Volatile Rates, Fragile Growth:

Global Financial Risk and Productivity Dynamics*

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February 6, 2025

PRELIMINARY - DO NOT CITE

Abstract

We demonstrate empirically that increased U.S. interest rate volatility forecasts a significant decline in the trends of GDP, consumption, and investment, with the drop being more pronounced in emerging economies than in advanced ones. Using a small open economy model with endogenous growth and financial crises, we rationalize this decline and the cross-country differences. In our model large adverse interest rate shocks disproportionally reduce firm values, tightening borrowing constraints and amplify economic slowdowns. This mechanism allows higher volatility to raise the likelihood of severe shocks, to stifle innovation, and slow trend growth. Our results reveal the critical role of global financial risk in shaping productivity trends, particularly in less financially developed emerging markets.

Keywords: Endogenous Growth, Stochastic Interest Rate Volatility, Financial Frictions, Long-Term Productivity Trends, Global Financial Risk Cycle

JEL Classifications: F32,F41,G15,O16

*The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or any other person associated with the Federal Reserve System. We thank Andrés Blanco, V.V. Chari, Nicolas Coeurdacier, Hal Cole, Luca Dedola, Jeremy Greenwood, Peter Karadi, Julian Kozlowski, Nels Lind, Enrique Mendoza, Pablo Ottonello, Diego Perez, Ricardo Reyes-Heroles, Juan Rubio-Ramírez, Cesar Sosa-Padilla, Martín Uribe, Vivian Yue, and seminar participants at the Philadelphia Federal Reserve-PIER-IER Conference in honor of Hal Cole, 2022 CEF, 2022 SEA, UAI, 2023 SED, VII Santiago Macro Workshop, 2024 International Macro Finance Workshop at Jackson Hole, 2024 Spring Midwest Macro, 2024 NASM, 2024 EEA-ESEM, Emory University, Sciences Po, the University of Nottingham, the ECB, Binghamtom University, Oxford, and LSE for their valuable comments. All remaining errors are our own.

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Policymakers from emerging market economies (EMEs) often express concerns about the effects of rising U.S. interest rates on their economies. However, it is not only the increase in interest rates that has contractionary effects on EMEs, but also the uncertainty surrounding global interest rates (Carrière-Swallow and Céspedes, 2013). The literature has long theorized that global risk in interest rates (Rey, 2015) contributes to the excess volatility in GDP and consumption observed in EMEs relative to developed economies. In this paper, we provide new insights by arguing that an increase in the volatility of U.S. interest rates not only affects business cycle fluctuations in EMEs but also influences the level and volatility of the long-term productivity trend that underpins their stationary fluctuations. We establish a novel link between U.S. interest rate dynamics and long-run trends in the small ipen economies, providing an explanation for the different low-frequency responses of advanced and emerging economies.

Using a multivariate time series model, we show that a rise in the stochastic volatility of U.S. real interest rates forecasts a decline in the growth rate of the productivity trend, with more pronounced effects in EMEs. We then introduce a parsimonious small open economy model featuring endogenous growth (Ates and Saffie, 2021) and occasionally binding borrowing constraints (Mendoza, 2010), which is subject to both stationary productivity shocks and first- and second-moment shocks to international interest rates (Fernández-Villaverde et al., 2011). Our model provides an endogenous link between interest rate volatility, firm dynamism, and growth that aligns with our empirical findings. Additionally, the model highlights a financial origin behind the key differences in low-frequency productivity dynamics between EMEs and advanced economies, bridging models of internal financial propagation (Garcia-Cicco et al., 2010) and exogenous trend dynamics (Aguiar and Gopinath, 2007). In our framework, stationary fluctuations endogenously trigger non-stationary dynamics in aggregate productivity via interactions with financial frictions. As a result, the heightened responsiveness of EMEs, relative to advanced economies, to U.S. interest rate risks can be attributed to deeper financial frictions in less developed economies.

Our empirical analysis estimates the impact of stochastic volatility shocks to U.S. interest rates on the trend growth rates of GDP, consumption, and investment. First, we apply the particle filter (Fernández-Villaverde et al., 2011) to estimate a time series of stochastic volatility for U.S. interest rates. Second, we impose restrictions common to models with balanced growth paths to isolate trend productivity dynamics from other exogenous stationary fluctuations. Third, we estimate

two panel VAR models to uncover the impact of U.S. interest rate volatility on the low-frequency dynamics of both emerging and advanced economies. A one-standard-deviation increase in U.S. interest rate volatility predicts a significantly larger decline in the trend growth rate of EMEs compared to advanced economies, with this differential effect persisting for several quarters.

To further explore this pass-through mechanism from interest rate volatility to trend growth, we develop a small open economy model that presents a microfoundation for this connection. In the model, firm entry and expansion in the intermediate goods sector drive growth (Klette and Kortum, 2004). The representative household owns all firms and uses the value of its investments as collateral to borrow from international markets. Because firm values are the present discounted value of future profits, an increase in interest rates reduces firm values, lowering the incentives for firm entry and expansion. Reduced business dynamism, in turn, slows economic growth. Consistent with endogenous sudden-stop models (Mendoza, 2010), a Fisherian deflation mechanism is triggered as lower firm values tighten collateral constraints, further raising the household's discount rate. Increases in interest rate volatility signal larger expected shocks in future rates. Due to the asymmetry of borrowing constraints and the predominant role of interest rate shocks in determining collateral values, large interest rate increases are more damaging to collateral and innovation incentives than equivalent decreases. Thus, second-moment shocks to interest rates immediately slow down firm dynamics and reduce the economy's growth rate, with these effects being more pronounced in the presence of severe financial frictions.

We calibrate the model to the Mexican economy to match long-run productivity growth, firm entry rates, and the relative size of incumbent firms. The interest rate and TFP processes from our empirical analysis guide the model's stochastic elements. Despite its simplicity, the model replicates the patterns observed in our empirical results. The occasionally binding borrowing constraint introduces state dependence in the effects of interest rate shocks. Specifically, moderate interest rate shocks have symmetric effects on firm values, whereas large shocks display strong asymmetry: the decline in firm values from a large interest rate increase is 40% larger than the increase from an equivalent decrease. Consequently, when interest rate volatility rises, agents expect larger absolute swings in future rates, reducing expected firm values and depressing firm entry and expansion efforts, thereby slowing endogenous trend growth. This mechanism helps explain the negative correlation between interest rates and TFP, commonly assumed in the EME

literature (Neumeyer and Perri, 2005; Uribe and Yue, 2006). Moreover, the model's focus on stationary shocks and financial frictions allows us to reconcile the findings of Aguiar and Gopinath (2007) and Garcia-Cicco et al. (2010).

Finally, in comparing advanced and emerging economies, we find that approximately three-fourths of the differential pass-through from interest rate volatility to trend growth can be attributed to lower financial development (i.e., lower pledgeability of assets), while the remaining portion is explained by higher financial intermediation costs faced by EMEs during periods of elevated interest rate volatility.

Related Literature: Our focus on the role of global interest rates and their volatility connects us to a large body of work in small open economy models. Mendoza (1991), Neumeyer and Perri (2005), and Uribe and Yue (2006) study the importance of changes in the level of interest rates for fluctuations in small open economies. In order to match the empirical effects of interest rates on output, this literature often assumes working capital constraints and an exogenous correlation between TFP and interest rate fluctuations. Our simple model abstracts from both elements, but the internal propagation of the model delivers both the importance of interest rate shocks and the correlation between productivity and interest rates. Fernández-Villaverde et al. (2011), Gruss and Mertens (2009), Carrière-Swallow and Céspedes (2013), Reyes-Heroles and Tenorio (2020), and Johri et al. (2022) provide evidence for the role of changes in interest rate volatility on the business cycle of emerging economies.[1] We contribute to this literature by showing that interest rate fluctuations not only affect short-run dynamics, but through their effect on trend productivity growth, also trigger hysteresis and low-frequency fluctuations.

Our study is closely related to the literature examining the role of trend shocks to productivity in EME business cycles, which has produced mixed findings (see, for instance, Aguiar and Gopinath (2007), Garcia-Cicco et al. (2010), Chang and Fernández (2013), and Boz et al. (2011)). By linking movements in trend productivity to stationary shocks, models with endogenous growth can reduce reliance on exogenous trend components to explain macroeconomic data. This micro-founded approach aligns more closely with the data and provides a richer framework for policy analysis (Gornemann et al., 2020).[2]

[1] Croce et al. (2012) show that a rise in fiscal uncertainty can lead to a growth slowdown in the U.S. economy.

[2] Some may question the applicability of innovation-driven endogenous growth models to emerging economies. However, these models define innovation broadly, encompassing any process that reduces the marginal cost of produc-

Incorporating endogenous growth improves the capacity of macroeconomic models to account for both business cycle and medium-term dynamics by allowing the productivity trend to respond to stationary shocks (Comin and Gertler, 2006). This addition has been particularly effective in explaining the volatile business cycles and hysteresis observed in EMEs (Cerra and Saxena, 2008). For example, Queralto (2020) demonstrates that a model combining financial frictions and endogenous productivity can account for the medium-term growth effects of the Asian financial crisis. Similarly, Ates and Saffie (2021) develop a creative-destruction model of endogenous growth (Aghion and Howitt, 1992) featuring firm selection dynamics, offering insights into firm entry selection during Chile's 1998 sudden stop. Meanwhile, Gornemann (2014) highlights the role of medium-term dynamics in the context of sovereign defaults in EMEs.

Although some studies have explored models with endogenous financial crises and occasionally binding financial constraints (Benguria et al., 2020; Ma, 2020; Matsumoto, 2022), they do not explicitly link international interest rates to financial collateral. Our model bridges this gap by showing how interest rate shocks influence collateral values, emphasizing the critical role of global risk shocks in shaping low-frequency productivity dynamics.[3]

Within the endogenous growth literature, our work aligns most closely with Aghion et al. (2004), Aghion et al. (2009), and Aghion et al. (2010), who develop theoretical models linking financial constraints to lower long-term growth and greater growth volatility. We build on this tradition by developing a tractable quantitative model that integrates stochastic interest rate volatility, financial development, and growth trend fluctuations into a dynamic stochastic general equilibrium framework suited for quantitative analysis.

In Section 1 we use time series analysis to suggest a link between US interest rate volatility and economic trends in small open economies. Section 2 discusses our economic model and its calibration. Section 3 presents results from this model while Section 4 concludes.

tion—be it through new technologies or process improvements. Moreover, our conclusions remain robust if innovation is interpreted as the costly, forward-looking adoption of frontier technologies. As long as this adoption is endogenous and shaped by interest rate volatility, the mechanisms in our model hold for emerging economies.

[3] Benigno et al. (2020) also connect interest rates to long-run productivity dynamics, arguing that declining interest rates in advanced economies reduce tradable goods production and slow global productivity growth. However, our focus is on stationary shocks rather than secular changes in interest rates.

1 Empirical Analysis

This section provides suggestive evidence linking changes in the volatility of global interest rates–approximated here as U.S. real rates–to persistent changes in economic activity in small open economies using time series models. Specifically, we demonstrate that this relationship is more pronounced for emerging market economies. After describing the data employed, we detail how we extract time series data for the volatility of U.S. real rates. We then outline the multivariate time series model applied to a set of emerging and developed small open economies and present the estimation results.

1.1 Data and Stochastic Volatility in U.S. Real Rates

For the United States, we utilize data on PCE and CPI inflation, real GDP, and Treasury interest rates (3-month, 2-year, 5-year) from FRED. Additionally, we incorporate data on the excess bond premium using the updated series from Gilchrist and Zakrajšek (2012). We pair this with time series data for GDP, consumption, and investment from the IMF's International Financial Statistics for our sample of emerging and developed economies. The period under study runs from 1993Q1 to 2019Q4, terminating before the COVID-19 pandemic affected macroeconomic conditions. Given missing data for some countries, our panel is unbalanced.

