Why so gradual? Disinflation under inflation targeting*

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Abstract

Central banks operating under inflation targeting often adopt gradual disinflationary policies. This paper studies the trade-offs between "Cold Turkey" (CT) and gradual disinflations using a New Keynesian model with imperfect credibility regarding the inflation target. In the baseline calibration, agents slowly learn about the target in both policies, which dampens inflation expectations decline. As a result, inflation is closer to the target under the gradual policy, allowing a relatively looser monetary policy and lower disinflation costs. CT is preferred when credibility is sufficiently high to promote a strong response of inflation expectations, as agents learn faster the immediate target decline.

Keywords: Credibility, inflation targeting, disinflation costs, gradualism, Cold Turkey.

Jel Classification: D8, E52, E58, E32

1 Introduction

Over the last few decades, several central banks have adopted inflation targeting (IT) as their monetary policy framework. Many countries adopted IT as part of disinflation policies, announcing targets below previous realized inflation rates. Figure 1 illustrates the evolution of headline consumer price inflation (solid blue line, starting four years before IT adoption) and inflation targets (dashed black lines) of 15 countries implementing disinflation upon adopting IT. Grey areas indicate disinflation periods.[1]

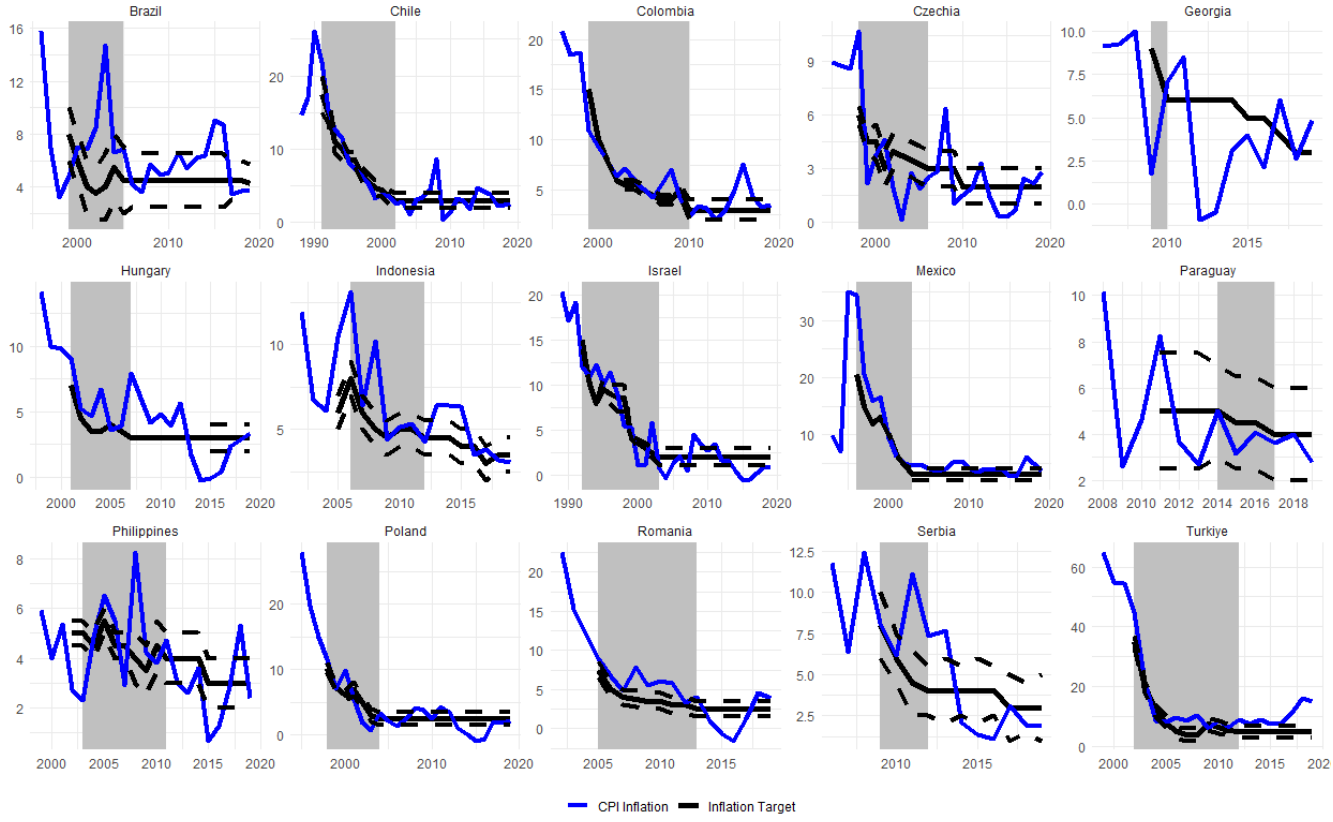
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[1]I use information from target path instead of the standard definition from Ball (1994b) based on the trend inflation. Specifically, a disinflation starts if the following year target is lower than the current target (usually, next

The figure reveals that disinflation processes are typically gradual. Overall, despite some volatility, inflation has fallen in line with the decrease in the targets.

Figure 1: Gradual changes in inflation targets and CPI inflation.



Note: CPI inflation in solid blue lines and dashed black lines represents inflation target (or mid-point of the target range) with their associated tolerance bands (when applies). Shaded areas represent disinflation episodes as described in the text.
Source: IMF IFS database for CPI and central banks websites for the inflation targets.

There has been a longstanding debate over the tradeoffs of CT and gradual policies.[2] This paper revisits this debate in the context of the inflation targeting regime and relates the disinflation strategy choice to the central bank's credibility regarding the target. As discussed by Fraga et al.

year targets are set in advance). It ends in the year that the target reaches a new lower level and remains constant for two subsequent years. Most countries do not monotonically decrease their targets. This definition considers the initial and final targets, allowing punctual increases in the target during disinflation. For instance, Colombia started with a 15% target in 1999 and ended the disinflation with a 3% target in 2010. In between, the target was constant between 2003-2004 (5.5%) and increased in 2009 to 5% (it was 4% in the previous two years) before setting to 3% in 2010.

[2] Gordon (1982) and Taylor (1983) suggest that gradual disinflation is less costly since it allows prices and wages to adjust to the tighter monetary policy in the presence of nominal rigidity and indexation. Sargent (1982) advocates that inflation inertia is associated with persistent inflationary policies. In that case, a CT disinflation generates lower disinflation costs.

(2003), credibility about the target is a crucial issue for the monetary authority of emerging countries when adopting IT.[3]

To do this, I propose a tractable model that allows immediate or gradual changes in the inflation target and imperfect credible announcements about the target. Using this model, I highlight the circumstances under which each policy is more beneficial and evaluate the benefits of introducing the IT regime regarding the disinflation sacrifice ratio (SR, henceforth).

The model is a standard New Keynesian model with imperfect credibility as in Erceg and Levin (2003). Agents know that the central bank follows a Taylor rule and pursues a time-varying target, but do not know the true target and infer it from interest rate decisions. I refer to that benchmark as the Opacity case.

The inflation targeting regime is represented by a noisy public signal about the target. Thus, agents infer the target from central bank actions and imperfectly credible communications about the target. This additional information helps to anchor inflation expectations toward the target depending on the IT regime's credibility. The variance of this public signal can be interpreted as the regime's (lack of) credibility. The larger the precision of the announcement, the more credible the IT regime and the stronger the inflation expectations anchoring. This holds for both disinflation policies.

The data depicted in Figure 1 allows calibrating the inflation target process for each country. I also calibrate the credibility of the IT regime based on: i) the size of the tolerance band around the targets (or target range) and ii) the compliance of the inflation target, i.e., the share of years that the inflation was within the band during the disinflation.

I show results for the median calibration. The median disinflation episode takes 7 years to fully reach its final target, decreasing its target by 0.6% per year, roughly a 4.75% drop from the initial to the final target. The tolerance band is $\pm 1\%$ around the target with 50% compliance during the disinflation period.

Indexation is a crucial aspect when studying the relationship between inflation and output, both in short- and long-run. In the NK model, when there is positive steady-state inflation and absence of indexation, staggered price adjustments generate steady-state price dispersion, which may lead to sizable output costs due to relatively small steady-state inflation (see Ascari and Sbordone; 2014, for a review). To deal with this issue, I consider an indexation scheme that avoids a long-run relationship between inflation and output to focus on the short-run disinflation costs.

In the baseline results, I focus on the case where agents fully index prices to their expectations about the target. That case replicates the well-known benchmark of a costless disinflation under CT when the central bank has full credibility (Ball; 1994a; Mankiw and Reis; 2002). Using this benchmark avoids explaining the gradual policies of the IT countries due to indexation to past

[3]See also Minella et al. (2003) for Brazil and Cespedes and Soto (2007) for Chile.

inflation and focuses on whether the credibility can explain why IT emerging countries follow such gradual policies as shown in Figure 1.

Consider the baseline with full credibility under CT. As firms know and trust the fall in the target, all firms adjust prices accordingly with the new target, as non-optimizing firms index by the target rate. Inflation expectations and nominal rates drop immediately along with the target, so real rates do not change. Thus, there is an immediate disinflation to the new target with zero output costs.

In contrast, gradual disinflation leads to output costs. As before, non-optimizing firms index their prices according to the current target while optimizing firms increase prices at a lower rate than the current target as they anticipate it will decrease over time. Inflation expectations fall more than nominal rates, leading to a weak hike in real rates and a recession.

Under imperfect credibility, there are additional disinflation costs. As agents gradually learn about the target, inflation expectations also adjust slowly so that the inflation does not drop as much as the target. While the nominal rate falls because it tracks the target, the central bank must tighten (relatively) its policy to fight the positive inflation gap (the difference between inflation and its target). Then, the real interest rate increases, leading to a recession.

In the baseline calibration under Opacity, after a 1% immediate drop in the target agents infer that the target has fallen by less than 0.2% on impact, and fully learning the target takes more than eight years.

Under gradualism, the target falls only 0.1% on impact and takes more than eight years to reach the 1% drop. On impact, agents expect a third of the actual drop in the target and fully learn in less than six years. Thus, gradual policy generates a more anchored disinflation: expectations about the target are closer to the actual target while the target is declining. A sensitivity analysis suggests that difference in learning holds for a large range of the key parameter values.

This mechanism is key for gradualism generating a lower SR than CT disinflation under Opacity. In the CT disinflation, as the target is less anchored, inflation expectations lag behind the actual target, opening a large positive inflation gap. Under gradualism, the inflation gap is much more muted. Therefore, the central bank under the gradual policy follows a relatively looser monetary policy than under CT. Real rates hike by much less in the former, leading to a much weaker recession than CT.

Under IT, agents observe a noisy public signal in addition to the interest rate decisions. This communication implies faster and more accurate learning about the new target, i.e., expectations are more anchored for both disinflation strategies. Under the IT calibration, the time to fully learn the target goes down to roughly four and six years for gradual and CT policies, respectively.

This implies that inflation expectations fall faster, requiring a less pronounced tightening from the central bank. This implies lower SR for IT compared to Opacity under both disinflation

strategies.

Interestingly, the model implies that adopting IT increases the relative benefit of pursuing a CT disinflation. Whether CT or gradualism generates lower SR depends on the IT credibility. The Opacity case is equivalent to the limiting case where the central bank communication has no credibility, i.e., it does not affect inflation expectations.

I show (numerically) that there is a credibility threshold such that the central bank has the same sacrifice ratio when adopting both policies. As credibility increases, learning about the target is faster, and more of the target fall is incorporated into inflation expectations. This benefits the CT disinflation relatively more than the gradual since the inflation gap is higher in the former. For a sufficiently high credibility level, both policies yield the same SR. Thus, a highly credible central bank is likely to opt for CT while central banks with credibility issues may choose a gradual policy.

The mechanism is consistent with the Sargent's (1982) case for CT when the policy is credible. The key difference is that the author evaluates disinflations under a drastic change of regime. Here, the monetary policy rule is known but agents do not know that the central bank is disinflating.

If there is a higher incentive for pursuing CT disinflation under IT, why do countries in Figure 1 choose such gradual policies according to the model? The credibility threshold is too stringent since it requires three times higher precision of the inflation target announcement than the calibrated value for the median country. Thus, gradualism has lower disinflation costs in the baseline calibration.

For concreteness, the central bank credibility calibration is based on $\pm 1\%$ tolerance band with 50% target compliance. The credibility threshold is consistent with: i) a $\pm 0.3\%$ tolerance band keeping the baseline compliance; or ii) 97.5% target compliance with the baseline target band.

The existence of such a threshold depends on some key parameters. For instance, when using an indexation scheme based on a combination of past inflation and their expectations about the target, for a sufficiently high indexation to past inflation, gradualism dominates CT in terms of SR for any degree of credibility. In that case, indexation to past inflation with nominal rigidity points towards gradualism even under full credibility. This is consistent with the Taylor's (1983) case for gradualism.

I also explore a credibility mechanism through which the central bank's response to inflation gap affects the disinflation costs when credibility is imperfect along the lines of Bordo et al. (2017).[4] Conditional on the disinflation strategy, as the central bank responds stronger to the inflation gap, the nominal interest rate becomes more informative about the target. Thus, as the central bank becomes more Hawkish, two opposing channels emerge: First, for a given inflation expectation, it raises more interest rates generating a larger recession (demand channel). Second, agents learn

[4] Bordo et al. (2017) study this channel for disinflation episodes in the US. This paper explores the same channel in a model that also includes announcements about the target and allows for gradual disinflation policies.

faster about the target, which leads to a stronger fall in inflation expectations (expectational channel). The former leads to more disinflation costs while the latter implies lower costs.

Keeping all other parameters in the baseline calibration under IT, the relation between central bank's response to the inflation gap and SR depends on the disinflation policy. Under gradualism, there is a weak demand channel as the real rate has a subdued rise and thus, the expectational channel dominates. Under CT, there is a non-monotonical relation: for sufficiently small responses to the inflation gap, SR increases in those responses and then decreases for sufficiently high responses. Intuitively, the demand channel is stronger under CT, as the central bank has to fight a much wider inflation gap because inflation is less anchored.

Overall, the model and the sensitivity analysis suggest that gradually decreasing inflation targets combined with a strong stance toward the inflation gap can generate relatively lower sacrifice ratios.

This paper is organized as follows. Section 1.1 discuss the related literature on disinflation costs, section 2 presents the model. Then, section 3 presents the results and section 4 concludes.

1.1 Literature review

This paper is related to a literature studying disinflation under imperfect credibility. Ball (1995) shows that imperfect credibility and staggered prices generates reasonable disinflation costs. Bonomo and Carvalho (2010) show that the lack of credibility is even more important when considering endogenous time-dependent pricing. In those papers, imperfect credibility is related with a probability that the central bank stop pursuing the announced lower path for the money growth.

Here I propose modeling credibility within the IT regime. Building on the Erceg and Levin (2003), agents know that the central bank follows a Taylor rule but cannot disentangle standard moretary shocks from changes in the target.[5] This literature models lack of credibility as an signal extraction problem regarding the target.

I extend this idea introducing noisy public signal about the inflation target whose precision controls the degree of credibility of the inflation targeting regime. Bonomo et al. (2024) also proposes this modeling device for credibility to study unanchored expectations in Brazil.

Gibbs and Kulish (2017) also study gradual and CT disinflations with imperfectly credible announcements within the NK model. The approach here differs in some important ways. First, they consider the target as an unknown parameter. Second, agents' expectations is a weighted average of the rational expectation and the adaptive learning. Their credibility parameter is the weight agents put on the rational expectations. They also focus on explaining the cross-country distribution of sacrifice ratio.

[5]This modeling device is standard when studying changes in the monetary policy rule (see also Ireland; 2007; Andolfatto et al.; 2008; Cogley et al.; 2010; Del Negro and Eusepi; 2011)

Also using learning, Eusepi and Preston (2010) study central bank communication in general and also for the inflation target. They focus on inflation expectations anchoring in terms of divergent learning dynamics. With the signal extraction approach, Taylor principle is sufficient for stability as agents know the monetary rule.

This paper is also related with the empirical literature on disinflations for IT countries. Gonçalves and Carvalho (2009) provide evidence that disinflations under IT have substantially lower sacrifice ratios than non-IT. However, Brito (2010) reevaluates the study by showing that the results are not robust to time control effects as well some reclassification of the disinflation episodes under IT. Roux and Hofstetter (2014) go further and shows that IT reduce SRs, but only for long disinflation episodes. This paper shows that both a gradual or immediate change in the target have similar duration to disinflate due to the slow learning about the target. This suggest some cautiousness on inferring the policy gradualism based on the path of the inflation as in the standard approach from Ball (1994b).

2 Model

The central bank sets the nominal interest rate according to the rule

$$r_t = \rho + \bar{\pi}_t + \phi_\pi(\pi_t - \bar{\pi}_t) + \phi_y y_t + \eta_t^r, \tag{1}$$

where r_t is the short-term nominal interest rate, π_t is current inflation, y_t is the real output in log-deviations from its steady-state and $\rho = 1/\beta - 1$ is the steady-state real interest rate. η_t^r denotes a standard exogenous monetary shock and $\bar{\pi}_t$ is the inflation target. To avoid indeterminacy issues, I assume that $\phi_\pi > 1$.

