Entering Recovery[⋆]

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Abstract

The entry of new firms and the introduction of new products are both pro-cyclical. Which of the two contributes most to business cycle fluctuations? I develop a dynamic stochastic general equilibrium model with endogenous entry that allows heterogeneous multi-product incumbent firms to invest in R&D to affect their probability of introducing a new product in the future. Firms are subject to aggregate productivity shocks that drive business cycles. I simulate the economy's recovery from a recession and compare the relative importance of entry to incumbent firms' responses. I find that potential entrants' responses account for the overwhelming majority (more than 90%) of the path of output during the recovery. Hence, a policymaker that wants to speed up the recovery from a recession might be advised to focus on encouraging new entry.

Key words: R&D, Economic Recovery, Product Innovation, Entry

JEL Codes: E30, O30, E22

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1. Introduction

The entry of new firms and the introduction of new products are strongly pro-cyclical. There are substantially more start-ups (in terms of absolute numbers, the proportion of existing firms, and their employment) when GDP is higher.[1] Likewise, more new products are introduced during economic upturns and booms compared to downturns and recessions, and these new products can account for up to 25% of output each year and nearly 50% of output over a five-year period.[2] [3] However, it is not clear which of these two factors, the entry of new firms or the introduction of new products, is more important in terms of driving (or responding to) business cycle fluctuations, particularly given that incumbent firms are responsible for the overwhelming majority of newly-introduced products.[4]

I examine the extent to which both factors affect fluctuations in (real) output over the business cycle, and recoveries from recessions, using a dynamic stochastic general equilibrium model of heterogeneous firms. In my model, multi-product intermediate goods firms can choose to invest in R&D to affect their probability of innovating a new product to add to their current set of products. Hence, incumbent intermediate goods firms are heterogeneous in terms of their current number of products. Potential entrants may choose to invest in R&D to increase their chances of developing a product and entering production next period. Intermediate goods firms are subject to exogenous fluctuations in aggregate output productivity, which drives the business cycle. I use this model to simulate an economy's recovery from a recession induced by this aggregate output productivity being temporarily reduced.

I find that entrants' decisions alone explain more than 90% of the path of output over the recovery from such a recession. When entrants can freely choose whether or not to

[1] See, for example, Sedlacek (2019) that finds a correlation of as much as 0.66 between the number of new firms and real GDP. Similarly, Sedlacek and Sterk (2017) finds a correlation of roughly 0.5 between employment by start-ups and real GDP.

[2] Broda and Weinstein (2010) finds that the creation of new products dropped during the 2001 recession and increased markedly during the recovery in 2002 and 2003. Similarly, Granja and Moreira (2021) finds that the Financial Crisis resulted in a large drop in the introduction of new products that only returned to its long-run level once the economy had started to recover.

[3] Granja and Moreira (2021) find that 33% of a firm's sales are obtained from new products, and Bernard, Redding and Schott (2010) find that firms that add new products account for roughly 80% of output. Hence, new products account for roughly 33%*80% $\approx$ 25% of output in each year.

[4] Broda and Weinstein (2010) find that incumbents account for 92% of the introduction of new products, with the remaining 8% accounted for by entrants.

attempt entry, and their initial size upon entering, lower aggregate TFP reduces potential entrants' incentive and ability to enter (through a lower value to entry and a reduction in the amount of money available for potential entrants to finance their entry). This reduces the number of firms that produce (as well as products and capital), such that output and other aggregate variables decrease relative to the steady-state. Only when aggregate TFP has increased sufficiently do potential entrants regain the incentive and ability to enter and output starts to increase. If, instead, entrants' decisions are fixed at the steady-state level, their incentive and ability to enter do not change over time, such that the number of products does not reach as low a trough and output recovers more quickly. Specifically, the response of entrants over the recovery lengthens the recovery by roughly ten years.

Conversely, I find that incumbents' R&D responses during the recession only account for about 20% of the path of output.[5] This is because incumbents generally do not spend a large proportion of their output on investing in R&D and it would take a large drop in aggregate productivity to substantially affect their ability and incentive to invest in R&D. As incumbents' R&D investments do not change substantially over time, then fixing those R&D investments at the steady-state level does not affect the path of the recovery substantially. My results imply that a policymaker caring about the speed of the recovery from a recession might wish to focus on encouraging more new entry rather than incentivise higher incumbent R&D.

I also find that there is substantial heterogeneity in the response of firms: firms' responses differ according to how many products they own. In particular, smaller firms (i.e. those with fewer products) respond much more to the drop in aggregate productivity in terms of their R&D investment, product introductions, and output. For example, the amount of R&D investment by the bottom quintile of firms in terms of number of products drops by three times as much as R&D investment by firms in the top quintile of products during the recession, with the result that smaller firms' output decreases by twice as much as that of larger firms. This reflects the facts that larger firms, through having a larger number of products and already making relatively large dividends, are better able to insulate themselves from the fall in aggregate productivity as well as benefit from the

[5] The contribution of each of entry and incumbent R&D responses is calculated by comparing the effect of turning off both entry and incumbent R&D responses against when each is only turned off by itself. As such, the "total" contribution of entry and incumbent R&D responses sums to more than 100% because the combined endogenous responses of entrants and incumbents ameliorates the depth and amount of time the recovery takes. This is discussed in more detail in Section 5.3.

increase in price of intermediate products (as the number of products and total output falls). Moreover, among incumbents, the larger firms contribute the most to output and the introduction of new products, such that their small reduction in aggregate R&D is the main driver of the path of aggregate R&D investment, such that fixing incumbents' decisions only has a relatively small effect on the path of the economy.

My paper contributes to three main strands of the literature. First, I contribute to the literature regarding the effect of the introduction of new products on business cycles. The closest work to mine in this area is Bilbiie, Ghironi and Melitz (2012), which uses a model of single-product firms and endogenous entry to examine the contributions of product creation (through the entry of new firms) and destruction to output over the business cycle. However, in their model, entry and the creation of new products are the same: there is no product creation by incumbent firms. My introduction of heterogeneity across firms means that I can also incorporate innovation by incumbent firms, thereby enabling the comparison between the relative importance of new entry versus innovation by incumbents that is not possible otherwise.

The relevance of multi-product firms is emphasised by empirical work by Bernard, Redding and Schott (2010) who find that almost 40% of firms own and produce more than one product, as well as by Hottman, Redding and Weinstein (2016)'s finding that some firms can have as many as one hundred different products. I also make use of the evidence reported by Bernard, Redding and Schott (2010) to target a novel set of moments related to the distribution of products over firms and the types of firms that introduce new products and find that, contrary to Bilbiie, Ghironi and Melitz (2012), it is new entry in and of itself, rather than the introduction of new products, that drives this channel of business cycles.