We construct a demeaned quarterly time series for the U.S. real interest rate, measured as the real 3-month T-bill rate, for the period of interest.[4] Following Fernández-Villaverde et al. (2011), we adjust nominal rates by the total change in the PCE index over the current and prior three quarters. We estimate the stochastic volatility of the U.S. real interest rate using the particle filter methodology described by Fernández-Villaverde et al. (2011). Specifically, we filter a sequence σ_t from the following model, estimated by maximum likelihood:

$$r_t - \bar{r} = \phi(r_{t-1} - \bar{r}) + \exp(\sigma^r)\varepsilon_t^r, \tag{1}$$

$$\sigma_t^r = (1 - \rho_\sigma^r)\mu_\sigma^r + \rho_\sigma^r \sigma_{t-1}^r + \sigma_r \nu_t^r, \tag{2}$$

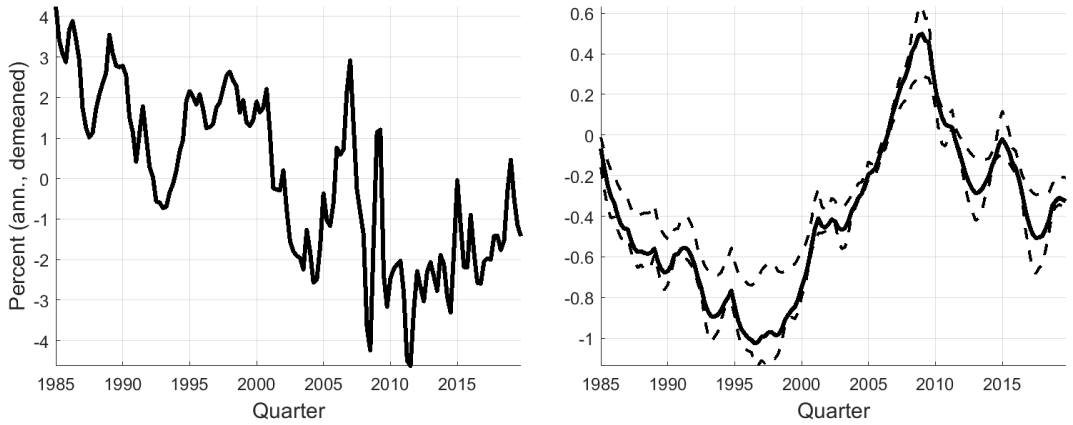
where $\bar{r}$ represents the sample mean of the real interest rate.

[4] Using the 2-year or 5-year interest rate instead does not significantly alter our core findings, as these series are highly correlated over the sample period.

Table 1 presents our parameter estimates.[5] Both the interest rate and its hidden volatility state are highly persistent. Figure 1 displays the real interest rate and the smoothed path of the volatility state.[6] Notably, the hidden volatility state shows substantial variation over time. Volatility declines from 1985 to 2000, corresponding to the relative stability in real rates during this period. Afterward, volatility rises over the next decade, paralleling a persistent decline in the average rate and significant fluctuations around this trend. Post-2010, real rates stabilize again, which is reflected in the stochastic volatility process.

ϕ	ρ_σ^r	$10 * \mu_\sigma^r$	$10 * \sigma_r$
0.941	0.967	1.60	0.50
(0.86,1.00)	(0.87,1.01)	(0.72,1.90)	(0.45,0.59)

Table 1: U.S. Interest Rate Model: Estimated Parameters



(a) U.S. Real Interest Rate (b) U.S. Real Interest Rate Stochastic Volatility

Figure 1: Estimated Series for U.S. Real Rates

1.2 Trend Growth Estimation and Stochastic Volatility Effects

Having derived our measure of time-varying volatility in U.S. interest rates, the next step is to estimate the trend component of GDP. To provide structure for identification, we assume that GDP, consumption, and investment share a common stochastic trend. This assumption mirrors

[5] Standard errors are derived from Monte Carlo simulations. While we later bound persistence parameters to be below 0.999, here we allow estimates to exceed 1.

[6] Error bands are generated via Monte Carlo simulation of the smoothed path. The asymmetry in the bands reflects the asymmetry in confidence intervals for the persistence parameters.

many growth models, both exogenous and endogenous, where these macroeconomic variables grow at the same rate once temporary disturbances subside. We estimate the growth rate of the stochastic trend a_t for a panel of emerging and advanced economies using a state-space model. Our sample spans 1993Q1 to 2019Q4, with country selection based on Vicondoa (2019).

Formally, we assume the joint process for logged GDP growth (Δy_t), logged consumption growth (Δc_t), and logged investment growth (Δi_t) is:

$$\begin{pmatrix} \Delta y_t \\ \Delta c_t \\ \Delta i_t \end{pmatrix} = \begin{pmatrix} \alpha \\ \alpha \\ \alpha \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} a_t + \begin{pmatrix} 1 \\ a \\ b \end{pmatrix} \Delta\lambda_t + \begin{pmatrix} \sigma_t & 0 & 0 \\ 0 & \sigma_c & 0 \\ 0 & 0 & \sigma_i \end{pmatrix} \epsilon_t^m$$

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \sigma_a \epsilon_t^a \\ \lambda_t &= \rho_\lambda \lambda_{t-1} + \sigma_\lambda \epsilon_t^\lambda. \end{aligned}$$

Here, α represents the joint deterministic trend, and a_t is a stochastic trend following an AR(1) process. λ_t is an AR(1) capturing persistent deviations around the trend, resembling typical business cycles. The parameters a and b gauge the responsiveness of consumption and investment relative to GDP, and σ_t allows for independent growth rate deviations.[7]

We then estimate two panel VARs that include (i) country trends, (ii) U.S. GDP growth, (iii) the excess bond premium, (iv) U.S. real rates, and (v) real rate stochastic volatility. Using a Cholesky decomposition assuming that all other variables do not respond immediately to stochastic volatility changes, we extract a shock to stochastic volatility and track its impact on the accumulated trend. Figure 2 shows the results. We observe a significant decline in the extracted trend for emerging economies following a one standard deviation change in stochastic volatility, while the impact on advanced economies is statistically insignificant.

[7] We tested temporary shocks to the levels of the series but found growth rate shocks fit better for most countries.

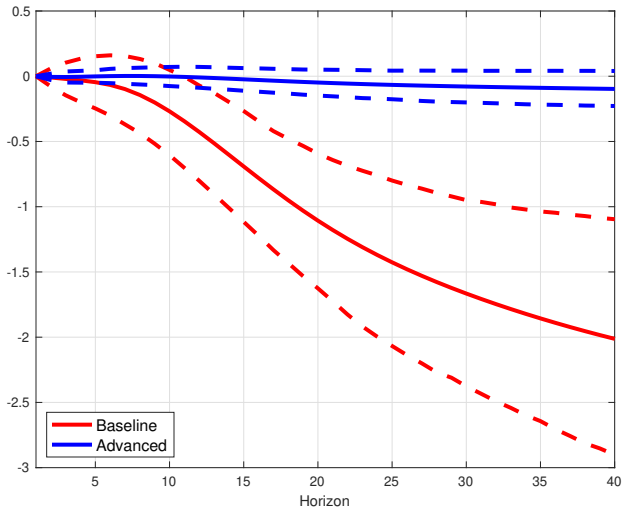


Figure 2: Trend Growth Rate Responses to Stochastic Volatility Shocks

Notes: This figure shows the accumulated response in a_t after a one standard deviation shock to stochastic volatility in the VAR described in the text for the emerging and advanced economy sample, i.e. how much is the level of y_t, c_t, i_t lower after x-quarters after a shock to stochastic volatility.

2 A Simple Model of Endogenous Trend Cycles

2.1 Model Overview

This section introduces a small open economy model that features an occasionally binding borrowing constraint and endogenous growth, driven by both aggregate supply shocks and first and second moment shocks to the international interest rate. The model builds upon traditional endogenous sudden stop frameworks, extending them in three key ways.

First, the representative household in this economy can post the value of intermediate good-producing firms as collateral when borrowing externally. Second, endogenous technical change links the value of this collateral to innovation decisions made by incumbent and entrant firms. Third, both the level (first moment) and volatility (second moment) of international interest rates influence firm dynamics, technological innovation, and the severity of financial frictions.

In summary, a representative household owns a continuum of intermediate good producers and can use their value as collateral to borrow from the global financial market, where the borrowing terms are influenced by a stochastic interest rate process. The entry, exit, and expansion dynamics of these intermediate producers, following the framework of Klette and Kortum (2004),

form the engine of long-run growth. The intermediate goods are sold to a representative final good producer, which operates under the influence of an aggregate efficiency shock. Importantly, the final good is the only tradable product in the economy.

2.2 Households

The representative household chooses consumption C and non-state-contingent bond holdings B each period. It inelastically supplies labor $\bar{L}$ at a wage w and receives lump-sum transfers T from incumbent firms, which it owns. The representative household solves the following problem:

$$\max_{C(s^t),B(s^t)^H} \sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[\frac{1}{1-\gamma}\left[C(s^t)\right]^{1-\gamma} | s_0\right] \tag{3}$$

subject to:

$$C(s^t) + \hat{q}(s^t)B^H(s^t) \leq w(s^t)\bar{L} + B^H(s^{t-1}) + T(s^t) \tag{4}$$

$$\hat{q}(s^t)B^H(s^t) \geq -\phi V(s^t), \tag{5}$$

where $\hat{q}$ denotes the price of debt, which is the reciprocal of the gross interest rate set by financial intermediaries in the economy. The financial constraint prevents households from borrowing more than a fraction ϕ of the value of their collateral, which is equal to the total value of incumbent firms in the economy, V.[8]

2.3 Financial Intermediaries

Households borrow from international financial intermediaries, which are competitive and risk-averse. These intermediaries have a gross funding cost R and lend to households at a price $\hat{q}$. The gross interest rate in international markets follows an AR(1) subject to stochastic volatility given

[8] A key deviation from the literature on endogenous financial crises (Bianchi and Mendoza, 2018) is the use of the household's investment in production as collateral. Using firms' values as collateral is consistent with Private Equity and Venture Capital financing practices. Because the firm's value summarizes future cash flows, this approach also aligns with recent research by Lian and Ma (2021) and Drechsel (2023). Moreover, even traditional bank lending determines the viability of credit based on interviews with entrepreneurs where they describe their projects.

by:

$$R(s^t) - \bar{R} = \rho_R \left(R(s^{t-1}) - \bar{R} \right) + \sigma(s^t)\omega(s^t), \quad \text{where} \quad \omega(s^t) \sim N(0,1), \tag{6}$$

$$\log(\sigma(s^t)) = (1-\rho_\sigma)\mu_\sigma + \rho_\sigma \log(\sigma(s^{t-1})) + \eta_\sigma \nu(s^t), \quad \text{with} \quad \nu(s^t) \sim N(0,1), \tag{7}$$

where $\bar{R}$ is the average cost of funds, ρ_R the persistence of the cost of funds, and σ the volatility of the process. The volatility of R follows an AR(1) with mean μ_σ, persistence ρ_σ, and volatility η_σ.

