International Equity Flows, Monetary Policy, and Time Consistency *

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Abstract

This paper shows that international equity flows create a time inconsistency problem for monetary policy. When foreign investors own domestic equity, the government is tempted to depart from full employment to reduce foreign investors' profits. Anticipating low returns, investors buy equity at reduced prices, leading to a reduction in domestic households' wealth. Moreover, we show that households sell too much equity in the competitive equilibrium and that this can give rise to coordination failures. Unlike existing theories, the model suggests the desirability of capital controls on equity inflows.

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1 Introduction

Over the past two decades, international equity flows have steadily expanded, driven by the rapid deepening and development of financial markets. A widely held view is that the increase in cross-border equity flows can contribute to international risk-sharing and raise welfare. In line with this view, the literature on international real business cycles has shown that allowing households to trade equity and other securities with foreign investors contributes to diversifying income risk and stabilizing consumption. However, there is a question of whether international equity flows remain desirable once we depart from a frictionless environment.

This paper explores the implications of international equity flows in a model featuring nominal frictions. Our analysis reveals that international equity flows introduce a time inconsistency problem for monetary policy, resulting in a coordination failure where households sell an excessive share of domestic equity to foreign investors. We show that when domestic agents sell their domestic equity, they create a temptation for the central bank to use monetary policy to diminish future profits, lowering equity prices and reducing household wealth. Our theory underscores that international equity flows can introduce an inefficiency where none had existed.

We formalize these insights in a simple open-economy New Keynesian model that allows households to trade domestic equity with deep-pocket foreign investors. To transparently illustrate our results, the baseline model abstracts from uncertainty, imperfections in capital markets, and domestic heterogeneity (although a model extension considers how equity can help relax borrowing constraints and improve intertemporal consumption smoothing). To understand our results, consider first the case where households own the entire share of domestic equity. As is standard in the literature, the optimal monetary policy induces full employment (or, in other words, a zero labor wedge or zero output gap). That is, the government raises employment until the point at which the marginal value of consumption resulting from one additional unit of labor compensates for the disutility of providing one more unit of labor.

In an economy where households own all domestic equity, an increase in real wages, given a level of employment, has no effects on the resource constraint. Changes in labor income and profits cancel out in the households' budget constraint. The split between labor income and profits is irrelevant. However, this is not the case when foreign investors own domestic equity. When foreign investors own domestic equity, higher real wages now translate into lower profits for foreign investors while the budget constraint of domestic households is relaxed. Although domestic households also face lower profits, this is more than compensated by higher salaries.

The split between labor income and profits is no longer irrelevant.

This observation has important implications for monetary policy. To the extent that monetary policy affects real wages, a central bank has incentives to target higher real wages in equilibrium when foreign investors own a share of domestic equity. That is, starting from a zero labor wedge, the central bank perceives a first-order gain from increasing real wages to reduce equity payouts for foreign investors.

The time inconsistency problem emerges because when foreign investors buy equity, they anticipate the central bank's incentives to erode their profits ex-post and, therefore, buy equity at a lower price. As a consequence, the inability of the central bank to commit translates into lower wealth for domestic households. When households sell equity, they do not internalize that a higher share of foreign-owned equity will prompt the central bank to lower profits, resulting in a fall in the price of equity today and lower welfare. Collectively, they therefore sell too much equity. Sales of domestic equity end up distorting output ex-post, leaving no benefits to households ex-ante. Furthermore, we show that multiple equilibria exist, such that the equilibrium without foreign ownership of domestic equity delivers the highest welfare for domestic households. The policy implication is that restricting purchases of equity by foreigners is optimal. In particular, when domestic households own initially the entire stock of equity, the optimal policy takes a simple form: banning all equity inflows is optimal.

When foreign investors initially own some equity, the optimal capital control faces a tradeoff. On one hand, repurchasing equity from foreign investors ensure that output is closer to the full employment. On the other hand, doing so maximizes the capital gains by foreign investors at the expense of domestic households. As a result, we show that it is optimal repurchase only a small share of the domestic equity from foreign investors.

In the context of a model with a single (tradable) good and sticky wages, the implication is that a contractionary monetary policy is optimal when foreign investors own a fraction of the domestic equity. That is, a tight monetary policy leads to an appreciated exchange rate and a higher real wage. As explained above, a higher real wage shifts resources from foreign investors to domestic households.

Extending the model to allow for non-tradable goods, our result that monetary policy departs from full employment remains unchanged, but now an important consideration is the relative price of non-tradables. While an appreciation lowers profits in firms in the non-tradable sector in units of domestic currency, as the price of non-tradable goods increases, profits in units of tradables may increase depending on the elasticity of substitution between tradables and non-tradables. We show that if the elasticity is above (below) one, it is

optimal to appreciate (depreciate) the exchange rate. When the elasticity is equal to one, full employment is optimal. In this case, changes in monetary policy do not affect wages or profits in terms of tradables, which implies that the time inconsistency problem of monetary policy is muted.

Related literature. Our paper is related to an extensive literature on monetary policy in open economies. A key theme in this literature, common with closed economy models, is about macro-stabilization. When prices or wages are rigid, monetary policy can help align relative prices toward the full-employment allocation. However, when a country fixes the exchange rate or joins a monetary union, it loses the ability to use the exchange rate as a shock absorber (Mundell, 1960; Friedman, 1953; Schmitt-Grohé and Uribe, 2001).

A vast number of studies have also examined channels by which a departure from full employment may be optimal. One strand of this literature is concerned with terms of trade manipulation. When countries have market power in the tradable goods they produce, central banks can use monetary policy to tilt the terms of trade on their favor (e.g., Obstfeld and Rogoff, 1995 and Corsetti and Pesenti, 2005). Another strand of this literature considers the case when foreign assets and liabilities are in domestic currency and analyzes how variations in exchange rates can generate valuation effects on the net foreign asset position. In an incomplete market economy with endogenous portfolios, Fanelli (2023) shows how the optimal monetary policy under commitment balances macro-stabilization and insurance while Ottonello and Perez (2019), Du, Pflueger and Schreger (2020) and Engel and Park (2022) study the tradeoffs for the optimal monetary policy without commitment. In contrast to these contributions, we consider a model where agents trade equity with foreign investors and show how this gives rise to a novel source of time inconsistency.

Our paper is also related to the literature on international portfolio choice.[1] A central theme in this literature is on understanding the diversification of capital flows in the context of real models, thus abstracting from the role of monetary policy. A notable exception is Engel and Matsumoto (2009). In a model with sticky prices, they show how a portfolio of home and foreign equities and a forward position in foreign exchange can achieve to a first-order the complete market allocations. However, they do not consider optimal monetary policy.

Our paper is also related to the literature on monetary unions, in particular, those emphasizing how joining a monetary union can deal with time inconsistency problems in

[1]See, for example, Atkeson, Heathcote and Perri (2022) Engel and Matsumoto (2009), Coeurdacier, Kollmann and Martin (2010), Devereux and Sutherland (2006), Tille and Van Wincoop (2010).

monetary policy. In Alesina and Barro (2002), joining a monetary reduces the inflationary bias generated by the time inconsistency problem of monetary policy, as stressed by Barro and Gordon (1983). Fornaro (2022) shows that a fixed exchange rate can help increase capital inflows in an environment where debt flows must be backed by non-tradable goods as collateral. He shows that the central bank has incentives to depreciate the exchange rate to reduce the value of collateral that foreign investors can recover. Different from these studies, the time inconsistency problem of monetary policy in this paper is related to international equity flows. We show that this channel provides distinctive implications for monetary policy and capital controls.

We also relate to the literature on monetary policy with heterogenous agents in closed economies. Much of the HANK literature assumes an equal share of profits between households or considers setups where equilibrium profits are zero, muting redistributive motives through equity holdings (e.g., Dávila and Schaab, 2023). Two exceptions are Bhandari, Evans, Golosov and Sargent (2021) and Acharya, Challe and Dogra (2020). However, these studies examine optimal policy under commitment. Moreover, they assume that fiscal policies are in place that effectively equalize marginal utilities of consumption at the steady state. Thus, their focus is on how monetary policy can provide insurance against aggregate shocks. In our open economy model, we examine a redistribution motive that emerges naturally because the government is unable to tax foreign investors and does not place any weight on their welfare.

Finally, we are also related to the literature on capital controls and capital flow management policies. This literature has developed a rationale for taxes on debt flows based on pecuniary externalities (e.g., Bianchi, 2011) and aggregate demand externalities (e.g., Schmitt-Grohé and Uribe, 2016; Farhi and Werning, 2016).[2] Here we provide a rationale for capital controls on equity flows based on a time inconsistency problem of monetary policy.

Outline. The remainder of the paper is organized as follows. Section 2 introduces the environment. Section 3 analyzes optimal monetary policy. Section 3.4 analyzes capital controls. Section 5.1 analyzes extensions of the baseline framework. Section 6 concludes. The proofs of all the lemmas and propositions stated in the paper are in the Appendix.

[2]See Bianchi and Lorenzoni (2021) for a review of the literature.

2 The Model

Consider a dynamic small open economy (SOE), with time indexed by $t = 0, 1...$, and without uncertainty. A continuum of identical households and firms of unit measures populates this SOE. The rest of the world economy is populated by a continuum of foreign investors, who trade domestic firms' shares with the SOE and foreign-denominated debt securities in international debt markets.

Firms. Firms produce a tradable good with labor hours using a standard production technology with decreasing marginal returns. Their problem is static, and consists on choosing labor hours to maximize profits Π_t, with

$$\Pi_t \equiv \max_{h_t \geq 0} \left\{ E_t F\left(h_t\right) - W_t h_t \right\} \ , \tag{1}$$

where E_t is the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency, W_t is the nominal wage in domestic currency, and $F\left(\cdot\right)$ is the production technology, which has the following properties: $F\left(0\right) = 0$, $F'\left(\cdot\right) > 0$, and $F''\left(\cdot\right) < 0$, $\lim_{h\to 0} F'(h) = \infty, \lim_{h\to\infty} F'(h) = 0$.

We assume the law of one price holds, and without loss of generality, we normalize the international good price to 1. Profit maximization implies a standard labor demand, given by

$$h_t = F'^{-1}\left(\frac{W_t}{E_t}\right) \ , \ \forall t \geq 0 \ . \tag{2}$$

Households. Households consume the tradable good and supply the labor hours. They have additively separable preferences between consumption and labor supply, given by

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v\left(l_t\right)\right] , \tag{3}$$

where $\beta \in (0, 1)$ is their subjective discount factor, $c_t \geq 0$ is their consumption, $l_t \geq 0$ is their labor supply. The functions $u\left(\cdot\right)$ and $v\left(\cdot\right)$ satisfy the following standard properties: $u'\left(\cdot\right) > 0$, $u''\left(\cdot\right) < 0$, $v'\left(\cdot\right) \geq 0$, and $v''\left(\cdot\right) \geq 0$. We also assume Inada conditions, $\lim_{c\to 0} u'(c) = \infty$, $\lim_{c\to\infty} u'(c) = 0$ and $\lim_{l\to 0} v'(l) = 0, \lim_{l\to\infty} v'(l) = \infty$.

Households trade domestic firms' shares and one-period bonds with foreign investors in international financial markets. Let $b_t \in \mathbb{R}$ and $\theta_t \geq 0$ respectively be their holdings at the

beginning of the period of the bonds and the shares. Then, their budget constraint is

$$c_t + b_{t+1} + Q_t\theta_{t+1} = \frac{W_t}{E_t}l_t + Rb_t + \left(\frac{\Pi_t}{E_t} + Q_t\right)\theta_t \ , \tag{4}$$

where R is the gross international interest rate and Q_t is the price of firms' shares in foreign currency in international equity markets. As standard, firms' shares entitle their owner to the counterpart share of firm profits, and the bonds and the shares are denominated in foreign currency, without loss of generality. Households are subject to financial frictions and these are such that debt issuance (i.e., negative bond holdings) is limited according to

$$b_{t+1} \geq -\underline{b} \ , \tag{5}$$

where parameter $\underline{b} \geq 0$ is interpreted as the degree of integration of the SOE in international debt markets. At period $t = 1$, in addition, households are subject to wage rigidity, which is such that without optimizing they must serve the aggregate quantity demanded of labor hours at the prevailing nominal wage. Formally,

$$l_1 = F'^{-1}\left(\frac{\overline{W}_1}{E_1}\right), \tag{6}$$

where $\overline{W}_1$ denotes an exogenous rigid wage. Notice that for simplicity we have assumed that the aggregate number of hours is evenly distributed across the households.

At every period households optimize over their consumption and portfolio of assets. In addition, they also optimize over the labor supply except at $t = 1$ where we have assumed that wages are sticky and the number of hours is determined by firms' labor demand. Let $V_t(b_t, \theta_t)$ be the value at the beginning of the period of a household with initial asset portfolio (b_t, θ_t). Then, the problem of households can be represented recursively as follows:

$$V_t(b_t, \theta_t) = \max_{c_t, l_t, b_{t+1} \geq -\underline{b}, \theta_{t+1} \geq 0} \{u(c_t) - v(l_t) + \beta V_{t+1}(b_{t+1}, \theta_{t+1})\} \tag{7}$$

subject to:

$$c_t + b_{t+1} + Q_t\theta_{t+1} = \frac{W_t}{E_t}l_t + Rb_t + \left(\frac{\Pi_t}{E_t} + Q_t\right)\theta_t$$

and (6) if $t = 1$. It is worth highlighting that if the return on bonds equals the return of firms' shares, the household will be indifferent between different composition of assets to the extent that they deliver the same amount of consumption and savings.

Foreign investors. The investors have deep pockets. Then, in a competitive equilibrium, the return on bonds must equal the return on firms' shares. That is, the price of firms' shares must satisfy

$$RQ_t = \frac{\Pi_{t+1}}{E_{t+1}} + Q_{t+1} \ , \ \forall t \geq 0 \ . \tag{8}$$

Competitive Equilibrium. We can now define the competitive equilibrium for a given exchange rate policy. We restrict attention to a symmetric equilibrium, in which all of the households always take the same consumption, labor, and portfolio decisions.

Definition 1. Given initial conditions (B_0, Θ_0), a rigid wage $\overline{W}_1$, and an exchange rate policy, $\{E_t\}_{t=0}^{\infty}$, a *competitive equilibrium* is a sequence of allocations $\{C_t, H_t\}_{t=0}^{\infty}$, portfolios $\{B_{t+1}, \Theta_{t+1}\}_{t=0}^{\infty}$ and prices $\{W_t, Q_t\}_{t=0}^{\infty}$ such that:

(i) Household optimize. That is, consumption, labor, and portfolios solve problem (7).

(ii) Firms choose employment optimally. That is, (2) holds.

(iii) Prices for firms' shares satisfies condition (8).