The monetary shock is a transitory shock that represents a change in the interest rate beyond the recommended by the rule. Its stochastic process is given by

$$\eta_t^r = \rho_r \eta_{t-1}^r + \varepsilon_t^r, \quad \varepsilon_t^r \sim \mathcal{N}(0, \sigma_r^2), \tag{2}$$

and the inflation target follows an unit-root process given by

$$\begin{aligned} \bar{\pi}_t &= \bar{\pi}_{t-1} + \eta_t^\pi \\ \eta_t^\pi &= \rho_\pi \eta_{t-1}^\pi + \varepsilon_t^\pi, \quad \varepsilon_t^\pi \sim \mathcal{N}(0, \sigma_\pi^2), \end{aligned} \tag{3}$$

where $\bar{\pi}_t = log(\bar{\Pi}_t)$ is the inflation target rate and $\bar{\Pi}_t$ the gross rate. Thus, any change in ε_t^π has a permanent effect on $\bar{\pi}_t$. Therefore, a target shift represents a permanent change in the conduct of monetary policy and in the rule itself.

Since the inflation target follows a unit-root process, nominal variables have a stochastic balanced growth path (BGP) that grows at the same rate of the inflation target. I discuss this point in more detail in section 2.1.

In the "Cold Turkey" strategy, I consider that $\rho_\pi = 0$, such that the target follows a random walk, such that shock in ε_t^π imply an immediate and permanent change in the target. $\rho_\pi > 0$ is consistent with a gradual disinflation consistent with the evidence shown in Figure 1.[6]

The inflation targeting regime requires not only following a target but also announcing the target to the public. Thus, I assume that a central bank that follows IT provides a noisy public signal about the inflation target, given by

$$s_t^{IT} = \bar{\pi}_t + u_t, \quad u_t \sim \mathcal{N}(0, \sigma_{IT}^2). \tag{4}$$

The public signal is a modeling device that represents the central bank communication about the target. In the following, I interpret σ_{IT} as the degree of credibility of the IT regime.

The intuition is the following. A central bank announces its target to the public but agents do not necessarily trust in the central bank's commitment to the target. The larger σ_{IT}, less credible is the central bank and less weight agents put on their communication when forming expectations about the actual target. Bonomo et al. (2024) uses the same notion of credibility in a simple model to study unanchored expectations in Brazil.

Other interpretations are possible. For instance, Faust and Svensson (2002) interpret the variance of such noise as the degree of transparency.[7] The key difference is that transparency can be also a choice of central bank while I also assume that σ_{IT} is a primitive and is beyond of the complete control of the central bank.

For simplicity, degree of central bank credibility is assumed to be exogenous and any strategic interaction between central banks communication and actions is abstracted.

When $\sigma_{IT} = 0$, the central bank is fully credible and agents observe the true inflation target: model is the same as under full information. For a sufficiently high σ_{IT} ($\sigma_{IT} \to \infty$), agents ignore the inflation target announcement and form expectations using the central bank actions only because the communication provides no useful information about the target.

[6] In general, the literature avoid using a unit root in the target, but use a highly persistent process in the target level as an approximation to preserve the steady-state inflation constant. In section 2.1, I discuss how to deal with this technicality.

[7] In their context, the public signal is about the inflation control error.

Households

There is a representative household whose utility function is given by

$$\sum_{s=0}^{\infty} \beta^s \left(log(C_{t+s}) - \frac{L_{t+s}^{1+\varphi}}{1+\varphi} \right), \tag{5}$$

where $0 < \beta < 1$ is the discount factor and φ is the inverse of the Frisch labor supply elasticity. C_t and L_t denote household's consumption and labor, respectively.

The composite consumption good is a CES aggregator

$$C_t = \left(\int_0^1 (C_{i,t})^{(\varepsilon-1)/\varepsilon} di \right)^{(\varepsilon-1)/\varepsilon},$$

where $C_{i,t}$ is the consumption of each variety i in period t and $\varepsilon > 1$ is the elasticity of substitution between varieties.

The household is subject to the budget constraint

$$P_t C_t + B_t \leq W_t L_t + R_{t-1} B_{t-1} + T_t, \tag{6}$$

in each period t. B_t is an one-period nominal bond, T_t denotes net transfers, W_t is the nominal wage and P_t is the price level of the consumption bundle.

Firms

There is a continuum $i \in [0, 1]$ of monopolistic competition firms that produce a differentiated good Y_{it} using a production function

$$Y_{it} = L_{it}^{\alpha}, \tag{7}$$

where $\alpha \in (0, 1]$.

As in Calvo (1983), in each period, there is a constant probability $(1 - \theta)$ that each firm will re-optimize its price. The firms that do not optimize their prices are assumed to adjust their previous price with the indexation rule

$$\Upsilon_t = (\Pi_{t-1})^{\iota} \left(E_t[\bar{\Pi}_t] \right)^{1-\iota}, \tag{8}$$

where $\iota \in [0, 1]$ controls the degree of indexation to past inflation, and $E_t[\cdot] \equiv E[\cdot|\mathcal{I}_t]$ is the expectation operator of firm i conditional on the current information set, $\mathcal{I}_t$.

The specification above extends the indexation rule from Smets and Wouters (2007) that allows indexation to past and steady-state inflation. Here, I include the expected inflation target instead

of the steady-state inflation due to two reasons. First, inflation has a BGP such that its path is the same as the inflation target. Second, it acknowledges that agents may not observe the target. Moreover, the indexation rule eliminates price dispersion in the BGP, which avoids a strong negative relationship between steady-state real output and inflation (see Ascari; 2004).

In the baseline model, I assume that $\iota = 0$, such that firms readjust by their expectation regarding the inflation target. The price rigidity and indexation rule imply that the evolution of the price index is given by

$$P_t = \left[\theta \left(\Upsilon_t P_{t-1}\right)^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{9}$$

where P_t^* denotes the optimal price from firms allowed to optimize.

Firms maximize the expected discounted value of their profit

$$E_t\left[\sum_{s=0}^{\infty} \theta^s \Lambda_{t,t+s}\left(\Upsilon_{t,t+s} P_t^* Y_{i,t+s|t} - P_{t+s} MC_{i,t+s|t}\right)\right], \tag{10}$$

subject to the market clearing condition, $Y_{i,t} = C_{i,t}$ and the firm individual firm demand. $Y_{i,t+s|t}$ and $MC_{i,t+s|t} = \frac{W_{t+s}}{P_{t+s}} Y_{i,t+s|t}^{1/\alpha}$ denote output and real marginal cost at period $t+s$ from firms that optimized their prices at period. $\Lambda_{t,t+s}$ is the stochastic discount factor from t to $t+s$ and $\Upsilon_{t,t+s}$ is the cumulative indexation adjustment from t to $t+s$ given by

$$\Upsilon_{t,t+s} = \begin{cases} \prod_{j=1}^{s} \Upsilon_{t+j} & \text{if} \quad s \geq 1 \\ 1 & \text{if} \quad s = 0. \end{cases}$$

which is consistent with the indexation rule (8).

I assume the following timing protocol. Every period t is divided into two stages. In stage 1, the inflation target, monetary shocks and the public signal realize, and central bank sets its interest rate. Firms set their price $P_{i,t}$ given their information, and (credibly) commit to satisfy the demand at the chosen prices. Households also decide their consumption $(C_{i,t}, C_t)$, labor supply L_t and demand for bonds, B_t, given their information. In stage 2, firms hire labor $L_{i,t}$ to produce and deliver the demanded quantity, $Y_{i,t} = C_{i,t}$. Wages adjust to ensure that labor market clears, $L_t = \int_0^1 L_{i,t} di$. Given the interest rate, the bond net supply is zero, $B_t = 0$.

2.1 Log-linearized model and solution

In the model, real variables are stationary, whereas nominal variables are not. Due to the inflation target exhibiting a unit root, the growth rates of nominal variables are non-stationary. To address this, the literature following Erceg and Levin (2003) typically assumes that the target follows an

AR(1) process with a parameter very close to one, which remains technically stationary.

In contrast, I assume a unit-root process for the inflation target and solve the model by stationarizing the nominal variables as deviations from the target, then working with this stationary version of the model. The approach is analogous to the literature that models TFP shocks as unit-root processes.

Detrending by the inflation target is similar to Cogley and Sbordone (2008), but they treat the target as an unobserved drifting parameter following a random walk and agents from expectations as in the learning literature (see Evans and Honkapohja; 2009).[8] Instead, I keep the signal extraction approach from Erceg and Levin (2003).

For linearization purposes, inflation is detrended as $\hat{\Pi}_t = \frac{\Pi_t}{\bar{\Pi}_t}$, such that $\hat{\Pi}_{ss} = 1$. Therefore, its log-linear deviation from steady-state is given by $\hat{\pi}_t = log(\hat{\Pi}_t) - log(\hat{\Pi}_{ss}) = log(\hat{\Pi}_t)$. The same applies for $\hat{r}_t = log(\hat{R}_t) - log(\hat{R}_{ss})$, where $\hat{R}_t = \frac{R_t}{\bar{\Pi}_t}$ and $\hat{R}_{ss} = 1/\beta$.

Thus, uppercase variables with hats denotes nominal variables detrend by target whereas lowercase variables denote the log-linear deviation from the steady-state.

Equilibrium

Despite the unit root in the inflation target, the model is quite standard. In the following, I present the equations in terms of actual inflation and nominal rates, instead of their stationary counterpart. Appendices A and B have the full derivation of stationary log-linearized equations (deviations from target) and how to recover the solution in terms of the non-stationary nominal variables.

The demand side of the model is given by the standard log-linearized Taylor rule and Euler equation below:

$$\tilde{r}_t = \bar{\pi}_t + \phi_\pi(\pi_t - \bar{\pi}_t) + \phi_y y_t + \eta^r_t \tag{11}$$

$$y_t = E_t\left[y_{t+1}\right] - (\tilde{r}_t - E_t\left[\pi_{t+1}\right]) \tag{12}$$

where $\tilde{r}_t = r_t - \rho$.

The log-linearized version of the price dynamics, indexation rule and optimal price are combined to derive a version of the New Keynesian Phillips Curve (NKPC) given by

$$\pi_t - \upsilon_t = \kappa y_t + \beta E_t\left[\pi_{t+1} - \upsilon_{t+1}\right] \tag{13}$$

where $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)\Psi}{\theta}$ is the slope of the NKPC and $\Psi \equiv \frac{1+\varphi}{\alpha+(1-\alpha)\epsilon}$ is a parameter that encompasses

[8] Their treatment of the target combined with partial indexation to past inflation implies that the coefficients of the New Keynesian Phillips Curve vary with the target. The assumption regarding the target and indexation rule used here shuts down this possibility.

the effects of output on wages via labor supply and the aggregation of individual marginal costs. υ_t follows the log-linearized version of the indexation rule (8) given by

$$\upsilon_t = \iota\pi_{t-1} + (1-\iota)E_t[\bar{\pi}_t] \tag{14}$$

In absence of inflation target changes ($\bar{\pi}_t = 0$ for all t), the equation can be further simplified to the "hybrid NKPC" with backward and forward terms: $\pi_t = \frac{\iota\pi_{t-1}+\kappa y_t+\beta\mathbb{E}_t[\pi_{t+1}]}{1+\beta\iota}$ with $\mathbb{E}_t[\cdot]$ denoting the expectation under full information.

When indexation is fully on inflation target expectations ($\iota = 0$), we have an NKPC on the expected inflation deviation from the target:

$$\pi_t - E_t[\bar{\pi}_t] = \kappa y_t + \beta E_t\left[\pi_{t+1} - \bar{\pi}_{t+1}\right].$$

Information and signal extraction

Firms and households do not observe the inflation target or the monetary shock. Agents infer about the inflation target through central bank decisions. They know the Taylor rule (1) and observe the interest rate r_t. Thus, the interest rate serves as a public signal.

Specifically, since agents observe inflation and output, they can compute the implied shock beyond what is expected from the rule (11) such that

$$r_t^{exo} \equiv \tilde{r}_t - \phi_\pi\pi_t - \phi_y y_t = -(\phi_\pi - 1)\bar{\pi}_t + \eta_t^r. \tag{15}$$

They observe this compound shock but cannot disentangle if the source is a monetary shock or a permanent fall in the inflation target.

In the following, I refer to this baseline as the "Opacity" case, i.e., when the central bank does not communicate about the target. Thus, agent's information set is given by[9]

$$\mathcal{I}_t^O = \{r_\tau^{exo} | \tau \leq t\}.$$

Under IT, agents also observe the public signal (4), implying the information set

$$\mathcal{I}_t^{IT} = \{r_\tau^{exo}, s_\tau^{IT} | \tau \leq t\}.$$

In other words, agents update their current view about the target by observing central banks'

[9]Inflation and output provide information about the monetary shock and target only via the monetary rule (11). Since information is common, the expectations do not provide any information about shocks. Thus, the Euler equation (12) implies that y_t has the same information content as i_t and the Philipps Curve (13) implies that π_t has the same information than y_t.

action, $\tilde{r}_t$ (r_t^{exo}), under Opacity. Under IT, they update both via its actions and communication, s_t^{IT}.

Since shocks and signals are normally distributed and the model is linear, rational expectations can be computed by the Kalman Filter. See details in Appendix C.

3 Disinflation policies

This section compares the impulse response functions (IRFs) from CT and gradual disinflation policies. First, it sets a baseline by showing the full credibility case. Then, it shows results under imperfect credibility – Opacity and IT.

The parameters are calibrated using standard values. The discount factor, $\beta = 0.99$, is such that the annual real interest rate is approximately 4%. The probability of not optimizing prices, $\theta = 2/3$, is set such that firms keep prices fixed by 3 quarters on average. The inverse Frisch elasticity is set to be consistent with the macro literature ($\varphi = 2.5$).

Moreover, the elasticity of substitution is chosen such that the firm's mark-up in the steady-state is 11% ($\varepsilon = 10$), following Woodford (2003). The labor share, α, is calibrated to 2/3.

For the monetary policy rule, I use standard values of $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$. I consider a monetary shock with small persistence $\rho_r = 0.5$ and volatility $\sigma_r = 0.15$. The latter is similar to the same choice for prior mean by Del Negro and Eusepi (2011) for the US economy. It generates a Kalman gain for the inflation target similar to the one calibrated by Erceg and Levin (2003) for the Volcker disinflation in the US.[10]

For gradualism, I set $\rho_\pi = 0.90$ and $\sigma_\pi = 0.025$ using the following strategy. First I compute the interpolated quarterly inflation target from Figure 1 for each country.[11] Then, I estimate ARIMA(1,1,0) for each country and take the median value.[12] Panel (a) from Figure 2 shows the point estimates of (ρ_π, σ_π) for all countries and the pair of calibrated values in dashed lines.

For the CT disinflation, I compute the standard deviation of the first difference of the same series for each country and take the median, which leads to $\sigma_\pi = 0.0525$.[13] Moreover, $\rho_\pi = 0.0$ by definition.

Measuring central bank's credibility is a daunting task. To find a calibration consistent with

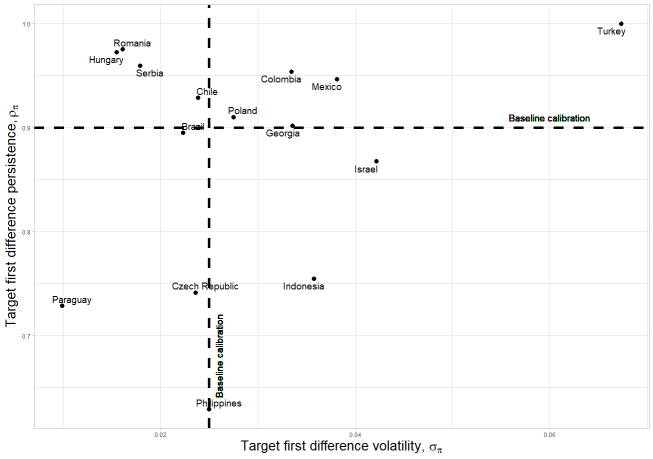
[10] Erceg and Levin (2003) calibrates σ_r to get a Kalman gain of 0.13. In this model, an analogous measure is $-(\phi_\pi - 1)\bar{k}_{\bar{\pi},\eta}$, where $\bar{k}_{\bar{\pi},r}$ is the Kalman gain from the inflation target with respect to the interest rate compound shock (15) in the CT policy under Opacity. This value is 0.1663 under the baseline calibration.

[11] I construct a quarterly series applying the respective target for the year to each quarter of that year. Then, I compute the 4-quarter rolling average to smooth the change over quarters and preserve the annual target value at the last quarter.

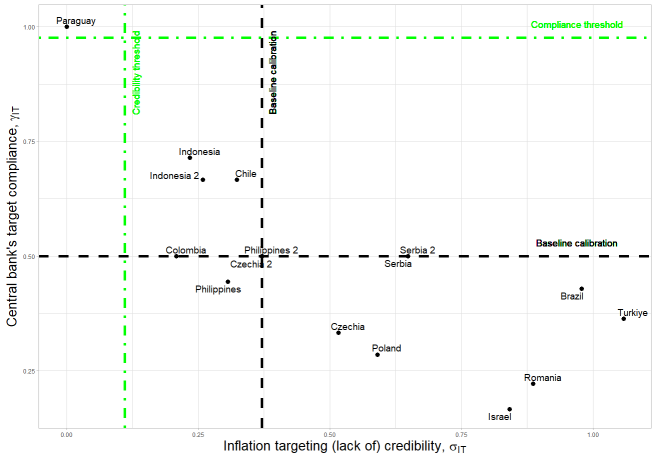
[12] The exact value for ρ_π is 0.91.

