Optimal FX Interventions with Limited Reserves in Shallow Markets*

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Abstract

We investigate the optimal time-consistent use of foreign exchange interventions (FXI) policy in a small open economy model driven by endowment and portfolio flow shocks. The model features endogenous FX market depth and is subject to a lower bound on FX reserves. In the competitive equilibrium, stronger portfolio outflows increase conditional exchange rate volatility and make FX markets more shallow. The central bank's optimal intervention policy is not solely targeted at contemporaneous portfolio flows but also incorporates a forward-looking element due to the risk of depleting reserves. Notably, the central bank should intervene aggressively in response to current portfolio outflows only if future outflows are anticipated to be relatively mild. When portfolio outflows are expected to intensify, maintaining a precautionary level of reserves becomes more important relative to the contemporaneous intervention motive. Implementing optimal time-consistent policy delivers substantial welfare gains, surpassing those achieved by alternative FXI rules considered despite the existence of a time inconsistency problem.

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1 Introduction

Volatility in international capital flows is a significant concern for policymakers in small open economies, especially in a more shock prone environment amid rising geopolitical tensions, as noted in a recent speech by the IMF's First Deputy Managing Director Gopinath (2024). In shallow FX markets, such volatility exacerbates exchange rate fluctuations, distorting external financing conditions and impeding international risk-sharing. Foreign exchange interventions (FXI) are a key policy tool that can be employed to mitigate these effects. Recent literature on open economy policy frameworks—such as Adrian et al. (2022), Itskhoki & Mukhin (2023), and Basu et al. (2020)—also emphasizes the role of FXI in stabilizing exchange rates. When deciding on contemporaneous interventions, policymakers must consider the risk of depleting reserves in the future, bringing intertemporal considerations to the forefront. How extensively should FXI offset current portfolio outflows, particularly when these risks are expected to abate or intensify? Is the effectiveness of FXI state-dependent in such circumstances and what is the average level of FX reserves that should be held when conducting FXI optimally?

We address these questions by studying the optimal time-consistent use of FXI in a real small open economy model with endogenous FX market depth akin to Itskhoki & Mukhin (2023) and subject to a lower bound on FX reserves. FX markets are shallow (i.e., not perfectly elastic) due to the existence of risk-averse financiers who are exposed to FX risk when intermediating funds, giving rise to endogenous violations in uncovered interest rate parity (UIP). In equilibrium, FX market depth is proportional to conditional exchange rate volatility. In particular, FX markets become more shallow in episodes of portfolio outflows. We show that in these episodes the central bank's optimal intervention policy is to lean against the wind by selling off reserves. However, the extent to which the central bank may want to offset contemporaneous portfolio outflows depends on the expected path of future portfolio outflows and the proximity to the lower bound on FX reserves. If outflows are expected to abate, the central bank should intervene more aggressively to contemporaneous outflows. Conversely, if outflows are expected to intensify, optimal policy prescribes maintaining a precautionary level of reserves rather than engaging in aggressive FX sales. This liquidity provision by the central bank reduces exchange rate volatility and, by extension, deepens FX markets. To ensure the flexibility to provide liquidity, the central bank must maintain a precautionary level of reserves on average. However, this reserve accumulation forces the economy to save more in aggregate. Optimal FXI policy balances this trade-off, yielding substantial welfare gains.

In line with the literature that we contribute to, FXI is effective in the model since it changes the amount intermediated by financiers and thus impacts the exchange rate. As illustration, consider a sterilized FX purchase i.e., the purchase of foreign currency bonds financed by the issuance of domestic currency bonds. Everything else equal, financiers will accommodate this increased foreign currency demand by selling foreign currency bonds and buying domestic currency bonds, which expands their net long FX exposure to the domestic currency. Since financiers are risk-averse, they need to be compensated for taking on this additional risk, resulting in an expected appreciation or contemporaneous depreciation of the domestic currency. Through a similar portfolio balance channel, exogenous changes in currency demand, which we refer to as portfolio flow shocks, affect the exchange rate. Besides this friction in international financial markets, the model is a standard real small open economy with a tradable and nontradable sector.[1]

Without the lower bound on FX reserves, it is optimal for the central bank to effectively take on the role of financiers since their intermediation is costly from a planner's perspective, as shown by Itskhoki & Mukhin (2023). Therefore, optimal FXI fully offsets portfolio flow shocks and eliminates ex-ante UIP deviations in the first best. While the exchange rate volatility due to portfolio flow shocks is eliminated, optimal policy still allows the exchange rate to fluctuate. More specifically, exchange rate adjustments due to changes in stochastic endowments allow for efficient expenditure switching. If FXI is potentially constrained by a lower bound on FX reserves, our results suggest that optimal policy becomes more nuanced. When intervening today, the central bank needs to take into account that it might run out of reserves in future periods. In other words, the intertemporal aspect becomes more important in the optimal, second-best FXI policy. Rather than eliminating UIP deviations in all periods, the policymaker optimally aims at smoothing out UIP deviations. With regards to the reaction to portfolio outflows, optimal policy prescribes an aggressive offset if outflows are relatively mild and expected to abate. As outflows become stronger, the focus shifts from offsetting them by selling FX reserves to maintaining a precautionary level of reserves. In the case of portfolio inflows, the lower bound on FX reserves is less of a constraint and the optimal policy in the second best resembles the first best i.e., optimal policy offsets portfolio inflows by buying FX reserves approximately one-for-one.

In the quantitative part of our analysis, we calibrate the model to Malaysia, a small open economy that actively uses FXI to manage the exchange rate. We back out the exogenous process for portfolio flows by applying the UIP condition from the model to the data. While

[1] A representative household receives stochastic endowments, consumes a bundle of tradable and nontradable goods and can smooth consumption by trading in a local currency bond.

the standard deviation of the estimated portfolio flow process amounts to 5% of annual GDP, the average level of FX reserves in the optimal policy regime is around 7% of GDP. Our quantitative results confirm that the FXI reaction is more aggressive to small than large outflows. We additionally find that the effectiveness of FXI is state-dependent: FX purchases depreciate the real exchange rate by less than equivalent FX sales. The reason is that FX purchases occur in periods of portfolio inflows which are associated with relatively deep FX markets while FX sales happen in periods of portfolio outflows in which FX markets are endogenously shallower. The standard deviation of the real exchange rate in the optimal policy regime is 24% lower than in the decentralized equilibrium without FXI. Associated with that, FX markets are also 18% deeper in the optimal policy regime. Finally, we compute the welfare gains relative to the observed Malaysian FXI regime. Under optimal policy, these gains amount to 0.44% of annual consumption, which exceeds the maximum welfare gains of 0.25% achieved by considered FXI rules designed to smooth UIP deviations. In contrast, committing to a policy of no FXI reduces welfare by 0.16%, while the first-best policy establishes an upper bound on welfare gains at 0.54%.

The primary contribution of this paper is the analytical and quantitative characterization of the optimal time-consistent FXI policy in a real small open economy with state-dependent FX market depth and a lower bound on FX reserves. While we incorporate frictions in international financial markets following Itskhoki & Mukhin (2023), our focus is on the optimal time-consistent policy. This focus is motivated by the fact that, as shown by Itskhoki & Mukhin (2023), the optimal policy under commitment becomes time inconsistent in the presence of an occasionally binding constraint on reserves. Instead of relying on a linear-quadratic approximation of the equilibrium system to derive results, we solve the exact nonlinear policy problem using a global, nonlinear solution algorithm. This approach enables us to calibrate a quantitative small open economy model that is both tractable and suitable for policy analysis while also distinguishing our work from Basu et al. (2018), who similarly examine the optimal use of FXI under limited reserves. Moreover, the fully nonlinear solution of the model allows us to account for the state-dependent nature of FX market depth, which is particularly relevant for evaluating the use of FXI, as market depth influences its effectiveness.[2]

Related Literature The theoretical foundation that gives rise to the effectiveness of FXI in open economy models is the incorporation of gross asset positions and segmentation in international financial markets. Within a two-country dynamic general equilibrium framework,

[2]In existing quantitative models (Itskhoki & Mukhin, 2021; Adrian et al., 2022; Davis et al., 2023, and others), FX market depth is a calibrated parameter and therefore constant. It is typically captured by $\omega\sigma^2$ or Γ when modeled according to Itskhoki & Mukhin (2021) or Gabaix & Maggiori (2015), respectively.

Devereux & Sutherland (2010) present a general approximation method for characterizing time-varying equilibrium portfolios. Gabaix & Maggiori (2015) and Itskhoki & Mukhin (2021) provide microfounded models of a portfolio balance channel in currency markets even though the idea dates back to Kouri (1976). The role of FXI in our model is to address this friction by stabilizing UIP deviations, thus smoothing external financing conditions. This view is consistent with the role of FXI in models with multiple policy tools such as Itskhoki & Mukhin (2023) and the IMF's work on the Intergrated Policy Framework (IPF) such as Adrian et al. (2022) and Basu et al. (2020). Other studies include Davis et al. (2023), Arce et al. (2019), Chang (2018), and Jeanne & Rancière (2011), which examine the macroprudential use of FXI to prevent sudden stops in emerging economies. Fanelli & Straub (2021) highlights an inequality motive for using FXI, while Cavallino (2019) and Basu et al. (2018) focus on capital flows, similar to our study. Moreover, FXI can function as an effective unconventional monetary policy tool when interest rates are at the effective lower bound (ELB) and appreciation pressures exist, as argued by Amador et al. (2020) and Cwik & Winter (2024).

The effectiveness of FXI in our model critically depends on FX market depth (i.e., the elasticity of currency demand), the estimation of which has been a key focus in the empirical literature (Beltran & He, 2024; Hertrich & Nathan, 2023; Adler et al., 2019; Fratzscher et al., 2019; Blanchard et al., 2015; Fatum & M. Hutchison, 2003, and others). A major limitation is the availability of non-confidential, high-frequency data on FXI. Progress has been made in this regard by Adler et al. (2024) who provide a comprehensive FXI data set on a monthly and quarterly basis which we leverage in calibrating our model. Methodologically, our paper connects to the literature that solves small open economies using global methods (Bianchi, 2011; Bianchi & Mendoza, 2018; Schmitt-Grohé & Uribe, 2016, 2021; Davis et al., 2023, and others)

Outline The remainder of this paper is structured as follows. Section 2 outlines the model in which the optimal FXI problem is then analyzed in Section 3. Section 4 presents and discusses the quantitative analysis. Finally, Section 5 concludes by summarizing the theoretical and numerical findings in the context of the existing literature.

2 Model

This section outlines a standard small open economy model with stochastic endowments extended with a segmented international financial sector that consists of financiers, portfolio

investors, and the central bank as in Itskhoki & Mukhin (2023). We first describe the decentralized equilibrium in which the central bank does not engage in FXI, before characterizing the constrained planner equilibrium in which the central bank follows a discretionary, time-consistent FXI policy. Finally, we provide the recursive representation of the decentralized equilibrium and the constrained planner equilibrium.

2.1 Outline

Utility Function Consider an economy that is populated by a large number of identical, infinitely-lived households with preferences described by the utility function

$$\sum_{t=0}^{\infty} \mathbb{E}_0 \left[\beta^t u \left(C_{T,t}, C_{N,t} \right) \right]$$

where

$$u(C_{T,t}, C_{N,t}) = \frac{1}{1-\sigma} \left(\underbrace{\left[\alpha \left(C_{T,t} \right)^{\frac{\xi-1}{\xi}} + (1-\alpha) \left(C_{N,t} \right)^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}}_{C_t} \right)^{1-\sigma}$$

Households derive utility from total consumption C_t that consists of consumption of tradable goods $C_{T,t}$ and consumption of nontradable goods $C_{N,t}$. Furthermore, $\beta \in (0,1)$ denotes the subjective discount factor, $1/\sigma$ is the intertemporal elasticity, ξ is the elasticity of substitution between tradable and nontradable goods and $\alpha \in (0,1)$ the share of tradables in total consumption.

Budget Constraint Each period t, households receive stochastic endowments of tradable and nontradable goods, denoted by $Y_{T,t}$ and $Y_{N,t}$, respectively. The endowments are exogenous and follow a first-order Markov process. Furthermore, households have access to a one period local currency bond B_t with gross interest rate I_t. The household's sequential budget constraint is then given by

$$P_{T,t} C_{T,t} + P_{N,t} C_{N,t} \leq P_{T,t} Y_{T,t} + P_{N,t} Y_{N,t} - B_t + B_{t-1} I_{t-1} + \Pi_{M,t} + \Pi_{F,t} + \Pi_{P,t}$$

where $\Pi_{M,t}$, $\Pi_{F,t}$ and $\Pi_{P,t}$ denote the profits of the central bank, financiers and portfolio investors, respectively. Note that this formulation of the budget constraint implies that the financial sector (i.e. financiers and portfolio investors) is fully domestically owned. Furthermore, the law of one price holds for tradable goods and the price level abroad is normalized to unity so that the price of tradables equals the exchange rate $P_{T,t} = \mathcal{E}_t$. On top of that, we

also normalize the price of nontradables to unity so that $\mathcal{E}_t$ can be interpreted as the relative price of tradable goods.

Households' Optimality The choice of B_t is characterized by the household Euler equation:

$$R_t \mathbb{E}_t \left[\Theta_{t+1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right] = 1 \tag{1}$$

where Θ_{t+1} denotes the stochastic discount factor (SDF) of domestic households:

$$\Theta_{t+1} = \beta \frac{u_1(C_{T,t+1}, C_{N,t+1})}{u_1(C_{T,t}, C_{N,t})} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{\frac{1-\sigma\xi}{\xi}} \left(\frac{C_{T,t+1}}{C_{T,t}} \right)^{-\frac{1}{\xi}}$$

In addition, combining the FOCs with respect to $C_{T,t}$ and $C_{N,t}$ allows us to obtain the equilibrium expenditure switching condition which pins down the exchange rate:

$$\mathcal{E}_t = \frac{u_1\left(C_{T,t}, C_{N,t}\right)}{u_2\left(C_{T,t}, C_{N,t}\right)} = \frac{\alpha}{1-\alpha} \left(\frac{C_{N,t}}{C_{T,t}} \right)^{\frac{1}{\xi}} \tag{2}$$

Finally, consumption of nontradables must equal the endowment in equilibrium:

$$C_{N,t} = Y_{N,t} \tag{3}$$

Financiers Financiers intermediate funds by holding a zero capital portfolio of foreign currency bonds $B^*_{F,t}$ and local currency bonds $B_{F,t}$ such that $B_{F,t} + \mathcal{E}_t B^*_{F,t} = 0$. They exhibit mean-variance preferences of the form:

$$\mathbb{E}_t \left[\Theta_{t+1} \tilde{R}^*_t B^*_{F,t} \right] + \frac{\omega}{2} var_t \left(\tilde{R}^*_t B^*_{F,t} \right) \text{ where } \tilde{R}^*_t = R^* - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$$

Note that the SDF of domestic households Θ_{t+1} enters the objective function of financiers and $\omega > 0$ measures the (additional) degree of risk aversion of financiers. Moreover, R^* is the (constant) world interest rate and $\tilde{R}^*_t$ denotes the carry trade return in foreign currency from period t to $t+1$. The FOC with respect to $B^*_{F,t}$ yields the modified UIP condition of the model:

$$R^* \mathbb{E}_t \left[\Theta_{t+1} \right] - R_t \mathbb{E}_t \left[\Theta_{t+1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right] = \overbrace{\omega \sigma_t^2 B^*_{F,t}}^{\text{Risk Sharing Wedge}} \quad \text{where } \sigma_t^2 = R_t^2 var_t \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right) \tag{4}$$

The ex-ante UIP deviation, referred to as the international risk sharing wedge by Itskhoki & Mukhin (2023), is driven by the intermediated amount of financiers $B^*_{F,t}$ and the depth of

FX markets $\omega\sigma_t^2 > 0$. Note that a higher (lower) $\omega\sigma_t^2$ implies shallower (deeper) FX markets. Importantly, FX market depth is state-dependent in the sense that its variation comes from changes in conditional exchange rate volatility σ_t^2. Finally, the domestic currency profits of financiers in period t are given by $\Pi_{F,t} = \left(R_{t-1} - R^*_{t-1}\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right) B_{F,t-1}$

Portfolio Investors Similar to financiers, portfolio investors hold a zero capital portfolio $(B^*_{P,t}, B_{P,t})$ such that $B_{P,t} + \mathcal{E}_t B^*_{P,t} = 0$. However, they are non-optimizing agents to the extent that they randomly buy (sell) foreign currency bonds $B^*_{P,t} > 0$ ($B^*_{P,t} < 0$) and sell (buy) domestic currency bonds $B_{P,t} < 0$ ($B_{P,t} > 0$). More precisely, $B^*_{P,t}$ is exogenous and follows a first-order Markov process. The profits of portfolio investors are $\Pi_{P,t} = \left(R_{t-1} - R^*_{t-1}\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right) B_{P,t-1}$

Bond Market Clearing Overall, market clearing in the domestic bond market requires

$$B_{F,t} + B_t + B_{P,t} = 0$$

and define the net foreign asset position in foreign currency as

$$B^*_t = B^*_{F,t} + B^*_{P,t} \tag{5}$$

which implies $B^*_t = B_t/\mathcal{E}_t$ by domestic bond market clearing and the balance sheet equations of the financial sector.