Disposable income and balance of payments. As we will see, a key equilibrium object for our analysis is the *disposable income net of foreign-owned firms' profits*. That is, formally,

$$I(H_t, \Theta_t) \equiv F(H_t) - (1 - \Theta_t) \underbrace{[F(H_t) - F'(H_t) H_t]}_{=\Pi_t / E_t} , \tag{9}$$

This expression follows from substituting labor demand (2) and firms' profits (1) into households' aggregate income $(W_t/E_t) H_t + (\Pi_t/E_t) \Theta_t$. Combining the income with households' budget constraint (4), one obtains a standard *balance of payment condition*,

$$I(H_t, \Theta_t) + (R-1)B_t - C_t = [B_{t+1} - (1 - \Theta_{t+1})Q_t] - [B_t - (1 - \Theta_t)Q_t], \tag{10}$$

which states that the current account (LHS) is given by the change in the net foreign asset position (RHS). Equations (9) and (10) then make clear that foreign holdings of domestic equity constitute a liability for the domestic economy. Specifically, to run a current account deficit, the economy must either reduce its bond positions or increase its liabilities by selling domestic equities.

3 International Equity Flows and Exchange Rate Policy

In this section, we characterize the equilibrium and analyze the implications of international equity flows for the optimal exchange rate policy.

To derive a tractable equilibrium, we assume $\beta R = 1$, which guarantees a stationary continuation equilibrium from period $t = 2$ onwards. In addition, to simplify the exposition, we first consider a case in which the debt limit is always slack, and then we study the case in which it is potentially binding.

Roadmap. Given these assumptions, we can solve the model following these steps. First, we characterize the steady-state equilibrium for $t \geq 2$. Second, we analyze the equilibrium and the optimal exchange rate policy at period $t = 1$, taking into account the continuation values for $t \geq 2$. Finally, we turn to period $t = 0$ and examine the scope for capital controls.

3.1 Steady-state equilibrium for $t \geq 2$

The assumption that nominal wages are flexible after $t \geq 2$ ensures monetary neutrality in the continuation equilibrium at period $t = 2$. Moreover, given that $\beta R = 1$, we have that all allocations are constant. We use the subscript "ss" to denote steady-state values. The following lemma characterizes this continuation equilibrium for any given exchange rate policy and for any initial portfolio (B_2, Θ_2).

Lemma 1 (Steady-state equilibrium)**.** *Given an initial portfolio (B_2, Θ_2), the continuation equilibrium at period $t = 2$ features constant consumption and labor. In particular, the steady-state values H_{ss} and C_{ss} are given by*

$$\frac{v'(H_{ss})}{u'(C_{ss})} = F'(H_{ss}) \tag{11}$$

$$C_{ss} = I(H_{ss}, \Theta_2) + (R-1)B_2 \tag{12}$$

and the sequence of portfolio positions $\{B_{t+1}, \Theta_{t+1}\}_{t\geq 2}$ satisfy

$$B_{t+1} - Q_{ss}(1 - \Theta_{t+1}) = B_2 - Q_{ss}(1 - \Theta_2) \tag{13}$$

for all $t \geq 2$ and where

$$Q_{ss} = \frac{F(H_{ss}) - F'(H_{ss})H_{ss}}{R-1}. \tag{14}$$

Proof. In Appendix A.1. □

Condition (11) is the standard intra-temporal condition between consumption and labor, which equates their marginal rate of substitution to their marginal rate of transformation. The expression for consumption, (12), results from a balanced current account in steady state. As equation (9) makes clear, the steady-state level of consumption is lower the larger is the share of the domestic equity owned by foreign investors. Related to this point, substituting disponsable income (9) and firms' share price (14) into consumption (12), one obtains the relatively more familiar expression for consumption,

$$C_{ss} = F(H_{ss}) + (R-1)[B_2 - (1-\Theta_2)Q_{ss}], \tag{15}$$

which highlights that consumption equals output plus the annuity value of the net foreign asset position.

Note that the portfolio position is undetermined in equilibrium, a result that holds because firms' shares and bonds offer the same return, in accord with (8). As indicated by (13), any portfolio that keeps the same net foreign asset position is consistent with an equilibrium.[3]

3.2 Continuation equilibrium at period $t = 1$

In period $t = 1$, the nominal wage is rigid, and consequently, the exchange rate does affect the equilibrium allocation. The below lemma describes the equilibrium outcome for any given exchange rate E_1 and initial portfolio (B_1, Θ_1).

Lemma 2 (Equilibrium at $t = 1$)**.** *Given an initial portfolio (B_1, Θ_1) and an exchange rate E_1, the continuation equilibrium at period $t = 1$ is such that*

i) H_1 is given by

$$\frac{\overline{W}_1}{E_1} = F'(H_1), \tag{16}$$

ii) $C_t = C_{ss}$ for all $t \geq 1$, $H_t = H_{ss}$ for all $t \geq 2$ where H_{ss} and C_{ss} is given by (11) and (12) and portfolios (B_2, Θ_2) satisfy

$$B_2 - (1-\Theta_2)Q_{ss} = B_1 - (1-\Theta_1)Q_{ss} + \frac{I(H_1, \Theta_1) - I(H_{ss}, \Theta_1)}{R}. \tag{17}$$

[3] In the case where there are potentially binding debt limits, the gross positions would have to be consistent with (5).

and $\{B_{t+1}, \Theta_{t+1}\}_{t\geq 2}$ *satisfy* (13).

Proof. In Appendix A.2. □

Condition (16) captures that when the nominal wage is rigid aggregate labor is determined by aggregate demand. Despite this rigidity, in equilibrium, households smooth consumption over time, because of the separability in their preferences between consumption and labor supply and the slackness in their debt limit. The rigidity in the nominal wage implies that disposable income $I(H_1, \Theta_1)$ may differ from its future counterpart $I(H_{ss}, \Theta_1)$. As a consequence, in period $t = 1$, the current account may not be balanced. For instance, as shown by expression (17), the current account is negative if the current income is below the future one, which naturally requires international capital inflows and a reduction in the net foreign asset position to sustain the relatively high consumption. As before, the portfolio composition is undetermined, but the net foreign asset position needs to be consistent with the disposable income and the consumption.

Until now we have considered an exogenous exchange rate. We next turn to the problem of a central bank that chooses an exchange rate policy to maximize households' welfare.

3.3 Optimal exchange rate policy

Using the equilibrium characterization in Lemma 2, we let $\{\mathcal{H}_1, \mathcal{H}_{ss}, \mathcal{C}_{ss}\}$ denote the equilibrium policies for aggregate labor and consumption, which are a function of the period 1 exchange rate and the initial portfolio positions $(E_1; B_1, \Theta_1)$.[4] The problem of the central bank can then be represented in dual form as follows:

$$\max_{E_1} \Big\{ u\left(\mathcal{C}_{ss}(E_1; B_1, \Theta_1)\right) - v\left(\mathcal{H}_1(E_1; B_1, \Theta_1)\right) + \frac{1}{R-1}\left[u\left(\mathcal{C}_{ss}(E_1; B_1, \Theta_1)\right) - v\left(\mathcal{H}_{ss}(E_1; B_1, \Theta_1)\right)\right] \Big\}, \quad (18)$$

where the first term in the objective is the period 1 utility flow and the second term is the present discount value of utility. Notice that this problem is dynamic because the exchange rate affects the continuation equilibrium at period $t = 2$ through its effect on portfolio position (B_2, Θ_2).

The following proposition provides a necessary condition for optimality.

[4] Recall that the equilibrium policies for consumption and labor are unique.

Proposition 1 (Necessary condition for optimality)**.** *Let* $\pi(H_t) \equiv F(H_t) - F'(H_t)H_t$ *denote* real profits *as a function of aggregate labor* H_t*. Then, the optimal exchange rate policy satisfies*

$$F'\left(\mathcal{H}_1\right) - \frac{v'\left(\mathcal{H}_1\right)}{u'\left(\mathcal{C}_{ss}\right)} - \left(1-\Theta_1\right)\left[\pi'\left(\mathcal{H}_1\right) + \frac{1}{R-1}\pi'\left(\mathcal{H}_{ss}\right)\frac{\partial\mathcal{H}_{ss}/\partial E_1}{\partial\mathcal{H}_1/\partial E_1}\right] = 0\,, \tag{19}$$

where

$$\frac{\partial\mathcal{H}_{ss}/\partial E_1}{\partial\mathcal{H}_1/\partial E_1} = \frac{u''\left(\mathcal{C}_{ss}\right)F'\left(\mathcal{H}_{ss}\right)\frac{R-1}{R}I_H\left(\mathcal{H}_1,\Theta_1\right)}{v''\left(\mathcal{H}_{ss}\right) - F''\left(\mathcal{H}_{ss}\right)u'\left(\mathcal{C}_{ss}\right) - u''\left(\mathcal{C}_{ss}\right)F'\left(\mathcal{H}_{ss}\right)\frac{1}{R}I_H\left(\mathcal{H}_{ss},\Theta_1\right)}. \tag{20}$$

Proof. In Appendix A.3 □

Notice first that if domestic households own all the firms' shares, $\Theta_1 = 1$, we recover the standard condition that a central bank would depreciate the exchange rate until the marginal rate of transformation and the marginal rate of substitution are equated. That is, the central bank would target a zero labor wedge.

When foreign investors own domestic firms' shares, however, there is an additional consideration. Now, the central bank may seek to deviate from a zero labor wedge to reduce the profits paid to foreigners. The RHS in this expression captures the change in the present discounted value of profits paid to foreigners upon a marginal increase in the exchange rate.[5] If by marginally depreciating the exchange rate it can reduce the profits (i.e., the third term on the LHS is negative), the central bank considers optimal to induce a negative labor wedge, whereas otherwise it prefers to induce a positive wedge. Whether the optimally induced labor wedge is negative or positive ultimately depends on the individual responses to the exchange rate of the different components of the profits.

Concerning the individual responses, $\pi'(H_t) = -F''(H_t)H_t > 0$ is always positive because an increase in aggregate labor H_t reduces real wage $F'(H_t)$. Current labor response $\partial\mathcal{H}_1/\partial E_1 > 0$ is always positive as well, because when the nominal wage is fixed, a depreciation of the exchange rate depresses the real wage and, consequently, boosts the aggregate labor demand. By contrast, the sign of future labor response $\partial\mathcal{H}_{ss}/\partial E_1$ is ambiguous, because the response of labor income $F'(H_t)H_t$ to labor hours H_t—and consequently also that of disposable income $I(H_t,\Theta_1)$—in general have an ambiguous sign.[6] In a case with a positive labor income response, that is, $F'(H_t) + F''(H_t)H_t > 0$, response $\partial\mathcal{H}_{ss}/\partial E_1 < 0$ is negative.

[5] More precisely, the change in present discounted value of the profits is the RHS times $\partial\mathcal{H}_1/\partial E_1$. The latter, however, is always positive, and it follows directly from (16).

[6] See equation (20) for details. The response of the latter income to the hours is $I_H(H_t,\Theta_1) = F'\left(H_t\right) + \left(1-\Theta_1\right)F''\left(H_t\right)H_t$.

This is because a depreciation of the exchange rate boosts labor income $F'(\mathcal{H}_1)\mathcal{H}_1$ and thus creates a positive wealth effect on impact that induces households to reduce their future labor supply. More importantly, in that case, starting from a situation without labor wedge [i.e., E_1 such that $\mathcal{H}_1(E_1; B_1, \Theta_1) = \mathcal{H}_{ss}(E_1; B_1, \Theta_1)$], marginally appreciating the exchange rate—and therefore inducing a positive labor wedge—uniquivocally improves social welfare. This holds because response $\partial\mathcal{H}_1/\partial E_1 > 0$ more than offsets its counterpart $\partial\mathcal{H}_{ss}/\partial E_1 < 0$ and the LHS on (19) is a monotonic transformation of the first-order derivative of objective (18).

The below corollary specifies a condition under which the optimally induced labor wedge when foreigners own domestic equities is always positive.

Corollary 1 (Positive labor wedge)**.** *Suppose that the necessary condition* (19) *is also sufficient for characterizing the optimal exchange rate policy. If labor income $F'(H_t)H_t$ is increasing in labor hours H_t, then the labor wedge when foreigners own domestic equities is always positive at the optimal exchange rate policy.*

Proof. In Appendix A.4. □

We note that if the functions $\{u(\cdot), v(\cdot), F(\cdot)\}$ are isoelastic, this guarantees that (19) is indeed sufficient for optimality. Furthermore, a production function that has diminishing returns to scale in labor guarantees that $F'(H_t)H_t$ is increasing in labor hours H_t. The case with an isoelastic production function, in particular, sheds further light on the mechanism. Under $F(h) = h^\alpha$, disposable income is given by

$$I(H_t, \Theta_1) = [\alpha + (1-\alpha)\Theta_1] F(H_t),$$

which implies that from the perspective of the SOE *foreign ownership of domestic equities is akin to a reduction in aggregate productivity.* One can then interpret the reduction in aggregate labor under the optimal exchange rate through this lens.

Takeaway. This section highlighted the implications of foreign ownership of domestic equities for the determination of the optimal exchange rate. A key insight is that the central bank has incentives to over-appreciate the exchange rate relative to what is required to eliminate the distortions from the rigid nominal wage to boost real wages and reduce the profits of firms accrued by foreign investors.

3.4 Equilibrium at period $t = 0$

So far, we have taken as given portfolio position $\{B_1, \Theta_1\}$, and given the portfolio, we have characterized the continuation equilibrium after $t \geq 1$. We will turn now to the determination of the initial portfolio $\{B_1, \Theta_1\}$. Crucially, the portfolio and the price of the firms' shares depend on the expected exchange rate policy.

The following lemma describes the equilibrium at the initial period for any given future exchange rate policy.

Lemma 3 (Equilibrium at $t = 0$)**.** *Given initial portfolios (B_0, Θ_0) and a period-one exchange rate policy $E_1 = \mathcal{E}(B_1, \Theta_1)$, the competitive equilibrium is characterized as follows:*

i) $H_t = H_{ss}$ for all $t \neq 1$ and $C_t = C_{ss}$ for all t. Moreover, C_{ss} H_{ss} and H_1 satisfy (11), (12), (16) given that the portfolio position $\{B_1, \Theta_1\}$ satisfies

$$B_1 - (1 - \Theta_1)Q_0 = B_0 - (1 - \Theta_0)Q_0 + \frac{R-1}{R}\frac{I(H_{ss}, \Theta_0) - I(H_1, \Theta_0)}{R}, \tag{21}$$

where Q_0 is given by

$$Q_0 = \frac{1}{R}\left[\pi(H_1) + \frac{1}{R-1}\pi(H_{ss})\right]. \tag{22}$$

In addition, (B_2, Θ_2) satisfy (17) and $\{B_{t+1}, \Theta_{t+1}\}_{t\geq 2}$ satisfy (13).