Intermediaries lend at a price $\hat{q}$, derived from the following optimization problem:

$$\max_{b(s^t)} \mathbb{E}\left[\hat{m}(s^{t+1}) b(s^t) \middle| s^t \right] - \hat{q}(s^t) b(s^t), \tag{8}$$

where $\hat{m}(s^t, s_{t+1})$ corresponds to the intermediaries' stochastic discount factor. To reflect lenders risk aversion, we follow Johri et al. (2022) and assume that the discount factor of financial intermediaries is given by $\hat{m}(s^{t+1}) = \exp(-(R(s^{t+1}) + \iota\sigma(s^{t+1})))$. This functional form allows international invetsors to be more risk averse when invetsing in emerging markets.[9] Thus, the solution to (8) yields the following pricing function:

$$\hat{q}(s^t) = \mathbb{E}\left[\exp(-(R(s^{t+1}) + \iota\sigma(s^{t+1}))) \middle| s^t \right], \tag{9}$$

where $\iota > 0$ reflects that lenders charge more during high volatility times. This pricing function can reflect risk aversion or increasing costs for financial intermediaries when volatility is high.[10]

2.4 Final Good Producer

The final good producer aggregates a mass Λ of differentiated intermediate goods according to the following technology:

$$\ln Y(s^t) = z(s^t) + \frac{1}{\Lambda} \int_0^\Lambda \ln y_i(s^t), di, \tag{10}$$

[9] The formulation for the stochastic discount factor can be seen as a reduced form structure of endogenous outcomes derived from models in the spirit of Gertler and Karadi (2011) and Gertler and Kiyotaki (2015) when allowing stochastic volatility on interest rates. In Appendix A we present a model with these features and show that increases in the volatility of the funding cost of financial intermediaries leads to higher loan interest rates.

[10] A similar structure for the discount factor of financial intermediaries has also been used in the literature. See also Arellano and Ramanarayanan (2012), Bianchi et al. (2018) and Hegarty et al. (2023).

where $Y(s^t)$ is the unique final good, $y_i(s^t)$ is the final good producer's demand for variety i, and $z(s^t)$ is an aggregate efficiency shock that follows:

$$\ln z(s^t) = \rho_z \ln z(s^{t-1}) + \epsilon_t, \quad \text{with} \quad \epsilon_t \sim N(0, \eta_z^2),$$

where ρ_z is the persistence of the aggregate efficiency shock and η_z its volatility.

2.5 Intermediate Good Producers

There is a continuum of intermediate goods indexed by i uniformly distributed in $[0, \Lambda]$. Each intermediate good is produced by the firm with the lowest marginal cost according to the following production function:

$$y_i(s^t) = q_i(s^t) l_i(s^t), \tag{11}$$

where q_i is the efficiency in the production of intermediate goods i, and l_i represents the labor used in production, respectively. The leader of each intermediate good is the producer with the lowest marginal cost, this producer enjoys monopoly rents and competes à la Bertrand with the producer with the second lowest marginal cost. A firm can own and produce several intermediate goods. Firms can expand by becoming leaders in other product lines, contract when they lose their advantage in producing certain intermediate goods, and even exit the market if they lose all their products.

2.5.1 Innovation by Incumbents

The productivity of each intermediate variety evolves endogenously with each innovation. Innovations arise when either an incumbent or a potential entrant successfully improves the existing technology for producing an intermediate good. Thus, when an innovation occurs in variety i the new technological leader has access to a productivity $q_i(s^{t+1}) = (1+\sigma)\tilde{q}_i(s^t)$, where $\tilde{q}_i(s^t)$ is the technology that the former leader had and $\sigma > 0$ is the innovation step size.

An incumbent firm can attempt to innovate and become the new leader in an intermediate product that she does not currently own. To model the innovation technology, we follow Ates

and Saffie (2021)'s discrete time version of Klette and Kortum (2004). In particular, a firm with n products can use research labor $L_r(s^t)$ to generate a success probability $x(s^t)$ per product line according to the following production function:

$$x(s^t) = \xi \left(\frac{L_r(s^t)}{n} \right)^{\theta} \equiv \xi \left(l_r(s^t) \right)^{\theta}. \tag{12}$$

The expansion and destruction probabilities are governed by binomial processes. Therefore, a firm with n products that optimally chooses an innovation rate $x(s^t)$ will win k new products with probability:

$$\mathbb{B}(k, n, x(s^t)) = \binom{n}{k} x(s^t)^k \left(1 - x(s^t)\right)^{n-k}, \tag{13}$$

and lose $\tilde{k}$ products when facing the aggregate creative destruction rate $\Delta(s^t)$ according to:

$$\mathbb{B}(\tilde{k}, n, \Delta(s^t)) = \binom{n}{\tilde{k}} \Delta(s^t)^{\tilde{k}} \left(1 - \Delta(s^t)\right)^{n-\tilde{k}}. \tag{14}$$

Lastly, an incumbent firm is subject to a fixed innovation cost $F(s^t)$, which is contingent upon achieving a successful innovation in a product line. The discrete time problem of an incumbent firm is given by:

$$\begin{aligned} V_n(s^t) &= \max_{x_n(s^t)} n \left[\pi(s^t) - w(s^t) \alpha \left(\frac{x_n(s^t)}{\xi} \right)^{\frac{1}{\theta}} - (1 - \alpha) x_n(s^t) F(s^t) \right] \\ &+ \mathbb{E} \left[m(s^{t+1}) \sum_{\tilde{k}=0}^{n} \mathbb{B}\left(\tilde{k}, n, \Delta(s^t)\right) \sum_{k=0}^{n} \mathbb{B}\left(k, n, x_n(s^t)\right) V_{n-\tilde{k}+k}(s^{t+1}) \middle| s_t \right], \end{aligned} \tag{15}$$

where $m(s^{t+1})$ denotes the stochastic discount factor of the household that owns the firms, and α is a parameter that determines the relative weight of variable and fixed costs of innovation.[11] We denote firm's profits by $\pi(s^t)$ and the equilibrium wage by $w(s^t)$. Note that, because firms are atomistic we can assume that the two binomial processes are independent and, therefore, separable. The combination of the two binomial processes characterizes the probability distribution of a firm with n products to transition in only one period to any number of products in $[0, 2n]$

[11] The innovation investment at time t materializes in product transitions at time $t+1$. Entry of new firms also takes one period to materialize.

instead.[12]

We also allow for an efficiency loss in each variety by depreciating the technology of leaders and followers in every product line at a rate of δ. This depreciation can capture either actual losses in production efficiency or changes in the taste of the consumer as the time passes. Because the depreciation affects every producer within a variety, the relative technological gap remains unaffected.

2.6 Entry of New Firms

To capture entry of new firms, define $M(s^t) \in [0,1]$ as the innovation effort to start a new business. We assume that when an entrepreneur invests $\kappa M(s^t)$ units of labor she succeeds with probability $M(s^t)^\nu$ in becoming a single-product firm next period. As with incumbent firms, there is also a fixed cost F associated to a successful innovation. The entrepreneur solves the following problem:

$$\max_{M(s^t)} \left(M(s^t)\right)^\upsilon \mathbb{E}\left[m(s^{t+1})V_1(s^{t+1})\middle| s_t\right] - \kappa\alpha M(s^t)w(s^t) - (1-\alpha)\left(M(s^t)\right)^\nu F(s^t). \tag{16}$$

The result of this optimization determines the number of new single-product firms in the economy.

2.7 Equilibrium Characterization

The parsimony of the model allows us to unveil the link between interest rate volatility, innovation, and the endogenous trend of productivity. In order to characterize the innovation decisions of firms we need to determine the relevant discounting and the production decision of intermediate good producers. To start, the stochastic discount factor of the household is given by

$$m(s^t, s_{t+1}) = \beta \frac{C(s_{t+1})^{-\gamma}}{C(s^t)^{-\gamma} - \phi\mu(s^t)}, \tag{17}$$

where $\mu(s^t)$ represents the Lagrange multiplier associated with the household's borrowing constraint (5). Household consumption is a function of the income of households, in addition to bond-holding allocations, which are influenced by the borrowing constraint that the representa-

[12] See Appendix B for a detailed description of how to calculate the firm size distribution of the economy.

tive household faces. The value of the collateral that households can post is a function of the value of firms in the economy.

Because the intermediate goods producers act as monopolists, we need the demand for intermediate goods in order to solve for the production decision of intermediate firms. The demand for each variety is given by:

$$y_i(s^t) = \frac{Y(s^t)}{\Lambda p_i(s^t)}. \tag{18}$$

Using this unit elastic demand and assuming Bertrand monopolistic competition at the intermediate good level, we can show that profits and labor per product are independent of the product-specific efficiency $q_i(s^t)$. In particular, we have:

$$\pi(s^t) = \frac{\sigma}{1+\sigma}\frac{Y(s^t)}{\Lambda}, \tag{19}$$

$$l(s^t) = \frac{Y(s^t)}{\Lambda w(s^t)\,(1+\sigma)}. \tag{20}$$

Because firm's decisions are independent of the productivity level of the intermediate products in their portfolio, this framework preserves the proportionality of the value function that has made the Klette and Kortum (2004) so popular in the creative destruction literature. In fact, we can guess and verify that $V_n(s^t) = nV_1(s^t)$. This means that every firm invests the same amount per product. By replacing the guess, factoring the future value of a product and using the properties of the binomial distribution, Equation (15) simplifies to:

$$\begin{aligned} V_1(s^t) &= \max_{x(s^t)} \pi(s^t) - w(s^t)\alpha\left(\tfrac{x(s^t)}{\xi}\right)^{\frac{1}{\theta}} - (1-\alpha)x(s^t)F(s^t) \\ &\quad + \mathbb{E}\left[m(s^{t+1})\left(1-\Delta(s^t)+x(s^t)\right)V_1(s^{t+1})\big|\, s_t\right]. \end{aligned} \tag{21}$$

Therefore, the optimal innovation effort per product line is independent of the total number of products that the firm has:

$$x^*(s^t) = \left[\frac{\theta}{\alpha}\xi^{\frac{1}{\theta}}\frac{\mathbb{E}\left[m(s^{t+1})V_1(s^{t+1}) - (1-\alpha)F(s^t)\big|\, s_t\right]}{w(s^t)}\right]^{\frac{\theta}{1-\theta}}. \tag{22}$$

Having characterized the value of each intermediate product line, we can solve for the mass of new firms:

$$M^*(s^t) = \left[\frac{\upsilon}{\alpha\kappa} \frac{\mathbb{E}\left[m(s^{t+1}) V_1(s^{t+1}) - (1-\alpha) F(s^t) \middle| s_t \right]}{w(s^t)} \right]^{\frac{1}{1-\upsilon}}, \tag{23}$$

The endogenous creative destruction rate is composed by the innovation of incumbent firms and new entrants, therefore:

$$\Delta(s^t) = \frac{\left(M^*(s^t)\right)^{\upsilon}}{\Lambda} + x^*(s^t) \tag{24}$$

Having the aggregate rate of creative destruction we can derive the endogenous productivity growth in the economy. In fact, replacing Equation (11) in Equation (10) we obtain:

$$Y(s^t) = e^{z(s^t)} A(s^t) l(s^t), \tag{25}$$

where $A(s^t)$ is the endogenous productivity level defined by:

$$\ln A(s^t) = \frac{1}{\Lambda} \int_0^{\Lambda} \ln q_i(s^t) di. \tag{26}$$

Endogenous productivity growth is generated by creative destruction. Moreover, the productivity of an intermediate good production only increases by a factor $(1+\sigma)$ when it is subject to the creative destruction rate $\Delta(s^t)$:

$$\begin{aligned} & \ln(A(s^t, s_{t+1})) - \ln(A(s^t)) = \Delta(s^t) \ln(1+\sigma) + \ln(1-\delta) \\ \text{or} \quad & \frac{A(s^t, s_{t+1})}{A(s^t)} = 1 + g\left(s^t, s_{t+1}\right) = (1+\sigma)^{\Delta(s^t)} (1-\delta), \end{aligned} \tag{27}$$

where $g\left(s^t, s_{t+1}\right)$ is the growth rate of the endogenous productivity index. When a shock triggers fluctuations in $\Delta(s^t)$, we will observe fluctuations in the endogenous productivity growth rate. Note that, endogenous productivity increases if and only if creative destruction overtakes the depreciation rate δ. Thus, this feature allows the model to potentially generate a negative growth rate.