Resource Constraint Consolidating the household's budget constraint by inserting the market clearing condition for nontradables and profits of the financial sector yields the resource constraint:

$$B^*_t - B^*_{t-1}R^* = Y_{T,t} - C_{T,t} \tag{6}$$

On aggregate, the domestic economy is borrowing in foreign currency at the world interest rate due to the assumption of domestic ownership of the financial sector.

Decentralized Equilibrium Given the exogenous process $\{B^*_{P,t}, Y_{T,t}, Y_{N,t}\}_{t=0}^{\infty}$ and initial condition B^*_{-1}, a competitive equilibrium is a sequence of prices $\{\mathcal{E}_t, R_t\}_{t=0}^{\infty}$ and implied $\{\sigma_t^2\}_{t=0}^{\infty}$, allocations $\{C_{T,t}, C_{N,t}\}_{t=0}^{\infty}$ and bond positions $\{B^*_t, B^*_{F,t}\}_{t=0}^{\infty}$ such that:

1. Households and financiers optimize (1), (2), (4)
2. Clearing in the goods markets (3), (6) and the domestic bond market (5)

3. Transversality condition on net foreign assets holds:

$$\lim_{T\to\infty} \frac{B^*_T}{(R^*)^T} = 0$$

FX Interventions The central bank engages in sterilized FXI by buying (selling) foreign reserves $B^*_{M,t}$ and selling (buying) domestic bonds $B_{M,t}$ such that $B_{M,t} + \mathcal{E}_t B^*_{M,t} = 0$. Crucially, it is subject to a non-negativity constraint on FX reserves (NNCR):

$$B^*_{M,t} \geq 0$$

The central bank's profits from its bond portfolio are given by $\Pi_{M,t} = \left(R_{t-1} - R^*_{t-1}\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right) B_{M,t-1}$. Given these additional bond holdings in the economy, the market clearing condition changes to

$$B_{F,t} + B_t + B_{P,t} + B_{M,t} = 0$$

and as a result, the net foreign asset position in foreign currency is now given by

$$B^*_t = B^*_{F,t} + B^*_{M,t} + B^*_{P,t} \tag{7}$$

Policy Problem Every period t, the central bank chooses FX reserves $B^*_{M,t}$ in a discretionary fashion. Consequently, the policy problem can be formulated as follows:

$$\max_{\{C_{T,t}, B^*_t, \mathcal{E}_t, R_t, B^*_{M,t}, \sigma^2_t\}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \left[u(C_{T,t+s}, Y_{N,t+s})\right] \tag{8}$$

subject to the constraints

$$R_{t+s}\mathbb{E}_{t+s}\left[\Theta_{t+s+1}\frac{\mathcal{E}_{t+s}}{\mathcal{E}_{t+s+1}}\right] = 1 \qquad \text{(Household Euler Equation)}$$

$$\mathcal{E}_{t+s} = \frac{u_1(C_{T,t+s}, Y_{N,t+s})}{u_2(C_{T,t+s}, Y_{N,t+s})} \qquad \text{(Expenditure Switching)}$$

$$B^*_{t+s} - B^*_{t+s-1}R^* = Y_{T,t+s} - C_{T,t+s} \qquad \text{(Resource Constraint)}$$

$$R^*\mathbb{E}_{t+s}\left[\Theta_{t+s+1}\right] = 1 + \overbrace{\omega\sigma^2_{t+s}\left(B^*_{t+s} - B^*_{M,t+s} - B^*_{P,t+s}\right)}^{\text{Risk Sharing Wedge}} \qquad \text{(International Risk Sharing)}$$

$$\sigma^2_{t+s} = R^2_{t+s} var_{t+s}\left(\frac{\mathcal{E}_{t+s}}{\mathcal{E}_{t+s+1}}\right) \qquad \text{(Conditional Exchange Rate Volatility)}$$

$$B^*_{M,t+s} \geq 0 \quad \text{(Non-Negativity Constraint on Reserves)}$$

Constrained Planner Equilibrium Given the exogenous process $\{B^*_{P,t}, Y_{T,t}, Y_{N,t}\}_{t=0}^{\infty}$ and

initial condition B^*_{-1}, a competitive equilibrium is a sequence of prices $\{\mathcal{E}_t, R_t\}_{t=0}^{\infty}$ and implied $\{\sigma_t^2\}_{t=0}^{\infty}$, allocations $\{C_{T,t}, C_{N,t}\}_{t=0}^{\infty}$, bond positions $\{B^*_t, B^*_{F,t}\}_{t=0}^{\infty}$ and FXI policy $\{B^*_{M,t}\}_{t=0}^{\infty}$ such that:

1. Households and financiers optimize (1), (2), (4)
2. The central bank solves the policy problem (8)
3. Clearing in the goods markets (3), (6) and the domestic bond market (7)
4. Transversality condition on net foreign assets holds:

$$\lim_{T\to\infty} \frac{B^*_T}{(R^*)^T} = 0$$

2.2 Recursive Representation

Using the standard notation, primes indicate next period values and $S = \{Y_T, Y_N, B^*_P\}$ denotes the exogenous states.

Decentralized Equilibrium The household takes the aggregate timepath for bonds as given by $B^{*'} = H(B^*, S)$ and solves the Euler equation

$$u_1(R^* b^* + Y_T - b^{*'}, Y_N) = \beta R \mathbb{E}_S \left[u_1(R^* b^{*'} + Y'_T - H(B^{*\prime}, S'), Y'_N) \right]$$

where u_1 denotes the marginal utility with respect to tradables and

$$R = \frac{R^* \mathbb{E}_S[\Theta'] - \omega\sigma^2 (B^* - B^*_P)}{\mathbb{E}_S\left[\Theta' \frac{\mathcal{E}}{\mathcal{E}'}\right]}, \quad \Theta' = \beta \left(\frac{C'}{C}\right)^{\frac{1-\sigma\xi}{\xi}} \left(\frac{C'_T}{C_T}\right)^{-\frac{1}{\xi}}$$

$$C_T = R^* B^* + Y_T - B^{*\prime}, \quad C = \left[\alpha C_T^{\frac{\xi-1}{\xi}} + (1-\alpha) Y_N^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}$$

$$\mathcal{E} = \frac{\alpha}{1-\alpha}\left(\frac{Y_N}{C_T}\right)^{\frac{1}{\xi}}, \quad \sigma^2 = \left(\mathbb{E}_S\left[\Theta' \frac{1}{\mathcal{E}'}\right]\right)^{-2} \left(\mathbb{E}_S\left[\left(\frac{1}{\mathcal{E}'}\right)^2\right] - \left(\mathbb{E}_S\left[\frac{1}{\mathcal{E}'}\right]\right)^2\right)$$

A decentralized equilibrium is a law of motion H such that the implied savings policy $b^{*'} = b^{*'}(b^*, B^*, S)$ verifies

$$b^{*'}(B^*, B^*, S) = H(B^*, S) \quad \text{for all } B^*, S$$

Constrained Planner The constrained planner solves

$$V(B^*, S) = \max_{B^{*\prime}} \left\{ u(R^* B^* + Y_T - B^{*\prime}, Y_N) + \beta \mathbb{E}_S V(B^{*'}, S') \right\}$$

subject to the market clearing conditions and the UIP constraint (i.e., UIP condition combined with NNCR)

$$\frac{1}{\omega \sigma^2} \left[R^* \mathbb{E}_S \left[\Theta' \right] - 1 \right] + B_P^* \leq B^{*\prime} \leq R^* B^* + Y_T$$

We see that the planner internalizes the impact of the savings choice $B^{*\prime}$ on consumption, the exchange rate and hence the UIP constraint. Once we have solved for the constrained planner's policy function for $B^{*\prime}$ we can simply back out the optimal FXI policy using the UIP condition

$$B_M^* = B^{*\prime} - B_P^* + \frac{1}{\omega \sigma^2} \left[1 - R^* \mathbb{E}_Y \left[\Theta' \right] \right]$$

3 Optimal FXI

We proceed by analytically studying the optimal time-consistent use of FXI in the model described in Section 2. We first characterize the first best and second, describe key results obtained from the second best.

First Best If households had direct, frictionless access to the foreign currency bond, the first order condition associated with that asset would be:

$$R^* \mathbb{E}_t \left[\Theta_{t+1} \right] = 1$$

In an economy without a NNCR, Itskhoki & Mukhin (2023) show that this condition holds in the Ramsey equilibrium and the associated optimal FXI policy is given by:

$$B_{M,t}^* = B_t^* - B_{P,t}^* \tag{9}$$

Consequently, optimal policy eliminates the international risk sharing wedge. In particular, the central bank makes costly intermediation provided by financiers redundant ($B_{F,t}^* = 0$). A corollary is that the first best optimal policy in an economy with portfolio shocks only is associated with no uncertainty about the exchange rate ($\sigma_t^2 = 0$) i.e., a peg.

Second Best In the existence of a NNCR, the central bank is generally unable to completely eliminate the risk-sharing wedge in all periods as in the first best. One can easily

imagine an equilibrium in which the economy holds a relatively large amount of debt and/or is confronted with considerable portfolio outflows such that $B_t - B^*_{M,t} < 0$ in which case the NNCR becomes binding. The characteristics of such a constrained equilibrium are described in the following proposition:

Proposition 1 *Consider an equilibrium in period $t = s$ with a binding NNCR ($B^*_s - B^*_{P,s} < 0$) and where portfolio outflows are associated with an improvement in the net foreign asset position ($\partial B^*_s / \partial B^*_{P,s} > 0$). Then, the equilibrium exhibits the following properties:*

(a) $\partial \sigma^2_s / \partial B^*_{P,s} > 0$: Portfolio outflows lead to an elevated conditional exchange rate volatility.

(b) $\partial \mathbb{E}_s \left[\Theta_{s+1}\right] / \partial B^*_{P,s} < 0$: The stochastic discount factor between period s and $s+1$ is expected to decrease with portfolio outflows.

The crucial relationship underpinning Proposition 1 is that we consider an economy in which portfolio outflows generate an improvement in the net foreign asset position. While this seems to be the empirically plausible case for most countries, we provide sufficient conditions in Appendix A for this relation to hold as well as the proof of Proposition 1. Since portfolio outflows $B^*_{P,t}$ are modeled equivalently as foreign reserves $B^*_{M,t}$, the relationship can be reinterpreted in terms of the crowding-out effect of foreign reserve accumulation. In particular, the crowding-out effect on private external savings must be less than one-for-one. If the conditions specified in Proposition 1 apply, state-dependent FX market depth $\omega \sigma^2_t$ magnifies the effect of portfolio outflows on the risk sharing wedge. Intuitively, portfolio outflows cause negative ex-ante UIP deviations not only because they directly shift the positioning in FX markets but FX risk also rises endogenously for which intermediaries, in turn, must be compensated. At the same time, the stochastic discount factor is expected to decrease in the constrained period: Households assign a higher marginal value to tradable consumption during the constrained period compared to the subsequent period.

Having the provided equilibrium characteristics in mind, we next formally study the second

best by setting up the corresponding Lagrangian:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \Bigg[& u\left(C_{T,t+s}, Y_{N,t+s}\right) \\ & - \lambda_{RC,t+s}\left(B^*_{t+s-1}R^* - B^*_{t+s} + Y_{T,t+s} - C_{T,t+s}\right) \\ & - \lambda_{IRS,t+s}\left(1 + \omega\sigma^2_{t+s}\left(B^*_{t+s} - B^*_{M,t+s} - B^*_{P,t+s}\right) - R^*\mathbb{E}_{t+s}\left[\Theta_{t+s+1}\right]\right) \\ & - \lambda_{EERV,t+s}\left(\sigma^2_{t+s}\beta^2\mathbb{E}_t\left[\frac{u_{2,t+s+1}}{u_{1,t+s}}\right]^2 - \mathbb{E}_{t+s}\left[\left(\frac{u_{2,t+s+1}}{u_{1,t+s+1}}\right)^2\right] + \mathbb{E}_{t+s}\left[\frac{u_{2,t+s+1}}{u_{1,t+s+1}}\right]^2\right) \\ & - \lambda_{NNCR,t+s}\left(-B^*_{M,t+s}\right)\Bigg]\end{aligned}$$

where $u_{j,t} \equiv u_j\left(C_{T,t}, Y_{N,t}\right) \forall t$ and $j \in \{1,2\}$. Note that we combined the expenditure switching condition and the household Euler equation with the conditional exchange rate volatility constraint following Itskhoki & Mukhin (2023). The FOCs of the Lagrangian with respect to $\{C_{T,t}, B^*_t, B^*_{M,t}, \sigma^2_t\}$ are given by:

$$u_{1,t} + \lambda_{RC,t} + \lambda_{IRS,t}R^*\left(-\frac{u_{11,t}}{u_{1,t}}\right)\mathbb{E}_t\left[\Theta_{t+1}\right] - \lambda_{EERV,t}\sigma^2_t\beta^2 2\left(-\frac{u_{11,t}}{u_{1,t}}\right)\mathbb{E}_t\left[\frac{u_{2,t+1}}{u_{1,t}}\right]^2 = 0 \quad (10)$$

$$\lambda_{RC,t} - \beta R^*\mathbb{E}_t\left[\lambda_{RC,t+1}\right] - \lambda_{IRS,t}\omega\sigma^2_t = 0 \quad (11)$$

$$\lambda_{IRS,t}\omega\sigma^2_t + \lambda_{NNCR,t} = 0 \quad (12)$$

$$-\lambda_{IRS,t}\omega\left(B^*_t - B^*_{M,t} - B^*_{P,t}\right) - \lambda_{EERV,t}\beta^2\mathbb{E}_t\left[\frac{u_{2,t+1}}{u_{1,t}}\right]^2 = 0 \quad (13)$$

Suppose FXI is unconstrained in period t ($\lambda_{NNCR,t} = 0$) but the NNCR is possibly binding in period $t+1$ ($\mathbb{E}_t\left[\lambda_{NNCR,t+1}\right] > 0$). By (12) and (13), we have $\lambda_{NNCR,t} = 0 \Rightarrow \lambda_{IRS,t} = \lambda_{EERV,t} = 0$. Then, combine (10) and (11) to obtain an Euler equation of the form:

$$\begin{aligned}u_{1,t} = \beta R^*\mathbb{E}_t\Bigg[& u_{1,t+1} + \lambda_{IRS,t+1}R^*\left(-\frac{u_{11,t+1}}{u_{1,t+1}}\right)\mathbb{E}_{t+1}\left[\Theta_{t+2}\right] \\ & - \lambda_{EERV,t+1}\sigma^2_{t+1}\beta^2 2\left(-\frac{u_{11,t+1}}{u_{1,t+1}}\right)\mathbb{E}_{t+1}\left[\frac{u_{2,t+2}}{u_{1,t+1}}\right]^2\Bigg]\end{aligned} \quad (14)$$

Next, express (14) in terms of $\lambda_{NNCR,t+1}$ using (12) and (13):

$$u_{1,t} = \beta R^* \mathbb{E}_t \Bigg[u_{1,t+1} - \lambda_{NNCR,t+1} \left(-\frac{u_{11,t+1}}{u_{1,t+1}} \right) \frac{R^*}{\omega \sigma^2_{t+1}} \mathbb{E}_{t+1} \left[\Theta_{t+2} \right] + \lambda_{NNCR,t+1} \left(-\frac{u_{11,t+1}}{u_{1,t+1}} \right) 2 \left(B^*_{P,t+1} - B^*_{t+1} \right) \Bigg] \tag{15}$$

Rearranging and substituting the IRS condition of period t and $t+1$ into (15) allows us to obtain our first result:[3]

Theorem 1 *If the use of FXI is unconstrained in period t ($\lambda_{NNCR,t} = 0$) but the NNCR is possibly binding in period $t+1$ ($\mathbb{E}_t \left[\lambda_{NNCR,t+1} \right] > 0$), the optimal discretionary FXI policy in period t is given by:*

$$B^*_{M,t} = B^*_t - B^*_{P,t} + \frac{\beta R^*}{\omega \sigma^2_t u_{1,t}} \mathbb{E}_t \left[\lambda_{NNCR,t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(3 \left(B^*_{P,t+1} - B^*_{t+1} \right) - \frac{1}{\omega \sigma^2_{t+1}} \right) \right] \tag{16}$$

which calls for a smaller (larger) intervention relative to the first best if the third term on the RHS is positive (negative).

