The international spillovers of corporate bonds purchases

Marco Garofalo* Philipp Poyntner†

This draft: January 31, 2025

[PRELIMINARY - PLEASE DO NOT SHARE WITHOUT PERMISSION FROM THE AUTHORS]

Abstract

This paper investigates the international economic spillovers of domestic corporate bond purchases conducted by the US Federal Reserve for the first time in response to the 2020 Covid crisis. We first document empirically the significantly positive effect of these spillovers on firm-level investment: foreign firms issuing USD-denominated bonds with similar features to the domestic ones purchased by the Fed exhibited more resilient investment then comparable peers. Investigating US mutual funds' holdings reveals that investors selling domestic bonds eligible for purchases by the Fed reallocated their demand also towards these similar, although ineligible, foreign bonds. We rationalise these new findings through the lens of a novel two-country model with international intermediaries and heterogeneous firms: Fed's purchases of eligible domestic bonds relax the leverage constraint of financial intermediaries, which replenish their holdings with similar foreign USD bonds. Foreign firms issuing these bonds thus have looser investment constraints than other non-US firms. Finally, we use our model to construct two scenarios. On the one hand, should the share of USD-denominated bonds issued outside of the US keep increasing to match the size of the US bond market, the Fed would not be able to stabilise the global USD bond market solely through domestic purchases, as done during the Covid pandemic. On the other hand, a more fragmented world with lower USD dominance in international debt market would lead to substantially smaller international spillovers of Fed's corporate bonds purchases. Both scenarios point to substantial risks to the global stabilisation powers of the Fed's monetary policy.

JEL Codes: E22, E32, E44, E52, E58, F41, F42
Key Words: International Spillovers; Monetary Policy; Corporate Bonds Purchases; Quantitative Easing; Firm-Level Data; Mutual funds; Macroeconomics

*University of Oxford, Bank of England and CfM, email: marco.garofalo@economics.ox.ac.uk.
†University of Salzburg, email: philipp.poyntner@plus.ac.at.
We are grateful to Andrea Chiavari and Francesco Zanetti for their valuable comments and suggestions. We would also like to thank for helpful discussions the participants of seminars at University of Oxford and Bank of England. All errors and omissions remain our own.
The views expressed in this paper are those of the author and not necessarily those of the Bank of England and its committees.

1 Introduction

As part of the broader package of monetary policy measures deployed to tackle the economic fallout from the Covid pandemic, in March 2020 the US Federal Reserve purchased corporate bonds for the first time in its history. While other central banks like the ECB (e.g. with the CSPP from 2016 to 2023) have been resorting to these unconventional monetary policy tools already for some years, this innovation from the Fed is particularly salient given the central role played by the US Dollar in the international financial system (Coppola et al., 2023; Das et al., 2022; Gopinath & Stein, 2021), which constitutes an important channel leading Fed's interventions to have international spillovers (Iacoviello & Navarro, 2019; Miranda-Agrippino & Nenova, 2022). Indeed it has been argued, for example, that the Fed's purchases of USD-denominated US corporate bonds was enough to stabilise the USD corporate bond market *outside* of the US as well (McCauley, 2023). Fed's interventions may not always be enough however, as the non-US USD corporate bond market keeps rapidly expanding beyond its current stock of more than half of the US market.[1] Nor they might retain this global stabilisation power, as global fragmentation threatens to undermine the dominance of the US Dollar in the international monetary and financial system.

Against this backdrop, this paper investigates the — so far unexplored to our knowledge — international economic spillovers of domestic corporate bond purchases conducted by the US Federal Reserve as part of its unconventional monetary policy toolkit.

First, we document empirically the significantly positive effect of these spillovers on foreign firm-level investment, and adduce using US mutual funds holdings data a potential portfolio channel for international investors, which we believe plays a role in these findings.

We identify the potential effects of Fed's purchases of US corporate bonds denominated in USD[2] on non-US USD bonds. Our hypothesis is that these interventions, while not targeting non-US bonds[3], can nonetheless increase their issuance and prices. This might be occurring because of a preferred habit behaviour from international investors, meaning the latter consider some non-US USD bonds as substitute to the ones issued by US firms; thus, as Fed's purchases increase the prices of the latter, investors seek out non-US bonds of similar characteristics. As a result, foreign firms issuing these bonds have looser financing constraints and thus their investment performs relatively better than similar other non-US firms which do not issue such bonds. To show this, we use a difference-in-difference approach which identifies "treated" non-US USD bonds as those that would have been eligible for the Fed 2020 purchase program as meeting all criteria except, of course, the one of US residence. We believe this approach is appropriate to control for confounding factors, including in particular the other contemporaneous interventions of the Fed and other central banks over 2020. Connecting bonds to businesses, we then investigate the spillovers on firm-level investment, where foreign firms with such bonds are the treated group, to be compared to a control group of businesses with similar characteristic but

[1] See on the BIS website here and here.

[2] Recall that non-USD denominated US corporate bond issues are not eligible for Fed purchases. (See here.)

[3] The eligibility criteria for the programme meant that for the Fed's secondary market corporate credit factibity, or SMCCF, targeted USD bonds up to five years in maturity for non-financial investment grade (IG) issuers with significant U.S. operations. It started buying IG and HY corporate bond ETFs on May 12 2020, and individual corporate bonds (IG) as of March 22 and not below BB- credit rating at time of purchase on June 16.

issuing "ineligible" bonds. Using firm-level data on listed and private companies from Orbis and Compustat, we thus document empirically the significantly positive effect of these spillovers on foreign firm-level investment: foreign firms issuing USD-denominated bonds with similar features to the domestic ones purchased by the Fed exhibited more resilience in terms of their investment compared to comparable peers.

Investigating US mutual funds' holdings from CRSP reveals that, on average investors decreased their holding of bonds targeted by the Fed's programme. Furthermore, those who sold domestic bonds eligible for purchases by the Fed reallocated their demand also towards ineligible foreign bonds. Specifically, we find that (real) outflows for holdings of securities bought in a given quarter of 2020 by the Fed decrease at the time and the following quarter of the registered purchase. Furthermore, we uncover evidence that the funds exposed to such purchases also on average increased their purchases of international bonds.

Second, in order to rationalise these empirical findings we build a novel two-country New Keynesian DSGE model where international intermediaries buy corporate bonds from heterogeneous firms, whose bonds can be either eligible or ineligible for purchases by their respective central banks.

Crucially, these institutional investors exhibit financial frictions in the form of a leverage requirement, which constrains the holdings of financial intermediaries not to exceed their continuation values. This creates scope for Fed's purchases of corporate bonds held by these intermediaries to relax this constraint and expand their holdings, in turn stimulating investment and demand in the economy. This is the mechanism through which this form of quantitative easing has real effects in the model.

The availability of foreign bonds in the financial intermediaries' holdings underpins the spillover mechanism in our model. As the Fed purchases domestic USD bonds lowering their yields and loosening intermediaries' constraint, it triggers a chain reaction among investors seeking to maximize returns. U.S. bonds become less attractive relative to non-U.S. bonds, particularly those that are perceived as substitutes. This is especially relevant when the characteristics of these bonds are nearly identical, including currency denomination, risk profile, and maturity terms. As a result, investors "search for yields" and shift their portfolios towards non-U.S. bonds that offer relatively higher yields. Through this channel, unconventional monetary policy in the Home country, i.e. the US, can have ripple effects, reshaping investor incentives and reallocating capital across borders.

These effects on financial cross-border flows transmit to real firm-level investment. Foreign firms face a loan-in-advance constraint which requires them to finance part of their investment through bond issuance. Those businesses that see their bonds purchased by international investors as considered potential substitutes to the US ones, have a looser investment constraint, meaning that their capital accumulation is more resilient in the model than other foreign firms.

Finally, we use our validated model to construct two scenarios. On the one hand, should the share of USD-denominated bonds issued outside of the US keep increasing to match the size of the US bond market, the Fed would not be able to stabilise the global USD bond market solely through domestic purchases, as done during the Covid pandemic. On the other hand, a more fragmented world with lower USD dominance in international debt market would lead to

substantially smaller international spillovers of Fed corporate bonds purchases. Both scenarios point to substantial risks to the global stabilisation powers of Fed monetary policy.

In the future, these risks might require precautionary intervention, such as the establishment of bespoken swap lines between the Fed and other central banks to provide the latter with sufficient US dollars to stabilise their USD corporate bond markets through bond purchases (McCauley, 2023), similarly to what done already for money markets (Bahaj & Reis, 2022a, 2022b).

Related literature With this work we build on and contribute to two main strands in the macroeconomic literature.

First, we contribute to the empirical literature on the effects of US corporate bonds purchases, which has so far focused on their effects on domestic bonds and firms (Boyarchenko et al., 2022; Darmouni & Siani, 2023; Garofalo, 2024; Gilchrist et al., 2021). We are the first, to our knowledge, to look at the international spillovers of Fed's corporate bonds purchases onto non-US firms emitting USD bonds. A closely related paper is Siani (2019), who studied the spillover impact of the ECB's CSPP on US firm USD bond issuance through a bond underwriting channel. We complement that work with our novel analysis of the spillovers from the US to the rest of the world. Another related strand of literature develops theoretical tools to discuss interventions in corporate credit markets on firms (Crouzet & Tourre, 2021). The authors show that in times of financial market disruptions, the positive effects of credit interventions on investment dominate the negative effects resulting from high leverage, a finding we corroborate.[4]

Second, our quantitative framework aims at taking further the modelling of the international spillovers of unconventional monetary policy tools such as asset purchases. With respect to recent works in this literature (Alpanda & Kabaca, 2020; Kolasa & Wesołowski, 2020; Wu et al., 2024b), we focus on corporate bonds purchases and draw out the important role played by firm heterogeneity, e.g. in the context of the aggregate impact of these policy tools, as shown in a close-economy setting by Garofalo (2024). Importantly, unlike those frameworks which assume foreign bonds are denominated in domestic currency, we specifically allow for both USD and non-USD foreign bonds in our framework. This is in line with observations in the real world, where it is usually firms rather than financial intermediaries to be currency-mismatched (Das et al., 2022).[5]

The remainder of this paper is organised as follows. Section 2 reports our new empirical findings. We proceed by describing our novel quantitative framework in Section 3. We then discuss our scenarios and their main insights in Section 4. Finally, Section 5 concludes. We relegate all additional details to the Appendix.

[4]There is also ample literature on the ECBs corporate asset purchase programme, CSPP: Abidi and Miquel-Flores (2018), Arce et al. (2021), De Santis and Zaghini (2021), De Santis et al. (2018), Grosse-Rueschkamp et al. (2019), Li et al. (2021), Poyntner (2023), and Zaghini (2019), as well as for the Bank of England corporate QE (D'Amico & Kaminska, 2019).

[5]Even when frameworks allow for Local Currency Pricing (LCP) such as Wu et al. (2024b), firms are still assumed to issue bonds in their domestic currency only.

2 Empirical evidence

The Fed deployed for the first time in its history corporate bond purchases programs as part of a swathe of policies in response to the economic crisis brought about by the Covid pandemic. First, in terms of eligibility criteria, the SMCCF aimed at USD bonds up to five years in maturity for non-financial investment grade (IG) issuers with significant U.S. operations. It started buying IG and HY corporate bond ETFs on May 12 2020, and individual corporate bonds (IG) as of March 22 and not below BB- credit rating at time of purchase) on June 16. Second, the PMCCF commenced its operations on June 29, but eventually closed with no purchases having been made. This is why in what follows I focus on the former. ETFs purchases concluded on July 23. As of December 31 2020, the SMCCF ceased purchases. On June 7 2021, the SMCCF began winding down the portfolio and, as of August 31 2021, all of its holdings of corporate bonds and ETFs have either matured or been sold. By the end of the purchase program, the Fed bought only $14 billion as opposed to the potential $750 billion made available (Darmouni & Siani, 2023). Furthermore, the SMCCF aimed at achieving a representative portfolio composition of the US corporate bonds market, as tracked by the Federal Reserve Bank of New York's Broad Market Index (Gilchrist et al., 2021).

For our empirical investigation, we aim at capitalising on this historic monetary intervention in order to estimate the spillover effect of the Fed's US corporate bond purchases on firms in other countries. Specifically, we are interested in firms' investment response. We identify the potential effects of Fed's purchases of US corporate bonds denominated in USD on non-US/Rest of the world USD bonds (calling those "ROW USD bonds"). Our hypothesis is that these interventions, while not targeting non-US bonds, can nonetheless increase their issuance and prices. This might be occurring because of a preferred habit behaviour from international investors, meaning the latter consider some non-US USD bonds as substitute to the ones issued by US firms; thus, as Fed's purchases increase the prices of the latter, investors seek out non-US bonds of similar characteristics. We show this using a difference-in-difference approach, by identifying "treated" ROW USD bonds as those that would have been eligible for the Fed 2020 purchase program as meeting all criteria except, of course, the one of being issued by a US business. This approach should be appropriate to control for confounding factors, including in particular the other contemporaneous interventions of the Fed and other central banks over 2020. Put simply, we compare firms that issue very similar bonds, the only difference being the currency of the bond.

We first describe our data, to then detail our empirical models and resulting findings.

2.1 Data

Bonds We source from Refinitiv data for every bond that meets the eligibility criteria of the Fed's corporate bond purchases expect US residency around the world. These amounts to 75,000 bonds with data on Issuer Name, Date of Incorporation, geographical information, parent information, issue and maturity dates, amount, currency and spread.

Balance sheets As we are interested in the spillovers on firm-level outcomes such as investment, we match this data with datasets including information on businesses' balance sheets. We consider two main sources: Orbis and Compustat.

The Orbis dataset provides detailed balance sheet data for firms around the world. While the frequency of the observations is exclusively annual, the coverage of the dataset is large, including a large sample of private businesses. For data cleaning, we follow the steps outlined by Kalemli-Ozcan et al. (2015). Specifically, we drop firms that report negative total assets, negative employment, employment larger than 2 million employees, negative sales, or negative tangible fixed assets. We focus on the period from 2010-2023. We only include firms where the main variable (investment) is never missing from 2015-2022. Finally, we winsorize the data for our main variable investment at the 1% level. More details on the data can be found in section A.1. After matching and cleaning, we obtain 67,806 firm-year observations for 4,845 firms.

Compustat include firm-level information on listed companies only. However, it provides quarterly series which allow for higher frequency analysis. We exclude all those that are not operating, with country of incorporation or address being the USA or a tax haven according to Hines (2010), whose country of the headquarters location and country of incorporation are not the same, and for which the currency of document (curd) is the domestic currency of the country in which the firm is located and incorporated (Adams & Verdelhan, 2022).

In addition, we carry out a series of adjustments common in the corporate finance literature, in particular studies using firm-level information to investigate issues in international macroeconomics and finance (Adams & Verdelhan, 2022; Hegarty et al., 2023). We exclude firms in utilities, finance and insurance, real estate and public sectors[6], which are likely to be too idiosyncratic to be treated in this general framework of analysis.

After selecting in this way a panel of companies, we source corresponding firm-level quarterly information on a series of variables of interest from Compustat North America and Global. In particular, notice how the capital stock of a firm is not readily available in Compustat. It must be constructed. We follow the vast literature on this (Clementi & Palazzo, 2019; Salinger & Summers, 1983), and use the perpetual inventory method (PIM) to build the capital stock of each firm in our sample from the book value of tangible capital.[7]

Furthermore, we source information on firm-level *outstanding* liabilities by currency and debt instrument from S&P Capital IQ. We convert debt into USD using the FX from the BIS.

For both datasets our variable of interest is the investment rate, defined as growth rate of tangible fixed assets plus depreciation rate. Investment represents how much the firm invested in tangible fixed assets, net of depreciation. This measure is useful in our cross-country setting since it abstracts from currency differences and firm size.

[6]For primary NAICS 2-digit sector codes: 22, 52, 53 and 92, respectively. For more details on NAICS classification, see here.

[7]See Appendix A.1 for more details. Recent studies have started to include also intangible capital in firm-level analyses of the capital stock (Peters & Taylor, 2017), and its response to monetary policy (Döttling & Ratnovski, 2021). Following most of the literature on the heterogeneous response of monetary policy across firms, in particular Ottonello and Winberry (2020), we consider solely tangible capital given the difficulties with dealing with intangible assets data at firm level.

Mutual funds We source information on quarterly bond holdings of US mutual funds from the CRSP mutual fund database, which is free from survivorship bias. We add also information on a fund's characteristics, such as its investment objective, fund family, type (e.g., equity, bond, hybrid), and founding date. We restrict the sample to consider only funds that mainly target international fixed income.

SMCCF purchases We match these holdings information with the list of the corporate bonds purchased by the SMCCF from the Fed Board website.[8] The latter dataset reflects how the SMCCF commenced buying individual corporate bonds on June 16 2020. As of December 31 2020, the SMCCF ceased purchasing eligible assets. There is no report on SMCCF from November 12 2021 onwards. The data from the Fed Board allow the identification of the firms which originally issued those bonds as well.

2.2 Mutual funds: sales and purchases

In this section we show how Fed's corporate bonds purchases led US mutual funds to (i) decrease their holding of those bonds and (ii) increase holdings of other international bonds.

We show (i) running the following local projections (Jordà, 2005):

$$Flows_{f,s,t+h} = \beta_h Purchased_{s,t} + \alpha + \delta_f + \delta_t + u_{i,t} \tag{1}$$

where

$$Flows_{f,s,t+h} = \frac{A_{f,s,t+h} - A_{f,s,t-1}}{A_{f,s,t-1}} \tag{2}$$

is the cumulative number of shares A of a security s held by mutual fund f in quarter t. This capture the *real* change in capital flows at horizon h with respect to period $t-1$. $Purchased_{s,t}$ is a dummy equal to 1 if security s has been bought by the Fed in quarter t, and 0 otherwise. Finally, we include a constant α, fund and time fixed effects, i.e. δ_f and δ_t, and we cluster at both fund and time level.

Figure 1 shows the results. As expected, the US mutual funds in our sample do appear to have significantly decreased their holdings of bonds that were bought by the Fed as part of its SMCCF programme at the quarter of purchase and the following one. Unsurprisingly given the short span of the programme, the effect seems to dissipate after that quarter.

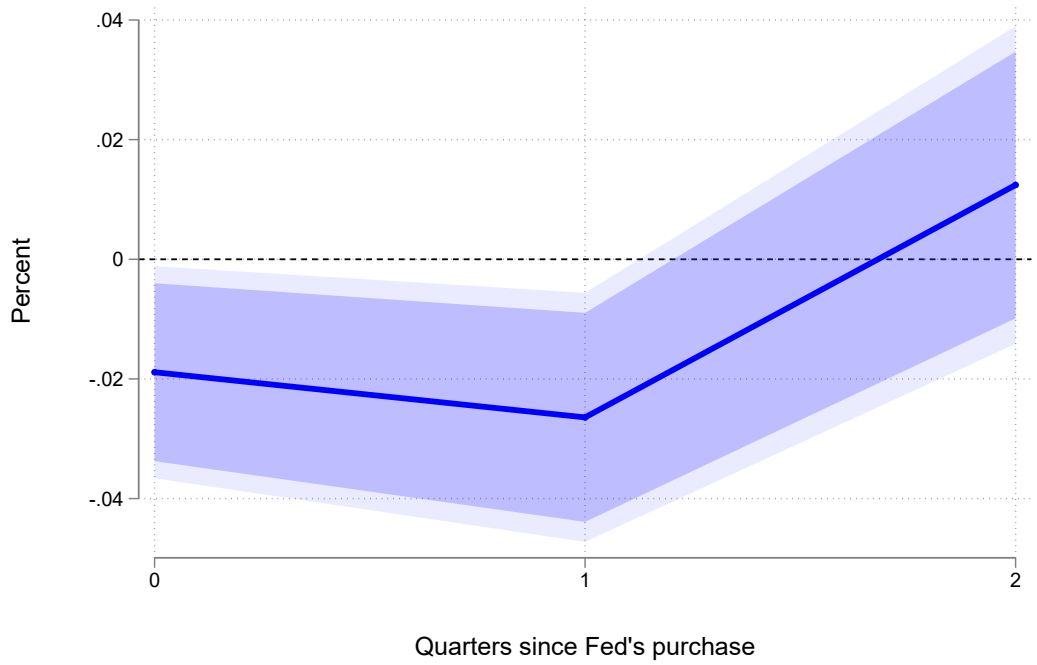
Again using local projections (Jordà, 2005), we provide evidence of (ii):

$$Flows_{f,s,t+h} = \beta_h Sold_{f,t} + \alpha + \delta_f + \delta_t + u_{i,t} \tag{3}$$

Now we restrict the sample to securities that have not been targeted by the Fed and that are issued by non-US entities. The explanatory variable is now $Sold_{f,t}$, which is a dummy equal to 1 if a fund has at least one security that was on the list of those bought by the Fed in quarter t, and 0 otherwise.

[8] `https://www.federalreserve.gov/publications/2020-reports-to-congress-in-response-to-covid-19.htm`.

Figure 1: (Real) change in capital flow - US funds' sales



Source: CRSP, data from Fed Board and authors' calculations. Notes: This chart graphically reports estimates of β_t of Equation 1. Capped spikes refer to the 95% confidence intervals.

Figure 2 shows that the (real) capital flows towards other securities increased for funds exposed to the SMCCF programme. This provides evidence of the programme freeing up capacity for these funds to expand their portfolio towards other suitable non-US bonds. This channel could then lead to foreign firms issuing these bonds to have looser investment constraints, and thus yield the results on the heterogeneous investment responses from the next sections. We will thus leverage on this mechanism in our quantitative analysis to bring out the aggregate consequences of these spillover mechanism, and provide suggestive scenario analyses of potential risks lying ahead for the global stabilisation powers of Fed's monetary policy.

2.3 Orbis: Empirical approach

Having demonstrated the real effect of the bond purchase programme at bond level, we turn to effects on the firm level.

