Global Price Shocks and International Monetary Coordination*

Veronica Guerrieri, Guido Lorenzoni, and Iván Werning

January 2025

When the central bank of an individual country is trying to contain domestic inflation, it mostly takes as given global supply conditions—which determine oil prices, prices of tradable intermediate goods, bottlenecks in global supply chains, and so on. However, the combined actions of all central banks matter for global demand, which matters for how global supply conditions translate into price and non-price pressures that tend to increase global inflation. This suggests the possibility that following a negative world supply shock, uncoordinated monetary policy may deliver excessively loose policy from a global perspective. We investigate this possibility in some simple models which explicitly model price determination in a global commodity market and show conditions under which there are gains from coordination.

1 Introduction

The inflation surge of 2021-2022 had two visible features: it had a global nature, as it played out in fairly similar way in many advanced and emerging economies, and it was strongly associated to symptoms of scarcity in the global supply of tradable goods, as captured, for example, by measures of supply chain pressures, increases in shipping costs, and, of course, spikes in energy prices.

These two observations have led many economists to view inflation as driven by global forces, mostly outside the control of individual central banks. However, if we think of the world as a whole, we need to recognize that global scarcity is always relative, in other words, it is always a symptom that supply is insufficient given the level of global aggregate demand. And global aggregate demand is eventually the outcome of the combined choices of individual countries' fiscal and monetary authorities. Each of these national authorities may take as given the level of world commodity prices and bottlenecks in the global supply of tradable intermediates. But their aggregate decisions contribute to determine these prices and the tightness of these supply constraints.

*Thanks...

A natural question is whether, in these circumstances, uncoordinated policy responses around the world will produce an optimal global response, or whether lack of coordination will produce a sub-optimal outcome.

A commonly held view, that goes back to the classic work of Canzoneri and Henderson (1991), is that in the fight against global inflation uncoordinated policy choices will lead to a contractionary bias. The argument is that each domestic central bank will have an incentive to tighten policy because tight policy would lead to a domestic currency appreciation, which reduces the domestic currency price of imports, and thus contributes to disinflation. But since all currencies cannot appreciate at the same time, this is a zero-sum game that only leads to excess tightening.

In this paper we revisit this conventional wisdom and show that, once we take into account the role of global commodity markets and other globally determined input prices, the conclusion above can be turned around, and lack of policy coordination may lead to excessively loose monetary policy at the global level.

The crucial ingredient in our exercise is an explicit model of a global commodity market in which the global price of the scarce commodity is determined. Another essential piece of the model, as we shall see, is the assumption of some form of wage rigidity, which matters because it introduces a trade-off between inflation stabilization and output gap stabilization.

Paper organization In Sections 2 to 5 we develop our argument in the context of the simplest model, with a homogenous tradable good, identical countries, and linear preferences in tradables. This model help us identify our externality in a transparent way because, due to the homogenous tradable good, we can abstract from term-of-trade manipulation motives.

In Section 6, we consider models where terms of trade considerations play a more prominent role and we show that the logic of our baseline model is still at work. In Section 7 we explore models with asymmetries between countries (large and small, energy exporters and energy importers), to evaluate how these asymmetries affect the direction and size of the externality.

2 The World Economy

The world is made of a continuum of small, identical countries. Time is discrete and infinite. In each country, there is a representative consumer with utility function

$$\sum_{t=0}^{\infty} \beta^t U(C_{Tt}, C_{Nt}, L_t),$$

where

$$U(C_{Tt}, C_{Nt}, L_t) = C_{Tt} + u(C_{Nt}) - v(L_t).$$

The variablesC_{Tt} and C_{Nt} are, respectively, consumption of tradable and non-tradable goods, and L_t is labor supply. The assumption of linear utility for tradable consumption simplifies the analysis but is in no way crucial for our argument. For ease of notation, we omit adding a subscript to denote each individual country. We focus on symmetric equilibria, so this will not cause confusion.

The representative consumer in each country receives an endowment $Y_T = 1$ of tradable goods and an endowment $\bar{X}_t$ of a commodity needed for production. To be concrete, we refer to this commodity as "oil", but we have in mind a broader set of inputs into production.

The model features no explicit treatment of uncertainty. We study a deterministic economy, we characterize the steady state, and then we introduce a one-time, unexpected, temporary shock to the supply of oil $\bar{X}_t$. The shock affects symmetrically all countries, so both in steady state and after the shock, in equilibrium, countries will not trade oil in the world market. But the world market is open and competitive, so each country will take the oil price as given and, in contemplating deviations from equilibrium behavior, each country's central bank will assume that the oil price is unaffected by its policy.

Non-tradable goods come in a continuum of varieties $[0, 1]$ that are aggregated according to a constant elasticity of substitution function with elasticity $\varepsilon > 1$,

$$C_{Nt} = \left(\int_0^1 C_{Nti}^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Each variety is produced by a firm with production function

$$Y_{Nit} = F\left(L_{it}, X_{it}\right),$$

where L_{it} is the labor input and X_{it} the oil input. The fact that oil enters the production of non-tradables and not of tradables is a simplification due to the fact that we assumed a tradable endowment, but it is not essential for our results. The function F has constant

returns to scale. The government of each country pays firms a proportional subsidy $\frac{\varepsilon}{\varepsilon-1}$ that exactly offsets the monopolistic distortion, financed with a lump-sum tax on the representative consumer. Given that there are no terms-of-trade effects, this subsidy is set at its individually optimal level for a benevolent fiscal authority in each country.

Each country has its own currency, there are floating exchange rates, and each country runs monetary policy independently. Prices in local currency for tradable goods and oil have perfectly flexible prices P_{Tt} and P_{Xt}.

Non-tradable goods prices are subject to a nominal rigidity which works as follows. In each period, a fraction λ of firms in the non tradable sector set their price at the beginning of the period, before the aggregate shock is realized, and $1-\lambda$ set their price after the aggregate shock is realized. The nominal wage rate is W_t. For wages, we consider two extreme cases: flexible wages and perfectly rigid nominal wage. Our focus in on the latter, but the former provides a point of comparison.

The budget constraint of the representative agent in each country is

$$P_{Tt}C_{Tt} + P_{Nt}C_{Nt} + P_{Xt}X_t = P_{Tt}\bar{Y}_T + W_tL_t + P_{Xt}\bar{X}_t + T_t,$$

where T_t includes any profits from monopolistic firms and transfers from the government sector.

The law of one price applies to tradable consumption goods and oil, so that

$$P_{Tt} = E_tP^W_{Tt},$$
$$P_{Xt} = E_tP^W_{Xt},$$

where P^W_{Tt} and P^W_{Xt} are world prices of tradable goods and oil, denominated in the world unit of account, and E_t is the nominal exchange rate, the price of the world unit of account in terms of local currency. We take the world unit of account to be a basket made of all countries' currencies, with equal weights.

We focus on symmetric equilibria where, in equilibrium, all currencies trade at the same exchange rate $E_t = 1$. But when domestic central banks choose policy, they will in general consider off-equilibrium policy deviations that would cause the exchange rate of their individual country to be different from 1.

3 Equilibrium and Policy Regimes

Despite the dynamic setting, the equilibrium can be studied period by period, so whenever there is no risk of confusion we drop time subscripts.

Assume that the world economy starts with all countries at a steady state with zero inflation, which, given our assumptions reaches the first best allocation. We normalize units so that in this steady state all prices, including the wage rate, are equal to 1.

Our objective is to study what happens when the world is hit by an completely unexpected, aggregate, temporary shock to the oil endowment $\bar{X}$.

Price setting Let $\tilde{P}_N$ denote the price of firms in the non-tradable sector who can adjust. Given the government subsidy to monopolistic non-tradable firms, the price $\tilde{P}_N$ is given by

$$\tilde{P}_N = \mathcal{M}(W, P_X), \tag{1}$$

where $\mathcal{M}$ is the marginal cost function defined as

$$\mathcal{M}\left(W, P_X\right) = \min\left\{WL + P_X X : F\left(L, X\right) = 1\right\}.$$

The firms who cannot adjust assign zero probability to the aggregate shock and set their price to the steady state value of 1.

