Does Growth Lead to Labor Reallocation? *†

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Abstract

This paper studies the relationship between growth and labor reallocation when producers innovate over products and are subject to demand shocks. Incumbent producers contribute to long-run growth by improving existing products or expanding to new product markets. These innovations lead to increased employment. However, job creation caused by demand shocks does not generate growth. The calibrated model using innovation survey data from Germany demonstrates that only 37% of job creation is attributable to innovation. Most growth stems from own-product improvements, which lead to minimal labor reallocation. Ignoring demand shocks in the calibration overestimates the role of product expansion and underestimates the impact of entrant innovation on growth. The model further predicts that while slow population growth modestly affects incumbent job creation, it substantially influences growth through own-product improvements and entrant innovation.

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1 Introduction

Productivity growth is often associated with large-scale labor reallocation across firms. This concept is rooted in the Schumpeterian view of technological progress, where new products and processes emerge as firms enter a product market and replace existing producers (Aghion and Howitt, 1992). Due to the Arrow replacement effect, incumbent producers face weaker incentives to innovate on products they currently produce. Consequently, job creation by growing firms and job destruction by shrinking firms have long been key metrics for assessing an economy's growth potential.[1]

This paper investigates the relationship between economic growth and labor reallocation when incumbent producers are subject to demand shocks. Building on Garcia-Macia et al. (2019), this study models how entrants and incumbents drive growth through product innovations—either by improving their own products or by expanding into new product markets.[2] The key contribution of the model is to introduce demand shocks that shift labor demand without contributing to growth.

In this setting, different innovations imply a different relationship between growth and labor reallocation. Product expansion and entrant innovation yield creative destruction of incumbent producers or the creation of new varieties. On the other hand, own-product improvements do not replace firms in product markets or create new product markets. Thus, for a given level of output growth, own-product improvements entail far less labor reallocation compared to product expansion or entrant innovation.

In order to decompose the contribution of demand shocks and different innovation types to growth and labor reallocation, this paper leverages novel data about establishment-level innovation activities from a German establishment panel survey. The survey directly asks whether each establishment has improved the quality of existing products, upgraded production processes, or introduced a new product variety. The survey responses help identify job creation by innovators and non-innovators. Additionally, the survey's structure naturally distinguishes different types of innovation. The survey data is merged with administrative employer-employee matched data to measure the impact of these innovation types. This process provides information on the number of employees at each establishment before and after the innovation events.

[1] In Davis et al. (1998), the job creation rate is defined by $2 \times \max\{E_t - E_{t-1}, 0\}/(E_t + E_{t-1})$, and the job destruction rate is defined by $2 \times \max\{E_{t-1} - E_t, 0\}/(E_t + E_{t-1})$, where E_t is the employment of an establishment or a firm at time t.

[2] In this context, a product is defined by substitutability. If the new product closely substitutes an existing one, it is considered another version of the old product. If it does not closely substitute the old product, it is treated as a distinct product.

Survey-based measures of innovation complement two widely used approaches in the Schumpeterian growth literature. First, previous studies such as Garcia-Macia et al. (2019) and Lentz and Mortensen (2008) use indirect inference to identify the nature of firm-level innovation. In their models, all changes at the firm level are attributed to technological shocks that permanently increase productivity. However, firm-level adjustments are also driven by temporary demand and productivity shocks. This paper uses survey answers to distinguish firm behavior associated with innovation activities. The calibrated model reveals that innovation accounts for less than 40% of incumbent job creation.

Furthermore, the survey-based measure of innovation effectively separates own-product improvements from product expansion. Garcia-Macia et al. (2019) primarily use the model structure to distinguish innovation types. They associate large increases in employment with product expansion and smaller increases with own-product improvements. The innovation survey data shows that the growth rate of employment after product expansion is only slightly higher than that of the average establishment and nearly equivalent to the growth after own-product improvements. Additionally, the distribution of size growth after product expansion is highly dispersed, resembling both the average firm's growth and that seen after own-product improvements. This suggests that it is difficult to isolate product expansion from other sources of large-scale job creation at the firm level.

The second approach uses patents as proxies, as in Akcigit and Kerr (2018). They use self-citation shares to distinguish patents for own-product improvements from patents for product expansion. However, the firms that own and make new patents are not representative of the firm population. 94.5% of US manufacturers do not own any patent, accounting for 48% of employment (Balasubramanian and Sivadasan, 2011). Even among firms that ever make patents, most firms are engaged in patent production for only one year in the UK (Geroski et al., 1997). As a result, patenting activity on the knowledge frontier does not necessarily reflect the overall composition of innovation associated with labor reallocation in the economy. Unlike patents, the survey data encompasses a representative sample of establishments across various characteristics, including age and size.

This paper uses the survey data merged with the administrative data to calibrate an endogenous growth model augmented with demand shocks. The economy sustains long-run growth through three types of innovation: own-product improvements, product expansion, and entrant innovation. The model extends Garcia-Macia et al. (2019) to include endogenous innovation choices and shocks to demand shifters at the product level. When the demand shifter follows a geometric random walk, the model still exhibits random growth in product demand, as in Garcia-Macia et al. (2019).

The calibrated model indicates that own-product improvements account for 78% of the Bal-

anced Growth Path (BGP) growth in the German economy. Product expansion accounts for only 1% of the growth on the BGP, and the remaining 21% stems from entrant innovation. The minimal contribution of product expansion to growth is due to its limited impact on employment growth. First, on the extensive margin, only 34% of surveyed establishments reported adding new products to their portfolios, compared to 54% that improved the quality of existing products or the productivity of production processes. Second, on the intensive margin, product-expanding establishments increased their employment growth rate by only 1.2 percentage points over four years after the innovation, relative to the average establishment. In contrast, establishments improving their existing products grew 2.6 percentage points faster than the average establishment.

In theory, product expansion requires significantly more labor reallocation across firms to achieve growth levels comparable to own-product improvements. Because product expansion replaces operating firms on product lines, the limited labor reallocation associated with it implies that firm-level replacements from product expansion contribute marginally to growth.

If the model is calibrated to match the gross job creation rate without accounting for demand shocks, the contribution of product expansion is overstated, while that of entrant innovation is understated. This happens because the calibrated magnitude of demand shocks is large. Due to the shocks, even non-innovators have a standard deviation of 17% in annual employment growth rates. Consequently, the demand shocks disproportionately drive large-size labor reallocation between incumbent producers.

The model reveals that growth and labor reallocation are not aligned when the economy experiences slow population growth. This misalignment occurs because own-product improvements significantly influence both growth and welfare. The decline in labor supply associated with slow population growth reduces incumbent innovation, which requires R&D inputs produced with labor. Own-product improvements make 47% of the reduction in the growth rate on the BGP and 76-92% of the welfare loss over the transition path from the fast-growing BGP to a slower one. While entrant innovation contributes substantially to the long-term decline in growth, its impact on welfare is relatively limited during the transition.

Related Literature

This paper makes three contributions. First, it complements the existing literature on endogenous growth models by introducing survey-based measures of innovation. Then, it finds that most labor reallocation among incumbents is unrelated to innovation, with most growth

driven by own-product improvements. A seminal paper by Aghion and Howitt (1992) models innovation as firms competing against each other to occupy a limited number of products. Most firm-level innovation papers, such as Klette and Kortum (2004), Lentz and Mortensen (2008), and Acemoglu et al. (2018), follow this tradition, assuming that all technological growth arises from product expansion and entrant innovation promoting competition across firms. As noticed by Acemoglu and Cao (2015) and Akcigit and Kerr (2018), incumbents can adopt less destructive innovations by improving their existing products. This paper uses a different calibration strategy using a more direct measure of innovation and demonstrates that the extent of incumbent-driven creative destruction is even smaller than previously thought in Garcia-Macia et al. (2019), with most creative destruction made by entrants.

Second, this paper contributes to discussions on the aggregate effect of innovation on employment. Theoretical studies offer varying predictions about whether destructive innovation redirects displaced workers to unemployment or to new employers (Postel-Vinay, 2002; Pissarides and Vallanti, 2007; Michau, 2013). Empirical results also give mixed evidence on the importance of innovation and growth to explain employment differences across countries and over time (Campbell, 1998; Davis and Haltiwanger, 1999; Hornstein et al., 2007; Faberman, 2017). In the literature on declining business dynamism, Decker et al. (2014, 2016) discuss that reduced entrant innovation in the U.S. is a primary factor behind slower labor reallocation. Using a more direct measure of innovation, the findings in this paper suggest that growth is largely orthogonal to employment because most innovation is driven by incumbents to improve their existing products.

Lastly, this paper incorporates survey-based measures of innovation into the structural macroeconomic analysis of economic growth. Economists frequently use indirect inference to infer innovation behavior and productivity shocks from establishment-level indicators such as employment, output, and wages (Lentz and Mortensen, 2008; Coles and Mortensen, 2016; Garcia-Macia et al., 2019). Other studies use the patent application, grant, or citation data to measure the innovation activities within sectors or firms, assuming that the new knowledge is exclusively attributed to the patent-owning firm (Aghion et al., 2005; Kogan et al., 2017; Akcigit and Kerr, 2018; Dorn et al., 2020; Kelly et al., 2021). Research on knowledge spillovers also uses patent data to measure innovation (Bottazzi and Peri, 2003; Hunt and Gauthier-Loiselle, 2010; Cai and Li, 2019).

This paper employs a survey-based measure of innovations that captures a broader range of innovation activities behind the knowledge frontier and makes it easier to distinguish various innovation activities.[3] Previous studies have also examined the employment effects

[3]Online Appendix 1.2 compares the numbers of external and internal patents, which Akcigit and Kerr (2018) consider as measures of product expansion and own-product improvements, respectively, with the

of survey-based measures of innovation. Using the European Community Innovation Survey, papers such as Harrison et al. (2014) and Lachenmaier and Rottmann (2011) distinguish product and process innovation and estimate their employment effects separately for each type of innovation.[4] They both find significantly positive but small employment effects of innovation in a panel regression framework. This paper builds on these insights by using survey data to identify parameters in an endogenous growth model and derive macroeconomic implications. This paper combines survey data with administrative employment records to address the limitations of survey-based measures, such as the inability to distinguish the quality and quantity of innovation. The merged data enables comparisons of employment growth after innovation with that of average establishments.

This paper is organized as follows. Section 2 builds a quantitative model of endogenous growth through innovation choices. Section 3 explains how model parameters are calibrated with the German survey data and compares the model with a model without demand shocks. Finally, Section 4 applies the calibrated model to analyze the quantitative effects of slower population growth.

2 Quality Ladder Model with Demand Shocks

2.1 Environment

The economy consists of representative households and firms.[5] The population of representative households is denoted by M_t, which grows at a constant rate, g_M. Representative households collect all labor income and business profits, supply labor, and consume homogeneous final goods. They also supply R&D inputs produced one-to-one with their labor.

Final goods are competitively produced by combining differentiated intermediate goods. Each variety of intermediate goods corresponds to a product line and is produced by intermediate goods producers operating on the line. Products within the same product line are perfect substitutes once quality differences are accounted for, while products across different product lines are imperfect substitutes. The production of intermediate goods requires labor

survey-based measure of innovations.

[4]For additional empirical studies using survey data, see Peters (2004, 2008); Hall et al. (2008) for European countries; Zuniga and Crespi (2013); Crespi et al. (2019) for Latin American Countries; and Mairesse and Wu (2019) for China.

[5]Throughout this paper, "firms" and "producers" are used interchangeably. "establishments" in the German data are considered firms in the analysis. Multiple production units in different locations belong to a single establishment if they share the same company, municipality, and economic class. See Section 3.1 for further discussion.

inputs. To keep a product line active, the firm has to hire ϕ overhead labor in addition to the production labor.

Firms producing intermediate goods are heterogeneous in their product portfolios. They can hold multiple product lines, with each product associated with a demand shifter: a. The shifter consists of two parts: the quality of the product, denoted by z, and the idiosyncratic preference, denoted by u. The idiosyncratic preference is common to all producers operating on a product line, whereas quality levels vary across producers.

The idiosyncratic preference for a product line follows a geometric random walk.

$$du = \varsigma dB_t.$$

In this equation, B_t represents the standard Wiener process, and ς governs the magnitude of the demand shocks. This shock component generates firm-level labor reallocation independently of the innovation processes.

Incumbent producers improve the quality of their existing products. If a producer innovates on a product line with a, the quality of the product line increases to $\lambda \times a$, where λ is drawn from $H(\lambda)$. They choose a Poisson rate for an own-product improvement, ξ, by paying $a \times c_{own}(\xi)$ units of R&D inputs per unit of time for a given product line. This R&D cost is convex in the rate of innovation, i.e., $c'_{own}(\xi), c''_{own}(\xi) > 0$, but increases linearly in $a = zu$. As a result, high-quality and high-demand products require more R&D efforts, but their returns to R&D are also larger in absolute terms with the multiplicative step size of innovation, λ.

Incumbent firms also expand to new product lines at a Poisson rate ζ by purchasing $c_{exp}(\zeta)$ units of R&D inputs per unit of time for each product line. The cost of innovation is also convex, i.e., $c'_{exp}(\zeta), c''_{exp}(\zeta) > 0$. When expanding, a firm randomly draws the quality of a product line from existing ones, learns from the producer with the highest quality, and increases the quality by a factor of $\lambda \sim H(\lambda)$. With a probability of ν_{exp}, the new product of the expanding producer replaces existing products on the drawn product line. With a probability of $1 - \nu_{exp}$, the firm creates a new variety. The producer cannot choose between creative destruction and variety creation. The quality of the new product is lower than the existing one by a factor of κ, as in Garcia-Macia et al. (2019). Then, conditional on drawing x and λ, the quality of the producer on the new line is $\lambda\kappa x$.

Since different products on a product line are perfect substitutes, only the producer with the highest quality takes on the product line. Consequently, the expanding firm always wins the product line, consistent with other Schumpeterian models in the literature (Aghion and Howitt, 1992; Garcia-Macia et al., 2019; Akcigit and Kerr, 2018). For simplicity, it

is assumed that the producer with the highest quality sets a monopolistic price on the differentiated intermediate goods.[6]

A firm loses a product line when either an expanding or entering firm comes to the product line. The Poisson rate of this event is denoted by $\tau(x)$. For tractability, this loss of a product line is assumed to be permanent. Thus, the second-best firm cannot re-enter the product line by improving its quality. Firms exit the market if their sole product line is poached by another firm or if its value becomes negative.

New firms enter the market continuously at a rate χN_t, where N_t is the mass of product lines at time t. All new firms start with a single product line. Upon entry, a firm first randomly draws a product line to learn and increases the quality of the matched product line with the step size $\lambda \sim H(\lambda)$. With probability $1 - \nu_{entry}$ upon entry, it drives out the incumbent producer on the matched product line. Conversely, with probability ν_{entry}, the entrant creates a new variety with a lower quality, reduced by a factor of κ. As in the case of product expansion, entrants cannot choose between driving out incumbents and creating new varieties. Following Garcia-Macia et al. (2019), entrants are directed toward product lines with lower demand. This assumption implies that products with a low demand shifter face higher rates of creative destruction.

2.2 Representative Household

The representative household maximizes the following lifetime utility:

$$\max_{C_t} \int_0^\infty e^{-\rho t} \left(\log C_t - \frac{\eta}{1+\eta} \phi H_t^{\frac{1+\eta}{\eta}} \right) dt.$$

subject to

$$C_t + \dot{D}_t = W_t H_t + \int_{\Omega_t} \Pi_{jt} dj + r_t D_t.$$

In this equation, C_t is the consumption, and D_t denotes the real bond holding with zero net supply. W_t is the wage, H_t is the labor supply of a household, and Π_{jt} is the profit (net of innovation costs) from product line j. Ω_t is the set of product lines available at time t.
The Euler equation of this maximization pins down the interest rate.

$$r_t = r = \rho + g_C, \tag{1}$$

[6]We can rationalize this setting by assuming that producers on a product line engage in a two-stage Bertrand Game, following Akcigit and Kerr (2018). In the first stage, producers decide whether to whether to pay $\varepsilon > 0$ to proceed to the second stage. In the second stage, only firms that paid in the first stage may set their price and sell in the market. In the Subgame Perfect Nash Equilibrium, only the producer with the highest quality pays in the first stage and sets the monopolist price. This assumption implies that, unlike Peters (2020), the economy does not exhibit misallocation of inputs.

where g_C is the consumption growth rate. The first-order conditions determine the labor supply per representative household:

$$\phi\left(H_t\right)^{\frac{1}{\eta}} = \frac{W_t}{C_t}. \tag{2}$$

Thus, on the BGP where H_t is constant, W_t grows at the same rate as C_t. With M_t denoting the mass of households at time t, the aggregate consumption and labor supply are given by $Y_t = M_t C_t$ and $L_t = M_t H_t$, respectively.

