# 3-D Gains from Trade*

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**Abstract**

Static trade models imply modest gains from trade. I quantify the gains from trade in a multi-country dynamic stochastic environment, taking into account the contributions to welfare of trade *across states* of the world, and *over time*, as well as trade *within* dates and states (3-D gains). For developing countries, which have volatile productivity, standard risk aversion implies that 3-D gains from trade are at least twice as big as static gains, even under financial autarky. Because productivity is less volatile for developed countries, their 3-D gains from trade are only modestly bigger than static gains, even under complete markets.

## 1 Introduction

The ability to trade internationally allows countries to consume a different bundle of goods and services from the bundle of goods and services they produce. This is true both at a point in time, and across dates and states of the world. Overall gains from trade, i.e. the welfare costs of moving from the current level of trade openness to full autarky, therefore depend not only on the contribution to welfare of trade within dates and states, but also on the contributions to welfare of trade across states of the world and trade over time. Static multicountry quantitative trade models imply that given the current level of trade openness, gains from trade are modest. I quantify the gains from trade taking into account all three dimensions of trade (3-D gains). For developing countries, which face volatile productivity, I find that gains from trade across states of the world and over time are large, even under

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financial autarky. As a result, 3-D gains from trade are at least twice as big as static gains. For developed countries, productivity is less volatile and more correlated across countries. For these countries 3-D gains from trade are only modestly bigger than static gains, even when international financial markets are complete.

I start by laying out a dynamic stochastic environment with multiple countries, inelastic labor supply, Armington specialization, iceberg costs of trade within dates and states of the world, and roundabout production. An exogenous stochastic process for productivity is the source of uncertainty. I assume constant relative risk aversion (CRRA) expected utility preferences, the same in all countries. I allow for very general frictions in international asset markets.

Within this environment, I generalize the formula for welfare gains from trade popularized by Arkolakis, Costinot and Rodriguez-Clare (2012). The first order welfare gain associated with infinitesimal changes in trade costs can be written as the sum of two terms. The first term is the weighted sum across all dates and states of changes in within-date-and-state shares of expenditure on own output, normalized by the negative of the trade elasticity. This echoes the formula in Arkolakis et al (2012). The second term is the weighted sum across all dates and states of changes in the share of total ex-ante expenditure devoted to consumption at a particular date and state, normalized by 1 minus the CRRA parameter. The weights in these sums are ratios of date- and state-specific value added to the value of total ex-ante expenditure on consumption. Under complete markets, once goods are appropriately defined, all countries face the same ex-ante prices. Under asset market frictions this is not in general true, but the formula remains the same as long as expenditures are valued using ex-ante prices from the perspective of the country whose welfare is being calculated.

This formula illustrates first, that 3-D gains from trade in a dynamic stochastic environment depend not just on the share of expenditure at each date and state that is domestically produced, but also on how consumption is distributed across dates and states of the world. This in turn depends on how the terms of trade and trade balance comove with output and productivity. Second, it shows that in a dynamic stochastic environment it is not possible to calculate ex-ante gains from trade using summary statistics. This is not only because gains from trade depend on the process for shocks rather than just realised outcomes. It is also because the change in welfare depends on expenditure shares that are not directly observed even for realized states. To calculate the 3-D gains from trade, it is necessary to calibrate the full model. This includes taking a stand on model parameters, the exogenous process for productivity, the nature of frictions impeding trade *across* dates and states of the world,

and resource costs of trade *within* dates and states.

To quantify the 3-D gains from trade, I therefore start by constructing a 50-year time-series for productivity for a large sample of countries in a model-consistent way. To do so, I first infer cross-sectional relative output prices and trade costs from data on bilateral import shares and expenditure PPPs by "inverting" the model. This does not require taking a stand on preferences for consumption across dates and states of the world, or on the nature of frictions in asset markets. I benchmark time-series variation in output prices using the US output price deflator from the Bureau of Economic Analysis (BEA). Using the resulting output price and trade cost series, I calculate labor productivity consistent with the model assumption of roundabout production.

I next use these time series to estimate a process for productivity that allows for (a) long-run growth that is the same for all countries, and equal to the US long-run growth rate (b) a rate of convergence to the productivity frontier that differs between developed and developing countries, (c) deviations of growth from trend that may be persistent, correlated across countries, and where the relevant parameters differ between developed and developing countries, and (d) a standard deviation of innovations to deviations of growth from trend that differs between developed and developing countries. The estimates imply that convergence is slower for developing than developed countries. Deviations of growth from trend are less correlated across developing and developed countries than they are within developed countries. Crucially, the standard deviation of innovations to deviations of growth from trend is higher for developing than for developed countries.

I then calculate ex-ante gains from trade from the perspective of 2019 for the 156 countries for which I have the required data, holding trade costs and population fixed at their 2019 levels. I assume an Armington elasticity of 4 and a coefficient of relative risk aversion of 4. I do this under two polar assumptions about international asset markets in the trade equilibrium: (a) financial autarky, and (b) complete markets with zero net foreign asset positions in 2019. Since I must recover consumption allocations in order to calculate prices and expenditure shares, I calculate exact changes in welfare rather than relying on the first order approximation. For small changes in welfare, the first order approximation yields an answer close to the exact welfare gain, but for large gains (and I find evidence of large gains) the first-order approximation performs poorly.

For developing countries, the 3-D gains from trade are considerably greater than static gains. In 2019, the static gains from trade under financial autarky imply that for the average developing country, being able to trade is equivalent to consumption that is permanently

17% higher than autarky consumption. But the 3-D gains under financial autarky imply that for the average developing country, being able to trade is equivalent to consumption that is permanently 58% higher than autarky consumption. For developed countries, the corresponding numbers are 21% and 22%.

Meanwhile, for developing countries, the 3-D gains from trade under complete markets are dramatically bigger than static gains. For the average developing country, being able to trade under complete markets is equivalent to consumption that is permanently 3 times greater than autarky consumption. For the average developed country, the corresponding number is 27% higher than autarky consumption.

Why are 3-D gains so big for developing countries? The key is that my model-consistent productivity series imply that developing countries face a lot of idiosyncratic risk. In the baseline calibration, agents are risk averse, so consumption in different states of the world is complementary. This implies that for developing countries, there is a lot of value to being able to engage in international trade across states of the world. And because the goods produced by different countries are imperfect substitutes, endogenous movements in the terms of trade provide risk sharing even under financial autarky, as in Cole and Obstfeld (1991). Of course, under complete markets, the amount of trade across states of the world is dramatically higher than under financial autarky, and the gains from trade are correspondingly bigger. And why are gains so much smaller for developed countries? For these countries productivity is less volatile and more correlated. In addition, developing countries are big. The risks they face are both modest, and mostly uninsurable aggregate risk. This implies that the gains from trade across states for these countries are limited, even under complete markets.

**Related literature**

This paper is closely related to Arkolakis et al (2012). They highlight that in a large class of quantitative multi-country trade models, gains from trade are a parsimonious function of the share of expenditure that is sourced domestically and the trade elasticity. Observed domestic expenditure shares and standard estimates of the trade elasticity imply modest gains from trade. Relative to their work, I show that redefining the commodity space to include goods indexed by state and time adds additional sources of gains from trade, and I quantify those gains. The role I identify for risk aversion (complementarity) in generating substantial gains from trade across states of the world is related to Ossa (2015), who notes the potential importance of sectors with very low trade elasticities in driving overall welfare gains in a static environment.

While much of the quantitative literature on the gains from trade focuses on static models, the literature on trade and endogenous growth examines dynamic environments. Sampson (2016), Buera and Oberfield (2020) and Perla, Tonetti and Waugh (2021) find additional sources of gains from trade in second best economies where productivity growth is endogenous to the extent of trade. In contrast to this literature, I assume productivity growth is stochastic, and exogenous to trade. Alvarez (2017) and Ravikumar, Santacreu and Sposi (2019) quantify the gains from reducing trade costs in deterministic neoclassical environments with trade costs and capital accumulation. They focus on the difference between steady state-to-steady state comparisons, and comparisons which take into account transition dynamics. Alessandria and Choi (2014), Alessandria, Choi and Ruhl (2021) and Boehm, Levchenko, Pandalai-Nayar and Toma (2024) examine how differences between short- and long-run trade elasticities may affect gains from trade. Like the literature on endogenous growth and trade, these papers do not address the possibility of gains from trade across time between countries with different long-run growth trajectories, nor do they incorporate gains from trade across states of the world into their frameworks.

An older theoretical literature e.g. Helpman and Razin (1978), Newbery and Stiglitz (1984), addresses potential gains from trade in environments with uncertainty. A more recent literature quantifies the relationship between trade and output volatility in environments with endogenous specialization, e.g. Caselli, Koren, Lisicky and Tenreyro (2019), Allen and Atkin (2022). However these papers do not try to measure the contribution to gains from trade of trade across states of the world.[1]

My work is closely related to a literature in international macroeconomics that calculates the welfare gains from moving from frictional asset markets to optimal international consumption risk sharing in environments with uncertainty, i.e. an *asset market* counterfactual rather than the *trade cost* counterfactual I perform here. See Van Wincoop (1999) and Heathcote and Perri (2014) for summaries, and Coeurdacier, Rey and Winant (2020) for a more recent example. This literature assumes costless trade, and focuses mostly on developed countries. An exception is Fitzgerald (2012). In that paper, I calculated ex-post gains from moving from historical asset markets to optimal international consumption risk sharing in an environment with international specialization and trade costs. The results I present here on ex-ante gains from trade are consistent with my previous finding that ex-post gains from moving from historical asset markets to optimal risk sharing are bigger for

[1]I assume that the volatility of productivity and the pattern of specialization are exogenous to trade openness in order to focus on gains from trade between countries with comparative advantage in different states of the world and at different time horizons.

developing countries, which face a lot of idiosyncratic risk, than for developed countries, which face mainly aggregate risk. Also related is Gourinchas and Jeanne (2006), who focus on the welfare gains from moving from incomplete asset markets to optimal intertemporal trade in the absence of idiosyncratic risk. They find small gains from optimal intertemporal trade between countries at different distances from steady state, consistent with the results presented here.

**Outline**

Section 2 lays out the model environment, and derives an expression for first order welfare changes due to changes in trade costs. Section 3 describes the data. Section 4 explains how historical productivity and trade costs are calculated. Section 5 describes the estimation of the productivity process. Section 6 quantifies ex-ante gains from trade under financial autarky and under complete markets. The final section concludes.

# 2 Model

**Summary**

There are $N$ countries in the world, indexed $i = 1, \ldots, N$. Labor in each country is inelastically supplied and deterministic. Each country produces a distinct tradeable intermediate good, also indexed $i$, using labor and materials. Productivity is stochastic. There are trade costs for intermediates which take the iceberg form. Intermediates are combined using a CES aggregator, the same in all countries, to produce a non-tradeable final good which is used for consumption and as the material input into the production of the intermediate. Agents have CRRA expected utility preferences over consumption of the final good. I first lay out the case of complete and frictionless asset markets and later generalize to the case of frictions in asset markets.

**Uncertainty**

In each period $t$, the world economy experiences one event, $s_t \in S$. Denote by $s^t$ the history of events from date 0 to date $t$, $\{s_0, \ldots, s_t\}$. The probability at date 0 that history $s^t$ will be realized at date $t$ is given by $\pi_{0,t}(s^t)$, with $\sum_{s^t} \pi_{0,t}(s^t) = 1$.

**Preferences**

Agents have identical CRRA expected utility. It is convenient to describe the problem as if

country $i$ had a single agent with date-0 (ex-ante) utility given by:

$$U_0^i = L_t^i \left( \sum_{t=0}^{\infty} \sum_{s^t} (1-\beta) \beta^t \pi_{0,t}\left(s^t\right) \left( \frac{C_t^i\left(s^t\right)}{L_t^i} \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{1}$$

where $C_t^i\left(s^t\right)$ is aggregate consumption in country $i$ at date $t$ given that history $s^t$ is realized, $L_t^i$ is deterministic population (and labor supply), so $C_t^i\left(s^t\right)/L_t^i$ is per capita consumption.

**Technology and trade costs**

Country $i$'s output of intermediate good $i$ at date $t$ after history $s^t$ is $Y_t^i\left(s^t\right)$, with:

$$Y_t^i\left(s^t\right) = A_t^i\left(s^t\right)\left(L_t^i\right)^{1-\mu} M_t^i\left(s^t\right)^{\mu} \tag{2}$$

where $A_t^i\left(s^t\right)$ is productivity, and $M_t^i\left(s^t\right)$ is material input. The production function for the final good, $X$ is:

$$X_t^i\left(s^t\right) = \left( \sum_{k=1}^{N} Z_t^{ki}\left(s^t\right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{3}$$

where $Z_t^{ki}\left(s^t\right)$ is absorption of the $k$-country intermediate in country $i$ at $s^t$. The Armington elasticity of substitution between different intermediates ($\eta$) determines the static trade elasticity. The resource constraint for the non-traded final good in country $i$ is:

$$X_t^i\left(s^t\right) = C_t^i\left(s^t\right) + M_t^i\left(s^t\right) \tag{4}$$

There are (deterministic) resource costs of trading intermediate goods. In order for one unit of $k$'s intermediate good to arrive in $i$ at $t$, $\tau_t^{ki} \geq 1$ units must be shipped, with $\tau_t^{kj}\tau_t^{ji} \geq \tau_t^{ki}$. Trade costs need not be symmetric, i.e. it may be the case that $\tau_t^{ik} \neq \tau_t^{ki}$. Trade costs are normalized to zero within a country, i.e. $\tau_t^{ii} = 1\ \forall i$. The resource constraint for each intermediate good must take account of the resource costs of trade:

$$Y_t^i\left(s^t\right) = \sum_{k=1}^{N} \tau_t^{ik} Z_t^{ik}\left(s^t\right) \tag{5}$$

**Markets for labor, intermediate and final goods**

The market for intermediate goods is perfectly competitive. Let $Q_{t,t}^k\left(s^t\right)$ be the price of 1 unit of the country-$k$ intermediate good in country $k$ at date $t$ after history $s^t$ in terms of

date-$t$ "dollars" (freely traded). The price of the country-$k$ intermediate in country $k$ is equal to the marginal cost of production:

$$Q_{t,t}^{k}\left(s^{t}\right)=\frac{1}{A_{t}^{k}\left(s^{t}\right)}\left(\frac{W_{t,t}^{k}\left(s^{t}\right)}{1-\mu}\right)^{1-\mu}\left(\frac{P_{t,t}^{k}\left(s^{t}\right)}{\mu}\right)^{\mu}$$

where $W_{t,t}^{k}\left(s^{t}\right)$ is the wage in country $k$ at $t$ after $s^{t}$ expressed in date-$t$ dollars, and $P_{t,t}^{k}\left(s^{t}\right)$ is the price of the final good in country $k$ at $t$ after $s^{t}$ expressed in date-$t$ dollars. Since $\tau_{t}^{ki}$ units of intermediate $k$ must be purchased in country $k$ in order for 1 unit to be delivered in country $i$, the price of 1 unit of the country-$k$ intermediate in country $i$ at date $t$ after history $s^{t}$ is $\tau_{t}^{ki}Q_{t,t}^{k}\left(s^{t}\right)$. Within date $t$ and state $s^{t}$, all agents face the same relative prices for trading claims to intermediate goods appropriately indexed by country of production and country of delivery.

