The Role of Central Bank Credibility in Anchoring Inflation Expectations

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Abstract

In a New Keynesian framework with imperfect inflation expectations, we introduce a central bank "credibility gap," representing agents' uncertainty about the bank's emphasis on growth and anticipated policy stance. Credibility is defined as the ability to align inflation expectations with the long-term target through policy actions. The "credibility gap" captures shifts in credibility, directly affecting the anchoring of inflation expectations. Our theoretical framework establishes the microfoundations of central bank credibility. Applying the framework to the United States and Turkey, we find that the Federal Reserve's credibility gap disappears post-pandemic, stabilizing expectations, while in Turkey, it has been volatile, undermining expectation anchoring over the past decade but recently improving with orthodox policy measures. Using Bayesian estimation, we quantify the influence of the credibility gap on shaping long-term expectations in both economies.

Keywords: *Monetary policy, central bank credibility, credibility gap, Bayesian inference.*
JEL Classification: E31, E52, E58, C11.

1 Introduction

Inflation has risen sharply worldwide following the pandemic, which has forced central banks to implement a variety of measures aimed at curbing inflation by effectively managing inflation expectations. Although some central banks, such as the Federal Reserve (Fed) in the United States, have been notably successful in reducing inflation, others, such as the Central Bank of the Republic of Turkey (CBRT), have faced significant challenges in anchoring inflation expectations and effectively guiding financial markets, despite adopting high interest rates.

Developing a theoretical perspective, this paper examines the varying effectiveness of central banks in anchoring inflation expectations through the lens of credibility. Our model is able to identify two key mechanisms to explain the differential performances of the Fed and CBRT. First, all else being equal, a central bank with weaker credibility faces greater challenges in aligning inflation expectations. Specifically, even if two central banks adopt equally hawkish monetary policy stances, the one with weaker credibility struggles more to shift expectations toward the inflation target. Second, a less credible central bank is generally less likely to adopt sufficiently hawkish policies, limiting its ability to curb demand and counteract inflationary pressures. While the underlying reasons for this reluctance are beyond the scope of this paper, existing literature highlights the role of political pressures in undermining independent monetary policy (See e.g. Çakmakli et al. 2024, and the references therein).

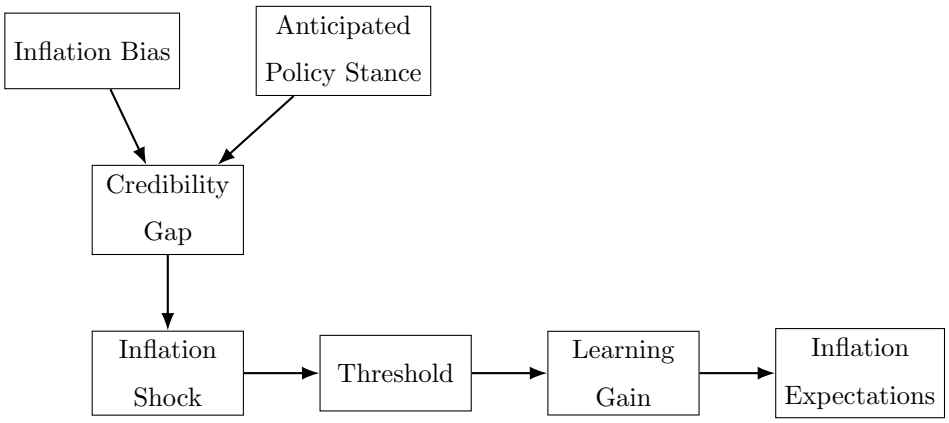
To identify and dissect the channels that influence central bank credibility, we extend the theoretical framework of Carvalho et al. 2023 to emphasize the role of informational asymmetries between economic agents and central banks. In this framework, agents are uncertain about the intended inflation target of the central bank, which may not align with the actual target. We also introduce an additional layer of uncertainty through the concept of 'inflation bias', allowing agents to question the relative emphasis of the central bank on price stability versus the output gap. This extension provides important microfoundations for central bank credibility, offering a more nuanced and theoretically grounded framework that links policy actions to the dynamics of inflation expectations. We estimate the model using datasets from the United States and Turkey, showcasing these two economies as polar cases. Lastly, we examine how changes in the credibility gap affect the monetary transmission mechanism in these two countries, leveraging the testable implications of our model to illustrate the underlying dynamics.

We define credibility as the central bank's control over inflation expectations through its policy actions. When expectations are already anchored,credibility is typically defined as the insensitivity of inflation expectations to news or policy changes, since the public remains confident in the bank's target. However, when expectations are unanchored, credibility requires the central bank's actions to encourage market participants to revise their inflation expectations toward the target. We model

the concept of a "credibility gap", which tracks changes in the central bank's ability to align inflation expectations based on current policy actions.

Figure 1 presents a flowchart illustrating the model. The "credibility gap" arises as a function of anticipated policy actions and agents' perceptions of the central bank's emphasis on growth over price stability, commonly referred to as "inflation bias." The anticipated policy stance reflects whether agents expect the central bank to tighten or ease policy, while the inflation bias captures how strongly agents believe the central bank prioritizes growth. The inflation bias is influenced by the size of the output gap and the central bank's historical performance in achieving price stability. Consequently, the same output gap may yield differing inflation biases depending on the central bank's credibility track record.

Figure 1: A Model of Central Bank Credibility



The interplay between inflation bias and the anticipated policy stance shapes both the magnitude and direction of the credibility gap, which in turn drives the inflation shock. The inflation shock reflects the divergence between inflation forecasts under symmetric and asymmetric information.[1]

A narrowing credibility gap mitigates the inflation shock, as agents gain confidence in the central bank's commitment to its inflation target, causing inflation bias to approach zero. Conversely, when inflation persistently deviates from the target and the central bank fails to correct course, agents revise their inflation expectations upward, amplifying the credibility gap and exacerbating inflationary pressures.

In the model, when the credibility gap widens and the inflation shock exceeds a critical threshold, $\bar{v}$, expectations become unanchored. In this unanchored state, agents place greater emphasis on short-term forecast errors and current policy actions when forming inflation expectations, leading to a higher "learning gain." Conversely, if the inflation shock remains below the threshold, the learning gain gradually declines over time, diminishing the impact of short-term fluctuations and reinforcing

[1]For instance, if agents with full knowledge of the central bank's reaction function predict 2 percent inflation, but incorporating asymmetric information and potential inflation bias raises the forecast to 2.5 percent, the resulting inflation shock is 0.5 percent.

the anchoring of long-term expectations. In this framework, credibility is achieved when the central bank successfully narrows the credibility gap by demonstrating a consistent commitment to price stability. As the central bank's capacity to guide inflation expectations toward the target improves, the credibility gap shrinks. Conversely, when this capacity weakens, the gap widens.

Our theoretical framework is versatile and applicable to both advanced economies with well-anchored inflation expectations and emerging markets with more variable outcomes. To demonstrate this adaptability, we employ Bayesian methods to estimate the model using data from the United States and Turkey, allowing for a comparative analysis. Our findings highlight that the Federal Reserve's credibility, strengthened significantly since the 1980s, has kept long-term inflation expectations firmly anchored, even in the face of the inflationary shock following the pandemic. During the pandemic, when the Federal Reserve implemented easing measures, the credibility gap temporarily widened, but the inflation shock remained below the critical threshold, preventing expectations from becoming unanchored. In scenarios where there is no inflation bias and agents trust the central bank's commitment to price stability, the credibility gap converges to zero. In such an environment, even expansionary policies can be implemented without unanchoring inflation expectations. For the United States, our empirical results demonstrate that enhancing central bank credibility improves the trade-off between inflation and output, enabling policy easing without triggering significant inflationary pressures. The model's predictions align closely with observed data on market-based inflation expectations, reinforcing the robustness and practical relevance of the theoretical framework proposed in this study.

For Turkey, where chronic inflation remains a persistent challenge, the pattern of central bank credibility has been notably volatile. Following the central banking reforms introduced after the 2001 crisis, the CBRT initially experienced an improvement in credibility. However, its ability to guide inflation expectations began to weaken after 2009, deteriorating further during the 2018 exchange rate crisis. The aggressive easing cycle implemented by the CBRT after September 2021—when the inflation rate was already at 20 percent—widened the credibility gap and further unanchored inflation expectations. As predicted by our model, when agents perceive that the central bank has an inflation bias, adopting an expansionary stance exacerbates the credibility gap, amplifies the inflation shock, and unanchors expectations. Conversely, if the central bank adopts a tighter policy stance, as occurred in Turkey after the general elections in May 2023, the credibility gap begins to narrow, and inflation expectations start to align more closely with the target. Our empirical findings confirm this dynamic, showing that the subsequent return to orthodox monetary policies reduced the credibility gap, signaling a partial restoration of central bank credibility.

In the final section of the paper, we analyze how central bank credibility dynamics influence the monetary transmission mechanism, particularly the ability to steer inflation expectations through policy rate adjustments. Our findings indicate that, despite an aggressive post-pandemic easing

cycle, the Federal Reserve has managed to keep long-term inflation expectations relatively well-anchored.

In contrast, CBRT has faced greater difficulty in maintaining control over inflation expectations in recent years. Following the shift to orthodox policies after the May 2023 elections and a cumulative 41.5 percentage point rate hike within nine months, inflation expectations in Turkey still remain loosely anchored. Within our framework, this reflects a persistently elevated and slow-moving learning gain curve. This limits CBRT's capacity to ease policy and increases the sacrifice ratio.

While hawkish policy actions have helped narrow the credibility gap, inflation shocks continue to surpass the critical threshold, keeping expectations sticky. Consequently, long-term inflation expectations remain highly sensitive to current inflation and policy moves, weakening CBRT's ability to realign them with its targets.

More recently, however, our findings illustrate stronger control over inflation expectations, as evidenced by successive rate hikes. This suggests that while credibility restoration is a gradual process, a sustained commitment to policy tightening can ultimately strengthen the central bank's ability to anchor expectations and enhance monetary policy effectiveness.

The rest of this paper is organized as follows. In the next section we briefly discuss the literature on central bank credibility and place our contribution in the literature. Section 3 lays out the theoretical framework. Section 4 shows the model's results for US and Turkey. Section 6 discusses the implications of our modeling framework on the monetary transmission mechanism. Section 7 concludes.

2 Previous Literature

The seminal work by Kydland and Prescott 1977 introduced the concept of time inconsistency, demonstrating how policymakers, despite setting optimal inflation targets, are tempted to stimulate short-term economic output by unexpectedly increasing inflation. Rational agents anticipate this behavior, leading to higher inflation expectations without any corresponding output gains. Barro and Gordon 1983 expanded on this by formalizing the trade-off, showing that discretionary monetary policy results in a persistent inflation bias when policymakers prioritize short-term goals over long-term credibility. The literature since then emphasizes the role of institutional solutions, such as central bank independence or rules-based policy frameworks, to mitigate this bias by anchoring expectations and enhancing policy credibility. This foundational theory underpins much of modern central banking practice, highlighting the tension between flexibility and commitment. Our emphasis on the inflation bias as the primary cause of the credibility gap builds upon this literature. We illustrate that introducing an asymmetry regarding the inflation bias and adding this uncertainty to inflation expectations produces forecasts that closely resemble survey data, allowing us a chance

to identify the role of central bank credibility in the formation of expectations and the monetary transmission mechanism.

By providing a framework that highlights the microfoundations of central bank credibility and its implications for inflation expectations, our paper offers a complementary perspective to the existing literature. Much of the literature focuses on the constraints of agents' rational decision-making that can contribute to inflation dynamics. For instance, Mankiw et al. 2003 discuss "sticky information," where agents adjust their expectations sluggishly, and Marcet and Nicolini 2003 explain recurring hyperinflation episodes through boundedly rational learning. Similarly, Milani 2014 attributes endogenous persistence in inflation to agents' evolving beliefs, while Malmendier and Nagel 2016 demonstrate how generational experiences shape responses to inflation surprises. Faust and Svensson 2001 argue that central banks lacking credibility face heightened inflation expectations, which prolongs the period of monetary tightening necessary to re-anchor these expectations. Furthermore, studies such as Cavallo et al. 2017 and Weber et al. 2023 show that individuals in high-inflation environments rely more heavily on prior beliefs and are more attentive to current inflation dynamics. These behaviors align with a wider credibility gap and a higher inflation shock, which, in our model, contribute to the unanchoring of inflation expectations.

Another contribution of our paper is bridging the gap between two branches of the literature on central bank credibility, which have evolved along distinct lines depending on whether the focus is on environments where price stability has largely been achieved or where it has not. Definitions of credibility that apply to one context often fail to translate effectively to the other. Empirical studies focusing on advanced economies, where inflation expectations are well-anchored, document muted market reactions to monetary policy actions (e.g., Lange et al. 2003; Gürkaynak et al. 2005; Cook and Hahn 1989; Kuttner 2001; Demiralp and Jorda 2004). Research by Christelis et al. 2020; Niizeki 2023; Cole et al. 2023 demonstrates that greater trust in the central bank aligns inflation expectations more closely with targets. As credibility strengthens, inflation expectations converge toward the central bank's target rather than reacting strongly to current policy actions, allowing price stability to be maintained primarily through expectations rather than frequent interventions (Tronzano 2005). Our approach frames these findings as a decline in the learning curve, emphasizing the central bank's role in anchoring inflation expectations and providing a theoretical foundation for empirical work on credibility.

Metrics developed for environments with anchored expectations are not entirely appropriate for countries facing chronic inflation. Many emerging markets struggle to anchor expectations (De Pooter et al. 2014) due to higher initial inflation, weaker institutions, and greater macroeconomic and political instability. In these contexts, expectations are more likely to be unanchored, driving volatility in both inflation and growth (Fraga et al. 2003). Credibility in such environments requires asymmetry: expectations should decline when inflation falls but remain relatively resistant

to increasing inflation. Our paper addresses this asymmetry by developing a model grounded in the micro-fundamentals of central bank credibility that applies to both anchored and unanchored environments. Our approach helps explain why some countries successfully anchor expectations while others do not, and how central bank actions can re-anchor expectations when they have drifted.

3 The Theoretical Framework

The concept central bank credibility is closely linked to agent's beliefs on the central bank actions. Therefore, our model heavily draws on the framework of Carvalho et al. 2023 which provides a stylized New Keynesian learning model with imperfect knowledge for agent's learning of central bank's inflation target and central bank's potential inflation bias. Accordingly, we define central bank credibility in this context where agents also learn about central bank's potential inflation bias. This leads to a measure of "credibility gap" that impact long-term inflation expectations, as described in this section. We first start to describe the production side. There is a continuum of monopolistic competitive firms with a quadratic price adjustment costs a la Rotemberg 1982 relative to a price index based on past inflation. Facing an exogenous elasticity of demand across goods, from a Constant Elasticity of Substitution (CES) demand function, firms set their optimal price $P_{f,t}$ to maximize their expected present discounted value of real revenues :

$$\hat{E}_t \sum_{t=0}^{\infty} \Lambda_t \left[\left(\frac{P_{f,t}}{P_t} - S_t \right) \left(\frac{P_{f,t}}{P_t} \right)^{-\Psi_t} Y_t - \frac{\Phi}{2} \left(\frac{P_{f,t}}{P_{f,t-1}} - e^{\gamma \pi_{t-1}} \right)^2 \right] \tag{1}$$

where P_t and Y_t are aggregate price and output, respectively, while S_t are real marginal costs. The elasticity of demand across goods is denoted as Ψ_t and Φ is the scale for the size of adjustment costs. γ is the degree of the price indexation on past inflation, π_{t-1}, where $\pi_t = \ln(P_t/P_{t-1})$. Future revenues are discounted by a stochastic discount factor Λ_t, which converges to β^{T-t} in the steady state. Optimal price decision can be obtained by computing the f.o.c. of (1). Taking a log-linear approximation of the optimal price equation around the zero-inflation steady state and aggregating in a symmetric equilibrium provides the following aggregate supply curve in response to the demand:

$$\pi_t - \gamma \pi_{t-1} = \mu_t + \hat{E}_t \sum_{t=0}^{\infty} (\alpha\beta)^t [\xi s_t + (1-\alpha)\beta(\pi_{t+1} - \gamma \pi_t)] \tag{2}$$

Symmetric equilibrium implies that firms set identical prices and hold the same beliefs. s_t is deviation of real marginal costs from the steady state value, and μ_t is a normally distributed and serially independent markup shock, which is a function of the demand elasticity[2]. Finally, α is the model's stable eigenvalue and $\xi = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$, which corresponds to the probability of the firms to optimize

[2]From the log-linearization of the firm optimal price equation, $\mu_t = -\xi \frac{\hat{\psi}_t}{\psi - 1}$, where $\hat{\psi}_t$ is the deviation of the demand elasticity from the steady state value, $\bar{\psi}$.

their prices in a Calvo type price setting to the first order. We assume that the production function depends solely on labor as in Carvalho et al. 2023, and therefore, we replace the real marginal cost s_t with the output gap x_t.

Given the economic environment as described in (2) the central bank implements optimal monetary policy under discretion with a target rule to achieve price stability as follows:

$$\pi_t - \gamma\pi_{t-1} + \lambda x_t = \phi_t \tag{3}$$

where λ is the weight that the central bank places on the stability of output gap relative to inflation, and:

$$\phi_t = \rho\phi_{t-1} + \epsilon_t$$

where ϕ_t shows the monetary policy stance. Giannoni and Woodford 2002 show that this criterion is the optimal policy under discretion, given the assumption that the central bank has a zero inflation target. In this context, monetary policy shock can be viewed as policy mistakes (misunderstanding of the underlying inflation and output gap) or due to the loosening and tightening of monetary policy. Specifically, positive values for the ϕ_t imply an expansionary policy while negative values reflect monetary policy contraction.