[13] Note that the unconditional standard deviations of the target process for both policies are quite similar ($0.025/\sqrt{1-0.9^2} = 0.0574$ under gradualism).

Figure 2: Calibration of key parameters



(a) Target process point estimates under gradualism



(b) Target compliance and credibility degree

the model, note that the implied confidence interval around the actual target based solely on the information from the public signal (4) is given by

$$b_{IT}/4 = |z_{(1-\gamma_{IT})/2}|\sigma_{IT}, \tag{16}$$

where $z_{(1-\gamma_{IT})/2}$ is the quantile of the standard normal with confidence level γ_{IT} and $b_{IT}/4$ is the tolerance band in quarterly rate. Thus, σ_{IT} is higher when the band is larger and the confidence level is lower.

Therefore, I consider measures for b_{IT} and γ_{IT} to compute the implied σ_{IT}. Most countries in Figure 1 adopt target tolerance bands.[14] The median annual band around the target is 1%.

To find a reasonable proxy for γ_{IT}, I use a simple measure of inflation targeting compliance during the disinflation episodes. Specifically, I compute the share of years that inflation was within the target tolerance bands during the disinflation period.

Note that the details of each country's IT regime differ substantially in terms of target horizon, inflation measure, penalties when not complying, and other relevant aspects. Treating them as homogeneous is only for tractability.

The y-axis from panel (b) of Figure 2 shows the values of the central bank compliance for all disinflation episodes.[15] The median value is 50% (horizontal black dashed line), i.e., the median central bank in the sample had inflation is within the tolerance bands half of the time during the disinflation episodes. Thus, I set $\gamma_{IT} = 0.5$. The x-axis of panel (b) of Figure 2 shows the σ_{IT} for each country and the calibrated value ($\sigma_{IT} = 0.37$) implied by $b_{IT} = 1$ and $\gamma_{IT} = 0.5$ in the

[14] The exception is Georgia.

[15] Czechia, Indonesia, Philippines and Serbia have two disinflation episodes shown in the shaded areas in Figure 1. Brazil's second disinflation episode was discarded as it started at the end of the sample.

vertical dashed line.

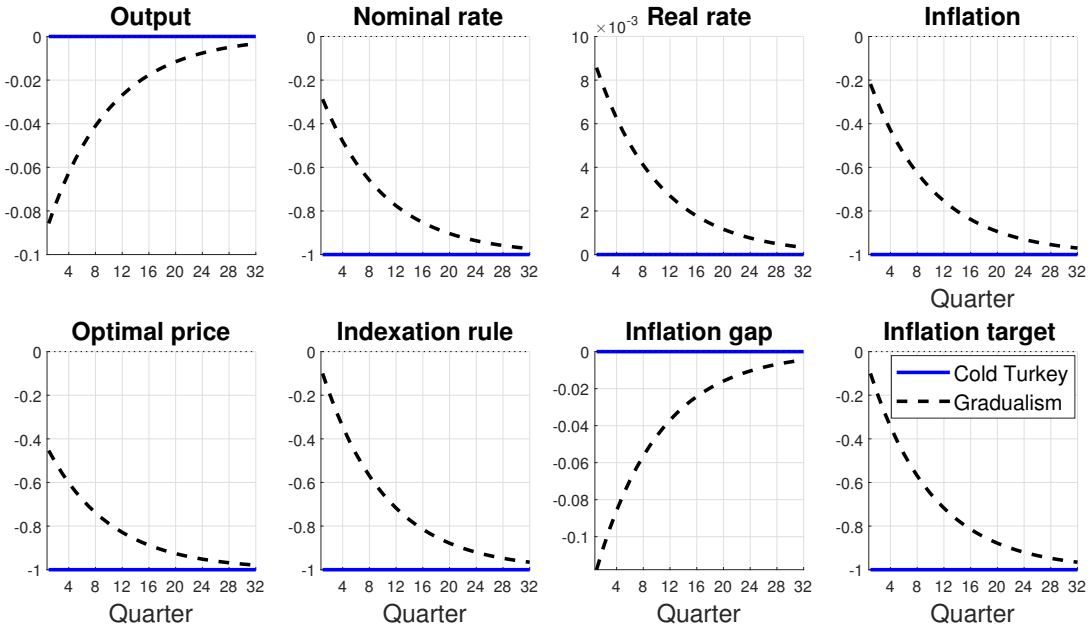
A disinflation policy is a permanent decrease in the inflation target. I compute the IRF after a shock with size $\varepsilon_t^\pi = -0.25(1-\rho_\pi)$ to compare both policies generating the same final inflation target. Under CT ($\rho_\pi = 0$) is an immediate 1% drop in the annual inflation target, whereas the gradual policy ($\rho_\pi = 0.9$) reaches the same new target over time.

3.1 Disinflation policies under full credibility

As a baseline, this section uses full indexation to the expectations of the inflation target, $\iota = 0$. This helps to focus on the channel related to credibility and avoid well-known disinflation costs due to indexation to past inflation. As it will be clear, this is a useful benchmark as it generates as it zero disinflation costs under full credibility. Section D discusses the case with full indexation to past inflation.

Figure 3 shows the impulse response functions (IRFs) from a 1% permanent decline in the inflation target for both policies.

Figure 3: "Cold turkey" vs gradual disinflation under full information



Note: All impulse response functions are multiplied by 4 to get annualized changes in percentage points.

Under full credibility, agents know about the change and trust the new target. It is equivalent to the Full Information (FI) case. Then, firms that do not optimize adjust prices by the target. In response, firms that can optimize also change prices aligned with the new target. Thus, inflation

(and expectations) perfectly track the new target, and the nominal rate instantly drops by the same amount as the target. The real interest rate stays at its steady-state level, generating zero output costs.

Now consider the black dashed lines that represent the disinflation under gradualism. Under the baseline calibration, the target decreases by 0.13% on impact and reaches 0.5% around six quarters afterward. It takes more than ten years to implement the 1% change fully. 95% of the decline in the target is done after seven years (the median disinflation duration in the data).

Firms that do not optimize gradually adjust prices by the current inflation target. Optimizing firms push price increases at a lower rate than the target fall as they anticipate that the target will decrease over time. This generates higher profits from higher demand due to better relative prices. This implies a slight negative inflation gap, which softens the central bank's response and leads to a small increase in the real rate. There are small costs in terms of output, but higher than the zero cost benchmark from CT policy.

Figure A1 from Appendix D shows the results under full indexation to past inflation ($\iota = 1$). In that case, the CT becomes more costly than gradualism. There is a major inflation gap as inflation adjusts slowly towards the target because of indexation. The central bank has to push the nominal rate in a relatively higher path, leading to a substantial hike in the real rate. This leads to a larger recession than the gradual policy. The combination of indexation with nominal rigidity points towards gradualism as in Taylor (1983). In the following, I shut down this channel to focus on the credibility issues as an explanation of gradualism in policy.

3.2 Disinflation policies under Opacity and inflation targeting

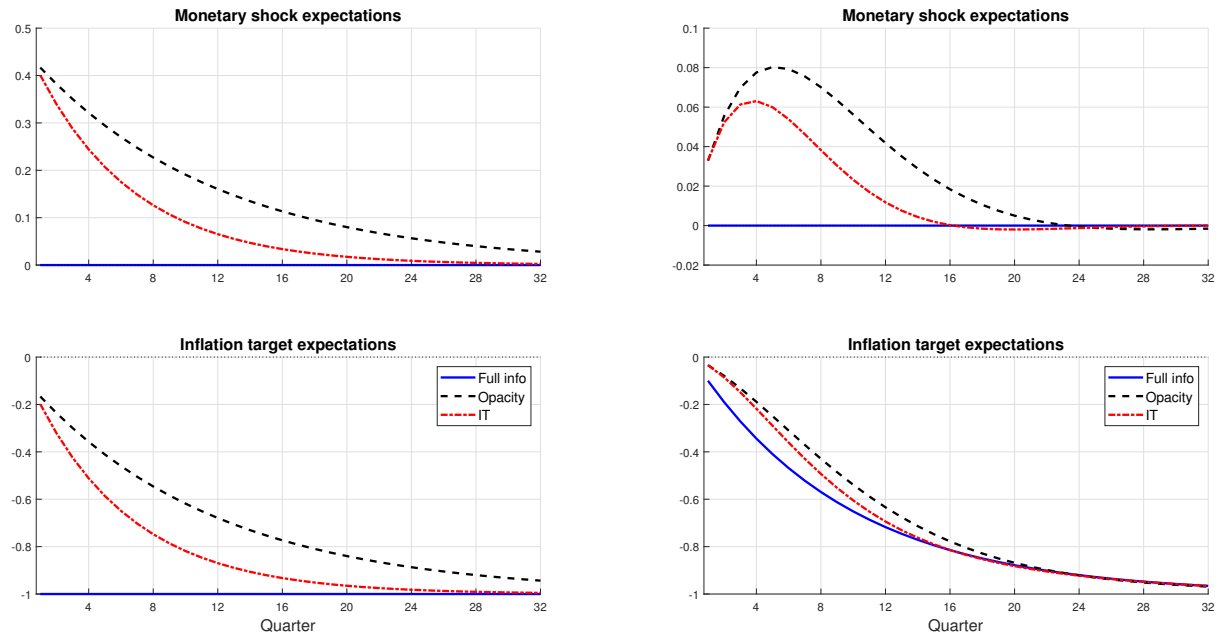
This section shows the results under imperfect credibility. The key difference from the full credibility benchmark is that agents must form expectations about the true inflation target. Figure 4 shows the learning process for each disinflation policy.

Consider the Opacity case (black dashed line). Agents observe the path of the interest rate, inflation and output. Thus, acknowledge the central bank's tightening, $r_t^{exo} > 0$, via equation (15). They do not know the source of the tightening: is it a temporary positive monetary shock or a permanent fall in the inflation target?

Panel (a) shows the rational expectations when the central bank is pursuing a CT disinflation. Agents expect a positive monetary shock of 0.4% and a fall of roughly 0.2% in the inflation target on impact, despite the actual target immediately dropping by 1%. As shown in the bottom plot, expectations about the target slowly evolve. Under the baseline calibration, learning about the target takes more than ten years (and most of the learning is already done after eight years).

The same policy has a faster learning process under IT (red dash-dotted line). This happens because the public signal (4) conveys additional information about the target, accelerating the

Figure 4: Learning in each disinflation policy



(a) "Cold turkey" ($\rho_\pi = 0.0$)

(b) Gradualism ($\rho_\pi = 0.9$)

Note: Target expectations impulse responses are multiplied by 4 to get annualized changes in percentage points.

learning process. It takes eight years to learn the actual fall of the target (but most of the learning is already done after four years). It is roughly twice as fast as the Opacity case.

Panel (b) shows the same plots under a gradual disinflation policy. As the inflation target decreases gradually, agents' expectations about the target are much closer to the truth than under CT. They are also less confused between shocks since they expect a monetary shock ten times lower (on impact) than under CT.

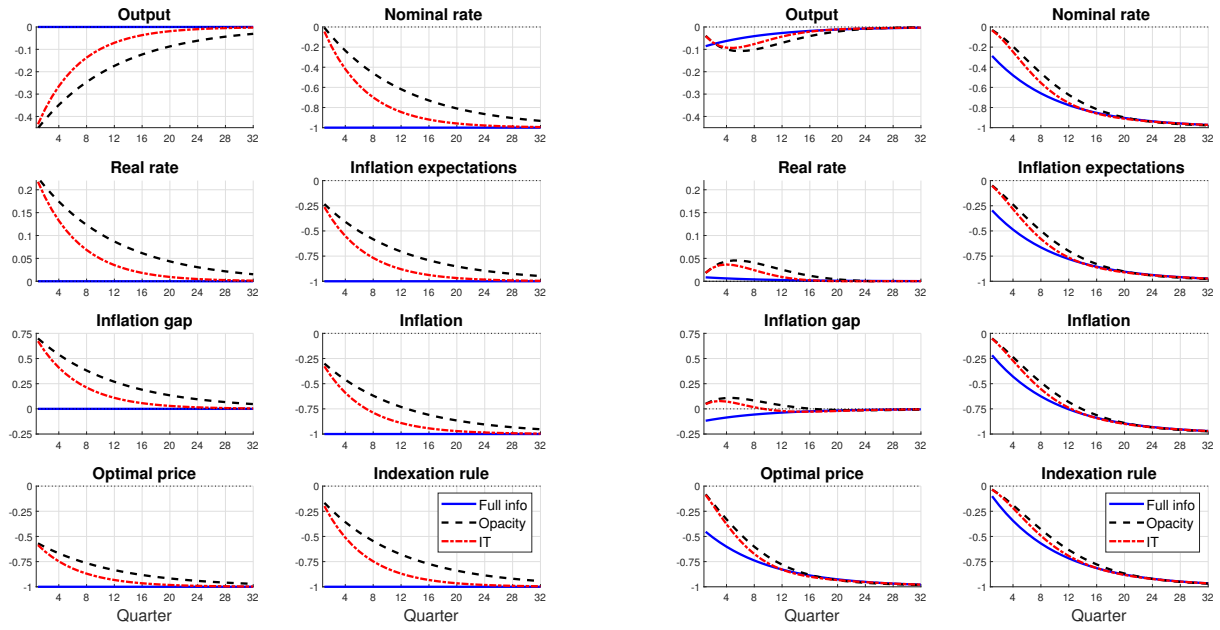
Agents fully learn the actual target after about six years, but their assessment of the target is much more accurate over time. The additional information from the IT regime provides even faster learning but is far more relevant in the CT case.

Therefore, by pursuing a gradual change in the target, expectations about the target are substantially more aligned with the actual target than under CT. This inflation target anchoring is the key mechanism to understand the output costs of both policies.

Figure 5 shows the IRFs for the two alternative disinflation policies. Consider CT disinflation (panel a). Unlike the FI benchmark (solid blue line), there are output costs from a CT disinflation under imperfect credibility.

As firms do not fully incorporate the change in the target, inflation expectations decrease

Figure 5: Disinflation policies under Opacity and IT with target indexation



(a) "Cold turkey" ($\rho_\pi = 0.0$)

(b) Gradualism ($\rho_\pi = 0.9$)

Note: All impulse response functions are multiplied by 4 to get annualized changes in percentage points.

roughly by a quarter of the target's fall on impact. Non-optimizing firms adjust prices according to their expectations about the target whereas optimizing firms decrease the inflation rate in their prices twice as much.

Overall, inflation falls much less than the actual target, i.e., there is a sharp rise in the inflation gap. This prevents the nominal rate from falling at the same pace as the target since the central bank reacts to the inflation gap. This implies a hike in real rates leading to a recession. This dynamics is qualitatively similar for Opacity and IT. The key difference is that agents learn faster about the inflation target when the central bank communicates about the target. Thus, inflation expectations fall faster, leading to lower output costs due to a smaller inflation gap.

Panel (b) from Figure 5 shows the dynamics under gradual disinflation. As discussed above, expectations about the target are much more accurate under gradualism for both Opacity and IT. This implies that inflation is much more aligned with the target. This generates a far weaker rise in the inflation gap, implying much lower real rates and weaker recession than under CT.

Surprisingly, either disinflation under CT or gradualism have similar duration under Opacity. In other words, it takes a similar time to fully implement the drop in *actual inflation* and its associated recession. Without target announcements, the learning process takes several years

under both strategies. Under IT, gradualism leads to a longer disinflation episode than under CT. However, given the major contrast in target paths, the disinflation duration difference among strategies is quite small. This underscores the empirical challenge of inferring disinflation strategies from inflation dynamics as in the standard approach from Ball (1994b).

Under IT, target expectations are more anchored toward the target than under Opacity. Thus, the inflation gap is even smaller, which allows the central bank to tighten real rates by less, leading to lower output costs.

For comparing both policies, I use the sacrifice ratio (SR), which computes the cumulative loss of output during the disinflation episode divided by the decrease in inflation. Since the model is log-linearized and IRFs are from a 1% decrease in the inflation target (and thus, inflation), we can compute the sacrifice ratio by

$$SR = -\sum_{t=0}^{T^*} y_t,$$

where T^* is the period in which inflation converges to its lower steady-state and output gap stabilizes back to zero.

Figure 6 summarizes how these differences in dynamics translate in terms of SR. The solid blue line and the dashed black line describe how costly the disinflation episode is as a function of IT credibility for CT and gradualism, respectively.