2.3 Final Goods

Homogeneous final goods are competitively produced by aggregating intermediate goods from different product lines with a constant elasticity of substitution:

$$Y_t = \left(\int_{\Omega_t} a_{jt}^{\frac{1}{\sigma}} y_{jt}^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution, and a_{jt} is the demand shifter for the intermediate good from product line j. The demand shifter increases with the quality z_{jt} and idiosyncratic preference u_{jt} of the product line:

$$a_{jt} = z_{jt} u_{jt}. \tag{3}$$

Incumbent producers on a product line face the same u_{jt} but differ in quality. Hence, under the Bertrand auction described in Section 2.1, only the firm with the highest quality produces on that line. The monopolistic firm then takes the demand for intermediate goods as given and sets a price to maximize the profit. Because the demand elasticity is constant, higher product quality shifts the demand curve rightward but does not affect the markup.
The price-setting problem is described by:

$$p_{jt} = \arg\max_{p_{jt}} \left\{ y_{jt} p_{jt} - W_t y_{jt} \right\},$$

subject to

$$y_{jt} = Y_t \, P_t^{\sigma} p_{jt}^{-\sigma} a_{jt}.$$

In this equation, $P_t := \left(\int_{\Omega_t} a_{jt} p_{jt}^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}$ is the quality-adjusted price index. Because the final good is the numeraire, P_t is one in equilibrium. Productivity is normalized to one on all product lines, so one unit of labor converts into one unit of intermediate goods.[7] The

[7]This specification is isomorphic to a case of productivity improvements. Consequently, both quality improvements and process innovations are included in the definition of own-product improvements used in the empirical analysis.

optimal price features a constant markup, i.e., $p_{jt} = \sigma/(\sigma-1)W_t$.
The zero-profit condition for final goods producers becomes

$$N_t A_t = W_t^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}, \tag{4}$$

where $N_t = N(\Omega_t)$ is the mass of available product lines at time t and $A_t = \int_{\Omega_t} a_{jt}dj/N_t$ is the average quality index. From Equation (4), W_t increases log-linearly in $N_t A_t$.
Aggregating production-labor demand yields

$$Y_t = L_t^P (N_t A_t)^{\frac{1}{\sigma-1}}, \tag{5}$$

where L_t^P is the production labor. Consequently, GDP per production-labor input, Y_t/L_t^P, also increases log-linearly in $N_t A_t$. Define $\bar{Y}_t = Y_t/N_t$ and $\bar{L}_t^P = L_t^P/N_t$ as the output and the production labor per unit of product lines.
Substituting into Equation (5) gives

$$\bar{Y}_t = \bar{L}_t^P (N_t A_t)^{\frac{1}{\sigma-1}} = \left(\frac{\sigma}{\sigma-1}\right)\bar{L}_t^P W_t. \tag{6}$$

Hence, in a BGP equilibrium—where employment per unit interval of product lines is fixed—W_t rises linearly in $\bar{Y}_t$.

The profit of a product line can be written as

$$\pi_t(z_{jt}, u_{jt}) = \frac{Y_t a_{jt}}{\sigma N_t A_t} = \bar{Y}_t\,\bar{\pi} x_{jt}.$$

In this equation, $x_{jt} := a_{jt}/A_t = z_{jt}u_{jt}/A_t$ is the relative demand shifter, and $\bar{\pi} = 1/\sigma$ is a marginal profit return to that shifter. This definition of x_{jt} implies that $\int x_{jt}dj = 1$. The production-labor demand is also linear in x_{jt}.

The model is solved on the balanced growth path (BGP) equilibrium, where the output, the wage, and the mass of product lines grow at constant rates. Notably, on the BGP, all profits and prices (including the wage) are homogeneous of degree one in $\bar{Y}_t$ conditional on x_{jt}. Moreover, x_{jt} has a stationary distribution. Therefore, in what follows, the subscript t is dropped whenever some variables or their distributions do not change on the BGP.

2.4 Values

The state variable of an intermediate goods producer is the set of relative demand shifters on product lines it occupies, $\{x_j\}_{j=1}^{n_i}$, where n_i is the number of product lines held by the firm. The firm's per-period revenue is $\sum_{j=1}^{n_i}\bar{Y}_t\bar{\pi}x_j$. Because all costs and revenues from the

product lines are additively separable, the total firm value is the sum of its product-line values:

$$\mathfrak{V}\left(\{x_j\}_{j=1}^{n_i}, W_t, \bar{Y}_t\right) = \sum_{j=1}^{n_i} \mathbf{V}(x_j, W_t, \bar{Y}_t).$$

Since the product-line value is sufficient to characterize the equilibrium, the j subscript is removed from x_j below. Furthermore, on the BGP, the value of a product line is a product between the aggregate output per mass of product lines and the time-invariant value that only depends on the relative demand:[8]

$$\mathbf{V}(x, W_t, \bar{Y}_t) = \bar{Y}_t \, v\left(x\right).$$

The stationary value function is then characterized by:

$$\begin{aligned}\left(\rho + \tau\left(x\right)\right) v\left(x\right) = \max_{\xi,\zeta \geq 0} \Bigg\{ & \bar{\pi} x - \phi w - g_A x \partial_x v\left(x\right) + \frac{x^2 \varsigma^2}{2} \partial_x^2 v\left(x\right) \\ & - w x c_{own}\left(\xi\right) + \xi \mathbb{E}_{H(\lambda)}\left[v\left(\lambda x\right) - v\left(x\right)\right] \\ & - w c_{exp}\left(\zeta\right) + \zeta \mathbb{E}_{H(\lambda)} \Bigg[\nu_{exp} \int V\left(\lambda x'\right) dF\left(x'\right) \\ & + \left(1 - \nu_{exp}\right) \int V\left(\lambda \kappa x'\right) dF\left(x'\right) \Bigg] \Bigg\},\end{aligned} \tag{7}$$

where ρ is the discount rate, and $\tau(x)$ is the creative destruction rate. $\tau(x)$ depends on x because new entrants direct their innovation toward product lines with lower x. g_A is the growth rate of A_t, and $w = W_t/\bar{Y}_t$ is the wage rate relative to the final output per mass of product lines. $\mathbb{E}_{H(\lambda)}[\cdot]$ denotes the expectation with respect to the distribution $H(\lambda)$ of step sizes λ. Lastly, $F(x)$ is the stationary distribution of x, which integrates into one. Unlike entrants, incumbent firms' expansion is undirected.

Let ζ denote the optimal expansion rate. Note that ζ does not depend on x because the return and the cost of product expansion are independent of x. Likewise, we define $\xi(x)$ as the rate of own-product improvement chosen by a firm on a product line with x. Because $\tau(x)$ varies with x, $\xi(x)$ also differs across x. Later, we introduce functional forms for $\tau(x)$ such that $\tau'(x) < 0$ and $\lim_{x\to\infty} \tau(x) = \zeta$. Then, when x is sufficiently large, $\xi(x)$ is almost constant, because the cost and return are (nearly) linear in x. Thus, along with preference shocks to x that follow a geometric random walk, the employment at the product line level approximately exhibits random growth as in many Schumpeterian growth models (Klette and Kortum, 2004; Garcia-Macia et al., 2019).

[8] Appendix A.1 presents the formulation of the firm-level value function and discusses multiplicative separability.

2.5 Entry

Entrants draw an existing product, learn from its quality, and enter the market as described in Section 2.1. Like own-product improvements and product expansions, entrant innovation also requires labor inputs. The free entry condition is given by

$$wc_{entry} \geq \iint \left(\left(1 - \nu_{entry}\right) v\left(\lambda x'\right) + \nu_{entry} v\left(\lambda \kappa x'\right) \right) dF_0(x') dH(\lambda). \tag{8}$$

Here, $F_0(x')$ is the probability distribution of x that applies when an entrant is matched to a product line. Note that F_0 differs from F with directed matching of entrants.

2.6 Distribution

The distribution of relative demand x is characterized by the following equation:

$$\begin{aligned} 0 = & \partial_x \left[g_A x f\left(x\right) \right] + \partial_x^2 \left[\frac{\varsigma^2 x^2}{2} f\left(x\right) \right] \\ & - \int \left(\xi\left(x\right) f\left(x\right) - \xi\left(\frac{x}{\lambda}\right) f\left(\frac{x}{\lambda}\right) \right) dH\left(\lambda\right) \\ & - \left(1 - \nu_{exp}\right) \zeta \left(\int \left(f\left(x\right) - f_0\left(\frac{x}{\lambda}\right) \right) dH\left(\lambda\right) \right) \\ & - \nu_{exp} \zeta \int \left(f\left(x\right) - f\left(\frac{x}{\kappa\lambda}\right) \right) dH\left(\lambda\right) \\ & - \left(1 - \nu_{entry}\right) \chi \left(\int \left(f_0\left(x\right) - f_0\left(\frac{x}{\lambda}\right) \right) dH\left(\lambda\right) \right) \\ & - \nu_{entry} \chi \int \left(f\left(x\right) - f_0\left(\frac{x}{\kappa\lambda}\right) \right) dH\left(\lambda\right) \\ & + \omega f\left(x\right), \end{aligned} \tag{9}$$

where $f(x) = dF(x)/dx$ and $f_0(x) = dF_0(x)/dx$. The term $\omega = \partial_x \left[\frac{\varsigma^2 x^2}{2} f_t\left(x\right) \right]_{x=\underline{x}}$ is the rate of exits due to the obsolescence of product lines (i.e., when x falls below the production cutoff $\underline{x}$). See Appendix A.2 for the derivation of Equation (9).

Integrating Equation (9) over $[0, x)$ and over $[0, \infty)$ gives the following expression for g_A:

$$\begin{aligned} g_A = & \omega + \iiint_{x' \leq x} \left(\xi\left(x'\right) f\left(x'\right) - \xi\left(\frac{x'}{\lambda}\right) f\left(\frac{x'}{\lambda}\right) \right) dH\left(\lambda\right) dx' dx \\ & + \left(1 - \nu_{exp}\right) \zeta \mathbb{E}\left[\lambda - 1\right] + \nu_{exp} \zeta \mathbb{E}\left[\kappa\lambda - 1\right] \\ & + \left(1 - \nu_{entry}\right) \chi \mathbb{E}\left[\left(\lambda - 1\right) \bar{x}_0\right] + \nu_{entry} \chi \mathbb{E}\left[\kappa\lambda \bar{x}_0 - 1\right], \end{aligned} \tag{10}$$

where $\bar{x}_0 = \int x dF_0\left(x\right)$ and $\mathbb{E}_H[\lambda - 1]$ is the expected step size of innovation. If the own-product improvement rate $\xi(x)$ is constant at $\bar{\xi}$, $\kappa = 1$, and $F(x) = F_0(x)$, the growth rate is simply $\left(\bar{\xi} + \zeta + \chi\right) \mathbb{E}_H[\lambda - 1]$.

2.7 Closing the Model

The creative destruction rate $\tau(x)$ can be written as

$$\tau(x) = (1 - \nu_{exp})\,\zeta + (1 - \nu_{entry})\,\chi f_0(x)/f(x). \tag{11}$$

With the directed matching of entering firms, $\tau(x)$ varies with x. Later, we parameterize $f_0(x)$ such that $f_0(x)/f(x)$ decreases in x. This setting provides additional incentives for firms to raise x and reduce the creative destruction rate through own-product improvements.

Next, let the population-to-variety ratio be $m_t := M_t/N_t$. On the BGP with a constant entry rate χ, its evolution is given by

$$\frac{dm_t}{dt} = (g_M + \omega - \zeta\nu_{exp} - \chi\nu_{entry})\,m_t.$$

Hence, on the BGP where m_t remains constant at m,

$$g_M = \zeta\nu_{exp} + \chi\nu_{entry} - \omega \tag{12}$$

Equation (12) implies that, on the BGP, slower population growth reduces the net variety creation rate. This dampens the growth of GDP per capita that increases linearly in $\bar{Y}_t$ on the BGP. In Section 4, we will quantify the effect of historical declines in population growth rates in the context of this model.

Finally, we describe the labor market clearing condition on the BGP. Rewriting Equation (2) gives the labor supply per unit interval of product lines, $\bar{L}$:

$$\bar{L} = \left(\frac{w}{\phi}\right)^{\eta} m^{1+\eta}. \tag{13}$$

The total demand for labor per unit interval is composed of production and R&D labor demands.

$$\bar{L} = \frac{\sigma - 1}{\sigma}\frac{1}{w} + \left(\int c_{own}(\xi(x))dF(x) + c_{exp}\zeta(x) + c_{entry}\chi\right). \tag{14}$$

2.8 The Balanced Growth Path Equilibrium

The Balanced Growth Path Equilibrium is defined as the value function $v(x)$; the innovation rates $\xi(x)$ and ζ; the product line distribution $f(x)$; the creative destruction rate $\tau(x)$; the entry rate χ; the growth rate of average quality, g_A; the population-to-variety ratio m; the constant wage rate w, and the constant labor supply per variety $\bar{L}$ such that the following conditions hold.[9]

[9]Given these equilibrium objects, the growth rate of GDP per capita follows from Equation (6): $g_C = (g_A + g_M)/(\sigma - 1)$. The real interest rate then follows from Equation (1).

1. The value functions and innovation rates solve the HJB equation (7).
2. The free entry condition (8) holds.
3. The distribution of product lines over x is time-invariant and satisfies Equation (9).
4. The growth rate of average quality, the creative destruction rate, and the population-to-variety ratio satisfy Equations (10), (11), and (13).
5. The labor market clears as specified in Equations (13) and (14).

3 Calibration

3.1 Data

The calibration draws on information about innovation activities from establishment-level survey data in Germany. The data come from the IAB Establishment Panel Survey (IAB-BP), a representative establishment survey conducted by the Research Data Centre (FDZ) in Germany. The data contains approximately 17,000 observations per year and is administered annually starting in 1993 for western Germany and 1996 for eastern Germany. It provides a wide range of establishment-level information about employment, hiring, separation, wages, business volume, investment, and innovation activities.

Innovation activities are measured from answers to the following survey questions.[10]

1. **Own-product Improvement**: In the last business year, did your establishment improve or further develop a product or service which had previously been part of your portfolio? (Quality Improvement) Or, did you develop or implement procedures in the last business year which have noticeably improved production processes or services? (Process Innovation)
2. **Product Expansion**: In the last business year, did your establishment start to offer a product/service that had been available on the market before? (Expansion to Existing Products) Or, have you started to offer a completely new product or service in the last year for which a new market had to be created? (Variety Creation)

The process innovation question in the survey is included to measure an own-product improvement since a productivity increase for a product line is isomorphic to a quality improvement in the model. These innovation variables have been surveyed consistently since 2008; because of this, the analysis includes only observations from 2007 (2008 survey) to 2017 (2018 survey).

[10]For a detailed description of the data, see Online Appendix 1.1.

One caveat of the German data is that the data is only available at the establishment level rather than the firm level. Despite this limitation, the notion of establishments in the German data is wider than the U.S. Census Bureau's concept of establishments. In the U.S., establishments are distinguished by their physical location. In Germany, however, a single establishment code is shared by multiple workplaces at different locations if they belong to the same company, are physically located in the same municipality, and fall under the same economic class. This paper interprets this German notion of establishments as units in which R&D decision is made.

Still, common ownership by a single company can involve different establishment codes across various municipalities and economic classes. Product expansion is reported in the survey when a company decides to swap products among establishments, regardless of whether new products are developed. Consequently, the degree of product expansion at the firm level can be smaller than indicated by this exercise. In the Online Appendix 1.3.1, we explore how changes in firm structure correlate with innovation choices and establishment performances after innovation.

3.2 Strategy

Previous analyses using Schumpeterian frameworks often calibrate models by focusing on cross-sectional firm growth differences without explicit conditioning on innovation. Specifically, Garcia-Macia et al. (2019) target ten moments to calibrate a Schumpeterian model with own-product improvements, product expansion, and entrant innovation: (1) the growth rate, (2) a minimum employment per firm of 1, (3) the cross-sectional standard deviation of log firm employment, (4) the employment share of entrants, (5 and 6) the job creation and destruction rates, (7) the share of job creation < 1, (8) the average employment for old firms relative to young ones, and (9 and 10) the exit rates for small and large firms.

The key moments informing the magnitudes of the three types of innovation are the entrants' employment share, the job creation rate, and the share of small-sized job creation (< 1). A low share of entrants' employment suggests that entrant innovation contributes relatively little to long-run growth. The overall importance of incumbent innovation comes from the job creation net of jobs created by entrants. Consequently, most job creation net of that by entrants is driven by incumbent innovation. The relative importance of own-product improvements can be inferred from the share of small-scale job creation by incumbents. Without turnover of the product lines across firms, own-product improvements are disproportionately related to small-scale labor reallocation toward innovators, compared with product expansion.

Other studies also use the unconditional distribution of input reallocation to infer the underlying innovation behavior. For example, Lentz and Mortensen (2008) set the correlation between employment growth and labor productivity as central targets in their estimation, whereas Acemoglu et al. (2018) compare employment growth between different age and size groups to draw inferences about input reallocation among heterogeneous firms.

The core assumption in this calibration strategy is that all adjustments at the firm level are driven by innovation that permanently improves the productivity of their product lines, thus contributing to long-run growth. Firms also adjust their employment in response to temporary shocks to demand or productivity. In the extreme case of Hopenhayn (1992), where productivity shocks are purely stationary, all growth may originate from entrant innovation.

To address this issue, we additionally target labor reallocation specifically among innovating firms. We focus on four moments derived from survey data on innovation activities: (1) the fraction of establishments improving their own products, (2) the fraction of establishments expanding to new products, (3) additional employment growth following an own-product improvement, and (4) additional growth following product expansion. Combined with the fraction of innovators, additional employment growth captures the degree of labor reallocation associated with each type of innovation.

A practical challenge arises because many establishments adopt multiple innovation types in the same year. In German data, 55% and 34% of establishments engage in own-product improvements and product expansion, respectively, in a given year, with 29% pursuing both activities. Moreover, innovations are serially correlated: roughly 60% of establishments that expand to new product markets do so again in the following year, and for own-product improvements, this figure is nearly 80%. To handle this issue and isolate the employment increase after each innovation, we estimate the following regression equation.

$$\text{Empl. Growth}_{i,t} = \sum_{s=0}^{3} \mathbf{1}^{a}_{i,t-s} \beta^{a}_{s} + \sum_{s=0}^{3} \mathbf{1}^{entry}_{i,t-s} \beta^{entry}_{s} + X_{i,t} + \varepsilon_{it}, \tag{15}$$

where Empl. Growth_{it} is the Davis-Haltiwanger employment growth rate of establishment i in year t. Entering and exiting establishments are also included in the regression with their employment growth rates equal to 2 and -2, respectively. For $a = own, exp$, $\mathbf{1}^{a}_{i,t-s}$ is an indicator that equals one if the establishment i implemented own-product improvements and product expansion in year $t-s$, respectively. Likewise, $\mathbf{1}^{entry}_{i,t-s}$ is an indicator whether the firm i entered in year $t-s$. Controls $X_{i,t}$ include sector and year fixed effects. In some specifications, X_{it} include dummies for other incumbent innovation events.