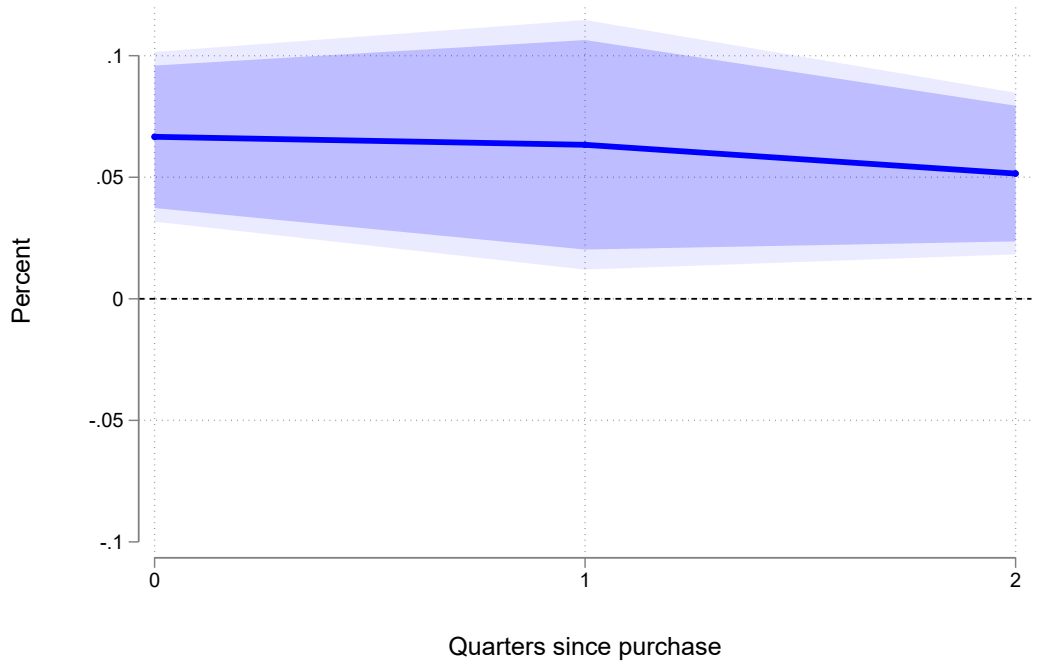
We employ a standard Difference-in-Differences approach as denoted in Equation 4:

$$investment_{i,t} = \sum_{t=1}^{T} \beta_t \times \text{Treated}_i + \gamma X_{i,t} + \alpha + u_{i,t} \tag{4}$$

where $investment_{i,t}$ denotes the investment rate for firm i in year t. *Treated* firms have outstanding bonds and notes in USD in 2019 "eligible" under the Fed's corporate bond purchase programme. The sample is restricted to firms that have "eligible" non-USD and USD bonds in 2019, so the control group is firms with only non-USD bonds.

We estimate Equation 4 via Fixed Effects, saturated with sector- and country-fixed effects.

Figure 2: (Real) change in capital flow - US funds' purchases



Source: CRSP, data from Fed Board and authors' calculations. Notes: This chart graphically reports estimates of β_t of Equation 3. Capped spikes refer to the 95% confidence intervals.

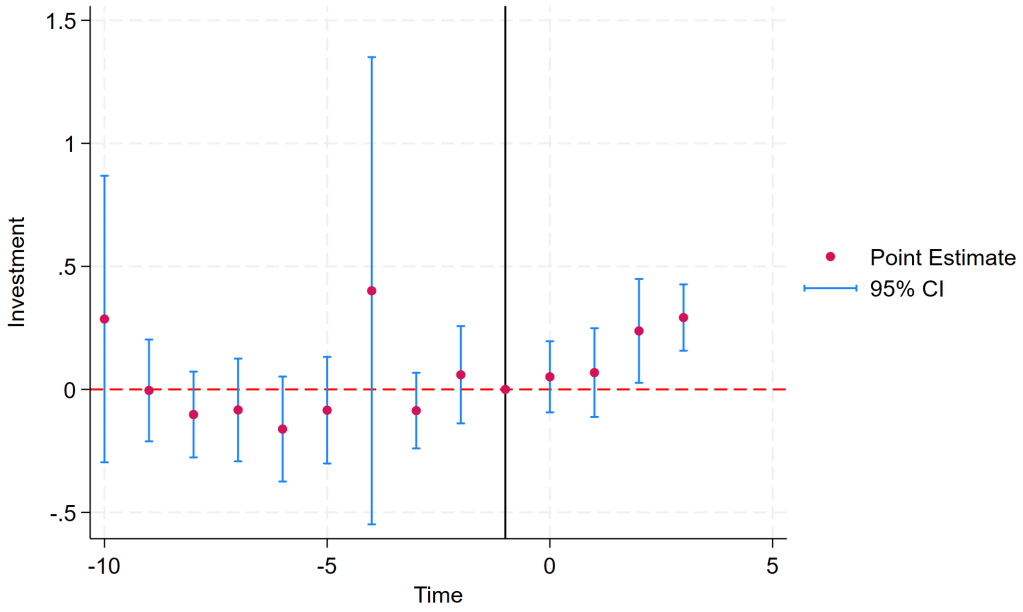
We include the following control variables X: tangible fixed assets, sales, added value. Standard errors are clustered by firm and year.

2.4 Baseline Results

Figure 3 shows the estimation results for β for this regression. We notice, first, that the parallel trends assumption is fulfilled, meaning that firms that emit targeted USD and non-USD bonds do not differ systematically in their investment rates. Second, we see a positive trend of investment for targeted firms after the 2020 intervention. Third, this effect is backloaded and economically significant, increasing investment rates by 15-30%. See Table 1 for details on two specifications: The first only includes year and firm fixed effects. There is a consistent positive association for the third lag. The second specification corresponds to Figure 3, where also the second lag exhibits a positive and statistically significant coefficient.

Overall, our empirical results point to a positive spillover effect of Fed corporate QE on the international USD bond market.

Figure 3: Spillover DiD estimation



Source: Orbis and authors' calculations. **Notes**: This chart graphically reports estimates of β_t of Equation 4. Capped spikes refer to the 95% confidence intervals.

Table 1: Investment FE estimation

VARIABLES	(1) firm FE	(2) FEs and controls
lead10	0.133	0.286
	(0.170)	(0.297)
lead9	-0.0890	-0.00423
	(0.0878)	(0.106)
lead8	-0.0781	-0.102
	(0.0879)	(0.0889)
lead7	-0.0446	-0.0836
	(0.0833)	(0.106)
lead6	-0.0897	-0.161
	(0.0919)	(0.109)
lead5	-0.107	-0.0845
	(0.101)	(0.110)
lead4	0.121	0.401
	(0.206)	(0.484)
lead3	0.0963	-0.0862
	(0.194)	(0.0784)
lead2	0.157	0.0595
	(0.121)	(0.101)
lag0	0.0566	0.0511
	(0.0747)	(0.0737)
lag1	-0.00482	0.0683
	(0.0762)	(0.0919)
lag2	0.0627	0.238**
	(0.0816)	(0.108)
lag3	0.155**	0.292***
	(0.0691)	(0.0687)
Constant	0.409***	0.427***
	(0.0266)	(0.0440)
Observations	54,499	32,915
R-squared	0.003	0.004
Number of ISSUER_ID_num	4,845	3,404
sector FE		yes
country FE		yes
controls		yes

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

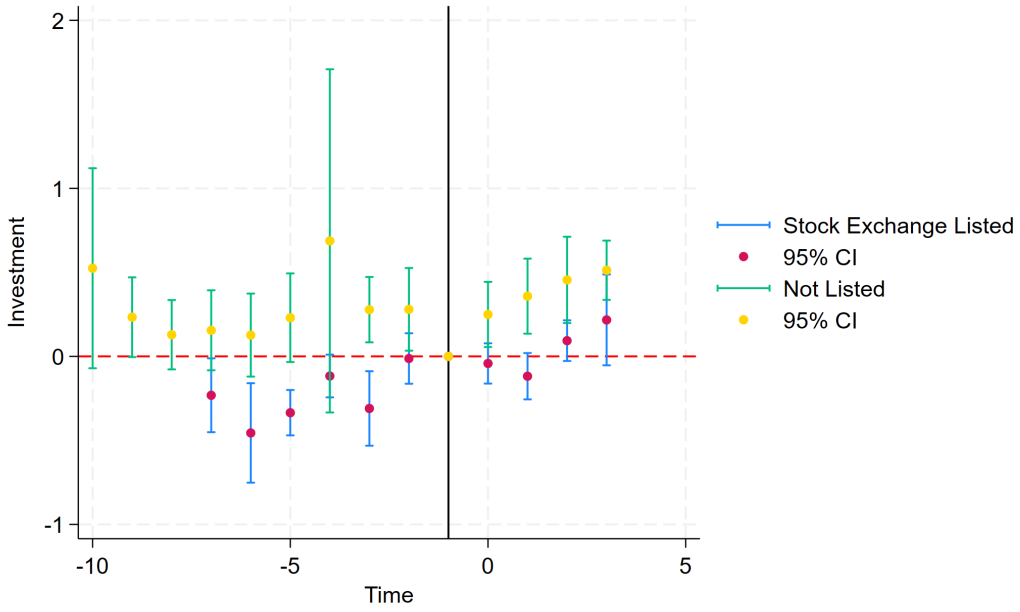
Source: Orbis and authors' calculations.

2.5 Heterogeneity analysis

Next, we turn to heterogeneity analysis.

First, we split the sample in public and private firms. Many studies analysing corporate bond issuance rely on listed firms. In our sample, 70% of firms are private. And indeed, as evident in Figure 4, the two groups have quite different investment reactions to the Fed's corporate QE. Albeit a lower number of observations, it seems that private firms' reaction is stronger than those of public firms. Overall, it does not seem warranted to exclude private firms categorically from the analysis.

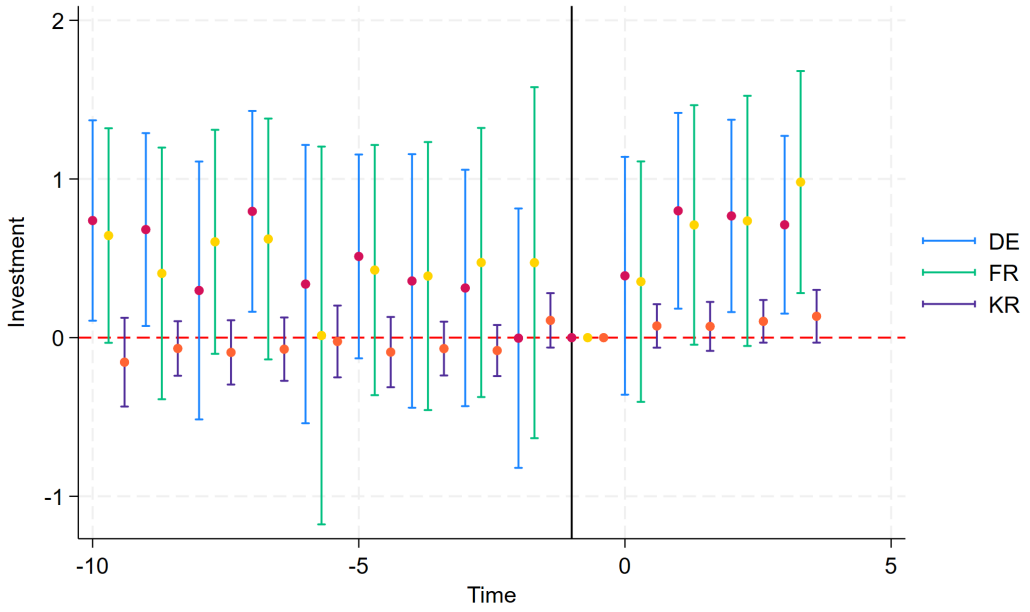
Figure 4: Public vs private firms



Source: Orbis and authors' calculations. **Notes**: This chart graphically reports estimates of β_t of Equation 4. Capped spikes refer to the 95% confidence intervals.

For some countries, the number of observations is big enough to obtain country-level estimation results. In Figure 5, we report estimation results for Germany, France, and South Korea. We see some evidence for positive spillovers even at country level with significantly lower number of observations. European firms seem to be more affected by spillovers than Asian firms.

Figure 5: Country-level estimations

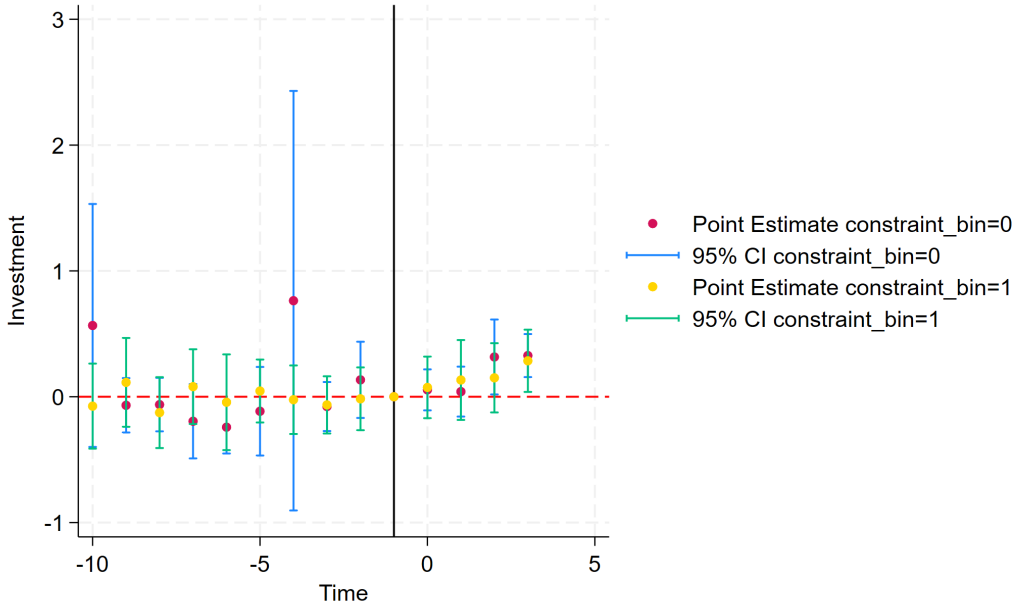


Source: Orbis and authors' calculations. **Notes**: This chart graphically reports estimates of β_t of Equation 4. Capped spikes refer to the 95% confidence intervals.

Next, we look into financially constrained and unconstrained firms. We consider a measure of financial constraint at the firm level. Specifically, we use the book-based indicator of Ferrando and Mulier, 2015 and apply it to ORBIS data as in Asdrubali et al., 2022. The indicator considers different combinations of investment, the financing gap (share of investment that is not financed by cash flow), and changes in debt and equity to classify firms as strongly financially constrained, relatively financially constrained, and unconstrained.

We split the sample in firms that are strongly/relatively financially constrained and firms that are financially unconstrained. In Figure 6, we see no discernible difference.

Figure 6: Estimations by financial constraint



Source: Orbis and authors' calculations. **Notes**. Capped spikes refer to the 95% confidence intervals.

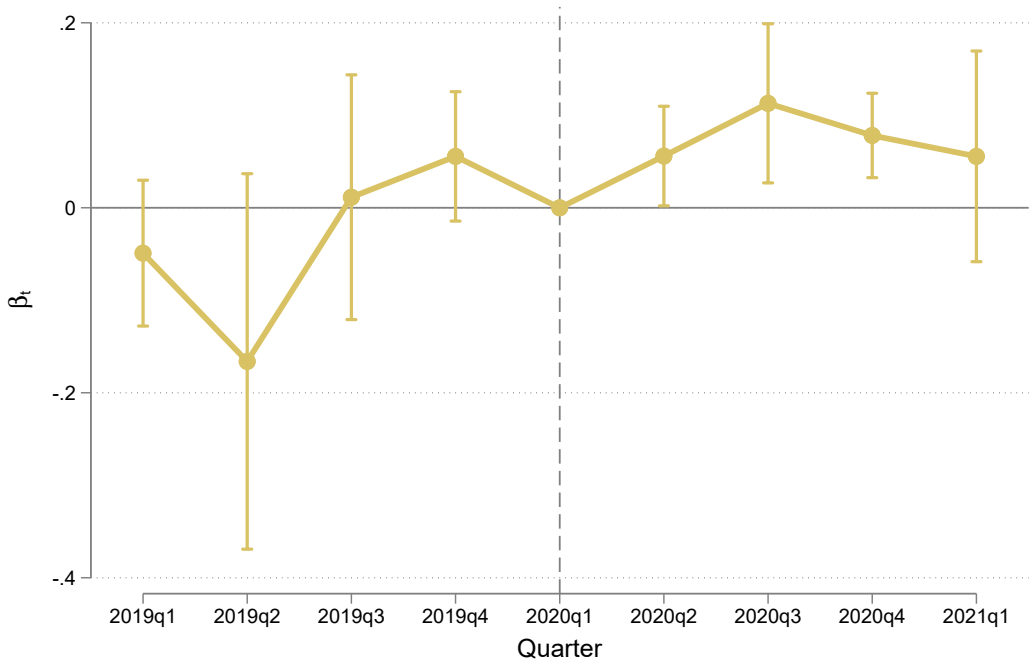
2.6 Compustat: Firm-level investment spillovers

We can run a similar analysis at higher frequency leveraging on the quarterly balance sheet information from the Compustat dataset.

$$d \ln k_{i,t} = \sum_{t=1}^{T} \beta_t \times Treated_i \times \delta_t + \gamma X_{i,t-1} + \alpha + \delta_{c,j,t} + u_{i,t} \tag{5}$$

Treatment here is having outstanding bonds and notes in USD in 2019. Sample is restricted to firms that have EUR and USD bonds in 2019, so here control group is firms with only EUR. Sample further restricted by excluding USA and EA countries. Event study saturated with country-sector-time and firm fixed effects. Errors clustered by firm and time.

Figure 7: SMCCF event study capital



Source: Compustat, data from Fed Board and authors' calculations. Notes: This chart graphically reports estimates of β_t of Equation 5. Capped spikes refer to the 95% confidence intervals.

Results in Figure 7 support the conclusion, in line with the findings from Figure 4, that positive spillovers were present as early as within the first year (2020) of the deployment of the Fed's corporate bonds purchases.

3 Model

In this section we set out the model we built to rationalise the empirical findings from the previous sections. We keep the discussion of the framework purposely parsimonious, and restrict our attention to those features that we identified to be essential for the characterisation of the main channel reconciling our results: the portfolio rebalancing towards foreign bonds perceived as substitutes. First, in Section 3.1 we briefly describe the main features and innovative com-

ponents of our framework, relegating a detailed exposition of the model and its equations to Appendix A.2. Finally, we show how the model matches our empirical findings in Section 3.2.

3.1 The model at a glance

Our model builds on the medium-scale open-macro DSGE New Keynesian framework with financial frictions, price and wage rigidities, and unconventional monetary policy tools proposed by Gertler and Karadi (2011, 2013), and in the recent work by Sims and Wu (2021a, 2021b).

Following those contributions, our setup allows for the joint interaction of households, labour unions, several types of production firms, financial intermediaries, a fiscal authority, and a central bank. Importantly, this economy exhibits two financial frictions which allow CBPs to have real effects. First, as common in the literature, we assume that wholesale firms must issue perpetual bonds (Woodford, 2001) to finance capital acquisition and obtain funds from intermediaries (Gertler & Karadi, 2011). This thus creates scope for external financing through bonds issuance. Second, intermediaries finance themselves through net worth and taking deposits (short-term debt), which they invest into central banks' reserves, long-term government bonds and, crucially, corporate bonds from wholesale firms. We also introduce a form of market segmentation preventing households from holding long-term bonds directly.[9] Because of a costly enforcement problem, a non-absconding constraint requires the value of continuing as an intermediary to be higher than the fraction of assets that can be retained in case of bankruptcy (Gertler & Karadi, 2011; Sims & Wu, 2021b). Then, CBPs by the central bank have real effects as they are able to swap assets with reserves, which are assumed not to be subject to this constraint.

On the other hand, our model exhibits some important innovations on the considered benchmarks, following the framework by Garofalo (2025) which in turn took Wu et al. (2024a). First, each economy is made up of *two* rather than just one sector (Garofalo, 2024). The two sectors are exactly the same in terms of their composition into firms and labour unions, and a final consumption entity allows the households to consume both goods, and produced in both countries. Second, wholesalers in sector 1 and 2 are identical, except that the bonds of the wholesale firms in sector 2 are assumed to bear, in steady-state, lower excess returns than the bonds of the wholesalers in the other sector. This has two consequences: (i) only the bonds from firms in sector 2 are eligible for central bank's purchases, as they are those with lower risk and thus higher credit rating; (ii) in steady-state, the model exhibits higher capital stock for firms in sector 2 than in sector 1. Finally, we assume that each intermediary can purchase bonds from firms in all sectors and from both countries.

As wholesale firms and intermediaries exhibit the main novel features of our framework, we focus on them here.[10]

[9] Other studies, such as Gertler and Karadi (2013), allow for households' direct securities and bonds holdings. Limited participation in asset markets by households and thus incomplete arbitrage are then achieved through the assumption of transactions costs. For simplicity and keep the model parsimonious, we do not allow households to hold directly bonds.

[10] We refer the reader to the appendix of Sims and Wu (2021b) for details on all the standard equations for these and the other parts of the model.

3.1.1 Wholesale firms

We include four types of production firms in each sector of the model. A competitive final good producer turns retail output into a final good to pass onto the bundler. A continuum of monopolistically competitive and price sticky retail firms sales wholesale output to the final good firm. And a representative investment good firm makes new physical capital from unused output through a convex adjustment cost technology. Finally, the sector's representative wholesale firm inputs in its Cobb-Douglas technology capital purchased from the investment good producer and labour from unions, which are subject to wage stickiness.

Crucially, each sector s's wholesale firm exhibits the following "loan in advance constraint" (Carlstrom et al., 2017; Gertler & Karadi, 2011):

$$\psi_s P^k_{s,t} I_{s,t+1} \leq Q_{s,t} F_{m,s,t} \quad \text{for} \quad s = 1, 2 \tag{6}$$

Equation 6 requires that, in order to finance a constant fraction, $\psi_s \in [0, 1]$, of its capital acquisition $I_{s,t+1}$ at price $P^k_{s,t}$, each sector's wholesaler must issue perpetual bonds $F_{m,s,t}$ at market price $Q_{s,t}$ with decaying coupon payments, whose decaying parameter is $\kappa \in [0, 1]$. $F_{m,s,t}$ is the total coupon liability due in period t, i.e. $F_{m,s,t-1} = CF_{m,s,t-1} + \kappa CF_{m,s,t-2} + \kappa^2 CF_{m,s,t-3} + ...$ The latter, through forward iteration, implies that one needs only keep track of the period t and $t-1$ total coupon liabilities, $CF_{m,s,t} = F_{m,s,t} - \kappa F_{m,s,t-1}$. This constraint creates scope for external financing, distorts the standard asset pricing decisions, and it is thus one of the main mechanisms through which QE type policies transmit to the real economy.

3.1.2 Financial intermediaries

We follow Garofalo (2025) and model financial intermediaries by taking the framework proposed by Wu et al. (2024a) to a two-sector set-up. Specifically, households own and hold deposits at both sectors' intermediaries. Each period and within each sector s a fraction $1 - \sigma_s$ with $\sigma_s \in [0, 1]$ stochastically exit and return their net worth $N_{s,t}$ to their household owner.[11] Once exited, they are replaced by an equal number of new intermediaries — these start up with X_s real funds from their household owner. As long as intermediaries keep operating, they finance their activities through their net worth $N_{s,t}$ and deposits $D_{s,t}$. These activities of each intermediary consist in investing into central bank reserves $RE_{s,t}$, government bonds $B_{s,t}$ and corporate bonds from the wholesale firms within their respective sector $F_{s,t}$, in order to maximise its expected terminal net worth where discounting by the stochastic discount factor of the household $\Lambda_{t,t+1}$:

$$V_{s,t} = \max(1 - \sigma)\mathbb{E}_t \sum_{j=1}^{\infty} \sigma^{j-1} \Lambda_{t,t+j} n_{s,t+j} \quad \text{for} \quad s = 1, 2 \tag{7}$$

where $n_{s,t} = N_{s,t}/P_t$ is real net worth, with P_t the price level of final output.