Proof. In Appendix A.5. □

As before, in equilibrium, we have perfect consumption smoothing for households. Moreover, because the marginal utility of consumption is constant over time, labor hours are constant except in period 1, where the nominal wage is sticky and hours are determined by firms' labor demand. This implies that disposable incomes at $t = 0$ and after $t \geq 2$ coincide as well.

Like before, we also have an indeterminacy of the portfolio. Although before the indeterminacy did not have any implications for real allocations, this is no longer the case.[7] The reason is that now the portfolios chosen in period 0 affect the exchange rate chosen by the government in period 1. As we saw above, when households choose to sell more of their firms' shares, the central bank will choose a more appreciated exchange rate. In turn, this has two

[7] To see more clearly the indeterminacy, notice that once we set Θ_0, there are four equations to determine 4 variables C_{ss}, H_{ss}, H_1, B_1. Moreover, different values for Θ_1 translate into different values for the exchange rate, $E_1(\Theta_1, B_1)$, and H_1 from (**??**). By contrast, in Lemmas 1 and 2, different values for the end-of period foreign shares do not affect employment.

effects. First, households' will earn higher real wages in period 1, while working fewer hours and obtaining lower profits. Second, as investors anticipate that they will obtain lower profits, this implies that households are able to sell their shares at a lower price. As equation (**??**) shows, this translates into a lower level of consumption.

Intuitively, the existence of multiple equilibria with different allocations reflects a coordination failure within the SOE. Specifically, at the initial period, households do not internalize the collective effect of their individual portfolio positions on the determination of the future exchange rate. When households sell more of their firms' shares, they generate a temptation for the central bank to distort output and reduce profits in period 1. Because foreign investors perfectly anticipate the future exchange rate policy, households end up selling the shares at lower prices. The outcome is that output is distorted and households' wealth is reduced.

3.5 Optimal capital control on international equity flows

The coordination failure uncovered in the previous section suggests a role for government policy. We show next how a policy of capital controls on equity inflows at period 0 can improve welfare.

Specifically, we consider a government that can directly set the equity holdings held by domestic households at $t = 0$. We denote by $\hat{\Theta}_1$ the choice for the government. Formally, households at $t = 0$ solve (7) with the key difference that Θ_{t+1} is restricted to be $\hat{\Theta}_{t+1}$.

To write the optimal policy problem we use that in line with Lemma 3, consumption is constant for all $t \geq 0$ and employment is constant for all $t \neq 1$. Let us denote by $\mathcal{B}_1(\hat{\Theta}_1)$ the equilibrium level of borrowing in period 0 as a function of the capital control policy $\hat{\Theta}_1$. It should be understood that $\mathcal{B}_1(\cdot)$ is also a function of the initial portfolio in the initial period (B_0, Θ_0) but we do not include it to avoid clutter. With some abuse of notation, we denote the equilibrium consumption as $\mathcal{C}_{ss}(\hat{\Theta}_1) = \mathcal{C}_{ss}(\mathcal{E}(\mathcal{B}_1(\hat{\Theta}_1), \hat{\Theta}_1); \mathcal{B}_1(\hat{\Theta}_1), \hat{\Theta}_1)$ and define analogously $\mathcal{H}_1$ and $\mathcal{H}_{ss}$. This notation reflects that a capital control policy Θ_1 determines the bond issuance B_1, and the two determine next period exchange rate E_1.

The optimal policy can then be formulated in dual form as follows:

$$\max_{\hat{\Theta}_1 \in [0,1]} \left\{ \frac{R}{R-1} u\left(\mathcal{C}_{ss}(\hat{\Theta}_1)\right) - \left[\left(\frac{R}{R-1} - \frac{1}{R} \right) v\left(\mathcal{H}_{ss}(\hat{\Theta}_1)\right) + \frac{1}{R} v\left(\mathcal{H}_1(\hat{\Theta}_1)\right) \right] \right\}. \tag{23}$$

Crucially, the government takes the decision in period 0 and lets the central bank choose the exchange rate policy in period 1.

The below proposition provides a necessary condition for optimality.

Proposition 2 (Necessary Condition)**.** *The optimal capital control satisfies*

$$F'\left(\mathcal{H}_1\right)-\frac{v'\left(\mathcal{H}_1\right)}{u'\left(\mathcal{C}_{ss}\right)}-\left(1-\Theta_0\right)\left[\pi'\left(\mathcal{H}_1\right)+R\left(\frac{R}{R-1}-\frac{1}{R}\right)\pi'\left(\mathcal{H}_{ss}\right)\frac{\partial\mathcal{H}_{ss}/\partial\hat{\Theta}_1}{\partial\mathcal{H}_1/\partial\hat{\Theta}_1}\right]=0, \quad (24)$$

where

$$\frac{\partial\mathcal{H}_{ss}/\partial\hat{\Theta}_1}{\partial\mathcal{H}_1/\partial\hat{\Theta}_1}=\frac{u''\left(\mathcal{C}_{ss}\right)F'\left(\mathcal{H}_{ss}\right)\frac{R-1}{R}\frac{1}{R}I_H(\mathcal{H}_1,\Theta_0)}{v''\left(\mathcal{H}_{ss}\right)-F''\left(\mathcal{H}_{ss}\right)u'\left(\mathcal{C}_{ss}\right)-u''\left(\mathcal{C}_{ss}\right)F'\left(\mathcal{H}_{ss}\right)\left(1-\frac{R-1}{R}\frac{1}{R}\right)I_H(\mathcal{H}_{ss},\Theta_0)}. \quad (25)$$

Proof. In Appendix A.6 □

As was the case for the optimal exchange rate policy, if households own all of the domestic firms' shares (i.e., $\Theta_0 = 1$), the optimal capital control implements an equilibrium with a null labor wedge. This requires $\hat{\Theta}_1 = 1$ to rule out distortionary incentives to the future exchange rate. In this case, the optimal capital control bans households from selling domestic equities to foreign investors. This is beneficial for the SOE because otherwise, before buying, the investors would correctly anticipate the future distortionary exchange rate, which would then only lead to a deterioration of the initial households' wealth.

As it was also the case for the optimal exchange rate policy, if households do not own all of the domestic equity, the optimal capital control is distortionary. This is because in this case foreign investors appropriate for themselves part of the efficiency gains from eliminating future distortionary inventives to the exchange rate via higher equity prices today. As the below corollary states, moreover, if condition (24) is both necessary and sufficient for optimality and if labor income $F'\left(H_t\right)H_t$ is increasing in labor hours H_t, then the optimal capital control satisfies $\hat{\Theta}_1 \in (\Theta_0, 1)$. That is, under those conditions, the optimal policy commands households to repurchase part—but not all—of the equity owned by foreigners.

Corollary 2. *If the necessary condition* (24) *is also sufficient for optimality and if labor income* $F'\left(H_t\right)H_t$ *is increasing in labor hours* H_t*, then if* $\Theta_0 < 1$ *the optimal capital control satisfies* $\hat{\Theta}_1 \in (\Theta_0, 1)$*.*

Proof. In Appendix A.7 □

4 Quantitative results

In this section, we provide a simple quantification to gauge the importance of equity flows in shaping the optimal exchange rate policy and the welfare implications of capital controls.

Parameter values. The time period is a year. We set the intertemporal elasticity of substitution to $\sigma = 1$, the inverse labor supply to $\psi = 3$, the labor share to $\alpha = 0.65$, and the gross interest rate $R = 1.04$. All values are standard in the literature.[8]

Exchange rate policy. Figure 1 presents the results for the optimal exchange rate policy at $t = 1$. The figure shows the exchange rate, the labor wedge, firms' profits, output, consumption, and welfare all as a function of the share of foreign-owned equity implied by the capital control. The values in the y-axis are all expressed in percentage deviation from an economy with flexible prices (or, equivalently, one where the central bank were to implement an economy with a zero labor wedge). To keep the net foreign asset position constant, we vary B_1 so that $B_1 = (1 - \Theta_1) Q^*_{ss}$, where Q^*_{ss} denotes the price of firms' shares with flexible wages.[9]

If foreigners do not own domestic firms' shares, we can see that the labor wedge is null under the optimal policy (panel [a]). Given that domestic households own all the firms' shares the central bank simply chooses the exchange rate to eliminate the distortions induced by the rigid wage. Namely, it chooses the exchange rate to align the real wage with the one corresponding to the flexible price allocation. In line with this, the figure shows that all allocations coincide with the flexible price benchmark for $1 - \Theta_1 = 0$.

As foreign investors' holdings of domestic firms' shares increase, the benefits for the SOE of boosting real wages and squeezing firms' profits are larger. Thus, when $1 - \Theta_1 > 0$, the central bank over appreciates the exchange rate relative to what is required to achieve the flexible price allocation and it induces a positive labor wedge. Both aggregate labor and aggregate output fall. Despite the increase in real wages, consumption falls as well, because labor income and firms' profits do so. For example, when foreign investors own half of the firms' shares, the central bank over appreciates the exchange rate by around 2%, which reduces both labor income and firms' profits by around 3%. Aggregate labor and aggregate output fall by 5.75% and 3.15%, respectively, while consumption does so by 0.1%. In addition, the appreciation of the nominal exchange rate generates an increase in real wages of about

[8] For the elasticity of labor supply, see in particular, Keane and Rogerson (2012)

[9] Proceeding in this way ensures that as we change Θ_1 all allocations would remain constant under flexible prices.

2%. As highlighted above, foreign holdings of firms' shares act like a negative productivity shock on the SOE, which then induces the central bank to implement a lower output level to save on disutility costs from supplying labor hours.

Panel (f) reports the welfare gains from implementing the optimal exchange rate policy relative to a policy that would keep a zero labor wedge. As the figure shows, there are sizable gains from departing from a zero-labor wedge allocation. When foreign investors own the entire share of firms, households are willing to give up 1% of period 1 consumption to shift to the optimal policy.

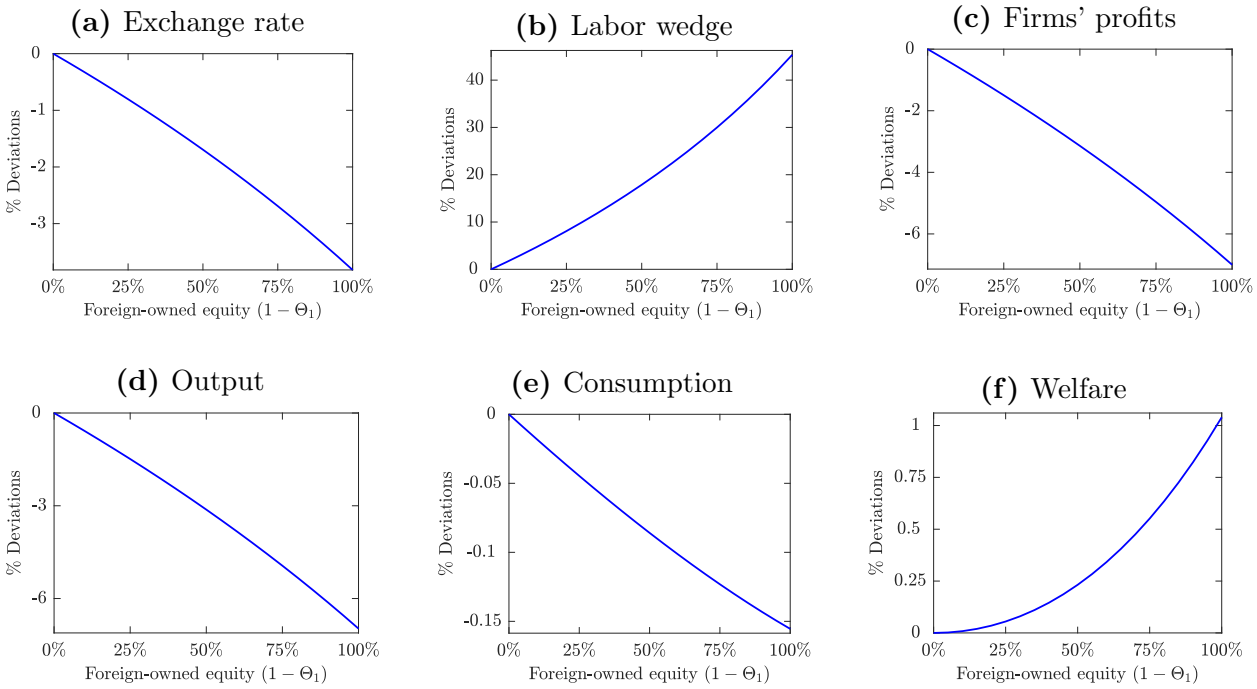


Figure 1: Continuation equilibrium at period $t = 1$ under the optimal exchange rate

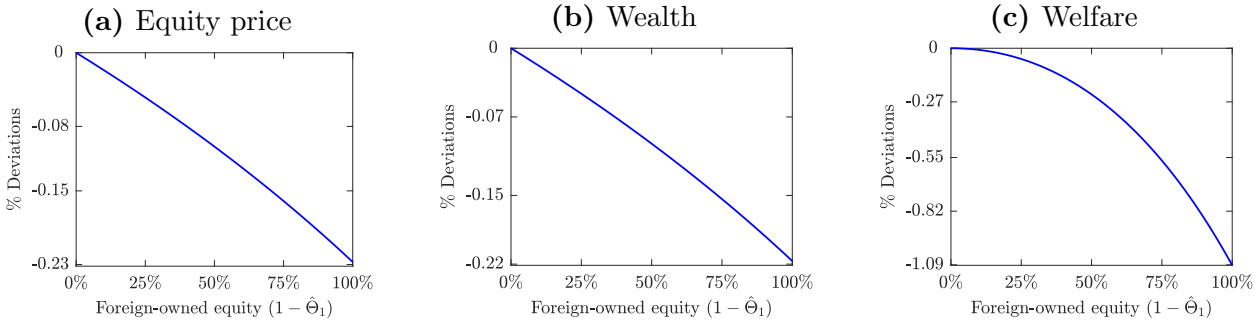
Notes: The figure plots the continuation equilibrium at period $t = 1$ under the optimal exchange rate as a function of the foreign-owned fraction of domestic firms' shares implied by the capital control. For any given $1 - \Theta_1$ external debt is such that $B_1 = (1 - \Theta_1) Q^*_{ss}$, where Q^*_{ss} is the price of firms' shares under flexible nominal wages, so that under those wages the net foreign asset position of the SOE is independent of the composition of the initial portfolio. Variables are expressed as percentage deviations from the benchmark continuation equilibrium with flexible nominal wages. Welfare is expressed in consumption equivalent.

Capital controls. Figure 2 presents the equity price, the households' wealth, and their welfare as a function of the foreign-owned share of domestic equities implied by the capital control. It does so for a case in which initially foreign investors do not own domestic equities and for a case with full foreign ownership of the equities.

