Accounting for Credibility: Monetary-Fiscal Interactions and the Credibility of Central Bank Mandates*

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Abstract

We develop a model for fiscal and monetary policy determination in the tradition of Sargent and Wallace (1981). Ex-ante, the government has incentives to commit to an inflation rate and delegate monetary policy to a central bank with an inflation targeting mandate. Ex-post, however, the government faces temptations to revoke the mandate to generate seigniorage revenues. The likelihood that the government will adhere to its commitment depends on shocks to fiscal fundamentals and the costs of reneging on the mandate. We interpret the latter as changes in institutions undertaken by governments. The economy endogenously transitions between a "fiscal-dominant" regime where the fiscal authority interferes with monetary policy to generate seigniorage revenues and inflation promises are not credible and a "monetary-dominant" regime where monetary policy adheres to its commitment. We use the model as a measurement device to interpret the pattern of fiscal and monetary policies in Latin American economies. Using the model's implications for the covariance between deficit and inflation, we study the relative contribution of fiscal consolidation efforts and institutional changes in successful disinflationary episodes.

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1 Introduction

In the decades that followed the inflation crises of the 1980s and early 1990s, many emerging-market governments attempted to stabilize prices by delegating monetary policy to newly independent central banks operating under explicit inflation-targeting mandates. While delegation can help anchor expectations before the fact, its effectiveness ultimately rests on whether fiscal authorities remain willing and able to respect the mandate once macroeconomic conditions change. In many instances, governments have revoked such mandates during crises. A canonical example is Argentina's decision in early 2002 to repeal the Convertibility Law and dismantle the currency-board regime, which was taken in the face of mounting debt servicing pressures.

This paper proposes a tractable framework for analyzing the credibility of inflation-targeting mandates in the presence of endogenous fiscal-monetary interactions. Building on the tradition of Sargent and Wallace, we consider a benevolent but time-inconsistent government that (i) commits ex ante to an inflation target by delegating policy to an independent central bank, yet (ii) may choose ex post to abrogate the mandate to generate seigniorage revenues. The decision to renege depends on two state variables: the level of fiscal fundamentals, summarized by outstanding public debt and the marginal utility of public spending, and a stochastic institutional cost of deviation that captures the legal, reputational, and political hurdles associated with overriding the mandate. Joint movements in these states lead the economy to transit endogenously between a monetary-dominant regime—where the target is honored—and a fiscal-dominant regime—where it is not. The model nests the Ramsey allocation, obtained when the cost of deviation is prohibitively large, and the Markov equilibrium, obtained when that cost is zero.

In the model, the government has incentives to reduce the amount of debt issued to reduce its temptation to abrogate the mandate in the following period. This incentive effect results in less debt issued relative to the Ramsey outcome. The effect is stronger if the temptation to switch is larger, which occurs when the marginal utility of spending is large, or the institutional costs are low.

The model has distinct predictions for disinflation that occur due to fiscal fundamentals versus institutional changes. The former is associated with a declining path of inflation and debt, whereas the latter is associated with a declining path of inflation and an increasing path of debt. The reason for this is that increased institutional costs decrease the incentive effect associated with debt issuance and result in higher debt levels.

We use the framework as a measurement device to understand prominent disinflation episodes in Latin America, focusing on Colombia and Chile. We calibrate the model to the

case of Colombia and then invert it to find the path of fiscal fundamental and institutional shocks that can rationalize the path of inflation and debt. Colombia saw a significant decline in inflation alongside an increase in debt after the introduction of the Colombian constitution in 1991, which guaranteed the independence of the central bank. The inversion exercise suggests that a declining path of fiscal fundamentals and an increasing institutional cost is necessary to rationalize the data. To highlight this, we feed a counterfactual path of institutional costs that remained low throughout the 1990s. While the decline path of fundamentals can help generate the declining inflation path, the model predicts that the debt level would have sharply declined. The exercise highlights the value of institutional changes alongside fiscal consolidation efforts.

Related literature

- Sargent and Wallace (1981), Lucas Jr and Stokey (1983), Nicolini (1998), Aiyagari, Marcet, Sargent, and Seppälä (2002), Calvo (1978), Chang (1998), Alvarez, Kehoe, and Neumeyer (2004), Espino, Kozlowski, Martin, Sánchez et al. (2023); Espino, Kozlowski, Martin, and Sánchez (2022), Chari and Kehoe (1999)
- Sargent, Williams, and Zha (2009)
- Sargent (1982), Kehoe and Nicolini (2022)
- Leeper (1991), Bianchi (2013); Bianchi and Ilut (2017); Bianchi, Faccini, and Melosi (2023), Witheridge (2024)
- Atkeson and Kehoe (2001); Atkeson, Chari, and Kehoe (2007), Dovis and Kirpalani (2021), Piguillem and Schneider (2013), Halac and Yared (2025)
- Passadore and Xandri (2024)
- Aguiar and Gopinath (2007), Bocola and Dovis (2019)

2 A Sargent-Wallace economy

2.1 Environment

Consider an economy that blends elements of Aiyagari et al. (2002) and Calvo (1978). Time is discrete and indexed by $t = 0, 1, ...$ The exogenous state of the economy is $s_t \in S$. We assume that s_t follows a Markov process with transition $\Pr(s_{t+1}|s_t)$. The economy is

composed of a stand-in household, competitive firms, and a benevolent government. The stand-in household supplies labor and has utility over private consumption $c\left(s^t\right)$, labor $l\left(s^t\right)$, real money balances $m\left(s^t\right)$, and public consumption $g\left(s^t\right)$ given by

$$\sum_t \sum_{s^t} \beta^t \Pr\left(s^t|s_0\right) \mathcal{U}\left(c\left(s^t\right), l\left(s^t\right), m\left(s^t\right), g\left(s^t\right), s_t\right) \tag{1}$$

where β is the stand-in household discount factor. We assume that

$$\mathcal{U}\left(c, l, m, g, s\right) = c - v\left(l\right) + v\left(m\right) + \theta\left(s\right) u\left(g\right) \tag{2}$$

where $v\left(l\right)$ is a strictly increasing and convex function, $\theta\left(s\right)$ is a preference shock to the marginal utility of government spending, $v\left(m\right)$ and $u\left(g\right)$ are strictly increasing and concave functions. We further assume that all functions are differentiable and v is such that $v'\left(l\right) l$ is convex.[1]

The resource constraint for the economy is

$$c\left(s^t\right) + g\left(s^t\right) \leq l\left(s^t\right). \tag{3}$$

The linear production technology is operated by competitive firms.

The government is benevolent but it may have a different discount factor than the stand-in household. Let $\hat{\beta} \leq \beta$ be the government's discount factor. The government finances government spending with linear taxes on labor income, by issuing real uncontingent debt, and by printing money injected into the economy via open market operations. We also allow the government to make lump-sum transfers to the stand-in household. The government budget constraint is

$$P\left(s^t\right) g\left(s^t\right) + P\left(s^t\right) B\left(s^{t-1}\right) + M\left(s^{t-1}\right) \leq \tau\left(s^t\right) W\left(s^t\right) l\left(s^t\right) + Q\left(s^t\right) B\left(s^t\right) + M\left(s^t\right) \tag{4}$$

where $P\left(s^t\right)$ is the nominal price level, $W\left(s^t\right)$ is the nominal wage, and $Q\left(s^t\right)$ is the price of real debt (in terms of money). (We omit the presence of lump-sum transfers.)

The household problem is to choose $\left\{c\left(s^t\right), l\left(s^t\right), M\left(s^t\right), B\left(s^t\right)\right\}$ to maximize (1) subject to the budget constraint

$$P\left(s^t\right) c\left(s^t\right) + Q\left(s^t\right) B\left(s^t\right) + M\left(s^t\right) \leq \left(1 - \tau\left(s^t\right)\right) W\left(s^t\right) l\left(s^t\right) + M\left(s^{t-1}\right) + B\left(s^{t-1}\right)$$

where real balances are given by $M\left(s^{t-1}\right)/P\left(s^t\right)$. We adopt the Nicolini (1998)-timing

[1] For example, this last assumption is satisfied if the labor disutility has a constant Frisch elasticity, $v\left(l\right) = \chi l^{1+\psi}/\left(1+\psi\right)$, or more generally if $2v''\left(l\right) + v'''\left(l\right) l \geq 0$.

where only money carried over from last period enters the utility function and not newly issued money. This timing convention creates costs to unexpected inflation and guarantees the existence of a Markov perfect equilibrium with positive real balances.

Given initial (M_0, B_0), an equilibrium is a set of allocations $\{c(s^t), l(s^t), M(s^t), B(s^t)\}$, policies $\{\tau(s^t), M(s^t), B(s^t)\}$, and prices $\{P(s^t), W(s^t), Q(s^t)\}$ such that i) the allocation solves household's maximization problem, ii) the government budget constraint holds, iii) $W(s^t) = P(s^t)$ and asset markets clear.

2.2 Implementable allocations

We next derive a set of implementability conditions that characterize the set of fiscal and monetary outcomes that can be supported as a competitive equilibrium. We focus on fiscal and monetary outcomes rather than allocations, deviating from the typical approach in the Ramsey literature, because these outcomes are central to our empirical analysis.

Optimality for the stand-in household's problem requires that the labor supply satisfies

$$(1 - \tau(s^t)) = v'(l(s^t)), \tag{5}$$

the Euler equation for money holdings holds

$$\frac{1}{P(s^t)} = \beta E_t \left[\frac{1}{P(s^{t+1})} + v' \left(\frac{M(s^t)}{P(s^{t+1})} \right) \frac{1}{P(s^{t+1})} \right], \tag{6}$$

and the price for real bonds is given by

$$\frac{Q(s^t)}{P(s^t)} = \beta. \tag{7}$$

Following the insight in Aiyagari (1989) and Aiyagari et al. (2002), we define the static indirect utility function over primary surpluses.[2] Define the real primary surplus as

$$\Delta(s^t) \equiv \tau(s^t) l(s^t) - g(s^t).$$

From the labor supply condition (5), we know that in any competitive equilibrium tax revenues must satisfy the following restriction

$$\tau(s^t) l(s^t) = (1 - v'(l(s^t))) l(s^t).$$

[2]See also Chari, Dovis, and Kehoe (2020).

We can define the static indirect utility function over surpluses as

$$U(\Delta, s) = \max_{c,l,g} c - v(l) + \theta(s)\, u(g)$$

subject to the resource constraint (3) and the static implementability constraint

$$\Delta \leq \left(1 - v'(l)\right) l - g.$$

This function is well-defined for all surplus values below the maximal surplus implied by the static Laffer curve, $\Delta \leq \bar{\Delta} \equiv \max_l \left(1 - v'(l)\right) l$. Under the assumption that $v(l)\, l$ is convex, we have:

Lemma 1. *The indirect utility function $U(\Delta, s)$ is decreasing and concave in Δ for all s. Moreover, $\lim_{\Delta \to \bar{\Delta}} U'(\Delta, s) = -\infty$ and $U'(\Delta, s) = 0$ for all $\Delta \leq g^*(s)$ implicitly defined by $\theta(s)\, u'(g^*(s)) = 1$.*

It is convenient to normalize all nominal variables by the amount of money inherited in each period. In particular, let the normalized price level be $p\left(s^t\right) = P\left(s^t\right) / M\left(s^{t-1}\right)$, the money growth rate be

$$\mu\left(s^t\right) \equiv M\left(s^t\right) / M\left(s^{t-1}\right),$$

and let

$$\phi\left(s^t\right) \equiv 1 / p\left(s^t\right) = M\left(s^{t-1}\right) / P\left(s^t\right)$$

be the inverse of the normalized price level – the price of money in terms of the private consumption good – or the real monetary balances $M\left(s^{t-1}\right) / P\left(s^t\right)$. Using these normalizations and defining

$$H(\phi) \equiv \phi + v'(\phi)\, \phi,$$

we can rewrite the money demand condition (6) as

$$\mu\left(s^t\right) \phi\left(s^t\right) = \beta \sum_{s_{t+q}} \Pr\left(s_{t+1} | s_t\right) H\left(\phi\left(s^{t+1}\right)\right) \tag{8}$$

and the government budget constraint (4) as

$$b\left(s^{t-1}\right) + \phi\left(s^t\right) = \Delta\left(s^t\right) + \beta b\left(s^t\right) + \mu\left(s^t\right) \phi\left(s^t\right) \tag{9}$$

