Distributional Consequences of Monetary Policy in India*

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Abstract

Studying monetary policy transmission in an economy like India, where traditional low-productive agricultural activities with volatile output prices account for the means of the employment of nearly half of the population, and as large as 62% of the households belong in hand-to-mouth category with marginal propensities to consume as high as 0.6, not only expands the existing literature on heterogeneous agent macroeconomics, it challenges the preconceptions of the magnitude of the impact. Overall consumption responses of a policy expansion of 0.5% point quarterly and 1% point yearly are found to vary between 1.26% and 8.5%, which is bigger than what is found in the US, four major economies in Europe: Germany, Spain, France, and Italy, and the Euro-area averages. This is also the first study to provide evidence that not accounting for the agricultural sector leads to an amplification of the consumption responses.

Keywords: Monetary policy; Agriculture; Redistribution; Heterogeneous households; India

JEL *classification:* D31; E12; E21; E24; E43; E52; E62

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1 Introduction

Monetary policy transmission into the real economy and its redistributive effects have gained increased attention in the past few decades (e.g., Werning (2015); Gornemann et al. (2016); Coibion et al. (2017); Kaplan et al. (2018); Auclert (2019); Bhandari et al. (2021); Acharya et al. (2023); Bilbiie et al. (2024)). However, only limited attempts have been made in evaluating such consequences in the Indian context[1]. Indian economy inherits a dichotomy between agricultural and non-agricultural sectors so drastically that the socio-economic conditions of Indian households are majorly influenced by their involvement in agriculture. As Gabriel et al. (2016) and Ghate et al. (2018) point out, India's large agricultural sector is prone to sector-specific distortions that transmit through changes in the relative prices and affect sector-specific wages, marginal costs, sector-specific inflation rates, generalized inflation, sectoral output gaps, and ultimately resource re-allocation.

Considering these facts, I develop a closed economy New Keynesian dynamic stochastic general equilibrium (NK-DSGE) model that features not only a continuum of heterogeneous households as in the fast-evolving heterogeneous agent New Keynesian (HANK) literature, it also features the dynamics between agricultural and non-agricultural households in India. Inspired by stylized facts from survey data, I incorporate three unique features in an otherwise standard sticky price incomplete market model; based on the realization of an exogenous Markov process, low-productivity households by default are employed in the agricultural sector, agricultural prices adjust frequently due to sector-specific distortions leaving these households more exposed to volatile income and consumption, and a comparatively higher proportion of non-hand-to-mouth households in the economy.

Along the line of Kaplan and Violante (2014); Gornemann et al. (2016); Debortoli and Gali (2017); Kaplan et al. (2018); McKay and Wolf (2023), I solve the model numerically. For which I calibrate model parameters using numerous data sources in Indian economy[2]. From the All India Debt and Investment Survey (AIDS), I obtain the

[1]The literature that examines the monetary policy propagation into the real economic variables in India predominantly uses time series econometric techniques on aggregate variables, e.g., Ahmed and Dua (2001); Bhattacharyya and Ray (2007); Aleem (2010); Singh (2011); Khundrakpam and Jain (2012); Mohanty (2012); Sengupta (2014); Das (2015); Mishra et al. (2016); Ghosh (2019); Goyal and Parab (2021).

[2]There is a complimentary yet parallel strand of HANK literature that analytically solves the model with simplifying assumption offering a great deal of traceability (e.g., Acharya and Dogra, 2018; Auclert, 2019; Bilbiie, 2008, 2020;

borrowing limits of households. The mean revision rate of the monetary policy shock is estimated using historical data on the short-term interest rate from the Reserve Bank of India (RBI). The transition probabilities between the two types of productivity states are obtained from the Indian Human Development Survey (IHDS) round I and II. Tax and Transfers parameters are calibrated using historical data from the RBI and the Ministry of Finance databases. Expenditure shares on agricultural and non-agricultural goods are obtained from the Household Consumer Expenditure Survey. These surveys also become handy in matching different model-generated moments with those in the data.

This study makes two major contributions. First, it discusses redistribution from two angles: through the continuum of heterogeneous households that quantifies the within-group variation and through the two-sector framework that highlights the between-group variation in agricultural and non-agricultural households at the steady state as well as in their responses to a monetary policy shock. Second, it provides conclusive evidence that in an economy with a large, low-productive agricultural sector prone to frequent price fluctuations, monetary policy transmission to households is smaller.

At the steady state, on average, per household consumption expenditure in proportion to GDP is 0.08 for agricultural households, which is close to 0.11 obtained from HCES data; similarly, for non-agricultural households, the model generated mean is 0.18, quite close to the data mean of 0.19. On the other hand, the probability of households being stuck in the same productivity level being really high leads to an accumulation of assets or debt, giving rise to a stark difference in steady state asset distribution between the two types of households.

I introduce a one-time monetary policy shock of magnitude -0.5 and -1 percentage points at quarterly and yearly time frames, with corresponding adjustments of model parameters. Due to the heterogeneous framework, I am able to illustrate how this shock propagates to each household in the entire joint distribution of assets and productivity. The average consumption response in agricultural households within a quarter varies between 1.26% to 2%, and in a year, it increases to 5.22% and 7.16%. Non-agricultural households respond between 2.2% to 2.8% in a quarter and 3.96% to 8.46% in a year. The average consumption responses at the bottom of the distribution are relatively

Patterson, 2019; Ragot, 2018; Werning, 2015).

higher than at the top.

Higher consumption responses among non-agricultural households stem from two micro-founded results of the model. First, the proportion of hand-to-mouth households in non-agri is higher than that in the agri, and second, poorer households in non-agri exhibit a much higher marginal propensity to consume than the poorer households in agri. This also leads to the fact that a higher proportion of agri households hold some positive assets as a cushion than non-agri households. The extremely high concentration of wealth among few non-agri households while the majority owns low or net-negative assets points out the extent of inequality that prevails in this sector.

With a comparative static featuring only one sector of production, I analyze whether the presence of the two production sectors employing two types of productive labor suppresses or amplifies the effect of policy intervention. In this case, both types of labor are employed by monopolistically competitive intermediate goods producers. The introduction of the policy shock leads to a higher consumption response at all levels of assets for both types of households. Consistent with the existing literature that higher price stickiness leads to higher real effects, an expansionary monetary policy leads to higher wage responses for all types of labor, leading to a higher increase in consumption. I also carry out a detailed comparison of these responses with developed economies like the USA and the Euro Area.

I also look at the economy's response to a total factor productivity reduction of a small magnitude in the agricultural sector that induces similar household consumption responses through output and income as to the impulse responses from monetary policy shock. Quantifying the impact and understanding monetary policy feedback in this scenario has immense policy implications.

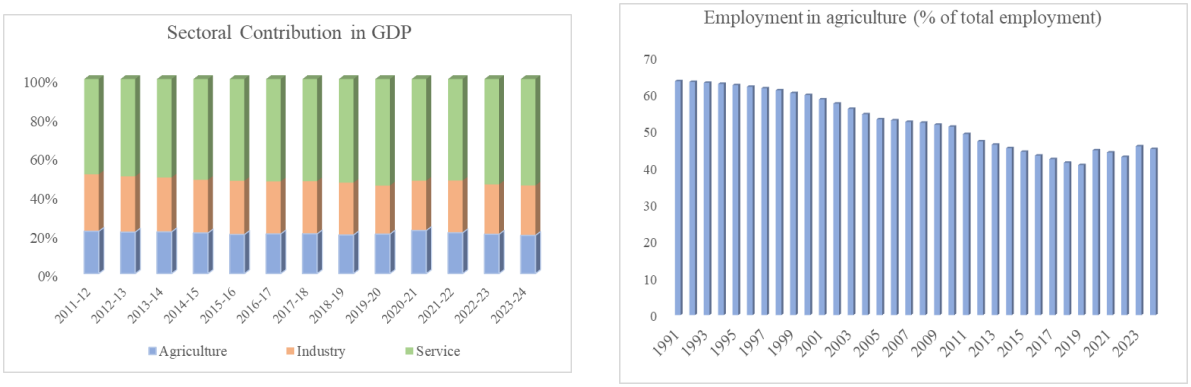
This is the first study that quantifies the redistribution effect on Indian households due to a monetary policy shock in a continuous heterogeneity New Keynesian model. It is also the first study to provide precise measures of the effect on predominantly agricultural and non-agricultural households subject to idiosyncratic productivity shocks. This study is also one of the few that showcases the importance of modeling the agricultural sector distinctly from the non-agricultural sector in order to understand redistribution in economies like India, where agriculture is traditional, yet the means of employment for a large population.

The remainder of the paper is organized as follows. Section 2 presents some stylized facts from data. Section 3 lays out the model featuring household heterogeneity and multiple sectors in production and discusses some theoretical results derived from this model. The numerical solution, calibration techniques, and a detailed discussion of the results are presented in section 4. And section 5 provides concluding remarks.

2 Stylized Facts

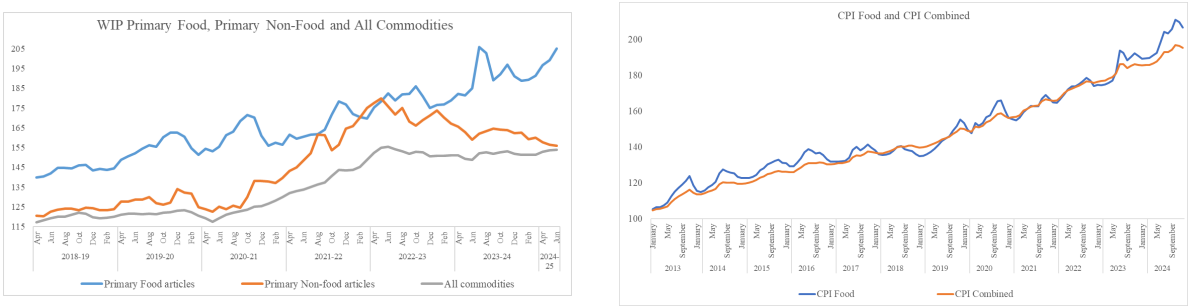
Before I dive into the theoretical model, it is crucial to keep in mind some of the stylized facts from survey data in India that will serve as the building blocks of the model and provide explanations for many of the findings. The sectoral contribution of agriculture

Figure 1: Sectoral Contribution of Agriculture in GDP and Share of Employment in Agriculture as Percentage of Total Employment Over the Years



Data source: Open Government Data (OGD) Platform, Government of India.

Figure 2: Wholesale and Consumer Price Indices

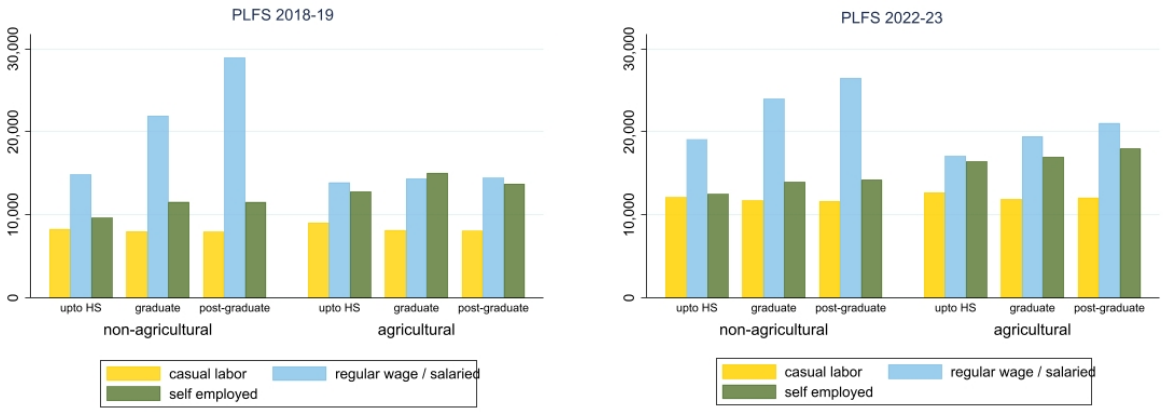


Data source: Ministry of Statistics and Programme Implementation (MoSPI), Government of India.