If initially foreigners do not own domestic equities, we can see that banning households

from internationally selling the equity maximizes social welfare in the SOE. Any other capital control would induce distortionary incentives to the future exchange rate and consequently it would depress the equity price today. This price fall would indeed be socially inefficient for the SOE because its households initially own all of the equity. A worst-case scenario with a full sale of the equities to the foreigners would have a quantitatively large impact on the equity price and the households' well-being. Specifically, the price would fall by around 20 basis points below its level under the optimal policy, while welfare would do so by around 1 percentage point in terms of annual consumption equivalent. By contrast, if foreigners initially own some of the equities, we can see that inducing a small equity repurchase slightly improves social welfare in the SOE relative to not repurchasing. Repurchasing a large share is not optimal for the SOE because the resulting increase in the equity price plays against its households. In effect, an scenario with a full equity repurchase would significantly increase the equity price, which would strongly reduce the households' welfare.

(A) Foreign investors initially do not own domestic equities (i.e., $\Theta_0 = 1$)



(B) Foreign investors initially own all domestic equities (i.e., $\Theta_0 = 0.5$)

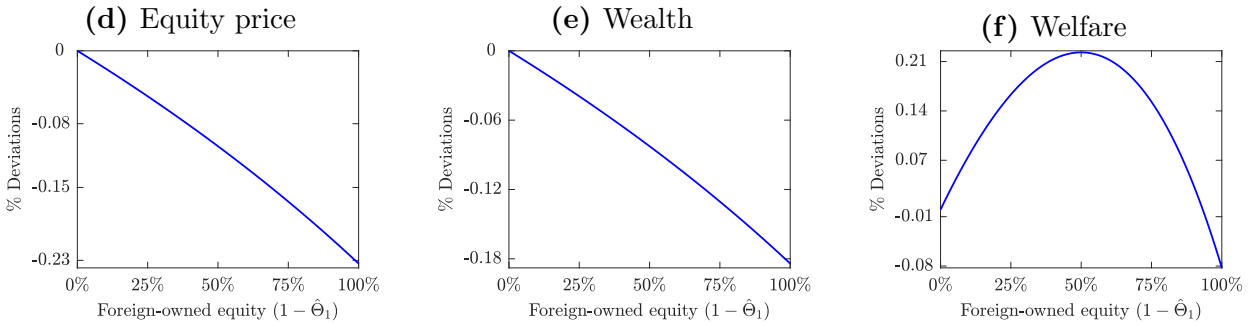


Figure 2: Equilibrium at period $t = 0$ for a given capital control

Notes: The figure plots the equilibrium at period $t = 0$ as a function of the foreign-owned fraction of domestic firms' shares at period $t = 1$ implied by the capital control. It does for a case without initial foreign ownership of domestic equities and for a case with. For any given $1 - \Theta_0$ external debt is such that $B_0 = (1 - \Theta_0) Q^*_{ss}$. Welfare is measured in consumption equivalent.

5 Extensions

This section extends our analysis along three dimensions:

5.1 The case with a binding debt limit

So far we have only considered the case with a slack debt limit. Next, we consider a case in which an international "sudden stop"—i.e., an unanticipated unwillingness of foreign investors to roll over external debt—can force the SOE to internationally sell its domestic equity to meet its debt obligations.

Define an international sudden stop as an scenario in which $B_0 < -\underline{b} < +\infty$. Put equivalently, the SOE cannot rollover over its external debt, because of an unanticipated tightening in its debt limit. Given capital control $\hat{\Theta}_1 \in [0,1]$, we say that the SOE is initially financially constrained if the equilibrium outcome in lemma 3 does not respect the debt limit. Put more precisely, for the given value of the control, the bond holding position consistent with (21) satisfies $\mathcal{B}_1 < -\underline{b}$. The following lemma describes the equilibrium allocation under a sudden stop.

Lemma 4. *If given $\hat{\Theta}_1 \in [0,1]$ the SOE is initially financially constrained, the equilibrium allocation is characterized as follows:*

i) Allocation at period $t = 0$:

$$\mathcal{C}_0 = I\left(\mathcal{H}_0, \Theta_0\right) + B_0 + \frac{1}{R}\underline{b} + \frac{1}{R}[\pi\left(\mathcal{H}_1\right) + \mathcal{Q}_{ss}]\left(\Theta_0 - \hat{\Theta}_1\right) < \mathcal{C}_1 \,, \tag{26}$$

where:

$$\frac{v'\left(\mathcal{H}_0\right)}{u'\left(\mathcal{C}_0\right)} = F'\left(\mathcal{H}_0\right) \,, \tag{27}$$

$\mathcal{Q}_{ss}$ is given by (14), $\mathcal{B}_1 = -\underline{b}$, and $\{\mathcal{H}_1, \mathcal{H}_{ss}, \mathcal{C}_1\}$ are determined below.

ii) Continuation allocation at period $t = 1$:

a. If at period $t = 1$ the SOE is instead not financially constrained, $\{\mathcal{H}_1, \mathcal{H}_{ss}, \mathcal{C}_1\}$ and $\mathcal{E}_1$ are jointly determined by the conditions in lemma 2 and proposition 1.

b. Otherwise (i.e., if in [a.] $\mathcal{B}_2 \geq -\underline{b}$ cannot hold), $\{\mathcal{C}_1, \mathcal{C}_{ss}\}$ are given by

$$\mathcal{C}_1 = I\left(\mathcal{H}_1, \hat{\Theta}_1\right) - \frac{R-1}{R}\underline{b} + \mathcal{Q}_{ss}\hat{\Theta}_1 < \mathcal{C}_{ss} \,, \tag{28}$$

$$\mathcal{C}_{ss} = I\left(H_{ss}, 0\right) - \frac{R-1}{R}\underline{b} \,, \tag{29}$$

where $\{\mathcal{H}_1, \mathcal{H}_{ss}\}$ *and* $\mathcal{E}_1$ *are jointly characterized by*

$$\frac{\overline{W}_1}{\mathcal{E}_1} = F'(\mathcal{H}_1), \tag{30}$$

$$\frac{v'(\mathcal{H}_{ss})}{u'(\mathcal{C}_{ss})} = F'(\mathcal{H}_{ss}), \tag{31}$$

$$0 = F'(\mathcal{H}_1) - \frac{v'(\mathcal{H}_1)}{u'(\mathcal{C}_1)} - \left(1 - \hat{\Theta}_1\right)\pi'(\mathcal{H}_1). \tag{32}$$

Proof. In Appendix A.8 □

When the SOE is financially constrained perfect consumption smoothing does not hold. Specifically, for the given capital control $\hat{\Theta}_1$, period-0 consumption $\mathcal{C}_0 < \mathcal{C}_1$ is below its period-1 counterpart because at the initial period the debt limit is binding. Perfect consumption smoothing may hold or not in the continuation equilibrium depending on whether the binding status of the limit. If it is slack, perfect consumption smoothing holds and consequently the continuation equilibrium is characterized by the system of equations already described in section 3. Otherwise, perfect consumption smoothing holds only from period $t = 2$ onwards, but not between periods $t = 1$ and $t = 2$. Put formally, $\mathcal{C}_t = \mathcal{C}_{ss}$ $\forall t \geq 2$, because from $t = 2$ onwards domestic production is stationary, whereas $\mathcal{C}_1 < \mathcal{C}_{ss}$ because at period $t = 1$ the debt limit is binding. In this case, the optimal exchange rate is characterized by (32), because decisions taken at period $t = 1$ do not affect the initial wealth at period $t = 2$. Lastly, the debt limit in the continuation equilibrium is slack if and only if condition $\mathcal{B}_2 \geq -\underline{b}$ holds for some $\Theta_2 \in [0, 1]$ under the assumption of the limit being slack.

We are now ready to derive the optimal capital control. If the SOE is initially financially constrained for at least one feasible capital control $\hat{\Theta}_1 \in [0, 1]$, then the optimal capital control solves:

$$\max_{\hat{\Theta}_1 \in [0,1]} \left\{ u(\mathcal{C}_0) - v(\mathcal{H}_0) + \frac{1}{R}\left[u(\mathcal{C}_1) - v(\mathcal{H}_1)\right] + \frac{1}{R}\frac{1}{R-1}\left[u(\mathcal{C}_{ss}) - v(\mathcal{H}_{ss})\right] \right\}, \tag{33}$$

where allocation $\{\mathcal{C}_t, \mathcal{H}_t\}$ is characterized as in lemma 3 if the SOE is not financially constrained or as in lemma 4 otherwise.

Figure 3 plots the optimal capital control and the resulting equilibrium outcome as a function of external debt $-B_0$ for the case in which the SOE initially owns all of the domestic equity (i.e., $\Theta_0 = 1$). If no international sudden stop occurs (i.e., $B_0 \geq -\underline{b}$), then SOE is initially financially unconstrained. Thus, as shown in subsection 3.4, the optimal capital control bans international equity inflows. Otherwise, the SOE is initially financially

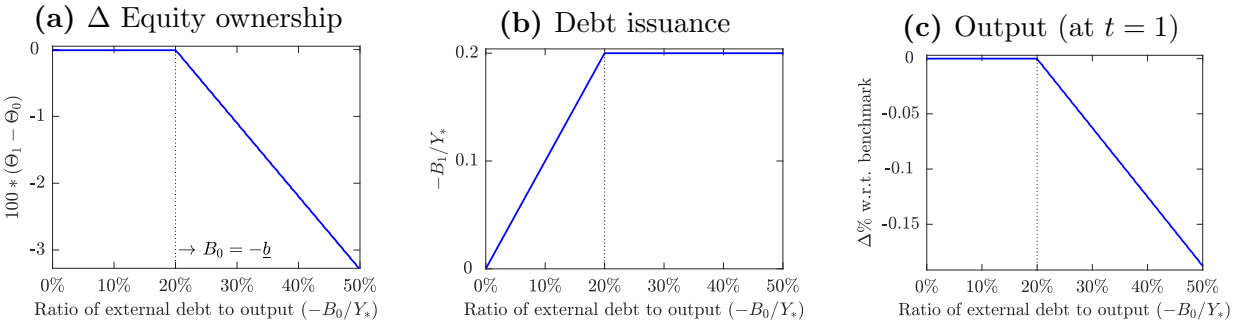


Figure 3: Optimal Capital Control when the SOE can be financially constrained

Notes: The figure plots the equilibrium outcome under the constrained optimal capital control as a function of external debt $-B_0$ for the case without foreign ownership of domestic equities (i.e., $\Theta_0 = 1$). Domestic production at period $t = 1$ (Panel c) is expressed as a percentage deviation from a benchmark continuation equilibrium with flexible nominal wages and without foreign ownership of the equity. The debt limit is such that $\underline{b} = -0.2Y_*$, where Y_* is the output level in the benchmark equilibrium.

constrained if—notably—the inflows are banned. In the latter case, the capital control thus faces a trade-off between sustaining perfect consumption smoothing and guaranteeing no distortionary incentives to the future exchange rate. For the isoelastic specification considered in the figure, the optimal capital control induces distortionary incentives, thus forcing the SOE to internationally sell its domestic equity at distress prices.

The key takeaways so far are that international equity inflows may induce distortionary incentives to exchange rate policy and that domestic capital controls should therefore curb the inflowns unless the SOE is severely financially constrained.

5.2 Model with tradable and non-tradable goods

So far, we have considered an environment with a single final good, assumed to be tradable internationally. In this section, we extend the analysis by introducing a non-tradable good and analyze the implications of international equity flows of non-tradable firms for monetary policy.

5.2.1 Environment

Households. Preferences are given again by

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v\left(l_t\right) \right], \tag{34}$$

but now consumption $c_t = c(c_t^T, c_t^N)$ is a linearly homogenous aggregator of the tradable good, c_t^T, and the non-tradable good, c_t^N. Equivalently, we will use $U(c_t^T, c_t^N)$ to denote the utility as a function of the two consumption goods, and σ_t the elasticity of substitution between the two goods. In addition, l_t is the total number of hours households work in the tradable and non-tradable sector. That is, $l_t = l_t^T + l_t^N$.

The budget constraint expressed in units of tradables is given by

$$c^T + \frac{P_t^N}{E_t} c_t^N + b_{t+1} + \sum_j Q_t^j \theta_{t+1}^j = \frac{W_t}{E_t} l_t + b_t R + \sum_j \left(\frac{\Pi_t^j}{E_t} + Q_t^j \right) \theta_t^j$$

where the sums are indexed by $j \in \{N, T\}$, Q_t^j is the price of equity of firms in sector j in units of domestic currency, and $P_t^N > 0$ is price of the non-tradable good in domestic currency as well. In the budget constraint, we have assumed implicitly that the wage is the same in both sectors, a result that follows because labor hours in both sectors are perfect substitutes for households.

In addition to the optimality conditions presented in Section 2, we have a condition equating the marginal rate of substitution between tradables and non-tradables to the relative price:

$$\frac{P_t^N}{E_t} = \frac{U_N(c^T, c_t^N)}{U_T(c_t^T, c_t^N)}.$$

where U_T and U_N denote the marginal utility with respect to the tradable and the non-tradable good.

We continue to assume that the wage in units of domestic currency is fixed at $\bar{W}$ at $t = 1$.[10] This implies that hours worked in both sectors are determined by firms' labor demand.

Firms. Firms in each sector choose labor hours to maximize profits. That is, firms solve

$$\Pi_t^j \equiv \max_{h_t^j \geq 0} \left\{ P_t^j F^j \left(h_t^j \right) - W_t h_t^j \right\} \text{ , for } j \in \{N, T\} . \tag{35}$$

where recall that by the law of one price and the fact that the foreign price is normalized to one, we have $P_t^T = E_t$.

Competitive equilibrium. We define now a competitive equilibrium in this environment.

[10] An analysis with sticky prices for non-tradables is available upon request.

Definition 2. Given initial conditions (B_0, Θ_0), a rigid wage $\overline{W}_1$, and an exchange rate policy, $\{E_t\}_{t=0}^{\infty}$, a *competitive equilibrium* is a sequence of allocations $\{C_t^T, C_t^N, H_t^T, H_t^N\}_{t=0}^{\infty}$, portfolios $\{B_{t+1}, \Theta_{t+1}^T, \Theta_{t+1}^N\}_{t=0}^{\infty}$ and prices $\left\{W_t, P_t^T, Q_t^T, Q_t^N\right\}_{t=0}^{\infty}$ such that:

(i) Households and firms optimize.

(ii) Market for non-tradable clears. That is, $C_t^N = F^N\left(H_t^N\right)$

(iii) Prices for firms' shares satisfy

$$RQ_t^j = \frac{\Pi_{t+1}^j}{E_{t+1}} + Q_{t+1}^j \text{ , for } j \in \{T, N\} . \tag{36}$$

Crucially, relative to the definition of competitive equilibrium in Section 2, an equilibrium also requires the market for the non-tradable good to clear domestically.