We can further characterize the equilibrium by combining Equations (25) and (20) to solve for the equilibrium wage:

$$w(s^t) = \frac{e^{z(s^t)} A(s^t)}{\Lambda (1+\sigma)}. \tag{28}$$

It remains to characterize the optimality conditions for the representative household's problem. The first order conditions yield the following equilibrium conditions:

$$C(s^t)^{-\gamma} = \mu(s^t) + \beta \hat{R}(s^t) \mathbb{E}\left[C(s^{t+1})^{-\gamma} | s^t \right] \tag{29}$$

$$C(s^t) + \hat{q}(s^t) B^H(s^t) \leq w(s^t)\bar{L} + B^H(s^{t-1}) + \Lambda e(s^t) - \kappa \alpha M(s^t) w(s^t) - (1-\alpha) \left(M(s^t) \right)^{\nu} F(s^t), \tag{30}$$

with $\hat{R}(s^t) = 1/\hat{q}(s^t)$. Equation (29) corresponds to the Euler equation of bond holdings and Equation (30) describes the budget constraint of the household replacing the expression for lump-sum transfers from firms in the economy. Note that Equation (21) implies $e(s^t) = \pi(s^t) - w(s^t)\alpha \left(\frac{x(s^t)}{\xi} \right)^{\frac{1}{\theta}} - (1-\alpha) x(s^t) F(s^t)$.

Lastly, the borrowing constraint faced by the representative household is

$$\hat{q}(s^t) B^H(s^t) \geq -\phi \Lambda V_1(s^t), \tag{31}$$

where the total value of firms in the economy V is equal to the mass of differentiated intermediate goods Λ times the value of owning one product line V_1. This is a consequence of the linearity of the value of a firm in the number of product lines it owns.

2.8 Trend Dynamics and Interest Rates

Equation (27) shows that stationary distortions affecting the endogenous rate of creative destruction (Δ) can trigger permanent losses in the level of the productivity index defined in Equation (26). Because that index determines the long-run growth rate of the economy, every non-stationary variable is normalized by it. Thus, fluctuations in the creative destruction rate generate hysteresis in output and consumption. In fact, in this class of models, stationary shocks can trigger non-stationary dynamics due to the internal propagation of the model. To understand the drivers of

these endogenous trend dynamics we need to study how different shocks affect the determinants of building blocks of creative destruction and, thus, how aggregate shocks drive firm entry and the expansion decision of incumbents. Equations (22) and (23) provide guidance when studying these drivers.

The dynamics of the two components of creative destruction are due to two endogenous objects. First, the dynamics of the stochastic discount factor of the household. Second the relative future value of a product normalized by the current wage of the economy. Let us focus first on the effect of a TFP shock. Because TFP shocks are very persistent, they have only fairly small effect on the household's stochastic discount factor. Consequently, TFP shocks typically do not significantly influence productivity growth through this channel. Regarding the second channel, Equation (28) shows that a TFP shock directly increases wages, increasing the denominator of the relative future value of a product normalized by the current wage. Nevertheless, as seen in Equation (21) the value of owning a product is given by the expected discounted value of profits nets of innovation costs, and both profits and R&D costs increase with a TFP shock. Once again, given the persistence of TFP shocks, the ratio of future profits and future output to current wages conditional on a TFP shock is relatively stable. Thus, TFP shocks are not a promising channel to generate endogenous trend dynamics.

In contrast to a TFP shock, an interest rate shock does not directly impact either profits or the cost of innovation.[13] Therefore, the numerator and denominator of the relative future value of a product normalized by the current wage of the economy are mostly unaffected by an interest rate shock. Nevertheless, the interest rate shock generates fluctuations in the stochastic discount factor of the household. Equation (29) illustrates this. Specifically, we see that increases in the financing cost faced by households leads to a decline in the term $\frac{C(s^{t+1})^{-\gamma}}{C(s^t)^{-\gamma}-\mu(s^t)}$, implying that firm will discount the future more heavily. Hence, the innovation intensity of incumbent and entrants fall (see equations (22) and (23)), negatively impacting the creative destruction rate of the economy. Thus, an interest rate shock has a direct impact on the innovation rate of the economy and constitutes the main driver of the endogenous productivity trend.

It is important to note that both TFP and interest rate shocks can influence the rate of creative

[13]Interest rate shocks do not affect profits or wages because we abstract from working capital constraints. With a working capital constraint, an interest rate shock affects wages and values similarly to a TFP shock. Consequently, the effect of interest rate shocks working through working capital constraints on endogenous trend dynamics is small.

destruction by triggering current or future tightening of the collateral constraint (31). When the constraint is binding, households are forced to delever, which reduces current consumption and increases the discounting of future returns. This, in turn, lowers the value of firms, further tightening the constraint and potentially leading to a Fisherian deflationary spiral. Thus, while it is interest rate shocks the main drivers of trend dynamics, large (TFP or interest rate) shocks that increase the likelihood of future crises can have permanent effects mediated by the discount factor and hence on the creative destruction rate.

In light of the importance of interest rate shocks and the potential triggering of Fisherian deflationary dynamics, we can understand the mechanism that links interest rate volatility and the trend of productivity. To understand this it is helpful to consider an initial situation where the borrowing constraint is slack. As volatility increases, households anticipate that future interest rate fluctuations will be larger in magnitude. While significant interest rate reductions increase the value of firms, larger interest rate hikes can decrease it by a larger magnitude (in absolute value). Specifically, interest rate reductions have a direct effect on the value of collateral, whereas interest rate hikes not only impact collateral value but can also trigger a Fisherian deflation mechanism. Therefore, even in the absence of concurrent interest rate shocks, and increase in interest rate volatility is sufficient to slow down innovation due the asymmetry between the effects of large interest rate hikes and reductions. this means, when the volatility of interest rates increases, the household understands that future interest rate shocks will be larger. Because increases in interest rates are more harmful than the benefit provided by a reduction of the same absolute magnitude, firm's value and productivity growth slow down in response to a pure volatility shock, even in absence of any first moment shock.

3 Quantitative Analysis

In this section, we demonstrate that, despite its simplicity, the model replicates the differential sensitivity of the productivity trend to interest rate volatility between emerging and advanced economies. First, we calibrate the model for both Mexico (an emerging economy) and a financially advanced version of this emerging economy. Importantly, all shock processes are common to both economies, with the only calibration difference being the level of financial frictions. Sec-

ond, using generalized impulse responses, we show that, consistent with the analysis in Section 1, the response of the productivity trend to interest rate risk in emerging markets is an order of magnitude larger than that in advanced economies. Third, we identify asymmetries in firms' value responses and in the endogenous trend to large interest rate shocks of the same absolute magnitude as the key driver of this differential response. Finally, we establish that the difference in asymmetry between emerging and advanced economies stems from their differing levels of financial development. As a result, the same aggregate financial risk can propagate very differently, particularly when international investors exhibit higher risk aversion towards emerging markets and the capacity to collateralize future cash flows is more constrained.

3.1 Calibration: Heterogeneous Financial Frictions

We calibrate our model to match features of Mexico, at a quarterly frequency. The model has 20 parameters to calibrate. We divide these parameters into two groups. The first set of 11 parameters is externally calibrated to common values in the literature and used to match time series dynamics of the Mexican economy. Table 2 shows the externally calibrated values. First, we estimate the process of the U.S. interest rate directly from the data, as described in Section 1. We set the average interest rate to 4% annually, which is equivalent to roughly 1% every quarter. We directly estimate the productivity process for Mexican data.

We set the innovation curvature of incumbents and entrants equal to 0.5, a value standard in the endogenous growth literature (Akcigit and Kerr, 2018). We assume a CRRA utility function with a curvature of $\gamma = 2$, which is also standard in the macro literature. The last externally calibrated parameter is the depreciation of ideas, which is set to be equal to 1.5% every quarter. From a growth perspective, this value implies a lower bound for productivity growth of -1.5% per quarter. This is, without any investment in the creation of new firms and expansion by incumbents, the efficiency index would shrink at rate δ.

Parameter	Symbol	Value	Source
Average US Interest Rate	$\bar{R}$	$(1.04)^{0.25}$	Estimation, US data
Persistence of Interest Rate	ρ_R	0.94	Estimation, US data
Average Interest Rate Volatility	μ_σ	0.16	Estimation, US data
Persistence of Interest Rate Volatility	ρ_σ	0.97	Estimation, US data
Volatility of Interest Rate Volatility	η_σ	0.05	Estimation, US data
Persistence of TFP Shock	ρ_z	0.95	Estimation, Mexican data
Volatility of TFP Shock	η_z	0.02	Estimation, Mexican data
Innovation curvature cost by incumbents	θ	0.50	Akcigit and Kerr (2018)
Innovation curvature cost by entrants	υ	0.50	Akcigit and Kerr (2018)
Utility function curvature	γ	2	Standard
Depreciation of ideas	δ	0.01	Max A contraction

Table 2: Externally Calibrated Parameters

Table 3 presents a summary of our 9 internally calibrated parameters. The step size σ is set to 0.23, which implies an average annualized growth rate close to 2%, consistent with Mexican data. We set the cost level of entry κ to 18.2, which generates an annualized 12% entry rate, consistent with Mexican empirical evidence. The R&D cost level ξ is set to 0.49 so that the average incumbent firm size relative to entrants is 3. Regarding the fixed cost structure of innovation, we set $\alpha = 0.8$ so that total labor R&D costs account for two-thirds of total R&D costs (Moris and Shackelford, 2020), and $F = 0.2$ so that total R&D expenses relative to firm value amount to 5%, where $F = F(s^t)/A(s^t)$. The mass of products Λ is normalized to 3, ensuring a unit mass of firms in the balanced growth path. We set the discount factor β to 0.95, generating an implied net-foreign-assets-to-GDP ratio close to 33% annually, in line with the long-run average for Mexico presented in Lane and Milesi-Ferretti (2017). We calibrate the collateral coefficient of the borrowing constraint ϕ to 0.67, so that the model produces an annual probability of a crisis close to 3%, consistent with the Sudden Stops literature.[14] Lastly, we set ι, the parameter governing the representative financial intermediary's discount factor, to 2.47 in order to generate an average spread of 3%, consistent with observed Mexican data.

Due to the nonlinear nature of a model featuring occasionally binding constraints, we rely on global methods to solve it. We employ a hybrid method that combines time iteration with value function iteration. Appendix C provides a detailed description of the solution method.

[14] We define a crisis or a Sudden Stop as an event where the current account-to-GDP ratio is two standard deviations above its long-run mean and where the borrowing constraint binds. This definition is standard and widely used in the literature (Mendoza, 2010; Bianchi et al., 2016; Mendoza and Rojas, 2019; Rojas and Saffie, 2022).