to the GDP over the past decade has remained close to 17% (Fig 1), while it persistently

absorbs almost $45-49\%$ (Fig 1) of the total employment. A number of factors, including small land holdings, reliance on traditional means of production, non-availability of easy loans, lack of new technology, and often lack of proper infrastructure for transport and preservation, contribute to the low productive state of Indian agriculture. Lack of infrastructure, along with seasonal disruptions and natural calamities, which are often more prevalent in some geographical regions than others, leads to the volatility of prices in agricultural commodities, causing food prices to be prone to frequent adjustments compared to other prices. Fig 2) exhibit the same, the wholesale price of primary food articles that include Cereals, Pulses, Fruits, Vegetables, Milk, Eggs, Meat, and fish, and primary non-food articles that include Oil seeds, Minerals, and Crude petroleum exhibit high fluctuations, compared to the combined WPI of all commodities. And this also translates into the consumer prices of food items, which fluctuate much more frequently than the CPI combined. Therefore, Households that are predominately involved in agricultural production not only earn less due to low productivity, but their income also fluctuates a lot. When I decompose individual earnings from agriculture and vice

Figure 3: Mean Monthly Earnings of Casual Labor, Regular Salaried and Self-Employed Individuals in Agricultural and Non-Agricultural Households for Different Education Levels



Data source: Periodic Labor Force Survey 2018-19 and 2022-23 by the National Sample Survey Organisation (NSSO), the Government of India.

versa non-agricultural over the status of employment like casual labor, self-employed, salaried, and over education levels, it shockingly reveals that even individuals with a post-graduate education are employed as casual labor in agriculture, and they are earn-

ing substantially less than their counterparts in the non-agricultural sector (Figure 3)[3]. Now, this leaves me curious about the debt-asset situations of these households. To

Table 1: Percentage of Households Reporting Asset Ownership and Average Value of Asset Holding

Components	Agricultural households		Non-agricultural households	
	% Ownership	Average Rs.'000 (proportion pc GDP)	% Ownership	Average Rs.'000 (proportion pc GDP)
Any asset	99.8	1590	98.7	2770
		(11.83)		(20.61)
Physical asset	97.3	1525	85.8	2466
		(11.34)		(18.34)
Financial asset	95.82	731	94.66	2520
		(5.44)		(18.75)
Bottom 50% physical asset		1090.3		776.4
		(8.12)		(5.78)
Bottom 50% financial asset		13		25.5
		(0.1)		(0.19)

Data source: All India Debt & Investment Survey 2019 by the National Sample Survey Organisation (NSSO), the Government of India.

my surprise, almost all households own some kind of asset (Table 1)[4]. The percentage of agricultural households holding physical assets is much higher than that of non-agricultural households. Among these physical assets, agricultural land and buildings hold the majority share, along with other assets like livestock, transport equipment, agricultural machinery, etc. However, the average value of these assets is smaller than that in non-agricultural households, even though their majority share is still in land and buildings. This indicates the disparity in the nature of land holdings and other physical assets owned by these households and the fact that the majority of agricultural households are small landholders with limited resources. To substantiate this claim, I include the statistics for the bottom 50% of physical and financial asset holders. The averages now fall drastically for both types of households, revealing the extent to which inequality of assets is prevalent. A quick calculation of the percentage of households

[3]Note that the mean regular wage / salaried earnings among non-agricultural households have reduced and over time while that among agricultural households have increased over time. To crosscheck this fact, the overall mean values are matched with the corresponding values in the PLFS reports.

[4]Note: Figures in this table are crosschecked with the report by the NSSO and made sure that these high percentages are constant with other survey data.

that do not own any assets or debt i.e., poor *'Hand-to-Mouth* (HtM hereafter), and households that hold only physical assets and no financial asset or debt i.e., rich HtM is also included, which is critical in understanding the composition of direct and indirect effects on household consumptions. A decomposition of household financial assets at

Table 2: Decomposition of Assets and Financial Debts in Proportion to Per-Capita GDP

Components	Agricultural households				Non-agricultural households			
	Observations	Mean	Std. Dev.	Maximum	Observations	Mean	Std. Dev.	Maximum
Deposits in banks	37,979	0.12	0.46	136.19	72,559	0.29	4.41	677.49
Deposits in post office	2,433	0.08	0.30	5.33	6,969	0.17	0.77	30.34
Other deposits	2,095	0.13	0.42	8.47	4,764	0.39	1.54	80.09
Institutional debt	38,396	0.67	2.02	357.68	74,407	1.77	7.36	406.47
Non-institutional debt	79,748	0.78	1.97	382.18	136,126	1.44	5.94	406.47

Data source: All India Debt & Investment Survey 2019 by the National Sample Survey Organisation (NSSO), the Government of India.

this point is useful to obtain the complete picture of the debt-asset situations. Both debt and financial asset holding among non-agricultural households are higher than those of agricultural households, as are the standard deviations. The number of households borrowing from non-institutional sources is leaps and bounds higher than those borrowing from institutions, indicating a lack of financial inclusion even among non-agricultural households[5].

Keeping these stylized facts in mind, the primary objective of this paper is to develop a model featuring the differences in earnings and debt-asset positions between agricultural and non-agricultural households, realistically quantify the redistributive effects of a monetary policy shock, and examine if the presence of two distinct types of heterogeneous households suppresses or amplifies the overall effect.

[5]On a separate note, (AIDS) 2019 reveals that indebted households, i.e., households having cash loans outstanding, account for 40.3% among cultivator households and 28.2% among non-cultivator households in rural India. The financial distress among rural households, especially among the cultivators, is acute. Not just indebtedness, in any given year, agricultural households suffer from loss of livelihood, inefficient supply chain, low-productivity, and stagnant or declining agricultural income, in many cases, leading to farmers' suicides indicating the severity of the agrarian distress in India (Bhoi and Dadhich (2019); Guha and Das (2022)).

3 A Heterogeneous Agent Model with Two Sectors of Production

There are four agents in the economy[6]: households, firms, a monetary policy authority, and a government. Households are heterogeneous in asset and productivity. There are two production sectors that produce two different types of goods: agricultural firms produce identical agricultural consumer goods, and modern intermediate monopolistically competitive firms produce a continuum of non-agricultural intermediate goods that are used as inputs to produce identical non-agricultural consumer goods by final goods producers in the non-agricultural sector. The government operates through proportional income tax and lump sum transfer. Monetary policy authority sets the short-term nominal interest rate following a Taylor-type rule.

3.1 Households

Households denoted by i are heterogeneous in their asset $b_{i,t}$ and labor productivity $z_{i,t}$. Labor productivity follows an exogenous stochastic Poisson process with two states: high (H) and low (L), with probability λ_1 of transition from state L to state H and probability λ_2 of transitioning from state H to state L. Time t is continuous. Households receive a flow of utility from consumption $c_t^i \geq 0$ and a flow of dis-utility from supplying labor $l_{i,t} \in [0, 1]$, i.e., total time endowment is normalized to 1. $l_{i,t}$ is the hours of work out of total time endowment. Households maximize expected lifetime utility at time 0,

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \left[\frac{(c_t^i)^{1-\sigma}}{1-\sigma} - \frac{(l_{i,t})^{1+\psi}}{1+\psi} \right] dt \tag{1}$$

[6]My model features a continuum of households indexed by their asset holding and idiosyncratic labor productivity by building upon the seminal work by Aiyagari (1994). Achdou et al. (2022) provides a useful extension of Aiyagari (1994) in a continuous time formulation. It also suggests a portable numerical solution method for the three pioneering heterogeneous agent models of income and asset distribution by Aiyagari (1994), Bewley (1986), and Huggett (1993). Aiyagari (1994) is one of the pioneering studies to incorporate 'a considerable amount of individual dynamics, uncertainty, and asset trading' as the main mechanism of individual consumption smoothening. Time is continuous in this model. A continuous time approach is advantageous while dealing with continuous heterogeneity, in two broad aspects: the computational efficiency and the handling of the borrowing constraint. The numerical solution employs a finite difference method that uses only consecutive values of the value function in the successive iteration. Due to the extremely sparse nature of the transition matrix, a continuous time solution to the heterogeneous agent model is more efficient than the discrete-time solution. The 'backward–forward MFG system' solves the household optimization problem and derives the evolution of the state. This approach follows the mathematical theory of 'Mean Field Games' (MFG) introduced in economic literature by Lasry and Lions (2007). On the other hand, in continuous time, the borrowing constraint binds only at the borrowing limit and nowhere else in the state space.

ρ is discount rate for future, $1/\sigma$ is intertemporal elasticity of substitution, $\sigma \geq 0$ and $1/\psi$ is Frisch elasticity of labour supply, $\psi \geq 0$. Households can save and borrow asset $b_{i,t}$ at real interest rate $r_{b,t}$. Households can borrow liquid assets up to an exogenous limit $\underline{b}_i$. Household asset holding evolves according to,

$$\dot{b}_{i,t} = (1-\tau)\, w_{i,t}\, z_{i,t}\, l_{i,t} + r_{b,t}\, b_{i,t} + T - c^i_t \tag{2}$$

where $\dot{b}_{i,t}$ is the changes in the household asset holding over time, i.e., $\dot{b}_{i,t} = \frac{\partial b_{i,t}}{\partial t}$, τ is the rate of proportional tax on labor income imposed by the government, $w_{i,t}$ is real wage rate for labor type i, T is a lump-sum direct transfer from the government to the household, c^i_t is real consumption expenditure. All the variables are expressed in real terms.

Purchase of household i, c^i_t, is an index of the agricultural goods $c^i_{A,t}$ and final non-agricultural goods $c^i_{M,t}$. Following Aoki (2001); this provides a straightforward formulation for including multiple consumer goods in the household consumption basket with constant budget share allocations.

$$c^i_t = \frac{(c^i_{M,t})^{\gamma}\,(c^i_{A,t})^{1-\gamma}}{(\gamma)^{\gamma}\,(1-\gamma)^{1-\gamma}} \tag{3}$$

γ is the share of non-agri goods in the consumer basket, and the elasticity of substitution between agri and non-agri goods is one. Heterogeneity among households leads to total consumption demand from each sector as a function of the sequence of equilibrium asset holding, productivity state, the return on asset, wage rate, tax rate, and transfers.

$$C_{A,t} = \sum_{i=L,H} \Big[\int c^i_{A,t}\left(b_{i,t}, z_{i,t}\,; \{\Gamma_t\}_{t\geq 0}\right)\, d\mu_t \Big] \tag{4}$$

$$C_{M,t} = \sum_{i=L,H} \Big[\int c^i_{M,t}\left(b_{i,t}, z_{i,t},\,; \{\Gamma_t\}_{t\geq 0})\right)\, d\mu_t \Big] \tag{5}$$

where $\Gamma_t = \{r_{b,t}, w_{L,t}, w_{H,t}, \tau, T\}$ is the sequence of the tax rate, transfer, equilibrium interest rate, and wage rates, μ_t is the joint distribution of asset and productivity $d\mu_t\left(db_t, dz_t; \{\Gamma_t\}_{t\geq 0}\right)$. Household i maximizes lifetime utility subject to equations of motion for asset, taking the equilibrium path of $\{w_{i,t}\}_{t\geq 0}$, $\{r_{b,t}\}_{t\geq 0}$, and $\{\tau,\, T\}_{t\geq 0}$ as given. This optimization provides decision rules for consumption (c^i_t), and labor supply $(l_{L,t}, l_{H,t})$. $\{\tau,\, T\}_{t\geq 0}$ are determined by the government, $\{r_{b,t}\}_{t\geq 0}$ and $\{w_{i,t}\}_{t\geq 0}$ are determined by market clearing conditions of asset and labor under the assumption

that factor markets are perfectly competitive. The household Euler equation for intertemporal consumption choice takes the form (derivation in Appendix 6.1),

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}(r_{b,t} - \rho). \tag{6}$$

3.2 Firms

There are two types of firms in the economy, both producing final consumption goods. However, their production technology, nature of the product, and price stickiness differ. This is motivated by the fact that in India, there is a distinction between how agri, agri-allied, and the relatively modern sectors operate. The so-called traditional agricultural and agri-allied sectors, which include forestry, fishery, poultry, etc., predominantly depend on manual labor to produce output, suffer from low productivity, and the prices of their produce are barely regulated, leading to frequent fluctuations in response to internal and external factors. On the other hand, the relatively modern non-agricultural sectors (broadly defined here as everything other than agri and agri-allied) depend heavily on physical capital along with high-skilled labor, which consistently shows higher productivity of labor, and the prices of their produce move slowly due to menu cost and other frictions.