Theorem 1 states that the optimal intervention in period t in anticipation of a binding NNCR in period $t+1$ can either be liquidity-providing in the sense that the central bank sells off more reserves compared to the first best ($B^*_{M,t} < B^*_t - B^*_{P,t}$) or liquidity-absorbing to the extent that the central bank purchases more reserves than in the first best ($B^*_{M,t} > B^*_t - B^*_{P,t}$). If the NNCR is expected not to bind in any possible states in period $t+1$, the optimal intervention in period t corresponds to the first best.

To grasp the intuition behind Theorem 1, recall that the friction in our model is the risk sharing wedge. While the wedge can always be completely eliminated in the first best, this is not feasible anymore in the second best. By definition, a period $t = s$ characterized by a binding NNCR is associated with negative intermediated funds $B^*_{F,t} = B^*_s - B^*_{P,s} < 0$ and consequently also a negative risk sharing wedge $\omega \sigma^2_s B^*_{F,t} < 0$. The aim of the second best policy in this context is to smooth out risk sharing wedges in an intertemporal sense. This can either be done by a liquidity-providing intervention $B^*_{M,s-1} < B^*_{s-1} - B^*_{P,s-1}$ and a resulting positive risk sharing wedge $\omega \sigma^2_{s-1} B^*_{F,s-1} > 0$ or a liquidity-absorbing intervention $B^*_{M,s-1} > B^*_{s-1} - B^*_{P,s-1}$ and a resulting negative risk sharing wedge $\omega \sigma^2_{s-1} B^*_{F,s-1} < 0$.[4]

[3] For the complete proof, see Appendix A.

[4] While the liquidity-providing intervention is described by Itskhoki & Mukhin (2023), their first order

Any given portfolio outflow shock has less of an effect on the risk sharing wedge in the constrained period $t = s$ if the conditional exchange rate volatility σ_s^2 is lower. Crucially, entering the constrained period with a more appreciated exchange rate dampens the conditional exchange rate volatility σ_s^2. Unlike households, the central bank internalizes this general equilibrium effect. By accumulating reserves beyond what the first best suggests in period $s-1$, the central bank forces the economy to save more on aggregate which results in a relatively more appreciated exchange rate in the constrained period s. This, in turn, decreases the absolute size of the risk sharing wedge $\omega\sigma_s^2 B_{F,t}^*$ due to a lower conditional exchange rate volatility σ_s^2. Overall, a liquidity-absorbing intervention $B_{M,s-1}^* > B_{s-1}^* - B_{P,s-1}^*$ helps to smooth risk sharing wedges over time because it compresses the absolute risk sharing wedge in the constrained period s at the cost of causing a negative risk sharing wedge in period $s-1$. Whether the liquidity-absorbing or liquidity-providing motive dominates describes the following proposition that we prove in Appendix A:

Proposition 2 *Suppose the use of FXI is unconstrained in period t ($\lambda_{NNCR,t} = 0$) but the NNCR is possibly binding in period $t+1$ ($\mathbb{E}_t[\lambda_{NNCR,t+1}] > 0$). Then, a precautionary level of reserves exists (i.e., the optimal level of reserves is higher relative to the first best) if and only if:*

$$R^*\mathbb{E}_t[\Theta_{t+2}] < \frac{2}{3} - \frac{cov\left(\lambda_{NNCR,t+1}\left(\frac{-u_{11,t+1}}{u_{1,t+1}}\right), R^*\mathbb{E}_{t+1}[\Theta_{t+2}]\right)}{\mathbb{E}_t\left[\lambda_{NNCR,t+1}\left(\frac{-u_{11,t+1}}{u_{1,t+1}}\right)\right]} \tag{17}$$

which is more likely to hold the lower the (expected) stochastic discount factor is between the potentially constrained period $t+1$ and period $t+2$.

According to Proposition 2, the liquidity-absorbing motive prevails if the stochastic discount factor is expected to drop sufficiently in the constrained period. In other words, if the economy is expected to experience a relatively severe drop in tradable consumption, it is optimal for the central bank to hold more reserves relative to the first best (i.e., absorb liquidity) before the potential crisis unfolds. The question remains how expected portfolio outflows and the strength of the liquidity-providing versus liquidity-absorbing motive interact:

Proposition 3 *Suppose the use of FXI is unconstrained in period t ($\lambda_{NNCR,t} = 0$) but the NNCR is possibly binding in period $t+1$ ($\mathbb{E}_t[\lambda_{NNCR,t+1}] > 0$). Then, sufficiently large (small) expected portfolio outflows in period $t+1$ calls for an optimal intervention that offsets the contemporaneous portfolio outflow less (more) than one-for-one.*

approximation of the equilibrium system fails to capture the liquidity-absorbing intervention.

Proposition 3 is obtained by combining Proposition 1(b), Theorem 1 and Proposition 2. It has two stark policy implications: First, the central bank should provide liquidity and intervene aggressively in response to contemporaneous portfolio outflows only if future outflows are anticipated to be relatively mild. Second, absorbing liquidity and maintaining a precautionary level of reserves is appropriate when severe future portfolio outflows are expected.

4 Quantitative Analysis

In this section, we begin by describing the calibration and then analyze the numerical results, highlighting the key insights in remarks.

4.1 Calibration

We calibrate the model to Malaysia, a small open emerging economy with a central bank that is an active user of FXI. We think that Malaysia is a fitting case to explore through the lens of our model since its central bank uses FXI continuously to stabilize the exchange rate, especially in periods of large and volatile two-way capital flows (Aziz, 2019).

Table 1: Calibration

Description	Value	Source/Target
World interest rate, quarterly	$R^* = 1.01$	Standard value DSGE-SOE
Relative risk aversion	$\sigma = 2$	Standard value DSGE-SOE
Elasticity of substitution of T-NT goods	$\xi = 0.83$	Conservative value, Bianchi (2011)
Weight on traded goods in CES aggregator	$\alpha = 0.39$	Malaysia's economy
Subjective discount factor, quarterly	$\beta = 0.9887$	NFA-GDP ratio, Malaysia's economy
Financiers' risk aversion	$\omega = 21$	FX market depth, Davis et al. (2023)

The parameter values of the model are listed in Table 1. We calibrate the model at a quarterly frequency. The annualized world interest rate is 4% and the relative risk aversion is set to $\sigma = 2$, both parameter values are standard in the small open economy literature. Regarding the crucial parameter guiding the elasticity of substitution of tradable to nontradable goods ξ, we choose a value $\xi = 0.83$ that is at the upper bound of the empirical estimates in the literature following Bianchi (2011). We calibrate the remaining structural parameters α, β and ω to Malaysia's economy. In particular, we set the weight on traded goods in the CES aggregator equal to the average share of the tradable component in GDP which is around

39%. Furthermore, the value of the subjective discount factor β targets the observed private NFA-to-GDP ratio and the financiers' risk aversion ω is set to have an average FX market depth $\omega\sigma^2 = 0.05$ in the decentralized economy without FXI consistent with Davis et al. (2023) and Adrian et al. (2022). Both of these targeted moments crucially depend on the FXI regime. For example, a higher level of reserves increases the net foreign asset position and an FXI regime that dampens real exchange rate volatility is associated with deeper FX markets in the model. Since Malaysia is a frequent user of FXI, we need to take its FXI regime into account when calibrating β and ω. In Appendix B, we provide details on the calibration.

For the endowment part of the exogenous driving forces in our model $\{Y_{T,t}, Y_{N,t}\}$, we follow the standard methodology in the literature and use the cyclical components of tradable and nontradable GDP at constant prices, retrieved from the central bank of Malaysia. We classify agriculture, mining and quarrying and manufacturing as tradables and classify the rest of the components of GDP as nontradables. To obtain the cyclical components, we remove a cubic trend and seasonality from the natural logarithm of tradables and nontradables following Schmitt-Grohé & Uribe (2016).

To estimate a process of the exogenous portfolio outflows $\{B^*_{P,t}\}$, we first calculate an empirical measure of the risk sharing wedge (or ex-ante UIP deviation) $\widehat{RSW}_t$ on a quarterly basis:

$$\widehat{RSW}_t = \hat{R}^*_t - \hat{R}_t \frac{\hat{\mathcal{E}}_t}{\widehat{\mathbb{E}\left[\mathcal{E}_{t+1}\right]}} \tag{18}$$

where $\hat{R}^*_t$ is the effective federal funds rate, $\hat{R}_t$ the Malaysia overnight policy rate, $\mathcal{E}_t$ the USD to Malaysian Ringgit (MYR) spot rate and $\widehat{\mathbb{E}\left[\mathcal{E}_{t+1}\right]}$ the Bloomberg composite forecast for USDMYR next quarter.[5] Next, we make use of the international risk sharing condition of the model in order to back out the exogenous process for portfolio outflows:

$$\hat{B}^*_{P,t} = \hat{B}^*_t - \hat{B}^*_{M,t} - \frac{\widehat{RSW}_t}{\omega\hat{\sigma}^2_t} \tag{19}$$

where we obtain the quarterly measure of Malaysia's NFA position $\hat{B}^*_t$ from the IMF IFS, the quarterly proxy for Malaysia's FXI from Adler et al. (2024)[6] and use the three month implied USDMYR volatility from Bloomberg to retrieve $\hat{\sigma}^2_t$.

We then incorporate the exogenous states in the model as an AR(1) process $\log \mathbf{S}_t =$

[5] We ignore the stochastic discount factor in the calculation.

[6] Since Adler et al. (2024) only provide FXI data in terms of first differences, we additionally use IMF IFS data to account for the level which is needed to compute $\hat{B}^*_{M,t}$.

$\rho \log \mathbf{S}_{t-1} + \varepsilon_t$ where $\mathbf{S}_t = [Y_{T,t}, Y_{N,t}, B_{P,t}]'$ are obtained as described above for the period 2010:Q1 to 2023:Q4. The error term $\varepsilon_t = [\varepsilon_{T,t}, \varepsilon_{N,t}, \varepsilon_{P,t}]'$ follows a trivariate normal distribution with zero mean and contemporaneous variance-covariance matrix $\mathbf{V}$ and ρ is a 3×3 matrix consisting of the autocorrelation terms:

$$\mathbf{V} = \begin{bmatrix} 0.0005571 & 0.0006227 & 0.0004702 \\ 0.0006227 & 0.0009429 & 0.0000765 \\ 0.0004702 & 0.0000765 & 0.0559356 \end{bmatrix}, \quad \rho = \begin{bmatrix} 0.8092629 & -0.3160263 & -0.0342763 \\ 0.1808142 & 0.3985611 & -0.0374895 \\ 0.3280863 & -0.8431225 & 0.4016298 \end{bmatrix}$$

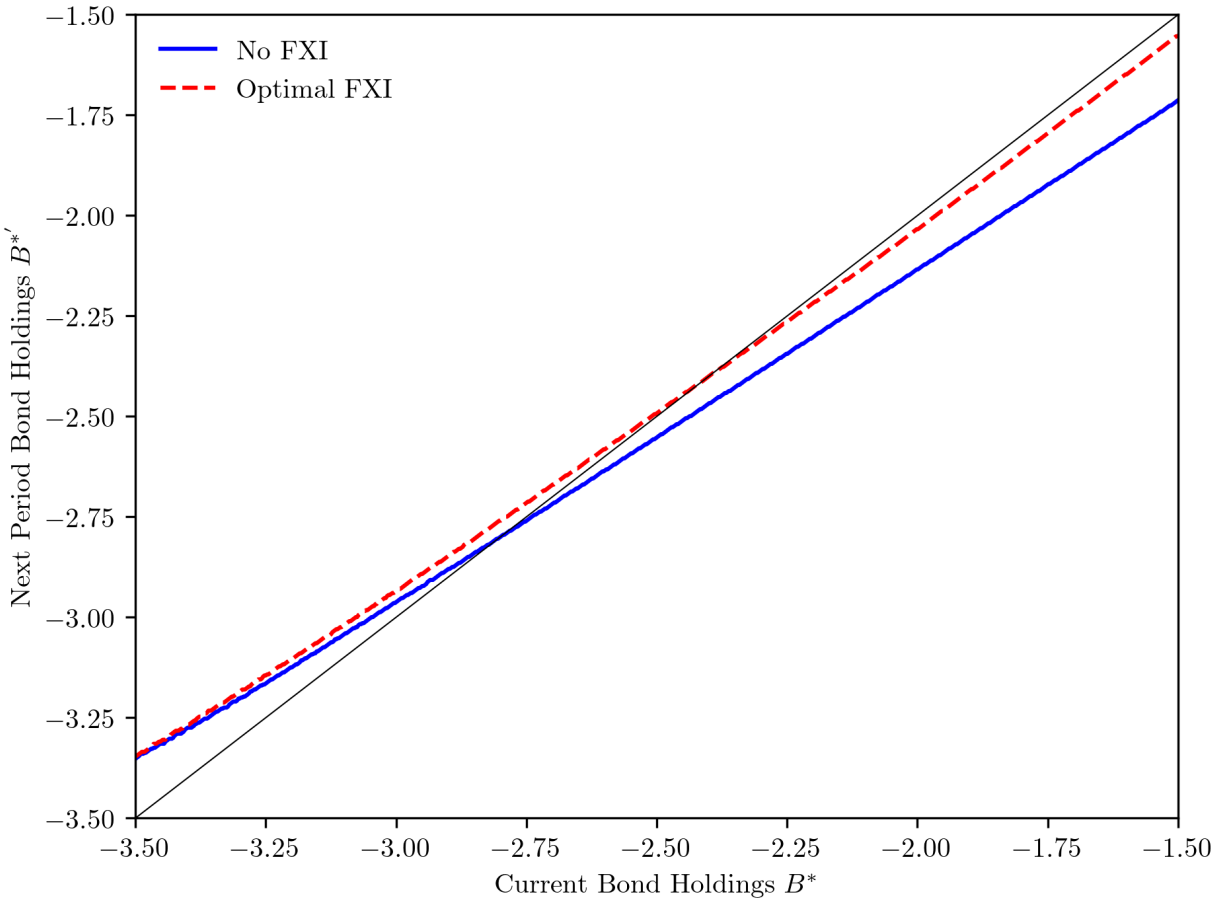
Without loss of generality, we normalize the mean of the endowment processes to one while the mean of portfolio outflows is $\bar{B}_P = -3.06$, indicating that Malaysia experiences portfolio inflows on average. Furthermore, the standard deviations in the data are $\sigma_{Y_T} = 0.029$, $\sigma_{Y_N} = 0.037$ and $\sigma_{B_P^*} = 0.567$. The tradable and nontradable endowment process are highly positively correlated with $\sigma_{Y_T,Y_N} = 0.795$ while portfolio outflows are weakly negatively correlated with the endowment process with $\sigma_{B_P^*,Y_N} = -0.092$ and $\sigma_{B_P^*,Y_N} = -0.192$. This weak correlation between portfolio outflows and the endowment process is a manifestation of the well-documented exchange rate disconnect i.e., the fact that exchange rates are to a large extent driven by factors other than the fundamentals of the economy.