Under rational expectations with full information of the agents on monetary policy, the equilibrium inflation can be obtained by combining the monetary policy into the aggregate supply curve as follows

$$\begin{aligned} \pi_t &= \gamma\pi_{t-1} + \tfrac{\xi}{\xi+\lambda(1-\beta\rho)}\rho\phi_{t-1} + \eta_t \\ \eta_t &= \tfrac{\lambda}{\lambda+\xi}\mu_t + \tfrac{\xi}{\xi+\lambda(1-\beta\rho)}\epsilon_t \end{aligned}$$

3.1 Imperfect knowledge

The inflation equilibrium as described in (3) involves the assumption that agents have perfect knowledge on the monetary policy. We depart from this assumption by considering agents' expectations when they have imperfect knowledge on monetary policy in two directions. First, while there is an explicit inflation target, the intended target is not necessarily equal to the explicit target depending on the central bank's implicit intended pace in approaching the explicit target. The model with imperfect knowledge only about the central bank's intended inflation target was introduced by Carvalho et al. 2023. Second, we extend the model to incorporate imperfect knowledge of agents about the importance that the central bank gives to output gap stability with respect to inflation. As we show in the following paragraphs, this gives room to introduce a definition of central bank credibility into the New Keynesian framework.

Under imperfect knowledge, instead, expectations are a function of agents' imperfect information

(nesting the case of rational expectations). Given the aggregate supply function and the functional form of monetary policy, agents form expectations on inflation, output gap and monetary policy stance, considering their belief (due to imperfect knowledge) on target rate, central banks' relative weight on output. First, they forecast inflation using

$$\pi_t = \gamma\pi_{t-1} + (1-\gamma)\bar{\pi}_t + \frac{\xi}{\xi + \bar{\lambda}_t(1-\beta\rho)}\rho\phi_{t-1} + f_t \tag{4}$$

where f_t is forecast error that we explicitly discuss in the next subsection. We assume that the expectations about the target inflation are equal to long-term inflation expectations:

$$\lim_{T\to\infty} \hat{E}_t \pi_T = \bar{\pi}_t. \tag{5}$$

We focus on the anticipated utility solution of the model, which implies that $\hat{E}_t\bar{\pi}_T = \bar{\pi}_t$ and $\hat{E}_t\bar{\lambda}_T^{-1} = \bar{\lambda}_t^{-1}$ for each $T > t$, see Cogley and Sargent 2008.[3] The expectations on the output gap is captured by

$$\hat{E}x_t = \hat{E}\frac{1}{\bar{\lambda}_t}\left(\phi_{t+1} - (\pi_{t+1} - \bar{\pi}_t) + \gamma(\pi_t - \bar{\pi}_t)\right) \tag{6}$$

together with $\hat{E}\phi_{t+1} = \rho\phi_t$

3.2 The Data Generating Process for Inflation

From the aggregate supply curve (2) and agents' expectations as described in (4)-(6), the data generating process for the inflation becomes:

$$\pi_t = \gamma\pi_{t-1} + (1-\gamma)\Gamma\bar{\pi}_t + f(\bar{\lambda}_t)\rho\phi_{t-1} + \tilde{\mu}_t \tag{7}$$

where $\Gamma = \frac{(1-\alpha)\beta\lambda}{(1-\alpha\beta)(\lambda+\xi)}$ and $\tilde{\mu}_t = \frac{\lambda}{\lambda+\xi}\mu_t$ and

$$f(\bar{\lambda}_t) = \frac{\xi}{\xi+\lambda} + \frac{\alpha\beta\rho\xi\lambda}{\bar{\lambda}_t(\lambda+\xi)(1-\alpha\beta\rho)} - \left(\frac{\alpha\beta\xi\lambda}{\bar{\lambda}_t(\xi+\lambda)} - \frac{\beta\lambda(1-\alpha)}{\xi+\lambda}\right)\frac{\xi\rho}{[\xi+\bar{\lambda}_t(1-\rho\beta)](1-\alpha\beta\rho)} \tag{8}$$

Please see Appendix B for the derivation of (7). (7) implies that agents' belief on the central banks monetary policy conduct in addition to the belief on inflation target is key for the evolution of inflation. First, as in the Carvalho et al. 2023, the beliefs on target inflation, $\bar{\pi}_t$, directly have an impact on inflation with the parameters determined by the central banks actual policy. Γ is an increasing function of λ taking the values in [0,1]. If the central bank concerns solely inflation stabilization, i.e. $\lambda = 0$ then agents' beliefs on target inflation is ineffective on inflation. Conversely,

[3] As $\bar{\lambda}_t^{-1}$ enters the expectations of agents in a nonlinear way, essentially $\hat{E}_t[\bar{\lambda}_t^{-1}] \geq [\hat{E}_t\bar{\lambda}_t]^{-1}$ due to Jensen's equality. However, the two expectations reach to quite similar values for potential values of λ_t which is typically larger than 0.5 with a high probability mass. We provide a brief analysis of this in Appendix E.

as $\lambda \to \infty$, Γ approaches to 1, implying that agents' beliefs on long run inflation expectations and thereby inflation target fully effective on the inflation process. Second, a function of the beliefs on monetary policy weight on output stabilization, $f(\bar{\lambda}_t)$, have an impact on inflation through the monetary policy stance of the central bank. Crucially, $f(\bar{\lambda}_t)$ is a positive (and decreasing) function of $\bar{\lambda}_t$. When agents have full knowledge on the central bank's policy, that is $f(\bar{\lambda}_t) = \lambda$, the model reduces to the rational expectations inflation equilibrium with

$$f(\lambda) = \frac{\xi}{\xi + \lambda(1 - \beta\rho)}$$

We display the function $f(\bar{\lambda}_t)$ on a range of $\bar{\lambda}_t$ for $\lambda = 0.75$ in Figure 2,

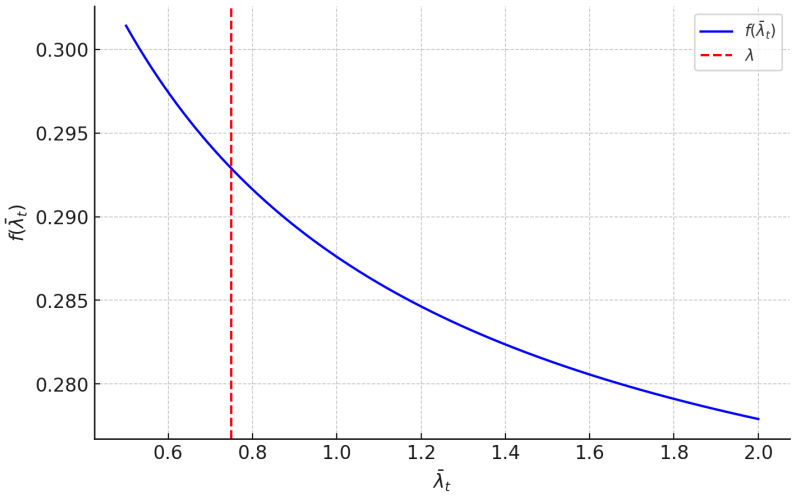


Figure 2: The evolution of $f(\bar{\lambda}_t)$ for $\bar{\lambda}_t \in [0.5, 2]$

As it can be seen from Figure 2, when $\bar{\lambda}_t < \lambda$, $\bar{\lambda}_t$ exceeds $f(\lambda)$, i.e. the case where the agents have full knowledge in central bank's conduct of monetary policy, agents believe that the central bank concerns more (less) about the stabilization of inflation then it actually does. In case of an anticipated contractionary monetary policy with $\rho\phi_{t-1} < 0$ to stabilize the inflation, this reduces the inflation more (less) than it suppose to reduced if agents had full knowledge. This impact already provides some information on how monetary policy shocks might have a differential impact on inflation depending on agents' beliefs about central bank's monetary policy conduct. We elaborate this further in the next subsection.

3.3 Learning Process and Central Bank Credibility

In the previous subsection, we showed that agents beliefs imperfect knowledge on the central bank inflation target and the policy weight on output stabilization has first order impact on the inflation process. In the following, we provide details on the learning process of these beliefs and we establish

the central bank credibility and this learning process.

3.3.1 Learning Process of $\bar{\pi}_t$

Following Carvalho et al. 2023, we assume that long-term inflation expectations are updated following a process similar to a random walk with the main difference on the incorporation of the most recent forecast errors, f_{t-1} as

$$\bar{\pi}_t = \bar{\pi}_{t-1} + K_t^{-1} f_{t-1}. \tag{9}$$

This learning gain, K_t, itself evolves according to either a decreasing function or a constant depending whether inflation expectations are anchored or not:

$$K_{t+1} = (K_t + 1) I_{\nu_t < \bar{\nu}} + \bar{g}^{-1}(1 - I_{\nu_t < \bar{\nu}}) \tag{10}$$

where ν_t is standardized forecast errors and $\bar{\nu}$ is the threshold for switching from a decreasing gain belief updating rule to a constant gain rule, with the gain $\bar{g}$. Expectations are considered as anchored when the standardized forecast error is below the threshold $\bar{\nu}$. In this case, the decreasing gain function leads the effect of the forecast errors vanish over time. This suggests that the agents are less attentive to the inflation surprises as anchoring the expectations would imply. Therefore, news about inflation and shock to monetary policy is increasingly irrelevant for agents' expectation formation, which remains stable at $\bar{\pi}_t$. This echoes the notion of credibility in well established central banks, where expectations are mostly anchored and, hence, credibility is typically defined as the case in which inflation expectations are insensitive to news releases and monetary policy actions. In case of the constant gain on the contrary, the effects of inflation surprises remain stable over time leading to inflation expectations unanchored from the inflation target. In this case, agents form their beliefs which might be distinct from the actual stance of the central bank. This divergence plagues the efficient conduct of the monetary policy especially during periods of economic turmoil.

Finally, the forecast error due to imperfect knowledge can be obtained by the difference between the data generating process of inflation as given in (7) and agents' forecasting equation for the inflation as given in (4), which yields the following:

$$f_t = (1 - \gamma)(\Gamma - 1)\bar{\pi}_t + g(\bar{\lambda}_t)\rho\phi_{t-1} + \tilde{\mu}_t + f(\bar{\lambda}_t)\epsilon_t \tag{11}$$

As stated above, central bank credibility implies anchored inflation expectations in developed economies. For developed countries, this point of view still boils down to the perfect alignment of the agents belief on the central bank conduct and actual conduct of monetary policy as in Carvalho et al. 2023. For developing countries with weak institutions, however, this underlying assumption of complete knowledge on the policy conduct of central bank is questionable. Therefore, this definition cannot

be generalized to developing countries, where inflation expectations are mostly indexed to current inflation, see for example Ha et al. 2019. In environments where inflation expectations are not anchored, credibility should be defined as the power to align expectations with the central bank's official inflation target. This implies that the credibility of the central bank is closely linked with alignment of agents' belief with the central banks' actions which might perhaps taken as granted for developed countries but not so for the emerging markets. The theoretical model helps us introduce the notion of central bank credibility by exploiting the behavior of $\bar{\lambda}_t$. Let us reconsider the expectations of the forecast error as the difference between the expectations under imperfect knowledge and perfect knowledge,

$$\nu_t = \frac{|(1-\gamma)(\Gamma-1)\bar{\pi}_t + g(\bar{\lambda}_t)\rho\phi_{t-1}|}{\sigma_t} \tag{12}$$

where we consider absolute value of the forecast error due to imperfect knowledge and σ_t is the standard error of the exogenous component $(f(\bar{\lambda}_t)\phi_t + \tilde{\mu}_t)$ following Carvalho et al. 2023. Essentially, the inflation shock is a function of the observable forecast errors. Furthermore, from equation (12), it can be noticed that it depends on long-term inflation expectations and the product of a function of agents' beliefs about how much weight the central bank places on growth and the inferred monetary policy stance. This last term, which we define as the "credibility gap" is the key factor determining the transition between anchored and unanchored expectations. Analyzing the "credibility gap" in more in detail:

$$g(\bar{\lambda}_t) = f(\bar{\lambda}_t) - \frac{\xi}{\xi + (1-\beta\rho)\bar{\lambda}_t} \tag{13}$$

is an increasing function of $\bar{\lambda}_t$, negative (positive) for lower (higher) values of $\bar{\lambda}_t$, with taking the value 0 when $\bar{\lambda}_t = \lambda$. We display the behavior of $g(\bar{\lambda}_t)$ in Figure 3

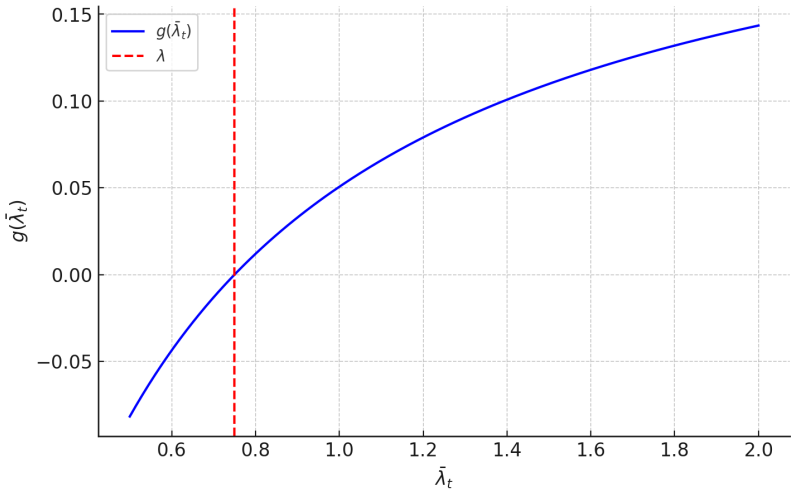


Figure 3: The evolution of $g(\bar{\lambda}_t)$ for $\bar{\lambda}_t \in [0.5, 2]$

Therefore, if agents believe that the central bank's focus on output stability is higher than its focus

on price stability,($g(\bar{\lambda}_t)$ positive) and there is an easing cycle (ϕ_{t-1} positive), this combination of beliefs and policy will make the central bank less credible and push expectations above the anchoring threshold, favoring unanchored expectations and higher and more volatile inflation forecasts. Note that ϕ_{t-1} is included in the definition of credibility gap, not ϵ_t, the current shock to monetary policy. Because ϕ_t is an AR(1) process, it captures the entire easing or tightening cycle. The current monetary policy shock, ϵ_t's impact on ϕ_t come indirectly with a lag.

It is important to notice that the credibility gap is not merely due to agents' beliefs regarding the inflation bias or monetary policy stance by themselves, but it is their product. Let's first consider an environment of unanchored expectations. If there are inflationary pressures, $\bar{\lambda}_t$ will increase, which will cause an increase in $g(\bar{\lambda}_t)$. Under these circumstances, the central bank will need to implement contractionary monetary policy to bring $g(\bar{\lambda}_t)$ down and maintain credibility. If there are deflationary pressures, however, people believe that the central bank is not sufficiently taking output stability into account and $\bar{\lambda}_t$ will be low. This time expansionary monetary policy will increase central bank credibility and avoid volatile unanchored long-term inflation expectations.

If there is already an environment of anchored inflation expectations, then $g(\bar{\lambda}_t)$ will be negligible, eliminating the credibility gap. Hence, the credibility gap that is defined in this manner can explain credibility dynamics in both developed and developing countries.

3.3.2 Learning Process of $\bar{\lambda}_T$

This section presents agents' learning process about the weight that the central bank places on the output gap stabilization, $\bar{\lambda}_t$. The agents develop their beliefs about λ as a weighted average of their previous beliefs and an updated estimate of λ. To recursively measure λ, agents behave as econometricians, being aware of the monetary policy rule structure. Specifically, they estimate λ from the previous period realization of inflation. Hence, the expectation of the weight on output gap ($\bar{\lambda}_t$) is computed as follows, where ω is a constant weight:

$$\bar{\lambda}_t = \omega\bar{\lambda}_{t-1} + (1-\omega)\tilde{\lambda}_t$$

where the updating term $\tilde{\lambda}_t$ is derived from a regression of the monetary policy rule on realized inflation and output gap, given agents' beliefs on long-term inflation:

$$\pi_{t-1} - \bar{\pi}_{t-1} = \gamma(\pi_{t-2} - \bar{\pi}_{t-1}) - \delta x_{t-1} + \phi_{t-1}$$

As we explain in Section 4, we calibrate $\tilde{\lambda}_t$ based on this regression.

3.4 Implications of the Credibility Gap on Central Bank Credibility

In this section, we summarize the key features of the model and discuss their implications for central bank credibility under different scenarios. Table 1 provides a matrix framework illustrating how public beliefs about the central bank's reaction function interact with its monetary policy stance. The columns represent opposing perceptions of the central bank's priorities. In column 1, there is an inflation bias ($g(\bar{\lambda}_t) > 0$), reflecting the belief that the central bank prioritizes output over price stability, leading to a perception of the central bank being "too dovish," which may occur if the bank overshoots its inflation target. In column 2, the public perceives the central bank as prioritizing price stability ($g(\bar{\lambda}_t) < 0$), leading to a "too hawkish" view, which might arise if the central bank undershoots its inflation target. The rows represent the monetary policy stance: an expansionary easing cycle ($\phi_t > 0$) or a contractionary tightening cycle ($\phi_t < 0$).