We can then state the implementability conditions:

Lemma 2. *A fiscal and monetary outcome $\left\{\Delta\left(s^t\right), b\left(s^t\right), \phi\left(s^t\right), \mu\left(s^t\right), \pi\left(s^t\right)\right\}$ is implementable as a competitive equilibrium given an initial level of normalized real debt b_0 if and only if it satisfies*

the normalized version of the government budget constraint (9), a NPG condition, the Euler equation for money holdings (8), the feasibility condition for the primary surplus $\Delta\left(s^t\right) \leq \bar{\Delta}$*, and inflation is given by*

$$1+\pi\left(s^{t+1}\right)=\frac{\mu\left(s^t\right)\phi\left(s^t\right)}{\phi\left(s^{t+1}\right)}. \tag{10}$$

The associated value for the government is given by

$$V_0=\sum_t\sum_{s^t}\hat{\beta}^t\Pr\left(s^t|s_0\right)\left[U\left(\Delta\left(s^t\right),s_t\right)+v\left(\phi\left(s^t\right)\right)\right]. \tag{11}$$

Note that we can combine the (9) with (8) and iterating forward and invoking the household's transversality conditions to obtain

$$b\left(s^{t-1}\right)+\phi\left(s^t\right)=\sum_{j=0}^{\infty}\sum_{s^{t+j}}\beta^j\Pr\left(s^{t+j}|s^t\right)\Delta\left(s^{t+j}\right)+\sum_{j=1}^{\infty}\sum_{s^{t+j}}\beta^j\Pr\left(s^{t+j}|s^t\right)h\left(\phi\left(s^{t+j}\right)\right) \tag{12}$$

where $h\left(\phi\right)\equiv v'\left(\phi\right)\phi$. That is, the real value of government's liabilities, $b\left(s^{t-1}\right)+\phi\left(s^t\right)$, equals the discounted value of current and future primary surpluses and future seigniorage revenues, $h\left(\phi\left(s^{t+j}\right)\right)$.

3 Policy determination

We want to study fiscal and monetary outcomes when the government cannot commit. A common approach is to consider a sustainable equilibrium, i.e., a subgame perfect equilibrium (SPE) of the policy game. This restriction however does not sufficiently narrow down the set of outcomes that can occur when the government lacks commitment. Following the logic in Abreu (1988) and Chari and Kehoe (1990), any fiscal and monetary outcome that is consistent with the implementabiliy conditions in Lemma 2 and a *sustainability constraint* that requires that the continuation value for the government after any history is above the value of the worst equilibrium is a valid sustainable equilibrium outcome.

The question is how to select, model, and think of ways the government tries to manage expectations and how private agents coordinate its punishment when there is a deviation.[3] Previous work considers either the best sustainable equilibrium or the Markov perfect equilibrium (typically the worst sustainable equilibrium).

We take a different approach motivated by the observation that several countries have recently adopted some form of an inflation targeting regime. We assume that the gov-

[3]This problem is beautifully explained in Sargent (2024).

ernment tries to manage expectations by promising to deliver an inflation target π^* next period. The government in the following period can choose whether to maintain the inflation target or to pay a random cost ξ to deviate from the target. This cost is exogenous and captures factors such as reputation losses, coordination failures leading to worse equilibria, institutional constraints, and the political costs faced by policymakers.

This exogenous cost allows us to model policy determination and expectation formation in a way that continuously nests both the Ramsey outcome and the Markov equilibrium. This flexible formulation enables us to confront the data and examine how changes in the credibility of promises — represented by changes in ξ— influence the dynamics of fiscal and monetary outcomes.

Recursive formulation We next set up the problem recursively. In addition to the exogenous state and the real value of debt, we also need to include the inflation target that was promised in the previous period, π^*. It is however more convenient and without loss to keep track of the promised value for real balances ϕ. In fact, consider a government in period t that promises an inflation target π^* for the next period. From (10) we know that

$$1 + \pi^* = \frac{\mu_t \phi_t}{\phi_{t+1}}$$

From period t's perspective – given the current money growth rate and ϕ_t – there is a one-to-one map between the inflation target π^* and the promised value for real money balances ϕ_{t+1}. Thus, we can keep track of promised real money balances and let the state be $x = (b, \phi, s)$. (We do not need to record additional state variables like the promised marginal utility for the stand-in households because we are considering an environment with quasi-linear preferences in consumption and so there is no time-inconsistency problem arising from interest rate manipulation.)

Note that we do not allow the promised value of real balances to be contingent on the next period's state. In principle, one could imagine that the delegation of monetary policy includes state-dependent escape clauses. However, a justification for our assumption is that it is difficult to detect deviations and impose penalties when promised inflation is state-dependent and information about the state is dispersed, as shown by Piguillem and Schneider (2013).

In any period, the economy can be in either a *monetary dominant* regime (denoted by md) where the government satisfies the inflation target and attains value $V_{md}(x)$, or in a *fiscal dominant* regime (denoted by fd) where the government deviates from the set target and

attains a value net of the cost ξ given by $V_{fd}(b,s)$. The value for the government is then

$$V(b,\phi,s) = \max\left\{V_{md}(b,\phi,s), V_{fd}(b,s) - \xi(s)\right\} \tag{13}$$

We let $\eta(x)$ be an indicator variable that takes value 1 if it is optimal to satisfy the target and be in the monetary dominant regime and 0 otherwise:

$$\eta\left(b',\phi',s'\right) = \begin{cases} 1 & \text{if } V_{md}(b,\phi',s') \geq V_{fd}(s') - \xi(s') \\ 0 & \text{if } V_{md}(b,\phi',s') < V_{fd}(s') - \xi(s') \end{cases}. \tag{14}$$

Finally, we let $J(b',\phi',s)$ be the expected marginal value of real balances the following period. As it will be clear, this is a function of newly issued debt b', the next period inflation target summarized by ϕ', and the current exogenous state s.

Monetary dominance The problem for the government when it respects the target is

$$V_{md}(b,\phi,s) = \max_{\Delta,b',\mu,\phi'} U(\Delta,\theta) + v(\phi) + \hat{\beta}\sum_{s'}\Pr\left(s'|s\right) V\left(b',\phi',s'\right) \tag{15}$$

subject to the budget constraint

$$\Delta = b + \phi - \beta b' - \mu\phi$$

and the money-demand condition

$$\mu\phi = J\left(b',\phi',s\right)$$

with $b' \in \left[0,\bar{b}\right]$.

Fiscal dominance The problem when the inflation target is not satisfied is similar but now the government can choose the current value for the real money balance ϕ:

$$V_{fd}(b,s) = \max_{\phi,\Delta,b',\mu,\phi'} U(\Delta,\theta) + v(\phi) + \hat{\beta}\sum_{s'}\Pr\left(s'|s\right) V\left(b',\phi',s'\right) \tag{16}$$

subject to

$$\Delta = b + \phi - \beta b' - \mu\phi$$

$$\mu\phi = J\left(b',\phi',s\right)$$

Note for later that in this case the value of real balances is determined by the static first order condition

$$-U(\Delta,\theta)=v'(\phi_{fd}). \tag{17}$$

That is, the marginal benefit of of real balances is equated to the marginal cost of the primary surplus. Thus, since money supply is pre-determined, the model predicts a higher price level (lower real balances) when the marginal cost of the surplus is high. This happens when either the value of public spending θ is high or when the amount of inherited debt is high and so the government must run a surplus just to stabilize the level of debt.

Equilibrium The expected marginal value for real balances next period is

$$J(b',\phi',s)=\beta\sum_{s'}\Pr(s'|s)\left[\eta(b',\phi',s')H(\phi')+(1-\eta(b',\phi',s'))H(\phi_{fd}(b',s'))\right] \tag{18}$$

We can then define an equilibrium as a set of value functions V, V_{md}, V_{fd}, policy functions $\phi_i,\Delta_i,b_i',\mu_i,\phi_i'$ for $i=md,fd$, a regime choice function η and J that satisfy (13), (15), (16), (18), and (14).

Note that our equilibrium notion nests the Ramsey outcome if the support of ξ is large enough so that $\eta=1$ in every period in which case $J(b',\phi',s)=\beta H(\phi')$ and the government in the current period controls the amount of inflation next period. The model also nests the Markov perfect equilibrium outcome if the cost ξ always equals 0 so that it is always better to be in the fiscal dominant regime, $\eta=0$, and $J(b',\phi',s)=\beta\sum_{s'}\Pr(s'|s)H(\phi_{fd}(b',s'))$.

4 Model dynamics and credibility of inflation targets

We now analyze the implied dynamics for debt, inflation targets, and inflation. Before doing so, we consider two benchmark allocations: the Ramsey outcome and the Markov outcome.

4.1 Ramsey outcome

The Ramsey outcome is the best allocation consistent with a competitive equilibrium. Given our definition of indirect utility $U(\Delta)$ and the characterization of implementable allocations in Lemma 2, we can write the Ramsey problem as choosing $\{\Delta(s^t),b(s^t),\phi(s^t),\mu(s^t)\}$ to maximize (11) subject to (8), (9) and $b(s^t)\leq\bar{b}$ given b_0.

Note that the Ramsey outcome, other than assuming commitment, allows for greater flexibility than the equilibrium considered here because the promised value $\phi\left(s^{t+1}\right)$ can be contingent on the state of the world in period $t+1$. This can be valuable because it allows the government's total real liabilities to be state-contingent and overcome the assumed market incompleteness for real debt.

To keep the analysis parallel with our recursive formulation of the full model above, we express the Ramsey problem in a quasi-recursive form. The Ramsey outcome from period 1 onward solves the following recursive problem:

$$V_R(b,\phi,s) = \max_{\Delta,b',\phi'(s')} U(\Delta,s) + v(\phi) + \hat{\beta}\sum_{s'}\Pr\left(s'|s\right) V_R\left(b',\phi'\left(s'\right),s'\right) \tag{19}$$

subject to

$$\Delta = b + \phi - \beta b' - \beta\sum_{s'}\Pr\left(s'|s\right) H\left(\phi'\left(s'\right)\right)$$

and $b' \leq \bar{b}$. In period 0, the Ramsey outcome solves (19) with $\phi_0 = \arg\max_{\phi} V_R(b_0,\phi,s_0)$.

Proposition 3. *Suppose $v(\phi) = \chi\frac{\phi^{1-\eta}}{1-\eta}$ for $\eta \in (0,1)$, $\chi > 0$, and there is a bound on real balances, $\phi\left(s^t\right) \leq \phi^*$. If the volatility of θ is small enough then the Ramsey outcome has $\phi'\left(s^t\right) = \phi^*$ for all $t \geq 1$ and s^t. If we impose the additional restriction that $\phi'(s') = \phi'$ for all s' then $\phi'\left(s^{t-1}\right) = \phi^*$ for all s^{t-1} for all process for θ_t.*

Under our assumptions, there is a trade-off between following the Friedman rule and making real debt state contingent.[4] If the volatility of the marginal value of government expenditures is sufficiently small, the benefits of making the real debt state contingent are small relative to the cost of anticipated inflation and it is optimal to set $\phi(s') = \phi^*$ for all s' next period. Thus, the constraint that the promised value of money must be constant across states in the next period is not binding.

Under the conditions in the proposition, the Ramsey outcome has a fixed inflation level given by[5]

$$1 + \pi_R = \frac{\beta H(\phi^*)}{\phi^*} = \beta\left(1 + \chi\left(\phi^*\right)^{-\eta}\right)$$

In particular, the inflation level does not depend on the fiscal fundamentals – the level of debt and θ. One way to implement the Ramsey outcome is to delegate monetary policy to an independent central bank with a mandate to target an inflation rate of π_R, while fiscal

[4] Chari and Kehoe (1999) show how in cash-credit good economy it is possible to simultaneously follow the Friedman rule and use inflation to generate state contingencies.

[5] The case with no bounds on real balances has $\phi^* \to \infty$ and so the inflation rate in the Ramsey outcome equals $\beta - 1$.

policy is determined by the treasury, which solves a problem similar to that in the real economy studied by Aiyagari et al. (2002), taking as given a constant flow of seigniorage revenues (which may be negative). The dynamics of surpluses and real debt must be consistent with the Euler equation

$$-U'(\Delta, s) = \frac{\hat{\beta}}{\beta} \sum_{s'} \Pr(s'|s) \left[-U\left(\Delta\left(s'\right), s'\right)\right] \tag{20}$$

and the budget constraint. This arrangement, while optimal, relies on the government's ability to commit. In the subsequent sections, we will see how the optimal mandate changes if the government cannot commit.