In what follows, we assume that $U(c^T, c^N) = \omega u(c^T) + (1-\omega)u(c^N)$. Together with the assumption that $\beta R = 1$, this ensures stationarity in the continuation equilibrium at period $t = 2$ and constant tradable consumption over time. We also restrict attention to the case in which the debt limit is slack.

5.2.2 Simple example

Consider first a simple example with isoelastic utility function u and isoelastic production functions $\{F^N, F^T\}$. Moreover, assume that in this simple economy labor hours in the tradable sector are fixed. Then, in the continuation equilibrium at period $t = 1$, for any given exchange rate E_1 and portfolio position (B_1, Θ_1), profits in the non-tradable sector are

$$\frac{\Pi_t^N}{E_t} = \underbrace{\frac{1-\omega}{\omega}\left(\frac{F^N\left(H_t^N\right)}{C_{ss}^T}\right)^{-1/\sigma}}_{=P_t^N/E_t} \underbrace{\left(1-\alpha^N\right)F^N\left(H_t^N\right)}_{=\pi^N\left(H_t^N\right)},$$

where tradable consumption satisfies $C_t^T = C_{ss}^T \ \forall t \geq 1$, with

$$\begin{aligned} C_{ss}^T = I^T\left(H_1^T, \Theta_1^T\right) + \frac{1}{R}[I^T\left(H_{ss}^T, \Theta_1^T\right) - I^T\left(H_1^T, \Theta_1^T\right)] + (R-1)B_1 + \\ - \left(1-\Theta_1^N\right)\left\{\frac{\Pi_1^N}{E_1} + \frac{1}{R}\left[\frac{\Pi_{ss}^N}{E_{ss}} - \frac{\Pi_1^N}{E_1}\right]\right\}. \end{aligned} \tag{37}$$

Relative to the environment in the model in Section 2, the only new element in the formula for tradable consumption C_{ss}^T is the last term, which captures the effect of foreign ownership of domestic equities in the non-tradable sector.

As shown by these equations, parameter σ—here interpreted as the elasticity of substitution between the goods—plays a critical role for the effect of the exchange rate on the relative price and the profits. Specifically, if $\sigma = 1$—meaning the goods are neither complementary nor substitutes—non-tradable production does not affect firm profits in the sector. This is because profits $\pi^N\left(H_t^N\right)$ are proportional to the production while relative price P_t^N/E_t is inversely proportional to it. As a result, given labor hours H_1^T and H_{ss}^T, the exchange rate cannot actually affect tradable consumption, which then eliminates any incentive to the optimal exchange rate policy to distort the allocation in the non-tradable sector relative to the flexible wage benchmark even when foreign investors own domestic shares in the sector.

If $\sigma < 1$ (i.e., the goods are complements), by contrast, the relative price responds more aggressively than proportionally to a change in the ratio of the quantities consumed. Thus, everything else the same, declines in non-tradable production boosts firm profits in the sector, whereas increases depress them. Given labor hours H_1^T and H_{ss}^T, then to boost tradable consumption at the expense of distorting the allocation in the non-tradable sector, the exchange rate policy can only consider optimal to over depreciate relative to the benchmark. Lastly, if $\sigma > 1$ (i.e., the goods are substitutes), the relative price responds less aggressively than proportionally. As a consequence, and in line with the model environment in Section 2, given the labor hours, the exchange rate policy can only consider optimal to over appreciate.

5.2.3 Optimal exchange rate policy

The above example illustrates the incentives of the optimal exchange rate policy to distort the allocation in a simple economy with isoelastic functional forms and with fixed labor hours in the tradable sector. Building on those insights the below proposition provides a necessary condition for policy optimality in the original environment.

Proposition 3 (Necessary condition for optimality)**.** *The optimal exchange rate policy satisfies*

$$\sum_j \frac{\mathcal{P}_1^j}{E_1}\left[F'^j\left(\mathcal{H}_1^j\right) - \frac{v'\left(\mathcal{H}_1^j + \mathcal{H}_1^{-j}\right)}{u'\left(\mathcal{C}_1^j\right)}\right]\frac{\partial \mathcal{H}_1^j}{\partial E_1} = \sum_j \left(1 - \Theta_1^j\right)\left[\frac{\partial\left(\Pi_1^j/E_1\right)}{\partial E_1} + \frac{1}{R-1}\frac{\partial\left(\Pi_{ss}^j/E_{ss}\right)}{\partial E_1}\right]. \tag{38}$$

Moreover, if foreigners do not own domestic firms' shares, the condition implies that

$$F'^{j}\left(\mathcal{H}_1^j\right) = \frac{v'\left(\mathcal{H}_1^j + \mathcal{H}_1^{-j}\right)}{u'_j\left(\mathcal{C}_1^j\right)}\,, \quad \textit{for } j \in \{N, T\}\,. \tag{39}$$

Proof. See Appendix A. □

As in the environement of the model in Section 2, the optimal exchange rate induces a domestic labor wedge equal to the marginal losses in domestic income that would have been born by the SOE had it owned the foreign-owned firm profits. This is what condition (38) reads. The only difference is that now both the labor wedge and the income losses are computed by summing across many (i.e., two) sectors. Also as in previously analyzed environment, if foreigners do not own domestic firms' shares, the optimal exchange rate only eliminates distortions from rigidity in nominal wages.

From Proposition 3 one can derive as a collorary the set of international equity portfolios that render the optimal exchange rate non-distortionary. This set can be derived by substituting condition (39) into condition (38), and consequently it can be characterized by Γ, with

$$\Gamma \equiv \left\{ \left(\Theta_1^N, \Theta_1^T\right) \in [\underline{\theta}, 1]^2 : \sum_j \left(1 - \Theta_1^j\right) \left[\frac{\partial\left(\Pi_1^j / E_1\right)}{\partial E_1} + \frac{1}{R-1} \frac{\partial\left(\Pi_{ss}^j / E_{ss}\right)}{\partial E_1} \right] = 0 \right\}.$$

Naturally, $\left(\Theta_1^N, \Theta_1^T\right) = (1, 1)$ belongs to the set. More interestingly, international equity portfolios with foreign ownership of domestic equities also belong to the set provided—notably—that the goods are complementary and their degree of complementation is sufficiently strong. A strong complementation is necessary for having firm profits in the non-tradable sector respond positively to changes in the exchange rate—as discussed in subsection **??**—and therefore for having opposite signs in the responses of firm profits between the two sectors. If the responses have opposite signs then there may exist a composition of international equity portfolios such that the marginal changes in foreign-owned firm profits induced by the optimal exchange rate offset each other between the sectors. Figure 4 illustrates the set for different degrees of complementation for the isoelastic specification.

The existence of such a class of international equity portfolios has the following two related implications for international equity flows and the optimal capital control. First, naturally, international equity inflows may not necessarily induce distortionary incentives to exchange rate policy. Second, the composition of domestic capital controls between firms' shares in

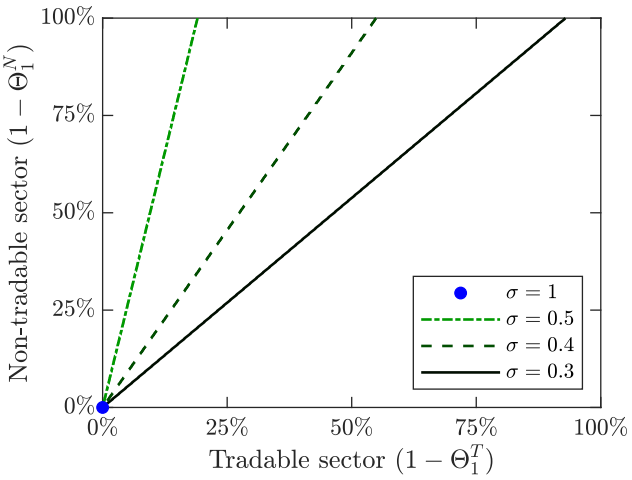


Figure 4: International equity portfolios that induce a non-distortionary exchange rate

Notes: The figure plots the set of international equity portfolios that induce a non-distortionary exchange. Parameter values are the same as in Figure **??** except for $\sigma = \{0.3, 0.4, 0.5, 1\}$ and $\omega = \{0.06, 0.1, 0.13, 0.25\}$. The target for parameter ω is the same as in Figure **??**.

the tradable and the non-tradable sector can be essential for reducing or even preventing distortionary incentives to the policy associated with the inflows.

5.3 Differences in portfolio holdings within the SOE

The analysis so far restricted attention to a case in which all of the households in the SOE initially have and afterwards always choose the same portfolio holdings. In this subsection we instead consider a case in which the households may differ in their initial portfolio holdings as well as may subsequently choose portfolio holdings with different compositions between external debt and domestic equity.

To ease exposition, we restrict attention to the environment with the single, tradable good, presented in Section 2. As in Section 3, we assume condition $\beta R = 1$, the regularity conditions on functions $\{u(\cdot), v(\cdot), F(\cdot)\}$ needed for characterizing optimality conditions with FOCs, and we only consider the case in which the financing constraints are slack.

Let $g_t(b, \theta) \geq 0$ denote the population density of households with portfolio positions (b, θ) at the beginning of period t. This population density function is exogenous at the initial period but endogenous at every other period. Naturally, it is always pre-determined as well as consistent with the following expressions:

$$\int dG_t(b, \theta) = 1 \,, \quad \int b \, dG_t(b, \theta) = B_t \,, \quad \int \theta \, dG_t(b, \theta) = \Theta_t \,, \quad \forall t \geq 0 \,,$$

where $G_t(\cdot,\cdot)$ is the population cumulative function associated with $g_t(\cdot,\cdot)$. To economize in notation, in what follows, we summarize with type $i \equiv (b,\theta)$ the class of households with the corresponding portfolio position. Proposition 4 is the counterpart in this more general treatment of the model to Proposition 1.

Proposition 4. *Under standard regularity conditions for functions* $\{u(\cdot), v(\cdot), F(\cdot)\}$*, the optimal exchange rate is unique and it imposes the following condition to the continuation equilibrium at period* $t = 1$ *:*

$$\int \left[F'(H_1) - \frac{v'(h_{i,1})}{u'(c_{i,1})}\right] \frac{\partial h_{i,1}}{\partial E_1} d\hat{G}_{i,1} = \int \left(\frac{h_{i,1}}{H_1} - \theta_i\right) d\hat{G}_{i,1} \left[\frac{\partial(\Pi_1/E_1)}{\partial E_1} + \frac{1}{R-1}\frac{\partial(\Pi_{ss}/E_{ss})}{\partial E_1}\right], \tag{40}$$

where $\hat{G}_{i,1}$ *is the cumulative function associated with density function* $\hat{g}_{i,1}$*, with:*

$$\hat{g}_{i,1} \equiv \frac{1}{\int u'(c_{i,1})\, dG_{i,1}} u'(c_{i,1})\, g_{i,1}\,. \tag{41}$$

Proof. In Appendix A □

Condition (40) is inherently the same as its counterpart condition in Proposition 1. The only difference is that in condition (40) households may differ in their consumption, labor supply schedule or holdings of domestic firms' shares. Formula (41) reveals a redistributional concern of the optimal exchange rate in light of the potential differences across households. Specifically, for any given density function $g_{i,1}$, the optimal exchange rate assigns a higher measurement weight $\hat{g}_{i,1}$ to households with lower consumption.

If all of the households end up at period $t = 1$ with the same consumption and labor supply schedule, then condition (40) reduces to condition (19). In equilibrium, individual consumption and labor supply are jointly determined by

$$c_{i,1} = c_{i,ss} = \frac{R-1}{R}\left[F'(H_1)\, h_{i,1} + \frac{1}{R-1} F'(H_{ss})\, h_{i,ss} + a_{i,1}\right], \tag{42}$$

and

$$\frac{v'(h_{i,1})}{u'(c_{i,1})} = \frac{\tilde{W}_1}{E_1}\,, \tag{43a}$$

$$\frac{v'(h_{i,ss})}{u'(c_{i,1})} = F'(H_{ss})\,, \tag{43b}$$

where financial wealth $a_{i,1} \in \mathbb{R}$ is defined as

$$a_{i,1} \equiv \left[\underbrace{\left[F\left(H_1\right) - F'\left(H_1\right) H_1\right]}_{=\Pi_1/E_1} + \frac{1}{R-1} \underbrace{\left[F\left(H_{ss}\right) - F'\left(H_{ss}\right) H_{ss}\right]}_{=\Pi_{ss}/E_{ss}} \right] \theta_i + b_i \ , \tag{44}$$

"fictitious" nominal wage $\tilde{W}_1 > 0$ is such that

$$H_1 = \int h_{i,1} dG_{i,1} \ , \quad \text{with} \quad F'\left(H_1\right) = \frac{\bar{W}_1}{E_1} \ , \tag{45}$$

and aggregate labor hours $H_{ss} > 0$ are given by

$$H_{ss} = \int h_{i,ss} dG_{i,1} \ . \tag{46}$$

These formulae directly reveal that consumption and labor supply are the same across households as long as financial wealth is. As a corollary one can then show that if $a_{i,1} = a_1 \, \forall i$ holds under the optimal exchange rate consistent with (19) for the given Θ_1 then heterogeneity in portfolio positions (b_i, θ_i) at period $t = 1$ does not influence exchange rate optimality (see the proof of Proposition 4 for details).

These results have the following two related implications on how heterogeneity in the domestic financial portfolio at the initial period in itself influences distortionary incentives to the future exchange rate. First, if it is such that for some $\Theta_1 \in [\underline{\theta}, 1]$ there is no variation across households in their resulting financial wealth under the optimal exchange rate consistent with (19), then the heterogeneity does not influence the optimal exchange rate for the given aggregate portfolio position. This holds because in the model there is no uncertainty and the future exchange rate policy is perfectly anticipated. The heterogeneity may be such nonetheless that the result holds under some but not all of the feasible aggregate positions. This may happen because the initial financial wealth (i.e., $a_{i,0} \in \mathbb{R}$) in equilibrium depends on the future exchange rate—as revealed by the below formula

$$a_{i,0} \equiv \left[\left[\frac{R}{R-1} - \frac{1}{R} \right] \underbrace{\left[F\left(H_0\right) - F'\left(H_0\right) H_0\right]}_{=\Pi_0/E_0=\Pi_{ss}/E_{ss}} + \frac{1}{R} \underbrace{\left[F\left(H_1\right) - F'\left(H_1\right) H_1\right]}_{=\Pi_1/E_1} \right] \theta_i + b_i \ . \tag{47}$$

Second, for the particular specification without heterogeneity in the initial domestic portfolio (i.e., Section 2 to Section 3.4), restricting attention to symmetric equilibria does not affect the determination of the optimal exchange rate. This is because initial financial wealth is

always the same across households regardless of the future exchange rate. In a related vein, the restriction does not affect the determination of the optimal capital control either.