3.2.1 Agriculture

Agricultural firms produce output $Y_{A,t}$ of identical goods using only low productive labor $L_{L,t}$.

$$Y_{A,t} = L_{L,t} \tag{7}$$

Agricultural goods are sold in a perfectly competitive market where prices adjust immediately. Labor is hired from a perfectly competitive labor market, and at equilibrium, total demand for agricultural goods equals total supply, i.e.,

$$Y_{A,t} = C_{A,t} \quad . \tag{8}$$

3.2.2 Non-Agriculture

The relatively modern non-agricultural sector features competitive final good producers who aggregate a continuum of intermediate outputs from monopolistically competitive

intermediate goods producers indexed by $j \in [0, 1]$.

Final goods producers: The aggregate output of the non-agri sector is a Dixit-Stiglitz index of the output from intermediate firms,

$$Y_{M,t} = \left(\int_0^1 {y_{M,t}^j}^{\frac{\epsilon-1}{\epsilon}} \, dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{9}$$

where $\epsilon > 0$ is the elasticity between intermediate non-agricultural goods, cost minimization (derivation in the appendix 6.3) implies demand for intermediate good j is,

$$y_{M,t}^j = \left(\frac{P_{M,t}^j}{P_{M,t}} \right)^{-\epsilon} Y_{M,t} \tag{10}$$

where the price of the final good follows the Dixit-Stiglitz formulation as an index of the prices of the intermediate goods,

$$P_{M,t} = \left(\int_0^1 \left(P_{M,t}^j \right)^{1-\epsilon} \, dj \right)^{\frac{1}{1-\epsilon}} \tag{11}$$

at equilibrium, aggregate demand for non-agricultural goods must be equal to the aggregate supply.

$$Y_{M,t} = C_{M,t} \, . \tag{12}$$

Equation 10 illustrates the relationship between the relative demand, relative price, and elasticity. An increase in the demand for final goods leading to an increase in production at equilibrium translates to an increase in the demand for intermediate goods by the aggregator firm at a given elasticity and prices. An increase in the relative price of the non-agri good j by the price setter intermediate firm j leads to a fall in the relative demand of its output at a given elasticity.

Intermediate goods producers: Intermediate non-agricultural goods producer j produces output $y_{M,t}^j$ of differentiated good j using high productive labor $l_{H,t}^j$ and capital k_t^j. Firms hire capital from households. Households save into financial assets at a given interest rate $r_{b,t}$, which they can invest into productive capital at a given interest rate $r_{k,t}$, and firms hire this capital at the given interest rate $r_{k,t}$ as in Aiyagari (1994). Therefore, at equilibrium, $r_{b,t} = r_{k,t} - \delta$ must hold, where δ is the rate of depreciation of capital. The production function of the non-agricultural good j takes the form,

$$y_{M,t}^j = (k_t^j)^{\alpha} \, (l_{H,t}^j)^{1-\alpha} \tag{13}$$

α is the share of capital. Intermediate non-agricultural goods are sold in a monopolistically competitive market where price $P_{M,t}^j$ adjust slowly. The wage rate and return on

capital equal their marginal revenue product under the assumption of perfectly competitive factor markets. Profit of firm j at time t is

$$Q_t^j = \left(\frac{P_{M,t}^j}{P_{M,t}} - MC_t \right) Y_{M,t} \tag{14}$$

where MC_t is the marginal cost of production. Intermediate non-agricultural goods producers can set prices at any instance of time. However, they face cost ${\Theta_t}^j$ every time they adjust the price. Following Rotemberg (1982) this cost is a quadratic function of the rate of inflation for good j denoted by $\Pi_{M,t}^j$ due to price adjustment, a parameter $\theta > 0$, and aggregate output of the non-agricultural sector $Y_{M,t}$.

$${\Theta_t}^j = \frac{\theta}{2} \left(\Pi_{M,t}^j \right)^2 Y_{M,t} \tag{15}$$

where $\Pi_{M,t}^j = \frac{\dot{P}_{M,t}^j}{P_{M,t}^j}$. Firm j, when it adjusts price, chooses $\{P_{M,t}^j\}_{t \geq 0}$ so as to maximize lifetime profit subject to the adjustment cost,

$$\int_0^\infty e^{-\int_0^t r_{k,s} ds} \left(Q_t^j - {\Theta_t}^j \right) dt \, . \tag{16}$$

$r_{k,t}$ is used for discounting instead of $r_{b,t}$ to satisfy no-arbitrage condition.

3.2.3 Inflation in the non-agricultural sector

Lemma 1: The solution to the optimization problem of firm j while setting price provides the expression for the evolution of inflation in the non-agricultural sector under symmetric equilibrium (*proof* in Appendix 6.2).

$$\dot{\Pi}_{M,t} + \frac{\epsilon}{\theta} \left(MC_t - MC_t^* \right) = \left(\frac{\dot{Y}_{M,t}}{Y_{M,t}} - r_{b,t} \right) \Pi_{M,t} \tag{17}$$

This is the New Keynesian Philips Curve (NKPC) for non-agricultural goods.

3.2.4 Aggregate Price Index

The aggregate price in the economy P_t is an index of the prices of agri and non-agri goods (follows from the household aggregation of the two consumer goods). The price of agricultural goods is flexible, whereas the price of the final non-agricultural goods, which is an index of the prices of the intermediate non-agricultural goods, is influenced by the underlying NKPC in the intermediate goods sector.

$$P_t = \left(P_{M,t} \right)^\gamma \left(P_{A,t} \right)^{1-\gamma} \tag{18}$$

where γ is the share of non-agricultural goods in the household consumer basket. Note that the total consumption expenditure of the households is, therefore, $P_t\, c_t^j$, where c_t^j is determined by equation 3. Note the households buy the two types of consumer goods in two different markets to make this consumption basket. Therefore, P_t can be perceived as the consumer price index in this economy. The aggregate inflation is obtained from equation 18.

$$\Pi_t = \gamma \Pi_{M,t} + (1-\gamma)\Pi_{A,t} \tag{19}$$

3.3 Monetary Policy Authority

Monetary policy authority sets the nominal interest rate on asset $i_{b,t}$ following a Taylor-type rule

$$i_{b,t} = \bar{r}_{b,t} + \beta \Pi_t + \zeta_t \tag{20}$$

where $\bar{r}_{b,t}$ is the steady state real interest rate on asset, $\beta > 0$ and $\zeta_t = 0$ at steady state. Given $i_{b,t}$ and Π_t real interest rate is determined by the Fisher equation $r_{b,t} = i_{b,t} - \Pi_t$ and $\bar{r}_{b,t}$ is the value of $r_{b,t}$ consistent with the equilibrium condition of asset market. I observe the economy's adjustment after a one-time shock ζ_0 that hits the economy at time $t = 0$ and mean reverts at rate η, i.e., $\zeta_t = e^{-\eta t}\zeta_0$.

3.4 Government

Government operates through proportional income tax at rate τ, lump-sum transfer T, where $\tau, T > 0$. The government budget always balances, therefore,

$$T = \sum_{i=L,H} \tau\, w_{i,t}\, z_{i,t} \int l_{i,t}\left(b_{i,t}, z_{i,t}\,; \{\Gamma_t\}_{t\geq 0}\right)\, d\mu_t \tag{21}$$

3.5 Equilibrium

Given the stochastic process of productivity shocks, the equilibrium of the economy is characterized by a sequence of consumption, hours worked, asset holding, and the prices of goods, wage rates, return on asset, return on capital, and measure μ_t such that households and firms objective functions are satisfied, the decisions satisfy aggregate consistency condition, govt budget balances and all markets clear.

The asset market clears when,

$$\sum_{i=L,H} \{ \int b_{i,t} \, d\mu_t \} = \int k_t^j \, dj \tag{22}$$

where $A_t = \sum_{i=L,H} \{ \int b_{i,t} \, d\mu_t \}$ is total household asset and $K_t = \int k_t^j \, dj$ is total capital input in non-agricultural intermediate production sector.

Markets for low- and high-productivity labor clear when, respectively,

$$\int l_{L,t} \left(b_{L,t}, z_{L,t} \, ; \{\Gamma_t\}_{t \geq 0} \right) \, d\mu_t = L_{L,t} \tag{23}$$

$$\int l_{H,t} \left(b_{H,t}, z_{H,t} \, ; \{\Gamma_t\}_{t \geq 0} \right) \, d\mu_t = \int l_{H,t}^j \, dj \tag{24}$$

Goods market clear when (8) and (12) satisfy.

3.6 Transmission Channels

The focal point of this paper is to quantify the transmission of an expansionary monetary policy shock on household consumption. At every instant of time, t the demand for agri and non-agri goods of each household is determined by the joint distribution of the two state variables, i.e., $c_{A,t}^i \left(b_{i,t}, z_{i,t}; \{\Gamma_t\}_{t \geq 0} \right)$ and $c_{M,t}^i \left(b_{i,t}, z_{i,t}; \{\Gamma_t\}_{t \geq 0} \right)$ where $\Gamma_t = \{ r_{b,t}, w_{L,t}, w_{H,t}, \tau, T \}$ is the sequence of equilibrium rates and prices that is given to the households. Therefore, the response of each of these prices and rates arising from a monetary policy shock leads to a separate channel of policy pass-through into household consumption. Taking the total derivative of consumption demand at $t = 0$ generates the following expression that holds for both types of goods,

$$dc_0^i = \frac{\partial c_0^i}{\partial r_{b,0}} \, dr_{b,0} + \frac{\partial c_0^i}{\partial w_{i,0}} \, dw_{i,0} + \frac{\partial c_0^i}{\partial \tau} \, d\tau + \frac{\partial c_0^i}{\partial T} \, dT \tag{25}$$

Kaplan et al. (2018) derive the exact expression for each of these components and compare their relative strengths. While quantifying these channels is necessary for that comparison, the transmission mechanism can be explained in spite of that. An expansionary monetary policy shock leads to an increase in consumption demand directly through intertemporal substitution as depicted in the consumption Euler equation 6. This increase in consumption demand leads to higher labor demand, pushing the wage rates upward. Which leads to an increase in wage earnings and a further increase in the consumption demand. Reduction in the real return on assets due to the shock leads to

re-adjustments in the government budget through tax revenues and, therefore, changes in the rate of tax and transfers, and household consumption demand responds to that. And a decrease in $r_{b,t}$ also benefits the borrowers at the cost of savers.

The price of the agricultural goods $P_{A,t}$ adjusts immediately. However, the prices in the intermediate goods market $P^j_{M,t}$ adjust slowly, which creates differences in the relative consumption demand for the two goods.

My model incorporates the following popular channels documented in the existing literature;

Direct channel: The difference in the marginal propensity to consume, which is a unique characteristic of the households, leads to different consumption responses through intertemporal substitution. This is one of the most prominent channels of redistribution in the Representative and limited heterogeneity New Keynesian models. However, in continuous heterogeneity models, as explored by Auclert (2019); Kaplan et al. (2018) among many others, the indirect channels, i.e., wage income, interest income, and tax and transfer channels, constitute a bigger share in the total monetary policy transmission into household consumption.

Income Channels: Households differ in terms of their primary source and level of income. Monetary policy has distributional effects as these different sources of income and levels of earnings react differently to monetary policy shocks.

Income composition channel: Richer households receive a larger share of their financial income, which is typically more responsive to monetary policy than labor income (Gornemann et al., 2016).

Earnings distribution channel: Households also differ in terms of where their earnings fall in the overall distribution. Wages and employment prospects of low-income households are typically more sensitive to monetary policy and business cycles (Heathcote and Perri, 2009). For instance, low-productivity, lower-income workers are more likely to lose their jobs during a recession.

Wealth Channels: Households differ not only by their wealth level (including whether their net wealth is positive or not) but also by what they own and what they owe (their wealth portfolio composition). By changing the value of the underlying assets and their return, monetary policy redistributes wealth among households.

Balance sheet composition channel: Household balance sheets differ substantially

both on the asset and the liability side. On the asset side, a fall in the interest rate affects different assets differently, depending on their type and duration. On the liability side, how unexpected reductions in interest rates affect debt service and balance of loans also depends on these liabilities' type and maturity.