Markets for final goods are also perfectly competitive, so price is equal to marginal cost. The price of 1 unit of the final good in country $i$ after history $s^{t}$ expressed in date-$t$ dollars is therefore:

$$P_{t,t}^{i}\left(s^{t}\right)=\left(\sum_{k=1}^{N}\left(\tau_{t}^{ki}Q_{t,t}^{k}\left(s^{t}\right)\right)^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{6}$$

Since the final good is non-traded, there is no additional condition relating final goods prices across countries.

The labor market is perfectly competitive and labor is paid its marginal product:

$$W_{t,t}^{k}\left(s^{t}\right)=\left(1-\mu\right)\mu^{\frac{\mu}{1-\mu}}A_{t}^{k}\left(s^{t}\right)^{\frac{1}{1-\mu}}Q_{t,t}^{k}\left(s^{t}\right)^{\frac{1}{1-\mu}}P_{t,t}^{i}\left(s^{t}\right)^{\frac{-\mu}{1-\mu}}$$

**Asset markets**

At date 0 after observing $s_0$, agents trade the full menu of Arrow-Debreu securities, which are indexed by country of production and country of delivery. All agents face the same prices for these securities. by Let $Q_{0,t}^{k}\left(s^{t}\right)$ be the price of 1 unit of intermediate good $k$ in country $k$ at date $t$ at $s^{t}$ in terms of date-0 dollars (date-0 price). Since $\tau_{t}^{ki}$ units must be purchased in country $k$ in order for 1 unit to be delivered in country $i$, the date-0 price of 1 unit of good $k$ in country $i$ at date $t$ at $s^{t}$ is $\tau_{t}^{ki}Q_{0,t}^{k}\left(s^{t}\right)$.

Meanwhile, $B_{0}^{i}$ is net wealth of country $i$ entering period 0 expressed in date-0 dollars.

Date-0 net wealth of the world as a whole is equal to zero:

$$\sum_{k=1}^{N} B_0^k = 0$$

This implies that the date-0 budget constraint for country $i$ can be written:

$$\sum_{t=0}^{\infty} \sum_{s^t} \sum_{k=1}^{N} \tau_t^{ki} Q_{0,t}^k \left(s^t\right) Z_t^{ki} \left(s^t\right) \leq B_0^i + \sum_{t=0}^{\infty} \sum_{s^t} Q_{0,t}^i \left(s^t\right) Y_t^i \left(s^t\right) \tag{7}$$

**Competitive equilibrium**

A competitive equilibrium is given by sequences of ex-ante prices $\left\{\left\{\left\{Q_{0,t}^i \left(s^t\right)\right\}_{i=1}^N\right\}_{s^t}\right\}_{t=0}^{\infty}$, intermediate goods absorption $\left\{\left\{\left\{\left\{Z_t^{ki} \left(s^t\right)\right\}_{k=1}^N\right\}_{i=1}^N\right\}_{s^t}\right\}_{t=1}^{\infty}$, materials $\left\{\left\{\left\{M_t^i \left(s^t\right)\right\}_{i=1}^N\right\}_{s^t}\right\}_{t=0}^{\infty}$ and consumption $\left\{\left\{\left\{C_t^i \left(s^t\right)\right\}_{i=1}^N\right\}_{s^t}\right\}_{t=0}^{\infty}$ such that:

1. Given ex-ante prices, country $i$ chooses absorption of each of the $N$ intermediate goods, materials, and consumption $C_t^i \left(s^t\right)$ for each date and state, to maximize welfare, subject to the country-$i$ final good resource constraints, and the country-$i$ ex-ante budget constraint.

2. All intermediate and final goods resource constraints are satisfied.

Alvarez and Lucas (2007) and Allen, Arkolakis and Takahashi (2020) provide conditions under which equilibrium exists and is unique in static one-sector gravity models with balanced trade. I conjecture that given the nested CES structure of the environment here, extensions of these results to multi-sector models with a continuum of sectors may be used to prove existence and uniqueness in this case, with each realization $\{t, s^t\}$ interpreted as corresponding to a "sector."

**Some first order conditions**

The nested CES form of the utility function and the intermediate aggregator function implies that several useful price indexes take a familiar form. Let $P_{0,t}^i \left(s^t\right)$ be the price of the aggregate good in country $i$ at $s^t$ in terms of date-0 dollars (i.e. the date-0 price). Then:

$$P_{0,t}^i \left(s^t\right) = \left[\sum_{k=1}^{N} \left(\tau_t^{ki} Q_{0,t}^k \left(s^t\right)\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

Let $\Pi_0^i$ be the price of $U_0^i$ in terms of date-0 dollars:

$$\Pi_0^i = \left[\sum_{t=0}^{\infty}\sum_{s^t}\left(\left(\left(1-\beta\right)\beta^t\right)\pi_{0,t}\left(s^t\right)\right)^{\rho}\left(P_{0,t}^i\left(s^t\right)\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

By writing the Lagrangian for country $i$ at different levels of aggregation, and taking the first order conditions with respect to those aggregates, one can derive the following expressions for expenditure shares as functions of date-0 relative prices (note that $\Pi_0^i$ is equal to the inverse of the multiplier on the date-0 budget constraint):

$$\lambda_{0,t}^{ki}\left(s^t\right) = \frac{\tau_t^{ki}Q_{0,t}^k\left(s^t\right)Z_t^{ki}\left(s^t\right)}{P_{0,t}^i\left(s^t\right)X_t^i\left(s^t\right)} = \left(\frac{\tau_t^{ki}Q_{0,t}^k\left(s^t\right)}{P_{0,t}^i\left(s^t\right)}\right)^{1-\eta} \tag{8}$$

$$\phi_{0,t}^i\left(s^t\right) = \frac{P_{0,t}^i\left(s^t\right)C_{0,t}^i\left(s^t\right)}{\Pi_0^i U_0^i} = \left(\left(\left(1-\beta\right)\beta^t\right)\pi_{0,t}\left(s^t\right)\right)^{\rho}\left(\frac{P_{0,t}^i\left(s^t\right)}{\Pi_0^i}\right)^{1-\rho} \tag{9}$$

Meanwhile, the first order condition with respect to $M_t^i\left(s^t\right)$ implies that expenditure on materials is a constant share of the value of output:

$$\frac{P_{0,t}^i\left(s^t\right)M_t^i\left(s^t\right)}{Q_{0,t}^i\left(s^t\right)Y_t^i\left(s^t\right)} = \mu \tag{10}$$

**First-order change in welfare**

With the price indexes and expenditure shares in hand, one can derive a first order approximation to the change in welfare induced by changes in trade costs or foreign productivity which relates to familiar expressions in the literature. Write the ex-ante budget constraint as:

$$\Pi_0^i U_0^i = B_0^i + \sum_{t=0}^{\infty}\sum_{s^t}\left(1-\mu\right)Q_{0,t}^i\left(s^t\right)A_t^i\left(s^t\right)\left(L_t^i\right)^{1-\mu}M_t^i\left(s^t\right)^{\mu}$$

Making use of the first order condition for $M_t^i\left(s^t\right)$ yields:

$$\Pi_0^i U_0^i = B_0^i + \sum_{t=0}^{\infty}\sum_{s^t}\left(1-\mu\right)\mu^{\frac{\mu}{1-\mu}}A_t^i\left(s^t\right)^{\frac{1}{1-\mu}}L_t^i Q_{0,t}^i\left(s^t\right)^{\frac{1}{1-\mu}}\left(P_{0,t}^i\left(s^t\right)\right)^{-\frac{\mu}{1-\mu}}$$

Normalize $\Pi_0^i = 1$ (this pins down what a dollar is). Assume $d\ln B_0^i = 0$ (initial net wealth is invariant to the change in trade costs or foreign productivity), $d\ln A_t^i\left(s^t\right) = 0$, and

$d \ln L_t^i = 0$. Then:

$$d \ln U_0^i = \sum_{t=0}^{\infty} \sum_{s^t} \frac{(1-\mu) Q_{0,t}^i (s^t) Y_t^i (s^t)}{\Pi_0^i U_0^i} \left( \frac{d \ln Q_{0,t}^i (s^t)}{1-\mu} - \frac{\mu d \ln P_{0,t}^i (s^t)}{1-\mu} \right)$$

Let

$$\theta_{0,t}^i \left( s^t \right) = \frac{(1-\mu) Q_{0,t}^i (s^t) Y_t^i (s^t)}{\Pi_0^i U_0^i} \tag{11}$$

i.e. $\theta_{0,t}^i (s^t)$ is the ratio of date-$t$ history-$s^t$ value added to date-0 expenditure on consumption at all dates and states, expressed in terms of date-0 prices. Making use of the expenditure shares defined above yields (see Appendix for details):

$$d \ln U_0^i = \sum_{t=0}^{\infty} \sum_{s^t} \theta_{0,t}^i \left( s^t \right) \left( \frac{d \ln \lambda_{0,t}^{ii} (s^t)}{(1-\mu)(1-\eta)} + \frac{d \ln \phi_{0,t}^i (s^t)}{(1-\rho)} \right) \tag{12}$$

This expression for the first order change in welfare induced by changes in trade costs or productivity in other countries generalizes the formula for welfare gains from trade in a static environment popularized by Arkolakis et al (2012). The first term, which is the weighted sum of changes in the domestic share in domestic expenditure normalized by the negative of the trade elasticity (and an adjustment for roundabout production), is familiar from Arkolakis et al (2012). It captures gains from trade within dates and states of the world. The second term, which is the weighted sum of changes in the share of ex-ante consumption expenditure devoted to consumption at a particular date and state of the world, normalized by 1 minus the CRRA parameter, captures gains from trade across states of the world and over time. The weights are given by ratios of value added at the relevant date and state to total ex ante consumption expenditure. Note that while the first order approximation provides a good guide for the impact of small changes in trade costs, moving to autarky is not a small change. For large changes in trade costs, higher order terms, including cross terms, matter. See the Appendix for more details.

The first key take-away from expression (12) and the corresponding expressions with higher order terms is that the gains from trade in a dynamic stochastic environment depend not just on the share of expenditure that is domestically produced, but on how consumption is distributed across dates and states of the world. Intuitively, within-date-and-state gains from trade are more valuable in dates and states where marginal utility is high than when it is low. This in turn depends on how the terms of trade and trade balance comove with productivity. If the terms of trade (price of output over price of consumption) is high when

productivity is low, this valuable in welfare terms. Negative net exports depress the domestic share in domestic expenditure, and it is therefor more valuable in welfare terms to run a trade deficit when productivity is low than when it is high.[2]

The second key take-away is that in contrast to the static case, it is not possible to calculate the first order welfare impact of small changes in trade costs using summary statistics alone. The domestic share in expenditure $\lambda_{0,t}^{ii}(s^t)$ is observed only for realized histories. Even for realized histories, the expenditure shares $\phi_{0,t}^{i}(s^t)$ and $\theta_{0,t}^{i}(s^t)$ are unobserved, because total ex-ante expenditure on consumption, $\Pi_0^i U_0^i$ is unobserved. This is also true of the corresponding shares under the counterfactual of infinite trade costs. As a result, the model must be calibrated and solved for both baseline and counterfactual in order to calculate the gains from trade. Of course, once the full baseline and counterfactual model have been solved, it is straightforward to calculate exact changes in welfare, and approximations are not necessary.

**Asset market frictions**

Under frictions in asset markets, different countries may face different marginal rates of substitution across dates and states. As long as an equilibrium consumption allocation exists and is known, date-0 shadow prices for each country $i$ can be computed based on country-$i$ marginal utilities at the equilibrium allocation. Let $Q_{0,t;i}^{k}(s^t)$ denote the date-0 shadow price for country $i$ of putting 1 unit of intermediate $k$ in country $k$ in period $t$ after history $s^t$. Using these shadow prices, a shadow date-0 budget constraint for country $i$ can be written:

$$\Pi_0^i U_0^i = B_0^i + \sum_{t=0}^{\infty} \sum_{s^t} (1-\mu)\, Q_{0,t;i}^{i}\left(s^t\right) A_t^i\left(s^t\right) \left(L^i\right)^{1-\mu} M_t^i\left(s^t\right)^{\mu}$$

Maximization of utility subject to this shadow budget constraint yields first order conditions which are satisfied by the equilibrium consumption allocation. Using this expression, and the expenditure shares $\lambda_{0,t;i}^{ii}(s^t)$, $\phi_{0,t;i}^{i}(s^t)$ and $\theta_{0,t;i}^{i}(s^t)$, which correspond to (8), (9) and (11) measured using the appropriate date-0 country-$i$ prices, one can derive an expression for first order changes in welfare similar to the expression in the complete markets case:

$$d\ln U_0^i = \sum_{t=0}^{\infty} \sum_{s^t} \theta_{0,t;i}^{i}\left(s^t\right) \left( \frac{d\ln \lambda_{0,t;i}^{ii}(s^t)}{(1-\mu)(1-\eta)} + \frac{d\ln \phi_{0,t;i}^{i}(s^t)}{(1-\rho)} \right) \tag{13}$$

Note that recovering the date-0 country-$i$-specific prices required to calculate $d\ln \lambda_{0,t;i}^{ii}(s^t)$,

[2]This is sensitive to the fact that there is no investment in the model.

$d \ln \phi^i_{0,t;i}(s^t)$ and $\theta^i_{0,t;i}(s^t)$ in all dates and states requires characterizing the nature of asset market frictions in order to construct the resulting equilibrium consumption allocation.

**Special case of asset market frictions: Financial autarky**

Financial autarky is a special case of asset market frictions where it is straightforward to construct the equilibrium consumption allocation. As long as goods are freely traded within dates and states subject to iceberg trade costs, then within dates and states all countries face the same relative prices for intermediate goods indexed appropriately by country of production and country of delivery. Date-by-date and state-by-state the value of output must equal the value of expenditure at these common prices:

$$Q^i_{t,t}\left(s^t\right) Y^i_t\left(s^t\right) = \sum_{k=1}^{N} Q^{ki}_{t,t}\left(s^t\right) Z^{ki}_t\left(s^t\right)$$

This condition is sufficient to construct the equilibrium consumption allocation. In general, shadow date-0 prices for delivery at $t > 0$ differ across countries. As noted above, these shadow prices may be obtained by evaluating marginal utilities at the equilibrium consumption allocation. Calculating the expenditure shares $\theta^i_{0,t;i}(s^t)$, $\lambda^{ii}_{0,t;i}(s^t)$ and $\phi^i_{0,t;i}(s^t)$ using the appropriate prices yields expression (13) for the first order change in welfare.

