Central Bank Predictions, Heterogeneous Expectations and Inflation Dynamics*

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Abstract

In a small-open-economy New Keynesian model, we explore the implications of both rational, fully informed agents and boundedly rational agents, coupled with central bank prediction errors, on inflation dynamics. Central bank prediction errors serve as monetary shocks, leading to suboptimal short-term inflation and output gap dynamics. Regardless of how agents process information and form their expectations, inflation persistence is linked to the persistence of the monetary shocks. However, differences emerge in inflation volatility and sensitivity to domestic slack due to disagreements between rational and boundedly rational agents regarding macroeconomic persistence. This affects inflation forecastability in small open economies. We support these findings with evidence from a set of emerging market economies, analyzing the role of judgment in forming expectations with survey-based forecasts from Consensus Economics(R) and the International Monetary Fund. Finally, we introduce the Divine Coincidence Curve, which illustrates conditions under which monetary policy can simultaneously reduce output gap and inflation volatility. The presence of boundedly rational agents alters this relationship, influencing policy effectiveness and its transmission mechanism.

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1 Introduction

In our globalized world economy, an increasing amount of information is dispersed and shared among different agents. These agents try to process this stock of information and shape their own expectations about the future path of relevant indicators in order to make decisions. Consistently, empirical evidence suggests that inflation expectations are neither homogeneous nor rational (see, for instance, Branch (2004), Pesaran and Weale (2006), Coibion, Gorodnichenko, and Kamdar (2018), and the references therein). In this context, it could be worth asking what role central bank's communications (e.g., inflation forecasts) play in its credibility and the formation of expectations, as well as in the dynamics of the inflation rate.[1]

We address the interplay among central bank predictions, credibility and inflation dynamics in a small-open-economy New Keynesian model with heterogeneous expectations. We approach the central bank's difficult task of forecasting and reporting predictions by assuming a monetary authority that needs to forecast the natural interest rate. This unobservable variable plays a crucial role in a New Keynesian framework and a central bank can obviously produce prediction errors in practice that, at the same time, might affect the policy rate decisions.[2] Likewise, we approximate the credibility of this central bank by the proportion of rational agents that live in the economy. The larger the fraction of rational agents compared to the fraction of boundedly-rational agents, the more effective the monetary policy is to curb the inflation rate and close the output gap.

More precisely, our objective is to examine the implications on the inflation dynamics of introducing boundedly rational agents—that follow adaptive expectations[3]—jointly with others that face a (lump-sum) cost of forming rational expectations. In this setup, the central bank errs in predicting the natural interest rate and these prediction errors lead to suboptimal levels of inflation and output gap in the short-run equilibrium. In this framework, first we show that the heterogeneous-expectations model for a small open economy is isomorphic to its counterpart for a closed economy. Put differently, our main results do not depend qualitatively on the degree of openness of the economy under analysis.

More importantly, we find that the inflation rate shows the same persistence regardless of the composition of the expectations. The autoregressive coefficient of the solution of the domestic inflation with only boundedly rational individuals is equal to that of the economy with a mix of rational and boundedly rational individuals. That said, we find differences in volatility. However, we show that significant differences in volatility and on the sensitivity of inflation to domestic slack (the Phillips curve relationship) appear due to disagreements between rational and boundedly rational agents about the underlying macroeconomic persistence. We argue that this has an impact among other things on the forecastability of inflation in small open economies and provide some supportive evidence of the role of judgement in forming expectations for a large group of 14 EMEs based on a novel assessment of Consensus Forecasts^{TM} and institutional forecasts

[1] Central bank communications in the form of either objectives, targets, and forecasts, seem to play an important role in the effectiveness of monetary policy (see, for example, the surveys by Blinder, Ehrmann, Fratzscher, De Haan, and Jansen (2008) and Moessner, Jansen, and De Haan (2017).

[2] Partly, this piece of our work is motivated by a recent literature that investigates whether central bank forecasts matter to determine inflation via public's expectations. Fujiwara (2005) shows that the diffusion of the Bank of Japan's inflation and growth forecasts affects those of professional forecasters and not vice versa. Hubert (2015a) reports evidence that central bank inflation forecasts influence private inflation forecasts in Sweden, the United Kingdom, Canada, Switzerland, and Japan. Hubert (2015b) extends that question for the European Central Bank and he obtains a positive answer as well. Pedersen (2015) finds that the forecasts of the central bank of Chile matter for the formation of private forecasts in the short run.

[3] Using surveys on inflation expectations, Branch (2004) finds that agents rely on simple rules such as adaptive or naive expectations.

from the IMF.

Our findings suggest that as more data become available over time to pin down the autoregressive coefficient (*à la* Brock and Hommes (1997)), the bounded rationality forecasts can become arbitrarily close to those obtained under rational expectations. Hence, in the limit, the benefits of rational expectations decline to the point that economic agents switch to forming their expectations under a simpler (less costly) adaptive mechanism if the cost of forming rational expectations is non-zero. This has significant implications for monetary policy in the event that changes in monetary policy that affect the persistence of the inflation rate (or the expectational errors) would not be quickly recognized until more data points and observations end up revealing that the persistence parameter has drifted away from what it had been. When there is growing evidence of a structural shift, the transition dynamics may conceivably mean that more agents adopt rational expectations as the disagreements become larger and more costly. This in turn has potential implication for learning and monetary policy communication strategies to be explored in subsequent work.

In addition, we introduce the Divine Coincidence Curve, which represents combinations of output gap volatility and inflation volatility for different values of the policy rate's response to inflation. While essentially a (degenerate) Taylor curve, it does not depict a trade-off but instead highlights the possibility of simultaneously reducing both volatilities. This curve also illustrates the conditions under which monetary policy achieves this dual stabilization, while the presence of boundedly rational agents alters the relationship, influencing the effectiveness of monetary policy.

Our work is related to several strands of the literature. To a certain extent, our main result contrasts with that of Muth (1960), which claims that adaptive expectations lead to an optimal prediction if the variable to be forecast follows a random walk. In our model, the solution entails a stationary process for the inflation rate and the optimal forecast is the same under adaptive expectations or both types of expectations. In the same vein, Gelain, Iskrev, K. Lansing, and Mendicino (2019) find that a hybrid-expectations version of the Smets-Wouters model provides superiority in generating an expected inflation series that more closely tracks expected inflation from the Survey of Professional Forecasters (SPF).

Another branch analyzes the macroeconomic effects of heterogeneous expectations similar to those adopted in our framework. Lines and Westerhoff (2010) study a traditional Phillips curve-output supply model for a closed economy with extrapolative and rational expectations to investigate endogenous fluctuations in macroeconomic variables. De Grauwe (2012) proposes a closed-economy New Keynesian model with fundamentalist (rational) and naive adaptive (although he calls them extrapolative) expectations to focus on how animal spirits may lead to inertia in inflation and business cycles. Branch and McGough (2010) formulate a closed-economy New Keynesian model similar to ours but with an expectations-based rule and conclude that periodic orbits and complex dynamics may arise. In our setup we consider prediction errors in the forecast of the natural interest rate and we focus on credibility as defined above and the different inflationary processes and forecasts.

The rest of the paper proceeds as follows. Section 2 briefly formulates the model, parameterization, and explains the economic intuition. In Section 3, we present and discuss the main results and robustness checks comparing our baseline model specification against a class of models augmented with learning. Section 4 provides a discussion of the implications of our main findings and the role of judgement in forming expectations. It also provides some supporting empirical evidence. Section 5 concludes with some final remarks. The Appendix provides all the relevant proofs and other technical derivations of the small-open-economy (SOE) New Keynesian model with heterogeneous beliefs that we propose in this paper.

2 Heterogenous-Expectations Model

There exists a growing literature on heterogenous beliefs exploring the role of mechanisms affecting the formation of expectations (Branch and McGough (2009), K. J. Lansing (2009), Assenza, Heemeijer, Hommes, and Massaro (2013), Gasteiger (2014), Di Bartolomeo, Pietro, and Giannini (2016), Gibbs (2017), and Pecora and Spelta (2017)), yet little work has been done on the implications for monetary policy in an open economy setting. We extend the workhorse small open-economy (SOE) New Keynesian model of Clarida, Gal, and Gertler (2001) and Galí and Monacelli (2005) to explicitly incorporate heterogenous expectations and bounded rationality.

The frictions in goods markets that introduce a short-run policy trade-off between inflation and real economic activity in our SOE model are the core New Keynesian assumptions of monopolistic competition and nominal price rigidities (staggered prices). The Home country is an open economy small in size relative to the economy of the Foreign country (rest of the world). Households in the Home country consume a Home and a Foreign good that are internationally traded at no cost and with no explicit trade tariffs. The Home and Foreign goods are imperfect substitutes of each other. The Home good is a bundle of a continuum of mass one of differentiated varieties, each produced by a monopolistically competitive firm located in the Home country. Similarly, the imported good from the Foreign country is a bundle of a continuum of differentiated varieties produced in the Foreign country.

Nominal prices are set on a staggered basis in the Home country, as in the Calvo (1983) model. Home firms changing prices in the current period choose an optimal price based on the expected path of current and future marginal costs, while Home firms not changing their price simply adjust their output to meet demand at previous period prices. We assume that the law of one price holds for individual varieties at all times (both for import and export prices), and that the bilateral nominal exchange rate is fully flexible. Furthermore, we allow households to perfectly share risks internationally and, accordingly, the terms of trade supporting risk-sharing will depend in equilibrium on both Home and Foreign shocks. However, under the small open economy assumption, shocks that originate at Home are absorbed by fluctuations in the terms of trade but do not otherwise influence the real and nominal dynamics of the Foreign country. Hence, Foreign variables—output and inflation in particular—are treated for simplicity as purely exogenous from the point of view of the Home country's economic agents.

In the remainder of the paper, we work directly with the log-linearized workhorse SOE model.[4] Accordingly, all endogenous and exogenous variables for the Home and Foreign country are expressed in logs and the model dynamics around its long-run deterministic steady state are described with a linear expectational system of equations. However, here is where we depart from the literature motivated by the log-linearized model of Galí and Monacelli (2005), since we abandon the widely-used assumption of homogenous expectations and full rationality. Following Branch and McGough (2009) (and the related work of Gasteiger (2014) and Di Bartolomeo, Pietro, and Giannini (2016)), we introduce heterogeneity in the formation of expectations across economic agent types while also allowing forms of bounded rationality for some of them.

Axiomatic approach. We assume generically a finite number of $n < +\infty$ economic agents indexed by $\tau = 1, ..., n$ that differ on the way they form their expectations about the future and define $0 \leq \lambda^{\tau} \leq 1$ to be the mass of type τ economic agents at time t (where $\sum_{\tau=1}^{n} \lambda^{\tau} = 1$ for all t). We denote $\mathcal{E}_t^{\tau}(g_{t+h})$

[4] To save on space, the interested reader is referred to Galí and Monacelli (2005) for the derivations of the log-linearized equations from first principles under the special case of homogenous and rational expectations. To facilitate comparability with Galí and Monacelli (2005), we also retain most of their notation as given.

the time-t subjective expectations formed by an economic agent of type τ on an h- periods-ahead generic variable g_{t+h}. We introduce expectations heterogeneity in the model following the axiomatic approach in Branch and McGough (2009) which assumes that:

(A1) Expectations operators fix observables;

(A2) If g_{t+1} is a variable forecasted by the economic agents and has steady state $\overline{g}$ then $\mathcal{E}_t^{\tau}(\overline{g}) = \mathcal{E}_t^{\tau'}(\overline{g}) = \overline{g}$ for all $\tau \neq \tau'$;

(A3) If g_{t+1}, h_{t+1}, $g_{t+1}+h_{t+1}$, and $\alpha_e g_{t+1}$ are variables forecasted by the economic agents, then $\mathcal{E}_t^{\tau}(g_{t+1}+h_{t+1}) = \mathcal{E}_t^{\tau}(g_{t+1}) + \mathcal{E}_t^{\tau}(h_{t+1})$ and $\mathcal{E}_t^{\tau}(\alpha_e g_{t+1}) = \alpha_e \mathcal{E}_t^{\tau}(g_{t+1})$ for any constant α_e and every τ;

(A4) If for all $h \geq 0$, g_{t+h} and $\sum_{h\geq 0}\beta^{t+h} g_{t+h}$ are forecasted by economic agents, then $\mathcal{E}_t^{\tau}\left(\sum_{h\geq 0}\beta^{t+h} g_{t+h}\right) = \sum_{h\geq 0}\beta^{t+h}\mathcal{E}_t^{\tau}(g_{t+h})$ for $0 < \beta \equiv 1/(1+\rho) < 1$ (subjective discount factor) and all τ;

(A5) $\mathcal{E}_t^{\tau}$ satisfies the law of iterated expectations: if g_{t+k} for $k > h \geq 0$ is a variable forecasted by economic agents at time t and time $t+h$, then $\mathcal{E}_t^{\tau} \circ \mathcal{E}_{t+h}^{\tau}(g_{t+k}) = \mathcal{E}_t^{\tau}(g_{t+k})$ for all τ;

(A6) If g_{t+k} for $k > h \geq 0$ is a variable forecasted by economic agents at time t and time $t+h$, then $\mathcal{E}_t^{\tau}\mathcal{E}_{t+h}^{\tau'}(g_{t+k}) = \mathcal{E}_t^{\tau}(g_{t+k})$ for all $\tau \neq \tau'$; and

(A7) All agents have common expectations on expected differences in limiting wealth.

As noted by Branch and McGough (2009), assumptions (*A6*) and (*A7*) are key to ensure a particular structure on higher-order beliefs that allows for aggregation. Assumption (*A6*) implies that agents' forecasts satisfy the law of iterated expectations at the aggregate level, while assumption (*A7*) rules out wealth distribution dynamics that otherwise could influence the forecasts by economic agent type and create difficulties for aggregation. Under these assumptions, the aggregate expectations operator $\mathcal{E}_t(\cdot)$ can simply be defined as a weighted average of the expectations across agent types, i.e., $\mathcal{E}_t(g_{t+1}) = \sum_{\tau=1}^{n}\lambda^{\tau}\mathcal{E}_t^{\tau}(g_{t+1})$ where $\sum_{\tau=1}^{n}\lambda^{\tau} = 1$.

The log-linearized model with heterogeneous expectations. The log-linear workhorse SOE model can be cast into an expectational system of two equations together with a definition for inflation and a perfect risk-sharing condition with which to pin down the output gap of the Home country (the small open economy), x_t, its producer price index (PPI) inflation, $\pi_{H,t}$, its consumer price index (CPI) inflation, π_t, and the terms of trade, s_t, conditional on the path of the Home country's nominal interest rate, i_t, and two stochastic forcing processes (for Home productivity, a_t, and for Foreign output, y_t^*).