Savings redistribution channel: Households differ in terms of their level of wealth. Some households are borrowers with negative net wealth, and others are savers with positive net wealth. Unexpected decreases in real interest rates favor borrowers and hurt savers. Further, borrowers typically are less patient and have a greater marginal propensity to consume.

The *inflation channel*, whereby the relevant dimension of household heterogeneity is cash holdings and nominally fixed debt (Erosa and Ventura (2002), Doepke and Schneider (2006)) and the *interest rate exposure channel*, whereby the measure of a household's balance sheet exposure to interest rates is the difference between all maturing assets and liabilities at a given point in time (Auclert, 2019) are also featured in my model. However, in my simplified framework, these two channels have to be classified as wealth channels as they relate to differences across households in their assets and liabilities and their net wealth positions.

4 Taking the Model to Data

The numerical solution for continuous-time heterogeneous agent models is based on a finite difference method. It solves two paired equations: the Hamilton–Jacobi–Bellman (HJB) and the Kolmogorov Forward (KF) or the Fokker–Planck equation. First, the HJB is solved for a given time path of prices, and then the KF is solved to obtain the evolution of the joint distribution of productivity and assets. Having solved the HJB, obtaining the time path of the distribution becomes easier as the KF is just an eigenvalue problem with the transpose of the same transition matrix as in HJB. In computation, the continuous time approach imparts a number of advantages over discrete time. The continuous-time solution with discretized state space is extremely sparse in nature, which gives computational advantages, and the algorithm becomes short and simple. Implementing this requires only a rudimentary understanding of matrix algebra and access to software packages that can solve sparse linear systems. Also, in continuous

time, the borrowing constraint binds only at the borrowing limit and nowhere else in the state space. The HJB and KF for this economy are, respectively,

$$\begin{aligned}\rho V(b_{i,t}, z_{i,t}) = max\ \ & u(c_t^i, l_{i,t}) \\ & + V_b\big[(1-\tau)\, w_{i,t}\, z_{i,t}\, l_{i,t} + r_{b,t}\, b_{i,t} + T - c_t^i\big] \\ & + \lambda_i(V_{-i}(b_{i,t}, z_{i,t}) - V_i(b_{i,t}, z_{i,t}))\end{aligned} \tag{26}$$

$$0 = -\frac{\partial}{\partial b_{i,t}}[s(b_{i,t}, z_{i,t})\, g(b_{i,t}, z_{i,t})] - \lambda_i g_i(b_{i,t}, z_{i,t}) + \lambda_{-i} g_{-i}(b_{i,t}, z_{i,t}) \tag{27}$$

where,

$$s(b_{i,t}, z_{i,t}) = (1-\tau)\, w_{i,t}\, z_{i,t}\, l_{i,t} + r_{b,t}\, b_{i,t} + T - c_t^i$$

$-i = L$ when $i = H$ and vice versa, $g(b_{i,t}, z_{i,t})$ is the density function corresponding to the state $\mu(b_{i,t}, z_{i,t})$. These two differential equations, along with the boundary conditions, characterize the steady-state equilibrium of the economy. HJB provides optimal consumption and saving, while KF provides the evolution of the joint distribution of assets and productivities.

4.1 Calibration

The structural parameters are generally calibrated using one of the following two strategies: either they are estimated using suitable data and econometric techniques that fit the real economy, or they are taken directly from the existing literature. In the latter case, the parameters must exhibit limited variability with changes in the model environment; therefore, they are considered to be standard in the literature. I employ a combination of these two strategies. I calibrate most of the parameters using data on the Indian economy; the empirical strategies are discussed below. I obtain a few parameters from studies specific to the Indian economy. Only two of the model parameters are included directly from the standard literature that do not require any change.

4.1.1 Household Assets

The National Sample Survey Office (NSSO), under the Ministry of Statistics and Programme Implementation, Government of India, conducts national-level surveys on various socioeconomic variables for households in India. I use their latest All India Debt & Investment Survey (AIDS) conducted between Jan-Dec 2019 to obtain the data on the

borrowing limit of the households. This survey reports the financial assets of households which include all household deposits in financial institutions (checking, saving, call, and money market accounts), government bonds, and corporate bonds net of revolving consumer credit, etc., and all household debts.

According to this data source, 84.4% of the rural population and 85.2% of the urban population aged 18 years and above have some financial assets in banks, and 35% households in rural India and 22.4% households in urban India hold some kind of financial debt, indicating decent access to financial markets in both rural and urban areas. On average, the amount of debt in rural households is Rs. $59,748$, and it is Rs. $1,20,336$ in urban households. The net (asset minus debt) household assets as a proportion of per capita GDP varies roughly between -2.98 and 3.26. Ignoring the extreme values beyond 10^{th} and 90^{th} percentile, net asset holding varies between -0.32 and 0.60. I calibrate the quarterly borrowing limit at -0.08, corresponding to the yearly borrowing limit of -0.32. I only target the mean while deriving the steady state distribution of net assets. Comparing the steady state moments with moments from survey data concludes that my model produces realistic values of this distribution.

One might think of setting different values of the borrowing limit for agricultural and non-agricultural households. Doing so seems redundant as the difference in the asset distribution between these two types of heterogeneous households arises endogenously from their labor-leisure and consumption choices. Besides, making one set of households more borrowing-constrained than the other will lead to exogenous manipulation in their choices, which is not necessary in this model.

4.1.2 Transition Rates

The rates of transition between the two-state productivity process are also estimated using household survey data. These rates contribute significantly to the determination of household asset holding. For example, a lower rate of transition may lead to the accumulation of assets in high-productivity non-agricultural households and a higher level of borrowing in low-productivity agricultural households.

Indian Human Development Survey (IHDS), I conducted in 2005 and II conducted in $2011-12$, provides panel data on a range of variables that include household and individual characteristics, their income sources, and detailed decomposition of income.

This helps me to track the households transitioning between agriculture as the main source of income and non-agriculture as the main source of income. Agricultural households can be either of these three types: cultivators, engaged in agri-allied activities like forestry, poultry, fishery, etc., or provide wage labor in agriculture. They constitute roughly 24%, 0.9%, and 11.8% on average in the total population. The rest of the households are considered to be non-agricultural households.

I calculate the fraction of households that transitioned between agricultural and non-agricultural households. The transition rate is calculated simply from the number of households that transitioned within the time period, converted to quarterly frequency. The rate of a household transitioning from low to high productivity status λ_1 is 0.02, and the rate of transition from high to low productivity status λ_2 is 0.04 corresponding to the yearly numbers of 0.08 and 0.16 respectively. Due to the unavailability of any other panel data on household income in India, this seems to be the only way to calculate these probabilities.

4.1.3 Tax and Transfer

The income tax rates vary widely depending on income slabs, age, and occupation. Therefore, I use the average income tax collection as a percentage of total taxable income for the last three years. The data is obtained from the RBI bulletin on the Union budget 2020-21 and 2022-23. The three-year average income tax rate is around 11% leading to a quarterly rate of 2.75%.

Ministry of Finance press release reports data on government direct benefit transfers to the households. The quarterly average of total direct benefit transfers from the state government and the central government to households as a percentage of the GDP in three consecutive pre-pandemic years ($2017-18$, $2018-19$, and $2019-20$) is obtained to be $T = 1.06\%$.

4.1.4 Share of Non-agricultural Goods in the Consumer Basket

I use the latest Consumer Expenditure Survey data by NSSO conducted in $2022-23$ to obtain the share of household consumer expenditure dedicated to food and other consumer items. To make an association between the components in the consumer basket with the broadly defined agri and non-agri goods in this model, I categorize all

food items as agri goods and everything else as non-agri goods. The average share of consumer expenditure on food alone by agricultural and non-agricultural households is 0.53 and 0.50. The expenditure shares in agri and non-agri households on food and non-food items barely differ. However, if I had to set two different parameter values for γ, say γ_L for agri households, and γ_H for non-agri households, then $\gamma_A = 0.47$ and $\gamma_H = 0.50$. This could be useful in future analysis if one wishes to decompose consumption responses due to monetary policy change into two different categories of food items. For now, γ is set at 0.48, a simple average of γ_L and γ_H.

4.1.5 Mean Reversion Rate of the Monetary Policy Shock

I calibrate the mean reversion rate of the monetary policy shock η using historical data on the weighted average call money rate (WACR). WACR is a short-term interest rate that is used as the operating target by the Central Bank of India (RBI). An operating target is an anchor that is kept as close as possible to the target policy rate (the repo rate) so as to minimize any internal lag within the monetary policy framework by controlling liquidity in the overnight interbank call money market. I obtain quarterly data on WACR from April 1991 to December 2023 that is available in the *Handbook of Statistics on Indian Economy* published by the RBI. I de-seasonalize the log WACR and test for the presence of unit roots. Then I fit an ARIMA model with appropriate AR and MA specifications along with the order of integration as 1. The quarterly mean revision rate is obtained as 0.45, leading to $\eta = 0.64$. I introduce a one-time expansionary shock to the monetary policy of 50 basis points, i.e., $\zeta_0 = -0.5\%$ points, and the shock dies at the rate $e^{-\eta t}$.

4.1.6 Others

For the rest of the parameters, I stay as close as possible to the parameterizations that are well-accepted in the New Keynesian literature. The parameter values, their descriptions, and sources are in the table 3.

4.2 Steady State Policy Functions

I obtain the steady state path of household consumption and savings following the numerical solution method mentioned above. Figure 4 depicts the steady state consump-

Table 3: Parameter Values, Description and Sources

Parameter	Value	Description	Source
Households			
ρ	0.98	Discount factor	Gabriel et al. (2016)
σ	1.99	Inverse of inter-temporal elasticity of substitution	Gabriel et al. (2016)
ψ	3	Inverse of Frisch elasticity of labour supply	Anand and Prasad (2010)
λ_1	0.02	Transition rate from agri to non-agri	Calculated by Author
λ_2	0.04	Transition rate from non-agri to agri	Calculated by Author
γ	0.48	Share of non-agricultural goods in consumer basket	Calculated by Author
$\underline{b}$	-0.08	Borrowing limit	Calculated by Author
Government			
τ	0.028	Rate of tax on labor income	Calculated by Author
T	0.0106	Lump-sum transfer by the government	Calculated by Author
Firms			
δ	0.069	Rate of depreciation of capital	Gabriel et al. (2016)
ϵ	10	Elasticity of substitution between modern goods	Anand and Prasad (2010)
α	0.32	Share of capital	Gabriel et al. (2016)
θ	100	Cost of adjustment parameter	Kaplan et al. (2018)
Monetary Policy			
β	1.25	Taylor rule parameter	Kaplan et al. (2018)
η	0.64	Rate of mean reversion of the shock	Calculated by Author

tion of households over different values of the assets. Consumption of high-productivity non-agricultural households is higher than that of the low-productivity agricultural households at all levels of household asset holdings. The differences, however, are not drastic, which is consistent with data. A slightly higher slope of the policy function of non-agri households indicates that consumption is more sensitive to asset holding for this group of households than agri households. The fact that high productivity non-agri households consume more at all levels of steady state assets might appear contradictory to our understanding that households with lower income or assets generally consume more with an increment in income or assets. Note that this consumption comprises both food and non-food consumption, like durable goods, medical care, education, conveyance, and conspicuous consumption of the households, which are generally higher in high income and wealthy households.