Parameter	Symbol	Value	Main Identification	Target
Step Size	σ	0.23	Annual Growth rate	2%
Cost level Entry	κ	18.2	Annual Entry rate	12%
Cost level R&D	ξ	0.49	Avg. Firm Size relative to entrants	3
Variable Cost Share R&D	α	0.80	R&D wage bill to total R&D costs	1/3
Fixed Cost R&D	F	0.20	Total R&D costs to firm value	5%
Mass of Products	Λ	3	Unit mass of firms	N/A
Discount Factor	β	0.95	Annual Debt-to-GDP Ratio	33%
Borrowing Constraint	ϕ	0.67	Annual Prob. of Crisis	3%
Financial Intermediary Discount Factor	ι	2.47	Avg. Spread	3%

Table 3: Internally Calibrated Parameters

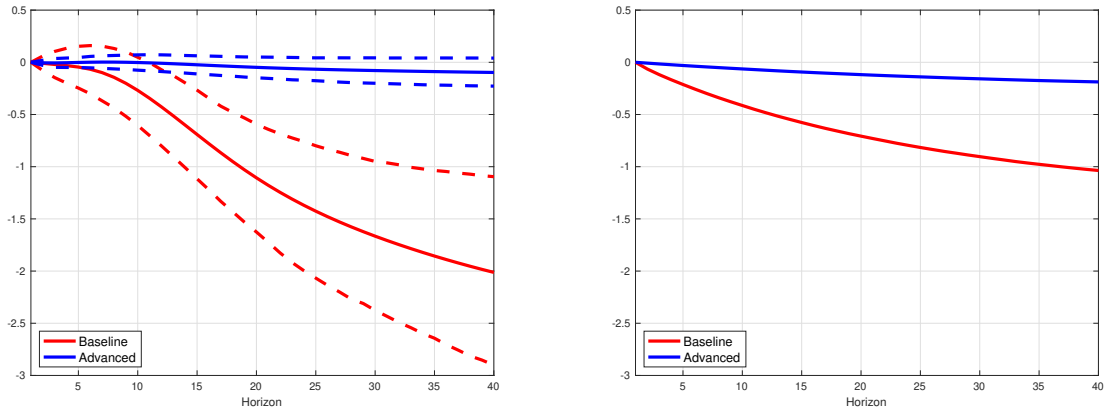
To highlight the importance of heterogeneous financial frictions, we recalibrate only two parameters from Table 3. Specifically, we recalibrate the parameter governing the risk aversion of international lenders (ι) and we jointly recalibrate the pledgeability of firms' values (ϕ). We set $\phi = 0.79$ so that the economy experiences a financial crisis with a 1% probability, and $\iota = 0.85$ so that the average spread is 1%, roughly consistent with targets used for developed economies (Bianchi and Mendoza, 2020). For concreteness, the reader can think of this open advanced economy as resembling Canada.

3.2 Model Meets Data: Stochastic Volatility and Endogenous Trend Dynamics

We use the main empirical fact described in Section 1 to validate the calibrated model. In particular, we estimate generalized impulse response functions from model-simulated data, following the approach of Gilchrist et al. (2014), where the shock is a one standard deviation increase in the volatility of the world interest rate (the model-equivalent of the U.S. real interest rate). We estimate the responses of the accumulated trend by simulating two economies: one that experiences a volatility shock in period T and one that does not, calculating the difference between the two time series. We simulate this exercise 20,000 times for 390 periods in each case, discarding the first 350 periods to eliminate any dependence on initial conditions. We perform this estimation with data simulated for both calibrations, using the same underlying sequence of shocks.

Figure 3 presents the impulse response functions of the trend component to a one standard deviation stochastic volatility shock, comparing the empirical and model-simulated approaches. The responses in panel (b) of Figure 3 reveal two key patterns: (1) the trend component in the developed economy (i.e., where the average spread is lower and the financial sector is more de-

veloped) experiences significantly less negative effects from a stochastic volatility shock than its counterpart, and (2) the magnitude of the decline for the advanced economy calibration is consistent with the empirical findings, while the decline for the baseline calibration is about 50% (at its trough) of its empirical counterpart. Because these responses were not a target of our calibration, we consider a success that our parsimonious model is able to explain the dynamics of trend growth in response to stochastic volatility shocks in U.S. interest rates.



(a) Empirical (b) Model

Figure 3: Impulse Response Functions

Notes: Panel (a) presents the impulse response functions of the accumulated trend to a one standard deviation stochastic volatility shock of the U.S. interest rate, for advanced and emerging economies. These responses are estimated following the approach described in Section 1. Panel (b) estimates responses following a similar approach but using model-simulated data.

3.3 Risk Matters: Asymmetric Responses to Interest Rate Shocks

The results shown in Figure 3 demonstrate that volatility shocks generate persistent effects on the economy's trend, which are more pronounced for emerging economies. The central question now is to understand the mechanism that links financial risk and trend dynamics. By definition, periods of high interest rate stochastic volatility are characterized by larger fluctuations in interest rates. Conditional on observing an interest rate hike (or drop), the average size of an interest rate increase (or decrease) is larger when volatility is higher.[15] In other words, during periods of high financial risk, interest rate shocks are larger on average, but symmetric (i.e., hikes and drops are

[15]To clarify, the average hike conditional on $\sigma(s^t)$ is $\mathbb{E}[\omega(s^t)\sigma(s^t)|\omega(s^t) > 0, \sigma(s^t), s^t]$. We use the same notation as in Section 2. Since $\omega(s^t) \sim N(0,1)$, we apply the truncated normal formula for a conditional expectation and find that $\mathbb{E}[\omega(s^t)\sigma(s^t)|\omega(s^t) > 0, s^t] = \sigma(s^t)\frac{\phi(0)}{1-\Phi(0)}$. Conditional on an interest rate hike, the expected increase is positively correlated with interest rate volatility $\sigma(s^t)$. The same reasoning applies to interest rate cuts.

of the same size in absolute terms). Interestingly, this symmetry in interest rate fluctuations does not necessarily result in symmetric responses within our framework. In fact, an interest rate hike may cause a much larger adjustment in the trend growth rate than an interest rate cut of the same absolute size.

Financial frictions are crucial to explaining the asymmetric response of the economy to large interest rate shocks. When the constraint binds, an interest rate hike triggers household deleveraging, which reduces present consumption, increases the discounting of future values, and ultimately decreases both the trend growth rate and the value of a product line. Our model incorporates a Fisherian deflationary mechanism that amplifies this cycle: as the value of the product line declines, so does the collateral value of the household. This further tightens the borrowing constraint, reinforcing the cycle and generating significant nonlinearities that lead to the aforementioned amplification. Conversely, when households experience an interest rate cut, the amplification mechanism is much weaker, as the multiplier is bounded by 0. Thus, a more volatile interest rate foreshadows a persistent period of large interest rate swings. Due to the asymmetric effects of these swings, the expected value of a firm decreases when the volatility regime changes, even without a concurrent interest rate shock. This decline in expected firm value explains the slowdown in innovation efforts by incumbents and entrants.

To measure the effect of symmetric interest rate changes, we simulate our model and identify periods in which interest rate changes (hikes or cuts) of one standard deviation occurred. We isolate these events and compute the changes between the period when the interest rate hike or cut occurred and the period preceding it for the endogenous trend growth, the expected discounted value of a product line, and the Lagrange multiplier of the borrowing constraint. It is important to note that the computed changes correspond to the model's endogenous outcomes. Table 4 presents the results of this exercise, which is performed for both our emerging and advanced economy calibrations.

The upper panel of Table 4 shows how the endogenous trend growth, the expected discounted value of a product line, and the Lagrange multiplier are affected in response to hikes or cuts in the interest rate for our baseline calibration.[16] We examine the responses of these endogenous variables under three volatility regimes. While interest rate hikes and drops have the same absolute

[16]Changes are standardized, so they represent deviations from their corresponding long-run means.

magnitude for a given regime, the changes are larger when volatility is high.

Focusing on the emerging economy's response, under a low volatility regime, interest rate drops are more beneficial for the endogenous productivity trend than the decreases caused by interest rate hikes. To understand this asymmetry, note that in a model without an occasionally binding constraint, the economy should benefit from dispersion in interest rate shocks. In fact, because the discount factor is concave with respect to the interest rate, by Jensen's inequality, interest rate hikes have lower absolute effects on the discount factor than interest rate drops of the same magnitude.

In contrast, under the high volatility regime, this asymmetry is reversed: interest rate hikes have larger absolute effects on the endogenous trend than interest rate drops of the same magnitude. Although the discount factor remains concave with respect to the interest rate for a given level of the Lagrange multiplier associated with the borrowing constraint, Jensen's inequality is overpowered by the Fisherian deflationary effect of higher interest rates. In fact, the Lagrange multiplier amplifies the impact of interest rate hikes on the stochastic discount factor, generating a symmetric effect of interest rates on firm values and growth.

Turning our attention to the bottom panel of Table 4, we can clearly see the predominant role of financial frictions by studying the dynamics of the advanced economy. Despite both economies being subject to the same shocks, the advanced economy is primarily influenced by Jensen's inequality in the low and medium volatility regimes. Even under the high volatility regime, the effects of interest rate shocks are practically symmetric across all growth-relevant endogenous variables.

Returning to the role of volatility shocks, when interest rate risk increases, the representative household understands that future interest rate shocks will be, on average, larger in absolute terms. For the emerging market, this means that future growth will be lower on average. Consequently, the household adjusts its consumption and savings behavior today, leading to an immediate decrease in the endogenous productivity trend, even when the economy does not experience any first-moment shock. This contrasts with much of the literature on uncertainty shocks (Bloom, 2009; Vavra, 2014), where second-moment shocks are often assumed to be correlated with first-moment realizations.

Emerging	Δg		$\Delta\mathbb{E}(mV_1)$		$\Delta\mu$		ΔR	
	Hike	Drop	Hike	Drop	Hike	Drop	Hike	Drop
Low Volatility	-0.03	0.12	0.00	0.02	-0.03	-0.05	0.65	-0.65
Average Volatility	-0.23	0.25	-0.13	0.14	0.01	-0.01	0.72	-0.72
High Volatility	-0.40	0.29	-0.24	0.16	0.03	0.01	0.78	-0.78
Advanced								
Low Volatility	-0.06	0.10	0.00	0.02	-0.04	0.00	0.65	-0.65
Average Volatility	-0.11	0.13	-0.04	0.04	-0.08	0.06	0.72	-0.72
High Volatility	-0.16	0.15	-0.06	0.05	-0.12	0.09	0.78	-0.78

Table 4: Interest Rate Fluctuations and Stochastic Volatility

Notes: Changes in the trend growth rate g, expected discounted value of a product line $\mathbb{E}(mV_1)$, Lagrange multiplier of the borrowing constraint μ, and world interest rate R are expressed in standard deviations. We identify periods with interest rate hikes and drops in our model-simulated data and average changes in the reported variables at the moment of the interest rate movement. We record events where the interest rate change is at least 0.5 (long-run) standard deviations. Low (high) volatility regimes are defined as those in which the volatility of the interest rate is lower (higher) than its long-run mean minus (plus) one standard deviation. Changes of variables are standardized, so they represent standard deviations from their corresponding long-run averages.

To further understand the role of volatility shocks in shaping the dynamics of emerging and advanced economies, we conduct an experiment where we turn off these shocks and set the volatility of the world interest rate to its ergodic mean. This allows us to isolate the effects of volatility on long-run growth moments and assess the extent to which financial systems in both types of economies are affected. Table 5 presents the selected long-run moments for our baseline and advanced economy calibration across the two volatility regimes.

Moment	Mexico		Advanced Mexico	
	SV	No SV	SV	No SV
$\sigma(g)/\mathbb{E}(g)$	0.19	0.16	0.06	0.05
$\sigma(CA/Y)$ %	3.57	3.52	0.90	0.89
$\mathbb{E}(B^H/Y)$ %	-32.75	-33.53	-43.87	-43.96
Spread %	3.00	3.00	1.00	1.00
Prob. Crisis %	2.52	2.78	0.62	0.62

Table 5: Volatile Rates and Volatile Trends: Deterministic Volatility

Notes: SV denotes the scenario with stochastic volatility in the world interest rate. In the No SV case, we set the variance of the world interest rate to be equal to its long-run average.