When the public believes the central bank is too dovish (column 1) and expects further easing (row 1), the credibility gap widens, weakening central bank credibility (row 1, column 1). Conversely, if the central bank shifts to a tightening stance, the credibility gap turns negative, reducing the inflation shock and aligning inflation expectations closer to long-term targets, thereby strengthening credibility (row 2, column 1).

Similarly, if the public perceives the central bank as too hawkish (column 2) and the central bank continues with a tightening cycle ($\phi_t < 0$), the credibility gap increases, and central bank credibility declines (row 2, column 2). On the other hand, if the central bank adopts easing measures to counteract deflationary pressures, the credibility gap turns negative, lowering the inflation shock and improving alignment with long-term inflation expectations, thereby enhancing credibility (row 1, column 2).

In the case of price stability (column 3), $g(\bar{\lambda}_t)$ is relatively low because the public perceives the central bank as balanced, without prioritizing growth over price stability. In this environment, the lagged effects of monetary policy shocks, represented by ϕ_{t-1}, have minimal impact on the credibility gap. Consequently, the inflation shock remains below the threshold, and inflation expectations remain anchored.

Table 1: Credibility Gap under Alternative Scenarios

	$g(\bar{\lambda}_t) > 0$	$g(\bar{\lambda}_t) < 0$	$g(\bar{\lambda}_t) = 0$
$\phi_t > 0$ (Easing cycle)	$\downarrow$	$\uparrow$	–
$\phi_t < 0$ (Tightening cycle)	$\uparrow$	$\downarrow$	–

4 Estimation

The model presented in Section 3 is estimated through Bayesian methodologies for the US and Turkey to illustrate the role of central bank credibility in the monetary transmission mechanism for a developed and emerging market economy. The data set used in estimation is described in the data appendix.

Since we build central bank credibility inside a New Keynesian model with imperfect knowledge as in Carvalho et al. 2023 (CEMP), we follow the authors' estimation procedure, with the needed adjustments to our extended specifications. These methods have become increasingly popular in macroeconomics in the last decades, at least since Primiceri 2006. However, only the learning process in CEMP assumes firm beliefs are updated using information that is not directly observable to the econometrician. We follow this assumption, including beliefs about the central bank weight on output stabilization as additional unobserved information.

We estimate, hence, a state-space model, whose observation equation is the following:

$$\begin{bmatrix} \pi_t \\ E_t^{st}\pi_{t+k} \end{bmatrix} = \pi^* + H_t' \begin{bmatrix} \bar{\pi}_t \\ \zeta_t \end{bmatrix} + R_t o_t, \tag{14}$$

where the left-hand side of the equation represents the observable data, π_t being the annualized CPI growth rate and $E_t^{st}\pi_{t+k}$ the vector of k-ahead short-term inflation expectations (four measures in the case of US, while only two for Turkey, as described in Section 7). Our state variables ζ_t are $(\eta_t, \phi_t, \phi_{t-1}, \pi_t)$, where $\eta_t = f(\bar{\lambda}_t)\epsilon_t + \tilde{\mu}_t$ includes the exogenous shocks of the model. Moreover, π^* is the mean inflation rate, and o_t is the measurement error vector attached to the data. Finally, the observation matrix H_t captures the model-implied true data-generating process of inflation and short-term forecasts, and it is consistent with the agents' expectation formation from the theory. Importantly, H_t is time-varying not only due to possible missing observed data (like R_t) but also because it is a function of $\bar{\lambda}_t$.

Due to the typology of agents's learning process, nonlinear estimation procedures are required. In particular, CEMP suggests separating the model states into a subset of nonlinear variables, i.e., $\bar{\pi}_t$ and k_t, and a subset of linear variables, ζ_t, writing the state space representation accordingly. This way, we can implement a marginalized particle filter as in Schon et al. 2005: the linear states are estimated using the linear Kalman filter, while the nonlinear states are estimated using the particle filter. As explained by CEMP, the prediction of the nonlinear state variables can be used to improve the estimates of the linear state variables thanks to their reciprocal dependence. After applying the marginalized particle filter, we complete the estimation using a marginalized smoother. For more detail on the state-space model representation and the estimation algorithm, please see Appendix C.

The marginalized particle filter allows us to approximate the posterior distribution of the main parameters of the model, as well as the variances of the measurement errors. We, then, draw from this distribution thanks to the Metropolis-Hastings algorithm[4].

Moreover, a few parameters are calibrated according to the main New Keynesian literature to simplify the estimation procedure. In particular, α, which would correspond to the probability that a firm cannot adjust its reset price in a Calvo setting, is set equal to 0.66, while the discount factor β and the marginal cost coefficient ξ are equal to 0.99 and 0.17, respectively. With this calibration, our mean estimate of Γ results similar to Carvalho et al. 2023 in the US case, although slightly smaller.

Finally, to keep the estimation procedure reasonably fast, we calibrate the new information agents receive to update their beliefs about λ. Hence, consistently with the theoretical framework, we compute the updates about λ through rolling regressions every two years of de-meaned inflation growth on its lag and the real output gap, as the agents would do in our model. This way, the absolute coefficient of the output gap is used to update agents' beliefs. Instead, the marginalized particle filter estimates the agents' learning process weight w (and λ). Thanks to this calibration, we avoid complex non-linearities in the estimation procedure without losing the explanation power of $\bar{\lambda}_t$.

Table 2 reports the estimation results for the case of the US, while the results for Turkey can be seen in Appendix D. For US, the estimation is pretty tight, implying a precise detection of the structural parameters. Moreover, the measurement errors are also small, which helps with parameter identification. In general, we obtain similar estimation results as in Carvalho et al. 2023, proving that the structural mechanism of inflation and expectations development provided by the authors is still in place. In particular, a small degree of price indexation is revealed, as is expected in more developed economies.

However, the constant learning gain parameter ($\bar{g}$) and the anchoring parameter ($\bar{\nu}$) exhibit the main difference with a model without inflation bias. In our results, in fact, the constant gain parameter is between 0.094 and 0.139 at the 90 percent credibility set (with a mean value of 0.118). The extension of an imperfect knowledge New Keynesian model to include inflation bias, hence, keeps the explanatory power of the simpler model but adds a possible explanation of what is at the basis of long-term forecast trends.

Moreover, the anchoring threshold $\bar{v}$ is larger than Carvalho et al. 2023's model without inflation bias. In a model without inflation bias, the smaller estimate for this parameter would imply a higher sensitivity of firms to turn to constant learning gain for a small deviation of long-term inflation expectations from the trend inflation π^*. In our model, instead, this implies that credibility gap

[4]We start the algorithm using a diagonal variance-covariance matrix of the parameters with prior variances and update it recursively through chains of 10 thousand draws until the matrix is stable. Then, we implement the Metropolis-Hastings algorithm with the obtained matrix for the final sample of 50 thousand draws. A step size of 0.2 gives a rejection rate of around 0.62 for the US data and around 0.65 for the Turkish data.

plays a more important role in shaping the characteristics of agents' expectations.

Furthermore, it is worth noticing that λ (row 7) is estimated to be between 0.724 and 0.955 in the 90 percent posterior credible interval, with a mean of 0.841, implying a higher relative weight on inflation stability than output. This can be viewed as evidence of the implementation of the Taylor principle in the US. Finally, the updating weight ω (row 6) is found to be, on average, around 0.9 consistent with persistent beliefs. This supports the idea of sticky agents' beliefs about the conduct of monetary policy, together with the lag between the change in policy and the update of expectations.

Table 2: Prior and Posterior Distribution of Structural Parameters - US

		Prior distribution			Posterior distribution			
		Distribution	Mean	SD	Mode	Mean	5^{th} Pct	95^{th} Pct
(1)	π^*	Normal	2.250	0.400	2.716	2.783	2.650	2.986
(2)	$\overline{\nu}$	Gamma	0.050	0.040	0.078	0.081	0.068	0.095
(3)	$\bar{g}$	Gamma	0.100	0.090	0.095	0.118	0.094	0.139
(4)	γ	Beta	0.500	0.265	0.156	0.166	0.125	0.203
(5)	ρ	Beta	0.500	0.200	0.897	0.894	0.867	0.916
(6)	ω	Normal	0.900	0.020	0.887	0.885	0.878	0.891
(7)	λ	Gamma	0.700	0.200	0.781	0.841	0.742	0.955
(8)	σ_ϵ	Inv.-Gamma	0.100	2.000	0.131	0.129	0.105	0.159
(9)	$\sigma_{\tilde{\mu}}$	Inv.-Gamma	0.100	1.000	0.328	0.317	0.280	0.356
(10)	$\sigma_{o,1}$	Inv.-Gamma	0.100	1.000	0.346	0.365	0.330	0.403
(11)	$\sigma_{o,2}$	Inv.-Gamma	0.100	1.000	0.039	0.038	0.029	0.047
(12)	$\sigma_{o,3}$	Inv.-Gamma	0.100	1.000	0.017	0.019	0.012	0.025
(13)	$\sigma_{o,4}$	Inv.-Gamma	0.100	1.000	0.069	0.071	0.062	0.082
(14)	$\sigma_{o,5}$	Inv.-Gamma	0.100	1.000	0.057	0.059	0.050	0.069

Note: The posterior distribution is obtained using a Metropolis-Hastings algorithm. Moreover, the remaining parameters are calibrated according to the main New Keynesian literature: $\alpha = 0.66$, $\beta = 0.99$, and, hence, $\xi = 0.17$.

Appendix D provides estimation results for Turkey. Here, it is sufficient to notice that the estimated structural parameters are found to feature higher variance due to the shorter sample period and the more volatile data. Moreover, in Turkey, expectations are found to be more indexed to past inflation , as is typical in emerging economies, and the weight on output stability with respect to inflation is larger than 1, implying less focus on price stability than in the case of the US and a violation of the Taylor principle. A mean value of 0.79 for $\bar{g}$ also implies a high weight on current forecast errors in updating long-term inflation expectations. Finally, ω results around 0.9, providing evidence for persistent beliefs in the case of Turkey as well.

Table D1 in Appendix D presents the estimation results for Turkey. Notably, the estimated structural parameters exhibit higher variance due to the shorter sample period and the greater volatility of the data. Additionally, expectations in Turkey are found to be more strongly indexed to past inflation, as shown by γ in row 4, a common characteristic of emerging economies. The

parameter λ in row 7 shows that the estimated weight on output stability relative to inflation exceeds 1, suggesting a weaker emphasis on price stability compared to the U.S. and indicating a violation of the Taylor principle. Furthermore, a mean value of 0.79 for $\bar{g}$ (row 3) suggests that agents place significant weight on current forecast errors when updating their long-term inflation expectations. Lastly, the parameter ω (row 6) is estimated to be around 0.9, providing further evidence of persistent belief dynamics in Turkey.

5 Results and Discussion

In this section, we present the estimation results for the United States and Turkey. For each country, we begin by comparing our model's forecasts with survey-based inflation expectations to evaluate its predictive accuracy. Next, we examine the learning gain and credibility gap, which shed light on the underlying dynamics driving the formation of inflation expectations within the model. Finally, we demonstrate how the forecasting performance of Carvalho et al. 2023 is enhanced by incorporating uncertainty related to the inflation bias.

5.1 The US case

5.1.1 Model Forecasts versus Survey Expectations

Figure 4 compares short-term inflation expectations from survey data with model predictions, including 95 percent credible intervals. The first three panels illustrate one-quarter-ahead and two-quarter-ahead forecasts (black line) plotted against the actual inflation rate (light gray line). One-quarter-ahead and two-quarter-ahead expectations from the Survey of Professional Forecasters (SPF) are depicted in red (panels a and b). Panel c shows six-month-ahead survey expectations from the two Livingston surveys, plotted in red and blue, against the model's two-quarter-ahead forecasts. The final panel compares the model-implied inflation rate to the actual inflation rate.

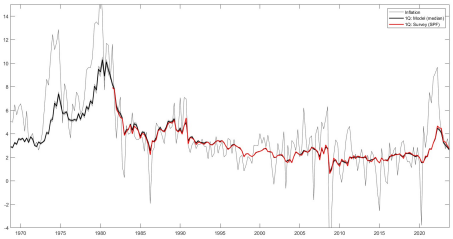
The first three panels indicate a close alignment between the model's fit and survey-based expectations, which is expected since these data are used to estimate the model parameters. The strong fit demonstrated in the fourth panel further underscores the robustness of the model's predictive capabilities and the efficacy of the estimation procedure. Notably, the model accurately captures the rise in inflation and short-term expectations during 2022 and 2023, as well as the subsequent decline toward the end of 2023.

Analyzing the model's predictive power for long-term survey expectations is even more critical because these expectations are not included in the estimation dataset, unlike short-term survey-based expectations. This allows us to explore the dynamics of credibility and its role in introducing an additional layer of uncertainty through the inflation bias's influence on agents' expectations.

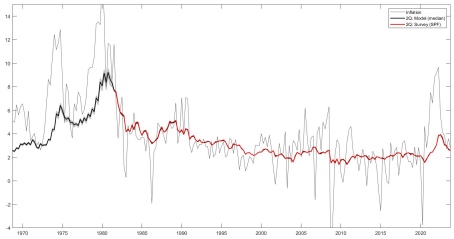
Figure 5 compares the model's predictions of average forecasts for the next 5 to 10 years (solid line) and 1 to 10 years (dashed line) against SPF's 5-to-10-year-ahead expectations (red dotted line) and Livingston's 1-to-10-year-ahead expectations (blue dotted line). The model effectively replicates trends observed in the surveys across the entire sample period. It captures the downward trend in forecasts during the 1990s and the prolonged flat trend that followed, with survey expectations consistently falling within the 95 percent credible interval. Moreover, the model reflects high levels of long-term inflation expectations during the 1970s and 1980s, prior to the availability of survey data, aligning with the actual inflation rates of that period.

Notably, while inflation rates increased in the post-pandemic period, model-based long-term inflation expectations remained stable, even as survey-based forecasts showed an upward trend. To explain the microfoundations behind this anchoring, the next section provides further insights into the dynamics of inflation expectations formation.

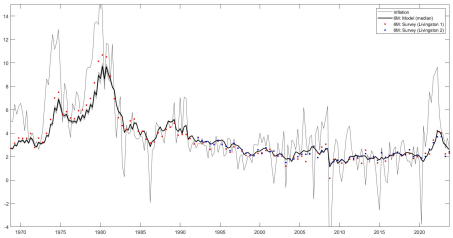
Figure 4: Short Run Model Forecasts against Survey-Based Expectations



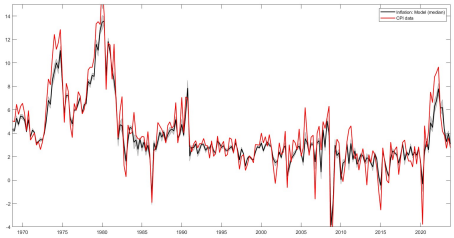
(a) 1Q-Ahead Inflation Expectations: Model vs. SPF Survey



(b) 2Q-Ahead Inflation Expectations: Model vs. SPF Survey



(c) 2Q-Ahead Inflation Expectations: Model vs. Livingstone Surveys



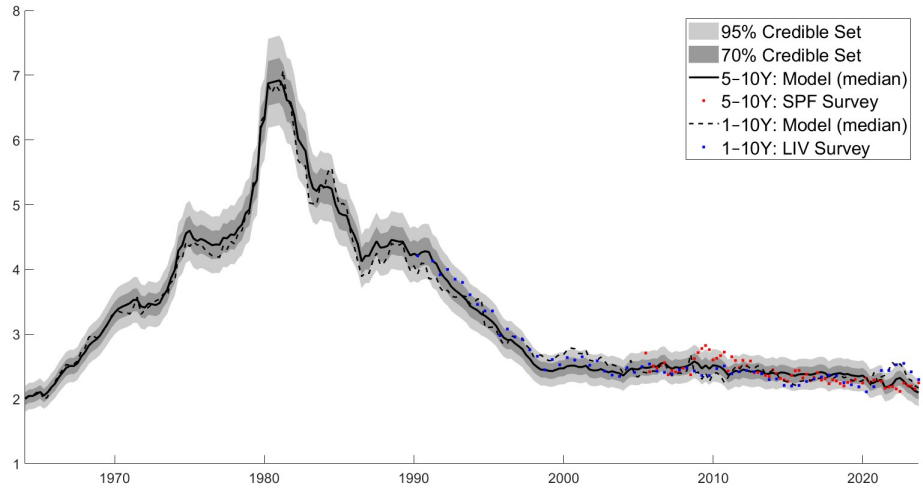
(d) Inflation Rate: Model vs. Data

Note: 95% credibility sets are shown.

5.1.2 Learning Gain and Credibility Gap

In the previous section, we demonstrated that long-term inflation forecasts remained largely anchored over the last couple of decades, even when actual inflation deviated from the Federal Reserve's target. Our modeling framework mimics Carvalho et al. 2023, which emphasizes that long-term

Figure 5: Long Run Model Forecasts against Survey Based Expectations



Note: The panel shows the evolution of five to ten years inflation expectations from SPF and one to ten years forecasts from LIV, compared with model predictions.