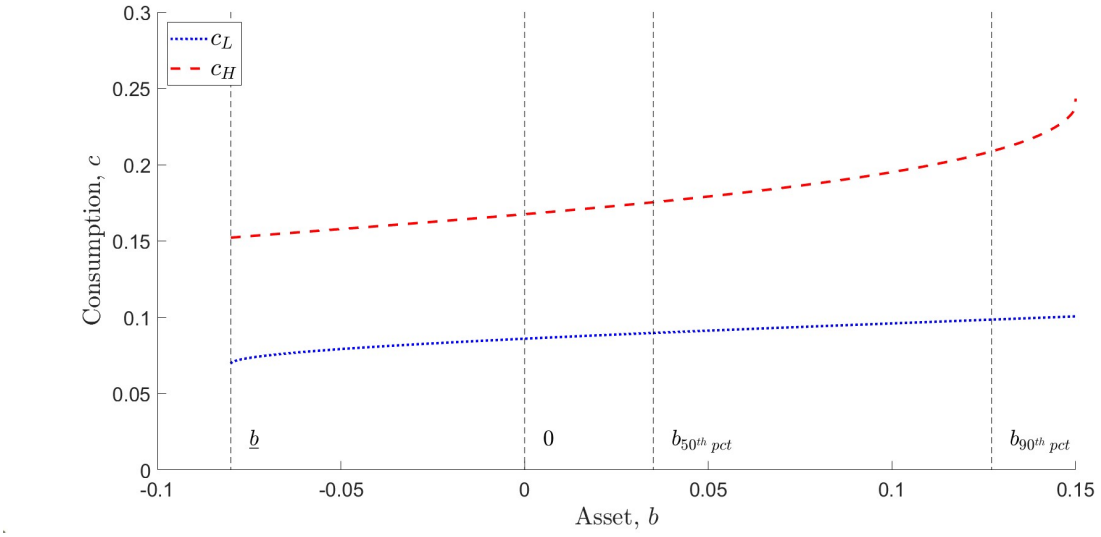


Figure 4: Consumption policy function

Figure 5 illustrates the steady-state distribution of assets. There is a high concentration of agri households at the borrowing limit, whereas non-agri households are highly concentrated at an extremely high level of assets. The higher density of agri households, between 0 and the borrowing limit, indicates that a large proportion of agri households are net borrowers at the steady state. However, the higher density of agri households, even at the higher percentiles, is puzzling. One of the reasons is that the transition rates between agri and non-agri households are really low, leading to a higher precautionary saving by the net savers agri households. However, due to income disparity, the high concentration of wealth is only observed in the high productive high income non-agri households.

While calibrating, I only target the overall mean of the steady state asset distribution. Table 4 showcases that the model-generated mean matches closely with the mean from the survey data, indicating the model is a good fit. I further compare different percentiles of the model-generated asset distribution with those obtained from the NSSO Household Debt & Investments Survey conducted in 2019. The model performs well except for the extreme values of the asset.

To examine if the model-generated values for consumption at the steady state are realistic, I match different model-generated consumption moments and percentiles with those from the consumption survey data. The newly released data on the Household Consumer Expenditure Survey (HCES) conducted by NSSO between August 2022 and

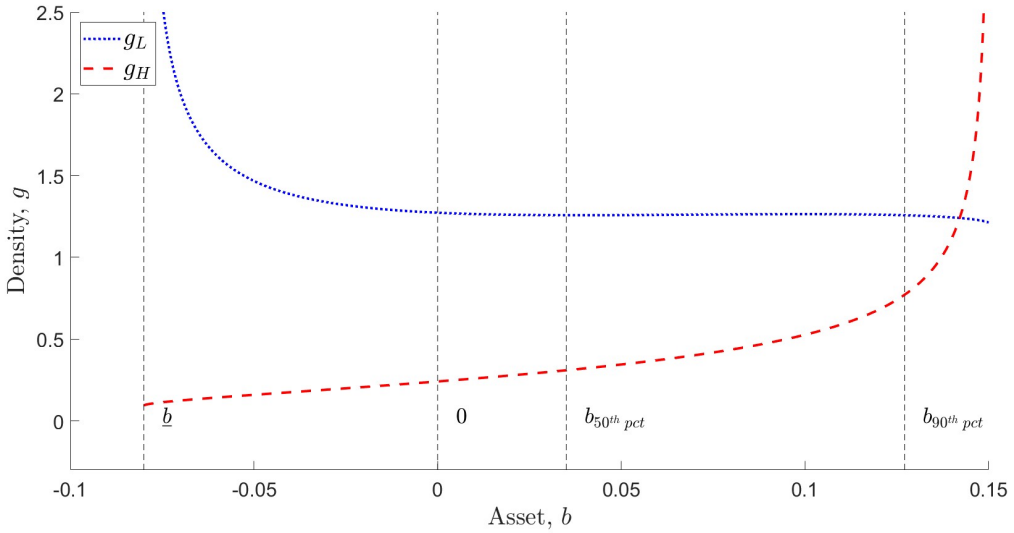


Figure 5: Distribution of Asset

Table 4: Moments and Percentiles of Asset Holding

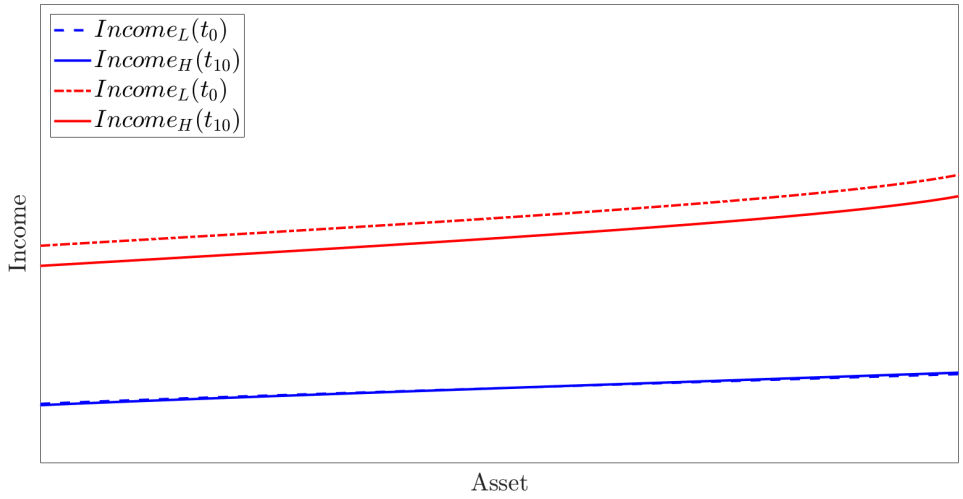
	Data	Model
Targeted		
Mean	0.05	0.04
Non-Targeted		
25^{st} Percentile	-0.06	-0.02
75^{th} Percentile	0.08	0.09
90^{th} Percentile	0.55	0.12

Table 5: Moments and Percentiles of Consumption

	c_L Data	c_L Model	c_H Data	c_H Model
Mean	0.11	0.08	0.19	0.18
10^{th} Percentile	0.05	0.07	0.09	0.15
25^{th} Percentile	0.07	0.08	0.12	0.16
50^{th} Percentile	0.09	0.09	0.17	0.17
75^{th} Percentile	0.13	0.10	0.23	0.20
90^{th} Percentile	0.19	0.18	0.31	0.21

July 2023 provides insights into the household monthly expenditure on different food and non-food items. Table 5 presents monthly household consumer expenditure as a proportion to per-capita GDP at constant prices. Comparing with the model-generated mean and percentiles confirms that the model is a decent fit for the real economy. A quick look at the income levels over assets might be useful here. Income from non-agricultural activities definitely lies above income from agriculture at all values of asset holding. The expansionary shock on monetary policy transcends into higher income for the non-agricultural household as monetary policy easing in the presence of sticky price leads to higher wage income. However, due to the high flexibility of agricultural prices, monetary policy shock barely effects the income of the households in this sector. This outright contributes to muted consumption responses in agri households. For a

Figure 6: Household Income Over Asset



deeper investigation into the micro mechanism behind the heterogeneity in consumption responses, let's look at the marginal propensity to consume.

4.3 MPC Heterogeneity

Recent studies have suggested considerable cross-sectional heterogeneity among households at different income levels. I plot The marginal propensity to consume of agricultural and non-agricultural households over income. Households at lower levels of income exhibit higher levels of MPCs, which is consistent with the existing literature, and as the income increases after a certain point, MPC remains unchanged. The highest MPC in agricultural households is much higher than the highest MPC in the non-agricultural household. However, note that even the lowest income levels for non-agricultural households lie substantially above the highest income among agricultural households. Household lying at the bottom of the income distribution among non-agri households still exhibits a much higher MPC than those at the top of the income distribution among agricultural households even though their income is higher. These heterogeneities in MPC substantially influence the consumption responses among households.

Figure 7: MPC

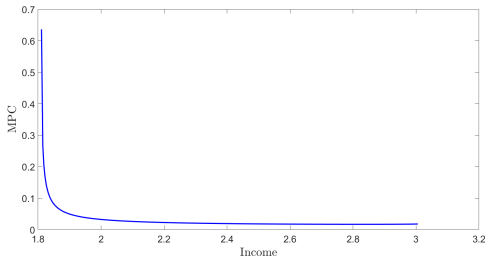
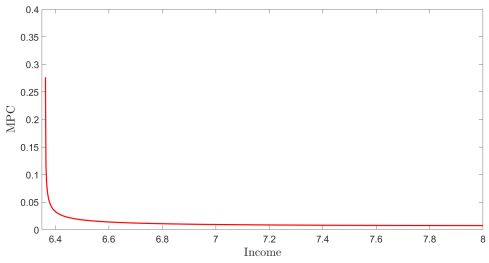


Figure 8: MPC



4.4 Hand-to-Mouth Matching

It is a standard practice in studies like these to match the number of hand-to-mouth households that the model generates with that in the household survey data. However, in India, finding a survey that can be used to calculate the percentage of Hand-to-Mouth households in the economy is a challenge. Kaplan and Violante (2014); Kaplan et al. (2018); Slacalek et al. (2020) suggest that a household is considered as 'hand-to-mouth' if

Net liquid wealth ≥ 0 & net liquid wealth $\leq$ biweekly (net) income OR

Net liquid wealth ≥ 0 & net liquid wealth $\leq$ biweekly (net) income - credit limit

A household is 'Poor hand-to-mouth' if it is hand-to-mouth and net illiquid wealth ≤ 0, it is 'Wealthy hand-to-mouth' if it is hand-to-mouth and net illiquid wealth > 0, Non-hand-to-mouth if it is not hand-to-mouth. Apart from a few minor differences between Kaplan and Violante (2014) and Slacalek et al. (2020), these are the definitions widely used in the calculation of hand-to-mouth households in the literature. This study considers only one type of wealth; therefore, there is no need to distinguish between poor and rich HtM. As is evident, I need data on household assets and income from survey data to calculate the same for the Indian economy. The Debt & Investment Survey reports households' debt and asset holdings; however, they do not report incomes. They report the usual household consumer expenditures. There are other household surveys like the household consumer expenditure survey, which reports consumer expenditure in a much more disintegrated way, however, does not report debt-asset situations, and there are periodic labor force surveys that ask detailed questions on household incomes; however, they do not provide any information on assets or debts. Therefore, the only way

to obtain an estimate of the percentage of hand-to-mouth households in the economy is to use the DIS data and replace income in the definition with consumption.

From the DIS data, by replacing income with consumption in the definition, the percentage of HtM among agricultural households is obtained to be 59%, and that among the non-agricultural household is 51%. The issue with these numbers is that a large chunk of households usually spend a lot more on the monthly consumption of goods and services than their monthly income. These households are essentially hand-to-mouth. The periodic labor force survey reports the usual monthly consumption spending of the households along with detailed information on individual and household incomes, sources of income, etc. Note all these surveys are nationally representative sampling surveys. Therefore, the households that were surveyed and the time of the survey are different from each other. Using PLFS data, I obtain 64% among agricultural households and 80% among the non-agricultural households who usually spend more on consumption in a month than their monthly total income.

Due to this severe limitation in the availability of appropriate data, unable to obtain an exact estimate of the HtM households, I consider the interval of $59\% - 64\%$ for agricultural households and $51\% - 80\%$ for the non-agricultural households to be the target. My model-generated percentages, respectively, are 62.2% and 62.6%.

4.5 Economy's Response to a Monetary Policy Shock

I evaluate the response of the economy to an unexpected one-time change in monetary policy stance. This shock hits the economy at time $t = 0$, with a magnitude of $\zeta_0 = -0.5$ percentage point i.e., 50 basis points expansionary shock on the monetary policy. And it mean reverts at rate η, i.e., $\zeta_t = e^{-\eta t}\zeta_0$. The magnitude of this monetary policy shock is consistent with the historical data on monetary policy stances by the RBI.