For emerging economies, the results in Table 5 show a notable reduction in the coefficient of variation of trend growth by about 16%, emphasizing the significant role that volatility shocks play in driving fluctuations. The advanced economy sees a similar change, indicating that volatility shocks have comparable effects on long-term growth in these economies. Why is it that shutting down interest rate volatility leads to similar effects on trend growth instability across emerging

and advanced economies? To answer this question notice that in the case of the baseline calibration, the stability gained by eliminating volatility shocks leads to more borrowing and increased exposure to financial crises. While the removal of volatility reduces the need for precautionary savings, this increased borrowing partially offsets the stability benefits by exposing the economy to higher financial risk. These findings suggest that the core challenge for emerging economies might not be the volatility itself but how they respond to it. The next section will explore in detail the role that financial factors play in understanding risk-taking behavior of agents in the economy.

3.4 Financial Risk and Financial Frictions

The previous section highlighted the asymmetrical responses of trend growth to volatility shocks in world interest rates across different economies. The main drivers of these asymmetries are the interaction between the tightness of borrowing constraints and borrowing costs (i.e., higher spreads). In this section, we aim to disentangle the individual contributions of these two factors to the dynamics of trend growth.

We begin by constructing an alternative economy that represents an intermediate case between the baseline (calibrated to an emerging economy) and the advanced economy calibration. Specifically, we relax the borrowing constraint in the baseline economy by setting the collateral coefficient, ϕ, equal to that of the advanced economy, while keeping all other parameters unchanged. We refer to this calibration as the "relaxed constraint" case. Thus, the key difference between the baseline and the relaxed constraint calibration is the increased collateral pledgeability in the latter, aligned with that of an advanced economy. This allows us to isolate the effects of the borrowing constraint on long-run economic outcomes. Any remaining differences between the long-run outcomes of the relaxed constraint and advanced economy scenarios can then be attributed to the impact of higher spreads.

Table 6 presents selected long-run moments of the three economies. Relaxing the borrowing constraint brings substantial changes to the long-run dynamics of the economy. Growth volatility, measured by the coefficient of variation, is reduced by 47%, and the volatility of the current account to GDP decreases by 50%. The External Financing Premium on Debt (EFPD) is defined as $\mathbb{E}\left[R^B(s^{t+1}) - \hat{R}(s^t)\right] = \mu(s^t)/\mathbb{E}[\lambda(s^{t+1})|s^t]$, a model-based measure of the spread faced by domestic households relative to the risk-free rate. Interestingly, while standard small open econ-

omy models typically assume a negative correlation between spreads and productivity growth (Neumeyer and Perri, 2005), our model generates this correlation endogenously for emerging markets. Moreover, the sign reverses when financial constraints are relaxed.

Moment	Mexico	Relaxed Constraint Mexico	Advanced Mexico
$\mathbb{E}(\text{Spread})$ %	3.00	3.00	1.00
$\sigma(g)/\mathbb{E}(g)$	0.19	0.09	0.06
$\sigma(CA/Y)$ %	3.57	1.75	0.90
$corr(EFPD, g)$	-0.41	0.44	0.58

Table 6: Financial Frictions and Business Cycles

When comparing the relative contributions of international risk aversion (ι) and differences in pledgeability (ϕ), we find that relaxing the borrowing constraint accounts for 77% of the difference in the coefficient of variation of the endogenous trend between emerging and advanced economies. Thus, while emerging markets exhibit cycles consistent with low-frequency fluctuations (Aguiar and Gopinath, 2007), stationary shocks coupled with financial frictions can also generate these dynamics (Garcia-Cicco et al., 2010). Moreover, our framework also provides a financial micro foundation for why returns are so risky in emerging economies even under a common global risk factor (David et al., 2014).

An important feature of our model, tied to the financial frictions within the framework, is the occurrence of Sudden Stops—episodes marked by sharp reversals in the current account, coupled with severe contractions in consumption and output. These events emerge from the presence of occasionally binding collateral constraints and endogenous collateral values, which trigger a Fisherian deflation mechanism during periods of stress (Mendoza, 2005; Mendoza, 2010). In contrast to typical business cycle fluctuations, Sudden Stops in our model have persistent effects on output due to the incorporation of endogenous growth. This persistence aligns with findings from other models that integrate financial frictions into growth dynamics (Queralto, 2020; Ates and Saffie, 2021). Appendix D presents event studies comparing Sudden Stops in emerging and advanced economies, calibrated to our model. Our results highlight the critical role of financial frictions in explaining why emerging markets experience more frequent and severe Sudden Stops, with more hysteresis than their advanced economy counterparts.

4 Conclusion

In this paper, we provide new empirical and theoretical evidence linking temporary surges in global interest rate uncertainty to permanent losses in economic activity, particularly in emerging market economies (EMEs). Our analysis shows that uncertainty shocks have a disproportionately larger impact on trend growth in EMEs compared to advanced economies (AEs), a disparity we attribute to financial frictions. To explain this, we develop a small open economy model with endogenous growth and occasionally binding borrowing constraints, where firms' intangible assets serve as collateral for borrowing. We differentiate EMEs from AEs in two key dimensions: (1) the fraction of assets that can be pledged as collateral, and (2) the degree of lender risk aversion.

Our model successfully replicates the asymmetric impact of interest rate volatility on growth trends observed in the data, without resorting to exogenous trend shocks or artificially correlating total factor productivity (TFP) with U.S. real interest rates. One of our main findings is that large interest rate hikes lead to significantly sharper declines in trend growth compared to rate decreases of a similar magnitude. This asymmetry becomes more pronounced during periods of heightened volatility, as firms and households, constrained by tightening borrowing limits, anticipate more extreme future financing costs.

By decomposing the drivers of this asymmetric response, we find that financial development accounts for roughly 75% of the volatility in trend growth between EMEs and AEs, with the remainder explained by the higher risk premiums (spreads) faced by EMEs. This suggests that rather than primarily worrying about global risk and U.S. rate volatility, emerging markets should focus on enhancing their financial development to improve resilience to such shocks.

Our findings emphasize the critical role of financial development and risk management in EMEs, highlighting the potential for macroprudential policies to bolster their stability in the face of uncertainty. Future research could investigate how macroprudential policy frameworks might differ between advanced and emerging economies, even in the context of shared global shocks. Additionally, this framework can also be used to explore how firm dynamics, such as economies with higher entry rates or with more disperse size distributions, could shape the heterogeneous responses of trend growth to global uncertainty shocks.

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Appendices

Appendix A A Simple Model with a Financial Accelerator

In this appendix we characterize a simple model following Gertler and Karadi (2011), Gertler and Kiyotaki (2015), and Akinci et al. (2022), where financial intermediaries in the economy face stochastic volatility in deposit rates. The rest of the features of the model are standard in this literature.

A.1 Model Setup

The optimization problem of an individual bank is

$$\begin{aligned} V_{i,t} &= \max_{L_{i,t}, D_{i,t}} E_t \left[\Lambda_{t,t+1} \left((1-\sigma) N_{i,t+1} + \sigma V_{i,t+1} \right) \right] \\ \text{subject to} \quad & V_{i,t} \geq \theta L_{i,t} \\ & N_{i,t} = R^l_{t-1} L_{i,t-1} - R^d_{t-1} D_{i,t-1} \\ & N_{i,t} + D_{i,t} = L_{i,t} \end{aligned}$$

We assume that the loan demand is given by $L(R^l_t) = A - B \cdot R^l_t$. The bond interest rate follows the stochastic process:

$$\begin{aligned} R^d_t &= R^d + \rho_1 (R^d_{t-1} - R^d) + \sigma_{t-1} \epsilon_t \\ \sigma_t &= \bar{\sigma} + \rho_2 (\sigma_t - \bar{\sigma}) + \bar{\sigma} \eta_t. \end{aligned}$$

Lastly, $\Lambda_{t,t+1}$ is the relevant discount factor and $\Lambda_{t,t+1} = \frac{1}{R^d_t}$.

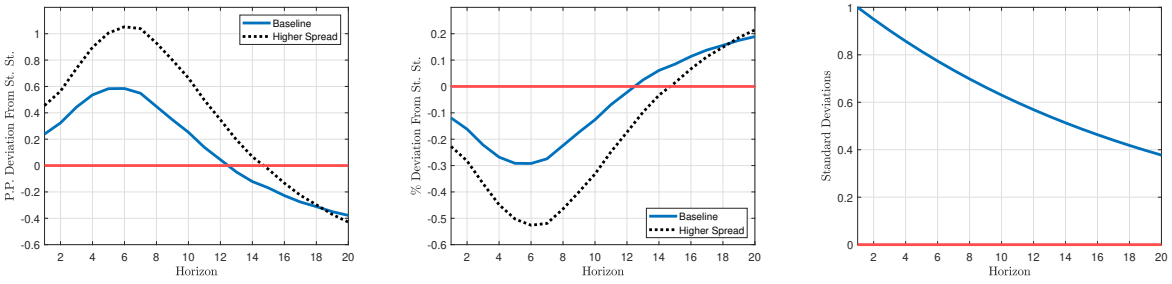
A.2 Quantitative Results

We calibrate the model to match features of the U.S. economy. Table 7 presents the externally calibrated parameters, along with their corresponding values and sources in the data.

Parameter	Symbol	Value	Source
Deposit Rate	R^d	1.005	US data
Demand Parameters	A, B	Internal	$L = 1$
Exit Probability	$1 - \sigma$	0.03	US data
Leverage Ratio	L/N	5	US data
Loan Rate	R^l	1.0075	US data

Table 7: Calibration Summary

The rest of the parameters, Ω, ψ, ν, and θ, are set internally such that equilibrium conditions are satisfied. Figure 4 presents the impulse response functions of the model in response to a one standard deviation increase in interest rate volatility. Responses appear as percentage point deviations from their corresponding steady state values. We also add the responses of an alternative calibration (black dotted line). This calibration assumes that the average loan-deposit rate spread is higher than in the baseline scenario ($R^L = 1.0085$). This assumption is consistent with an economy that features a larger value of the parameter ι, which rules the average spread faced by agents in the economy.



Loan Interest Rate R^l | Loans L_t | Stochastic Volatility σ_t

Figure 4: Impulse Response Functions to a Stochastic Volatility Shock

The impulse response functions of the baseline case presented in Figure 4 show that an increase in interest rate volatility leads to an increase in the borrowing costs faced by agents in the economy, which ultimately leads to a decrease in borrowing. Interestingly, the exercise also shows that an economy that has a larger average loan-deposit spread is also more sensitive to a volatility shock: loan rates increases and lending declines are more pronounced relative to the baseline.