When the preferences over real money balances take a different form then the iso-elastic one considered in the proposition, we cannot prove that a version of the Friedman rule is optimal. For example, if $v(m)$ is quadratic, the optimal value for ϕ in the Ramsey outcome will typically be different from the satiation point and dependent on fiscal fundamentals, but, in numerically simulations, we find that such variations are small and delegation of monetary policy to an independent central bank with a fixed inflation targeting is, loosely speaking, approximately optimal under commitment.

4.2 Markov outcome

We now turn to the polar opposite case in which the government has no way to commit to inflation and consider a Markov equilibrium outcome. This is a special case of our environment with $\xi(s) = 0$ for all s in which case the fiscal dominant regime is always optimal. Consequently, we can drop ϕ' as a choice in (16) since it has no effect on the value. The problem reduces to

$$V_M(b, s) = \max_{\phi, \Delta, b'} U(\Delta, \theta) + v(\phi) + \hat{\beta} \sum_{s'} \Pr(s'|s) V_M\left(b', s'\right) \tag{21}$$

subject to

$$\Delta = b + \phi - \beta b' - \beta \sum_{s'} \Pr(s'|s) H\left(\phi_M\left(b', s'\right)\right).$$

The optimum is then characterized by a static optimality condition that relates the surplus to the level of inflation, (17), and an intertemporal optimality condition,

$$-U'(\Delta,\theta)\left[1+\sum_{s'}\Pr(s'|s)\,H'\left(\phi_M(b',s')\right)\frac{\partial\phi_M(b',s')}{\partial b'}\right]=\frac{\hat{\beta}}{\beta}\sum_{s'}\Pr(s'|s)\,V'_M(b',s') \quad (22)$$
$$=\frac{\hat{\beta}}{\beta}\sum_{s'}\Pr(s'|s)\left[-U'(\Delta',s')\right]$$

The term $\sum_{s'}\Pr(s'|s)\,H'\left(\phi_M(b',s')\right)\frac{\partial\phi_M(b',s')}{\partial b'}$ captures the incentive effect of debt issuance. The current government can influence the choices of next period government only indirectly by affecting the amount of debt inherited by the next period government. Since $\partial\phi_M(b',s')/\partial b' < 0$ and $H' > 0$, this term is negative and acts a tax on debt issuance relative to the Ramsey Euler equation (20). This is because the next period value of real balances is too low from perspective of the current government since it would like to commit to the Ramsey level ϕ_R. The current government then tries to increase the value of real balances next period by reducing the amount of debt inherited by the subsequent government thereby inducing it to choose a larger ϕ'. Therefore, debt issuances are distorted downward relative to the Ramsey outcome.

4.3 Full model

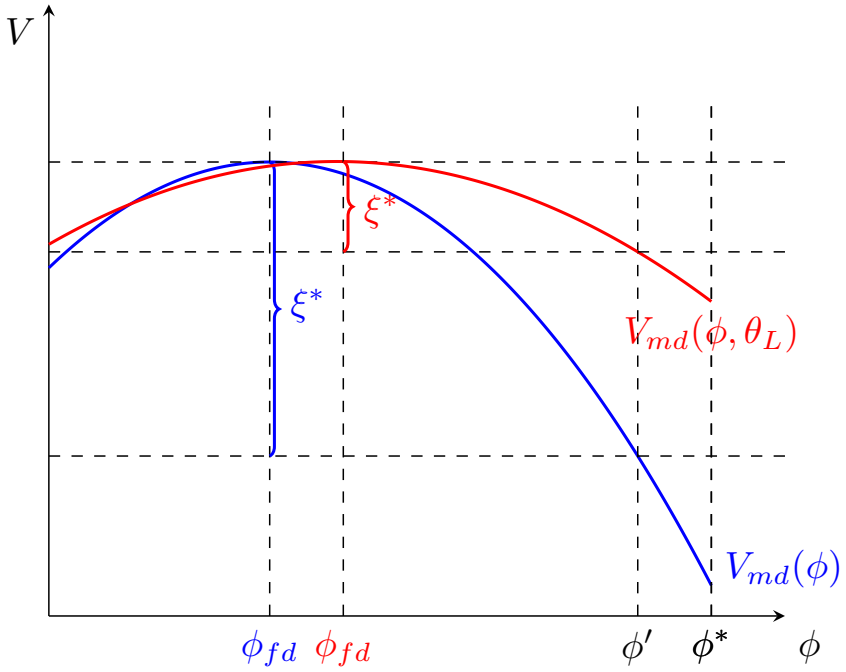
Credibility of mandates and optimal inflation target We now analyze the credibility of an inflation target summarized by a promised ϕ. The inflation target is satisfied if and only if

$$\xi \geq \xi^* = V_{fd}(b,s) - V_{md}(b,\phi,s) = \max_{\phi_{fd}} V_{md}\left(b,\phi_{fd},s\right) - V_{md}(b,\phi,s)$$

As illustrated in Figure 1, deviating from the target allows the government to attain the maximum utility possible net of the cost ξ. Thus, a cost ξ greater than the cutoff ξ^* is required for the target to be sustained and for $\eta = 1$. In particular, the cutoff ξ is larger the more ambitious the inflation target. That is, inflation targets closer to the Ramsey value ϕ^* are harder to achieve than lower inflation targets.

The inflation target is easier to achieve when the marginal utility of government expenditure is lower. This is because the value of reducing the distortions associated with positive surpluses is smaller when θ is low and government expenditures are less valuable. A similar argument applies to the level of inherited debt: a lower initial stock of debt b reduces the minimum cost required to satisfy the target, thereby making it more likely that the target can be met.

Figure 1: Credibility of inflation targets



The next lemma summarizes the above discussion:

Lemma 4. *The cutoff $\xi^*(b, \phi, s)$ is increasing in b, ϕ, and θ. That is, the target is less likely to be satisfied if b, ϕ and θ are large.*

We now characterize the optimal inflation target. For this, we assume that ξ follows a continuous distribution $f(\xi'|\xi)$. If the optimal inflation target is interior, it must satisfy the following necessary condition whenever the equilibrium objects are differentiable:

$$-U'(\Delta, s)\frac{\partial J(b', \phi', s')}{\partial \phi'} + \hat{\beta}\sum_{s'}\Pr(s'|s)\frac{\partial V(b', \phi', s')}{\partial \phi'} = 0$$

where

$$\begin{aligned}\frac{\partial J(b', \phi', s)}{\partial \phi'}/\beta = &\sum_{s'}\Pr(s'|s)\,\eta(b', \phi', s')\,H'(\phi') \\ &-\sum_{\theta'}\Pr(\theta'|\theta)\frac{\partial \xi^*}{\partial \phi'}\left[H(\phi') - H(\phi_{fd}(b', s'))\right] f(\xi^*|\xi)\end{aligned}$$

and

$$\sum_{s'}\Pr(s'|s)\frac{\partial V(b', \phi', s')}{\partial \phi'} = \sum_{s'}\Pr(s'|s)\,\eta(b', \phi', s')\left[v'(\phi') + \frac{\partial V(b', \phi', s')}{\partial b'}\right]$$

which can be combined to obtain

$$0 = -U'(\Delta, s)\beta H'(\phi') + \hat{\beta}\left[v'(\phi') + \mathbb{E}\left(\frac{\partial V(b', \phi', s')}{\partial b'}|\eta' = 1\right)\right] \tag{23}$$
$$+ \frac{U'(\Delta, s)}{N(b', \phi', s)} \sum_{\theta'} \Pr(\theta'|\theta) \frac{\partial \xi^*}{\partial \phi'} \left[H(\phi') - H\left(\phi_{fd}(b', s')\right)\right] f(\xi^*|\xi)$$

where $N(b', \phi', s) \equiv \sum_{s'} \Pr(s'|s)\, \eta(b', \phi', s')$ is the probability that the target is satisfied next period.

The first line in (23) is the optimality condition that must be satisfied in the Ramsey outcome (with $\eta = 1$ always). It equates the marginal benefits of increasing promised ϕ' coming from higher resources raised today, $-U'(\Delta, s)\beta H'(\phi')$, and a higher value of real balances tomorrow, $\hat{\beta} v'(\phi')$, to the marginal cost of having higher total real liabilities, captured by the term $\hat{\beta}\mathbb{E}\left(\frac{\partial V(b', \phi', s')}{\partial b'}|\eta' = 1\right)$. The term in the second line represents the additional marginal cost of increasing ϕ' due to the incentive provision. The term is negative since $\partial \xi^* / \partial \phi' > 0$, $H(\phi') \geq H\left(\phi'_{fd}\right)$ and $U' < 0$. A lower ϕ' increases incentives to respect the target ($V_{md} > V_{fd} - \xi'$). This in turn increases the expected marginal value of money as $\phi' > \phi'_{fd}$.

The presence of the last term makes it optimal to choose a lower promised ϕ' than under the Ramsey outcome, in order to better incentivize the next period's government not to deviate from the plan.[6] For example, it may not be optimal to promise the Friedman rule under the condition in Proposition 3. Note further that a lower ϕ' implies a lower inflation target as the realized inflation if the promised ϕ' is delivered is $1 + \pi' = \frac{\mu\phi}{\phi'} = \frac{\beta J(\phi')}{\phi'}$ which is decreasing in ϕ'.

However, if we have CRRA preferences with a maximal amount of real balances, ϕ^*, the constraint $\phi' \leq \phi^*$ can be binding if

$$-U'(\Delta, s)\beta H'(\phi^*) + \hat{\beta}\left[v'(\phi^*) + \mathbb{E}\left(\frac{\partial V(b', \phi^*, s')}{\partial b'}|\eta' = 1\right)\right]$$
$$> -\frac{U'(\Delta, s)}{N(b', \phi^*, s)} \sum_{\theta'} \Pr(\theta'|\theta) \frac{\partial \xi^*}{\partial \phi'} \left[H(\phi^*) - H\left(\phi_{fd}(b', s')\right)\right] f(\xi^*|\xi)$$

in which case $\phi' = \phi^*$.

Realized inflation and money growth rate Realized inflation differs from the announced inflation target because of the possibility that the government ignores the target and chooses the statically optimal inflation level. Using the definition of J in (10), we can express the

[6] This is similar to the logic in Dovis and Kirpalani (2021).

realized inflation level in the monetary-dominant regime as

$$1+\pi\left(b,\phi,s_{-}\right)=\frac{J\left(b,\phi,s_{-}\right)}{\phi}<\frac{\beta H\left(\phi\right)}{\phi}$$

Note that realized inflation in a given period depends on the inherited debt b, the promised value of money holdings, and the state of the economy in the previous period, s_{-}, (and not the current state s). Moreover, for a given promised ϕ, uncertainty about satisfying the regime actually decreases the realized inflation relative to the commitment case.

If instead we are in the fiscal-dominant regime then inflation depends on the current state as well:

$$1+\pi_{fd}\left(b,\phi,s_{-},s\right)=\frac{J\left(b,\phi,s_{-}\right)}{\phi_{fd}\left(b,s\right)}.$$

This is because s affects the current fiscal surpluses and therefore the chosen $\phi_{fd}\left(b,s\right)$. Since in equilibrium $\phi_{fd}\left(b,s\right)<\phi$, the realized inflation is higher in the fiscal dominant regime than in the monetary dominant regime. [7]

The model also has implications about the growth rate of money supply described in

$$\mu=J\left(b',\phi',s\right)/\phi.$$

If there is uncertainty about respecting the target in the future and $J\left(b',\phi',s\right)$ falls, the government must follow a much tighter monetary policy to implement its target ϕ.

Debt dynamics We next consider the optimal debt issuance. The necessary condition for an optimal interior b' is

$$\begin{aligned}-U'\left(\Delta,s\right)\left(1+\frac{\partial J\left(b',\phi',s\right)}{\partial b'}/\beta\right)&=-\frac{\hat{\beta}}{\beta}\sum_{s'}\Pr\left(s'|s\right)\frac{\partial V\left(b',\phi',s'\right)}{\partial b'}\\&=\frac{\hat{\beta}}{\beta}\sum_{s'}\Pr\left(s'|s\right)\left[-U'\left(\Delta\left(s'\right),s'\right)\right]\end{aligned}\tag{24}$$

[7] Note that

$$1+\pi\left(b',\phi',s,s'\right)=\frac{J\left(b',\phi',s\right)}{\phi_{fd}\left(b',s'\right)}>\frac{\beta\sum_{s'}\Pr\left(s'|s\right)H\left(\phi_{fd}\left(b',s'\right)\right)}{\phi_{fd}\left(b',s'\right)}$$

so realized inflation can be higher than in a Markov equilibrium.