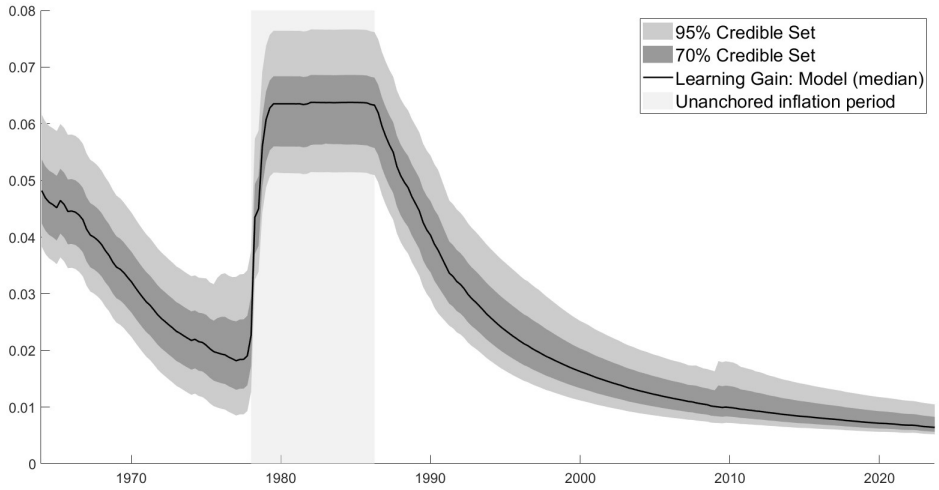
inflation forecasts are closely tied to agents' beliefs about the central bank's inflation target, updated recursively through the learning gain. When expectations are anchored, current inflation news have only a limited impact on long-term forecasts. This mechanism is replicated in our theoretical framework.However, our extended model goes further by providing a microfoundation for why these expectations remained anchored during this period by introducing the concept of the credibility gap. In this section, we present the estimation results for these two key pillars of credibility in our model.

The learning gain curve in Figure 6 illustrates the relative emphasis agents place on current forecast errors, where a decline in the learning gain reflects improvements in central bank credibility. The vertically shaded area in Figure 6 highlights the period of unanchored inflation expectations, occurring when the inflation shock exceeds the threshold. This corresponds to the inflationary shocks of the 1970s, which were eventually contained through a successful disinflation program under Chairman Volcker. His aggressive tightening policies were instrumental in defeating inflation and restoring the Federal Reserve's credibility. The subsequent improvement in credibility is captured by the gradual decline in the learning gain, following the sharp reduction in inflation.

The credibility gap (solid line) in Figure 7 moves in tandem with the learning gain, particularly during periods of significant changes in credibility, as seen in the 1980s. However, it is notably more volatile than the learning gain because only substantial shifts in the credibility gap trigger the learning gain to transition from an anchored state to an unanchored one.

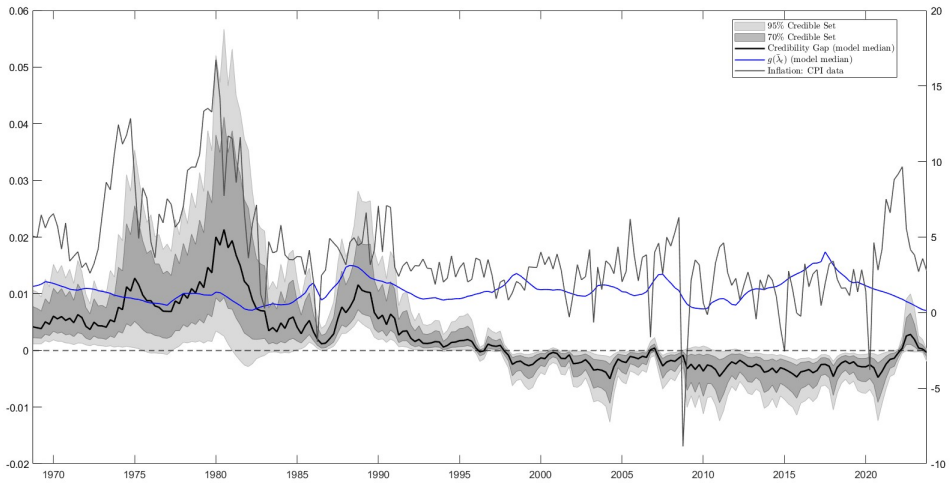
The figure further illustrates that the credibility gap correlates positively with the inflation rate. Figure 7 illustrates this co-movement between the credibility gap, the inflation rate, and the inflation

Figure 6: Learning Gain for the United States



Note: The shaded area shows the period of unanchored inflation expectations from 1978 to 1986.

Figure 7: Credibility Gap for the United States



Note: The panel shows the central bank credibility gap and $g(\bar{\lambda}_t)$ as defined by the model (left axis), as well as the inflation rate (right axis).

bias (blue line).

The graph highlights the sharp rise in inflation during the 1970s and the subsequent decline in the Federal Reserve's credibility, as noted by Bryan 2013, Lopez and Wolla 2012, and Bernanke 2010. This is evidenced by simultaneous spikes in both the learning gain and the credibility gap. The trajectory of the inflation bias ($g(\bar{\lambda}_t)$) in the post-1980 period is particularly notable. The Fed-

induced recession of the early 1980s caused a significant economic contraction. As a result, despite persistently high headline inflation, $g(\bar{\lambda}_t)$ declined, contributing to a reduction in the credibility gap.

In the period following the late 1980s, the lagged benefits of Volcker period became apparent. The achievement of price stability and the anchoring of inflation expectations are reflected in the credibility gap's downward trend, signaling an overall improvement in credibility. In the early 1990s, there was a temporary rise in the credibility gap due to an increase in inflation and the inflation bias. However, this decline in credibility was not severe enough to unanchor inflation expectations, as indicated by the stability in the learning gain.

In the more recent post-pandemic period, the inflationary environment caused a temporary rise in the credibility gap. Supply chain disruptions during this time generated stagflationary shocks reminiscent of those in the 1970s. While the decline in output led $g(\bar{\lambda}_t)$ to trend downward, it remained positive. The Federal Reserve's accommodative policies initially exacerbated the credibility gap, which only began to narrow following the adoption of a more hawkish monetary stance. The subsequent containment of the credibility gap ensured that the inflation shock remained below the threshold, thereby keeping expectations anchored, as evidenced by the learning gain.

Our findings for the pandemic period reaffirm that temporary fluctuations in the credibility gap do not necessarily unanchor inflation expectations immediately. The model effectively captures the mechanism that maintained anchored inflation expectations in the United States during the post-pandemic era, even amid rising inflation. This observation aligns with what is emphasized by Powell 2024 and Ball et al. 2022, further emphasizing the resilience of expectations in the face of temporary shocks.

Before we end this section, it is worth emphasizing that it is the trend of the credibility gap rather than its level that signals credibility. Starting from a level of well-established credibility, upward movement in the credibility gap is a sign of relative deterioration in credibility, while downward movement in the credibility gap is a sign of improvement. Hence, it is not possible to compare the credibility gap charts for any two countries and conclude that one has more central bank credibility relative to the other.

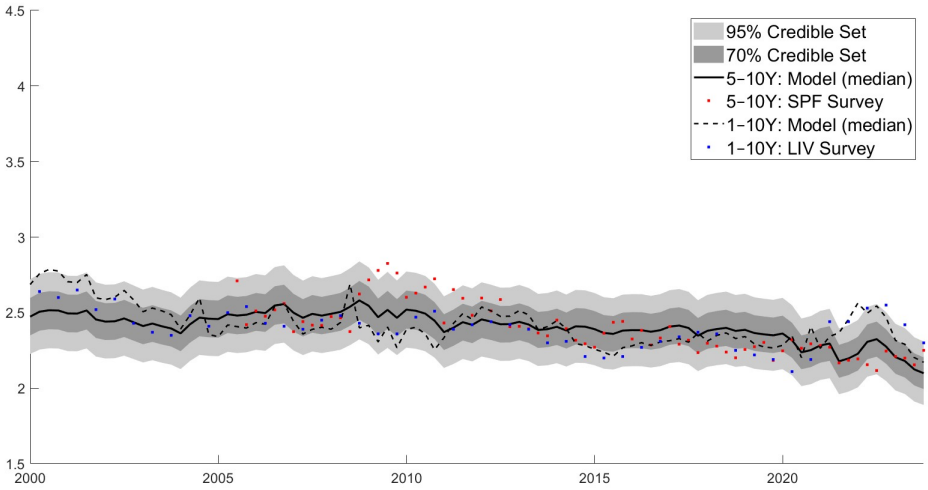
5.1.3 The Role of Inflation Bias

In order to illustrate how we contribute to the modeling framework that is laid out by Carvalho et al. 2023, we compare the performance of our model in predicting long-term inflation expectations against Carvalho et al. 2023 who assume perfect information about the parameter λ in the central bank decision rule.

The comparison is done for the period after 2000. Figure 8 shows that adding an inflation bias component to the model (panel (a)) reduces the volatility of forecasts obtained in Carvalho et al. 2023 (panel (b)). Furthermore, the inflation bias is better able to capture survey-based expectations

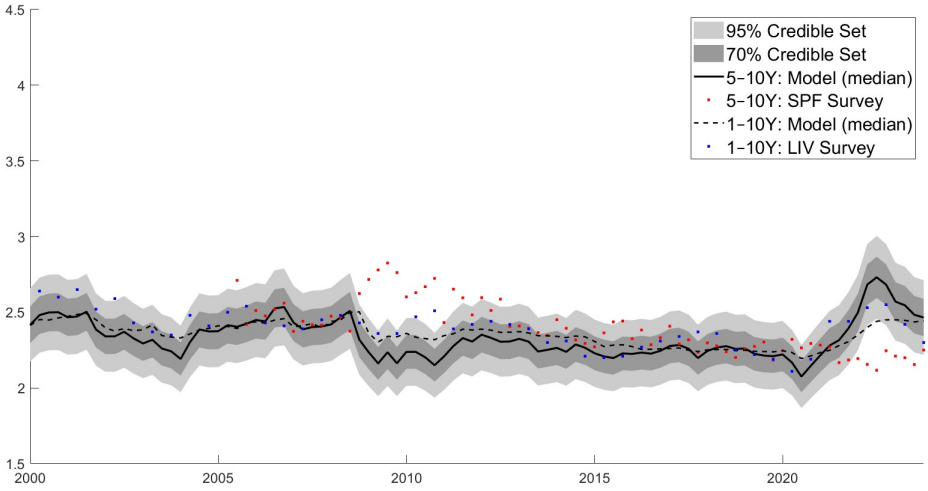
around 2010 and after 2020 (as shown by the blue and red dots). This exercise illustrates that adding inflation bias is a step in the right direction in improving Carvalho et al. 2023, bringing the model's predictions closer in line with market expectations, mimicking the expectations formation process more successfully.

Figure 8: Long-run Inflation Expectations: The Role of Inflation Bias



(a) Model with Inflation Bias

Note: The panel shows the evolution of five to ten years inflation expectations from SPF and one to ten years forecasts from LIV, compared with model predictions.



(b) Model without Inflation Bias (Carvalho et al.,2023)

Note: The panel shows the evolution of five to ten years inflation expectations from SPF and one to ten years forecasts from LIV, compared with model predictions.

We observe that adding uncertainty regarding inflation bias works as an element that further

decreases the weight of current forecast errors affecting long-term forecasts and keeps expectations stable. This can also be seen in equation 11 of the model, where the weights on the shocks depend on agents' beliefs. Hence, even though Carvalho et al. 2023's model that already incorporates uncertainty regarding the central bank's policy stance also keeps expectations anchored, including another layer regarding uncertainty about inflation bias improves their theoretical framework even further delivers a better fit in the presence of the post-pandemic price shocks.

Table 3: Model Comparison

US	$ln\ p(Y^{st})$	$ln\ p(Y^{st}, Y^{lt})$	$ln\ p(Y^{lt}\|Y^{st})$
Model with inflation bias	-676.74	-803.30	-126.56
Model without inflation bias	-679.94	-809.51	-129.57
Rational Expectations	-701.47	-842.83	-141.36
Turkey			
Model with inflation bias	-778.45	-929.24	-150.79
Model without inflation bias	-783.37	-936.58	-153.21
Rational Expectations	-821.92	-992.19	-170.27

Note: Comparison of marginal likelihood for the model with and without inflation bias, and rational expectation model.

5.2 The case of Turkey

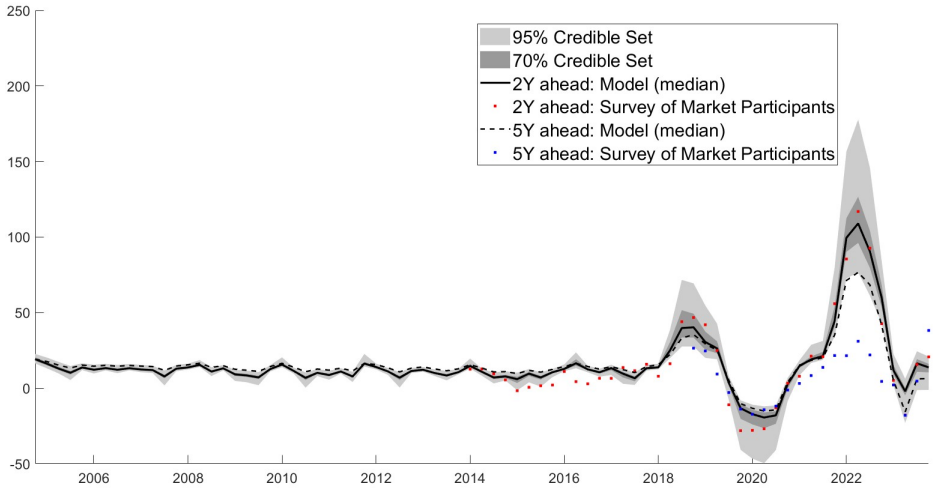
After estimating our model with the US data and illustrating the role of credibility gap in improving the forecasting performance as well as anchoring inflation expectations, we now consider another illustration from a country with weaker central bank credibility. Turkey is one such country that suffers from a chronic inflation problem.

Short-term inflation forecasts are provided in Appendix D. As shown in the appendix, the model produces a pretty good fit of data, even though the data is more volatile.

Figure 9 compares the model's long-term forecast predictions with and without inflation bias against survey data from the CBRT. Notably, long-term inflation expectations have exhibited significantly higher volatility since 2018, following the surge in inflation triggered by the exchange rate crisis. Similar to the U.S. results, the model incorporating inflation bias (upper panel) aligns more closely with the survey data. This finding highlights the critical role of credibility in anchoring expectations within an emerging market context, where inflation expectations are more strongly linked to current inflation dynamics.

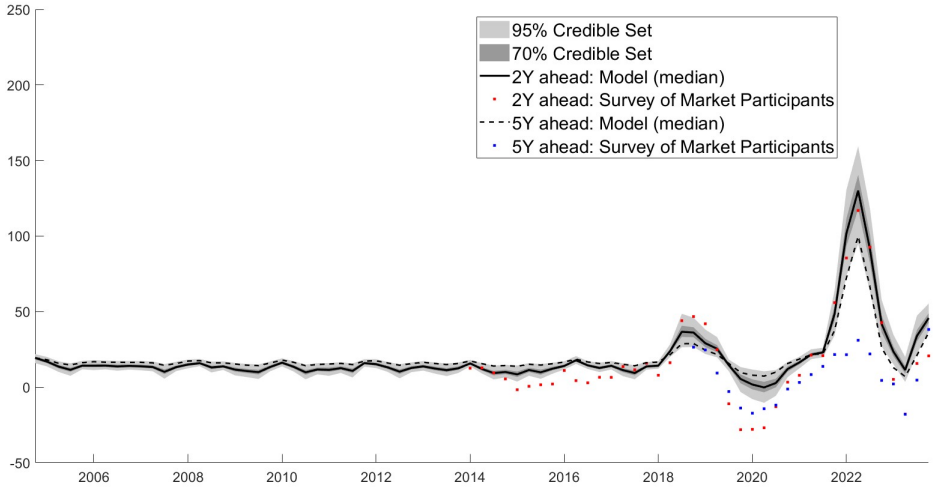
Figure 10 illustrates the evolution of the learning gain since 2005, with the lightly shaded area indicating periods when expectations were unanchored. While the estimation exhibits considerable volatility due to the nature of the observable data and the relatively short sample period, a gradual trend toward anchoring expectations is evident in the early 2000s. However, after 2009, forecasts

Figure 9: Long-run Inflation Expectations with and without Inflation Bias



(a) Model with Inflation Bias

Note: The panel shows the evolution of two and five years ahead inflation expectations from CBRT's survey of market participants, compared with model predictions.



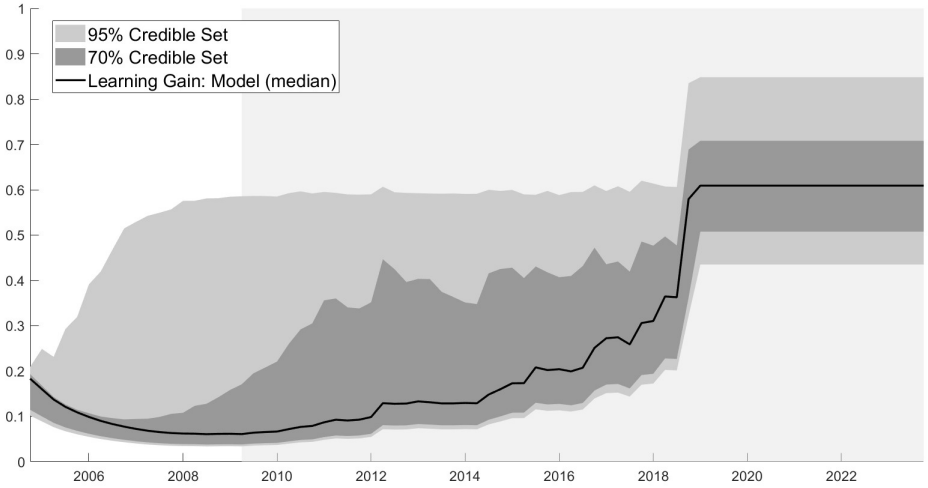
(b) Model without Inflation Bias (Carvalho et al, 2023)

Note: The panel shows the evolution of two and five years ahead inflation expectations from CBRT's survey of market participants, compared with model predictions.

become unanchored, signaling a shift in expectation dynamics. The exchange rate crisis in 2018 and the subsequent surge in inflation are reflected as a pronounced jump in the learning gain curve, reinforcing the already unanchored nature of expectations at that time.