Table 1 presents the estimation results. Two years after innovation, establishments grow sig-

Table 1: Regression Results: Employment Growth

	(1)	(2)	(3)	(4)	(5)	(6)
Periods	2 Years	2 Years	2 Years	4 Years	4 Years	4 Years
Own-product	5.611***		4.963***	3.051***		2.508***
Improvement	(0.307)		(0.333)	(0.278)		(0.329)
Product		3.760***	1.952***		2.733***	1.103***
Expansion		(0.316)	(0.343)		(0.304)	(0.360)
N.	58132	58024	58019	58132	58024	58019
Other Innov.			✓			✓

Notes: This table presents estimation results of Equation (15). Columns (1), (2), (4), and (5) exclude other incumbent innovation dummies, whereas Columns (3) and (6) include them. All specifications include entrant innovation dummies, sector fixed effects, and year fixed effects. Columns (1)-(3) and (4)-(6) exhibit $\sum_{s=0}^{1}\beta_s^a$ and $\sum_{s=0}^{3}\beta_s^a$, respectively, where $a = own$ for own-product improvements and $a = exp$ for product expansion. Samples are weighted by sampling weights adjusted by the average of employment in years t and $t-1$. Entering and exiting establishments are included. All regressions include sector and year fixed effects. The numbers in parentheses are standard errors of the estimates.
*** - Significant at 1%, ** - Significant at 5%, * - Significant at 10%.

nificantly faster than non-innovators in the same sector. This effect remains even when controlling for other innovation events, including entries. A firm that undertakes own-product improvements in a given year grows 5 percentage points faster over the following two years, while a firm expanding to new product markets grows 2 percentage points faster. After two additional years, these numbers diminish to 2.5 percentage points for own-product improvements and 1.1 percentage points for product expansion. Because firms may temporarily increase employment to promote new products or make new investments, we use the four-year additional growth rates as our calibration targets.

We summarize the targeted moments in Table 2. Additional calibration targets follow Garcia-Macia et al. (2019). The last two columns compare empirical moments in German data with those in Garcia-Macia et al. (2019) for the US economy. Despite differences in how business units are defined, as well as cross-country differences in labor market institutions, these moments remain broadly comparable between the U.S. and Germany. Nonetheless, there exists differences in the share of small job creation, the entrant employment share, and the size dispersion across firms. This arises because the U.S. data encompass firms with multiple establishments in different municipalities and sectors. When an existing firm opens a new establishment in a different municipality, German data classify it as a new business unit, whereas the U.S. data treat it as job creation by an incumbent unit.

Table 2: Targeted Moments

Moment	Baseline	No Shock	Data	GHK
TFP Growth Rate	1.34%	1.34%	1.34%	1.32%
Employment Growth Rate	0.3%	0.3%	0.3%	0.5%
Job Creation Rate	33.1%	32.2%	32.3%	32.5%
Share of Small Job Creation	31.0%	22.4%	27.6%	36.3%
Entrant Employment Share	20.8%	19.9%	21.3%	15.5%
Employment Incumbents/Entrants	2.2	2.1	2.1	3.2
Std. of Log Employment	1.61	0.88	1.80	1.28
Large/Small Relative Exit Rate	0.50	0.41	0.61	0.65
Minimum/Average Employment	0.06	0.05	0.05	0.04
Fraction of Own-Product Improvement	54.3%	-	55.0%	-
Fraction of Product Expansion	34.0%	-	33.7%	-
Employment Growth (Own-Improvement)	2.4ppt	-	2.6ppt	-
Employment Growth (Expansion)	1.2ppt	-	0.8ppt	-

Notes: This table lists the calibration targets and compares them with German data and with the calibration targets of Garcia-Macia et al. (2019). Column Baseline displays the moments generated by the full model, including demand shocks and permanent demand differences. Column No shock shows moments generated by a model without them. The model without shocks is calibrated without the last four moments. Columns Data and GHK correspond to German data and Garcia-Macia et al. (2019), respectively.

For calibration, we assume the following isoelastic innovation cost functions.

$$c_{own}(\xi) = \psi_{own}\frac{\epsilon}{\epsilon+1}\xi^{\frac{\epsilon+1}{\epsilon}}.$$
$$c_{exp}(\zeta) = \psi_{exp}\frac{\epsilon}{\epsilon+1}\zeta^{\frac{\epsilon+1}{\epsilon}}.$$

As ϵ grows, these cost functions become less convex, so innovation choices respond more elastically to changes in returns to innovation. Thus, we refer to ϵ as the elasticity of innovation.

We also introduce a parameter q that governs the probability of a successful product expansion associated with a positive survey response for product expansion. We need this parameter to reconcile the small employment gains observed after product expansion. Under the Klette and Kortum (2004) assumption that the number of product lines is integer-valued, adding one new product line can translate into a disproportionately large employment increase (e.g., doubling if the establishment initially has only one line). However, the data show that establishments expanding to new products grow by only 0.8 percentage points more than the average. We interpret q as an auxiliary parameter that aligns the data with the integer-line assumption: in practice, one reported expansion event in the survey corresponds to q new product lines rather than exactly one.

The entrant distribution $f_0(x)$ is parameterized as follows:

$$f_0(x) \propto \frac{f(x)}{1+e^{\upsilon(x-\underline{x})}}. \tag{16}$$

Plugging this into Equation (11) gives $(\tau(x)-\zeta) \propto (1/1+e^{\upsilon(x-\underline{x})})$. Thus, this specification implies that entrants' creative destruction is skewed toward product lines with lower x. This feature provides firms with low x to innovate and avoid being displaced. Calibrating ν is equivalent to calibrating $\bar{x}_{entry}$, the average quality of an entrant's product line.

In the quantitative version of the model, we also introduce permanent differences in demand shifters across product lines. This extension does not alter the innovation rates when the returns to innovation, innovation costs, and revenues are all assumed to be homogeneous of degree one in the permanent level of demand shifters. More details appear in Online Appendix 2.1. We assume the permanent demand level is log-normally distributed with mean zero and standard deviation θ.

We calibrate the model twice. First, under Column Baseline, the full model—including demand shocks and permanent demand differences—is calibrated to match all the moments in Table 2, including those based on innovation survey responses. Second, under Column No Shock, we exclude demand shocks as well as permanent demand dispersion, and we

Table 3: Values of Calibrated Parameters

Parameter	Meaning	Baseline	No Shock
A. External Moments			
ρ	Discount Rate	1.66%	
ϵ	Innovation Elasticity	1.00	
B. Internal Moments			
σ	Elasticity of Substitution	3.11	3.62
α	Average Innovation Step Size	2.8%	8.9%
χ	Entry Rate	9.0%	6.9%
q	Probability of Successful Expansion	5.6%	-
ν_{exp}	Variety Creation (Incumbent)	5.0%	0.6%
ν_{entry}	Variety Creation (Entrant)	8.7%	4.7%
κ	Relative Quality of New Variety	0.247	0.232
$\bar{x}_{entry}$	Mean Quality of Entrants	0.530	0.577
ϕ	Overhead Labor	0.022	0.047
ς	Demand Shock	0.049	-
ψ_{own}	Own-Product Improvement Cost	25	139
ψ_{exp}	Product Expansion Cost	226	39
θ	Demand Dispersion	1.296	-

Notes: This table presents the calibration results. Column Baseline shows parameters of the full model with demand shocks and permanent demand differences, matching the moments under Column Baseline in Table 2. Column No Shock shows parameter values of a model without them, which matches the moments under Column No Shock in Table 2.

do not target the four survey-based innovation moments. Aside from allowing endogenous innovation choices and adopting a continuous-time framework, this version of the model and its calibration targets align with those of Garcia-Macia et al. (2019).

3.3 Results

Table 3 presents the calibrated values of the model's parameters. The moments in the first panel are transferred directly into the model. The discount rate of the representative household, ρ, is calibrated such that the annual interest rate, $\exp(12 \times r) - 1 = \exp(12 \times (\rho + g_C)) - 1 \approx 12 \times r$, is equal to 3%. The innovation elasticity ϵ is taken from the literature. Estimating the elasticity of innovation is challenging; therefore, previous studies, such as

Akcigit and Kerr (2018) and Acemoglu et al. (2018), use the curvature of the innovation cost function estimated in Blundell et al. (2002). They observe patent production at the firm level and estimate the quadratic innovation cost function, implying a unit elasticity of innovation. The wage level and the population-to-variety ratio are also normalized to one on the baseline BGP.

The parameters in the second panel of Table 3 are calibrated to match model-generated moments to their empirical counterparts in Table 2. The baseline model is exactly identified with 13 moments to match with 13 parameters. Despite the restrictions of the model, the calibrated model closely fits these moments.

The elasticity of substitution across product lines, σ, is calibrated to exactly match the long-run growth rate given the quality growth rate, g_A, implied by the model. When σ is higher, a given amount of quality growth is translated into large output growth. In the baseline case, σ is calibrated at 3.11, which implies that the relative demand of a product line depreciates by 2.8% per year on average. This value is slightly lower than the value of 4 used by Garcia-Macia et al. (2019). In German data, the five-year job destruction rate is 31%, while non-innovating establishments grow at only 1.3 percentage points below the average establishment's rate. This observation suggests that most job destruction occurs independent of technological obsolescence.

The average step size of an own-product improvement, α, is chosen to match the additional employment growth following an own-product improvement. α is calibrated to 2.8%. This figure is larger than the 1.3% step size of product expansion in Acemoglu et al. (2018), yet smaller than the 6% average step size of innovation in Garcia-Macia et al. (2019) and the 5.1% step size of internal innovation in Akcigit and Kerr (2018). We use the same step size parameter for all the other innovation types.

Given α, the cost shifters of own-product improvements (ψ_{own}) and product expansion (ψ_{exp}) are calibrated to match the fraction of establishments undertaking each type of innovation. Under this calibration, 43% of product lines experience own-product improvements, and 13% expand to new products. The annual entry rate, χ, is 9%.

We set q, the probability of successful product expansion given a positive survey response, to align with the observed additional employment growth. A positive response to the product expansion question corresponds to just 0.06 additional product lines, inferred from the modest growth of product-expanding firms relative to non-innovators. For simplicity, if there is no entry or exit, the average continuing establishment's growth rate is near zero on the BGP. An establishment with one product line that successfully expands sees a Davis-Haltiwanger growth rate of about 67%; with two product lines, successful expansion yields 50% growth.

In the data, establishments that expand products grow about 0.8 percentage points faster than the average. This small difference implies that product expansion in the survey typically involves only a small number of new product lines, thus limiting product-line reallocation among incumbents.

In Garcia-Macia et al. (2019), the annual probability of own-product improvement is 78%, and the probabilities of creative destruction by incumbents and entrants are 6% and 16%, respectively. By contrast, our calibrated model suggests that only 43% of product lines undergo own-product improvements, fewer than 0.8% experience creative destruction by incumbents, and entrants' creative destruction affects up to 8.2% of product lines. These low innovation probabilities reflect limited labor reallocation associated with innovation activities. The model thus indicates that most reallocation among incumbents stems from random shocks. Indeed, removing random shocks from the baseline parameters ($\varsigma = 0$)reduces incumbent job creation by 63%.

The probabilities of incumbent and entrant variety creation—ν_{exp} and ν_{entry}—are calibrated to match the job destruction rate and the size of incumbents relative to entrants. Higher variety creation probabilities reduce the job destruction rate, given ζ and χ. In addition, if ν_{exp} rises, incumbents become larger relative to entrants. We set $\nu_{exp} = 0.05$ and $\nu_{entry} = 0.087$, implying that 0.04% and 0.78% of new varieties are created by incumbents and entrants, respectively, each year. By contrast, Garcia-Macia et al. (2019) find that incumbents create 0% and entrants 3%.

The relative quality of new variety, κ, and the mean quality of entrants, $\bar{x}_{entry}$, affect the entrant employment share and the share of small job creation. If new varieties are lower quality than average, entrants that create these varieties will have a lower employment share, but the share of small job creation will be higher. We calibrate κ to 0.53—modestly higher than 0.3 calibrated in Garcia-Macia et al. (2019)— and set $\nu = 0.86$. Under this calibration, the average quality of product lines that an entrant meets is 53% of the average, close to 59% in Garcia-Macia et al. (2019).

We calibrate the overhead labor cost at 0.022, so that the average quality of exiting products is 1.9% of the mean. This exit threshold aligns closely with the 2.5% threshold reported by Garcia-Macia et al. (2019).

The last two parameters in Table 3 capture the new components of the model. The demand shock parameter is 0.05, implying that annual growth rates have a standard deviation of 0.17. The permanent demand dispersion parameter is 1.3. These results suggest that a significant fraction of incumbent growth and size dispersion are independent of innovation. Indeed, when the demand shock is omitted ($\varsigma = 0$), job creation by incumbents falls by 63%.

Table 4: Contribution to Growth and Job Creation

	Own-Product Improvement	Product Expansion	Entrant Innovation	Demand Shock
A. Growth				
Baseline	78.0%	1.1%	21.0%	-
No Shock	76.9%	8.3%	14.8%	-
B. Job Creation				
Baseline	11.2%	5.9%	66.4%	16.6%
No Shock	5.3%	26.8%	56.8%	0%

Notes: This table compares growth and job creation decompositions. Rows Baseline and No Shock report results based on parameter values calibrated to their respective columns in Table 2. Growth and job creation decompositions are derived in Equations (18) and (19), respectively.

Similarly, setting the demand-dispersion parameter to zero reduces firm size dispersion to 72% of its baseline value.

3.4 Comparison with the Conventional Approach

To compare this model's implications with those of a conventional approach in Garcia-Macia et al. (2019), we recalibrate the model without demand shocks or permanent demand dispersion (i.e., $\varsigma = 0$ and $\theta = 0$). The second column, No Shock, in Table 3 shows the calibrated parameters when the model is set to match the moments in Table 2—except for the last four moments from the innovation survey. These calibration moments are equivalent to those targeted in Garcia-Macia et al. (2019), and the targeted values are reported in the second column of Table 2.

Table 4 compares the implications of the two models. In the first panel, both imply that most growth stems from own-product improvements. However, they differ in the relative contributions of product expansion and entrant innovation. The baseline model, which targets modest employment growth following product expansion, attributes a smaller share of growth to expansion and a larger share to entrant innovation than does the model without demand shocks. The latter yields a growth-decomposition result similar to Garcia-Macia et al. (2019), where product expansion and entrant innovation account for 6.2% and 19.8% of growth, respectively.

The second panel of Table 4 decomposes instantaneous job creation into the innovation and

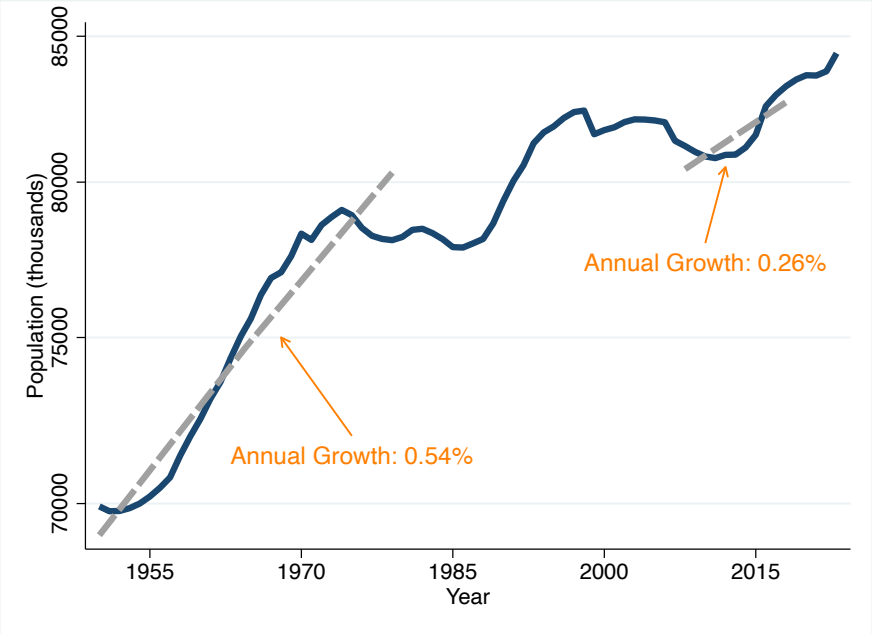


Figure 1: German Population Growth

Notes: This figure plots the population in Germany from 1950 to 2019 using data from the United Nations. The y-axis is shown in log scale.

demand-shock margins. In both models, most new jobs are created by entrants. Nevertheless, the model without demand shocks overstates the contribution of product expansion. With a large ς, demand shocks primarily replace product expansion in job creation.

4 Application: Slower Population Growth

We use the calibrated model to assess how slower population growth affects growth and labor reallocation.[11]. This exercise follows Karahan et al. (2024) and Hopenhayn et al. (2022), which extend the standard firm dynamics model in Hopenhayn (1992) to examine the impact of slower population growth on firm entry. In our model, slower population growth translates into slower expansion in product varieties, causing the entry rate to decline as entrants are the primary source of new varieties.

As in most developed countries, Germany's population growth has slowed down considerably (see Figure 1). From 2008 to 2018, Germany's annual population growth rate was 0.26%, the same as in our baseline scenario. By contrast, during the 1950s and 1960s, Germany's population grew at twice that rate. Thus, we compute the BGP for the 1950s and 1960s by

[11]We also analyze the effects of an R&D tax incentive program introduced in Germany in 2020, as well as the Hartz reform—a major labor-market change in mid-2000s Germany—on innovation behavior and long-run growth rates. See Online Appendix 3 4 for further application exercises.