The term $\frac{\partial J(b',\phi',s)}{\partial b'}/\beta$ is negative and effectively acts as a tax on debt issuance, leading to lower debt levels relative to the Ramsey Euler equation (20). To see this, note that

$$\frac{\partial J\left(b',\phi',s\right)}{\partial b'}/\beta = \sum_{s'} \Pr\left(s'|s\right)\left(1-\eta\left(b',\phi',s'\right)\right) H'\left(\phi_{fd}\left(b',s'\right)\right)\frac{\partial \phi_{fd}\left(b',s'\right)}{\partial b'} \tag{25}$$
$$-\sum_{s'}\frac{\partial \xi^*}{\partial b'}\left[H\left(\phi'\right)-H\left(\phi_{fd}\left(b',s'\right)\right)\right] \leq 0$$

The first term captures the incentive effect described in the Markov equilibrium, which operates for next-period states where it is optimal to deviate from the inflation target and enter the fiscal dominant regime ($\eta' = 0$). This term is negative since $H' > 0$ and $\partial \phi_{fd}/\partial b' < 0$. The second term is also negative because, as explained above, since the cutoff ξ^* is increasing in the amount of inherited debt, and $H\left(\phi'\right) > H\left(\phi_{fd}\left(b',s'\right)\right)$.

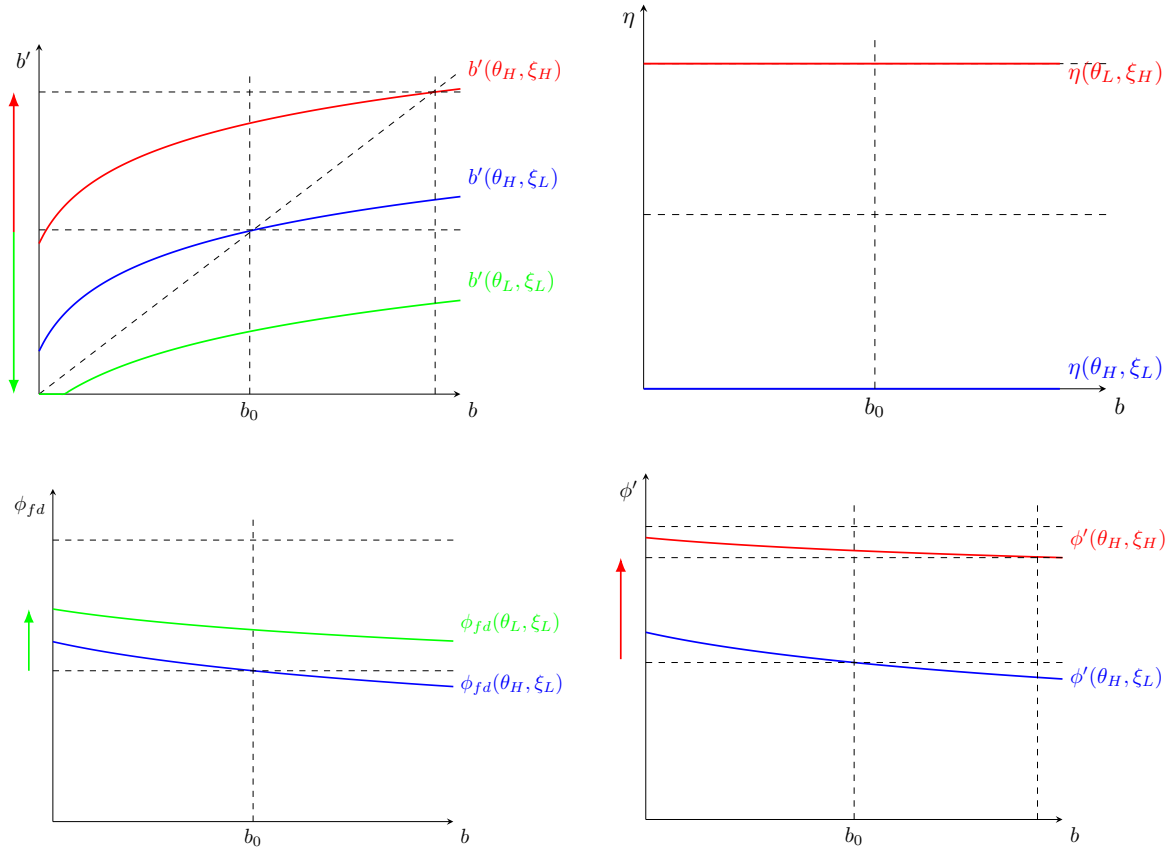
It is therefore optimal to reduce debt issuance in order to induce a higher ϕ_{fd} in the event of a switch to the fiscal dominant regime in the next period, and to strengthen the next-period government's incentive to adhere to the inflation target. This incentive to limit indebtedness becomes stronger as the probability of switching to the fiscal dominant regime increases. Heuristically, if there are two distributions for ξ' and one first-order stochastically dominates the other, we should expect that the wedge in the Euler equation (24), $|\frac{\partial J(b',\phi',s)}{\partial b'}/\beta|$, is larger for the dominated distribution. That is, the higher the expected future costs of switching to the fiscal dominant regime, the smaller the wedge becomes. In the limiting case where only the monetary dominant regime is possible next period—as in the Ramsey outcome—the wedge disappears entirely, and there are no downward distortions to debt issuance. This basic intuition is central to understanding the different responses of debt to shocks to θ or ξ, which we study next.

5 Two types of disinflation

We now describe the two ways in which the model can generate a reduction in the level of realized inflation. Inflation can decline either due to a reduction in the marginal value of government spending, θ, or due to an increase in the (expected) cost of deviating from the promised inflation. We will refer to the former as *fundamental disinflation* and to the latter as *institutional disinflation*. We will show that these two distinct paths have different implications for the dynamics of public debt.[8]

[8]In Appendix B we prove for a deterministic economy with $\beta = \hat{\beta}$ that a Markov equilibrium outcome is characterized by a higher level of inflation and a lower level of debt relative to the Ramsey outcome that starts with the same level of inherited debt. This result illustrate how increase in the ability to commit allows

Figure 2: Two types of disinflation



Fundamental disinflation Consider a path in which the realization of ξ_t is low enough so that it is always optimal to be in the fiscal dominant regime. Along this path, the value of real money balances (and inflation) is determined by the static condition (17) and is therefore closely tied to fiscal considerations, as in a Markov equilibrium.

Suppose next that the desirability of government expenditures falls from θ_H to θ_L in period t_0 but it remains optimal to stay in the fiscal dominant regime. As illustrated in the third panel of Figure (2), the reduction in θ shifts the policy function $\phi_{fd}(b, \theta)$ upward: for any level of real debt, the government finds it optimal to choose a higher value for real balances, reflecting the lower marginal value of relaxing its budget constraint when government spending is less valuable. This has the effect of reducing the level of realized inflation.

The optimal policy for debt issuance, by contrast, shifts downward (see the first panel of the figure). This occurs because the government now has stronger precautionary saving motives and therefore chooses to reduce its debt issuance. As a result, a decline in θ while keeping ξ at a low level (e.g. $\xi = 0$) leads to an increase in real money balances (i.e., lower inflation) and a decrease in real debt.

Figure 3 plots the impulse response function (IRF) to a reduction in θ in our calibrated model (see below for details). The realized shocks are shown in the first two panels. Following the shock, the debt-to-GDP ratio declines, while the inflation rate initially drops sharply in the period when θ falls, reflecting the lower marginal cost of generating primary surpluses. Inflation then continues to decline gradually, tracking the falling path of debt, which further reduces the marginal cost of surpluses.

Institutional disinflation We now consider the effects of an increase in the cost of deviating from the promised inflation target. Specifically, we begin with the same path of low ξ and high θ considered above, and suppose that at t_0, the realized cost of deviation permanently rises from ξ_L to ξ_H, with ξ_H high enough to make it optimal to switch to the monetary dominant regime, as shown in the second panel of Figure 2. In this case, the reduction in inflation and the increase in the realized value for real balances is driven by the change in the regime. The realized ϕ now equals to the promised value of real balances which is higher than the statically optimal level ϕ_{fd}. Critically, if the process for ξ is persistent, an increase in current ξ implies an increase in the expected value for ξ'. Thus, as argued in the previous section, the government now has lower incentives to reduce the amount of debt it issues because the wedge in (24) given by (25) is smaller in absolute value. Thus, the government increases its debt issuance as shown in the first panel of Figure 2.

for low inflation and higher debt as in a fundamental disinflation.