Again, for the sake of clarity, we focus here on the different financial constraints of our set-up with respect to the benchmarks in the literature, and relegate all the equations and details to the Appendix A.2.

[11] Notice we have dropped the single intermediary index i here to simplify the notation, as all intermediaries behave in the same way and are subject to the same constraints within each sector.

Crucially, intermediaries in Home and Foreign economies are equally constrained by a non-absconding requirement (Gertler & Karadi, 2011), which prevents them from declaring bankruptcy and retain the entirety of their balance sheet. They can only divert away from households a stochastic and exogenous fraction θ_t of their assets $\Delta Q_t^2 b_{H,t}^{2,FI} + Q_t^1 b_{H,t}^{1,FI}$ and $\left[\Delta Q_t^{*2} b_{F,t}^{2,FI} + Q_t^{*1} b_{F,t}^{1,FI}\right] \mathcal{Q}_t$, where $\mathcal{Q}_t$ is the real exchange rate, b are the real (divided by price level P_t) private bond holdings respectively, and $0 \leq \Delta \leq 1$ is the steady state difference in recoverability between the two types of bonds.[12] As a result of this, for households to be still willing to lend funds, intermediaries must guarantee (also in steady-state) excess returns of long term private eligible and non-eligible bonds over the deposit rate, with the ones on the former being lower due to the higher recoverability through Δ. Crucially, this set-up allows CBPs to have real effects by swapping assets with reserves not subject to constraint, and thus relaxing the requirement for intermediaries. However, in line with the observed features of the SMCCF, we assume that only the bonds from sector 2 are eligible for CBPs. This means that the action of the central bank will be targeted to a group of intermediaries and, in turn, firms rather than the whole economy. The constraints for the two types of intermediaries are thus:

$$V_t \geq \theta_t \left\{ \Delta Q_t^g b_{H,t}^{g,FI} + Q_t^b b_{H,t}^{b,FI} + \left[\Delta Q_t^{*g} b_{F,t}^{g,FI} + Q_t^{*b} b_{F,t}^{b,FI}\right] \mathcal{Q}_t \right\} \tag{8}$$

3.2 Quantitative results

In this section we calibrate and solve[13] the model to produce impulse responses to an exogenous increase in corporate bond purchases by the Home central bank. Our objective is to use those to assess and understand the channels driving their spillovers to the Foreign economy.

3.2.1 Calibration

TBU

3.2.2 The international spillovers of corporate bond purchases

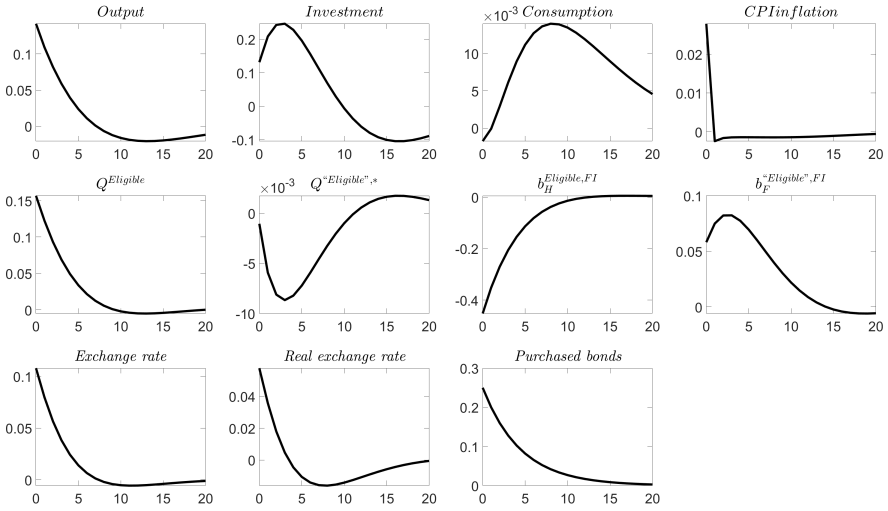
In order to quantify the international spillovers of Home central bank's corporate bond purchases, we use the model to produce impulse responses to an increase in CBPs. Looking at the Home economy (Figure 8), the peak effect on output is around 0.15%, then gradually declining towards steady state.

In the model central bank corporate bond purchases operate by relaxing the costly enforcement problem 8. Because of that, these bond purchases do not just crowd out intermediary bond purchases swapping them for reserves, but increase the total demand for bonds of sector 2. In turn, this leads to higher bond prices, which loosen the loan-in-advance constraint 6 and spur wholesale firms in sector 2 to increase their investment. This process eventually boosts aggregate investment and output, as well as consumption and prices as showed in Figure 8.

[12] This adjustment captures in a stylised way the fact that the bank's private non-eligible loan portfolio is arguably more exposed to bank malfeasance than its eligible bond portfolio. This is the case because depositors monitor the performance of the latter with more difficulty than the former.

[13] We solve the model with Dynare via a linear approximation about the non-stochastic steady state.

Figure 8: IRFs for Home macroeconomic variables from Home QE shock



Importantly, corporate bond purchases from the domestic central bank have spillover effects onto the Foreign economy.

First, notice how domestic intermediary expand their holdings of bonds issued by foreign firms, which are similar to those issuing domestic eligible bonds, and thus in this sense "eligible" substitute to target for portfolio expansion. This is in line with our empirical findings in section 2.

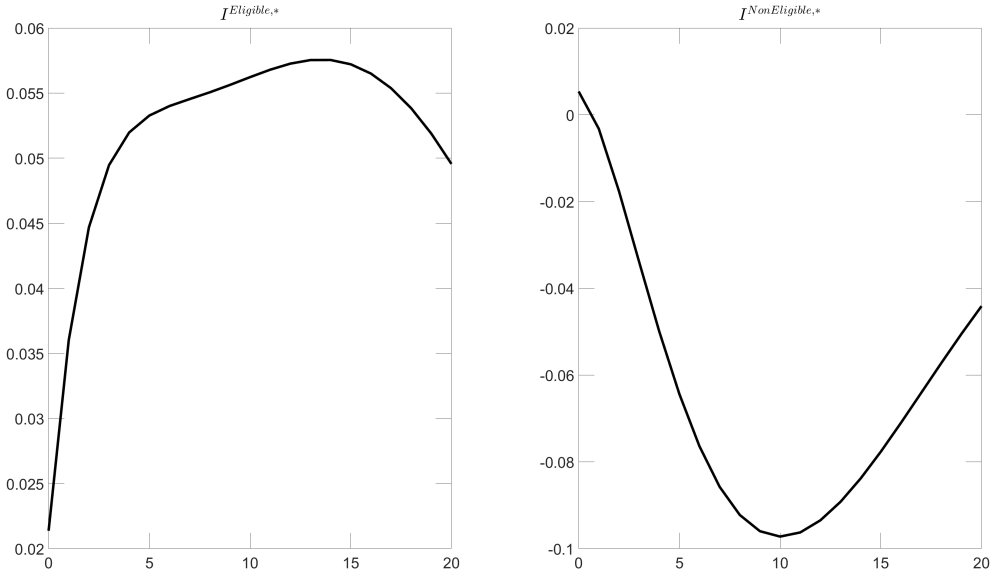
Second, comparing capital accumulation of eligible and non-eligible firms in the Foreign economy shows how these purchases have unequal cross-border effects (Figure 9). The increase in purchases of eligible foreign bonds by domestic intermediaries makes the yield curve faced by foreign eligible firms when financing less steep and thus stimulate their capital accumulation. This occurs through the reduction in their *real long term yield*, i.e. the overall spread capturing the difference between the net long term private yield and the net deposit rate in sector 2, plus the real net deposit rate, i.e. the difference between the net deposit rate and expected inflation. As in Sims and Wu (2021b), this variable is the relevant metric for assessing monetary policy transmission to the firms in the model.[14] Through this channel, our model is able to match the differential spillover dynamics we have uncovered in our empirical analysis in section 2.

4 Counterfactual / scenario

TBU

[14]Sims and Wu (2021b) highlight that having long term yields as the central driver in the economy is in line with (Wu & Zhang, 2016, 2019) and the other studies in the literature using the shadow rate of Wu and Xia (2016) as a summary statistic for the effects of unconventional policies.

Figure 9: Firm-level IRFs for CBPs



5 Conclusion

In this paper we explored the global economic ripple effects of the U.S. Federal Reserve's first-ever corporate bond purchasing program, introduced in response to the 2020 Covid crisis. Our empirical analysis reveals a notably positive impact on firm-level investment: foreign companies issuing USD-denominated bonds with characteristics similar to those acquired by the Fed demonstrated greater investment resilience compared to similar firms. Examining the portfolios of U.S. mutual funds, we find that investors who sold domestic bonds eligible for the Fed's program also redirected their capital toward similar, yet ineligible, foreign bonds.

To explain these findings, we develop a two-country model incorporating international financial intermediaries and heterogeneous firms. According to our model, the Fed's purchase of eligible U.S. corporate bonds eases financial intermediaries' leverage constraints, allowing them to replenish their portfolios with comparable foreign USD-denominated bonds. As a result, foreign firms issuing such bonds experience more relaxed investment constraints than other foreign firms.

We then use our framework to analyze two potential future scenarios. First, if the proportion of USD-denominated bonds issued outside the U.S. continues to grow to a scale matching the U.S. bond market, the Fed would struggle to stabilize the global USD bond market through domestic purchases alone, as it did during the Covid crisis. Second, in a world with reduced USD dominance in international debt markets, the international spillover effects of the Fed's corporate bond purchases would diminish significantly. Both cases underscore potential threats to the Fed's ability to stabilize global markets through its monetary policy interventions.

References

Abidi, N., & Miquel-Flores, I. (2018). *Who benefits from the corporate QE? a regression discontinuity design approach* (tech. rep.). European Central Bank.

Adams, P., & Verdelhan, A. (2022). Exchange Rate Risk in Public Firms.

Alpanda, S., & Kabaca, S. (2020). International Spillovers of Large-Scale Asset Purchases. *Journal of the European Economic Association*, *18*(1), 342–391.

Arce, O., Mayordomo, S., & Gimeno, R. (2021). Making room for the needy: The credit-reallocation effects of the ECB's corporate QE. *Review of Finance*, *25*(1), 43–84.

Asdrubali, P., Hallak, I., Harasztosi, P., et al. (2022). *Financial constraints of eu firms a sectoral analysis* (tech. rep.). Directorate General Economic and Financial Affairs (DG ECFIN), European ...

Bahaj, S., & Reis, R. (2022a). The Economics of Liquidity Lines Between Central Banks [_eprint: https://doi.org/10.1146/annurev-financial-111620-022146]. *Annual Review of Financial Economics*, *14*(1), 57–74.

Bahaj, S., & Reis, R. (2022b). Central Bank Swap Lines: Evidence on the Effects of the Lender of Last Resort. *The Review of Economic Studies*, *89*(4), 1654–1693.

Boyarchenko, N., Cox, C., Crump, R. K., Danzig, A., Kovner, A., Shachar, O., & Steiner, P. (2022, July). The Primary and Secondary Corporate Credit Facilities.

Carlstrom, C. T., Fuerst, T. S., & Paustian, M. (2017). Targeting Long Rates in a Model with Segmented Markets. *American Economic Journal: Macroeconomics*, *9*(1), 205–242.

Clementi, G. L., & Palazzo, B. (2019). Investment and the Cross-Section of Equity Returns. *The Journal of Finance*, *74*(1), 281–321.

Coppola, A., Krishnamurthy, A., & Xu, C. (2023). Liquidity, Debt Denomination, and Currency Dominance.

Crouzet, N., & Tourre, F. (2021). Can the cure kill the patient? Corporate credit interventions and debt overhang. *SSRN Electronic Journal*.

D'Amico, S., & Kaminska, I. (2019). Credit easing versus quantitative easing: Evidence from corporate and government bond purchase programs.

Darmouni, O., & Siani, K. (2023, September). Bond Market Stimulus: Firm-Level Evidence.

Das, M., Gopinath, G., Kim, T., & Stein, J. C. (2022, December). Central Banks as Dollar Lenders of Last Resort: Implications for Regulation and Reserve Holdings.

De Santis, R. A., Geis, A., Juskaite, A., Cruz, L. V., et al. (2018). The impact of the corporate sector purchase programme on corporate bond markets and the financing of euro area non-financial corporations. *Economic Bulletin Articles*, *3*.

De Santis, R. A., & Zaghini, A. (2021). Unconventional monetary policy and corporate bond issuance. *European Economic Review*, *135*, 103727.

Döttling, R., & Ratnovski, L. (2021, August). *Monetary Policy and Intangible Investment* (SSRN Scholarly Paper No. 3612304). Social Science Research Network. Rochester, NY.

Ferrando, A., & Mulier, K. (2015). Firms' financing constraints: Do perceptions match the actual situation? *The Economic and Social Review*, *46*(1, Spring), 87–117.

Garofalo, M. (2024). Granular monetary policy.

Garofalo, M. (2025). Greenback? sustainable investment and the international transmission of monetary policy.

Gertler, M., & Karadi, P. (2011). A model of unconventional monetary policy. *Journal of Monetary Economics*, *58*(1), 17–34.

Gertler, M., & Karadi, P. (2013). QE 1 vs. 2 vs. 3. . . : A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool. *International Journal of Central Banking*, *9*, 49.

Gilchrist, S., Wei, B., Yue, V. Z., & Zakrajšek, E. (2021). The Fed takes on corporate credit risk: An analysis of the efficacy of the SMCCF, 47.

Gopinath, G., & Stein, J. C. (2021). Banking, Trade, and the Making of a Dominant Currency*. *The Quarterly Journal of Economics*, *136*(2), 783–830.

Grosse-Rueschkamp, B., Steffen, S., & Streitz, D. (2019). A capital structure channel of monetary policy. *Journal of Financial Economics*, *133*(2), 357–378.

Hegarty, C., Moretti, M., Ottonello, P., & Perez, D. J. (2023). Global Borrowing Costs and Firms' Risk in Open Economies.

Hines, J. R. (2010). Treasure Islands. *Journal of Economic Perspectives*, *24*(4), 103–126.

Iacoviello, M., & Navarro, G. (2019). Foreign effects of higher U.S. interest rates. *Journal of International Money and Finance*, *95*, 232–250.

Jordà, Ò. (2005). Estimation and Inference of Impulse Responses by Local Projections. *American Economic Review*, *95*(1), 161–182.

Kalemli-Ozcan, S., Sorensen, B., Villegas-Sanchez, C., Volosovych, V., & Yesiltas, S. (2015, September). *How to Construct Nationally Representative Firm Level Data from the Orbis Global Database: New Facts and Aggregate Implications* (tech. rep. No. w21558). National Bureau of Economic Research. Cambridge, MA.

Kolasa, M., & Wesołowski, G. (2020). International spillovers of quantitative easing. *Journal of International Economics*, *126*, 103330.

Li, F., Mercatanti, A., Mäkinen, T., & Silvestrini, A. (2021). A regression discontinuity design for ordinal running variables: Evaluating central bank purchases of corporate bonds. *The Annals of Applied Statistics*, *15*(1), 304–322.

McCauley, R. N. (2023). Bond Market Crises and the International Lender of Last Resort. In R. Z. Aliber, M. Gudmundsson, & G. Zoega (Eds.), *Fault Lines After COVID-19* (pp. 261–277). Springer Nature Switzerland.

Miranda-Agrippino, S., & Nenova, T. (2022). A tale of two global monetary policies. *Journal of International Economics*, *136*, 103606.

OECD (Ed.). (2009). *Measuring capital: OECD manual 2009* (2. ed). OECD.

Ottonello, P., & Winberry, T. (2020). Financial Heterogeneity and the Investment Channel of Monetary Policy. *Econometrica*, *88*(6), 2473–2502.

Peters, R. H., & Taylor, L. A. (2017). Intangible capital and the investment-q relation. *Journal of Financial Economics*, *123*(2), 251–272.

Poyntner, P. (2023). Firm-level Consequences of Corporate Quantitative Easing.

Salinger, M., & Summers, L. H. (1983). Tax Reform and Corporate Investment: A Microeconometric Simulation Study. In *Behavioral Simulation Methods in Tax Policy Analysis* (pp. 247–288). University of Chicago Press.

Schmitt-Grohé, S., & Uribe, M. (2003). Closing small open economy models. *Journal of International Economics*, *61*(1), 163–185.

Siani, K. (2019, December). Global Demand Spillovers: When the Central Bank Buys Corporate Bonds.

Sims, E., & Wu, J. C. (2021a). Wall Street QE vs. Main Street Lending, 32.

Sims, E., & Wu, J. C. (2021b). Evaluating Central Banks' tool kit: Past, present, and future. *Journal of Monetary Economics*, *118*, 135–160.

Woodford, M. (2001). Fiscal Requirements for Price Stability [Publisher: [Wiley, Ohio State University Press]]. *Journal of Money, Credit and Banking*, *33*(3), 669–728.

Wu, J. C., & Xia, F. D. (2016). Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound. *Journal of Money, Credit and Banking*, *48*(2-3), 253–291.

Wu, J. C., Xie, Y., & Zhang, J. (2024a). The role of international financial integration in monetary policy transmission.

Wu, J. C., Xie, Y., & Zhang, J. (2024b). The Role of International Financial Integration in Monetary Policy Transmission. *IMF Economic Review*.

Wu, J. C., & Zhang, J. (2016, November). A Shadow Rate New Keynesian Model.

Wu, J. C., & Zhang, J. (2019). Global effective lower bound and unconventional monetary policy. *Journal of International Economics*, *118*, 200–216.

Zaghini, A. (2019). The cspp at work: Yield heterogeneity and the portfolio rebalancing channel. *Journal of Corporate Finance*, *56*, 282–297.

Online Appendix for:

The international spillovers of corporate bonds purchases

by Marco Garofalo and Philipp Poyntner

A.1 Data

A.1.1 Data sources

Orbis For the firm balance sheet from Orbis, we proceeded as follows. First, the download was conducted from April 2024 to October 2024. Second, we downloaded all relevant companies for some countries in our sample (Switzerland, Germany, Austria, Argentina, China) using the following search steps in Orbis:

1. Status: Aktive, Inactive, Unknown
2. Industry classifications: NACE Rev. 2. → Add all except: 35 Electricity, gas, steam and air conditioning supply (Section D); 36-39 (Section E), 64 Financial service activities (Part of Section K)
3. Country (iterative): Switzerland, Austria, Germany, Argentina, China
4. Tangible Fixed assets: All companies with a known value 2020 SW, DE, AT
5. Total assets (TOAS): All companies with a known value 2020 Argentina, China

We only download firms that have observations for their assets in 2020, since we are not able to use them in our empirical investigation otherwise.

For the rest of the data, we directly use Orbis' batch search tool and match by firm name and country. Table A.1 shows the number of firms and number of bonds issued by countries in our sample and how many could be matched by firm name.

National Accounts We calculate depreciation rates following Ottonello and Winberry (2020) from BEA Interactive Data Tables - National Data - Fixed Assets Accounts Tables.[15] From "Section 3 - Private Fixed Assets by Industry", the Fixed Assets Tables provide data on the stock, investment, and depreciation of (i) equipment, (ii) structures, and (iii) intellectual property products. These data are disaggregated by NAICS industries. From the stock (in billions of dollars) and the depreciation (in billions of dollars), one can calculate the rate as the ratio of the latter over the former. These series, which are industry specific, are at annual frequency, starting from 1947 and ending in 2021 (the latest year available). To go from annual to quarterly frequency, one can assume the same depreciation figures within each year, and use the relationship linking the quarterly (δ_{ind}) to the annual ($\delta_{ind,ann}$) depreciation rate:

$$\delta_{ind} = 1 - (1 - \delta_{ind,ann})^{(1/4)} \tag{9}$$

[15] `https://apps.bea.gov/iTable`.

Table A.1: Matching between bond data and firm data (Orbis)

Country	Unique firms	number of firms matched	Unique Issues
Argentina	253	240	654
Australia	62	55	228
Austria	75	73	241
Azerbaijan	1	0	1
Barbados	1	0	1
Belgium	45	34	159
Bermuda	10	8	40
Botswana	1	1	1
Brazil	694	669	1764
Bulgaria	1	1	2
Cambodia	1	0	1
Canada	280	227	2029
Cayman Islands	9	4	63
Chile	75	59	419
China	2767	2056	14009
Colombia	15	12	36
Costa Rica	7	5	17
Croatia	7	7	12
Curacao	1	1	2
Cyprus	3	3	3
Czech Republic	32	31	56
Denmark	37	35	138
Dominican Republic	5	4	13
El Salvador	1	0	4
Estonia	5	5	6
Faroe Islands	1	0	1
Finland	41	40	99
France	226	194	1381
French Guiana	1	0	1
Georgia	1	0	2
Germany	356	349	847
Ghana	1	0	65
Greece	15	11	22
Guatemala	1	0	6
Guernsey	3	0	7
Honduras	1	0	5
Hong Kong	59	25	484
Hungary	47	45	66
Iceland	15	12	19
India	858	812	2830
Indonesia	136	120	639
Ireland; Republic of	15	15	70
Isle of Man	1	0	2
Israel	118	105	306
Italy	517	491	857
Jamaica	3	3	6
Japan	829	770	3822
Jersey	1	0	7
Jordan	2	1	5
Kazakhstan	14	11	22
Korea; Republic (S. Korea)	3824	3394	10639
Latvia	3	2	4
Liechtenstein	5	4	8
Lithuania	2	2	2
Luxembourg	39	34	278
Macau	3	0	41
Malaysia	81	78	379
Malta	21	21	34
Mauritius	1	1	1
Mexico	104	89	380
Monaco	2	2	8
Mongolia	1	0	1
Morocco	23	17	272
NA	55		87
Namibia	1	1	1
Netherlands	65	60	360
New Zealand	17	15	58
Nigeria	5	4	6
Norway	138	136	1247
Oman	2	2	2
Panama	43	41	268
Paraguay	52	45	430
Peru	36	35	94
Philippines	23	18	71
Poland	74	73	219
Portugal	36	33	73
Puerto Rico	1	0	2
Romania	12	11	20
Russia	252	193	682
Singapore	56	30	144
Slovak Republic	17	17	29
Slovenia	5	5	6
South Africa	28	26	323
Spain	74	71	197
Sri Lanka	2	2	4
Sweden	174	165	1334
Switzerland	114	109	295
Taiwan	107	99	624
Thailand	194	183	1502
Tunisia	1	1	5
Turkey	93	88	697
Ukraine	8	5	13
United Arab Emirates	7	6	21
United Kingdom	204	182	626
Uruguay	15	14	53
Uzbekistan	1	0	2
Vietnam	122	101	291
Virgin Islands; British	4	2	4

which just comes from

$$1 - \delta_{ind,ann} = (1 - \delta_{ind})^4 \tag{10}$$

i.e. annual *un-depreciation* equals the product of quarterly *un-depreciations*.

How to build the capital stock at firm level. There are several ways to build capital stocks at firm level, but a very popular one, as commonly used also by National Statistics Authorities to construct National Accounts (OECD, 2009), is the perpetual inventory method (PIM).