Aggregating the prices of adjusters and non-adjusters, the price index for non-tradable goods is

$$P_N = \left(\lambda + (1-\lambda)\tilde{P}_N^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}. \tag{2}$$

All firms use the same oil-to-labor ratio in equilibrium. Therefore, we can aggregate the input demands of all firms, and obtain the following aggregate resource constraint, which includes the distortionary effects of price dispersion, captured by the function $\Delta\left(P_N\right)$,

$$\Delta\left(P_N\right) \cdot C_N = F(L, X). \tag{3}$$

Here we allow the country's oil consumption X to be in general different from $\bar{X}$ as we want to study countries' potential deviations from the symmetric equilibrium. The dis-

tortion function $\Delta(P_N)$ can be derived following standard steps[1] and is

$$\Delta(P_N) = P_N^{\varepsilon}\left[\lambda + (1-\lambda)^{\frac{1}{1-\varepsilon}}\left(P_N^{1-\varepsilon} - \lambda\right)^{-\frac{\varepsilon}{1-\varepsilon}}\right].$$

When $P_N = 1$ there is zero inflation, all firms sell at the same price, and $\Delta(P_N) = 1$. When $P_N \neq 1$ we have $\Delta(P_N) > 1$, as inputs are misallocated across firms selling at different prices. This is the traditional distortionary effect of inflation in sticky price models.

Equilibrium Conditions Four more equations complete our characterization of an equilibrium within a period.

Optimal demand for tradable and non-tradable goods implies

$$u'(C_N) = \frac{P_N}{P_T}. \tag{4}$$

The optimal factor mix in non-tradable production implies

$$\frac{F_X(L,X)}{F_L(L,X)} = \frac{P_X}{W}. \tag{5}$$

The country consolidated budget constraint is

$$C_T = 1 - Q\left(X - \bar{X}\right), \tag{6}$$

where Q is the world relative price of oil, expressed in terms of tradable goods

$$Q = \frac{P_X^W}{P_T^W}.$$

Here, we are assuming that whenever a country buys oil in excess of its endowment, it pays for it by reducing the consumption of tradable goods in the same period. In general a country could borrow and pay partly in the future, but given linear preferences the timing

[1] For a given level of consumption of non-tradables C_N, the demand for sticky goods is $(1/P_N)^{-\varepsilon} C_N$ and the demand for flexible goods is $\left(\tilde{P}_N/P_N\right)^{\varepsilon} C_N$, so total demand is

$$\left[\lambda\left(\frac{1}{P_N}\right)^{-\varepsilon} + (1-\lambda)\left(\frac{\tilde{P}_N}{P_N}\right)^{-\varepsilon}\right] C_N.$$

Inverting condition (2) gives

$$\tilde{P}_N = \left(\frac{P_N^{1-\varepsilon} - \lambda}{1-\lambda}\right)^{\frac{1}{1-\varepsilon}}.$$

Substituting in the previous expression gives the expression for $\Delta(P_N)$ in the text.

of oil payments does not matter. It is easy to extend the model to the case of non-linear, separable utility in tradable goods, in which countries will optimally spread payments over all future periods. All our results below go through in that environment.

The law of one price implies that the relative price of oil in tradable goods is equal in all countries and equal to Q,

$$\frac{P_X}{P_T} = Q. \tag{7}$$

We look at two cases for wages. For most of the analysis, we assume rigid nominal wages

$$W = 1.$$

But we will also consider the case of flexible wages, satisfying the optimal labor supply condition

$$u'(C_N)\frac{W}{P_N} = v'(L).$$

Equilibrium definitions Let us focus on the rigid wage case. We have seven equilibrium conditions (1)-(7) and eight endogenous variables

$$\left(C_N, C_T, L, X, P_T, \tilde{P}_N, P_N, P_X\right),$$

so the central bank has one degree of freedom. Throughout the paper, we will describe the central bank's problem in terms of choosing C_N and letting equations (1)-(7) determine all other variables.

In our formal analysis, we describe monetary policy simply in terms of choosing C_N and we do not explicitly keep track of the nominal exchange rate E, but it is perfectly equivalent to let the central bank choose the nominal interest rate and use the consumer Euler equation and the UIP condition to derive endogenously the values of the exchange rate E and of consumption C_N. In the derivations below, we discuss what happens to the nominal exchange rate where it helps to give intuition for the results.

Assume that the domestic central bank in each country is benevolent and maximizes the utility of the representative consumer.

We compare two regimes for international monetary policy coordination. In the "no coordination" regime, each central banks acts independently and our equilibrium notion is the Nash equilibrium of the game in which each central bank chooses C_N, taking as given the choice of all other central banks. In the "coordination" regime, a world planner chooses the same level of C_N for all countries, to maximize the utility of the representative world consumer.

In the case of no coordination, since the countries are all identical, we focus on a symmetric Nash equilibrium.

Definition 1. *(No coordination, Nash equilibrium)* A symmetric Nash equilibrium is a vector

$$\left(C_N, C_T, L, X, P_T, \tilde{P}_N, P_N, P_X\right)$$

that satisfies two conditions:

1. *Country's optimization.* Prices and quantities satisfy the equilibrium conditions (1)-(7) and, among all vectors of prices and quantities that satisfy those conditions, they maximize the utility of the representative household in each country, taking Q as given.

2. *Market clearing in the world oil market.* The price Q is such that $X = \bar{X}$.

In the case of coordination, we look at prices and quantities chosen by a social planner that internalizes the market clearing condition in the world oil market. Since all countries are identical, the global social planner chooses identical prices and quantities in all countries.

Definition 2. *(Coordination)* An equilibrium with coordination is a vector

$$\left(C_N, C_T, L, X, P_T, \tilde{P}_N, P_N, P_X, Q\right)$$

that maximizes the utility of the representative household, taking as given conditions (1)-(7), plus the condition $X = \bar{X}$.

The crucial difference is that Q is taken as given in the no coordination case, while it is a choice variable in the coordination case.

4 Phillips Curves at the Country and World Level

In this section we derive Phillips curves, i.e., equilibrium relations between real activity and inflation. First we do it at the country level, taking the world oil price as given. Next, we do it at the world level, taking the world oil supply as given and taking into account the endogenous determination of the world oil price. This allows us to show in what cases the Phillips curve perceived by an individual country is flatter than the world Phillips curve, a condition that will play a crucial role in the normative analysis.

4.1 Country Phillips curve

The world economy is in steady state, with constant endowments of tradables and oil and constant nominal prices (zero inflation). Consider the effect of a temporary negative shock to oil supply that reduces oil supply by $\bar{x} < 0$. All lowercase variables denote log deviations from the initial steady state. Suppose for now that the effect of the shock is to increase the world oil price by $q > 0$. We linearize the model near its steady state and use π_N to denote non-tradable inflation in the period in which the shock occurs.[2] Derivations in Appendix A.1 show that the central bank of any individual country faces the country-level Phillips curve

$$\pi_N = \frac{(1-\lambda)\, s_X}{1-(1-\lambda)\, s_X}\left(\frac{1}{\epsilon} c_N + q\right), \tag{8}$$

where ϵ is the steady-state elasticity of substitution between tradable and non-tradable goods, and s_X is the steady-state share of oil in total costs. Consumption in the non-tradable sector is our measure of aggregate activity because tradable output is a fixed endowment. We focus on inflation in the non-tradable sector because tradable inflation does not have distortionary effects in our environment. It is easy to obtain a similar relation between c_N and inflation in the consumer price index (CPI), which includes both tradables and non-tradables.[3]

From the point of view of the individual country, the oil supply shock $q > 0$ causes an upward shift of the Phillips curve, outside the control of the domestic central bank. The central bank can partly or completely offset the effect of the oil shock on inflation, by accepting a contraction in domestic activity $c_N < 0$.

Why does a contraction in domestic activity help contain inflation? Since nominal wages are fixed, the labor market channel is muted here. The only reason why monetary policy is effective is through the exchange rate channel. Contractionary monetary policy produces an appreciation of the domestic currency, which makes oil cheaper since $P_X = EP_X^W$. This reduces nominal marginal costs and lowers non-tradable price inflation. The price of traded goods P_T also falls, and it falls in proportion to the nominal appreciation, due to $P_T = EP_T^W$. Because of wage rigidities and price stickiness, non-tradable prices fall by less than tradable prices, so non-traded goods become relatively more expensive.

[2] Given our assumptions and notational conventions, $\pi_N = p_N$.

[3] Just combine (8) and the following relation between CPI inflation π and non-tradable inflation π_N:

$$\pi = \omega \frac{1}{\epsilon} c_N + \pi_N,$$

where ω is the steady state share of spending in tradable goods.