Financial autarky additionally implies that $\phi^i_{0,t;i}(s^t) = \theta^i_{0,t;i}(s^t)$, where

$$1 = \sum_{t=0}^{\infty} \sum_{s^t} \phi^i_{0,t;i}\left(s^t\right)$$

and therefore

$$0 = \sum_{t=0}^{\infty} \sum_{s^t} \phi^i_{0,t;i}\left(s^t\right) d \ln \phi^i_{0,t;i}\left(s^t\right)$$

so the first order change in welfare reduces to:

$$d \ln U^i_0 = \sum_{t=0}^{\infty} \sum_{s^t} \phi^i_{0,t;i}\left(s^t\right) \frac{d \ln \lambda^{ii}_{0,t;i}(s^t)}{(1-\mu)(1-\eta)}$$

This looks like a straightforward generalization of Arkolakis et al (2012). Note however that higher order terms involving $d \ln \lambda^{ii}_{0,t;i}(s^t)$ and $d \ln \phi^i_{0,t;i}(s^t)$ do not cancel. For large changes in trade costs, the change in welfare therefore depends on $d \ln \phi^i_{0,t;i}(s^t)$ as well as $d \ln \lambda^{ii}_{0,t;i}(s^t)$.

**Special case of preferences: Log utility**

Under log utility, the share of ex-ante expenditure devoted to each date and state (i.e. $\phi^i_{0,t;i}(s^t)$) is constant and equal to $((1-\beta)\beta^t)\pi_{0,t}(s^t)$. Therefore $d\ln\phi^i_{0,t;i}(s^t) = 0$. This implies that the first order change in welfare can be written:

$$d\ln U^i_0 = \sum_{t=0}^{\infty}\sum_{s^t}\theta^i_{0,t;i}\left(s^t\right)\frac{d\ln\lambda^{ii}_{0,t;i}(s^t)}{(1-\mu)(1-\eta)}$$

Under log utility *and* financial autarky, consumption shares and output shares are equal, so the first order change in welfare can be written:

$$d\ln U^i_0 = \sum_{t=0}^{\infty}\sum_{s^t}\left((1-\beta)\beta^t\right)\pi_{0,t}\left(s^t\right)\frac{d\ln\lambda^{ii}_{0,t;i}(s^t)}{(1-\mu)(1-\eta)}$$

Of course for large changes in trade costs, the change in welfare depends on higher powers of $d\ln\lambda^{ii}_{0,t;i}(s^t)$, even under log utility.

# 3 Data

Since a sufficient statistics approach is not possible, computing welfare requires information on all details of the environment, including trade costs, initial productivity, and a process for productivity. I require the following data to construct trade costs and productivity consistent with the model: (a) national income accounts expressed in current US$ at market exchange rates (b) the matrix of bilateral trade flows expressed in current US$ at market exchange rates, (c) labor supply, proxied by population, (d) relative expenditure prices across countries, and (e) time series of real gross output for the U.S. I also make use of data on the ratio of gross output to value added to calibrate the model. I construct an unbalanced panel dataset with variables (a)-(d) covering 160 countries and the period 1970-2019. Subject to some restrictions, I include countries for which there are at least 10 consecutive years of complete data on the required variables. The number of countries present in each cross-section ranges from 85 in 1970 to 160 in the 2010s, with 156 countries present in 2019. I also construct a Rest of the World (ROW) aggregate. Full details of data coverage are reported in the Appendix.

## 3.1 Data sources

### 3.1.1 National income accounts

I obtain national income accounts data (GDP, exports, and imports measured in current US$ at market exchange rates, along with the merchandise share in total balance of payments imports) for most countries from the World Development Indicators (WDI). I supplement this with data from the Organization for Economic Cooperation and Development (OECD) for a number of countries for which OECD time-series coverage is better. I use data from the OECD on national income accounts for the former West Germany, 1970-1990. I obtain data for Taiwan directly from the Taiwanese statistical agency. For details, see the data section in the Appendix.

### 3.1.2 Bilateral trade

I obtain bilateral merchandise trade data for 1970-1994 from Feenstra and Lipsey's NBER-United Nations Trade Data. I obtain bilateral merchandise trade data for 1995-2019 from the CEPII's Baci dataset. I obtain bilateral services trade data for 1995-2004 from WTO-OECD BaTiS, version BPM5. I obtain bilateral services trade data for 2005-2019 from WTO-OECD BaTiS, version BPM6. All of these data sets measure bilateral import flows in current US$ at market exchange rates.

### 3.1.3 Labor supply

I use population as a proxy for labor supply. I obtain data on population for most countries from the WDI. I supplement this with OECD data for a number of countries for which coverage is better. I use the Penn World Tables 5.6 to obtain population for the former West Germany, 1970-1990. I obtain population data for Taiwan directly from the Taiwanese statistical agency.

### 3.1.4 Expenditure prices

I obtain expenditure PPPs from the Penn World Tables, version 10.01. The variable I use is "pl_da."

### 3.1.5 US real GDP

I obtain time series data on US constant price real gross output, used to benchmark output growth over time, from the Bureau of Economic Analysis (BEA).

### 3.1.6 Gross output to value added ratio

I use the OECD's national accounts data to obtain the ratio of nominal gross output to nominal value added for a subsample of the countries and years from the full data set. This guides my choice for the parameter governing $\mu$, i.e. the materials share in gross output.

## 3.2 Construction of the working dataset

The average ratio of gross output to value added in the OECD national accounts data is approximately 2, and time-variation in this ratio is modest for most covered countries. Guided by this fact and by equation (10), I use the value $\mu = 0.5$ for the materials share.

I then use the data on GDP to calculate the value of gross output of country $i$ at date $t$ using $OUT_t^i = GDP_t^i/\mu$. For Singapore, Hong Kong, and Luxembourg this value implies that exports exceed gross output in at least some years, so I drop these countries from the sample.[3] I measure a country's imports from itself as gross output less total exports from national income accounts:

$$IM_t^{ii} = OUT_t^i - EXna_t^i$$

I measure a country's total absorption as gross output less exports from national income accounts plus imports from national income accounts:

$$ABS_t^i = OUT_t^i - EXna_t^i - IMna_t^i$$

For the period 1970-1994 I use bilateral merchandise trade data to calculate bilateral import shares in total imports:

$$\frac{IM_t^{ki}}{\sum_{j\neq i} IM_t^{ji}} = \frac{Merchim_t^{ki}}{\sum_{j\neq i} Merchim_t^{ji}}$$

For the period 1995-2019, for countries for which I observe the share of merchandise in BOP imports, I construct bilateral import shares in total imports as follows:

$$\frac{IM_t^{ki}}{\sum_{j\neq i} IM_t^{ji}} = \frac{Merchim_t^{ki}}{\sum_{j\neq i} Merchim_t^{ji}} \frac{BOPmerchim_t^i}{BOPim_t^i} + \frac{Servim_t^{ki}}{\sum_{j\neq i} Servim_t^{ji}} \left(1 - \frac{BOPmerchim_t^i}{BOPim_t^i}\right)$$

If the share of merchandise in BOP imports is not available, I continue to use bilateral merchandise import shares to measure bilateral import shares. Shares in absorption are

[3]The OECD reports 21 years of gross output data for Luxembourg, and there is a marked downward trend in the ratio of value added to gross output over this period.

then calculated by multiplying the bilateral import shares in total imports by the ratio of imports from national income accounts to absorption:

$$\frac{IM_t^{ki}}{ABS_t^i} = \frac{IMna_t^i}{ABS_t^i} \frac{IM_t^{ki}}{\sum_{j \neq i} IM_t^{ji}}$$

I construct a rest of the world (ROW) aggregate composed of all countries not included in the baseline sample using WDI data on world GDP and bilateral trade data on world imports and exports. The ROW aggregate accounts for more than 10% of world GDP in the 1970s, falling to just over 2% from 2000. It accounts for around 1% of imports of in-sample countries in the 1970s and early 1980s, falling to between 0.5% and 0.6% of imports of in-sample countries for the remainder of the sample period. Of course the importance of the ROW as a trade partner differs across countries.

Expenditure PPPs from the PWT are used to measure the price of the expenditure basket in each country at date $t$ relative to that in the US:

$$\frac{\hat{P}_t^i}{\hat{P}_t^{US}} = \frac{PPP_t^i}{PPP_t^{US}}$$

# 4 Constructing productivity and trade costs

In order to recover historical trade costs and productivity consistent with the model, I need to take a stand on the values of $\mu$, the materials share, and $\eta$, the elasticity of substitution between intermediates produced by different countries. As explained in the previous section, guided by data on the ratio of gross output to value added, I set $\mu = 0.5$. Following the trade literature, I choose a baseline value of $\eta = 4$, implying a trade elasticity of 3.

**Recovering output prices and trade costs**

The first step in constructing trade costs and productivity is recovering time-series for trade costs and output prices consistent with the model. Remember the model tells us (dropping state-contingent notation and using prices expressed in date-$t$ dollars) that:

$$\frac{IM_t^{ki}}{ABS_t^i} = \frac{\tau^{ki} Q_t^k Z_t^{ki}}{P_t^i X_t^i} = \frac{\left(\tau_t^{ki} Q_t^k\right)^{1-\eta}}{\sum_{j=1}^N \left(\tau_t^{ji} Q_t^j\right)^{1-\eta}} \tag{14}$$

and

$$\left(P_t^i\right)^{1-\eta} = \sum_{j=1}^N \left(\tau_t^{ji} Q_t^j\right)^{1-\eta}$$

These expressions imply that given information on own import shares and expenditure prices, the vector of relative output prices can be recovered using:

$$\left(\hat{Q}_t^i\right)^{1-\eta} = \frac{IM_t^{ii}}{ABS_t^i}\left(\hat{P}_t^i\right)^{1-\eta}$$

Then, given the vector of output prices for each country, bilateral trade costs can be recovered from bilateral import shares:

$$\left(\hat{\tau}_t^{ki}\right)^{1-\eta} = \frac{IM_t^{ki}}{ABS_t^i}\left(\frac{\hat{P}_t^i}{\hat{Q}_t^k}\right)^{1-\eta}$$

The bilateral trade costs recovered using this approach and the data described above on import shares and expenditure prices do not always satisfy the restriction $\hat{\tau}_t^{ki} \geq 1$. This is problematic for counterfactuals, as it implies that output can be created costlessly by shipping intermediate $k$ from $k$ to $i$. I take the stand that these negative implied trade costs are the result of systematic mismeasurement of expenditure prices. This is strongly suggested by the fact that negative trade costs are clustered in developing countries importing from developed countries in time periods in the neighborhood of strong shifts in the developing country expenditure price. To deal with this issue, I apply an adjustment to the measured expenditure PPPs for problematic importer country-time pairs. This adjustment imposes $\tau_t^{ki} \geq 1$ for all partners $k \neq i$, with equality for the pair for which $\hat{\tau}_t^{ki}$ is smallest (i.e. most negative trade costs):

$$\left(\tilde{P}_t^i\right)^{1-\eta} = \left(\hat{P}_t^i\right)^{1-\eta} \max_k \left\{ \frac{ABS_t^i}{IM_t^{ki}}\left(\frac{\hat{P}_t^i}{\hat{Q}_t^k}\right)^{\eta-1} \right\}$$

I then construct output prices and iceberg trade costs using the adjusted expenditure prices:

$$\left(\tilde{Q}_t^i\right)^{1-\eta} = \frac{IM_t^{ii}}{ABS_t^i}\left(\tilde{P}_t^i\right)^{1-\eta}$$

$$\left(\tau_t^{ki}\right)^{1-\eta} = \frac{IM_t^{ki}}{ABS_t^i}\left(\frac{\tilde{P}_t^i}{\tilde{Q}_t^k}\right)^{1-\eta}$$

This procedure recovers trade costs and cross-sectional relative output prices, but does not pin down how output prices (and therefore real quantities) evolve over time. To do this, I use time series data on US real output from the BEA. I recover time-series variation in the

US output price using:

$$\frac{Q_t^{US}}{Q_{1970}^{US}} = \frac{OUT_t^{US}/Realout_t^{US}}{OUT_{1970}^{US}/Realout_{1970}^{US}}$$

I normalize $Q_{1970}^{US} = 1$. The output prices used to calculate productivity are then constructed using:

$$\left(Q_t^i\right)^{1-\eta} = \left(\frac{\tilde{Q}_t^i}{\tilde{Q}_t^{US}}\frac{Q_t^{US}}{Q_{1970}^{US}}\right)^{1-\eta}$$

Corresponding expenditure prices are constructed using

$$\left(P_t^i\right)^{1-\eta} = \frac{ABS_t^i}{IM_t^{ii}}\left(Q_t^i\right)^{1-\eta}$$

**Productivity**

Real output is obtained by dividing the current dollar value of output by the appropriate output price:

$$Y_t^i = \frac{OUT_t^i}{Q_t^i}$$

Given the value of $\mu$, materials inputs to production are calculated using:

$$M_t^i = \mu\frac{Q_t^k Y_t^k}{P_t^i}$$

Using these implied materials inputs to production, and data on labor input (i.e. population), productivity is given by:

$$A_t^i = \frac{Y_t^i}{\left(L_t^i\right)^{1-\mu}\left(M_t^i\right)^{\mu}}$$

**Results**

Figure 1 plots the resulting time-series of productivity for all countries in the sample, with US productivity normalized to 1 in 1970. As this figure shows, the productivity series for countries other than the US are quite volatile. For countries which do not use the dollar, output measured in current dollars moves around a lot relative to US output, and this is not fully offset by the output prices recovered through the lens of the model, nor by the implied movements in materials inputs to production. Volatile productivity is the only way in which the model can be consistent with the data. Some of the largest productivity swings, and most of the cases where productivity is above that of the US, are associated with oil-producing

countries.

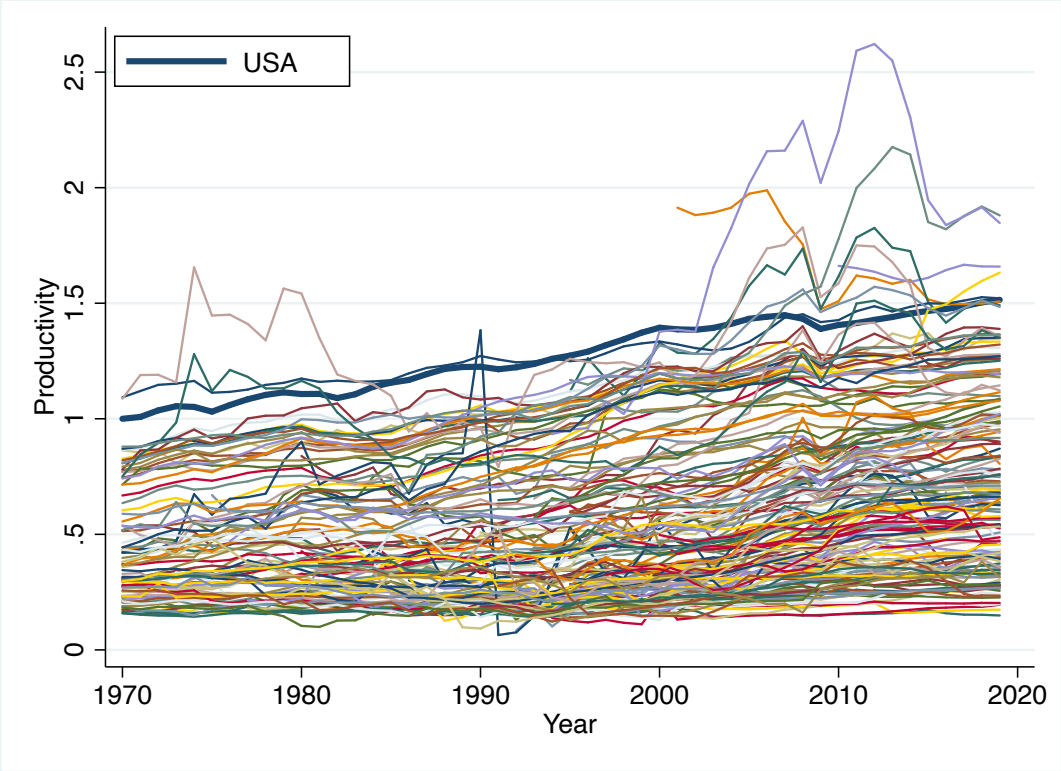



Figure 1: Model-consistent productivity

Endogenous risk sharing through movements in the terms of trade plays an important role in governing 3-D gains from trade under financial autarky. To explore what the historical data implies about this, I calculate correlations between the measured terms of trade and productivity. In particular, I calculate the correlation between log changes in the terms of trade defined as the price of output over the price of expenditure ($\Delta \ln (Q_t^i/P_t^i)$), and log changes in productivity ($\Delta \ln A_t^i$), country-by-country. Figure 2 plots the histogram of these correlations. The mean correlation across OECD countries is 0.12, while the mean correlation across non-OECD countries is -0.29. A negative correlation implies that in good times (high productivity) the price of a country's output falls relative to the price of consumption, precisely the correlation necessary for endogenous risk sharing. Clearly, this type of correlation is more prevalent for developing than it is for developed countries in the historical data.