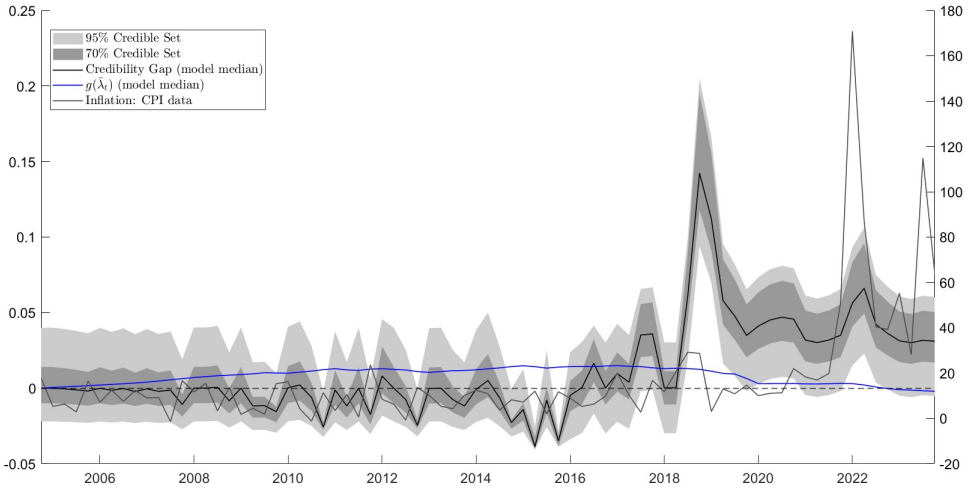
Turning to the credibility gap in Figure 11, we observe that it remains relatively stable until 2016. However, following the exchange rate crisis in 2018, which triggered a sharp rise in inflation

Figure 10: Learning Gain for Turkey



Note: The lighly shaded area correspons to unanchored expectations

Figure 11: Credibility Gap for Turkey



Note: The panel shows the central bank credibility gap and $g(\bar{\lambda}_t)$ as defined by the model (left axis), as well as the inflation rate (right axis).

(gray line), the credibility gap (bold line) spikes significantly. In the context of our model, this surge in inflation leads agents to believe that the central bank's commitment to price stability has weakened. Consequently, they revise their expectations, assuming a higher value for the parameter λ, as indicated by the blue line.

In response to the inflationary pressures of 2018, the CBRT implemented a substantial rate hike,

which temporarily restored some credibility. However, the subsequent replacement of the CBRT governor with more dovish successors led to renewed increases in the credibility gap. It was only after the May 2023 elections, with the appointment of a more hawkish team, that the credibility gap began to decline gradually.

A comparison of Figures 8 and 9 reinforces the well-known perception that credibility is hard to build but easy to lose. We observe that short-term fluctuations in central bank credibility are clearly reflected in the credibility gap. However, such temporary improvements are insufficient to firmly anchor inflation expectations. True anchoring requires sustained efforts, where only a prolonged and consistently tight monetary policy can restore and maintain credibility over the long term.

6 The role of credibility gap in monetary policy transmission

The model estimates in the previous sections illustrated how credibility interacts with the anchoring of inflation expectations both in the US and in Turkey. In this section, we take a look at the role of credibility in the monetary transmission mechanism in these two countries. Specifically, we focus on the days of interest rate decisions and investigate how the central banks' control over inflation expectations changes over time. The interpretation of our findings mimic changes in central bank credibility, as it is mapped out by the "credibility gap" analysis.

In order to analyze how the central bank can shape inflation expectations through its policy actions, we focus on changes in inflation expectations in the days prior to policy decisions. For measuring changes in the inflation expectations at the exact day of the central bank's decision, we use long term interest rates as a proxy for inflation expectations and control for a rich set of variables that can affect long term interest rates to filter out their impact on bond rates and focus purely on changes in inflation expectations.

In the context of a regression model, this framework can be expressed as follows:

$$\Delta i_t^{LT} \quad = \quad \beta \Delta i_t^{Policy} + \gamma Z_t + \varepsilon_t \tag{15}$$

where Δi_t^{LT} is the change in the long-term interest rate as of time t, Δi_t^{Policy} is the change in the policy rate, and Z_t is a vector of control variables that may affect the long rate on day t.

One critical factor that affects long term interest rates related to a particular policy decision is its implications for future policy actions. We control for this factor by including changes in short term interest rates as well as the order of the rate change in the policy cycle. If the rate change is perceived to be a relatively early move in an easing or a tightening cycle such that there are a series of anticipated future policy moves in the same direction, the overall weight of expected rate changes could pull the long term rate in that direction. If a rate change comes later in the cycle, then it may not have as much of an impact on the long term rate.

After we filter out any potential factors that might affect long term interest rates following a policy change (details are provided in the next sub-section), the corresponding effect, which measures the response due to changes in inflation expectations, is captured by the coefficient β in equation (15).

The dependent variable (i_t^{LT}) in equation (15) is the 10-year USD/TL swap rate for Turkey and 10 year bond rate for the US.[5] In most emerging markets with underdeveloped government bond markets, the swap market is more complete than the government bond market, and is thus used as the long term rate. Another reason for using the swap rate for Turkey is that the data goes back to 2005 while 10-year government bond rate data only starts in 2010.

Equation (15) is estimated for a sample of policy easings and policy tightenings separately. As noted in the earlier literature, on the day of a policy action, financial markets only respond to the unanticipated component of the policy action because the response to the anticipated component already takes place in the days prior to the action (See e.g. Carpenter and Demiralp 2006, Lange et al. 2003). Hence, if we restrict our analysis to the day of the policy action and find a limited response, we would inevitably neglect the response to the anticipated policy and reach the potentially improper conclusion that CBRT cannot control the long rate. Thus, to capture the full response of long rates to monetary policy, we need to focus on the period before the policy action as well as the day of the policy change.

In order to capture anticipated policy actions before a rate change (Δi_t^{Policy}), we look at the change in the one month forward implied yield in Turkey and one-month OIS rate in the US, in the 25 business day period prior to a policy action on day t.[6] Both the implied forward rate as well as the OIS rate reflect the average expected policy changes for the next 30 days.[7]

If the credibility of the central bank is stable over time, one can estimate equation (15) on the days of policy changes (se e.g. Kuttner, 2001, Demiralp and Jorda, 2004). Once we incorporate changes in the credibility gap, however, we depart from the existing studies and allow the model parameters to change over time. Because policy meetings are not necessarily arranged on a regular schedule and there is not necessarily a rate change at each policy meeting, the time series of the policy rate changes are not observed in fixed intervals. Nevertheless, by ordering the policy changes over time, we allow the parameters to change following a random walk process.

$$\begin{aligned} \Delta i_\tau^{LT} &= \beta_\tau \Delta i_\tau^{Policy} + \gamma Z_\tau + \varepsilon_\tau \\ \beta_\tau &= \beta_{\tau-1} + \eta_\tau \end{aligned} \tag{16}$$

[5]The rate on a 10 year swap is the fixed rate that the borrower demands in return for the uncertainty of having to pay the floating rate over ten years. For example, if a Turkish company borrows USD, it makes interest payments based on 3-month Libor rate. In exchange, it lends TL for which it charges the fixed rate which is the 10-year interest rate on TL.

[6]Implied yields are annualized interest rates for the given currency and tenor, derived from the covered interest rate parity theorem. They are derived from the prevailing spot and forward rates for the Turkish Lira versus the US Dollar for the 30-day period, along with the US interest rate for the same period.

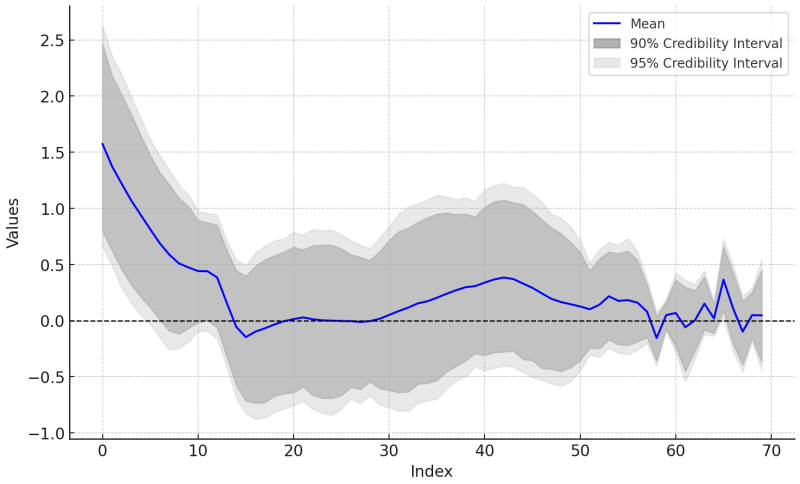
[7]There is not a futures market for the interbank market in Turkey to allow us to extract market expectations of a policy move a la Kuttner 2001. Similarly, OIS instrument is only available for the 2007-2013 period but for maturities longer than one year, which rules out their use as a proxy for short term policy expectations.

where time index τ is used to indicate the order of the meetings with a rate change decision over time. ε_τ and η_τ are error terms that follow standard bivariate Normal distribution. Z_τ includes six-month government bond rate, the order of the policy rate change, [8] and 5-year CDS rate. For the estimation for Turkey, we also include the USD/TL exchange rate and US 10-year Treasury Bond rate, in order to capture the additional risks and pressures on bond rates faced by an emerging market economy. [9]

Equation (16) implies that, a priori, we do not expect a change in the response of the long rate, β_τ, but this could change smoothly over time depending on the shocks to credibility. The model specified in equation (16) can easily be cast into the state space form because it is a special case of the unobserved components model. We further proceed with Bayesian inference in order to take the uncertainty in the unobserved components and the parameters jointly into account. This is crucial in our analysis given the limited number of observations, which aggravates the uncertainties on the parameters as well as the unobserved components. We use the Gibbs sampler for drawing a sample from the joint posterior of the model parameters and unobserved components. Because the state space model is linear and Gaussian, the inference on the unobserved components is carried out using a Kalman filter coupled with a simulation smoother (see e.g. Carter and Kohn 1994).

Equation (16) is estimated for Turkey and US. Before we go into regression results, we provide a brief background of monetary policy in each country as well as the data that is utilized.

Figure 12: Monetary policy easings

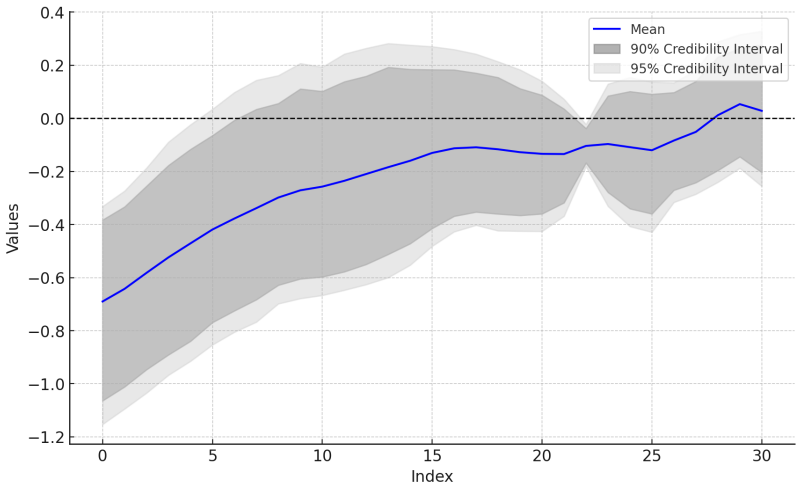


Note: The figure shows the response of long rates to an decrease in the policy rate for expansionary cycles. 95% and 90% credibility sets are shown.

[8] This variable is constructed such that it is set to 1 at the beginning of each easing or tightening cycle and increases with each consecutive change such that it reflects the order of each rate change in a given cycle. Note that this is a real time measure and does not contain any information about the cumulative number of changes by the end of the cycle.

[9] US 10 year yield enters with one lag to account for the time gap between US and Turkey.

Figure 13: Monetary policy tightening



Note: The figure shows the response of long rates to an increase in the policy rate for contractionary cycles. 95% and 90% credibility sets are shown.

6.1 Monetary Policy Frameworks in Turkey and US

Central Bank of the Republic of Turkey (CBRT) implements an interest rate corridor. Until May 2010, the policy rate was set to the lower bound of the interest rate corridor. After April 2010, the CBRT started using all three rates of the corridor with potential asymmetry. [10] Prior to May 2010, it is rather straightforward to categorize the policy cycles by tracking changes in the lower bound of the interest rate corridor, which was the announced policy rate. After April 2010, we track changes in the band values as well as changes in the weighted average funding rate that is determined by the CBRT to detect policy cycles.

Our sample for Turkey starts in June 2005, determined by the availability of 10-year swap rate, and extends through January 2025. Despite the inflation rate that has been persistently above its target, the CBRT has been rather hesitant with rate hikes and generous with rate cuts during that time, with 70 rate cuts and only 31 rate hikes. The vast majority of rate cuts may signal the CBRT's bias towards policy easings which partly explains its poor performance in achieving the inflation target.

The Federal Reserve uses the federal funds rate as its main policy instrument. Our sample for the US starts in June 2005, and extends through 2025.

[10] In the environment after GFC, the reaction of many emerging market economies to surging capital inflows was to keep policy rates at low levels in order to avoid excessive appreciation of domestic currencies. At the same time, they engaged in macro prudential tightening to curb the rapid credit growth.

6.2 Regression Results

Table 1 illustrated a matrix that showed how central bank's policy actions affected the credibility gap, conditional on the perceived inflation bias. The regression results in this section illustrate what they imply with respect to the monetary transmission mechanism following central bank actions.

When the central bank cuts its policy rate (row 1 in Table 1), its goal is to convince the public that the dovish move is consistent with declining inflationary pressures in the long run. The market's response to central bank action, on the other hand, depends on the perceptions regarding the inflation bias. An improvement in inflation expectations following a rate cut is consistent with a central bank that convinces the markets that it is a "hawkish cut" such that the easing is consistent with the inflationary goals, narrowing the credibility gap (column 2). If the markets interpret the rate cut as part of the inflation bias, however, the credibility gap increases, pushing inflation expectations further away from the target (column 1). If there is well established credibility and inflation expectations are anchored, then long term rates should not respond to the policy action (column 3).

This theoretical framework provides testable implications. If the inflation expectations exceed the target, then aligning inflation expectations with rate cuts necessitate a positive relationship between the policy rate and inflation expectations. In our regression framework, as β_τ gets more positive, $g(\bar{\lambda}_t)$ declines such that the agents begin to view the central bank prioritize price stability over growth. Consequently, the credibility gap narrows down and inflation expectations decline following rate cuts (row 1, column 2). If the rate cut is considered to be too dovish and triggers inflation expectations, then β_τ gets less positive, $g(\bar{\lambda}_t)$ increases and the credibility gap widens.

Figure 10 shows the estimation results for the easing cycles in Turkey. We include the 95% (and 90%) Highest Posterior Density Interval (HPDI) covering the posterior mean of the response parameter, β_t. We note that β_t is positive at the beginning of the sample. Following a 1 percentage point rate cut, long term rates declined over 1 percent at the beginning of our sample. During that time, inflation expectations are anchored at the long term target, as suggested by the low values of the learning gain curve (Figure 11).

Over the course of our sample, however, there is a significant decline in the size of β_t. Following the discussion in Table 1, we interpret the decline in beta to be consistent with perceptions of dovish rate cuts that correspond to a decline in credibility. When the central bank's ability to lower inflation expectations with successive rate cuts declines, it is a sign that the perceived growth bias increases. The increase in $g(\bar{\lambda}_t)$, particularly in the period after 2016 is consistent with these findings. This increases the credibility gap and the cumulative decline in credibility reveals itself around 2018: Although the earlier stages of deterioration in central bank credibility does not have immediate impact on unanchoring expectations, as the threshold tolerance limit gets larger, it exceeds the threshold as shown by the jump in the learning curve after 2018. This reduces the central bank's ability to anchor inflation expectations with its interest rate decisions.

The last leasing cycle that is included in our analysis covers the period from September 2021 through May 2023. During that time, CBRT reduced the policy rate from 20 percent to 8.5 percent while the inflation rate increased from 20 percent to a peak of 85 percent, only to stabilize at 40 percent before the general elections in May 2023. In our framework, one could expect this easing cycle to reduce central bank credibility further because the easings were not justified by inflation dynamics. Nevertheless, the set of regulations blur the picture by increasing bond demand. During that time bond rates were repressed via macroprudential measures that demanded banks to hold government bonds as part of their reserve requirements. The modest increase in beta through the end of the sample thus reflects regulatory restrictions and not entirely attributable to a decline in inflation expectations.