Investment as $\Delta log(k_{i,t})$, where $k_{i,t}$ is capital stock of firm i at the end of period t. In first period reported in Compustat set value of $k_{i,t}$ at the level of gross plant, property, and equipment (ppegtq). After that, cumulative changes of net plant, property, and equipment (ppentq), as this measure of (net of depreciation) investment reports many more observations than ppegtq. One needs to resort to linear interpolation if there are missing data points between two non-missing observations; no other imputation is carried out.

Finally, as in Ottonello and Winberry (2020), we consider only spells for firms with 40 consecutive quarters, or more, in order to estimate precisely the fixed effects in our regressions.

A.1.2 Data cleaning

Following the literature, in particular Ottonello and Winberry (2020), we apply a series of cleaning steps to our dataset.

With the aim to exclude extreme observations or unreasonable values from the final dataset, we make sure that the firm-quarter observations in my sample do not satisfy any of the following conditions:

- Negative capital or assets
- Acquisitions larger than 5% of assets. Incidentally, this requires that my sample starts in 1983q3, when the data on acquisitions becomes available.
- Investment rate is in the top and bottom 0.5% of the distribution.
- Investment spell is shorter than 40 quarters.
- Net current assets as a share of total assets higher than 10 or below -10.
- Leverage higher than 10 or negative.
- Quarterly real sales growth above 1 or below -1.
- Negative sales or liquidity.

Finally, we winsorise observations of leverage and distance to default at the top and bottom 0.5% of the distribution.

A.2 Details on model

In what follows, we detail the parts of the two-country model used in our analysis. Throughout the exposition recall that: (i) starred variables correspond to the Foreign economy; (ii) the two countries are symmetric bar for their size and parameter values. Notice also that superscript g indicates firms whose bonds are *eligible*, while b the *ineligible* for central bank purchases.

A.2.1 Households - Intratemporal decision

Home households consume bundle

$$C_t = \Bigg\{ [\nu\Upsilon]^{\frac{1}{\vartheta}} (C^b_{H,t})^{\frac{\vartheta-1}{\vartheta}} + [(1-\nu)\Upsilon]^{\frac{1}{\vartheta}} (C^b_{F,t})^{\frac{\vartheta-1}{\vartheta}} + [\nu(1-\Upsilon)]^{\frac{1}{\vartheta}} (C^g_{H,t})^{\frac{\vartheta-1}{\vartheta}} + [(1-\nu)(1-\Upsilon)]^{\frac{1}{\vartheta}} (C^g_{F,t})^{\frac{\vartheta-1}{\vartheta}} \Bigg\}^{\frac{\vartheta}{\vartheta-1}} \tag{11}$$

while Foreign households

$$C^*_t = \Bigg\{ [\nu^*\Upsilon^*]^{\frac{1}{\vartheta}} (C^{*b}_{H,t})^{\frac{\vartheta-1}{\vartheta}} + [(1-\nu^*)\Upsilon^*]^{\frac{1}{\vartheta}} (C^{*b}_{F,t})^{\frac{\vartheta-1}{\vartheta}} + [\nu^*(1-\Upsilon^*)]^{\frac{1}{\vartheta}} (C^{*g}_{H,t})^{\frac{\vartheta-1}{\vartheta}} + [(1-\nu^*)(1-\Upsilon^*)]^{\frac{1}{\vartheta}} (C^{*g}_{F,t})^{\frac{\vartheta-1}{\vartheta}} \Bigg\}^{\frac{\vartheta}{\vartheta-1}} \tag{12}$$

where $C^b_{H,t}$ ($C^{*b}_{H,t}$) and $C^b_{F,t}$ ($C^{*b}_{F,t}$) represent Home (Foreign) consumption of Home and Foreign ineligible final goods respectively, while the corresponding green variables are $C^g_{H,t}$ ($C^{*g}_{H,t}$) and $C^g_{F,t}$ ($C^{*g}_{F,t}$). The weight of the Home (Foreign) good in the Home (Foreign) bundle, ν (ν^*), captures households' preferences for the domestic good, and it might differ between Home and Foreign economies. Specifically, $\nu = 1-(1-\gamma)v$ and $\nu^* = \gamma v$, where $\gamma \in [0,1]$ and $(1-\gamma)$ measures the (relative) population size of the Home (Foreign) country, while $v \in [0,1]$ and $(1-v)$ measures the degree of trade openness. Notice that the framework nests a closed-economy set-up retrieved when $v = 0$; when $v = 1$ there is no home bias, while for $0 < v < 1$ the bias is towards home goods. Υ (Υ^*) is the share of brown goods in Home (Foreign) consumption. Finally, ϑ is the elasticity of substitution between Home and Foreign goods.

For both Home and Foreign households the intratemporal problem requires to maximise nominal consumption net of the costs of the Home and Foreign goods bundled together, that is optimally picking the allocation of consumption expenditure between domestic and foreign goods:

$$\max_{\{C^b_{H,t}, C^b_{F,t}, C^g_{H,t}, C^g_{F,t}\}} P_t C_t - P^b_{H,t} C^b_{H,t} - P^b_{F,t} C^b_{F,t} - P^g_{H,t} C^g_{H,t} - P^g_{F,t} C^g_{F,t}$$

$$\max_{\{C^{*b}_{H,t}, C^{*b}_{F,t}, C^{*g}_{H,t}, C^{*g}_{F,t}\}} P^*_t C^*_t - P^{*b}_{H,t} C^{*b}_{H,t} - P^{*b}_{F,t} C^{*b}_{F,t} - P^{*g}_{H,t} C^{*g}_{H,t} - P^{*g}_{F,t} C^{*g}_{F,t}$$

where $P^b_{H,t}$ ($P^{*b}_{H,t}$) is the price of the Home ineligible goods in Home (Foreign), while $P^b_{F,t}$ ($P^{*b}_{F,t}$) is the price of the Foreign ineligible goods in Home (Foreign), all in the corresponding domestic currency and at time t. Similarly, the prices for Home and Foreign eligible goods in Home (Foreign) are $P^g_{H,t}$ ($P^{*b}_{H,t}$) and $P^b_{F,t}$ ($P^{*b}_{F,t}$). respectively. P_t (P^*_t) is the CPI index of the domestic (foreign) economy, to be defined shortly below.

Using the definition of consumption bundles above (11) and (12), the FOCs for $C^b_{H,t}$, $C^b_{F,t}$, $C^g_{H,t}$, $C^g_{F,t}$, $C^{*b}_{H,t}$, $C^{*b}_{F,t}$, $C^{*g}_{H,t}$ and $C^{*g}_{F,t}$ yield respectively:

$$C^b_{H,t} = \left(\frac{P^b_{H,t}}{P_t}\right)^{-\vartheta} \nu\Upsilon C_t \tag{13}$$

$$C^b_{F,t} = \left(\frac{P^b_{F,t}}{P_t}\right)^{-\vartheta} (1-\nu)\Upsilon C_t \tag{14}$$

$$C^g_{H,t} = \left(\frac{P^g_{H,t}}{P_t}\right)^{-\vartheta} \nu(1-\Upsilon) C_t \tag{15}$$

$$C^g_{F,t} = \left(\frac{P^g_{F,t}}{P_t}\right)^{-\vartheta} (1-\nu)(1-\Upsilon) C_t \tag{16}$$

$$C^{*b}_{H,t} = \left(\frac{P^{*b}_{H,t}}{P^*_t}\right)^{-\vartheta} \nu^*\Upsilon^* C^*_t \tag{17}$$

$$C^{*b}_{F,t} = \left(\frac{P^{*b}_{F,t}}{P^*_t}\right)^{-\vartheta} (1-\nu^*)\Upsilon^* C^*_t \tag{18}$$

$$C^{*g}_{H,t} = \left(\frac{P^{*g}_{H,t}}{P^*_t}\right)^{-\vartheta} \nu^*(1-\Upsilon^*) C^*_t \tag{19}$$

$$C^{*g}_{F,t} = \left(\frac{P^{*g}_{F,t}}{P^*_t}\right)^{-\vartheta} (1-\nu^*)(1-\Upsilon^*) C^*_t \tag{20}$$

$$\tag{21}$$

One can plug these demand equations into (11) and (12) to obtain the CPI index P_t and P^*_t for Home and Foreign, respectively:

$$P_t = \left[\nu\Upsilon(P^b_{H,t})^{1-\vartheta} + (1-\nu)\Upsilon(P^b_{F,t})^{1-\vartheta} + \nu(1-\Upsilon)(P^g_{H,t})^{1-\vartheta} + (1-\nu)(1-\Upsilon)(P^g_{F,t})^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}} \tag{22}$$

$$P^*_t = \left[\nu^*\Upsilon^*(P^{*b}_{H,t})^{1-\vartheta} + (1-\nu^*)\Upsilon^*(P^{*b}_{F,t})^{1-\vartheta} + \nu^*(1-\Upsilon^*)(P^{*g}_{H,t})^{1-\vartheta} + (1-\nu^*)(1-\Upsilon^*)(P^{*g}_{F,t})^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}} \tag{23}$$

At this point, I assume that the law of one price (LOOP) holds, that is the prices of Home and Foreign differentiated goods in the Home economy are equal to prices in Foreign converted in domestic currency: for brown (green) prices $P^b_{H,t}(h) = \mathcal{E}_t P^{*b}_{H,t}(h)$ ($P^g_{H,t}(h) = \mathcal{E}_t P^{*g}_{H,t}(h)$) and $P^b_{F,t}(f) = \mathcal{E}_t P^{*b}_{F,t}(f)$ ($P^g_{F,t}(f) = \mathcal{E}_t P^{*g}_{F,t}(f)$), with $\mathcal{E}_t$ being the nominal exchange rate, i.e. the price of one unit of Foreign currency in terms of Home currency. This also implies:

$$P^b_{H,t} = \mathcal{E}_t P^{*b}_{H,t} \tag{24}$$
$$P^g_{H,t} = \mathcal{E}_t P^{*g}_{H,t} \tag{25}$$
$$P^b_{F,t} = \mathcal{E}_t P^{*b}_{F,t} \tag{26}$$
$$P^g_{F,t} = \mathcal{E}_t P^{*g}_{F,t} \tag{27}$$

When $v < 1$, that is when there is not full openness and thus home bias exists, purchasing power parity does not hold: $P_t \neq \mathcal{E}_t P^*_t$. This means that the real exchange rate is different from one, and defined as

$$\mathcal{Q}_t \equiv \mathcal{E}_t \frac{P^*_t}{P_t} \tag{28}$$

Thus, an increase in $\mathcal{E}_t$ indicates a nominal depreciation of the Home currency, and an increase in $\mathcal{Q}_t$ a real depreciation.

Because the CPI indexes are non-stationary, I must parse them out from the expressions for the real exchange rate:

$$\mathcal{E}_t = \mathcal{Q}_t \frac{P^*_t}{P_t} \quad \Rightarrow \quad \frac{\mathcal{Q}_t}{\mathcal{Q}_{t-1}} = \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \frac{P^*_t}{P^*_{t-1}} \frac{P_{t-1}}{P_t} \quad \Rightarrow \quad \frac{\mathcal{Q}_t}{\mathcal{Q}_{t-1}} = \Delta\mathcal{E}_t \frac{\Pi^*_t}{\Pi_t}$$

where

$$\Pi_t \equiv P_t / P_{t-1} \tag{29}$$
$$\Pi^*_t \equiv P^*_t / P^*_{t-1} \tag{30}$$

are (gross) Home and Foreign CPI inflation respectively, and $\Delta\mathcal{E}_t \equiv \mathcal{E}_t / \mathcal{E}_{t-1}$ is the gross rate of nominal de/appreciation of the domestic currency. Furthermore, (gross) Home and Foreign PPI inflation are respectively:

$$\Pi^b_{H,t} \equiv P^b_{H,t} / P^b_{H,t-1} \tag{31}$$
$$\Pi^g_{H,t} \equiv P^g_{H,t} / P^g_{H,t-1} \tag{32}$$
$$\Pi^{*b}_{F,t} \equiv P^{*b}_{F,t} / P^{*b}_{F,t-1} \tag{33}$$
$$\Pi^{*g}_{F,t} \equiv P^{*g}_{F,t} / P^{*g}_{F,t-1} \tag{34}$$

Finally, terms of trade are defined as the ratio between the price of imports and the price of exports (and using LOOP):

$$\mathcal{T}_t^{bb} \equiv \frac{P_{F,t}^b}{P_{H,t}^b} = \frac{P_{F,t}^{*b}}{P_{H,t}^{*b}} \tag{35}$$

$$\mathcal{T}_t^{gg} \equiv \frac{P_{F,t}^g}{P_{H,t}^g} = \frac{P_{F,t}^{*g}}{P_{H,t}^{*g}} \tag{36}$$

$$\mathcal{T}_t^{bg} \equiv \frac{P_{F,t}^b}{P_{H,t}^g} = \frac{P_{F,t}^{*b}}{P_{H,t}^{*g}} \tag{37}$$

$$\mathcal{T}_t^{gb} \equiv \frac{P_{F,t}^g}{P_{H,t}^b} = \frac{P_{F,t}^{*g}}{P_{H,t}^{*b}} \tag{38}$$

A.2.2 Households - Intertemporal decision

Households in each economy solve a utility maximisation problem to allocate consumption over time. They have the same preferences over consumption and labour supply:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[u(C_t) - g(L_t) \right]$$

where

$$u(C_t) = \frac{(C_{t+j} - bC_{t+j-1})^{1-\zeta} - 1}{1-\zeta}$$
$$g(L_t) = \chi \frac{L_{t+j}^{1+\eta}}{1+\eta}$$

with $\beta \in (0,1)$ being the households' subjective discount factor, $b < 1$ captures the degree of internal habit formation, $\zeta > 0$ is the inverse of elasticity of intertemporal substitution, $\eta \geq 0$ the inverse of the Frisch elasticity of labour supply, and χ the scaling parameter on the disutility of labour.

Home households

Home households solves the maximisation problem:

$$\max_{\{C_t, L_t^g, L_t^b, D_t\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[\frac{(C_{t+j} - bC_{t+j-1})^{1-\zeta} - 1}{1-\zeta} - \chi^g \frac{(L_{t+j}^g)^{1+\eta}}{1+\eta} - \chi^b \frac{(L_{t+j}^b)^{1+\eta}}{1+\eta} \right]$$

subject to the budget constraint in nominal terms

$$P_t C_t + D_t \leq MRS_t^g L_t^g + MRS_t^b L_t^b - R_{t-1}^d D_{t-1} + DIV_t - P_t X - P_t T_t \tag{39}$$

where C_t is consumption, L_t^g (L_t^b) labour supply paid at nominal wage MRS_t^g (MRS_t^b) in domestic currency by the labour union working with the eligible (ineligible) wholesaler, D_t holdings of one-period deposits denominated in domestic currency and yielding nominal interest rate from t to $t+1$, DIV_t nominal dividends from the domestic intermediary and firms, X is

the real (start-up) transfer to the financial intermediary, and finally T_t is a real lump-sum tax paid to the domestic government.[16]

I rewrite the budget constraint deflating it by P_t

$$C_t + d_t \leq mrs_t L_t - \frac{R^d_{t-1}}{\Pi_t} d_{t-1} + div_t - X - T_t$$

where I have defined $d_t \equiv D_t/P_t$, $div_t \equiv DIV_t/P_t$, $mrs_t \equiv MRS_t/P_t$, I used the definition of inflation π_t to get rid of the (non-stationary) price levels.

Then the Lagrangian for Home households' maximisation problem is

$$\mathcal{L}_t = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \Bigg\{ \left(\frac{(C_{t+j} - bC_{t+j-1})^{1-\zeta} - 1}{1-\zeta} - \chi^g \frac{(L^g_{t+j})^{1+\eta}}{1+\eta} - \chi^b \frac{(L^b_{t+j})^{1+\eta}}{1+\eta} \right) - \mu_t \left[C_t + d_t - mrs^g_t L^g_t - mrs^b_t L^b_t + \frac{R^d_{t-1}}{\Pi_t} d_{t-1} - div_t + X_t + T_t \right] \Bigg\}$$

where μ_t is the lagrangian multiplier on the households' budget constraint and the FOCs $\forall t = 0, ..., \infty$ for consumption, labour supply and deposits are:

$$\frac{\partial \mathcal{L}_t}{\partial C_t} : \quad \mu_t = (C_t - bC_{t-1})^{-\zeta} - b\beta \mathbb{E}_t (C_{t+1} - bC_t)^{-\zeta} \tag{40}$$

$$\frac{\partial \mathcal{L}_t}{\partial L^g_t} : \quad mrs^g_t \mu_t = \chi^g (L^g_t)^{\eta} \tag{41}$$

$$\frac{\partial \mathcal{L}_t}{\partial L^b_t} : \quad mrs^b_t \mu_t = \chi^b (L^b_t)^{\eta} \tag{42}$$

$$\frac{\partial \mathcal{L}_t}{\partial d_t} : \quad 1 = R^d_t \mathbb{E}_t \left[\Lambda_{t,t+1} \Pi^{-1}_{t+1} \right] \tag{43}$$

where

$$\Lambda_{t,t+1} = \beta \frac{\mu_{t+1}}{\mu_t} \tag{44}$$

is households' real stochastic discount factor.

Finally, as it will be clear from the derivations below, the expression for households' dividends received from retailers, wholesalers, capital producers, labour unions and financial intermediaries is:

$$div_t \equiv \frac{DIV_t}{P_t} = \frac{P^b_{H,t}}{P_t} Y^b_t + \frac{P^g_{H,t}}{P_t} Y^g_t - mrs^g_t L^g_t - mrs^b_t L^b_t - I_t + (1-\sigma)\lfloor_t + Q^g_t (b^g_t - \kappa b^g_{t-1} \Pi^{-1}_t) - b^g_{t-1} \Pi^{-1}_t + Q^b_t (b^b_t - \kappa b^b_{t-1} \Pi^{-1}_t) - b^b_{t-1} \Pi^{-1}_t \tag{45}$$

Foreign households

[16] Notice how every element of the budget constraint needs to be in domestic price/currency. For example, consumption C_t is in units of CPI bundle, hence I multiply it by P_t.

Foreign households solve the same maximisation problem as Home households:

$$\max_{\{C_t^*, L_t^{*g}, L_t^{*b}, D_t^*\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[\frac{(C_{t+j}^* - bC_{t+j-1}^*)^{1-\zeta} - 1}{1-\zeta} - \chi^g \frac{(L_{t+j}^{*g})^{1+\eta}}{1+\eta} - \chi^b \frac{(L_{t+j}^{*b})^{1+\eta}}{1+\eta} \right]$$

subject to the budget constraint in nominal terms

$$P_t^* C_t^* + D_t^* \leq MRS_t^{*g} L_t^{*g} + MRS_t^{*b} L_t^{*b} - R_{t-1}^{*d} D_{t-1}^* + DIV_t^* - P_t^* X^* - P_t^* T_t^* \tag{46}$$

Furthermore, similarly to what done for Home household, I rewrite the budget constraint deflating it by P_t^*:

$$C_t^* + d_t^* \leq mrs_t^{*g} L_t^{*g} + mrs_t^{*b} L_t^{*b} - \frac{R_{t-1}^{*d}}{\Pi_t^*} d_{t-1}^* + div_t^* - X^* - T_t^*$$

Then the Lagrangian for Foreign households' maximisation problem is

$$\begin{aligned} \mathcal{L}_t^* = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \Bigg\{ & \left(\frac{(C_{t+j}^* - bC_{t+j-1}^*)^{1-\zeta} - 1}{1-\zeta} - \chi^g \frac{(L_{t+j}^{*g})^{1+\eta}}{1+\eta} - \chi^b \frac{(L_{t+j}^{*b})^{1+\eta}}{1+\eta} \right) \\ & - \mu_t^* \left[C_t^* + d_t^* - mrs_t^{*g} L_t^{*g} - mrs_t^{*b} L_t^{*b} + \frac{R_{t-1}^{*d}}{\Pi_t^*} d_{t-1}^* - div_t^* + X_t^* + T_t^* \right] \Bigg\} \end{aligned}$$

and the FOCs $\forall t = 0, ..., \infty$ are:

$$\frac{\partial \mathcal{L}_t^*}{\partial C_t^*} : \quad \mu_t^* = (C_t^* - bC_{t-1}^*)^{-\zeta} - b\beta \mathbb{E}_t (C_{t+1}^* - bC_t^*)^{-\zeta} \tag{47}$$

$$\frac{\partial \mathcal{L}_t^*}{\partial L_t^{*g}} : \quad mrs_t^{*g} \mu_t^* = \chi^g (L_t^{*g})^{\eta} \tag{48}$$

$$\frac{\partial \mathcal{L}_t^*}{\partial L_t^{*b}} : \quad mrs_t^{*b} \mu_t^* = \chi^b (L_t^{*b})^{\eta} \tag{49}$$

$$\frac{\partial \mathcal{L}_t^*}{\partial d_t^*} : \quad 1 = R_t^{*d} \mathbb{E}_t \left[\Lambda_{t,t+1}^* (\Pi_{t+1}^*)^{-1} \right] \tag{50}$$

where

$$\Lambda_{t,t+1}^* = \beta \frac{\mu_{t+1}^*}{\mu_t^*} \tag{51}$$

Also, dividends are:

$$div_t^* \equiv \frac{DIV_t^*}{P_t^*} = \frac{P_{F,t}^{*b}}{P_t^*} Y_t^{*b} + \frac{P_{F,t}^{*g}}{P_t^*} Y_t^{*g} - mrs_t^{*g} L_t^{*g} - mrs_t^{*b} L_t^{*b} - I_t^* + (1-\sigma)\lfloor_t^* + Q_t^{*g}(b_t^{*g} - \kappa b_{t-1}^{*g}(\Pi_t^*)^{-1}) - b_{t-1}^{*g}(\Pi_t^*)^{-1} + Q_t^{*b}(b_t^{*b} - \kappa b_{t-1}^{*b}(\Pi_t^*)^{-1}) - b_{t-1}^{*b}(\Pi_t^*)^{-1} \tag{52}$$

A.2.3 Firms - Final-good producers

At the consumer-facing end of the supply chain for the Home economy lies a green and a brown representative firms producing final-good output Y_t^g and Y_t^b respectively through the CES technologies:

$$Y_t^g = \left(\int_0^{\gamma(1-\phi_g)} Y_t(h)^{\frac{e_p-1}{e_p}} dh \right)^{\frac{e_p}{e_p-1}} \tag{53}$$

$$Y_t^b = \left(\int_{\gamma(1-\phi_g)}^{\gamma} Y_t^b(h)^{\frac{e_p-1}{e_p}} dh \right)^{\frac{e_p}{e_p-1}} \tag{54}$$

where e_p is the labour elasticity of substitution, $Y_t^g(h)$ ($Y_t^b(h)$) are domestic eligible (ineligible) intermediate goods, whose price is $P_{H,t}^g(h)$ ($P_{H,t}^b(h)$). Differentiated output is provided by retail firms indexed by $h \in [0, \gamma)$. Of these, a fraction $1 - \phi_g$, where $0 \leq \phi_g \leq 1$, provides differentiated output using intermediate goods from eligible wholesalers, while a fraction ϕ_g sales goods produced with inputs from ineligible intermediate-good producers.