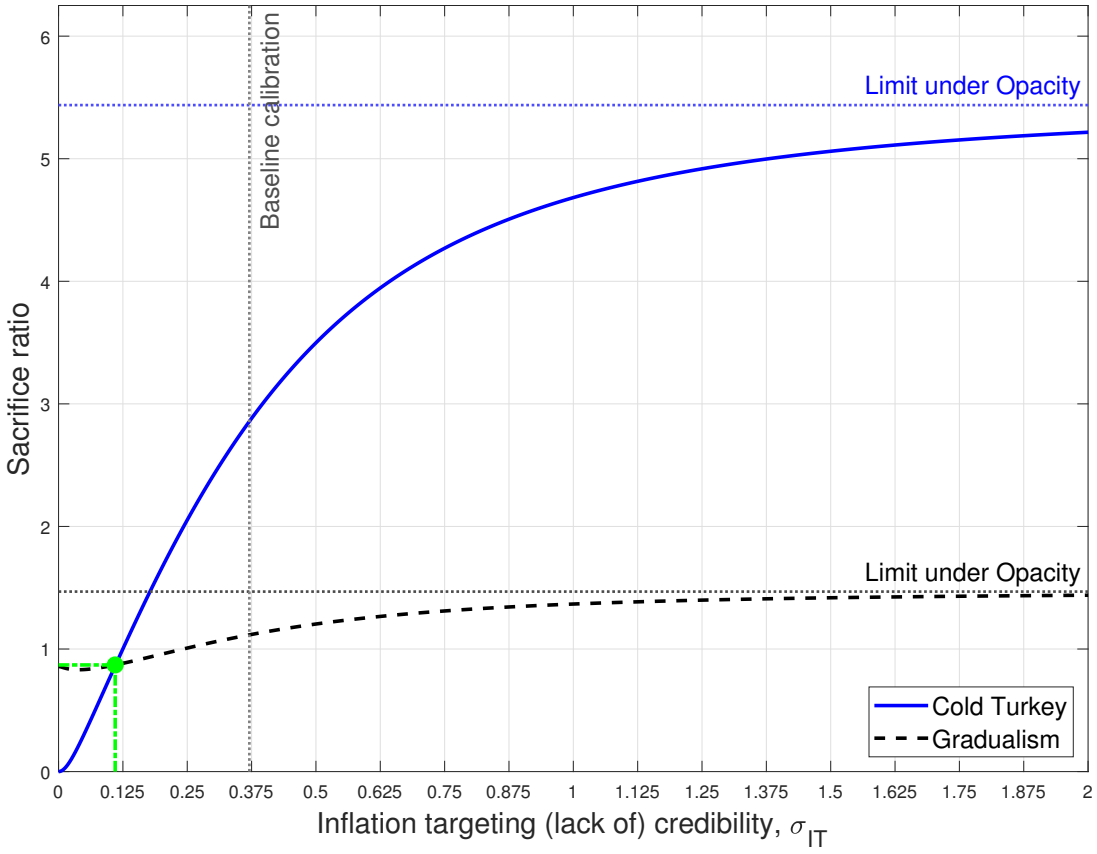
When $\sigma_{IT} = 0$, the central bank announces the inflation target with full credibility. Thus, CT has no disinflation costs and gradualism has an SR slightly lower than 1, consistent with the description in section 3.1. As $\sigma_{IT} \to \infty$, the IT converges to the Opacity case (dotted horizontal lines), where agents learn about the target only by the central bank's actions.

In the baseline calibration, the SR under Opacity for the gradual policy is roughly 1.5; under CT it is more than 3 times higher, reaching 5.37. The vertical black dotted line on $\sigma_{IT} = 0.37$ represents the calibrated value IT credibility. This line crosses a region where the gradual policy has economically significantly lower SR than under CT. Specifically, a gradual disinflation has an SR of 1.1, whereas it implies a 2.8 under CT.

Comparing the solid blue with the dashed black lines reveals two key results. First, it confirms that CT disinflation benefits substantially more than gradualism as credibility increases. Second, the SR range as a function of the credibility under gradualism is very tight (from 0.85 to 1.46) whereas CT has a quite wide range (from 0 to 5.37). Therefore, credibility has a potentially larger impact on countries pursuing CT than a gradual policy. However, for a reasonable degree of credibility, the gradual policy yields a smaller inflation-output trade-off.

Therefore, the baseline calibration suggests that IT countries prefer to pursue gradual disinfla-

Figure 6: Sacrifice ratio depending on IT credibility



Note: The dotted blue and blue black correspond to the Opacity sacrifice ratio under both policies, which is equivalent to $\sigma_{IT} \to \infty$. The vertical black dotted line is the baseline calibration, $\sigma_{IT} = 0.37$. The green dot is the credibility threshold (defined below in the text).

tion policies as in Figure 1 if the sacrifice ratio is a key determinant when deciding the disinflation strategy.

Appendix D shows the same results under full indexation to past inflation ($\iota = 1$). In that case, gradualism is even more beneficial in terms of SR. This happens because non-optimizing firms now change prices by past inflation, which is higher than the target expectation (the baseline indexation scheme). This slows down the inflation adjustment towards the new target, increasing output costs (see Figure A2). This issue is more pronounced in the CT disinflation as it immediately decreases the target.

Under indexation to past inflation, gradualism has a lower SR than CT for any degree of credibility (see Figure A3). Intuitively, even under full credibility, the slow response of inflation due to the indexation implies a large positive inflation gap under the CT strategy. This leads to

higher costs in output than the gradual policy that better aligns the target with the inflation path.

3.3 Inspecting the mechanism: inflation target expectations

The benchmark calibration suggests that gradualism generates lower sacrifice ratios than "Cold Turkey". The key driver of this result is the fact that the gradual policy induces a more accurate learning process.

In this section, I evaluate if this result is robust to changes in the key parameters determining the signal extraction. Recall that agents use two signals given by

$$r_t^{exo} = -(\phi_\pi - 1)\bar{\pi}_t + \eta_t^r$$
$$s_t = \bar{\pi}_t + u_t.$$

for $\phi_\pi > 1$.

When learning about the target, the monetary shock works as a confounding factor in the policy rule. Thus, the learning process depends on (ρ_r, σ_r). Moreover, if the central bank is more responsive to inflation (higher ϕ_π), it also reacts more to changes in the target. Thus, the central bank reveals more about the target when it is more hawkish. Finally, when the central bank is more credible, the public signal from the IT regime is more informative.

Figure 7 shows how agents learn about the target depending on those parameters for both disinflation policies under IT. The black dashed line depicts the baseline calibration. Red dotted lines represent high parameter values while gray dot-dashed lines are small ones.

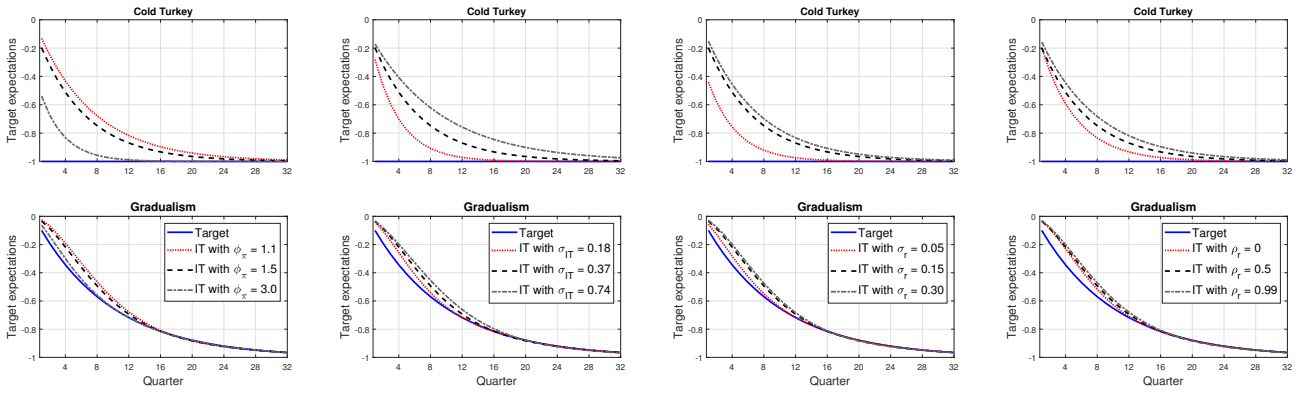
In general, the learning process under gradualism is similar when varying all parameters. While there is some gain in terms of how fast agents learn about the target, the gap between the expected and actual target is small irrespective of the parameters used.

Learning under CT is more sensitive to changes in parameters. As the central bank is more responsive to inflation and more credible the learning about the target might change substantially. For instance, for high response to inflation ($\phi_\pi = 3$), agents fully learn about the target after roughly three years (two times faster than the baseline). For a credibility degree two times higher ($\sigma_{IT} = 0.18$), learning about the target takes roughly four years.

Regarding the two parameters related to the monetary shock, the monetary shock volatility matters more than the persistence for the target expectations.

The results above are for the IT regime. Figure A6 in Appendix E shows the same results under Opacity. In general, learning is more sensitive to changes in the parameters of the Taylor rule and monetary shocks, particularly for CT. This happens because agents form expectations using the interest rate only. Under IT, if the interest rate becomes less informative, this is balanced with the information from the central bank communication.

Figure 7: Inflation target expectations as function of key parameters



(a) Taylor rule (b) IT Credibility (c) MP shock volatility (d) MP shock persistence

Note: Target expectations impulse responses are multiplied by 4 to get annualized changes in percentage points. CT disinflation uses $\rho_\pi = 0$ and $\sigma_\pi = 0.0525$ whereas Gradualism uses $\rho_\pi = 0.9$ and $\sigma_\pi = 0.025$.

Changing key parameters does not change key the mechanism: target expectations are more anchored in a gradual than an abrupt change in the target.

3.4 Credibility threshold

Despite gradualism implying a better SR under the baseline calibration, Figure 6 also shows the CT policy can be more beneficial in terms of SR. Under the baseline calibration, the vertical dot-dashed green line represents the credibility level such that gradualism and CT have the same sacrifice ratio. I call this point the credibility threshold, σ^*_{IT}. For $\sigma_{IT} < \sigma^*_{IT}$, the IT regime is sufficiently credible such that CT yields a lower SR than gradualism. The opposite holds for $\sigma_{IT} > \sigma^*_{IT}$.

The case where $\sigma_{IT} < \sigma^*_{IT}$ is consistent with the Sargent (1982) case for CT when the policy is credible. The key difference is that the author studies substantial changes in the policy regime. The idea is that drastic changes in the policy regime would generate credibility and could immediately change inflation expectations without generating output costs.

In the baseline calibration, $\sigma^*_{IT} = 0.11$, which is less than a third of the calibrated value for σ_{IT}. According to equation (16), the credibility threshold implies either a lower tolerance band or higher confidence for the same band.

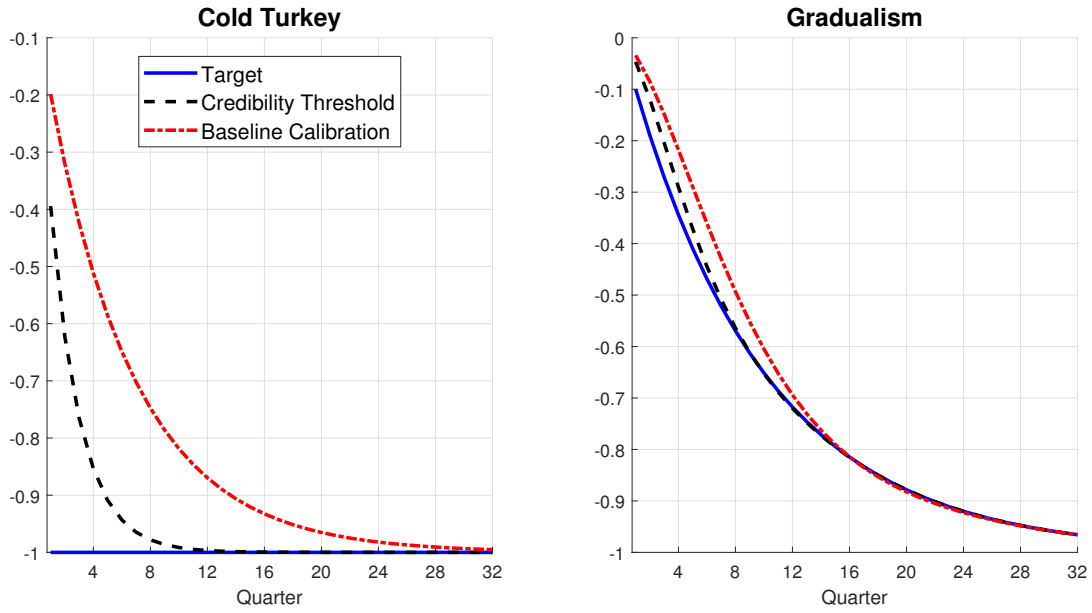
Keeping the calibrated γ_{IT}, one can find a band consistent with the credibility threshold and the baseline compliance, which I denote b^*_{IT}. This implies $b^*_{IT} = 0.3$. Analogously, keeping the calibrated b_{IT}, one can compute the central bank's threshold compliance, $\gamma^*_{IT} = 0.976$.

The values of σ^*_{IT} and γ^*_{IT} given the baseline calibration are the green dot-dashed vertical and

horizontal lines in Panel (b) of Figure 2. Among the sixteen disinflation episodes (12 countries), only Paraguay has measures lower than the threshold. Consistently with the model intuition, Paraguay has a much faster disinflation than the median country. It took three years to decrease its target from 5% to 4%.

Figure 8 shows the learning under the credibility threshold and compare with the baseline calibration for both policies.

Figure 8: Inflation target anchoring: Baseline vs Credibility Threshold



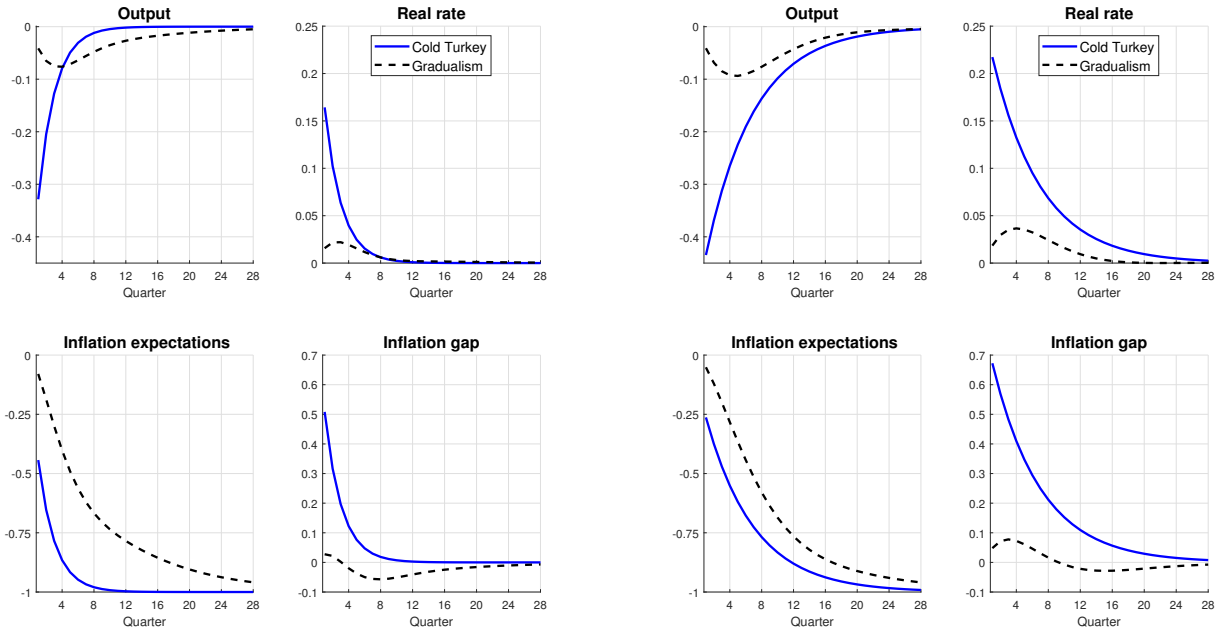
Note: Target expectations impulse responses are multiplied by 4 to get annualized changes in percentage points.

For the CT policy, under the credibility threshold agents expectations fall twice as much than the baseline and fully incorporate the fall in the target in four years (in comparison with eight years in the baseline). Most of the fall is acknowledged within one to two years. For gradualism, learning speed up by one year, but still quite similar to the baseline. Thus, CT benefits much more credibility than gradualism.

Figure 9 compares the IRFs under the credibility threshold (panel a) and the baseline calibration (panel b). By comparing the gradual disinflation under the two different credibility levels, it is clear that their difference is much smaller than for their CT counterparts. This is consistent with the difference in learning discussed above and that the SR does not vary much with the credibility level in Figure 6.

Inflation expectations fall much faster for the CT disinflation than the gradual one (panel a). At the same time, this difference is much less pronounced under the baseline calibration (panel b),

Figure 9: IRFs depending on IT credibility



(a) Under the credibility threshold ($\sigma_{IT} = 0.11$) (b) Under baseline calibration ($\sigma_{IT} = 0.37$)

Note: All impulse response functions are multiplied by 4 to get annualized changes in percentage points.

in line with the target expectations in Figure 8.

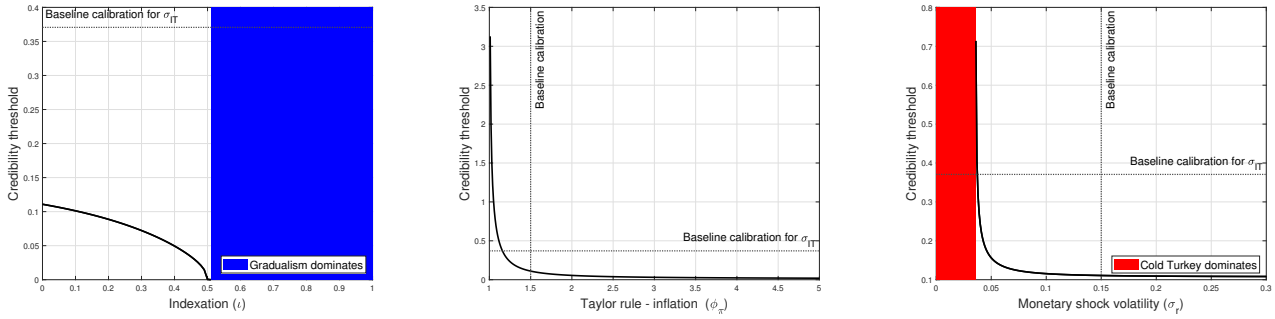
In the baseline calibration, recession after CT is deeper and longer-lasting than under gradualism. With credibility equal to the threshold, the CT recession is weaker and sufficiently fast such that the cumulative loss in output is equal to the even smaller but longer recession under gradualism.

