E}_t \Theta_{t+j} \left(\prod_{k=0}^{j-1} (1 + \tau^c_{t+k}) \right) tb_{t+j}$$

Then, since $\Theta_{t+j} = \beta \frac{C_t}{C_{t+j}}$, we can use the expressions for consumption (17) and the trade balance (20) to write the above as:

$$\frac{B_{Ft-1}}{P_t} = PVTB\Big(\{Q_{t+j}, \tau^c_{t+j}\}_{j=0}^{\infty} \Big) \tag{31}$$

where $PVTB$ stands for the present value of trade balances. Equation (31) states that the present value of trade surpluses is, in equilibrium, a function of current and future real exchanges rates and capital inflow taxes only. From (30) we know that current and expected equilibrium real exchange rates are, in turn, a function of current and future capital controls shocks $\{\mathbb{E}_t \varepsilon^K_{t+j}\}_{j=0}^{\infty}$. Thus, by choosing the sequence $\{\mathbb{E}_t \varepsilon^K_{t+j}\}_{j=0}^{\infty}$, capital account policy can effectively pin down the entire right-hand side of (31), that is, the present value of trade surpluses. The price level P_t—being the only free variable left in (31)—will adjust to equate the real value of net foreign liabilities to the expected present value of trade surpluses *set by capital account policy*. Fiscal policy will then take this price level, and the sequence of capital controls, as given and set the sequence of lump-sum taxes $\{\tau_{t+j}\}_{j=0}^{\infty}$ to satisfy (22).

To understand the mechanisms at work, it is useful to linearize the intertemporal current account condition:

$$\hat{\tilde{b}}_{Ft-1} - \hat{\pi}_t = \underbrace{\frac{\omega}{b_F} \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t \hat{Q}_{t+j}}_{\text{PV of trade balances}} - \underbrace{(1-\beta)\beta \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t [\hat{Q}_{t+j+1} - \hat{Q}_{t+j}]}_{\text{PV of real interest rates}}$$

where I define the real value of *current* NFLs as $\tilde{b}_{Ft} = \frac{B_{Ft}}{P_t}$. As this shows, the present value of trade surpluses can be decomposed into two terms. The first term reflects how the real exchange rate affects the trade balance while the second term reflects how real exchange *changes* affect the

domestic real interest rate at which these trade balances are discounted. The above allows us to express equilibrium inflation as:

$$\hat{\pi}_t = \hat{\tilde{b}}_{Ft-1} - \frac{\omega}{b_F}\sum_{j=0}^{\infty}\beta^j \mathbb{E}_t \hat{Q}_{t+j} + (1-\beta)\beta\sum_{j=0}^{\infty}\beta^j \mathbb{E}_t[\hat{Q}_{t+j+1} - \hat{Q}_{t+j}] \tag{32}$$

since, from (30), we know that current and future real exchange rates are a function of the exogenous shocks $\{\varepsilon^K_{t+j}\}$. Hence, an unexpected increase in τ^c_t (i.e. $\varepsilon^K_t > 0$) reduces equilibrium inflation through two separate channels. The first channel relates to how capital inflow taxes affect exchange rates and therefore the trade balance. Since $\varepsilon^K_t > 0$ increases the tax on foreign bond purchases, foreign households demand less domestic bonds and capital flows into the SOE fall. As argued above, this causes the real exchange rate Q_t to rise to generate a higher trade balance. Higher Q_t also causes the real value of domestic endowment $\frac{P_{Ht}}{P_t}y = Q_t^{-\frac{\gamma}{1-\gamma}}y$ to fall as the relative price of the domestic good fell. Lower relative income and higher demand for goods from abroad imply domestic consumption falls as reflected in the aggregate resource constraint $C_t + tb_t = \frac{P_{Ht}}{P_t}y$. The unexpected increase in τ^c_t therefore lowers domestic inflation by reducing domestic demand through a 'crowding-out' of domestic consumption and a reduction of real household income.

The second channel is related to how the capital inflow tax affects domestic real interest rates. When the tax that foreign agents must pay to purchase domestic bonds increases, foreign demand for these bonds falls. This excess supply in the bond market drives real bond prices down and their real return up. Higher real bond returns mean higher real interest rates in the SOE which, in turn, induce domestic households to increase their savings and reduce consumption. The higher domestic demand for bonds eliminates the excess supply in the bond market but creates an excess supply in the goods market as domestic demand for goods falls. Finally, a lower aggregate demand for goods and a constant supply y implies inflation falls to clear the goods market.

As (32) shows, monetary and fiscal surprises have no effect on today's equilibrium inflation in Regime K. This reflects the 'passiveness' of these policies as they must accommodate to ensure government and current account solvency taking the price level and exchange rate chosen by capital account policy as given. As I will discuss in the next section, monetary policy shocks will have an effect on expected future inflation $\mathbb{E}_t\pi_{t+j}$, $j \geq 1$. Higher domestic rates increase demand for domestic bonds, therefore household real wealth (both domestic and foreign) will increase next period. Expected inflation rises as agents foresee everybody trying to convert their higher financial wealth into consumption. In equilibrium, the increase in future inflation fully offsets the increase

in households' financial wealth and consumption is constant.

4.2.2 NFLs and Inflation Expectations

Inflation expectations under Regime K will be closely related to the dynamics of external debt. To keep the exposition simple, assume $\{\varepsilon^K_{t+j}\}$ are i.i.d and mean-zero shocks. Then, by rolling (32) forward and taking expectation conditional on information at t we can obtain the entire sequence of equilibrium expected inflation:

$$\mathbb{E}_t\hat{\pi}_{t+j} = \mathbb{E}_t\hat{\tilde{b}}_{Ft+j-1}, \quad \forall j \geq 1 \tag{33}$$

Inflation expectations have therefore a one-to-one relationship with the path of NFLs (i.e. external debt). This shows the importance of NFL stability in preserving price stability under Regime K. Now use the fact that outstanding NFLs are related to current NFLs by $\hat{b}_{Ft} = \hat{\tilde{b}}_{Ft-1} - \hat{\pi}_t$ to write the above condition as:

$$\mathbb{E}_t\hat{b}_{Ft+j} = 0, \quad \forall j \geq 1 \tag{34}$$

That is, in equilibrium, outstanding NFLs must always be expected to be constant at the steady state.[18] This condition formalizes the intuition in the previous discussion of how, when capital account policy is active, household optimal behavior leads to diverging NFLs if they deviate from steady state. Since diverging NFLs are inconsistent with equilibrium (violates condition (12)), it follows that deviations of NFLs from steady state cannot exist in an equilibrium under active capital account policy, which is precisely what condition (34) states.

Importantly, condition (33) implies that, in Regime K, changes in current and future foreign demand for domestic bonds must be perfectly offset by changes in expected inflation. This suggests that capital account policy can also control inflation expectations by adjusting future bond tax rates as this would directly impact foreign demand for domestic bonds and therefore inflation expectations.

An interesting implications of these simple theoretical relationships is that inflation under Regime K will be highly dependent on foreign investor sentiments since changes in foreign prefer-

[18] This quite stringent constraint on equilibrium NFLs follows from the assumption that $\{\varepsilon^K_t\}$ are i.i.d. In the more general case where shocks are allowed to be persistent, condition (34) will impose that NFLs must be expected to return to steady state by the time the initial shock has died out. Condition (34) can therefore be interpreted as a more restrictive version of limiting condition (12) that arises from active capital account policy behavior.

ences for domestic bonds would have to be fully offset by changes in inflation expectations. This gives a rationale for how real exchange rate targeting policies could have engendered episodes of high and volatile inflation in SOEs such as Colombia in 1990 and Mexico in 1987. Moreover, this framework could also provide a simple theoretical basis to explain the connection between high inflation during external debt crises. As (33) shows, concerns about the sustainability of a SOE's external debt would create higher expected inflation which would then, in turn, feed into higher inflation today.

Since monetary and fiscal policies must be consistent with these inflation expectations, they set the sequences $\{\tau_{t+j}, R_{t+j}\}_{j=0}^{\infty}$ to ensure that all current and expected future inflation rates (chosen by capital account policy) are consistent with intertemporal current account and government solvency. In other words, given the choices of capital account policy that determine all current and future $\{P_{t+j}, Q_{t+j}, \tau_{t+j}^{c}\}$, fiscal policy must the sequence of lump-sum taxes τ_{t+j} to obey (22) while monetary policy must the sequence of interest rates R_{t+j} to obey (2) in each period.

As I argue in the next section, what passive monetary-fiscal policies do in practice is 'manage' (i.e. offset or allow for) changes in household real wealth to accommodate to capital account policy's real exchange objective. Importantly, this will result in monetary-fiscal policies effectively stabilizing NFLs and government liabilities.

4.3 Why is a Joint Monetary-Fiscal Backing Necessary?

Since under Regime K capital account policy determines equilibrium P_t and Q_t, monetary and fiscal policy must accommodate to ensure these prices are consistent with all equilibrium conditions. In the spirit of Aiyagari and Gertler, 1985 I refer to this behavior as policy providing the 'necessary backing' for capital account policy. Unlike previous definitions for closed economies, the necessary backing in a SOE must be a *joint* backing from two policy authorities. The reason is that the government budget constraint and the current account condition impose two independent constraints on the equilibrium. They are independent because the government's ability to borrow from abroad introduces the concept of external solvency in addition to government solvency. Since both conditions must be satisfied in equilibrium, it follows that two policy authorities (or two independent policy instruments) are necessary to ensure an equilibrium exists. I state this result as a corollary of Proposition 1.

Corollary 1. *Equilibrium determinacy in a SOE model always requires one active policy and two passive policies.*

In Regime K, joint monetary-fiscal backing means agents expect equilibrium policies to eliminate wealth effects that would cause $\frac{B_{Ft-1}}{P_t}$ or $\frac{B_{Ht-1}}{P_t}$ to enter unstable paths. Cross-country ownership of government bonds implies that there will generally be two types of wealth effects: on domestic and on foreign bond holders. The necessity of a joint backing from two policies arises from the fact that fiscal policy is only able to eliminate *domestic* wealth effects since only domestic households pay domestic taxes. By strongly adjusting taxes in response to changes in the real value of government debt ($|\phi_b| > 1$), passive fiscal policy is able to offset domestic wealth effects and thus stabilize domestic demand for bonds (B_{Ht}). *Foreign* wealth effects must then be offset by monetary policy who—unlike fiscal policy—has the ability to affect foreign household wealth. By weakly reacting to inflationary pressures ($|\phi_\pi| < 1$), monetary policy can create inflation surprises that directly impact the ex-post real return of domestic bonds. This will affect the wealth of both domestic *and* foreign households who hold these bonds. Passive monetary policy is thus able to offset foreign wealth effects and, in doing so, stabilize total demand for domestic bonds ($B_t = B_{Ht} + B_{Ft}$). Finally, since bond market clearing implies that NFLs are $B_{Ft} = B_t - B_{Ht}$, it is clear that a *joint* monetary-fiscal backing is necessary to ensure NFL are stable. The passive behavior of both policy authorities is what ensures that an equilibrium exists under active capital account policy.