In other words, the nominal appreciation causes a real appreciation. This relative price effect reduces non-tradable consumption. Therefore, the currency appreciation achieves lower inflation but reduces real activity.[4]

4.2 World Phillips curve

Turning to the world Phillips curve, it is useful to first take an intermediate step and ask how does contractionary monetary policy in a given economy, all else equal, affect that economy's oil consumption. Here there are two opposing effects. On the one hand, by reducing domestic consumption, contractionary policy reduces oil demand. On the other hand, by making oil cheaper in domestic currency it increases the quantity of oil demanded. The first effect dominates if the following inequality holds

$$\eta s_L < \epsilon \left[1 - (1-\lambda) s_X\right], \tag{9}$$

where η is the steady-state elasticity of substitution between labor and oil and $s_L = 1 - s_X$ is the steady-state labor share in total costs. The inequality holds if the elasticity of substitution η is low enough, which seems the empirically relevant configuration.[5]

Let $X = \mathcal{X}(C_N, Q)$ denote the consumption of oil of a country as a function of its level of aggregate activity, measured in terms of non-tradable consumption C_N, and of the world price of oil. This mapping is derived from equations (1)-(7). The following result formalizes the result in the previous paragraph. Throughout the paper we use a subscript to denote the partial derivative of a function, e.g., $\mathcal{X}_{C_N}$ is the same as $\partial \mathcal{X} / \partial C_N$.

Proposition 1. *The elasticity of domestic oil consumption to domestic activity C_N, at the steady state allocation, is*

$$\mathcal{X}_{C_N}(C_N, Q) \frac{C_N}{X} = 1 - \frac{\eta}{\epsilon} \frac{s_L}{1 - (1-\lambda) s_X},$$

and is positive if and only if inequality (9) holds.

[4]The exchange rate elasticity of P_T is 1, while the exchange rate elasticity of P_N is $(1-\lambda) s_X < 1$. The elasticity of the relative price P_N / P_T is then $(1-\lambda) s_X - 1 < 0$, and that of non-tradable consumption $\epsilon (1 - (1-\lambda) s_X) > 0$. Therefore, a 1% nominal appreciation gives a $(1-\lambda) s_X \cdot 1\%$ reduction in non-tradable inflation at the cost of a $(1 - (1-\lambda) s_X) \epsilon \cdot 1\%$ reduction in non-tradable consumption. This gives an exact intuition for the slope of the Phillips curve 8.

An equivalent (but algebraically more cumbersome) way of reaching this result is to derive the domestic interest rate increase necessary to obtain the same exchange rate appreciation and to use the consumer Euler equation to derive the effect on consumer spending.

[5]Following up on the intuition in footnote 4, the reduction in consumption caused by a 1% appreciation is $(1 - (1-\lambda) s_X) \epsilon \cdot 1\%$, and it causes a proportional reduction in oil demand. The price effect of the appreciation, due to substitution between oil and labor, is $\eta s_L \cdot 1\%$. Therefore, the two terms on the right-hand side of the inequality capture exactly the two effects described in the text.

From an individual country perspective, under assumption (9), a contraction in C_N leads to a contraction in the country's oil demand X. If a single country chooses a lower C_N, since all countries are atomistic, the effect on world oil demand is negligible. From a world general equilibrium perspective though, if all countries contract spending at the same time, the price of oil needs to decrease to keep the oil market in equilibrium. We formalize this observation in the last part of the next proposition, which is proved in Appendix A.1. From the reasoning avobe, the same inequality (9) that determines the sign of the response of X to a local change in C_N, determines the general equilibrium response of the price Q to a world-level change in C_N.

Let $Q = \mathcal{Q}(\bar{X}, C_N)$ denote the general equilibrium mapping that gives the world oil price as a function of oil supply $\bar{X}$ and world non-tradable spending C_N, assuming all countries choose the same C_N.

Proposition 2. *At the zero-inflation steady state, the function $\mathcal{Q}$ satisfies*

$$\mathcal{Q}_{\bar{X}}(\bar{X}, C_N)\frac{\bar{X}}{Q} = \zeta_{\bar{X}} = -\frac{1}{\eta}\frac{1-(1-\lambda)s_X}{s_L},$$
$$\mathcal{Q}_{C_N}(\bar{X}, C_N)\frac{C_N}{Q} = \zeta_{C_N} = \frac{1}{\eta}\frac{1-(1-\lambda)s_X}{s_L} - \frac{1}{\epsilon}.$$

The coefficient $\zeta_{\bar{X}}$ is negative. The coefficient ζ_{C_N} is positive iff inequality (9) holds.

It follows that when a global supply shock hits, the oil price response, conditional on a given response of world spending c_N, can be written as follows, in terms of log-deviations from the initial steady state:

$$q = \zeta_{\bar{X}}\bar{x} + \zeta_{C_N}c_N.$$

Substituting the last equation in the country-level Phillips curve (8), the world Phillips curve takes the form:

$$\pi_N = \frac{(1-\lambda)s_X}{1-(1-\lambda)s_X}\left[\left(\frac{1}{\epsilon} + \zeta_{C_N}\right)c_N + \zeta_{\bar{X}}\bar{x}\right], \tag{10}$$

and we obtain the following result, which, once again, depends on our inequality (9).

Proposition 3. *The world Phillips curve is steeper than the country-level Phillips curve iff $\zeta_{C_N} > 0$, that is, if inequality (9) holds.*

In other words, whenever the world oil price is an increasing function of consumer spending in all countries, the world Phillips curve is steeper. The result is intuitive: when (9) holds, by running a coordinated world monetary expansion, world oil demand increases, and we end up with a higher oil price in general equilibrium which causes higher world inflation.

The results so far are illustrated in Figure 1, where we plot country-level Phillips curves and the world Phillips curve in a case with $\zeta_{C_N} > 0$. Country-level Phillips curves are indexed by the level of world demand c_N^W, which determines q: higher world demand pushes up the country-level curve. When countries act in coordination, c_N and c_N^W move together and we move along the world Phillips curve shown in the figure.

Simply inspecting Figure 1, one would be tempted to immediately conclude that in the uncoordinated Nash equilibrium countries tend to underestimate the disinflationary benefit of a contraction in spending, leading to excessive inflation from a world planner point of view. However, as we will see in the next section, when we consider carefully welfare, the objectives of the individual country and of the social planner take a different form, so it will take a bit more work to figure out under what conditions we get excess inflation.

5 Nash Equilibrium and the Gains from Coordination

In this section, we analyze the decision problem of the central bank of a given country and characterize equilibria with and without coordination. This will give us conditions for the uncoordinated equilibrium to display an excessive level of real activity and inflation.

5.1 No coordination

Using the equilibrium conditions (1)-(7), we can express the levels of domestic endogenous variables in any given individual country as functions of the level of local activity C_N and of the world oil price Q. We already introduced the mapping $X = \mathcal{X}(C_N, Q)$ constructed in this way for oil consumption. In this section, we will also need the following analogous mappings for employment and the price of non-tradables:

$$L = \mathcal{L}(C_N, Q), \qquad P_N = \mathcal{P}_N(C_N, Q).$$

In a world without coordination, the domestic central bank's best response, that is, the level of activity C_N chosen for a given world oil price Q, is characterized compactly as the solution to the following problem:

$$\mathcal{C}_N(Q) = \arg\max_{C_N} U\left(1 - Q \cdot \left(\mathcal{X}(C_N, Q) - \bar{X}\right), C_N, \mathcal{L}(C_N, Q)\right). \tag{11}$$

Substituting this best response in the oil demand function, and imposing market clearing

in the world oil market, gives

$$\mathcal{X}\left(\mathcal{C}_N\left(Q\right), Q\right) = \bar{X}.$$

These last two conditions fully characterize a Nash equilibrium.

To further analyze the tradeoff faced by the individual central bank, let us derive the first order condition of the maximization problem in (11). Omitting the functions' arguments for ease of notation, and, again, using subscripts to denote partial derivatives, we have

$$U_{C_N} + U_L \cdot \mathcal{L}_{C_N} - Q \cdot U_{C_T} \cdot \mathcal{X}_{C_N} = 0.$$

To interpret this expression and similar expressions to come, it is useful to recall that $U_L < 0$, capturing the marginal disutility of labor.