Table 5: Balanced Growth Path with Rapid Population Growth

	Rapid Population Growth			Baseline
Output Growth Rate		1.53%		1.34%
Own-Product Improvement		1.13%		1.04%
Product Expansion		0.02%		0.01%
Entrant Innovation		0.38%		0.28%
Job Creation Rate		38.3%		33.1%
Incumbents		11.7%		12.1%
Entrants		26.6%		21.0%
Labor Supply Elasticity	$\eta = 0.1$	$\eta = 1$	$\eta = 3$	
Population-Product Ratio	1.196	1.185	1.179	1.000
Wage	1.021	1.011	1.006	1.000
Labor Supply	0.929	0.937	0.943	0.927

Notes: This table shows how innovations contribute to output growth rates and how incumbents and entrants contribute to job creation across different BGP equilibria. The output growth rate is annual; the job-creation rate is measured over five years. The growth-rate decomposition follows Equation (18). Wages are expressed relative to GDP per capita, and labor supply is per household.

doubling the population growth rate in the baseline. Thus, we compute a balanced growth path (BGP) for the 1950s and 1960s by doubling the net variety growth rate in the baseline. Next, we calculate the transition path from this earlier, higher-growth BGP to the baseline BGP, beginning with an unexpected reduction in the population growth rate at $t = 0$. The transition path to the new BGP is computed using equations in Online Appendix 2.2.

4.1 Balanced Growth Path

Table 5 compares the BGP moments between an economy with rapid population and net variety growth and the baseline economy. We consider three values of the Frisch elasticity of labor supply for representative households: $\eta = 0.1$, 1, and 3. Because the population-product ratio m appears only with η in Equation (13), other BGP variables on the BGP do not change with η. Instead, η affects only the population-product ratio, the wage relative to output per capita, and the labor supply per household, given the effective wage w and labor per unit interval of product line, $\bar{L}$.

Compared with an economy with rapid population growth, the baseline economy exhibits a lower growth rate on the BGP. Both own-product improvements and entrant innovation contribute to this difference, but population growth influences these two margins of innovation

via different channels. When the population grows more slowly, own-product improvement decreases because labor supply per product variety declines, and the R&D cost, proportional to w, increases. Entrant innovation, on the other hand, is directly reduced by the equilibrium force that equates the net variety growth rate to the population growth rate (Equation (12)). Consequently, entrant innovation responds more elastically to the underlying demographic change. Although product expansion is also influenced, its small share in overall growth makes its effect on the BGP growth rate negligible.

The second panel of Table 5 breaks down job creation by incumbents and entrants. With a lower entry rate, the economy with slower population growth experiences fewer jobs created by entrants. Since own-product improvements do not involve product-line turnover, the effect of increased own-product improvements on job creation is minimal—consistent with Karahan et al. (2024), who find that labor reallocation among incumbents remains stable over time.

The last panel of Table 5 summarizes the wage, labor supply, and population-product ratio m for three values of η. The effective wage w rises from 0.85 to 1 because reduced creative destruction increases the entry value, elevating w in equilibrium per the free entry condition 8. Consequently, labor demand per unit mass of product lines $\bar{L}$ declines, in line with the labor demand equation (14). From Equation (13), m decreases as w grows and $\bar{L}$ falls. This further reduces the labor supply per household, $\bar{L}/m$, and the ratio of wage to output per capita, $w \times m$. Overall, a smaller η leads to a larger increase in w and a smaller increase in H, but the elasticity of labor supply does not alter firm behavior on the BGP. In contrast, as the economy transitions to the new BGP below, the speed at which m converges depends on η.

4.2 Transition Path

Unlike the BGP—where the population and varieties grow at the same rate—population-to-product ratio evolves over transition, which alters the wages, entries, and incumbent innovation. We thus compute the transition path from the faster-growth BGP to the baseline BGP by introducing a one-time, unexpected change in the path of population growth at $t = 0$ (shown by the dotted line in Figure 2).

Table 6 reports the output responses and welfare implications of slower population growth along the transition path for three values of η. In each case, output initially falls because more labor is used for entrant innovation. During the first few years following the slowdown, entrant innovation exceeds its level under the old BGP, while incumbent innovation is relatively lower. This early surge in entrant innovation accelerates the convergence of the

population-to-product ratio. When labor supply is more elastic, these effects are amplified, resulting in a larger initial decline in output but also a faster transition to the new BGP, which features a smaller population-to-product ratio.

Although the average quality level, A_t, grows more slowly, output per capita can temporarily exceed that under the old, high-growth BGP. This is more likely to happen when η is smaller, and the population-to-product ratio converges more slowly. For example, when $\eta = 0.1$, output reaches the level 1.9% above the old BGP at the peak and remains higher for 34 years. When $\eta = 1$, the economy is only 0.2% higher than the old BGP at the highest and maintains the status for 10 years. When $\eta = 3$, the economy stays permanently below the old BGP.

These output dynamics lead to changes in the representative household's lifetime utility The second panel of Table 6 decomposes the output-related welfare changes into own-product improvements, product expansion, entrant innovation, and adjustments in production labor.[12] The slowdown of population growth yields welfare changes equivalent to permanent consumption reductions of 0.47%, 0.7%, and 0.74%, respectively. With a temporary boost to entrant innovation, most welfare losses arise from reduced own-product improvements. For higher η, the slowdown shifts more labor from production to entrant innovation in the early transition, magnifying the welfare decline.

Figure 2 shows the five-year job creation rates from simulated firm panels under both the old BGP and the transition path. The transitional economy exhibits higher job creation rates for roughly 25 years after the shock, mainly driven by the increased entry of new firms. Once the population growth rate settles at 60% of its former level, this pattern reverses; the entry rate eventually converges toward its new BGP level, which is about 25% below the old BGP one.

By contrast, incumbent job creation declines for the first five years following the shock due to higher wages and decreased incumbent innovation. As the economy converges to the new BGP, incumbent job creation stabilizes near its new BGP level, which remains close to that of the old BGP. Although incumbent innovation accounts for a large fraction of welfare changes, its contribution to job creation remains unchanged. This is because own-product improvements are a dominant form of incumbent innovation. Unlike product expansion and entrant innovation, own-product improvements do not require large-scale labor reallocation among producers. Consequently, even with minimal changes in labor reallocation, own-product improvements continue to drive output growth via the quality ladder.

[12]Differences in labor supply over the transition path also affect welfare. Higher values of η automatically reduce the utility cost of labor. As a result, welfare losses from labor disutility amount to 2%, 1.3%, and 0.8% declines in the consumption schedule of the BGP with rapid growth for $\eta = 0.1$, 1, and 3, respectively.

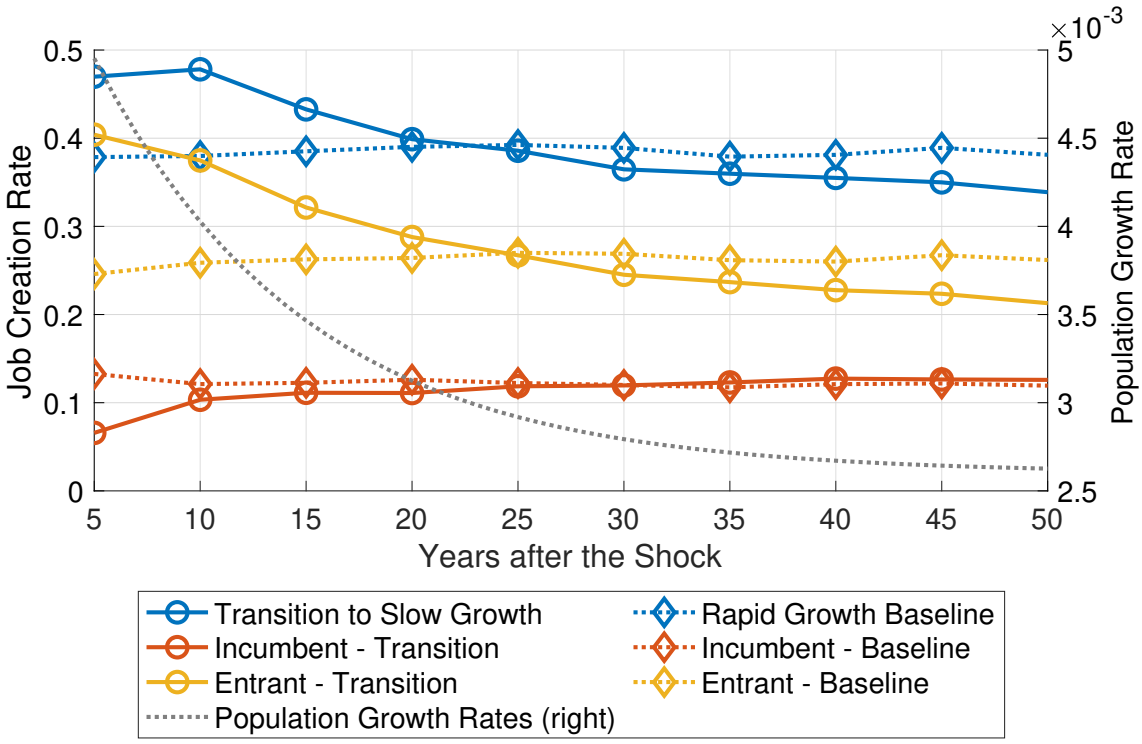


Figure 2: Job Creation and Destruction Rates over the Transition Path

Notes: This figure displays job creation and destruction rates on the BGP with faster population growth and along the transition path to a new BGP with slower population growth. The five-year job creation and job destruction rates are calculated from simulated panels and plotted on the left axis. The right axis shows annual population growth rates over the transition.

Table 6: Effect of Slow Population Growth on the Transition Path

Labor Supply Elasticity	$\eta = 0.1$	$\eta = 1$	$\eta = 3$
Initial Output Change	-4.70%	-10.39%	-10.53%
Peak Relative Output	1.90%	0.20%	-0.46%
Years above the Rapid-Growth BGP	33.5	10.4	-
Consumption Equivalent Variation	-0.470%	-0.694%	-0.743%
Own-Product Improvement	-0.434%	-0.565%	-0.565%
Product Expansion	-0.003%	-0.004%	-0.004%
Entrant Innovation	-0.070%	-0.029%	-0.042%
Production Labor	0.037%	-0.096%	-0.132%

Notes: This table reports changes in output and welfare along the transition path relative to a faster-population-growth BGP for different labor-supply elasticities η. The first panel summarizes the output path; the second panel decomposes the contributions of innovation and production labor to welfare. See Online Appendix 2.2 for further details on the derivations.

5 Concluding Remarks

This paper analyzes how labor reallocation relates to growth when demand shocks exist. Both innovation and demand shocks reallocate labor across firms, but only innovation contributes to growth. We distinguish these two sources of labor reallocation by incorporating survey-based measures of establishment-level innovation. These measures enable us to identify innovation-driven labor reallocation and separately track own-product improvements and product expansions through self-reported survey responses.

Our findings show that the majority of incumbent job creation arises from idiosyncratic demand shocks. Only 37% stems from innovation. Among incumbent innovation margins, own-product improvements generate substantially more job creation and growth than product expansion. Entrant innovation accounts for most job creation, but its contribution to growth is 21%.

We also find that ignoring demand shocks in the calibration overstates the role of product expansion and understates the role of entrant innovation. Under the assumption that all job creation arises from innovation-related firm changes, the model attributes 8% of growth to product expansion instead of 1% in the baseline calibration. This discrepancy largely reflects the modest share of product-expansion-related labor reallocation in the data. The survey data suggests that most large-scale job creation events occur independently of firms' innovation status.

The results in this paper underscore the value of direct innovation measures in quantitative macroeconomic analysis. Temporary shocks that affect employment, investment, or revenue at the firm level pose a challenge for accurately diagnosing innovation activities. This paper proposes using survey-based innovation measures to address this problem. As more innovation surveys appear and gain popularity, such data offer a promising avenue for future macroeconomic research.

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A Appendix

A.1 Derivation of the Value Function

On the BGP, the firm value function is characterized by the following HJB equation:

$$
\begin{aligned}
r\mathfrak{V}\left(\{x_j\}_{j=1}^{N_i}, W_t, \bar{Y}_t\right) = \max_{\{\xi_j,\zeta_j\}\geq 0} \Bigg\{ & \sum_{j=1}^{N_i} \left(\bar{Y}_t \bar{\pi} x_j - \phi W_t\right) \\
& - \sum_{j=1}^{N_i} g_A x_j \partial_{x_j} \mathfrak{V}\left(\{x_{j'}\}_{j'=1}^{N_i}, W_t, \bar{Y}_t\right) \\
& + g_{\bar{Y},t} \bar{Y}_t \partial_{\bar{Y}_t} \mathfrak{V}\left(\{x_{j'}\}_{j'=1}^{N_i}, W_t, \bar{Y}_t\right) \\
& + g_w W_t \partial_{W_t} \mathfrak{V}\left(\{x_{j'}\}_{j'=1}^{N_i}, W_t, \bar{Y}_t\right) \\
& \sum_{j=1}^{N_i} \frac{\varsigma^2 x_j^2}{2} \partial_{x_j}^2 \mathfrak{V}\left(\{x_{j'}\}_{j'=1}^{N_i}, W_t, \bar{Y}_t\right) \\
& - W_t \sum_j x_j c_{own}\left(\xi_j\right) - W_t \sum_j c_{exp}\left(\zeta_j\right) \\
& + \sum_j \xi_j \Bigg(\int \mathbb{E}_{H(\lambda)} \left[\mathfrak{V}_t\left(\{x_{j'}\}_{j'\neq j} \cup \{\lambda x_j\}, W_t, \bar{Y}_t\right)\right] \\
& \quad - \mathfrak{V}_t\left(\{x_{j'}\}_{j'=1}^{N_i}, W_t, \bar{Y}_t\right) \Bigg) \\
& + \sum_j \zeta_j \Bigg(\nu_{exp} \int \mathbb{E}_{H(\lambda)} \left[\mathfrak{V}_t\left(\{x_{j'}\}_{j'=1}^{N_i} \cup \{\lambda x\}, W_t, \bar{Y}_t\right)\right] dF\left(x\right) \\
& \quad + \left(1-\nu_{exp}\right) \int \mathbb{E}_{H(\lambda)} \left[\mathfrak{V}_t\left(\{x_{j'}\}_{j'=1}^{N_i} \cup \{\lambda \kappa x\}, W_t, \bar{Y}_t\right)\right] dF\left(x\right) \\
& \quad - \mathfrak{V}_t\left(\{x_{j'}\}_{j'=1}^{N_i}, W_t, \bar{Y}_t\right) \Bigg) \\
& + \sum_j \tau\left(x_j\right) \Bigg(\mathfrak{V}_t\left(\{x_{j'}\}_{j'=1}^{N_i} \setminus \{x_j\}, W_t, \bar{Y}_t\right) \\
& \quad - \mathfrak{V}_t\left(\{x_{j'}\}_{j'=1}^{N_i}, W_t, \bar{Y}_t\right) \Bigg) \Bigg\},
\end{aligned}
$$

where ∂_y and ∂_y^2 are the operators that take the first- and second-order partial derivatives of a function with respect to a variable y, respectively. g_y denotes the stationary growth rate of a variable y.

First, guess and verify that $\mathfrak{V}\left(\{x_j\}_{j=1}^{N_i}, W_t, \bar{Y}_t\right) = \sum_{j=1}^{N_i} \mathbf{V}\left(x_j, W_t, \bar{Y}_t\right)$, where

$$\begin{aligned}(r+\tau(x))\,\mathbf{V}\left(x, W_t, \bar{Y}_t\right) = \max_{\xi,\zeta\geq 0}\Bigg\{ & \bar{Y}_t\bar{\pi}x - \phi W_t \\ & - g_A x\partial_x \mathbf{V}\left(x, W_t, \bar{Y}_t\right) + \frac{x^2\varsigma^2}{2}\partial_x^2 \mathbf{V}\left(x, W_t, \bar{Y}_t\right) \\ & + g_{\bar{Y}}\partial_{\bar{Y}_t}\mathbf{V}\left(x, W_t, \bar{Y}_t\right) + g_w W_t \partial_{W_t}\mathbf{V}\left(x, W_t, \bar{Y}_t\right) \\ & - W_t x c_{own}(\xi) + \xi\Big(\mathbb{E}_{H(\lambda)}\left[\mathbf{V}\left(\lambda x, W_t, \bar{Y}_t\right)\right] - \mathbf{V}\left(x, W_t, \bar{Y}_t\right)\Big) \\ & - W_t c_{exp}(\zeta) + \zeta\int \mathbb{E}_{H(\lambda)}\Bigg[\nu_{exp}\mathbf{V}\left(\lambda x', W_t, \bar{Y}_t\right) \\ & \quad + (1-\nu_{exp})\,\mathbf{V}\left(\lambda\kappa x', W_t, \bar{Y}_t\right)\Bigg] dF(x')\Bigg\}.\end{aligned}$$

Next, plug in $\mathbf{V}(x, W_t, \bar{Y}_t) = \bar{Y}_t v(x)$ and verify that Equation (7) holds on the BGP where $W_t/\bar{Y}_t$ is constant at w.

A.2 Derivation of the Distribution Equation

We start from the Kolmogorov forward equation augmented with the jump process related to innovation. The equation characterizes the evolution of $\mathbf{f}_t(x)$, the density of product lines with relative demand x at time t. The density function integrates into N_t, growing over time due to the creation of varieties.

$$\begin{aligned}\frac{d\mathbf{f}_t(x)}{dt} = & \partial_x\left[g_A x\mathbf{f}_t(x)\right] + \partial_x^2\left[\frac{\varsigma^2 x^2}{2}\mathbf{f}_t(x)\right] \\ & - \xi(x)\,\mathbf{f}_t(x) + \int \xi\left(\frac{x}{\lambda}\right)\mathbf{f}_t\left(\frac{x}{\lambda}\right) dH(\lambda) \\ & - (1-\nu_{exp})\,\zeta\left(\int\left(\mathbf{f}_t(x) - \mathbf{f}_t\left(\frac{x}{\lambda}\right)\right) dH(\lambda)\right) \\ & + \nu_{exp}\zeta\int \mathbf{f}_t\left(\frac{x}{\kappa\lambda}\right) dH(\lambda) \\ & - (1-\nu_{entry})\,\chi N_t\left(\int\left(f_0(x) - f_0\left(\frac{x}{\lambda}\right)\right) dH(\lambda)\right) \\ & + \nu_{entry}\chi N_t\int f_0\left(\frac{x}{\kappa\lambda}\right) dH(\lambda)\end{aligned}$$

Define $f(x) = \frac{\mathbf{f_t}(x)}{N_t}$ and apply the chain rule. Note that $dN_t/dt = N_t\left(\nu_{exp}\zeta + \nu_{entry}\chi - \omega\right)$.