Appendix B Calculating the Firm Size Distribution

Because there is a continuum of products we can use the law of large numbers to track the distribution of firms. In particular, denote by $\Omega_n(s^t)$ the mass of firms with n products. The law of motion of this distribution is characterized by a system of dynamic equations that only depend on the innovation rate of incumbents and the mass of entrants:

$$\begin{aligned}
\Omega_1(s^t) &= \left(M^*(s^{t-1})\right)^v + \sum_{n=1}^{\infty} \Omega_n(s^{t-1}) \sum_{k=0}^{1} \mathbb{B}\left(k, n, x^*(s^{t-1})\right) \mathbb{B}\left((k+n-1, n, \Delta(s^{t-1})\right) \\
\Omega_{\tilde{n}>1}(s^t) &= \sum_{n=\mathbb{I}^+(\frac{\tilde{n}}{2})}^{\tilde{n}} \left\{ \Omega_n(s^{t-1}) \sum_{k=\tilde{n}-n}^{n} \mathbb{B}\left(k, n, x^*(s^{t-1})\right) \mathbb{B}\left(k-(\tilde{n}-n), n, \Delta(s^{t-1})\right) \right\} \\
&+ \sum_{n=\tilde{n}+1}^{\infty} \left\{ \Omega_n(s^{t-1}) \sum_{k=0}^{\tilde{n}} \mathbb{B}\left(k, n, x^*(s^{t-1})\right) \mathbb{B}\left(k-(\tilde{n}-n), n, \Delta(s^{t-1})\right) \right\},
\end{aligned}$$

where $\mathbb{I}^+(a)$ refers to the integer closest to a such that $\mathbb{I}^+(a) \geq a$.[17] The first equation has two terms, the first one tracks the entry of new firms with one product while the second term tracks firms that used to have more than 1 product and contracted to exactly 1 product. The second line shows an analogous law of motion for categories of firms with more than one product where the first component are firms that started with less than $\tilde{n}$ products and had net gains that left them at $\tilde{n}$, while the second term reflects firms that were above $\tilde{n}$ and experienced net losses that left them exactly at $\tilde{n}$.[18] The BGP distribution can be found by iterating in these laws of motion until the mass of firms in each product category is constant.

[17] A firm with n products can become a firm with $n' \in [0, 2n]$ in one period, so, at most it can double its size in one period.

[18] The condition that ensures that the mass of products is always equal to one is given by $\sum_{n=1}^{\infty} n\Omega_n(s^t) = \Lambda$.

Appendix C Solution Method

Our solution algorithm for the decentralized equilibrium follows a combination of the time-iteration method employed in Bianchi et al. (2016) with value function iteration. We extend it in order to update guesses for the continuation value of a product line.

We discretize the processes for aggregate efficiency z, interest rate R and volatility σ_r using following Tauchen (1986). We consider grids of 7 points for aggregate efficiency, 21 points for the interest rate, and 9 points for interest rate volatility. Our bond holdings grid consists of 60 points, and is skewed towards larger debt holdings.

In what follows we drop the superscript H from bond holdings B^H in order to save notation. All functions presented below are stationary, unless otherwise noted. We start with a conjecture for the bond holdings policy function, B', defined over the state space (z, R, σ_r, B).[19] We also make a guess for innovation intensity $x(z, R, \sigma_r, B)$.

The steps of the solution algorithm are the following:

1. Start iteration j with a guess for $B'_j(z, R, \sigma_r, B)$ and innovation intensity $x_j(z, R, \sigma_r, B)$. Using these guesses construct:

$$M_j(z, R, \sigma_r, B) = \left[\frac{x_j(z, R, \sigma_r, B)^{\frac{1-\theta}{\theta}}}{\xi^{\frac{1}{\theta}}\theta} \frac{\nu}{\kappa} \right]^{\frac{1}{1-\nu}} \tag{32}$$

$$g_j(z, R, \sigma_r, B) = (1+\sigma)^{x_j(z,R,\sigma_r,B) + \frac{M_j(z,R,\sigma_r,B)^\nu}{\Lambda}} (1-\delta) - 1 \tag{33}$$

$$\Delta_j(z, R, \sigma_r, B) = x_j(z, R, \sigma_r, B) + \frac{M_j(z, R, \sigma_r, B)^\nu}{\Lambda} \tag{34}$$

$$l_{rj}(z, R, \sigma_r, B) = \left(\frac{x_j(z, R, \sigma_r, B)}{\xi} \right)^{\frac{1}{\theta}} \tag{35}$$

$$l_j(z, R, \sigma_r, B) = \frac{(1 - \kappa M_j(z, R, \sigma_r, B))}{\Lambda} - l_{rj}(z, R, \sigma_r, B) \tag{36}$$

$$Y_j(z, R, \sigma_r, B) = z l_j(z, R, \sigma_R, B) \tag{37}$$

[19] Note that this guess corresponds to a matrix with dimensions $N_z \times N_R \times N_\sigma \times N_B$, where N_z, N_R, N_σ and N_B correspond to the number of elements in the grid of aggregate efficiency, interest rate, volatility of interest rate and debt, respectively.

$$\pi(z, R, \sigma_r, B) = \frac{\sigma}{\Lambda(1+\sigma)} Y_j(z, R, \sigma_r, B) \tag{38}$$

$$\begin{aligned} t_j(z, R, \sigma_r, B) = \Lambda(\pi_j(z, R, \sigma_r, B) - \alpha w(z) l_{rj}(z, R, \sigma_r, B) - (1-\alpha) F x_j(z, R, \sigma_r, B)) \\ - \alpha\kappa M_j(z, R, \sigma_r, B) w(z) - (1-\alpha) M_j(z, R, \sigma_r, B)^{\nu} F \end{aligned} \tag{39}$$

$$w(z) = \frac{\exp(z)}{\Lambda(1+\sigma)} \tag{40}$$

$$c_j(z, R, \sigma_r, B) = w(z) + B - B'_j(z, R, \sigma_r, B)(1 + g_j(z, R, \sigma_r, B))\hat{q}(R, \sigma_r) + t_j(z, R, \sigma_R, B) \tag{41}$$

Lastly, compute the discounted expected marginal utility

$$\beta \frac{\beta}{\hat{q}(R, \sigma_r)} \mathbb{E}\left[u_j(z', R', \sigma'_r, B'_j(z, R, \sigma_r, B))\right] (1 + g_j(z, R, \sigma_r, B))^{-\gamma}, \tag{42}$$

where $u_j(z, R, \sigma_r, B) = c_j(z, R, \sigma_r, B)^{-\gamma}$.

2. Using the above guesses compute guess for the value of a product line and the stochastic discount factor. For the value or a product line we iterate over the following value function:

$$\begin{aligned} V_{1,k+1}(z, R, \sigma_r, B) = \pi_j(z, R, \sigma_r, B) - \alpha w(z) \left(\frac{x_j(z, R, \sigma_r, B)}{\xi}\right)^{\frac{1}{\theta}} - (1-\alpha) x_j(z, R, \sigma_r, B) F \\ + \mathbb{E}\left[m_j(z, R, \sigma_r, B)(1 - \Delta_j(z, R, \sigma_r, B) + x_j(z, R, \sigma_r, B))\left(1 + g_j(z, R, \sigma_r, B)\right) V_{1,k}(z, R, \sigma_r, B)\right] \end{aligned} \tag{43}$$

Until $||V_{1,k+1}(z, R, \sigma_r, B) - V_{1,k}(z, R, \sigma_r, B)|| <$ tol. $V_{1,j}(z, R, \sigma_r, B)$ is the converged value function.

3. Assume the borrowing constraint binds. Note that when the constraint binds we have that consumption is

$$c_{j+1}(z, R, \sigma_r, B) = w(z) + B + \phi\Lambda(V_{1,j}(z, R, \sigma_r, B)) + t_j(z, R, \sigma_R, B) \tag{44}$$

We then check whether this assumption holds by calculating the residual of the Euler equa-

tion:

$$\mathcal{R}(z,R,\sigma_r,B) = u_{j+1}(z,R,\sigma_r,B)$$
$$-\beta\frac{\beta}{\hat{q}(R,\sigma_r)}\mathbb{E}\left[u_j(z',R',\sigma_r',B_j'(z,R,\sigma_r,B))\right](1+g_j(z,R,\sigma_r,B))^{-\gamma}. \tag{45}$$

If $\mathcal{R}(z,R,\sigma_r,B) > 0$, we keep the values for $c_{j+1}(z,R,\sigma_r,B)$, and we set $B'_{j+1}(z,R,\sigma_r,R) = -\frac{\phi\Lambda V_{1,j}(z,R,\sigma_r,B)}{(1+g_j(z,R,\sigma_r,B))\hat{q}(R,\sigma_r)}$, $\mu_{j+1}(z,R,\sigma_r,B) = \mathcal{R}(z,R,\sigma_r,B)$. Otherwise, the constraint does not bind for that point of the state space and we discard $c_{j+1}(z,R,\sigma_r,B)$ and $B'_{j+1}(z,R,\sigma_r,R)$. We then numerically solve for the value of $c_{j+1}(z,R,\sigma_r,B)$ that satisfies

$$u_{j+1}(z,R,\sigma_r,B) = \beta\frac{\beta}{\hat{q}(R,\sigma_r)}\mathbb{E}\left[u_j(z',R',\sigma_r',B_j'(z,R,\sigma_r,B))\right](1+g_j(z,R,\sigma_r,B))^{-\gamma} \tag{46}$$

We then set $B'_{j+1}(z,R,\sigma_r,B) = \frac{w(z)+B+t_j(z,R,\sigma_R,B)-c_{j+1}(z,R,\sigma_r,B)}{(1+g_j(z,R,\sigma_r,B))\hat{q}(R,\sigma_r)}$ and $\mu_{j+1}(z,R,\sigma_r,B) = 0$.

4. Using the updated guesses $c_{j+1}(z,R,\sigma_R,B)$ and $B'_{j+1}(z,R,\sigma_R,B)$, compute an updated guess for the stochastic discount factor:

$$m_{j+1}(z,R,\sigma_r,B) = \beta\frac{c_{j+1}(z,R,\sigma_r,B'(z,R,\sigma_r,B))^{-\gamma}}{c_{j+1}(z,R,\sigma_r,B)^{-\gamma}-\phi\mu_{j+1}(z,R,\sigma_r,B)}(1+g_j(z,R,\sigma_r,B))^{-\gamma} \tag{47}$$

5. Recompute the value of a product line using the updated stochastic discount factor. This is, use value function iteration over the value of a product line:

$$V_{1,k+1}(z,R,\sigma_r,B) = \pi_j(z,R,\sigma_r,B) - \alpha w(z)\left(\frac{x_j(z,R,\sigma_r,B)}{\xi}\right)^{\frac{1}{\theta}} - (1-\alpha)x_j(z,R,\sigma_r,B)F$$
$$+\mathbb{E}\left[m_{j+1}(z,R,\sigma_r,B)(1-\Delta_j(z,R,\sigma_r,B)+x_j(z,R,\sigma_r,B))\left(1+g_j(z,R,\sigma_r,B)\right)V_{1,k}(z,R,\sigma_r,B)\right] \tag{48}$$

Until $||V_{1,k+1}(z,R,\sigma_r,B) - V_{1,k}(z,R,\sigma_r,B)|| < \text{tol}$. $V_{1,j+1}(z,R,\sigma_r,B)$ is the converged value function.

6. Update guess for innovation intensity of incumbents:

$$x_{j+1}(z,R,\sigma_r,B) = \left[\frac{\theta}{\alpha}\xi^{\frac{1}{\theta}}\frac{\mathbb{E}\left[m_{j+1}(z,R,\sigma_r,R)\left(1+g_j(z,R,\sigma_r,R)\right)V_{1,j+1}(z,R,\sigma_R,B)-(1-\alpha)F\middle|\, s_t\right]}{w(z)}\right]^{\frac{\theta}{1-\theta}} \tag{49}$$

7. Check for convergence. If $||B'_{j+1}(z,R,\sigma_r,B) - B'_j(z,R,\sigma_r,B)|| < \epsilon$ and $||x_{j+1}(z,R,\sigma_r,B) - x_j(z,R,\sigma_r,B)|| < \epsilon$ then the problem is solved. Otherwise, discard $B'_j(z,R,\sigma_r,B)$ and $x'_j(z,R,\sigma_r,B)$, and use $B'_{j+1}(z,R,\sigma_r,B)$ and $x'_{j+1}(z,R,\sigma_r,B)$ as new guesses for the problem (go back to step 1).