The exogenous state variables S are discretized into a first-order Markov process with four grid points for each variable while the endogenous state variable B^* contains 1500 grid points. Finally, we solve the decentralized equilibrium and the constrained planner's problem using time iteration.

4.2 Policy Functions

Figure 1 shows the policy functions for bond holdings B^* separately for the decentralized equilibrium without FXI as shown by the blue solid line and the constrained planner's solution (optimal FXI) as shown by the red dotted line. Since bond holdings B^* are equivalent to the net foreign asset position (NFA) of the country, Figure 1 shows the evolution of the NFA conditional on the current NFA. For low levels of the current NFA position, the economy increases its NFA position next period while the opposite is true for a relatively high current NFA position. As a result, both lines intersect the 45 degree line and a stationary equilibrium exists in both economies. Intuitively, the financiers' risk aversion gives rise to an upward supply of funds to the domestic economy which limits the amount of external borrowing in equilibrium. In addition, Figure 1 reveals that the constrained planner's solution exhibits a

higher next period NFA position $B^{*\prime}$ conditional on the current NFA position B^*, indicating the importance of the level of FX reserves for the steady state NFA position.



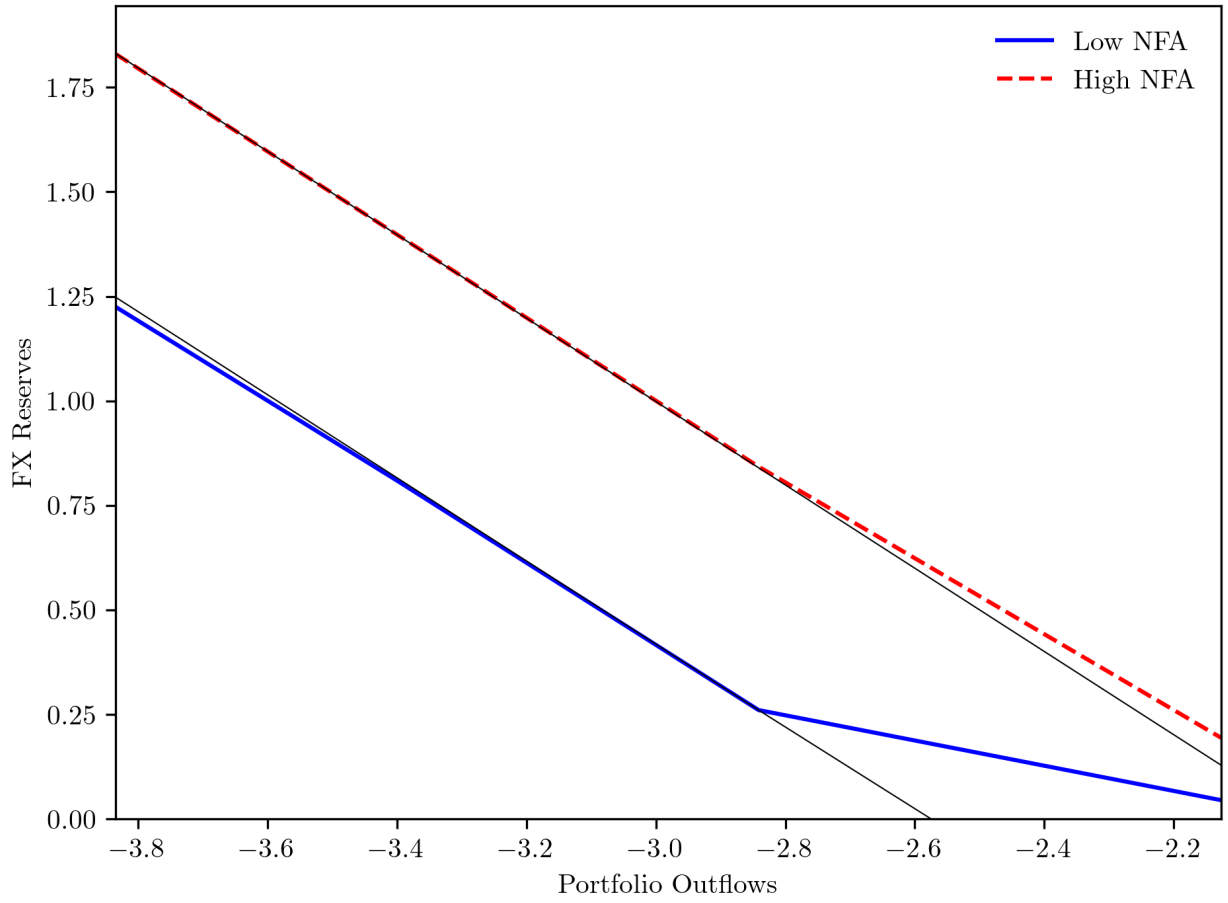
Notes: The plot shows the policy function for bond holdings of the competitive equilibrium without FXI (blue solid line) and the constrained planner's solution (red dotted line) for the exogenous state $\{Y_T, Y_N, B_P^*\} = \{1.0, 1.0, -2.1\}$.

Figure 1: Policy function for bond holdings B^*

To further shed light on the characteristics of optimal FXI policy, Figure 2 shows the optimal level of FX reserves B_M^* as a function of portfolio outflows B_P^*, distinguishing by states with a relatively low and high NFA position. The key analytical results derived in Section 3, are quantitatively confirmed in the policy functions shown. More specifically, for a relatively high NFA position, the optimal policy offsets portfolio outflows approximately one-for-one as dictated by the first best policy. The implied first best policy is depicted by the black solid lines. In the optimal constrained policy regime, a relatively high NFA position is associated with a considerable level of FX reserves, implying that the lower bound on reserves is less relevant in this scenario and as a result, the reaction to portfolio flows resembles the first best. In contrast, the lower bound on reserves becomes relevant if the economy finds itself

in a state with a relatively low NFA and FX reserve position as shown by the blue line. As described in Section 3, the constrained optimal policy is offsetting portfolio outflows more than one-for-one if they are relatively mild. This changes if portfolio outflows become severe, in which case the optimal policy is guided by maintaining a precautionary stock of reserves as suggested by Proposition 3 in Section 3 rather than aggressively offsetting portfolio flows. This leads us to our first remark which is closely related to Proposition 3.

Remark 1 *Conditional on the net foreign asset position, the degree to which optimal FXI policy offsets contemporaneous portfolio outflows diminishes as their magnitude increases.*

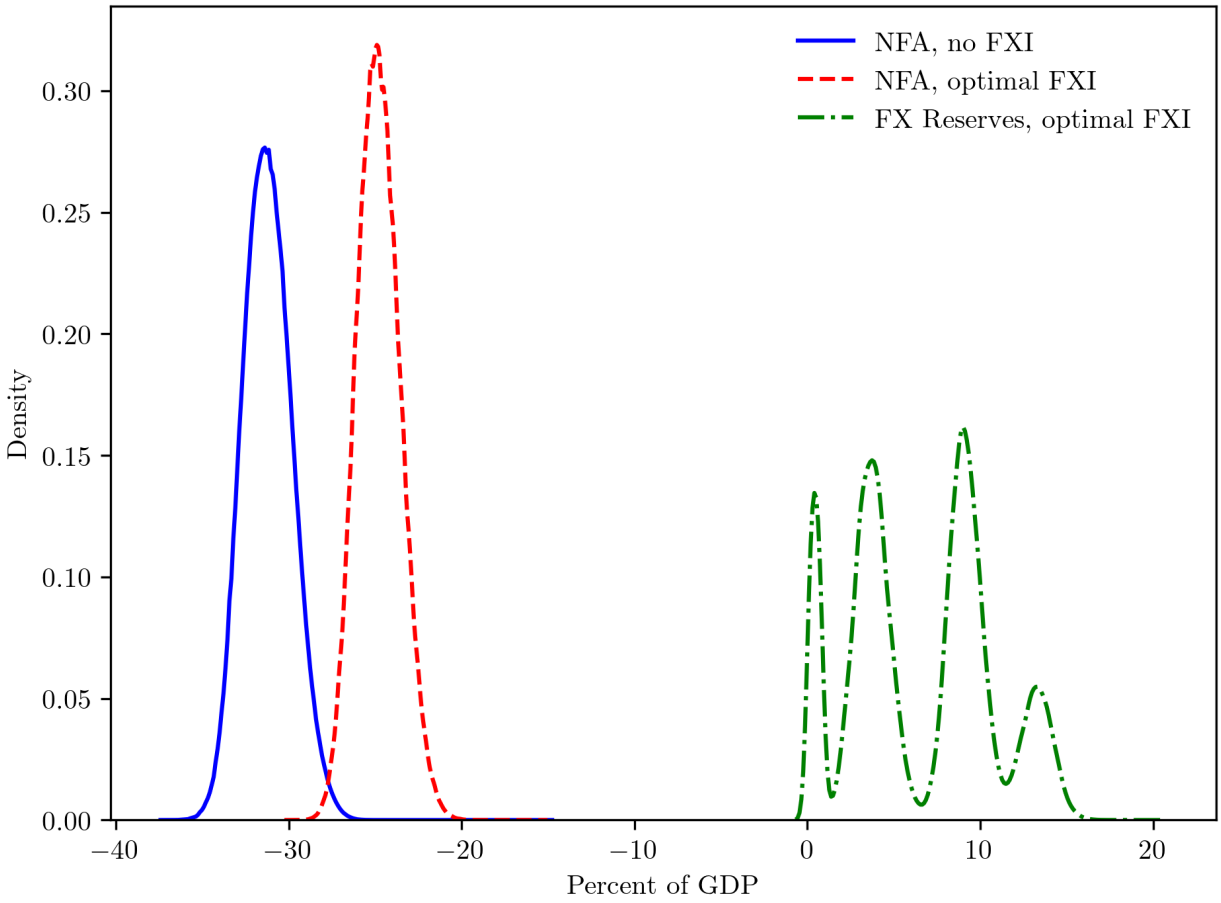


Notes: The plot shows the policy function for FX reserves of the constrained planner's solution conditional on relatively high current bond holdings (red dotted line) and relativley low current bond holdings (blue solid line). The black solid lines depict the corresponding policy functions under the first best for comparison.

Figure 2: Policy function for FX reserves

4.3 Simulations

We next simulate the model for 2×10^6 quarters in an economy without FXI and optimal FXI. More precisely, we first obtain a sequence of the exogenous state variables $\{Y_{T,t}, Y_{N,t}, B^*_{P,t}\}_{t=1}^{2\times 10^6}$ based on the estimated Markov process as described in Section 4.1. Using this exogenous sequence and the policy functions, we then compute the equilibrium variables for the economy without FXI and optimal FXI.

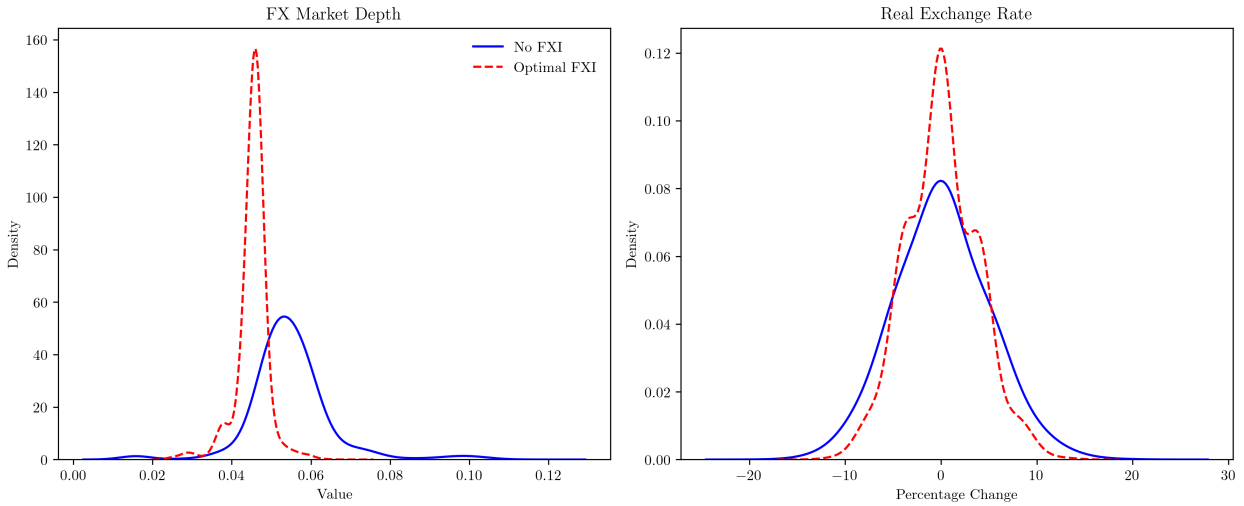


Notes: The plot shows the densities for the net foreign asset positions and FX reserves obtained from a simulation based on 2×10^6 quarters.

Figure 3: Densities for the net foreign asset position and FX reserves

Figure 3 shows the unconditional distribution of the net foreign asset position under no FXI and optimal FXI as well as of FX reserves under optimal FXI. Strikingly, the optimal policy regime changes the steady state of the economy: Under optimal FXI, the economy is saving more on average, resulting in a higher net foreign asset position. Furthermore, the variance of the net foreign asset position under optimal FXI is higher. Intuitively, optimal policy forces the economy to save more due to a positive level of reserves while relaxing the

friction in international financial markets by providing liquidity when needed. This enables households to better smooth consumption in response to endowment shocks since they are effectively more flexible in their savings decisions. Finally, the distribution of FX reserves is truncated at the lower bound on reserves. This indicates that it is optimal to run out of reserves in certain states.



Notes: The plot shows the densities for the FX market depth and the real exchange rate obtained from a simulation based on 2×10^6 quarters. The real exchange rate is computed as $\left[\alpha^{\xi} + (1-\alpha)^{\xi} \left(\frac{1}{\mathcal{E}_t}\right)^{1-\xi}\right]^{-\frac{1}{1-\xi}}$.

Figure 4: FX market conditions

How FX market conditions vary in the two economies is illustrated in Figure 4. It shows the unconditional distribution under the two policy regimes for FX market depth $\omega\sigma_t^2$ (left panel) and the real exchange rate (right panel). On both variables, optimal FXI policy has a stabilizing effect by decreasing the variance. By counteracting portfolio flows, the central bank reduces the part of the exchange rate fluctuations that are inefficient from the planner's perspective. However, it allows for efficient exchange rate fluctuations originating from endowment shocks, explaining why there is still sizable exchange rate variability even in the optimal policy regime. Looking at the left panel, we can also see that the optimal policy eliminates spikes in FX market depth. As we will explain in the next section, these spikes arise endogenously in episodes of portfolio outflows. Under the optimal policy regime, the central bank leans against portfolio outflows in these episodes by selling off reserves which has a stabilizing effect on FX market depth. Overall, Remark 2 summarizes key steady state characteristics of the optimal FXI policy.