A.3 Equations for the Growth Rates

Integrate Equation (9) over $[\underline{x}, x)$.

$$\begin{aligned} g_A x f(x) = & -\partial_x \left[\frac{\varsigma^2 x^2}{2} f(x) \right] + \psi F(x) \\ & + \iint_{x' \leq x} \left(\xi(x') f(x') - \xi\left(\frac{x'}{\lambda}\right) f\left(\frac{x'}{\lambda}\right) \right) dH(\lambda) dx' \\ & + (1 - \nu_{exp}) \zeta \left(\int \left(F(x) - F_0\left(\frac{x}{\lambda}\right) \right) dH(\lambda) \right) \\ & + \nu_{exp} \zeta \int \left(F(x) - F\left(\frac{x}{\kappa\lambda}\right) \right) dH(\lambda) \\ & + (1 - \nu_{entry}) \chi \left(\int \left(F_0(x) - F_0\left(\frac{x}{\lambda}\right) \right) dH(\lambda) \right) \\ & + \nu_{entry} \chi \int \left(F(x) - F_0\left(\frac{x}{\kappa\lambda}\right) \right) dH(\lambda) \end{aligned} \tag{17}$$

Next, integrate Equation (17) over $[\underline{x}, \infty)$ to get Equation (10). Use that $\int_{x \geq 0} (1 - G(x))\, dx = \int_{x \geq 0} x dG(x)$ for a cumulative distribution function $G(x)$.

The growth rate for the GDP per capita is decomposed in the following equation:

$$\begin{aligned} g_C &= (g_A + g_M)/(\sigma - 1) \\ &= \underbrace{\left(\iiint_{x' \leq x} \left(\xi(x') f(x') - \xi\left(\frac{x'}{\lambda}\right) f\left(\frac{x'}{\lambda}\right) \right) dH(\lambda) dx' dx \right) / (\sigma - 1)}_{\text{Own-product improvements}} \\ &\quad + \underbrace{\left((1 - \nu_{exp}) \zeta \mathbb{E}[\lambda - 1] + \nu_{exp} \zeta \mathbb{E}[\kappa\lambda] \right) / (\sigma - 1)}_{\text{Product expansion}} \\ &\quad + \underbrace{\left((1 - \nu_{entry}) \chi \mathbb{E}[(\lambda - 1)\bar{x}_0] + \nu_{entry} \chi \mathbb{E}[\kappa\lambda\bar{x}_0 - 1] \right) / (\sigma - 1)}_{\text{Entrant innovation}}. \end{aligned} \tag{18}$$

A.4 Job Creation Decomposition

Within a short time interval Δ, jobs are created by either own-product improvements, product expansion, entrant innovation, or positive demand shocks as in the following equation:

$$JC(\Delta) \propto \underbrace{\int \Delta\xi\left(x\right) \int (\lambda - 1) dH(\lambda) dF\left(x\right)}_{\text{Own-product improvements}} + \underbrace{\Delta\zeta\left(1 - \nu_{exp} + \nu_{exp}\kappa\right) \int \lambda dH(\lambda)}_{\text{Product Expansion}} + \underbrace{\Delta\chi\left(1 - \nu_{entry} + \nu_{entry}\kappa\right) \bar{x}_0 \int \lambda dH(\lambda)}_{\text{Entrant Innovation}} + \underbrace{\exp\left\{\int_0^\infty s d\Phi(s; 0, \Delta\varsigma^2)\right\}}_{\text{Demand Shock}} \tag{19}$$

Online Appendix
Does Growth Lead to Labor Reallocation? *

Hyejin Park†

January 28, 2025

1 Data Appendix

1.1 Innovation Measures

The IAB-BP survey asks whether an establishment implemented the following four types of innovation activities in the year prior to the survey. Two questions measure own-product improvements, and two measure product expansion.

1. **Quality Improvement**: In the last business year, did your establishment improve or further develop a product or service which had previously been part of your portfolio? (classified as own-product improvements in the paper)
2. **Process Innovation**: Did you develop or implement procedures in the last business year which have noticeably improved production processes or services? (classified as own-product improvements in the paper)
3. **Product Expansion**: In the last business year, did your establishment start to offer a product/service that had been available on the market before? (classified as product expansion in the paper)
4. **Variety Creation**: Have you started to offer a completely new product or service in the last year for which a new market had to be created? (classified as product

*This study uses the LIAB longitudinal model, version 1975-2017, of the Linked-Employer-Employee Data (LIAB) from the IAB. Data access was provided via on-site use at the Research Data Centre (FDZ) of the German Federal Employment Agency (BA) at the Institute for Employment Research (IAB) and subsequently remote data access. DOI: 10.5164/IAB.LIABLM7517.de.en.v1.

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expansion in the paper)

Table 1.1: Innovation Frequency across Size (%)

	Quality Improv.	Product Expansion	New Variety	Process Innov.	Any Innov.	N. Obs.
All	44.57	25.49	10.68	21.67	52.63	49,411
Small	34.13	19.61	6.62	13.23	42.16	29,886
Medium	43.93	24.71	9.97	20.64	52.68	12,596
Large	56.47	32.64	15.79	31.83	63.86	6,929

Notes: This table presents the fraction of establishments that implemented one of the four innovation types surveyed in IAB-BP. Establishments are classified by their employment size two years before the survey and weighted by cross-sectional weights adjusted by the two-year-lagged number of employees.

Table 1.2: Innovation Frequency across Age (%)

	Quality Improv.	Product Expansion	Variety Creation	Process Innov.	Any Innov.	N. Obs.
All	44.57	25.49	10.68	21.67	52.63	49,411
Young	44.31	26.35	9.81	21.37	52.95	18,054
Medium	45.68	25.71	11.18	22.02	52.99	18,735
Old	43.55	24.15	11.18	21.64	51.78	12,622

Notes: This table presents the fraction of establishments that implemented one of the four innovation types surveyed in IAB-BP. Establishments are classified by their age and weighted by cross-sectional weights adjusted by the two-year-lagged number of employees.

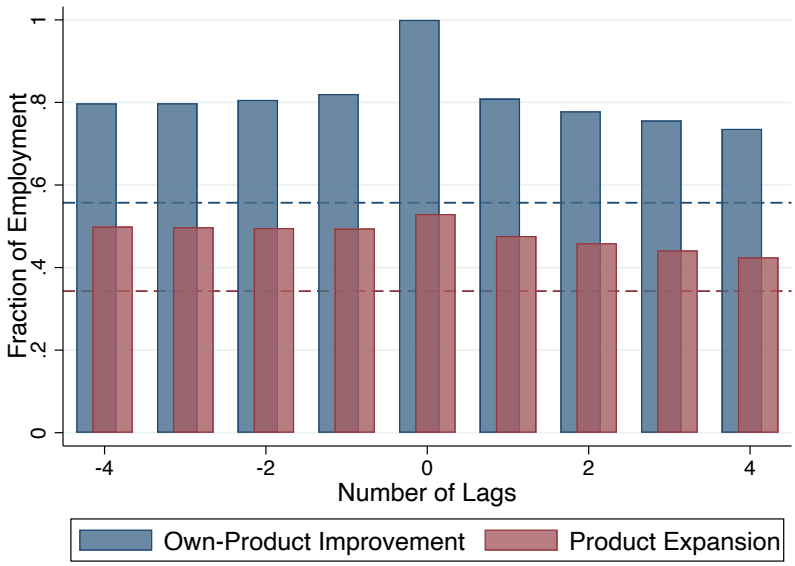
Table 1.1 shows the frequencies of innovation separately for the four questions across different establishment size groups. with equal numbers of workers. Each observation consists of an establishment in a given year, and all samples are pooled over different years. Samples are weighted by 2-year lagged employment from the survey year multiplied by cross-sectional weights from the data provider in calculating innovation frequency. Quality improvements are the most common type of innovation in the survey, followed by product expansion and process innovation. Variety creation is the least common form of innovation in the survey. For all types of innovation, larger establishments are more likely to innovate.

Table 1.2 shows the innovation frequencies across establishment age groups with equal numbers of workers. Old establishments are almost as likely to innovate as young establishments. This result seems to contrast with previous studies using patent measures, such as Acemoglu et al. (2018) who emphasize higher intensities of young establishments

to innovate. Below, however, we see that establishments indeed reduce the innovation frequency as they get older, but labor reallocation happens disproportionately more toward innovating establishments among older establishments.

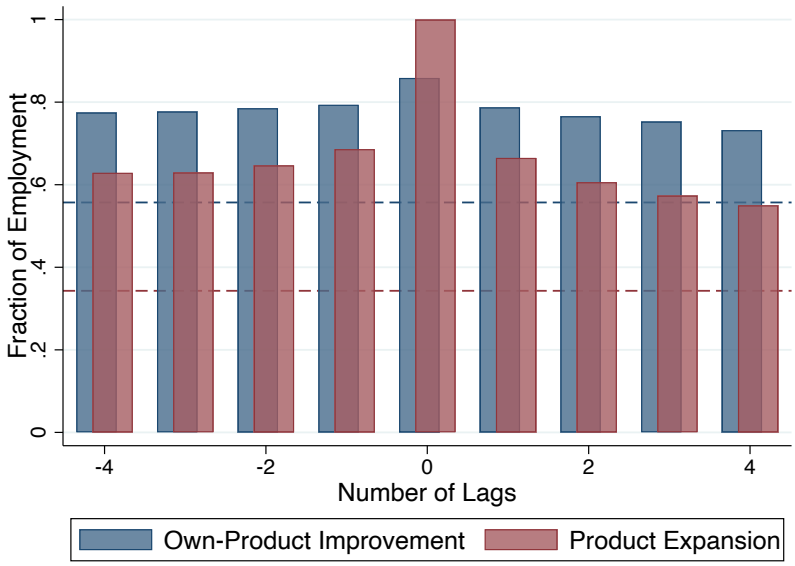
In all empirical exercises, including the two tables above, all the establishment samples are pooled across different years. As a result, one establishment appears multiple times over different years. To gain insight into how an establishment innovates over time in the data, Figures 1.1 and 1.2 show the autocorrelation of innovation. First, both types of innovation have a positive auto-correlation. Own-product improvements and product expansion in year 0 predict higher probabilities of own-product improvements and product expansion in other years relative to the averages, indicated by the navy dashed line (see Figure 1.1) for own-product improvements and the maroon dashed line (see Figure 1.2) for product expansion. Second, own-product improvements and product expansion are positively correlated with each other. Own-product improvements in year 0 are associated with overall higher probabilities of product expansion in all other periods. Lastly, the forward correlation is slightly smaller than the backward correlation; innovation in year t predicts a slightly higher probability of innovation before year t than after.Along with the marginal increase in the number of innovating establishments in establishment age in Table 1.2, this result implies that establishments reduce innovation activities as they grow. However, employment growth is more concentrated around innovative establishments as they get older.

Figure 1.1: Innovation Intensities Conditional on Own-Product Improvement



Notes: This figure shows the probabilities of innovation before and after conditional on implementing an own-product improvement in year 0.

Figure 1.2: Innovation Intensities Conditional on Product Expansion



Notes: This figure shows the probabilities of innovation before and after conditional on implementing product expansion in year 0.

1.2 Comparison to Patent-Based Measures

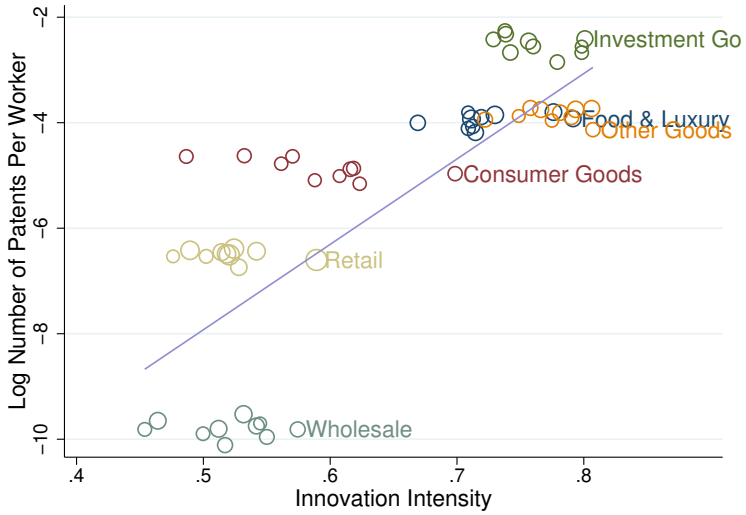
This appendix compares the survey-based measures of innovation with the patent-based measures, which have been widely used in the innovation literature. Unfortunately, it is impossible to match patents to establishments in IAB-BP because of the anonymization. As a result, the technological classes of patents are matched to the industrial sectors to construct a sectoral panel. The number of patents granted to German entities by the United States Patent and Trademark Office (USPTO) in each sector is made using the raw patent data parsed and managed by `patentsview.org`. The cooperative patent class endowed by the USPTO is translated to the 3-digit North America Industry Classification System (NAICS) Code of 2007 using probabilistic crosswalks constructed by Goldschlag et al. (2019). The 3-digit NAICS code is manually matched to the sectoral specification of IAB-BP.

The patent-based measure of innovation does not exist for agriculture, mining, utility, or, more importantly, service sectors except retail and wholesale. Patent measures exist primarily for manufacturing sectors where measuring new knowledge is more tangible, whereas innovations made in the service sector are often harder to patent. In this exercise, only sectors having both types of measures are used for comparison.

Specifically, the average number of patents per worker is compared to the fraction of es-

tablishments doing own-product improvements and/or product expansion in each year and sector[1]. In Figure 1.3, the fraction of innovating establishments in each year and sector is positively correlated with the number of patents per worker in that year and sector in the pooled sample, and the correlation coefficient is 0.81. This positive correlation, however, comes more from sectoral heterogeneities than time-series variations within each sector. Indeed, within each sector, the log number of patents per worker is almost flat over the fraction of innovating establishments. Incorporating sectoral heterogeneities is preferred in this case because there can be lags between the appearance of frontier knowledge, which is proxied by the patent data, and the adoption of new knowledge, which appears as a fraction of establishments implementing innovations.

Figure 1.3: Innovation Frequency and the Number of Patents



Notes: This figure presents the fraction of establishments implementing innovation and the number of patents at the sector and year level.

In Figures 1.4 and 1.5, the survey measures of own-product improvements and product expansion are separated. External and internal patents are also distinguished in a way similar to Akcigit and Kerr (2018), wherein internal patents are defined as patents that cite other patents granted to their assignees for more than half of the total citations. External patents are all patents other than internal patents. Akcigit and Kerr (2018) use internal versus external patents to measure innovation over firms' previous product lines versus innovation made over others' product lines. Patents are classified by the previously written criterion, and the numbers of internal and external patents are recalculated

[1]Each establishment sample is weighted by year with the employment of the survey year multiplied by the cross-sectional weights.

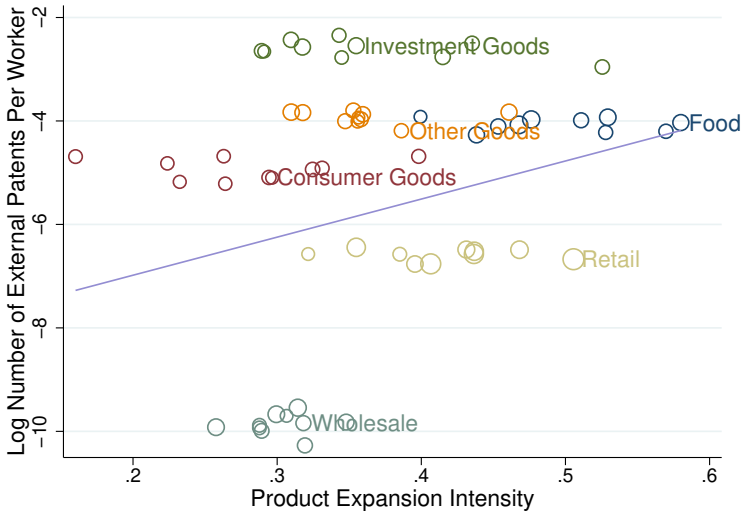
in each sector and year separately. Figures 1.4 and 1.5 show the scatter plots of the number of internal and external patents per worker against the fraction of establishments with own-product improvements and product expansion, respectively.

Figure 1.4: Own-Product Improvement Frequency and the Number of Internal Patents



Notes: This figure presents the fraction of establishments implementing own-product improvements and the number of internal patents at the sector and year level.

Figure 1.5: Product Expansion Frequency and the Number of External Patents



Notes: This figure presents the fraction of establishments implementing product expansion and the number of external patents at the sector and year level.

In Figure 1.4, the frequency of own-product improvements at the sector by year level is

still positively associated with the number of internal patents per worker, with a correlation coefficient of 0.72. Again, this positive correlation stems more from the sectoral heterogeneities in the average frequency of own-product improvements and the number of internal patents than from within-sector variations. Meanwhile, in Figure 1.5, the frequency of product expansion is weakly correlated with the number of external patents per worker, with a correlation coefficient of 0.22. For example, the retail sector has a relatively higher fraction of product-expanding establishments than the investment goods sectors, but its number of external patents is lower. This contrasts with the own-product improvement case, where the retail sector has both a lower frequency of own-product improvements than most manufacturing establishments and a lower number of internal patents per worker.

1.3 Surveyed Innovation and Establishment Performance

This appendix shows the estimation results of Equation (15) in the paper on additional variables. First, the dependent variables are defined in the following list. Growth rates are defined in a way similar to Davis et al. (1998). $L.$ is the yearly lag operator.