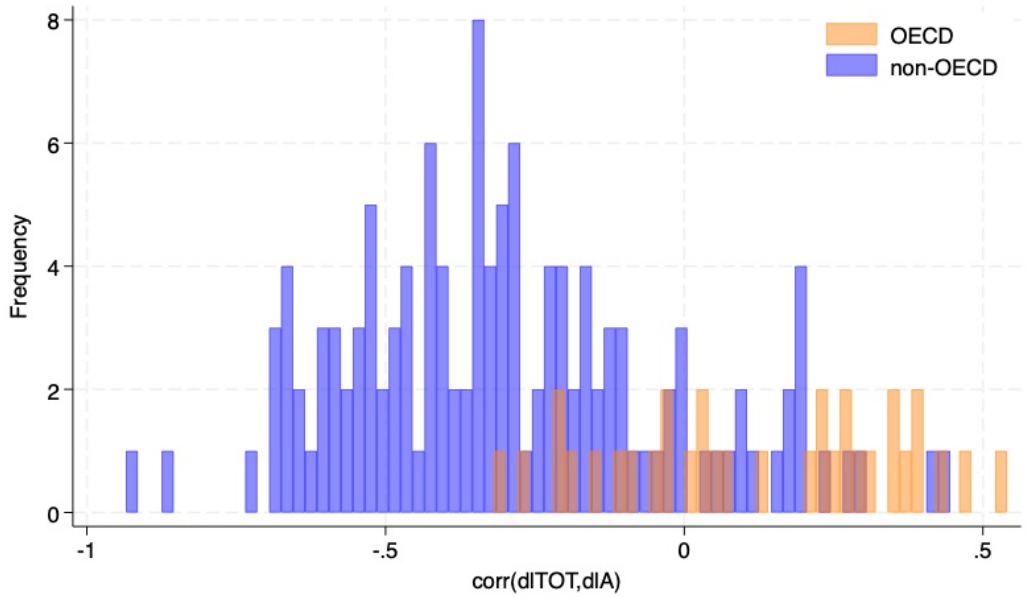



Figure 2: Correlation of log change in terms of trade & log change in productivity

# 5 Productivity process

With the productivity series in hand, the next step is to estimate a stochastic process for productivity. The ideal process should be flexible enough to account for (a) long-run growth, (b) conditional convergence at rates that may differ across countries, (c) persistent deviations of growth from trend, (d) correlations in deviations of growth from trend across countries, and (e) heterogeneous volatility of deviations of growth from trend across countries.

I implement this by assuming that the US is the world technology leader. Let $a_t^i = \ln A_t^i$. I assume that $a_t^{US}$ grows at constant rate $g^{US}$. Actual US productivity growth equals trend growth $g^{US}$, plus deviations from trend $\varepsilon_t^{US}$, which may be persistent, i.e.

$$\Delta a_t^{US} = g^{US} + \varepsilon_t^{US}$$

where

$$\varepsilon_t^{US} = \psi^{US} \varepsilon_{t-1}^{US} + \sigma^{US} \nu_t^{US}$$

with $\nu_t^{US} \sim N(0,1)$.

Meanwhile, trend productivity in $i \neq US$ grows at the same average rate as the US (i.e. the rate of growth of the technology frontier) plus a term reflecting convergence to the

frontier. Actual productivity growth again equals trend growth, plus deviations from trend.

$$\Delta a_t^i = g^{US} + \gamma^i \left( a_{t-1}^{US} - a_{t-1}^i \right) + \varepsilon_t^i$$

Here, $\gamma^i$ governs the speed of convergence to the frontier. In the case of $i \neq US$, deviations from trend may depend on lagged deviations of US productivity growth from trend, as well as innovations to US deviations of growth from trend:

$$\varepsilon_t^i = \psi^i \varepsilon_{t-1}^i + \psi^{i,US} \varepsilon_{t-1}^{US} + \sigma^i \nu_t^i + \sigma^{i,US} \nu_t^{US}$$

with $\nu_t^i \sim N(0,1)$. This dependence of deviations from trend on deviations of US productivity growth from trend induces correlation in deviations of productivity growth from trend across non-US countries.

To implement this in a parsimonious fashion, I divide countries into three groups: the US, non-US OECD and non-OECD. The Appendix lists which are the OECD members. I assume $\gamma^i = \gamma^O$ for non-US OECD members, while $\gamma^i = \gamma^N$ for non-members of the OECD. I assume $\psi^i = \psi^O$ and $\psi^{i,US} = \psi^{O,US}$ for non-US OECD members, while $\psi^i = \psi^N$ and $\psi^{i,US} = \psi^{N,US}$ for non-OECD countries. Finally, I assume $\sigma^i = \sigma^O$ and $\sigma^{i,US} = \sigma^{O,US}$ for non-US OECD members, while $\sigma^i = \sigma^N$ and $\sigma^{i,US} = \sigma^{N,US}$ for non-OECD countries. I first estimate the parameters of the US productivity process. I then estimate the process for the remaining countries by maximum likelihood, conditioning on $\left\{ g^{US}, \psi^{US}, \sigma^{US} \right\}$. Details are reported in the Appendix. I restrict the estimation sample to the 80 countries (plus the US) for which I have a balanced panel of data. The Appendix lists the countries included in this sample.

The parameter estimates are reported in Table 1. The process for productivity differs substantially between developed and developing countries. Productivity in developed countries converges faster to that of the US. Deviations of growth from trend are comove more with US deviations of growth from trend. And the standard deviation of idiosyncratic innovations to deviations from trend is substantially lower in developed than it is in developing countries.

Table 1: Parameters of estimated productivity process

| Parameter | | Value |
|---|---|---|
| US LR growth rate | $g^{US}$ | 0.0085 |
| Autocorrelation of deviations from trend, US | $\psi^{US}$ | 0.31 |
| SD of innovations to deviations from trend, US | $\sigma^{US}$ | 0.002 |
| Rate of convergence to US, OECD | $\gamma^{O}$ | 0.0102 |
| Autocorrelation of deviations from trend, OECD | $\psi^{O}$ | 0.11 |
| Correlation with lagged US deviations from trend, OECD | $\psi^{O,US}$ | 0.38 |
| SD of innovations to deviations from trend, OECD | $\sigma^{O}$ | 0.02 |
| Correlation with US innovations to deviations from trend, OECD | $\sigma^{O,US}$ | 0.001 |
| Rate of convergence to US, non-OECD | $\gamma^{N}$ | 0.0027 |
| Autocorrelation of deviations from trend, non-OECD | $\psi^{N}$ | -0.04 |
| Correlation with lagged US deviations from trend, non-OECD | $\psi^{N,US}$ | 0.24 |
| SD of innovations to deviations from trend, non-OECD | $\sigma^{N}$ | 0.09 |
| Correlation with US innovations to deviations from trend, non-OECD | $\sigma^{N,US}$ | 0.001 |

# 6 Gains from trade

Calculating the ex-ante gains from trade in a dynamic stochastic environment requires taking a stand on trade costs and asset markets in the trade equilibrium. I assume trade costs are fixed at their levels in 2019, the last year in my sample, and the year for which I calculate ex-ante gains from trade. The literature on international macroeconomics has not converged on a way of modeling frictions in asset markets that performs well in matching international comovements at a business cycle frequency. Clearly from the fact that net exports deviate from zero, some asset trade is possible. Yet Heathcote and Perri (2002) conclude that financial autarky does a better job of matching features of international business cycles in developed countries than either complete markets, or trade in a single non-contingent nominal bond, another popular benchmark. With this in mind, I focus on two polar cases, financial autarky, and complete markets, which have the advantage of being straightforward to compute, leaving analysis of gains from trade under alternative asset market structures to future research.

To compute the gains from trade I must therefore solve for equilibrium consumption allocations and prices under three different scenarios:

1. Infinite trade costs (full autarky) at all dates and states, including 2019. Because trade costs are infinite, there is no incentive to trade assets, and the structure of asset markets is irrelevant for allocations.

2. Trade costs held fixed at their 2019 levels at all dates and states; financial autarky in all dates and states, including 2019.

3. Trade costs held fixed at their 2019 levels at all dates and states; complete and frictionless asset markets with zero net wealth ($B_0^i = 0$) for all countries in 2019.

In all three scenarios, productivity in 2018 and 2019 is held fixed at the levels calculated in Section 4. From 2020 forward, productivity is assumed to follow the process estimated in Section 5. In addition, labor supply is held fixed at 2019 levels.

Under scenarios (1) and (2), there are no dynamic decisions, so the problem can be treated as if all choices are made at date 0. Under scenario (3), re-opening markets at any date or state does not lead countries to change their asset positions vis-a-vis the choices made at date 0. Given this, I can approximate the solution to the date-0 problem under each scenario by simulating $S$ paths of length $T$ from the process for $\mathbf{a}_t = \ln \mathbf{A}_t$, starting at $t = 2019$, and conditioning on $\{\mathbf{a}_{2019}, \Delta\mathbf{a}_{2019}\}$. I then solve for the equilibrium from a date-0 perspective under the assumption that each of these $S$ paths has equal probability. I now describe how equilibrium is calculated in each of the three scenarios.

**1. Autarky equilibrium**

In autarky, the sum of consumption and materials equals output in each country at each date and state. Given the optimal choice of materials, the consumption allocation is given by:

$$C_t^{i,aut}\left(s^t\right) = (1-\mu)\,\mu^{\frac{\mu}{1-\mu}} A_t^i\left(s^t\right)^{\frac{1}{1-\mu}} L^i$$

and this allows $U_0^{i,aut}$ to be calculated. In autarky, the country-$i$ output price is equal to the country-$i$ expenditure price at each date and state. Given the consumption allocation, the prices of output and expenditure at date $t$ after history $s^t$ relative to the price of ex-ante utility can be recovered from the first order condition for consumption:

$$\frac{Q_{0,t;i}^{i,aut}\left(s^t\right)}{\Pi_{0;i}^{i,aut}} = \frac{P_{0,t;i}^{i,aut}\left(s^t\right)}{\Pi_{0;i}^{i,aut}} = \left(\frac{1-\beta}{1-\beta^{T+1}}\right)\beta^t \pi_{0,t}\left(s^t\right)\left(\frac{C_t^{i,aut}\left(s^t\right)}{U_0^{i,aut}}\right)^{-\frac{1}{\rho}}$$

**2. Trade equilibrium with financial autarky**

I use a tatonnement algorithm to solve for the competitive equilibrium quantities and prices under financial autarky. I guess a vector $\left\{Q_{t,t}^{k,fa}\left(s^t\right)\right\}_{k=1}^{N}$ of output prices for each interme-

diate at each date and state, and use excess demands for each intermediate at each date and state to update the output price vector, increasing prices when excess demand is positive and reducing them when excess demand is negative, and iterating until excess demands converge to zero. Excess demand is given by:

$$ED_t^{k,fa}\left(s^t\right) = \sum_{i=1}^{N} \tau_{2019}^{ki} Z_t^{ki,fa}\left(s^t\right) - Y_t^{k,fa}\left(s^t\right)$$

where

$$Y_t^{k,fa}\left(s^t\right) = L^k\left(A_t^k\left(s^t\right)\right)^{\frac{1}{1-\mu}} \left(\frac{\mu Q_{t,t}^{k,fa}\left(s^t\right)}{P_{t,t}^{k,fa}\left(s^t\right)}\right)^{\frac{\mu}{1-\mu}}$$

and

$$Z_t^{ki,fa}\left(s^t\right) = \left(\frac{P_{t,t}^{i,fa}\left(s^t\right)}{\tau_{2019}^{ki} Q_{t,t}^{k,fa}\left(s^t\right)}\right)^{\eta} \left(C_t^{i,fa}\left(s^t\right) + \mu^{\frac{1}{1-\mu}} \left(\frac{Q_{t,t}^{i,fa}\left(s^t\right) A_t^i\left(s^t\right)}{P_{t,t}^{i,fa}\left(s^t\right)}\right)^{\frac{1}{1-\mu}} L_{2019}^i\right)$$

and, imposing financial autarky:

$$C_t^{i,fa}\left(s^t\right) = (1-\mu)\frac{Q_{t,t}^{i,fa}\left(s^t\right) Y_t^{i,fa}\left(s^t\right)}{P_{t,t}^{i,fa}\left(s^t\right)}$$

Given the resulting consumption allocation, date-0 expenditure prices for country $i$ relative to $\Pi_{0;i}^{i,fa}$ can be recovered from the the first order condition for consumption:

$$\frac{P_{0,t;i}^{i,fa}\left(s^t\right)}{\Pi_{0;i}^{i,fa}} = \left(\frac{1-\beta}{1-\beta^{T+1}}\right)\beta^t \pi_{0,t}\left(s^t\right)\left(\frac{C_t^{i,fa}\left(s^t\right)}{U_0^{i,fa}}\right)^{-\frac{1}{\rho}}$$

and date-0 output prices for country $i$ can be recovered using:

$$\frac{Q_{0,t;i}^{i,fa}\left(s^t\right)}{\Pi_{0;i}^{i,fa}} = (1-\mu)\frac{P_{0,t;i}^{i,fa}\left(s^t\right)}{\Pi_{0;i}^{i,fa}}\frac{C_t^{i,fa}\left(s^t\right)}{Y_t^{i,fa}\left(s^t\right)}$$