Let $x_t \equiv y_t - y_t^n$ be the domestic output gap, where y_t is the actual output, and y_t^n is the natural level of output. The dynamic IS equation for the Home country in terms of the domestic output gap is given by:

$$x_t = \mathcal{E}_t(x_{t+1}) - \frac{1}{\sigma_\alpha}\left[i_t - \mathcal{E}_t(\pi_{H,t+1}) - r_t^n\right], \tag{1}$$

where the composite coefficients $\sigma_\alpha \equiv \frac{\sigma}{(1-\alpha)+\alpha\omega} > 0$ and $\omega \equiv \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$ are functions of the inverse of the intertemporal elasticity of substitution $\sigma > 0$, the share of the imported Foreign good in the Home consumption basket (degree of openness) $0 \leq \alpha < 1$, the degree of substitutability between Home and Foreign goods in the Home consumption basket $\eta > 0$, and the degree of substitutability between imported

varieties produced in the Foreign country (from the perspective of the Home consumer) $\gamma > 0$. The Home country's natural interest rate is denoted as r_t^n.

The dynamics of PPI inflation in the Home country, defined as the rate of change in the Home good price index, i.e., $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$, are given by a New Keynesian Phillips Curve in terms of domestic slack, x_t, which can be expressed as:

$$\pi_{H,t} = \beta \mathcal{E}_t \left(\pi_{H,t+1} \right) + \kappa_\alpha x_t, \tag{2}$$

where $\kappa_\alpha \equiv \left(\frac{(1-\beta\theta)(1-\theta)}{\theta} \right) (\sigma_\alpha + \varphi)$ is the slope of the small open economy New Keynesian Phillips curve, $0 < \beta < 1$ is the intertemporal discount factor, $0 < \theta < 1$ is the Calvo (1983) price stickiness parameter, and $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply. It follows that CPI inflation—the rate of change in the price index of the Home consumption budget, i.e., $\pi_t \equiv p_t - p_{t-1}$—and PPI inflation, $\pi_{H,t}$, are related as follows:

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t, \tag{3}$$

where $\Delta s_t \equiv s_t - s_{t-1}$ is the rate of change on the terms of trade, s_t.

Furthermore, from the perfect risk-sharing condition, the terms of trade, s_t, in equilibrium are proportional to the differential between Home aggregate output, y_t, and Foreign aggregate output, y_t^*, as follows:

$$\begin{aligned} s_t &= \sigma_\alpha \left(y_t - y_t^* \right) \\ &= s_t^n + \sigma_\alpha x_t, \end{aligned} \tag{4}$$

The second equality is derived from the definition of the Home output gap ($x_t \equiv y_t - y_t^n$), and the Home natural terms of trade, s_t^n, defined subsequently in (6).

The frictionless allocation which arises under perfectly flexible prices and no monopolistic competition distortion pins down the potential output of the Home country, y_t^n, as well as its corresponding natural terms of trade, s_t^n, and natural interest rate, r_t^n, in the following terms:

$$y_t^n \equiv \Omega + \Gamma a_t + \alpha \Psi y_t^*, \tag{5}$$

$$s_t^n \equiv \sigma_\alpha \left(y_t^n - y_t^* \right), \tag{6}$$

$$r_t^n \equiv \rho - \sigma_\alpha \Gamma \mathcal{E}_t \left(\Delta a_{t+1} \right) + \alpha \sigma_\alpha \left(\Theta + \Psi \right) \mathcal{E}_t \left(\Delta y_{t+1}^* \right), \tag{7}$$

where the composite coefficients are $\Omega \equiv \frac{\upsilon - \mu}{\sigma_\alpha + \varphi}$, $\Gamma \equiv \frac{1+\varphi}{\sigma_\alpha + \varphi} > 0$, $\Psi \equiv -\frac{\Theta \sigma_\alpha}{\sigma_\alpha + \varphi} < 0$, and $\Theta \equiv \omega - 1$. Moreover, $\upsilon \equiv -\ln(1 - \tau_e)$ with τ_e being an employment subsidy, $\mu \equiv \left(\frac{\varepsilon}{\varepsilon - 1} \right)$ corresponds to the log of the (gross mark-up) in steady state with $\varepsilon > 1$ being the elasticity of substitution among varieties produced locally in the Home country, and $\rho \equiv \beta^{-1} - 1$ is the real interest rate in steady state.

The frictionless allocation given by (5) − (7) is unobservable and depends entirely on two exogenous random processes for Home productivity, a_t, and for Foreign output, y_t^*. The bivariate process of the exogenous driving processes is modeled as follows:

$$a_t = \rho_a a_{t-1} + \epsilon_t^a, \tag{8}$$

$$y_t^* = \rho_{y^*} y_{t-1}^* + \epsilon_t^{y^*}, \tag{9}$$

with the standard deviation of each innovation given by $\sigma_a > 0$ and $\sigma_{y^*} > 0$, and the correlation of their

innovations by $-1 < \rho_{a,y^*} < 1$.

The core dynamics are set by the system of expectational equations in (1) and (2) for the Home output gap, x_t, and Home PPI inflation, $\pi_{H,t}$, coupled with the Home natural rate of interest, r_t^n, given by (7) and with the bivariate shock processes for Home productivity, a_t, and for Foreign output, y_t^*, in (8) − (9), and conditional on the path of the nominal interest rate in the Home country, i_t. Then, equations in (3) and (4) are used to map the path for x_t and $\pi_{H,t}$ together with the path of potential output, y_t^n, and the natural terms of trade, s_t^n, in (5) − (6) into CPI inflation, π_t, and the terms of trade, s_t. This core dynamic system of the small open economy has clear parallels with the standard closed economy New Keynesian model (Woodford (2003)) as the next proposition claims.

Proposition 1 *The dynamic IS equation in* (1) *and the New Keynesian Phillips curve in* (2) *are isomorphic to those of a closed economy model. In turn, the natural rate of interest,* r_t^n*, in* (7) *adds an extra term not present in the closed economy model that depends on expected Foreign output growth.*

Proposition 1 follows naturally by comparing equations (1) − (2) and (7) with their closed economy counterparts. Furthermore, notice that the closed economy model is nested into the small open economy framework presented here. Whenever the import share is set to $\alpha = 0$ (or in the special case where $\sigma = \frac{1}{\gamma}$ and $\gamma = \eta$) the slope of the small open economy dynamic IS curve in (1) reduces to $\frac{1}{\sigma}$ and the slope of the New Keynesian Phillips curve in (2) to $\left(\frac{(1-\beta\theta)(1-\theta)}{\theta}\right)(\sigma + \varphi)$, which are the standard slopes in the closed economy case. From (3) it follows that PPI and CPI inflation are the same whenever $\alpha = 0$, as in the closed economy model. Moreover, for $\alpha = 0$, the natural rate of interest in (7) drops the expected Foreign growth component and reduces to its counterpart in the closed economy case.

Expectations formation and bounded rationality. In the variant of the log-linear SOE model described in this paper, there are three economic agents in the Home country differentiated by their role in the economy and the decisions they make. Households make intertemporal decisions on the investment-savings margin (and their aggregate expectations are incorporated in the dynamic IS equation in (1)), firms make intertemporal decisions when setting prices (and their aggregate expectations appear on the New Keynesian Phillips curve in (2)), and the central bank sets monetary policy. Implicitly, we assume that households own a portfolio of all the domestic firms and share equally in all the firm's per-period profits or losses (lump-sum). We also implicitly assume that the management of the firms is aligned with that of their owners (the households) and, therefore, reflect in their expectations the same heterogeneity of beliefs as households—accordingly, we use the same aggregate expectations operator $\mathcal{E}_t(\cdot)$ in both (1) and (2).[5] We discuss the distinct role of the Home central bank and its expectations in greater detail later on, as part of our overall discussion of monetary policy implementation.

To keep the model simple and tractable, we assume heterogeneous forecasts for private economic agents in the Home country (households and firms) may differ on how they form their expectations in two ways (i.e., $n = 2$). Two distinct types of forecasting heterogeneity also suffice to introduce partial bounded rationality in the SOE model. A fraction $0 \leq \lambda^R \leq 1$ of private economic agents indexed by $\tau = R$ has rational expectations, while the remaining fraction $0 \leq \lambda^B \equiv \left(1 - \lambda^R\right) \leq 1$ indexed by $\tau = B$ form expectations according to a bounded rationality mechanism. In the limit where $\lambda^R \to 1$ for all t, we naturally recover the

[5] There is growing awareness about the differences between consumer and business surveys of inflation expectations (see, e.g., Coibion, Gorodnichenko, and Kumar (2018)). We leave this issue for future research.

solution of the homogeneous and rational expectations (log-linear) model of Galí and Monacelli (2005) as a special case.

We assume that rational expectations agents ($\tau = R$) form their expectations with all available information up to time t, i.e., $\mathcal{E}_t^R(g_{t+1}) = \mathbb{E}_t(g_{t+1})$ where $\mathbb{E}_t(\cdot)$ is the corresponding mathematical conditional expectations operator. In turn, private economic agents under bounded rationality form their beliefs about the future on the basis of a perceived or subjective linear law of motion whereby $g_t = \phi_g g_{t-1}$. The parameter ϕ_g implies that $\mathcal{E}_t^B(g_{t+1}) = \phi_g g_t$ so it is a form of adaptive expectations ($\phi_g < 1$), na{ï}ve or random-walk expectations ($\phi_g = 1$), or extrapolative expectations ($\phi_g > 1$).[6] Notice that the linear operator ϕ_g is indexed by the generic variable g which permits differences between the perceived linear law of motion for domestic output gap, x_t, and that for PPI inflation, $\pi_{H,t}$, in the dynamic IS equation in (1).

Given this structure of heterogeneous expectations, we can re-express the core system of expectational equations in (1) and (2) in matrix form as:

$$\begin{pmatrix} \left(1-\left(1-\lambda^R\right)\phi_x\right) & -\left(1-\lambda^R\right)\frac{\phi_{\pi_H}}{\sigma_\alpha} \\ -\kappa_\alpha & \left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right) \end{pmatrix} \begin{pmatrix} x_t \\ \pi_{H,t} \end{pmatrix} = \lambda^R \begin{pmatrix} 1 & \frac{1}{\sigma_\alpha} \\ 0 & \beta \end{pmatrix} \begin{pmatrix} \mathbb{E}_t(x_{t+1}) \\ \mathbb{E}_t(\pi_{H,t+1}) \end{pmatrix} + \begin{pmatrix} -\frac{1}{\sigma_\alpha}(i_t - r_t^n) \\ 0 \end{pmatrix}, \tag{10}$$

whenever $0 < \lambda^R \leq 1$. In other words, the SOE model with heterogeneous beliefs described in this paper can be recast in the form of a modified log-linear SOE model under rational expectations as long as there is a non-zero mass of private economic agents that form their expectations with all available information under full rationality ($\lambda^R > 0$). Whenever $\lambda^R = 0$, the core equations in (1) and (2) become a deterministic system of difference equations:

$$\begin{pmatrix} 1-\phi_x & -\frac{\phi_{\pi_H}}{\sigma_\alpha} \\ -\kappa_\alpha & 1-\phi_{\pi_H}\beta \end{pmatrix} \begin{pmatrix} x_t \\ \pi_{H,t} \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sigma_\alpha}(i_t - r_t^n) \\ 0 \end{pmatrix}, \tag{11}$$

whose solution implies that:

$$\begin{aligned} \begin{pmatrix} x_t \\ \pi_{H,t} \end{pmatrix} &= \frac{1}{\kappa_\alpha \phi_{\pi_H} - \sigma_\alpha(1-\phi_x)(1-\beta\phi_{\pi_H})} \begin{pmatrix} -\sigma_\alpha(1-\beta\phi_{\pi_H}) & -\phi_{\pi_H} \\ -\sigma_\alpha\kappa_\alpha & -\sigma_\alpha(1-\phi_x) \end{pmatrix} \begin{pmatrix} -\frac{1}{\sigma_\alpha}(i_t - r_t^n) \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} \frac{1-\beta\phi_{\pi_H}}{\kappa_\alpha\phi_{\pi_H} - \sigma_\alpha(1-\phi_x)(1-\beta\phi_{\pi_H})} \\ \frac{\kappa_\alpha}{\kappa_\alpha\phi_{\pi_H} - \sigma_\alpha(1-\phi_x)(1-\beta\phi_{\pi_H})} \end{pmatrix} (i_t - r_t^n). \end{aligned} \tag{12}$$

Both the rational expectations modified SOE model in (10) when $0 < \lambda^R \leq 1$ as well as the bounded rationality SOE model solution in (12) when $\lambda^R = 0$ show that the core dynamics for domestic output gap, x_t, and for PPI inflation, $\pi_{H,t}$, are conditional on the path of the Home country's nominal interest rate, i_t, and that, in principle, the exogenous driving forces operate on them solely through their impact on the natural rate of interest, r_t^n.

Monetary (and fiscal) policy with heterogeneous beliefs. As shown by Rotemberg and Woodford (1998), a constant employment subsidy that neutralizes in steady state the markup distortion arising from monopolistic competition (i.e., $\tau_e = \frac{1}{\varepsilon}$ implying that $\upsilon = \mu$ and $\Omega = 0$) suffices to ensure that the optimal

[6] For empirical support, see Pesaran (1987) and Brock and Hommes (1997), among others.

monetary policy set in terms of the Home country's nominal interest rate, i_t, achieves the first-best allocation under flexible prices and perfect competition in the closed economy case ($\alpha = 0$). The intuition for this is well-known—with the optimal labor subsidy $\tau_e = \frac{1}{\varepsilon}$ in place, the only distortion remaining for monetary policy to correct is that arising from sticky prices. In turn, in an open economy setting—as indicated by Galí and Monacelli (2005) for the SOE framework—there are two distortions that monetary policy has to deal with even when domestic markups are subsidized with the optimal employment subsidy: the Home central bank has an incentive to fight inflation to mitigate the costs of price stickiness while also distorting terms of trade to benefit the domestic households when domestic and imported goods are imperfect substitutes.