If we substitute for $\mathcal{L}_{C_N}$ in the expression above, we obtain the following decomposition that is easier to interpret (the necessary steps are in the appendix):

$$\underbrace{\left(U_{C_N} + \frac{\Delta}{F_L} U_L\right)}_{\text{Labor wedge}} + \underbrace{U_L \frac{\Delta'\left(P_N\right) C_N}{F_L}}_{\text{Inflation distortion}} \cdot \mathcal{P}_{N,C_N} + \underbrace{\left(-U_L \frac{F_X}{F_L} - Q \cdot U_{C_T}\right)}_{\text{Oil wedge}} \cdot \mathcal{X}_{C_N} = 0. \tag{12}$$

The first two terms are standard. The first captures a notion of output gap. When it is positive, there is a negative output gap and increasing output is welfare improving as the marginal utility of consumption exceeds the marginal labor cost. The second term captures the welfare cost of inflation, due to its distortionary effect captured by Δ. With positive inflation the expression $\Delta'\left(P_N\right)$ is positive: increasing output increases inflation distortions (the partial derivative $\mathcal{P}_{N,C_N}$ is always positive).

The last term is novel and captures the gap between the domestic social benefits of higher oil consumption and the cost of importing oil. Suppose we are in the case discussed in Proposition (1), where an expansion causes oil imports to increase, $\mathcal{X}_{C_N} > 0$. Oil imports are socially beneficial, as more oil reduces the need to use of labor in production in proportion to the marginal rate of substitution between oil and labor F_X/F_L. The cost of oil is given by the loss of tradable consumption, as the country needs to pay for the oil. The difference between these two expressions captures the net social benefit of stimulating oil demand through expansionary policy. We call this term the "oil wedge". The sign of this wedge can in general be positive or negative.

Wage rigidity plays an important role in our analysis, as we shall see shortly. We already see its importance here: with flexible wages the oil wedge is always zero. With flexible wages the real wage in tradable goods W/P_T is equalized to the ratio $-U_L/U_{C_T}$ by workers' optimality. Since the ratio of the factor prices QP_T/W is equal to the factors'

marginal rate of substitution by firms' optimality, we get

$$-\frac{U_L}{U_{C_T}} = \frac{W}{P_T} = \frac{F_L}{F_X}Q,$$

which immediately implies a zero oil wedge.

By a similar line of reasoning, when wages are rigid, the oil wedge in our model always has the opposite sign of the expression:

$$\frac{W}{P_T} + \frac{U_L}{U_{C_T}},$$

which is the distance between the equilibrium real wage and its' flexible price counterpart. When this expression is positive firms perceive an over-priced labor input and tend to use too much oil from a social perspective. In this case, the oil wedge is negative and containing oil consumption is socially beneficial from the central bank's perspective.

5.2 Coordination and inefficiency

Let's turn to the case of coordinated macroeconomic policies.

The world social planner solves the same problem of an individual country's central bank, but with two differences: the quantity of oil used in the economy cannot be changed through imports, and the oil price is endogenously determined. This leads to the following problem

$$\max_{C_N, Q} \left\{ U\left(1, C_N, \mathcal{L}\left(C_N, Q\right)\right) | Q = \mathcal{Q}\left(\bar{X}, C_N\right) \right\}.$$

The fact that we have $C_T = 1$ instead of $C_T = 1 - Q \cdot (X - \bar{X})$ captures the fact that the world as a whole cannot import oil.

Proceeding as in the case of the individual country, the planner first order condition is

$$U_{C_N} + U_L \cdot \mathcal{L}_{C_N} + U_L \cdot \mathcal{L}_Q \cdot \mathcal{Q}_{C_N} = 0$$

which, after substituting for $\mathcal{L}_{C_N}$ and $\mathcal{L}_Q$ can be rewritten as follows (steps in the appendix):

$$\underbrace{\left(U_{C_N} + \frac{\Delta}{F_L}U_L\right)}_{\text{Labor wedge}} + \underbrace{U_L \frac{\Delta'\left(P_N\right)C_N}{F_L}}_{\text{Inflation distortion}} \cdot \left(\mathcal{P}_{N,C_N} + \mathcal{P}_{N,Q}\mathcal{Q}_{C_N}\right) = 0. \tag{13}$$

Comparing the optimality conditions (12) and (13) shows two differences between the world planner and the individual central bank.

First, individual countries fail to internalize the benefit of reducing aggregate spend-

ing on inflation, as they ignore the expression

$$\mathcal{P}_{N,Q}\mathcal{Q}_{C_N}$$

in the last term in parenthesis in (13). This expression captures exactly the different slope of the world Phillips curve, so the discussion in the previous section is immediately relevant. As shown in Proposition 3, condition (9) implies $\mathcal{P}_{N,Q}\mathcal{Q}_{C_N} > 0$ and countries underestimate the social benefit of world-wide restrictive policy.

However, there is a second difference between the world planner and country optimization, captured in the third term in (12), the oil wedge, which is absent in (13). Individual countries can increase oil imports, and due to price rigidites, the domestic planner can perceive a net positive or negative benefit of importing more oils.

In examples we'll show below, there is a negative oil wedge, which pushes the domestic central bank to be more contractionary in reaction to the oil shock. The logic of this negative wedge was discussed above: the real wage is above its natural level, so firms tend to import too much oil from a social welfare point of view, to substitute for labor. Contracting the economy is a way for the domestic central bank to partly correct for this bias, by reducing aggregate oil imports.

Whether the Nash equilibrium displays excessively expansionary policy or the opposite, depends on the sign and magnitude of the two differences just discussed, as shown in the following proposition.

Proposition 4. *(Excess world inflation) At the Nash equilibrium with no coordination, the level of real activity C_N is inefficiently high and world inflation is inefficiently high iff*

$$\underbrace{U_L \frac{\Delta'(P_N)\, C_N}{F_L}}_{\text{Inflation distortion}} \cdot \underbrace{\mathcal{P}_{N,Q}\mathcal{Q}_{C_N}}_{\text{Extra slope of the world P.C.}} < \underbrace{\left(-U_L \frac{F_X}{F_L} - Q \cdot U_{C_T}\right)}_{\text{Oil wedge}} \cdot \mathcal{X}_{C_N}.$$

When the inequality above holds, the marginal welfare effect of increasing inflationary distortions at the world level is more strongly negative than the oil wedge perceived by the individual central bank. A world planner would direct all countries to reduce output below its Nash equilibrium level.

5.3 Flexible prices and flexible wages

The combination of price and wage rigidities is crucial to produce a need for international coordination in our model. If we allow for either flexible prices or flexible wages the following propostion shows that in equilibrium the expression above is zero.

Proposition 5. *(Divine coincidence, no coordination benefits.) With either flexible prices or flexible wages, the Nash equilibrium yields the first best allocation and coincides with the world planner optimum.*

Notice that this result is inependent of the slopes of the world and country-level Phillips curves: both intersect at the natural allocation with zero inflation, so the slope of the trade off is irrelevant.[6]

5.4 Examples

[...]

6 A Model with Terms of Trade Manipulation

[...]

7 Asymmetries Across Countries

[...]

[6]Notice also that this result is only meaningful if some nominal rigidity is present in the model either on the price or wage side. If both prices and wages are perfectly flexible, monetary policy no longer has the power to affect the real allocation, so the analysis above does not apply.

A Appendix

A.1 First-order properties of the equilibrium

The system of equations (1)-(7) characterizes an equilibrium for given C_N and Q. In this section, we use differential notation to derive first-order properties of this system of equations. These properties will be used for two purposes:

1. Derive a linear approximation of the equilibrium near the initial steady state;
2. Derive the derivatives of the mappings $\mathcal{L}(C_N, Q)$ and $\mathcal{X}(C_N, Q)$ to analyze the efficiency properties of the equilibrium.

The Phillips curves in Section 4 are a side produce of point 1.

For a generic variable X, we use the following notation

$$X = X_o e^x,$$

so X_o denotes the steady state level and x log deviations from the steady state.