Figure 3: Fundamental disinflation

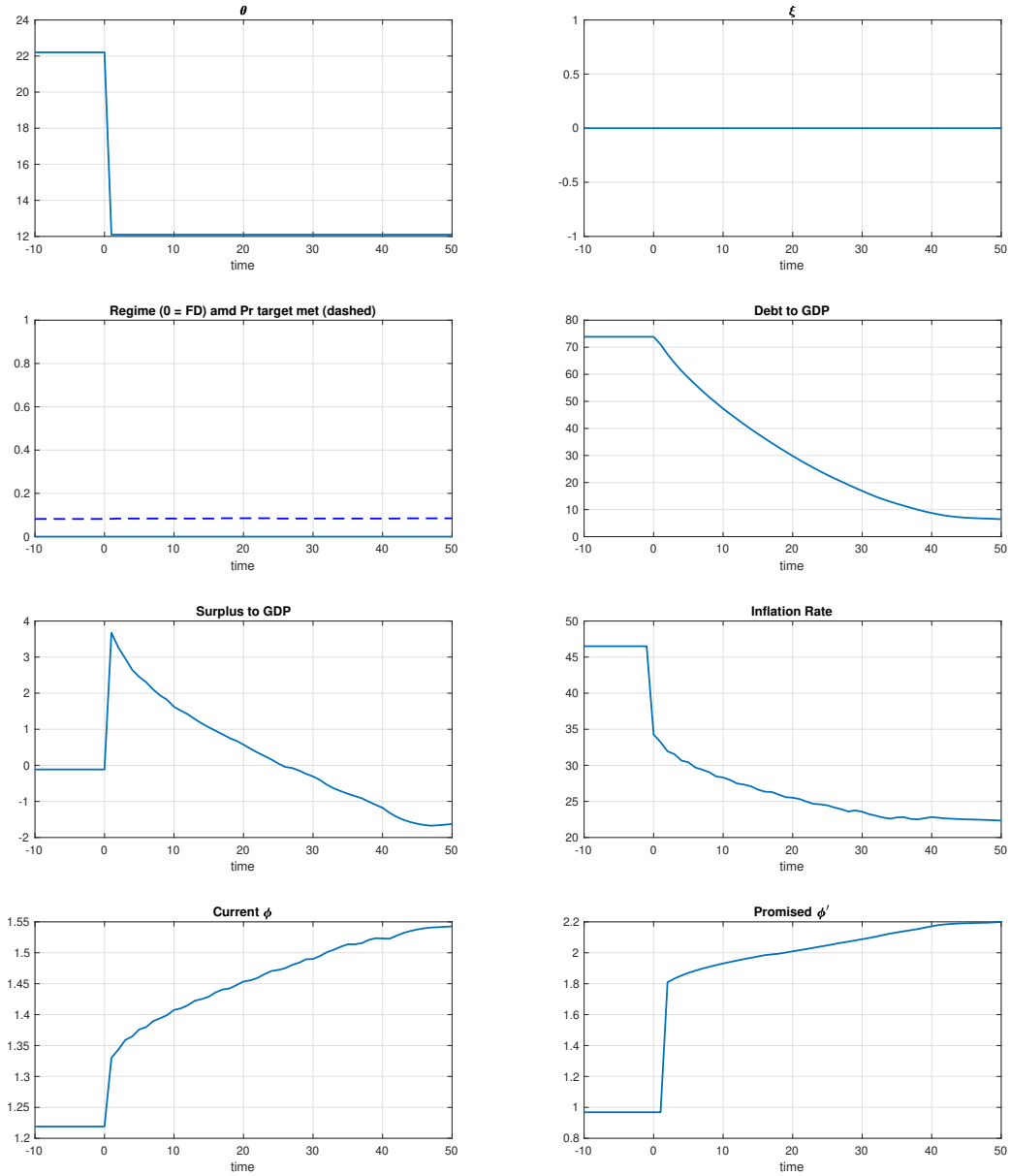


Figure 4: Institutional disinflation

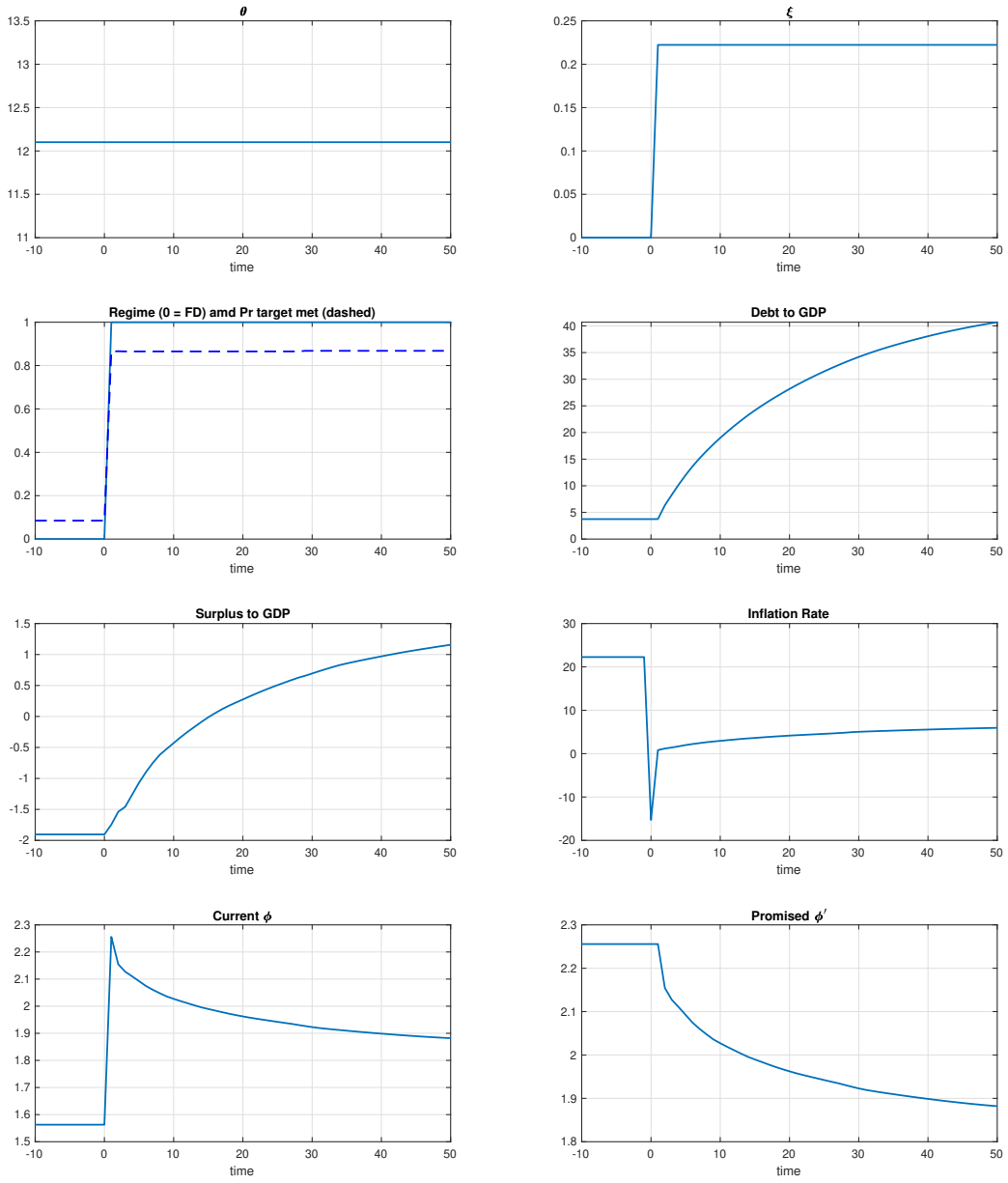


Figure 4 plots an IRF to an increase in ξ for our calibrated model. The path of real government debt is increasing as argued above. The inflation rate instead initially jumps downward at t_0. The level of inflation may overshoot its value a few years out because as the level of debt increases the government may find it optimal to reduce the promised value for real balances (increase the inflation target) to ensure that the target is satisfied.

Figures 3 and 4 illustrate the different debt dynamics associated with the two types of disinflation episodes. Fundamental disinflations generate a positive correlation between inflation and the level of government debt, whereas institutional disinflations produce a negative correlation between the two. We will use this contrasting behavior to assess the contribution of institutional changes to the evolution of inflation in the data.

6 Quantitative analysis

[Preliminary and incomplete] In this Section we fit the model to data from Colombia and Chile and study the implied path for $\{\theta_t, \xi_t\}$ that account for the observed outcomes.

6.1 Calibration

We assume the following functional form for the preferences of the stand-in household:

$$v(l) = \chi \frac{l^{1+\psi}}{1+\psi}, \quad v(m) = \kappa m - \eta m^2, \quad u(g) = \frac{g^{1-\sigma}}{1-\sigma}.$$

The fundamental shock θ_t follows an AR(1) process given by:

$$\theta_{t+1} = (1-\rho_\theta)\overline{\theta} + \rho_\theta \theta_t + \sigma_\theta \varepsilon_{t+1}.$$

The process for ξ is modeled as follows: we assume that it has a persistent component ξ_1 and two iid components, ξ_{md} and ξ_{fd}, so

$$\xi = \xi_1 + \xi_{fd} - \xi_{md}$$

The iid components follow a Gumbel$(\mu, 1/\lambda)$ with mean zero so $\mu + \gamma/\lambda = 0 \to \mu = -\gamma/\lambda$ for $\lambda > 0$ where γ is the Euler-Mascheroni constant.[9] The persistent component follows a Markov process. We assume that $\xi_1 \in [0, \overline{\xi}]$, where $\overline{\xi}$ is a large upper bound. The transition probabilities are:

[9] We introduce the iid shock to ensure convergence of the solution algorithm.

$$\Pr(\xi_1' = 0 \mid \xi_1) = \alpha_l, \quad \Pr(\xi_1' = \xi_1 \mid \xi_1) = \alpha,$$

and with probability $1 - \alpha_l - \alpha$, ξ_1' is drawn uniformly from the interval $[0, \overline{\xi}]$. The model solution is described in Appendix C.

The model is parameterized by the preference parameters $(\beta, \hat{\beta}, \chi, \psi, \kappa, \eta, \sigma)$ and the parameters governing the stochastic processes for θ_t and ξ_t, $(\rho_\theta, \sigma_\theta, \bar{\theta})$ and $(\alpha_l, \alpha, \bar{\xi}, \lambda)$. We set a subset of parameters to some predetermined values and we calibrate the remaining to match a set of moments related to fiscal and monetary variables for Colombia from 1960-2017. We calibrate the model at an annual frequency. Table 1 reports the parameters value.

We set the inverse of the Frisch elasticity, ψ, to 1 and the inverse of the elasticity of substitution, σ, for government expenditures to the common parameter of 2. We set the stand-in household's discount factor, β, to 0.95 so that the real interest rate is approximately 5 percent.

The other parameters are set so that the model matches the average inflation in the first quartile (Q1), the average inflation in the fourth quartile (Q4), the probabilities that inflation remains in either the top or bottom quartile from one period to the next, the average real money balances, the persistence, mean, and variance of primary surpluses. We summarize the target moments in Table 2.

Informally, the persistence, mean, and variance of primary surpluses are informative for the parameters governing θ_t, $(\rho_\theta, \sigma_\theta, \bar{\theta})$. The persistence of inflation in Q1 and Q4 are informative about the persistence of the costs of deviating from the target, α_l and α. The remaining parameters χ, η, and κ are informed by the average inflation in the first quartile (Q1), the average inflation in the fourth quartile (Q4), and the average real money balances.

6.2 Typical outcomes

Figure 5 plots a representative path of outcomes generated by our model. In this scenario, a sequence of shocks to θ and ξ leads the economy to transition from the MD regime to the FD regime, resulting in a significant reduction in both debt and inflation. It is instructive to compare these outcomes with those under the Ramsey and Markov equilibria for the same sequence of θ shocks; Figure 6 illustrates this comparison. We observe that the Markov equilibrium sustains almost no debt and exhibits high and volatile inflation.[10] In contrast, the Ramsey equilibrium supports substantially higher debt levels along with lower and

[10] This finding echoes the result we prove for a deterministic economy with $\beta = \hat{\beta}$ in Appendix B.

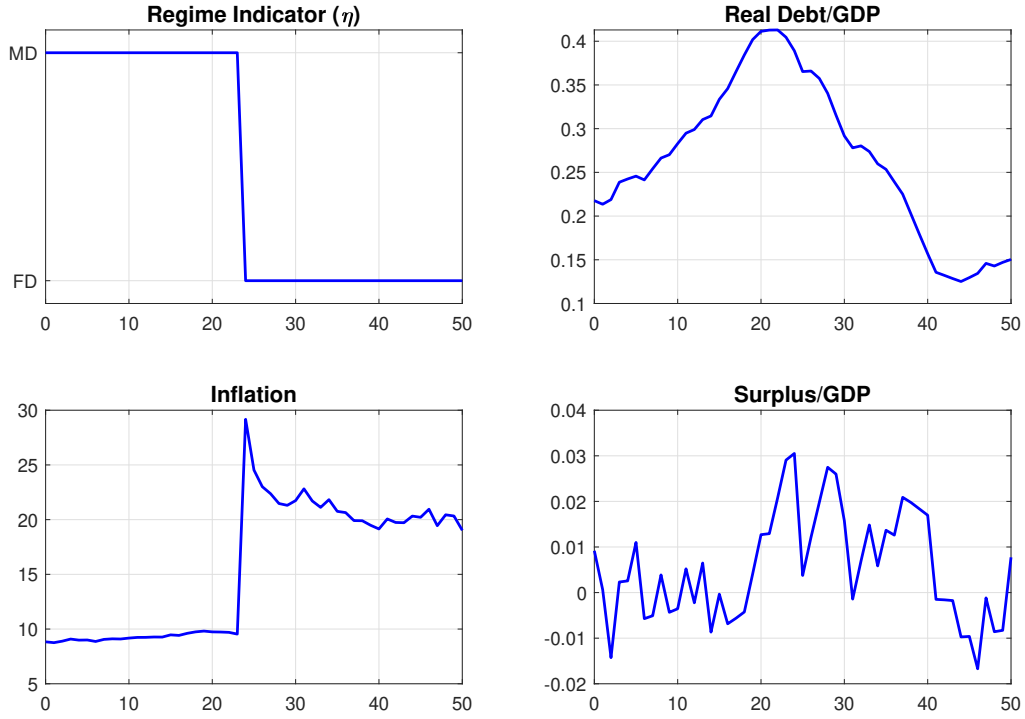
Table 1: Calibration: Parameters

Parameter	Description	Value
β	Households' discount factor	0.950
ψ	Inverse Frisch elasticity	1
σ	Inverse EIS gov't expenditures	2
$\hat{\beta}$	Gov'ts discount factor	0.946
χ	Labor disutility parameter	0.15
κ	Money-demand parameter	0.9
η	Money-demand parameter	0.1
$\bar{\theta}$	Mean θ	110
ρ_θ	Persistence of θ	0.8
σ_θ^2	Variance of θ	15
α_l	Probability $\xi_1' = 0$	0.1
α	Probability $\xi_1' = \xi_1$	0.7

Table 2: Calibration: Moments

Moment	Data	Model
Avg. inflation in Q1	4.00%	7.08%
Avg. inflation in Q4	27.00%	24.30%
Prob. of staying in Q1	0.80	0.67
Prob. of staying in Q4	0.67	0.64
Average debt-to-GDP	19.00%	23.54%
Avg. real money balances-to-GDP	6.70%	6.33%
Variance of primary surplus	0.9000	1.6350
Autocorr. of primary surplus	0.5700	0.7291
Avg. of primary surplus-to-GDP	0.50%	0.30%

Figure 5: Typical outcome



more stable inflation.