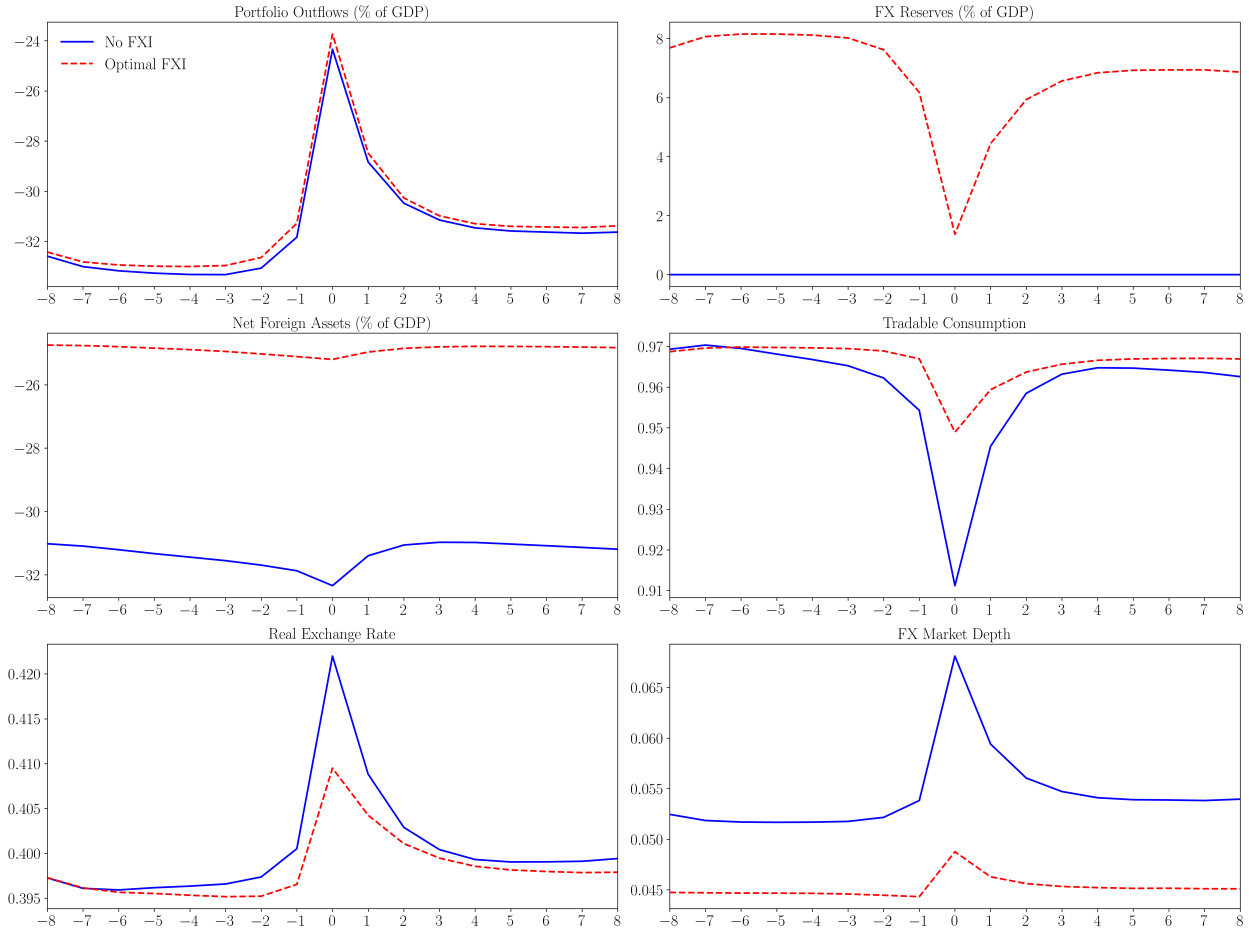
Remark 2 *The stochastic steady state of the optimal policy regime is characterized by:*

(a) a precautionary level of FX reserves and thus higher net foreign asset position

(b) deeper FX markets as the central bank acts as an FX liquidity provider

4.4 Dynamics

After examining the distributional characteristics, we now shift our focus to the model's dynamics. The subject of interest in this regard are episodes of portfolio outflows or inflows and the differential response of the economy due to the optimal use of FXI. To compute episodes of portfolio outflows, we gather all non-overlapping samples of a length of 17 quarters from the simulations in which the economy experiences the peak of outflows in the middle period of the sample. We then compute the average of relevant macroeconomic variables such as the real exchange rate or consumption over all samples.[7]



Notes: The plot depicts the average response in the model in episodes of portfolio outflows. One period corresponds to a quarter and period 0 coincides with the peak of outflows.

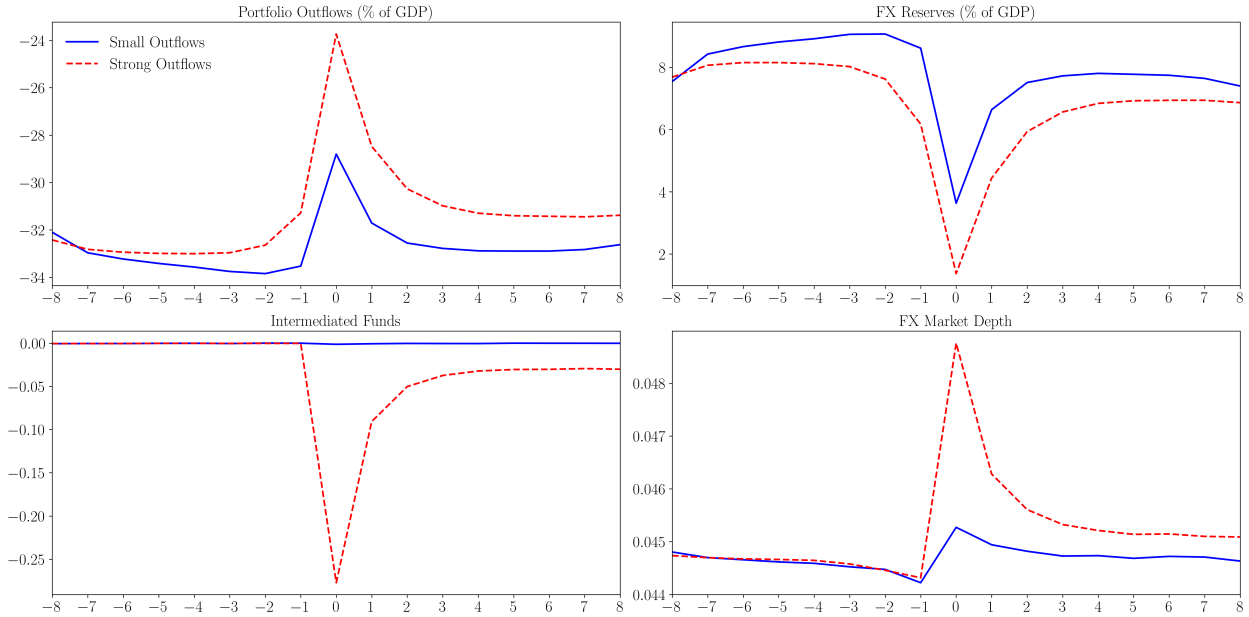
Figure 5: Episodes of portfolio outflows

[7]For portfolio inflows, we proceed analogously.

Figure 5 depicts episodes of portfolio outflows computed as described above. In particular, the peak of portfolio outflows, reaching magnitudes of more than 8% of GDP, corresponds to period $t = 0$ with window periods of 8 quarters so that the first period depicted corresponds to $t = -8$ and the last one to $t = 8$. The optimal FXI reaction mirrors the portfolio outflows, although the offset is only partial. At the peak of portfolio outflows in period $t = 0$, the central bank sold off on average around 6% of GDP in reserves which is smaller than the average 8% of GDP portfolio outflows. Nevertheless, this policy reaction stabilizes the economy to the extent that it dampens the depreciation and therefore also limits the deterioration in external financing conditions. As a result, the increase in the net foreign asset position and decrease in consumption is lower under the optimal policy regime. Besides stabilizing the exchange rate, optimal FXI policy also prevents spikes in FX market depth. Since shallow FX markets magnify the adverse effect of portfolio outflows, FX sales are especially effective under these circumstances.

The extent to which the optimal FXI policy offsets contemporaneous portfolio outflows depends on the size of (expected) portfolio outflows. While this offset is partial in Figure 5, it can also be more aggressive as Proposition 3 in Section 3 states. We emphasize this point in Figure 6 where we distinguish between episodes of relatively small and large portfolio outflows. While the depicted episodes of strong portfolio outflows are around five percentage points larger in terms of GDP than the episodes of relatively small outflows, the optimal FXI reaction are much more similar. This asymmetry is reflected in intermediated funds, denoted in terms of foreign curreny $B^*_{F,t} = B^*_t - B^*_{M,t} - B^*_{P,t}$. For strong portfolio outflows, financiers increase their lending in domestic currency (and an equivalent increase in foreign currency borrowing) since portfolio outflows are only partially absorbed by FX sales of the central bank. This effect vanishes in the case for small portfolio outflows; the central bank's FX sales fully offset the portfolio outflows such that the net demand for domestic currency is unchanged.

While the previous figures were concerned with portfolio outflows, Figure 7 depicts the average response in the model in episodes of portfolio inflows. The central bank offsets contemporaneous portfolio inflows almost one-for-one and is therefore able to follow the first best policy more closely. The reason being that the lower bound on reserves becomes less relevant in episodes of inflows and as a result, the policy response approaches the unconstrained case. The economy is almost fully stabilized as the real exchange rate appreciation is muted. Note that unlike portfolio outflows, portfolio inflows deepen FX markets as the drop in the bottom right panel of Figure 2 indicates. To understand the intuition, recall that FX market depth crucially depends on conditional exchange rate volatility. In par-



Notes: The plot depicts the average response in the model with optimal FXI in episodes of relatively small and strong portfolio outflows. One period corresponds to a quarter and period 0 coincides with the peak of outflows.

Figure 6: Optimal response to small and large portfolio outflows

ticular, the exchange rate fluctuates less for a fixed change in the relative consumption of tradables versus nontradables if it is in a more appreciated state.[8] As a result, FX markets are generally deeper if the exchange rate is more appreciated.

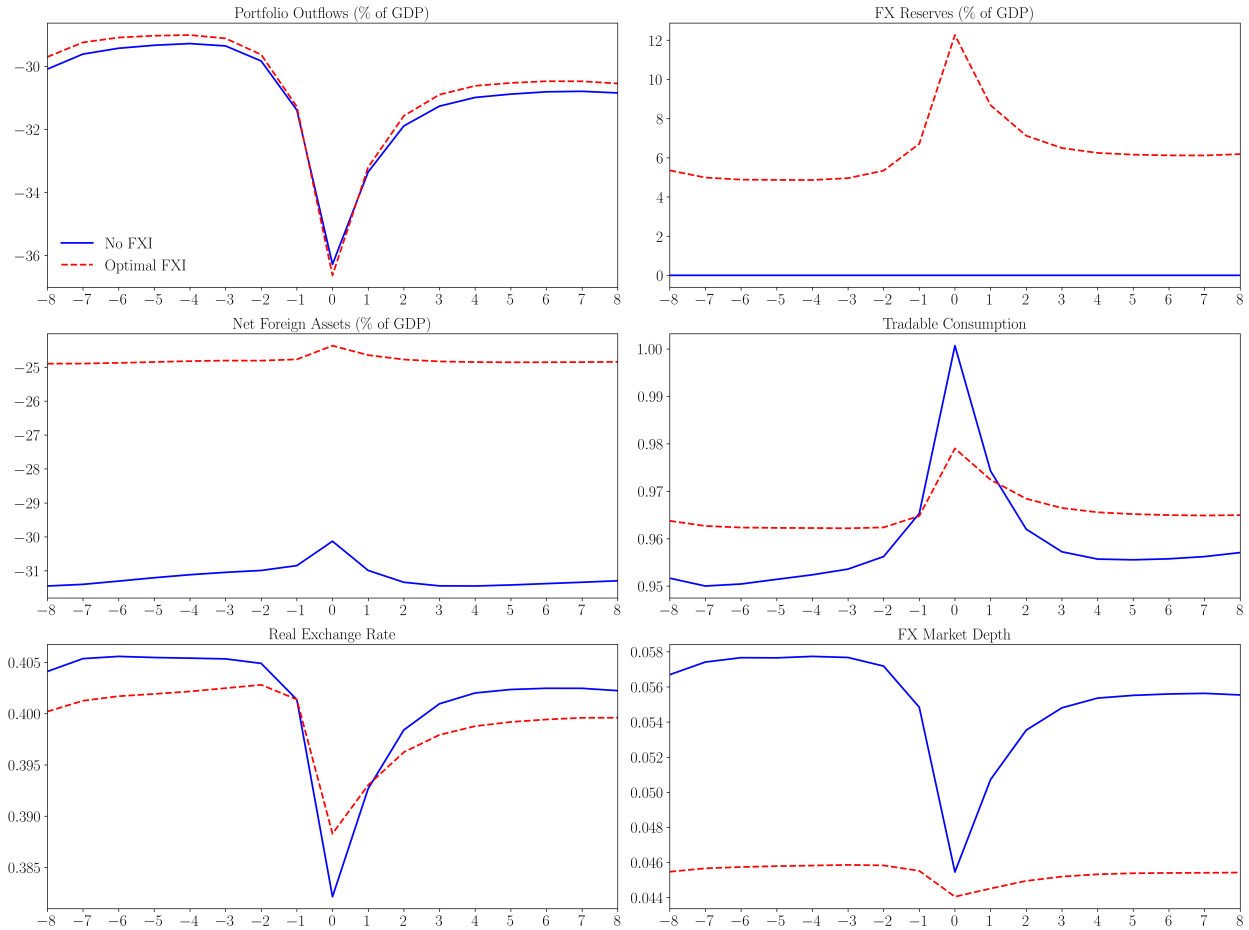
A corollary of this observation is that FXI under the optimal policy regime is more effective in episodes of portfolio outflows than inflows as FX markets are generally shallower in the former case. Comparing Figures 5 and 7, the central bank conducts, in both cases, FX sales or purchases of approximately similar absolute magnitudes.[9] However, in the case of outflows, the real exchange rate is 3.1% more appreciated under optimal FXI compared to the decentralized equilibrium at the peak in period $t = 0$, while it is only around 2.5% more depreciated in the case of inflows. Therefore, FX sales are around 40% more effective than FX purchases in terms of their impact on the real exchange rate. We highlight this state-dependency of FXI effectiveness in the next remark.

Remark 3 *Under the optimal policy regime, FX sales are more effective than FX purchases due to shallower FX markets.*

[8]The more technical explanation is that the exchange rate is an increasing convex function of the relative price of nontradables versus tradables if the elasticity of substitution is less than one. This condition is satisfied as we set $\xi = 0.83$ (see Table 1).

[9]More precisely, the change in FX reserves from period $t = -8$ to $t = 0$ is -6.3% of GDP in epsiodes of portfolio outflows while it is 6.9% of GDP in episodes of inflows.

Finally, we address the question to what extent FXI is used to offset fundamental shocks, captured by stochastic endowments in our model. Figure C.1 shows recessionary episodes in the tradable sector, where a modest FX sale occurs under the optimal policy regime. This outcome arises partly because, by design, the exogenous processes for tradable endowment and portfolio outflows are slightly negatively correlated (see Section 4.1). Additionally, tradable endowment shocks cause inefficient fluctuations in the risk-sharing wedge, which necessitates exchange rate stabilization–even though, in general, exchange rate adjustment is desirable due to efficient expenditure switching.



Notes: The plot depicts the average response in the model in episodes of portfolio inflows. One period corresponds to a quarter and period 0 coincides with the peak of inflows.

Figure 7: Episodes of portfolio inflows

4.5 Welfare Comparison

So far, we have seen how optimal FXI policy is beneficial in reducing excessive exchange rate volatility due to portfolio flows and stabilizing the economy. In this section, we compute the welfare gain of this policy in terms of consumption equivalence and compare it to other possible policy regimes.

Let κ be the additional fraction of consumption that households in the benchmark economy b will have to receive to make them indifferent to the economy with the alternative policy ap. Formally, κ can be expressed as

$$\kappa = \left(\frac{\tilde{V}_{ap}\left(\overline{B}^*, S\right)}{\tilde{V}_{p}\left(\overline{B}^*, S\right)} \right)^{\frac{1}{1-\sigma}} - 1 \tag{20}$$

where V_i is the value function of policy regime i weighted by the stationary distribution of the exogenous state process and $\overline{B}^*$ are the steady state bond holdings of the benchmark economy. Appendix C provides the derivation of this welfare measure. Since it is conditional on the steady state bond holdings of the benchmark economy, the welfare gain κ takes into account the transition dynamics and thus is not simply based on an unconditional comparison of steady states.

The benchmark economy is the version of the model calibrated to Malaysia which accounts for the observed FX interventions conducted by the central bank of Malaysia. We consider five alternative policy regimes which can be thought of as counterfactual scenarios: First, an economy in which the central bank does not conduct any FXI and second, the optimal time-consistent FXI policy as described in Section 3. Third and fourth, we consider policies in which the central bank commits to an FXI rule. Finally, we compute the first best which provides an upper bound with regards to welfare gains.