Simplifying assumptions. As shown by Galí and Monacelli (2005) with homogeneous and rational expectations, in the special case of a small open economy ($\alpha > 0$) where $\sigma = \frac{1}{\gamma}$ and $\gamma = \eta$, setting the optimal employment subsidy τ_e to satisfy that $(1-\tau_e)(1-\alpha) = 1 - \frac{1}{\varepsilon}$ (implying that $\upsilon = \mu + \ln(1-\alpha)$) is sufficient to guarantee that monetary policy can be used to deliver the flexible-price, perfect-competition first-best equilibrium allocation. Whenever $\sigma \neq \frac{1}{\gamma}$ or $\gamma \neq \eta$, an export subsidy or some other equivalent tariff on imports might be needed to complement monetary policy in order to attain a first-best equilibrium allocation. While we consider the issue on the choice of fiscal instruments to be very important, they are not central to the purpose of the paper. Therefore, from here onwards we implicitly assume the special case where $\sigma = \frac{1}{\gamma}$ and $\gamma = \eta$, unless otherwise noted. In this special case, strict domestic PPI inflation targeting (i.e., $x_t = \pi_{H,t} = 0$) is indeed the optimal monetary policy to achieve the first-best allocation, as shown by Galí and Monacelli (2005).

How does the Home central bank implement a policy of PPI inflation targeting? Stabilizing the domestic output gap as well as PPI inflation at zero for all t implies that $i_t = r_t^n$ will also hold in equilibrium for all t and output and terms of trade will be set at their natural levels $y_t = y_t^n$ and $s_t = s_t^n$ for all t as well. However, setting the policy rate of the Home country, i_t, to match the country's natural interest rate, r_t^n, does not guarantee that the Home country will achieve full stabilization and its first-best allocation. As can be inferred from either (10) when $0 < \lambda^R \leq 1$ or (11) when $\lambda^R = 0$, the full stabilization would not be unique. Hence, stationary sunspot fluctuations could arise. As shown by Galí and Monacelli (2005), indeterminacy problems can be avoided with a monetary policy rule that tracks the natural rate, r_t^n, while requiring the policy rate, i_t, to respond to domestic PPI inflation, $\pi_{H,t}$, and/or the domestic output gap, x_t, when they deviate from their (zero) target values.

Accordingly, the central bank's optimal monetary policy rule discussed by Galí and Monacelli (2005) is given as:

$$i_t = r_t^n + \psi_{\pi_H} \pi_{H,t} + \psi_x x_t, \tag{13}$$

for which a necessary and sufficient condition for uniqueness of the optimal allocation requires the policy parameters, $\psi_{\pi_H}, \psi_x \geq 0$, to satisfy that $\kappa_\alpha (\psi_{\pi_H} - 1) + (1-\beta)\psi_x > 0$. Hence, as explained by Galí and Monacelli (2005), "stabilization of the output gap and inflation requires a credible threat by the central bank to vary the interest rate sufficiently in response to any deviations of inflation and/or the output gap from the target; yet, the very existence of that threat makes its effective application unnecessary."

Central bank's prediction errors. The challenge of implementing strict PPI inflation targeting as in (13) arises in part because the central bank does not always observe $\pi_{H,t}$ and y_t (or π_t, Δs_t, and y_t) in real time. Another interpretation is that the central bank's PPI inflation target might deviate from the optimal zero target—at least temporarily—even when all other variables are either observed or can be recovered

without error. In turn, we favor the view that even if some data is observable in a timely manner and the central bank is committed to its zero PPI inflation target, the central bank still has to confront the estimation problem that arises with (13) because the frictionless allocation is unobservable.

Indeed, the central bank needs to rely on its own estimates (expectations) of the natural rate of interest, r_t^n, and output potential, y_t^n. Hence, expectational errors can get embedded in a monetary policy rule such as (13) as follows:

$$i_t = r_t^n + \psi_{\pi_H} \pi_{H,t} + \psi_x x_t + m_t, \tag{14}$$

where m_t is a expectational error given by $m_t \equiv \left(\mathcal{E}_t^{CB}\left(r_t^n\right) - r_t^n\right) + \psi_{\pi_H}\left(\mathcal{E}_t^{CB}\left(\pi_{H,t}\right) - \pi_{H,t}\right) + \psi_x\left(\mathcal{E}_t^{CB}\left(x_t\right) - x_t\right)$ where $\mathcal{E}_t^{CB}(\cdot)$ is the central bank's expectations operator. Even in the most favorable case where $\pi_{H,t}$ is observed in real time and incorporated into the central bank's expectations (i.e., $\mathcal{E}_t^{CB}\left(\pi_{H,t}\right) = \pi_{H,t}$) and $\psi_x = 0$, expectations errors can occur because the natural rate of interest is unobservable and has to be estimated by the central bank.[7]

Rather than modeling explicitly the expectations formation mechanism for the central bank in the Home country, we are going to follow an indirect approach assuming that expectational errors follow an autoregressive process of the following form:

$$m_t = \rho_m m_{t-1} + \epsilon_t^m, \tag{15}$$

where the volatility of its innovations is given by $\sigma_m > 0$, autoregressive coefficient $0 < \rho_m < 1$, and their correlations with the innovations of domestic productivity and Foreign output by $-1 < \rho_{m,a} < 1$ and $-1 < \rho_{m,y^*} < 1$, respectively. We take the view that this is a more flexible parameterization of the expectational errors that encompasses potentially different expectation formation mechanisms at play for the central bank. It also provides a plausible description of how those expectational errors might behave including transitory versus persistent errors and allowing for the errors to be more or less strongly correlated with the driving processes (domestic productivity and Foreign output). We often will refer to m_t interchangeably as policy expectational errors or monetary shocks.

How should we interpret the implementation of monetary policy under the PPI targeting? We should first note that the rule in (14) is suboptimal when $m_t = 0$ for some t, even though in the special-case-parameterization of the heterogenous beliefs SOE model that we explore here strict PPI inflation targeting as in (13)—if possible—would achieve full stabilization and a first-best allocation. There are certain issues, however, that are worth highlighting:

First, PPI inflation targeting is observationally equivalent to a more common monetary policy rule targeting CPI inflation and some measure of change in international relative prices (changes in terms of trade in this case). Notice that implementing strict PPI inflation targeting as in (13) can be combined with the definition of domestic inflation in (3) as follows:

$$i_t = r_t^n + \psi_{\pi_H} \pi_t - \alpha \psi_{\pi_H} \Delta s_t + \psi_x x_t. \tag{16}$$

This alternative monetary policy rule achieves the same first-best allocation, yet it does so with a more familiar form. Rather than responding directly to deviations of PPI inflation from its zero target, the

[7] Notice that using assumption (*A6*) of Branch and McGough (2009), the central bank's expectations of the natural rate of interest can be written as $\mathcal{E}_t^{CB}\left(r_t^n\right) \equiv \rho - \sigma_\alpha \Gamma \mathcal{E}_t^{CB}\left(\Delta a_{t+1}\right) + \alpha \sigma_\alpha \left(\Theta + \Psi\right) \mathcal{E}_t^{CB}\left(\Delta y_{t+1}^*\right)$. Implicitly, the assumption is that both the private agents and the central bank are not uncertain about the structure of the economy. However, they may still differ on their expectations about future domestic productivity growth, Δa_{t+1}, and future Foreign growth, Δy_{t+1}^*.

central bank of the small open economy does it indirectly responding to deviations of CPI inflation, π_t, and deviations of terms of trade changes, Δs_t, from zero. Monetary policy rules that combine CPI inflation and terms of trade changes (or related real exchange rate changes) are quite commonplace as descriptions of monetary policy behavior among many small open economies.

Second, monetary policy deviations can arise also from deviations in the monetary policy objectives. Under strict PPI targeting, the optimal targets for PPI inflation, $\pi_{H,t}$, and the output gap, x_t, are given by $\pi_H^T = x^T = 0$. Monetary policy deviations such as those captured by (15) can, therefore, arise even when the frictionless allocation—the natural rate, r_t^n, and output potential, y_t^n, in particular—are known to both private economic agents as well as to the central bank. This is the case when the central bank's monetary policy framework allows for flexible PPI inflation targeting as follows:

$$i_t = r_t^n + \psi_{\pi_H}\left(\pi_{H,t} - \pi_{H,t}^T\right) + \psi_x\left(x_t - x^T\right), \tag{17}$$

where the central bank's output gap target remains at zero but its (possibly time-varying) inflation target, $\pi_{H,t}^T$, is a convex combination of the optimal target, $\pi_H^T = 0$, and the central bank's inflation forecast, $\mathcal{E}_t^{CB}(\pi_{H,t})$, i.e., $\pi_{H,t}^T = \lambda^{CB}\pi_H^T + (1-\lambda^{CB})\mathcal{E}_t^{CB}(\pi_{H,t})$ with a constant weight $0 \leq \lambda^{CB} \leq 1$. This scenario allows (albeit somewhat loosely) a discussion of the "anchoring of inflation expectations." Whenever $\lambda^{CB} \approx 1$, the central bank's inflation target approximates its optimal target of zero PPI inflation and we observe near full inflation stabilization. However, more generally, this implies that the implementation of monetary policy is suboptimal as in (14) with $m_t = -\psi_{\pi_H}\left(\pi_{H,t}^T - \pi_H^T\right) = -\psi_{\pi_H}(1-\lambda^{CB})\left(\mathcal{E}_t^{CB}(\pi_{H,t}) - \pi_H^T\right)$.

In here, we can think of λ^{CB} as some measure of central bank independence—the more independent the central bank is, the more likely it is to pursue full inflation stabilization ($\lambda^{CB} \approx 1$); while the less independent the central bank becomes, the more is expected to accommodate temporary bouts of inflation not to harm economic activity in the process ($\lambda^{CB} < 1$). What's really interesting about this scenario is that it is implicitly tied to central bank credibility and, in a way, can induce a self-fulfilling suboptimal equilibrium. The reason why it is conceptually tied to the notion of central bank credibility is because when the threat of punishing deviations from the optimal target is diminished by allowing partial accommodation of temporary fluctuations in inflation expectations, the effect this has is precisely to accommodate some inflation (something that would become apparent in the derivations in the next section) fulfilling the central bank's prophecy. As a result, private agents' belief in the target simply erodes, generating inflation and a suboptimal outcome.[8]

Third, we can reinterpret policy deviations from strict PPI targeting as arising from forecasting disagreements between the central bank and the private economic agents. Going back to the benchmark interpretation of equation (14) given earlier where $\pi_{H,t}$ is observed in real time and incorporated into the central bank's expectations (i.e., $\mathcal{E}_t^{CB}(\pi_{H,t}) = \pi_{H,t}$) and $\psi_x = 0$, we can write the forecasting errors $m_t \equiv \left(\mathcal{E}_t^{CB}(r_t^n) - r_t^n\right)$ as reflecting a fundamental disagreement between the central bank and the private sector. We consider two different, yet plausible scenarios: The first scenario is one where the central bank's aim is shifted from the optimal objective of tracking the short-term natural rate, r_t^n, to that of tracking a long-run natural rate (the

[8] We'll show subsequently that adaptive expectations suffice to obtain optimal forecasts of PPI inflation and of the output gap. Whenever $\lambda^{CB} \approx 1$, near full stabilization is achieved and inflation expectations are given by the central bank's target. Inflation expectations, even if optimally predicted with an adaptive model, would differ from the central bank's target when $\lambda^{CB} < 1$. Hence, when private agents' inflation expectations differ from the central bank's inflation target, that should signal according to our theory that monetary policy is deviating from the optimal strict PPI inflation targeting rule in (13) and this might be because of declining central bank independence and monetary policy credibility.

steady state real rate), $r^n = \rho$, i.e.,[9] $\mathcal{E}_t^{CB}(r_t^n) = r^n = \rho$ and

$$m_t \equiv (r^n - r_t^n) = \sigma_\alpha \Gamma \mathcal{E}_t(\Delta a_{t+1}) - \alpha\sigma_\alpha(\Theta + \Psi)\mathcal{E}_t\left(\Delta y_{t+1}^*\right), \tag{18}$$

implicitly assuming that both private economic agents and the central bank know (and agree on) the structure of the economy on which they operate. The second scenario simply assumes that the Home country's central bank has its own forecasts about the short-term natural rate, r_t^n, which might be at odds with the aggregate forecasts of the private agents on account of the natural rate's fundamental drivers, i.e.,

$$m_t \equiv \left(\mathcal{E}_t^{CB}(r_t^n) - r_t^n\right) = \sigma_\alpha \Gamma\left(\mathcal{E}_t(\Delta a_{t+1}) - \mathcal{E}_t^{CB}(\Delta a_{t+1})\right) - \alpha\sigma_\alpha(\Theta + \Psi)\left(\mathcal{E}_t\left(\Delta y_{t+1}^*\right) - \mathcal{E}_t^{CB}\left(\Delta y_{t+1}^*\right)\right). \tag{19}$$

Domestic productivity in our environment is just labor productivity, $a_t = y_t - n_t$, and both output (y_t) and employment (n_t) are viewed as observables. In turn, Foreign output is much harder to measure and perhaps more likely to result in persistent disagreements between private agents and the central bank.[10] We could, therefore, envision a situation where $\mathcal{E}_t(\Delta a_{t+1}) = \mathcal{E}_t^{CB}(\Delta a_{t+1})$, $\mathcal{E}_t\left(\Delta y_{t+1}^*\right) = (\rho_{y^*} - 1)y_t^*$, and $\mathcal{E}_t^{CB}\left(\Delta y_{t+1}^*\right) = (\rho_{y^*}^{CB} - 1)y_t^*$, where $\rho_{y^*} \neq \rho_{y^*}^{CB}$ would imply that $m_t \equiv -\alpha\sigma_\alpha(\Theta + \Psi)\left(\rho_{y^*} - \rho_{y^*}^{CB}\right)y_t^*$. This would be the case, for instance, if private economic agents learn about the persistence of Foreign output, ρ_{y^*}, in (9) but the central bank's forecasts are systematically biased.[11]

Both scenarios in (18) and (19) come to illustrate that deviations from strict PPI inflation targeting as those modelled in (14) and (15) can arise also because of disagreements on the appropriate concept of the natural rate to use (the short-run versus the long-run natural rate) or because of disagreements between private economic agents and the central bank on the fundamental drivers of the natural rate (domestic productivity, a_t, and Foreign output, y_t^*).

[9] In the special case of the SOE model that we consider here supported by an optimal employment subsidy, the deterministic zero-PPI inflation steady state is the same as the frictionless steady state.

[10] Notice that the foreign output, y_t^*, can be backed out from the observable output, y_t, and the observable terms of trade, s_t, of the Home country using the implications of the perfect risk-sharing condition in (4).