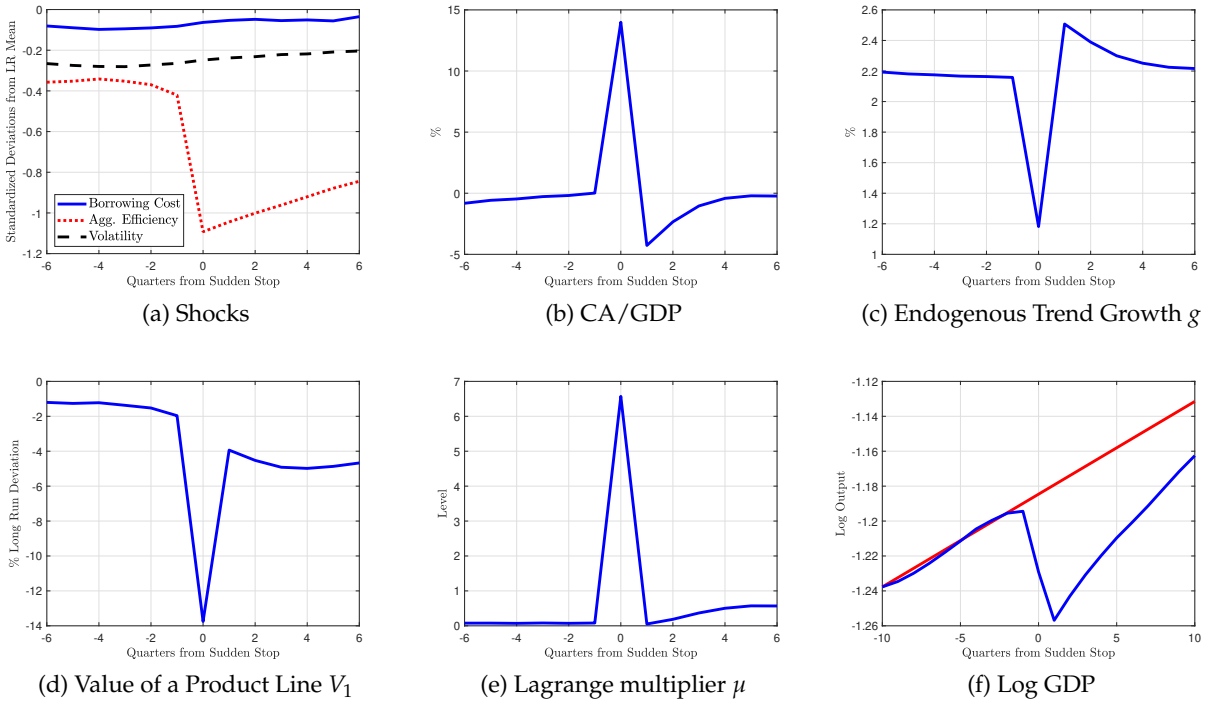
Appendix D Persistent Effects of Sudden Stops

We conduct an event analysis around Sudden Stops endogenously generated by the model under both the baseline and advanced economy calibrations. Using a time window of 10 quarters before and after each event, we average relevant time series.[20] Figure 5 illustrates the Sudden Stop dynamics for the baseline scenario.

Our focus is on the dynamics of five key variables: the value of a product line, the Lagrange multiplier, the current account-to-GDP ratio, the endogenous trend, and log GDP (in levels). Panel (a) shows the evolution of the model's three shocks around the event. In our framework, Sudden Stops typically occur under three conditions: (1) borrowing costs are below their long-run average, (2) interest rate volatility is low, and (3) a sudden, significant drop in aggregate efficiency takes place. Low and stable borrowing costs promote excessive borrowing, as households do not fully internalize the consequences of their borrowing decisions, leading to overborrowing.[21] These episodes are particularly pronounced when borrowing conditions are favorable. Because efficiency shocks are persistent, the combination of high leverage and a protracted decline in productivity triggers sharp adjustments in borrowing, as depicted in panel (b), where a steep current account reversal follows.

[20] We define a Sudden Stop as an episode where the current account-to-GDP ratio exceeds its long-run average by two standard deviations, coinciding with a binding borrowing constraint.

[21] This behavior is characteristic of models with occasionally binding borrowing constraints and endogenous collateral values. See Bianchi (2011) and Bianchi and Mendoza (2018) for a detailed analysis of pecuniary externalities in endowment and production models.



(a) Shocks (b) CA/GDP (c) Endogenous Trend Growth g

(d) Value of a Product Line V_1 (e) Lagrange multiplier μ (f) Log GDP

Figure 5: Sudden Stop Dynamics - Emerging Economy

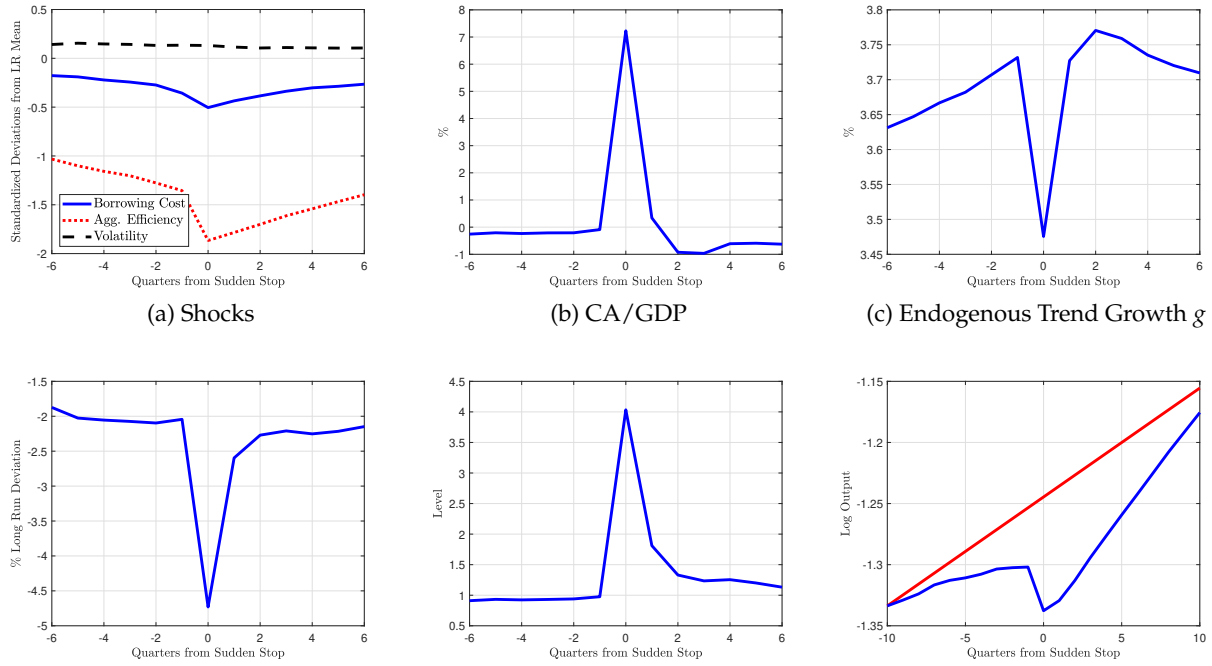
Notes: Sudden Stops are defined as events where the borrowing constraint binds and where the current account-to-GDP ratio is two standard deviations above its long-run mean. The three shocks of the model are standardized by their corresponding long-run means and standard deviations. CA/GDP denotes the current account-to-GDP ratio, and log GDP corresponds to the log of GDP in levels. The red solid line in the log GDP panel denotes the average trend, which is constructed by averaging the linear growth using the trend growth rate 10 quarters before a Sudden Stop.

Panel (c) depicts the trend growth response: a decline of approximately 1 percentage point (annualized), followed by a rapid recovery. However, as shown in panel (d), the value of a product line experiences a sharp drop and fails to return to its pre-Sudden Stop level. This persistent decline is driven by both the sustained low aggregate efficiency and the increased likelihood of a binding borrowing constraint post-event (panel (e)). A depressed product line value also implies that collateral values remain below their long-run average, contributing to the sluggish recovery.[22] Finally, panel (f) clearly demonstrates the long-lasting consequences, or hysteresis, of a Sudden Stop: output suffers an abrupt and permanent deviation from its pre-event trend.

To assess the role of financial frictions in the severity of Sudden Stops, we conduct a similar event analysis using the advanced economy calibration. Figure 6 presents the Sudden Stop

[22]Our framework is notable for generating both slow recoveries and borrowing constraints that bind for several periods after the Sudden Stop, a novel feature typically seen in models with capital accumulation, where capital is used as collateral for borrowing.

dynamics for this scenario.



(a) Shocks (b) CA/GDP (c) Endogenous Trend Growth g

(d) Value of a Product Line V_1 (e) Lagrange multiplier μ (f) Log GDP

Figure 6: Sudden Stop Dynamics - Advanced Economy

Notes: Sudden Stops are defined as events where the borrowing constraint binds and where the current account-to-GDP ratio is two standard deviations above its long-run mean. The three shocks of the model are standardized by their corresponding long-run means and standard deviations. CA/GDP denotes the current account-to-GDP ratio, and log GDP corresponds to the log of GDP in levels. The red solid line in the log GDP panel denotes the average trend, which is constructed by averaging the linear growth using the trend growth rate 10 quarters before a Sudden Stop.

This analysis contrasts the dynamics of Sudden Stops between advanced and emerging economies, highlighting both the likelihood and severity of these events. The results in Figure 6 reveal that Sudden Stops in advanced economies are triggered by a much more adverse sequence of shocks compared to the baseline scenario. Specifically, the aggregate efficiency level that leads to a Sudden Stop is nearly 2 standard deviations below its long-run average, almost twice as large as in the emerging economy case. Moreover, Sudden Stops in advanced economies also occur when borrowing costs are lower than their long-run average, and lower than in the baseline calibration. These findings highlight the stronger resilience of advanced economies: not only are financial crises less frequent, but they require a significantly worse deterioration in economic conditions to materialize.

In terms of severity, Sudden Stops in advanced economies are far milder than in emerging

economies. While the current account reversal is substantial, it is about half the size of that seen in emerging economies. Trend growth experiences a smaller decline—approximately 0.3 percentage points (annualized)—at the moment of the Sudden Stop, followed by a rapid recovery. As in the baseline scenario, the value of a product line declines and recovers slowly, but the magnitude is roughly one-third of that observed in emerging economies. Finally, although GDP declines are also milder, the persistent drop in aggregate efficiency causes long-run GDP losses to start accumulating several quarters before the Sudden Stop occurs.

Overall, our results demonstrate that financial frictions play a critical role in shaping both the likelihood and severity of Sudden Stops across advanced and emerging economies. The framework not only captures realistic crisis dynamics but also illustrates how financial crises can have lasting effects on output and the economy's long-run trend.

It is important to distinguish between the crises driven by U.S. interest rate volatility and the Sudden Stops analyzed in this appendix. In the main text, we focus on how increased volatility in U.S. interest rates negatively impacts growth, as agents anticipate larger future interest rate shocks. This is because adverse rate shocks tend to be more damaging than rate reductions are beneficial for growth in emerging economies. However, as shown here, Sudden Stops are different. These events involve a sharp current account reversal and are typically triggered by large negative TFP shocks. Not every instance of a binding borrowing constraint leads to a Sudden Stop; the multiplier can bind without causing a major crisis. Sudden Stops are more closely linked to deep contractions in aggregate efficiency and borrowing needs for consumption smoothing, whereas the main text centers on the dynamics of interest rate volatility and its asymmetric effects on growth due to a binding constraint, without requiring a sharp current account reversal.