Dekle, Jeong and Kiyotaki (2014) and Dekle et al. (2021) integrate the importance of multi-product firms in the business cycle by using models and regressions involving multi-product firms to investigate the links between aggregate exports / imports and changes in real exchange rates for Japan. These papers assume that firms can innovate a new product by paying a fixed cost to gain a fixed probability of getting a new product. Similarly, Garcia-Macia, Hsieh and Klenow (2019) assume that a firm's introduction of new products is determined by an exogenous probability, while Minniti and Turino (2013) assume that firms have a free choice over the number of products they can offer without needing to undertake costly R&D in order to innovate and introduce a new

product. In contrast, firms in my model can alter their probability of innovating a new product by changing how much they spend on R&D (rather than just choosing to pay a fixed fee or not), providing both an extensive and intensive margin for firms to alter their R&D spending.

The second contribution of my paper is to the literature regarding the importance of entrants in affecting the business cycle. Both Sedlacek and Sterk (2017) and Sterk, Sedlacek and Pugsley (2021) show that new (and young) firms are crucial contributors to the performance of the economy over the business cycle. In particular, Sedlacek and Sterk (2017) show that there is a positive correlation between growth in output and the total employment accounted for by entrants, while Sterk, Sedlacek and Pugsley (2021) shows that (ex-ante) heterogeneity across entrants (in terms of their productivity) has large aggregate effects. Each paper models incumbent firms as producing only a single-product. By incorporating multi-product firms in my model, I show that entry still plays an important role in affecting output over the business cycle even when incumbents are able to innovate as well.

Similar to Sedlacek (2019)'s finding that the decline in firm entry during the Great Recession lengthened the time taken for the economy to recover in a model of single-product firms subject to frictional labour markets, I find that the recovery from recovery from a recession caused by a drop in aggregate productivity is also increased. However, I find that the cause of this increase in recovery time is not due solely to the presence of fewer entrants but also due to the fact this induces incumbent firms to reduce their R&D and introduction of new products further than they would if entry was fixed. In addition, the contribution of the change in the number and distribution of entrants to the drop in output found in my model is substantially more than that found by Ayres and Raveendranathan (2023) who investigate the importance of entry in a model of firms subject to financial frictions in the form of a collateral constraint (they find that changes in entry, and exit, account for roughly 20% of the fall in output in their model, as compared to the over 90% contribution of entry in my model).

My paper also contributes to the literature regarding the heterogeneous responses of firms over the business cycle. Khan and Thomas (2013) looks at the impact of some firms being financially constrained compared to other firms and the effect that has on recoveries from recessions, while Jo (2021) incorporates the impact of the US firm-size distribution within this framework. I validate my model by showing that it can get

close to the Pareto shape of the firm-size distribution despite using only (log) normal idiosyncratic productivity shocks (Jo (2021) matches the firm-size distribution using Pareto productivity shocks). In addition, Moreira (2016) (as well as the aforementioned Sedlacek and Sterk (2017)) shows that heterogeneity in the timing of when firms entered (particularly in terms of whether a firm entered during a boom or a recession) can affect firms' responses to business cycle fluctuations. I show that firm heterogeneity in terms of the number of products they own also matters for the responses of firms, and the aggregate economy, during business cycle fluctuations.[6]

The rest of this paper proceeds as follows. Section 2 presents some stylised facts and indicative evidence regarding the pro-cyclicality and potential relative importance of both the innovation of new products and the entry of new firms. Section 3 then presents my model of multi-product firms with endogenous entry, while Section 4 describes how I calibrate and solve the model non-linearly. Section 5 contains the results of a simulated recession and recovery, first examining the aggregate effects and differences between a model with an endogenous response by entrants and one without that response, and then investigating the heterogeneous response among firms. Section 6 concludes and briefly describes some potential avenues for future work.

2. New products and new entry are pro-cyclical

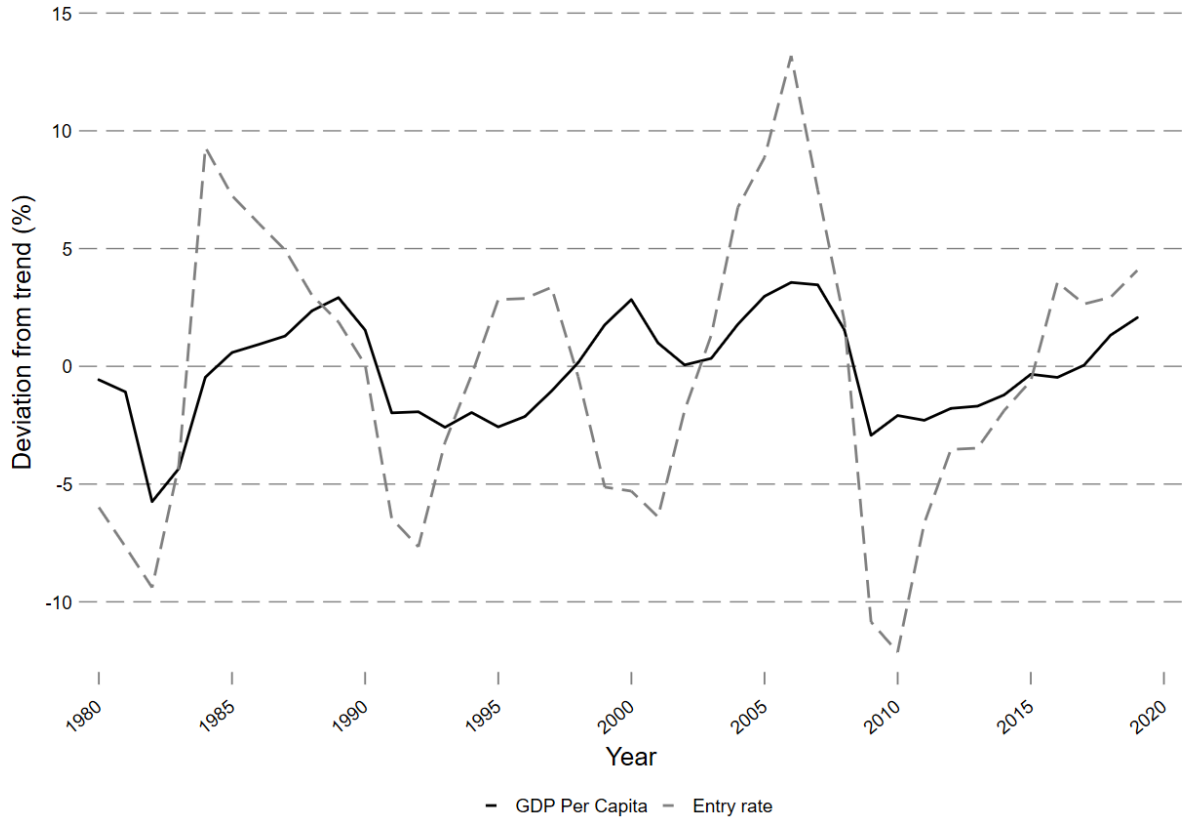
Both the entry rate of new firms, and the introduction of new products is strongly pro-cyclical. Figure 1 shows the (de-trended) paths of GDP per capita (the solid black line) and the entry rate (the dashed grey line) for the US since 1978. The cyclical component of the entry rate closely follows the cyclical component of GDP per capita, with a correlation of 0.62.[7]

Similarly, the introduction of new products is pro-cyclical. Figure 2 reproduces Figure 1B from Broda and Weinstein (2010) and shows the path of the creation rate of new products (the solid grey line) and the path the growth rate of total sales (the dashed black line)

[6] Argente, Lee and Moreira (2018) incorporates this heterogeneity within a model of endogenous growth to examine the effect of the introduction of new products on growth, whereas I focus on the effect of this heterogeneity within the context of business cycle fluctuations.