Finally, note that it is because monetary and fiscal policies are geared toward ensuring government and current account solvency that capital account policy is free to determine the real exchange rate Q_t and the price level P_t. This is the same idea as in a conventional, closed-economy, monetary dominance equilibrium where monetary policy is free to determine inflation *because* fiscal policy is geared toward ensuring government solvency.

4.4 Impulse Responses under Capital Account Dominance

The above results and intuition can be summarized by computing model-implied impulse response functions to different policy shocks. Figures 2-4 show equilibrium responses after a monetary contraction ($\varepsilon_t^M > 0$), a fiscal expansion ($\varepsilon_t^F < 0$), and an unexpected increase in capital inflow taxes ($\varepsilon_t^K > 0$) under Regime K.[19]

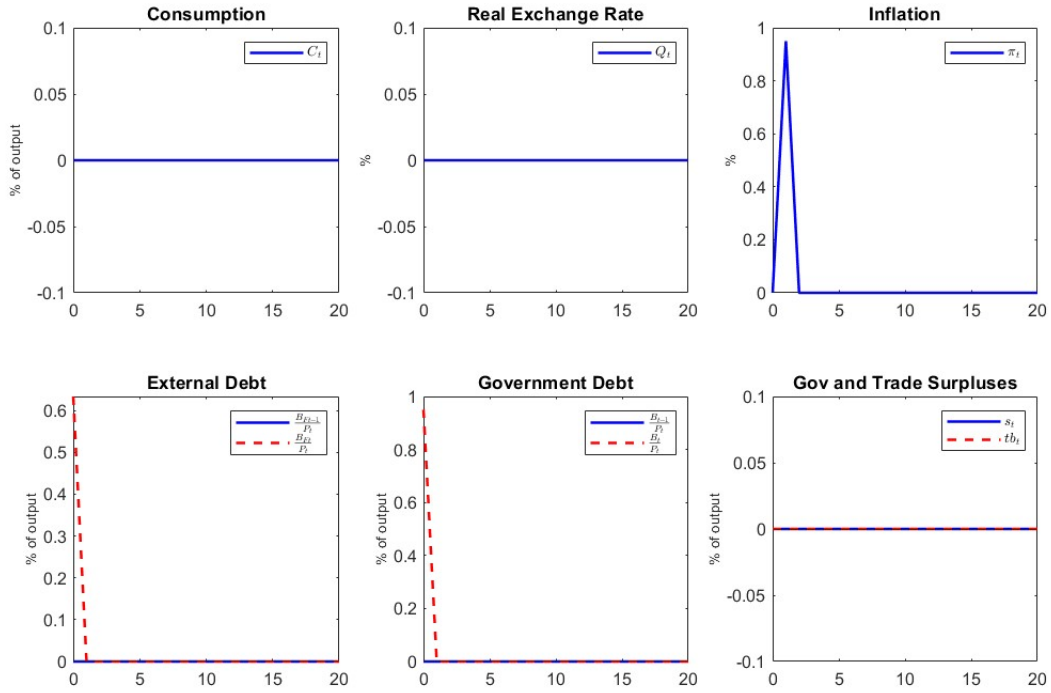
First, it is clear that capital account policy targets the real exchange rate as only capital control shocks have an effect on equilibrium Q_t. Figure 2 shows a monetary tightening has no effect on

[19] Responses are obtained using the following parameter values:

$$\beta = 0.95, \quad \gamma = 0.4, \quad z = 1.05, \quad \phi_\pi = 0, \quad \phi_b = 1.5, \quad \phi_k = -0.5$$

These imply $\frac{b_F}{y} = 0.67$ and $\frac{tb}{y} = 0.033$.

Figure 2: Impulse responses to a monetary contraction ($\varepsilon_t^M > 0$) under Regime K

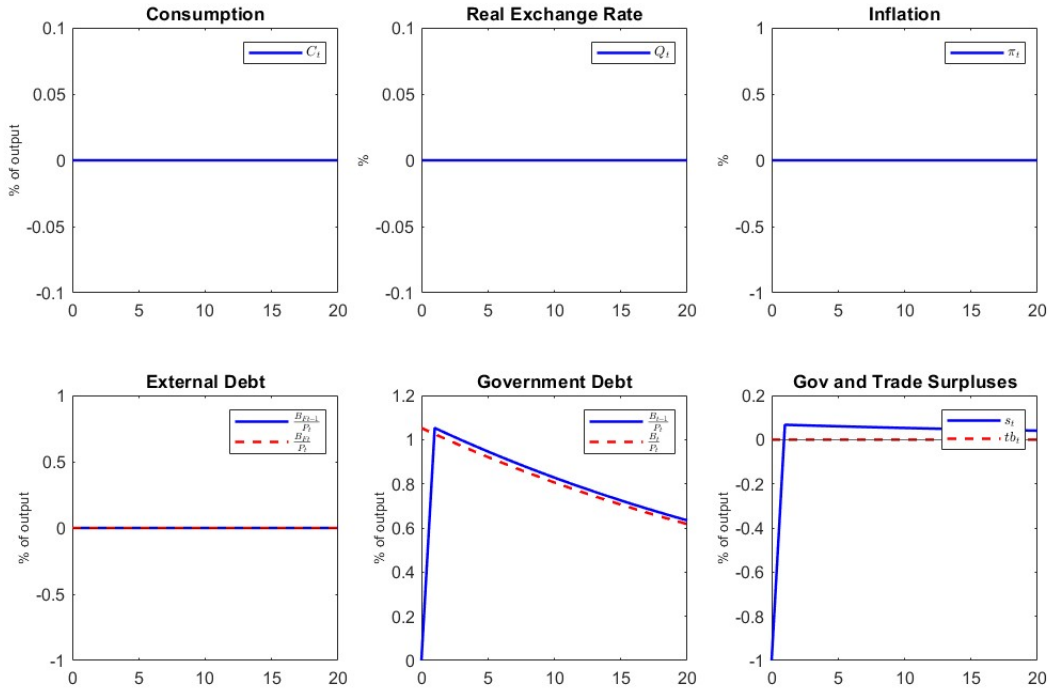


Notes: Figure shows equilibrium responses after a 100 basis point increase in R_t under Regime K. s_t are primary government surpluses and tb_t are trade surpluses.

current inflation and increases future inflation. This result echoes the 'unpleasant monetarist arithmetic' of Sargent and Wallace, 1981 where higher interest rates increase future inflation whenever they lack the necessary fiscal backing (see section 5.2). Inflation's response in Figure 2 stems from the fact that this monetary tightening lacks capital account backing—since capital account policy is active and therefore does not accommodate to monetary policy. As explained above, inflation rises at $t+1$ as foreign agents, in response to higher domestic rates, increase their demand for domestic bonds. In equilibrium, higher real returns are eroded away by higher expected inflation. If this was not the case, active capital account policy would lead NFLs to diverge in its attempt to stabilize the real exchange rate. Current inflation is unchanged as agents, due to higher inflation expectations, experience no change in their expected real wealth and therefore do not alter their consumption plan.

In Figure 3 we can see that the effects of a debt-financed fiscal expansion are fully offset by fiscal policy's passive behavior. After the initial expansion, the fiscal authority generates surpluses and slowly retires the higher level of real debt. Since foreign households experienced no effect from

Figure 3: Impulse responses to a fiscal expansion ($\varepsilon_t^F < 0$) under Regime K

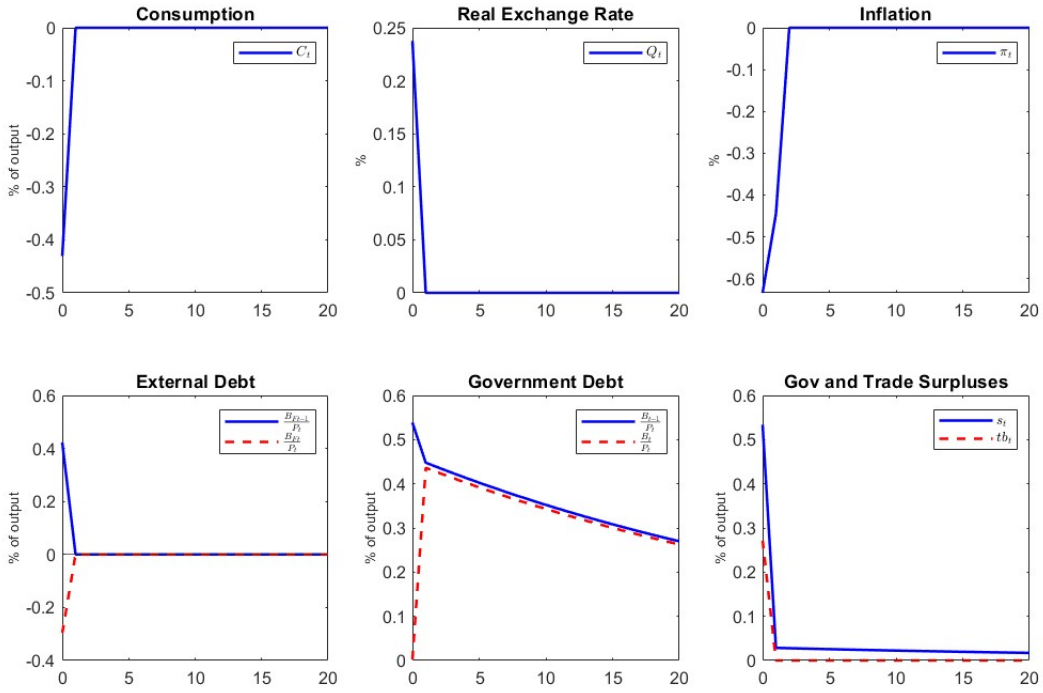


Notes: Figure shows equilibrium responses after a 1% of output decrease in τ_t under Regime K. s_t are primary government surpluses and tb_t are trade surpluses.

this shock (their tax liabilities did not change), they absorb none of the newly issued bonds, that is, NFLs are unchanged. Since there are no foreign wealth effects and domestic wealth effects are offset by higher future taxes, current and future inflation are both unchanged. Capital account policy targets the real exchange rate by fixing τ_t^c which means the present value of trade surpluses is constant.