For credibility levels higher than that, the fast learning about the target pushes inflation expectations down, which mitigates the recession and accelerates the recovery. In that case, CT disinflation has better inflation-output than gradualism.

Does the credibility threshold, σ^*_{IT}, always exist? No. In general, its existence depends on the parameters. For instance, as discussed in section 3.2, CT has higher disinflation costs than gradualism for any degree of credibility. Figure A3 from Appendix D provides an example when such threshold does not exist for the case when agents have full indexation to inflation. In that case, independently of the credibility degree, gradualism yields lower SR than CT.

Now we explore how sensitive the credibility threshold is to changes in parameters. Figure 10 shows how the credibility threshold depends on the key parameters of the model, keeping all other parameters fixed at the baseline calibration.

Figure 10: Credibility threshold as function of key parameters



(a) Indexation to past inflation (b) Taylor rule (c) MP shock volatility

Panel (a) shows that as the indexation parameter, ι, increases the credibility threshold decreases. For a sufficiently high indexation, gradualism dominates CT in terms of sacrifice ratio (blue-shaded area). Figure 6 depicts the credibility threshold in the extreme where $\iota = 0$.[16]

Intuitively, as the economy has more indexation to past inflation, gradualism has a relatively lower SR than CT for a given credibility level. Then, a central bank must be more credible to justify pursuing a CT policy.

Panel (b) shows that the credibility threshold is decreasing in the policy rule response to inflation. Interestingly, the credibility threshold becomes very large in comparison with the baseline calibration for σ_{IT} when ϕ_π becomes close to one. In other words, if the central bank has a sufficiently weak response to inflation the CT disinflation has lower disinflation costs even under the baseline calibration for credibility. The next section further explores this point.

Finally, panel (c) shows that the credibility threshold is decreasing with monetary shock volatility. As the volatility increases, the interest rate becomes less informative about the target. As shown in panel (c) from Figure 4, it affects more the learning under CT than gradualism. Then, it requires more credibility for being indifferent between CT than gradualism.

On the other hand, for a sufficiently low volatile environment for the monetary shock, CT dominates gradualism for any degree of credibility as the target becomes easily inferred by monetary policy actions. This is far from the calibrated value for the monetary shock, whose calibration is based on the US. As monetary policy tends to be less predictable in emerging countries, this value likely is a lower bound for σ_r.

Even considering reasonable changes in the parameters does not change the key message remains: the credibility degree from emerging IT countries seems to be far from the level required to prefer choosing a CT disinflation if the key objective function is the sacrifice ratio.

[16]Figure A3 highlights that there is no credibility threshold for $\iota = 1$.

3.5 Does a Hawk generates greater disinflation costs?

A central bank may choose how "Hawkish" towards inflation deviations from the target in addition to the disinflation strategy. This section explores this route.

The response to inflation in the monetary rule has two contrasting channels on the inflation-output trade-off in the FI model. First, as the central bank becomes more hawkish, it raises real interest rates more strongly when the inflation gap increases, which leads to a deeper recession,*for a given inflation expectation*.

Second, as agents know that the central bank is more reactive to inflation, inflation expectations fall more strongly for the same fall in the target. This leads to a smaller inflation gap such that the central bank does not have to increase real rates by as much. The first is a demand channel that increases disinflation costs whereas the second is an expectation channel that mitigates them. How strong those channels are depends on calibration.

Figure 11 shows the sacrifice ratio as a function of ϕ_π for both disinflation strategies. Consider the FI benchmark (panel a). A CT disinflation has no costs independently of the monetary rule response to inflation. Intuitively, as inflation perfectly tracks the new target in equilibrium, there is no role for a stronger response to the inflation gap. For gradualism, there is a small role for the demand channel as the inflation gap is muted and negative. Thus, the expectational channel dominates: the SR decreases monotonically with ϕ_π.

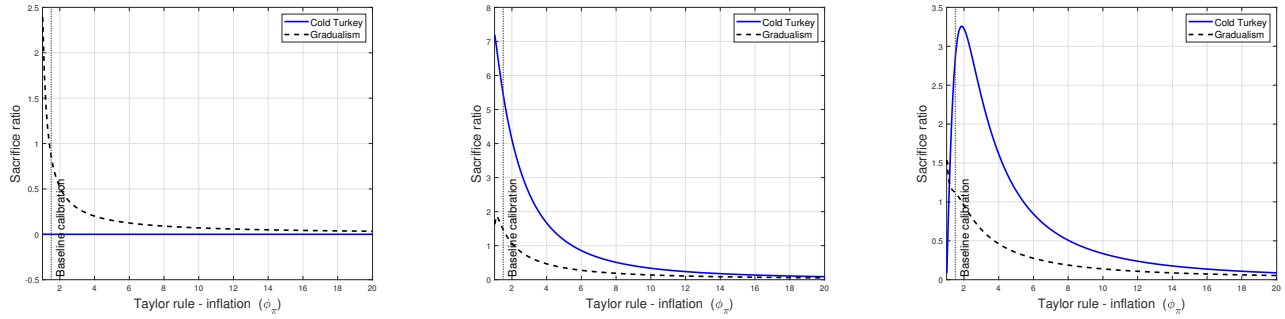
In general, imperfect credibility mitigates the expectation channel: inflation expectations evolve slowly as agents do not fully incorporate the fall in the inflation target. However, the response to inflation has an additional channel when there is imperfect credibility.

As the central bank becomes more Hawkish it also responds more to changes in the inflation target. Thus, the interest rate becomes a more informative signal about the target as shown in Figure 7. Therefore, agents learn faster about the target and thus, inflation expectations adjust accordingly. I refer as a learning channel. This channel is also studied by Bordo et al. (2017) in a fully-fledged DSGE model, but abstract from gradual disinflation and public announcements about the target.

Under Opacity (panel b), agents learn about the target only through the compound shock from the Taylor rule (15). Thus, if ϕ_π is very close to 1, the interest rate provides little information about the target, which raises the SR. This is true for both CT and gradualism, but this channel is more important for the former as the inflation gap increases substantially when learning is slow. As ϕ_π increases, learning about the target improves, enhancing the expectational channel.

Under IT, the expectational channel also dominates for gradualism. The key difference comes from the CT disinflation. In that case, SR is non-monotonic in ϕ_π. For a sufficiently low ϕ_π (lower than 1.15, keeping all other parameters fixed), the CT disinflation can generate even lower SR than gradualism. The sacrifice quickly increases to its maximum (roughly 3.25 for $\phi_\pi = 1.75$) and

Figure 11: Sacrifice ratio depending on central bank's response to inflation



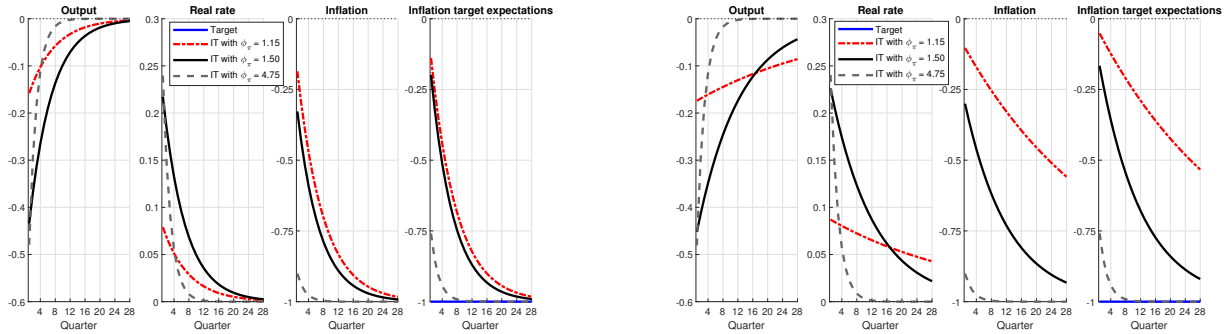
(a) Full information (b) Opacity (c) Inflation targeting

Note: The vertical black dotted line is the baseline calibration, $\phi_\pi = 1.5$.

then monotonically decreases with ϕ_π. Interestingly, the baseline calibration ($\phi_\pi = 1.5$) has an SR close to the maximum. Moreover, the non-monotonicity happens within a range of reasonable values for ϕ_π.

Figure 12 sheds some light on the relative importance of the channels and how they explain the non-monotonicity. It presents the IRFs for the CT policy under IT for $\phi_\pi = 1.15$ (when CT and gradualism have the same SR) and $\phi_\pi = 4.75$ (has roughly the same SR as $\phi_\pi = 1.15$ for the CT disinflation). It also compares with the baseline calibration ($\phi_\pi = 1.5$), which has a higher SR than both under CT.

Figure 12: IRFs under CT disinflation for different responses to inflation



(a) Inflation targeting (b) Opacity

Note: All impulse response functions are multiplied by 4 to get annualized changes in percentage points.

Under IT (panel a), agents learn from the central bank communication and the interest rate. For $\phi_\pi = 1.15$ (dot-dashed red line), agents can learn at a similar pace to the baseline (solid black line). This contrasts with the slow learning under Opacity (panel b) for the same degree of

response to inflation.

Thus, despite a sizable inflation gap, the real rate rises less than under the baseline as the central bank is less responsive. There is a milder recession and agents eventually learn about the target. In other words, learning and expectational channels are similar to the baseline but there is a weaker demand channel. This explains the rising part of panel (c) in Figure 11.

Now consider the case for $\phi_\pi = 4.75$ (dashed gray line). Since the central bank is so responsive to the inflation gap, agents learn quite fast about the change in the target. While the central bank responds more than 3 times than the baseline, the real raises quite similarly to the baseline on impact and falls very quickly to levels lower than for $\phi_\pi = 1.15$ after about a year. This happens because fast learning about the target triggers a strong inflation expectations response. While there is a steeper recession on impact, it reverts to a weaker recession than for $\phi_\pi = 1.15$ after one year. Those two recessions lead to similar sacrifice ratios.

In other words, learning and expectational channels are quite strong, attenuating the effects of the demand channel. This explains the decreasing part of panel (c) in Figure 11.

Interestingly, those results suggest that a combination of CT strategy with a "Dovish" central bank under IT may generate lower disinflation costs.

However, the results are not robust to the introduction of full indexation to past inflation ($\iota = 1$). As shown in Figure A4 in Appendix D, the SR is higher under CT than gradualism for any reasonable calibration for ϕ_π, even when the parameter is close to one under IT. There is still a region where SR decreases with the response to inflation, but it is much smaller and quickly becomes increasing in ϕ_π in contrast with the baseline. While the learning is the same, the key difference is that the expectation channel is mitigated as firms that do not optimize respond to past inflation. This amplifies the demand channel as the real interest rate rises sharply on the impact, generating a deep recession in the first year (see Figure A5).

Considering all these factors, a gradual policy in terms of change of the target aligned with a Hawk stance on the inflation gap seems to deliver lower disinflation costs.

It is worth comparing the results with the ones presented by Bordo et al. (2017). They use a quantitative NK model with imperfect credibility to study the Volcker disinflation. They find that a more aggressive stance on inflation than the one estimated during the period would have led to lower disinflation costs as inflation expectations would adjust faster due to the learning channel discussed above. Their results are more comparable with the Opacity case under CT from this paper.[17] Results from panel (b) in Figure 11 are consistent with their results as the SR is decreasing in ϕ_π for both policies.[18] Here I emphasize that the communication about the target

[17] Their specification differs from the one used here as they include a smoothing parameter and, more importantly, the intercept varies with the current inflation instead of the current target. The latter may imply a gradual response of the interest rate in response to the target.

[18] For the range of parameters they consider, the SR is decreasing in our model when considering full indexation

can generate benefits as agents do not learn about the target only via interest rate decisions, which attenuates the benefits of being a Hawk from the learning perspective.

Including indexation leads to results consistent with the medium-scale FI model from Ascari and Ropele (2012). The authors find that a more Hawkish central bank ($\phi_\pi = 3$ vs $\phi_\pi = 1.5$) generates a higher sacrifice ratio for the US. In their setting, the real and nominal frictions dampen the expectational channel.

4 Concluding remarks

This paper contributes to the ongoing debate on the effectiveness and trade-offs of "Cold Turkey" and gradual disinflation strategies under inflation targeting frameworks.

By employing an NK model that integrates imperfect credibility and noisy inflation target announcements., I study under which circumstances gradualism is beneficial under inflation targeting. The findings suggest that while CT disinflation can theoretically achieve a lower SR under high credibility, gradual disinflation results in lower costs when credibility is limited, which is likely to be a major issue for emerging market central banks.

Inflation targeting promotes more precise information about the inflation pursued by the central bank, enhancing the learning process and better anchoring the expectations during a disinflation. This allows a more moderate contractionary stance in the monetary policy during the disinflation, which decreases the sacrifice ratio.

Furthermore, this study underscores the role of central banks' responses to the inflation gap in shaping disinflation outcomes. When central banks adopt a stronger stance, it makes policy actions more informative about the true goals of the central bank. The results suggest that the benefit of the inflation expectations channel exceeds the potential costs from the demand channel, particularly when using a gradual policy.

This paper evaluates the trade-offs of disinflation based on the sacrifice ratio. Future research could further explore the optimal disinflation based on social welfare function. Moreover, any strategic interaction between the central bank's communication and actions is abstracted, which may be also an interesting venue.

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to past inflation ($\iota = 1$), but eventually becomes increasing for higher values of ϕ_π. See panel (b) in Figure A4.

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Appendix

A Model and log-linearization

A.1 First-order conditions

Households

The household problem is standard. First, she chooses her optimal allocation of consumption between varieties by maximizing the consumption bundle, C_t, subject to the consumption budget.

Therefore, the demand for each good i is given by

$$C_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} C_t, \tag{17}$$

where the price level of the composite good is given by $P_t = \left(\int_0^1 (P_{i,t})^{(1-\varepsilon)} di\right)^{1/(1-\varepsilon)}$. Second, the household chooses her optimal allocation between bonds, labor, and consumption that maximizes the utility function subject to the household budget constraint.

The first-order conditions for this problem can be written as

$$\frac{W_t}{P_t} = C_t L_t^{\varphi} \tag{18}$$

$$\frac{1}{C_t} = \beta E_t \left[\frac{R_t}{\Pi_{t+1} C_{t+1}}\right] \tag{19}$$

Firms

Substituting the production function (7) into firms problem (10) leads to

$$\max_{P_t^*} E_t \left[\sum_{s=0}^{\infty} (\beta\theta)^s \left(\frac{\Upsilon_{t,t+s} P_t^* Y_{i,t+s|t} - W_{t+s} Y_{i,t+s|t}^{1/\alpha}}{P_{t+s}}\right)\right]. \tag{20}$$

The first order condition is given by

$$E_t \left[\sum_{s=0}^{\infty} (\beta\theta)^s \frac{1}{P_{t+s}} \left(\Upsilon_{t,t+s} Y_{i,t+s|t} - \Upsilon_{t,t+s} P_t^* \varepsilon \frac{Y_{i,t+s|t}}{P_t^*} + W_{t+s} \frac{\varepsilon}{\alpha} \frac{Y_{i,t+s|t}}{P_t^*} \left(Y_{i,t+s|t}\right)^{1/\alpha - 1}\right)\right] = 0$$

which rearranging leads to

$$P_t^* = \frac{E_t \left[\sum_{s=0}^{\infty} (\beta\theta)^s Y_{i,t+s|t} \mu MC_{i,t+s|t}\right]}{\sum_{s=0}^{\infty} (\beta\theta)^s E_t \left[\frac{\Upsilon_{t,t+s} Y_{i,t+s|t}}{P_{t+s}}\right]} \tag{21}$$

where $\mu = (\varepsilon - 1)/\varepsilon$ is the mark-up and $MC_{i,t+s|t} = \frac{W_{t+s}}{P_{t+s}} Y_{i,t+s|t}^{\frac{1-\alpha}{\alpha}}$ is the real marginal cost at period $t+s$ that optimized their prices at period t.

A.2 Detrending and log-linearizing

Since the inflation target has a unit-root, we need to detrend inflation and the nominal interest rate or any growth rate of a nominal variable before log-linearizing.

For linearization purposes, we detrend inflation as $\hat{\Pi}_t = \frac{\Pi_t}{\bar{\Pi}_t}$, in balanced growth path, inflation follows the inflation target and thus $\hat{\Pi}_{ss} = 1$. Variables with 'hats' denotes detrended variables.

Lowercase variables are the log-linear version of the $\hat{\pi}_t = log(\hat{\Pi}_t) - log(\hat{\Pi}_{ss}) = log(\hat{\Pi}_t)$. The same applies for $\hat{r}_t = log(\hat{R}_t) - log(\hat{R}_{ss})$, where $\hat{R}_t = \frac{R_t}{\bar{\Pi}_t}$ and $\hat{R}_{ss} = 1/\beta$.