Focusing on the eligible final-good producer, her problem involves maximising the present discounted value of real profits using the household's stochastic discount factor:

$$\max_{Y_t^g, \left\{Y_t^g(h)\right\}_{h\in[0,\gamma(1-\phi_g))}} \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left[\frac{P_{H,t+j}^g}{P_{t+j}} Y_{t+j}^g - \int_0^{\gamma(1-\phi_g)} \frac{P_{H,t+j}^g(h)}{P_{t+j}} Y_{t+j}^g(h) dh \right]$$

subject to the technology (53).

Simplifying the problem to a choice of inputs $Y_t^g(h)$ only, by plugging this constraint into the objective function, the FOC for a generic $Y_t^g(h)$ is (notice I simplified out the common deflator P_t)

$$P_{H,t}^g \left(\int_0^{\gamma(1-\phi_g)} Y_t^g(h)^{\frac{e_p-1}{e_p}} dh \right)^{\frac{1}{e_p-1}} Y_t^g(h)^{-\frac{1}{e_p}} - P_{H,t}^g(h) = 0$$

$$\Rightarrow P_{H,t}^g (Y_t^g)^{\frac{1}{e_p}} Y_t^g(h)^{-\frac{1}{e_p}} = P_{H,t}^g(h)$$

$$\Rightarrow Y_t^g(h) = \left(\frac{P_{H,t}^g(h)}{P_{H,t}^g} \right)^{-e_p} Y_t^g \tag{55}$$

where the last equation is the downward-sloping demand function for the differentiated good h faced by eligible retailers.

The aggregate green price index P_t^g satisfies:

$$(P_{H,t}^g)^{1-e_p} = \frac{1}{\gamma(1-\phi_g)} \int_0^{\gamma(1-\phi_g)} (P_{H,t}^g(h))^{1-e_p} dh \tag{56}$$

Similar equations apply for ineligible retailers

$$Y_t^b(h) = \left(\frac{P_{H,t}^b(h)}{P_{H,t}^b} \right)^{-e_p} Y_t^b \tag{57}$$

$$(P_{H,t}^b)^{1-e_p} = \frac{1}{\gamma\phi_g} \int_{\gamma(1-\phi_g)}^{\gamma} (P_{H,t}^b(h))^{1-e_p} dh \tag{58}$$

and in the Foreign economy:

$$Y_t^{*g}(f) = \left(\frac{P_{F,t}^{*g}(f)}{P_{F,t}^{*g}} \right)^{-e_p} Y_t^{*g} \tag{59}$$

$$(P_{F,t}^{*g})^{1-e_p} = \frac{1}{(1-\gamma)(1-\phi_g)} \int_{\gamma}^{1-(1-\gamma)\phi_g} (P_{F,t}^{*g}(h))^{1-e_p} dh \tag{60}$$

$$Y_t^{*b}(f) = \left(\frac{P_{F,t}^{*b}(f)}{P_{F,t}^{*b}} \right)^{-e_p} Y_t^{*b} \tag{61}$$

$$(P_{F,t}^b)^{1-e_p} = \frac{1}{(1-\gamma)\phi_g} \int_{1-(1-\gamma)\phi_g}^{1} (P_{F,t}^b(h))^{1-e_p} dh \tag{62}$$

A.2.4 Firms - Retail producers

Here I first focus on the eligible retailers, as the derivation for the ineligible ones is similar.

There is a continuum of Home green retailers indexed by $h \in [0, \gamma(1-\phi_g))$, each operating with a simple "repackaging" technology represented by the production function:

$$Y_t^g(h) = Y_{w,t}^g(h),$$

where $Y_{w,t}^g(h)$ is the intermediate good produced by eligible wholesalers.

With this technology, the nominal profits the retail firm tries to maximise are:

$$DIV_{R,t}^g(h) = (P_{H,t}^g(h) - P_{w,t}^g) Y_t^g(h)$$

While intermediate bundles (and goods) markets are perfectly competitive, the retail market is not: firms operate under monopolistic competition, that is they are able to set the price of their retail goods given the demand for those from the final good firms, i.e. $Y_t^g(h) = \left(\frac{P_{H,t}^g(h)}{P_{H,t}^g} \right)^{-e_p} Y_t^g$. This allows firms to select prices that could be different from the flexible prices / competitive equilibrium outcome. This decision is the result of the firm's maximisation problem below and is affected by nominal rigidities in the form of Calvo pricing, that is the firm can adjust its prices in each period with probability $1-\phi_p$, where $0 \leq \phi_p \leq 1$.

Thus, each retailer able to re-optimise wants to maximise the present discounted value of its (real) profits (using the discount factor of the household Λ):

$$\max_{P^g_{H,t}(h)} \mathbb{E}_t \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+1} P_{t+j}^{-1} \left[(P^g_{H,t}(h))^{1-e_p} (P^g_{H,t+j})^{e_p} Y^g_{t+j} - P^g_{w,t+j} (P^g_{H,t}(h))^{-e_p} (P^g_{H,t+j})^{e_p} Y^g_{t+j} \right]$$

where I have substituted in the demand function for $Y^g_t(h)$, so that the problem involves picking only the price.

The FOC is:

$$\frac{\partial \mathcal{L}_{R,t}}{\partial P^g_{H,t}(h)} : \quad (e_p - 1)(P^g_{H,t}(h))^{-e_p} \mathbb{E}_t \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+j} P_{t+j}^{-1} (P^g_{H,t+j})^{e_p} Y^g_{t+j} = e_p P^g_{H,t}(h)^{-e_p-1} \mathbb{E}_t \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+j} P_{t+j}^{-1} P^g_{H,w,t+j} (P^g_{H,t+j})^{e_p} Y^g_{t+j}$$

Define the auxiliary variables:

$$X^g_{1,t} \equiv \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+j} P_{t+j}^{-1} P^g_{w,t+j} (P^g_{H,t+j})^{e_p} Y^g_{t+j}$$
$$X^g_{2,t} \equiv \sum_{j=0}^{\infty} \phi_p^j \Lambda_{t,t+j} P_{t+j}^{-1} (P^g_{H,t+j})^{e_p} Y^g_{t+j}$$

Equivalently, in the more convenient recursive form (grouping $p^g_{w,t} Y^g_t$ and Y^g_t respectively):

$$X^g_{1,t} \equiv p^g_{w,t} (P^g_{H,t})^{e_p} Y^g_t + \phi_p \mathbb{E}_t \Lambda_{t,t+1} (P^g_{H,t+1})^{e_p} X^g_{1,t+1}$$
$$X^g_{2,t} \equiv P_t^{-1} (P^g_{H,t})^{e_p} Y^g_{H,t} + \phi_p \mathbb{E}_t \Lambda_{t,t+1} P_{t+1}^{-1} (P^g_{H,t+1})^{e_p} X^g_{2,t+1}$$

where $p^g_{w,t} \equiv \frac{P^g_{w,t}}{P_t}$.

I now divide the first variable by $(P^g_{H,t})^{e_p}$ and the second by $P_t^{-1}(P^g_{H,t})^{e_p}$ to define $x^g_{1,t}$ and $x^g_{2,t}$ (also $x^g_{1,t} = X^g_{1,t}/(P^g_{H,t})^{e_p} \Rightarrow x^g_{1,t+1} = X^g_{1,t+1}/(P^g_{H,t+1})^{e_p}$ and $x^g_{2,t} = X^g_{2,t}/(P_t^{-1}(P^g_{H,t})^{e_p}) \Rightarrow x^g_{2,t+1} = X^g_{2,t+1}/(P_{t+1}^{-1}(P^g_{H,t+1})^{e_p})$):

$$x^g_{1,t} \equiv p^g_{w,t} Y^g_t + \phi_p \mathbb{E}_t \Lambda_{t,t+1} (\Pi^g_{H,t+1})^{e_p} x^g_{1,t+1} \tag{63}$$
$$x^g_{2,t} \equiv Y^g_t + \phi_p \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{-1} (\Pi^g_{H,t+1})^{e_p} x^g_{2,t+1} \tag{64}$$

Consider a symmetric equilibrium where firms pick the same price, then $P^g_{H,t}(h) = P^{g,o}_{H,t}$ and the FOC simplifies to:

$$p^{g,o}_{H,t} = \frac{e_p}{e_p - 1} \frac{x^g_{1,t}}{x^g_{2,t}} \tag{65}$$

where $p^{g,o}_{H,t} \equiv \frac{P^{g,o}_{H,t}}{P_t}$.

At this point I can integrate the demand function for retailers:

$$\int_0^{\gamma(1-\phi_g)} Y^g_t(h)dh = \int_0^{\gamma(1-\phi_g)} \left(\frac{P^g_{H,t}(h)}{P^g_{H,t}}\right)^{-e_p} Y^g_t dh$$

Because each retailer produces the same amount, i.e. an equal share of the total output of the domestic country as shown by the demand function for each $Y^g_t(h)$ (and thus use equal amounts of $Y^g_{w,t}(h)$ as inputs), and given technology $Y^g_t(h) = Y^g_{w,t}$, the left-hand side of the equation above sums up to a fraction of the total output for intermediate output $Y^g_{w,t}$: $\int_0^{\gamma(1-\phi_g)} Y^g_t(h)dh = \int_0^{\gamma(1-\phi_g)} Y^g_{w,t}(h)dh = \gamma(1-\phi_g)Y^g_{w,t}$.

$$\gamma(1-\phi_g)Y^g_{w,t} = \int_0^{\gamma(1-\phi_g)} \left(\frac{P^g_{H,t}(h)}{P^g_{H,t}}\right)^{-e_p} Y^g_t dh \quad \Rightarrow \quad Y^g_{w,t} = v^{g,p}_t Y^g_t \tag{66}$$

where $v^{g,p}_t = \frac{1}{\gamma(1-\phi_g)} \int_0^{\gamma(1-\phi_g)} \left(\frac{P^g_{H,t}(h)}{P^g_{H,t}}\right)^{-e_p} dh$ is a measure of price dispersion.

One can use the Calvo assumption to split $v^{g,p}_t$ noting that $(1-\phi_p)$ of retailers can update setting the same price $P^{g,o}_{H,t}$, and the rest can only retain the price from the previous period:

$$v^{g,p}_t = \frac{1}{\gamma(1-\phi_g)} \int_0^{\gamma(1-\phi_g)(1-\phi_p)} \left(\frac{P^{g,o}_{H,t}}{P^g_{H,t}}\right)^{-e_p} dh + \frac{1}{\gamma(1-\phi_g)} \int_{\gamma(1-\phi_g)(1-\phi_p)}^{\gamma(1-\phi_g)} \left(\frac{P^g_{H,t-1}(h)}{P^g_{H,t}}\right)^{-e_p} dh \tag{67}$$

which becomes by multiplying and dividing by $P_t^{-e_p}$ the first term and by $(P^g_{H,t-1})^{-e_p}$ the second term on the right-hand side respectively, and rearranging terms out of the integrals:

$$v^{g,p}_t = (1-\phi_p)(p^{g,o}_{H,t})^{-e_p} P_t^{-e_p} (P^g_{H,t})^{e_p} + \frac{1}{\gamma(1-\phi_g)} (P^g_{H,t})^{e_p} (P^g_{H,t-1})^{-e_p} \int_{\gamma(1-\phi_g)(1-\phi_p)}^{\gamma(1-\phi_g)} \left(\frac{P^g_{H,t-1}(h)}{P^g_{H,t-1}}\right)^{-e_p} dh$$

For the definition of gross inflation and the price dispersion term, the expression simplifies to:

$$v^{g,p}_t = (1-\phi_p)(p^{g,o}_{H,t})^{-e_p} P_t^{-e_p} (P^g_{H,t})^{e_p} + \frac{\gamma(1-\phi_g)\phi_p}{\gamma(1-\phi_g)} (\Pi^g_{H,t})^{e_p} v^{g,p}_{t-1}$$

$$\Rightarrow \quad v^{g,p}_t = (1-\phi_p)(p^{g,o}_{H,t} P_t / P^g_{H,t})^{-e_p} + \phi_p (\Pi^g_{H,t})^{e_p} v^{g,p}_{t-1} \tag{68}$$

For the same Calvo price properties, the price index (56) can be rewritten as:

$$(P^g_{H,t})^{1-e_p} = (1-\phi_p)(P^{g,o}_{H,t})^{1-e_p} + \frac{1}{\gamma(1-\phi_g)} \int_{\gamma(1-\phi_g)(1-\phi_p)}^{\gamma(1-\phi_g)} (P^g_{H,t-1}(h))^{1-e_p} dh$$

$$\Rightarrow \quad (P^g_{H,t})^{1-e_p} = (1-\phi_p)(P^{g,o}_{H,t})^{1-e_p} + \phi_p (P^g_{H,t-1})^{1-e_p}$$

$$\Rightarrow \quad 1 = (1-\phi_p)(p^{g,o}_{H,t} P_t / P^g_{H,t})^{1-e_p} + \phi_p (\Pi^g_{H,t})^{e_p - 1} \tag{69}$$

As for final-good firms, the same equations apply for ineligible retailers

$$p_{H,t}^{b,o} = \frac{e_p}{e_p - 1} \frac{x_{1,t}^b}{x_{2,t}^b} \tag{70}$$

$$x_{1,t}^b \equiv p_{w,t}^b Y_t^{*b} + \phi_p \mathbb{E}_t \Lambda_{t,t+1} (\Pi_{H,t+1}^b)^{e_p} x_{1,t+1}^b \tag{71}$$

$$x_{2,t}^b \equiv Y_t^b + \phi_p \mathbb{E}_t \Lambda_{t,t+1} (\Pi_{t+1})^{-1} (\Pi_{H,t+1}^b)^{e_p} x_{2,t+1}^b \tag{72}$$

$$Y_t^b = v_t^{b,p} Y_t^b \tag{73}$$

$$v_t^{b,p} = (1 - \phi_p)(p_{H,t}^{b,o} P_t / P_{H,t}^b)^{-e_p} + \phi_p (\Pi_{H,t})^{e_p} v_{t-1}^{b,p} \tag{74}$$

$$1 = (1 - \phi_p)(p_{H,t}^{b,o} P_t / P_{H,t}^b)^{1-e_p} + \phi_p (\Pi_{H,t})^{e_p - 1} \tag{75}$$

and the Foreign economy. In particular:

$$p_{F,t}^{*g,o} = \frac{e_p}{e_p - 1} \frac{x_{1,t}^{*g}}{x_{2,t}^{*g}} \tag{76}$$

$$x_{1,t}^{*g} \equiv p_{w,t}^{*g} Y_t^{*g} + \phi_p \mathbb{E}_t \Lambda_{t,t+1}^* (\Pi_{F,t+1}^{*g})^{e_p} x_{1,t+1}^{*g} \tag{77}$$

$$x_{2,t}^{*g} \equiv Y_t^{*g} + \phi_p \mathbb{E}_t \Lambda_{t,t+1}^* (\Pi_{t+1}^*)^{-1} (\Pi_{F,t+1}^{*g})^{e_p} x_{2,t+1}^{*g} \tag{78}$$

$$Y_t^{*g} = v_t^{*g,p} Y_t^{*g} \tag{79}$$

$$v_t^{*g,p} = (1 - \phi_p)(p_t^{*g,o} P_t^* / P_{F,t}^{*g})^{-e_p} + \phi_p (\Pi_{F,t}^*)^{e_p} v_{t-1}^{g,p} \tag{80}$$

$$1 = (1 - \phi_p)(p_{F,t}^{*g,o} P_t^* / P_{F,t}^{*g})^{1-e_p} + \phi_p (\Pi_{F,t-1}^*)^{e_p - 1} \tag{81}$$

$$p_{F,t}^{*b,o} = \frac{e_p}{e_p - 1} \frac{x_{1,t}^{*b}}{x_{2,t}^{*b}} \tag{82}$$

$$x_{1,t}^{*b} \equiv p_{w,t}^{*b} Y_t^{*b} + \phi_p \mathbb{E}_t \Lambda_{t,t+1}^* (\Pi_{F,t+1}^{*b})^{e_p} x_{1,t+1}^{*b} \tag{83}$$

$$x_{2,t}^{*b} \equiv Y_t^{*b} + \phi_p \mathbb{E}_t \Lambda_{t,t+1}^* (\Pi_{t+1}^*)^{-1} (\Pi_{F,t+1}^{*b})^{e_p} x_{2,t+1}^{*b} \tag{84}$$

$$Y_t^{*b} = v_t^{*b,p} Y_t^{*b} \tag{85}$$

$$v_t^{*b,p} = (1 - \phi_p)(p_t^{*b,o} P_t^* / P_{F,t}^{*b})^{-e_p} + \phi_p (\Pi_{F,t}^*)^{e_p} v_{t-1}^{b,p} \tag{86}$$

$$1 = (1 - \phi_p)(p_t^{*b,o} P_t^* / P_{F,t}^{*b})^{1-e_p} + \phi_p (\Pi_{F,t-1}^*)^{e_p - 1} \tag{87}$$

A.2.5 Firms - Eligible and ineligible capital producer

In each country a representative eligible (ineligible) capital producer takes as input unconsumed final eligible (ineligible) output I_t^g (I_t^b), where

$$I_t = I_t^g + I_t^b \tag{88}$$

and similarly in Foreign

$$I_t^* = I_t^{*g} + I_t^{*b}, \tag{89}$$

and uses the following technology to create new physical eligible (ineligible) capital:

$$\hat{I}_t^g = \left[1 - S\left(\frac{I_t^g}{I_{t-1}^g}\right)\right] I_t^g \tag{90}$$

$$\hat{I}_t^b = \left[1 - S\left(\frac{I_t^b}{I_{t-1}^b}\right)\right] I_t^b \tag{91}$$

with convex function of adjustment cost $S(I_t/I_{t-1}) = \frac{\kappa}{2}(I_t/I_{t-1} - 1)^2$.
Similarly for Foreign:

$$\hat{I}_t^{*g} = \left[1 - S\left(\frac{I_t^{*g}}{I_{t-1}^{*g}}\right)\right] I_t^{*g} \tag{92}$$

$$\hat{I}_t^{*b} = \left[1 - S\left(\frac{I_t^{*b}}{I_{t-1}^{*b}}\right)\right] I_t^{*b} \tag{93}$$

Each capital producer maximises the present discounted value of its dividends

$$DIV_{k,t} = P_t^k \hat{I}_t - P_t I_t = P_t^k \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - P_t I_t$$

or in real terms

$$div_{k,t} = p_t^k \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t - I_t$$

using the household's stochastic discount factor:

$$\max_{I_t} \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left\{ p_{t+j}^k \left[1 - S\left(\frac{I_{t+j}}{I_{t+j-1}}\right)\right] I_{t+j} - I_{t+j} \right\}$$

Then, the FOCs for eligible and ineligible Home producers are:

$$1 = p_t^{g,k} \left[1 - S\left(\frac{I_t^g}{I_{t-1}^g}\right) - S'\left(\frac{I_t^g}{I_{t-1}^g}\right)\frac{I_t^g}{I_{t-1}^g}\right] + \mathbb{E}_t \Lambda_{t,t+1} p_{t+1}^{g,k} S'\left(\frac{I_{t+1}^g}{I_t^g}\right)\left(\frac{I_{t+1}^g}{I_t^g}\right)^2 \tag{94}$$

$$1 = p_t^{b,k} \left[1 - S\left(\frac{I_t^b}{I_{t-1}^b}\right) - S'\left(\frac{I_t^b}{I_{t-1}^b}\right)\frac{I_t^b}{I_{t-1}^b}\right] + \mathbb{E}_t \Lambda_{t,t+1} p_{t+1}^{b,k} S'\left(\frac{I_{t+1}^b}{I_t^b}\right)\left(\frac{I_{t+1}^b}{I_t^b}\right)^2 \tag{95}$$

Similarly for Foreign:

$$1 = p_t^{*g,k} \left[1 - S\left(\frac{I_t^{*g}}{I_{t-1}^{*g}}\right) - S'\left(\frac{I_t^{*g}}{I_{t-1}^{*g}}\right)\frac{I_t^{*g}}{I_{t-1}^{*g}}\right] + \mathbb{E}_t \Lambda_{t,t+1}^* p_{t+1}^{*g,k} S'\left(\frac{I_{t+1}^{*g}}{I_t^{*g}}\right)\left(\frac{I_{t+1}^{*g}}{I_t^{*g}}\right)^2 \tag{96}$$

$$1 = p_t^{*b,k} \left[1 - S\left(\frac{I_t^{*b}}{I_{t-1}^{*b}}\right) - S'\left(\frac{I_t^{*b}}{I_{t-1}^{*b}}\right)\frac{I_t^{*b}}{I_{t-1}^{*b}}\right] + \mathbb{E}_t \Lambda_{t,t+1}^* p_{t+1}^{*b,k} S'\left(\frac{I_{t+1}^{*b}}{I_t^{*b}}\right)\left(\frac{I_{t+1}^{*b}}{I_t^{*b}}\right)^2 \tag{97}$$

A.2.6 Firms - Wholesalers

In each country there are two types of wholesale firms: eligible g and ineligible b. Each wholesaler produces intermediate output $Y_{w,t}$ using a Cobb-Douglas technology with productivity A_t and combining capital K_t purchased from the investment-good producer and labour $L_{d,t}$ from unions:

$$Y_{w,t}^{g} = A_t^{g}(u_t^{g}K_t^{g})^{\alpha}(L_{d,t}^{g})^{1-\alpha} \tag{98}$$

$$Y_{w,t}^{b} = A_t^{b}(u_t^{b}K_t^{b})^{\alpha}(L_{d,t}^{b})^{1-\alpha} \tag{99}$$

where $0 < \alpha < 1$ is the capital share in production. Similarly in Foreign:

$$Y_{w,t}^{*g} = A_t^{*g}(u_t^{*g}K_t^{*g})^{\alpha}(L_{d,t}^{*g})^{1-\alpha} \tag{100}$$

$$Y_{w,t}^{*b} = A_t^{*b}(u_t^{*b}K_t^{*b})^{\alpha}(L_{d,t}^{*b})^{1-\alpha} \tag{101}$$

Notice u_t reflects the degree of capital utilization, which shows up both in production and in capital accumulation:

$$K_{t+1}^{g} = \hat{I}_t^{g} + (1-\delta(u_t^{g}))K_t^{g}, \tag{102}$$

$$K_{t+1}^{b} = \hat{I}_t^{b} + (1-\delta(u_t^{b}))K_t^{b}, \tag{103}$$

with $\hat{I}_t$ as new physical capital purchased from the capital producer and $\delta(u_t)$ as a functional form linking capital utilisation to its depreciation. In Foreign similarly:

$$K_{t+1}^{*g} = \hat{I}_t^{*g} + (1-\delta(u_t^{*g}))K_t^{*g}, \tag{104}$$

$$K_{t+1}^{*b} = \hat{I}_t^{*b} + (1-\delta(u_t^{*b}))K_t^{*b}, \tag{105}$$

Each wholesale firm exhibits the following "loan in advance constraint" (Carlstrom et al., 2017; Gertler & Karadi, 2011; Sims & Wu, 2021b):

$$\psi P_t^{g,k}\hat{I}_t^{g} \leq Q_t^{g}(B_t^{g} - \kappa B_{t-1}^{g}) \tag{106}$$

$$\psi P_t^{b,k}\hat{I}_t^{b} \leq Q_t^{b}(B_t^{b} - \kappa B_{t-1}^{b}) \tag{107}$$

Dividing by P_t and rearranging (by multiplying and dividing by P_{t-1}) so to feature inflation terms:

$$\psi p_t^{g,k}\hat{I}_t^{g} \leq Q_t^{g}\left(\frac{B_t^{g}}{P_t} - \kappa\frac{B_{t-1}^{g}}{P_{t-1}}\Pi_t^{-1}\right) \tag{108}$$

$$\psi p_t^{b,k} \hat{I}_t^b \leq Q_t^b \left(\frac{B_t^b}{P_t} - \kappa \frac{B_{t-1}^b}{P_{t-1}} \Pi_t^{-1} \right) \tag{109}$$

where $p_t^{g,k} \equiv P_t^{g,k}/P_t$ and $p_t^{b,k} \equiv P_t^{b,k}/P_t$. In Foreign:

$$\psi p_t^{*g,k} \hat{I}_t^{*g} \leq Q_t^{*g} \left(\frac{B_t^{*g}}{P_t^*} - \kappa \frac{B_{t-1}^{*g}}{P_{t-1}^*} (\Pi_t^*)^{-1} \right) \tag{110}$$

$$\psi p_t^{*b,k} \hat{I}_t^{*b} \leq Q_t^{*b} \left(\frac{B_t^{*b}}{P_t^*} - \kappa \frac{B_{t-1}^{*b}}{P_{t-1}^*} (\Pi_t^*)^{-1} \right) \tag{111}$$

Equation (108) and (109) requires that, in order to finance a constant fraction, $\psi \in (0, 1]$, of its investment $\hat{I}_t$ at price P_t^k, each wholesaler must issue bonds as perpetuities $B_t - \kappa B_{t-1}$ at market price Q_t with decaying coupon payments, whose decaying parameter is $\kappa \in (0, 1)$ (Woodford, 2001). This means that one unit of bonds today yields a coupon payment of one unit of the domestic currency next period, but this payment decays at rate κ. Then B_{t-1} is the total coupon liability due in period t, i.e. $B_{t-1} = CB_{t-1} + \kappa CB_{t-2} + \kappa^2 CB_{t-3} + ...$ with CB as the new nominal issuance in each period. The latter, through forward iteration, implies that one needs only keep track of the period t and $t-1$ total coupon liabilities, $CB_t = B_t - \kappa B_{t-1}$.