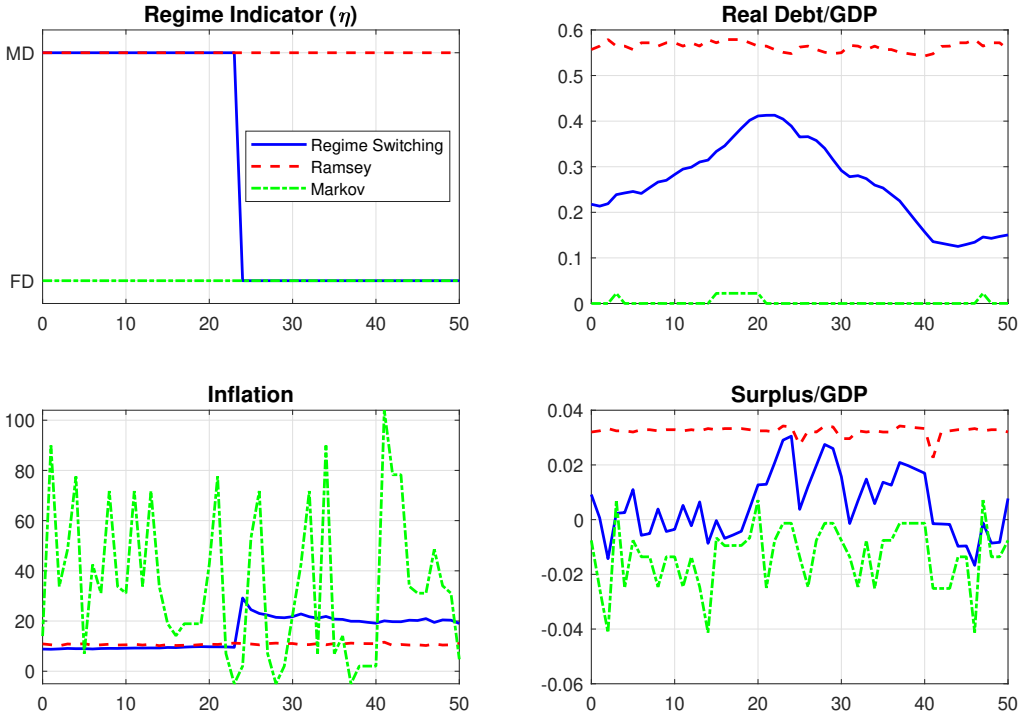
6.3 Case studies

Colombia

We now use our model to understand the disinflation episode in Colombia during the 1990s. In 1991, Colombia instituted a new constitution that granted substantial independence to its central bank, Banco de la República, explicitly mandating price stability as its primary objective and significantly insulating monetary policy from political influence. As written in Perez-Reyna and Osorio-Rodríguez (2017)

> The central bank was given technical independence as to the instruments employed to achieve its main task, which was defined solely as the control of inflation. In addition, the monetary board was replaced by a board of governors in which the minister of finance had only one vote (of seven) and no veto power. Finally, the constitution prescribed that any direct loan from the central bank to the central government would require unanimous approval by the members of

Figure 6: Comparison between models



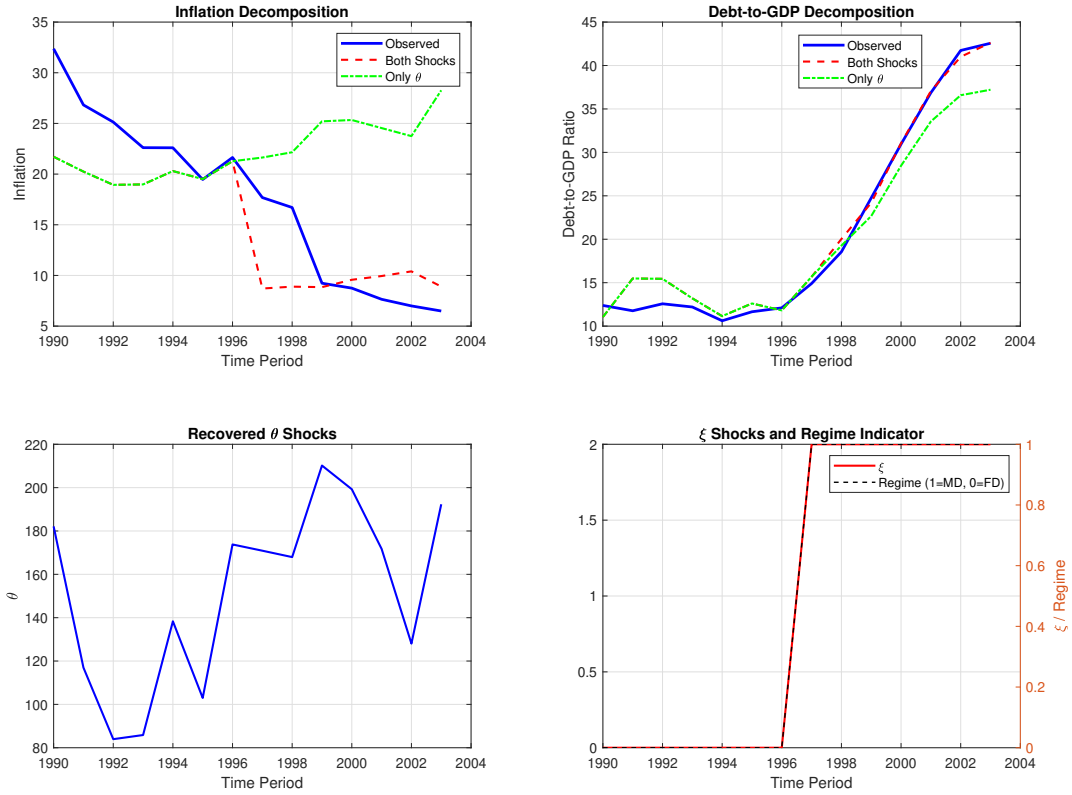
the board, thus all but forbidding monetary financing under this guise. To date, the independent central bank has never granted any direct loans to the central government.

In 2001, Colombia adopted an explicit inflation targeting regime with a long-term inflation goal of 3%. Prior to the 1991 reform, the central bank lacked autonomy, often making monetary policy susceptible to government pressures. As a result Colombia suffered from persistent high inflation despite the relatively low level of debt.

Motivated by this institutional reform, we use our model to identify sequences of θ and ξ shocks that can rationalize the observed paths of inflation and debt in Colombia following the constitutional change in 1991. The data is displayed in the first two panels of Figure 7. Starting in 1991, inflation fell.

Specifically, we find the sequence $\{\theta_t, \xi_t\}$ that minimizes the distance between observed data and a weighted average of model-generated debt and inflation paths. The results are presented in Figure 7. The first two panels show that the model implied paths for inflation and debt (the red dashed lines) closely match the observed trajectories of debt and inflation (solid blue lines), in particular the series for the debt-to-GDP ratio. The two panels at the

Figure 7: Colombia



bottom display the resulting series of θ and ξ shocks.

The model accounts for the reduction in inflation in the 1990s with an increase in the cost of deviating from the inflation target in 1997. Such an increase results in a persistent shift to a fiscal-dominant regime from 1997 onward. The observed increase in the debt-to-GDP ratio from 1994 to 2002 is driven by the switch to a monetary-dominant regime that allows for greater debt issuances and by higher-than-average realizations of θ_t.[11] The model can account for the stable and low inflation (at least from 1997 onward) and the increasing path for the debt-to-GDP ratio because in the MD regime, fiscal considerations – θ_t and the inherited debt – have little effect on inflation. Monetary policy is credibly delegated, and there is little spillover to monetary variables from the fiscal side. It is then possible to find a path of θ_t to match the debt dynamics.

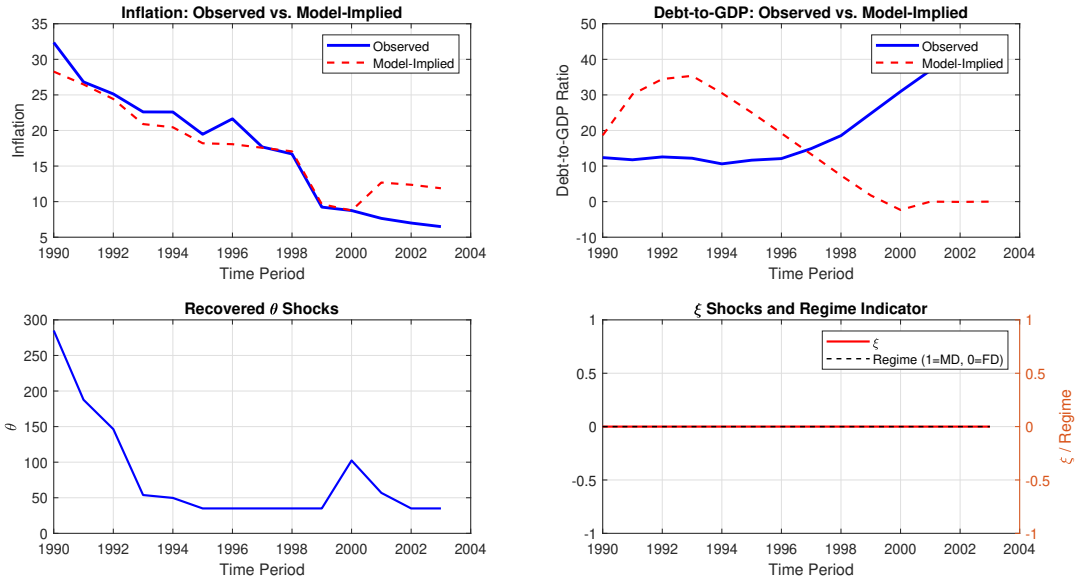
To understand the importance of the detected change in the regime, we also conduct a counterfactual exercise in which we retain the computed sequence of θ shocks but hold $\xi_t = 0$ throughout the entire period, so it is optimal to always be in the FD regime. This corresponds to the green lines in the first panel. This scenario aims to replicate conditions had Colombia not undertaken a *credible* constitutional reform. As illustrated, without this institutional change, debt would have similarly increased, driven by the high realizations of θ_t, but inflation would have remained constant or even risen during the latter half of the decade. This result underscores the crucial role credible institutional reforms played in simultaneously achieving higher debt levels and declining inflation in Colombia during this period.

To further illustrate this point, we now compute a path of θ shocks that minimizes the distance between observed and model implied paths for inflation when $\xi_t = 0$ for the entire period. As Figure 8 shows, the model can find a path for θ_t shocks that closely replicates the declining path of inflation but without the corresponding institutional changes, this would have eventually led to a declining path of debt.

These two counterfactuals underscore the tight link between fiscal and monetary variables when there is no credible delegation of monetary policy. A declining path for fiscal needs θ_t can account for a declining inflation path, but also implies a declining path for debt. Vice versa, an increasing path for θ_t can account for an increasing path for debt but not for inflation. Thus, when we observe an increase in the debt-to-GDP ratio during a disinflation episode, through the lens of the model, it must be associated with an increase in the credibility of the government inflation targeting.

[11] The increasing path for θ_t is consistent with the observation that in Colombia the "size of the government almost doubled between 1991 and 1999, as the ratio of central government expenditures to GDP increased from 8.9 percent to 16.9 percent , Perez-Reyna and Osorio-Rodríguez (2017)."

Figure 8: Colombia



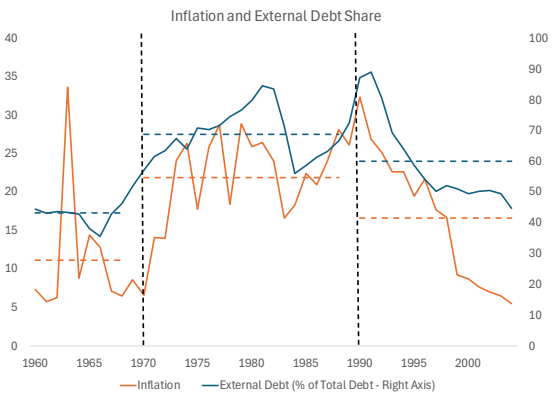
Our finding that the delegation of monetary policy was credible starting in 1997 is corroborated by other variables, indicating that the government is now less worried about constraining future governments by affecting the state variables in the next period. In the paper, we make the case for real debt, but a similar case can be made for the share of debt in domestic currency and the maturity of such debt. For example, a government worried by the credibility of its inflation promised would have a higher incentive to issue more debt in foreign currencies to reduce the incentives of future governments to inflate away domestic currency debt.[12] As shown in Figure 9, the path for the share of foreign currency government debt is highly correlated inflation in Colombia.[13] Such a share is very high from 1996 onward, consistent with our recovered path for ξ_t.