We solve for the first best policy by imposing that FXI is characterized by (9) and thereby ignore the NNCR (i.e., the central bank is able to borrow in FX reserves). Note that the associated equilibrium is stationary since households are impatient ($\beta R^* < 1$) and there is a lower bound on the net foreign asset position guaranteeing that the transversality condition is satisfied. The first rule, which we refer to as the UIP rule, is inspired by the first best policy with the crucial distinction that the central bank is subject to the NNCR

$$B^*_{M,t} = \max\left(B^*_t - B^*_{P,t}, 0\right) \tag{21}$$

Note that the UIP rule eliminates ex-ante UIP deviations whenever feasible. Next, we consider a simple UIP rule which only offsets the exogenous portfolio flow position

$$B^*_{M,t} = \max\left(-B^*_{P,t}, 0\right) \tag{22}$$

The choice of these two FXI rules is interesting for two reasons. First, the optimal policy closely resembles the UIP rule, albeit with some differences, as outlined in Section 3. A quantitative analysis of these policy regimes helps us evaluate the significance of these differences. Second, comparing the UIP rule to the simple UIP rule allows us to assess the extent to which UIP deviations are driven by the endogenous component (i.e., the net foreign asset position B^*_t) versus the exogenous component (i.e., the portfolio flow position $B^*_{P,t}$), as well as the resulting welfare implications.

Table 2: Welfare Comparison

	No FXI	Optimal FXI	UIP Rule	Simple UIP Rule	First Best
Welfare Gain	-0.16	0.44	0.25	0.12	0.54

Notes: The welfare gain is computed according (20), annualized and expressed in percentages. The UIP rule and simple UIP rule is defined by (21) and (22), respectively.

Table 2 lists the annualized welfare gain in percentages for the five mentioned policy regimes. Interestingly, households are 0.16% worse off in the policy regime without FXI relative to the benchmark economy. In contrast, the optimal time-consistent FXI regime improves welfare by 0.44%. The rules-based policy regimes also enhance welfare, albeit to a lesser extent, with the UIP and simple UIP rules yielding welfare gains of 0.25% and 0.12%, respectively. Unsurprisingly, the highest welfare gain is provided by the first best with 0.54% of annual consumption. Overall, the discretionary use of FXI outperforms the considered rules and is only slightly worse in terms of welfare compared to the first best.

Delving into the unconditional characteristics of the various policy regimes, Table 3 presents their first and second moments as obtained from the simulations. One point to highlight is that the average FX reserves as a percentage of annual GDP varies in the different policy regimes. It is 7% under optimal FXI, 4% under the UIP rule, 30% under the simple UIP rule and -4% in the first best. The average reserves are considerably higher under the simple UIP rule than under the UIP rule, since it only offsets the exogenous portfolio outflows and is not proportional to the endogenous net foreign asset position. As a result of the calibration, portfolio outflows are negative on average (and thus portfolio inflows) while households are impatient and therefore tend to borrow. In the first best, the central bank does not need

to accumulate precautionary reserves in the form of FX reserves since its use of FXI is unconstrained. In fact, it borrows in reserves in order to accommodate the borrowing needs of households. These differences in average FX reserves among the different policy regimes result in similarly different steady states among the policy regimes. However, the volatility of the real exchange rate is comparable among the FXI regimes and is around 3.3%. Similarly, the use of FXI also unconditionally reduces the variations in FX market depth and the risk sharing wedge. Under optimal FXI, the central bank depletes its reserves in 1.4% of periods, compared to 16.8% under the UIP rule and never under the simple UIP rule. Finally, Figure C.2 compares how the economy behaves under the different policy regimes during episodes of portfolio outflows.

Table 3: Model Moments

	No FXI		Optimal FXI		UIP Rule		Simple UIP Rule		First Best	
Variable Name	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
Output	1.658	0.070	1.652	0.061	1.653	0.062	1.633	0.058	1.660	0.059
Portfolio Flows to GDP	-0.312	0.046	-0.309	0.047	-0.310	0.047	-0.304	0.047	-0.312	0.049
NFA to GDP	-0.312	0.014	-0.248	0.012	-0.270	0.015	-0.011	0.005	-0.350	0.012
FX Reserves to GDP	0.000	0.000	0.066	0.041	0.043	0.038	0.303	0.047	-0.038	0.049
Real Exchange Rate	0.401	0.017	0.399	0.013	0.399	0.013	0.392	0.013	0.402	0.013
FX Depth	0.055	0.010	0.045	0.004	0.047	0.005	0.041	0.003	0.034	0.007
Risk Sharing Wedge	-0.013	0.113	-0.009	0.024	-0.008	0.039	-0.016	0.008	0.000	0.000
Consumption	0.981	0.014	0.983	0.014	0.983	0.013	0.993	0.014	0.979	0.015
Binding NNCR	1.000		0.014		0.168		0.000			

Notes: The moments are obtained from a simulation based on 2×10^6 quarters. The portfolio flows, NFA and FX reserves are expressed as a fraction of annual GDP. The risk sharing wedge is annualized. Binding NNCR shows the fraction of periods in which the non-negativity constraint on FX reserves is binding.

While we document substantial welfare benefits for the discretionary policy, it is worth pointing out that discretionary policy is not equivalent to optimal policy under full commitment due to time inconsistency. As Itskhoki & Mukhin (2023) discuss, there is a "forward guidance" motive in the optimal use of FXI that requires commitment. More precisely, the optimal policy under commitment prescribes a lower level of FX reserves relative to the time-consistent policy when reserves were depleted in the previous period. Time inconsistency therefore becomes a problem if reserves are at or very close to the lower bound. Since the initial condition that we use to compute the welfare gain is based on the observed level of FX reserves for Malaysia (equal to around 30% of GDP), time inconsistency is quantitatively negligible in our calibration. Nevertheless, if we change the initial condition to a hypothetical benchmark economy in which the central bank holds no reserves and commits to no FXI, the welfare gains of the time-consistent policy indeed become negative, reflecting the time inconsistency problem. This point is illustrated in Figure C.3. Our final remark concludes

our discussion on the welfare comparison.

Remark 4 *Optimal time-consistent FXI yields significant welfare gains, surpassing those of alternative FXI rules and falling only slightly short of the first-best outcome. However, this result critically hinges on the benchmark economy exhibiting a positive level of FX reserves; otherwise, time inconsistency issues can emerge, reducing the welfare gains.*

5 Conclusion

This paper analyzes the optimal use of foreign exchange intervention (FXI) in a small open economy with endogenous FX market depth and a lower bound on FX reserves. We find that optimal time-consistent FXI can effectively reduce exchange rate volatility caused by portfolio flow shocks, thereby stabilizing deviations from uncovered interest parity (UIP) and improving market depth. The optimal response to FXI hinges on the anticipated path of portfolio flows: when outflows are moderate and expected to ease, aggressive intervention is optimal, whereas in the face of larger, likely persistent outflows, preserving a precautionary reserve level becomes essential.

Our quantitative analysis further reveals that the effectiveness of FXI in terms of its impact on the exchange rate is state-dependent in the optimal policy regime. Specifically, FX purchases tend to be less effective as they occur during periods of inflows and deeper FX markets, while FX sales are relatively more effective, as they take place amid portfolio outflows and shallower markets. Comparing welfare, we find that optimal FXI is associated with substantial welfare gains if the economy starts out with a positive level of reserves. Committing to FXI rules that aim to smooth UIP deviations improves welfare, albeit to a lesser extent, whereas reducing reserves to zero and refraining from any FXI is detrimental to welfare.

Overall, our analysis provides a tractable quantitative framework to analyze the use of FXI. It highlights the importance of the precautionary motive and state dependency of FX markets in the conduct of FXI policy. Future theoretical research could build on this framework and incorporate additional relevant financial frictions in the model while it would be interesting to explore the state-dependent effectiveness of FXI empirically.

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Appendix

A Proofs

Proof of Proposition 1 By the resource constraint, note that $\partial B_t^*/\partial B_{P,t}^* = -\partial C_{T,t}/\partial B_{P,t}^*$. Next we proceed by proving the three properties:

(a) The conditional exchange rate volatility can be rewritten as:

$$\begin{aligned}
\sigma_t^2 &= R_t^2 var_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) \\
&= \left(\frac{1}{\mathbb{E}_t\left[\Theta_{t+1}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right]}\right)^2 var_t\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) \\
&= \left(\frac{1}{\mathbb{E}_t\left[\Theta_{t+1}\frac{1}{\mathcal{E}_{t+1}}\right]}\right)^2 var_t\left(\frac{1}{\mathcal{E}_{t+1}}\right) \\
&= \left(\frac{1}{\mathbb{E}_t\left[\beta\frac{u_{1,t+1}}{u_{1,t}}\frac{1}{\mathcal{E}_{t+1}}\right]}\right)^2 var_t\left(\frac{1}{\mathcal{E}_{t+1}}\right) \\
&= \frac{(u_{1,t})^2}{\left(\mathbb{E}_t\left[\beta\frac{u_{1,t+1}}{\mathcal{E}_{t+1}}\right]\right)^2} var_t\left(\frac{1}{\mathcal{E}_{t+1}}\right)
\end{aligned}$$

where we have used the Euler equation in the second line. The partial derivative with respect to consumption in period t is given by

$$\frac{\partial \sigma_t^2}{\partial C_{T,t}} = 2\left(\frac{u_{11,t}}{u_{1,t}}\right)\frac{(u_{1,t})^2}{\left(\mathbb{E}_t\left[\beta\frac{u_{1,t+1}}{\mathcal{E}_{t+1}}\right]\right)^2} var_t\left(\frac{1}{\mathcal{E}_{t+1}}\right) = 2\left(\frac{u_{11,t}}{u_{1,t}}\right)\sigma_t^2$$

Now, take the partial derivative with respect to $B_{P,t}^*$ and use the previous equations to get the result:

$$\frac{\partial \sigma_t^2}{\partial B_{P,t}^*} = \frac{\partial \sigma_t^2}{\partial C_{T,t}}\frac{\partial C_{T,t}}{\partial B_{P,t}^*} = 2\overbrace{\left(\frac{u_{11,t}}{u_{1,t}}\right)}^{<0}\sigma_t^2\overbrace{\left(-\frac{\partial B_t^*}{\partial B_{P,t}^*}\right)}^{<0} > 0$$

(b) Suppose by contradiction that $\frac{\partial \mathbb{E}_t[\Theta_{t+1}]}{\partial B^*_{P,t}} \geq 0$ which then implies:

$$\frac{\partial \mathbb{E}_t\left[\Theta_{t+1}\right]}{\partial B^*_{P,t}} = \frac{\partial \mathbb{E}_t\left[\Theta_{t+1}\right]}{\partial C_{T,t}} \frac{\partial C_{T,t}}{\partial B^*_{P,t}} = \left(-\frac{u_{11,t}}{u_{1,t}}\right) \mathbb{E}_t\left[\Theta_{t+1}\right] \frac{\partial C_{T,t}}{\partial B^*_{P,t}} \geq 0$$
$$\Rightarrow \frac{\partial C_{T,t}}{\partial B^*_{P,t}} \geq 0 \Rightarrow \frac{\partial B^*_t}{\partial B^*_{P,t}} \leq 0$$

which violates our assumption $\frac{\partial B^*_t}{\partial B^*_{P,t}} > 0$ and thus we must have $\frac{\partial \mathbb{E}_t[\Theta_{t+1}]}{\partial B^*_{P,t}} < 0$. ■

Derivation of $\partial B^*_t / \partial B^*_{P,t}$ Take the partial derivative with respect to $B^*_{P,t}$ of the IRS condition:

$$\omega \frac{\partial \sigma^2_t}{\partial B^*_{P,t}} \left(B^*_t - B^*_{P,t}\right) + \omega \sigma^2_t \left(\frac{\partial B^*_t}{\partial B^*_{P,t}} - 1\right) = R^* \frac{\partial \mathbb{E}_t\left[\Theta_{t+1}\right]}{\partial B^*_{P,t}}$$

next insert the expressions for $\frac{\partial \sigma^2_t}{\partial B^*_{P,t}}$ and $\frac{\partial \mathbb{E}_t[\Theta_{t+1}]}{\partial B^*_{P,t}}$ obtained in the proof of proposition 1:

$$\omega 2 \left(\frac{u_{11,t}}{u_{1,t}}\right) \sigma^2_t \left(-\frac{\partial B^*_t}{\partial B^*_{P,t}}\right) \left(B^*_t - B^*_{P,t}\right) + \omega \sigma^2_t \left(\frac{\partial B^*_t}{\partial B^*_{P,t}} - 1\right) = -R^* \left(-\frac{u_{11,t}}{u_{1,t}}\right) \mathbb{E}_t\left[\Theta_{t+1}\right] \frac{\partial B^*_t}{\partial B^*_{P,t}}$$

insert the IRS condition $R^* \mathbb{E}_t\left[\Theta_{t+1}\right] = 1 + \omega \sigma^2_t \left(B^*_t - B^*_{P,t}\right)$ on the RHS to obtain:

$$\omega 2 \left(\frac{u_{11,t}}{u_{1,t}}\right) \sigma^2_t \left(-\frac{\partial B^*_t}{\partial B^*_{P,t}}\right) \left(B^*_t - B^*_{P,t}\right) + \omega \sigma^2_t \left(\frac{\partial B^*_t}{\partial B^*_{P,t}} - 1\right) = -\left(-\frac{u_{11,t}}{u_{1,t}}\right) \frac{\partial B^*_t}{\partial B^*_{P,t}} \left(1 + \omega \sigma^2_t \left(B^*_t - B^*_{P,t}\right)\right)$$

rearrange to get:

$$\omega \sigma^2_t \frac{\partial B^*_t}{\partial B^*_{P,t}} \left[\left(-\frac{u_{11,t}}{u_{1,t}}\right) \left(3\left(B^*_t - B^*_{P,t}\right) + \frac{1}{\omega \sigma^2_t}\right) + 1\right] = \omega \sigma^2_t$$

Finally, rewrite the equation as an expression for $\frac{\partial B^*_t}{\partial B^*_{P,t}}$:

$$\frac{\partial B^*_t}{\partial B^*_{P,t}} = \frac{1}{1 - \left(-\frac{u_{11,t}}{u_{1,t}}\right) \left(3\left(B^*_{P,t} - B^*_t\right) - \frac{1}{\omega \sigma^2_t}\right)}$$

and therefore the sufficient condition for $\frac{\partial B^*_t}{\partial B^*_{P,t}} > 0$ is given by:

$$\frac{\partial B^*_t}{\partial B^*_{P,t}} > 0 \Leftrightarrow \left(-\frac{u_{11,t}}{u_{1,t}}\right) \left(3\left(B^*_{P,t} - B^*_t\right) - \frac{1}{\omega \sigma^2_t}\right) < 1$$

Proof of Theorem 1 Insert the period $t+1$ IRS condition $R^* \mathbb{E}_{t+1}\left[\Theta_{t+2}\right] = 1 + \omega \sigma^2_{t+1} \left(B^*_{t+1} - B^*_{P,t+1}\right)$

into (15) to get:

$$u_{1,t} = \beta R^* \mathbb{E}_t \Bigg[u_{1,t+1} - \lambda_{NNCR,t+1} \left(-\frac{u_{11,t+1}}{u_{1,t+1}} \right) \left(\frac{1}{\omega \sigma_{t+1}^2} + \left(B_{t+1}^* - B_{P,t+1}^* \right) \right) + \lambda_{NNCR,t+1} \left(-\frac{u_{11,t+1}}{u_{1,t+1}} \right) 2 \left(B_{P,t+1}^* - B_{t+1}^* \right) \Bigg]$$

which can be further simplified to:

$$u_{1,t} = \beta R^* \mathbb{E}_t \left[u_{1,t+1} + \lambda_{NNCR,t+1} \left(-\frac{u_{11,t+1}}{u_{1,t+1}} \right) \left(3 \left(B_{P,t+1}^* - B_{t+1}^* \right) - \frac{1}{\omega \sigma_{t+1}^2} \right) \right]$$

next divide by $u_{1,t}$ on both sides and rewrite:

$$1 - R^* \mathbb{E}_t \left[\Theta_{t+1} \right] = \frac{\beta R^*}{u_{1,t}} \mathbb{E}_t \left[\lambda_{NNCR,t+1} \left(-\frac{u_{11,t+1}}{u_{1,t+1}} \right) \left(3 \left(B_{P,t+1}^* - B_{t+1}^* \right) - \frac{1}{\omega \sigma_{t+1}^2} \right) \right]$$