### 3. Trade equilibrium with complete and frictionless asset markets

I use a tatonnement algorithm to solve for the competitive equilibrium quantities and prices under complete markets. I first guess a vector $\left\{\left\{\left\{Q_{0,t}^{k,cm}\left(s^t\right)\right\}_{k=1}^{N}\right\}_{s^t\in S}\right\}_{t=1}^{T}$ of date-0 output prices for each intermediate at each date and state. I use this vector to calculate expenditure

prices in each country at each date and state, and therefore $\Pi_0^{i,cm}$ as outlined in Section 2. I then use excess demands for each intermediate at each date and state to update the output price vector, increasing prices when excess demand is positive and reducing them when excess demand is negative, and iterating until excess demands converge to zero. Excess demand is given by:

$$ED_t^{k,cm}\left(s^t\right) = \sum_{i=1}^{N} \tau_{2019}^{ki} Z_t^{ki,cm}\left(s^t\right) - Y_t^{k,cm}\left(s^t\right)$$

where

$$Y_t^{k,cm}\left(s^t\right) = L^k\left(A_t^k\left(s^t\right)\right)^{\frac{1}{1-\mu}} \left(\frac{\mu Q_{0,t}^{k,cm}\left(s^t\right)}{P_{0,t}^{k,cm}\left(s^t\right)}\right)^{\frac{\mu}{1-\mu}}$$

and

$$Z_t^{ki,cm}\left(s^t\right) = \left(\frac{P_{0,t}^{i,cm}\left(s^t\right)}{\tau_{2019}^{ki} Q_{0,t}^{k,cm}\left(s^t\right)}\right)^{\eta} \left(C_t^{i,cm}\left(s^t\right) + \mu^{\frac{1}{1-\mu}} \left(\frac{Q_{0,t}^{i,cm}\left(s^t\right) A_t^i\left(s^t\right)}{P_{0,t}^{i,cm}\left(s^t\right)}\right)^{\frac{1}{1-\mu}} L_{2019}^i\right)$$

and making use of the CES structure of preferences, complete markets and $B_0^i = 0$:

$$C_t^{i,cm}\left(s^t\right) = \left(\frac{\Pi_0^{i,cm}\left(\frac{1-\beta}{1-\beta^{T+1}}\right)\beta^t \pi_{0,t}\left(s^t\right)}{P_{0,t}^{i,cm}\left(s^t\right)}\right)^{\rho} \left(\frac{(1-\mu)\sum_{t=1}^{\infty}\sum_{s^t} Q_{0,t}^{i,cm}\left(s^t\right) Y_t^i\left(s^t\right)}{\Pi_0^{i,cm}}\right)$$

The solution to this problem delivers both equilibrium allocations and date-0 prices.

**Welfare gains from trade**

I use the equilibrium consumption allocations under autarky and trade (with the two different configurations of asset markets) to calculate the exact welfare gains from trade. I measure welfare gains in country $i$ as the factor $\kappa^i$ by which country-$i$ autarky consumption would have to be increased at each date and state in order for $i$ to be indifferent between autarky consumption and consumption in the trade equilibrium. This notion of welfare corresponds to the measure of welfare costs of business cycles in Lucas (1987). Due to the homotheticity of the utility function, $\kappa^i$ is given by:

$$\kappa^i = \frac{\left(\sum_{t=0}^{T}\sum_{s^t}\left(\frac{1-\beta}{1-\beta^{T+1}}\right)\beta^t \pi_{0,t}\left(s^t\right)\left(C_t^i\left(s^t\right)^{tr}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}}{\left(\sum_{t=0}^{T}\sum_{s^t}\left(\frac{1-\beta}{1-\beta^{T+1}}\right)\beta^t \pi_{0,t}\left(s^t\right)\left(C_t^i\left(s^t\right)^{aut}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}}$$

where $tr \in \{fa, cm\}$.

The first order change in welfare using the formula derived in Section 2, can implemented as follows:

$$\ln \tilde{\kappa}^i = \sum_{t=0}^{T} \sum_{s^t} \theta_{0,t;i}^{i,aut}\left(s^t\right) \left( \frac{\ln \lambda_{0,t;i}^{ii,tr}\left(s^t\right) - \ln \lambda_{0,t;i}^{ii,aut}\left(s^t\right)}{\left(1-\mu\right)\left(1-\eta\right)} + \frac{\ln \phi_{0,t;i}^{i,tr}\left(s^t\right) - \ln \phi_{0,t;i}^{i,aut}\left(s^t\right)}{\left(1-\rho\right)} \right)$$

This turns out to be a poor approximation to exact changes in welfare, so I focus on exact changes.[4]

**Parameters**

As already noted, it is necessary to assume values for $\eta$ and $\mu$ in order to calculate productivity and trade costs, and the baseline values I choose are $\eta = 4$ and $\mu = 0.5$. For the counterfactuals, values for $\{\beta, \rho\}$ are also required. I set $\beta = 0.96$, and $\rho = 1/4$. I set $T = 100$ and $S = 200$.

**Results**

Table 2 reports statistics on the distribution of 3-D gains from trade, for the case of financial autarky and for the case of complete markets. Statistics are reported for the full sample, and for the OECD and non-OECD subsamples. The table also reports statistics on the static gains from trade for 2019 under financial autarky.[5] The results are also illustrated in Figures 3 and 4, which plot 3-D gains under financial autarky and 3-D gains under complete markets against static gains from trade evaluated in 2019.

Table 2: Distribution of gains from trade

| | Static gains | | | 3-D gains | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2019 | | | Financial autarky | | | Complete markets | | |
| | All | Rich | Poor | All | Rich | Poor | All | Rich | Poor |
| Mean | 1.18 | 1.21 | 1.17 | 1.50 | 1.22 | 1.58 | 2.78 | 1.27 | 3.19 |
| Med | 1.15 | 1.15 | 1.14 | 1.39 | 1.16 | 1.46 | 2.72 | 1.21 | 2.89 |
| Min | 1.04 | 1.05 | 1.04 | 1.05 | 1.05 | 1.11 | 1.06 | 1.06 | 1.87 |
| Max | 2.22 | 1.65 | 2.22 | 4.54 | 1.69 | 4.54 | 7.54 | 1.78 | 7.54 |
| Pop-wgt mean | 1.09 | 1.12 | 1.08 | 1.25 | 1.13 | 1.28 | 1.35 | 1.17 | 2.60 |

[4] I calculate the approximation up to third order. For small changes in welfare, a first order approximation is sufficient. For large changes, even a third order approximation performs poorly.

[5] In the Appendix, I report static gains from trade for 2019 under observed trade balances. For countries which run trade deficits in 2019, gains from trade under observed trade balances are larger than under balanced trade. For countries which run surpluses in 2019, gains from trade are smaller than under balanced trade.

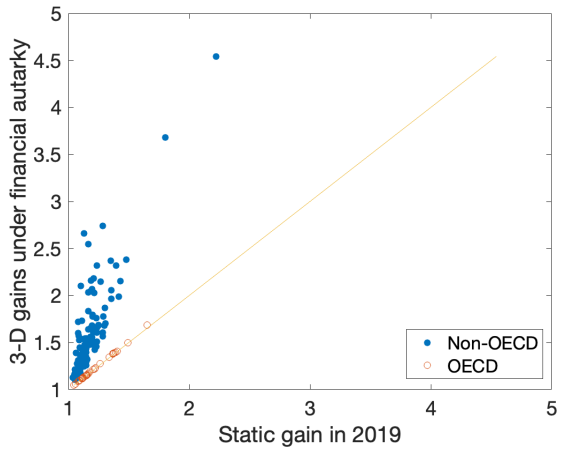



Figure 3: 3-D gains: financial autarky

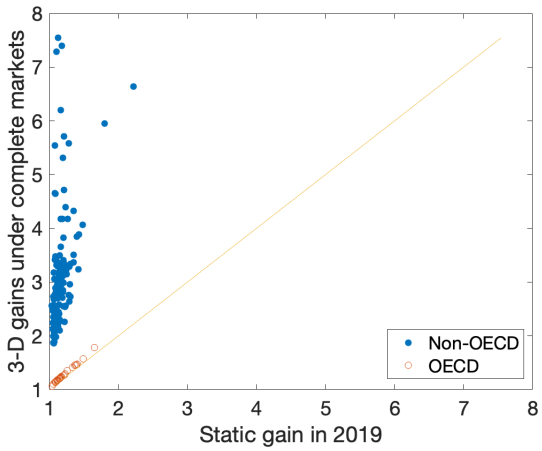



Figure 4: 3-D gains: complete markets

Table 2 and Figures 3 and 4 show that 3-D gains from trade are bigger than static gains, especially for developing countries, and especially under complete markets. For OECD countries, 3-D gains under financial autarky are 1.005 times static gains on average, while 3-D gains under complete markets are 1.05 times static gains. Meanwhile, for non-OECD countries, 3-D gains under financial autarky are 1.33 times static gains. This implies that 3-D gains from trade are more than double static gains. Under complete markets, 3-D gains for non-OECD countries are on average 2.71 times static gains. In the Appendix, I provide scatter plots of the first, second and third order approximations of changes in welfare against exact changes, showing that the the first approximation performs well for small changes in welfare, but poorly for large changes. Indeed, even the third order approximation performs poorly for the large welfare changes under complete markets.

**Decomposing ex-ante gains**

Since the first-order approximation of changes in welfare performs poorly for large changes, it does not provide a useful way of decomposing overall gains into contributions from trade within dates and states, and contributions from trade across states of the world and over time. So to better understand the sources of overall gains from trade, I perform an alternative exercise based on decomposing consumption under the different scenarios into different components.

Consumption in country $i$ under world productivity history $s \in S$ and market scenario

$mkt \in \{aut, fa, cm\}$ (i.e. autarky, financial autarky or complete markets) can be written:

$$C_t^i(s)^{mkt} = C_0^{i,mkt} \exp\left(g^i(s)^{mkt} \cdot t\right) \exp\left(\varepsilon_t^i(s)^{mkt}\right)$$

where

$$g^i(s)^{mkt} = \frac{1}{T}\sum_{t=1}^{T} \Delta \ln C_t^i(s)^{mkt}$$

Using this decomposition of consumption, the overall gains from trade $\kappa^i$ can be decomposed into two terms:

$$\kappa^i = \frac{C_0^{i,tr}}{C_0^{i,aut}} \frac{\left(\sum_{t=0}^{T}\sum_s \left(\frac{1-\beta}{1-\beta^{T+1}}\right)\beta^t \pi(s)\left(\exp\left(g^i(s)^{tr}\cdot t\right)\exp\left(\varepsilon_t^i(s)^{tr}\right)\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}}{\left(\sum_{t=0}^{T}\sum_s \left(\frac{1-\beta}{1-\beta^{T+1}}\right)\beta^t \pi(s)\left(\exp\left(g^i(s)^{aut}\cdot t\right)\exp\left(\varepsilon_t^i(s)^{aut}\right)\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}} = \kappa_1^i \kappa_2^i$$

Table 3 reports statistics on the distribution of $\kappa_1^i$ and $\kappa_2^i$.[6] For developed countries, most of the gains from trade are due to static gains, even under complete markets. For developing countries however, there are substantial gains from trade across dates and states, under financial autarky as well as under complete markets.

In addition, the following objects can be calculated based on the decomposition of consumption:

$$\kappa_3^i = \frac{\left(\sum_{t=0}^{T}\sum_s \left(\frac{1-\beta}{1-\beta^{T+1}}\right)\beta^t \pi(s)\exp\left(g^i(s)^{tr}\cdot t\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}}{\left(\sum_{t=0}^{T}\sum_s \left(\frac{1-\beta}{1-\beta^{T+1}}\right)\beta^t \pi(s)\exp\left(g^i(s)^{aut}\cdot t\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}}$$

and

$$\kappa_4^i = \frac{\left(\sum_{t=0}^{T}\sum_s \left(\frac{1-\beta}{1-\beta^{T+1}}\right)\beta^t \pi(s)\exp\left(\varepsilon_t^i(s)^{tr}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}}{\left(\sum_{t=0}^{T}\sum_s \left(\frac{1-\beta}{1-\beta^{T+1}}\right)\beta^t \pi(s)\exp\left(\varepsilon_t^i(s)^{aut}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}}$$

These measures reflect the contributions to welfare of smoothing average growth rates across alternative histories, and of smoothing deviations of growth from trend within histories respectively. Note however that $\kappa_2^i$ cannot be written as the product of $\kappa_3^i$ and $\kappa_4^i$. The bottom two panels of Table 3 report statistics on the distribution of $\kappa_3^i$ and $\kappa_4^i$. These statistics indicate that both dimensions contribute to gains from trade across dates and states of the world.

[6] In the case of financial autarky, $\kappa_1^i$ is identical to the static gains for 2019 reported in Table 2.

Table 3: Decomposing exact gains from trade

| | Financial autarky | | | Complete markets | | |
|---|---|---|---|---|---|---|
| | All | Rich | Poor | All | Rich | Poor |
| | Static gains: $\kappa_1$ | | | | | |
| Mean | 1.18 | 1.21 | 1.17 | 1.27 | 1.23 | 1.27 |
| Median | 1.15 | 1.15 | 1.14 | 1.23 | 1.18 | 1.24 |
| Min | 1.04 | 1.05 | 1.04 | 0.98 | 1.02 | 0.98 |
| Max | 2.22 | 1.65 | 2.22 | 2.43 | 1.62 | 2.43 |
| | Gains from trade across dates and states: $\kappa_2$ | | | | | |
| Mean | 1.26 | 1.00 | 1.33 | 2.19 | 1.03 | 2.50 |
| Median | 1.20 | 1.00 | 1.25 | 2.18 | 1.04 | 2.33 |
| Min | 1.00 | 1.00 | 1.05 | 0.97 | 0.97 | 1.54 |
| Max | 2.36 | 1.02 | 2.36 | 6.55 | 1.10 | 6.55 |
| | Gains from trend consumption: $\kappa_3$ | | | | | |
| Mean | 1.14 | 1.00 | 1.18 | 1.29 | 1.02 | 1.36 |
| Median | 1.06 | 1.00 | 1.09 | 1.18 | 1.02 | 1.24 |
| Min | 1.00 | 1.00 | 1.01 | 0.98 | 0.98 | 1.03 |
| Max | 2.83 | 1.01 | 2.83 | 4.87 | 1.06 | 4.87 |
| | Gains from deviations from trend: $\kappa_4$ | | | | | |
| Mean | 1.12 | 1.01 | 1.15 | 1.92 | 1.01 | 2.17 |
| Median | 1.11 | 1.01 | 1.13 | 2.03 | 1.01 | 2.12 |
| Min | 1.00 | 1.00 | 1.04 | 0.97 | 0.97 | 1.71 |
| Max | 1.43 | 1.02 | 1.43 | 3.45 | 1.05 | 3.45 |

**The role of the process for productivity growth**

To examine the role of the estimated process for productivity in driving the results, I redo the welfare calculations under the assumption that developing countries face the same process as developed countries. I hold fixed productivity levels in 2018 and 2019, as well as trade costs. I also hold fixed the productivity process for the US. The implied distribution of gains is reported in the top panel of Table 4. With this modification, the gains from trade for both developed and developing developing countries under financial autarky are almost exactly equal to the static gains from trade. Gains from trade under complete markets for both developed and developing countries are marginally higher than static gains from trade. For developed countries, these gains are close to those under the estimated productivity process, but for developing countries they are dramatically lower.