[7] The (raw) number of entrants is also strongly pro-cyclical, with a correlation between the cyclical components of the number of entrants and GDP of 0.64.

Figure 1: The entry rate is pro-cyclical



Note: The solid black line shows GDP per capita in the USA, while the dashed grey line shows the rate of entry of new firms (calculated as firms of age zero divided by the total number of firms, both according to BDS data). Both GDP per capita and the entry rate have been detrended using a Hodrick-Prescott filter with a smoothing parameter of 400.

for all products covered by AC Neilsen's Homescan data during the period 2000-2003. There is a strong co-movement between both lines, indicating that the introduction of new products closely follows the total sales of all products (with the latter being a strong indicator of aggregate output).

Finally, R&D more generally is also pro-cyclical. Although total private R&D also includes expenditure on areas unrelated to the development and introduction of new products (such as the research and development of new process to improve productivity), the path of total R&D over the business cycle can still provide some information. In particular, as R&D is required in order to develop new products R&D spending is itself potentially correlated with the introduction of new products.[8] As such, Figure 3 shows

[8] Although it is possible that all cyclicality in R&D spending could come through changes in R&D

Figure 2: The introduction of new products is pro-cyclical

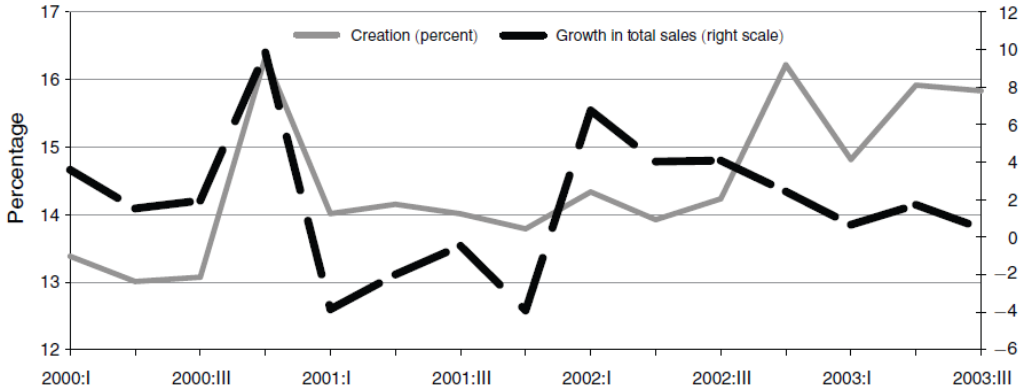


FIGURE 1B. SALES GROWTH AND CREATION (*Four-quarter growth rates*)

Note: Taken from Broda and Weinstein (2010). The solid grey line shows the percentage sales attributed to newly-created products in a particular quarter, while the dashed black line shows the growth in total sales for products covered by AC Neilsen's Homescan data.

the (de-trended) paths of GDP per capita (the solid black line) and total private R&D spending (the dashed grey line) for the US since 1980.

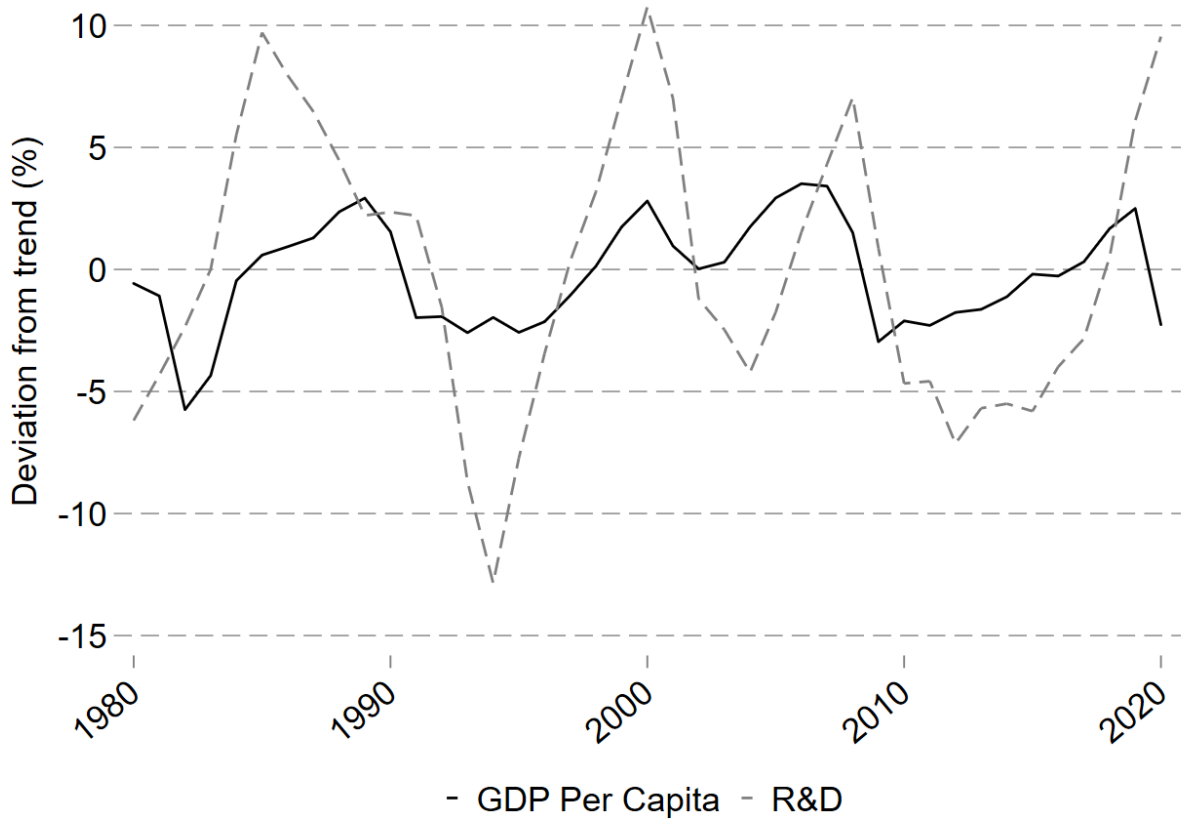
The cyclical component of private R&D spending is pro-cyclical; its correlation with GDP is 0.36. This is slightly less than the correlation between GDP and the entry rate (or new firms), which hints at R&D perhaps being less important than the entry of new firms at determining business cycles. However, as there are many other factors that could affect these simple correlations it is necessary to develop a model to isolate the true effect of both entry and the introduction of new products on business cycles.

3. Model

In this Section I set out a discrete-time dynamic stochastic general equilibrium model and characterise agents' optimisation problems. There are three categories of agents in my model: an infinitely-lived representative household, a single perfectly competitive final goods producer, and a mass of monopolistically-competitive heterogeneous multi-product intermediate goods producers. The representative household has preferences

spending unrelated to new products, this seems unlikely given the aforementioned pro-cyclicality of the introduction of new products.