Then, one can recover the dynamics of inflation by

$$\begin{aligned}\pi_t &= log(\Pi_t) = log(\hat{\Pi}_t) + log(\bar{\Pi}_t) \\ &= \hat{\pi}_t + \bar{\pi}_t\end{aligned}$$

The same idea applies to the nominal interest rate $\tilde{r}_t = r_t - \rho = \hat{r}_t + \bar{\pi}_t$.

Monetary policy

The monetary rule (1) is reproduced below for convenience:

$$r_t = \rho + \bar{\pi}_t + \phi_\pi(\pi_t - \bar{\pi}_t) + \phi_y y_t + \eta_t^r. \tag{22}$$

Since the inflation target has a unit-root, we need to detrend inflation and the nominal interest rate. Thus, the stationary version of the rule is simply

$$\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_y y_t + \eta_t^r. \tag{23}$$

where the left hand side uses $\hat{r}_t = log(\hat{R}_t) - log(\hat{R}_{ss}) = log(R_t) - log(\bar{\Pi}_t) - \rho = r_t - \bar{\pi}_t - \rho$.

Households

The labor supply (18) is straightfoward to log-linearize, which is given by

$$\hat{w}_t = c_t + \varphi l_t. \tag{24}$$

where $\hat{W}_t = \frac{W_t}{P_t}$ is the real wage.

For the Euler equation (19), first it is necessary to stationarize the nominal variables by the

following:

$$\frac{1}{C_t} = \beta E_t \left[\frac{R_t \bar{\Pi}_t \bar{\Pi}_{t+1}}{\bar{\Pi}_t \bar{\Pi}_{t+1} \Pi_{t+1} C_{t+1}} \right]$$
$$\frac{1}{C_t} = \beta E_t \left[\frac{\hat{R}_t \bar{\Pi}_t}{\bar{\Pi}_{t+1} \hat{\Pi}_{t+1} C_{t+1}} \right]$$
$$\frac{1}{C_t} = \beta E_t \left[\frac{\hat{R}_t}{e^{\eta_t^\pi} \hat{\Pi}_{t+1} C_{t+1}} \right],$$

where the first line multiplies and divided by $\bar{\Pi}_t \bar{\Pi}_{t+1}$ the term within the expectation, the second uses the definition of $\hat{R}_t$ and $\hat{\Pi}_{t+1}$ and the last one uses the target definition.

Log-linearizing both sides implies

$$1 - c_t = E_t \left[1 + \hat{r}_t - \eta_{t+1}^\pi - \hat{\pi}_{t+1} - c_{t+1} \right]$$

which can be rearranged to

$$c_t = E_t \left[c_{t+1} \right] + E_t \left[-\hat{r}_t + \hat{\pi}_{t+1} + \eta_{t+1}^\pi \right] \tag{25}$$

Note that $E_t \left[\hat{r}_t \right] = E_t \left[r_t - \bar{\pi}_t \right] = r_t - E_t \left[\bar{\pi}_t \right] = r_t - \bar{\pi}_t + \bar{\pi}_t - E_t \left[\bar{\pi}_t \right] = \hat{r}_t + \bar{\pi}_t - E_t \left[\bar{\pi}_t \right]$. Substituting into the equation above implies

$$c_t = E_t \left[c_{t+1} \right] - \left(\hat{r}_t + \bar{\pi}_t - E_t \left[\hat{\pi}_{t+1} + \bar{\pi}_{t+1} \right] \right), \tag{26}$$

where the second expectation term uses that $\bar{\pi}_{t+1} = \bar{\pi}_t + \eta_{t+1}^\pi$.

Indexation rule index

Consider the indexation rule (8), reproduced below by convenience

$$\Upsilon_t = \left(\Pi_{t-1} \right)^\iota \left(E_t [\bar{\Pi}_t] \right)^{1-\iota}.$$

Since the inflation target has a unit-root, the indexation rule must be also stationarized by dividing by the target.

Dividing both sides by the target leads to

$$\begin{aligned}\frac{\Upsilon_t}{\bar{\Pi}_t} &= \left(\frac{\Pi_{t-1}}{\bar{\Pi}_{t-1}}\frac{\bar{\Pi}_{t-1}}{\bar{\Pi}_t}\right)^{\iota}\left(\frac{E_t[\bar{\Pi}_t/\Pi_{t-1}]}{\bar{\Pi}_t/\Pi_{t-1}}\right)^{1-\iota} \\ &= \left(\frac{\Pi_{t-1}}{\bar{\Pi}_{t-1}}\frac{\bar{\Pi}_{t-1}}{\bar{\Pi}_t}\right)^{\iota}\left(\frac{\Pi_{t-1}}{\bar{\Pi}_{t-1}e^{\eta_t^{\pi}}}E_t\left[\frac{\bar{\Pi}_{t-1}e^{\eta_t^{\pi}}}{\Pi_{t-1}}\right]\right)^{1-\iota}\end{aligned}$$

where in the first equality is also multiplied by $\bar{\Pi}_{t-1}/\bar{\Pi}_{t-1}$ the first term and the second term by Π_{t-1}/Π_{t-1}. I also used the fact that Π_{t-1} is known by firms in period t that allows including it within the expectation operator. The second equality uses the definition of the target, $\bar{\Pi}_t = \bar{\Pi}_{t-1}e^{\eta_t^{\pi}}$ that follows from equation (3).

$$\hat{\Upsilon}_t = \left(\frac{\hat{\Pi}_{t-1}}{e^{\eta_t^{\pi}}}\right)^{\iota}\left(\frac{\hat{\Pi}_{t-1}}{e^{\eta_t^{\pi}}}E_t\left[\frac{e^{\eta_t^{\pi}}}{\hat{\Pi}_{t-1}}\right]\right)^{1-\iota}.$$

where $\hat{\Upsilon}_t = \Upsilon_t/\bar{\Pi}_t$.

Now all variables have a defined steady-state. Log-linearizing equation above leads to

$$\begin{aligned}\hat{\upsilon}_t &= \iota(\hat{\pi}_{t-1} - \eta_t^{\pi}) + (1-\iota)\left(\hat{\pi}_{t-1} - \eta_t^{\pi} - E_t[\hat{\pi}_{t-1} - \eta_t^{\pi}]\right) \\ &= \iota(\hat{\pi}_{t-1} - \eta_t^{\pi}) + (1-\iota)\left(E_t[\bar{\pi}_t] - \bar{\pi}_t\right) \qquad (27)\end{aligned}$$

where the last equality uses that $\hat{\pi}_{t-1} - E_t[\hat{\pi}_{t-1}] = -log(\bar{\Pi}_{t-1}) + E_t[log(\bar{\Pi}_{t-1})] = -\left(\bar{\pi}_{t-1} - E_t[\bar{\pi}_{t-1}]\right)$ since $\hat{\pi}_{t-1} = log(\Pi_{t-1}) - log(\bar{\Pi}_{t-1})$ and Π_{t-1} is known.

Price index

Consider the price index law of motion (9), reproduced below by convenience

The price index is given by

$$P_t = \left[\theta\left(\Upsilon_t P_{t-1}\right)^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Dividing both sides by $P_{t-1}^{1-\varepsilon}$ by leads to

$$\Pi_t^{1-\varepsilon} = \theta\Upsilon_t^{1-\varepsilon}di + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon} \qquad (28)$$

Dividing by the target, we can stationarize the equation such that

$$\left(\frac{\Pi_t}{\bar{\Pi}_t}\right)^{1-\varepsilon} = \theta\left(\frac{\Upsilon_t}{\bar{\Pi}_t}\right)^{1-\varepsilon} + (1-\theta)\left(\frac{\frac{P_t^*}{P_{t-1}}}{\bar{\Pi}_t}\right)^{1-\varepsilon} \tag{29}$$

$$\hat{\Pi}_t^{1-\varepsilon} = \theta\hat{\Upsilon}_t^{1-\varepsilon} + (1-\theta)\left(\hat{P}_t^*\right)^{1-\varepsilon} \tag{30}$$

where $\hat{P}_t^* = \frac{P_t^*}{P_{t-1}\bar{\Pi}_t}$ is the stationary optimal relative price.

Note that $\hat{P}_{ss}^* = 1$, since $\hat{\Pi}_{ss}^* = \hat{\Upsilon}_s^{ss} = 1$. Thus, log-linearizing the equation above leads to

$$1-(1-\varepsilon)\hat{\pi}_t = \theta\left(1-(1-\varepsilon)\hat{\upsilon}_t\right) + (1-\theta)\left(1-(1-\varepsilon)\hat{p}_t^*\right),$$

which can be rearranged to

$$\hat{\pi}_t = \theta\hat{\upsilon}_t + (1-\theta)\hat{p}_t^* \tag{31}$$

Production function and marginal cost

The production function (7) is standard and its log-linear version is given by

$$y_{i,t} = \alpha l_{i,t}, \tag{32}$$

which the aggregate is simply $y_t = \alpha l_t$. Given this production function the real marginal cost of any firm i is given by

$$MC_{i,t} = \frac{W_t}{P_t} Y_{i,t+s}^{\frac{1-\alpha}{\alpha}}$$

Its log-linearized version is given by

$$mc_{i,t} = \hat{w}_t + \frac{1-\alpha}{\alpha} y_{i,t}$$

where $\hat{w}_{t+s} = log(\hat{W}_t) - log(\hat{W}_{ss})$ and $\hat{W}_t = W_t/P_t$. Thus, the average marginal cost at period $t+s$ is given by

$$mc_{t+s} = w_{t+s} + \frac{1-\alpha}{\alpha} y_{t+s} \tag{33}$$

Moreover, the marginal cost at period $t+s$ from a firm that chose its price at period t is given

by

$$mc_{i,t+s|t} = w_{t+s} + \frac{1-\alpha}{\alpha} y_{t+s|t}.$$

One can rewrite the individual marginal cost as a function of the average marginal cost and the difference between their demand and aggregate demand:

$$mc_{i,t+s|t} = mc_{t+s} + \frac{1-\alpha}{\alpha}(y_{t+s|t} - y_{t+s}). \tag{34}$$

Therefore, we need to find the log-linear version of the individual demand. The individual demand at period $t+s$ for a firm that did not choose prices optimally since period t is given by

$$Y_{i,t+s|t} = \left(\frac{\Upsilon_{t,t+s} P_t^*}{P_{t+s}}\right)^{-\varepsilon} Y_{t+s}$$

Dividing and multiplying by $P_{t-1}\bar{\Pi}_t \Pi_{j=1}^{s} \bar{\Pi}_{t+j}$

$$\begin{aligned} Y_{i,t+s|t} &= \left(\frac{P_t^*}{P_{t-1}\bar{\Pi}_t} \frac{\Upsilon_{t,t+s}}{\Pi_{j=1}^{s}\bar{\Pi}_{t+j}} \frac{\bar{\Pi}_t P_{t-1} \Pi_{j=1}^{s} \bar{\Pi}_{t+j}}{P_{t+s}}\right)^{-\varepsilon} Y_{t+s} \\ &= \left(\frac{\hat{\Upsilon}_{t,t+s} \hat{P}_t^*}{\hat{\Pi}_{t-1,t+s}}\right)^{-\varepsilon} Y_{t+s} \end{aligned}$$

where $\hat{\Pi}_{t-1,t+s} = \frac{\Pi_{t-1,t+s}}{\Pi_{j=0}^{s}\bar{\Pi}_{t+j}}$, $\Pi_{t-1,t+s} = \frac{P_{t+s}}{P_{t-1}} = \Pi_{j=0}^{s}\Pi_{t+j}$ and

$$\hat{\Upsilon}_{t,t+s} = \begin{cases} \Pi_{j=1}^{s} \hat{\Upsilon}_{t+j} & \text{if} \quad s \geq 1 \\ 1 & \text{if} \quad s = 0. \end{cases} \tag{35}$$

Note that their steady-state level are $\hat{\Upsilon}_s^{ss} = \hat{\Pi}_{-1,s}^{ss} = 1$ for any s. Thus, log-linearizing the demand leads to

$$y_{i,t+s|t} = y_{t+s} - \epsilon\left(\hat{p}_t^* + \hat{v}_{t,t+s} - \hat{\pi}_{t-1,t+s}\right) \tag{36}$$

where $\hat{\pi}_{t-1,t+s} = \sum_{j=0}^{s} \hat{\pi}_{t+j}$ and $\hat{v}_{t,t+s} = \sum_{j=1}^{s} \hat{v}_{t+j}$ for $s > 1$ and 0 for $s = 0$.

Substituting equation (36) into the marginal cost (34) leads to

$$mc_{i,t+s|t} = mc_{t+s} - \frac{(1-\alpha)\epsilon}{\alpha}\left(\hat{p}_t^* + \hat{v}_{t,t+s} - \hat{\pi}_{t-1,t+s}\right) \tag{37}$$

This equation writes the individual marginal costs of a firm that updated its price optimally

at period t as a function of aggregates and the optimal price from period t.

Optimal price

The optimal price (21) can be rewritten as

$$\sum_{s=0}^{\infty}(\beta\theta)^s P_t^* E_t\left[\frac{\Upsilon_{t,t+s}Y_{i,t+s|t}}{P_{t+s}}\right] = E_t\left[\sum_{s=0}^{\infty}(\beta\theta)^s Y_{i,t+s|t}\mu MC_{i,t+s|t}\right].$$

Dividing and multiplying by $P_{t-1}\bar{\Pi}_t\Pi_{t-1}$ outside the expectation and by $\Pi_{j=0}^{s}\bar{\Pi}_{t+j}$ within the expectation

$$\sum_{s=0}^{\infty}(\beta\theta)^s \frac{P_t^* P_{t-1}\bar{\Pi}_t\Pi_{t-1}}{P_{t-1}\bar{\Pi}_t\Pi_{t-1}} E_t\left[\frac{\Upsilon_{t,t+s}Y_{i,t+s|t}}{\Pi_{j=0}^{s}\bar{\Pi}_{t+j}}\frac{\Pi_{j=0}^{s}\bar{\Pi}_{t+j}}{P_{t+s}}\right] = E_t\left[\sum_{s=0}^{\infty}(\beta\theta)^s Y_{i,t+s|t}\mu MC_{i,t+s|t}\right]$$

$$\sum_{s=0}^{\infty}(\beta\theta)^s \frac{P_t^*}{P_{t-1}\bar{\Pi}_t}\frac{\bar{\Pi}_{t-1}e^{\eta_t^{\pi}}}{\Pi_{t-1}} E_t\left[Y_{i,t+s|t}\frac{\Pi_{t-1}\Upsilon_{t,t+s}}{\bar{\Pi}_t\Pi_{j=1}^{s}\bar{\Pi}_{t+j}}\frac{P_{t-1}\Pi_{j=0}^{s}\bar{\Pi}_{t+j}}{P_{t+s}}\right] = E_t\left[\sum_{s=0}^{\infty}(\beta\theta)^s Y_{i,t+s|t}\mu MC_{i,t+s|t}\right]$$

$$\sum_{s=0}^{\infty}(\beta\theta)^s \frac{\hat{P}_t^* e^{\eta_t^{\pi}}}{\hat{\Pi}_{t-1}} E_t\left[Y_{i,t+s|t}\frac{\hat{\Pi}_{t-1}\hat{\Upsilon}_{t,t+s}}{e^{\eta_t^{\pi}}\hat{\Pi}_{t-1,t+s}}\right] = E_t\left[\sum_{s=0}^{\infty}(\beta\theta)^s Y_{i,t+s|t}\mu MC_{i,t+s|t}\right]$$

where the second line uses the definition of the target and the fact that Π_{t-1} and P_{t-1} are known. The third equality uses the definitions of $\hat{\Upsilon}_{t,t+s}$, $\hat{\Pi}_{t-1,t+s}$ and $\bar{\Pi}_t$.

Note also that $MC_{ss} = 1/\mu$. Then, the stationarized equation for the optimal price can be rewritten as

$$\sum_{s=0}^{\infty}(\beta\theta)^s exp(\hat{p}_t^* - \hat{\pi}_{t-1} + \eta_t^{\pi}) E_t\left[Y_{ss}exp\left(y_{i,t+s|t} + \hat{\upsilon}_{t,t+s} + \hat{\pi}_{t-1} - \eta_t^{\pi} - \hat{\pi}_{t-1,t+s}\right)\right] =$$
$$\sum_{s=0}^{\infty}(\beta\theta)^s E_t\left[Y_{ss}\mu\bar{MC}exp\left(y_{i,t+s|t} + mc_{i,t+s|t}\right)\right]$$

Using the steady-states, the equation above simplifies to

$$\sum_{s=0}^{\infty}(\beta\theta)^s exp(\hat{p}_t^* - \hat{\pi}_{t-1} + \eta_t^{\pi}) E_t\left[exp\left(y_{i,t+s|t} + \hat{\upsilon}_{t,t+s} + \hat{\pi}_{t-1} - \eta_t^{\pi} - \hat{\pi}_{t-1,t+s}\right)\right] =$$
$$\sum_{s=0}^{\infty}(\beta\theta)^s E_t\left[exp\left(y_{i,t+s|t} + mc_{i,t+s|t}\right)\right]$$

Loglinearizing leads to:

$$\sum_{s=0}^{\infty}(\beta\theta)^s(1+\hat{p}_t^*-\hat{\pi}_{t-1}+\eta_t^\pi)E_t\left[1+y_{i,t+s|t}+\hat{\upsilon}_{t,t+s}+\hat{\pi}_{t-1}-\eta_t^\pi-\hat{\pi}_{t-1,t+s}\right]=$$
$$\sum_{s=0}^{\infty}(\beta\theta)^s E_t\left[1+y_{i,t+s|t}+mc_{i,t+s|t}\right]$$
$$\sum_{s=0}^{\infty}(\beta\theta)^s\left(\hat{p}_t^*-\hat{\pi}_{t-1}+\eta_t^\pi+E_t\left[\hat{\upsilon}_{t,t+s}+\hat{\pi}_{t-1}-\eta_t^\pi-\hat{\pi}_{t-1,t+s}\right]\right)=\sum_{s=0}^{\infty}(\beta\theta)^s E_t\left[mc_{i,t+s|t}\right]$$
$$\frac{\hat{p}_t^*-\hat{\pi}_{t-1}+\eta_t^\pi}{1-\beta\theta}=\sum_{s=0}^{\infty}(\beta\theta)^s E_t\left[mc_{i,t+s|t}-\hat{\pi}_{t-1}+\eta_t^\pi+\hat{\pi}_{t-1,t+s}-\hat{\upsilon}_{t,t+s}\right]$$
$$\frac{\hat{p}_t^*+\bar{\pi}_t-E_t\left[\bar{\pi}_t\right]}{1-\beta\theta}=\sum_{s=0}^{\infty}(\beta\theta)^s E_t\left[mc_{i,t+s|t}+\hat{\pi}_{t-1,t+s}-\hat{\upsilon}_{t,t+s}\right]$$

where the last equality uses that $\hat{\pi}_{t-1}+\eta_t^\pi-E_{h,t}[\hat{\pi}_{t-1}+\eta_t^\pi]=-\left(\bar{\pi}_t-E_{h,t}[\bar{\pi}_t]\right)$.