[11] The bivariate processes for domestic productivity, a_t, and Foreign output, y_t^*, are given in (8) – (9). From this process, we should recognize that the frictionless allocation described in (7) would be distorted unless $\mathcal{E}_t(\Delta a_{t+1}) = \mathbb{E}_t(\Delta a_{t+1})$ and $\mathcal{E}_t\left(\Delta y_{t+1}^*\right) = \mathbb{E}_t\left(\Delta y_{t+1}^*\right)$, i.e., unless all private agents form their expectations rationally in the frictionless case. One possible explanation for why biases may arise through private agents instead is this: the central bank may aim to achieve an efficient allocation tackling the one distortion that is specific to our SOE model with heterogeneous beliefs—the distortion in private agents' expectations about the natural interest rate. In that scenario, the central bank would aim to target the rational expectations natural rate: $r_t^n \equiv \rho - \sigma_\alpha \Gamma \mathbb{E}_t(\Delta a_{t+1}) + \alpha\sigma_\alpha(\Theta + \Psi)\mathbb{E}_t\left(\Delta y_{t+1}^*\right)$ in an environment where there is a non-trivial share of agents operating under bounded rationality with possibly different views on expected foreign output (or even on domestic productivity). The discrepancy arises only in the presence of boundedly rational agents and ultimately reflects the disagreement between rational expectations forecasts and boundedly rational forecasts about the underlying drivers of the natural rate of interest. It is worth emphasizing again that this example also reveals the deeper issue that—with heterogenous beliefs and bounded rationality—we are introducing an additional distortion in the formation of expectations into the SOE model that materializes in the alluded forecasting disagreements.

3 Main Results

3.1 Determinacy Region

Given the monetary policy implementation laid out by (14) and (15), the core system of expectational equations re-expressed in rational expectations terms in (10) becomes:[12]

$$\begin{pmatrix} \frac{1}{\sigma_\alpha}\psi_x+\left(1-\left(1-\lambda^R\right)\phi_x\right) & \frac{1}{\sigma_\alpha}\psi_{\pi_H}-\left(1-\lambda^R\right)\frac{\phi_{\pi_H}}{\sigma_\alpha} \\ -\kappa_\alpha & \left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right) \end{pmatrix}\begin{pmatrix} x_t \\ \pi_{H,t} \end{pmatrix}=\lambda^R\begin{pmatrix} 1 & \frac{1}{\sigma_\alpha} \\ 0 & \beta \end{pmatrix}\begin{pmatrix} \mathbb{E}_t\left(x_{t+1}\right) \\ \mathbb{E}_t\left(\pi_{H,t+1}\right) \end{pmatrix}-\begin{pmatrix} \frac{1}{\sigma_\alpha}m_t \\ 0 \end{pmatrix}, \tag{20}$$

whenever $0<\lambda^R\leq 1$. We can express the transformed rational expectations model in (20) as follows,

$$\begin{aligned}\begin{pmatrix} x_t \\ \pi_{H,t} \end{pmatrix}=\lambda^R\Lambda_\lambda\begin{pmatrix} 1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta & \frac{1}{\sigma_\alpha}\left(1-\psi_{\pi_H}\beta\right) \\ \kappa_\alpha & \kappa_\alpha\frac{1}{\sigma_\alpha}+\left(\frac{1}{\sigma_\alpha}\psi_x+\left(1-\left(1-\lambda^R\right)\phi_x\right)\right)\beta \end{pmatrix}\begin{pmatrix} \mathbb{E}_t\left(x_{t+1}\right) \\ \mathbb{E}_t\left(\pi_{H,t+1}\right) \end{pmatrix}-\ldots \\ \Lambda_\lambda\begin{pmatrix} \left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\frac{1}{\sigma_\alpha} \\ \kappa_\alpha\frac{1}{\sigma_\alpha} \end{pmatrix}m_t,\end{aligned} \tag{21}$$

or, in first-order canonical form, as,

$$z_t=A_\lambda\mathbb{E}_t\left(z_{t+1}\right)+B_\lambda m_t, \tag{22}$$

where the vector $z_t\equiv\left(x_t,\pi_{H,t}\right)^T$ includes domestic slack (x_t) and PPI inflation ($\pi_{H,t}$) for the Home country and the driving process which is given by the monetary shock process m_t. The matrices of structural parameters are defined as

$$A_\lambda\equiv\lambda^R\Lambda_\lambda\begin{pmatrix} 1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta & \frac{1}{\sigma_\alpha}\left(1-\psi_{\pi_H}\beta\right) \\ \kappa_\alpha & \Phi_\lambda \end{pmatrix},$$

$$B_\lambda\equiv-\Lambda_\lambda\begin{pmatrix} \left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\frac{1}{\sigma_\alpha} \\ \kappa_\alpha\frac{1}{\sigma_\alpha} \end{pmatrix}$$

where

$\Lambda_\lambda\equiv\frac{1}{\left(\frac{1}{\sigma_\alpha}\psi_x+\left(1-\left(1-\lambda^R\right)\phi_x\right)\right)\left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)+\left(\frac{1}{\sigma_\alpha}\psi_{\pi_H}-\left(1-\lambda^R\right)\frac{\phi_{\pi_H}}{\sigma_\alpha}\right)\kappa_\alpha}$, and

$\Phi_\lambda\equiv\kappa_\alpha\frac{1}{\sigma_\alpha}+\left(\frac{1}{\sigma_\alpha}\psi_x+\left(1-\left(1-\lambda^R\right)\phi_x\right)\right)\beta.$

The transformed linear rational expectations model in (22) characterizes the dynamics of the transformed rational expectations core model. Time breaks, if any, might arise from the variation of the share of rational agents λ^R treated as a possible externality shifter in our framework.

Under the assumption that m_t in (15) is stationary and given that λ^R is bounded ($0\leq\lambda^R\leq 1$), then (22) has a unique nonexplosive solution in which the vector $z_t\equiv\left(x_t,\pi_{H,t}\right)^T$ is stationary whenever both eigenvalues of the matrix A_λ are inside the unit circle. The eigenvalues corresponding to the matrix A_λ can be written as

$$\lambda_1^{eig}\equiv\frac{1}{2}\lambda^R\Lambda_\lambda\left(\Phi_\lambda-\sqrt{\left(\Phi_\lambda\right)^2-4\frac{\beta}{\Lambda_\lambda}}\right),\ \lambda_2^{eig}\equiv\frac{1}{2}\lambda^R\Lambda_\lambda\left(\Phi_\lambda+\sqrt{\left(\Phi_\lambda\right)^2-4\frac{\beta}{\Lambda_\lambda}}\right), \tag{23}$$

where $\Lambda_\lambda>0$ and $\Phi_\lambda>0$, as defined above, hold whenever the bounded rationality parameters $\phi_{\pi_H}\geq 0$ and

[12] A related analysis of the workhorse New Keynesian model in a two-country setting (albeit only under fully rational expectations) can be found in Martínez-García (2017).

$\phi_x \geq 0$ given that $\kappa_\alpha > 0$ and the policy coefficients satisfy that $\psi_{\pi_H} > 0$ and $\psi_x \geq 0$. For standard parameterizations of the transformed SOE model such that $(\Phi_\lambda)^2 - 4\frac{\beta}{\Lambda_\lambda} \geq 0$, it follows that $0 < \lambda_1^s \leq \lambda_2^s$. Therefore, both eigenvalues of A_t would lie inside the unit circle when $\lambda_2^{eig} \equiv \frac{1}{2}\lambda^R\Lambda_\lambda\left(\Phi_\lambda + \sqrt{(\Phi_\lambda)^2 - 4\frac{\beta}{\Lambda_\lambda}}\right) < 1$. This inequality holds, in turn, whenever $\lambda^R\Lambda_\lambda\sqrt{(\Phi_\lambda)^2 - 4\frac{\beta}{\Lambda_\lambda}} < 2 - \lambda^R\Lambda_\lambda\Phi_\lambda$. Taking the square on both sides of the inequality—i.e., $\left(\lambda^R\Lambda_\lambda\right)^2\left((\Phi_\lambda)^2 - 4\frac{\beta}{\Lambda_\lambda}\right) < \left(\lambda^R\Lambda_\lambda\Phi_\lambda - 2\right)^2$ if $(\Phi_\lambda)^2 - 4\frac{\beta}{\Lambda_\lambda} \geq 0$, $\lambda^R \geq 0$ and $\Lambda_\lambda > 0$—and then, re-arranging terms, the inequality can be rewritten as: $\lambda^R\Lambda_\lambda\left(\Phi_\lambda - \lambda^R\beta\right) < 1$. Similarly, if $\Lambda_\lambda < 0$ we obtain that $\lambda^R\Lambda_\lambda\left(\Phi_\lambda - \lambda^R\beta\right) > 1$. From here it follows that $\lambda_2^{eig} < 1$ when $\Lambda_\lambda > 0$ and $\lambda^R\frac{1+\frac{1}{\sigma_\alpha}(\beta\psi_x+\kappa_\alpha)+\left(1-\lambda^R\right)\beta\left(1-\left(\phi_{\pi_H}+\phi_x\right)\right)}{(1-(1-\lambda^R)\phi_x)\left(1-(1-\lambda^R)\phi_{\pi_H}\beta\right)+\frac{1}{\sigma_\alpha}\left(\psi_x+\kappa_\alpha\psi_{\pi_H}-(1-\lambda^R)\phi_{\pi_H}(\kappa_\alpha+\beta\psi_x)\right)} < 1$ or when $\Lambda_\lambda < 0$ and $\lambda^R\frac{1+\frac{1}{\sigma_\alpha}(\beta\psi_x+\kappa_\alpha)+\left(1-\lambda^R\right)\beta\left(1-\left(\phi_{\pi_H}+\phi_x\right)\right)}{(1-(1-\lambda^R)\phi_x)\left(1-(1-\lambda^R)\phi_{\pi_H}\beta\right)+\frac{1}{\sigma_\alpha}\left(\psi_x+\kappa_\alpha\psi_{\pi_H}-(1-\lambda^R)\phi_{\pi_H}(\kappa_\alpha+\beta\psi_x)\right)} > 1$. After further algebraic manipulations, we conclude that $\lambda_2^{eig} < 1$ when it can be verified that:

$$\begin{cases} \psi_{\pi_H} + \left(\frac{1-\lambda^R\beta}{\kappa_\alpha}\right)\psi_x > \lambda^R + \left[\lambda^R - \left(1-\left(1-\lambda^R\right)\phi_x\right)\left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\right]\left(\frac{\sigma_\alpha}{\kappa_\alpha}\right) + ... \\ \quad \left(1-\lambda^R\right)\left[\lambda^R\beta\left(\frac{\sigma_\alpha}{\kappa_\alpha}\right)\left(1-\left(\phi_{\pi_H}+\phi_x\right)\right) + \phi_{\pi_H}\left(1+\frac{\beta}{\kappa_\alpha}\psi_x\right)\right], \text{ if } \Lambda_\lambda > 0, \\ \psi_{\pi_H} + \left(\frac{1-\lambda^R\beta}{\kappa_\alpha}\right)\psi_x < \lambda^R + \left[\lambda^R - \left(1-\left(1-\lambda^R\right)\phi_x\right)\left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\right]\left(\frac{\sigma_\alpha}{\kappa_\alpha}\right) + ... \\ \quad \left(1-\lambda^R\right)\left[\lambda^R\beta\left(\frac{\sigma_\alpha}{\kappa_\alpha}\right)\left(1-\left(\phi_{\pi_H}+\phi_x\right)\right) + \phi_{\pi_H}\left(1+\frac{\beta}{\kappa_\alpha}\psi_x\right)\right], \text{ if } \Lambda_\lambda < 0, \end{cases} \tag{24}$$

which is derived under the implicit assumption that the bounded rationality parameters satisfy that $0 \leq \phi_{\pi_H}, \phi_x \leq 1$ (although this can be generalized to extrapolative expectations where these parameters are close but above one). Notice that we also require to ensure determinacy that $\Lambda_\lambda > 0$ which, in turn, would imply that $\frac{1}{\sigma_\alpha}\left(\left(1-\lambda^R\right)\phi_{\pi_H}\left(\kappa_\alpha+\beta\psi_x\right) - \left(\psi_x+\kappa_\alpha\psi_{\pi_H}\right)\right) < \left(1-\left(1-\lambda^R\right)\phi_x\right)\left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)$.

Proposition 2 *A small open-economy (SOE) variant of the Taylor principle with a mixture of rational and boundedly rational is needed to ensure the uniqueness and existence of a nonexplosive solution for transformed system of expectational equations in (21). We require that* $(\Phi_\lambda)^2 - 4\frac{\beta}{\Lambda_\lambda} \geq 0$ *where* $\Lambda_\lambda \equiv \frac{1}{(1-(1-\lambda^R)\phi_x)\left(1-(1-\lambda^R)\phi_{\pi_H}\beta\right)+\frac{1}{\sigma_\alpha}\left(\psi_x+\kappa_\alpha\psi_{\pi_H}-(1-\lambda^R)\phi_{\pi_H}(\kappa_\alpha+\beta\psi_x)\right)}$ *and* $\Phi_\lambda \equiv 1+\beta-\left(1-\lambda^R\right)\beta\left(\phi_{\pi_H}+\phi_x\right)+\frac{1}{\sigma_\alpha}\left(\beta\psi_x+\kappa_\alpha\right) > 0$ *for the eigenvalues in (23) to be real-valued. The Taylor principle then requires that, for any given share of rational expectations private economic agents* $\lambda^R \in [0,1]$*, condition in (24) must hold and be verified. In the limiting case where all private economic agents are fully rational (*$\lambda^R = 1$*),* $\Lambda_\lambda > 0$ *trivially and the condition in (24) reduces to the standard closed-economy Taylor principle that requires* $\psi_{\pi_H} + \left(\frac{1-\beta}{\kappa_\alpha}\right)\psi_x > 1$*. In the limiting case where bounded rationality prevails among all private economic agents (*$\lambda^R = 0$*), it follows that* $\Lambda_\lambda > 0$ *if* $\left(\frac{\phi_{\pi_H}\kappa_\alpha-\left(1-\beta\phi_{\pi_H}\right)\psi_x-\kappa_\alpha\psi_{\pi_H}}{\sigma_\alpha(1-\phi_x)\left(1-\phi_{\pi_H}\beta\right)}\right) < 1$ *and condition (24) reduces to the same inequality (that is, to require that* $\left(\frac{\phi_{\pi_H}\kappa_\alpha-\left(1-\beta\phi_{\pi_H}\right)\psi_x-\kappa_\alpha\psi_{\pi_H}}{\sigma_\alpha(1-\phi_x)\left(1-\phi_{\pi_H}\beta\right)}\right) < 1$*).*

Proposition 2 shows the nontrivial impact that the inclusion of boundedly rational private agents has on determinacy. Existence and uniqueness of an equilibrium to the heterogeneous-beliefs SOE model depends on the policy parameters $\psi_{\pi_H} > 0$ and $\psi_x \geq 0$. In the case where $\psi_x = 0$, the basic Taylor principle requiring $\psi_{\pi_H} > 1$ holds as-is only in the scenario where all agents form their expectations in a fully rational manner. If $\psi_x > 0$, then the Taylor principle under rational expectations becomes $\psi_{\pi_H} + \left(\frac{1-\beta}{\kappa_\alpha}\right)\psi_x > 1$ and this in turn permits specifications where ψ_{π_H} might be close but below one. In other words, whenever the monetary

policy rule involves a positive response to the domestic output gap ($\psi_x > 0$), then the standard Taylor principle can be relaxed whereby $\psi_{\pi_H} > 0$ can fall to some extent below one and still be consistent with determinacy given that $\left(\frac{1-\beta}{\kappa_\alpha}\right) > 0$.