- Employment growth rate = $(E - L.E)/(\frac{1}{2}E + \frac{1}{2}L.E)$
- Exit = 1 if $E = 0$ and $L.E > 0$, $= 0$ if $E > 0$ and $L.E > 0$
- Wage, Revenue, and Investment growth rates are defined similarly.
- Hire rate = $2 \times Hire/(\frac{1}{2}E + \frac{1}{2}L.E)$
- Separation rate = $2 \times Separation/(\frac{1}{2}E + \frac{1}{2}L.E)$

where

- E = Employment of an establishment as of June 30 in the survey year from the administrative data
- $Hire$ = Total number of newly hired workers in the first half of the survey year from the survey
- $Separation$ = Total number of separated workers in the first half of the survey year from the survey
- $Wage$ = Imputed daily wage of full-time employees as of June 30 in the survey year from the administrative data
- $Revenue$ = Business volume in the previous year from the survey

- $Investment$ = Sum of investment expenditures in the previous year from the survey

As in the paper, the following lead-lag regression specification, similar to the event study specification, is used to estimate dynamic correlations of innovation dummies with dependent variables before and after the innovation events.

$$Y_{i,t} = \sum_{s=0}^{3} \mathbf{1}^{a}_{i,t-s} \beta^{a}_{s} + \sum_{s=0}^{3} \mathbf{1}^{entry}_{i,t-s} \beta^{entry}_{s} + X_{i,t} + \varepsilon_{i,t}, \tag{1.1}$$

where $Y_{i,t}$ is the dependent variable of establishment i in year t. $\mathbf{1}^{a}_{i,t-s}$ is an indicator of innovation type a, either own-product improvements (own) or product expansion (exp). Likewise, $\mathbf{1}^{entry}_{i,t-s}$ is an indicator whether the firm i entered in year $t-s$. Controls $X_{i,t}$ include sector and year fixed effects.

All samples from 2005 are used in this regression because the year 2005 is the three years before the beginning of the innovation modules. $\mathbf{1}^{a}_{it-s} = 0$ if the sample is surveyed in year $t-s$ and responds that it did not do innovation type a or if the sample entered after $t-s$.

The change of the innovation year is defined as the change made between the innovation year and the year before. Since the survey asks about innovation activities in the previous year, while the wage expense and the employment variables are defined as of June of the survey year, the growth rate after the innovation may be the one between the innovation year and the previous year of the innovation. The same definition applies to other rates regarding hires, separations, investment growth, and revenue growth. The lagged terms in the regression equation are defined in the regression accordingly.

Similarly, because the numbers of hires and separations are defined over the first six months of the survey year, the hire and the separation rates of the year are defined as the rates of the innovation year if the innovation activities of the previous year took place in the survey of the current year. The survey asks about the revenue and investments in the previous year of the survey. Thus, the profits and the investments of the survey year are the revenue and the investments of the innovation year if the innovation dummies are one in the survey of the current year. The lagged terms in the regression equations are defined in the regression accordingly.

The following specification is further considered where all the innovation types surveyed in the data are included. This specification controls for any spurious correlation of indi-

vidual innovation types to outcome variables Y_{it} through other types of innovation[2].

$$Y_{i,t} = \sum_{s=0}^{3} \mathbf{1}_{i,t-s}^{own} \beta_s^{own} + \sum_{s=0}^{3} \mathbf{1}_{i,t-s}^{exp} \beta_s^{exp} + \sum_{s=0}^{3} \mathbf{1}_{i,t-s}^{entry} \beta_s^{entry} + X_{i,t} + \varepsilon_{it}. \tag{1.2}$$

This equation controls for contemporaneous and lagged behavior of the other innovation type.

Table 1.3 shows estimation results on employment-related variables. The first panel on employment growth also appears in the paper. Own-product improvements predict an increase in employment growth by 2.5 percentage points one year after the innovation year, and the correlation becomes weaker two years later. Product expansion also correlates significantly with increased employment growth one year after the innovation year but with a smaller magnitude than own-product improvements. This correlation also becomes weaker in two years (compare the third to the sixth columns), from 1.95 percentage points in the innovation year to 1.1 percentage points two years after.

These two types of innovation, however, feature opposite changes in gross hires and separations. Additional employment growth after own-product improvements entirely comes from fewer separations. Hire rates also decrease after own-product improvements. On the other hand, product expansion is associated with more hires and separations. This suggests that own-product improvements require the existing workforce to stay and use the human capital accumulated in the firm, whereas product expansion makes firms try new workers and separate if their productivity turns out to be low.

[2]For employment growth rates, entering and exiting establishments are included in the samples. For all the other variables, only continuing establishments are included.

Table 1.3: Regression Results

Dependent Variable: Employment Growth Rates

	(1)	(2)	(3)	(4)	(5)	(6)
Periods	2 Years	2 Years	2 Years	4 Years	4 Years	4 Years
Own-product	5.611***		4.963***	3.051***		2.508***
Improvement	(0.307)		(0.333)	(0.278)		(0.329)
Product		3.760***	1.952***		2.733***	1.103***
Expansion		(0.316)	(0.343)		(0.304)	(0.360)
N.	58132	58024	58019	58132	58024	58019
Other Innov.			✓			✓

Dependent Variable: Hire Rates

	2 Years	2 Years	2 Years	4 Years	4 Years	4 Years
Own-product	0.694*		0.145	-0.415		-1.461***
Improvement	(0.381)		(0.411)	(0.338)		(0.399)
Product		0.904**	1.143***		1.223***	2.049***
Expansion		(0.389)	(0.421)		(0.369)	(0.436)
N.	51082	51033	51028	51082	51033	51028
R-squared	0.068	0.069	0.069	0.068	0.069	0.069
Other Innov.			✓			✓

Dependent Variable: Separation Rates

	2 Years	2 Years	2 Years	4 Years	4 Years	4 Years
Own-product	-0.812**		-1.151***	-0.974***		-1.621***
Improvement	(0.332)		(0.359)	(0.295)		(0.349)
Product		0.066	0.679*		0.330	1.293***
Expansion		(0.340)	(0.368)		(0.322)	(0.382)
N.	51108	51060	51055	51108	51060	51055
R-squared	0.044	0.044	0.045	0.044	0.044	0.045
Other Innov.			✓			✓

Notes: This table presents estimation results of Equations (1.1) and (1.2). Columns (1), (2), (4), and (5) exclude other incumbent innovation dummies, whereas Columns (3) and (6) include them. All specifications include entrant innovation dummies, sector fixed effects, and year fixed effects. Columns (1)-(3) and (4)-(6) exhibit $\sum_{s=0}^{1}\beta_s^a$ and $\sum_{s=0}^{3}\beta_s^a$, respectively, where $a = own$ for own-product improvements and $a = exp$ for product expansion. Samples are weighted by sampling weights adjusted by the average of employment in years t and $t-1$. Entering and exiting establishments are included only for employment growth rates. All regressions include sector and year fixed effects. The numbers in parentheses are standard errors of the estimates.

*** - Significant at 1%, ** - Significant at 5%, * - Significant at 10%.

Table 1.4 presents regression results for additional variables, including wage, revenue, and investment growth rates. Neither own-product improvements nor product expansion is significantly correlated with wage increases, as indicated in the first panel. This suggests that workers do not share in the returns to innovation. Revenue increases significantly one year after both own-product improvements and product expansion. However, own-product improvements are not linked to changes in revenue growth two years later. In contrast, product expansion shows a significant and cumulative correlation with revenue increases over time. Both innovation types result in substantial investment growth two years after the innovation, though their effects diminish considerably four years post-innovation.

Table 1.4: Regression Results

Dependent Variable: Wage Growth Rates

	2 Years	2 Years	2 Years	4 Years	4 Years	4 Years
Own-product	0.044		0.038	0.116		0.079
Improvement	(0.140)		(0.151)	(0.124)		(0.146)
Product		0.094	0.065		0.093	0.052
Expansion		(0.142)	(0.154)		(0.134)	(0.159)
N.	45974	45926	45923	45974	45926	45923
R-squared	0.002	0.003	0.003	0.002	0.003	0.003
Other Innov.			✓			✓

Dependent Variable: Revenue Growth Rates

	2 Years	2 Years	2 Years	4 Years	4 Years	4 Years
Own-product	1.425***		1.017**	1.027***		0.439
Improvement	(0.378)		(0.404)	(0.328)		(0.386)
Product		1.100***	0.807**		1.486***	1.174***
Expansion		(0.378)	(0.407)		(0.358)	(0.419)
N.	35573	35552	35549	35573	35552	35549
R-squared	0.042	0.042	0.042	0.042	0.042	0.042
Other Innov.			✓			✓

Dependent Variable: Investment Growth Rates

	2 Years	2 Years	2 Years	4 Years	4 Years	4 Years
Own-product	19.450***		16.641***	2.932*		3.707*
Improvement	(1.994)		(2.137)	(1.775)		(2.077)
Product		16.545***	12.604***		0.931	-1.534
Expansion		(1.997)	(2.151)		(1.922)	(2.247)
N.	37389	37362	37359	37389	37362	37359
R-squared	0.016	0.015	0.018	0.016	0.015	0.018
Other Innov.			✓			✓

Notes: This table presents estimation results of Equations (1.1) and (1.2). Columns (1), (2), (4), and (5) exclude other incumbent innovation dummies, whereas Columns (3) and (6) include them. All specifications include entrant innovation dummies, sector fixed effects, and year fixed effects. Columns (1)-(3) and (4)-(6) exhibit $\sum_{s=0}^{1} \beta_s^a$ and $\sum_{s=0}^{3} \beta_s^a$, respectively, where $a = own$ for own-product improvements and $a = exp$ for product expansion. Samples are weighted by sampling weights adjusted by the average of employment in years t and $t-1$. Entering and exiting establishments are excluded. All regressions include sector and year fixed effects. The numbers in parentheses are standard errors of the estimates.
*** - Significant at 1%, ** - Significant at 5%, * - Significant at 10%.

1.3.1 Robustness Check: Integration of New Business Units and Innovation

Product expansion by an incumbent establishment does not necessarily signify an improvement in an existing product variety; the establishment may simply adopt a new product without any modifications following mergers or acquisitions at the company level. Furthermore, employment growth resulting from the integration of other business units may not reflect the labor reallocation processes that enhance aggregate productivity. If most employment expansion activities post-innovation arise from company-level integration, and such integration does not contribute to productivity growth, the calibrated model may overstate the role of product expansion in growth and labor reallocation.[3] This analysis accounts for such structural changes and demonstrates the robustness of the empirical results.

Establishments with company-level organizational changes are identified using the following questions in the IAB-BP.

1. Were parts of this establishment closed down or relocated with other company units between 1 July of the previous year and 30 June of the survey year, or were they separated and continued as independent businesses?

2. Conversely, were there any organizational developments that resulted in the integration of other establishments or establishment units into your company?

The second question is used to identify establishments with newly integrated units at the company level and exclude them in the regression analysis in Online Appendix 1.3. Establishments are more likely to improve their own products (74%) and expand to new product varieties (48%) when their companies integrate new business units. However, only 1% of establishments, which account for 3.6% of employment in the entire sample, report that their companies integrated new business units. Larger establishments are more likely to fall into this category, with 2.6% of establishments experiencing business integration in the third largest employment group compared to only 0.6% of establishments in the third smallest group. Thus, the results do not change significantly when these establishments are excluded in the estimation below.

[3] Previous studies about mergers and acquisitions, such as Jovanovic and Rousseau (2002), assume that mergers and acquisitions reallocate capital from less efficient firms to more efficient firms, generating productivity gains. If mergers and acquisitions are a medium for firms and establishments to expand and innovate existing product varieties, then including integrated establishments is more desirable. Thus, this view is chosen for the baseline.

Table 1.5: Regression Results

Dependent Variable: Employment Growth Rates

	2 Years	2 Years	2 Years	4 Years	4 Years	4 Years
Own-product	5.477***		4.880***	2.928***		2.406***
Improvement	(0.310)		(0.335)	(0.279)		(0.330)
Product		3.512***	1.763***		2.652***	1.080***
Expansion		(0.319)	(0.346)		(0.309)	(0.365)
N.	56155	56054	56049	56155	56054	56049
R-squared	0.743	0.741	0.742	0.743	0.741	0.742
Other Innov.			✓			✓

Dependent Variable: Wage Growth Rates

	2 Years	2 Years	2 Years	4 Years	4 Years	4 Years
Own-product	-0.018		-0.049	0.117		0.067
Improvement	(0.143)		(0.154)	(0.126)		(0.149)
Product		0.128	0.125		0.115	0.084
Expansion		(0.145)	(0.157)		(0.138)	(0.163)
N.	44319	44271	44268	44319	44271	44268
R-squared	0.002	0.003	0.003	0.002	0.003	0.003
Other Innov.			✓			✓

Notes: This table presents estimation results of Equations (1.1) and (1.2). Columns (1), (2), (4), and (5) exclude other incumbent innovation dummies, whereas Columns (3) and (6) include them. All specifications include entrant innovation dummies, sector fixed effects, and year fixed effects. Columns (1)-(3) and (4)-(6) exhibit $\sum_{s=0}^{1}\beta_s^a$ and $\sum_{s=0}^{3}\beta_s^a$, respectively, where $a = own$ for own-product improvements and $a = exp$ for product expansion. Samples are weighted by sampling weights adjusted by the average of employment in years t and $t-1$. Entering and exiting establishments are included only for employment growth rates. All regressions include sector and year fixed effects. The numbers in parentheses are standard errors of the estimates.
*** - Significant at 1%, ** - Significant at 5%, * - Significant at 10%.

Tables 1.5 show the results on employment-related variables with sub-samples of establishments that report no company-level integration in the current year and the previous year. The coefficient estimates on employment growth stay almost the same as in Table 1.3 because most establishments do not experience firm-level integration in the data.

2 Numerical Appendix

2.1 Model with Permanent Demand Differences

The full model in the quantitative analysis of the paper contains a permanent difference in demand for each product $\bar{d} \sim$ Lognormal$(-\theta^2/2, \theta^2)$, which implies $\mathbb{E}[\bar{d}] = 1$ and $\mathbb{V}[\bar{d}] = \theta^2$. This demand shifter is constant across time and between producers that enter the product market. In order to simplify the analysis, the model is deliberately made to be homogeneous of degree one in $\bar{d}$. Specifically, the HJB equation for a producer on a product line with $(x, \bar{d})$ is given by the following equation:

$$\begin{aligned}\left(r + \tau\left(x\right)\right)\mathbf{V}\left(x, \bar{d}, W_t, \bar{Y}_t\right) = \max_{\xi,\zeta\geq 0}\Bigg\{ & \bar{Y}_t\bar{\pi}\bar{d}x - \phi\bar{d}W_t \\ & - g_A x\partial_x\mathbf{V}\left(x, \bar{d}, W_t, \bar{Y}_t\right) + \frac{x^2\varsigma^2}{2}\partial_x^2\mathbf{V}\left(x, \bar{d}, W_t, \bar{Y}_t\right) \\ & + g_{\bar{Y}}\partial_{\bar{Y}_t}\mathbf{V}\left(x, \bar{d}, W_t, \bar{Y}_t\right) + g_w W_t\partial_{W_t}\mathbf{V}\left(x, \bar{d}, W_t, \bar{Y}_t\right) \\ & - W_t\bar{d}xc_{own}\left(\xi\right) + \xi\Bigg(\mathbb{E}_{H(\lambda)}\left[\mathbf{V}\left(\lambda x, \bar{d}, W_t, \bar{Y}_t\right)\right] - \mathbf{V}\left(x, \bar{d}, W_t, \bar{Y}_t\right)\Bigg) \\ & - W_t\bar{d}c_{exp}\left(\zeta\right) + \zeta\bar{d}\iint\mathbb{E}_{H(\lambda)}\Bigg[\nu_{exp}\mathbf{V}\left(\lambda x', \bar{d}', W_t, \bar{Y}_t\right) \\ & \quad + \left(1-\nu_{exp}\right)\mathbf{V}\left(\lambda\kappa x', \bar{d}', W_t, \bar{Y}_t\right)\Bigg]dF\left(x'\right)dG\left(\bar{d}'\right)\Bigg\}.\end{aligned} \tag{2.1}$$

This implies that the size of the permanent demand shifter, $\bar{d}$, increases all the revenue, the fixed cost of operation, the R&D costs, and the returns to innovation linearly. Notably, the demand shifter after an own-product improvement becomes $\bar{d}\lambda x$ from $\bar{d}x$. As for product expansion, $\bar{d}$ increases the frequency of product expansion linearly to guarantee the homogeneity. Then, we can show that, on the BGP,

$$\mathbf{V}\left(x, \bar{d}, W_t, \bar{Y}_t\right) = \bar{Y}_t\bar{d}v\left(x\right). \tag{2.2}$$

It follows that the innovation rates $\xi(x)$ and ζ are unaffected by $\bar{d}$ but depend only on x, despite the dispersion of employment size associated with $\bar{d}$.