Substituting firm i's marginal cost (37) into equation above implies that

$$\hat{p}_t^*+\bar{\pi}_t-E_t\left[\bar{\pi}_t\right]=$$
$$(1-\beta\theta)\sum_{s=0}^{\infty}(\beta\theta)^s E_t\left[mc_{t+s}-\frac{(1-\alpha)\epsilon}{\alpha}\left(\hat{p}_t^*+\hat{\upsilon}_{t,t+s}-\hat{\pi}_{t-1,t+s}\right)+\hat{\pi}_{t-1,t+s}-\hat{\upsilon}_{t,t+s}\right]$$

Note that

$$\begin{aligned}E_t[\hat{p}_t^*]&=E_t\left[log\left(\frac{P_t^*}{P_{t-1}\bar{\Pi}_t}\right)-log(\hat{P}_{ss}^*)\right]\\&=log\left(P_t^*\right)-log\left(P_{t-1}\right)-E_t\left[log\left(\bar{\Pi}_t\right)\right]\\&=log\left(P_t^*\right)-log\left(P_{t-1}\right)-\bar{\pi}_t+\bar{\pi}_t-E_t\left[\bar{\pi}_t\right]\\&=\hat{p}_t^*+\bar{\pi}_t-E_t\left[\bar{\pi}_t\right]\end{aligned}$$

where the second equality uses that $\hat{P}_{ss}^*=1$ and firms know their own and past prices. The last uses the definition of $\hat{p}_t^*$ and $s_{it}^{\bar{\pi}}$.

Using this relation in the optimal price equation leads to

$$\hat{p}_t^*+\bar{\pi}_t-E_t\left[\bar{\pi}_t\right]=(1-\beta\theta)\sum_{s=0}^{\infty}(\beta\theta)^s E_t\left[\Theta mc_{t+s}+\hat{\pi}_{t-1,t+s}-\hat{\upsilon}_{t,t+s}\right]$$

where $\Theta=\frac{\alpha}{\alpha+(1-\alpha)\epsilon}$. Also using the fact that $\hat{\upsilon}_{t,t+s}=\sum_{j=1}^{s}\hat{\upsilon}_{t+j}$ and $\hat{\pi}_{t-1,t+s}=\sum_{j=0}^{s}\hat{\pi}_{t+j}$, the

optimal price becomes

$$\hat{p}_t^* + \bar{\pi}_t - E_t\left[\bar{\pi}_t\right] = (1-\beta\theta)\sum_{s=0}^{\infty}(\beta\theta)^s E_t\left[\Theta mc_{t+s} + \sum_{j=0}^{s}\hat{\pi}_{t+j} - \sum_{j=1}^{s}\hat{\upsilon}_{t+j}\right]$$

Now we use the usual steps to rewrite $\hat{p}_t^*$ recursively. First, taking the expectation of the optimal price one-period ahead leads to

$$E_t[\hat{p}_{t+1}^*] = (1-\beta\theta)\sum_{s=0}^{\infty}(\beta\theta)^s E_t\left[\Theta mc_{t+s+1} + \sum_{j=0}^{s}\hat{\pi}_{t+j+1} - \sum_{j=1}^{s}\hat{\upsilon}_{t+j+1}\right]$$

where the equality uses Law of Iterated Expectations. Second, we can rearrange the optimal price equation as

$$\begin{aligned}
&\hat{p}_t^* + \bar{\pi}_t - E_t\left[\bar{\pi}_t\right] = (1-\beta\theta)\sum_{s=0}^{\infty}(\beta\theta)^s E_t\left[\Theta mc_{t+s} + \sum_{j=0}^{s}\hat{\pi}_{t+j} - \sum_{j=1}^{s}\hat{\upsilon}_{t+j}\right] \\
&= (1-\beta\theta)(\Theta mc_t + E_t\left[\hat{\pi}_t\right]) + (1-\beta\theta)\sum_{s=1}^{\infty}(\beta\theta)^s E_t\left[\Theta mc_{t+s} + \sum_{j=0}^{s}\hat{\pi}_{t+j} - \sum_{j=1}^{s}\hat{\upsilon}_{t+j}\right] \\
&= (1-\beta\theta)(\Theta mc_t + E_t\left[\hat{\pi}_t\right]) + (1-\beta\theta)\sum_{s=0}^{\infty}(\beta\theta)^{s+1} E_t\left[\Theta mc_{t+s+1} + \sum_{j=0}^{s+1}\hat{\pi}_{t+j} - \sum_{j=1}^{s+1}\hat{\upsilon}_{t+j}\right] \\
&= (1-\beta\theta)(\Theta mc_t + E_t\left[\hat{\pi}_t\right]) + (1-\beta\theta)\beta\theta\sum_{s=0}^{\infty}(\beta\theta)^s E_t\left[\Theta mc_{t+s+1} + \hat{\pi}_t + \sum_{j=1}^{s+1}\hat{\pi}_{t+j} - \left(\hat{\upsilon}_{t+1} + \sum_{j=2}^{s+1}\hat{\upsilon}_{t+j}\right)\right] \\
&= (1-\beta\theta)\Theta mc_t + E_t\left[\hat{\pi}_t\right] - \beta\theta E_t\left[\hat{\upsilon}_{t+1}\right] + (1-\beta\theta)\beta\theta\sum_{s=0}^{\infty}(\beta\theta)^s E_t\left[\Theta mc_{t+s+1} + \sum_{j=0}^{s}\hat{\pi}_{t+j+1} - \sum_{j=1}^{s}\hat{\upsilon}_{t+j+1}\right]
\end{aligned}$$

where the second equality separate the $s=0$ term from the summation and the third redefines the summation counter as $s = s-1$. The fourth equality separate the first term from the summations inside the brackets, and the fifth redefines the summation counter as $j = j-1$ and solve the infinity sum for terms that do not depend on s.

Thus, the optimal price can be written as

$$\hat{p}_t^* + \bar{\pi}_t = (1-\beta\theta)\Theta mc_t + E_t\left[\hat{\pi}_t + \bar{\pi}_t\right] + \beta\theta E_t\left[\hat{p}_{t+1}^* - \hat{\upsilon}_{t+1}\right] \tag{38}$$

B Log-linearized model

B.1 Stationary equations

The equilibrium is given by equations (23-27), (31-33), (38) and the market clearing $c_t = y_t$ such that

$$\begin{aligned}
\hat{r}_t &= \phi_\pi \hat{\pi}_t + \phi_y y_t + \eta_t^r \\
y_t &= E_t\left[y_{t+1}\right] - \left(\hat{r}_t - E_t\left[\hat{\pi}_{t+1} + \eta_{t+1}^\pi\right]\right) \\
mc_t &= \hat{w}_t + \frac{1-\alpha}{\alpha} y_t \\
\hat{w}_t &= y_t + \varphi l_t \\
y_t &= \alpha l_t \\
\hat{\pi}_t &= \theta \hat{\upsilon}_t + (1-\theta)\hat{p}_t^* \\
\hat{\upsilon}_t &= \iota(\hat{\pi}_{t-1} - \eta_t^\pi) + (1-\iota)\left(E_t[\bar{\pi}_t] - \bar{\pi}_t\right) \\
\hat{p}_t^* + \bar{\pi}_t &= (1-\beta\theta)\Theta mc_t + E_t\left[\hat{\pi}_t + \bar{\pi}_t\right] + \beta\theta E_t\left[\hat{p}_{t+1}^* - \hat{\upsilon}_{t+1}\right]
\end{aligned}$$

Using the labor supply (24) , the production function (32) into the marginal cost and substituting into the optimal price leads to

$$\begin{aligned}
\hat{p}_t^* + \bar{\pi}_t &= (1-\beta\theta)\Psi y_t + E_t\left[\hat{\pi}_t + \bar{\pi}_t\right] + \beta\theta E_t\left[\hat{p}_{t+1}^* - \hat{\upsilon}_{t+1}\right] \\
\hat{p}_t^* &= (1-\beta\theta)\Psi y_t + \hat{\pi}_t + \beta\theta E_t\left[\hat{p}_{t+1}^* - \hat{\upsilon}_{t+1}\right] \qquad (39)
\end{aligned}$$

where $\Psi = \Theta\frac{1+\varphi}{\alpha} = \frac{1+\varphi}{\alpha+(1-\alpha)\epsilon}$ and the last equality uses that $E_t\left[\hat{\pi}_t + \bar{\pi}_t\right] = \hat{\pi}_t + \bar{\pi}_t$, since $\pi_t = \hat{\pi}_t + \bar{\pi}_t$ is known.

As usual, the New Keynesian Phillips curve combines the inflation dynamics (31) with the optimal price (38) such that

$$\begin{aligned}
\frac{\hat{\pi}_t - \theta\hat{\upsilon}_t}{1-\theta} &= (1-\beta\theta)\Psi y_t + \hat{\pi}_t + \beta\theta E_t\left[\frac{\hat{\pi}_{t+1} - \theta\hat{\upsilon}_{t+1}}{1-\theta} - \hat{\upsilon}_{t+1}\right] \\
\hat{\pi}_t - \theta\hat{\upsilon}_t &= (1-\beta\theta)(1-\theta)\Psi y_t + (1-\theta)\hat{\pi}_t + \beta\theta E_t\left[\hat{\pi}_{t+1} - \hat{\upsilon}_{t+1}\right] \\
\hat{\pi}_t - \hat{\upsilon}_t &= \kappa y_t + \beta E_t\left[\hat{\pi}_{t+1} - \hat{\upsilon}_{t+1}\right]
\end{aligned}$$

where $\kappa = \frac{(1-\beta\theta)(1-\theta)\Psi}{\theta}$. The last equality uses that $E_t\left[\hat{\pi}_t + \bar{\pi}_t\right] = \hat{\pi}_t + \bar{\pi}_t$, since $\pi_t = \hat{\pi}_t + \bar{\pi}_t$ is known.

Therefore, the model equilibrium of the real and nominal stationary variables can be described

by the following equations:

$$\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_y y_t + \eta_t^r \tag{40a}$$

$$y_t = E_t\left[y_{t+1}\right] - \left(\hat{r}_t - E_t\left[\hat{\pi}_{t+1} + \eta_{t+1}^\pi\right]\right) \tag{40b}$$

$$\hat{\upsilon}_t = \iota(\hat{\pi}_{t-1} - \eta_t^\pi) + (1-\iota)\left(E_t[\bar{\pi}_t] - \bar{\pi}_t\right) \tag{40c}$$

$$\hat{\pi}_t - \hat{\upsilon}_t = \kappa y_t + \beta E_t\left[\hat{\pi}_{t+1} - \hat{\upsilon}_{t+1}\right] \tag{40d}$$

B.2 Recovering the model in terms of non-stationary nominal variables

The variables with hats are deviations from the target. We can recover the dynamics from the variables by simply adding back target.

Using the fact $\hat{r}_t = r_t + \bar{\pi}_t$ and that households know the interest rate, we have that $E_t\left[\hat{r}_t\right] = E_t\left[r_t - \bar{\pi}_t\right] = r_t - E_t\left[\bar{\pi}_t\right]$. Using the latter and $\bar{\pi}_{t+1} = \bar{\pi}_t + \eta_{t+1}^\pi$, into the Euler equation (40b) leads to

$$\begin{aligned} y_t &= E_t\left[y_{t+1}\right] - \left(E_t\left[\hat{r}_t\right] - E_t\left[\hat{\pi}_{t+1} + \eta_{t+1}^\pi\right]\right) \\ y_t &= E_t\left[y_{t+1}\right] - \left(r_t - E_t\left[\pi_{t+1}\right]\right) \end{aligned}$$

Analogously, the indexation rule (40c) in the following way

$$\begin{aligned} \hat{\upsilon}_t &= \iota(\hat{\pi}_{t-1} - \eta_t^\pi) + (1-\iota)\left(E_t[\bar{\pi}_t] - \bar{\pi}_t\right) \\ \hat{\upsilon}_t + \bar{\pi}_t &= \iota(\hat{\pi}_{t-1} + \bar{\pi}_t - \eta_t^\pi) + (1-\iota)E_t[\bar{\pi}_t] \\ &= \iota(\hat{\pi}_{t-1} + \bar{\pi}_{t-1}) + (1-\iota)E_t[\bar{\pi}_t] \\ \upsilon_t &= \iota\pi_{t-1} + (1-\iota)E_t[\bar{\pi}_t] \end{aligned}$$

where $\upsilon_t = log(\Upsilon_t) = \hat{\upsilon}_t + \bar{\pi}_t$

The inflation equation from the price index can be readily written as

$$\pi_t = \theta\upsilon_t + (1-\theta)p_t^*$$

by summing $\bar{\pi}_t$ in both sides and rearranging. Similarly, defining $p_t^* = log(P_t^*/P_{t-1}) = \hat{p}_t^* + \bar{\pi}_t$, the optimal price equation (39) can be rearranged as

$$\begin{aligned} \hat{p}_t^* + \bar{\pi}_t &= (1-\beta\theta)\Psi mc_t + \hat{\pi}_t + \bar{\pi}_t + \beta\theta E_t\left[\hat{p}_{t+1}^* + \bar{\pi}_{t+1} - \bar{\pi}_{t+1} - \hat{\upsilon}_{t+1}\right] \\ p_t^* &= (1-\beta\theta)\Psi mc_t + \pi_t + \beta\theta E_t\left[p_{t+1}^* - \upsilon_{t+1}\right] \end{aligned}$$

where the first line adds $\bar{\pi}_t$ in both sides and sums and subtracts $\bar{\pi}_{t+1}$ conveniently. The second equality uses the definition of υ_t, π_t and p_t^*. Finally, using the definition of $\hat{r}_t$ and $\hat{\pi}_t$ into the Taylor rule (40a)leads to

$$r_t = \bar{\pi}_t + \phi_\pi (\pi_t - \bar{\pi}_t) + \phi_y y_t + \eta_t^r$$

Similarly, the NKPC (40d) can be rearranged as

$$\begin{aligned}\hat{\pi}_t + \bar{\pi}_t - \bar{\pi}_t - \hat{\upsilon}_t - &= \kappa y_t + \beta E_t [\hat{\pi}_{t+1} + \bar{\pi}_{t+1} - \bar{\pi}_{t+1} - \hat{\upsilon}_{t+1}] \\ \pi_t - \upsilon_t &= \kappa y_t + \beta E_t [\pi_{t+1} - \upsilon_{t+1}]\end{aligned}$$

Therefore, the model equilibrium of the real and nominal non-stationary variables can be described by the following equations:

$$\begin{aligned}r_t &= \bar{\pi}_t + \phi_\pi (\pi_t - \bar{\pi}_t) + \phi_y y_t + \eta_t^r \\ y_t &= E_t [y_{t+1}] - (r_t - E_t [\pi_{t+1}]) \\ \upsilon_t &= \iota \pi_{t-1} + (1-\iota) E_t[\bar{\pi}_t] \\ \pi_t - \upsilon_t &= \kappa y_t + \beta E_t [\pi_{t+1} - \upsilon_{t+1}]\end{aligned}$$

C Signal extraction

Since shocks and signals are normally distributed and the model is linear, rational expectations can be computed by the Kalman Filter. The state equation is given by

$$\begin{pmatrix} \eta_t^r \\ \eta_{t+1}^{\bar{\pi}} \\ \bar{\pi}_t \end{pmatrix} = \begin{pmatrix} \rho_r & 0 & 0 \\ 0 & \rho_\pi & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} \eta_{t-1}^r \\ \eta_t^{\bar{\pi}} \\ \bar{\pi}_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t^r \\ \varepsilon_{t+1}^{\bar{\pi}} \\ 0 \end{pmatrix},$$

and the measurement equations are

$$\begin{aligned}r_t^{exo} &= -(\phi_\pi - 1)\bar{\pi}_t + \eta_t^r \\ s_t^{IT} &= \bar{\pi}_t + u_t\end{aligned}$$

where r_t^{exo} is defined in equation (15). Under Opacity, agents use only r_t^{exo} as measurement whereas they use (r_t^{exo}, s_t^{IT}) under IT.