In turn, implementing a policy rule where $\psi_x > 0$ presents the central bank with the problem that domestic slack depends on the unobserved output potential and this, in turn, lends itself to introduce forecasting disagreements between the central bank and private agents unless all form their expectations rationally. Furthermore, we see that, under rational expectations, the determinacy region does not depend on the degree of openness α of the small-open economy—the Taylor principle is the same one as in the closed-economy case ($\alpha = 0$)—for all plausible parameterizations of the model (not just for the special case we focus on elsewhere where strict PPI inflation targeting is optimal).

However, when the economy is populated by a mixture of rational expectations private agents and boundedly rational ones, the determinacy region will keenly depend on the share of rational agents λ^R. Proposition 2 provides a detailed derivation of the implications for determinacy. Intuitively, what happens in this scenario is that the central bank needs to up its threat to punish deviations from the full stabilization targets (notably its response to inflation deviations) because the dissuasionary effect of that threat now operates only on part of the private agents but otherwise does not factor into the behavior of that portion of the private agents that mechanically forms their expectations under bounded rationality. This is a case where central banks might gain more flexibility to operate if more agents adopt fully rational expectations—bank communication, in this sense, can be part of the central bank's toolkit aimed at aligning private expectations more generally with those of the central bank. Central bank communication, to the extent that it provides a wealth of information about the economy and reduces the costs of forming rational expectations, may (in doing so) contribute to giving more room for monetary policy to operate without violating the Taylor principle.

3.2 Model Solution

Whenever $\lambda^R = 0$, given the monetary policy implementation laid out by (14) and (15), the deterministic core system of difference equations in (11) becomes:

$$\begin{pmatrix} \frac{1}{\sigma_\alpha}\psi_x + (1-\phi_x) & \frac{\psi_{\pi_H}-\phi_{\pi_H}}{\sigma_\alpha} \\ -\kappa_\alpha & 1-\phi_{\pi_H}\beta \end{pmatrix} \begin{pmatrix} x_t \\ \pi_{H,t} \end{pmatrix} = -\begin{pmatrix} \frac{1}{\sigma_\alpha} m_t \\ 0 \end{pmatrix}, \tag{25}$$

whose solution implies that:

$$\begin{aligned} \begin{pmatrix} x_t \\ \pi_{H,t} \end{pmatrix} &= \frac{1}{\left(\frac{1}{\sigma_\alpha}\psi_x + (1-\phi_x)\right)(1-\phi_{\pi_H}\beta) + \left(\frac{\psi_{\pi_H}-\phi_{\pi_H}}{\sigma_\alpha}\right)\kappa_\alpha} \begin{pmatrix} 1-\phi_{\pi_H}\beta & \frac{\phi_{\pi_H}-\psi_{\pi_H}}{\sigma_\alpha} \\ \kappa_\alpha & \frac{1}{\sigma_\alpha}\psi_x + (1-\phi_x) \end{pmatrix} \begin{pmatrix} -\frac{1}{\sigma_\alpha} m_t \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} -\left(\frac{1-\phi_{\pi_H}\beta}{\left(\frac{1}{\sigma_\alpha}\psi_x+(1-\phi_x)\right)\left(1-\phi_{\pi_H}\beta\right)+\left(\frac{\psi_{\pi_H}-\phi_{\pi_H}}{\sigma_\alpha}\right)\kappa_\alpha}\right)\frac{1}{\sigma_\alpha} m_t \\ -\left(\frac{\kappa_\alpha}{\left(\frac{1}{\sigma_\alpha}\psi_x+(1-\phi_x)\right)\left(1-\phi_{\pi_H}\beta\right)+\left(\frac{\psi_{\pi_H}-\phi_{\pi_H}}{\sigma_\alpha}\right)\kappa_\alpha}\right)\frac{1}{\sigma_\alpha} m_t \end{pmatrix}. \end{aligned} \tag{26}$$

which shows that failing to achieve full stabilization and a first-best allocation is the result of central bank's expectations errors propagating when implementing the optimal monetary policy—that is, when implement-

ing strict PPI inflation targeting.

The heterogenous-expectations SOE model developed in this paper provides a framework with which to investigate how disagreements in the formation of expectations affect and even define monetary policy in a small open economy setting. Under the monetary policy laid out by (14) and (15), we have shown that the bounded rationality law of motion implied by (26) is self-sustaining. Similarly, we find that the law of motion implied by transformed core system of rational expectations equations in (21) is also self-sustaining.

Proposition 3 *(a) The solution to the deterministic core system of difference equations in (26) when all private economic agents are subject to bounded rationality inherits the autoregressive specification of the monetary shock process in (15) and is discoverable.*

(b) Similarly, the solution to the core system of transformed rational expectations equations in (10) that allows a time-varying fraction of private economic agents $0 \leq \lambda^R \leq 1$ to form rational expectations also inherits the autorregressive specification of the monetary shock process in (15) and is discoverable.

Moreover, there is no difference in the first-order autorregressive coefficient on the PPI inflation process irrespective of the share of private economic agents that rely on rational expectations which is equal to the persistence of the monetary shock process ρ_m. Therefore, monetary policy cannot influence the next period's inflation expectations of boundedly rational private agents nor, importantly, the expectations formed for private agents under rational expectations unless it were to affect ρ_m in doing so.

Part (a) of Proposition 3 follows by simple algebraic manipulation given that (26) implies that both domestic slack, x_t, and PPI inflation, $\pi_{H,t}$, are linear transformations of the monetary shock process given by (15). It easily follows from here that:

$$\begin{aligned} x_t &= \left(\tfrac{1-\phi_{\pi_H}\beta}{\kappa_\alpha}\right)\pi_{H,t}, \\ \pi_{H,t} &= \rho_m \pi_{H,t-1} - \left(\frac{\kappa_\alpha}{\left(\frac{1}{\sigma_\alpha}\psi_x+(1-\phi_x)\right)\left(1-\phi_{\pi_H}\beta\right)+\left(\frac{\psi_{\pi_H}-\phi_{\pi_H}}{\sigma_\alpha}\right)\kappa_\alpha}\right)\tfrac{1}{\sigma_\alpha}\epsilon_t^m. \end{aligned} \tag{27}$$

Accordingly, if private economic agents under bounded rationality were to use an adaptive expectations mechanism such that $0 < \phi_{\pi_H} = \phi_x < 1$, they could not be convinced by the evidence on PPI inflation to do anything different to produce their forecasts. In this setting, there would be no effective way in which monetary policy influences the formation of expectations (although monetary policy parameters ψ_{π_H} and ψ_x can still affect the volatility of inflation).

Part (b) of Proposition 3 follows from the solution to (10) whenever the monetary shock process is given by (15). It easily follows from here that:

$$\begin{aligned} x_t &= \left(\frac{\left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\left(1-\lambda^R\Lambda_\lambda\rho_m\Phi_\lambda\right)+\lambda^R\Lambda_\lambda\rho_m\kappa_\alpha\frac{1}{\sigma_\alpha}\left(1-\psi_{\pi_H}\beta\right)}{\kappa_\alpha}\right)\pi_{H,t}, \\ \pi_{H,t} &= \rho_m\pi_{H,t-1} - \left(\frac{\kappa_\alpha}{\frac{1}{\Lambda_\lambda}-\lambda^R\left[\left(1-(1-\lambda^R)\phi_{\pi_H}\beta\right)(1-\lambda^R\Lambda_\lambda\rho_m\Phi_\lambda)+\Phi_\lambda+\lambda^R\Lambda_\lambda\rho_m\kappa_\alpha\frac{1}{\sigma_\alpha}\left(1-\psi_{\pi_H}\beta\right)\right]\rho_m}\right)\frac{1}{\sigma_\alpha}\epsilon_t^m, \end{aligned} \tag{28}$$

where $\Lambda_\lambda \equiv \frac{1}{\left(\frac{1}{\sigma_\alpha}\psi_x+(1-(1-\lambda^R)\phi_x)\right)\left(1-(1-\lambda^R)\phi_{\pi_H}\beta\right)+\left(\frac{1}{\sigma_\alpha}\psi_{\pi_H}-(1-\lambda^R)\frac{\phi_{\pi_H}}{\sigma_\alpha}\right)\kappa_\alpha}$ and

$\Phi_\lambda \equiv \kappa_\alpha\frac{1}{\sigma_\alpha} + \left(\frac{1}{\sigma_\alpha}\psi_x + \left(1-\left(1-\lambda^R\right)\phi_x\right)\right)\beta$. This holds true for any $0 \leq \lambda^R \leq 1$. The derivations of this solution can be found in the Appendix.

Notice that whenever $\lambda^R = 0$, the solution in (28) reduces to the solution in (27) when all are boundedly rational agents. In turn, in the polar opposite case where all private agents rely on rational expectations and $\lambda^R = 1$, it follows from (28) that:

$$\begin{aligned} x_t &= \left(\tfrac{1-\rho_m\beta}{\kappa_\alpha}\right)\pi_{H,t}, \\ \pi_{H,t} &= \rho_m\pi_{H,t-1} - \left(\frac{\kappa_\alpha}{\left(\frac{1}{\sigma_\alpha}\psi_x+(1-\rho_m)\right)(1-\rho_m\beta)+\left(\frac{\psi_{\pi_H}-\rho_m}{\sigma_\alpha}\right)\kappa_\alpha}\right)\tfrac{1}{\sigma_\alpha}\epsilon_t^m, \end{aligned} \tag{29}$$

Once again, an adaptive expectations mechanism such that $\phi_{\pi_H} = \phi_x = \rho_m$ cannot be refuted through learning. Economic agents forming expectations under bounded rationality could not be convinced by the evidence on PPI inflation or by using all available information to form rational forecasts to do anything different than following that rule-of-thumb for forecasting. This follows from the observation that under the rule-of-thumb $\phi_{\pi_H} = \phi_x = \rho_m$, the solution to the model populated solely by boundedly rational private agents in (27) is identical to the one in (29) that corresponds to an economy populated solely by rational expectations private agents.

Calibration and some quantitative results. In this section we follow the calibration of the SOE model of Galí and Monacelli (2005). The main assumptions underlying this baseline calibration set, $\sigma = \eta = \gamma = 1$, are consistent with the special case for which strict PPI inflation targeting is optimal as noted earlier. We assume that the inverse of the Frisch elasticity of labor supply is set at $\varphi = 3$, while $\varepsilon = 6$ is set to ensure that the steady state markup is $\mu = 1.2$. The Calvo (1983) parameter θ takes the conventional value of 0.75 consistent with an average duration in quarterly data of one year between price adjustments. We also set $\beta = 0.99$, consistent with a riskless annual return of about 4% in steady state. These parameter values allow us to compute the slope of the dynamic IS equation and of the New Keynesian Phillips curve to be $\sigma_\alpha = 1$ and $\kappa_\alpha \equiv \left(\frac{(1-\beta\theta)(1-\theta)}{\theta}\right)(1+\varphi) = 0.3433$ irrespective of the degree of openness α. We keep Galí and Monacelli (2005)'s baseline value for α (the degree of openness) of 0.4 based on Canadian data. For the calibration of the interest rate rules, we follow Taylor (1993) and set $\psi_{\pi_H} = 1.5$ and $\psi_x = 0.5$. We also use the estimates of the domestic productivity and Foreign output from Galí and Monacelli (2005) for (8) − (9) and a standard calibration of the monetary shock process, i.e., $\rho_a = 0.66$, $\rho_{y^*} = 0.86$, $\rho_m = 0.8$, $\sigma_a = 0.0071$, $\sigma_{y^*} = 0.0078$, $\sigma_m = 0.0025$, and $\rho_{a,y^*} = 0.3$.

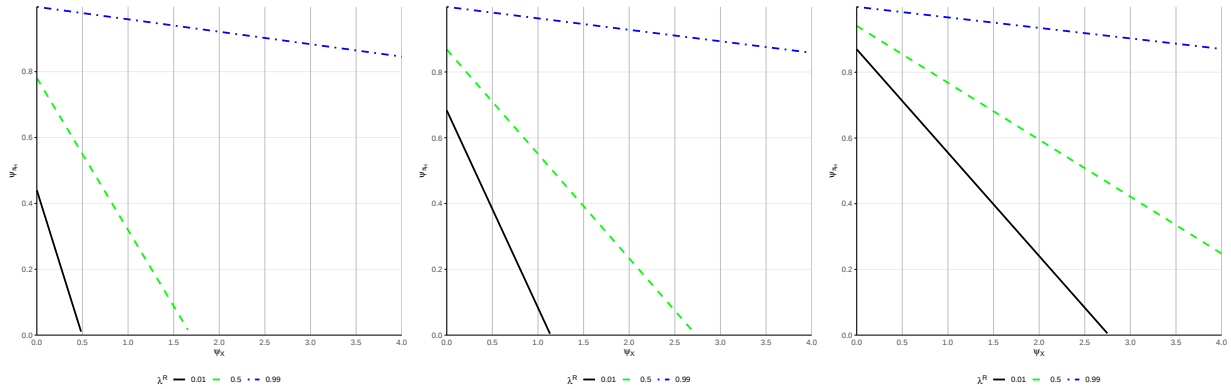
Table 1. Calibration of the Key Model Parameters

Structural parameters		**Parameterization**
Intertemporal discount factor	$0 < \beta < 1$	0.99
Intert. elasticity of substitution	$\sigma > 0$	1
Inverse Frisch elasticity of labor supply	$\varphi > 0$	3
Elast. of subst. Home & Foreign	$\eta > 0$	1
Elast. of subst. Foreign varieties	$\gamma > 0$	1
Cons. share of foreign goods	$0 \leq \alpha < 1$	0.4
Calvo price stickiness	$0 < \theta < 1$	0.75
Elasticity of substitution among varieties	$\varepsilon > 1$	6
Monetary policy parameters		
Response to PPI inflation	$\psi_{\pi_H} > 1$	1.5
Response to output gap	$\psi_x \geq 0$	0.5
Adaptive expectations	$\phi_x = \phi_{\pi_H}$	–
Shock parameters		**Parameterization**
Productivity shock		
Persistence	$-1 < \rho_a < 1$	0.66
Volatility	$\sigma_a > 0$	0.0071
Foreign output shock		
Persistence	$-1 < \rho_{y^*} < 1$	0.86
Volatility	$\sigma_{y^*} > 0$	0.0078
Correlation of innovations	$-1 < \rho_{a,y^*} < 1$	0.3
Monetary policy shock		
Persistence	$-1 < \rho_m < 1$	0.8
Volatility	$\sigma_m > 0$	0.0025
Correlation of innovations	$-1 < \rho_{m,a}, \rho_{m,y^*} < 1$	0

Notice that under the special case we are considering here, the solution to the model with heterogenous beliefs given generically in (28) does not depend on the degree of openness α because this parameter drops from the slopes of the IS curve and of the Phillips curve (that is, it drops from σ_α and κ_α).