2.2 Transition Path: Details

This appendix describes the detailed equations necessary to derive the equilibrium over the transition path. First, the length of the time interval, Δ, is fixed to approximate the continuous-time equations. The subscript t represents the object evaluated at time t. The value function along the transition path is then governed by the following equation:

$$\begin{aligned}\left(r_t - g_{\bar{Y}_t} + \tau_t(x)\right) v_t(x) = \max_{\xi,\zeta \geq 0} \Bigg\{ & \pi_t(x) - \phi w_t \\ & - g_{x,t} x v_t'(x) + \frac{v_{t+\Delta}(x) - v_t(x)}{\Delta} \\ & - w_t c_\xi(\xi) + \xi \left(\int v_t(\lambda x)\, dH(\lambda) - v_t(x) \right) \\ & - w_t c_\zeta(\zeta) + q\zeta \iint \Big[\nu_{exp} v_t\left(\lambda x'\right) \\ & \quad + \left(1 - \nu_{exp}\right) v_t\left(\lambda \kappa x'\right) \Big] dH(\lambda)\, dF\left(x'\right) \Bigg\}. \end{aligned} \tag{2.3}$$

Then, the optimal innovation rates are given by

$$\xi_t(x) = \left(\frac{v_t(\lambda x) - v_t(x)}{w_t c_{own}} \right)^{\epsilon}, \tag{2.4}$$

$$\zeta_t = \left(\frac{\mathbb{E}_{H(\lambda),F(x')} \left[\nu_{exp} v_t\left(\lambda x'\right) + \left(1 - \nu_{exp}\right) v_t\left(\lambda \kappa x'\right) \right]}{w_t c_{exp}} \right)^{\epsilon}. \tag{2.5}$$

In this equation system, $w_t := W_t / \bar{Y}_t$ is not constant on the transition path.
The sequences of prices w_t and r_t satisfy the free entry condition and the Euler equation, respectively.

$$w_t = \frac{1}{c_{entry}} \iint \left(\left(1 - \nu_{entry}\right) v_t\left(\lambda x'\right) + \nu_{entry} v_t\left(\lambda \kappa x'\right) \right) dF_{0,t}(x') dH(\lambda). \tag{2.6}$$

$$r_t = g_{C,t} + \rho. \tag{2.7}$$

Then, the labor market clearing condition pins down the entry rate, χ_t.

$$m_t^{\eta+1} \left(\frac{w_t}{\phi} \right)^{\eta} = \frac{1}{w_t} \frac{\sigma - 1}{\sigma} + \left(\int c_{own}\left(\xi_t(x)\right) dF_t(x) + c_{exp}\left(\zeta_t\right) + c_{entr} \chi_t + \phi \right). \tag{2.8}$$

The distribution along the transition path is expressed in the following equation:

$$\begin{aligned}\frac{df_{t+\Delta}(x)-f_t(x)}{dt} &= \partial_x\left[g_A x f_t(x)\right]+\partial_x^2\left[\frac{\varsigma^2x^2}{2}f_t(x)\right] \\ &-\int \xi_t\left(\xi_t(x)f_t(x)-\left(\frac{x}{\lambda}\right)f_t\left(\frac{x}{\lambda}\right)\right)dH(\lambda) \\ &-\left(1-\nu_{exp}\right)\zeta_t\left(\int\left(f_t(x)-f_t\left(\frac{x}{\lambda}\right)\right)dH(\lambda)\right) \\ &-\nu_{exp}\zeta_t\int\left(f_t(x)-f_t\left(\frac{x}{\kappa\lambda}\right)\right)dH(\lambda) \\ &-\left(1-\nu_{exp}\right)\chi_t\left(\int\left(f_{0,t}(x)-f_{0,t}\left(\frac{x}{\lambda}\right)\right)dH(\lambda)\right) \\ &-\nu_{exp}\chi_t\int\left(f_t(x)-f_{0,t}\left(\frac{x}{\kappa\lambda}\right)\right)dH(\lambda) \\ &-\omega_t f_t(x)\end{aligned} \tag{2.9}$$

In this equation, ω_t and $f_{0,t}(x)$ are defined as follows.

$$\omega_t = \partial_x\left[\frac{\varsigma^2x^2}{2}f_t(x)\right]_{x=\underline{x}_t}, \tag{2.10}$$

$$f_{0,t}(x) \propto \frac{f_t(x)}{1+e^{\upsilon(x-\underline{x}_t)}}, \tag{2.11}$$

$$\tau_t(x) = \zeta_t + \chi_t\frac{\int\left(1+e^{\upsilon(x'-\underline{x}_t)}\right)dx'}{1+e^{\upsilon(x-\underline{x}_t)}}. \tag{2.12}$$

Lastly, given the sequence of population growth rates, $g_{M,t}$, the endogenous growth rates can be expressed as the following equations.

$$g_{N,t} = \nu_{exp}\zeta_t + \nu_{entry}\chi_t - \omega_t \tag{2.13}$$

$$\begin{aligned} g_{A,t} = \omega_t + \iiint_{x' \leq x} \left(\xi_t\left(x'\right) f_t\left(x'\right) - \xi_t\left(\frac{x'}{\lambda}\right) f_t\left(\frac{x'}{\lambda}\right) \right) dH\left(\lambda\right) dx' dx \\ + \left(1-\nu_{exp}\right)\zeta_t \mathbb{E}\left[\lambda - 1\right] + \nu_{exp}\zeta_t \mathbb{E}\left[\kappa\lambda - 1\right] \\ + \left(1-\nu_{ent}\right)\chi_t \mathbb{E}\left[\lambda - 1\right]\bar{x}_{0,t} + \nu_{ent}\chi_t \mathbb{E}\left[\kappa\lambda\bar{x}_{0,t} - 1\right] \end{aligned} \tag{2.14}$$

$$g_{Y,t} = g_{A,t} + g_{N,t} \tag{2.15}$$

$$g_{C,t} = g_{Y_t} - g_{M,t} \tag{2.16}$$

Then, $m_t \equiv M_t/N_t$ evolves according to the following equation: $m_{t-\Delta} \times (1 + \Delta g_{M,t-\Delta} - \Delta g_{N,t-\Delta})$. Unlike on the BGP, m_t is a state variable over the transition path and determines the speed of convergence to the new BGP.

The equilibrium on the transition path is defined as the sequence of the value function $v_t(x)$; the policy functions $\xi_t(x)$ and ζ_t; the product line distribution $f_t(x)$; the creative destruction rate $\tau_t(x)$; the growth rates $g_{M,t}$, $g_{A,t}$, $g_{Y,t}$, and $g_{C,t}$; the entry rate χ_t; as well as the paths for wages w_t and interest rates r_t such that

1. The value function and the policy function satisfy the limit of Equations (2.3)-(2.5) when $\Delta \to 0$.
2. The distribution function follow the limit of Equations (2.9)-(2.12) when $\Delta \to 0$.
3. The entry rate and the price schedule meet the limit of Equations (2.6) and (2.8).
4. The growth rates are consistent with the limit of Equations (2.13) and (2.16).

The algorithm to find the equilibrium along the transition path is as follows:

1. Fix T. T is set to 1000 years. This will be where the economy converges (approximately) to the new BGP.
2. Start with an arbitrary sequence for w_t, r_t, $g_{A,t}$, $g_{N,t}$, and $f_t(x)$.
3. Given the rest, update $v_t(x)$, $\xi_t(x)$, $\zeta_t(x)$, and χ_t backward from $t = T$ to $t = 0$. Notice that the value at $t = T + \Delta$ will be the value at the new BGP.
4. Given the updated policy function, update $f_t(x)$, $\tau_t(x)$, and χ_t forward from $t = 0$ to $t = T$. Notice that the initial f_t at $t = 0$ is the distribution at the old BGP.
5. Given the updated policy and distribution function, update w_t, r_t, $g_{A,t}$, and $g_{N,t}$

respectively. Go back to step 3 until they converge.

After the computation, given $Y_0^{\bar{BS}} = 1$, one gets the sequence of output per unit mass of product lines $\{\bar{Y}_t^{BS}\}_{t=0}^T$ and $\{\bar{Y}_t^{TR}\}_{t=0}^T$ on the baseline BGP and transition path. Then, the welfare of each household can be calculated with the sequences of output per capita $\{\tilde{Y}_t^{BS}\}_{t=0}^T = \{\bar{Y}_t^{BS}/m_t^{BS}\}_{t=0}^T$ and $\{\tilde{Y}_t^{TR}\}_{t=0}^T = \{\bar{Y}_t^{TR}/m_t^{TR}\}_{t=0}^T$.

Let W_0^{BS} and W_0^{TR} denote the output-related welfare, i.e., without disutility of labor supply, of the representative household on the baseline and the transition paths, respectively. Then, the consumption-equivalent variation (CEV) is derived by the following equations:

$$\begin{aligned} W_0^{TR} &= \int_0^\infty e^{-\rho t} \log\left(\left(1+CEV\right)\tilde{Y}_t^{BS}\right) dt \\ &= W_0^{BS} + \log\left(1+CEV\right)\int_0^\infty e^{-\rho t} dt \\ &= W_0^{BS} + \frac{\log\left(1+CEV\right)}{\rho}. \\ \Rightarrow CEV &:= \exp\left(\rho\left(W_0^{TR} - W_0^{BS}\right)\right) - 1. \end{aligned}$$

The change in welfare is further decomposed into own-product improvement, product expansion, and entrant innovation margins. To do this, decompose the growth of per-capita output as in the following equation.

$$\begin{aligned} g_{\tilde{Y},t} &= g_{A,t} + g_{N,t} - g_{M_t} \\ &= \underbrace{\iiint_{x' \le x} \left(\xi_t\left(x'\right) f_t\left(x'\right) - \xi_t\left(\frac{x'}{\lambda}\right) f_t\left(\frac{x'}{\lambda}\right)\right) dH\left(\lambda\right) dx' dx}_{=: g_t^{own}} \\ &\quad + \underbrace{\left(1-\nu_{exp}\right)\zeta_t \mathbb{E}\left[\lambda\right] + \nu_{exp}\zeta_t \mathbb{E}\left[\kappa\lambda\right]}_{=: g_t^{exp}} \\ &\quad + \underbrace{\left(1-\nu_{ent}\right)\chi_t \mathbb{E}\left[\lambda\right]\bar{x}_{0,t} + \nu_{ent}\chi_t \mathbb{E}\left[\kappa\lambda\right]\bar{x}_{0,t}}_{=: g_t^{entry}} \end{aligned}$$

Then, the welfare and the change in welfare will be expressed in the following way:

$$
\begin{aligned}
W_0 &:= \int_0^\infty e^{-\rho t} \log\left(e^{\int g_{\tilde{Y},s} ds} \tilde{Y}_0\right) dt \\
&= \int_0^\infty e^{-\rho t} \int_0^t g_{\tilde{Y},s} ds dt + \int_0^\infty e^{-\rho t} \log \tilde{Y}_0 dt \\
&= \int_0^\infty e^{-\rho t} \int_0^t g_s^{own} ds dt + \int_0^\infty e^{-\rho t} \int_0^t g_s^{exp} ds dt \\
&\quad + \int_0^\infty e^{-\rho t} \int_0^t g_s^{entry} ds dt + \text{other terms}. \\
CEV &\approx \rho\left(W_0^{TR} - W_0^{BS}\right) \\
&= \rho\left(\int_0^\infty e^{-\rho t} \int_0^t \left(g_s^{own,TR} - g_s^{own,BS}\right) ds dt\right) \\
&\quad + \rho\left(\int_0^\infty e^{-\rho t} \int_0^t \left(g_s^{exp,TR} - g_s^{exp,BS}\right) ds dt\right) \\
&\quad + \rho\left(\int_0^\infty e^{-\rho t} \int_0^t \left(g_s^{entry,TR} - g_s^{entry,BS}\right) ds dt\right) \\
&\quad + \text{Contribution of Production Labor}.
\end{aligned}
$$

In this equation, superscripts BS and TR indicate that variables are evaluated at the baseline BGP and over the transition path, respectively.

Then, the metrics in Table 3.2 are defined in the following way, where Δg_0^{own}, Δg_0^{exp}, Δg_0^{entry} are the contributions of own-product improvements, product expansion, and entrant innovation to the welfare change, respectively:

$$
\begin{aligned}
\Delta g_0^{own} &:= \rho\left(\int_0^\infty e^{-\rho t} \int_0^t \left(g_s^{own,TR} - g_s^{own,BS}\right) ds dt\right). \\
\Delta g_0^{exp} &:= \rho\left(\int_0^\infty e^{-\rho t} \int_0^t \left(g_s^{exp,TR} - g_s^{exp,BS}\right) ds dt\right). \\
\Delta g_0^{entr} &:= \rho\left(\int_0^\infty e^{-\rho t} \int_0^t \left(g_s^{entry,TR} - g_s^{entry,BS}\right) ds dt\right).
\end{aligned}
$$

The residual comes from changes in production labor.

3 Application: 2020 R&D Tax Incentive

3.1 Background

We use the calibrated model to quantify the impact of an R&D tax incentive program that started in Germany in 2020. This exercise is in line with Atkeson and Burstein (2019), who use a quantitative endogenous growth model to evaluate the effect of R&D policies on aggregate growth and welfare. According to the OECD R&D Tax Incentive database, the implied R&D tax subsidy rate jumped from -2% to 19% between 2019 and 2020 in Germany[4]. This policy is modeled as an increase in the R&D subsidy rate by 21 percentage points.[5]

The baseline BGP is assumed to have zero subsidy rates on R&D expenditure, and the new R&D tax incentive is modeled as a positive 21% subsidy on R&D expenditures. Then, the new R&D costs become $0.79 \times W_t c_{own}(\xi)$, $0.79 \times W_t c_{exp}(\zeta)$, and $0.79 \times W_t c_{entry}$ for own-product improvements, product expansion, and entrant innovation, respectively. These R&D subsidies are financed by a lump-sum tax on representative households. However, the labor market clearing condition remains the same as before. Labor demand takes the full R&D labor along with the production labor to determine the wage level.

As in the paper, the new BGP with the R&D tax incentive is compared to the baseline BGP. Then, we evaluate the changes in output and welfare over the transition path. The economy starts from the baseline BGP and introduces an unexpected change in the R&D subsidy rate at $t = 0$. The deterministic transition path to the new BGP is computed using equations in Online Appendix 2.2.

3.2 Balanced Growth Path

Table 3.1 compares the BGP moments before and after the new R&D tax incentive. Column (1) summarizes the baseline BGP, while column (5) summarizes the BGP after the R&D tax incentive. Columns (2)-(4) decompose the effect of R&D subsidies only on one of the three innovation types. Although not very realistic, having a separate subsidy for only one type of innovation helps to distinguish how the two types of innovation affect

[4]This tax subsidy rate is defined as one minus B index, where the B index is a *'measure of the level of pre-tax profit a "representative" company needs to generate to break even on a marginal, unitary outlay on R&D'*, OECD Measuring Tax Support for R&D and Innovation: Measurement (`https://www.oecd.org/sti/rd-tax-definition-and-measurement.htm`).

[5]A recent paper by Chen et al. (2021) find that firms relabel expenses as R&D to benefit from R&D tax credits. This paper takes the stance that the R&D tax credits give the incentive to label more expenses as R&D and increase all expenses related to innovation. If relabeling incurs economic costs, then the estimated effect of R&D on welfare will be lower.

Table 3.1: Balanced Growth Path Moments before and after R&D Incentive

Moment	(1) Baseline	(2) Own Subsidy	(3) Expansion Subsidy	(4) Entry Subsidy	(5) Full Subsidy
Output Growth Rate	1.34%	1.52%	1.31%	1.15%	1.31%
Own-Product Improvement	1.04%	1.22%	1.02%	0.86%	1.03%
Product Expansion	0.01%	0.01%	0.02%	0.01%	0.01%
Entrant Innovation	0.28%	0.28%	0.27%	0.27%	0.27%
Job Creation Rate	33.1%	33.2%	34.3%	34.1%	32.7%
Incumbents	12.1%	12.8%	13.7%	13.3%	12.3%
Entrants	21.0%	20.4%	20.6%	20.8%	20.4%
Population-Product Ratio	1.000	1.006	0.995	0.816	0.832
Wage	1.000	1.017	1.002	1.020	1.031
Labor Supply	0.927	0.943	0.929	0.946	0.956

Notes: This table shows how innovations contribute to output growth rates and how incumbents and entrants contribute to job creation across different BGP equilibria. The output growth rate is annual; the job-creation rate is measured over five years. The growth-rate decomposition follows Equation (18) of the paper. Wages are expressed relative to GDP per capita, and labor supply is per household. Labor supply elasticity $\eta = 1$.

the economy differently.

Subsidizing own-product improvements is more important in accelerating growth than subsidizing product expansion across different BGPs. This is because own-product improvements constitute the majority of growth and own-product improvements do not yield business-stealing effects. In columns (3) and (4), when business-stealing product expansion and entrant innovation are subsidized, the growth rate on BGP declines, mainly because producers improve their existing products less frequently. Having universal subsidies for all types of innovation, therefore, implies a decrease in the growth rate of the new BGP. Due to more demand for R&D activities, wage increases in all cases.

Labor reallocation rates increase by more when product expansion and entrant innovation are subsidized in columns (3) and (4). When only own-product improvements are subsidized in column (2), the job creation rate among incumbents increases by less than 0.1 percentage points despite the increased growth rate. On the other hand, in columns (3) and (4), the job creation rates increase by 1.2 and 1 percentage points, respectively, despite negative changes in growth rates.

Table 3.2: Effect of the R&D Incentive over the Transition Path

Moment	(1) Own Subsidy	(2) Expansion Subsidy	(3) Entry Subsidy	(4) Full Subsidy
Initial Output Change	3.91%	-1.18%	-19.22%	-13.45%
Consumption Equivalent Variation	1.348%	-0.183%	-0.841%	0.093%
Own-Product Improvement	1.563%	-0.162%	-0.917%	0.163%
Product Expansion	0.001%	0.018%	-0.014%	0.001%
Entrant Innovation	-0.090%	-0.013%	0.454%	0.404%
Production Labor	-0.127%	-0.027%	-0.364%	-0.475%

Notes: This table reports changes in output and welfare along the transition path relative to a BGP without R&D incentives for labor supply elasticity $\eta = 1$. The first panel summarizes the output path; the second panel decomposes the contributions of innovation and production labor to welfare. See Online Appendix 2.2 for further details on the derivations.

3.3 Transition Path

Table 3.2 describes output responses and welfare effects of the R&D tax incentives over the transition path. In all cases except in column (1), the output drops initially because more labor inputs are used for R&D and less for production. In column (1), the output experiences an initial increase of 4% because entrant innovation decreases right after the R&D incentives. With more own-product improvements, firms are less likely to hit the demand cutoff and exit, lowering the entry rate in equilibrium. Consistent with the BGP comparison in Online Appendix 3.2, product expansion and entrant innovation absorb production labor without having sizeable growth effects.