The transition dynamics of consumption and asset distribution in response to the shock are depicted in figure 9 and 10. Consumption responses seem quite small from the figure. Therefore, I calculate the percentage deviation of household consumption in proportion to per capita GDP at the mean and different percentiles observed in the first quarter when the shock is introduced (i.e., $t = 0$), compared to the steady state. Table 6 showcases the results. An expansionary monetary policy stimulates consumption at all levels for both types of households. The magnitude of the response

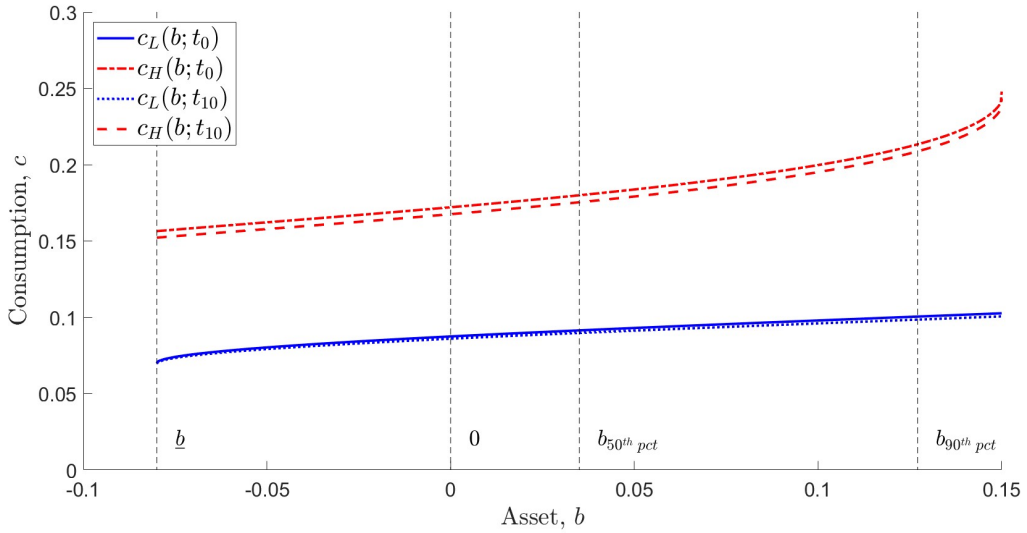


Figure 9: Transition of consumption policy function

in non-agri households is higher at all levels than that of agri households. Note that this consumption response includes household non-food expenditures like healthcare, education, durables, etc. A higher response in the non-agri than agri also stems from the fact that real effects are higher with higher price rigidities. Higher price rigidity in the non-agri sector leads to a higher income through higher output, leading to higher consumption responses.

A negative monetary policy shock reduces the density of agricultural households at all positive values of net assets, i.e., the density of net savers among agri households decreases while the density of net borrowers remains seemingly indifferent. For non-agricultural households, density at different values of net asset holding remains the same. This again stems from the fact that net saver agri households are driven by precautionary motives and respond substantially to even small changes in the interest rates because these households having persistently low income, and maintaining some precautionary savings means sacrificing consumption at present to secure the future. A lower return, therefore, discourages them more than anyone else.

Being able to quantify the effect on consumption for the entire asset distribution and over two types of households is one of the most crucial contributions of this paper. The impulse responses in figure 16 show that, as the shock mitigates, economy returns to the steady state.

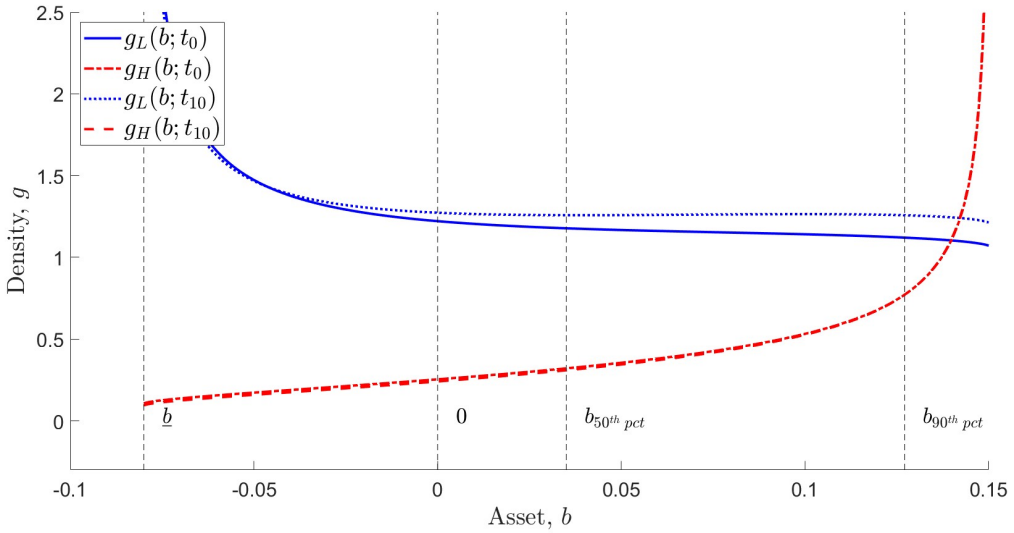


Figure 10: Transition of asset distribution

Table 6: Consumption Redistribution After the Shock

	Steady-state c_L	c_L at quarter 1	% increase	Steady-state c_H	c_H at quarter 1	% increase
Mean	0.0889	0.0905	1.7681	0.1793	0.1839	2.5279
10^{th} Percentile	0.0779	0.0789	1.2639	0.1565	0.1609	2.7980
25^{th} Percentile	0.0832	0.0846	1.5862	0.1630	0.1675	2.7504
50^{th} Percentile	0.0898	0.0914	1.8600	0.1754	0.1800	2.6144
75^{th} Percentile	0.0953	0.0972	1.9902	0.1922	0.1969	2.4054
90^{th} Percentile	0.0985	0.1005	2.0217	0.2087	0.2133	2.2070

4.6 Yearly Time Frame

The quarterly time frame is associated with small values of some of the calibrated parameters that are crucial in determining the magnitude of redistribution, like the transition rates, borrowing limit, and policy shock. One experiment to analyze the persistence of the results can be through expanding the time frame to yearly frequency and then comparing the results with that obtained in the quarterly time frame. Therefore, I repeat the exercise by first calibrating parameters accordingly (they are discussed above in the calibration section, along with the quarterly calibrations).

Figure 11 depicts the steady state consumption of households over different values of the assets. Similar to the quarterly results, the steady state consumption of high-productivity households is higher than that of the low-productivity households at all

levels of household asset holdings, the slope of the policy functions at the steady state, households with higher asset holdings consume more and consumption increases

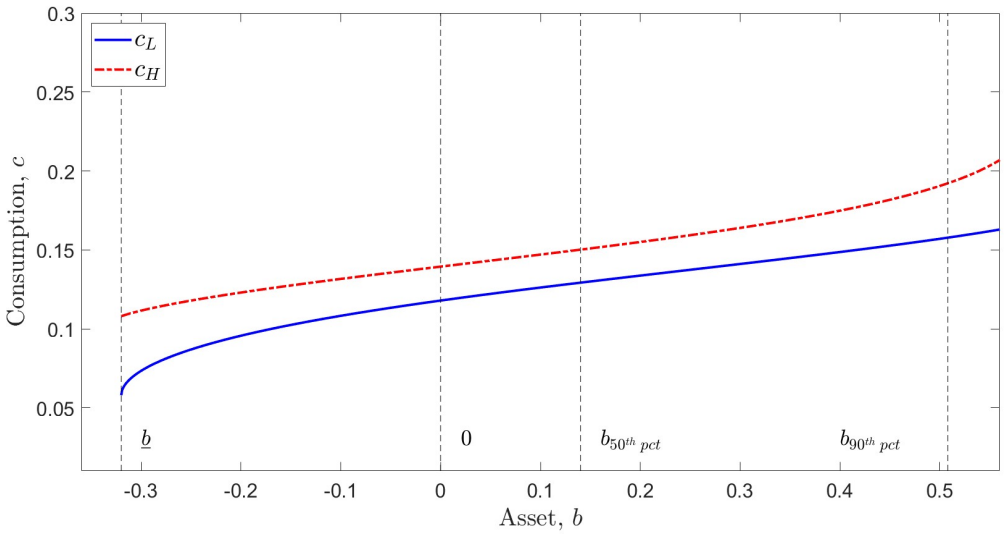


Figure 11: Consumption policy function

Figure 12 illustrates the steady-state distribution of assets. Similar to the quarterly analysis and consistent with the AIDS survey, the density of agricultural households at all levels of net borrowing is higher than that of non-agricultural households. And the density of non-agri households the extremely high net positive value to assets is much higher than that of agri households. These similarities in findings between quarterly and yearly results indicate the reliability of the calibrated parameters.

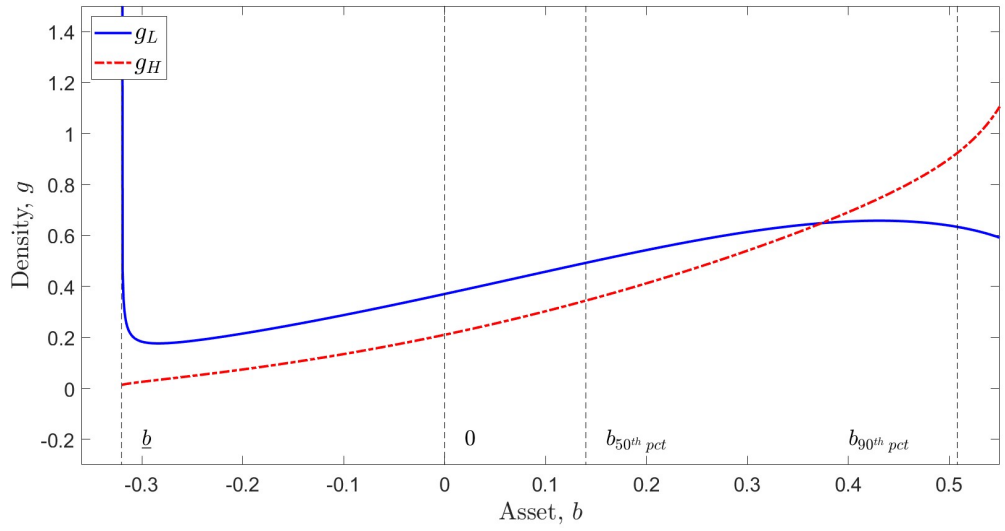


Figure 12: Distribution of Asset

Table 7 shows that the model-generated mean matches exactly with the mean from the survey data, indicating how well the model fits the economy. I also compare different percentiles of the model-generated asset distribution with those obtained from AIDS. Except for the two extreme ends, the values generated by the model match the data closely.

Table 7: Moments and Percentiles of Asset Holding

	Data	Model
Targeted		
Mean	0.14	0.14
Non-Targeted		
1^{st} Percentile	-0.31	-1.94
10^{th} Percentile	-0.23	-0.18
90^{th} Percentile	0.51	0.62
99^{th} Percentile	0.60	3.60

Table 8: Moments and Percentiles of Consumption

	c_L Data	c_L Model	c_H Data	c_H Model
Mean	0.11	0.11	0.19	0.15
10^{th} Percentile	0.05	0.08	0.09	0.11
25^{th} Percentile	0.07	0.10	0.12	0.12
50^{th} Percentile	0.09	0.12	0.17	0.14
75^{th} Percentile	0.13	0.13	0.23	0.16
90^{th} Percentile	0.19	0.15	0.31	0.19

Again, I match the model generated steady state consumption at different percentiles with the monthly household consumer expenditure as a proportion to per-capita GDP at constant prices obtained from HCES. The numbers match closely, even at the higher percentiles.

At the time $t = 0$, I introduce the one-time shock that hits the economy with a magnitude of $\zeta_0 = -1$ percentage point, i.e., 100 basis points easement of monetary policy which mean reverts following $\zeta_t = e^{-\eta t}\zeta_0$. This magnitude of yearly change in monetary policy is consistent with the historical monetary policy stance in India. The mean reversion rate of the monetary policy shock is estimated with the same variables, i.e., WACR, as in the quarterly analysis, only with a change in the frequency of the time series. Log WACR is deseasonalized and tested for unit roots and appropriate auto-regressive and moving-average specifications. The ARIMA model generates the mean reversion rate $\eta = 0.69$.