Result 1. Determinacy regions. We illustrate the role of boundedly rational agents on the determinacy of the solution in Figure 1 under the assumption that adaptive and rational expectations of inflation coincide and take the value of 0.8 (as suggested by the estimates of Rudebusch (2006)), i.e., $\phi_x = \phi_{\pi_H} \lessgtr \rho_m = 0.8$. We use expression (24) and assume $\phi_x = \phi_{\pi_H} = 0.7$ (left panel), $\phi_x = \phi_{\pi_H} = 0.8$ (center panel), and $\phi_x = \phi_{\pi_H} = 0.9$ (right panel). The figure illustrates the lower bound (or lower contour) of the determinacy region when $\Lambda_\lambda > 0$ and $\lambda^R = 0.01$ (black solid line), $\Lambda_\lambda > 0$ and $\lambda^R = 0.5$ (green solid line), and $\Lambda_\lambda > 0$ and $\lambda^R = 0.99$ (light blue solid line). This shows that the presence of boundedly rational agents in the model may, in fact, contribute to mitigating the magnitude of the threat the central bank makes to ensure a unique solution exists.

Figure 1. Determinacy Region Depending on the Share of Rational Agents



Note: The figure illustrates the lower bound (or lower contour) of the determinacy region when $\Lambda_\lambda > 0$ and $\lambda^R = 0.01$ (black solid line), $\Lambda_\lambda > 0$ and $\lambda^R = 0.5$ (green dashed line), and $\Lambda_\lambda > 0$ and $\lambda^R = 0.99$ (light blue dashed-dotted line). We assume $\phi_x = \phi_{\pi_H} = 0.7$ (left panel), $\phi_x = \phi_{\pi_H} = 0.8$ (center panel), and $\phi_x = \phi_{\pi_H} = 0.9$ (right panel).

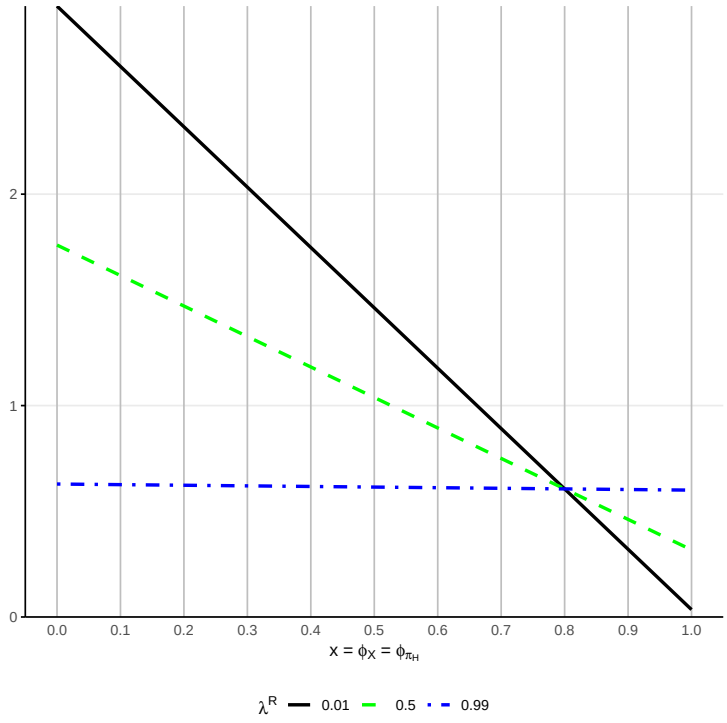
It is also fairly straightforward to show that for any plausible parameterization of the model (including the calibration we adopt here), it must be the case that the trade-off between domestic slack and PPI inflation for the polar cases with only rationally bounded private agents and with only rational expectations agents satisfies that: $\left(\frac{1-\phi_{\pi_H}\beta}{\kappa_\alpha}\right) > \left(\frac{1-\rho_m\beta}{\kappa_\alpha}\right)$ if and only if $\rho_m > \phi_{\pi_H}$. In other words, when boundedly rational private agents forecast future inflation believing it to be less persistent than what they would conclude under rational expectations, any given change in domestic slack would be accompanied by a smaller change in PPI inflation than under rational expectations.

Result 2. The trade-off between PPI inflation and output gap. We also illustrate in Figure 2 the impact on the policy trade-off between PPI inflation and domestic slack from equation (28), as given by the composite parameter

$$\left(\frac{\left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\left(1-\lambda^R\Lambda_\lambda\rho_m\Phi_\lambda\right)+\lambda^R\Lambda_\lambda\rho_m\kappa_\alpha\frac{1}{\sigma_\alpha}\left(1-\psi_{\pi_H}\beta\right)}{\kappa_\alpha}\right),$$

when $x = \phi_x = \phi_{\pi_H}$, and $\rho_m = 0.8$, and $\lambda^R = 0.01$ (black solid line), $\lambda^R = 0.5$ (green solid line), $\lambda^R = 0.99$ (light blue solid line). According to this, any given change in PPI inflation would be accompanied by a smaller change in domestic slack if $x = \phi_x = \phi_{\pi_H}$ is higher, but the slope of the relationship depends on the fraction of agents that are rational (that is, depends on λ^R). Moreover, the trade-off would be smaller the higher λ^R is if $x = \phi_x = \phi_{\pi_H} < \rho_m = 0.8$, while the opposite holds true whenever $x = \phi_x = \phi_{\pi_H} > \rho_m = 0.8$. This opens up a very interesting area of research as it shows that the slope of the Phillips curve that we can detect in the comovement between PPI inflation and domestic slack in the data would reflect both the actual share of rational agents in the economy (λ^R) as well as the sign and magnitude of the disagreement between rational and non-rational agents on macroeconomic persistence ($x - \rho_m$).

Figure 2. Trade-off between PPI Inflation and Domestic Slack Depending on the Share of Rational Agents



Note: Trade-off coefficient when $x = \phi_x = \phi_{\pi_H}$, and $\rho_m = 0.8$, and $\lambda^R = 0.01$ (black solid line), $\lambda^R = 0.5$ (green dashed line), $\lambda^R = 0.99$ (light blue dashed-dotted line).

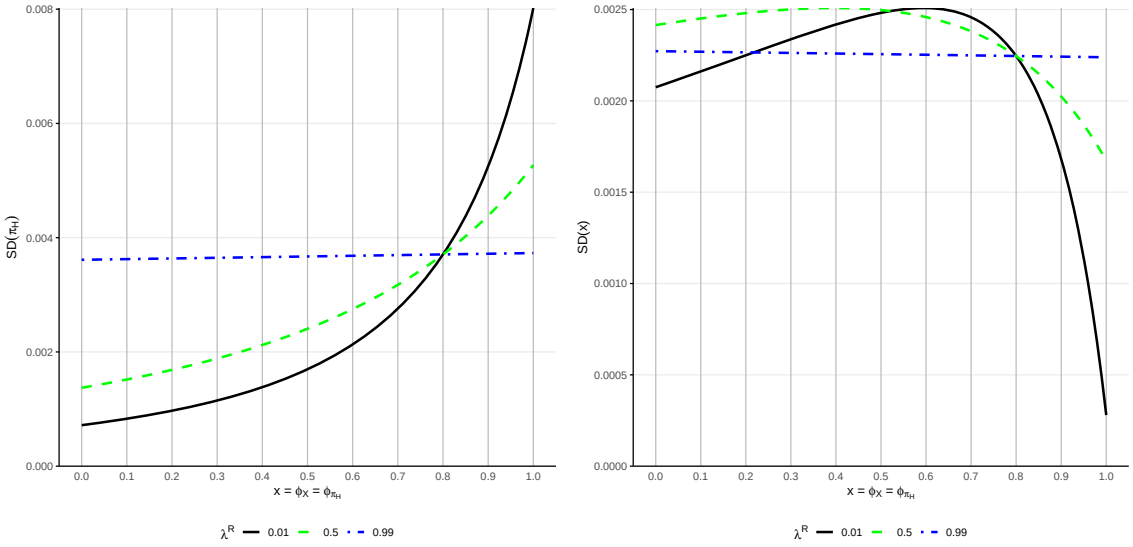
Result 3. The volatility of PPI inflation and the output gap. The volatility of PPI inflation is also going to be at variance in both polar cases ((27) under bounded rationality versus (29) under rational expectations). Figure 3 below illustrates this by plotting the coefficient

$$\left(\frac{\kappa_\alpha}{\frac{1}{\Lambda_\lambda} - \lambda^R \left[\left(1 - \left(1 - \lambda^R\right) \phi_{\pi_H} \beta\right) \left(1 - \lambda^R \Lambda_\lambda \rho_m \Phi_\lambda\right) + \Phi_\lambda + \lambda^R \Lambda_\lambda \rho_m \kappa_\alpha \frac{1}{\sigma_\alpha} \left(1 - \psi_{\pi_H} \beta\right) \right] \rho_m} \right) \frac{1}{\sigma_\alpha}$$

under the baseline calibration of Galí and Monacelli (2005). To be more precise, the figure illustrates the inflation volatility in (28) when $x = \phi_x = \phi_{\pi_H}$, $\rho_m = 0.8$, and $\lambda^R = 0.01$ (black solid line), $\lambda^R = 0.5$ (green solid line), and $\lambda^R = 0.99$ (light blue solid line). From here, it easily follows that (at least under this baseline calibration), when boundedly rational private agents forecast future inflation believing it to be less persistent than what they would conclude under rational expectations ($\rho_m > \phi_x = \phi_{\pi_H}$), we observe lower volatility in the dynamics of PPI inflation. PPI inflation volatility would be larger the larger the share of rational agents λ^R is if $x = \phi_x = \phi_{\pi_H} < \rho_m = 0.8$, while the opposite holds true whenever $x = \phi_x = \phi_{\pi_H} > \rho_m = 0.8$. This in turn showcases that the volatility of PPI inflation in the data reflects both the actual share of rational agents in the economy (λ^R) as well as the sign and magnitude of the disagreement between rational and non-rational agents on macroeconomic persistence ($x - \rho_m$). In addition, Figure 3 shows that the volatility

of the output gap per equation (28) follows a non-linear pattern under the presence of boundedly rational agents.

Figure 3. Volatility of PPI Inflation and Output Gap Depending on the Share of Rational Agents



Note: Inflation (left) and output gap (right) volatility according to (28) when $x = \phi_x = \phi_{\pi_H}$, $\rho_m = 0.8$, and $\lambda^R = 0.01$ (black solid line), $\lambda^R = 0.5$ (green dashed line), and $\lambda^R = 0.99$ (light blue dashed-dotted line).

Result 4. The Divine Coincidence curve. The model solution allows us to derive what we refer to as the **Divine Coincidence Curve**—a curve representing combinations of output gap volatility and PPI inflation volatility for different values of the policy rate's response to inflation (ψ_{π_H}). These curves are essentially (degenerate) Taylor curves (see Figure 4), but instead of illustrating a trade-off between the two volatilities, they highlight the favorable possibility of simultaneously reducing the variability of both indicators.

In the plot on the right of Figure 4, when the fraction of boundedly rational agents is small, their expectations appear to have little impact. However, the most notable finding is that when non-rational agents constitute a significant fraction (i.e., $\lambda^R = 0.01$ or $\lambda^R = 0.5$), they influence the slope of the Divine Coincidence curve, shaping how monetary policy affects output gap and inflation volatilities.

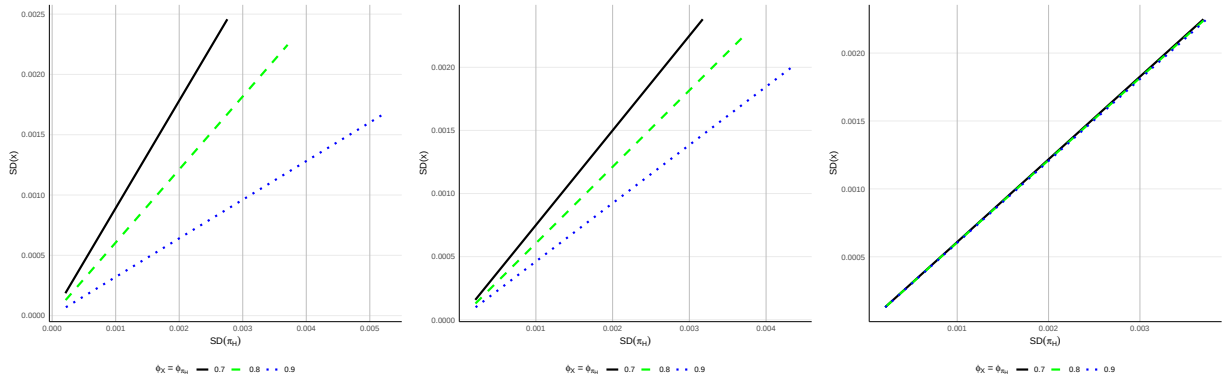
The other two plots (left and center) illustrate that a more aggressive monetary policy response to inflation reduces inflation volatility, as expected. However, this reduction leads to an even larger decline in output gap volatility if boundedly rational agents expect persistence below 0.8, whereas the opposite occurs if their expectations exceed 0.8. This effect is reflected in the slope of the lines.

Most importantly, the presence of non-rational agents implies that when expected persistence is below 0.8, the ratio of output gap volatility to inflation volatility is always higher than in the case where agents correctly perceive persistence at 0.8. This pattern becomes more pronounced as the share of non-rational agents increases, particularly when rational agents expect persistence above 0.8.