6 Conclusion

In this paper, we have examined the implications of international equity flows for monetary policy. We present a model where foreign investors own a share of domestic equity and characterize the optimal monetary policy. We show that international equity flows generate a time inconsistency for monetary policy: when domestic agents sell their domestic equity, they create a temptation for the central bank to use monetary policy to diminish future profits, which in turn leads to lower stock prices and reduced household wealth. In contrast to existing theory of prudential capital flow management policies, the model calls for capital controls on equity flows.

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A Proofs

A.1 Proof of Lemma 1.

Let's conjecture $V_t(b,\theta) = V_{ss}(b,\theta)\ \forall t \geq 2$. Then, value $V_{ss}(b_2,\theta_2)$ satisfies

$$V_{ss}(b_2,\theta_2) = \max_{\{c,l,b',\theta'\}} \left\{ u(c) - v(l) + \frac{1}{R} V_{ss}(b',\theta') \right\} \tag{A.1}$$

subject to:

$$c + b' + Q_{ss}\theta' = w_{ss} l + R b_2 + (\pi_{ss} + Q_{ss})\theta_2 \ , \tag{A.2}$$

where it has already been imposed that (i) $\beta R = 1$; (ii) $W_t/E_t = w_{ss}$, $\Pi_t/E_t = \pi_{ss}$ and $Q_t = Q_{ss}\ \forall t \geq 2$; (iii) $Q_{ss} = \frac{1}{R-1}\pi_{ss}$ is consistent with (8); and (iv) $b' \geq -\underline{b}$ and $\theta' \geq 0$ are slack.

The first-order condition of the problem together with its envelop condition imply that: (i) $v'(l)/u'(c) = w_{ss}$; (ii) households are indifferent between holding bonds b' and holding firms' shares θ'; and (iii) perfectly smoothing consumption over time is optimal. Thus, the solution to the problem is a vector $\{c_{ss}, l_{ss}, b_{ss}, \theta_{ss}\}$, with

$$w_{ss} = \frac{v'(l_{ss})}{u'(c_{ss})},$$
$$c_{ss} = w_{ss} l_{ss} + \pi_{ss}\theta_2 + (R-1)b_2,$$

where $(b',\theta') = (b_{ss},\theta_{ss})$ is any portfolio position that satisfies budget constraint (A.2) given $c = c_{ss}$ and $l = l_{ss}$. Value $V_{ss}(\cdot,\cdot)$ is given by

$$V_{ss}(b_2,\theta_2) = \frac{R}{R-1}\left[u(c_{ss}) - v(l_{ss})\right] .$$

Combining these optimality conditions with firm optimality condition $F'(h_{ss}) = w_{ss}$ and with market clearing for labor hours, one gets that in equilibrium, aggregate consumption is given by

$$C_{ss} = I(H_{ss},\Theta_2) + (R-1)B_2 \ ,$$

where aggregate labor satisfies

$$\frac{v'(H_{ss})}{u'(C_{ss})} = F'(H_{ss}) \ .$$

The set of admissible portfolio positions is characterized by

$$B_{t+1} - Q_{ss}(1 - \Theta_{t+1}) = B_2 - Q_{ss}(1 - \Theta_2)$$

Note that $V_t(b, \theta) = V_{ss}(b, \theta) \ \forall t \geq 2$ holds because the equilibrium allocation is stationary.

A.2 Proof of Lemma 2.

Value $V_1(b_1, \theta_1)$ satisfies

$$\begin{aligned} V_1(b_1, \theta_1) \quad = \quad & \max_{\{c_1, b_2, \theta_2\}} \left\{ u(c_1) - v(l_1) + \frac{1}{R} V_{ss}(b_2, \theta_2) \right\} \\ & \text{subject to:} \\ & c_1 + \frac{1}{R} b_2 + Q_{ss}\theta_2 = \frac{\bar{W}_1}{E_1} l_1 + b_1 + \left(\frac{\Pi_1}{E_1} + Q_{ss} \right) \theta_1 \ , \end{aligned}$$

where we have already imposed that: (i) $l_1 = F'^{-1}\left(\frac{\bar{W}_1}{E_1}\right)$; (ii) $Q_1 = \frac{1}{R}(\pi_{ss} + Q_{ss}) = Q_{ss}$; and (iii) restrictions $b_2 \geq -\underline{b}$ and $\theta_2 \geq 0$ are slack.

The first-order condition of the problem together with its envelop condition imply that households are indifferent between holding bonds b' and holding firms' shares θ' and that perfect consumption smoothing is optimal. Thus, $c_1 = c_{ss}$, which implies

$$c_1 = \frac{R-1}{R} \left(\frac{\bar{W}_1}{E_1} n_1 + \frac{\Pi_1}{E_1} \theta_1 \right) + \frac{1}{R} (w_{ss} l_{ss} + \pi_{ss} \theta_1) + (R-1) b_1 \ ,$$

where labor hours l_{ss} are given by $v'(l_{ss})/u'(c_1) = w_{ss}$. Any portfolio position $\{b_2, \theta_2\}$ that is consistent with both the budget constraint and the optimal consumption rule is optimal. Value $V_1(b_1, \theta_1)$ can be reduced to

$$V_1(b_1, \theta_1) = \frac{R}{R-1} u(c_1) - v(l_1) - \frac{1}{R-1} v(l_{ss}) \ .$$

Combining these optimality conditions with firm optimality condition $F'(h_1) = \bar{W}_1/E_1$ and with market clearing for labor hours, one gets that in equilibrium, aggregate consumption is given by $C_1 = C_{ss}$, with

$$C_{ss} = I(H_1, \Theta_1) + \frac{I(H_{ss}, \Theta_1) - I(H_1, \Theta_1)}{R} + (R-1) B_1 \ ,$$

where aggregate labor hours H_1 and H_{ss} satisfy

$$\frac{\bar{W}_1}{E_1} = F'(H_1)\,, \qquad\qquad \frac{v'(H_{ss})}{u'(C_{ss})} = F'(H_{ss})\,.$$

The set of admissible portfolios is then characterized by

$$B_2 - B_1 + Q_{ss}(\Theta_2 - \Theta_1) = \frac{I(H_1, \Theta_1) - I(H_{ss}, \Theta_1)}{R}\,.$$

A.3 Proof of Proposition 1

The first-order condition of the problem of the central bank is

$$\frac{R}{R-1} u'(\mathcal{C}_{ss}) \frac{\partial}{\partial E_1}\mathcal{C}_{ss} - v'(\mathcal{H}_1)\frac{\partial}{\partial E_1}\mathcal{H}_1 - \frac{1}{R-1} v'(\mathcal{H}_{ss})\frac{\partial}{\partial E_1}\mathcal{H}_{ss} = 0\,.$$

From (??) it follows that

$$\begin{aligned}\frac{\partial}{\partial E_1}\mathcal{C}_{ss} \;=\; & \frac{R-1}{R}\left[F'(\mathcal{H}_1) - (1-\Theta_1)\pi'(\mathcal{H}_1)\right]\frac{\partial}{\partial E_1}\mathcal{H}_1 + \\ & + \frac{1}{R}\left[F'(\mathcal{H}_{ss}) - (1-\Theta_1)\pi'(\mathcal{H}_{ss})\right]\frac{\partial}{\partial E_1}\mathcal{H}_{ss}\,.\end{aligned}$$

Condition (19) follows from substituting the above derivative for $\frac{\partial}{\partial E_1}\mathcal{C}_{ss}$ and equation (??) into the above first-order condition. Condition (20) follows from applying the Implicit Function Theorem to equation (??) with respect to E_1 and combining the resulting equation with the above condition for $\frac{\partial}{\partial E_1}\mathcal{C}_{ss}$.

A.4 Proof of Corollary 1

If condition (19) is both neccesary and sufficient for characterizing the optimal exchange rate policy, then the optimal exchange rate is unique. Moreover, if labor income $F'(H_t)H_t$ is increasing in labor hours H_t, then starting from E_1 such that $\mathcal{H}_1(E_1; B_1, \Theta_1) = \mathcal{H}_{ss}(E_1; B_1, \Theta_1)$, marginally appreciating the exchange rate is welfare improving. From these results directly follows that the optimally induced labor wedge when foreigners own domestic equities is always positive.

A.5 Proof of Lemma 3

Value $V_0(b_0, \theta_0)$ satisfies

$$\begin{aligned} V_0(b_0,\theta_0) \quad \equiv \quad & \max_{\{c_0,l_0,b_1,\theta_1\}} \left\{ u(c_0) - v(l_0) + \frac{1}{R} V_1(b_1,\theta_1) \right\}, \\ & \text{subject to:} \\ & c_0 + \frac{1}{R} b_1 + Q_0\theta_1 = \frac{W_0}{E_0} l_0 + b_0 + \left(\frac{\Pi_0}{E_0} + Q_0 \right) \theta_0 \,, \end{aligned}$$

where we have already imposed that price $Q_0 = \frac{1}{R}(\Pi_1/E_1 + Q_{ss})$ is consistent with condition (8), and where initial portfolio positions $\{b_0, \theta_0\}$ are exogenous and thus taken as given. The optimization problem on the RHS already incorporates that restrictions $b_0 \geq -\underline{b}$ and $\theta_0 \geq \underline{\theta}$ are slack.

The first-order condition of the problem together with its envelop condition imply that households are indifferent on the margin between consumption and supply of labor hours. Moreover, the conditions imply that households are indifferent between holding bonds and holding firms' shares and that they perfectly smooth consumption over time. Thus, consumption satisfies $c_0 = c_1 = c_{ss}$, with

$$c_0 = \frac{R-1}{R} \left(\frac{W_0}{E_0} l_0 + \frac{\Pi_0}{E_0} \theta_0 + b_0 \right) + \frac{1}{R} \left[\frac{R-1}{R} \left(\frac{\bar{W}_1}{E_1} l_1 + \frac{\Pi_1}{E_1} \theta_0 \right) + \frac{1}{R} \left(w_{ss} l_{ss} + \pi_{ss} \theta_0 \right) \right],$$

and labor hours l_0 and l_{ss} satisfy

$$\frac{v'(l_0)}{u'(c_0)} = \frac{W_0}{E_0} \,, \qquad\qquad \frac{v'(l_{ss})}{u'(c_0)} = w_{ss} \,.$$

Any portfolio position $\{b_1, \theta_1\}$ that is consistent with the budget constraint and the above optimality conditions is optimal.

As in the previously analyzed periods, combining these optimality conditions with firm optimality condition $F'(h_0) = W_0/E_0$ and with market clearing for labor hours, one gets that in equilibrium, aggregate consumption is given by $C_0 = C_1 = C_{ss}$, with

$$C_{ss} = I(H_{ss}, \Theta_0) + \frac{R-1}{R} \frac{I(H_1, \Theta_0) - I(H_{ss}, \Theta_0)}{R} + (R-1) B_0,$$

where

$$\begin{aligned} \frac{\overline{W}_1}{E_1} &= F'(H_1), \\ \frac{v'(H_{ss})}{u'(C_{ss})} &= F'(H_{ss}). \end{aligned} \tag{A.5}$$

The set of admissible portfolio positions $\{B_1, \Theta_1\}$ is given by

$$B_1 - (1-\Theta_1)Q_0 = B_0 - (1-\Theta_0)Q_0 + \frac{R-1}{R}\frac{I(H_{ss},\Theta_0) - I(H_1,\Theta_0)}{R},$$

where Q_0 is given by

$$Q_0 = \frac{1}{R}\left[F(H_1) - F'(H_1)H_1 + \frac{1}{R-1}[F(H_{ss}) - F'(H_{ss})H_{ss}]\right].$$

A.6 Proof of Proposition 2.

The first-order condition of the problem is

$$\frac{R}{R-1}u'(\mathcal{C}_{ss})\frac{\partial}{\partial\Theta_1}\mathcal{C}_{ss} - \left[\left(\frac{R}{R-1} - \frac{1}{R}\right)v'(\mathcal{H}_{ss})\frac{\partial}{\partial\Theta_1}\mathcal{H}_{ss} + \frac{1}{R}v'(\mathcal{H}_1)\frac{\partial}{\partial\Theta_1}\mathcal{H}_1\right] = 0.$$

From (**??**) it follows that

$$\frac{\partial}{\partial\Theta_1}\mathcal{C}_{ss} = I_H(\mathcal{H}_{ss},\Theta_0)\frac{\partial}{\partial\Theta_1}\mathcal{H}_{ss} + \frac{1}{R}\frac{R-1}{R}\left[I_H(\mathcal{H}_1,\Theta_0)\frac{\partial}{\partial\Theta_1}\mathcal{H}_1 - I_H(\mathcal{H}_{ss},\Theta_0)\frac{\partial}{\partial\Theta_1}\mathcal{H}_{ss}\right],$$

where recall that

$$I_H(H_t,\Theta_t) = F'(H_t) - (1-\Theta_t)\pi'(H_t).$$

Condition (24) follows from substituting the above derivative for $\frac{\partial}{\partial\Theta_1}\mathcal{C}_{ss}$ and equation (**??**) into the above first-order condition. Condition (25) follows from applying the Implicit Function Theorem to equation (**??**) with respect to Θ_1 and combining the resulting equation with the above condition for $\frac{\partial}{\partial\Theta_1}\mathcal{C}_{ss}$.

A.7 Proof of Corollary 2.

Assume condition (24) is both necessary and sufficient for optimality and labor income $F'(H_t)H_t$ is increasing in labor hours H_t. Consider $\Theta_0 < 1$. Then,

1. $\Theta_1 = 1$ is not optimal, because otherwise conditions (19) and (24) would not simultaneously hold.

2. $\Theta_1 \leq \Theta_0$ is not optimal either, because from combining conditions (19) and (24), it follows that

$$\frac{1-\Theta_1}{1-\Theta_0} < 1 \Leftrightarrow R > 1.$$

A.8 Proof of Lemma 4

First, we derive the continuation equilibrium at period $t = 1$. If at $t = 1$ given $B_1 = -\underline{b}$ and Θ_1 the debt limit is slack, then allocation $\{H_1, H_{ss}, C_1\}$ and exchange rate E_1 are jointly determined by the conditions in lemma 2 and proposition 1. The debt limit is slack is there exists B_{t+1} such that $B_{t+1} \leq -\underline{b}$. Otherwise, if that condition does not hold, allocation $\{H_1, H_{ss}, C_1, C_{ss}\}$ and exchange rate E_1 are jointly determined

$$\begin{aligned}
C_1 &= I\left(H_1, \Theta_1\right) - \frac{R-1}{R}\underline{b} + Q_{ss}\left(\Theta_1 - \underline{\theta}\right) < C_{ss}, \\
C_{ss} &= I\left(H_{ss}, \underline{\theta}\right) - \frac{R-1}{R}\underline{b}, \\
\frac{\overline{W}_1}{E_1} &= F'\left(H_1\right), \\
\frac{v'\left(H_{ss}\right)}{u'\left(C_{ss}\right)} &= F'\left(H_{ss}\right), \\
0 &= F'\left(H_1\right) - \frac{v'\left(H_1\right)}{u'\left(C_1\right)} - \left(1-\Theta_1\right)\pi'\left(H_1\right),
\end{aligned}$$

where the first-order condition associated with the determination of the optimal exchange rate policy (i.e., the last equation above) is static, because allocation $\{H_{ss}, C_{ss}\}$ does not depend on Θ_1.