Plugging in the the period t IRS condition $R^* \mathbb{E}_t \left[\Theta_{t+1} \right] = 1 + \omega \sigma_t^2 \left(B_t^* - B_{P,t}^* - B_{M,t}^* \right)$:

$$\omega \sigma_t^2 \left(B_{P,t}^* + B_{M,t}^* - B_t^* \right) = \frac{\beta R^*}{u_{1,t}} \mathbb{E}_t \left[\lambda_{NNCR,t+1} \left(-\frac{u_{11,t+1}}{u_{1,t+1}} \right) \left(3 \left(B_{P,t+1}^* - B_{t+1}^* \right) - \frac{1}{\omega \sigma_{t+1}^2} \right) \right]$$

Finally, rewrite the equation in terms of $B_{M,t}^*$:

$$B_{M,t}^* = B_t^* - B_{P,t}^* + \frac{\beta R^*}{\omega \sigma_t^2 u_{1,t}} \mathbb{E}_t \left[\lambda_{NNCR,t+1} \left(-\frac{u_{11,t+1}}{u_{1,t+1}} \right) \left(3 \left(B_{P,t+1}^* - B_{t+1}^* \right) - \frac{1}{\omega \sigma_{t+1}^2} \right) \right]$$

which completes the proof. ■

Proof of Proposition 2 By Theorem 1 the optimal level of reserves is higher relative to

the first best if and only if:

$$
\begin{aligned}
& B^*_{M,t} > B^*_t - B^*_{P,t} \\
\Leftrightarrow & \frac{\beta R^*}{\omega \sigma_t^2 u_{1,t}} \mathbb{E}_t \left[\lambda_{NNCR,t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(3 \left(B^*_{P,t+1} - B^*_{t+1} \right) - \frac{1}{\omega \sigma^2_{t+1}} \right) \right] > 0 \\
\Leftrightarrow & \mathbb{E}_t \left[\lambda_{NNCR,t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(3 \left(B^*_{P,t+1} - B^*_{t+1} \right) - \frac{1}{\omega \sigma^2_{t+1}} \right) \right] > 0 \\
\Leftrightarrow & \mathbb{E}_t \left[\lambda_{NNCR,t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(-3 B^*_{F,t+1} - \frac{1}{\omega \sigma^2_{t+1}} \right) \right] > 0 \\
\Leftrightarrow & \mathbb{E}_t \left[\lambda_{NNCR,t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(-3 B^*_{F,t+1} - \frac{1}{\omega \sigma^2_{t+1}} \right) \right] > 0 \\
\Leftrightarrow & \mathbb{E}_t \left[\lambda_{NNCR,t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(-\omega \sigma^2_{t+1} B^*_{F,t+1} - \frac{1}{3} \right) \right] > 0 \\
\Leftrightarrow & \mathbb{E}_t \left[\lambda_{NNCR,t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(\omega \sigma^2_{t+1} B^*_{F,t+1} + \frac{1}{3} \right) \right] < 0
\end{aligned}
$$

where we have made use of $B^*_{F,t} = B^*_{t+1} - B^*_{P,t+1}$ on the fourth line and multiplied by $\omega \sigma^2_{t+1}/3$ on the sixth line. Next, use the IRS condition $R^* \mathbb{E}_{t+1} [\Theta_{t+2}] - 1 = \omega \sigma^2_{t+1} B^*_{F,t+1}$ to obtain:

$$
\begin{aligned}
& \mathbb{E}_t \left[\lambda_{NNCR,t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \left(R^* \mathbb{E}_{t+1} [\Theta_{t+2}] - \frac{2}{3} \right) \right] < 0 \\
& \quad \Leftrightarrow \mathbb{E}_t \left[\lambda_{NNCR,t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) R^* \mathbb{E}_{t+1} [\Theta_{t+2}] \right] < \frac{2}{3} \mathbb{E}_t \left[\lambda_{NNCR,t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \right] \\
& \Leftrightarrow \mathbb{E}_t \left[\lambda_{NNCR,t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \right] \mathbb{E}_t \left[R^* \mathbb{E}_{t+1} [\Theta_{t+2}] \right] + cov \left(\lambda_{NNCR,t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right), R^* \mathbb{E}_{t+1} [\Theta_{t+2}] \right) \\
& \qquad\qquad < \frac{2}{3} \mathbb{E}_t \left[\lambda_{NNCR,t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \right]
\end{aligned}
$$

Finally, divide by $\mathbb{E}_t \left[\lambda_{NNCR,t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \right]$ on both sides and use the law of iterated expectations to get:

$$
R^* \mathbb{E}_t [\Theta_{t+2}] < \frac{2}{3} - \frac{cov \left(\lambda_{NNCR,t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right), R^* \mathbb{E}_{t+1} [\Theta_{t+2}] \right)}{\mathbb{E}_t \left[\lambda_{NNCR,t+1} \left(\frac{-u_{11,t+1}}{u_{1,t+1}} \right) \right]}
$$

which completes the proof. ■

The third term on the RHS of (16) is positive if $-3B^*_{F,t} - 1/\omega \sigma^2_t > 0$ for periods t in which $\lambda_{NNCR,t} > 0$ (and $B^*_{M,t} = 0$) where we have used $B^*_{F,t} = B^*_t - B^*_{P,t}$. This condition can be

rewritten as:

$$\omega\sigma_t^2 B_{F,t}^* < -\frac{1}{3} \tag{A.1}$$

Furthermore, the modified UIP condition (4) and Euler equation (1) can be combined to:

$$R^*\mathbb{E}_t\left[\Theta_{t+1}\right] - 1 = \omega\sigma_t^2 B_{F,t}^*$$

Use the equation above and (A.1) to obtain:

$$R^*\mathbb{E}_t\left[\Theta_{t+1}\right] < \frac{2}{3}$$

Derivation $B_{F,t}^*$ Combine the FOCs (10) and (11) to obtain:

$$u_{1,t} + \lambda_{IRS,t}R^*\left(-\frac{u_{11,t}}{u_{1,t}}\right)\mathbb{E}_t\left[\Theta_{t+1}\right] - \lambda_{EERV,t+1}\sigma_{t+1}^2\beta^2 2\left(-\frac{u_{11,t+1}}{u_{1,t+1}}\right)\mathbb{E}_{t+1}\left[\frac{u_{2,t+2}}{u_{1,t+1}}\right]^2 + \lambda_{IRS,t}\omega\sigma_t^2 =$$
$$\beta R^*\mathbb{E}_t\left[u_{1,t+1} + \lambda_{IRS,t+1}R^*\left(-\frac{u_{11,t+1}}{u_{1,t+1}}\right)\mathbb{E}_{t+1}\left[\Theta_{t+2}\right] - \lambda_{EERV,t+1}\sigma_{t+1}^2\beta^2 2\left(-\frac{u_{11,t+1}}{u_{1,t+1}}\right)\mathbb{E}_{t+1}\left[\frac{u_{2,t+2}}{u_{1,t+1}}\right]^2\right]$$

Simplify by using (12), (13) and $B_{F,t}^* = B_t^* - B_{P,t}^* - B_{M,t}^*$:

$$u_{1,t} - \lambda_{NNCR,t}\left(-\frac{u_{11,t}}{u_{1,t}}\right)\left(3B_{F,t}^* + \frac{1}{\omega\sigma_t^2} + 1\right) =$$
$$\beta R^*\mathbb{E}_t\left[u_{1,t+1} - \lambda_{NNCR,t+1}\left(-\frac{u_{11,t+1}}{u_{1,t+1}}\right)\left(3B_{F,t+1}^* + \frac{1}{\omega\sigma_{t+1}^2}\right)\right]$$

Divide by $u_{1,t}$ and make use of the IRS condition in period t:

$$1 - \frac{\lambda_{NNCR,t}}{u_{1,t}}\left(-\frac{u_{11,t}}{u_{1,t}}\right)\left(3B_{F,t}^* + \frac{1}{\omega\sigma_t^2} + 1\right) =$$
$$1 + \omega\sigma_t^2 B_{F,t}^* - \beta R^*\mathbb{E}_t\left[\frac{\lambda_{NNCR,t+1}}{u_{1,t}}\left(-\frac{u_{11,t+1}}{u_{1,t+1}}\right)\left(3B_{F,t+1}^* + \frac{1}{\omega\sigma_{t+1}^2}\right)\right]$$

Finally, write the equation in terms of $B_{F,t}^*$:

$$B_{F,t}^* = \frac{-\frac{\lambda_{NNCR,t}}{u_{1,t}}\left(-\frac{u_{11,t}}{u_{1,t}}\right)\left(\frac{1}{\omega\sigma_t^2} + 1\right) + \beta R^*\mathbb{E}_t\left[\frac{\lambda_{NNCR,t+1}}{u_{1,t}}\left(-\frac{u_{11,t+1}}{u_{1,t+1}}\right)\left(3B_{F,t+1}^* + \frac{1}{\omega\sigma_{t+1}^2}\right)\right]}{\frac{\lambda_{NNCR,t}}{u_{1,t}}\left(-\frac{u_{11,t}}{u_{1,t}}\right)3 + \omega\sigma_t^2} \tag{A.2}$$

We can now start to characterize $B^*_{F,t}$. Note that the denominator on the RHS of (A.2) is strictly positive and the first term in the numerator is strictly negative. Furthermore, the second term in the numerator is also negative if $B^*_{F,t+1} < -1/3\omega\sigma^2_{t+1}$ for states in which the NNCR is binding in period $t+1$. For now, suppose that $\mathbb{E}_t\left[\lambda_{NNCR,t+1}\right] = 0$ so that the aforementioned term is zero and (A.2) simplifies to:

$$B^*_{F,t} = \frac{-\frac{\lambda_{NNCR,t}}{u_{1,t}}\left(-\frac{u_{11,t}}{u_{1,t}}\right)\left(\frac{1}{\omega\sigma^2_t}+1\right)}{\frac{\lambda_{NNCR,t}}{u_{1,t}}\left(-\frac{u_{11,t}}{u_{1,t}}\right)3+\omega\sigma^2_t}$$

Derivation Absolute Risk Aversion The marginal utility with respect to tradables is given by:

$$u_{1,t} = \alpha C_t^{-\sigma}\left(\frac{C_t}{C_{T,t}}\right)^{\frac{1}{\xi}} = \alpha C_t^{\frac{1-\sigma\xi}{\xi}} C_{T,t}^{-\frac{1}{\xi}}$$

Taking the derivative with respect to tradables again, yields:

$$u_{11,t} = \alpha C_t^{\frac{1-\sigma\xi}{\xi}} C_{T,t}^{-\frac{1}{\xi}}\left(\frac{1-\sigma\xi}{\xi}C_t^{-1}\frac{\partial C_t}{\partial C_{T,t}} - \frac{1}{\xi}C_{T,t}^{-1}\right) = \alpha C_t^{\frac{1-\sigma\xi}{\xi}} C_{T,t}^{-\frac{1}{\xi}}\left(\frac{1-\sigma\xi}{\xi}C_t^{-1}\alpha\left(\frac{C_t}{C_{T,t}}\right)^{\frac{1}{\xi}} - \frac{1}{\xi}C_{T,t}^{-1}\right)$$

which allows us to characterize the absolute risk aversion as:

$$\frac{-u_{11,t}}{u_{1,t}} = \frac{\sigma\xi-1}{\xi}\alpha C_t^{\frac{1-\xi}{\xi}} C_{T,t}^{-\frac{1}{\xi}} + \frac{1}{\xi}C_{T,t}^{-1}$$

Derivation Price Index Rewrite the intratemporal optimality condition as

$$C_{T,t} = \left(\frac{\alpha}{1-\alpha}\right)^{\xi} C_{N,t}\mathcal{E}_t^{-\xi}$$

Define the price index P_t as the price of one (aggregated) consumption good C_t. Making use of the equation above, we can then write

$$\left(\frac{\alpha}{1-\alpha}\right)^{\xi} C_{N,t}\mathcal{E}_t^{1-\xi} + C_{N,t} = P_tC_t \Leftrightarrow C_{N,t} = \frac{P_tC_t}{\left[\left(\frac{\alpha}{1-\alpha}\right)^{\xi}\mathcal{E}_t^{1-\xi}+1\right]}$$

and similarly

$$C_{T,t}\mathcal{E}_t + C_{T,t}\left(\frac{\alpha}{1-\alpha}\right)^{-\xi}\mathcal{E}_t^{\xi} = P_tC_t \Leftrightarrow C_{T,t} = \left(\frac{\alpha}{1-\alpha}\right)^{\xi}\mathcal{E}_t^{-\xi}\frac{P_tC_t}{\left[\left(\frac{\alpha}{1-\alpha}\right)^{\xi}\mathcal{E}_t^{1-\xi}+1\right]}$$

Next, plug the expression for tradable and nontradable consumption in the consumption aggregator

$$\begin{aligned}
C_t &= \left[\alpha \left(C_{T,t}\right)^{\frac{\xi-1}{\xi}} + (1-\alpha)\left(C_{N,t}\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}} \\
&= \frac{P_t C_t}{\left[\left(\frac{\alpha}{1-\alpha}\right)^{\xi} \mathcal{E}_t^{1-\xi} + 1\right]} \left[\left(\frac{\alpha}{1-\alpha}\right)^{\xi} (1-\alpha)\, \mathcal{E}_t^{1-\xi} + (1-\alpha)\right]^{\frac{\xi}{\xi-1}} \\
&= \frac{P_t C_t}{\left[\left(\frac{\alpha}{1-\alpha}\right)^{\xi} \mathcal{E}_t^{1-\xi} + 1\right]} \left[(1-\alpha)\left(\left(\frac{\alpha}{1-\alpha}\right)^{\xi} \mathcal{E}_t^{1-\xi} + 1\right)\right]^{\frac{\xi}{\xi-1}} \\
&= P_t C_t (1-\alpha)^{\frac{\xi}{\xi-1}} \left[\left(\frac{\alpha}{1-\alpha}\right)^{\xi} \mathcal{E}_t^{1-\xi} + 1\right]^{\frac{1}{\xi-1}} \\
&= P_t C_t \left[\alpha^{\xi} \mathcal{E}_t^{1-\xi} + (1-\alpha)^{\xi}\right]^{\frac{1}{\xi-1}}
\end{aligned}$$

The price index is thus given by

$$P_t = \left[\alpha^{\xi} \mathcal{E}_t^{1-\xi} + (1-\alpha)^{\xi}\right]^{\frac{1}{1-\xi}}$$

Derivation Real Exchange Rate Define the real exchange rate as

$$\mathcal{Q} = \frac{\mathcal{E}_t P_t^*}{P_t}$$

Using the expression for the domestic price index and the fact that the price level abroad is normalized to unity, we obtain the following expression for the real exchange rate

$$\mathcal{Q} = \left[\alpha^{\xi} + (1-\alpha)^{\xi} \left(\frac{1}{\mathcal{E}_t}\right)^{1-\xi}\right]^{-\frac{1}{1-\xi}}$$

B Calibration Details

Observed FXI regime While we do not know the exact reaction function for FXI of the central bank of Malaysia, we can account for their FXI policy in the model by treating the observed FX interventions $\hat{B}_{M,t}$ as an exogenous process in the model similar to the portfolio outflows $\hat{B}_{M,t}$. For this purpose, we define a new variable that is the sum of portfolio outflows and FX interventions $\hat{B}_{PM,t} = \hat{B}_{P,t} + \hat{B}_{M,t}$ and replace $B_{P,t}$ by $B_{PM,t}$ as

the third exogenous state variable. We can make this simplification since the only difference between $B_{P,t}$ and $B_{M,t}$ from the model's perspective, namely that $B_{P,t}$ is exogenous and $B_{M,t}$ is a policy variable, ceases to exit. Given the changed exogenous state variables, we estimate an AR(1) process $\log \tilde{\mathbf{S}}_t = \tilde{\rho} \log \tilde{\mathbf{S}}_{t-1} + \tilde{\varepsilon}_t$ where $\tilde{\mathbf{S}}_t = [Y_{T,t}, Y_{N,t}, B_{PM,t}]'$. The error term $\tilde{\varepsilon}_t = [\varepsilon_{T,t}, \varepsilon_{N,t}, \varepsilon_{PM,t}]'$ follows a trivariate normal distribution with zero mean and contemporaneous variance-covariance matrix $\tilde{\mathbf{V}}$ and $\tilde{\rho}$ is a 3×3 matrix consisting of the autocorrelation terms:

$$\tilde{\mathbf{V}} = \begin{bmatrix} 0.0005452 & 0.0006076 & -0.0009553 \\ 0.0006076 & 0.0009243 & -0.0017925 \\ -0.0009553 & -0.0017925 & 0.0740447 \end{bmatrix}, \quad \tilde{\rho} = \begin{bmatrix} 0.8024195 & -0.386813 & -0.0342064 \\ 0.1747354 & 0.3177107 & -0.0381535 \\ -0.2865547 & -1.4977 & 0.3445864 \end{bmatrix}$$

In the data, FX interventions are negatively correlated with portfolio outflows at $\sigma_{B_P^*, B_M^*} = -0.19$ which is directionally consistent with the optimal policy. Interestingly, FX interventions are also negatively correlated with the endowment process, particularly so with the nontradable component as $\sigma_{Y_N, B_M^*} = -0.4531$ and $\sigma_{Y_T, B_M^*} = -0.3720$. The observed FXI regime not only counteracts portfolio outflows but also supports the nontradable sector in a recession by depreciating the exchange rate. From the perspective of optimal policy, this latter reaction is unwarranted since it hinders efficient expenditure switching.