I repeat this exercise changing only one feature of the non-OECD productivity process at a time. Results are reported in the lower panels of 4. They shows that the key feature of the non-OECD productivity process is the standard deviation of innovations to deviations of growth from trend (second panel). Changing only this feature of the process largely

eliminates gains from trade across dates and states of the world for developing countries.

Table 4: Distribution of gains from trade under alternative productivity processes

| | Static gains | | | 3-D gains | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2019 | | | Financial autarky | | | Complete markets | | |
| | All | Rich | Poor | All | Rich | Poor | All | Rich | Poor |
| | OECD productivity process for Non-OECD | | | | | | | | |
| Mean | 1.18 | 1.21 | 1.17 | 1.18 | 1.22 | 1.17 | 1.23 | 1.25 | 1.22 |
| Med | 1.15 | 1.15 | 1.14 | 1.14 | 1.16 | 1.14 | 1.19 | 1.20 | 1.19 |
| Min | 1.04 | 1.05 | 1.04 | 1.04 | 1.05 | 1.04 | 1.06 | 1.06 | 1.08 |
| Max | 2.22 | 1.65 | 2.22 | 2.23 | 1.69 | 2.23 | 2.29 | 1.75 | 2.29 |
| | OECD innovation variance for Non-OECD | | | | | | | | |
| Mean | 1.18 | 1.21 | 1.17 | 1.18 | 1.21 | 1.18 | 1.22 | 1.25 | 1.21 |
| Med | 1.15 | 1.15 | 1.14 | 1.15 | 1.16 | 1.14 | 1.18 | 1.20 | 1.17 |
| Min | 1.04 | 1.05 | 1.04 | 1.04 | 1.05 | 1.04 | 1.05 | 1.05 | 1.06 |
| Max | 2.22 | 1.65 | 2.22 | 2.25 | 1.68 | 2.25 | 2.30 | 1.74 | 2.30 |
| | OECD comovement with US for Non-OECD | | | | | | | | |
| Mean | 1.18 | 1.21 | 1.17 | 1.50 | 1.22 | 1.58 | 2.78 | 1.27 | 3.19 |
| Med | 1.15 | 1.15 | 1.14 | 1.39 | 1.16 | 1.46 | 2.73 | 1.21 | 2.90 |
| Min | 1.04 | 1.05 | 1.04 | 1.05 | 1.05 | 1.11 | 1.06 | 1.06 | 1.87 |
| Max | 2.22 | 1.65 | 2.22 | 4.55 | 1.69 | 4.55 | 7.47 | 1.78 | 7.47 |
| | OECD convergence for Non-OECD | | | | | | | | |
| Mean | 1.18 | 1.21 | 1.17 | 1.28 | 1.22 | 1.30 | 1.85 | 1.27 | 2.01 |
| Med | 1.15 | 1.15 | 1.14 | 1.20 | 1.16 | 1.23 | 1.80 | 1.22 | 1.87 |
| Min | 1.04 | 1.05 | 1.04 | 1.05 | 1.05 | 1.06 | 1.07 | 1.07 | 1.48 |
| Max | 2.22 | 1.65 | 2.22 | 3.01 | 1.69 | 3.01 | 4.11 | 1.79 | 4.11 |
| | OECD persistence for Non-OECD | | | | | | | | |
| Mean | 1.18 | 1.21 | 1.17 | 1.85 | 1.22 | 2.02 | 4.85 | 1.28 | 5.80 |
| Med | 1.15 | 1.15 | 1.14 | 1.58 | 1.16 | 1.75 | 4.27 | 1.22 | 4.93 |
| Min | 1.04 | 1.05 | 1.04 | 1.05 | 1.05 | 1.14 | 1.07 | 1.07 | 2.41 |
| Max | 2.22 | 1.65 | 2.22 | 7.30 | 1.69 | 7.30 | 23.3 | 1.90 | 23.3 |

**Robustness to alternative values of key parameters**

The two key parameters in the model are the Armington elasticity $\eta$, and the CRRA parameter $\rho$. I examine the sensitivity of 3-D gains from trade to alternative values for each. For $\eta$ I consider values in the range $[1.5, 8]$, which runs from the lower end to the upper end of values used in the trade literature.[7] For $\rho$ I consider values in the range $[0.1, 1.5]$ which

[7]International macroeconomics often uses values for $\eta$ on the interval $[0, 1]$, but values less than or equal to 1 do not make sense in the context of a model with iceberg trade costs, as they imply that import shares in expenditure are insensitive to or increasing in iceberg trade costs.

corresponds to values of the CRRA parameter in the range $[0.75, 10]$, i.e. from less risk averse than log utility to risk aversion of 10. The upper panel of Table 5 reports the mean welfare gains for different values of $\eta$, holding $\rho$ fixed at its baseline value, while the lower panel reports the mean welfare gains holding $\eta$ fixed and varying $\rho$. Note that trade costs and the productivity process depend on $\eta$, so when I change $\eta$, I recompute trade costs and productivity and re-estimate the productivity process before redoing the counterfactuals. In the Appendix, I report the estimated parameters of the productivity process corresponding to each value of $\eta$.

As is well known, holding fixed trade openness, static gains from trade are decreasing in the value of $\eta$. This is reflected in the top left panel of Table 5. In addition to this effect, in the 3-D gains calculation there are two additional offsetting effects. First, endogenous risk sharing through the terms of trade increases as $\eta \to 1$.[8] Second, lower $\eta$ implies higher trade costs are necessary to rationalize observed trade in 2019, and as a result, trade costs are higher throughout. This reduces the ability of trade to provide optimal smoothing of consumption across dates and states of the world. The net effect of these two offsetting forces is that 3-D gains from trade under financial autarky are always greater than static gains, but the contribution of trade across dates and states of the world (i.e. $\kappa_2^i$) to welfare gains does not vary much with $\eta$, and 3-D gains overall are decreasing in $\eta$. In the case of complete markets, 3-D gains are always greater than static gains, and for developing countries there is a U-shaped relationship between 3-D gains and $\eta$.

Turning to $\rho$, which governs risk aversion and the elasticity of intertemporal substitution, higher risk aversion (lower $\rho$) implies higher gains from trade, both under financial autarky and complete markets. The increase in gains from trade with higher risk aversion is most dramatic for developing countries, which have productivity that is more volatile and less correlated with aggregate risk than the productivity of developed countries. A commonly used value of the intertemporal elasticity of substitution corresponds to $\rho = 0.5$. For this value of $\rho$, 3-D gains are only modestly higher than static gains, even for developing countries. Risk aversion of 8, corresponding to a value of $\rho = 0.125$ is not out of line in the finance literature. For this value of $\rho$, 3-D gains for developing countries are dramatically higher than static gains, even under financial autarky. For intuition, note that risk aversion implies that consumption is *complementary* across dates and states. This relates to the work of Ossa (2015) which shows that sectors with low elasticities of substitution can play an important role in magnifiying static gains from trade.

[8] Cole and Obstfeld (1991) consider the case where $\eta = 1$, where movements in the terms of trade perfectly offset shocks to productivity. In an environment with iceberg trade costs, values of $\eta \leq 1$ do not make sense.

Table 5: Average gains from trade under alternative parameter values

| | | Static gains | | | 3-D gains | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 2019 | | | Financial autarky | | | Complete markets | | |
| $\eta$ | $\rho$ | All | Rich | Poor | All | Rich | Poor | All | Rich | Poor |
| 1.5 | 0.25 | 3.21 | 3.59 | 3.11 | 5.04 | 3.65 | 5.42 | 6.69 | 3.81 | 7.47 |
| 3 | 0.25 | 1.28 | 1.33 | 1.27 | 1.62 | 1.34 | 1.70 | 2.57 | 1.40 | 2.88 |
| **4** | **0.25** | **1.18** | **1.21** | **1.17** | **1.50** | **1.22** | **1.58** | **2.78** | **1.27** | **3.19** |
| 5 | 0.25 | 1.13 | 1.15 | 1.13 | 1.45 | 1.16 | 1.52 | 3.08 | 1.23 | 3.57 |
| 6 | 0.25 | 1.10 | 1.12 | 1.10 | 1.43 | 1.13 | 1.50 | 3.47 | 1.21 | 4.08 |
| 8 | 0.25 | 1.07 | 1.08 | 1.07 | 1.39 | 1.09 | 1.47 | 4.26 | 1.21 | 5.07 |
| 4 | 1.5 | 1.18 | 1.21 | 1.17 | 1.18 | 1.21 | 1.17 | 1.29 | 1.23 | 1.30 |
| 4 | 0.75 | 1.18 | 1.21 | 1.17 | 1.20 | 1.21 | 1.20 | 1.47 | 1.25 | 1.53 |
| 4 | 0.5 | 1.18 | 1.21 | 1.17 | 1.23 | 1.21 | 1.23 | 1.63 | 1.26 | 1.73 |
| **4** | **0.25** | **1.18** | **1.21** | **1.17** | **1.50** | **1.22** | **1.58** | **2.78** | **1.27** | **3.19** |
| 4 | 0.125 | 1.18 | 1.21 | 1.17 | 1.97 | 1.22 | 2.17 | 7.79 | 1.29 | 9.54 |
| 4 | 0.1 | 1.18 | 1.21 | 1.17 | 2.03 | 1.22 | 2.25 | 9.88 | 1.29 | 12.18 |

**Discussion**

The most striking finding in Table 2 is that under financial autarky, 3-D gains from trade for developing countries are more than twice as big as static gains from trade. The decomposition of gains into static gains and gains from trade across dates and states, as well as examination of the role of the productivity process, and the relationship between values of $\eta$ and $\rho$ and implied gains, suggests that this is because developing countries are subject to a lot of idosyncratic risk, endogenous movements in the terms of trade provide insurance against this risk, and this insurance is valued by risk averse agents. Focusing only on the static gains from trade ignores this benefit of trade openness.

The role of the terms of trade in risk sharing is illustrated by the negative correlation (-0.93) between log changes in the terms of trade (price of output over the price of consumption) and log changes in productivity under financial autarky. This mechanism is exactly that highlighted by Cole and Obstfeld (1991). Notice that although there is more consumption risk sharing under complete markets, the correlation between the terms of trade and output is less negative in this scenario (-0.84), consistent with risk sharing being provided at least partially through non-zero trade balances.

For developed countries, which are relatively open, the static gains from trade are larger on average than they are for developing countries. But gains from trade across dates and states of the world are substantially smaller, implying smaller 3-D gains from trade both under financial autarky and complete markets. Why are 3-D gains less dramatic for devel-

oped than for developing countries? The results on the role of productivity process in Table 4 show that this can be accounted for in large part by the fact that the standard deviation of innovations to productivity growth is much lower for developed than developing countries. With relatively little idiosyncratic risk, gains from trade across states of the world are modest.

# 7 Conclusion

Static multi-country models of international trade imply relatively modest losses associated with moving from the current level of trade openness to full autarky. But static models ignore a potentially important role for international trade in allowing countries to smooth consumption across states and over time. Under standard CRRA preferences with a reasonable degree of risk aversion, access to this channel of smoothing is potentially welfare-increasing. The extent of welfare gains from access to trade across states and over time depends on the volatility and correlation of productivity across countries, endogenous risk sharing through movements in the terms of trade, and access to financial markets.

I quantify the gains from trade taking into account gains from trade across states and over time, as well as gains from trade within dates and states (3-D gains). For developing countries, which have volatile productivity growth which is not very correlated with that of developed countries, there are substantial gains from trade across states of the world and over time. This is true even under financial autarky. For developing countries, 3-D gains from trade under financial autarky are more than twice as big as static gains from trade. Intuitively, endogenous movements in the terms of trade (as in Cole and Obstfeld (1991)) provide some insurance even under financial autarky, as long as the goods produced by different countries are substitutes, but not perfect substitutes. This insurance is valued if agents are risk averse.

For developed countries, which have less volatile productivity, and face mainly aggregate risk, both because they are big, and because their productivity growth is more highly correlated, gains from trade under financial autarky are very close to static gains. Under complete markets, the 3-D gains from trade for developing countries are dramatically bigger than static gains. However for developed countries, even under complete markets, 3-D gains are only modestly bigger than static gains.

Future work will examine the robustness of these results to alternative specifications of the productivity process, to allowing for capital accumulation, and potentially to non-expected

utility and disaster risk.

# References


[1] Alessandria, G. and Choi, H. (2014), "Establishment Heterogeneity, Exporter Dynamics, and the Effect of Trade Liberalization," *Journal of International Economics* 94(2), 207-223.

[2] Alessandria, G., Choi, H., & Ruhl, K. J. (2021), "Trade adjustment dynamics and the welfare gains from trade," *Journal of International Economics*, 131, 103458.

[3] Allen, T., C. Arkolakis and Y. Takahashi (2020), "Universal Gravity," *Journal of Political Economy* 128(2), 393-433.

[4] Allen, T., and D. Atkin (2022), "Volatility and the Gains from Trade," *Econometrica*, 90(5), 2053-2092.

[5] Alvarez, F. and R. Lucas, (2007), "General equilibrium analysis of the Eaton-Kortum model of international trade," *Journal of Monetary Economics*, 54(6), 1726-1768.

[6] Alvarez, F. (2017), "Capital Accumulation and International Trade," *Journal of Monetary Economics*, 91, 1-18.

[7] Arkolakis, C., A. Costinot and A. Rodriguez-Clare, (2012), "New Trade Models, Same Old Gains?," *American Economic Review*, 102(1), 94-130.

[8] Boehm, C., Levchenko, A. A., Pandalai-Nayar, N., & Toma, H. (2024), "Dynamic Models, New Gains from Trade?" NBER WP 32565.

[9] Buera, F. J., & Oberfield, E. (2020), "The Global Diffusion of Ideas," *Econometrica*, 88(1), 83-114.

[10] Caselli, F., Koren, M., Lisicky, M., & Tenreyro, S. (2020), "Diversification Through Trade," *Quarterly Journal of Economics*, 135(1), 449-502.

[11] Coeurdacier, N., Rey, H. and Winant, P. (2020), "Financial Integration and Growth in a Risky World," *Journal of Monetary Economics* 112, 1-21.

[12] Cole, H. and M. Obstfeld, (1991), "Commodity Trade and International Risk Sharing," *Journal of Monetary Economics* 28, 3-24.

[13] Fitzgerald, D., (2012), "Trade Costs, Asset Market Frictions and Risk Sharing" *American Economic Review*, 102(6), 2700-2733.

[14] Gourinchas, P. and O. Jeanne, (2006), "The Elusive Gains from International Financial Integration," *Review of Economic Studies* 73, 1-27.

[15] Heathcote, J. & Perri, F. (2002), "Financial Autarky and International Business Cycles," *Journal of Monetary Economics* 49, 601-627.

[16] Heathcote, J., & Perri, F. (2014), "Assessing International Efficiency," in *Handbook of International Economics* (Vol. 4, pp. 523-584). Elsevier.

[17] Helpman, E., & Razin, A. (1978), *A Theory of International Trade under Uncertainty*, Academic Press.