Figure 3: Private R&D spending is pro-cyclical



Note: The solid black line shows GDP per capita in the USA, while the dashed grey line shows total private R&D spending (obtained from the NSF). Both GDP per capita and total private R&D spending have been detrended using a Hodrick-Prescott filter with a smoothing parameter of 400.

over a single consumption good C and leisure $1 - L$ and owns the firms (so receives all profits).[9]

The final goods producer takes the output of each product from the intermediate goods producers and aggregates them into the single consumption good using a CES aggregator with an elasticity of substitution between each good of θ. The price it pays for each intermediate good is p.

Each multi-product intermediate goods producer is characterised by its (endogenous) measure of products N that only it can produce, its (endogenous) stock of capital

[9] Households also have access to a full set of state-contingent claims, but as the household is representative in my model, these are in zero net supply in equilibrium. As such, I do not explicitly model these claims here for convenience.

k, its persistent idiosyncratic (exogenous) research productivity ε_r, and its persistent idiosyncratic (exogenous) output productivity ε_o, forming a distribution μ over these four characteristics. Each persistent idiosyncratic shock is an independent continuous process that follows an AR(1) process with a zero mean (in logs), persistence ρ_o and a disturbance standard error of σ_o for the output shock, and persistence ρ_r and a disturbance standard error of σ_r for the research productivity shock. The amount of each intermediate good that is produced depends on the stock of physical capital and idiosyncratic output productivity of the intermediate goods firms that owns that product. As shown in Section 3.2, this means that the price p of a particular intermediate good is a function of the owning intermediate goods firm's capital and output productivity.

Each period, incumbent firms face a probability π_d of exogenous exit and produce output y of each product using labour ℓ in combination with its capital stock. Firms that did not receive the exogenous exit shock choose their capital stock for the next period k', and how much R&D investment R to make in order to gain the possibility of innovating a new product. This probability of innovation is a function $\Phi(R)$ (which is increasing in R and bounded between 0 and 1).[10] Note that a successful innovation only affects a firm's stock of products; it does not affect a firm's idiosyncratic output or research productivities (which are exogenous).

An incumbent firm's stock of products depreciates exogenously at rate δ_N each period. This reflects the lifecycle of individual products, whereby a new product makes most of its revenues (and profits) for a firm in the first few years of its introduction.[11] The depreciation rate of physical capital is δ_k.

There is also a unit mass of potential intermediate goods entrants that differ according to their cash endowment m_e as well as their research and output productivities that can choose to pay a fixed cost of entry ξ_e and invest in their starting stock of physical capital as well as the possibility of innovating a new product and starting to produce next period.

[10] This varying probability of innovating a new product is what necessitates my use of a heterogeneous firm framework. Even absent any idiosyncratic output and research productivity across firms, each period some firms would be successful at innovating a new product and some firms would not be successful. This naturally creates a distribution of firms across the number of products they own, thereby creating firms that are heterogeneous.

[11] See, for example, Argente, Lee and Moreira (2021), which shows that new products lose about 50% of their sales (on average) within two or three years of their introduction, with products only making about 30% of their initial sales as little as six years after their introduction.

If an incumbent firm, or potential entrant, wishes to make an investment in R&D, it must pay a fixed cost ξ_r before doing so.[12] Finally, firms are subject to aggregate uncertainty in the form of an aggregate productivity shock z that varies over time (transition probabilities given by π_z) and affects the productivity of intermediate goods producers and the cash endowment available to potential entrants.

3.1. Households

The infinitely-lived representative household has preferences over the single consumption good C and leisure $1 - L$ (with L representing the household's labour supply), with a subjective rate of time preference β. It earns wages $w(z, \mu)$ for each unit of labour it supplies and owns shares (represented by the vector λ) in the firms.

I denote the ex-dividend price of shares in a firm with products N, capital stock k, output productivity ε_o, and research productivity ε_r be $\rho_x(N', k', \varepsilon_o', \varepsilon_r'; z', \mu)$; and the share price including dividend be $\rho_i(N, k, \varepsilon_o, \varepsilon_r; z, \mu)$, households maximise lifetime discounted utility via their choice of consumption, labour, bond, and share-holdings. In other words, households maximise Equation (1) subject to the household budget constraint Equation (2)

$$V^h(\lambda; z, \mu) = \max_{C, L, \lambda'} U(C, 1 - L) + \beta E_{\pi_z} V^h(\lambda'; z', \mu') \tag{1}$$

$$\begin{aligned} C + \int \rho_x(N', k', \varepsilon_o', \varepsilon_r'; z', \mu) \lambda'(d[N' \times \varepsilon_r')] \\ = w(z, \mu) L + \int \rho_i(N, k, \varepsilon_o, \varepsilon_r, \mu) \lambda(d[N \times \varepsilon_r) \end{aligned} \tag{2}$$

This household problem is very similar to that in Khan and Thomas (2013), albeit adapted for the environment of multi-product firms. Hence, given the dividend-inclusive value of

[12] Hence, both the entry and R&D costs in this model are in terms of units of the consumption good. This follows the likes of Akcigit and Kerr (2018),Clementi and Palazzo (2016), Dekle et al. (2021), Hamano and Zanetti (2017), Klette and Kortum (2004), and Minniti and Turino (2013) in this area. An alternative formulation would scale the entry and R&D costs by the prevailing real wage (an approach taken by, for example, Aghion and Howitt (1992) andAyres and Raveendranathan (2023)). The impact of this modelling choice are outside the scope of this paper.

their current shares and the real wage, households determine their current consumption hours worked, and the number of new shares priced ex-divided. In equilibrium, the wage w is determined by the household's marginal rates of substitution between consumption and leisure such that $w(z,\mu) = \frac{D_2U(C,1-L)}{D_1U(C,1-L)}$. Finally, the household's share optimisation problem establishes the firm's stochastic discount factor due to the fact that household owns all firms. Hence, following Khan and Thomas (2013), I can re-write the firm problem to incorporate that stochastic factor in a more parsimonious way. In other words the firm problem is written with any current period profit scaled by $q(z,\mu) = D_1U(C,1-L)$.

I use Rogerson-Hansen preferences such that $U(C,1-L) = \log C + B(1-L)$ where B is the household's disutility of working.

3.2. Final goods producers

The perfectly competitive final goods producer uses a CES aggregator to produce the single aggregate output good $Y(z,\mu)$ according to equation (3).

$$Y(z,\mu) = \left[\int^{\bar{N}} y(k,\varepsilon_o,z,\mu)^{\frac{\theta-1}{\theta}} dn\right]^{\frac{\theta}{\theta-1}} \tag{3}$$

It maximises profits by choosing how much output $y(k,\varepsilon_o,z,\mu)$ of each of the N intermediate products produced by an intermediate goods firm with capital stock k and idiosyncratic output productivity ε_o to purchase at price $p(k,\varepsilon_o,z,\mu)$ in order to make aggregate output $Y(z,\mu)$, with $\bar{N}$ representing the total number of products in the economy.[13] In other words it solves equation (4) subject to equation (3).

$$\max_{Y,y} P(z,\mu)Y(z,\mu) - \int^{\bar{N}} p(k,\varepsilon_o,z,\mu)y(k,\varepsilon_o,z,\mu) \tag{4}$$

[13] N is the measure of products owned by an individual firm while $\bar{N}$ is the total across the entire economy. In other words, $\bar{N}$ is obtained by summing up N over all firms in the economy. Hence, summing over all products present in the economy is the equivalent to first summing the number of products each firm has and then adding up the number of firms weighted by their number of products.