(Estimation results for the US easings are going to be added as the right panel of Figure 10 in due course)

6.2.1 Central Bank Credibility after Policy Tightenings

The definition of credibility is asymmetric between a rate cut and a rate hike. Following a rate hike, expected future hikes would move the long term rate upwards while a potentially downwards revision in inflation expectations, moves the long rate downwards.

Once again, we are interested in the response of long rates after we filter out the effects of the expected policy path. If our starting point is an inflationary environment, a decline in long rates, after keeping near term policy expectations constant, is consistent with credibility, and captured by a negative β. In contrast, if the market participants assess the rate hike to be "too little, too late" and maintain an inflation bias (g (lambda)¿0), this would be a sign of weak credibility, which limits the decline in the long rate.

In terms of the testable implications, if the inflation expectations exceed the target, then aligning inflation expectations with rate hikes necessitate a negative relationship. As β gets more negative, g(lambda) declines such that the agents begin to view the central bank's weight on price stability increases. Consequently, the credibility gap narrows down and inflation expectations decline following rate hikes.

Figure 11 shows the estimation results for Turkey for rate hikes. A deterioration in credibility is associated with an increase in the size of β_t, which becomes less negative as the credibility declines. If the erosion in credibility follows a parallel path between easings and tightenings, the figure for easings should mirror the figure for tightenings. What we observe is highly consistent with this prediction. At the beginning of our sample, a 1 percentage point anticipated rate hike is associated with an almost one-to-one decline in long rates, which gradually diminishes throughout the sample and approaches zero. That is, CBRT's ability to lower long-term interest rates with rate hikes declines over time, consistent with the erosion in its credibility. We observe a brief period of improvement

in central bank credibility as shown by the narrowing of the error bands, which corresponds to the tightening cycle adopted by Naci Agbal, who had replaced a dowish governor, Murat Uysal and considered a series of rate hikes. Our sample period also captures the tightening cycle after the general elections. The new central bank took over an inflation rate of 40 percent in May 2023 and increased interest rates from 8.5 percent to 50 percent from June 2023 to March 2024 and held it there until December 2024. We note that such a substantial hiking cycle has offsetting effects on long term interest rates. At the beginning of an aggressive tightening cycle, anticipated future rate hikes push the long term rate upwards. If the central bank is credible and the tightenings are perceived to be a succesful fight against inflation, then the downwards revision in expectations may eventually outweigh the policy expectations channel. If the central bank is not viewed to be credible and the rate hikes are perceived to be too little and too late, then the downward revision in inflation expectations would be limited which would result in an increase in the long rate. What we observe in Figure 11 is that at the decline in β_t starts only recently as the tightening cycle approaches its end and the markets cautiously gain trust in sustainability of the hawkish stance.

(Estimation results for the US easings are going to be added as the right panel of Figure 10 in due course)

7 Conclusions

We extend a New Keynesian model with imperfect knowledge, allowing us to track changes in central bank credibility. Our paper contributes to the theoretical literature by introducing a comprehensive, micro-founded measure of credibility based on individuals' beliefs about the central bank's priorities—balancing price stability and growth—as well as their expectations regarding monetary policy. The framework is general enough to apply to both developed and emerging economies.

We test the empirical implications of our model for the US and Turkey, offering insights into why inflation expectations have remained largely anchored in the US while exhibiting greater volatility in Turkey. The "credibility gap"—a combination of agents' perceptions of the central bank's inflation bias and the anticipated policy stance—effectively captures the anchoring mechanism, shaping forecasts and influencing control over inflation expectations over time. Our findings highlight the Federal Reserve's ability to implement easing measures without unanchoring long-term inflation expectations in the post-pandemic era, while also underscoring the CBRT's challenges in restoring its credibility.

References

Ball, Laurence, Daniel Leigh, and Prachi Mishra (2022). "Understanding US inflation during the COVID-19 era". In: *Brookings Papers on Economic Activity* 2022.2, pp. 1–80.

Barro, Robert J and David B Gordon (1983). "Rules, discretion and reputation in a model of monetary policy". In: *Journal of monetary economics* 12.1, pp. 101–121.

Bernanke, Ben S (2010). *Monetary policy objectives and tools in a low-inflation environment: a speech at the Revisiting Monetary Policy in a Low-Inflation Environment Conference, Federal Reserve Bank of Boston, Boston, Massachusetts, October 15, 2010*. Tech. rep.

Bryan, Michael (2013). "The great inflation". In: *Federal Reserve History* 22.

Çakmakli, Cem, Selva Demiralp, and Gökhan Şahin Güneş (2024). "Do Financial Markets Respond to Populist Rhetoric?" In: *Oxford Bulletin of Economics and Statistics* 86.3, pp. 541–567.

Carpenter, Seth B and Selva Demiralp (2006). "Anticipation of monetary policy and open market operations". In: *International Journal of Central Banking*.

Carter, Chris K and Robert Kohn (1994). "On Gibbs sampling for state space models". In: *Biometrika* 81.3, pp. 541–553.

Carvalho, Carlos, Stefano Eusepi, Emanuel Moench, and Bruce Preston (2023). "Anchored inflation expectations". In: *American Economic Journal: Macroeconomics* 15.1, pp. 1–47.

Cavallo, Alberto, Guillermo Cruces, and Ricardo Perez-Truglia (2017). "Inflation expectations, learning, and supermarket prices: Evidence from survey experiments". In: *American Economic Journal: Macroeconomics* 9.3, pp. 1–35.

Christelis, Dimitris, Dimitris Georgarakos, Tullio Jappelli, and Maarten van Rooij (2020). "Trust in the central bank and inflation expectations". In: *International Journal of Central Banking* 16.6, pp. 1–37.

Cogley, Timothy and Thomas J Sargent (2008). "Anticipated utility and rational expectations as approximations of Bayesian decision making". In: *International Economic Review* 49.1, pp. 185–221.

Cole, Stephen, Enrique Martinez-Garcia, and Eric R Sims (2023). *Living up to expectations: Central bank credibility, the effectiveness of forward guidance, and inflation dynamics post-global financial crisis*. Tech. rep. National Bureau of Economic Research.

Cook, Timothy and Thomas Hahn (1989). "The effect of changes in the federal funds rate target on market interest rates in the 1970s". In: *Journal of Monetary Economics* 24.3, pp. 331–351.

De Pooter, Michiel, Patrice Robitaille, Ian Walker, and Michael Zdinak (June 2014). "Are Long-Term Inflation Expectations Well Anchored in Brazil, Chile, and Mexico?" In: *International Journal of Central Banking* 10.2, pp. 337–400.

Demiralp, Selva and Oscar Jorda (2004). "The response of term rates to Fed announcements". In: *Journal of Money, Credit and Banking*, pp. 387–405.

Faust, Jon and Lars EO Svensson (2001). "Transparency and credibility: Monetary policy with unobservable goals". In: *International Economic Review* 42.2, pp. 369–397.

Fernández-Villaverde, Jesús and Juan F Rubio-Ramírez (2007). "Estimating macroeconomic models: A likelihood approach". In: *The Review of Economic Studies* 74.4, pp. 1059–1087.

Fraga, Arminio, Ilan Goldfajn, and Andre Minella (2003). "Inflation targeting in emerging market economies". In: *NBER macroeconomics annual* 18, pp. 365–400.

Galí, Jordi (2015). *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*. Princeton University Press.

Giannoni, Marc P and Michael Woodford (2002). *Optimal interest-rate rules*. Citeseer.

Gürkaynak, Refet, Brian Sack, and Eric Swanson (2005). "The sensitivity of long-term interest rates to economic news: Evidence and implications for macroeconomic models". In: *American economic review* 95.1, pp. 425–436.

Ha, Jongrim, M Ayhan Kose, and Franziska Ohnsorge (2019). "Understanding inflation in emerging and developing economies". In: *World Bank Policy Research Working Paper* 8761.

Kitagawa, Genshiro (1996). "Monte Carlo filter and smoother for non-Gaussian nonlinear state space models". In: *Journal of computational and graphical statistics* 5.1, pp. 1–25.

Kuttner, Kenneth N (2001). "Monetary policy surprises and interest rates: Evidence from the Fed funds futures market". In: *Journal of monetary economics* 47.3, pp. 523–544.

Kydland, Finn E and Edward C Prescott (1977). "Rules rather than discretion: The inconsistency of optimal plans". In: *Journal of political economy* 85.3, pp. 473–491.

Lange, Joe, Brian Sack, and William Whitesell (2003). "Anticipations of monetary policy in financial markets". In: *Journal of Money, Credit and Banking*, pp. 889–909.

Lindsten, Fredrik, Thomas B Schön, et al. (2013). "Backward simulation methods for Monte Carlo statistical inference". In: *Foundations and Trends® in Machine Learning* 6.1, pp. 1–143.

Lopez, David A and Scott A Wolla (2012). "The great inflation: a historical overview and lessons learned". In: *Page One Economics Newsletter*, pp. 1–4.

Malmendier, Ulrike and Stefan Nagel (2016). "Learning from inflation experiences". In: *The Quarterly Journal of Economics* 131.1, pp. 53–87.

Mankiw, N Gregory, Ricardo Reis, and Justin Wolfers (2003). "Disagreement about inflation expectations". In: *NBER macroeconomics annual* 18, pp. 209–248.

Marcet, Albert and Juan P Nicolini (2003). "Recurrent hyperinflations and learning". In: *American Economic Review* 93.5, pp. 1476–1498.

Milani, Fabio (2014). "Learning and time-varying macroeconomic volatility". In: *Journal of Economic Dynamics and Control* 47, pp. 94–114.

Niizeki, Takeshi (2023). "Trust in the central bank and inflation expectations: Experimental evidence". In: *Economics Letters* 231, p. 111296.

Powell, Jerome H. (2024). *As Long as That Remains True, Inflation Expectations Remain Well Anchored*. Accessed: 2024-12-31. URL: https://www.federalreserve.gov/newsevents/speech/powell20240930a.htm.

Primiceri, Giorgio E (2006). "Why inflation rose and fell: policy-makers' beliefs and US postwar stabilization policy". In: *The Quarterly Journal of Economics* 121.3, pp. 867–901.

Rotemberg, Julio J (1982). "Monopolistic price adjustment and aggregate output". In: *The Review of Economic Studies* 49.4, pp. 517–531.

Schon, Thomas, Fredrik Gustafsson, and P-J Nordlund (2005). "Marginalized particle filters for mixed linear/nonlinear state-space models". In: *IEEE Transactions on signal processing* 53.7, pp. 2279–2289.

Tronzano, Marco (2005). "Inflation Targeting and Credibility: a note on the recent empirical literature". In: *Economia Internazionale/International Economics* 58.4, pp. 489–506.

Weber, Michael, Bernardo Candia, Hassan Afrouzi, Tiziano Ropele, Rodrigo Lluberas, Serafin Frache, Brent H Meyer, Saten Kumar, Yuriy Gorodnichenko, Dimitris Georgarakos, et al. (2023). *Tell me something i don't already know: Learning in low and high-inflation settings*. Tech. rep. National Bureau of Economic Research.

Appendices

Appendix A. The Data

We use data both for model estimation and for testing its implications on the monetary transmission mechanism. This section details the data sources used for each component of the analysis.

Data used for Model Estimation

In the case of the US, four survey measures of CPI inflation forecasts are used: one- and two-quarter-ahead forecasts from the *Survey of Professional Forecasters* (SPF)[11] and two six-month-ahead forecast measures from the *Livingston Survey* (LIV)[12]. The first LIV measure is the growth rate between the forecasted CPI level six months ahead and the last available monthly price level at the time of the survey, while the second is the growth rate between the forecasted CPI level six months ahead and the forecasted current CPI level[13]. This ensures that the model is estimated using a sufficiently rich dataset. Finally, the inflation series is drawn from the seasonally adjusted *Consumer Price Index for All Urban Consumers: All Items in U.S. City Average*, provided by the US *Bureau of Labor Statistics*[14].

Data are quarterly and span from 1955Q1 to 2023Q4, though each survey dataset covers different sample periods and frequencies. SPF measures are available quarterly from 1981Q3 onward, while LIV data are biannual, with the first six-month-ahead forecast available for the full sample and the second beginning in 1992Q2.

As for long-term inflation expectations, we again rely on SPF and LIV survey data. Specifically, from SPF, we take the five-to-ten-year-ahead annual average CPI inflation rate forecast[15]. From LIV, we use the one-to-ten-year-ahead inflation rate forecast[16].

For Turkey, inflation and forecast data are sourced from the *Central Bank of the Turkish Republic* (TCMB) database[17]. Short-term expectations include two-month- and twelve-month-ahead inflation forecasts from market participants, while long-term expectations consist of twenty-four-month- and five-year-ahead forecasts[18]. All inflation measures are in annualized terms, with the two-month-ahead forecast serving as a proxy for one-quarter-ahead expectations. For model estimation in the

[11] Available at https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/survey-of-professional-forecasters. The one- and two-quarter-ahead forecasts correspond to variables *CPI3* and *CPI4*, respectively, and represent the mean forecast across individual professional forecasters.

[12] Available at https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/livingston-survey.

[13] The first forecast measure corresponds to variable *G-BP-To-6M* from the LIV dataset *Growth of Mean Forecast for the Levels of Survey Variables*, while the second measure is variable *G-ZM-To-6M*.

[14] Available at https://fred.stlouisfed.org/series/CPIAUCSL.

[15] Variable *CPIF5*, estimated as the mean across professional forecasters and available at a quarterly frequency from 2005Q3 to 2023Q4.

[16] Variable *CPI_10Y*, computed as the mean across professional forecasters and available biannually from 1990Q2 to 2023Q4.

[17] Data from TCMB are available at https://evds2.tcmb.gov.tr/index.php?/evds/serieMarket.

[18] The corresponding TCMB variables are *TP.PKAUO.S01.C.U_3*, *TP.PKAUO.S01.E.U_3*, *TP.PKAUO.S01.F.U_3*, and *TP.PKAUO.S01.G.U_3*.

Turkish case, the sample period spans from 2005Q1 to 2023Q4, though inflation forecasts are only available from 2014Q1 (or from 2018Q4 in the case of five-year-ahead expectations).

Finally, we use real GDP data to calibrate the updated process of $\bar{\lambda}_t$. US GDP, measured in billions of chained 2017 dollars, is obtained from the *Federal Reserve Bank of St. Louis* database (FRED)[19]. For Turkey, GDP in chain-linked volume by the expenditure approach (thousand Turkish liras) is retrieved from the TCMB database. Both series are available for all related sample periods.

Data used for Monetary Transmission Mechanism

For the US, we use the 10-year bond yield from the *Federal Reserve Bank of St. Louis* database (FRED), the Federal Funds rate target from the Federal Reserve Bank of New York's website, and the five-year CDS premium and one-month OIS spread from Bloomberg.

For Turkey, we use the 10-year swap rate, five-year CDS premium, and one-month forward rates from Bloomberg, along with the policy rate and exchange rate from the CBRT website.

Appendix B. Details on the model

As for the demand side, the representative household maximizes their present discounted value of utility over consumption (C_t) and labour (L_t):

$$\hat{E}_t \sum_{t=0}^{\infty} \beta^t \left[ln(C_t - L_t\right]$$

s.t.

$$P_t C_t + B_t \leq R_{t-1} B_{t-1} + W_t L_t + \Pi_t$$

and

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\Psi_t - 1}{\Psi_t}} dj\right]^{\frac{\Psi_t}{\Psi_t - 1}}$$

where B_t, R_t, W_t, and Π_t are one-period bonds, the interest rate on bonds, wages, and profits from owning firm shares, respectively.

The household problem is like in a textbook New Keynesian model[20], where total consumption C_t is a CES function over the continuum of differentiated goods. Hence, the log-linearized Euler equation is:

$$\hat{c}_t = \hat{E}_t \left[\hat{c}_{t+1} - (i_t - \pi_{t+1})\right]$$

where $\hat{c}_t$ is log linearized consumption.

[19] Variable *GDPC1*, available at https://fred.stlouisfed.org.
[20] See Galí 2015.

The consequent labour supply equation is derived as follows:

$$\frac{W_T}{P_T} = w_T = C_T$$

7.1 Equilibrium

Output is produced by the following production function where the only input is labour (L_t) and productivity A_t is Hicks-neutral, to keep computation light:

$$Y_t = A_t L_t$$

It also corresponds to total consumption, net of aggregate adjustment costs:

$$Y_t + \frac{\Phi}{2}\left(\frac{P_t}{P_{t-1}} - e^{\gamma\pi_{t-1}}\right) = C_t$$

Finally, in this framework, the marginal costs are proportional to consumption:

$$S_t = \frac{w_t}{A_t} = \frac{C_t}{A_t}$$

where w_t is the real wage.

Therefore, log-linearizing the above equality, we derive the following path for marginal costs, which implies that real marginal costs are constant under flexible prices:

$$\hat{s}_t = \hat{c}_t - \hat{a}_t = \hat{y}_t - \hat{a}_t$$

where $\hat{a}_t$ is productivity, $\hat{y}_t$ is output. We can, then, assume that $\hat{s}_t$ in the aggregate supply curve is equal to the output gap, denoted by x_t.

Further details about the model

In the following, we derive the true data generation process of the theoretical model.