Lastly, Figure 4 shows how, consistent with expression (30), $\varepsilon_t^K > 0$ causes the real exchange rate to rise (depreciate). After the increase in τ_t^c, which lowers inflation on impact, fiscal policy generates future surpluses to payoff a higher level of real debt caused by lower inflation. Inflation falls due to the contractionary effects of higher τ_t^c on domestic consumption discussed in the previous section. Future inflation also falls in equilibrium since NFLs B_{Ft} fall by more than inflation which must be offset by lower inflation at $t+1$ since the present value of trade surpluses from $t+1$ onward did not change.

Figure 4: Impulse responses to an increase in the capital inflow tax ($\varepsilon_t^K > 0$) under Regime K



Notes: Figure shows equilibrium responses in deviations from steady state after a 1 percentage point increase in the capital inflow tax rate τ_t^{c*} under Regime K. s_t are primary government surpluses and tb_t are trade surpluses.

5 Inflation Targeting in a Small Open Economy

As mentioned above, the first two regimes in Proposition 1 generalize Leeper's 1991 monetary and fiscal regimes to a SOE framework. In this section I discuss how the 'open economy' monetary dominance regime (Regime M*) highlights the fact that commonly-used New-Keynesian SOE models implicitly make the important assumption that capital account policy is always passive. Though this assumption might be close to reality in many cases, it has important implications for the theoretical conditions under which monetary policy can target inflation in a SOE and offers useful advice for policymakers in these countries.

5.1 When Can a Central Bank Target Inflation in a SOE?

A very common practice in the SOE and open economy literature is to assume a monetary authority that targets inflation. This assumption relies on imposing the so-called 'Taylor principle' whereby the nominal interest rate responds to changes in inflation by more than one-to-one (i.e. $\phi_\pi > 1$).

This amounts to assuming that monetary policy is always active. The Taylor principle underpins the results of any New-Keynesian model as it is a key assumption for these models to deliver a (locally) unique equilibrium.

As many authors in the monetary-fiscal interactions literature have argued before, models that follow the Taylor principle implicitly assume a passive fiscal authority. Similarly, Proposition 1 implies that SOE New-Keynesian models that impose the Taylor principle (i.e. assume monetary policy is active) are also implicitly assuming a passive capital account policy. An important policy implication of this is that SOE central banks who want to successfully target inflation will need the support of both fiscal *and* capital account policies. In other words, Proposition 1 suggests there exist restrictions on what equilibrium fiscal and capital account policies can be in a SOE model where the central bank is assumed to target inflation. In particular, it is not enough for a SOE to have accommodating fiscal policies to make an inflation target a sustainable policy objective, accommodating capital account policies are also necessary. These policies being accommodative means they are constrained by the government and current account intertemporal conditions (22) and (23) (i.e. are passive).

Specifically, fiscal policy being constrained by the government solvency condition (22) means—taking the price level and the exchange rate as given—it must set current and future taxes such that their present value is consistent with (22). For instance, if the central bank raises interest rates, this increases government liabilities since lower bond prices force the government to have to issue more debt to finance a given amount of spending. Then the fiscal authority must raise future taxes by enough to pay for the higher level of debt and the higher future interest spending that will accrue (hence why the condition $\phi_b > 1$ is needed). Notice that this behavior is equivalent to a closed-economy passive fiscal policy.

Similarly, capital account policy being constrained by condition (23) means—also taking the price level and exchange rate as given—it must set capital inflow taxes to ensure that the present value of trade balances is consistent with (23). For instance, if the central bank raises interest rates, this also increases the SOE's net foreign liabilities as some of the newly issued bonds are absorbed by foreign households. Then, the capital account authority must raise future capital inflow taxes to induce higher future trade balances to support the higher level of NFLs.[20] To see why higher

[20]This statement assumes that PPP holds in the long run as this fixes the long run real exchange rate Q. Relaxing this assumption i.e. allowing for Q to jump after a shock, creates the possibility for a monetary tightening to lower inflation on impact without a capital account tightening if capital account policy keeps the capital inflow tax constant (i.e. $\phi_k = 0$).

capital inflow taxes induce higher trade balances consider the fact that, intuitively, higher inflow taxes will lower capital inflows to the SOE. Then, as the flow of capital into the country drys out the real exchange rate depreciates. Finally, a more depreciated exchange rate brings higher trade surpluses as the domestic good becomes relatively cheaper.

The above implies that a monetary tightening requires a specific fiscal and capital account policy response (i.e. both fiscal and capital account authorities tightening in the future) to actually lower inflation on impact. This is formally stated as a second corollary to Proposition 1.

Corollary 2. *Assuming PPP holds in the long run, a monetary tightening in a SOE must be accompanied by a future fiscal and capital account tightening to deliver lower inflation on impact*

As mentioned above, the assumption of long-run PPP is only relevant in the special case of $\phi_k = 0$. See footnote 21 below for a description of how higher interest rates can lower inflation in the absence of a capital account tightening if the long-run real exchange rate is allowed to jump. In the next section I illustrate how the lack of either the fiscal or capital account backing results in a tighter money causing higher inflation in the future as emphasized by Sargent and Wallace, 1981.

5.2 Some Unpleasant Monetarist Arithmetic in a SOE

To illustrate how the effects of monetary policy on inflation critically depend on the stance of capital account (and fiscal) policy, in this section I follow Ljungqvist and Sargent, 2018 and simplify the model presented in section 3 to be able to reduce equilibrium dynamics to two 'periods'. The current ($t = 0$) and the future period ($t \geq 1$). Using this simplified framework, I show how the lack of either fiscal or capital account backing replicates Sargent and Wallace's, 1981 'unpleasant' monetarist arithmetic result whereby tighter money today fails to reduce current inflation and raises inflation in the future.

Following Ljungqvist and Sargent, 2018, I focus on a stationary equilibrium where:

$$\frac{P_t}{P_{t-1}} \equiv \pi_t = \pi \quad ; \quad Q_t = Q \qquad \forall t \geq 1$$

$$\frac{B_{Ht}}{P_t} \equiv b_{Ht} = b_H \qquad \forall t \geq 0$$

thus allowing P_0, Q_0 to differ from their long-run values. Also, I specialize policy to be such that:

$$\begin{aligned} &\tau_t = \tau \quad ; \quad \tau_t^c = \tau^c \quad ; \quad R_t = R && \forall t \geq 1 \\ &\frac{B_t}{P_t} \equiv b_t = b && \forall t \geq 0 \end{aligned}$$

thus allowing $\tau_0 \neq \tau, \tau_0^c \neq \tau^c, R_0 \neq R$. Without loss of generality for my results, I make the simplifying assumption that domestic endowment y is in units of the domestic consumption basket instead of in units of the domestic good

Imposing these assumption and the foreign resource constraint $C_t^* = y^* \ \forall t$, the Euler equations at $t = 0$ and $t \geq 1$ become:

$$\begin{aligned} \frac{1}{R_0} = \beta \frac{y - Q_0 \gamma^* y^*}{y - Q \gamma^* y^*} \frac{1}{\pi} \quad ; \quad \frac{1 + \tau_0^c}{R_0} = \beta \frac{Q_0}{Q} \frac{1}{\pi} && \quad t = 0 \\ \frac{1}{R} = \frac{\beta}{\pi} \quad ; \quad \frac{1 + \tau^c}{R} = \frac{\beta}{\pi} && \quad t \geq 1 \end{aligned}$$

Combining the two Euler equations at $t = 0$ yields:

$$1 + \tau_0^c = \frac{y - Q \gamma^* y^*}{y - Q_0 \gamma^* y^*} \frac{Q_0}{Q} \tag{35}$$

which shows that there will always be a inverse relationship between the equilibrium real depreciation rate $\frac{Q}{Q_0}$ and the capital inflow tax τ_0^c. Moreover, combining the two Euler equations at $t \geq 1$ implies $\tau^c = 0$. That is, capital inflow taxes must be zero in the long-run.

Next, we can impose the equilibrium assumptions above on the government budget constraint (10) and current account condition (18) to obtain the four equations that, together with the Euler equations above, describe the equilibrium:

$$\begin{aligned} \frac{B_{-1}}{P_0} &= \tau_0 + \beta \frac{Q_0}{Q} \frac{\tau_0^c b_F}{1 + \tau_0^c} + \beta \frac{Q_0}{Q} \frac{1}{1 + \tau_0^c} \frac{b}{\pi} && \text{(Current GBC)} \\ \frac{b}{\pi} &= \frac{\tau}{1 - \beta} && \text{(Future GBC)} \\ \frac{B_{F-1}}{P_0} &= \frac{Q_0 \gamma^* y^* - \gamma y}{1 - \gamma} + \beta \frac{Q_0}{Q} \frac{b_F}{\pi} && \text{(Current CA)} \\ \frac{b_F}{\pi} &= \frac{1}{1 - \beta} \frac{Q \gamma^* y^* - \gamma y}{1 - \gamma} && \text{(Future CA)} \end{aligned}$$

Below I use this framework to describe the effects of an open market operation (OMO)—defined an

increase in the supply of bonds that raises the nominal interest rate i.e. a monetary tightening—under two different assumptions regarding the backing of capital account policy.

5.2.1 An OMO with fiscal and capital account backing

By assumption, the OMO increases R_0 by increasing B_0. First, this will increase long-run real government indebtedness $b = \frac{B_0}{P_0}$. Fiscal policy provides backing for monetary policy by raising future taxes τ to pay for this higher level of government debt. Since τ increases by enough to ensure the government budget constraint holds for all $t \geq 1$, (Future GBC) implies long-run inflation π is unchanged. This reflects the fact that agents will save more today expecting a tax hike in the future and will use their higher future wealth to pay for higher future tax liabilities. Since higher savings today implies lower demand for goods, the OMO drives today's price level P_0 down.

Second, higher domestic rates and a lower price level also increase the SOE's foreign indebtedness $b_F = \frac{B_{F0}}{P_0}$ since some of the bonds issued in the OMO are bought by foreign households. Capital account policy also provides backing for monetary policy by raising the tax that foreign households must pay when purchasing domestic bonds τ_0^c. A higher tax on these bond purchases offsets the effect of lower bond prices. Capital account policy therefore effectively prevents the surge in capital inflows from higher domestic rates by increasing the tax on foreign capitals. This, in turn, prevents the long-run real exchange rate Q from increasing (depreciating) as it prevents future foreign household wealth from increasing. Since the real exchange rate Q reflects the relative purchasing power of the foreign country relative to the SOE, higher capital inflow taxes and higher lump-sum taxes imply Q is constant as relative household wealth does not change.

Now, we know from the household's Euler equations that a higher capital inflow tax must lower the real depreciation rate Q/Q_0 as domestic bonds are less attractive to foreign investors i.e. capital inflows fall. However, from the above, we also know the long-run real exchange rate Q is constant. Therefore, today's real exchange rate Q_0 must increase (a real depreciation of the domestic currency). A more depreciated real exchange rate implies higher trade balances for the SOE as the domestic domestic good becomes relatively cheaper. A higher present value of trade surpluses thus supports a higher level of real external indebtedness $\frac{B_{F-1}}{P_0}$ caused by the drop in the price level P_0.