Solving this maximisation problem and normalising the price $P(z, \mu)$ of the single aggregate output good to 1 means that the price of each intermediate good for a firm with capital stock k and idiosyncratic output productivity ε_o is given by equation (5).

$$p(k, \varepsilon_o, z, \mu) = \left(\frac{Y(z, \mu)}{y(k, \varepsilon_o, z, \mu)} \right)^{\frac{1}{\theta}} \tag{5}$$

This price of an intermediate good depends on both the output of the intermediate good and the level of aggregate output, but this price will itself affect how much of each individual product an intermediate goods firms produces. In other words, there is a circular relationship between aggregate output and the output of an intermediate product. Hence, firms need to forecast current aggregate output when deciding how much to produce within a particular period, requiring a second intra-period forecasting rule alongside the forecasting rule for consumption when solving the model (this is discussed in more detail in Section 4.2.

Note that the fact that the idiosyncratic output productivity ε_o and the stock of k differs over firms means that the price of intermediate goods p will differ across firms, but the proportional effect of changes in aggregate output Y and z will be the same across all firms (such that changes in z will result in the same deviation of the price per product from its steady-state value across all firms). In other words, different values of ε_o and k simply shift the level of the price per product and do not affect its response to changes in z and Y, with firms that have less capital or a lower idiosyncratic output productivity having a higher price for their product.

3.3. Intermediate goods producers

Each incumbent intermediate goods producer learns whether or not they will be affected by the exogenous death shock at the start of the period, at the same time as they learn their persistent idiosyncratic research and output productivities (with high values of research productivity meaning that the same amount of investment in R&D can be made more cheaply for those firms than it can for firms with low research productivity). They also learn the level of the aggregate productivity shock at this time.

After these shocks are observed each period, the intermediate goods firms hire labour, and produce output of each product they own via a decreasing returns to scale Cobb-Douglas production function with weight γ on labour and α on capital. A firm is assumed to use all of its capital stock in the production of each of the products it owns; in other words, the firm does not split its capital across products but uses all of it producing each one at the same time (hence, the capital stock per product and a firm's aggregate capital stock are both represented by just k). The profit for each individual intermediate good is given by $\pi(k, \varepsilon_o, z, \mu) = \max_{\ell,p} p(k, \varepsilon_o, z, \mu) y(k, \varepsilon_o, z, \mu) - w(z, \mu)\ell$ subject to $p(k, \varepsilon_o, z, \mu) = \left(\frac{Y(z,\mu)}{y(k,\varepsilon_o,z,\mu)}\right)^{\frac{1}{\theta}}$ where $y(k, \varepsilon_o, z, \mu) = z\varepsilon_o k^{\alpha} \ell^{\gamma}$ is the output of that individual product and $p(k, \varepsilon_o, z, \mu)$ is given by equation (5).[14] Note that each product owned by a firm is produced independently of any other product owned by a firm such that the production function of an individual product does not depend on the number of other products a firm owns.[15]

After production has occurred, those firms that did not receive the exogenous death shock choose how much to invest in physical capital and in R&D to affect the possibility of innovating a new product. Firms (including those that received the exogenous death shock) then pay out any profits as dividends to the household. Each period there is also a unit mass of potential entrants that can choose to pay a fixed cost of entry ξ_e and then how much to invest in the possibility of developing a new product and being able to produce it next period. Therefore, the timing within a period is as follows:

1. Incumbents find out their idiosyncratic research and output productivities for the current period, and whether or not they are subject to the exogenous death shock. Potential entrants discover their idiosyncratic cash endowment as well as their research and output productivities.
2. All incumbents hire labour and produce output
3. Incumbents continuing onto the next period, and potential entrants that decide to pay the fixed entry cost, choose how much to invest physical capital and in the possibility of innovating a new product. Firms that are subject to the exogenous exit shock sell off all of their undepreciated capital. All incumbent firms then pay any dividends to the household.

[14] This means that a firms revenue (in terms of its inputs) is $Y(z,\mu)^{\frac{1}{\theta}} (z\varepsilon_o)^{\frac{\theta-1}{\theta}} k^{\alpha\frac{\theta-1}{\theta}} l^{\gamma\frac{\theta-1}{\theta}}$ such that a firm's returns to scale with respect to inputs is given by $\alpha\frac{\theta-1}{\theta} + \gamma\frac{\theta-1}{\theta}$.

[15] Moreover, since a firm's research productivity only affects the effectiveness of its R&D in developing new products, a firm's research productivity has no effect on the output of products that already exist.

3.3.1. Incumbent firms

Let W represent the value of an incumbent firm at the start of the period and W^1 be the value of a firm that does not receive the exogenous death shock. The incumbent firm's optimisation problem can be defined recursively as

$$\begin{aligned} W(N,k,\varepsilon_o,\varepsilon_r;z,\mu) = \pi_d \left[\max_{\ell,p} q(z,\mu)[N\pi(k,\varepsilon_o,z,\mu)]\right] \\ + (1-\pi_d)W^1(N,k,\varepsilon_o,\varepsilon_r;z,\mu) \end{aligned} \tag{6}$$

Equation (6) states that the firm takes the probability of an exogenous death into account. If the firm receives the exogenous death shock, it chooses its labour to maximise profits across all of its products and pays out dividends to the household.[16] If the firm does not receive the exogenous death shock, then it hires labour to maximise output, then chooses its capital stock next period, R&D stock next period, and debt next period to maximise W^1, which is given below:

$$\begin{aligned} W^1(N,k,\varepsilon_o,\varepsilon_r;z,\mu) = \max_{\ell,p,R,k'} q(z,\mu)[N\pi(k,\varepsilon_o,z,\mu) + (1-\delta_k)k - \frac{R}{\varepsilon_r} - \mathbb{I}_r\xi_r - k'] \\ + \beta\Phi(R)E_{\pi_z}E_{\pi_{\varepsilon_r}}E_{\pi_{\varepsilon_o}}W((1-\delta_N)N+1,k',\varepsilon_o',\varepsilon_r';z',\mu') \\ + \beta(1-\Phi(R))E_{\pi_z}E_{\pi_{\varepsilon_r}}E_{\pi_{\varepsilon_o}}W((1-\delta_N)N,k',\varepsilon_o',\varepsilon_r';z',\mu') \end{aligned} \tag{7}$$

subject to

$$\begin{aligned} R &\geq 0 \\ 0 &\leq N\pi(k,\varepsilon_o,z,\mu)_i - \frac{R}{\varepsilon_r} - \mathbb{I}_r\xi_r - k' + (1-\delta_k)k \\ p(k,\varepsilon_o,z,\mu) &= \left(\frac{Y(z,\mu)}{y(k,\varepsilon_o,z,\mu)}\right)^{\frac{1}{\theta}} \\ \mu' &= \Gamma(\mu) \end{aligned} \tag{8}$$