The labor share and the oil share are

$$s_L = \frac{LF_L(L,X)}{F(L,X)}, \quad s_X = \frac{XF_X(L,X)}{F(L,X)}. \tag{14}$$

Combining (1) and (2) gives

$$P_N^{1-\varepsilon} = \lambda + (1-\lambda)\left(\mathcal{M}(1, P_X)\right)^{1-\varepsilon}.$$

Differentiating both sides and rearranging we obtain

$$dp_N = h \cdot dp_X \tag{15}$$

where

$$h = (1-\lambda)\left(\frac{\tilde{P}_N}{P_N}\right)^{1-\varepsilon} \frac{P_X \mathcal{M}_{P_X}}{\mathcal{M}}.$$

Rearranging (2) we have

$$1 - \lambda P_N^{\varepsilon-1} = (1-\lambda)\left(\frac{\tilde{P}_N}{P_N}\right)^{1-\varepsilon},$$

and standard properties of the cost function imply

$$\frac{P_X \mathcal{M}_{P_X}}{\mathcal{M}} = s_X.$$

We can then rewrite h as a function of P_N and s_X:

$$h = \left(1 - \lambda P_N^{\varepsilon-1}\right) s_X. \tag{16}$$

Differentiating the resource constraint (3) gives

$$C_N \Delta' (P_N)\, dP_N + \Delta (P_N)\, dC_N = F_L(L, X) dL + F_X(L, X) dX,$$

dividing, side by side, by (3) and rearranging gives

$$\delta (P_N)\, dp_N + dc_N = s_L dl + s_X dx, \tag{17}$$

where the function δ is the elasticity of the distortion function

$$\delta (P_N) = \frac{P_N \Delta' (P_N)}{\Delta (P_N)}. \tag{18}$$

Differentiating (4)-(7) gives

$$dc_N = -\epsilon (dp_N - dp_T), \tag{19}$$

$$dx - dl = -\eta \cdot dp_X, \tag{20}$$

$$dc_T = -\frac{Q (X - \bar{X})}{C_T} dq - \frac{QX}{C_T} dx, \tag{21}$$

$$dp_X = dq + dp_T, \tag{22}$$

where ϵ is the elasticity of substitution between tradable and non-tradable goods in preferences

$$\epsilon = -\frac{C_N u''(C_N)}{u' (C_N)},$$

and η is the elasticity of substitution between labor and oil in the production function

$$\eta = \frac{F_L \cdot F_X}{F_{XL} \cdot F}.$$

Substituting (19) and (22) in (15) gives

$$dp_N = h \left(dq + dp_N + \frac{1}{\epsilon} dc_N \right)$$

and solving for the non-tradable price

$$dp_N = \frac{h}{1-h} \frac{1}{\epsilon} dc_N + \frac{h}{1-h} dq. \tag{23}$$

Computing this expression at the steady state gives the country Phillips curve (8) in the

text. Substituting back in (19) and (22) we get the other two prices,

$$dp_T = \frac{1}{1-h}\frac{1}{\epsilon}dc_N + \frac{h}{1-h}dq,$$
$$dp_X = \frac{1}{1-h}\frac{1}{\epsilon}dc_N + \frac{1}{1-h}dq. \tag{24}$$

Rewrite (17) as

$$dl = \delta \cdot dp_N + dc_N - s_X\left(dx - dl\right),$$

and, using (20) and (24), we obtain

$$dx - dl = -\eta\left(\frac{1}{1-h}dq + \frac{1}{1-h}\frac{1}{\epsilon}dc_N\right)$$

and

$$dl = \delta\left(\frac{h}{1-h}dq + \frac{h}{1-h}\frac{1}{\epsilon}dc_N\right) + dc_N + \eta s_X\left(\frac{1}{1-h}dq + \frac{1}{1-h}\frac{1}{\epsilon}dc_N\right),$$

which, grouping terms, give

$$dl = \left(1 + \frac{1}{\epsilon}\frac{\eta s_X + \delta h}{1-h}\right)dc_N + \frac{\eta s_X + \delta h}{1-h}dq, \tag{25}$$
$$dx = \left(1 - \frac{1}{\epsilon}\frac{\eta s_L - \delta h}{1-h}\right)dc_N - \frac{\eta s_L - \delta h}{1-h}dq. \tag{26}$$

A.2 Proof of Proposition (1)

From the conditions (25)-(26) derived above, we immediately derive the following properties of the mappings $\mathcal{L}$ and $\mathcal{X}$:

$$\mathcal{L}_{C_N} = \left(1 + \frac{1}{\epsilon}\frac{\eta s_X + \delta h}{1-h}\right)\frac{L}{C_N},$$
$$\mathcal{L}_Q = \frac{\eta s_X + \delta h}{1-h}\frac{L}{Q},$$
$$\mathcal{X}_{C_N} = \left(1 - \frac{1}{\epsilon}\frac{\eta s_L - \delta h}{1-h}\right)\frac{X}{C_N},$$
$$\mathcal{X}_Q = -\frac{\eta s_L - \delta h}{1-h}\frac{X}{Q}.$$

A.3 Derivation of equations (12) and (13)

Totally differentiating the resource constraint

$$\Delta\left(P_N\right)C_N = F\left(L, X\right),$$

and using the notation $\mathcal{P}_N(C_N, Q)$ to denote the equilibrium mapping for the non-tradable price, yields

$$\Delta'(P_N) C_N \mathcal{P}_{N,C_N} + \Delta = F_L(L, X) \mathcal{L}_{C_N} + F_X \mathcal{X}_{C_N},$$

which can be rearranged to give

$$\mathcal{L}_{C_N} = \frac{1}{F_L} \Delta'(P_N) C_N \mathcal{P}_{N,C_N} + \frac{1}{F_L} \Delta - \frac{F_X}{F_L} \mathcal{X}_{C_N}.$$

Similar steps, differentiating with respect to Q, give

$$\mathcal{L}_Q = \frac{1}{F_L} \Delta'(P_N) C_N \mathcal{P}_{N,Q} - \frac{F_X}{F_L} \mathcal{X}_Q.$$

Substituting for $\mathcal{L}_{C_N}$ in the country optimality condition yields

$$U_{C_N} + \frac{1}{F_L} U_L \left(\Delta'(P_N) C_N \mathcal{P}_{N,C_N} + \Delta - F_X \mathcal{X}_{C_N} \right) - Q \cdot U_{C_T} \cdot \mathcal{X}_{C_N} = 0$$

which, after rearranging, yields (12) in the text.

Substituting for $\mathcal{L}_{C_N}$ and $\mathcal{L}_Q$ in the planner optimality condition, yields

$$\begin{aligned} U_{C_N} + \frac{1}{F_L} U_L \left(\Delta'(P_N) C_N \mathcal{P}_{N,C_N} + \Delta - F_X \mathcal{X}_{C_N} \right) + \\ + U_L \cdot \left(\frac{1}{F_L} \Delta'(P_N) C_N \mathcal{P}_{N,Q} - \frac{F_X}{F_L} \mathcal{X}_Q \right) \cdot \mathcal{Q}_{C_N} = 0. \end{aligned}$$

Differentiating the market clearing condition in the oil market gives $\mathcal{X}_{C_N} + \mathcal{X}_Q \mathcal{Q}_{C_N} = 0$, and we obtain expression (13).

A.4 Proof of Proposition 2

The equilibrium oil price function $\mathcal{Q}(\bar{X}, C_N)$ satisfies, by definition,

$$\mathcal{X}(C_N, \mathcal{Q}(\bar{X}, C_N)) = \bar{X}.$$

Differentiating both sides with respect to $\bar{X}$, using the expressions for $\mathcal{X}_{C_N}$ and $\mathcal{X}_Q$ derived in A.1, and rearranging gives

$$\mathcal{Q}_X \frac{\bar{X}}{Q} = -\frac{1-h}{\eta s_L - \delta h}.$$

Similarly, differentiating w.r.t. C_N, and rearranging gives

$$\mathcal{Q}_{C_N} \frac{C_N}{Q} = \frac{1-h}{\eta s_L - \delta h} - \frac{1}{\epsilon}.$$

At the steady state with zero inflation we have $\delta = 0$ and $h = (1-\lambda) s_X$. Substituting in the expressions above we obtain the expressions for $\zeta_{\bar{X}}$ and ζ_{C_N} in the proposition.