In sum, we have that a monetary tightening with fiscal *and* capital account backing lowers current inflation as is the conventional result in New-Keynesian models.

5.2.2 An OMO with fiscal backing but without capital account backing

Consider the same experiment but now capital account policy does not provide backing. In particular, capital account policy has its own objective of stabilizing (targeting) the real exchange rate.

As before, by assumption the OMO increases R_0 by increasing B_0 which, at initial P_0, increases government real indebtedness b. Also, higher domestic rates increase foreign demand for bonds b_F. Now, however, capital account policy **does not** provide backing for this OMO as it follows its own objective i.e. stabilize the real exchange rate. Capital account policy therefore **lowers** τ_0^c to prevent Q_0 from depreciating after the OMO.[21] Long-run NFLs b_F will therefore increase due to higher domestic rates and due to a lower tax rate τ_0^c. In equilibrium, this must be supported by higher long-run trade surpluses through a more depreciated currency (higher Q). However, higher Q prompts capital account policy to lower τ_0^c even more to again prevent Q_0 from rising as equation (35) shows. This would, in turn, increase NFLs even more and so on. It is easy to see how these dynamics are not consistent with an equilibrium where NFLs are finite. In equilibrium, $\frac{b_F}{\pi}$ must remain unchanged through higher future inflation π. This is achieved by fiscal policy **not** increasing future taxes τ and thus allowing future inflation to erode the higher future household wealth caused by the OMO. Note that higher inflation affects domestic and foreign wealth as both types of household hold part of their wealth in local currency bonds. Unchanged $\frac{b_F}{\pi}$ means relative foreign household wealth is unchanged which implies a constant long-run real exchange rate Q. This implies capital account policy does not lower τ_0^c in the first place and allows for an equilibrium with stable NFLs. Since domestic and foreign wealth is kept constant by higher future inflation, agents do not adjust their current consumption plans. Thus, today's price level P_0 is also constant.

In sum, a monetary tightening **without** the appropriate capital account backing replicates Sargent and Wallace's, 1981 result whereby tighter monetary policy today not only fails to reduce current inflation, but actually raises future inflation.

[21] The case of a constant τ_0^{c*} after the OMO ($\phi_k = 0$) can still be considered as capital account policy providing backing (i.e. being passive). In this case, the long-run real exchange rate Q jumps up to satisfy (Future CA) without the need for future inflation π to rise. In particular, Q increases to raise the long-run trade balance $tb = \frac{Q\gamma^* c^* - \gamma y}{1-\gamma}$ and match the increase in NFLs caused by higher domestic rates. Fiscal policy, on the other hand, increases τ to eliminate domestic wealth effects so that π is unchanged. Constant τ_0^c implies a constant rate of real depreciation Q/Q_0. Since we know Q is increasing, this implies Q_0 must be increasing by the same amount. Finally, higher Q_0 implies a higher current trade balance which will induce lower domestic consumption and lower P_0. Thus, as in the case where capital account policy provides explicit backing by raising τ_0^c, the OMO lowers the current price level P_0.

5.2.3 Discussion

The two cases analyzed above illustrate how capital account backing is a necessary condition for a central bank to be able to reduce inflation by increasing nominal interest rates. This exercise extends the results of Sargent and Wallace, 1981 to a SOE by showing that the conventional monetarist result wherein a monetary tightening reduces inflation on impact critically depends on the stance of fiscal policy—as emphasized by Sargent and Wallace, 1981—but also on the stance of capital account policy.[22]

Importantly, the 'unpleasant' increase in future inflation with no reduction in current inflation in the second case happens despite the fact that fiscal policy is willing to accommodate to monetary policy. This carries important implications for policy-making in SOEs as it illustrates the *necessity* of a three-way coordination by policy authorities in SOEs to be able to implement an inflation-targeting framework—as widely recommended by economists and international organizations. As argued in the introduction, most previous work has focused on the coordination between monetary and fiscal policies. However, as the above examples illustrate, including capital account policy into the analysis is central to understand inflation and exchange rates respond to policy changes. The above is an extreme (but useful) example whereby, even though the monetary and fiscal authorities coordinate towards a common objective (i.e. an inflation target), the lack capital account policy cooperation makes monetary policy powerless to control current inflation.

6 Conclusion

While leaving capital account policy unmodeled does not affect a SOE model's ability to produce a unique equilibrium, it restricts the set of possible equilibria that the model *is able to produce*. As shown in this paper, doing so not only rules out an entire class of equilibria where capital account policy determines inflation (Regime K), but also ignores the *necessary* passive behavior of capital account policy required to ensure an equilibrium where the central bank targets inflation exists. The resurgence of capital account measures and other forms of financial repression (see Reinhart et al., 2011), as well as recent changes in the IMF's position toward such policies (see Adrian et al., 2022) suggests the importance of capital account policies in the future is likely to increase. Understanding the role that monetary, fiscal, *and* capital account policy interactions

[22]The case of an OMO with capital account backing and no fiscal backing is essentially equivalent to the discussion in Sargent and Wallace, 1981.

play in determining equilibrium inflation and exchange rates thus becomes critical for effective policy-making.

The central role that capital account—as well as fiscal—policy plays for the conditions under which monetary policy is able to control inflation not only highlights the dependence of a central bank's effective monetary policies on the stance of fiscal and capital account policy, but also points to the delicate nature of an equilibrium where the central bank can actually control inflation. As shown in this paper, uncooperative fiscal *or* capital account policies cause monetary policy to have 'perverse' effects on inflation. Central bankers in SOEs ought to carefully consider how fiscal *and* capital account policies are being conducted before assuming that raising interest rates will lower inflation.

The literature of monetary-fiscal interactions has long argued the importance of explicitly modeling fiscal policy to understand the sources of inflation in a country like the United States. In the same way, the main message of this paper is arguing that capital account policy is equally important to understand the sources of inflation in a SOE.

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Appendix

A Rest of the World and Trade Openness

In the spirit of Sutherland, 2005 and De Paoli, 2009, I assume the openness parameters γ, γ^* are related as follows:[23]

$$\gamma^* = \frac{y}{y^*}\gamma z \tag{A.1}$$

The scaling parameter z allows for deviations from the (rather strict) assumption that 'relative trade openness'—captured by γ^*/γ—is exactly proportional the relative size of the domestic economy—reflected by y/y^*. Assumption (A.1) reflects the fact that for a SOE (i.e. $\frac{y}{y^*} \to 0$) we have that $\gamma^* \to 0$. That is, the rest of the world will be essentially closed to the SOE. Notice that, since $\frac{y}{y^*} \to 0$, (A.1) places no restrictions γ and thus on how open the SOE can be.

The parameter z allows us to consider different steady state trade balances tb—and net foreign asset position since $b_F = \frac{tb}{1-\beta}$—while maintaining the assumption that PPP holds in steady state $(Q = 1)$[24]. Goods market clearing in the domestic and foreign countries imply:

$$C = \frac{y - \gamma^* C^*}{1 - \gamma} \quad ; \quad C^* = y^*$$

Using assumption (A.1), it is straightforward to then show that the steady state trade balance is:

$$tb \equiv Q\gamma^* C^* - \gamma C = \frac{\gamma y}{1-\gamma}(z-1)$$

Thus, $z > 1$ amounts to assuming a positive trade balance in steady state (positive NFLs) and, conversely, $z < 1$ amounts to assuming a negative trade balance in steady state (negative NFLs). A common practice is to linearize around a steady state where the trade balance (and NFLs) are zero. This can be captured here by $z = 1$. There are restrictions on the values parameter z can take. First, since $\gamma^* \in (0, 1)$, by definition, we have that $z > 0$. Second, using (A.1) in the expression for

[23] De Paoli, 2009 and Sutherland, 2005 use $1 - \nu$ to denote domestic households' preference for the foreign good (γ) and ν^* to denote foreign households' preference for the domestic good (γ^*). They specify $1 - \nu = (1 - n)\lambda$ and $\nu^* = n\lambda$ where n is the SOE's relative size in the world economy and λ is 'openness'. Relative trade openness is thus $\frac{\nu^*}{1-\nu} = \frac{n}{1-n}$. In my case, relative openness is $\frac{\gamma^*}{\gamma} = \frac{y}{y^*}z = \frac{y/Y}{y^*/Y}z = \frac{n}{1-n}z$ where I use Y to denote world output. Hence, assumption (A.1) is equivalent to these papers when $z = 1$.

[24] Considering long-run violations of PPP i.e. $Q \neq 1$ would only introduce another channel through which the SOE's trade balance is not zero in the long-run as $tb = \frac{\gamma y}{1-\gamma}(zQ - 1)$. Assuming $Q = 1$ only changes the threshold value from $z = Q^{-1}$ to $z = 1$.

steady state consumption above we obtain:

$$C = \frac{y - \gamma^* C^*}{1 - \gamma} = \frac{1 - \gamma z}{1 - \gamma} y$$

Therefore, a strictly positive steady state consumption ($C > 0$), due to an infinite marginal utility of consumption at that point, requires $z < \frac{1}{\gamma}$. Hence, $z \in (0, \frac{1}{\gamma})$.

Assumption (A.1) also allows us to rationalize the two goods market clearing conditions above in a general equilibrium setting. From the foreign households' expenditure minimization problem we have

$$C^*_{Ft} = (1 - \gamma^*) \left(\frac{P^*_{Ft}}{P^*_t} \right)^{-1} C^*_t$$

which, as $\gamma^* \to 0$, implies $C^*_{Ft} \to C^*_t$ since $P^*_t = {P^*_{Ht}}^{\gamma^*} {P^*_{Ft}}^{1-\gamma^*} \to P^*_{Ft}$. The foreign goods market clearing condition can be written as:

$$\frac{y^*_t}{C^*_t} = \underbrace{\frac{C_{Ft}}{C^*_t}}_{\to 0} + \underbrace{\frac{C^*_{Ft}}{C^*_t}}_{\to 1} \to 1$$

That is, foreign goods market clearing implies $C^*_t \to y^*_t$. In other words, the foreign country is essentially a closed economy. Using $C^*_t = y^*$, we have that, in steady state, $\gamma^* C^* = y \gamma z$ even though $\gamma^* \to 0$. This is an important implication of making γ^* a function of y/y^* as it prevents the SOE's steady state exports $\gamma^* C^*$ from going to zero (or infinity) as we make the domestic country infinitesimally small (or the foreign country infinitely large).