Figure 13 depicts the effect of the shock on steady-state consumption. As the interest rate on assets decreases, consumption increases multi-fold for both agri and non-agri households. Table 9 summarizes the moments of consumption redistribution. The percentage increase in consumption is much bigger than the quarterly analysis at all

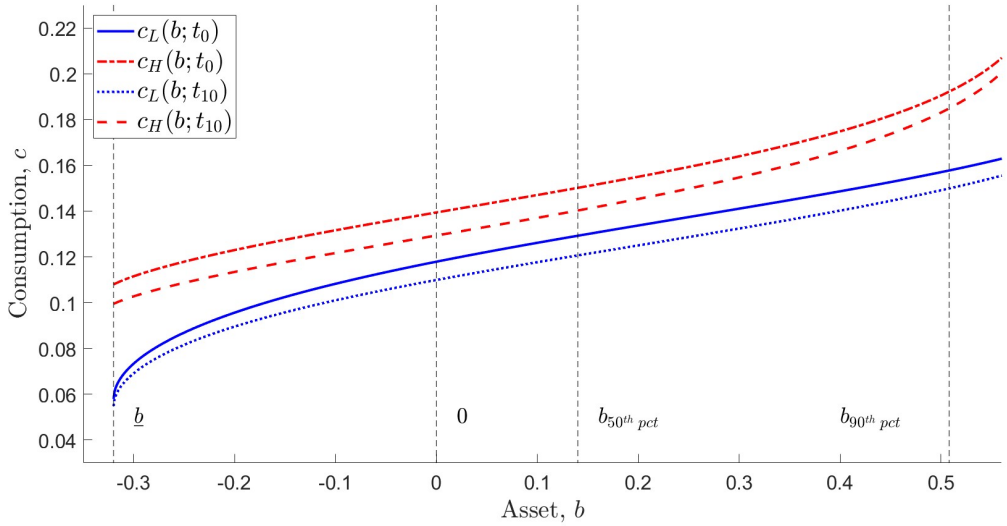


Figure 13: Transition of consumption policy function

percentiles. The responses are relatively higher in the lower percentiles.

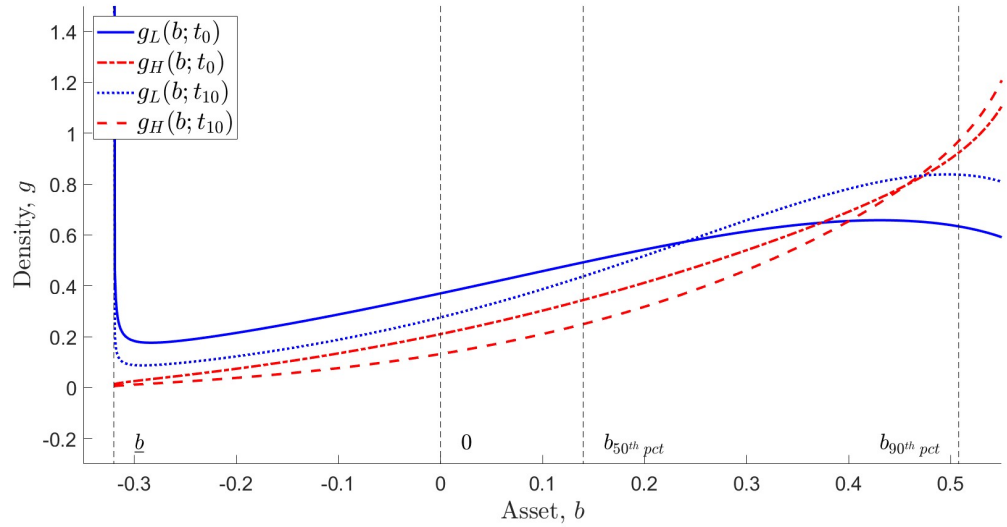


Figure 14: Transition of asset distribution

Figure 14 illustrates the effect of the shock on the asset distributions. The density of agricultural households with positive net borrowing increases sharply, with such non-agricultural households responding faintly. Redistribution of assets in the yearly time frame is similar yet much more drastic than in the quarterly time frame.

Table 9: Consumption Redistribution After the Shock

	Steady-state c_L	c_L at year 1	% increase	Steady-state c_H	c_H at year 1	% increase
Mean	0.1189	0.1265	6.41	0.1446	0.1536	6.27
10^{th} Percentile	0.0854	0.0911	6.66	0.1108	0.1202	8.46
25^{th} Percentile	0.1020	0.1093	7.16	0.1225	0.1324	8.12
50^{th} Percentile	0.1206	0.1292	7.12	0.1402	0.1501	7.05
75^{th} Percentile	0.1378	0.1463	6.19	0.1625	0.1712	5.38
90^{th} Percentile	0.1499	0.1577	5.22	0.1848	0.1921	3.96

4.7 Aggregate Responses

The impulse responses of aggregate consumption to yearly and quarterly analysis are reported respectively in figure 16 and 17. The increase in aggregate consumption for both types of households systematically falls over time till it returns to the steady state figures in around 6^{th} quarter and 7^{th} year. Inflation in the non-agricultural sector is stable. The economy returns to the steady state when the shock mitigates.

4.8 Comparison with Other Economies

There are four critical features of this model: two assumptions based on the stylized facts in the Indian economy - prices of agricultural commodities fluctuate often, labor productivity in this sector is lower than in the non-agricultural sector, and two observations from the simulated results - a higher proportion of non-agricultural households are hand-to-mouth (see section 4.4), the marginal propensity to consume for low-income non-agricultural households is much higher than that of the high-income agricultural household although the absolute income of low-income non-agri households is much higher than the low-income agri households (see 7 and 8). While most of these features are typical of the Indian economy, comparing the consumption responses with those from some developed economies can be insightful. Slacalek et al. (2020) quantifies the effect of an initial 100-bp monetary policy easing on consumption in the Euro Area using Eurosystem Household Finance and Consumption Survey, wave 2. Figure 18 and 19 show a decomposition of the effects of an initial 100-basis-point shock to interest rates on consumption from a structural VAR model and HANK-based back-of-the-envelope

Figure 15: Impulse Responses of Monetary Policy Shock on Aggregate Variables

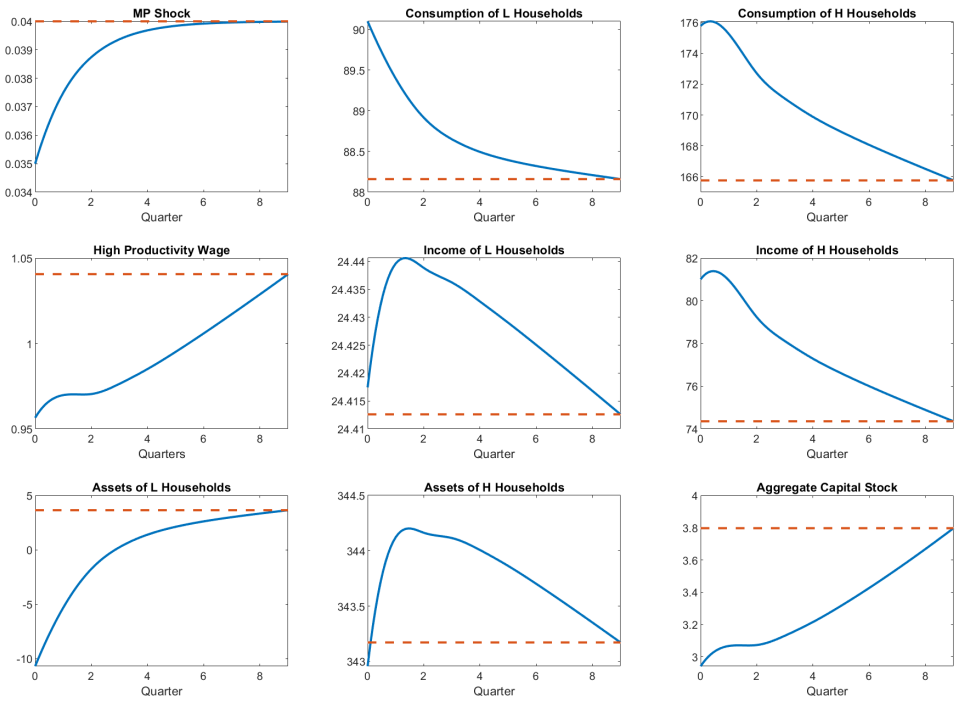


Figure 16: Impulse response to quarterly shock

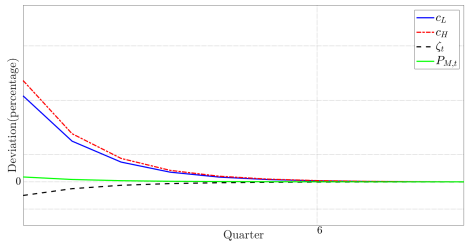
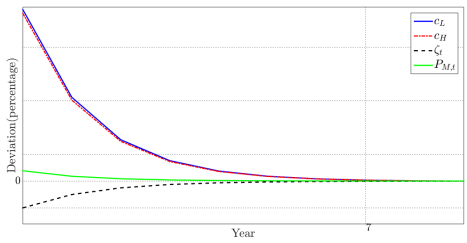


Figure 17: Impulse response to annual shock



calculations. The total consists of six parts. The direct effect of intertemporal substitution (IES) and the net interest rate exposure (NIE), and the indirect effect due to labor income, net nominal positions (Fisher), collateral/housing wealth, and stock market wealth, the sizes of these effects vary depending on households hand-to-mouth status.

Comparing these Euro-area results to the consumption responses of Indian house-

Figure 18: Effects of Monetary Policy Easing on Consumption in Euro Area from Slacalek et al. (2020)

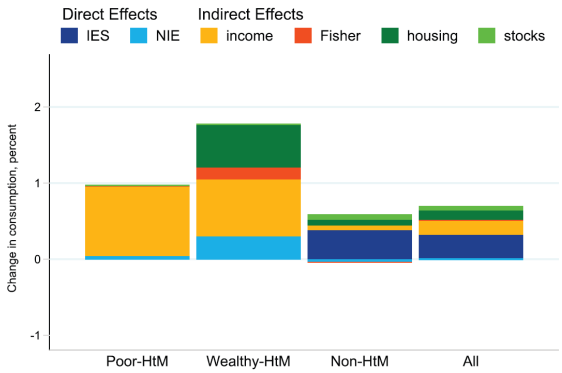
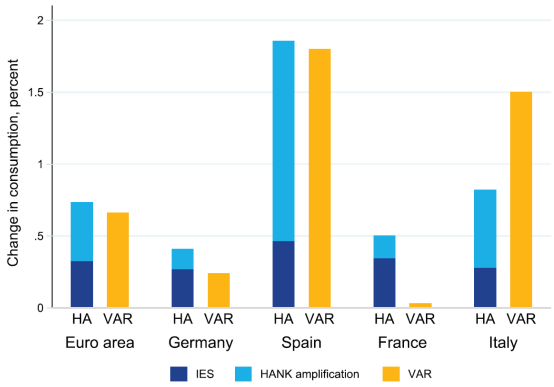


Figure 19: Effects on Consumption: VAR vs HANK Decomposition from Slacalek et al. (2020)



holds to a 100 basis point rate cut in a yearly time frame (see detail in table 9), I observe that the effect are much higher for both agri and non-agri households in India. This occurs due to the fact that the percentage of hand-to-mouth households (see section 4.4) and the average MPC (see figure 8) for different percentiles of income are higher in India than in the Euro-area. Slacalek et al. (2020) reports from the literature that an average annual MPC out of income windfalls for the US economy is around 0.02, and they set the MPC out of transitory income for (both poor and wealthy) hand-to-mouth households at 0.50 and for non-hand-to-mouth at 0.05. Compared to that, the MPC for low-income households in India is around 0.62. Additionally, in the euro area, about 10% of households are classified as poor hand-to-mouth, and about 12% of households are wealthy hand-to-mouth. The overall share of hand-to-mouth households, therefore, stands at 22% compared to 62% in India.

Slacalek et al. (2020) also provides the estimates of consumption responses for four major countries in the Euro-area: Germany, France, Italy, and Spain. Out of which, wealthy hand-to-mouth households in Spain seem to be an outlier with a consumption response of roughly 6% similar to the response obtained in this paper. One of the noteworthy features of Spanish households that leads to this distinction is that 83% of the Spanish households are homeowners. Although my paper does not model illiquid wealth separately from liquid wealth, the fact that around $85\% - 97\%$ households in India hold some kind of physical asset (see table 1), of which ownership of buildings is

the largest contributor, also explains the relatively high consumption responses similar to that of wealthy hand to mouth households in Spain.