Calibration of the subjective discount factor β In the model with an exogenous state process that accounts for the observed FXI regime (see above), we choose β such that the model matches the mean of the observed net foreign asset position in the data which is $B_t^* = 0.2$ or around 2% of annual GDP. The value of the subjective discount factor that matches this moment is $\beta = 0.9887$.

Calibration of the financiers' risk aversion ω The target for the calibration of the financiers' risk aversion is an average FX market depth of $\omega\sigma^2 = 0.05$ in the decentralized equilibrium without FXI. This condition holds for $\omega = 21$. Note that changing the financiers' risk aversion not only has implications for the endogenous variables in the model but is also associated with a changed (exogenous) portfolio outflow process $B_{P,t}^*$ since ω enters the risk sharing wedge. As is evident from (19), a higher financiers' risk aversion ω makes FX markets more shallow and therefore decreases the impact that the observed risk sharing wedge $\widehat{RSW}$ has on $\hat{B}_t^*$. Intuitively, FX market shallowness magnifies the effect of portfolio outflows on the risk sharing wedge. Therefore, a given level of fluctuations in the risk sharing wedge can either be explained by more volatile portfolio outflows and deeper FX markets or less volatile portfolio outflows and more shallow FX markets.

C Simulation Details

Derivation Consumption Equivalence Let κ be the additional fraction of consumption that households in the benchmark economy b will have to receive to make them indifferent to the economy with the alternative policy ap:

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t u\left((1+\kappa)C_t^b\right) = \sum_{t=0}^{\infty} \beta^t u\left((1+\kappa)C_t^{ap}\right) \\
\Leftrightarrow &\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma}\left((1+\kappa)C_t^b\right)^{1-\sigma} = \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma}\left(C_t^{ap}\right)^{1-\sigma} \\
\Leftrightarrow &(1+\kappa)^{1-\sigma} \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma}\left(C_t^b\right)^{1-\sigma} = \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma}\left(C_t^{ap}\right)^{1-\sigma}
\end{aligned}$$

Next use

$$V_i\left(B^*, S\right) = \sum_{t=0}^{\infty} \beta^t u\left(C_t^i\right)$$

for $i \in \{b, ap\}$ to obtain

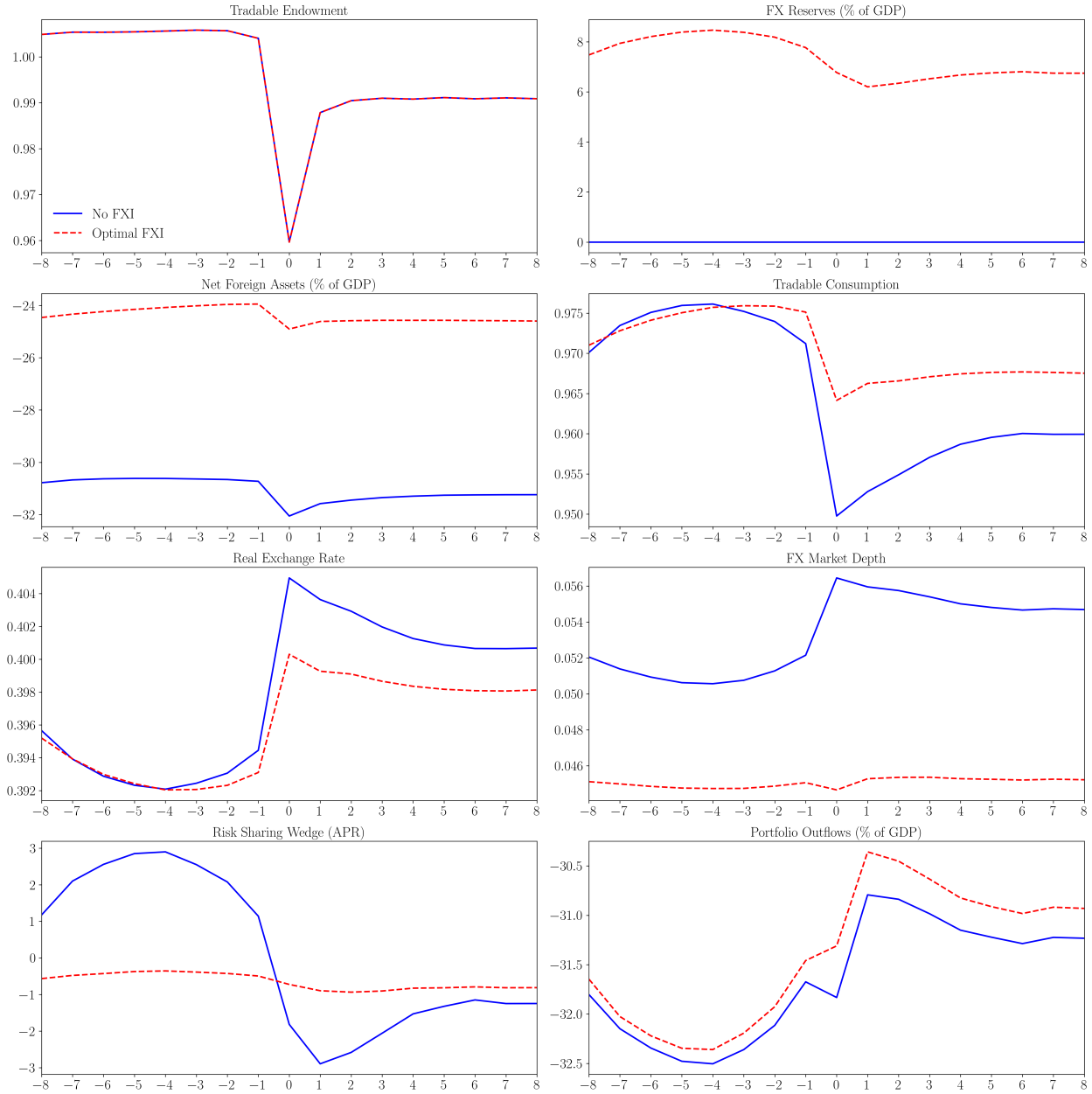
$$\kappa = \left(\frac{V_{ap}\left(B^*, S\right)}{V_b\left(B^*, S\right)}\right)^{\frac{1}{1-\sigma}} - 1$$

We weight the value functions with respect to the stationary distribution of the exogenous state process

$$\tilde{V}_i\left(B^*, \mathcal{S}\right) = \sum_{S \in \mathcal{S}} \psi_S V_i\left(B^*, S\right)$$

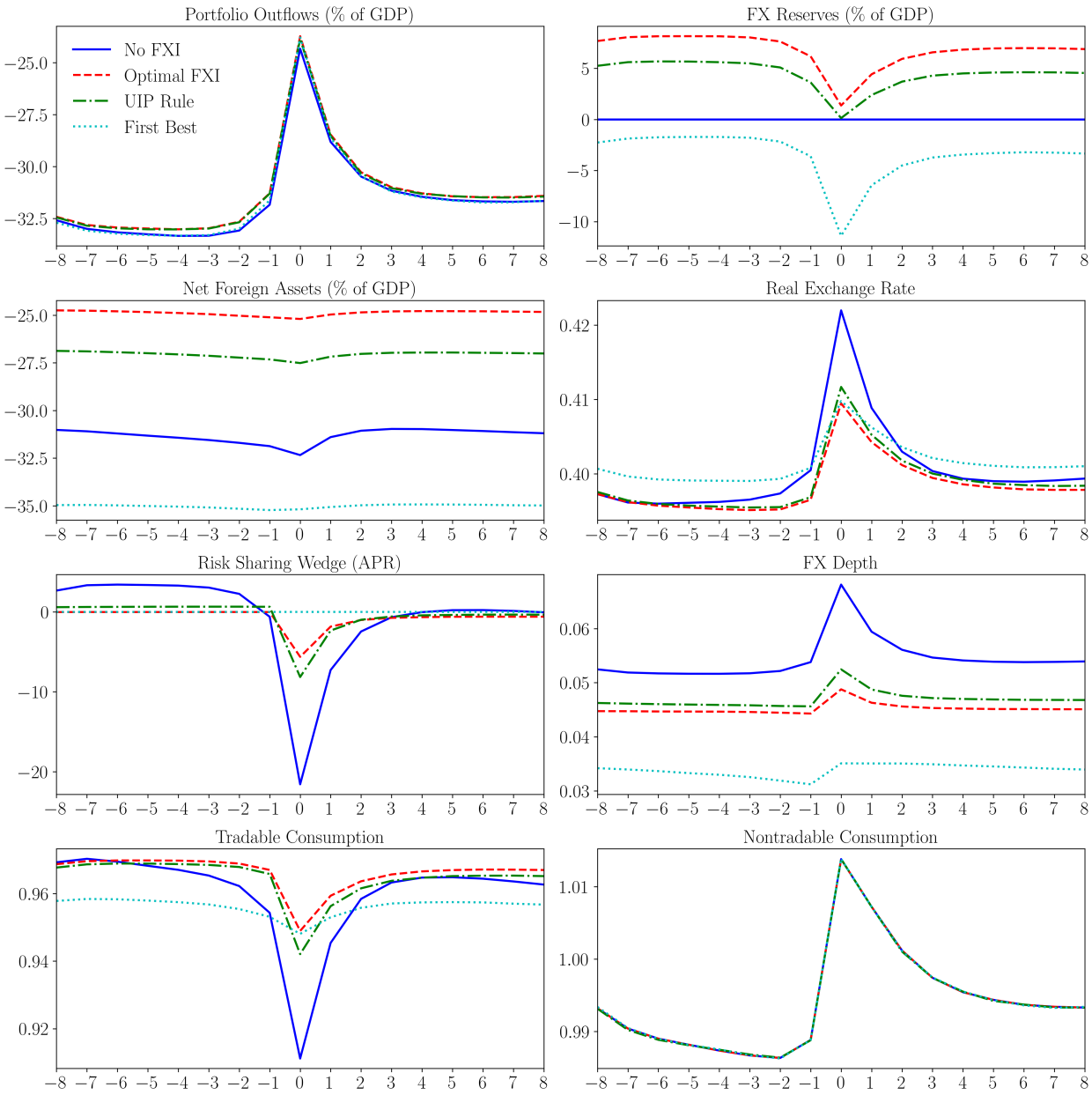
where $\mathcal{S}$ is the set of all exogenous states S and the ψ_S is the probability of state S according to the stationary distribution and $\sum_{S \in \mathcal{S}} \psi_S = 1$. Finally, we impose the steady state net foreign asset position of the benchmark economy $B^* = \overline{B}^*$ as initial condition to get

$$\kappa = \left(\frac{\tilde{V}_{ap}\left(\overline{B}^*, \mathcal{S}\right)}{\tilde{V}_b\left(\overline{B}^*, \mathcal{S}\right)}\right)^{\frac{1}{1-\sigma}} - 1$$



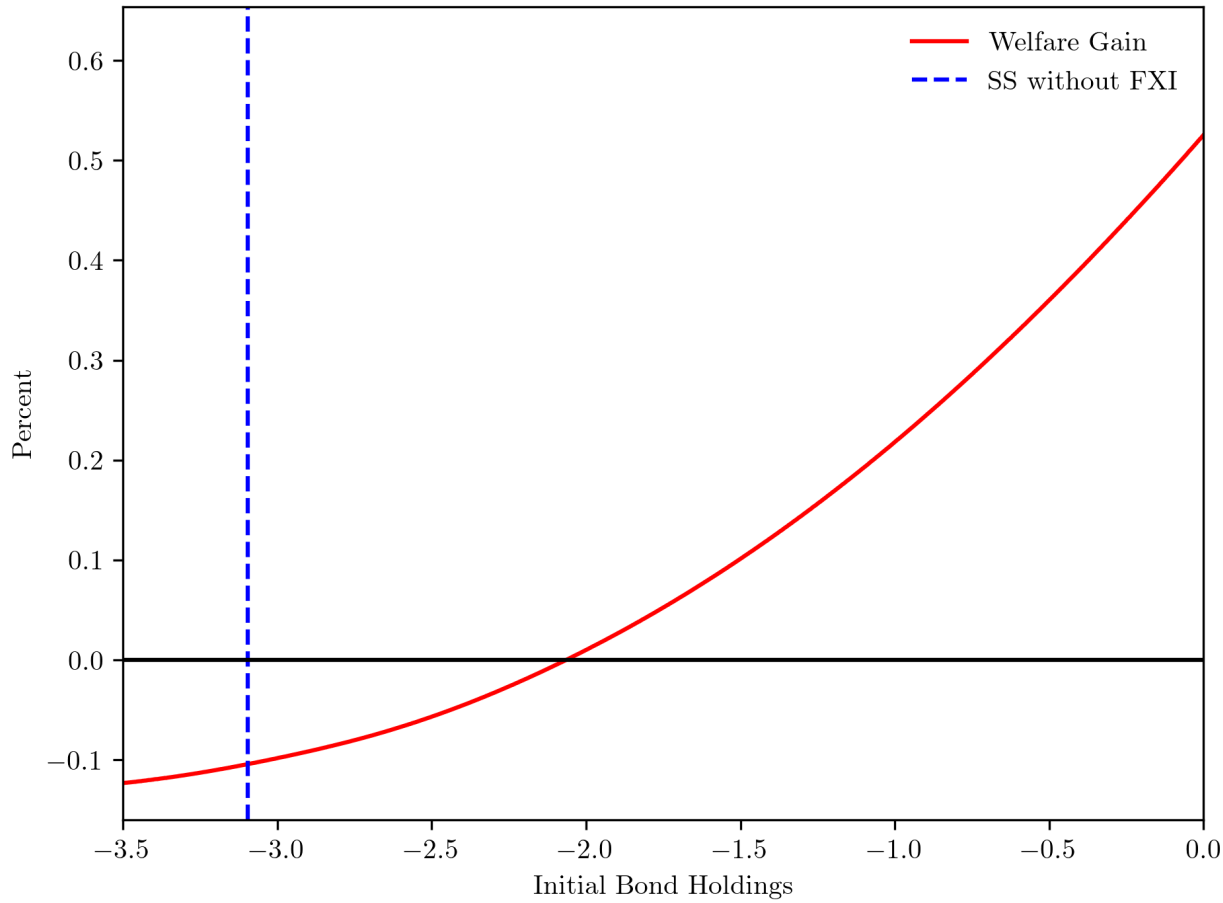
Notes: The plot depicts the average response in the model in episodes of tradable endowment decreases. One period corresponds to a quarter and period 0 coincides with the minimum in tradable endowment.

Figure C.1: Recessionary episodes in the tradable sector



Notes: The plot depicts the average response in the model in episodes of portfolio outflows for different policy regimes. One period corresponds to a quarter and period 0 coincides with the peak of outflows.

Figure C.2: Episodes of portfolio outflows in different policy regimes



Notes: The red solid line shows the welfare gain as a function of initial bond holdings, computed according to (20), where the benchmark economy is no FXI and the alternative policy is optimal time-consistent FXI. The blue dashed vertical line depicts the steady state bond holdings in the economy with no FXI.

Figure C.3: Welfare gain of optimal time-consistent policy relative to no FXI