B Deterministic Steady State

I consider a symmetric zero-inflation steady state ($\pi = \pi^* = 1$) where purchase power parity (PPP) holds ($Q = 1$). The steady state domestic and foreign Euler equations (2), (3), and (4) are:

$$1 = \beta R$$

$$1 = \beta R^*$$

$$1 + \tau^c = \beta R$$

The first and second equations imply:

$$R = R^* = \beta^{-1} \tag{B.1}$$

from which we obtain that $\tau^c = 0$ must be true in steady state. The steady state goods market clearing conditions are

$$y = Q^{\frac{\gamma}{1-\gamma-\gamma^*}}(1-\gamma)C + Q^{\frac{1-\gamma^*}{1-\gamma-\gamma^*}}\gamma^* C^*$$
$$y^* = Q^{-\frac{1-\gamma}{1-\gamma-\gamma^*}}\gamma C + Q^{-\frac{\gamma^*}{1-\gamma-\gamma^*}}(1-\gamma^*)C^*$$

Using the fact that $\gamma^* \to 0$ but $\gamma^* C^* \not\to 0$, and that $Q = 1$, these become:

$$y = (1-\gamma)C + \gamma^* C^*$$
$$y^* = \gamma C + (1-\gamma^*)C^*$$

Dividing the second market clearing condition through by C^* and using $C/C^* = 0$ we recover the foreign aggregate resource constraint:

$$C^* = y^* \tag{B.2}$$

Then use $\gamma^* C^* = z\gamma y$ so that the first condition implies:

$$C = \frac{y - \gamma^* C^*}{1-\gamma} = \frac{1-z\gamma}{1-\gamma} y \tag{B.3}$$

The steady state bond market clearing condition in term of domestic consumption units is:

$$b = b_H + b_F \tag{B.4}$$

The steady state household budget constraint is:

$$C + \frac{b_H}{R} = Q^{-\frac{\gamma}{1-\gamma}} y - \tau + \frac{b_H}{\pi}$$

Using $R = \beta^{-1}$ and the fact that $\pi = \pi^* = Q = 1$ we can write:

$$C = y - \tau + (1 - \beta)b_H \tag{B.5}$$

The steady state government budget constraint is:

$$\tau + \frac{b}{R} + \frac{b_F}{R}\tau^c = \frac{b}{\pi}$$

Again, $R = \beta^{-1}$ and $\pi = \pi^* = Q = 1$ and the fact that $\tau^c = 0$ to write:

$$\tau = b(1 - \beta) \tag{B.6}$$

The steady state current account is:

$$-\left(\frac{b_F}{R}(1 + \tau^c) - \frac{b_F}{\pi}\right) = nx = Q\gamma^* C^* - \gamma C$$

Since $1 + \tau^c = 1$ we have:

$$-\left(\frac{b_F}{R} - \frac{b_F}{\pi}\right) = Q\gamma^* C^* - \gamma C$$

Using $R = \beta^{-1}$ and the facts that $\pi = \pi^* = Q = 1$:

$$\gamma^* C^* - \gamma C = -(1 - \beta)b_F \tag{B.7}$$

Equations (B.1)-(B.7) fully describe the steady state as they pin down unique values for $C, C^*, b_H, b_F, b, R, R^*, \tau^c$ given values for y, y^*, τ and parameters β, γ, z.

C Derivation of Linearized System

C.1 Linearized Equilibrium Conditions

The linearized domestic and foreign Euler equations (2),(3),(4) are:

$$\begin{aligned} i_t &= \mathbb{E}_t[\hat{\pi}_{t+1}] + \mathbb{E}_t[\hat{C}_{t+1} - \hat{C}_t] \\ i_t^* &= \mathbb{E}_t[\hat{\pi}_{t+1}^*] + \mathbb{E}_t[\hat{C}_{t+1}^* - \hat{C}_t^*] \\ i_t - \widehat{1+\tau_t^c} &= \mathbb{E}_t[\hat{\pi}_{t+1}^*] + \mathbb{E}_t[\hat{C}_{t+1}^* - \hat{C}_t^*] + \mathbb{E}_t[s_{t+1} - s_t] \end{aligned}$$

where $i_t \equiv \hat{R}_t = \log R_t/R$, $s_t = \log \mathcal{E}_t$. The last two pricing equations imply the domestic UIP relationship:

$$i_t - i_t^* = \mathbb{E}_t[s_{t+1} - s_t] + \widehat{1+\tau_t^c}$$

Deviation from UIP will be driven by capital account policy. Using the definition of the real exchange rate implies $\mathbb{E}_t[\hat{Q}_{t+1} - \hat{Q}_t] = \mathbb{E}_t[\hat{\pi}_{t+1}^*] - \mathbb{E}_t[\hat{\pi}_{t+1}] + \mathbb{E}_t[\hat{\epsilon}_{t+1}]$, we can combine the two pricing equations for domestic bonds (first and third equations) to obtain the domestic household's risk-sharing condition:

$$\mathbb{E}_t[\hat{C}_{t+1}^* - \hat{C}_t^*] = \mathbb{E}_t[\hat{C}_{t+1} - \hat{C}_t] - \left[\mathbb{E}_t[\hat{Q}_{t+1} - \hat{Q}_t] + \widehat{1+\tau_t^c}\right] \tag{C.1}$$

Using the facts that $C/C^* \to 0$ but $\gamma^* C^* \not\to 0$ and that $C_t^* = y^*$, the linearized domestic goods market clearing condition (16) is:

$$\hat{C}_t = -\frac{\gamma + \kappa}{1-\gamma}\hat{Q}_t \quad ; \quad \hat{C}_t^* = 0 \tag{C.2}$$

where $\kappa = \frac{\gamma^* C^*}{(1-\gamma)C}$. Aggregate consumption in the SOE will move inversely with the real exchange rate. Consumption in the foreign country will be constant at $C^* = y^*$.

The linearized policy rules (13)-(15) are:

$$\begin{aligned} i_t &= \phi_\pi \hat{\pi}_t + \varepsilon_t^M \\ \hat{\tau}_t &= \phi_b \hat{b}_t + \varepsilon_t^F \\ \widehat{1+\tau_t^c} &= \phi_k \hat{b}_{Ft} + \varepsilon_t^K \end{aligned}$$

Combining the linearized equilibrium conditions above, we can reduce the model's equilibrium dynamics to a four-equation dynamic system in q_t, π_t, b_t, b_{Ft}.

First, substitute the goods market clearing conditions (C.2) and the linearized capital controls rule into the risk-sharing condition (C.1) to obtain the equilibrium law of motion for the real exchange rate:

$$\mathbb{E}_t[\hat{Q}_{t+1}] = \hat{Q}_t - \frac{1-\gamma}{1+\kappa}\left(\phi_k \hat{b}_{Ft} + \varepsilon_t^K\right) \tag{C.3}$$

This difference equation shows that, in the absence of capital controls ($\phi_k = 0$), the real exchange rate Q_t will follow a random walk.

Next, combine the first Euler equation with the linearized monetary policy rule and the goods market clearing conditions (C.2) to get:

$$\mathbb{E}_t[\hat{\pi}_{t+1}] = \phi_\pi \hat{\pi}_t + \varepsilon_t^M + \frac{\gamma+\kappa}{1-\gamma}\mathbb{E}_t[\hat{Q}_{t+1} - \hat{Q}_t]$$

Then substitute (C.3) into the above and simplify terms to obtain the equilibrium law of motion for inflation:

$$\mathbb{E}_t[\hat{\pi}_{t+1}] = \phi_\pi \hat{\pi}_t + \varepsilon_t^M - \frac{\gamma+\kappa}{1+\kappa}(\phi_k \hat{b}_{Ft} + \varepsilon_t^K) \tag{C.4}$$

Next, take the linearized current account condition (18) and impose goods market clearing (C.2), monetary and capital controls policy rules to obtain the equilibrium law of motion for outstanding NFLs as a share of output:

$$\hat{b}_{Ft+1} = \left(\frac{1}{\beta} - \phi_k\right)\hat{b}_{Ft} + \phi_\pi \hat{\pi}_t - \hat{\pi}_{t+1} - \frac{\omega}{\beta b_F}\hat{Q}_t + \varepsilon_t^M - \varepsilon_t^K \tag{C.5}$$

where $\omega = \frac{\gamma^* C^* + \gamma C \frac{\gamma+\kappa}{1-\gamma}}{y}$. Linearizing the government budget constraint (10) imposing market clearing and the policy rules we obtain the equilibrium law of motion for outstanding government debt as a share of output:

$$\hat{b}_{t+1} = \frac{1-(1-\beta)\phi_b}{\beta}\hat{b}_t - \frac{b_F}{b}\phi_k \hat{b}_{Ft} + \phi_\pi \hat{\pi}_t - \hat{\pi}_{t+1} + \varepsilon_t^M - \frac{1-\beta}{\beta}\varepsilon_t^F - \frac{b_F}{b}\varepsilon_t^K \tag{C.6}$$

D Proof of Proposition 1

The linearized system (27)-(29) can be written in matrix form as:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{\Gamma}_0} \begin{bmatrix} \hat{\pi}_{t+1} \\ \hat{b}_{t+1} \\ \hat{Q}_{t+1} \\ \hat{b}_{Ft+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \phi_\pi & 0 & 0 & -\frac{\gamma+\kappa}{1+\kappa}\phi_k \\ \phi_\pi & \frac{1-(1-\beta)\phi_b}{\beta} & 0 & -\phi_k \\ 0 & 0 & 1 & -\frac{1-\gamma}{1+\kappa}\phi_k \\ \phi_\pi & 0 & -\frac{\omega}{\beta b_F} & \frac{1}{\beta} - \phi_k \end{bmatrix}}_{\mathbf{\Gamma}_1} \begin{bmatrix} \hat{\pi}_t \\ \hat{b}_t \\ \hat{Q}_t \\ \hat{b}_{Ft} \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 & -\frac{\gamma+\kappa}{1+\kappa} \\ 1 & -\frac{1-\beta}{\beta} & -1 \\ 0 & 0 & -\frac{1-\gamma}{1+\kappa} \\ 1 & 0 & -1 \end{bmatrix}}_{\mathbf{\Psi}} \begin{bmatrix} \varepsilon_t^M \\ \varepsilon_t^F \\ \varepsilon_t^K \end{bmatrix}$$

$$+ \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}}_{\mathbf{\Phi}} \begin{bmatrix} \eta_{t+1}^\pi \\ \eta_{t+1}^Q \end{bmatrix}$$

and η_{t+1}^x is variable x's endogenous one-step-ahead forecast error defined as $\eta_{t+1}^x = x_{t+1} - \mathbb{E}_t[x_{t+1}]$. Premultiplying both sides by $\mathbf{\Gamma}_0^{-1}$:

$$\begin{bmatrix} \hat{\pi}_{t+1} \\ \hat{b}_{t+1} \\ \hat{Q}_{t+1} \\ \hat{b}_{Ft+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \phi_\pi & 0 & 0 & -\frac{\gamma+\kappa}{1+\kappa}\phi_k \\ 0 & \frac{1-(1-\beta)\phi_b}{\beta} & 0 & -\phi_k\frac{1-\gamma}{1+\kappa} \\ 0 & 0 & 1 & -\phi_k\frac{1-\gamma}{1+\kappa} \\ 0 & 0 & -\frac{\omega}{\beta b_F} & \frac{1}{\beta} - \phi_k\frac{1-\gamma}{1+\kappa} \end{bmatrix}}_{\mathbf{\Gamma}} \begin{bmatrix} \hat{\pi}_t \\ \hat{b}_t \\ \hat{Q}_t \\ \hat{b}_{Ft} \end{bmatrix} + \begin{bmatrix} 1 & 0 & -\frac{\gamma+\kappa}{1+\kappa} \\ 0 & -\frac{1-\beta}{\beta} & -\frac{1-\gamma}{1+\kappa} \\ 0 & 0 & -\frac{1-\gamma}{1+\kappa} \\ 0 & 0 & -\frac{1-\gamma}{1+\kappa} \end{bmatrix} \begin{bmatrix} \varepsilon_t^M \\ \varepsilon_t^F \\ \varepsilon_t^K \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \eta_{t+1}^\pi \\ \eta_{t+1}^Q \end{bmatrix}$$