A.5 General equilibrium linearized

The following are linearized relations that hold in the global equilibrium, that is, taking into account the endogenous determination of Q. Differentiating the equilibrium condition

$$\mathcal{X}\left(C_N, Q\right) = \bar{X}$$

yields

$$\left(1 - \frac{1}{\epsilon}\frac{\eta s_L - \delta h}{1-h}\right) dc_N - \frac{\eta s_L - \delta h}{1-h} dq = d\bar{x}$$

and rearranging we obtain

$$dq = \left(\frac{1-h}{\eta s_L - \delta h} - \frac{1}{\epsilon}\right) dc_N - \frac{1-h}{\eta s_L - \delta h} d\bar{x}.$$

At the zero inflation steady state this becomes

$$dq = \left(\frac{1}{\eta}\frac{s_L + \lambda s_X}{s_L} - \frac{1}{\epsilon}\right) dc_N - \frac{1}{\eta}\frac{s_L + \lambda s_X}{s_L} d\bar{x} \tag{27}$$

Also at the zero inflation steady state, equilibrium conditions imply:

$$dl = \frac{1}{s_L}\left(dc_N - s_X d\bar{x}\right), \tag{28}$$

$$dp_X = \frac{1}{\eta}\left(dl - d\bar{x}\right) = \frac{1}{\eta}\frac{1}{s_L}\left(dc_N - d\bar{x}\right),$$

$$dp_N = \left(1-\lambda\right) s_X dp_X = \frac{1}{\eta}\frac{s_X}{s_L}\left(dc_N - d\bar{x}\right). \tag{29}$$

A.6 Steady state, natural allocation, divine coincidence

We assume that the economy starts in steady state, at a first best efficient allocation with

$$U_N(1, C_N, L) F_L(L, \bar{X}) + U_L(1, C_N, L) = 0 \tag{30}$$

and the world price of oil is

$$Q = \frac{U_{C_N}}{U_{C_T}} F_X. \tag{31}$$

Totally differentiating the resource constraint gives

$$C_N \Delta'\left(P_N\right) dP_n + \Delta\left(P_N\right) dC_N = F_L(L, X) dL + F_X\left(L, X\right) dX.$$

Therefore, at an allocation with $P_N = 1$—which implies $\Delta\left(P_N\right) = 1$ and $\Delta'\left(P_N\right) = 0$—we have

$$1 = F_L(L, X) \mathcal{L}_{C_N} + F_X\left(L, X\right) \mathcal{X}_{C_N} \tag{32}$$

and

$$0 = F_L(L,X)\mathcal{L}_Q + F_X(L,X)\,\mathcal{X}_Q. \tag{33}$$

Computing the optimality condition for the Nash equilibrium and using (30) and (31) implies

$$U_{C_N} + U_L\mathcal{L}_{C_N} - QU_{C_T}\mathcal{X}_{C_N} = U_{C_N} - U_{C_N}F_x\mathcal{L}_{C_N} - U_{C_N}F_x\mathcal{X}_{C_N} = 0,$$

where the last equality follows from (32).

Consider now the gap between the planner's and the private first order conditions in expression in the text. The expression in parenthesis is

$$QU_{C_T} - U_L\frac{\mathcal{L}_Q}{\mathcal{X}_Q} = U_{C_N}F_X + U_L\frac{F_X}{F_L} = \frac{F_X}{F_L}\left(U_{C_N}F_L + U_L\right) = 0,$$

where the first equality follows from (33) and the second from (30).

B Example

Preferences are

$$C_T + \ln C_N - \frac{\Psi}{1+\phi}L^{1+\phi},$$

and the production function is CES

$$F(L,X) = \left(a_L^{\frac{1}{\eta}}L^{1-\frac{1}{\eta}} + a_X^{\frac{1}{\eta}}X^{1-\frac{1}{\eta}}\right)^{\frac{1}{1-\frac{1}{\eta}}}$$

with $a_L + a_X = 1$. The parameter Ψ in the utility function is set to $\Psi = a_L^{-\phi}$, and the endowment in steady state is $\bar{X} = a_X$. These are normalizations that ensure that all equilibrium prices are 1. The cost function is

$$\mathcal{C}(W,P_X) = \left(a_L W^{1-\eta} + a_X P_X^{1-\eta}\right)^{\frac{1}{1-\eta}}.$$

Marginal productivities are

$$F_L = a_L^{\frac{1}{\eta}}C_N^{\frac{1}{\eta}}L^{-\frac{1}{\eta}},$$
$$F_X = a_X^{\frac{1}{\eta}}C_N^{\frac{1}{\eta}}X^{-\frac{1}{\eta}}.$$

Marginal utilities

$$U_{C_N} = C_N^{-1}, \quad U_L = -a_L^{-\phi}L^{\phi}.$$

B.1 Steady state

In steady state we have quantities

$$L_o = a_L, \bar{X} = a_X, C_{No} = 1,$$

marginal utilities

$$U_{C_T o} = 1, U_{C_N o} = 1, U_{Lo} = -1,$$

and prices

$$P_{To} = 1, P_{No} = 1, W_o = 1, P_{Xo} = 1, Q = 1.$$

B.2 Algorithms

To compute Nash equilibrium. Given Δ, C_N:

1. Solve for L from
$$\Delta \cdot C_N = \left(a_L L^{1-\frac{1}{\eta}} + a_X X^{1-\frac{1}{\eta}}\right)^{\frac{1}{1-\frac{1}{\eta}}}$$

2. Solve for $P_X, \tilde{P}_N, P_N, P_T, Q$ iteratively from
$$\begin{aligned} P_X &= \left(\frac{a_X}{a_L}\frac{L}{X}\right)^{\frac{1}{\eta}}, \\ \tilde{P}_N &= \left(a_L + a_X P_X^{1-\eta}\right)^{\frac{1}{1-\eta}}, \\ P_N &= \left(\lambda + (1-\lambda)\tilde{P}_N^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}, \\ P_T &= P_N C_N, \\ Q &= \frac{P_X}{P_T}. \end{aligned}$$

3. Solve for
$$\Delta^{+1} = P_N^{\varepsilon}\left[\lambda + (1-\lambda)^{\frac{1}{1-\varepsilon}}\left(P_N^{1-\varepsilon} - \lambda\right)^{-\frac{\varepsilon}{1-\varepsilon}}\right]$$
check if Δ^{+1} close to Δ, set $\Delta = \Delta^{+1}$ and iterate steps 1-3 until convergence.

4. Compute s_L, s_X, h from
$$\begin{aligned} \frac{s_L}{1-s_L} &= \left(\frac{a_L}{a_X}\right)^{\frac{1}{\eta}}\left(\frac{L}{X}\right)^{1-\frac{1}{\eta}}, \\ s_X &= 1 - s_L, \\ h &= \left(1 - \lambda P_N^{\varepsilon-1}\right) s_X. \end{aligned}$$

5. Substitute in FOC of Nash and coordinated equilibrium, find zeros.

To compute allocation for single country given C_N and Q:

1. Define a function that given P_N computes

$$\begin{aligned} P_T &= C_N P_N, \\ \tilde{P}_N &= \left(a_L + a_X \left(Q P_T\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}, \\ f &= \left(\lambda + (1-\lambda)\tilde{P}_N^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} - P_N, \end{aligned}$$

   and find p_N such that $f = 0$.

2. Compute

$$\Delta = P_N^{\varepsilon}\left[\lambda + (1-\lambda)^{\frac{1}{1-\varepsilon}}\left(P_N^{1-\varepsilon} - \lambda\right)^{-\frac{\varepsilon}{1-\varepsilon}}\right],$$

   compute X/L ratio ξ

$$\xi = \frac{a_X}{a_L}\left(P_T Q\right)^{-\eta},$$

   and find L from

$$\Delta C_N = \left(a_L + a_X \xi^{1-\frac{1}{\eta}}\right)^{\frac{1}{1-\frac{1}{\eta}}} L$$

   and $X = \xi L$.