We now turn to the equilibrium at period $t = 0$. The characterization of the equilibrium allocation in the lemma follows directly from imposing a binding debt limit $B_1 = -\underline{b}$.

A.9 Proof of Proposition ??

In an economy with a single insulated sector and without foreign ownership of domestic equities in that sector, the optimal exchange rate is unique and it imposes the following

condition to the continuation equilibrium at period $t = 1$:

$$\left[F_j'\left(H_1^j\right) - \frac{v'\left(H_1^j + H_1^{-j}\right)}{u_j'\left(C_1^j\right)}\right]\frac{\partial H_1^j}{\partial E_1} = \left(1 - \Theta_1^j\right)\left[\frac{\partial\left(\Pi_1^j/E_1\right)}{\partial E_1} + \frac{1}{R-1}\frac{\partial\left(\Pi_{ss}^j/E_{ss}\right)}{\partial E_1}\right]\frac{E_1}{P_1^j}$$

Proof. To prove this proposition we proceed according to the following three steps:

1. Solve the problem of households in the continuation equilibrium at period $t = 2$ and derive that continuation equilibrium.
2. Do the same as in Step 1. but for the continuation equilibrium at period $t = 1$.
3. Solve the problem of the central bank in the continuation equilibrium at period $t = 1$.

Step 1. Let $V_{ss}\left(b_2; \theta_2^N, \theta_2^T\right)$ be the value at the beginning of the period of a household with bonds b_2 and pair of firms' shares $\left(\theta_2^N, \theta_2^T\right)$. This value satisfies

$$V_{ss}\left(b_2; \theta_2^N, \theta_2^T\right) \equiv \max_{\left\{c_N, c_T, n, b', \theta_N', \theta_T'\right\}} \left\{u_N\left(c_N\right) + u_T\left(c_T\right) - v\left(n\right) + \frac{1}{R}V_{ss}\left(b'; \theta_N', \theta_T'\right)\right\} \quad \text{(A.6)}$$

subject to:

$$\sum_j p_{ss}^j c_j + \frac{1}{R}b' + \sum_j Q_{ss}^j \theta_j' = w_{ss}n + b_2 + \sum_j \left(\pi_{ss}^j + Q_{ss}^j\right)\theta_2^j \quad \text{(A.7)}$$

where $P_t^j/E_t = p_{ss}^j$, $W_t/E_t = w_{ss}$, $\Pi_t/E_t = \pi_{ss}$ and $Q_t = Q_{ss}$ $\forall t \geq 2$ hold because of stationarity and where $V_t\left(\cdot;\cdot\right) = V_{ss}\left(\cdot;\cdot\right)$ $\forall t \geq 2$ is for the moment conjectured but later verified. Price p_{ss}^T is already given by $p_{ss}^T = 1$. Value $V_{ss}\left(\cdot;\cdot\right)$ already incorporates assumption $\beta R = 1$ and prices $Q_{ss}^j = \frac{1}{R-1}\pi_{ss}^j$ are already consistent with pricing condition (8). The optimization problem on the RHS already incorporates that the financing limits are slack. $S_t^N = S_t^T = \tau_t = 0$ $\forall t \geq 2$ because in this continuation equilibrium nominal wages are flexible.

At the solution to the optimization problem, households consume the goods according to $u_N'(c_N)/u_T'(c_T) = p_{ss}^N$ and supply labor hours according to $v'(n)/u_T'(c_T) = w_{ss}$. Moreover, households are indifferent between holding bonds b' and holding firms' shares θ_N' or θ_T', as well as they perfectly smooth consumption of both goods over time. These results follow from the FOCs and the envelope condition of the problem. Thus, the solution is a vector $\{c_{ss}^N, c_{ss}^T, n_{ss}, b_{ss}, \theta_{ss}^N, \theta_{ss}^T\}$, with

$$c_{ss}^T = w_{ss}n_{ss} - p_{ss}^N c_{ss}^N + \sum_j \pi_{ss}^j \theta_2^j + \frac{R-1}{R}b_2\ , \quad \frac{u_N'\left(c_{ss}^N\right)}{u_T'\left(c_{ss}^T\right)} = p_{ss}^N\ , \quad \frac{v'\left(n_{ss}\right)}{u_T'\left(c_{ss}^T\right)} = w_{ss}\ , \quad \text{(A.8)}$$

where $(b', \theta'_N, \theta'_T) = (b_{ss}, \theta^N_{ss}, \theta^T_{ss})$ is any portfolio position that satisfies budget constraint (A.7) given $c_j = c^j_{ss}$ and $n = n_{ss}$. Value $V_{ss}(\cdot;\cdot)$ is given by

$$V_{ss}\left(b_2; \theta^N_2, \theta^T_2\right) = \frac{R}{R-1}\left[u_N\left(c^N_{ss}\right) + u_T\left(c^T_{ss}\right) - v\left(n_{ss}\right)\right] . \tag{A.9}$$

Combining these optimality conditions with firm optimality conditions $p^j_{ss} F'_j\left(h^j_{ss}\right) = w_{ss}$ and with market clearing conditions $N_{ss} = H^N_{ss} + H^T_{ss}$ and $C^N_{ss} = F_N\left(H^N_{ss}\right)$, one gets that in equilibrium, aggregate consumption of the tradable good is given by

$$C^T_{ss} = Y_{ss}\left(\Theta^N_2, \Theta^T_2\right) + \frac{R-1}{R} B_2 , \tag{A.10}$$

where $Y_t\left(\Theta^N, \Theta^T\right)$ is defined as

$$Y_t\left(\Theta^N, \Theta^T\right) \equiv F_T\left(H^T_t\right) - \sum_j \left(1 - \Theta^j\right) \underbrace{\frac{P^j_t}{E_t}\left[F_j\left(H^j_t\right) - F'_j\left(H^j_t\right) H^j_t\right]}_{=\Pi^j_t / E_t} , \tag{A.11}$$

and where the following conditions hold:

$$p^T_{ss} = 1 , \tag{A.12}$$

$$p^N_{ss} = \frac{u'_N\left[F_N\left(H^N_{ss}\right)\right]}{u'_T\left(C^T_{ss}\right)} , \tag{A.13}$$

$$\frac{v'\left(H^N_{ss} + H^T_{ss}\right)}{u'_T\left(C^T_{ss}\right)} = p^j_{ss} F'_j\left(H^j_{ss}\right) . \tag{A.14}$$

The set of admissible portfolio positions is characterized by

$$\frac{1}{R}\left(B_{ss} - B_2\right) + \sum_j \underbrace{\frac{1}{R-1}\pi^j_{ss}}_{=Q^j_{ss}}\left(\Theta^j_{ss} - \Theta^j_2\right) = 0 , \text{ with } B_{ss} \geq -\underline{b} \text{ and } \Theta^j_{ss} \geq \underline{\theta} . \tag{A.15}$$

Step 2. Let $V_1(\cdot;\cdot)$ be the counterpart value at period $t = 1$ to value $V_{ss}(\cdot;\cdot)$. The former value satisfies

$$V_1\left(b_1; \theta^N_1, \theta^T_1\right) \equiv \max_{\left\{c^N_1, c^T_1, b_2, \theta^N_2, \theta^T_2\right\}} \left\{u_N\left(c^N_1\right) + u_T\left(c^T_1\right) - v\left(n_1\right) + \frac{1}{R} V_{ss}\left(b_2; \theta^N_2, \theta^T_2\right)\right\} \tag{A.16}$$

subject to:

$$\sum_j \frac{P^j_1}{E_1} c^j_1 + \frac{1}{R} b_2 + \sum_j Q^j_{ss} \theta^j_2 = \frac{\bar{W}_1}{E_1} n_1 - \tau_1 + b_1 + \sum_j \left(\frac{\Pi^j_1}{E_1} + Q^j_{ss}\right) \theta^j_1 \tag{A.17}$$

This value already incorporates both results $V_2(\cdot;\cdot) = V_{ss}(\cdot;\cdot)$ and $Q_1^j = \frac{1}{R}(\pi_{ss}^j + Q_{ss}^j) = Q_{ss}^j$ as well as the slackness of the financing limits. Labor hours n_1 are taken as given in the optimization problem on the RHS because nominal wages $\bar{W}_1$ are rigid.

At the solution to the optimization problem, households consume the goods according to $u_N'(c_1^N)/u_T'(c_1^T) = P_1^N/E_1$. Moreover, households are indifferent between holding bonds b_2 and holding firms' shares θ_2^N or θ_2^T, as well as they perfectly smooth consumption of the tradable good over time. Therefore, $c_1^T = c_{ss}^T$, which implies

$$c_1^T = \frac{R-1}{R}\left(\frac{\bar{W}_1}{E_1}n_1 - \tau_1 - \frac{P_1^N}{E_1}c_1^N + \sum_j \frac{\Pi_1^j}{E_1}\theta_1^j + b_1\right) + \frac{1}{R}\left(w_{ss}n_{ss} - p_{ss}^N c_{ss}^N + \sum_j \pi_{ss}^j \theta_1^j\right), \tag{A.18}$$

where $\{c_1^N, c_{ss}^N, n_{ss}\}$ are given by $u_N'(c_1^N)/u_T'(c_1^T) = P_1^N/E_1$, $u_N'(c_{ss}^N)/u_T'(c_1^T) = p_{ss}^N$, and $v'(n_{ss})/u_T'(c_1^T) = w_{ss}$. Any portfolio position $\{b_2, \theta_2^N, \theta_2^T\}$ that is consistent with both budget constraint (A.17) and consumption rule (A.18) is optimal. Value $V_1(\cdot;\cdot)$ can then be reduced to

$$V_1\left(b_1;\theta_1^N,\theta_1^T\right) = \frac{R}{R-1}u_T\left(c_1^T\right) + u_N\left(c_1^N\right) - v\left(n_1\right) + \frac{1}{R-1}\left[u_N\left(c_{ss}^N\right) - v\left(n_{ss}\right)\right]. \tag{A.19}$$

Combining these optimality conditions with firm optimality conditions $\left(1+S_1^j\right)P_1^j F_j'\left(h_1^j\right) = \bar{W}_1$, market clearing conditions $N_1 = H_1^N + H_1^T$ and $C_1^N = F_N\left(H_1^N\right)$, and balanced-budget condition $\tau_1 = \sum_j S_1^j P_1^j F_j\left(H_1^j\right)$, one gets that in equilibrium aggregate consumption of the tradable good is given by

$$C_1^T = C_{ss}^T = Y_1\left(\Theta_1^N,\Theta_1^T\right) + \frac{Y_{ss}\left(\Theta_1^N,\Theta_1^T\right) - Y_1\left(\Theta_1^N,\Theta_1^T\right)}{R} + \frac{R-1}{R}B_1, \tag{A.20}$$

where the following conditions hold:

$$\frac{P_t^T}{E_t} = 1, \tag{A.21}$$

$$\frac{P_t^N}{E_t} = \frac{u_N'\left[F_N\left(H_t^N\right)\right]}{u_T'\left(C_1^T\right)}, \tag{A.22}$$

$$\left(1+S_1^j\right)\frac{P_1^j}{E_1}F_j'\left(H_1^j\right) = \frac{\bar{W}_1}{E_1}, \tag{A.23}$$

$$\frac{v'\left(H_{ss}^N + H_{ss}^T\right)}{u_T'\left(C_1^T\right)} = p_{ss}^j F_j'\left(H_{ss}^j\right). \tag{A.24}$$

Any aggregate portfolio position $\{B_2, \Theta_2^N, \Theta_2^T\}$ that is consistent with both $B_2 \geq -\underline{b}$ and

$\Theta_2^j \geq \underline{\theta}$ and that in addition satisfies the following current-account condition is admissible:

$$\frac{1}{R}\left(B_2 - B_1\right) + \sum_j \underbrace{\frac{1}{R-1}\pi_{ss}^j}_{=Q_{ss}^j}\left(\Theta_2^j - \Theta_1^j\right) = \frac{Y_1\left(\Theta_1^N, \Theta_1^T\right) - Y_{ss}\left(\Theta_1^N, \Theta_1^T\right)}{R}\,. \tag{A.25}$$

For any given exchage rate $E_1 > 0$ this continuation equilibrium admits a unique allocation but multiple portfolio positions.

Step 3. In the continuation equilibrium at period $t = 1$ the central bank solves

$$\max_{E_1}\left\{\frac{R}{R-1}u_T\left(C_1^T\right) + u_N\left[F_N\left(H_1^N\right)\right] - v\left(H_1^N + H_1^T\right) + \frac{u_N\left[F_N\left(H_{ss}^N\right)\right] - v\left(H_{ss}^N + H_{ss}^T\right)}{R-1}\right\}, \tag{A.26}$$

where equilibrium quantities $\{C_1^T, H_1^j, H_{ss}^j\}$ should be regarded here as functions of exchange rate E_1—as specified by formulae (A.20)-(A.25).