The model does not identify the increase in the credibility of the reform in 1992, the first year after the reform, but only in 1997. This is because inflation was still relatively high, and the level of debt only started to increase in that period. This may be driven by the fact that it took time for the government to convince private agents that the new institutional arrangement was credible and not just a cosmetic adjustment. Our measure thus bring a complementary perspective to purely de-jure measure of central bank credibility like

[12]For example, see Du, Pflueger, and Schreger (2020).

[13]As written by Perez-Reyna and Osorio-Rodríguez (2017) "early in the decade of the 1990s, the government decided to turn to the domestic financial market to finance its increasing primary deficit through the use of debt securities (TES). These securities boosted the development of domestic money markets and became the predominant source of government finance until the present." This is not special for Colombia. For example, Du and Schreger (2016) show that emerging market are increasingly borrowing in their own currency rather than in dollars.

Figure 9: Inflation and share of external debt



Grilli, Masciandaro, and Tabellini (1991); Cukierman (2008); Romelli (2022). For example, the credibility index for Colombia in Romelli (2022) increases in 1992 and stays constant at this level. Furthermore, our measure can be used to provide guidance for deeper models for the evolution of credibility and to identify patterns that made reforms credible.

Chile

[TBA]

Brazil

[TBA]

7 Connection to deeper models of reputation building

8 Conclusion

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Appendix

A Omitted proofs

A.1 Proof of Proposition 3

Consider the Ramsey problem

$$V_R(b,\phi,s) = \max_{\Delta,b',\phi'} U(\Delta,s) + v(\phi) + \hat{\beta}\sum_{s'} \Pr(s'|s)\, V_R(b',\phi',s')$$

subject to

$$\Delta = b + \phi - \beta b' - \beta H(\phi')$$

The first order conditions and the envelope condition imply

$$U'(\Delta,s) = \frac{\hat{\beta}}{\beta}\sum_{s'} \Pr(s'|s)\, U'(\Delta'(s'),s') \tag{26}$$

and

$$\hat{\beta} v'(\phi') - \beta U'(\Delta,s)\, h'(\phi') = 0 \tag{27}$$

Under the assumption $v(\phi) = \frac{\phi^{1-\eta}}{1-\eta}$ we have

$$h(\phi) = \phi^{1-\eta}, \quad h'(\phi) = (1-\eta)\phi^{-\eta} = (1-\eta)\, v'(\phi)$$

Suppose by way of contradiction that $v'(\phi(s')) > 0$. Condition (27) then implies

$$U'(\Delta,s) = \hat{\beta}/\beta \frac{v'(\phi')}{h'(\phi')} = \frac{\hat{\beta}/\beta}{1-\eta} \tag{28}$$

Clearly, if $\eta \in (0,1)$ we have a contradiction since $U'(\Delta) < 0$ so the left side of the equation above is negative while the right side is positive.

Thus, it must be that $v'(\phi(s')) = 0$ for all s'. *Q.E.D.*

B Debt and inflation in Markov and Ramsey

In this section we provide an analytical result that makes clear the comparison between inflation and level of real debt in the Markov equilibrium and in a Ramsey outcome. To do

so, we consider a deterministic economy with $\beta = \hat{\beta}$. We assume that $v(\phi) = \phi - \frac{1}{2}\phi^2$.

Proposition. *Consider a Markov equilibrium outcome* $\{\phi_{Mt}, \Delta_{Mt}, \pi_{Mt+1}, b_{Mt+1}\}$ *and a Ramsey outcome* $\{\phi_{Rt}, \Delta_{Rt}, \pi_{Rt+1}, , b_{Rt+1}\}$ *starting from a common inherited level of real debt* b_0*. As* $t \to \infty$, $b_{Mt} \to 0$ *while* $b_{Rt} = b_R > 0$ *for all* $t \geq 1$*. Moreover, if* b_0 *is small enough then* $\phi_{Rt} > \phi_{Mt}$ *and* $\pi_{Rt+1} < \pi_{Mt+1}$ *for all* t*,*

Proof. Consider first the Ramsey equilibrium. Letting $B = b + \phi$ be the total value for real government liabilities, we can write the recursive problem that characterizes the *continuation* Ramsey problem as

$$W_R(B) = \max_{\Delta, B', \phi'} U(\Delta) + \hat{\beta}\left[v\left(\phi'\right) + W_R\left(B'\right)\right]$$

subject to

$$\begin{aligned} \Delta &= B - \beta\left(h\left(\phi'\right) + B'\right) \\ \phi' &\leq B' \leq \bar{b} + \phi' \end{aligned}$$

In period 0 the Ramsey problem is

$$V_{R0}(b_0) = \max_{\phi, \Delta, b'_G, \phi'} U(\Delta) + v(\phi) + \hat{\beta}\left[v\left(\phi'\right) + W_R\left(B'\right)\right]$$

subject to

$$\begin{aligned} \Delta &= b_0 + \phi - \beta\left(h\left(\phi'\right) + B'\right), \\ \phi' &\leq B' \leq \bar{b} + \phi' \end{aligned}$$

where $\bar{b}$ is the upper bound of real debt. When $\beta = \hat{\beta}$, ϕ_t is constant from $t \geq 1$ and surpluses are always constant. So the Ramsey outcome is fully characterized by $(\Delta_R, \phi_{R0}, \phi_{R1})$ that solve[14]

$$\begin{aligned} v'(\phi_{R0}) &= -U'(\Delta_R) \\ v'(\phi_{R1}) &= U'(\Delta_R) h'(\phi_{R1}) \\ b_0 + \phi_{R0} &= \frac{\Delta_R}{1-\beta} + \frac{\beta h(\phi_{R1})}{1-\beta} \end{aligned}$$

[14]Note that we need $h'(\phi') < 0$ for an interior solution. With quadratic utility for real money balances we have $h'(\phi) = v'(\phi) + v''(\phi)\phi = 1 - 2\phi$ and $h'(\phi) < 0$ if $\phi > 1/2$.

The Markov equilibrium outcome solves the following problem

$$V_M(b) = \max_{\phi,\Delta,b'} U(\Delta) + v(\phi) + \hat{\beta} V\left(b', \theta'\right)$$

subject to

$$\begin{aligned} b + \phi &= \Delta + \beta b' + \beta H\left(\phi_M\left(b'\right)\right) \\ 0 &\leq b' \leq \bar{b} \end{aligned}$$

taking as given $\phi_M(\cdot)$. The Markov outcome $\{\Delta_t, \phi_t, b_{t+1}\}$ satisfies

$$\begin{aligned} v'(\phi_t) &= -U'(\Delta_t) \\ -U'(\Delta_t)\left[1 + H'\left(\phi_M\left(b_{t+1}\right)\right)\frac{\partial \phi_M\left(b_{t+1}\right)}{\partial b'}\right] &= -U'(\Delta_{t+1}) \\ b_t + H\left(\phi_M\left(b_{t+1}\right)\right) &= \frac{1}{\beta}\left[b_t + \phi_t - \Delta_t\right] \end{aligned}$$

Note that

$$H(\phi) = \phi + h(\phi) \to H'(\phi) = 1 + h'(\phi) = 1 + (1 - 2\phi) = 2(1 - \phi) > 0$$

Thus, since $\frac{\partial \phi_M(b_{t+1})}{\partial b'} < 0$ we have

$$\begin{aligned} -U'(\Delta_{t+1}) &= -U'(\Delta_t)\left[1 - H'\left(\phi_M\left(b_{t+1}\right)\right)\left|\frac{\partial \phi_M\left(b_{t+1}\right)}{\partial b'}\right|\right] \\ &< -U'(\Delta_t) \end{aligned}$$

so $\{\Delta_t\} \downarrow$. If the above Euler is always interior $\{\Delta_t\}$ is a decreasing sequence and $\{\phi_t\}$ is an increasing sequence in a bounded set. Thus, they must converge so we have that as $t \to \infty$

$$b_{M\infty} = \frac{\Delta_{M\infty}}{1-\beta} + \frac{\beta h\left(\phi_{M\infty}\right)}{1-\beta} - \phi_{M\infty} \geq 0.$$

Clearly, it must be that $b_{M\infty} = 0$. We next argue that the corresponding level of debt must be zero.[15] Suppose by way of contradiction that there exists $b > 0$ such that $b'_M(b) \geq b$. We

[15] Using the envelope condition we can write

$$-V'_M\left(b_{Mt}\right) < -\hat{\beta} V'_M\left(b_{Mt+1}\right)$$

so if V_M is concave then $b_{t+1} < b_t$ and so $\{b_{Mt}\} \to 0 = b_{M\infty}$. The argument in the text does not assume concavity of V_M.

have established that $\{\Delta_t\}$ is strictly decreasing if not constrained by $b' \geq 0$. Then, starting at $b_0 = b$ we have

$$b_0 = -f_M(\Delta_0) + \sum_{t=0}^{\infty} \beta^t \Delta_t + \sum_{t=1}^{\infty} \beta^t h(f_M(\Delta_t))$$
$$b'_M(b_0) = -f_M(\Delta_1) + \sum_{t=1}^{\infty} \beta^{t-1} \Delta_t + \sum_{t=2}^{\infty} \beta^{t-1} h(f_M(\Delta_t))$$

where we define

$$f_M(\Delta) \equiv (v')^{-1}(-U'(\Delta)) \to \phi = f_M(\Delta)$$

with $f'_M(\Delta) < 0$. Since $\{\Delta_t\}$ is decreasing and strictly so between 0 and 1 since $b'_M(b) \geq b \geq 0$ by the contradiction hypothesis so the lower bound is not binding, $f'_M < 0$ and $h' < 0$ then

$$\begin{aligned} b_0 &= -f_M(\Delta_0) + \sum_{t=0}^{\infty} \beta^t \Delta_t + \sum_{t=1}^{\infty} \beta^t h(f_M(\Delta_t)) \\ &> -f_M(\Delta_1) + \sum_{t=0}^{\infty} \beta^t \Delta_{t+1} + \sum_{t=1}^{\infty} \beta^t h(f_M(\Delta_{t+1})) \\ &= b'_M(b_0) \end{aligned}$$

yielding a contradiction. Thus, $b'_M(b) < b$ for all $b > 0$ and by continuity of the policy function $b_M(\cdot)$ it follows that $b_{Mt} \to 0$ as $t \to \infty$.

We next turn to compare the value of real money balances in the two equilibria. Iterating the budget constraint in the Markov equilibrium outcome we have

$$b_0 + \phi_{M0} = \sum_{t=0}^{\infty} \beta^t \Delta_{Mt} + \sum_{t=1}^{\infty} \beta^t h(f_M(\Delta_{Mt}))$$

Define $f_R(\Delta)$ as the implicit solution to

$$v'(f_R(\Delta)) = U'(\Delta) h'(f_R(\Delta)) \to \phi = f_R(\Delta)$$

with

$$f'_R(\Delta) = \frac{U'' h'}{v'' - U' h''}$$

moreover we know that

$$f_M(\Delta) < f_R(\Delta)$$

since $h'(\phi) = 1 - 2\phi \to -h' \leq 1$ for all ϕ.