Similarly, because of the assumed perpetual bond structure, the price in t of bonds issued in $t-j$ is $\kappa^j Q_t$ for $j \geq 0$, and thus knowing the price for the current issue allows to know the entire sequence of prices for past issues too. This keeps the model tractable, as for example the value of the (outstanding) bond holdings in period t can be expressed as $Q_t B_t = Q_t CB_t + \kappa Q_t CB_{t-1} + \kappa^2 Q_t CB_{t-2} + ...$ If in period t one sells $Q_{t-1} B_{t-1}$ worth of bonds from $t-1$, the realised return would be the sum of the total coupon liability due in t and the value of the bond portfolio from $t-1$ in t (thus at price κQ_t) divided by the value of that same portfolio in $t-1$: $R_t = \frac{B_{t-1} + \kappa Q_t B_{t-1}}{Q_{t-1} B_{t-1}} \Rightarrow R_t = \frac{1 + \kappa Q_t}{Q_{t-1}}$. Thus we have:

$$R_t^g = \frac{1 + \kappa Q_t^g}{Q_{t-1}^g} \tag{112}$$

$$R_t^b = \frac{1 + \kappa Q_t^b}{Q_{t-1}^b} \tag{113}$$

and in Foreign:

$$R_t^{*g} = \frac{1 + \kappa Q_t^{*g}}{Q_{t-1}^{*g}} \tag{114}$$

$$R_t^{*b} = \frac{1 + \kappa Q_t^{*b}}{Q_{t-1}^{*b}} \tag{115}$$

Crucially, constraints (108) and (109) create scope for external financing, distort the standard asset pricing decisions, and in turn represent the main mechanism through which corporate bonds purchases transmit to the real economy.

Each wholesaler provides the bundling firm with intermediate goods within a competitive market, and maximise their present discounted value (using households' stochastic discount

factor) of dividends

$$DIV^g_{w,t} = P^g_{w,t} A^g_t (u^g_t K^g_t)^\alpha L^g_{d,t})^{1-\alpha} - W^g_t L^g_{d,t} - P^{g,k}_t \hat{I}^g_t - B^g_{t-1} + Q^g_t (B^g_t - \kappa B^g_{t-1})$$

$$DIV^b_{w,t} = P^b_{w,t} A^b_t (u^b_t K^b_t)^\alpha L^b_{d,t})^{1-\alpha} - W^b_t L^b_{d,t} - P^{b,k}_t \hat{I}^b_t - B^b_{t-1} + Q^b_t (B^b_t - \kappa B^b_{t-1})$$

In real terms (dividing by P_t and rearranging by multiplying and dividing by P_{t-1}) so to feature inflation terms):

$$div^g_{w,t} = p^g_{w,t} A^g_t (u^g_t K^g_t)^\alpha (L^g_{d,t})^{1-\alpha} - w^g_t L^g_{d,t} - p^{g,k}_t \hat{I}^g_t - \frac{B^g_{t-1}}{P_{t-1}} \Pi^{-1}_t + Q^g_t \left(\frac{B^g_t}{P_t} - \kappa \frac{B^g_{t-1}}{P_{t-1}} \Pi^{-1}_t \right)$$

$$div^b_{w,t} = p^b_{w,t} A^g_t (u^b_t K^b_t)^\alpha (L^b_{d,t})^{1-\alpha} - w^b_t L^b_{d,t} - p^{b,k}_t \hat{I}^b_t - \frac{B^b_{t-1}}{P_{t-1}} \Pi^{-1}_t + Q^b_t \left(\frac{B^b_t}{P_t} - \kappa \frac{B^b_{t-1}}{P_{t-1}} \Pi^{-1}_t \right)$$

with real wages $w^g_t \equiv W^g_t / P_t$ and $w^b_t \equiv W^b_t / P_t$, and output prices $p^g_{w,t} \equiv P^g_{w,t} / P_t$ and $p^b_{w,t} \equiv P^b_{w,t} / P_t$.

Notice how I have already substituted in the production technology (98) and (99), and that wholesalers take into account their costs for labour, past financing and capital accumulation, net of the part that is paid with external financing (leading to future liabilities to be paid and reducing dividends in the following periods).

The maximisation for the green wholesaler is carried out considering the process for capital accumulation (102) and loan-in-advance constraint (108), and thus the lagrangian for the problem is:

$$\begin{aligned} \mathcal{L}^g_{w,t} = \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \Bigg\{ & p^g_{w,t+j} A^g_{t+j} (u^g_{t+j} K^g_{t+j})^\alpha (L^g_{d,t+j})^{1-\alpha} - w^g_{t+j} L^g_{d,t+j} - p^{g,k}_{t+j} \hat{I}^g_{t+j} - \frac{B^g_{t+j-1}}{P_{t+j-1}} \Pi^{-1}_{t+j} \\ & + Q^g_t \left(\frac{B^g_{t+j}}{P_{t+j}} - \kappa \frac{B^g_{t+j-1}}{P_{t+j-1}} \Pi^{-1}_{t+j} \right) + \nu^g_{1,t+j} \left[\hat{I}^g_{t+j} + (1 - \delta(u^g_{t+j})) K^g_{t+j} - K^g_{t+j+1} \right] \\ & + \nu^g_{2,t+j} \left[Q^g_{t+j} \left(\frac{B^g_{t+j}}{P_{t+j}} - \kappa \frac{B^g_{t+j-1}}{P_{t+j-1}} \Pi^{-1}_{t+j} \right) - \psi p^{g,k}_{t+j} \hat{I}^g_{t+j} \right] \Bigg\} \end{aligned}$$

where $\nu^g_{1,t}$ and $\nu^g_{2,t}$ are the lagrangian multipliers on constraints (102) and (108), respectively.

The FOCs $\forall t = 0, ..., \infty$ for consumption, labour supply and deposits are:

$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial L^g_{d,t}}: \quad p^g_{w,t}(1-\alpha)A^g_t(u^g_t K^g_t)^\alpha (L^g_{d,t})^{-\alpha} - w^g_t = 0$$

$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial \hat{I}^g_t}: \quad -p^{g,k}_t + \nu^g_{1,t} - \psi p^{g,k}_t \nu^g_{2,t} = 0$$

$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial u^g_t}: \quad \alpha p^g_{w,t} A^g_t (u^g_t K^g_t)^{\alpha-1} K^g_t (L^g_{d,t})^{1-\alpha} - \nu^g_{1,t}\delta'(u^g_t)K^g_t = 0$$

$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial K^g_{t+1}}: \quad \mathbb{E}_t \Lambda_{t,t+1}\left[\alpha p^g_{w,t+1} A^g_{t+1}(u^g_{t+1}K^g_{t+1})^{\alpha-1} u_{t+1}(L^g_{d,t+1})^{1-\alpha} + \nu^g_{1,t+1}(1-\delta(u^g_{t+1}))\right] - \nu^g_{1,t} = 0$$

$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial B_t}: \quad (1+\nu^g_{2,t})\frac{Q^g_t}{P_t} - \mathbb{E}_t \Lambda_{t,t+1}\left[\frac{1}{P_t}\Pi^{-1}_{t+1} + \kappa \frac{Q^g_{t+1}}{P_t}\Pi^{-1}_{t+1} + \nu^g_{2,t+1}\kappa\frac{Q^g_{t+1}}{P_t}\Pi^{-1}_{t+1}\right] = 0$$

Defining $M^g_{1,t} = 1 + \psi\nu^g_{2,t}$ and $M^g_{2,t} = 1 + \nu^g_{2,t}$ so that:

$$\frac{M^g_{1,t} - 1}{M^g_{2,t} - 1} = \psi, \tag{116}$$

the FOCs simplify to:

$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial L^g_{d,t}}: \quad w^g_t = p^g_{w,t}(1-\alpha)A^g_t(u^g_t K^g_t)^\alpha (L^g_{d,t})^{-\alpha} \tag{117}$$

$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial \hat{I}^g_t}: \quad \nu^g_{1,t} = p^{g,k}_t M^g_{1,t} \tag{118}$$

$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial u^g_t}: \quad p^k_t M^g_{1,t}\delta'(u^g_t) = \alpha p^g_{w,t} A^g_t (u^g_t K^g_t)^{\alpha-1}(L^g_{d,t})^{1-\alpha} \tag{119}$$

$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial K^g_{t+1}}: \quad p^{g,k}_t M^g_{1,t} = \mathbb{E}_t \Lambda_{t,t+1}\left[\alpha p^g_{w,t+1} A^g_{t+1}(u^g_{t+1})^\alpha (K^g_{t+1})^{\alpha-1}(L^g_{d,t+1})^{1-\alpha} + (1-\delta(u^g_{t+1}))p^{g,k}_{t+1}M^g_{1,t+1}\right] \tag{120}$$

$$\frac{\partial \mathcal{L}^g_{w,t}}{\partial B_t}: \quad Q^g_t M^g_{2,t} = \mathbb{E}_t \Lambda_{t,t+1}\Pi^{-1}_{t+1}\left[1 + \kappa Q^g_{t+1}M^g_{2,t+1}\right] \tag{121}$$

For the brown wholesaler we have:

$$w^b_t = p^b_{w,t}(1-\alpha)A^b_t(u^b_t K^b_t)^\alpha (L^b_{d,t})^{-\alpha} \tag{122}$$

$$\nu^b_{1,t} = p^{b,k}_t M^b_{1,t} \tag{123}$$

$$p^k_t M^b_{1,t}\delta'(u^b_t) = \alpha p^b_{w,t} A^b_t (u^b_t K^b_t)^{\alpha-1}(L^b_{d,t})^{1-\alpha} \tag{124}$$

$$p^k_t M^b_{1,t} = \mathbb{E}_t \Lambda_{t,t+1}\left[\alpha p^b_{w,t+1} A^b_{t+1}(u^b_{t+1})^\alpha (K^b_{t+1})^{\alpha-1}(L^b_{d,t+1})^{1-\alpha} + (1-\delta(u^b_{t+1}))p^{b,k}_{t+1}M^b_{1,t+1}\right] \tag{125}$$

$$Q^b_t M^b_{2,t} = \mathbb{E}_t \Lambda_{t,t+1}\Pi^{-1}_{t+1}\left[1 + \kappa Q^b_{t+1}M^b_{2,t+1}\right] \tag{126}$$

$$\frac{M^b_{1,t} - 1}{M^b_{2,t} - 1} = \psi \tag{127}$$

Similarly in Foreign:

$$w_t^{*g} = p_{w,t}^{*g}(1-\alpha)A_t^{*g}(u_t^{*g}K_t^{*g})^{\alpha}(L_{d,t}^{*g})^{-\alpha} \tag{128}$$

$$\nu_{1,t}^{*g} = p_t^{*g,k}M_{1,t}^{*g} \tag{129}$$

$$p_t^{*k}M_{1,t}^{*g}\delta'(u_t^{*g}) = \alpha p_{w,t}^{*g}A_t^{*g}(u_t^{*g}K_t^{*g})^{\alpha-1}(L_{d,t}^{*g})^{1-\alpha} \tag{130}$$

$$p_t^{*k}M_{1,t}^{*g} = \mathbb{E}_t\Lambda_{t,t+1}^{*}\left[\alpha p_{w,t+1}^{*g}A_{t+1}^{*g}(u_{t+1}^{*g})^{\alpha}(K_{t+1}^{*g})^{\alpha-1}(L_{d,t+1}^{*g})^{1-\alpha} + (1-\delta(u_{t+1}^{*g}))p_{t+1}^{*g,k}M_{1,t+1}^{*g}\right] \tag{131}$$

$$Q_t^{*g}M_{2,t}^{*g} = \mathbb{E}_t\Lambda_{t,t+1}^{*}(\Pi_{t+1}^{*})^{-1}\left[1+\kappa Q_{t+1}^{*g}M_{2,t+1}^{*g}\right] \tag{132}$$

$$\frac{M_{1,t}^{*g}-1}{M_{2,t}^{*g}-1} = \psi \tag{133}$$

$$w_t^{*b} = p_{w,t}^{*b}(1-\alpha)A_t^{*b}(u_t^{*b}K_t^{*b})^{\alpha}(L_{d,t}^{*b})^{-\alpha} \tag{134}$$

$$\nu_{1,t}^{*b} = p_t^{*b,k}M_{1,t}^{*b} \tag{135}$$

$$p_t^{*k}M_{1,t}^{*b}\delta'(u_t^{*b}) = \alpha p_{w,t}^{*b}A_t^{*b}(u_t^{*b}K_t^{*b})^{\alpha-1}(L_{d,t}^{*b})^{1-\alpha} \tag{136}$$

$$p_t^{*k}M_{1,t}^{*b} = \mathbb{E}_t\Lambda_{t,t+1}^{*}\left[\alpha p_{w,t+1}^{*b}A_{t+1}^{*b}(u_{t+1}^{*b})^{\alpha}(K_{t+1}^{*b})^{\alpha-1}(L_{d,t+1}^{*b})^{1-\alpha} + (1-\delta(u_{t+1}^{*b}))p_{t+1}^{*b,k}M_{1,t+1}^{*b}\right] \tag{137}$$

$$Q_t^{*b}M_{2,t}^{*b} = \mathbb{E}_t\Lambda_{t,t+1}^{*}(\Pi_{t+1}^{*})^{-1}\left[1+\kappa Q_{t+1}^{*b}M_{2,t+1}^{*b}\right] \tag{138}$$

$$\frac{M_{1,t}^{*b}-1}{M_{2,t}^{*b}-1} = \psi \tag{139}$$

A.2.7 Financial intermediaries

As international financial markets are integrated, in each country a representative intermediary can hold both Home and Foreign private bonds issued by (the two types of) wholesale firms. I model financial intermediaries by taking the framework proposed by Wu et al. (2024b) to a two-firm set-up within each country.

Each period financial intermediaries pay a fraction $1-\sigma$ (with $\sigma \in [0,1]$) of their beginning balance (to be defined shortly) to and receive X real funds from their household owners. As long as intermediaries keep operating, they finance their activities through their net worth N_t and deposits D_t, exclusively from domestic households. These activities of each intermediary consist in investing into domestic central bank reserves RE_t and corporate bonds from the two types of wholesale firms from Home and Foreign, i.e. $B_{H,t}^{g,FI}$, $B_{H,t}^{b,FI}$, $B_{F,t}^{g,FI}$ and $B_{F,t}^{b,FI}$. The resulting balance sheet condition is:

$$Q_t^g B_{H,t}^{g,FI} + Q_t^b B_{H,t}^{b,FI} + Q_t^{*g} B_{F,t}^{g,FI}\mathcal{E}_t + Q_t^{*b} B_{F,t}^{b,FI}\mathcal{E}_t + RE_t = D_t + N_t \tag{140}$$

where Q_t^g, Q_t^b, Q_t^{*g} and Q_t^{*b} are the prices of the (holdings of the) bonds. Foreign bonds are denominated in the currency of the country of issuance, and thus their value in the balance sheet condition is converted into domestic currency using the nominal exchange rate $\mathcal{E}_t$.

In real terms:

$$Q_t^g b_{H,t}^{g,FI} + Q_t^b b_{H,t}^{b,FI} + Q_t^{*g} b_{F,t}^{g,FI} \mathcal{E}_t + Q_t^{*b} b_{F,t}^{b,FI} \mathcal{E}_t + re_t = d_t + n_t \tag{141}$$

The beginning balance in each period t is thus:

$$\mathcal{B}_t = (R_t^g - R_{t-1}^d) Q_{t-1}^g B_{H,t-1}^{g,FI} + (R_t^b - R_{t-1}^d) Q_{t-1}^b B_{H,t-1}^{b,FI} \tag{142}$$

$$+ \left(R_t^{*g} \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \left[1 - \omega/2 (b_{F,t-1}^{g,FI} / b_F^{g,FI} - 1)^2 \right] - R_{t-1}^d \right) Q_{t-1}^{*g} B_{F,t-1}^{g,FI} \mathcal{E}_{t-1} \tag{143}$$

$$+ \left(R_t^{*b} \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \left[1 - \omega/2 (b_{F,t-1}^{b,FI} / b_F^{b,FI} - 1)^2 \right] - R_{t-1}^d \right) Q_{t-1}^{*b} B_{F,t-1}^{b,FI} \mathcal{E}_{t-1} \tag{144}$$

$$+ (R_{t-1}^{re} - R_{t-1}^d) RE_{t-1} + R_{t-1}^d N_{t-1} \tag{145}$$

where returns are defined in (112), (113), (114) and (115), while R_{t-1}^{re} is the nominal interest rate on reserves of the domestic central bank. The balance sums up the excess returns over the cost of deposits of the domestic eligible and ineligible bonds, as well as the foreign ones. Importantly, I assume quadratic transaction costs for (real) Foreign bonds, whose generated revenues are paid to the government. I define $b_{F,t}^{g,FI} \equiv B_{F,t}^{g,FI} / P_t^*$ and $b_{F,t}^{b,FI} \equiv B_{F,t}^{b,FI} / P_t^*$ with $b_F^{g,FI}$ and $b_F^{b,FI}$ their steady-state values. Also notice the smaller ω, the higher the degree of financial openness of a country. This additional friction is a common way in the literature to achieve the existence of a determinate steady state and a stationary solution in open-economy models with domestic holdings of Foreign bonds whose returns are exogenously determined abroad (Schmitt-Grohé & Uribe, 2003). The last term in the balance sheet condition measures savings from using own net worth as opposed to deposits as source of financing.

Intermediaries have thus net worth

$$N_t = \sigma \mathcal{B}_t + P_t X \tag{146}$$

and want to maximise the present discounted value of their future dividends discounting by the stochastic discount factor of the household $\Lambda_{t,t+1}$:

$$V_t = \max (1 - \sigma) \mathbb{E}_t \sum_{j=1}^{\infty} \Lambda_{t,t+j} \lfloor_{t+j} \tag{147}$$

where $\lfloor_t \equiv \mathcal{B}_t / P_t$ and $(1 - \sigma) \lfloor_t$ are real initial balance and dividend payment, respectively. Notice the real-term expression for (146) is:

$$n_t = \sigma \lfloor_t + X \tag{148}$$

and thus

$$n_t = \sigma\Pi_t^{-1}\Bigg\{(R_t^g - R_{t-1}^d)Q_{t-1}^g b_{H,t-1}^{g,FI} + (R_t^b - R_{t-1}^d)Q_{t-1}^b b_{H,t-1}^{b,FI} \tag{149}$$

$$+\left(R_t^{*g}\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\left[1-\omega/2(b_{F,t-1}^{g,FI}/b_F^{g,FI}-1)^2\right]-R_{t-1}^d\right)Q_{t-1}^{*g}b_{F,t-1}^{g,FI}\mathcal{Q}_{t-1} \tag{150}$$

$$+\left(R_t^{*b}\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\left[1-\omega/2(b_{F,t-1}^{b,FI}/b_F^{b,FI}-1)^2\right]-R_{t-1}^d\right)Q_{t-1}^{*b}b_{F,t-1}^{b,FI}\mathcal{Q}_{t-1} \tag{151}$$

$$+(R_{t-1}^{re} - R_{t-1}^d)re_{t-1} + R_{t-1}^d n_{t-1}\Bigg\} + X \tag{152}$$

This problem can be rewritten in recursive form:

$$V_t = \max\left[(1-\sigma)\mathbb{E}_t\Lambda_{t,t+1}\lfloor_{t+1} + \mathbb{E}_t\Lambda_{t,t+1}V_{t+1}\right] \tag{153}$$

and substituting in the expression for $\lfloor_{t+1}$ (notice $\lfloor_{t+1} = \mathcal{B}_{t+1}/P_{t+1}$ and thus inflation terms appear in the expression below for the elements within $\mathcal{B}_{t+1}$ that are lagged - through multiplying and dividing by P_t):

$$V_t = \max\Bigg\{(1-\sigma)\mathbb{E}_t\Lambda_{t,t+1}\Pi_{t+1}^{-1}\Bigg\{(R_{t+1}^g - R_t^d)Q_t^g b_{H,t}^{g,FI} + (R_{t+1}^b - R_t^d)Q_t^b b_{H,t}^{b,FI}$$

$$+\left(R_{t+1}^{*g}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\left[1-\omega/2(b_{F,t}^{g,FI}/b_F^{g,FI}-1)^2\right]-R_t^d\right)Q_t^{*g}b_{F,t}^{g,FI}\mathcal{Q}_t$$

$$+\left(R_{t+1}^{*b}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\left[1-\omega/2(b_{F,t}^{b,FI}/b_F^{b,FI}-1)^2\right]-R_t^d\right)Q_t^{*b}b_{F,t}^{b,FI}\mathcal{Q}_t$$

$$+(R_t^{re} - R_t^d)re_t + R_t^d n_t\Bigg\} + \mathbb{E}_t\Lambda_{t,t+1}V_{t+1}\Bigg\} \tag{154}$$

where $b_{H,t}^{g,FI} = B_{H,t}^{g,FI}/P_t$, $b_{H,t}^{b,FI} = B_{H,t}^{b,FI}/P_t$, $n_t = N_t/P_t$, $d_t = D_t/P_t$ and $re_t = RE_t/P_t$.