[16] Note that a firm makes a profit of $\pi(k,\varepsilon_o,z,\mu)$ and has N products, such that its total operating profits from all of its products is simply $N\pi(k,\varepsilon_o,z,\mu)$. This means that a firm is indifferent between making a profit on one of its products compared to another one of its products. In other words, there is no cannibalisation of sales (or operating profits) within a firm's own portfolio of goods beyond that which occurs via the aggregate output CES function.

where $\Phi(R)$ is the probability of innovating a new product given the firm's choice of investment in R&D (R) such that the firm would have $(1-\delta_N)N+1$ products next period if its R&D investments are successful.[17] Hence, $1-\Phi(R)$ therefore represents the probability that the firm does not make a successful innovation so would have only $(1-\delta_N)N$ products next period. In this way a firm's choice of R&D investment R affects the number of products that it will have next period, such that this number of products is an endogenous individual state variable. $\mathbb{I}_r$ is an indicator function taking the value zero when a firm makes zero investment in R&D and a value of one when it makes a positive R&D investment; hence $\mathbb{I}_r\xi_r$ represents the fixed cost of being able to carry out some non-zero amount of R&D investment.

I use a logit function for this probability given an investment in R&D. In other words, $\Phi(R) = \frac{e^{\chi R}}{1+e^{\chi R}}$ where the parameter χ will be calibrated to match target moments. The firm must choose its stock of capital for the next period k' today, without knowing whether it will be successful in innovating a new product. As such, there is a close interplay between a firm's decision to invest in R&D and physical capital. Note that the constraints mean a firm cannot make negative investments in R&D and it cannot equity finance any R&D or capital investments (i.e. it must always pay non-negative dividends). Finally, $\mu' = \Gamma(\mu)$ represents the law of motion of the distribution of firms.

3.3.2. Potential entrants

There is also a mass M of potential entrants each period, each of which is endowed with an idiosyncratic level of cash m_e drawn from a distribution H.[18] The cash endowment allows me to capture the fact that some potential entrants are more financially constrained than are others, such that some cannot afford to enter even if the value from doing so is greater than the value of not doing so. This cash endowment also depends on aggregate productivity such that the draw from the distribution H is multiplied by the level of z in a particular period for all potential entrants. This fluctuation of the (mean of the)

[17] The assumption that a firm can only add one product (at most) per year allows me to capture Luttmer (2011)'s finding that small firms grow much more quickly than do large firms: in my model a one-product firm adding one product doubles their stock of product (and, hence, increases their sales and employment by a lot) whereas a, say, five-product firm adding one product increases their number of products by 20% and so increases their sales and employment by much less relatively than does the smaller firm.

[18] The parameters of the distribution are chosen so as to target the relative importance of new entrants in the aggregate economy. This is discussed in more detail in Section 4.1.

distribution over the business cycles reflects the fact that potential entrants' financial constraints are lower during booms and tighter during recessions (and also allows me to obtain an entry rate that is pro-cyclical).[19] [20]

Each potential entrant also draws their persistent idiosyncratic research and output productivities (each is drawn from the ergodic distributions of the respective shock). If the potential entrant chooses to try to enter, they must fund the fixed cost of entry ξ_e, along with their choice of R&D investment R and physical capital investment k' from their cash endowment. Any of the cash endowment that is not used is returned lump-sum to households. If the potential entrant's innovation is successful then they will start next period with one product (i.e. $N' = 1$ for successful potential entrants).

Hence, the value of entry W^E is given by

$$W^E(m_e, \varepsilon_o, \varepsilon_r; z, \mu) = \max_{R,k'} \beta\Phi(R) E_{\pi_z} E_{\pi_{\varepsilon_r}} W(1, k', \varepsilon_o', \varepsilon_r'; z', \mu') \tag{9}$$

subject to

$$\begin{aligned} R &\geq 0 \\ zm_e &\geq \frac{R}{\varepsilon_r} + \xi_e + k' + \mathbb{I}_r \xi_r \\ \mu' &= \Gamma(\mu) \end{aligned} \tag{10}$$

with potential entrants only entering if $W^E > 0$. The right-hand side of this expression is zero rather than the value of the potential entrant's idiosyncratic cash endowment due to any unused cash endowment being returned to households (such that if the potential entrant does not try to enter, then it simply returns its entire cash endowment to the household).

[19] For more detail on the impact of various constraints, including financial ones, on potential entrants see, for example, Sedlacek and Sterk (2017) and Sterk, Sedlacek and Pugsley (2021).

[20] Without the distribution fluctuating over the business cycle the model would not obtain sufficient variation in the entry rate over the business cycle to get close to the data. Given the importance of capturing the real-life contribution of entry to the business cycle, by letting the distribution of potential entrants' cash endowments to vary over the business cycle the model can more closely reflect the correlation between the number of firms and GDP .

Potential entrants have the same form of $\Phi(R)$ as do incumbent firms. Note that potential entrants must also pay the fixed cost of conducting some R&D if they wish to make a non-zero investment in it. This maximisation problem means that there are some entrants that pay this fixed cost, alongside the fixed cost of entry and invest in physical capital (and R&D) but do not actually develop a product and enter next period as the probability function $\Phi(R)$ is such that they are not guaranteed to enter and start producing next period. This reflects the reality that a number of potential firms invest in start-up costs but never end up producing; one such (infamous) example of a firm in this category would be Theranos.

3.4. Competitive equilibrium

Let $\mathbb{I}^O$ represent those firms in the distribution that are continuing incumbent firms and $\mathbb{I}^{\mathbb{N}}$ denote potential entrants that decide to enter, a Recursive Competitive Equilibrium is a set of prices p, w, q, ρ_i, ρ_x, and functions for quantities ℓ, R, k', Y, C, L, λ', R^E, k^E and values V^h, W, W^1, W^E that solve the firm and household problems and clear markets as described below:

1. W and W^1solve Equations (6)and (7), and ℓ, R, and k' are the associated policy functions for incumbent firms
2. V^h solves Equation (1) with C, L, λ being the household's associated policy functions
3. W^Esolves Equation (9), with $R^E$and k^E as the associated policy functions for potential entrants
4. The distribution of firms over $N \times k \times \varepsilon_o \times \varepsilon_r$ is given by μ and evolves according to $\mu' = \Gamma(\mu)$
5. Aggregate output is given by $Y = \left[\int^{\bar{N}} y^{\frac{\theta-1}{\theta}} dn\right]^{\frac{\theta}{\theta-1}}$
6. The labour market clears $L = \int N\ell(N, k, \varepsilon_o, \varepsilon_r; z, \mu)\mu(d[N \times k \times \varepsilon_o \times \varepsilon_r)$
7. The goods market clears $C = Y - \int \Bigg(\mathbb{I}^O[\frac{R}{\varepsilon_r} + \mathbb{I}_r\xi_r + k'] + \mathbb{I}^{\mathbb{N}}[\frac{R^N}{\varepsilon_r} + \xi_e + \mathbb{I}_r\xi_r + k^N] \Bigg)\mu(d[N \times k \times \varepsilon_o \times \varepsilon_r]) + \int \Big((1-\delta_k)k \Big)\mu(d[N \times k \times \varepsilon_o \times \varepsilon_r])$
8. Prices ρ_i and ρ_x clear the market for shares

where N is the measure of products owned by an individual firm while $\bar{N}$ is the total across the entire economy ($\bar{N}$ is obtained by summing up N over all firms in the economy such that summing over all products present in the economy is the equivalent to first summing the number of products each firm has and then adding up the number of firms weighted by their number of products). Note that potential entrants' idiosyncratic cash endowment does not appear in the aggregate resource constraint because any of it that is used is represented by entrants' total spending of $\frac{R^N}{\varepsilon_r} + \xi_e + \mathbb{I}_r \xi_r + k^N$ and any of it that is unused is returned to households.