3. Compute utility

B.3 Conditions for inefficiency

The country optimality condition can be written as

$$1 - a_L^{-\phi} L^{1+\phi}\left(1 + \frac{\eta s_X + \delta h}{1-h}\right) - \left(1 - \frac{\eta s_L - \delta h}{1-h}\right) QX = 0,$$

or

$$\left(1 - a_L^{-\phi} L^{1+\phi} - QX\right) - \frac{\eta s_L - \delta h}{1-h}\left(\frac{\eta s_X + \delta h}{\eta s_L - \delta h} a_L^{-\phi} L^{1+\phi} - QX\right) = 0. \tag{34}$$

The planner optimality condition can be written as

$$1 - a_L^{-\phi} L^{1+\phi}\left(1 + \frac{\eta s_X + \delta h}{1-h}\right) - a_L^{-\phi} L^{1+\phi}\frac{\eta s_X + \delta h}{\eta s_L - \delta h}\left(1 - \frac{\eta s_L - \delta h}{1-h}\right) = 0.$$

The expression

$$\mathcal{X}_{C_N}\left(U_L \frac{\mathcal{L}_Q}{\mathcal{X}_Q} - U_T Q\right)$$

has same sign as

$$\left(1 - \frac{\eta s_L - \delta h}{1-h}\right)\left(\frac{\eta s_X + \delta h}{\eta s_L - \delta h} a_L^{-\phi} L^{1+\phi} - QX\right)$$

and is positive (excess inflation) if

$$1-\frac{\eta s_L-\delta h}{1-h}>0$$

and

$$\frac{\eta s_X+\delta h}{\eta s_L-\delta h}a_L^{-\phi}L^{1+\phi}>QX \tag{35}$$

Our objective is to linearize (34) w.r.t. C_N and $\bar{X}$, at the steady state, using equilibrium conditions and conditions (27)-(29), and to check when (35) holds. We proceed in steps.

Linearizing the expression

$$1-a_L^{-\phi}L^{1+\phi}-QX$$

gives

$$-\left(1+\phi\right)a_L l-a_X\left(q+\bar{x}\right)$$

which becomes

$$-\left(1+\phi\right)\left(c_N-a_X\bar{x}\right)-a_X\left[\left(\frac{1}{\eta}\frac{a_L+\lambda a_X}{a_L}-1\right)c_N-\frac{1}{\eta}\frac{a_L+\lambda a_X}{a_L}\bar{x}+\bar{x}\right]$$

or

$$-\left(1+\phi\right)\left(c_N-a_X\bar{x}\right)-a_X\left(\frac{1}{\eta}-1+\frac{\lambda}{\eta}\frac{a_X}{a_L}\right)\left(c_N-\bar{x}\right).$$

The next step is to linearize the expression

$$\frac{\eta s_L-\delta h}{1-h}\left(a_L^{-\phi}L^{1+\phi}\frac{\eta s_X+\delta h}{\eta s_L-\delta h}-Q\bar{X}\right).$$

Since the term in parenthesis is zero at the steady state, this is equivalent to linearizing

$$\eta\frac{a_L}{a_L+\lambda a_X}\left(a_L^{-\phi}L^{1+\phi}\frac{\eta s_X+\delta h}{\eta s_L-\delta h}-Q\bar{X}\right).$$

To do so we derive some preliminary results. From

$$\frac{s_L}{s_X}=\frac{F_L L}{F_X X}=\frac{a_L}{a_X}\left(\frac{L}{X}\right)^{1-\frac{1}{\eta}}$$

we get

$$\frac{ds_L}{s_L}-\frac{ds_X}{s_X}=\left(1-\frac{1}{\eta}\right)\left(l-x\right)$$

which combined with

$$c_N-x=s_L\left(l-x\right)$$

gives

$$\frac{ds_L}{s_L}-\frac{ds_X}{s_X}=\left(1-\frac{1}{\eta}\right)\frac{1}{a_L}\left(c_N-\bar{x}\right)$$

Differentiating twice the distortion function (**??**) gives

$$\Delta'(P_N) = \varepsilon P_N^{\varepsilon-1}\left[\lambda + (1-\lambda)^{\frac{1}{1-\varepsilon}}\left(P_N^{1-\varepsilon} - \lambda\right)^{-\frac{\varepsilon}{1-\varepsilon}}\right] + \\ - \varepsilon(1-\lambda)^{\frac{1}{1-\varepsilon}}\left(P_N^{1-\varepsilon} - \lambda\right)^{-\frac{1}{1-\varepsilon}}$$

and

$$\Delta''(P_N) = \varepsilon(\varepsilon-1)P_N^{\varepsilon-2}\left[\lambda + (1-\lambda)^{\frac{1}{1-\varepsilon}}\left(P_N^{1-\varepsilon} - \lambda\right)^{-\frac{\varepsilon}{1-\varepsilon}}\right] + \\ -\varepsilon^2 P_N^{\varepsilon-1}(1-\lambda)^{\frac{1}{1-\varepsilon}}\left(P_N^{1-\varepsilon} - \lambda\right)^{-\frac{\varepsilon}{1-\varepsilon}-1} P_N^{-\varepsilon} + \\ +\varepsilon(1-\lambda)^{\frac{1}{1-\varepsilon}}\left(P_N^{1-\varepsilon} - \lambda\right)^{-\frac{1}{1-\varepsilon}-1} P_N^{-\varepsilon}$$

which computed at the steady state give

$$\Delta'(1) = 0, \quad \Delta''(1) = \varepsilon\frac{\lambda}{1-\lambda}.$$

From the definition of δ and $\Delta'(P_N) = 0$ we have

$$\left[\frac{d\delta}{dP_N}\right]_{P_N=1} = \left[P_N\frac{\Delta''(P_N)}{\Delta(P_N)} + \frac{\Delta'(P_N)}{\Delta(P_N)} - P_N\left(\frac{\Delta'(P_N)}{\Delta(P_N)}\right)^2\right]_{P_N=1} = \varepsilon\frac{\lambda}{1-\lambda}$$

and, using (29), we can write

$$d\delta = \varepsilon\frac{\lambda}{1-\lambda}p_N = \varepsilon\frac{\lambda}{\eta}\frac{a_X}{a_L}(c_N - \bar{x}). \tag{36}$$

Using the results just derived, (**??**) and (36), we can linearize the expression

$$\frac{\eta s_X + \delta h}{\eta s_L - \delta h}$$

as follows

$$\frac{a_X}{a_L}\left(\frac{ds_X}{s_X} + \frac{1-\lambda}{\eta}d\delta - \frac{ds_L}{s_L} + \frac{1-\lambda}{\eta}\frac{a_X}{a_L}d\delta\right) = \frac{a_X}{a_L}\left(\frac{ds_X}{s_X} - \frac{ds_L}{s_L} + \frac{1-\lambda}{\eta}\frac{1}{a_L}d\delta\right) = \\ = \frac{a_X}{a_L^2}\left(\frac{1}{\eta} - 1 + \varepsilon\frac{\lambda(1-\lambda)}{\eta^2}\frac{a_X}{a_L}\right)(c_N - \bar{x}). \tag{37}$$

Linearizing $Q\bar{X}$ gives

$$a_X(q + \bar{x}) = a_X\left(\frac{1}{\eta} - 1 + \frac{\lambda}{\eta}\frac{a_X}{a_L}\right)(c_N - \bar{x}).$$

Combining all previous results, the linearized first order condition of the individual country is

$$\begin{aligned}
-(1+\phi)(c_N - a_X\bar{x}) - a_X\left(\frac{1}{\eta} - 1 + \frac{\lambda}{\eta}\frac{a_X}{a_L}\right)(c_N - \bar{x}) + \\
-\frac{\eta a_X}{a_L + \lambda a_X}(1+\phi)(c_N - a_X\bar{x}) + \\
-\frac{\eta a_X}{a_L + \lambda a_X}\left(\frac{1}{\eta} - 1 + \varepsilon\frac{\lambda(1-\lambda)}{\eta^2}\frac{a_X}{a_L}\right)(c_N - \bar{x}) + \\
+\frac{\eta a_X}{a_L + \lambda a_X}a_L\left(\frac{1}{\eta}\frac{a_L + \lambda a_X}{a_L} - 1\right)(c_N - \bar{x}) = 0,
\end{aligned}$$

or

$$\begin{aligned}
(1+\phi)(c_N - a_X\bar{x}) + a_X\left(\frac{1}{\eta}\frac{a_L + \lambda a_X}{a_L} - 1\right)(c_N - \bar{x}) + \\
+\frac{\eta a_X}{a_L + \lambda a_X}\left\{(1+\phi)(c_N - a_X\bar{x}) + \left[\left(\frac{1}{\eta} - 1\right)a_X - \frac{\lambda}{\eta}a_X + \varepsilon\frac{\lambda(1-\lambda)}{\eta^2}\frac{a_X}{a_L}\right](c_N - \bar{x})\right\} = 0.
\end{aligned}$$