Intermediaries in both countries are constrained by a non-absconding requirement (Gertler & Karadi, 2011), which prevents them from declaring bankruptcy and retain the entirety of their balance sheet. While reserves at the domestic central bank are fully recoverable by depositors, intermediaries can only divert away from households a stochastic and exogenous fraction θ_t of their real holdings of green and brown domestic (foreign) bonds $\Delta Q_t^g b_{H,t}^{g,FI} + Q_t^b b_{H,t}^{b,FI} + \left[\Delta Q_t^{*g} b_{F,t}^{g,FI} + Q_t^{*b} b_{F,t}^{b,FI}\right]\mathcal{Q}_t$, where $0 \leq \Delta \leq 1$ is the steady state difference in recoverability between those bonds. As a result of this, for households to be still willing to lend funds, intermediaries must guarantee (also in steady-state) excess returns of long term private bonds over the deposit rate, with the spread between eligible and ineligible bonds calibrated through Δ.

Crucially, this set-up allows CBPs to have real effects by swapping assets with reserves not subject to constraint, and thus relaxing the requirement for intermediaries. However, in line with the observed features of the SMCCF, we assume that only the bonds from sector 2 are eligible for CBPs. This means that the action of the central bank will be targeted to a group of intermediaries and, in turn, firms rather than the whole economy.

The non-absconding constraint for intermediaries is thus:

$$V_t \geq \theta_t \left\{ \Delta Q_t^g b_{H,t}^{g,FI} + Q_t^b b_{H,t}^{b,FI} + \left[\Delta Q_t^{*g} b_{F,t}^{g,FI} + Q_t^{*b} b_{F,t}^{b,FI} \right] \mathcal{Q}_t \right\} \tag{155}$$

The maximisation problem in (154) subject to (155) has Lagrangian

$$\begin{aligned}
\mathcal{L}_{Int,t} = \max (1+\lambda_t) \mathbb{E}_t \Bigg\{ & (1-\sigma) \Lambda_{t,t+1} \Pi_{t+1}^{-1} \Bigg\{ (R_{t+1}^g - R_t^d) Q_t^g b_{H,t}^{g,FI} + (R_{t+1}^b - R_t^d) Q_t^b b_{H,t}^{b,FI} \\
& + \left(R_{t+1}^{*g} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \left[1 - \omega/2 (b_{F,t}^{g,FI} / b_F^{g,FI} - 1)^2 \right] - R_t^d \right) Q_t^{*g} b_{F,t}^{g,FI} \mathcal{Q}_t \\
& + \left(R_{t+1}^{*b} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \left[1 - \omega/2 (b_{F,t}^{b,FI} / b_F^{b,FI} - 1)^2 \right] - R_t^d \right) Q_t^{*b} b_{F,t}^{b,FI} \mathcal{Q}_t \\
& + (R_t^{re} - R_t^d) re_t + R_t^d n_t \Bigg\} + \Lambda_{t,t+1} V_{t+1} \Bigg\} \\
& - \lambda_t \theta_t \Bigg\{ \Delta Q_t^g b_{H,t}^{g,FI} + Q_t^b b_{H,t}^{b,FI} + \left[\Delta Q_t^{*g} b_{F,t}^{g,FI} + Q_t^{*b} b_{F,t}^{b,FI} \right] \mathcal{Q}_t \Bigg\}
\end{aligned}$$

where λ_t is the Lagrangian multiplier on constraint (155).
The FOCs are:

$$\begin{aligned}
\frac{\partial \mathcal{L}_{Int,t}}{\partial b_{H,t}^g} : \quad & (1+\lambda_t) \mathbb{E}_t \left\{ (1-\sigma) \Lambda_{t,t+1} \Pi_{t+1}^{-1} \left\{ (R_{t+1}^g - R_t^d) Q_t^g \right\} + \Lambda_{t,t+1} \frac{\partial V_{t+1}}{\partial \lfloor_{t+1}} \right\} - \lambda_t \theta_t \Delta Q_t^g = 0 \\
\frac{\partial \mathcal{L}_{Int,t}}{\partial b_{H,t}^b} : \quad & \nu_{1,t}^g = p_t^k M_{1,t}^g \\
\frac{\partial \mathcal{L}_{Int,t}}{\partial b_{F,t}^{*g}} : \quad & w_t^g = p_{w,t}^g (1-\alpha) A_t^g (u_t^g K_t^g)^\alpha (L_{d,t}^g)^{-\alpha} \\
\frac{\partial \mathcal{L}_{Int,t}}{\partial b_{F,t}^{*b}} : \quad & \nu_{1,t}^g = p_t^k M_{1,t}^g \\
\frac{\partial \mathcal{L}_{Int,t}}{\partial re_t} : \quad & \nu_{1,t}^g = p_t^k M_{1,t}^g
\end{aligned}$$

Defining the auxiliary variable $\Omega_{t+1} \equiv 1 - \sigma + \frac{\partial V_{t+1}}{\partial \lfloor_{t+1}}$, the FOCs simplify to:

$$\frac{\lambda_t}{1+\lambda_t}\theta_t\Delta = \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi_{t+1}^{-1}(R_{t+1}^g - R_t^d) \tag{156}$$

$$\frac{\lambda_t}{1+\lambda_t}\theta_t = \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi_{t+1}^{-1}(R_{t+1}^b - R_t^d) \tag{157}$$

$$\frac{\lambda_t}{1+\lambda_t}\theta_t\Delta = \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi_{t+1}^{-1}\left\{R_t^{*g}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\left[1-\frac{\omega}{2}(b_{F,t}^{g,FI}/b_F^{g,FI}-1)(3b_{F,t}^{g,FI}/b_F^{g,FI}-1)\right]-R_t^d\right\} \tag{158}$$

$$\frac{\lambda_t}{1+\lambda_t}\theta_t = \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi_{t+1}^{-1}\left\{R_t^{*b}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\left[1-\frac{\omega}{2}(b_{F,t}^{b,FI}/b_F^{b,FI}-1)(3b_{F,t}^{b,FI}/b_F^{b,FI}-1)\right]-R_t^d\right\} \tag{159}$$

$$0 = \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi_{t+1}^{-1}(R_t^{re} - R_t^d) \tag{160}$$

When the constraints binds and thus $\lambda_t > 0$, the FOCs show that the model generates excess returns of investing in bonds while financing through deposits. These returns get closer across types of bonds as Δ approaches one, which increases the required returns of eligible bonds. As in Wu et al. (2024b), I assume that (155) always binds, so that the slope of the yield curve is never zero and always positive, as usually the case in reality.

Defining

$$\phi \equiv V/(\sigma\theta\lfloor_t) \tag{161}$$

and by definition of Ω_{t+1}, I get the expression:

$$\Omega_t = 1-\sigma+\sigma\theta\phi_t \tag{162}$$

By definition of ϕ the binding constraint (155) implies:

$$\phi = \frac{\Delta Q_t^g b_{H,t}^{g,FI} + Q_t^b b_{H,t}^{b,FI} + \left[\Delta Q_t^{*g} b_{F,t}^{g,FI} + Q_t^{*b} b_{F,t}^{b,FI}\right]\mathcal{Q}_t}{\sigma\lfloor_t} \tag{163}$$

Combining this with the FOCs and the beginning balance (145):

$$\mathbb{E}_t\left[\Lambda_{t,t+1}\Omega_{t+1}\lfloor_{t+1}\right] = \sigma\frac{\lambda_t}{1+\lambda_t}\theta\phi_t\lfloor_t + \mathbb{E}_t\left[\Lambda_{t,t+1}\Omega_{t+1}\Pi_{t+1}^{-1}\right]R_t^d n_t + \Phi_t \tag{164}$$

with

$$\Phi_t = \omega\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi_{t+1}^{-1}R_{t+1}^{*g}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\left(\frac{b_{F,t}^{g,FI}}{b_F^{g,FI}}-1\right)\frac{b_{F,t}^{g,FI}}{b_F^{g,FI}}Q_t^{*g}b_{F,t}^{g,FI}\mathcal{Q}_t \tag{165}$$

$$+\,\omega\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi_{t+1}^{-1}R_{t+1}^{*b}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\left(\frac{b_{F,t}^{b,FI}}{b_F^{b,FI}}-1\right)\frac{b_{F,t}^{b,FI}}{b_F^{b,FI}}Q_t^{*b}b_{F,t}^{b,FI}\mathcal{Q}_t \tag{166}$$

Rewriting (153) through (161), (162) and (164), and dividing by $\lfloor_t$, I get the expression

for:

$$\phi_t = \frac{1+\lambda_t}{\sigma\theta}\left\{\mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\Pi_{t+1}^{-1}R_t^d\frac{n_t}{\lfloor_t} + \frac{\Phi_t}{\lfloor_t}\right\} \tag{167}$$

The financial intermediary pays domestic households real dividends $div_t^{FI} = (1-\sigma)\lfloor_t$. Similarly for the foreign economy we have:

$$\frac{\lambda_t^*}{1+\lambda_t^*}\theta_t^*\Delta = \mathbb{E}_t\Lambda_{t,t+1}^*\Omega_{t+1}^*(\Pi_{t+1}^*)^{-1}(R_{t+1}^{*g} - R_t^{*d}) \tag{168}$$

$$\frac{\lambda_t^*}{1+\lambda_t^*}\theta_t^* = \mathbb{E}_t\Lambda_{t,t+1}^*\Omega_{t+1}^*(\Pi_{t+1}^*)^{-1}(R_{t+1}^{*b} - R_t^{*d}) \tag{169}$$

$$\frac{\lambda_t^*}{1+\lambda_t^*}\theta_t^*\Delta = \mathbb{E}_t\Lambda_{t,t+1}^*\Omega_{t+1}^*(\Pi_{t+1}^*)^{-1}\left\{R_t^g\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\left[1-\frac{\omega}{2}(b_{H,t}^{*g,FI}/b_H^{*g,FI}-1)(3b_{H,t}^{*g,FI}/b_H^{*g,FI}-1)\right] - R_t^{*d}\right\} \tag{170}$$

$$\frac{\lambda_t^*}{1+\lambda_t^*}\theta_t^* = \mathbb{E}_t\Lambda_{t,t+1}^*\Omega_{t+1}^*(\Pi_{t+1}^*)^{-1}\left\{R_t^b\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\left[1-\frac{\omega}{2}(b_{H,t}^{*b,FI}/b_H^{*b,FI}-1)(3b_{H,t}^{*b,FI}/b_H^{*b,FI}-1)\right] - R_t^{*d}\right\} \tag{171}$$

$$0 = \mathbb{E}_t\Lambda_{t,t+1}^*\Omega_{t+1}^*(\Pi_{t+1}^*)^{-1}(R_t^{*re} - R_t^{*d}) \tag{172}$$

$$Q_t^{*g}b_{F,t}^{*g,FI} + Q_t^{*b}b_{F,t}^{*b,FI} + Q_t^g b_{H,t}^{*g,FI}\mathcal{E}_t^{-1} + Q_t^b b_{H,t}^{*b,FI}\mathcal{E}_t^{-1} + re_t^* = d_t^* + n_t^* \tag{173}$$

$$n_t^* = \sigma\lfloor_t^* + X^* \tag{174}$$

$$n_t^* = \sigma(\Pi_t^*)^{-1}\Bigg\{(R_t^{*g} - R_{t-1}^{*d})Q_{t-1}^{*g}b_{F,t-1}^{*g,FI} + (R_t^{*b} - R_{t-1}^{*d})Q_{t-1}^{*b}b_{F,t-1}^{*b,FI} \tag{175}$$

$$+\left(R_t^g\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}\left[1-\omega/2(b_{H,t-1}^{*g,FI}/b_H^{*g,FI}-1)^2\right] - R_{t-1}^{*d}\right)Q_{t-1}^g b_{H,t-1}^{*,FI}/\mathcal{Q}_{t-1} \tag{176}$$

$$+\left(R_t^b\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}\left[1-\omega/2(b_{H,t-1}^{*b,FI}/b_H^{*b,FI}-1)^2\right] - R_{t-1}^{*d}\right)Q_{t-1}^b b_{H,t-1}^{*b,FI}/\mathcal{Q}_{t-1} \tag{177}$$

$$+(R_{t-1}^{*re} - R_{t-1}^{*d})re_{t-1}^* + R_{t-1}^{*d}n_{t-1}^*\Bigg\} + X^* \tag{178}$$

$$\Omega_t^* = 1 - \sigma + \sigma\theta^*\phi_t^* \tag{179}$$

$$\phi^* = \frac{\Delta Q_t^{*g}b_{F,t}^{*g,FI} + Q_t^{*b}b_{F,t}^{*b,FI} + \left[\Delta Q_t^g b_{H,t}^{*g,FI} + Q_t^b b_{H,t}^{*b,FI}\right]/\mathcal{Q}_t}{\sigma\lfloor_t^*} \tag{180}$$

$$\Phi_t^* = \omega \mathbb{E}_t \Lambda_{t,t+1}^* \Omega_{t+1}^* (\Pi_{t+1}^*)^{-1} R_{t+1}^g \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \left(\frac{b_{H,t}^{*g,FI}}{b_H^{*g,FI}} - 1 \right) \frac{b_{H,t}^{*g,FI}}{b_H^{*g,FI}} Q_t^g b_{H,t}^{*g,FI} / \mathcal{Q}_t \tag{181}$$

$$+ \omega \mathbb{E}_t \Lambda_{t,t+1}^* \Omega_{t+1}^* (\Pi_{t+1}^*)^{-1} R_{t+1}^b \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \left(\frac{b_{H,t}^{*b,FI}}{b_H^{*b,FI}} - 1 \right) \frac{b_{H,t}^{*b,FI}}{b_H^{*b,FI}} Q_t^b b_{H,t}^{*b,FI} / \mathcal{Q}_t \tag{182}$$

$$\phi_t^* = \frac{1 + \lambda_t^*}{\sigma \theta^*} \left\{ \mathbb{E}_t \Lambda_{t,t+1}^* \Omega_{t+1}^* (\Pi_{t+1}^*)^{-1} R_t^{*d} \frac{n_t^*}{\lfloor_t^*} + \frac{\Phi_t^*}{\lfloor_t^*} \right\} \tag{183}$$

A.2.8 Labour market

The labour market in each country exhibits a similar system as the one of final-good firms and retailers, but separately for agents facing labour demand from eligible and ineligible wholesalers.

There are two labour packers which combines differentiated labour from the labour unions into final labour used in production through the CES technology:

$$L_{d,t}^g = \left(\int_0^{\gamma(1-\phi_g)} L_{d,t}^g(l)^{\frac{e_w - 1}{e_w}} dl \right)^{\frac{e_w}{e_w - 1}} \tag{184}$$

$$L_{d,t}^b = \left(\int_{\gamma(1-\phi_g)}^{\gamma} L_{d,t}^b(l)^{\frac{e_w - 1}{e_w}} dl \right)^{\frac{e_w}{e_w - 1}} \tag{185}$$

where e_w is the labour elasticity of substitution, $L_{d,t}^g(l)$ ($L_{d,t}^b(l)$) is differentiated labour from suppliers of green (brown) packers whose price is nominal wage $W_t^g(l)$ ($W_t^b(l)$). Differentiated labour is provided by labour unions indexed by $l \in [0, \gamma)$. Of these, a fraction $1 - \phi_g$, where $0 \leq \phi_g \leq 1$, provides differentiated labour which is combined to satisfy labour demand of eligible wholesalers. Conversely, a fraction ϕ_g would go to ineligible packers and, in turn, ineligible intermediate-good producers.

The maximisation problem for the labour packers is:

$$\max_{L_{d,t}^g, \left\{ L_{d,t}(l) \right\}_{l \in [0, \gamma(1-\phi_g))}} \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left[\frac{W_{t+j}^g}{P_{t+j}} L_{d,t+j}^g - \int_0^{\gamma(1-\phi_g)} \frac{W_{t+j}^g(l)}{P_{t+j}} L_{d,t+j}^g(l) dl \right]$$

subject to technology (184), and

$$\max_{L_{d,t}^b, \left\{ L_{d,t}(l) \right\}_{l \in [\gamma(1-\phi_g), \gamma)}} \mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left[\frac{W_{t+j}^b}{P_{t+j}} L_{d,t+j}^b - \int_{\gamma(1-\phi_g)}^{\gamma} \frac{W_{t+j}^b(l)}{P_{t+j}} L_{d,t+j}^b(l) dl \right]$$

subject to technology (185).

Simplifying the problem to a choice of inputs $L_{d,t}(l)$ only, by plugging the technology functions into the objective function, the FOC for a generic $L_{d,t}(l)$ is (notice I simplified out the common deflator P_t)

$$W_t^g \left(\int_0^{\gamma(1-\phi_g)} L_{d,t}^g(l)^{\frac{e_w-1}{e_w}} dl \right)^{\frac{1}{e_w-1}} L_{d,t}^g(l)^{-\frac{1}{e_w}} - W_t^g(l) = 0$$

$$\Rightarrow W_t^g (L_{d,t}^g)^{\frac{1}{e_w}} L_{d,t}^g(l)^{-\frac{1}{e_w}} = W_t^g(l)$$

and thus

$$L_{d,t}^g(l) = \left(\frac{W_t^g(l)}{W_t^g} \right)^{-e_w} L_{d,t}^g \tag{186}$$

where the last equation is the downward-sloping demand function for the differentiated labour faced by unions. Similarly for ineligible wholesalers' demand:

$$L_{d,t}^b(l) = \left(\frac{W_t^b(l)}{W_t^b} \right)^{-e_w} L_{d,t}^b \tag{187}$$

The aggregate wage levels W_t^g and W_t^b satisfy:

$$(W_t^g)^{1-e_w} = \frac{1}{\gamma(1-\phi_g)} \int_0^{\gamma(1-\phi_g)} W_t^g(l)^{1-e_w} dl \tag{188}$$

$$(W_t^b)^{1-e_w} = \frac{1}{\gamma\phi_g} \int_{\gamma(1-\phi_g)}^{\gamma} W_t^b(l)^{1-e_w} dl \tag{189}$$

Unions buy labour $L_t^g(l)$ and $L_t^b(l)$ from households at price MRS_t^g and MRS_t^b respectively. They operate with a simple "repackaging" technology represented by the production function $L_{d,t}^g(l) = L_t^g(l)$ and $L_{d,t}^b(l) = L_t^b(l)$. With this technology and the demand functions above, the nominal profits the union tries to maximise are:

$$DIV_{L,t}^g(l) = [W_t^g(l) - MRS_t^g](W_t^g(l))^{-e_w}(W_t^g)^{e_w} L_{d,t}^g$$

$$DIV_{L,t}^b(l) = [W_t^b(l) - MRS_t^b](W_t^b(l))^{-e_w}(W_t^b)^{e_w} L_{d,t}^b$$

In real terms (dividing by P_t and rearranging so to feature real wages w_t^g and w_t^b):

$$div_{L,t}^g(l) = (W_t^g(l))^{1-e_w} P_t^{e_w-1} (w_t^g)^{e_w} L_{d,t}^g - mrs_t^g (W_t^g(l))^{-e_w} P_t^{e_w} (w_t^g)^{e_w} L_{d,t}^g$$

$$div_{L,t}^b(l) = (W_t^b(l))^{1-e_w} P_t^{e_w-1} (w_t^b)^{e_w} L_{d,t}^b - mrs_t^b (W_t^b(l))^{-e_w} P_t^{e_w} (w_t^b)^{e_w} L_{d,t}^b$$

In this market there are Calvo-type wage rigidities, that is the union can adjust its wages in each period with probability $1-\phi_w$, where $0 \leq \phi_w \leq 1$.

Thus, each union able to re-optimise wants to maximise the present discounted value of its (real) profits using the discount factor of the household. I develop here the case for the eligible labour union, but the derivation for the others is similar.

$$\max_{W_t^g(l)} \mathbb{E}_t \sum_{j=0}^{\infty} \phi_w^j \Lambda_{t,t+j} \left[(W_t^g(l))^{1-e_w} P_{t+j}^{e_w-1} (w_{t+j}^g)^{e_w} L_{d,t+j}^g - mrs_{t+j}^g (W_t^g(l))^{-e_w} P_{t+j}^{e_w} (w_{t+j}^g)^{e_w} L_{d,t+j}^g \right]$$

The FOC is:

$$\frac{\partial \mathcal{L}_{L,t}^g}{\partial W_t^g(l)} : \quad (e_w - 1) W_t^g(l)^{-e_w} \mathbb{E}_t \sum_{j=0}^{\infty} \phi_w^j \Lambda_{t,t+j} P_{t+j}^{e_w-1} (w_{t+j}^g)^{e_w} L_{d,t+j}^g = e_w (W_t^g(l))^{-e_w-1} \mathbb{E}_t \sum_{j=0}^{\infty} \phi_w^j \Lambda_{t,t+j} mrs_{t+j}^g P_{t+j}^{e_w} (w_{t+j}^g)^{e_w} L_{d,t+j}^g$$

Define the auxiliary variables:

$$F_{1,t}^g \equiv \sum_{j=0}^{\infty} \phi_w^j \Lambda_{t,t+j} mrs_{t+j}^g P_{t+j}^{e_w} (w_{t+j}^g)^{e_w} L_{d,t+j}^g$$
$$F_{2,t}^g \equiv \sum_{j=0}^{\infty} \phi_w^j \Lambda_{t,t+j} P_{t+j}^{e_w-1} (w_{t+j}^g)^{e_w} L_{d,t+j}^g$$

Equivalently, in the more convenient recursive form:

$$F_{1,t}^g \equiv mrs_t^g P_t^{e_w} (w_t^g)^{e_w} L_{d,t}^g + \phi_w \mathbb{E}_t \Lambda_{t,t+1} F_{1,t+1}^g$$
$$F_{2,t}^g \equiv P_t^{e_w-1} (w_t^g)^{e_w} L_{d,t}^g + \phi_w \mathbb{E}_t \Lambda_{t,t+1} F_{2,t+1}^g$$

I now divide the first variable by $P_t^{e_w}$ and the second by $P_t^{e_w-1}$ to define $f_{1,t}^g$ and $f_{2,t}^g$ (also $f_{1,t}^g = F_{1,t}^g / P_t^{e_w} \Rightarrow f_{1,t+1}^g = F_{1,t+1}^g / P_{t+1}^{e_w}$ and $f_{2,t}^g = F_{2,t}^g / P_t^{e_w-1} \Rightarrow f_{2,t+1}^g = F_{2,t+1}^g / P_{t+1}^{e_w-1}$):

$$f_{1,t}^g \equiv mrs_t^g (w_t^g)^{e_w} L_{d,t}^g + \phi_w \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{e_w} f_{1,t+1}^g \tag{190}$$
$$f_{2,t}^g \equiv (w_t^g)^{e_w} L_{d,t}^g + \phi_w \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{e_w-1} f_{2,t+1}^g \tag{191}$$

Consider a symmetric equilibrium where unions pick the same wage, then $W_t^g(l) = W_t^{g,o}$ and the FOC simplifies to:

$$w_t^{g,o} = \frac{e_w}{e_w - 1} \frac{f_{1,t}^g}{f_{2,t}^g} \tag{192}$$

where $w_t^{g,o} \equiv \frac{W_t^{g,o}}{P_t}$.