[18] Lucas, R. E. (1987). *Models of business cycles*. New York: Basil Blackwel

[19] Newbery, D. M., & Stiglitz, J. E. (1984), "Pareto Inferior Trade," *Review of Economic Studies*, 51(1), 1-12.

[20] Ossa, R. (2015), "Why Trade Matters After All," *Journal of International Economics*, 97(2), 266-277.

[21] Perla, J., Tonetti, C., & Waugh, M. E. (2021), "Equilibrium Technology Diffusion, Trade, and Growth," *American Economic Review*, 111(1), 73-128.

[22] Ravikumar, B., A.M. Santacreu and M. Sposi (2019), "Capital Accumulation and Dynamic Gains from Trade," *Journal of International Economics* 119, 93-110.

[23] Sampson, T. (2016), "Dynamic Selection: An Idea Flows Theory of Entry, Trade, and Growth," *Quarterly Journal of Economics*, 131(1), 315-380.

[24] van Wincoop, E., (1999), "How Big Are Potential Welfare Gains From International Risksharing," *Journal of International Economics*, 47, 109-135.

# A Derivation of welfare formula

The budget constraint for country $i$ is:

$$\Pi_0^i U_0^i = W_0^i + \sum_{t=0}^{\infty} \sum_{s^t} (1-\mu)\, \mu^{\frac{\mu}{1-\mu}} A_t^i \left(s^t\right)^{\frac{1}{1-\mu}} L^i Q_{0,t}^i \left(s^t\right)^{\frac{1}{1-\mu}} P_{0,t}^i \left(s^t\right)^{-\frac{\mu}{1-\mu}}$$

where $\Pi_0^i$, $P_{0,t}^i\left(s^t\right)$ and $Q_{0,t}^i\left(s^t\right)$ are date-0 prices from the perspective of country $i$. Fix $W_0^i$. Normalize $\Pi_0^i = 1$ (this pins down what a dollar is). This implies:

$$U_0^i d\ln U_0^i = \sum_{t=0}^{\infty} \sum_{s^t} (1-\mu)\, Q_{0,t}^i \left(s^t\right) Y_t^i \left(s^t\right) \left( \frac{d\ln Q_{0,t}^i\left(s^t\right)}{1-\mu} - \frac{\mu d\ln P_{0,t}^i\left(s^t\right)}{1-\mu} \right)$$

so

$$d\ln U_0^i = \sum_{t=0}^{\infty} \sum_{s^t} \frac{(1-\mu)\, Q_{0,t}^i\left(s^t\right) Y_t^i\left(s^t\right)}{\Pi_0^i U_0^i} \left( \frac{d\ln Q_{0,t}^i\left(s^t\right)}{1-\mu} - \frac{\mu d\ln P_{0,t}^i\left(s^t\right)}{1-\mu} \right)$$

Defining

$$\theta_{0,t}^i\left(s^t\right) = \frac{(1-\mu)\, Q_{0,t}^i\left(s^t\right) Y_t^i\left(s^t\right)}{\Pi_0^i U_0^i}$$

we can write this as:

$$d\ln U_0^i = \sum_{t=0}^{\infty} \sum_{s^t} \theta_{0,t}^i\left(s^t\right) \left( \frac{d\ln Q_{0,t}^i\left(s^t\right)}{1-\mu} - \frac{\mu d\ln P_{0,t}^i\left(s^t\right)}{1-\mu} \right)$$

Now make use of the expression for the domestic share in date-$t$, history $s^t$ expenditure, $\lambda_{0,t}^{ii}\left(s^t\right)$, to get:

$$d\ln \lambda_{0,t}^{ii}\left(s^t\right) = (1-\eta)\left(d\ln Q_{0,t}^i\left(s^t\right) - d\ln P_{0,t}^i\left(s^t\right)\right)$$

Rearrange:

$$d\ln Q_{0,t}^i\left(s^t\right) = \frac{d\ln \lambda_{0,t}^{ii}\left(s^t\right)}{1-\eta} + d\ln P_{0,t}^i\left(s^t\right)$$

and therefore

$$\frac{d\ln Q_{0,t}^i\left(s^t\right)}{1-\mu} = \frac{d\ln \lambda_{0,t}^{ii}\left(s^t\right)}{(1-\mu)(1-\eta)} + \frac{d\ln P_{0,t}^i\left(s^t\right)}{1-\mu}$$

Substitute this back into the welfare expression to get:

$$d\ln U_0^i = \sum_{t=0}^{\infty} \sum_{s^t} \theta_{0,t}^i\left(s^t\right) \left( \frac{d\ln \lambda_{0,t}^{ii}\left(s^t\right)}{(1-\mu)(1-\eta)} + d\ln P_{0,t}^i\left(s^t\right) \right)$$

Now make use of the expression for $\phi_{0,t}^{i}\left(s^{t}\right)$ to get:

$$d\ln\phi_{0,t}^{i}\left(s^{t}\right)=\left(1-\rho\right)\left(d\ln P_{0,t}^{i}\left(s^{t}\right)-d\ln\mathcal{P}_{0,t}^{i}\right)$$

Rearrange:

$$d\ln P_{0,t}^{i}\left(s^{t}\right)=\frac{d\ln\phi_{0,t}^{i}\left(s^{t}\right)}{\left(1-\rho\right)}+d\ln\mathcal{P}_{0,t}^{i}$$

Substitute this back into the welfare expression to get:

$$d\ln U_{0}^{i}=\sum_{t=0}^{\infty}\sum_{s^{t}}\theta_{0,t}^{i}\left(s^{t}\right)\left(\frac{d\ln\lambda_{0,t}^{ii}\left(s^{t}\right)}{\left(1-\mu\right)\left(1-\eta\right)}+\frac{d\ln\phi_{0,t}^{i}\left(s^{t}\right)}{\left(1-\rho\right)}+d\ln\mathcal{P}_{0,t}^{i}\right)$$

Now use the expression for $\omega_{0,t}^{i}$, and the normalization $\Pi_{0}^{i}=1$ to get:

$$d\ln\omega_{0,t}^{i}=\left(1-\rho\right)d\ln\mathcal{P}_{0,t}^{i}$$

$$d\ln\mathcal{P}_{0,t}^{i}=\frac{d\ln\omega_{0,t}^{i}}{\left(1-\rho\right)}$$

Substituting this into the welfare expression, we get:

$$d\ln U_{0}^{i}=\sum_{t=0}^{\infty}\sum_{s^{t}}\theta_{0,t}^{i}\left(s^{t}\right)\left(\frac{d\ln\lambda_{0,t}^{ii}\left(s^{t}\right)}{\left(1-\mu\right)\left(1-\eta\right)}+\frac{d\ln\phi_{0,t}^{i}\left(s^{t}\right)}{\left(1-\rho\right)}+\frac{d\ln\omega_{0,t}^{i}}{\left(1-\rho\right)}\right)$$

This generalizes the formula for static welfare gains from trade popularized by Arkolakis, Costinot and Rodriguez-Clare (2012). Note that the derivation does not assume that different countries face the same relative prices across dates or states of the world. This implies that conditional on fixing initial net wealth $W_{0}^{i}$, it is independent of what is assumed about international financial markets.

## A.1 Second order approximation

Normalizing $\Pi_{0}^{i}=1$, taking a second order approximation of

$$\Pi_{0}^{i}U_{0}^{i}=W_{0}^{i}+\sum_{t=0}^{\infty}\sum_{s^{t}}\left(1-\mu\right)\mu^{\frac{\mu}{1-\mu}}A_{t}^{i}\left(s^{t}\right)^{\frac{1}{1-\mu}}L^{i}Q_{0,t}^{i}\left(s^{t}\right)^{\frac{1}{1-\mu}}P_{0,t}^{i}\left(s^{t}\right)^{-\frac{\mu}{1-\mu}}$$

yields:

$$U_0^i d\ln U_0^i = \sum_{t=0}^{\infty}\sum_{s^t}(1-\mu)\,Q_{0,t}^i\left(s^t\right)Y_t^i\left(s^t\right)\left[\frac{d\ln Q_{0,t}^i\left(s^t\right)}{1-\mu}-\frac{\mu d\ln P_{0,t}^i\left(s^t\right)}{1-\mu}+\frac{1}{2}\left(\frac{d\ln Q_{0,t}^i\left(s^t\right)^2}{(1-\mu)^2}-\frac{2\mu d\ln Q_{0,t}^i\left(s^t\right)d\ln P_{0,t}^i\left(s^t\right)}{(1-\mu)^2}+\frac{\mu^2 d\ln P_{0,t}^i\left(s^t\right)^2}{(1-\mu)^2}\right)\right]$$

Since

$$d\ln Q_{0,t}^i\left(s^t\right)=\frac{d\ln\lambda_{0,t}^{ii}\left(s^t\right)}{(1-\eta)}+\frac{d\ln\phi_{0,t}^i\left(s^t\right)}{(1-\rho)}$$

and

$$d\ln P_{0,t}^i\left(s^t\right)=\frac{d\ln\phi_{0,t}^i\left(s^t\right)}{(1-\rho)}$$

then

$$d\ln U_0^i = \sum_{t=0}^{\infty}\sum_{s^t}\theta_{0,t}^i\left(s^t\right)\left[\frac{d\ln\lambda_{0,t}^{ii}\left(s^t\right)}{(1-\mu)(1-\eta)}+\frac{d\ln\phi_{0,t}^i\left(s^t\right)}{(1-\rho)}+\frac{1}{2}\left(\left(\frac{d\ln\lambda_{0,t}^{ii}\left(s^t\right)}{(1-\mu)(1-\eta)}\right)^2+2\frac{d\ln\lambda_{0,t}^{ii}\left(s^t\right)}{(1-\mu)(1-\eta)}\frac{d\ln\phi_{0,t}^i\left(s^t\right)}{(1-\rho)}+\left(\frac{d\ln\phi_{0,t}^i\left(s^t\right)}{(1-\rho)}\right)^2\right)\right]$$

where

$$\theta_{0,t}^i\left(s^t\right)=\frac{(1-\mu)\,Q_{0,t}^i\left(s^t\right)Y_t^i\left(s^t\right)}{\Pi_0^i U_0^i}$$

Higher order approximations are derived analogously. Figures **??** and **??** illustrate the performance of first, second and third order approximations to the gains from trade under financial autarky and complete markets respectively.

# B Data

Tables 6, 7, and 8 list the countries in the sample used to recover productivity, along with the dates for which they are present. They also note whether the country is present in the balanced panel used to estimate the productivity process, and whether data is present for 2019 (countries included in the sample used for welfare calculations). OECD membership is also listed.

Table 6: Data coverage I

| Country | 2019 | Balanced | OECD | Country | 2019 | Balanced | OECD |
|---|---|---|---|---|---|---|---|
| Albania | Yes | No | No | Chile | Yes | Yes | Yes |
| Algeria | Yes | Yes | No | China | Yes | Yes | No |
| Angola | Yes | No | No | Colombia | Yes | Yes | No |
| Argentina | Yes | No | No | Comoros | Yes | No | No |
| Armenia | Yes | No | No | Congo, Dem Rep | Yes | No | No |
| Aruba | Yes | No | No | Congo | Yes | Yes | No |
| Australia | Yes | Yes | Yes | Costa Rica | Yes | Yes | No |
| Austria | Yes | Yes | Yes | Cote d'Ivoire | Yes | Yes | No |
| Azerbaijan | Yes | No | No | Croatia | Yes | No | No |
| Bahamas | Yes | No | No | Cyprus | Yes | No | No |
| Bahrain | Yes | No | No | Czech Republic | Yes | No | Yes |
| Bangladesh | Yes | No | No | Denmark | Yes | Yes | Yes |
| Barbados | Yes | No | No | Dominica | No | No | No |
| Belarus | Yes | No | No | Dominican Republic | Yes | Yes | No |
| Belgium | Yes | Yes | Yes | Ecuador | Yes | Yes | No |
| Belize | Yes | No | No | Egypt | Yes | Yes | No |
| Benin | Yes | Yes | No | El Salvador | Yes | Yes | No |
| Bermuda | Yes | No | No | Equatorial Guinea | Yes | No | No |
| Bhutan | Yes | No | No | Estonia | Yes | No | Yes |
| Bolivia | Yes | Yes | No | Eswatini | Yes | No | No |
| Bosnia & Herzegovina | Yes | No | No | Ethiopia | Yes | No | No |
| Botswana | Yes | No | No | Fiji | Yes | No | No |
| Brazil | Yes | No | No | Finland | Yes | Yes | Yes |
| Brunei | Yes | No | No | France | Yes | Yes | Yes |
| Bulgaria | Yes | No | No | Gabon | Yes | Yes | No |
| Burkina Faso | Yes | Yes | No | Gambia | Yes | Yes | No |
| Burundi | Yes | Yes | No | Georgia | Yes | No | No |
| Cabo Verde | Yes | No | No | Germany | Yes | Yes | Yes |
| Cambodia | Yes | No | No | Ghana | Yes | Yes | No |
| Cameroon | Yes | Yes | No | Greece | Yes | Yes | Yes |
| Canada | Yes | Yes | Yes | Guatemala | Yes | Yes | No |
| Central African Republic | Yes | Yes | No | Guinea | Yes | No | No |
| Chad | Yes | Yes | No | Guinea-Bissau | Yes | Yes | No |

Table 7: Data coverage II

| Country | 2019 | Balanced | OECD | Country | 2019 | Balanced | OECD |
|---|---|---|---|---|---|---|---|
| Guyana | No | No | No | Mexico | Yes | Yes | Yes |
| Haiti | Yes | No | No | Moldova | Yes | No | No |
| Honduras | Yes | Yes | No | Mongolia | Yes | No | No |
| Hungary | Yes | No | Yes | Montenegro | Yes | No | No |
| Iceland | Yes | Yes | Yes | Morocco | Yes | Yes | No |
| India | Yes | Yes | No | Mozambique | Yes | No | No |
| Indonesia | Yes | Yes | No | Namibia | Yes | No | No |
| Iran | Yes | No | No | Nepal | Yes | Yes | No |
| Iraq | Yes | Yes | No | Netherlands | Yes | Yes | Yes |
| Ireland | Yes | Yes | Yes | New Zealand | Yes | Yes | Yes |
| Israel | Yes | Yes | Yes | Nicaragua | Yes | No | No |
| Italy | Yes | Yes | Yes | Niger | Yes | Yes | No |
| Jamaica | Yes | Yes | No | Nigeria | Yes | No | No |
| Japan | Yes | Yes | No | North Macedonia | Yes | No | No |
| Jordan | Yes | No | No | Norway | Yes | Yes | Yes |
| Kazakhstan | Yes | No | No | Oman | Yes | No | No |
| Kenya | Yes | Yes | No | Pakistan | Yes | Yes | No |
| Korea | Yes | Yes | Yes | Panama | Yes | Yes | No |
| Kuwait | Yes | Yes | No | Paraguay | Yes | Yes | No |
| Kyrgyz Republic | Yes | No | No | Peru | Yes | No | No |
| Laos | No | No | No | Philippines | Yes | No | No |
| Latvia | Yes | No | No | Poland | Yes | No | Yes |
| Lebanon | Yes | No | No | Portugal | Yes | Yes | Yes |
| Lesotho | Yes | No | No | Qatar | Yes | No | No |
| Lithuania | Yes | No | No | Romania | Yes | No | No |
| Macao | Yes | No | No | Russia | Yes | No | No |
| Madagascar | Yes | Yes | No | Rwanda | Yes | Yes | No |
| Malawi | Yes | No | No | Saudi Arabia | Yes | Yes | No |
| Malaysia | Yes | Yes | No | Senegal | Yes | Yes | No |
| Mali | Yes | Yes | No | Serbia | Yes | No | No |
| Malta | Yes | Yes | No | Seychelles | Yes | No | No |
| Mauritania | Yes | Yes | No | Sierra Leone | Yes | Yes | No |
| Mauritius | Yes | No | No | Slovak Republic | Yes | No | Yes |