Write this equation compactly as

$$B_1(c_N - a_X\bar{x}) + B_2(c_N - \bar{x}) + \xi(B_1(c_N - a_X\bar{x}) + B_3(c_N - \bar{x})) = 0 \tag{38}$$

where

$$\xi = \frac{\eta a_X}{a_L + \lambda a_X}$$

and

$$\begin{aligned}
B_1 &= 1 + \phi, \\
B_2 &= a_X\left(\frac{1}{\eta} - 1\right) + \frac{\lambda}{\eta}\frac{a_X^2}{a_L}, \\
B_3 &= a_X\left(\frac{1}{\eta} - 1\right) + \varepsilon\frac{\lambda(1-\lambda)}{\eta^2}\frac{a_X}{a_L} - \frac{\lambda}{\eta}a_X.
\end{aligned}$$

We now analyze under what conditions $\bar{x} < 0$ implies

$$B_1(c_N - a_X\bar{x}) + B_2(c_N - \bar{x}) < 0, \tag{39}$$

since this inequality implies a Nash equilibrium in which

$$1 - a_L^{-\phi}L^{1+\phi} - QX > 0$$

which, from 34, implies

$$\frac{\eta s_X + \delta h}{\eta s_L - \delta h}a_L^{-\phi}L^{1+\phi} > QX.$$

Proposition 6. *Following a (small) negative supply shock $\bar{x} < 0$, at the Nash equilibrium, world output and world inflation are inefficiently high if $\lambda > 0$, $a_X > 0$, and*

$$\eta < 1 + \lambda \frac{a_X}{a_L}, \quad \eta < \varepsilon (1 - \lambda).$$

Proof. Solving (38) gives

$$c_N = \frac{a_X (1 + \xi) B_1 + B_2 + \xi B_3}{(1 + \xi) B_1 + B_2 + \xi B_3} \bar{x}.$$

Condition (39) becomes

$$\left((B_1 + B_2) \frac{a_X (1 + \xi) B_1 + B_2 + \xi B_3}{(1 + \xi) B_1 + B_2 + \xi B_3} - (a_X B_1 + B_2) \right) \bar{x} < 0$$

and, with $\bar{x} < 0$, we need

$$(B_1 + B_2) \frac{a_X (1 + \xi) B_1 + B_2 + \xi B_3}{(1 + \xi) B_1 + B_2 + \xi B_3} > a_X B_1 + B_2. \tag{40}$$

The denominator on the left-hand side is positive because it can be written as

$$\left(1 + \frac{\eta a_X}{a_L + \lambda a_X}\right) \left(1 + \phi + a_X \frac{1 - \eta}{\eta}\right) + \frac{\lambda}{\eta} \frac{a_X}{a_L} + \frac{\eta a_X}{a_L + \lambda a_X} \left(\varepsilon \lambda (1 - \lambda) \frac{1}{\eta^2} \frac{a_X}{a_L} - \frac{\lambda}{\eta} a_X \right),$$

and

$$1 + \phi + a_X \frac{1 - \eta}{\eta} = a_L + \phi + a_X \frac{1}{\eta} > 0$$

and the last term is parenthesis is positive if $\eta < \varepsilon (1 - \lambda)$. Therefore, we can rearrange (40) and obtain

$$(a_X B_1 + B_3) (B_1 + B_2) > (a_X B_1 + B_2) (B_1 + B_3)$$

which is equivalent to

$$B_3 > B_2,$$

or

$$\varepsilon \lambda (1 - \lambda) \frac{1}{\eta^2} \frac{a_X}{a_L} - \frac{\lambda}{\eta} a_X > \frac{\lambda}{\eta} \frac{a_X^2}{a_L},$$

As long as $\lambda > 0, a_X > 0$ this is equivalent towhich is equivalent to

$$\varepsilon \frac{1 - \lambda}{\eta} > 1.$$

□

B.4 Numerical example with inefficiently low inflation

Set parameter values to

$$\eta = 1.2, \varepsilon = 2$$
$$\frac{a_X}{a_L} = 2, \lambda = 0.5$$

so that world Phillips curve is steeper:

$$\eta = 1.2 < 2 = 1 + \lambda \frac{a_X}{a_L}$$

but

$$\varepsilon \frac{1-\lambda}{\eta} = 2\frac{0.5}{1.2} < 1.$$

B.5 Decomposition in example

From

$$\underbrace{\left(U_{C_N} + \frac{\Delta}{F_L} U_L\right)}_{\text{Labor wedge}} + \underbrace{U_L \frac{\Delta'(P_N) C_N}{F_L}}_{\text{Inflation distortion}} \cdot \frac{\partial \mathcal{P}_N}{\partial C_N} + \underbrace{\left(-U_L \frac{F_X}{F_L} - Q \cdot U_{C_T}\right)}_{\text{Oil imports wedge}} \cdot \frac{\partial \mathcal{X}}{\partial C_N} = 0$$

The first term is

$$U_{C_N} + \frac{\Delta}{F_L} U_L = \frac{1}{C_N} + \frac{U_L L}{C_N} \frac{1}{s_L}.$$

Using

$$\frac{\partial \mathcal{P}_N}{\partial C_N} = \frac{P_N}{C_N} \frac{h}{1-h}$$

the second term is

$$U_L \frac{\Delta'(P_N) C_N}{F_L} \cdot \frac{\partial \mathcal{P}_N}{\partial C_N} = \frac{U_L L}{C_N} \frac{1}{s_L} \frac{\delta h}{1-h}.$$

Using

$$\frac{\partial \mathcal{X}}{\partial C_N} = \frac{X}{C_N} \left(1 - \frac{\eta s_L - \delta h}{1-h}\right)$$

the third term is

$$-\left(U_L \frac{F_X}{F_L} + Q \cdot U_{C_T}\right) \cdot \frac{\partial \mathcal{X}}{\partial C_N} = -\left(\frac{U_L L}{C_N} \frac{s_X}{s_L} + \frac{QX}{C_N}\right) \left(1 - \frac{\eta s_L - \delta h}{1-h}\right).$$

The planner's extra term is

$$U_L \frac{\Delta'(P_N) C_N}{F_L} \cdot \mathcal{P}_{N,Q} \mathcal{Q}_{C_N},$$

where

$$\mathcal{P}_{N,Q} = \frac{P_N}{Q}\frac{h}{1-h}, \quad \mathcal{Q}_{C_N} = \frac{Q}{C_N}\left(\frac{1-h}{\eta s_L - \delta h} - 1\right),$$

so

$$U_L \frac{\Delta'(P_N) C_N}{F_L} \cdot \mathcal{P}_{N,Q}\mathcal{Q}_{C_N} = \frac{U_L L}{C_N}\frac{1}{s_L}\frac{\delta h}{1-h}\left(\frac{1-h}{\eta s_L - \delta h} - 1\right).$$

Is there a connection between labor wedge and oil wedge?
Labor wedge can be written as

$$\left(U_{C_N} + \frac{\Delta}{F_L}U_L\right) = U_L\left(\frac{U_{C_N}}{U_L} + \frac{\Delta}{F_L}\right) = U_L\frac{\Delta}{F_L}\left(\frac{F_L}{\Delta}\frac{U_{C_N}}{U_L} + 1\right),$$

the oil wedge as

$$\begin{aligned} -\left(U_L\frac{F_X}{F_L} + Q\cdot U_{C_T}\right) &= -\left(U_L\frac{Q}{W/P_T} + Q\cdot U_{C_T}\right) = -U_L Q\frac{P_T}{W}\left(1 + \frac{WU_{C_T}}{U_L P_T}\right) \\ &= -U_L Q\frac{P_T}{W}\left(1 + \frac{W}{P_N}\frac{U_{C_N}}{U_L}\right). \end{aligned}$$

With sticky prices and fixed nominal wages we tend to have after supply shock a positive labor wedge

$$1 < \frac{F_L}{\Delta}\frac{U_{C_N}}{-U_L}$$

and a negative oil wedge because

$$\frac{W}{P_N} > \frac{F_L}{\Delta}$$

implies

$$\frac{W}{P_N}\frac{U_{C_N}}{-U_L} > \frac{F_L}{\Delta}\frac{U_{C_N}}{-U_L} > 1$$

Natural output

$$(1+\phi)(c_N - a_X\bar{x}) + a_X\frac{1-\eta}{\eta}(c_N - \bar{x}) = 0$$

$$c_N = a_X\frac{\phi + \frac{1}{\eta}}{\phi + a_L + a_X\frac{1}{\eta}}\bar{x}$$