The matrix $\mathbf{\Gamma} = \mathbf{\Gamma}_0^{-1}\mathbf{\Gamma}_1$ can be written in block form as:

$$\mathbf{\Gamma} = \begin{bmatrix} A & B \\ 0 & D \end{bmatrix}$$

where

$$A = \begin{bmatrix} \phi_\pi & 0 \\ 0 & \frac{1-(1-\beta)\phi_b}{\beta} \end{bmatrix} \quad ; \quad B = \begin{bmatrix} 0 & -\phi_k\frac{\gamma+\kappa}{1+\kappa} \\ 0 & -\phi_k\frac{1-\gamma}{1+\kappa} \end{bmatrix} \quad ; \quad D = \begin{bmatrix} 1 & -\phi_k\frac{1-\gamma}{1+\kappa} \\ -\frac{\omega}{\beta b_F} & \frac{1}{\beta} - \phi_k\frac{1-\gamma}{1+\kappa} \end{bmatrix}$$

Since $\mathbf{\Gamma}$ is upper block triangular, its eigenvalues correspond to the eigenvalues of submatrices A and D. Since submatrix A is diagonal, its eigenvalues are easily obtained as the diagonal elements:

$$\lambda_1 = \phi_\pi \quad ; \quad \lambda_2 = \frac{1-(1-\beta)\phi_b}{\beta} \tag{D.1}$$

The two eigenvalues of D can be obtained from solving the quadratic equation:

$$\lambda^2 - (d_{11}+d_{22})\lambda + (d_{11}d_{22} - d_{12}d_{21}) = 0$$

where d_{ij} is the ij element of submatrix D. That is:

$$\lambda_3 = \frac{d_{11}+d_{22}+\sqrt{(d_{11}+d_{22})^2 - 4(d_{11}d_{22}-d_{12}d_{21})}}{2} \tag{D.2}$$

$$\lambda_4 = \frac{d_{11}+d_{22}-\sqrt{(d_{11}+d_{22})^2 - 4(d_{11}d_{22}-d_{12}d_{21})}}{2} \tag{D.3}$$

Solving for the conditions when the first eigenvalues are inside the unit circle (i.e. $|\lambda_1| < 1, |\lambda_2| < 1$) from (D.1) is straightforward:

$$|\phi_\pi| < 1 \quad ; \quad |\phi_b| > 1$$

To obtain the conditions under which the second two eigenvalues are inside the unit circle (i.e. $|\lambda_3| < 1, |\lambda_4| < 1$) it is convenient to define:

$$x_1 \equiv d_{11} + d_{22} = 1 + \beta^{-1} - \phi_k \frac{1-\gamma}{1+\kappa} \tag{D.4}$$

$$x_2 \equiv -d_{12}d_{21} = -\frac{\phi_k}{b_F}\frac{1-\gamma}{1+\kappa}\frac{\omega}{\beta} \tag{D.5}$$

Then we can write λ_3 and λ_4 as:

$$\lambda_3 = \frac{x_1 + \sqrt{x_1^2 - 4(x_1 - 1 + x_2)}}{2}$$

$$\lambda_4 = \frac{x_1 - \sqrt{x_1^2 - 4(x_1 - 1 + x_2)}}{2}$$

Notice that the term inside the square root can be negative or positive depending on parameter values. This implies eigenvalues λ_3, λ_4 can be real or complex numbers.

Consider the condition for the eigenvalues being complex numbers (i.e. the term in the square

root is negative):

$$x_1^2 - 4(x_1 - 1 + x_2) < 0$$

Using the definitions of x_1 and x_2 and rearranging, the above condition becomes:

$$\left(\frac{1-\gamma}{1+\kappa}\right)^2 \phi_k^2 - 2\left((\beta^{-1}-1)\frac{1-\gamma}{1+\kappa} - 2\frac{1-\gamma}{1+\kappa}\frac{\omega}{\beta b_F}\right)\phi_k + (\beta^{-1}-1)^2 < 0 \tag{D.6}$$

The roots of this quadratic equation will give the threshold values ϕ_k^D and ϕ_k^U for which this condition is satisfied:

$$\phi_k^D = \frac{(\beta^{-1}-1)\frac{1-\gamma}{1+\kappa} - 2\frac{1-\gamma}{1+\kappa}\frac{\omega}{\beta b_F} - 2\sqrt{\left(\frac{1-\gamma}{1+\kappa}\right)^2\left(\frac{\omega}{\beta b_F}\right)^2 - (\beta^{-1}-1)\left(\frac{1-\gamma}{1+\kappa}\right)^2\frac{\omega}{\beta b_F}}}{\left(\frac{1-\gamma}{1+\kappa}\right)^2}$$

$$\phi_k^U = \frac{(\beta^{-1}-1)\frac{1-\gamma}{1+\kappa} - 2\frac{1-\gamma}{1+\kappa}\frac{\omega}{\beta b_F} + 2\sqrt{\left(\frac{1-\gamma}{1+\kappa}\right)^2\left(\frac{\omega}{\beta b_F}\right)^2 - (\beta^{-1}-1)\left(\frac{1-\gamma}{1+\kappa}\right)^2\frac{\omega}{\beta b_F}}}{\left(\frac{1-\gamma}{1+\kappa}\right)^2}$$

We can show that ϕ_k^D and ϕ_k^U are always strictly negative for all $\gamma \in (0,1)$ and $\beta \in (0,1)$. To see this, it is useful to consider the special case of $\beta \to 1$:

$$\lim_{\beta\to 1}(\phi_k^D, \phi_k^U) = \frac{-2\frac{1-\gamma}{1+\kappa}\frac{\omega}{b_F} \pm 2\frac{1-\gamma}{1+\kappa}\frac{\omega}{b_F}}{\left(\frac{1-\gamma}{1+\kappa}\right)^2} = \left(-\frac{4\frac{\omega}{b_F}}{\frac{1-\gamma}{1+\kappa}}, 0\right)$$

Since $\omega, \kappa > 0$, and we assumed $b_F > 0$, we know that $\phi_k^D, \phi_k^U < 0$ for all $\gamma \in (0,1)$ when $\beta \to 1$. Moreover, we can show that ϕ_k^D, ϕ_k^U are both increasing in β which implies that $\phi_k^D, \phi_k^U < 0$ for all $\gamma, \beta \in (0,1)$. Finally, since the coefficient on the quadratic term $\left(\frac{1-\gamma}{1+\kappa}\right)^2$ is always positive, we have that the condition for complex eigenvalues is:

$$\phi_k \in (\phi_k^D, \phi_k^U) \tag{D.7}$$

With condition (D.6) we can analyze the sub-cases of complex and real eigenvalues. In all subsequent subcases, I will always assume that $b_F > 0$.

First consider λ_3:

Case 1.1: When λ_3 is complex, that is when $\phi_k \in (\phi_k^D, \phi_k^U)$, we can write $\lambda_3 = a + bi$ where $a = \frac{x_1}{2}$ is the real part and $b = \frac{\sqrt{4(x_1-1+x_2)-x_1^2}}{2}$ the complex part. Then, the modulus of λ_3 is:

$$|\lambda_3| = \sqrt{a^2+b^2} = \frac{1}{2}\sqrt{x_1^2 + 4(x_1 - 1 + x_2) - x_1^2} = \frac{1}{2}\sqrt{4(x_1 - 1 + x_2)} = \sqrt{x_1 - 1 + x_2}$$

Therefore the condition for the eigenvalue to be outside the unit circle reduces to:

$$\sqrt{x_1 - 1 + x_2} > 1 \iff x_1 - 1 + x_2 > 1 \iff x_1 + x_2 > 2$$

Substituting for x_1, x_2 from (D.4) and (D.5) and reordering implies $|\lambda_3| > 1$ if and only if:

$$\phi_k < \frac{\beta^{-1} - 1}{\frac{1-\gamma}{1+\kappa}(\frac{\omega}{\beta b_F} + 1)} > 0$$

Since $\phi_k^U < 0$ for all $\gamma, \beta \in (0, 1)$, this condition is always satisfied. Therefore, $|\lambda_3| > 1$ whenever λ_3 is a complex number.

Case 1.2: When λ_3 is a real number, that is when $\phi_k \notin (\phi_k^D, \phi_k^U)$, then,

$$|\lambda_3| = \Big|\frac{x_1 + \sqrt{x_1^2 - 4(x_1 - 1 + x_2)}}{2}\Big|$$

Since the term inside the absolute value is, by assumption, always positive and a real number:

$$\begin{aligned} |\lambda_3| = \frac{x_1 + \sqrt{x_1^2 - 4(x_1 - 1 + x_2)}}{2} > 1 &\iff x_1 + \sqrt{x_1^2 - 4(x_1 - 1 + x_2)} > 2 \\ &\iff \sqrt{x_1^2 - 4(x_1 - 1 + x_2)} > 2 - x_1 = 1 - \beta^{-1} + \phi_k \frac{1-\gamma}{1+\kappa} \end{aligned}$$

- Case 1.2.1: If $\phi_k < 0$, the right-hand side of the inequality is always negative since $\beta < 1$. Therefore:

$$|\lambda_3| > 1 \iff \sqrt{x_1^2 - 4(x_1 - 1 + x_2)} > 0 > 1 - \beta^{-1} + \phi_k(1 - \delta_F \rho)$$

is always true. Thus, $|\lambda_3| > 1$ whenever $\phi_k < 0$.