The second panel of Table 3.2 summarizes welfare responses to the subsidies and the contribution of each innovation margin. The welfare effect of subsidizing own-product improvements is positive, whereas the welfare effects of subsidizing the other two are negative. The negative effect of expansion and entry subsidies on welfare comes from their stronger business-stealing effect, as emphasized in Atkeson and Burstein (2017). All three types of innovation have positive externalities because future innovation is built on top of the current frontier knowledge of product lines. However, these effects are ignored when firms make innovation decisions. At the same time, innovations have negative externalities from two types of business stealing. First, the relative demand for a product line decreases if others innovate. Second, especially for product expansion and entrant innovation, innovation causes direct creative destruction of existing producers on product lines. Thus, product expansion and entrant innovation have larger negative externalities, and households are worse off under the subsidies of these two. The welfare effect of

universal subsidies is only modestly positive because of the substantial negative effect of entry subsidies.

4 Application: the Hartz Reform and Innovation

4.1 Hartz Reform and Aggregate Moments

This appendix explains the details of the Hartz reform and presents the aggregate time series before and after the reform before the model application in Section 4.2. The Hartz reform was a large-scale labor-market reform launched in the early 2000s, implemented in four stages (Hartz I, II, III, and IV). The first stage took effect on January 1, 2003, and the last stage began on January 1, 2005.

The Hartz reform had three cornerstones (Jacobi and Kluve, 2006). First, it improved the efficiency of government employment services, which were restructured during the reform. At the same time, a market-based mechanism was introduced, enabling workers to choose private employment agencies. Second, it required unemployed workers to actively seek jobs, reorganized the benefits system, and demanded proactive job-search efforts as a condition for receiving benefits. Additionally, low-income wage earners received subsidies under the reform. Third, it deregulated the labor market by expanding temporary work contracts and reducing firm-size restrictions on layoffs.

Because it restructured public employment agencies as well as the unemployment benefit system and employment-protection laws, the Hartz reform is widely considered one of the most significant postwar changes in Germany's labor market institutions. However, most research examines its static impact on employment, wages, and welfare. Krause and Uhlig (2012) assess the Hartz reform's effectiveness using a search-and-matching model, finding a 2.8-percentage-point reduction in the steady-state unemployment rate. Recent research (Launov and Wälde, 2013, 2016; Bradley and Kügler, 2019; Hochmuth et al., 2021) also uses frictional labor-market models to provide quantitative assessments.

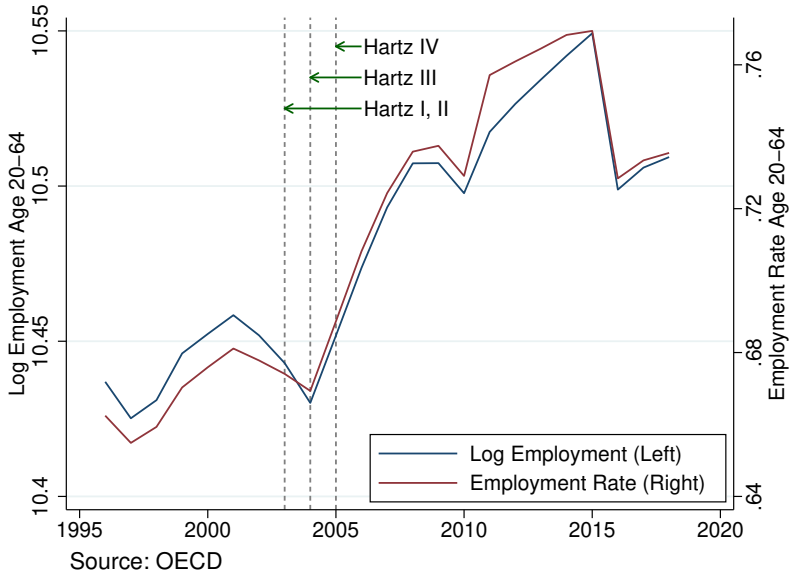
In this paper, we treat the Hartz reform as a driver of both output growth and broader economic welfare. Besides adding production inputs in the short run, the labor-market reform can raise R&D returns by boosting labor supply and easing labor-market rigidities. In turn, rising innovation may enhance the representative household's welfare via positive innovation externalities.

Following the reform, Germany's economy enjoyed a substantial increase in employment, output per capita, and labor productivity. As Figure 4.1 shows, employment growth among those aged 20–64 surged. The rise in employment rates was all the more pronounced because the working-age population declined steadily during this period. Meanwhile, after the mid-2000s (when Hartz I, II, III, and IV were implemented), Germany's economy outpaced its early-2000s growth performance in terms of both per capita output

and labor productivity (see Figure 4.2).

Yet, contrary to Schumpeterian predictions that faster growth would come from establishment-level replacements[6], job destruction rates actually fell during the high aggregate-growth phase after the reform (Figure 4.3).[7]. In line with overall employment growth, job creation rose. Even after employment peaked and growth slowed in the post-2014 period, growth rates remained high, and job destruction rates stayed low. This pattern suggests that the rapid economic growth seen in the 2010s stemmed largely from incumbent firms improving existing product portfolios, which required minimal job destruction at the establishment level.

Figure 4.1: Employment before and after the Hartz Reform

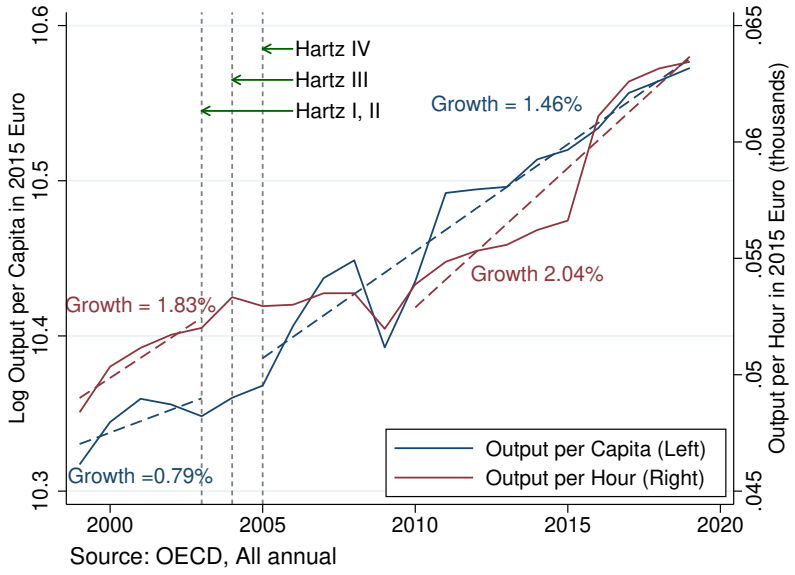


Notes: This figure presents the number of employees aged 20-64 before and after the Hartz reform in Germany.

[6]For more discussions, see Caballero (2010).

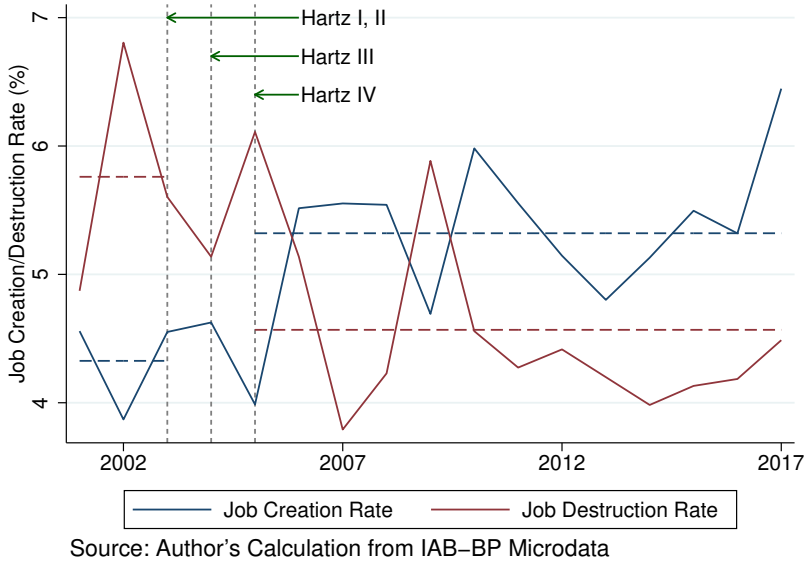
[7]In the extensive margin, establishment exit rates also decreased during the same period.

Figure 4.2: GDP per capita before and after the Hartz Reform



Notes: This figure presents the GDP per capita before and after the Hartz reform in Germany.

Figure 4.3: Job Creation and Destruction Rates before and after the Hartz Reform



Notes: This figure presents the job creation and destruction rates before and after the Hartz reform in Germany.

4.2 Interpretation of the Hartz Reform

We interpret the institutional features of the Hartz reform from Appendix 4.1 within our framework. First, the reform manifests as an exogenous increase in labor supply, modeled as a decline in the labor wedge within the representative household's optimization problem. This approach aligns with modernizing federal employment agencies and improving work incentives. The household's labor-supply decision now faces a wedge, ω_L, consistent with a linear labor-income tax redistributed via lump-sum transfers:

$$\phi(H_t^s)^{\frac{1}{\eta}} C_t = (1-\omega_L) W_t.$$

We assume the labor wedge was strictly positive before the Hartz reform and zero thereafter. Its value in the pre-reform period is set to match the 10-percentage-point rise in labor supply between the early 2000s and the late 2010s. According to the OECD's Annual Labour Statistics, Germany's national employment of the population aged 20–64 rose from 34 million (2000–2004) to 37 million (7.6%) in the late 2010s, even though this age cohort shrank by 2.5%, from 51 million to 50 million. We interpret the additional increase in employment as the Hartz reform's effect. Under our baseline assumption that the Frisch elasticity of labor supply is 1, the implied labor wedge prior to the reform is 17%.

Second, the reform is assumed to reduce the cost of unexpected product-line closures—associated with severance pay and administrative firing costs—via the following closure-cost parameter, κ in the value function.

$$\begin{aligned}
\left(\rho + \tau\left(x\right)\right) v\left(x\right) = \max_{\xi,\zeta \geq 0} \Bigg\{ & \bar{\pi} x - \phi \tilde{w} - g_A x \partial_x v\left(x\right) + \frac{\varsigma^2 x^2}{2} \partial_x^2 v\left(x\right) \\
& - \tilde{w} x c_{own}\left(\xi\right) + \xi \mathbb{E}_{H(\lambda)} \left[v\left(\lambda x, s\right) - v\left(x\right) \right] \\
& - \tilde{w} c_{exp}\left(\zeta\right) + \zeta \nu_i \mathbb{E}_{H(\lambda)} \left[\int v\left(\lambda x'\right) dF\left(x'\right) \right] \\
& + \zeta \left(1 - \nu_i\right) \mathbb{E}_{H(\lambda)} \left[\int v\left(\lambda \kappa x'\right) dF\left(x'\right) \right] \\
& - \tau\left(x\right) \underbrace{\kappa x}_{\text{line closure cost}} \Bigg\}.
\end{aligned}$$

For simplicity, it is assumed that the closure cost increases linearly in x and does not depend on the actual employment[8]. This cost is summarized by κ in the model. The

[8] If the cost of unexpected product line closures depends on the actual employment, the establishment's dynamic problem would need employment as an additional state variable, which complicates the analysis. This assumption ignores additional employment gains from the Hartz reform for establishments to hire

severance pay is around 50% of the monthly wage per tenure year, and the mean tenure of a German worker is 10 years. Few studies estimated firing costs in the German economy, but Bentolila and Bertola (1990) estimate that the firing costs amount to 75% of annual wages. A product line with x hires $\frac{\sigma-1}{\sigma}x/w$ units of production. Consequently, we set $\kappa = (10 \times 0.5 + 0.75 \times 12) \times L \approx 4.99$ before the reform and $\kappa = 0$ after the reform.

4.3 Balanced Growth Path

Table 4.1 compares BGP outcomes before and after the Hartz reform. The first column represents the pre-reform BGP, and the fourth column represents the BGP post-reform. In order to isolate the effects of the two components, the second column shows a new BGP with the labor wedge removed but firing costs intact. The third column retains the positive labor wedge but sets κ to zero.

Reducing firing costs has a counterintuitive effect on the new BGP, as shown in column (2): the growth rate declines mainly because of decreased own-product improvements. Product expansion and entrant innovation rates likewise drop modestly, stemming from a larger effective wage implied by the free-entry condition. The effective wage w becomes $1.1/1.41 = 0.77$ from $1.1/1.1 = 1$. In the long run, this wage effect raises R&D costs, slowing growth under the full reform.

In column (3), a drop in the labor wedge lowers wages firms pay and increases labor supply. Both the magnitude and composition of growth barely change from the baseline, and job creation remains constant across BGPs. In the long run, the population-to-product ratio absorbs all the increase in labor supply. With more labor supply, varieties increase relative to the population.

more production labor at the same quality level.

Table 4.1: Effect of the Hartz Reform on the BGP

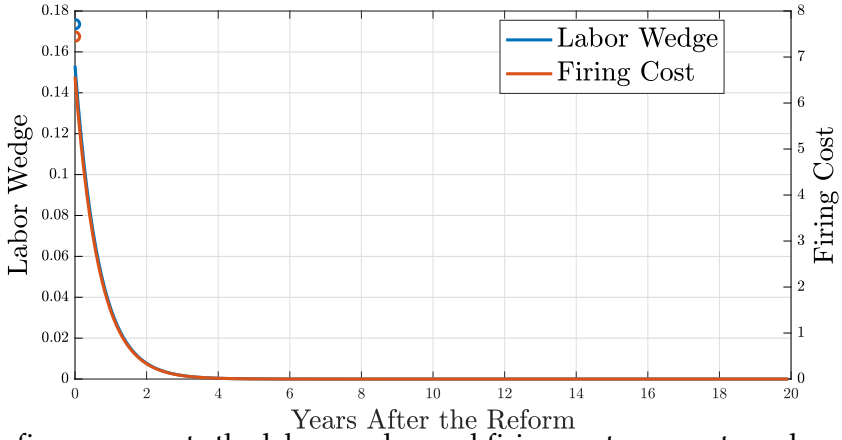
Moment	(1) Before Reform	(2) Firing Cost Reform	(3) Labor Wedge Reform	(4) Full Reform
Output Growth Rate	1.50%	1.34%	1.50%	1.34%
Own-Product Improvement	1.18%	1.04%	1.18%	1.04%
Product Expansion	0.02%	0.01%	0.02%	0.01%
Entrant Innovation	0.30%	0.28%	0.30%	0.28%
Job Creation Rate	34.1%	33.4%	34.1%	33.1%
Incumbents	12.5%	12.4%	12.5%	12.1%
Entrants	21.7%	20.9%	21.7%	21.0%
Population-Product Ratio	1.411	1.100	1.282	1.000
Wage	1.081	1.100	0.983	1.000
Labor Supply	0.828	0.843	0.911	0.927

Notes: This table shows how innovations contribute to output growth rates and how incumbents and entrants contribute to job creation across different BGP equilibria. The output growth rate is annual; the job-creation rate is measured over five years. The growth-rate decomposition follows Equation (18) of the paper. Wages are expressed relative to GDP per capita, and labor supply is per household. Labor supply elasticity $\eta = 1$.

4.4 Transition Path

Along the transition path, labor wedge and firing costs are assumed to drop initially and then exponentially decrease after the unexpected policy introduction at $t = 0$. The initial labor wedge and the firing cost parameters at $t = 0$ are set to $\exp(-0.12) \approx 88\%$ of the BGP level. Motivated by the fact that Hartz I-IV was implemented over three years, we let the parameters decline log-linearly such that the values of the labor wedge and the firing cost parameters in the 36th month become 1% of their pre-reform level. This implies a monthly decay rate of about 12%. Figure 4.4 describes their path.

Figure 4.4: Labor Wedge and Firing Cost under the Hartz Reform Transition



Notes: This figure presents the labor wedge and firing cost parameters along the transition path.

Table 4.2 shows how the Hartz reform influences welfare through output changes relative to the pre-reform BGP. Reducing firing costs impairs the welfare of the economy because of fewer own-product improvements. Lower firing costs increase wages and entry rates in the first few years after the transition, which crowds out incumbent innovation. With business-stealing effects of entrant innovation, this increase in entrant innovation is suboptimal.

With the increase in labor supply, output jumps up initially after the reform in time 0 and grows further with the decaying labor wedge. The increase in labor supply increases production labor and entrant innovation without discouraging incumbent innovation. The entrant innovation margin also contributes to welfare improvement after the reform. The labor wedge margin cancels out the negative welfare effect of firing cost reform and contributes to the welfare improvement after the full reform in column (3).

The third column of Table 4.2 shows that the welfare effect of the Hartz reform is equivalent to a permanent 0.3% increase in the consumption of the representative household in the pre-reform BGP. Entrant innovation is the most important factor driving the welfare change because the increases in labor supply are primarily used for entrant innovation and then for production. Increases in entrants' creative destruction crowded out own-product improvements.

Table 4.2: Effect of the Hartz Reform over the Transition Path

Moment	(1) Firing Cost Reform	(2) Labor Wedge Reform	(3) Full Reform
Initial Output Change	-6.86%	0.17%	0.82%
Consumption Equivalent Variation	-0.724%	0.911%	0.299%
Own-Product Improvement	-0.839%	0.001%	-0.732%
Product Expansion	-0.006%	-0.000%	-0.006%
Entrant Innovation	0.461%	0.233%	0.656%
Production Labor	-0.340%	0.719%	0.381%

Notes: This table reports changes in output and welfare along the transition path relative to a pre-reform BGP for labor supply elasticity $\eta = 1$. The first panel summarizes the output path; the second panel decomposes the contributions of innovation and production labor to welfare. See Online Appendix 2.2 for further details on the derivations. Labor supply elasticity $\eta = 1$.

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