4. Quantitative strategy

In this Section I describe how I solve my model and the calibration of its parameters. There are 16 internally calibrated parameters chosen to match 18 target moments, such that the model is over-identified. Among these targets are a number of novel moments taken from the literature regarding the distribution of products over firms and the introduction of new products by incumbents and entrants. There are also two model parameters that are set externally.

4.1. Steady-state

In order to solve the (non-stochastic) steady-state of the model, I iterate on guesses of aggregate consumption (which affects the wage) and output (which affects the price intermediate goods firms charge for their output) until both converge. Conditional on the guesses of consumption and output, I first find the optimum choice of capital given a choice of R&D investment assuming that a firm can afford this optimum level. I then find the optimum choice of R&D assuming that a firm is able to afford both the optimum choice of R&D and the implied optimum choice of physical capital. These optimum choices depend just on a firm's starting number of goods and their idiosyncratic research and output productivities. In other words, the unconstrained optimum levels of R&D and physical capital are independent of a firm's starting level of physical capital.

Next, I see if the firm can actually afford these unconstrained optimum choices without violating the condition that they must pay non-negative dividends. If a firm can afford these unconstrained optimums then that is what they choose. If choosing the

uncosntrained optimums would imply the firm paying negative dividends (i.e. having to equity finance) then I re-solve the firm's optimum choice of R&D and physical capital. Specifically, I find a constrained firm's optimum choice of R&D ensuring that such a firm's choice of physical capital is given by $k' = N\pi(z,\mu)_i - \frac{R}{\varepsilon_r} - \mathbb{I}_r\xi_r + (1-\delta_k)k$. Using these optimum choices of R&D and physical capital, alongside the optimum decisions of entrants, I iterate on the distribution of firms until it converges and then find the resulting new values of consumption and output, which are used to update the original guesses of consumption and output using Broyden's algorithm until convergence. More details regarding this solution algorithm are presented in Appendix A.1.

Table 1 shows the model and targeted steady-state moments that relate to aspects of the model other than the distribution of firms over products. Although the parameters are all determined jointly, the rows of the table show what parameters are most-closely associated with each target. The capital : output ratio is predominantly affected by the exponent α on capital in intermediate firms' production function and the persistence ρ_o and standard deviation v_o of the idiosyncratic output productivity shock. The relevant target is the average ratio of private non-residential fixed assets to the sum of non-durable and services consumption plus private non-residential investment, taken over the period 1954-2007.

The labour share is determined by γ the exponent on labour in the intermediate goods firms' Cobb-Douglas production function, while the household disutility of working B targets the proportion of time the household spends working. The elasticity of substitution θ in the final goods producers' CES aggregator function is set to target the midpoint between the 1976 and 2006 mark-ups found by Loecker, Eeckhout and Unger (2020). Finally, the entry rate is determined by the exogenous chance of death π_d while the relative average size of entrants compared to incumbents and the proportion of total employment by entrants are determined by the distribution of potential entrants' cash endowment H (which follows a Pareto distribution and has parameters $m_{e,low}$ for the lower bound, $m_{e,high}$ for the upper bound and Ξ for its shape) and the level of the fixed cost of entry ξ_e.

Turning to the target moments that are novel for a DSGE model (i.e. those relating to the distribution of firms over number of products and introduction of new products), these are shown in Table 2 and are governed predominantly by the logit probability parameter χ, the product depreciation rate δ_N, and the fixed cost of making a positive investment

Table 1: Model's steady-state moments for standard aggregate measures

Moment	Main parameter	Source	Model	Data
Capital : Output Ratio	Exponent on capital α, Idiosyncratic shock terms for ε_o	NIPA data	1.73	1.74
Labour Share	Cobb-Douglas Exponent γ	Cooley & Prescott (1995)	60%	60%
% of time worked	Household disutility of working B	Literature	33%	33%
Aggregate mark-up	Elasticity of substitution θ	De Loecker, Eeckhout & Unger (2020)	36.25%	36.25%
R&D investment as % of output	Idiosyncratic shock terms for ε_r, fixed cost of positive R&D investment ξ_r	NSF	1.98%	1.8%
Proportion of firms with 0 R&D investment	Idiosyncratic shock terms for ε_r, fixed cost of positive R&D investment ξ_r	Compustat 1978-2006	49.6%	53.7%
Entry rate	Exogenous death probability π_d	BDS	10.6%	10.6%
Relative average size of entrants vs incumbents	Distribution of potential entrants' cash endowment H, Fixed cost of entry ξ_e	BDS	24.6%	25.6%
Proportion of total employment by entrants	Distribution of potential entrants' cash endowment H, Fixed cost of entry ξ_e	BDS	2.8%	3.0%

in R&D ξ_r, with the targets for these moments taken from Bernard, Redding and Schott (2010).[21]

My model performs well in matching this over-identified set of targets. In particular, it exactly matches standard aggregate moments such as the capital-to-output ratio and hours worked. It also matches the relative importance of new entrants compared to

[21] Specifically, they affect the mean number of products across firms; the proportion of firms that are single-product; the proportion of firms that add a product; the proportion of output accounted for by firms that add a product; the proportion of output accounted for by multi-product firms; the proportion of multi-product firms that add a product; the proportion of output from multi-product firms that is accounted for by those multi-product firms that add a product; the proportion of "large" firms that add a new product; and the proportion of output by "large" firms that is accounted for by those large firms that add a new product (where a "large" firm uses the same definition as that in Bernard, Redding and Schott (2010) that have output above the seventy-fifth percentile).