- Case 1.2.2: If $\phi_k > 0$, we need to consider two subcases. If $\phi_k \geq \frac{\beta^{-1}-1}{\frac{1-\gamma}{1+\kappa}} > 0$ such that the right-hand side of the inequality is still negative, the same argument as above applies and we

can easily show that $|\lambda_3| > 1$ is always true. If $\phi_k < \frac{\beta^{-1}-1}{\frac{1-\gamma}{1+\kappa}}$ such that the right-hand side of the inequality is positive, then we have:

$$|\lambda_3| > 1 \iff x_1^2 - 4(x_1 - 1 + x_2) > (1 - \beta^{-1} + \phi_k \frac{1-\gamma}{1+\kappa})^2$$

Using the definitions of x_1, x_2 and of ω, κ we can simply the above to:

$$4\phi_k \frac{1-\gamma}{1+\kappa} \frac{\omega}{\beta b_F} > 0$$

Since $\phi_k > 0$ by assumption and $\omega, \kappa > 0$ and $\gamma \in (0, 1)$, this condition is always satisfied. Combining these two subcases yields that $|\lambda_3| > 1$ whenever $\phi_k > 0$.

Finally, combining cases 1.2.1 and 1.2.2 implies that $|\lambda_3| > 1$ for all values of ϕ_k and $\beta, \gamma \in (0, 1)$.

Now consider λ_4:

Case 2.1: When λ_4 is complex, that is, when $\phi_k \in (\phi_k^D, \phi_k^U)$ we can use the fact that λ_3 and λ_4 are complex conjugates and therefore $|\lambda_3| = |\lambda_4|$. As we have already shown, $|\lambda_3| > 1$ for all values of ϕ_k such that λ_3 is complex. Thus, $|\lambda_4| > 1$ for all values of ϕ_k such that λ_4 is complex.
Case 2.2: When λ_4 is real, that is, when $\phi_k \notin (\phi_k^D, \phi_k^U)$, then:

$$|\lambda_4| = \Big| \frac{x_1 - \sqrt{x_1^2 - 4(x_1 - 1 + x_2)}}{2} \Big|$$

Since λ_4 can be positive or negative depending on parameter values, two more subcases need to be examined.
Case 2.2.1: If $\lambda_4 > 0$ then we have:

$$\begin{aligned} |\lambda_4| > 1 &\iff \frac{x_1 - \sqrt{x_1^2 - 4(x_1 - 1 + x_2)}}{2} > 1 \\ &\iff \sqrt{x_1^2 - 4(x_1 - 1 + x_2)} < x_1 - 2 = \beta^{-1} - 1 - \phi_k \frac{1-\gamma}{1+\kappa} \end{aligned}$$

- If $\phi_k < 0$: In this case the right-hand side of the inequality is always positive. Thus, we can

square both sides to write:

$$\begin{aligned}
|\lambda_4| > 1 &\iff x_1^2 - 4(x_1 - 1 + x_2) < (x_1 - 2)^2 = x_1^2 - 4x_1 + 4 \\
&\iff -x_2 < 0 \\
&\iff \phi_k \frac{1-\gamma}{1+\kappa} \frac{\omega}{\beta b_F} < 0
\end{aligned}$$

Since $\omega, \kappa, \beta, b_F > 0$ and $\gamma \in (0, 1)$ this reduces to

$$\phi_k < 0$$

which is true by assumption. Thus, $|\lambda_4| > 1$ whenever $\lambda_4 > 0$ and $\phi_k < 0$.

• If $\phi_k > 0$: In this case there two possible cases: $\phi_k \frac{1-\gamma}{1+\kappa} < \beta^{-1} - 1$ and $\phi_k \frac{1-\gamma}{1+\kappa} > \beta^{-1} - 1$. In the first case, the right-hand side of the inequality is still positive. Therefore, we can write again:

$$\begin{aligned}
|\lambda_4| > 1 &\iff x_1^2 - 4(x_1 - 1 + x_2) < (x_1 - 2)^2 = x_1^2 - 4x_1 + 4 \\
&\iff -x_2 < 0 \\
&\iff \phi_k \frac{1-\gamma}{1+\kappa} \frac{\omega}{\beta b_F} < 0 \\
&\iff \phi_k < 0
\end{aligned}$$

Now, however, $\phi_k > 0$ by assumption. Thus, this condition is never satisfied implying that $|\lambda_4| < 1$.

In the second case, the right-hand side turns negative:

$$\beta^{-1} - 1 - \phi_k \frac{1-\gamma}{1+\kappa} < 0$$

but, since λ_4 is a real number, we know that:

$$\sqrt{x_1^2 - 4(x_1 - 1 + x_2)} > 0$$

Therefore, the condition

$$\sqrt{x_1^2 - 4(x_1 - 1 + x_2)} < \beta^{-1} - 1 - \phi_k \frac{1-\gamma}{1+\kappa}$$

is never satisfied, implying that $|\lambda_4| < 1$. Combining these two subcases, we have that $|\lambda_4| < 1$

whenever $\lambda_4 > 0$ and $\phi_k > 0$.

Case 2.2.2: If $\lambda_4 < 0$ then we have:

$$\begin{aligned}
|\lambda_4| > 1 &\iff \frac{x_1 - \sqrt{x_1^2 - 4(x_1 - 1 + x_2)}}{2} < -1 \\
&\iff \sqrt{x_1^2 - 4(x_1 - 1 + x_2)} > 2 + x_1 = 3 + \beta^{-1} - \phi_k \frac{1-\gamma}{1+\kappa} > 0 \\
&\iff x_1^2 - 4(x_1 - 1 + x_2) > (2 + x_1)^2 = 4 + 4x_1 + x_1^2 \\
&\iff -x_2 > 2x_1 \\
&\iff \phi_k \frac{1-\gamma}{1+\kappa} \frac{\omega}{\beta b_F} > 2(1 + \beta^{-1} - \phi_k \frac{1-\gamma}{1+\kappa})
\end{aligned}$$

Solving for ϕ_k we obtain the condition:

$$\phi_k > \frac{2(1+\beta^{-1})}{\frac{1-\gamma}{1+\kappa}(\frac{\omega}{\beta b_F} + 2)} \equiv \overline{\phi_k}$$

Thus, when $\lambda_4 < 0$, $|\lambda_4| > 1$ if and only if $\phi_k > \overline{\phi_k}$.

In sum, from all the above cases we have shown that λ_3 is outside the unit circle (i.e. $|\lambda_3| > 1$) for all values of ϕ_k and $\beta, \gamma \in (0, 1)$. Moreover:

(i) $|\lambda_4| > 1$ if and only if $\phi_k < 0$ or $\phi_k > \overline{\phi_k}$

(ii) $|\lambda_4| < 1$ if and only if $\phi_k \in [0, \overline{\phi_k}]$

where $\overline{\phi_k} = \frac{2(1+\beta^{-1})}{\frac{1-\gamma}{1+\kappa}(\frac{\omega}{\beta b_F} + 2)}$.

Finally, these results, together with the conditions for $|\lambda_1|$ and $|\lambda_2|$, imply three regions of the parameter space where the condition for existence of a unique equilibrium (i.e. two eigenvalues inside the unit circle and two outside) is satisfied:

1. $|\lambda_1|, |\lambda_3| > 1$; $|\lambda_2|, |\lambda_4| < 1 \iff |\phi_\pi| > 1$, $|\phi_b| > 1$, and $\phi_k \in [0, \overline{\phi_k}]$

2. $|\lambda_2|, |\lambda_3| > 1$; $|\lambda_1|, |\lambda_4| < 1 \iff |\phi_\pi| < 1$, $|\phi_b| < 1$, and $\phi_k \in [0, \overline{\phi_k}]$

3. $|\lambda_3|, |\lambda_4| > 1$; $|\lambda_1|, |\lambda_2| < 1 \iff |\phi_\pi| < 1$, $|\phi_b| > 1$, and $\phi_k \notin [0, \overline{\phi_k}]$

with $\overline{\phi_k} = \frac{2(1+\beta^{-1})}{\frac{1-\gamma}{1+\kappa}(\frac{\omega}{\beta b_F} + 2)}$. as stated in Proposition 1. ■

E Proof of Proposition 2

Using the method of undetermined coefficients, guess that the equilibrium real exchange rate is a function of current and future capital account policy shocks:

$$\hat{Q}_t = \sum_{k=0}^{\infty} \lambda_k \mathbb{E}_t[\varepsilon^K_{t+k}]$$

Substitute this guess into the risk sharing condition (26):

$$\mathbb{E}_t[\hat{Q}_{t+1}] = \hat{Q}_t - \frac{1-\gamma}{1+\kappa}\left(\phi_k \hat{b}_{Ft} + \varepsilon^K_t\right)$$

to obtain:

$$\sum_{k=0}^{\infty} \lambda_k \mathbb{E}_t[\varepsilon^K_{t+k+1}] = \sum_{k=0}^{\infty} \lambda_k \mathbb{E}_t[\varepsilon^K_{t+k}] - \frac{1-\gamma}{1+\kappa}\left(\phi_k \hat{b}_{Ft} + \varepsilon^K_t\right)$$

Grouping terms and solving for b_{Ft} we obtain:

$$\hat{b}_{Ft} = \frac{1+\kappa}{1-\gamma}\frac{1}{\phi_k}(\lambda_0 - \frac{1-\gamma}{1+\kappa})\varepsilon^K_t + \frac{1+\kappa}{1-\gamma}\frac{1}{\phi_k}\sum_{k=0}^{\infty}(\lambda_{k+1} - \lambda_k)\mathbb{E}_t[\varepsilon^K_{t+k+1}]$$

Now use equation (27) in the current account condition to write:

$$\mathbb{E}_t[\hat{b}_{Ft+1}] = \left(\frac{1}{\beta} - \phi_k \frac{1-\gamma}{1+\kappa}\right)\hat{b}_{Ft} - \frac{\omega}{\beta b_F}\hat{Q}_t - \frac{1-\gamma}{1+\kappa}\varepsilon^K_t$$