This pattern suggests that monetary policy may be more effective in controlling inflation volatility without necessarily having the same impact on real variables. This supports the argument that, while monetary policy

appears to be more effective, its transmission mechanism changes due to the presence of boundedly rational agents, similar to those that seem to characterize the Great Moderation period.

Figure 4. Divine Coincidence Curves Depending on the Share of Rational Agents



Note: The figure shows the standard deviation of the output gap (horizontal axis) and inflation (vertical axis) for different values of the policy rate's response to inflation ($\psi_{\pi_H} \geq 1.5$), when $\phi_x = \phi_{\pi_H} = 0.7$ (black solid line), $\phi_x = \phi_{\pi_H} = 0.8$ (green dashed line), and $\phi_x = \phi_{\pi_H} = 0.9$ (light blue dashed-dotted line). We assume $\Lambda_\lambda > 0$ and $\lambda^R = 0.01$ (left panel), $\lambda^R = 0.5$ (center panel), and $\lambda^R = 0.99$ (right panel).

4 Some Policy Implications

What role is there for λ^R? This parameter matters for monetary policy because disagreements between rational and boundedly rational agents can influence the volatility of the domestic output gap, x_t, and of PPI inflation, $\pi_{H,t}$, for any given expectational error process for monetary policy. The influence of λ^R will depend on whether economic agents rely on rational expectations or on a given perceived or subjective law of motion under bounded rationality. The impact they have on the dynamics of inflation also will crucially depend on the sign and magnitude of the disagreement between rational and boundedly rational agents on the macroeconomic persistence of the data. The anticipatory real effects of monetary policy are limited due to the failure of economic agents to fully incorporate signals regarding the future trajectory of monetary policy or regarding changes in the systematic component of the monetary policy rule when forming their expectations about future economic conditions.

Primarily, our analysis reveals that when a significant proportion of economic agents exhibits bounded rationality, the economy tends to be characterized by heightened inflation volatility alongside a weakened trade-off between Producer Price Index (PPI) inflation and domestic slack. This phenomenon may stem from the increasingly transient nature of monetary shocks, which influence the persistence of PPI inflation but are not fully acknowledged or factored into the expectations formation process of boundedly rational agents. Consequently, a perceived breakdown in both inflation volatility and persistence in the empirical data, potentially indicating a reduced responsiveness of inflation to domestic slack, can be challenging for inflation forecasting models that rely on the conventional Phillips curve relationship.

To evaluate the reliability of inflation forecasts over time, we conduct a comparative analysis between the random walk (RW-AO) forecasting specification, following the methodology outlined by Atkeson and Ohanian (2001) and Faust and Wright (2013), and forecasts derived from both private sources, such as Consensus Economics$^{(R)}$, and institutional sources, including the International Monetary Fund's World Economic Outlook (IMF WEO) database. Our examination aims to discern the influence of subjective judgment relative to straightforward, break-robust strategies like the RW-AO model. We utilize a dataset comprising seasonally-adjusted, average-quarterly series for 14 Emerging Market Economies (EMEs), spanning the period from $1980:Q1$ to $2016:Q4$. The selected EMEs include Chile, China, Colombia, Hungary, Indonesia, India, Malaysia, Mexico, Nigeria, Peru, Philippines, South Africa, Thailand, and Turkey.

Faust and Wright (2013) find that subjective forecasts tend to exhibit higher accuracy compared to model-based forecasts when predicting inflation within advanced economies. This assertion is further supported by Mandalinci (2017) and Duncan and Martínez-García (2019), who conducted a study revealing the superiority of institutional and even private forecasts over model-based counterparts for numerous EMEs. Mandalinci (2017)'s findings are based on institutional forecasts sourced from the International Monetary Fund's World Economic Outlook (IMF-WEO) while Duncan and Martínez-García (2019) also include private forecasts from Consensus Economics$^{(R)}$. Duncan and Martínez-García (2019) also argue that inflation in many EMEs is characterized by significant structural breaks, making the RW-AO break-robust specification clearly preferable to conventional Phillips curve-based inflation forecasting models.

Our primary results, outlined in Table 2, indicate that professional forecasts generally outperform the RW-AO-based forecasts across the majority of EMEs. These findings underscore the potential benefits of incorporating subjective judgment and possibly leveraging additional supplementary sources beyond the scope of a given model to detect structural shifts, thereby enhancing inflation forecasts relative to our preferred benchmark (and improving the performance of RW-AO forecasts). The effectiveness of the RW-AO model, in practice, stems from the challenges associated with encompassing all pertinent economic features within a forecasting framework while accommodating changes over time. Notably, when accounting for heterogeneous beliefs and bounded rationality, inflation may exhibit varying degrees of persistence that is difficult to capture with Phillips curve-based models (or related specifications) that leave no room for structural change.

In our theoretical analysis, we demonstrate that structural breaks can originate from shifts in the monetary shock process and discrepancies between rational and boundedly rational agents regarding the underlying macroeconomic persistence. The challenge of quantifying disagreements among agents in their inflation forecasting approaches complicates the task of predicting inflation in EMEs. Notably, professional forecasters, represented by survey data from Consensus Economics$^{(R)}$ and the International Monetary Fund's World Economic Outlook (IMF WEO), often yield comparable or superior Root Mean Square Prediction Errors (RMSPEs) relative to the RW-AO model. This observation suggests that the integration of subjective judgment can enhance forecast accuracy beyond break-robust models like the RW-AO model by promptly incorporating additional information concerning significant structural shifts, particularly where it relates to the monetary policy framework. Hence, our findings underscore the pivotal role of expectations formation in driving macroeconomic dynamics and structural breaks.

Table 2 - Performance of RW-AO Model Relative to Consensus Forecasts™ and IMF World Economic Outlook (WEO): Forecasting Annual Inflation Rates

	Relative RMSPE: RW-AO vs. Consensus Forecast™				Relative RMSPE: RW-AO vs. IMF WEO			
	1991m1-2016m12		2003m1-2016m12		1991m1-2016m12		2003m1-2016m12	
	Current Year	Next Year	Current Year	Next Year	Current Year	Next Year	Current Year	Next Year
Chile	3.337	3.167	1.716	1.423	3.445	2.108	2.969	1.618
China	3.587	2.817	3.065	1.811	1.994	1.594	2.798	1.862
Colombia	4.242	3.137	2.395	1.771	1.603	1.302	2.694	1.686
Hungary	3.539	1.636	4.201	1.506	3.190	1.300	3.311	1.465
India	1.488	1.489	1.612	1.431	1.296	1.196	1.542	1.379
Indonesia	3.615	1.777	2.543	2.363	4.365	1.701	2.262	2.209
Malaysia	1.873	1.736	2.249	1.543	1.440	1.132	1.658	1.145
Mexico	4.350	4.308	3.028	3.627	9.394	3.271	4.382	2.864
Nigeria	1.572	2.400	1.529	1.459	2.130	1.102	2.240	1.191
Peru	408.143	249.622	1.470	1.298	20.149	12.762	1.911	1.144
Philippines	1.999	1.728	1.943	1.603	2.243	1.229	1.781	1.455
South Africa	2.637	1.961	2.528	1.659	1.130	1.827	2.625	1.550
Thailand	2.431	1.697	3.812	1.488	2.229	1.519	3.088	1.228
Turkey	3.796	1.740	12.189	8.370	1.902	0.936	10.531	5.197
Mean	31.901	19.944	3.163	2.240	4.036	2.356	3.128	1.857
Median	3.438	1.869	2.462	1.573	2.180	1.410	2.659	1.508
#<1	0	0	0	0	0	1	0	0
	Directional accuracy: RW-AO vs. Consensus Forecast™				Directional accuracy: RW-AO vs. IMF WEO			
	1991m1-2016m12		2003m1-2016m12		1991m1-2016m12		2003m1-2016m12	
	Current Year	Next Year	Current Year	Next Year	Current Year	Next Year	Current Year	Next Year
Chile	0.409	0.400	0.696	0.737	0.468	0.465	0.667	0.737
China	0.634	0.806	0.720	0.737	0.681	0.795	0.750	0.700
Colombia	0.304	0.432	0.370	0.600	0.364	0.462	0.435	0.571
Hungary	0.560	0.634	0.880	1.111	0.560	0.667	0.880	1.176
India	1.160	0.840	0.933	0.714	0.972	0.794	0.667	0.625
Indonesia	0.820	0.951	0.741	0.800	0.891	1.000	0.833	0.870
Malaysia	0.718	0.722	0.800	0.783	0.700	0.813	0.769	0.857
Mexico	0.522	0.571	0.696	0.842	0.490	0.571	0.667	0.842
Nigeria	0.875	1.095	0.778	1.143	0.674	1.069	0.583	1.143
Peru	0.615	0.765	0.952	1.222	0.565	0.718	0.870	1.222
Philippines	0.721	0.816	0.769	0.833	0.700	0.795	0.800	0.800
South Africa	0.614	0.892	0.571	0.957	0.646	0.881	0.571	0.917
Thailand	0.737	0.848	0.667	0.762	0.700	0.778	0.640	0.842
Turkey	0.634	0.722	0.741	0.857	0.651	0.757	0.800	0.900
Mean	0.666	0.750	0.737	0.864	0.647	0.755	0.709	0.872
Median	0.634	0.786	0.741	0.817	0.663	0.786	0.708	0.850
#<1	13	13	14	11	14	12	14	11

Notes: The RW-AO model for the annual inflation rate is implemented as a simple moving average of the last four observations available (forecasts for current year (t) and next year (t+1) are given by an unweighted mean of the observed annual inflation rates between t-4 and t-1). Columns in the top panel report the ratio of root mean squared prediction error (RMSPE) from the RW-AO model relative to the RMSPE of both Consensus Forecasts (CF) and International Monetary Fund World Economic Outlook (IMF WEO) predictions. Values less than one imply that the RW-AO model has a lower RMSPE than does the competitive benchmark. The row #<1 reports the number of economies (out of 14) that show relative RMSPE lower than 1 for the annual inflation rate for a particular forecast horizon. Columns in the bottom panel report the ratio of success in directional accuracy from the RW-AO model relative to the ratio of directional accuracy of both CF and IMF WEO predictions. Values less than one imply that the RW-AO model has a lower success ratio than does the competitive benchmark. The row #<1 in the bottom panel reports the number of economies (out of 14) that show relative ratio of success lower than 1 for a particular forecast horizon for the annual inflation rate. CF forecasts come at a higher frequency (often with monthly or bi-monthly releases) than those of the IMF WEO which are updated twice a year only--for comparability, we only retain the CF predictions that most closely correspond to the April/May and September/October updates of the IMF WEO. Forecasts of the annual inflation rate from CF and IMF WEO that overlap with each other start in: April 1990 for Mexico; April 1991 for Indonesia, Thailand, and Malaysia; May 1991 for Hungary; May 1993 for Peru, Colombia, and Chile; September 1993 for South Africa; April 1995 for the Philippines, India, and China; May 1995 for Turkey; and April 1999 for Nigeria.

5 Concluding Remarks

In our study, we investigate the intricate relationship among central bank predictions, credibility, and inflation dynamics within a small-open-economy New Keynesian model incorporating heterogeneous expectations. Our approach delves into the challenging task faced by central banks in forecasting and communicating predictions, operating under the assumption of a monetary authority tasked with tracking the natural interest rate—an essential, albeit unobservable, variable within the New Keynesian framework that policymakers must infer. In practice, central banks may introduce prediction errors in the process of recovering the natural rate of interest when private agents' expectations are heterogenous, which can subsequently influence policy rate decisions. Moreover, we approximate the credibility of the central bank by considering the proportion of rational agents within the economy. A higher ratio of rational agents to boundedly-rational agents enhances the effectiveness of monetary policy in mitigating inflationary pressures and narrowing output gaps.

Within this conceptual New Keynesian framework, first, our analysis demonstrates that the model of heterogeneous expectations in a small open economy exhibits an isomorphic relationship with its closed economy counterpart. However, our key finding pertains to the persistence of inflation, which remains unaffected by the composition of different processes to incorporate information and form expectations. Specifically, we observe that the autoregressive coefficient governing the domestic inflation dynamics, when considering solely boundedly rational individuals, equals that derived from a scenario involving a mixture of rational and boundedly rational agents.

Consequently, differences between short-run and long-run inflation forecasts derived from models exclusively comprising boundedly rational agents versus those incorporating a mixture of agent types are negligible. By contrast, despite this common source of persistence, discernible differences emerge in volatility. While inflation variance remains constant in the former case, it might become time-varying in the latter if the proportion of rational agents in the economy varies exogeneously over time.

Our findings also indicate that as the data expands over time, facilitating a more precise estimation of the autoregressive coefficient, forecasts based on bounded rationality gradually converge towards those generated by rational expectations. Hence, the benefits associated with rational expectations diminish to the extent that economic agents may opt for a simpler, less costly adaptive mechanism for forming expectations, particularly if the costs associated with processing full information under rational expectations are non-trivial.

That last implication carries significant implications for monetary policy, particularly concerning the recognition of changes in policy affecting the inflation rate persistence (or the persistence of the expectational errors). Such changes may not be promptly identified until enough additional data and observations become available revealing a shift in the persistence parameter. That shift has an effect on the perceived slope of the Phillips curve and, therefore, on the trade-offs that policymakers ultimately face. However, as evidence of a structural shift accumulates, we also anticipate a greater proportion of economic agents in the model adopting rational expectations, potentially outweighing associated costs and facilitating a more rapid internalization of the shift. We leave the exploration of those "learning" dynamics and other related issues for future research.