Suppose that $b_0 \approx b_{M\infty}$ then the Markov outcome is already in steady state and $\Delta_{M\infty}$ solves

$$\Phi_M(\Delta_{M\infty}) = 0$$

with

$$\Phi_M(\Delta) = b_{M\infty} + f_M(\Delta) - \frac{\Delta}{1-\beta} - \frac{\beta h(f_M(\Delta))}{1-\beta}$$

The surplus in the Ramsey outcome instead solves

$$\Phi_R(\Delta_R) = 0$$

with

$$\Phi_R(\Delta) = b_{M\infty} + f_M(\Delta) - \frac{\Delta}{1-\beta} - \frac{\beta h(f_R(\Delta))}{1-\beta}$$

Since h is decreasing in the relevant range and $f_M' < 0$ then $\Phi_M(\Delta)$ is strictly decreasing in Δ and $f_R(\Delta) > f_M(\Delta)$ for all Δ then

$$\Phi_R(\Delta) > \Phi_M(\Delta) \quad \forall \Delta$$

and so

$$\Delta_R > \Delta_{M\infty}$$

Then it must be that

$$\begin{aligned}\frac{\beta}{1-\beta} h(\phi_{M\infty}) &= b_{M\infty} + f_M(\Delta_{M\infty}) - \frac{\Delta_{M\infty}}{1-\beta} \\ &> b_{M\infty} + f_M(\Delta_R) - \frac{\Delta_R}{1-\beta} \\ &= \frac{\beta}{1-\beta} h(\phi_{R1})\end{aligned}$$

Thus, since h is decreasing in the relevant range then

$$\phi_{R1} > \phi_{M\infty}.$$

Since $\{\phi_{Mt}\}$ is an increasing sequence then

$$\phi_{R1} > \phi_{M\infty} \geq \phi_{Mt} \quad \forall t \tag{29}$$

Finally, note that in each regime

$$1+\pi_t=\frac{\beta H\left(\phi_t\right)}{\phi_t}=\frac{\beta\left[\phi_t+\left(1-\phi_t\right)\phi_t\right]}{\phi_t}=\beta\left[2-\phi_t\right]$$

Therefore (29) implies that for all $t \geq 1$

$$1+\pi_{Rt}=\beta\left[2-\phi_{R1}\right]<\beta\left[2-\phi_{Mt}\right]=1+\pi_{Mt}$$

Q.E.D.

The result is true also if $v\left(m\right)=m^{1-\eta}/\left(1-\eta\right)$ with $m \leq m^*$. The proof for this result is even more straightforward since in the Ramsey outcome is optimal to have $\phi_1 = m^*$ while in the Markov equilibrium it is optimal to have $\phi_{Mt} < m^*$ if m^* is large enough and government expenditures are sufficiently valuable.

The result remains true also if we allow for the government to be more impatient than the stand-in household. If $\beta > \hat{\beta}$, the Ramsey outcome features an increasing path for primary surpluses with $\Delta_{Rt} \to \Delta_{max}$, the maximal surplus implied by the static Laffer curve. In the Markov equilibrium outcome, if $\hat{\beta}$ is sufficiently close to β, the incentive effects still dominate the impatience effect and the level of debt still converges to 0 with a decreasing sequence $\{\Delta_{Mt}\}$. Thus, the result in the proposition still holds provided that the gap between β and $\hat{\beta}$ is sufficiently small.

C Model solution

Here we describe how we solved the full model. First, note that we can reduce the number of endogenous state variables to one by considering the problem of the government after ϕ is chosen. In this case, the unique state variable is the level of real government liabilities, $B = b + \phi$. Denote by $W\left(B,s\right)$ the value function at this stage. Then

$$V_{md}\left(b,\phi,s\right)=W\left(b+\phi,s\right)+v\left(\phi\right) \tag{30}$$

$$V_{fd}\left(b,s\right)=\max_{\phi_{fd}}\left\{W\left(b+\phi_{fd}\right)+v\left(\phi_{fd}\right)\right\} \tag{31}$$

$$\phi_{fd}\left(b,s\right)=\arg\max_{\phi_{fd}}\left\{W\left(b+\phi_{fd}\right)+v\left(\phi_{fd}\right)\right\} \tag{32}$$

and

$$\eta\left(b,\phi,s\right)=\begin{cases}1 & \text{if } W\left(b+\phi,s\right)+v\left(\phi\right)\geq\max_{\phi_{fd}}\left\{W\left(b+\phi_{fd}\right)+v\left(\phi_{fd}\right)\right\}-\xi\left(s\right)\\ 0 & \text{otherwise}\end{cases} \tag{33}$$

The value function W solves the Bellman equation

$$\begin{aligned}W\left(B,s\right)=&\max_{\Delta,b',\phi'}U\left(\Delta,\theta\right) \\ &+\hat{\beta}\sum_{s'}\Pr\left(s'|s\right)\max\left\{W\left(b'+\phi',s\right)+v\left(\phi\right),\max_{\phi_{fd}}\left\{W\left(b'+\phi_{fd}\right)+v\left(\phi_{fd}\right)\right\}-\xi\left(s\right)\right\}\end{aligned} \tag{34}$$

subject to

$$\Delta=B-\beta b'-\beta\sum_{s'}\Pr\left(s'|s\right)\left\{\eta\left(b',\phi',s'\right)H\left(\phi'\right)+\left(1-\eta\left(b',\phi',s'\right)\right)H\left(\phi_{fd}\left(b',s'\right)\right)\right\}$$

with ϕ_{fd} and η given by (32) and (33).

Algorithm:

- Let the state space: $B,B'\in\left[0,\bar{B}\right]$, $\phi,\phi'\in\left[0,\phi^{*}\right]$ and $b,b'\in\left[0,\bar{b}\right]$ where $\bar{B}=\bar{b}+\phi^{*}$
- Start with a guess W_0 (starting from the Ramsey value)
- For any n, calculate ϕ_{fd} and η associated with candidate W_n using (32) and (33)
- Compute W_{n+1} using the Bellman operator (34)
- Iterate until convergence of the value function.

Note that if $s=(\theta,\xi)$ and θ and ξ are independent and ξ is a continuous variable then the Bellman equation is

$$\begin{aligned}W\left(B,s\right)=&\max_{\Delta,b',\phi'}U\left(\Delta,\theta\right) \\ &+\hat{\beta}\sum_{\theta'}\Pr\left(\theta'|\theta\right)\int_{\xi'\leq\xi^{*}(b',\phi',\theta')}\left[\max_{\phi_{fd}}\left\{W\left(b'+\phi_{fd},s'\right)+v\left(\phi_{fd}\right)\right\}-\xi'\right]f\left(\xi'|\xi\right)d\xi' \\ &+\hat{\beta}\sum_{\theta'}\Pr\left(\theta'|\theta\right)\int_{\xi'\geq\xi^{*}(b',\phi',\theta')}\left[W\left(b'+\phi,s'\right)+v\left(\phi\right)\right]f\left(\xi'|\xi\right)d\xi'\end{aligned}$$

subject to

$$\Delta = B - \beta b' - \beta \sum_{\theta'} \Pr\left(\theta'|\theta\right) \int_{\xi' \leq \xi^*(b',\phi',\theta')} H\left(\phi_{fd}\left(b',s'\right)\right) f\left(\xi'|\xi\right) d\xi'$$
$$- \beta \sum_{\theta'} \Pr\left(\theta'|\theta\right) \int_{\xi' \geq \xi^*(b',\phi',\theta')} H\left(\phi'\right) f\left(\xi'|\xi\right) d\xi'$$

We make the following assumption on ξ: we assume that it has a persistent component ξ_1 that follows a Markov chain $\Gamma\left(\xi_1'|\xi_1\right)$ and two iid components ξ_{md} and ξ_{fd} that follow Gumbel$(\mu, 1/\lambda)$ and

$$\xi = \xi_1 + \xi_{fd} - \xi_{md}$$

We will assume that the expectation of the Gumbel random variables are zero and so $\mu + \gamma/\lambda = 0 \rightarrow \mu = -\gamma/\lambda$ for $\lambda > 0$ where γ is the Euler-Mascheroni constant ($\approx$0.5772).

Given a state $s_1 = (\theta, \xi_1)$, the expected value over ξ_{fd} and ξ_{md} is

$$\begin{aligned}\Omega\left(b,\phi,s_1\right) &\equiv \int_{\xi_{md}} \int_{\xi_{fd}} \max\left\{V_{md}\left(b,\phi,s_1\right) + \xi_{md}, V_{fd}\left(b,s_1\right) - \xi_1 + \xi_{md}\right\} f\left(\xi_{fd}\right) f\left(\xi_{md}\right) d\xi_{fd} d\xi_{md} \\ &= \frac{1}{\lambda} \log\left(\exp\left(\left[V_{md}\left(b,\phi,s_1\right) + \mu\right]\lambda\right) + \exp\left(\left[V_{fd}\left(b,s_1\right) - \xi_1 + \mu\right]\lambda\right)\right) + \frac{\gamma}{\lambda} \\ &= \frac{1}{\lambda} \log\left(\exp\left(V_{md}\left(b,\phi,s_1\right)\lambda\right) + \exp\left(\left[V_{fd}\left(b,s_1\right) - \xi_1\right]\lambda\right)\right) + \mu + \frac{\gamma}{\lambda}\end{aligned}$$

and

$$\begin{aligned}\bar{\eta}\left(b,\phi,s_1\right) &\equiv \Pr\left(V_{md}\left(b,\phi,s_1\right) + \xi_{md} \geq V_{fd}\left(b,s_1\right) - \xi_1 - \xi_{fd}\right) \\ &= \frac{\exp\left(V_{md}\left(b,\phi,s_1\right)\lambda\right)}{\exp\left(V_{md}\left(b,\phi,s_1\right)\lambda\right) + \exp\left(\left[V_{fd}\left(b,s_1\right) - \xi_1\right]\lambda\right)} \\ &= \frac{1}{1 + \exp\left(-\lambda\left[V_{md}\left(b,\phi,s_1\right) - V_{fd}\left(b,s_1\right) + \xi_1\right]\right)}\end{aligned}$$

Thus, the Bellman equation is

$$W\left(B,s_1\right) = \max_{\Delta,b',\phi'} U\left(\Delta,\theta\right) + \hat{\beta} \sum_{s_1'} \Pr\left(s_1'|s_1\right) \Omega\left(b',\phi,s_1'\right)$$

subject to

$$\Delta = B - \beta b' - \beta \sum_{s_1'} \Pr\left(s_1'|s_1\right) \left[\bar{\eta}\left(b',\phi',s_1\right) H\left(\phi'\right) + \left(1 - \bar{\eta}\left(b',\phi',s_1'\right)\right) H\left(\phi_{fd}\left(b',s_1'\right)\right)\right]$$

The focs for ϕ' and b' are:

$$0 \leq \sum_{s_1'} \Pr\left(s_1'|s_1\right) \left[\frac{-U'\left(\Delta,\theta\right)\beta H'\left(\phi'\right) + \hat{\beta}\left[W'\left(b'+\phi',s_1'\right)+v'\left(\phi'\right)\right]}{1+\exp\left(-\lambda\Delta V\left(b',\phi',s_1'\right)\right)}\right]$$
$$- U'\left(\Delta,\theta\right)\beta \sum_{s_1'} \Pr\left(s_1'|s_1\right) \left[\frac{\exp\left(-\lambda\Delta V\left(b',\phi',s_1'\right)\right)\lambda}{\left[1+\exp\left(-\lambda\Delta V\left(b',\phi',s_1'\right)\right)\right]^2} \frac{\partial V_{fd}\left(b',\phi',s_1'\right)}{\partial \phi'}\right] \left[H\left(\phi'\right) - H\left(\phi_{fd}\left(b',s_1'\right)\right)\right]$$

$$0 = -U'\left(\Delta,\theta\right)\beta + \sum_{s_1'} \Pr\left(s_1'|s_1\right)\hat{\beta} \frac{\exp\left(V_{md}\left(b',\phi',s_1'\right)\lambda\right)\frac{\partial V_{md}}{\partial b'} + \exp\left(\left[V_{fd}\left(b,s_1\right)-\xi_1\right]\lambda\right)\frac{\partial V_{fd}}{\partial b'}}{\exp\left(V_{md}\left(b,\phi,s_1\right)\lambda\right)+\exp\left(\left[V_{fd}\left(b,s_1\right)-\xi_1\right]\lambda\right)}$$
$$- U'\left(\Delta,\theta\right)\beta \sum_{s_1'} \Pr\left(s_1'|s_1\right) \left[\frac{\exp\left(-\lambda\Delta V\left(b',\phi',s_1'\right)\right)\lambda}{\left[1+\exp\left(-\lambda\Delta V\left(b',\phi',s_1'\right)\right)\right]^2} \frac{\partial \Delta V\left(b',\phi',s_1'\right)}{\partial b'}\right] \left[H\left(\phi'\right) - H\left(\phi_{fd}\left(b',s_1'\right)\right)\right]$$
$$- U'\left(\Delta,\theta\right)\beta \sum_{s_1'} \Pr\left(s_1'|s_1\right) \left[\frac{\exp\left(-\lambda\Delta V\left(b',\phi',s_1'\right)\right)}{1+\exp\left(-\lambda\Delta V\left(b',\phi',s_1'\right)\right)} H'\left(\phi_{fd}\left(b',s_1'\right)\right)\right] \frac{\partial \phi_{fd}\left(b',s_1'\right)}{\partial b'}$$