Substitute the expression above:

$$\begin{aligned}
\frac{1+\kappa}{1-\gamma}\frac{1}{\phi_k}(\lambda_0 - \frac{1-\gamma}{1+\kappa})\mathbb{E}_t[\varepsilon^K_{t+1}] + \frac{1+\kappa}{1-\gamma}\frac{1}{\phi_k}\sum_{k=0}^{\infty}(\lambda_{k+1} - \lambda_k)\mathbb{E}_t[\varepsilon^K_{t+k+2}] = \\
\left(\frac{1}{\beta} - \phi_k \frac{1-\gamma}{1+\kappa}\right)\left(\frac{1+\kappa}{1-\gamma}\frac{1}{\phi_k}(\lambda_0 - \frac{1-\gamma}{1+\kappa})\varepsilon^K_t + \frac{1+\kappa}{1-\gamma}\frac{1}{\phi_k}\sum_{k=0}^{\infty}(\lambda_{k+1} - \lambda_k)\mathbb{E}_t[\varepsilon^K_{t+k+1}]\right) \\
-\frac{\omega}{\beta b_F}\sum_{k=0}^{\infty}\lambda_k \mathbb{E}_t[\varepsilon^K_{t+k}] - \frac{1-\gamma}{1+\kappa}\varepsilon^K_t
\end{aligned}$$

Matching coefficients for ε_t^K:

$$0 = \left(\frac{1}{\beta} - \phi_k \frac{1-\gamma}{1+\kappa}\right)\left(\frac{1+\kappa}{1-\gamma}\frac{1}{\phi_k}\left(\lambda_0 - \frac{1-\gamma}{1+\kappa}\right)\right) - \frac{\omega}{\beta b_F}\lambda_0 - \frac{1-\gamma}{1+\kappa}$$

Canceling terms and simplifying:

$$\left[\frac{1+\kappa}{1-\gamma}\frac{1}{\beta\phi_k} - 1 - \frac{\omega}{\beta b_F}\right]\lambda_0 = \frac{1}{\beta\phi_k}$$

Solving for λ_0

$$\lambda_0 = \frac{\frac{1}{\beta\phi_k}}{\frac{1+\kappa}{1-\gamma}\frac{1}{\beta\phi_k} - 1 - \frac{\omega}{\beta b_F}} = \frac{1}{\frac{1+\kappa}{1-\gamma} - \phi_k(\beta + \frac{\omega}{b_F})}$$

Notice $\lambda_0 > 0$ whenever $\phi_k < 0$ and $b_F > 0$ implying that an increase in capital controls will always increase (depreciate) the real exchange rate.

Matching coefficients for $\mathbb{E}_t[\varepsilon_{t+1}^K]$

$$\left[\left(\frac{1}{\beta} - \phi_k \frac{1-\gamma}{1+\kappa}\right)\frac{1+\kappa}{1-\gamma}\frac{1}{\phi_k} - \frac{\omega}{\beta b_F}\right]\lambda_1 = \frac{1+\kappa}{1-\gamma}\frac{1}{\phi_k}(\lambda_0 - \frac{1-\gamma}{1+\kappa}) + \left(\frac{1}{\beta} - \phi_k\frac{1-\gamma}{1+\kappa}\right)\frac{1+\kappa}{1-\gamma}\frac{1}{\phi_k}\lambda_0$$

and solving for λ_1 we obtain:

$$\lambda_1 = \frac{\frac{1+\kappa}{1-\gamma}\frac{1}{\phi_k}(1+\beta^{-1} - \phi_k\frac{1-\gamma}{1+\kappa})\lambda_0 - \frac{1}{\phi_k}}{(\beta^{-1} - \phi_k\frac{1-\gamma}{1+\kappa})\frac{1+\kappa}{1-\gamma}\frac{1}{\phi_k} - \frac{\omega}{\beta b_F}}$$

Matching coefficients for $\mathbb{E}_t[\varepsilon_{t+2}^K]$:

$$\frac{1+\kappa}{1-\gamma}\frac{1}{\phi_k}(\lambda_1 - \lambda_0) = \left(\frac{1}{\beta} - \phi_k\frac{1-\gamma}{1+\kappa}\right)\frac{1+\kappa}{1-\gamma}\frac{1}{\phi_k}(\lambda_2 - \lambda_1) - \frac{\omega}{\beta b_F}\lambda_2$$

Which implies:

$$\lambda_2 = \frac{\Phi_1\lambda_1 - \Phi_0\lambda_0}{\Omega}$$

where

$$\Phi_1 = \frac{1+\kappa}{1-\gamma}\frac{1}{\phi_k}(1+\beta^{-1} - \phi_k\frac{1-\gamma}{1+\kappa}) \quad ; \quad \Phi_0 = \frac{1+\kappa}{1-\gamma}\frac{1}{\phi_k} \quad ; \quad \Omega = \frac{1+\kappa}{1-\gamma}\frac{1}{\phi_k}(\beta^{-1} - \phi_k\frac{1-\gamma}{1+\kappa}) - \frac{\omega}{\beta b_F}$$

Since the matching terms are the same for all $\mathbb{E}_t[\varepsilon_{t+k}]$, $k \geq 2$, we can write the general expression:

$$\lambda_k = \frac{\Phi_1 \lambda_{k-1} - \Phi_0 \lambda_{k-2}}{\Omega}, \quad \forall k \geq 2$$

Therefore,

$$\begin{aligned} Q_t &= \sum_{k=0}^{\infty} \lambda_k \mathbb{E}_t[\varepsilon_{t+k}^K] \\ &= \lambda_0 \varepsilon_t^K + \lambda_1 \mathbb{E}_t[\varepsilon_{t+1}^K] + \frac{1}{\Omega} \sum_{k=2}^{\infty} \left(\Phi_1 \lambda_{k-1} - \Phi_0 \lambda_{k-2}\right) \mathbb{E}_t[\varepsilon_{t+k}^K] \end{aligned}$$

F Generalizing Proposition 1 to $b_F \leq 0$

Considering the case of a SOE who is a net lender ($b_F < 0$) in steady state makes the region of the parameter space where a unique equilibrium exist more complicated to describe but does not change the overall message. In particular, Proposition 1 can be generalized as follows.

Proposition 3. *Assuming the SOE has a non-zero net foreign asset position ($b_F \neq 0$), the system of equations* (26)-(29) *has a (locally) unique and bounded equilibrium if and only if policy parameters ϕ_π, ϕ_b, ϕ_k satisfy one of the following mutually exclusive conditions:*

1. *$|\phi_\pi|, |\phi_b| > 1$ and $\phi_k \in \mathcal{S}_1$ (Regime M*)*
2. *$|\phi_\pi|, |\phi_b| < 1$ and $\phi_k \in \mathcal{S}_1$ (Regime F*)*
3. *$|\phi_\pi| < 1$, $|\phi_b| > 1$ and $\phi_k \in \mathcal{S}_2$ (Regime K)*

where

$$\mathcal{S}_1 = \begin{cases} \left[0, \overline{\phi_k}\right] & \text{if} \quad \frac{b_F}{y} > 0 \\ (-\infty, 0] \cup [\phi_k', \infty) & \text{if} \quad -\frac{\gamma}{(1-\gamma)^2} \leq \frac{b_F}{y} < 0 \\ (-\infty, 0] \cup [\phi_k'', \infty) & \text{if} \quad -\frac{\gamma}{(1-\gamma)^2} > \frac{b_F}{y} \end{cases} \quad ; \quad \mathcal{S}_2 = \begin{cases} \mathcal{S}_1^{\complement} & \text{if} \quad \frac{b_F}{y} > 0 \\ \mathcal{S}_1^{\complement} & \text{if} \quad -\frac{\gamma}{(1-\gamma)^2} \leq \frac{b_F}{y} < 0 \\ (0, \phi_k''') & \text{if} \quad -\frac{\gamma}{(1-\gamma)^2} > \frac{b_F}{y} \end{cases}$$

with $0 < \phi_k''' < \phi_k'' < \phi_k'$ functions of model parameters.

Importantly, in the case when $b_F = 0$, Regime K no longer exists. To see why an equilibrium is not possible under active capital account policy when $b_F = 0$, recall the linearized intertemporal current account condition:

$$\hat{\tilde{b}}_{Ft-1} - \hat{\pi}_t = \frac{\omega}{b_F} \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t \hat{Q}_{t+j} - (1-\beta)\beta \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t [\hat{Q}_{t+j+1} - \hat{Q}_{t+j}]$$

Use the fact that $\hat{\tilde{b}}_{Ft-1} \approx \frac{\tilde{b}_{Ft-1} - b_F}{b_F}$ and multiply across by b_F:

$$\tilde{b}_{Ft-1} - b_F - \hat{\pi}_t b_F = \omega \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t \hat{Q}_{t+j} - b_F(1-\beta)\beta \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t [\hat{Q}_{t+j+1} - \hat{Q}_{t+j}]$$

Now impose $b_F = 0$ to obtain:

$$\tilde{b}_{Ft-1} - b_F = \omega \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t \hat{Q}_{t+j}$$

Since $\tilde{b}_{Ft-1}$ is predetermined, this condition will determine a unique real exchange rate that is consistent with external solvency. Hence, capital account is not able to target a desired level of the real exchange rate. Furthermore, even if active capital account policy chooses the shock sequence $\{\varepsilon^K_{t+j}\}$ to conform with the above condition, notice that inflation would be indeterminate.

It is important to point out that this happens strictly because local currency external debt (B_{Ft}) is zero in steady state, not because the country's NFLs are zero. This is important because under the latter assumption—a common choice of steady state to linearize around—Regime K still exists. To see it, allow domestic households to hold foreign debt (denoted B^*_{Ht}) such that NFLs were defined as $D_t = B_{Ft} - \mathcal{E}_t B^*_{Ht}$. Then, the intertemporal current account condition can be written:

$$\frac{b_F}{d}(\hat{\tilde{b}}_{Ft-1} - \hat{\pi}_t) - \frac{b^*_H}{d}(\hat{\tilde{b}}^*_{Ht-1} + \Delta\hat{Q}_t) = \frac{\omega}{d} \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t \hat{Q}_{t+j} - (1-\beta)\beta \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t [\hat{Q}_{t+j+1} - \hat{Q}_{t+j}]$$

Multiplying across by d gives:

$$b_F(\hat{\tilde{b}}_{Ft-1} - \hat{\pi}_t) - b^*_H(\hat{\tilde{b}}^*_{Ht-1} + \Delta\hat{Q}_t) = \omega \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t \hat{Q}_{t+j} - d(1-\beta)\beta \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t [\hat{Q}_{t+j+1} - \hat{Q}_{t+j}]$$

Hence, even if $d = 0$, capital account policy can set the sequences of $\{Q_{t+j}\}$ and, in doing so, uniquely determine inflation. It is only when there is exactly zero local currency external debt ($b_F = 0$) that there is no mechanism through which inflation can revalue the SOE's NFLs to equal the present value of trade surpluses that capital account policy sets by targeting the real exchange rate.

The assumption of $b_F = 0$ is not only very restrictive but also very far removed from the data. As the database and estimates collected by Arslanalp and Tsuda, 2014 on the currency composition of external debt reveals, the stock of local currency external debt in most SOEs is (and has long been) far from zero. Therefore, I argue that this extremely special case wherein Regime K is absent does not undermine its relevance or applicability.