At this point I can integrate the demand function (186):

$$\int_0^{\gamma(1-\phi_g)} L_{d,t}^g(l) dl = \int_0^{\gamma(1-\phi_g)} \left(\frac{W_t^g(l)}{W_t^g} \right)^{-e_w} L_{d,t}^g dl$$

Because each union provides the same amount, i.e. an equal share of the total $L_{d,t}^g$ as shown

by the demand function (and thus use equal amounts of $L_t^g(l)$ as inputs), and given unions' technology $L_{d,t}^g(l) = L_t^g(l)$, the left-hand side of the equation above sums up to a fraction of the total supply L_t: $\int_0^{\gamma(1-\phi_g)} L_{d,t}^g(l)dl = \int_0^{\gamma(1-\phi_g)} L_t^g(l)dl = \gamma(1-\phi_g)L_t^g$.

$$\gamma(1-\phi_g)L_t^g = \int_0^{\gamma(1-\phi_g)} \left(\frac{W_t^g(l)}{W_t^g}\right)^{-e_w} L_{d,t}^g dl \quad \Rightarrow \quad L_t^g = v_t^g L_{d,t}^g \tag{193}$$

where $v_t^g = \frac{1}{\gamma(1-\phi_g)} \int_0^{\gamma(1-\phi_g)} \left(\frac{W_t^g(l)}{W_t^g}\right)^{-e_w} dl$ is a measure of wage dispersion.

One can use the Calvo assumption to split v_t^g noting that $(1-\phi_g)$ of unions can update setting the same wage $w_t^{g,o}$, and the rest can only retain the wage from the previous period:

$$v_t^g = \frac{1}{\gamma(1-\phi_g)} \int_0^{\gamma(1-\phi_g)(1-\phi_w)} \left(\frac{W_t^{g,o}}{W_t^g}\right)^{-e_w} dl + \frac{1}{\gamma(1-\phi_g)} \int_{\gamma(1-\phi_g)(1-\phi_w)}^{\gamma(1-\phi_g)} \left(\frac{W_{t-1}^g(l)}{W_t^g}\right)^{-e_w} dl$$

which becomes by multiplying and dividing by $P_t^{-e_w}$ the first term and by $(W_{t-1}^g)^{-e_w}$ the second term on the right-hand side respectively, and rearranging terms out of the integrals:

$$v_t^g = (1-\phi_w)\left(\frac{w_t^{g,o}}{w_t^g}\right)^{-e_w} + \frac{1}{\gamma(1-\phi_g)}(W_t^g)^{e_w}(W_{t-1}^g)^{-e_w} \int_{\gamma(1-\phi_g)(1-\phi_w)}^{\gamma(1-\phi_g)} \left(\frac{W_{t-1}^g(l)}{W_{t-1}^g}\right)^{-e_w} dl$$

For the definition of gross inflation and the wage dispersion term, the expression simplifies to:

$$v_t^g = (1-\phi_w)\left(\frac{w_t^{g,o}}{w_t^g}\right)^{-e_w} + \phi_w \Pi_t^{e_w} \left(\frac{w_t^g}{w_t^g}\right)^{-e_w} v_{t-1}^g \tag{194}$$

For the same Calvo price properties, the wage level (188) can be rewritten as:

$$\begin{aligned}
(W_t^g)^{1-e_w} &= (1-\phi_w)(W_t^{g,o})^{1-e_w} + \frac{1}{\gamma(1-\phi_g)} \int_{\gamma(1-\phi_g)(1-\phi_w)}^{\gamma(1-\phi_g)} W_{t-1}^g(l)^{1-e_w} dl \\
\Rightarrow \quad (W_t^g)^{1-e_w} &= (1-\phi_w)(W_t^{g,o})^{1-e_w} + \phi_w (W_{t-1}^g)^{1-e_w} \\
\Rightarrow \quad (w_t^g)^{1-e_w} &= (1-\phi_w)(w_t^{g,o})^{1-e_w} + \phi_w \Pi_t^{e_w-1} (w_{t-1}^g)^{1-e_w}
\end{aligned} \tag{195}$$

Similar equations apply for unions linked to the ineligible packer:

$$w_t^{b,o} = \frac{e_w}{e_w - 1} \frac{f_{1,t}^b}{f_{2,t}^b} \tag{196}$$

$$f_{1,t}^b = mrs_t (w_t^b)^{e_w} L_{d,t}^b + \phi_w \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{e_w} f_{1,t+1}^b \tag{197}$$

$$f_{2,t}^b = (w_t^b)^{e_w} L_{d,t}^b + \phi_w \mathbb{E}_t \Lambda_{t,t+1} \Pi_{t+1}^{e_w - 1} f_{2,t+1}^b \tag{198}$$

$$L_t^b = v_t^b L_{d,t}^b \tag{199}$$

$$v_t^b = (1 - \phi_w) \left(\frac{w_t^{b,o}}{w_t^b} \right)^{-e_w} + \phi_w \Pi_t^{e_w} \left(\frac{w_t^b}{w_t^b} \right)^{-e_w} v_{t-1}^b \tag{200}$$

$$(w_t^b)^{1-e_w} = (1 - \phi_w)(w_t^{b,o})^{1-e_w} + \phi_w \Pi_t^{e_w - 1} (w_{t-1}^b)^{1-e_w} \tag{201}$$

And similar equations apply in Foreign:

$$w_t^{*g,o} = \frac{e_w}{e_w - 1} \frac{f_{1,t}^{*g}}{f_{2,t}^{*g}} \tag{202}$$

$$f_{1,t}^{*g} = mrs_t^* (w_t^{*g})^{e_w} L_{d,t}^{*g} + \phi_w \mathbb{E}_t \Lambda_{t,t+1}^* \Pi_{t+1}^{*e_w} f_{1,t+1}^{*g} \tag{203}$$

$$f_{2,t}^{*g} = (w_t^{*g})^{e_w} L_{d,t}^{*g} + \phi_w \mathbb{E}_t \Lambda_{t,t+1}^* (\Pi^*)_{t+1}^{e_w - 1} f_{2,t+1}^{*g} \tag{204}$$

$$L_t^{*g} = v_t^{*g} L_{d,t}^{*g} \tag{205}$$

$$v_t^{*g} = (1 - \phi_w) \left(\frac{w_t^{*g,o}}{w_t^{*g}} \right)^{-e_w} + \phi_w \Pi_t^{*e_w} \left(\frac{w_t^{*g}}{w_t^{*g}} \right)^{-e_w} v_{t-1}^{*g} \tag{206}$$

$$(w_t^{*g})^{1-e_w} = (1 - \phi_w)(w_t^{*g,o})^{1-e_w} + \phi_w (\Pi_t^*)^{e_w - 1} (w_{t-1}^{*g})^{1-e_w} \tag{207}$$

$$w_t^{*b,o} = \frac{e_w}{e_w - 1} \frac{f_{1,t}^{*b}}{f_{2,t}^{*b}} \tag{208}$$

$$f_{1,t}^{*b} = mrs_t^* (w_t^{*b})^{e_w} L_{d,t}^{*b} + \phi_w \mathbb{E}_t \Lambda_{t,t+1}^* \Pi_{t+1}^{*e_w} f_{1,t+1}^{*b} \tag{209}$$

$$f_{2,t}^{*b} = (w_t^{*b})^{e_w} L_{d,t}^{*b} + \phi_w \mathbb{E}_t \Lambda_{t,t+1}^* (\Pi^*)_{t+1}^{e_w - 1} f_{2,t+1}^{*b} \tag{210}$$

$$L_t^{*b} = v_t^{*b} L_{d,t}^{*b} \tag{211}$$

$$v_t^{*b} = (1 - \phi_w) \left(\frac{w_t^{*b,o}}{w_t^{*b}} \right)^{-e_w} + \phi_w \Pi_t^{*e_w} \left(\frac{w_t^{*b}}{w_t^{*b}} \right)^{-e_w} v_{t-1}^{*b} \tag{212}$$

$$(w_t^{*b})^{1-e_w} = (1 - \phi_w)(w_t^{*b,o})^{1-e_w} + \phi_w (\Pi_t^*)^{e_w - 1} (w_{t-1}^{*b})^{1-e_w} \tag{213}$$

A.2.9 Monetary policy

Home and Foreign central banks follow a Taylor rule in setting the domestic nominal (policy) interest rate R_t^{re}, targeting domestic PPI inflation and output fluctuations:

$$\left(\frac{R_t^{re}}{R^{re}}\right) = \left(\frac{R_{t-1}^{re}}{R^{re}}\right)^{\rho_r} \left[\left(\frac{\Pi_{H,t}}{\Pi_H}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_y}\right]^{1-\rho_r} \exp\left(s_r \varepsilon_t^r\right) \tag{214}$$

$$\left(\frac{R_t^{*re}}{R^{*re}}\right) = \left(\frac{R_{t-1}^{*re}}{R^{*re}}\right)^{\rho_r} \left[\left(\frac{\Pi_{H,t}^{*}}{\Pi_H^{*}}\right)^{\phi_\pi} \left(\frac{Y_t^{*}}{Y_{t-1}^{*}}\right)^{\phi_y}\right]^{1-\rho_r} \exp\left(s_{*r} \varepsilon_t^{*r}\right) \tag{215}$$

with monetary policy shocks $\varepsilon_t^r \sim N(0, s_r^2)$ and $\varepsilon_t^{*r} \sim N(0, s_{*r}^2)$. The variables without subscript are the corresponding steady-state targets followed by the central banks. The parameter $0 < \rho_r < 1$ measures the degree of inertia of the nominal interest rate adjustment, while $\phi_\pi > 1$ and $\phi_y > 0$ modulates the reaction of the policy rate to domestic inflation and output, respectively.

The central bank in each country issues reserves to finance its purchases of corporate bonds, the form of unconventional monetary policy included in this setting

$$Q_t^g B_t^{g,CB} + Q_t^b B_t^{b,CB} = RE_t \tag{216}$$

$$Q_t^{*g} B_t^{*g,CB} + Q_t^{*b} B_t^{*b,CB} = RE_t^{*} \tag{217}$$

is the central bank's balance-sheet condition, while the exogenous rules governing changes in domestic (real) bond holdings are

$$b_t^{g,CB} - b^{g,CB} = \rho_g (b_t^{g,CB} - b^{g,CB}) + s_g \varepsilon_t^{g,CB} \tag{218}$$

$$b_t^{b,CB} - b^{b,CB} = \rho_b (b_t^{b,CB} - b^{b,CB}) + s_b \varepsilon_t^{b,CB} \tag{219}$$

$$b_t^{*g,CB} - b^{*g,CB} = \rho_g (b_t^{*g,CB} - b^{*g,CB}) + s_{*g} \varepsilon_t^{*g,CB} \tag{220}$$

$$b_t^{*b,CB} - b^{*b,CB} = \rho_b (b_t^{*b,CB} - b^{*b,CB}) + s_{*b} \varepsilon_t^{*b,CB} \tag{221}$$

where I define $b_t^{g,CB} \equiv B_t^{g,CB}/P_t$, $b_t^{b,CB} \equiv B_t^{b,CB}/P_t$, $b_t^{*g,CB} \equiv B_t^{*g,CB}/P_t^*$ and $b_t^{*b,CB} \equiv B_t^{*b,CB}/P_t^*$. Innovations $\varepsilon_t^{g,CB} \sim N(0, s_g^2)$, $\varepsilon_t^{*g,CB} \sim N(0, s_{*g}^2)$, $\varepsilon_t^{b,CB} \sim N(0, s_b^2)$, $\varepsilon_t^{*b,CB} \sim N(0, s_{*b}^2)$ are policy shocks. The variables without subscript are again the corresponding steady-state values, and the parameters $0 < \rho_g, \rho_b < 1$ are autoregressive coefficients.

The profit made by the central bank on its balance sheet, which is then transferred to the government, is

$$T_t^{CB} = (1 + \kappa Q_t^g)\Pi_t^{-1} b_{t-1}^{g,CB} + (1 + \kappa Q_t^b)\Pi_t^{-1} b_{t-1}^{b,CB} - R_{t-1}^{re}\Pi_t^{-1} re_{t-1} \tag{222}$$

For Foreign a similar expression holds:

$$T_t^{*CB} = (1+\kappa Q_t^{*g})(\Pi_t^*)^{-1} b_{t-1}^{*g,CB} + (1+\kappa Q_t^{*b})(\Pi_t^*)^{-1} b_{t-1}^{*b,CB} - R_{t-1}^{*re}(\Pi_t^*)^{-1} re_{t-1}^* \tag{223}$$

For simplicity, there is no government spending nor debt, which means that the government's budget constraint simplifies to:

$$0 = P_t T_t + P_t T_t^{CB} \tag{224}$$
$$+ R_t^{*g} \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \omega/2 (b_{F,t-1}^{g,FI}/b_F^{g,FI} - 1)^2 Q_{t-1}^{*g} B_{F,t-1}^{g,FI} \mathcal{E}_{t-1} \tag{225}$$
$$+ R_t^{*b} \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \omega/2 (b_{F,t-1}^{b,FI}/b_F^{b,FI} - 1)^2 Q_{t-1}^{*b} B_{F,t-1}^{b,FI} \mathcal{E}_{t-1} \tag{226}$$

which means that the profit remittance from central bank's asset holdings and from the transaction cost on foreign assets are paid to domestic households in the form of subsidies, i.e. negative taxes.

Similarly for Foreign:

$$0 = P_t^* T_t^* + P_t^* T_t^{*CB} \tag{227}$$
$$+ R_t^{g} \frac{\mathcal{E}_{t-1}}{\mathcal{E}_t} \omega/2 (b_{H,t-1}^{*g,FI}/b_H^{*g,FI} - 1)^2 Q_{t-1}^{g} B_{H,t-1}^{*g,FI}/\mathcal{E}_{t-1} \tag{228}$$
$$+ R_t^{b} \frac{\mathcal{E}_{t-1}}{\mathcal{E}_t} \omega/2 (b_{H,t-1}^{*b,FI}/b_H^{*b,FI} - 1)^2 Q_{t-1}^{b} B_{H,t-1}^{*b,FI}/\mathcal{E}_{t-1} \tag{229}$$

A.2.10 Market clearing

In equilibrium the resources constraint must be satisfied and markets must clear.

As far as domestic goods are concerned, it must hold that the sum of domestic and foreign consumption of home goods together with domestic investment equals domestic output. Thus, the aggregate resource constraint for the each economy is:

$$Y_t = C_{H,t} + C_{H,t}^* + I_t \tag{230}$$
$$Y_t^* = C_{F,t}^* + C_{F,t} + I_t^* \tag{231}$$

Market clearing for bonds:

$$b_t^g = b_{H,t}^{g,FI} + \frac{1-\gamma}{\gamma} b_{H,t}^{*g,FI} + b_t^{g,GB} \tag{232}$$

$$b_t^b = b_{H,t}^{b,FI} + \frac{1-\gamma}{\gamma} b_{H,t}^{*b,FI} + b_t^{b,GB} \tag{233}$$

$$b_t^{*g} = b_{F,t}^{*g,FI} + \frac{\gamma}{1-\gamma} b_{F,t}^{g,FI} + b_t^{*g,GB} \tag{234}$$

$$b_t^{*b} = b_{F,t}^{*b,FI} + \frac{\gamma}{1-\gamma} b_{F,t}^{b,FI} + b_t^{*b,GB} \tag{235}$$

A.2.11 Exogenous processes

The model exhibits three types of exogenous shocks within each economy: the conventional monetary policy shock, the eligible and ineligible asset purchase shocks, and a productivity shock. The latter features in the AR(1) process for productivity of eligible and ineligible wholesalers A_t and A_t^* in logarithmic form:

$$\ln A_t^g = \rho_A \ln A_{t-1}^g + s_A \varepsilon_t^{A,g} \tag{236}$$

$$\ln A_t^b = \rho_A \ln A_{t-1}^b + s_A \varepsilon_t^{A,b} \tag{237}$$

$$\ln A_t^* = \rho_A \ln A_{t-1}^* + s_{*A} \varepsilon_t^{*A,g} \tag{238}$$

$$\ln A_t^* = \rho_A \ln A_{t-1}^* + s_{*A} \varepsilon_t^{*A,b} \tag{239}$$

A.2.12 Equilibrium

I collect here the equilibrium conditions of the model.

Financial intermediaries

Home

(112), (113), (156), (157), (158), (159), (160), (141), (148), (152), (162), (163), (166), (167).

Foreign

(114), (115), (168), (169), (170), (171), (172), (173), (174), (178), (179), (180), (182), (183).

Households: Intratemporal decision

Home

All: (22).
Eligible: (15), (16).
Ineligible: (13), (14).

Foreign

All: (23).
Eligible: (19), (20).
Ineligible: (17), (18).

Households: Intertemporal decision

Home

(40), (41), (42), (43), (44), (39), (45).

Foreign

(47), (48), (49), (50), (51).

Production - Wholesalers

Home

Eligible: (98), (102), (108), (117), (119), (120), (121), (116).
Ineligible: (99), (103), (109), (122), (124), (125), (126), (127).

Foreign

Eligible: (100), (104), (110), (128), (130), (131), (132), (133).
Ineligible: (101), (103), (109), (122), (124), (125), (126), (127).

Production - Retailers

Home

Eligible: (65), (63), (64), (66), (68), (69).
Ineligible: (70), (71), (72), (73), (74), (75).

Foreign

Eligible: (76), (77), (78), (79), (80), (81).
Ineligible: (82), (83), (84), (85), (86), (87).

Production - Capital producer

Home

Eligible: (90), (94).
Ineligible: (91), (95).
Eligible + Ineligible: (88).

Foreign

Eligible: (92), (96).
Ineligible: (93), (97).
Eligible + Ineligible: (89).

Labour market

Home

Eligible: (192), (190), (191), (193), (194), (195).
Ineligible: (196), (197), (198), (199), (200), (201).

Foreign

Eligible: (202), (203), (204), (205), (206), (207).
Ineligible: (208), (209), (210), (211), (212), (213).

Government and central bank

Home

(226), (216), (222), (214).

Foreign

(229), (217), (223), (215).

Market clearing

Home

(230), (232), (233).

Foreign

(231), (234), (235).

Whole economy

Home

(29), (32), (31).

Foreign

(30), (34), (33).

Cross economies

(24), (26), (25), (27), (28), (35), (36), (37), (38).

Exogenous processes

Home

(218), (219), (236), (237).

Foreign

(220), (221), (238), (239).

There are 160 equations for 160 endogenous variables

$$\{R_t^g, Q_t^g, R_t^b, Q_t^b, R_t^{re}, R_t^d, \Pi_t, \Lambda_{t,t+1}, \Omega_t, \lambda_t, \phi_t, \mu_t, C_t, L_t, mrs_t, w_t^{g,o}, f_{1,t}^g, f_{2,t}^g, w_t^{b,o}, f_{1,t}^b, f_{2,t}^b, w_t^g, w_t^b, L_{d,t}^g, L_{d,t}^b,$$
$$p_{H,t}^o, x_{1,t}, x_{2,t}, \tilde{p}_{m,t}, p_{w,t}^g, p_{w,t}^b, Y_t, Y_{m,t}, Y_{w,t}^g, Y_{w,t}^b, u_t^g, K_t^g, p_t^{g,k}, M_{1,t}^g, M_{2,t}^g, I_t^g, \hat{I}_t^g, u_t^b, K_t^b, p_t^{b,k}, M_{1,t}^b, M_{2,t}^b, I_t^b, \hat{I}_t^b,$$
$$I_t, b_t^g, b_t^b, T_t, T_t^{CB}, b_t^{g,CB}, b_t^{b,CB}, re_t, v_t^p, v_t^{g,w}, v_t^{b,w}, d_t, n_t, A_t^g, A_t^b, C_{H,t}, C_{F,t}, b_{H,t}^{g,FI}, b_{H,t}^{b,FI}, b_{F,t}^{g,FI}, b_{F,t}^{b,FI},$$
$$p_{H,t}, p_{F,t}, \Pi_{H,t}, P_t, \Phi_t, \lfloor_t,$$
$$R_t^{*g}, Q_t^{*g}, R_t^{*b}, Q_t^{*b}, R_t^{*re}, R_t^{*d}, \Pi_t^*, \Lambda_{t,t+1}^*, \Omega_t^*, \lambda_t^*, \phi_t^*, \mu_t^*, C_t^*, L_t^*, mrs_t^*, w_t^{*g,o}, f_{1,t}^{*g}, f_{2,t}^{*g}, w_t^{*b,o}, f_{1,t}^{*b}, f_{2,t}^{*b}, w_t^{*g}, w_t^{*b},$$
$$L_{d,t}^{*g}, L_{d,t}^{*b}, p_{H,t}^{*o}, x_{1,t}^*, x_{2,t}^*, \tilde{p}_{m,t}^*, p_{w,t}^{*g}, p_{w,t}^{*b}, Y_t^*, Y_{m,t}^*, Y_{w,t}^{*g}, Y_{w,t}^{*b}, u_t^{*g}, K_t^{*g}, p_t^{*g,k}, M_{1,t}^{*g}, M_{2,t}^{*g}, I_t^{*g}, \hat{I}_t^{*g}, u_t^{*b}, K_t^{*b}, p_t^{*b,k},$$
$$M_{1,t}^{*b}, M_{2,t}^{*b}, I_t^{*b}, \hat{I}_t^{*b}, I_t^*, b_t^{*g}, b_t^{*b}, T_t^*, T_t^{*CB}, b_t^{*g,CB}, b_t^{*b,CB}, re_t^*, v_t^{*p}, v_t^{*g,w}, v_t^{*b,w}, d_t^*, n_t^*, A_t^{*g}, A_t^{*b}, C_{H,t}^*, C_{F,t}^*,$$
$$b_{H,t}^{*g,FI}, b_{H,t}^{*b,FI}, b_{F,t}^{*g,FI}, b_{F,t}^{*b,FI}, p_{H,t}^*, p_{F,t}^*, \Pi_{H,t}^*, P_t^*, \Phi_t^*, \lfloor_t^*,$$
$$\mathcal{T}_t, \mathcal{E}_t, \mathcal{Q}_t, div_t, \}$$

and 8 exogenous shock $\{\varepsilon_t^A; \varepsilon_t^r; \varepsilon_t^{g,CB}; \varepsilon_t^{b,CB}; \varepsilon_t^{*A}; \varepsilon_t^{*r}; \varepsilon_t^{*g,CB}; \varepsilon_t^{*b,CB}\}$.

A.2.13 Calibration

TBU