The shocks' expectations can be computed by standard Kalman filter recursions using the

following state space model:

$$x_t = Ax_{t-1} + \varepsilon_t \tag{41}$$

$$S_t = Cx_t + Du_t \tag{42}$$

where $x_t = \begin{bmatrix} \eta_t^r & \eta_{t+1}^{\bar{\pi}} & \bar{\pi}_t \end{bmatrix}'$, $\varepsilon_t = \begin{bmatrix} \varepsilon_t^r & \varepsilon_{t+1}^{\bar{\pi}} & 0 \end{bmatrix}'$, $A = \begin{pmatrix} \rho_r & 0 & 0 \\ 0 & \rho_\pi & 0 \\ 0 & 1 & 1 \end{pmatrix}$ and $\Sigma_\varepsilon = diag(\sigma_r^2 \quad \sigma_\pi^2 \quad 0)$, where $diag(x)$ denotes matrix with diagonal equal to x and zero for other elements.

Under Opacity, one can see that: $S_t = r_t^{exo}$, $C = \begin{pmatrix} 1 & 0 & -(\phi_\pi - 1) \end{pmatrix}$ and $D = 0$. In contrast, $S_t = \begin{bmatrix} r_t^{exo} & s_t^{IT} \end{bmatrix}'$, $C = \begin{pmatrix} 1 & 0 & -(\phi_\pi - 1) \\ 0 & 0 & 1 \end{pmatrix}$ and $D = [0 \quad 1]'$ under IT.

Expectations. Using the Kalman filter with the state equation as (41) and observational equation (42), one can write the update equation as

$$E_t(x_t) = E_{t-1}(x_t) + K_t[S_t - E_{t-1}(S_t)], \tag{43}$$

where the Kalman gain, K_t, and mean square error of the one-step ahead prediction error, $P_{t+1|t}$, have the following dynamics:

$$\begin{aligned} K_t &= P_{t|t-1}C' \left[CP_{t|t-1}C' + DD'\sigma_u^2\right]^{-1} \\ P_{t+1|t} &= A(I_n - K_tC)P_{t|t-1}A' + \Sigma_\varepsilon. \end{aligned} \tag{44}$$

where $n = 3$.

I follow the literature by focusing on stationary equilibrium. In this equilibrium, agents update their forecasts based on the steady-state Kalman gain.

If such solution exists, one can set $\bar{P} = P_{t+1|t} = P_{t|t-1}$, obtaining the Riccati equation:

$$\bar{P} = A\left[\bar{P} - \bar{P}C'\left[C\bar{P}C' + DD'\sigma_u^2\right]^{-1} C\bar{P}\right]A' + \Sigma_\varepsilon, \tag{45}$$

which implies the steady-state Kalman gain $\bar{K} = \bar{P}C'\left[C\bar{P}C' + DD'\sigma_u^2\right]^{-1}$.

Usually, the literature resorts to stationarity of x_t to guarantee the existence of the invariant Kalman filter. In this application, x_t is non-stationary due to the unit-root in the inflation target. There are weaker conditions for the existence of a steady-state filter.

If the system (41-42) is detectable and stabilizable, then $\lim_{t\to\infty} P_{t|t-1} = \bar{P}$ (Anderson and Moore; 1979, section 4.4).

Since $\phi_\pi > 1$, it is possible to show that the system satisfies controllability and observability for both Opacity and IT cases. Those two properties imply detectability and stabilizability, guaranteeing the existence of the steady-state filter. Thus, the expectation formation is given by

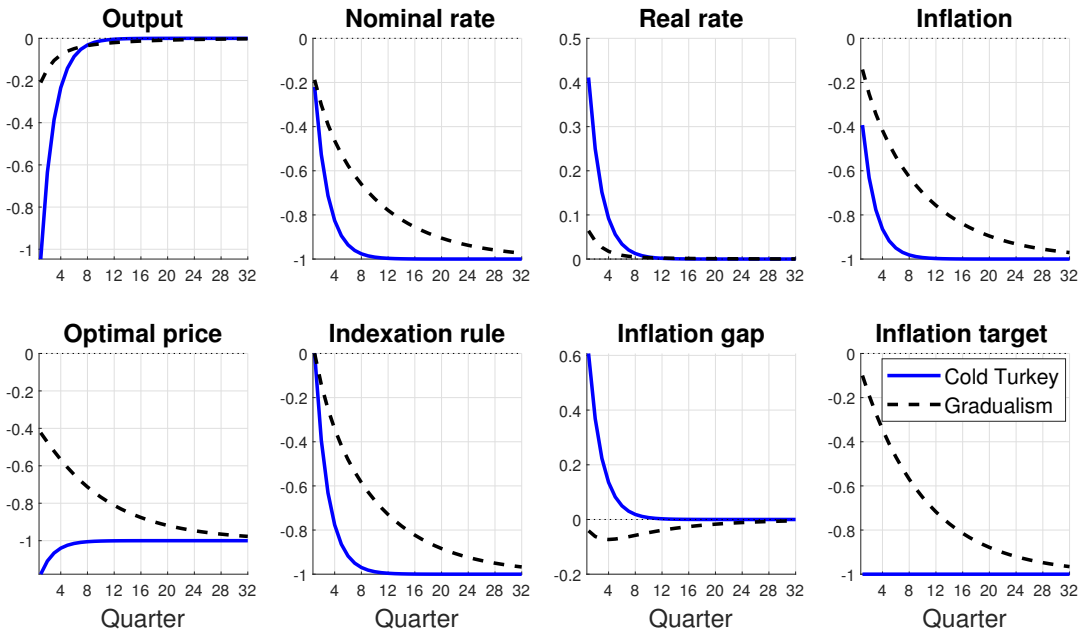
$$E_t(x_t) = E_{t-1}(x_t) + \bar{K}[S_t - E_{t-1}(S_t)] \tag{46}$$

where $\bar{K}$ is defined above.

D Results for full indexation to past prices ($\iota = 1$)

IRFs under CT and gradualism. Figure 3 shows IRFs under both policies under full credibility for the baseline indexation scheme. Here I show the analogous results for full indexation to past prices ($\iota = 1$).

Figure A1: "Cold turkey" vs gradual disinflation under full information

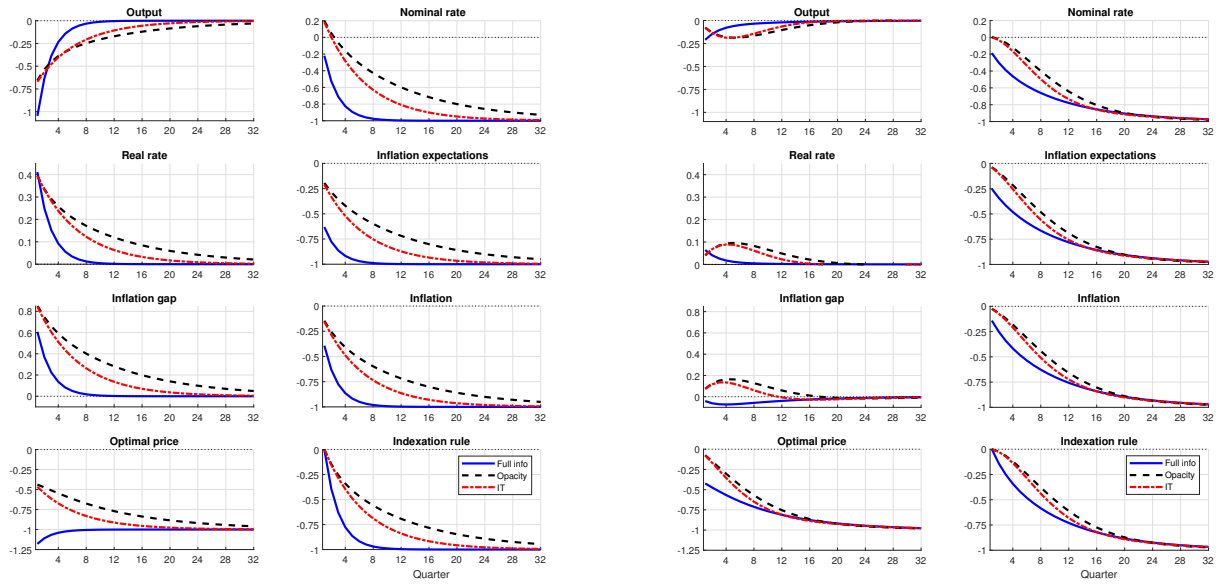


Note: All impulse response functions are multiplied by 4 to get annualized changes in percentage points.

Figure 5 shows the same IRFs under imperfect credibility for the baseline indexation scheme. Here I show the analogous results for full indexation to past prices ($\iota = 1$).

Sacrifice ratio and Credibility threshold. Figure A3 shows sacrifice ratio as a function of credibility of the IT regime (σ_{IT}) in the baseline indexation scheme. Here I show the analogous

Figure A2: Disinflation policies under Opacity and IT with past indexation

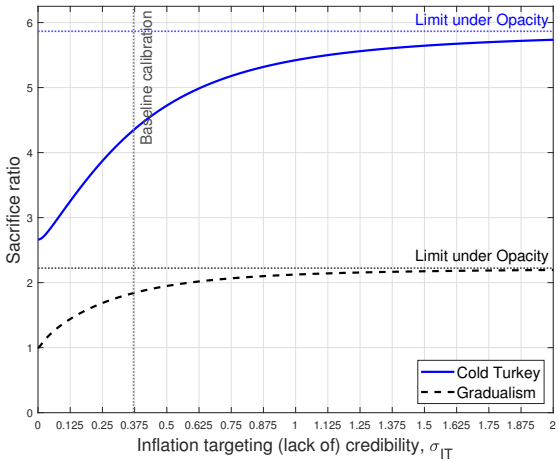


(a) "Cold turkey" ($\rho_\pi = 0.0$) (b) Gradualism ($\rho_\pi = 0.9$)

Note: All impulse response functions are multiplied by 4 to get annualized changes in percentage points.

results for full indexation to past prices ($\iota = 1$).

Figure A3: Sacrifice ratio depending on IT credibility



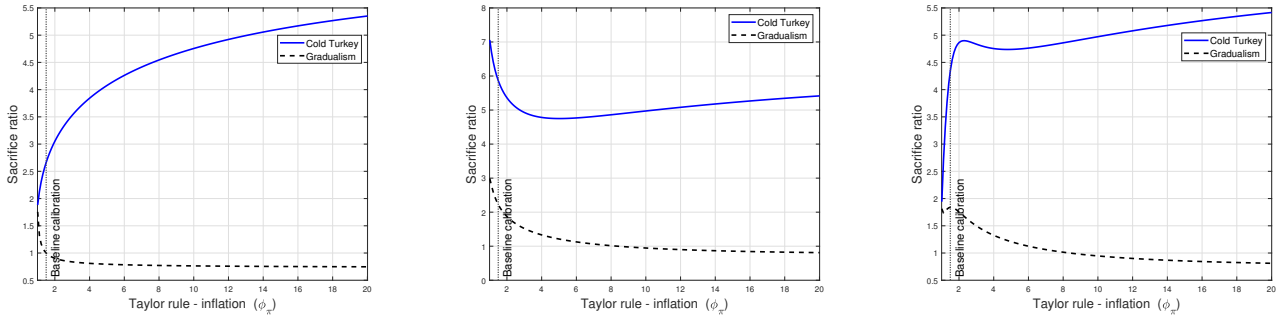
Note: The dotted blue and blue black correspond to the Opacity sacrifice ratio under both policies, which is equivalent to $\sigma_{IT} \to \infty$. The vertical black dotted line is the baseline calibration, $\sigma_{IT} = 0.37$.

In this case, there is no credibility threshold, since the CT policy has a higher SR than gradu-

alism in the full information benchmark.

Sacrifice ratio and central bank response to inflation. Figure 11 shows sacrifice ratio as a function of central bank response to inflation (ϕ_π) in the baseline indexation scheme. Here I show the analogous results for full indexation to past prices ($\iota = 1$).

Figure A4: Sacrifice ratio depending on central bank's response to inflation



(a) Full information (b) Opacity (c) Inflation targeting

Note: The vertical black dotted line is the baseline calibration, $\phi_\pi = 1.5$.

The key difference from Figure 11 is that the SR becomes (at some point) increasing in ϕ_π under CT (strictly for full information and only for sufficiently high ϕ_π for Opacity and IT). This happens because indexation to past inflation increases the demand channel and weakens the expectational channel as inflation react slowly due to the indexation to past prices. There is a wider inflation gap under CT such that the demand channel is stronger than the expectational channel.

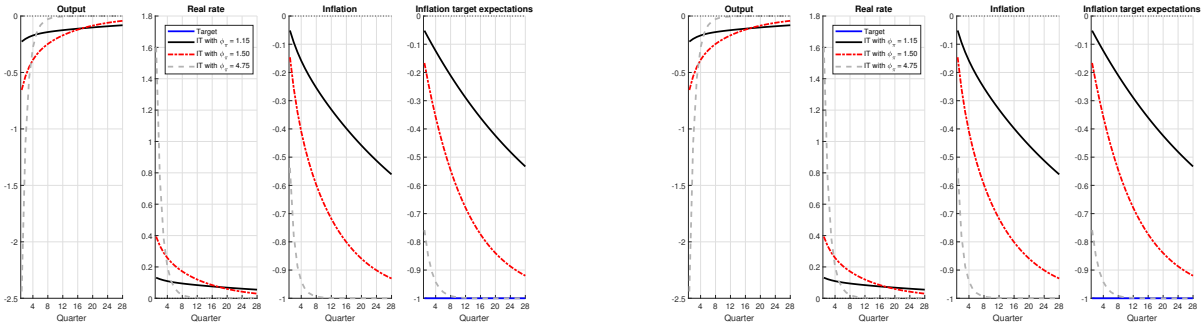
Under Opacity, the learning about the target is very responsive to ϕ_π, specially when ϕ_π is close to one. This happens because gents learn only through the interest rate and the it becomes uninformative about the target when $\phi_\pi \to 1$ (see equation 15). This explain the decreasing part for small values of ϕ_π. Under IT, there is a non-monotonicity for the same reason as before (see section 3.5). The difference is that for high values of ϕ_π, the demand channel dominates in contrast with the baseline indexation scheme.

Under gradualism, the relation between SR and ϕ_π is qualitatively similar because inflation target also decrease slowly, implying a relatively looser monetary policy. This makes the demand channel weaker such that the expectational channel dominates.

E Other results under Opacity

Inflation target expectations as function of key parameters under Opacity. Figure 7 shows learning about the inflation target as function of selected parameters under the IT regime.

Figure A5: IRFs under CT disinflation for different responses to inflation

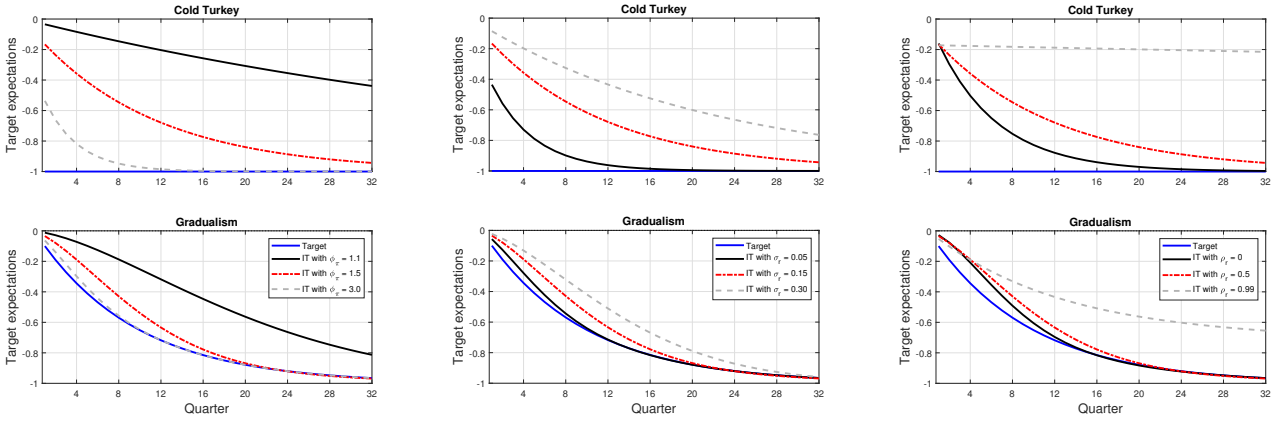


(a) Inflation targeting (b) Opacity

Note: All impulse response functions are multiplied by 4 to get annualized changes in percentage points.

Here I show the analogous results under Opacity.

Figure A6: Inflation target expectations as function of key parameters



(a) Taylor rule (b) MP shock volatility (c) MP shock persistence

Note: Target expectations impulse responses are multiplied by 4 to get annualized changes in percentage points. CT disinflation uses $\rho_\pi = 0$ and $\sigma_\pi = 0.0525$ whereas Gradualism uses $\rho_\pi = 0.9$ and $\sigma_\pi = 0.025$.

In general, both gradualism and CT learning are more sensitive to changes in parameters of the Taylor rule and monetary shocks. This happens because agents form expectations using the interest rate only. Under IT, if the interest rate becomes less informative, this is balanced with the information from the central bank communication.