Consider technical conditions for functions $\{u_j(\cdot), v(\cdot), F_j(\cdot)\}$ such that in problem (A.26) the FOC characterizes the maximum. Isoelastic functional forms—for instance—satisfy those conditions. The FOC of the problem is

$$\begin{aligned} 0 \;=\; & \frac{R}{R-1}u_T'\left(C_1^T\right)\frac{\partial C_1^T}{\partial E_1} + u_N'\left[F_N\left(H_1^N\right)\right]F_N'\left(H_1^N\right)\frac{\partial H_1^N}{\partial E_1} - v'\left(H_1^N + H_1^T\right)\left(\frac{\partial H_1^N}{\partial E_1} + \frac{\partial H_1^T}{\partial E_1}\right) \\ & + \frac{1}{R-1}\left\{u_N'\left[F_N\left(H_{ss}^N\right)\right]F_N'\left(H_{ss}^N\right)\frac{\partial H_{ss}^N}{\partial E_1} - v'\left(H_{ss}^N + H_{ss}^T\right)\left(\frac{\partial H_{ss}^N}{\partial E_1} + \frac{\partial H_{ss}^T}{\partial E_1}\right)\right\}, \end{aligned} \tag{A.27}$$

where

$$\begin{aligned} \frac{\partial C_1^T}{\partial E_1} \;=\; & \frac{R-1}{R}\left[F_T'\left(H_1^T\right)\frac{\partial H_1^T}{\partial E_1} - \sum_j\left(1 - \Theta_1^j\right)\frac{\partial\left(\Pi_1^j/E_1\right)}{\partial E_1}\right] + \\ & + \frac{1}{R}\left[F_T'\left(H_{ss}^T\right)\frac{\partial H_{ss}^T}{\partial E_1} - \sum_j\left(1 - \Theta_1^j\right)\frac{\partial\left(\Pi_{ss}^j/E_1\right)}{\partial E_1}\right]. \end{aligned} \tag{A.28}$$

Substituting $\{(A.21), (A.22), (A.24), (A.28)\}$ into (A.27) one gets

$$\sum_j \frac{P_1^j}{E_1}\left[F_j'\left(H_1^j\right) - \frac{v'\left(H_1^j + H_1^{-j}\right)}{u_j'\left(C_1^j\right)}\right]\frac{\partial H_1^j}{\partial E_1} = \sum_j\left(1 - \Theta_1^j\right)\left[\frac{\partial\left(\Pi_1^j/E_1\right)}{\partial E_1} + \frac{1}{R-1}\frac{\partial\left(\Pi_{ss}^j/E_{ss}\right)}{\partial E_1}\right]. \tag{A.29}$$

Let $-j$ denote the insulated sector. Then

$$S_1^{-j} = \frac{\bar{W}_1}{E_1} \frac{u_T'\left(C_1^T\right)}{v'\left(H_1^{-j} + H_1^j\right)} - 1 \,, \tag{A.30}$$

$$S_1^j = 0 \,. \tag{A.31}$$

Substituting $\{(A.21), (A.22), (A.23), (A.30)\}$ and $\Theta_1^{-j} = 1$ into (A.29) one gets

$$\left[F_j'\left(H_1^j\right) - \frac{v'\left(H_1^j + H_1^{-j}\right)}{u_j'\left(C_1^j\right)}\right] \frac{\partial H_1^j}{\partial E_1} = \left(1 - \Theta_1^j\right) \left[\frac{\partial\left(\Pi_1^j/E_1\right)}{\partial E_1} + \frac{1}{R-1} \frac{\partial\left(\Pi_{ss}^j/E_{ss}\right)}{\partial E_1}\right] \frac{E_1}{P_1^j} \,. \tag{A.32}$$

□

A.10 Proof of Proposition 3.

In the economy with $S_t^N = S_t^T = \tau_t = 0$, the optimal exchange rate is unique and it imposes the following condition to the continuation equilibrium at period $t = 1$:

$$\sum_j \frac{P_1^j}{E_1} \left[F_j'\left(H_1^j\right) - \frac{v'\left(H_1^j + H_1^{-j}\right)}{u_j'\left(C_1^j\right)}\right] \frac{\partial H_1^j}{\partial E_1} = \sum_j \left(1 - \Theta_1^j\right) \left[\frac{\partial\left(\Pi_1^j/E_1\right)}{\partial E_1} + \frac{1}{R-1} \frac{\partial\left(\Pi_{ss}^j/E_{ss}\right)}{\partial E_1}\right] . \tag{A.33}$$

Moreover, if foreigners do not own domestic firms' shares, the condition implies that

$$F_j'\left(H_1^j\right) = \frac{v'\left(H_1^j + H_1^{-j}\right)}{u_j'\left(C_1^j\right)} \,, \quad \text{for } j \in \{N, T\} \,. \tag{A.34}$$

Proof. Condition (A.33) is derived in the proof of Proposition **??** [see equation (A.29)]. If $\Theta_1^j = 1$ for $j \in \{N, T\}$ then condition (A.33) reduces to

$$\sum_j \frac{P_1^j}{E_1} \left[F_j'\left(H_1^j\right) - \frac{v'\left(H_1^j + H_1^{-j}\right)}{u_j'\left(C_1^j\right)}\right] \frac{\partial H_1^j}{\partial E_1} = 0 \,. \tag{A.35}$$

Substituting $S_1^j = 0$ and $\{(A.21), (A.22), (A.23), (A.35)\}$ into (A.35) one gets

$$\frac{P_1^j}{E_1} \left[F_j'\left(H_1^j\right) - \frac{v'\left(H_1^j + H_1^{-j}\right)}{u_j'\left(C_1^j\right)}\right] \sum_j \frac{\partial H_1^j}{\partial E_1} = 0 \,, \text{ for } j \in \{N, T\} \,, \tag{A.36}$$

which in turn implies from (A.20)-(A.23) that

$$F_j'\left(H_1^j\right) = \frac{v'\left(H_1^j + H_1^{-j}\right)}{u_j'\left(C_1^j\right)}\ , \quad \text{for } j \in \{N, T\}\ . \tag{A.37}$$

□

A.11 Proof of Proposition 4

Under standard regularity conditions for functions $\{u(\cdot), v(\cdot), F(\cdot)\}$, the optimal exchange rate is unique and it imposes the following condition to the continuation equilibrium at period $t = 1$:

$$\int \left[F'(H_1) - \frac{v'(h_{i,1})}{u'(c_{i,1})}\right] \frac{\partial h_{i,1}}{\partial E_1} d\hat{G}_{i,1} = \int \left(\frac{h_{i,1}}{H_1} - \theta_i\right) d\hat{G}_{i,1} \left[\frac{\partial(\Pi_1/E_1)}{\partial E_1} + \frac{1}{R-1}\frac{\partial(\Pi_{ss}/E_{ss})}{\partial E_1}\right], \tag{A.38}$$

where $\hat{G}_{i,1}$ is the cumulative function associated with density function $\hat{g}_{i,1}$, with:

$$\hat{g}_{i,1} \equiv \frac{1}{\int u'(c_{i,1})\, dG_{i,1}} u'(c_{i,1})\, g_{i,1}\ . \tag{A.39}$$

Proof. To prove this proposition we proceed according to the following three steps:

1. Solve the problem of households in the continuation equilibrium at period $t = 2$ and derive that continuation equilibrium.
2. Do the same as in Step 1. but for the continuation equilibrium at period $t = 1$.
3. Solve the problem of the central bank in the continuation equilibrium at period $t = 1$.

Step 1. The problem of households is the same as that already laid out in Section 3. Using the notation in Subsection 5.3, one can express the solution to the problem as follows:

$$c_{i,ss} = w_{ss} h_{i,ss} + \frac{R-1}{R} a_{i,ss}\ , \tag{A.40}$$

and

$$\frac{v'(h_{i,ss})}{u'(c_{i,ss})} = w_{ss}\ , \tag{A.41}$$

where financial wealth $a_{i,ss} \in \mathbb{R}$ is defined as

$$a_{i,ss} \equiv \frac{R}{R-1}\pi_{i,ss}\theta_i + b_i \ , \tag{A.42}$$

and where $(b_i', \theta_i') = (b_{i,ss}, \theta_{i,ss})$ is any portfolio position that satisfies

$$\frac{R}{R-1}\pi_{ss}\theta_{i,ss} + b_{i,ss} = a_{i,ss} \ . \tag{A.43}$$

Combining these optimality conditions with firm optimality condition $F'(H_{ss}) = w_{ss}$ and with market clearing for labor hours, one gets that the equilibrium is characterized by

$$c_{i,ss} = F'(H_{ss})\, h_{i,ss} + \frac{R-1}{R} a_{i,ss} \ , \tag{A.44}$$

and

$$\frac{v'(h_{i,ss})}{u'(c_{i,ss})} = F'(H_{ss}) \ , \tag{A.45}$$

where financial wealth $a_{i,ss}$ is given by

$$a_{i,ss} = \frac{R}{R-1}\left[F(H_{ss}) - F'(H_{ss})\, H_{ss}\right]\theta_i + b_i \ , \tag{A.46}$$

and aggregate labor hours H_{ss} are given by

$$H_{ss} = \int h_{i,ss} dG_{i,2} \ . \tag{A.47}$$

Cumulative distribution function $G_{i,2}$ evolves according to

$$g_{ss}(b_{i,ss}, \theta_{i,ss}) = g_2(b_i, \theta_i) \ , \tag{A.48}$$

with

$$\int dG_{i,ss} = 1 \ . \tag{A.49}$$

Step 2. The solution to the problem of households can be expressed as follows:

$$c_{i,1} = c_{i,ss} = \frac{R-1}{R}\left[\frac{\bar{W}_1}{E_1} h_{i,1} + \frac{1}{R-1} w_{ss} h_{i,ss} + a_{i,1}\right] , \tag{A.50}$$

and

$$\frac{v'(h_{i,1})}{u'(c_{i,1})} = w_{ss} \ , \tag{A.51}$$

where financial wealth $a_{i,1} \in \mathbb{R}$ is defined as

$$a_{i,1} \equiv \left(\frac{\Pi_1}{E_1} + \frac{1}{R-1} \pi_{ss} \right) \theta_i + b_i \ , \tag{A.52}$$

where portfolio position $(b_{i,2}, \theta_{i,2})$ is given by

$$\underbrace{\frac{R}{R-1} \pi_{ss} \theta_{i,2} + b_{i,2}}_{= a_{i,2}} = \frac{\bar{W}_1}{E_1} h_{i,1} - w_{ss} h_{i,ss} + a_{i,1} \ . \tag{A.53}$$

Combining these optimality conditions with firm optimality condition $F'(H_1) = \bar{W}_1 / E_1$ and with market clearing for labor hours, one gets that the equilibrium is characterized by

$$c_{i,1} = c_{i,ss} = \frac{R-1}{R} \left[F'(H_1) h_{i,1} + \frac{1}{R-1} F'(H_{ss}) h_{i,ss} + a_{i,1} \right] , \tag{A.54}$$

and

$$\frac{v'(h_{i,1})}{u'(c_{i,1})} = \frac{\tilde{W}_1}{E_1} \ , \tag{A.55a}$$

$$\frac{v'(h_{i,ss})}{u'(c_{i,1})} = F'(H_{ss}) \ , \tag{A.55b}$$

where financial wealth $a_{i,1}$ is given by

$$a_{i,1} \equiv \left[\left[F(H_1) - F'(H_1) H_1 \right] + \frac{1}{R-1} \left[F(H_{ss}) - F'(H_{ss}) H_{ss} \right] \right] \theta_i + b_i \ , \tag{A.56}$$

"fictitious" nominal wage $\tilde{W}_1 > 0$ is such that

$$H_1 = \int h_{i,1} dG_{i,1} \ , \quad \text{with} \quad F'(H_1) = \frac{\bar{W}_1}{E_1} \ , \tag{A.57}$$

and aggregate labor hours $H_{ss} > 0$ are given by

$$H_{ss} = \int h_{i,ss} dG_{i,1} \ . \tag{A.58}$$

Cumulative distribution function $G_{i,1}$ evolves according to

$$g_2(b_{i,2}, \theta_{i,2}) = g_1(b_i, \theta_i) \ , \tag{A.59}$$

with

$$\int dG_{i,2} = 1 \ . \tag{A.60}$$

Step 3. In the continuation equilibrium at period $t = 1$ the central bank solves

$$\max_{E_1} \int \left[\frac{R}{R-1} u\left(c_{i,1}\right) - \left[v\left(h_{i,1}\right) + \frac{1}{R-1} v\left(h_{i,ss}\right) \right] \right] dG_{i,1} \ . \tag{A.61}$$

where equilibrium quantities $\{c_{i,1}, h_{i,1}, h_{i,ss}\}$ should be regarded here as functions of exchange rate E_1—as specified by the above formulae.

Consider technical conditions for functions $\{u(\cdot), v(\cdot), F(\cdot)\}$ such that in problem (A.61) the FOC characterizes the maximum. Isoelastic functional forms—for instance—satisfy those conditions. The FOC of the problem is

$$\int \left[\frac{R}{R-1} u'\left(c_{i,1}\right) \frac{\partial c_{i,1}}{\partial E_1} - v'\left(h_{i,1}\right) \frac{\partial h_{i,1}}{\partial E_1} - \frac{1}{R-1} v'\left(h_{i,ss}\right) \frac{\partial h_{i,ss}}{\partial E_1} \right] dG_{i,1} = 0 \ , \tag{A.62}$$

where

$$\begin{aligned} \frac{\partial c_{i,1}}{\partial E_1} \quad = \quad & \frac{R-1}{R} \left[F''\left(H_1\right) \frac{\partial H_1}{\partial E_1} h_{i,1} + F'\left(H_1\right) \frac{\partial h_{i,1}}{\partial E_1} \right] + \\ & \frac{1}{R} \left[F''\left(H_{ss}\right) \frac{\partial H_{ss}}{\partial E_1} h_{i,ss} + F'\left(H_{ss}\right) \frac{\partial h_{i,ss}}{\partial E_1} \right] + \\ & - \left[\frac{R-1}{R} F''\left(H_1\right) H_1 \frac{\partial H_1}{\partial E_1} + \frac{1}{R} F''\left(H_{ss}\right) H_{ss} \frac{\partial H_{ss}}{\partial E_1} \right] \theta_i \ . \end{aligned} \tag{A.63}$$

Substituting (A.63) into (A.62) one gets

$$\int \left[F'\left(H_1\right) - \frac{v'\left(h_{i,1}\right)}{u'\left(c_{i,1}\right)} \right] \frac{\partial h_{i,1}}{\partial E_1} d\hat{G}_{i,1} = \int \left(\frac{h_{i,1}}{H_1} - \theta_i \right) d\hat{G}_{i,1} \left[\frac{\partial \left(\Pi_1 / E_1\right)}{\partial E_1} + \frac{1}{R-1} \frac{\partial \left(\Pi_{ss} / E_{ss}\right)}{\partial E_1} \right] , \tag{A.64}$$

with

$$\hat{g}_{i,1} \equiv \frac{1}{\int u'\left(c_{i,1}\right) dG_{i,1}} u'\left(c_{i,1}\right) g_{i,1} \ , \tag{A.65}$$

where recall that

$$\frac{\partial \left(\Pi_t / E_t\right)}{\partial E_1} = -F''\left(H_t\right) H_t \frac{\partial H_t}{\partial E_1} \ . \tag{A.66}$$

As an aside, note that if $a_{i,1} = A_1 \ \forall i$ then $c_{i,1} = C_1$ and $h_{i,1} = H_1 \ \forall i$ as well, which allows

one to reduce (A.64) as follows

$$\left[F'\left(H_1\right)-\frac{v'\left(H_1\right)}{u'\left(C_1\right)}\right]\underbrace{\int\frac{\partial h_{i,1}}{\partial E_1}dG_{i,1}}_{=\partial H_1/\partial E_1}=\left(1-\underbrace{\int\theta_i dG_{i,1}}_{=\Theta_1}\right)\left[\frac{\partial\left(\Pi_1/E_1\right)}{\partial E_1}+\frac{1}{R-1}\frac{\partial\left(\Pi_{ss}/E_{ss}\right)}{\partial E_1}\right]. \tag{A.67}$$

□