Table 8: Data coverage III

| Country | 2019 | Balanced | OECD |
|---|---|---|---|
| Slovenia | Yes | No | Yes |
| South Africa | Yes | No | No |
| Spain | Yes | Yes | Yes |
| Sri Lanka | Yes | Yes | No |
| Sudan | Yes | Yes | No |
| Sweden | Yes | Yes | Yes |
| Switzerland | Yes | Yes | Yes |
| Syria | Yes | Yes | No |
| Taiwan | Yes | Yes | No |
| Tajikistan | Yes | No | No |
| Tanzania | Yes | No | No |
| Thailand | Yes | Yes | No |
| Togo | Yes | Yes | No |
| Tunisia | Yes | Yes | No |
| Turkey | Yes | Yes | Yes |
| Turkmenistan | Yes | No | No |
| Uganda | Yes | Yes | No |
| Ukraine | Yes | No | No |
| UAE | Yes | No | No |
| UK | Yes | Yes | Yes |
| USA | Yes | Yes | Yes |
| Uruguay | Yes | Yes | No |
| Uzbekistan | Yes | No | No |
| Venezuela | No | No | No |
| Vietnam | Yes | No | No |
| West Bank & Gaza | Yes | No | No |
| Yemen | No | No | No |
| Zambia | Yes | No | No |
| Zimbabwe | Yes | No | No |

# C Estimation of the productivity process

Let $\mathbf{a}_t = \left[\ln A_t^{US}, A_t^{O_1}, \ldots, A_t^{O_{N_O}}, A_t^{N_1}, \ldots, A_t^{N_{N_N}}\right]'$ be the vector of log productivity for all countries in the estimation sample at date $t$. The first country is the US, then $N_O$ OECD countries, followed by $N_N$ non-OECD countries. The productivity process to be estimated can be written:

$$(\mathbf{a}_t - \mathbf{a}_{t-1}) = \mathbf{g} + \Gamma \mathbf{a}_{t-1} + \boldsymbol{\varepsilon}_t$$

where

$$\boldsymbol{\varepsilon}_t = \Psi \boldsymbol{\varepsilon}_{t-1} + \Omega \boldsymbol{\eta}_t$$

and

$$\mathbf{g} = \begin{bmatrix} g^{US} \\ \begin{bmatrix} g^{US} \\ \vdots \\ g^{US} \end{bmatrix} \\ \begin{bmatrix} g^{US} \\ \vdots \\ g^{US} \end{bmatrix} \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} [0] & \begin{bmatrix} 0 & \ldots & 0 \end{bmatrix} & \begin{bmatrix} 0 & \ldots & 0 \end{bmatrix} \\ \begin{bmatrix} \gamma^O \\ \vdots \\ \gamma^O \end{bmatrix} & \begin{bmatrix} -\gamma^O & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & -\gamma^O \end{bmatrix} & \begin{bmatrix} 0 & \ldots & 0 \\ 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 0 \end{bmatrix} \\ \begin{bmatrix} \gamma^N \\ \vdots \\ \gamma^N \end{bmatrix} & \begin{bmatrix} 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 0 \end{bmatrix} & \begin{bmatrix} -\gamma^N & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & -\gamma^N \end{bmatrix} \end{bmatrix}$$

$$\Psi = \begin{bmatrix} \left[\psi^{US}\right] & \begin{bmatrix} 0 & \ldots & 0 \end{bmatrix} & \begin{bmatrix} 0 & \ldots & 0 \end{bmatrix} \\ \begin{bmatrix} \psi_O^{US} \\ \vdots \\ \psi_O^{US} \end{bmatrix} & \begin{bmatrix} \psi_O^{O} & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \psi_O^{O} \end{bmatrix} & \begin{bmatrix} 0 & \ldots & 0 \\ 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 0 \end{bmatrix} \\ \begin{bmatrix} \psi_N^{US} \\ \vdots \\ \psi_N^{US} \end{bmatrix} & \begin{bmatrix} 0 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 0 \end{bmatrix} & \begin{bmatrix} \psi_N^{N} & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \psi_N^{N} \end{bmatrix} \end{bmatrix}$$

$$\Omega = \begin{bmatrix} \begin{bmatrix} \sigma^{US} \end{bmatrix} & \begin{bmatrix} 0 & \dots & 0 \end{bmatrix} & \begin{bmatrix} 0 & \dots & 0 \end{bmatrix} \\ \begin{bmatrix} \sigma_O^{US} \\ \vdots \\ \sigma_O^{US} \end{bmatrix} & \begin{bmatrix} \sigma_O^{O} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_O^{O} \end{bmatrix} & \begin{bmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{bmatrix} \\ \begin{bmatrix} \sigma_N^{US} \\ \vdots \\ \sigma_N^{US} \end{bmatrix} & \begin{bmatrix} 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{bmatrix} & \begin{bmatrix} \sigma_N^{N} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_N^{N} \end{bmatrix} \end{bmatrix}$$

Condition on $\mathbf{a}_1$. Then at $t = 2$:

$$(\mathbf{a}_2 - \mathbf{a}_1) = \mathbf{g} + \Gamma \mathbf{a}_1 + \boldsymbol{\varepsilon}_2$$

and

$$\boldsymbol{\varepsilon}_2 = \Psi \boldsymbol{\varepsilon}_1 + \Omega \boldsymbol{\eta}_2$$

so

$$(\mathbf{a}_2 - \mathbf{a}_1) = \mathbf{g} + \Gamma \mathbf{a}_1 + \Psi \boldsymbol{\varepsilon}_1 + \Omega \boldsymbol{\eta}_2$$

We don't observe $\boldsymbol{\varepsilon}_1$, but we do know its unconditional distribution:

$$\boldsymbol{\varepsilon}_1 \sim N\left(0, (I - \Psi)^{-1} \Omega \Omega' \left((I - \Psi)^{-1}\right)'\right).$$

Meanwhile, $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\eta}_2$ are independent. So

$$(\mathbf{a}_2 - \mathbf{a}_1) | \mathbf{a}_1 \sim N\left(\mathbf{g} + \Gamma \mathbf{a}_1, \Omega \Omega' + \Psi (I - \Psi)^{-1} \Omega \Omega' \left((I - \Psi)^{-1}\right)' \Psi'\right).$$

In general, for $t > 2$

$$(\mathbf{a}_t - \mathbf{a}_{t-1}) = \mathbf{g} + \Gamma \mathbf{a}_{t-1} + \boldsymbol{\varepsilon}_t$$

while

$$\boldsymbol{\varepsilon}_t = \Psi \boldsymbol{\varepsilon}_{t-1} + \Omega \boldsymbol{\eta}_t$$

and

$$\boldsymbol{\varepsilon}_{t-1} = (\mathbf{a}_{t-1} - \mathbf{a}_{t-2}) - \mathbf{g} - \Gamma \mathbf{a}_{t-2}$$

so

$$(\mathbf{a}_t - \mathbf{a}_{t-1}) = (I - \Psi) \mathbf{g} + (\Gamma + \Psi) \mathbf{a}_{t-1} - \Psi (I + \Gamma) \mathbf{a}_{t-2} + \Omega \boldsymbol{\eta}_t$$

and

$$(\mathbf{a}_t - \mathbf{a}_{t-1}) | \mathbf{a}_1, \ldots, \mathbf{a}_{t-1} \sim N\left((I - \Psi)\,\mathbf{g} + (\Gamma + \Psi)\,\mathbf{a}_{t-1} - \Psi\,(I + \Gamma)\,\mathbf{a}_{t-2}, \Omega\Omega'\right)$$

Let $\boldsymbol{\theta} = \{\mathbf{g}, \Gamma, \Psi, \Omega\}$.The log likelihood conditional on $\mathbf{a}_1$ is given by:

$$L\left(\Delta\mathbf{a}_T, \ldots, \Delta\mathbf{a}_2 | \mathbf{a}_1; \boldsymbol{\theta}\right) = -\sum_{t=2}^{T} \frac{1}{2} \left[ \begin{array}{c} \left(\Delta\mathbf{a}_t - \boldsymbol{\mu}_t\left(\mathbf{a}_1, \ldots, \mathbf{a}_{t-1}; \boldsymbol{\theta}\right)\right)' \Sigma_t\left(\boldsymbol{\theta}\right)^{-1} \left(\Delta\mathbf{a}_t - \boldsymbol{\mu}_t\left(\mathbf{a}_1, \ldots, \mathbf{a}_{t-1}; \boldsymbol{\theta}\right)\right) \\ \\ - \ln\left(\left|\Sigma_t\left(\boldsymbol{\theta}\right)^{-1}\right|\right) + N_t \ln\left(2\pi\right) \end{array} \right]$$

with

$$\boldsymbol{\mu}_t\left(\mathbf{a}_1, \ldots, \mathbf{a}_{t-1}; \boldsymbol{\theta}\right) = \begin{cases} \mathbf{g} + \Gamma\mathbf{a}_1 & t = 2 \\ (I - \Psi)\,\mathbf{g} + (\Gamma + \Psi)\,\mathbf{a}_{t-1} - \Psi\,(I + \Gamma)\,\mathbf{a}_{t-2} & t > 2 \end{cases}$$

and

$$\Sigma_t\left(\boldsymbol{\theta}\right) = \begin{cases} \Omega\Omega' + \Psi\left(I - \Psi\right)^{-1}\Omega\Omega'\left(\left(I - \Psi\right)^{-1}\right)'\Psi' & t = 2 \\ \Omega\Omega' & t > 2 \end{cases}$$

The idea is to choose $\boldsymbol{\theta} = \{\mathbf{g}, \Gamma, \Psi, \Omega\}$ to maximize the log likelihood $L\left(\Delta\mathbf{a}_T, \ldots, \Delta\mathbf{a}_2 | \mathbf{a}_1; \boldsymbol{\theta}\right)$. This is done in two steps. First, the parameters $\boldsymbol{\theta}^{US} = \left\{g^{US}, \psi^{US}, \sigma^{US}\right\}$ are chosen to maximize $L\left(\Delta a_T^{US}, \ldots, \Delta a_2 | a_1; \boldsymbol{\theta}^{US}\right)$. Then the remaining parameters $\left\{\boldsymbol{\theta}^O, \boldsymbol{\theta}^N\right\}$ are chosen to maximize the log likelihood conditional on the estimates of $\boldsymbol{\theta}^{US}$. That is, what is maximized is $L\left(\Delta\mathbf{a}_T^{nonUS}, \ldots, \Delta\mathbf{a}_2^{nonUS} | \mathbf{a}_1^{nonUS}, \boldsymbol{\theta}^{US}; \boldsymbol{\theta}^O, \boldsymbol{\theta}^N\right)$.

# D Additional results

Table 9: Static gains from trade under observed trade balances (2019)

| | All | Rich | Poor |
|---|---|---|---|
| Mean | 1.21 | 1.18 | 1.21 |
| Med | 1.17 | 1.14 | 1.19 |
| Min | 0.62 | 1.05 | 0.62 |
| Max | 2.11 | 1.74 | 2.11 |

Table 10: Parameters of productivity process for alternative values of $\eta$

| | Value of $\eta$ | | | | | |
|---|---|---|---|---|---|---|
| Parameter | 1.5 | 3 | 4 | 5 | 6 | 8 |
| $g^{US}$ | 0.0085 | 0.0085 | 0.0085 | 0.0085 | 0.0085 | 0.0085 |
| $\psi^{US}$ | 0.31 | 0.31 | 0.31 | 0.31 | 0.31 | 0.31 |
| $\sigma^{US}$ | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| $\gamma^{O}$ | 0.0053 | 0.0089 | 0.0102 | 0.0108 | 0.0122 | 0.0128 |
| $\psi^{O}$ | 0.11 | 0.15 | 0.11 | 0.10 | 0.05 | 0.04 |
| $\psi^{O,US}$ | 0.26 | 0.32 | 0.38 | 0.45 | 0.53 | 0.64 |
| $\sigma^{O}$ | 0.03 | 0.02 | 0.02 | 0.03 | 0.03 | 0.04 |
| $\sigma^{O,US}$ | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| $\gamma^{N}$ | 0.0025 | 0.0026 | 0.0027 | 0.0027 | 0.0022 | 0.0014 |
| $\psi^{N}$ | -0.08 | -0.05 | -0.04 | -0.05 | -0.04 | -0.04 |
| $\psi^{N,US}$ | 0.05 | 0.17 | 0.24 | 0.27 | 0.23 | 0.21 |
| $\sigma^{N}$ | 0.10 | 0.09 | 0.09 | 0.10 | 0.10 | 0.10 |
| $\sigma^{N,US}$ | 0.001 | 0.001 | 0.001 | 0.000 | 0.000 | 0.000 |

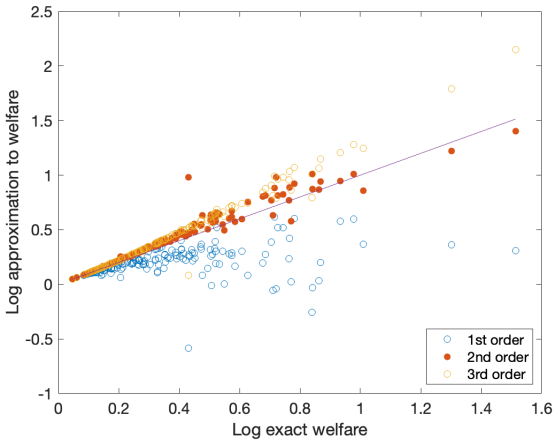



Figure 5: Exact vs approx: fin autarky

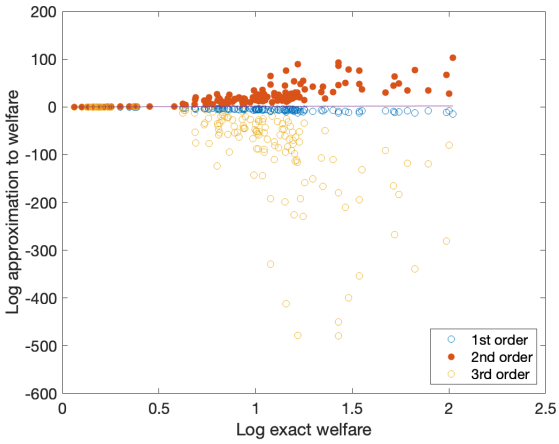



Figure 6: Exact vs approx: complete mkts