As shown in equation 2 of the main text, the aggregate supply is:

$$\pi_t - \gamma\pi_{t-1} = \mu_t + \hat{E}_t \sum_{t=0}^{\infty} (\alpha\beta)^t [\xi s_t + (1-\alpha)\beta(\pi_{t+1} - \gamma\pi_t)]$$

Taking out s_t from the summation and substituting it with the central bank policy rule, as well as inserting expectations about future policy rules in the summation:

$$\pi_t - \gamma\pi_{t-1} = \frac{\lambda}{\lambda+\xi}\mu_t + \frac{\xi}{\lambda+\xi}\phi_t + \frac{\xi\lambda}{(\lambda+\xi)\bar{\lambda}_t}\hat{E}_t\sum_{t=0}^{\infty}(\alpha\beta)^t[\alpha\beta(\phi_{t+1} - (\pi_{t+1} - \bar{\pi}_t) + \gamma(\pi_t - \bar{\pi}_t))] + \frac{\lambda\beta(1-\alpha)}{\lambda+\xi}\hat{E}_t\sum_{t=0}^{\infty}(\alpha\beta)^t(\pi_{t+1} - \gamma\pi_t)$$

Then, aggregating π_t on the left hand side, we get:

$$\left[1 - \frac{\alpha\beta\gamma\xi\lambda}{(\lambda+\xi)\bar{\lambda}_t} + \frac{(1-\alpha)\beta\gamma\lambda}{\lambda+\xi}\right]\pi_t - \gamma\pi_{t-1} = \frac{\lambda}{\lambda+\xi}\mu_t + \frac{\xi}{\lambda+\xi}\phi_t + \frac{\xi\lambda}{(\lambda+\xi)\bar{\lambda}_t}\hat{E}_t\sum_{t=0}^{\infty}(\alpha\beta)^t\alpha\beta\phi_{t+1} + \frac{\lambda}{\lambda+\xi}\hat{E}_t\sum_{t=0}^{\infty}(\alpha\beta)^t\left[-(1-\alpha\beta\gamma)(\frac{\alpha\beta\xi}{\bar{\lambda}_t} - (1-\alpha)\beta)\pi_{t+1}\right] + \frac{\xi\lambda}{(\lambda+\xi)\hat{\lambda}_t}\hat{E}_t\sum_{t=0}^{\infty}(\alpha\beta)^t[\alpha\beta(1-\gamma)\bar{\pi}_t]$$

Given that ϕ_t is a known AR(1) process, $\bar{\pi}_t$ is constant in expectations (anticipated utility solution), and substituting the expectation of future inflations π_{t+1} with the imperfect knowledge forecast of agents, as shown below:

$$\hat{E}_t\sum_{t=0}^{\infty}(\alpha\beta)^t\pi_{t+1} = \hat{E}_t\sum_{t=0}^{\infty}(\alpha\beta)^t\left[\gamma\pi_t + (1-\gamma)\bar{\pi}_t + \frac{\xi}{\xi + \hat{\lambda}_t(1-\beta\rho)}\rho\phi_t\right]$$

$$\hat{E}_t\sum_{t=0}^{\infty}(\alpha\beta)^t\pi_{t+1} = \hat{E}_t\sum_{t=0}^{\infty}(\alpha\beta)^t(1-\gamma)\bar{\pi}_t + \hat{E}_t\sum_{t=0}^{\infty}(\alpha\beta\rho)^t\frac{\xi}{\xi + \hat{\lambda}_t(1-\beta\rho)}\phi_t + \sum_{t=0}^{\infty}(\alpha\beta)^t\gamma\pi_t$$

$$\hat{E}_t\sum_{t=0}^{\infty}(\alpha\beta)^t\pi_{t+1} = \hat{E}_t\sum_{t=0}^{\infty}(\alpha\beta)^t(1-\gamma)\bar{\pi}_t + \hat{E}_t\sum_{t=0}^{\infty}(\alpha\beta\rho)^t\frac{\xi}{\xi + \hat{\lambda}_t(1-\beta\rho)}\phi_t + \alpha\beta\sum_{t=0}^{\infty}(\alpha\beta)^t\gamma\pi_{t+1} + \gamma\pi_t$$

$$\hat{E}_t\sum_{t=0}^{\infty}(\alpha\beta)^t\pi_{t+1} = \frac{1}{1-\alpha\beta\gamma}\left[\gamma\pi_t + \frac{1-\gamma}{1-\alpha\beta}\bar{\pi}_t + \frac{\xi\rho}{(1-\alpha\beta\rho)(\xi + (1-\rho\beta)\hat{\lambda}_t)}\phi_t\right]$$

the true data generation process is found as in the main text.

Appendix C. The estimation algorithm

Model notation

The model is estimated using Bayesian techniques based on US and Turkish data. Given the non-linearities in the model, marginalized particle filter and smoother are implemented. From the main text, the model equations are the following:

$$\pi_t = (1-\gamma)\Gamma\bar{\pi}_t + \gamma\pi_{t-1} + f(\bar{\lambda}_t)\phi_t + \tilde{\mu}_t$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t^{-1} \times f_{t-1}$$

$$k_t = (k_{t-1}+1)I(\bar{\pi}_{t-1}, \phi_{t-2}) + \bar{g}^{-1}(1 - I(\bar{\pi}_{t-1}, \phi_{t-2}))$$

$$f_t = (1-\gamma)(\Gamma-1)\bar{\pi}_t + g(\bar{\lambda}_t)\rho\phi_{t-1} + f(\bar{\lambda}_t)\epsilon_t + \tilde{\mu}_t$$

$$\phi_t = \rho\phi_{t-1} + \epsilon_t,$$

where the functions $I(\bar{\pi}_{t-1}, \phi_{t-2})$ and $I(\bar{\lambda}_{t-1})$ derive from the following anchoring thresholds:

$$I(\bar{\pi}_{t-1}, \phi_{t-2}) = \begin{cases} 1, & \text{if } |(1-\gamma)(\Gamma-1)\bar{\pi}_{t-1} + g(\bar{\lambda}_{t-1})\rho\phi_{t-2}| \leq \bar{\nu}\sigma \\ 0, & \text{otherwise} \end{cases}.$$

$$I(\bar{\lambda}_{t-1}) = \begin{cases} 1, & \text{if } |\bar{\lambda}_{t-1} - \lambda| \leq \bar{\tau}\sigma_\lambda \\ 0, & \text{otherwise} \end{cases}.$$

To better describe the estimation algorithm subsequently, the model can also be written in matrix notation, separating linear and non linear variables as follows. The linear states are represented by ζ_t:

$$\zeta_t = f_\zeta(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2}) + A_\zeta(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2})\zeta_{t-1} + S_{\zeta,t}\begin{bmatrix}\epsilon_t \\ \mu_t\end{bmatrix},$$

where

$$\zeta_t = \begin{bmatrix} \eta_t \\ \phi_t \\ \phi_{t-1} \\ \pi_t \end{bmatrix}; \quad \eta_t = f(\bar{\lambda}_t)\epsilon_t + \tilde{\mu}_t;$$

and

$$f_\zeta(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2}) = \begin{bmatrix} 0_{3\times 1} \\ (1-\gamma)\Gamma f_{\bar{\pi}}(.) \end{bmatrix};$$

$$A_\zeta(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2}) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 \\ 0 & 1 & 0 & 0 \\ (1-\gamma)\Gamma f_k(.)^{-1} & \rho & (1-\gamma)\Gamma f_k(.)^{-1} g(\bar{\lambda}_{t-1})\rho & \gamma \end{bmatrix};$$

$$S_{\zeta,t} = \begin{bmatrix} f(\bar{\lambda}_t) & 1 \\ 1 & 0 \\ 0 & 0 \\ f(\bar{\lambda}_t) & 1 \end{bmatrix};$$

while

$$f_k(.) = f_k(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2}) = (k_{t-1}+1)I(\bar{\pi}_{t-1}, \phi_{t-2}) + \bar{g}^{-1}(1 - I(\bar{\pi}_{t-1}, \phi_{t-2}));$$
$$f_{\bar{\pi}}(.) = f_{\bar{\pi}}(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2}) = \left[1 - (1-\Gamma)(1-\gamma) f_k(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2})^{-1}\right] \bar{\pi}_{t-1}.$$

Whereas the nonlinear states are the following functions:

$$k_t = f_k(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2})$$

and

$$\bar{\pi}_t = f_{\bar{\pi}}(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2}) + A_{\bar{\pi}}(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2})\zeta_{t-1},$$

where

$$A_{\bar{\pi}}(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2}) = \begin{bmatrix} f_k(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2})^{-1} \\ 0 \\ f_k(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2})^{-1} g(\bar{\lambda}_{t-1})\rho \\ 0 \end{bmatrix}'.$$

Finally, we can unify notation for both linear and nonlinear variables in a more compact way:

$$k_t = f_k(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2})$$
$$\begin{bmatrix} \bar{\pi}_t \\ \zeta_t \end{bmatrix} = f(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2}) + A(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2})\zeta_{t-1} + \begin{bmatrix} 0 \\ S_{\zeta,t} \end{bmatrix} \begin{bmatrix} \epsilon_t \\ \mu_t \end{bmatrix}$$

where

$$f(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2}) = \begin{bmatrix} f_{\bar{\pi}}(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2}) \\ f_\zeta(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2}) \end{bmatrix},$$

$$A(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2}) = \begin{bmatrix} A_{\bar{\pi}}(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2}) \\ A_{\zeta}(\bar{\pi}_{t-1}, k_{t-1}, g(\bar{\lambda}_{t-1})\rho\phi_{t-2}) \end{bmatrix},$$

and

$$\Sigma = E\left(\begin{bmatrix} \epsilon_t \\ \mu_t \end{bmatrix}\begin{bmatrix} \epsilon_t \\ \mu_t \end{bmatrix}'\right)$$

is the variance-covariance matrix of the exogenous shocks.

Algorithm of the marginalized particle filter

Given that our CBC model is an extension of the imperfect knowledge New Keynesian model as described by Carvalho et al. 2023, we follow their marginalized particle filter methodology, applied to our model case. This procedure is based on Schon et al. 2005 and Kitagawa 1996. The marginalized particle filter allows us to compute the probability of a sequence of states during the sample period and, based on that, the likelihood function and the posterior probability of the estimated parameters. As explained in the main text, to keep computation simpler, the updating sequence of $\bar{\lambda}_t$ is calibrated from the data, as well as α, β, and ξ are taken from the related literature. In the following, we describe the algorithm in the case of our CBC model. For more discussion and proofs, see Online Appendix of Carvalho et al. 2023.

For all the estimations, we set the number of particles $N = 2500$. As suggested by Carvalho et al. 2023 and discussed deeply in Fernández-Villaverde and Rubio-Ramírez 2007, we keep the following innovations constant, to avoid extra randomness in the likelihood calculation: initial conditions for the nonlinear state variables; draws to compute shocks in the nonlinear prediction step; and draws in the resampling step.

Algorithm:

1. Initialization. $\bar{\pi}^{(i)}_{1|0}, k^{(i)}_{1|0}$ are drawn from the normal distribution and $U(0, \bar{g}^{-1})$, respectively, while $\zeta^{(i)}_{1|0}$, $P^{(i)}_{1|0} = [\zeta_0, P_{1|0}]$, where $P_{1|0}$ denotes the initial precision matrix in the linear Kalman filter.

2. Measurement equation. For each $t = 1...T$, we compute:

$$\Omega_t = H_t' P_{t|t-1} H_t + R_t$$

and its inverse.

It is important to note that H_t is time-varying not only due to possible missing values in the

observed data but also because it is a function of $\bar{\lambda}_t$. For $i = 1, \ldots, N$, the importance weights are:

$$q_t^{(i)} = p(y_t|\bar{\pi}_{t|t-1}^{(i)}, \zeta_{t|t-1}^{(i)}) = \mathcal{N}(h_{0,t} + h_{\bar{\pi},t}\bar{\pi}_{t|t-1}^{(i)} + H_t'\zeta_{t|t-1}^{(i)}, H_t'P_{t|t-1}H_t + R_t)$$

More precisely:

$$q_t^{(i)} = w_{t|t-1}^{(i)}|\Omega_t|^{-1/2} \times \exp\left\{-\frac{1}{2}\left(y_t - h_{0,t} - h_{\bar{\pi},t}\bar{\pi}_{t|t-1}^{(i)} - H_t'\zeta_{t|t-1}^{(i)}\right)' \Omega_t^{-1}\left(y_t - h_{0,t} - h_{\bar{\pi},t}\bar{\pi}_{t|t-1}^{(i)} - H_t'\zeta_{t|t-1}^{(i)}\right)\right\}$$

where $w_{t|t-1}^{(i)}$ denotes the particle weight from the previous period.

3. Re-sampling. If the effective sample size (ESS), computed as:

$$ESS = \frac{1}{\sum (w_t^{(i)})^2},$$

falls below the threshold ($ESS < 0.75 \times N$), we resample the nonlinear variable through a systematic re-sampling where:

$$p\left(\left[\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}\right] = \left[\bar{\pi}_{t|t-1}^{(j)}, k_{t|t-1}^{(j)}\right]\right) = \frac{q_t^{(j)}}{\sum q_t^{(j)}}$$

After resampling, particles feature a discrete distribution with equal weights:

$$w_t^{(i)} = \frac{1}{N} \quad \text{for } i = 1, \ldots, N.$$

In case of not resampling, the weights are:

$$w_t^{(i)} = \frac{q_t^{(i)}}{\sum q_t^{(j)}}.$$

4. Linear Kalman filter. For $i = 1, \ldots, N$, we compute:

$$\zeta_{t|t}^{(i)} = \zeta_{t|t-1}^{(i)} + K_t\left(y_t - h_{0,t} - h_{\bar{\pi},t}\bar{\pi}_{t|t}^{(i)} - H_t'\zeta_{t|t-1}^{(i)}\right)$$

where

$$K_t = P_{t|t-1}H_t\Omega_t^{-1}$$

$$P_{t|t} = P_{t|t-1} - K_tH_t'P_{t|t-1}$$

5. Particle filter prediction. For $i = 1, \ldots, N$, we evaluate:

$$k_{t+1|t}^{(i)} = f_k(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)})$$

and draw $\bar{\pi}_{t+1|t}^{(i)}$ from the distribution:

$$p(\bar{\pi}_{t+1} \mid Y_t, \bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)}) = \mathcal{N}\Big(f_{\bar{\pi}}(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)}) + f_k(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)})^{-1}\eta_{t|t}^{(i)} + g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)}, \quad f_k(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)})^{-2} P_{t|t}^{[\eta,\eta]} \Big)$$

where $P_{t|t}^{[i,j]} = P_{t|t}(i,j)$.

6. Model prediction of linear states.

$$\tilde{\zeta}_{t|t}^{(i)} = \zeta_{t|t}^{(i)} + \tilde{K}_t^{(i)} \left(\bar{\pi}_{t+1|t}^{(i)} - f_{\bar{\pi}}(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)}) - f_k(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)})^{-1}\eta_{t|t}^{(i)} + g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)} \right)$$

$$\zeta_{t+1|t}^{(i)} = f_\zeta(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)}) + A_\zeta(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)})\tilde{\zeta}_{t|t}^{(i)}$$

$$P_{t+1|t} = Q_{\zeta,t} + \tilde{P}_{t|t}$$

where

$$Q_{\zeta,t} = S_{\zeta,t}\Sigma S_{\zeta,t}'$$

$$\begin{aligned}
\tilde{K}_t^{(i)} &= P_{t|t}A_{\bar{\pi}}'(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)}) \left(A_{\bar{\pi}}(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)}) P_{t|t} A_{\bar{\pi}}'(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)}) \right)^{-1} \\
&= f_k(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)}) \begin{bmatrix} 1 \\ \frac{P_{t|t}^{[\eta,\phi]}}{P_{t|t}^{[\eta,\eta]}} \\ 0 \\ \frac{P_{t|t}^{[\eta,\pi]}}{P_{t|t}^{[\eta,\eta]}} \end{bmatrix}
\end{aligned}$$

and

$$\tilde{P}_t|_t = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \iota_1\rho^2 & \iota_1\rho & \iota_1\rho^2 f_t(\bar{\lambda}_t) + \iota_2\rho\gamma \\ 0 & \iota_1\rho & \iota_1 & \iota_1\rho f_t(\bar{\lambda}_t) + \iota_2\gamma \\ 0 & \iota_1\rho^2 f_t(\bar{\lambda}_t) + \iota_2\rho\gamma & \iota_1\rho f_t(\bar{\lambda}_t) + \iota_2\gamma & \rho f_t(\bar{\lambda}_t)(\iota_1\rho f_t(\bar{\lambda}_t) + \iota_2\gamma) + \gamma(\rho f_t(\iota_2\bar{\lambda}_t) + \iota_3\gamma) \end{bmatrix}$$

where

$$\iota_1 = \left[P_{t|t}^{[\phi,\phi]} - \frac{\left(P_{t|t}^{[\eta,\phi]}\right)^2}{P_{t|t}^{[\eta,\eta]}} \right]$$

$$\iota_2 = \left[P_{t|t}^{[\phi,\pi]} - \frac{P_{t|t}^{[\eta,\pi]} P_{t|t}^{[\eta,\phi]}}{P_{t|t}^{[\eta,\eta]}} \right]$$

$$\iota_3 = \left[P_{t|t}^{[\pi,\pi]} - \frac{\left(P_{t|t}^{[\eta,\pi]}\right)^2}{P_{t|t}^{[\eta,\eta]}} \right]$$

$P_{t+1|t}$ would be particle dependent given that it is a function of $g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)}$. However, since $g(\bar{\lambda}_t)$ takes values close to zero, we can approximate this element to zero when computing $P_{t+1|t}$ in the algorithm, to allow a fast evaluation of the likelihood and the posterior distribution without affecting the solution of the model close to the equilibrium where $g(\bar{\lambda}_t) = 0$.