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Appendix

A Proofs

We conjecture that the solution for the endogenous variables $(x_t, \pi_{H,t}, i_t)$ for the transformed rational expectations model in (20) given the monetary policy rule in (14) and (15) can be expressed as,

$$\pi_{H,t} = \chi_1 \pi_{H,t-1} + \eta_t,\ \eta_t \sim i.i.d.\left(0, \sigma_\eta^2\right), \tag{30}$$

$$x_t = \frac{1}{\chi_0}\pi_{H,t}, \tag{31}$$

where the nominal short-term interest rate is given by $i_t = r_t^n + \psi_{\pi_H}\pi_{H,t} + \psi_x x_t + m_t$. We can express the transformed model in (20) as follows,

$$\begin{pmatrix} x_t \\ \pi_{H,t} \end{pmatrix} = \lambda^R \Lambda_\lambda \begin{pmatrix} 1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta & \frac{1}{\sigma_\alpha}\left(1-\psi_{\pi_H}\beta\right) \\ \kappa_\alpha & \kappa_\alpha\frac{1}{\sigma_\alpha} + \left(\frac{1}{\sigma_\alpha}\psi_x + \left(1-\left(1-\lambda^R\right)\phi_x\right)\right)\beta \end{pmatrix} \begin{pmatrix} \mathbb{E}_t\left(x_{t+1}\right) \\ \mathbb{E}_t\left(\pi_{H,t+1}\right) \end{pmatrix} - ...$$
$$\Lambda_\lambda \begin{pmatrix} \left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\frac{1}{\sigma_\alpha} \\ \kappa_\alpha\frac{1}{\sigma_\alpha} \end{pmatrix} m_t,$$
$$m_t = \rho_m m_{t-1} + \epsilon_t^m,\ \epsilon_t^m \sim i.i.d.\left(0, \sigma_m^2\right),$$

or, simply,

$$x_t = \lambda^R\Lambda_\lambda\left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\mathbb{E}_t\left(x_{t+1}\right) + \lambda^R\Lambda_\lambda\frac{1}{\sigma_\alpha}\left(1-\psi_{\pi_H}\beta\right)\mathbb{E}_t\left(\pi_{H,t+1}\right) - \Lambda_\lambda\left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\frac{1}{\sigma_\alpha}m_t,$$
$$\pi_{H,t} = \lambda^R\Lambda_\lambda\kappa_\alpha\mathbb{E}_t\left(x_{t+1}\right) + \lambda^R\Lambda_\lambda\left(\kappa_\alpha\frac{1}{\sigma_\alpha} + \left(\frac{1}{\sigma_\alpha}\psi_x + \left(1-\left(1-\lambda^R\right)\phi_x\right)\right)\beta\right)\mathbb{E}_t\left(\pi_{H,t+1}\right) - \Lambda_\lambda\kappa_\alpha\frac{1}{\sigma_\alpha}m_t,$$
$$m_t = \rho_m m_{t-1} + \epsilon_t^m,\ \epsilon_t^m \sim i.i.d.\left(0, \sigma_m^2\right),$$

where $\Lambda_\lambda \equiv \frac{1}{\left(\frac{1}{\sigma_\alpha}\psi_x + (1-(1-\lambda^R)\phi_x)\right)\left(1-(1-\lambda^R)\phi_{\pi_H}\beta\right) + \left(\frac{1}{\sigma_\alpha}\psi_{\pi_H} - (1-\lambda^R)\frac{\phi_{\pi_H}}{\sigma_\alpha}\right)\kappa_\alpha}$ while the forcing process m_t is characterized elsewhere in the paper.

Step 0. We replace the conjecture for $x_t = \frac{1}{\chi_0}\pi_{H,t}$ in each of the two expectational difference equations of the transformed core model so that we can express them in terms of inflation alone as,

$$\frac{1}{\chi_0}\pi_{H,t} = \lambda^R\Lambda_\lambda\left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\mathbb{E}_t\left(\frac{1}{\chi_0}\pi_{H,t+1}\right) + \lambda^R\Lambda_\lambda\frac{1}{\sigma_\alpha}\left(1-\psi_{\pi_H}\beta\right)\mathbb{E}_t\left(\pi_{H,t+1}\right) - ...$$
$$\Lambda_\lambda\left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\frac{1}{\sigma_\alpha}m_t,$$
$$\pi_{H,t} = \lambda^R\Lambda_\lambda\kappa_\alpha\mathbb{E}_t\left(\frac{1}{\chi_0}\pi_{H,t+1}\right) + \lambda^R\Lambda_\lambda\left(\kappa_\alpha\frac{1}{\sigma_\alpha} + \left(\frac{1}{\sigma_\alpha}\psi_x + \left(1-\left(1-\lambda^R\right)\phi_x\right)\right)\beta\right)\mathbb{E}_t\left(\pi_{H,t+1}\right) - \Lambda_\lambda\kappa_\alpha\frac{1}{\sigma_\alpha}m_t,$$

or, simply,

$$\pi_{H,t} = \lambda^R\Lambda_\lambda\left[\left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right) + \chi_0\frac{1}{\sigma_\alpha}\left(1-\psi_{\pi_H}\beta\right)\right]\mathbb{E}_t\left(\pi_{H,t+1}\right) - \chi_0\Lambda_\lambda\left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\frac{1}{\sigma_\alpha}m_t,$$
$$\pi_{H,t} = \lambda^R\Lambda_\lambda\left[\kappa_\alpha\left(\frac{1}{\chi_0}+\frac{1}{\sigma_\alpha}\right) + \left(\frac{1}{\sigma_\alpha}\psi_x + \left(1-\left(1-\lambda^R\right)\phi_x\right)\right)\beta\right]\mathbb{E}_t\left(\pi_{H,t+1}\right) - \Lambda_\lambda\kappa_\alpha\frac{1}{\sigma_\alpha}m_t.$$

Step 1. We replace the conjecture for $\pi_{H,t} = \chi_1 \pi_{H,t-1} + \eta_t$, $\eta_t \sim i.i.d.\left(0, \sigma_\eta^2\right)$ to express the PPI inflation expectations in terms of current PPI inflation,

$$\begin{aligned}
\pi_{H,t} &= \lambda^R \Lambda_\lambda \left[\left(1 - \left(1-\lambda^R\right)\phi_{\pi_H}\beta\right) + \chi_0 \frac{1}{\sigma_\alpha}\left(1 - \psi_{\pi_H}\beta\right) \right] \chi_1 \pi_{H,t} - \chi_0 \Lambda_\lambda \left(1 - \left(1-\lambda^R\right)\phi_{\pi_H}\beta\right) \frac{1}{\sigma_\alpha} m_t, \\
\pi_{H,t} &= \lambda^R \Lambda_\lambda \left[\kappa_\alpha \left(\frac{1}{\chi_0} + \frac{1}{\sigma_\alpha} \right) + \left(\frac{1}{\sigma_\alpha}\psi_x + \left(1 - \left(1-\lambda^R\right)\phi_x\right) \right)\beta \right] \chi_1 \pi_{H,t} - \Lambda_\lambda \kappa_\alpha \frac{1}{\sigma_\alpha} m_t,
\end{aligned}$$

and then re-write the system of equations as follows,

$$\begin{aligned}
\pi_{H,t} &= -\left[\frac{\chi_0 \Lambda_\lambda \left(1 - \left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\frac{1}{\sigma_\alpha}}{1 - \lambda^R \Lambda_\lambda \left[\left(1 - \left(1-\lambda^R\right)\phi_{\pi_H}\beta\right) + \chi_0 \frac{1}{\sigma_\alpha}\left(1 - \psi_{\pi_H}\beta\right) \right] \chi_1} \right] m_t, \\
\pi_{H,t} &= -\left[\frac{\Lambda_\lambda \kappa_\alpha \frac{1}{\sigma_\alpha}}{1 - \lambda^R \Lambda_\lambda \left[\kappa_\alpha \left(\frac{1}{\chi_0} + \frac{1}{\sigma_\alpha} \right) + \left(\frac{1}{\sigma_\alpha}\psi_x + \left(1 - \left(1-\lambda^R\right)\phi_x\right) \right)\beta \right] \chi_1} \right] m_t, \\
m_t &= \rho_m m_{t-1} + \epsilon_t^m,\ \epsilon_t^m \sim i.i.d.\left(0, \sigma_m^2\right).
\end{aligned}$$

Step 2. We replace the linear mappings between PPI inflation and the monetary shock into the monetary shock process in order to obtain that,

$$\begin{aligned}
\pi_{H,t} &= \rho_m \pi_{H,t-1} - \left[\frac{\chi_0 \Lambda_\lambda \left(1 - \left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\frac{1}{\sigma_\alpha}}{1 - \lambda^R \Lambda_\lambda \left[\left(1 - \left(1-\lambda^R\right)\phi_{\pi_H}\beta\right) + \chi_0 \frac{1}{\sigma_\alpha}\left(1 - \psi_{\pi_H}\beta\right) \right] \chi_1} \right] \epsilon_t^m,\ \epsilon_t^m \sim i.i.d.\left(0, \sigma_m^2\right), \\
\pi_{H,t} &= \rho_m \pi_{H,t-1} - \left[\frac{\Lambda_\lambda \kappa_\alpha \frac{1}{\sigma_\alpha}}{1 - \lambda^R \Lambda_\lambda \left[\kappa_\alpha \left(\frac{1}{\chi_0} + \frac{1}{\sigma_\alpha} \right) + \left(\frac{1}{\sigma_\alpha}\psi_x + \left(1 - \left(1-\lambda^R\right)\phi_x\right) \right)\beta \right] \chi_1} \right] \epsilon_t^m,\ \epsilon_t^m \sim i.i.d.\left(0, \sigma_m^2\right).
\end{aligned}$$

Step 3. We then apply the method of matching coefficients to equate the two formulas obtained in the previous step with the conjecture above for the dynamics of PPI inflation, imposing at the same time enough restrictions to ensure that both solutions are identical,

$$\begin{aligned}
&\chi_1 = \rho_m, \\
&\left(\frac{\chi_0 \Lambda_\lambda \left(1 - \left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\frac{1}{\sigma_\alpha}}{1 - \lambda^R \Lambda_\lambda \left[\left(1 - \left(1-\lambda^R\right)\phi_{\pi_H}\beta\right) + \chi_0 \frac{1}{\sigma_\alpha}\left(1 - \psi_{\pi_H}\beta\right) \right] \rho_m} \right) \\
&\qquad = \left(\frac{\Lambda_\lambda \kappa_\alpha \frac{1}{\sigma_\alpha}}{1 - \lambda^R \Lambda_\lambda \left[\kappa_\alpha \left(\frac{1}{\chi_0} + \frac{1}{\sigma_\alpha} \right) + \left(\frac{1}{\sigma_\alpha}\psi_x + \left(1 - \left(1-\lambda^R\right)\phi_x\right) \right)\beta \right] \rho_m} \right), \\
&\eta_t = -\left(\frac{\Lambda_\lambda \kappa_\alpha \frac{1}{\sigma_\alpha}}{1 - \lambda^R \Lambda_\lambda \left[\kappa_\alpha \left(\frac{1}{\chi_0} + \frac{1}{\sigma_\alpha} \right) + \left(\frac{1}{\sigma_\alpha}\psi_x + \left(1 - \left(1-\lambda^R\right)\phi_x\right) \right)\beta \right] \rho_m} \right) \epsilon_t^m.
\end{aligned}$$

The implicit formula for χ_0 comes down to,

$$\chi_0 = \begin{cases} \frac{\kappa_\alpha}{\left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\left(1 - \lambda^R \Lambda_\lambda \rho_m \left(\kappa_\alpha \frac{1}{\sigma_\alpha} + \left(\frac{1}{\sigma_\alpha}\psi_x + \left(1-\left(1-\lambda^R\right)\phi_x\right)\right)\beta\right)\right) + \lambda^R \Lambda_\lambda \rho_m \kappa_\alpha \frac{1}{\sigma_\alpha}\left(1 - \psi_{\pi_H}\beta\right)}, & \text{if } 0 \leq \lambda^R < 1, \\ \frac{\kappa_\alpha}{1 - \Lambda_\lambda \left[1 + \frac{1}{\sigma_\alpha}\left(\kappa_\alpha \psi_{\pi_H} + \psi_x\right)\right]\beta\rho_m}, & \text{if } \lambda^R = 1. \end{cases}$$

Then, it holds that,

$$\left(\frac{\frac{\Lambda_\lambda \kappa_\alpha \frac{1}{\sigma_\alpha}}{1-\lambda^R\Lambda_\lambda\left[\kappa_\alpha\left(\frac{1}{\chi_0}+\frac{1}{\sigma_\alpha}\right)+\left(\frac{1}{\sigma_\alpha}\psi_x+(1-(1-\lambda^R)\phi_x)\right)\beta\right]\rho_m}\right)$$
$$=\left(\frac{\frac{\Lambda_\lambda \kappa_\alpha \frac{1}{\sigma_\alpha}}{1-\lambda^R\Lambda_\lambda\left[\left(1-(1-\lambda^R)\phi_{\pi_H}\beta\right)(1-\lambda^R\Lambda_\lambda\rho_m\Phi_\lambda)+\Phi_\lambda+\lambda^R\Lambda_\lambda\rho_m\kappa_\alpha\frac{1}{\sigma_\alpha}\left(1-\psi_{\pi_H}\beta\right)\right]\rho_m}\right),$$

where $\Phi_\lambda \equiv \kappa_\alpha \frac{1}{\sigma_\alpha} + \left(\frac{1}{\sigma_\alpha}\psi_x + \left(1-\left(1-\lambda^R\right)\phi_x\right)\right)\beta$. Hence, we can summarize the results as follows,

$$\begin{aligned}
\chi_0 &= \frac{\kappa_\alpha}{\left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\left(1-\lambda^R\Lambda_\lambda\rho_m\Phi_\lambda\right)+\lambda^R\Lambda_\lambda\rho_m\kappa_\alpha\frac{1}{\sigma_\alpha}\left(1-\psi_{\pi_H}\beta\right)},\\
\chi_1 &= \rho_m,\\
\eta_t &= -\left(\frac{\Lambda_\lambda\kappa_\alpha\frac{1}{\sigma_\alpha}}{1-\lambda^R\Lambda_\lambda\left[\left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\left(1-\lambda^R\Lambda_\lambda\rho_m\Phi_\lambda\right)+\Phi_\lambda+\lambda^R\Lambda_\lambda\rho_m\kappa_\alpha\frac{1}{\sigma_\alpha}\left(1-\psi_{\pi_H}\beta\right)\right]\rho_m}\right)\epsilon_t^m.
\end{aligned}$$

Notice that $\mathbb{E}\left[\eta_t\right]=0$ and

$$\sigma_\eta^2 \equiv \mathbb{V}\left[\eta_t\right] = \left(\frac{\Lambda_\lambda\kappa_\alpha\frac{1}{\sigma_\alpha}}{1-\lambda^R\Lambda_\lambda\left[\left(1-\left(1-\lambda^R\right)\phi_{\pi_H}\beta\right)\left(1-\lambda^R\Lambda_\lambda\rho_m\Phi_\lambda\right)+\Phi_\lambda+\lambda^R\Lambda_\lambda\rho_m\kappa_\alpha\frac{1}{\sigma_\alpha}\left(1-\psi_{\pi_H}\beta\right)\right]\rho_m}\right)^2\sigma_m^2,$$

where σ_m^2 is the variance of the underlying monetary policy shock innovation.