4.9 Modern Agriculture

An interesting way to understand if the presence of the distinction between agricultural and non-agricultural households amplifies or suppresses the redistributive effect will be through the introduction of a comparative static where the households are heterogeneous in asset holding and productivity while they are all employed in the non-agricultural sector producing the intermediate goods. The fundamental change required in the theoretical model to accommodate this is that the wage rate for low- and high-productivity households is now the same. However, their productivity difference makes the total wage earnings different.

Table 10: Consumption Redistribution After the Shock in One Production Sector Model

	Steady-state c_L	c_L at quarter 1	% increase	Steady-state c_H	c_H at quarter 1	% increase
Mean	0.1382	0.1433	3.7140	0.1994	0.2055	3.0743
10^{th} Percentile	0.1284	0.1326	3.2457	0.1829	0.1895	3.5981
25^{th} Percentile	0.1331	0.1379	3.5752	0.1877	0.1942	3.4773
50^{th} Percentile	0.1388	0.1442	3.8386	0.1968	0.2031	3.2042
75^{th} Percentile	0.1439	0.1495	3.9279	0.2089	0.2148	2.8146
90^{th} Percentile	0.1469	0.1526	3.9177	0.2203	0.2257	2.4483

Table 10 summarizes the quarterly consumption redistribution in the new framework. Comparing with table 6 indicates that the responses are higher at all percentiles. The presence of two sectors producing final consumption goods with different levels of stickiness leads to a suppression of consumption response. A plethora of literature has evidenced that sectors with higher price stickiness exhibit higher real effects. Therefore, in the comparative static, when an expansionary monetary policy shock is introduced, both types of labor employed in the monopolistically competitive market experience a higher wage response due to the assumption of a perfectly competitive factor market. This leads to a higher consumption response compared to the two production sector economies. In other words, the presence of an agricultural sector with relatively flexi-

ble prices pertains to a smaller real effect, leading to a suppressed overall response in consumption.

4.10 Productivity Shock in Agriculture

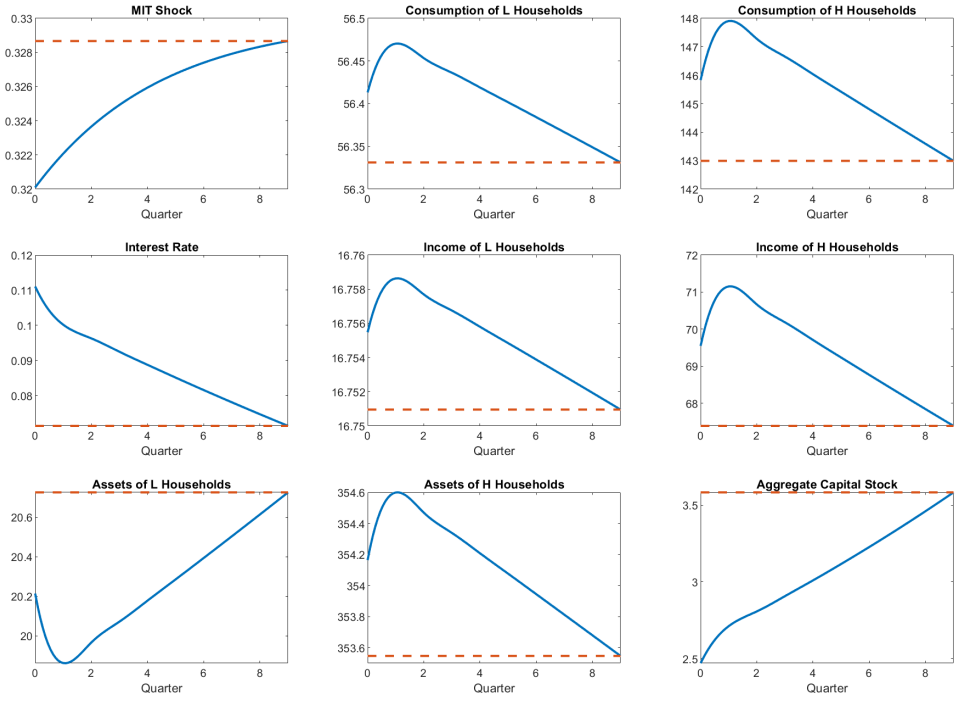
By far, the agricultural production function did not account for a total factor productivity (TFP); the low productivity was only induced by the low-productive households being employed in agricultural activities. Let us introduce a traditional technology parameter (A_t) in the production of agricultural goods

$$Y_{A,t} = A_t \, L_{L,t} \tag{28}$$

The calibration of A_t is simplistic, as agriculture absorbs around 50% of the total workforce while contributing only 16% to the country's GDP; TFP stands roughly at 0.33.

Let us now consider a small TFP shock of magnitude 0.01, i.e., the TFP in the shock period stands at 0.32; a shock of this magnitude is observed quite frequently in the agricultural sector in the form of seasonal disruptions, natural calamities, etc. Looking at the impulse responses of aggregate variables to the shock (Fig 20), it is evident that the consumption responses are similar to that of the MP shocks.

Figure 20: Impulse Responses of Productivity Shock in the Agricultural Sector on Aggregate Variables



5 Conclusion

I evaluate the redistributive effects of monetary policy shocks on household consumption, saving, and financial assets in India by developing a general equilibrium model with continuous heterogeneity. I introduce an idiosyncratic labor productivity process with the assumption that at each state of this process, labor is hired in different sectors of the economy to incorporate the unique feature that in India, the traditional low-productive agricultural sector employs almost half of the population. This structure of the theoretical model produces realistic stationary policy functions of household consumption-saving behavior and generates a realistic distribution of assets. A number of parameters are calibrated using household surveys and aggregate data on the Indian economy. The paper focuses on the economy's response to a one-time deterministic monetary policy shock.

The heterogeneous structure enables me to quantify the effect for each household and display the redistributive effect over the entire joint distribution of assets and produc-

tivity. Household consumption and asset holding respond substantially to a monetary policy easing. The effects vary widely among agricultural and non-agricultural households. Consumption is more sensitive towards the lower end of the asset distribution. A comparative static with only the non-agricultural sector employing all the labor reveals that omitting the agricultural sector severely amplifies the policy impact. This study, for the first time, provides a clear and detailed evaluation of the redistributive effects of a monetary policy shock on agricultural and non-agricultural households in India, featuring a continuum of heterogeneous households.

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Declaration of competing interest

None.

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6 Appendix

6.1 Derivation of Euler Equation

Households maximize lifetime utility

$$\int_0^\infty e^{-\rho t} \left[\frac{(c_t^i)^{1-\sigma}}{1-\sigma} - \frac{(l_{i,t})^{1+\psi}}{1+\psi} \right] dt$$

subject to

$$\dot{b}_{i,t} = (1-\tau)\, w_{i,t}\, z_{i,t}\, l_{i,t} + r_{b,t}\, b_{i,t} + T - c_t^i$$

$$b_{i,t} \geq -\underline{b}$$

household's Hamilton Jacobi Bellman (HJB) equation therefore is,

$$\begin{aligned}\rho V(b_{i,t}, z_{i,t}) = max\ \ & u(c_t^i, l_{i,t}) \\ & + V_b\big[(1-\tau)\, w_{i,t}\, z_{i,t}\, l_{i,t} + r_{b,t}\, b_{i,t} + T - c_t^i\big] \\ & + \lambda_i(V_{-i}(b_{i,t}, z_{i,t}) - V_i(b_{i,t}, z_{i,t}))\end{aligned}$$

and the Kolmogorov Forward equation is,

$$0 = -\frac{\partial}{\partial b_{i,t}}[s(b_{i,t}, z_{i,t})\, g(b_{i,t}, z_{i,t})] - \lambda_i g_i(b_{i,t}, z_{i,t}) + \lambda_{-i} g_{-i}(b_{i,t}, z_{i,t})$$

where,

$$s(b_{i,t}, z_{i,t}) = (1-\tau)\, w_{i,t}\, z_{i,t}\, l_{i,t} + r_{b,t}\, b_{i,t} + T - c_t^i$$

$-i = L$ when $i = H$ and vice versa, $g(b_{i,t}, z_{i,t})$ is the density function corresponding to the state $\mu(b_{i,t}, z_{i,t})$. From the first order and envelop conditions of the optimization problem 26, I obtain,

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\sigma}(r_{b,t} - \rho)$$

this is the consumption Euler equation.

6.2 Derivation of Philips Curve

Non-agricultural firms set prices so as to maximize lifetime profit minus adjustment cost,

$$\int_0^\infty e^{-\int_0^t r_{k,s} ds}\, (Q_t^j - {\Theta_t}^j)\, dt$$

where,

$${\Theta_t}^j = \frac{\theta}{2}\big(\Pi_{M,t}^j\big)^2\, Y_{M,t}$$

and

$$Q_t^j = \big(P_{M,t}^j - MC_t\big)\, \left(\frac{P_{M,t}^j}{P_{M,t}}\right)^{-\epsilon} Y_{M,t}$$

the optimization problem in recursive form,

$$\begin{aligned} r_{b,t} J = max\ \ & \big(P_{M,t}^j - MC_t\big) \left(\frac{P_{M,t}^j}{P_{M,t}}\right)^{-\epsilon} Y_{M,t} \\ & - \frac{\theta}{2}\big(\Pi_{M,t}^j\big)^2 Y_{M,t} + J_p P_{M,t}^j \Pi_{M,t}^j + J_t \end{aligned} \tag{29}$$

first-order condition,

$$J_p = \frac{\theta \Pi^j_{M,t} Y_{M,t}}{P^j_{M,t}} \tag{30}$$

and envelop condition is,

$$\begin{aligned}(r_{b,t} - \Pi^j_{M,t}) J_p = & \left(\frac{P^j_{M,t}}{P_{M,t}}\right)^{-\epsilon} \frac{Y_{M,t}}{P_{M,t}} \\ & - \epsilon \left(\frac{P^j_{M,t}}{P_{M,t}} - MC_t\right) \left(\frac{P^j_{M,t}}{P_{M,t}}\right)^{-\epsilon} \frac{Y_{M,t}}{P_{M,t}} \\ & + J_{pp} P^j_{M,t} \Pi^j_{M,t} + J_{tp}\end{aligned} \tag{31}$$

differentiating 30 with respect to time, substituting into 31 and simplifying under symmetric equilibrium condition $P^j_{M,t} = P_{M,t}$, I obtain the New Keynesian Philips Curve,

$$\dot{\Pi}_{M,t} + \left(\frac{\dot{Y}_{M,t}}{Y_{M,t}} - r_{b,t}\right) \Pi_{M,t} = \frac{\epsilon}{\theta} (1 - MC_t) - \frac{1}{\theta} \tag{32}$$

substituting the flexible price markup $\frac{1}{MC^*_t} = \frac{1-\epsilon}{\epsilon}$,

$$\dot{\Pi}_{M,t} + \frac{\epsilon}{\theta} (MC_t - MC^*_t) = \left(\frac{\dot{Y}_{M,t}}{Y_{M,t}} - r_{b,t}\right) \Pi_{M,t} \, .$$

6.3 Derivation of Demand for Intermediate Goods

Competitive final goods producers aggregate output of firm j to produce output following

$$Y_{M,t} = \left(\int_0^1 {y^j_{M,t}}^{\frac{\epsilon-1}{\epsilon}} \, dj\right)^{\frac{\epsilon}{\epsilon-1}} \, .$$

Monotonic transformation implies;

$$Y_{M,t} = \int_0^1 {y^j_{M,t}}^{\frac{\epsilon-1}{\epsilon}} \, dj$$

Minimizing cost $x_{M,t}$ subject to output where

$$x_{M,t} = \int_0^1 y^j_{M,t} \, p^j_{M,t} \, .$$

$$\frac{y_{i,t}}{y_{j,t}} = \left(\frac{p_{j,t}}{p_{i,t}}\right)^{\epsilon}$$

simplifying provides,

$$y^j_{M,t} = \left(\frac{P^j_{M,t}}{P_{M,t}}\right)^{-\epsilon} Y_{M,t}$$