7. The log-likelihood is then approximated by the particles weigths:

$$L(\cdot) = \sum_{t=1}^{T} \log p(y_t|Y_{t-1}) = \sum_{t=1}^{T} \log \left(\sum_{i=1}^{N} q_t^{(i)} \right)$$

Finally, it must be noticed that the algorithm is implemented with the constraint that $\zeta_{2,t} = \zeta_{3,t+1}$ since the current monetary policy state is the lagged monetary policy state in the following period.

Algorithm of the marginalized smoother

Also in the case of the particle smoother, we apply to our extended model a procedure similar to the one implemented by Carvalho et al. 2023, who apply joint backward simulation Rao-Blackwellised particle smoother as in Lindsten, Schön, et al. 2013. The full distribution of states and parameters is computed using Carter and Kohn 1994.

We start drawing $\bar{\pi}_{T|T}^{(j)}$, $k_{T|T}^{(j)}$ from the empirical distribution of $\{\pi_{t|t}^{(k)}, k_{t|t}^{(k)}\}_{k=1}^{N}$ at the end of the above forward filter, where each particle has weight $w_t^{(k)}$. Moreover, conditional on the draw $\bar{\pi}_{T|T}^{(j)}$, $k_{T|T}^{(j)}$ and the constraint on $\zeta_{3,T|T}$, we draw the linear state $\zeta_{T|T}^{(j)}$ from the normal distribution $\mathcal{N}(\zeta_{T|T}^{(j)}, P_{T|T})$. Given such initialization, we compute the following algorithm.

Algorithm:

For $t = T-1$ to 1 (backward step):

For each $i = 1 \dots N$, the particle weight is

$$w_{t|t+1}^{(i,j)} = \frac{w_t^{(i)} p(\bar{\pi}_{t+1|t+1}^{(j)}, k_{t+1|t+1}^{(j)}, \zeta_{t+1|t+1}^{(j)} | \pi_{t|t}^{(i)}, k_{t|t}^{(i)}, \zeta_{3,t|t}^{(i)}, Y_t)}{\sum_k w_t^{(k)} p(\bar{\pi}_{t+1|t+1}^{(j)}, k_{t+1|t+1}^{(j)}, \zeta_{t+1|t+1}^{(j)} | \pi_{t|t}^{(k)}, k_{t|t}^{(k)}, \zeta_{3,t|t}^{(k)}, Y_t)}$$

The probability distribution above can be expressed as:

$$p(\bar{\pi}_{t+1|T}^{(j)}, k_{t+1|T}^{(j)}, \zeta_{t+1|T}^{(j)} | \pi_{t|t}^{(i)}, k_{t|t}^{(i)}, \zeta_{3,t|t}^{(i)}, Y_t) =$$
$$p(\bar{\pi}_{t+1|T}^{(j)}, \zeta_{t+1|T}^{(j)} | \bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, \zeta_{3,t|t}^{(i)}, Y_t) p(k_{t+1|T} | \bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, \zeta_{3,t|t}^{(i)}, Y_t)$$

where $k_{t+1|t} = f_k(\bar{\pi}_t, k_t, g(\bar{\lambda}_t)\rho\zeta_{3,t})$. We, hence, evaluate

$$p(\bar{\pi}_{t+1|T}^{(j)}, \zeta_{t+1|T}^{(j)} | \bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, \zeta_{3,t|t}^{(i)}, Y_t) \times \mathbf{1}_{k_{t+1}^{(j)} = f_k(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)})} =$$
$$\begin{cases} p_{t|t+1}^{j,i}, & \text{if } k_{t+1|T}^{(j)} = f_k(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)}) \\ 0, & \text{otherwise} \end{cases}$$

where

$$p_{t|t+1}^{j,i} \propto \exp\left\{ -\frac{1}{2} \ln |\tilde{\Omega}_t^{(i)}| - \frac{1}{2} (\tau_{t+1}^{j,i})' \times (\tilde{\Omega}_t^{(i)})^{-1} (\tau_{t+1}^{j,i}) \right\}$$

$$\tau_{t+1}^{j,i} = \begin{bmatrix} \bar{\pi}_{t+1|t+1}^{(j)} \\ \zeta_{t+1|t+1}^{(j)} \end{bmatrix} - \left[f(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)}) + A(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)}) \zeta_{t|t}^{(i)} \right]$$

$$\tilde{\Omega}_t^{(i)} = Q_t + A(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)}) P_{t|t} A(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)})'$$

$$Q_t = \begin{bmatrix} 0 & 0 \\ 0 & S_{\zeta,t} \Sigma S_{\zeta,t}' \end{bmatrix}$$

and

$$\mathbf{1}_{k_{t+1}^{(j)} = f_k(\bar{\pi}_{t|t}^{(i)}, k_{t|t}^{(i)}, g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)})} = \mathbf{1}_{k_{t+1}^{(j)} = \bar{g}} \times \mathbf{1}_{|(1-\gamma)(\Gamma-1)\bar{\pi}_{t|t}^{(i)} + g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)}| > \bar{\nu}\sigma} +$$
$$(1 - \mathbf{1}_{k_{t+1}^{(j)} = \bar{g}}) \times \mathbf{1}_{|(1-\gamma)(\Gamma-1)\bar{\pi}_{t|t}^{(i)} + g(\bar{\lambda}_t)\rho\zeta_{3,t|t}^{(i)}| \leq \bar{\nu}\sigma} \times \mathbf{1}_{k_{t+1}^{(j)} - 1 = k_{t|t}^{(i)}}$$

and taking into account the constraint on $\zeta_{2,t+1|T}$.

Therefore, we draw the nonlinear variables $\bar{\pi}_{t|T}^{(j)}, k_{t|T}^{(j)}$ from$\{\bar{\pi}_{t|t}^{(k)}, k_{t|t}^{(k)}\}_{k=1}^N$ using the newly computed set of weights $\{w_{t|t+1}^j(k)\}_{k=1}^N$. Conditional on this draw, we finally sample from (subject to the mentioned constraints):

$$p(\zeta_t | \bar{\pi}_{1:t}^j, k_{1:t}^j, \zeta_{t+1}, \bar{\pi}_{t+1}, Y_t).$$

In particular, we draw $\zeta_{t|T}^{(j)}$ from the distribution

$$\mathcal{N}(\zeta_{t|t}^{(j)} + \Delta_t^{(j)} \left(\left[\bar{\pi}_{t+1|T}^{(j)}, \zeta_{t+1|T}^{(j)} \right]' - f(\bar{\pi}_{t|T}^{(j)}, \zeta_{t|T}^{(j)}) - A(\bar{\pi}_{t|T}^{(j)}, \zeta_{t|T}^{(j)}) \zeta_{t|t}^{(j)}), \Lambda_{t|t}^{(j)} \right)$$

where $\zeta_{t|t}^{(j)}$ corresponds to the same draw j from which the particles $\bar{\pi}_{t|T}^{(j)}$, $k_{t|T}^{(j)}$ are obtained, and where

$$\Delta_t^{(j)} = P_{t|t} A(\bar{\pi}_{t|T}^{(j)}, k_{t|T}^{(j)}, g(\bar{\lambda}_t) \rho \zeta_{3,t|T}^{(j)})' (Q_t + A(\bar{\pi}_{t|T}^{(j)}, k_{t|T}^{(j)}, g(\bar{\lambda}_t) \rho \zeta_{3,t|T}^{(j)}) P_{t|t} A(\bar{\pi}_{t|T}^{(j)}, k_{t|T}^{(j)}, g(\bar{\lambda}_t) \rho \zeta_{3,t|T}^{(j)})')^{-1}$$

$$\Lambda_{t|t}^{(j)} = P_{t|t} - \Delta_t^{(j)} A(\bar{\pi}_{t|T}^{(j)}, k_{t|T}^{(j)}, g(\bar{\lambda}_t) \rho \zeta_{3,t|T}^{(j)}) P_{t|t}.$$

Appendix D. Details on the Turkish case

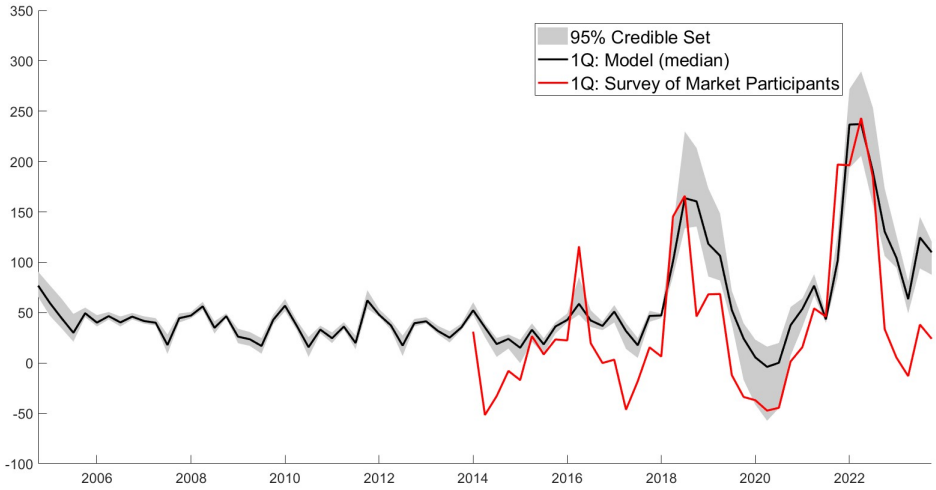
Table D1 presents the results from the Bayesian inference for Turkey. The model's predictions for one- and four-quarters-ahead inflation, compared to survey data, are illustrated in Figures D1, D2, and D3, respectively. Despite the high volatility of Turkish data, our model incorporating inflation bias demonstrates a strong fit, capturing the underlying patterns effectively.

Table D1: Prior and Posterior Distribution of Structural Parameters - Turkey

		Prior distribution			Posterior distribution			
		Distribution	Mean	SD	Mode	Mean	5^{th} Pct	95^{th} Pct
(1)	π^*	Normal	18.800	2.000	18.817	19.108	15.790	22.425
(2)	$\bar{\nu}$	Gamma	0.200	0.040	0.189	0.186	0.127	0.245
(3)	$\bar{g}$	Gamma	0.800	0.090	0.768	0.796	0.651	0.938
(4)	γ	Beta	0.500	0.265	0.500	0.848	0.364	0.970
(5)	ρ	Beta	0.500	0.200	0.985	0.673	0.220	0.968
(6)	ω	Normal	0.900	0.020	0.903	0.906	0.873	0.944
(7)	λ	Gamma	2.000	0.200	2.006	2.002	1.740	2.318
(8)	σ_ϵ	InvGamma	1.000	2.000	1.326	2.245	0.339	7.807
(9)	$\sigma_{\tilde{\mu}}$	InvGamma	1.000	1.000	0.998	2.605	0.598	4.131
(10)	$\sigma_{o,1}$	InvGamma	5.000	1.000	4.402	4.449	3.558	5.394
(11)	$\sigma_{o,2}$	InvGamma	9.000	1.000	11.934	11.775	10.368	13.651
(12)	$\sigma_{o,3}$	InvGamma	8.000	1.000	8.732	8.648	7.104	10.294

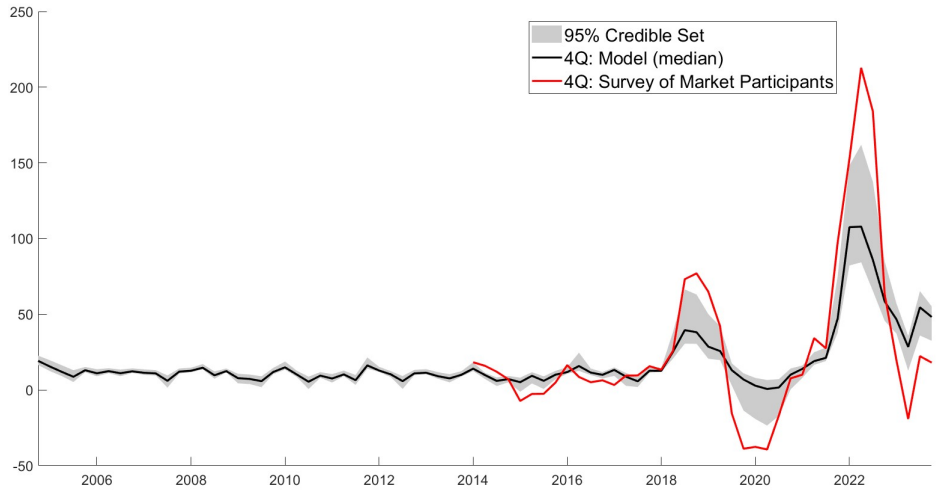
Note: The posterior distribution is obtained using a Metropolis-Hastings algorithm. Moreover, the remaining parameters are calibrated according to the main New Keynesian literature: $\alpha = 0.66$, $\beta = 0.99$, and, hence, $\xi = 0.17$.

Figure D1: Short-run Inflation Expectations: Model vs Data



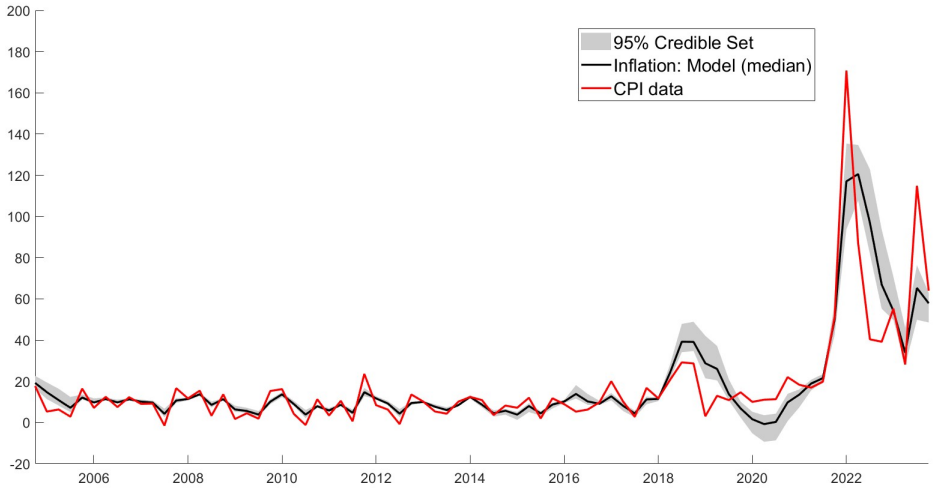
Note: The panel shows the evolution of one-quarter-ahead inflation expectations from CBRT's survey of market participants, compared with model predictions.

Figure D2: Short-run Inflation Expectations: Model vs Data



Note: The panel shows the evolution of four-quarters-ahead inflation expectations from CBRT's survey of market participants, compared with model predictions.

Figure D3: Inflation: Model vs Data



Note: The panel shows the evolution of inflation from CBRT's survey of market participants, compared with model predictions.

Appendix E. The Effect of Nonlinearity in Expectations

Since $\bar{\lambda}_t^{-1}$ enters the expectations of agents in a nonlinear way, essentially $\hat{E}_t[\bar{\lambda}_t^{-1}] \geq [\hat{E}_t \bar{\lambda}_t]^{-1}$ due to Jensen's equality. However, the two expectations reach to quite similar values for potential values of λ_t which is typically larger than 0.5 with a high probability mass. In Figure **??** we show these expectations for the two cases. In the first case, the mean of the $\bar{\lambda}_t^{-1}$ is taking values in a range $[0, 2]$ but with a fixed standard deviation of 0.2. In the second case, we increase the standard deviation to 0.5 all else remaining equal. As can be seen, for potential values, the two expectations yields similar results for potential range of $\bar{\lambda}_t^{-1}$ values with high probability mass.

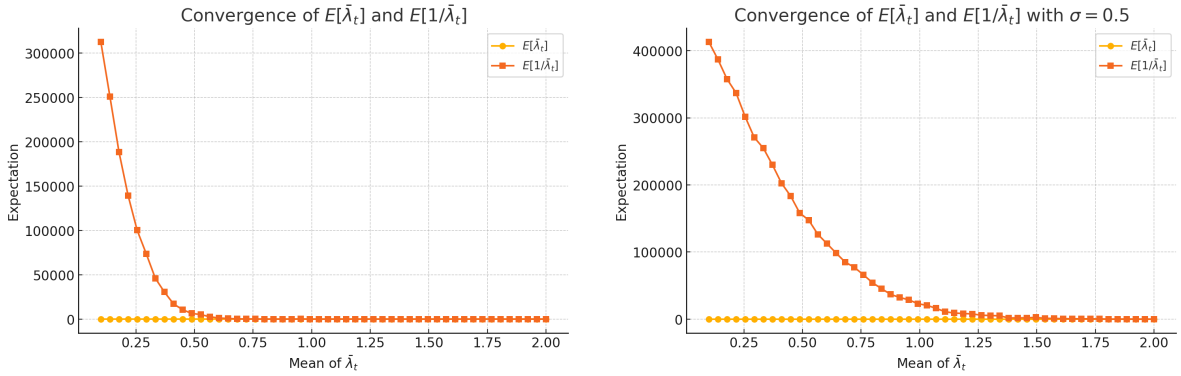


Figure 17: Caption