Table 2: Model's steady-state moments for the distribution of firms over number of products

Moment	Model	Data
Mean number of products per firm	2.01	1.975
% of firms that are single-product	48.5%	61%
Proportion of firms that add a new product	53%	39%
Proportion of output accounted for by firms adding a product	89%	78%
Proportion of output accounted for by multi-product firms	97%	87%
Proportion of multi-product firms adding a product	67%	68%
Proportion of multi-product firm output accounted for by multi-product firms adding a product	88%	87%
Proportion of large firms adding a product	88%	44%
Proportion of large firm output accounted for by large firms adding a product	89%	80%

Note: Although all moments in my model are determined jointly by all parameters, the specific parameters that predominantly determine the moments shown in this table are the logit probability parameter χ, the product depreciation rate δ_N, and the fixed cost of making a positive investment in R&D ξ_r. The targets for all of these moments are obtained from Bernard, Redding and Schott (2010)

incumbent firms. Moreover, it gets very close to the novel moments regarding the distribution of firms over products and the frequency of firms adding new products: my model almost exactly matches the average number of products per firm as found by Bernard, Redding and Schott (2010) and only slightly overstates the importance of multi-product firms. Importantly, it captures the proportion of firms adding new products, both in total and split by subgroups of firm, relatively well. Moreover, even though not explicitly targeted, my model gets close to Broda and Weinstein (2010)'s finding that roughly 25% of products in any one year are "new".[22] The only target moment in which the model substantially misses is the proportion of large firms that add a new product, but as my model gets very close to matching the proportion of output by large firms that is accounted for by those large firms that add a new product, my model still captures the relative importance of large firms adding a new product in the

[22] For my model, this number can be obtained by multiplying the steady-state number of firms plus the unit mass of potential entrants (4.92+1.00) by the average probability of an incumbent or entrant firm developing a new product (53%) to get 2.59 new products introduced each year. This is then divided by the total number of products in the economy (obtained by multiplying the average number of products per firm (2.01) by the number of incumbent firms). As such, the proportion of products that are new in any one year is (5.92*0.53)/(4.92*2.01) = 31.7%.

aggregate.

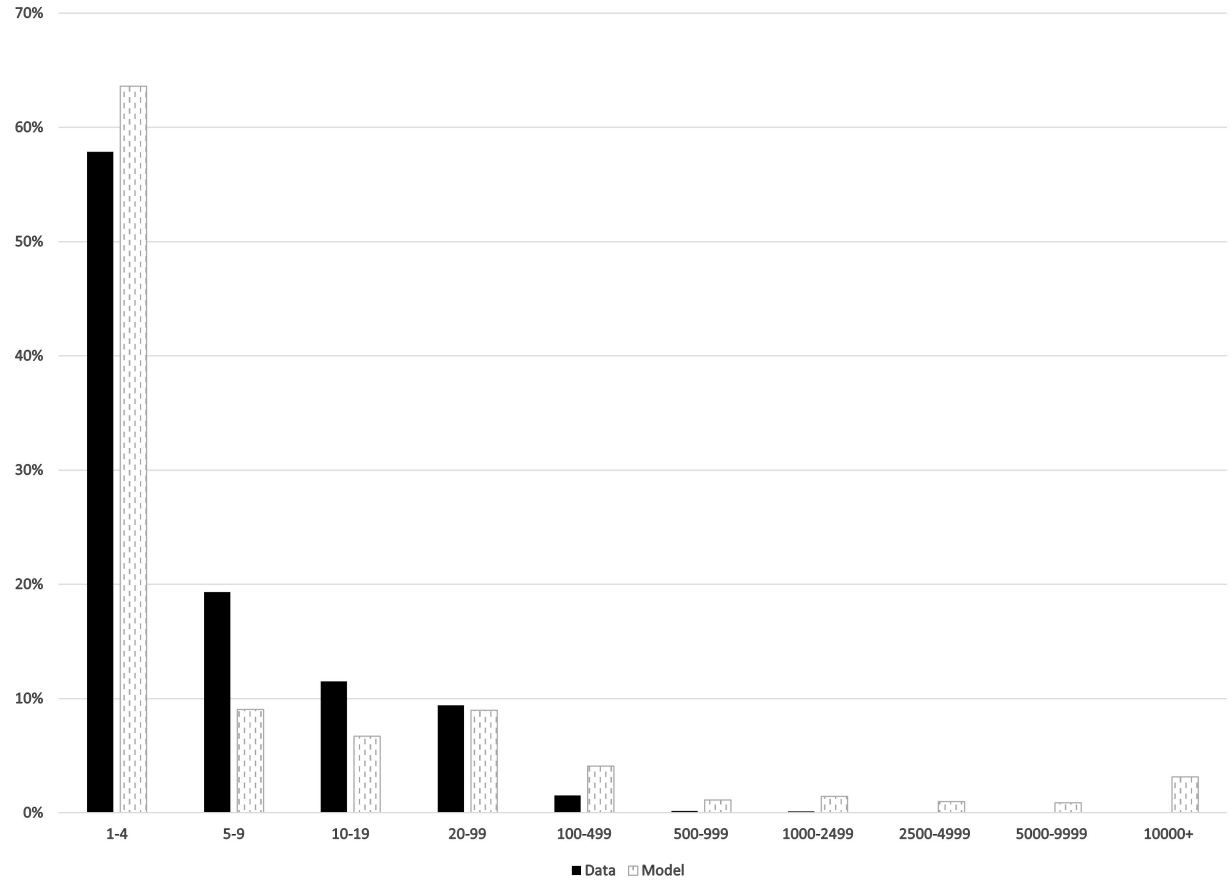
Crucially for the analysis of the relative importance of new entry compared to the innovation of new products presented subsequently, my model closely matches the target moments related to both of these channels. As such, switching off one of these channels will capture the relative importance of that channel in affecting the economy and path of aggregate variables during the recovery from a recession.

As external validation of my model, I also examine the extent to which it can capture the firm-size distribution, which is shown in Figure 4. In this graph, the solid black bars represent the distribution of firms over bins defined using the number of people they employ obtained from BDS data averaged over the period 1980-2006. The grey patterned bars show the distribution of firms over these bins that result from my model, with those bins defined using the same approach as Jo (2021).[23] My model is able to get very close to the Pareto shape of this (untargeted) distribution despite using only (log-) normally distributed idiosyncratic productivity shocks.

My model also captures the fact that entrants grow much more quickly than do old firms, as found by Luttmer (2011). In particular, Figure 5 plots the average sales (the left panel, defined as $Np(k, \varepsilon_o, z, \mu)y(k, \varepsilon_o, z, \mu)$) and employment (the right panel) for a simulated cohort of entrant in the years after they enter. The y axis shows the value of these variables relative to the value they took in the year the cohort entered, while the x axis shows the number of years since entry. Both panels show that (conditional on surviving, i.e. not being hit with the exogenous death shock) the average new entrant grows quickly until it has been around for roughly 50 years, at which point its growth starts to slow down. In other words, smaller, younger firms grow quickly relative to larger, older firms, exactly capturing the path of firm growth shown in Luttmer (2011). Moreover, the rapid increase in the average (surviving) entrants' sales in the first few years of its existence captures the path of sales in the first 3-4 years as found by Argente et al. (2021). As such, my assumption that firms can only add one product (at most) in any one year ensures that my model capturing the rapid growth of small, young firms and the relatively slower growth of larger, older firms.

[23] Jo (2021) first uses the cumulative proportion of employment in each bin to convert the "real life" employment range of each bin to something that can be used within the model. Then the proportion of firms that falls within each of these model-defined bins is calculated to obtain the overall firm-size distribution.

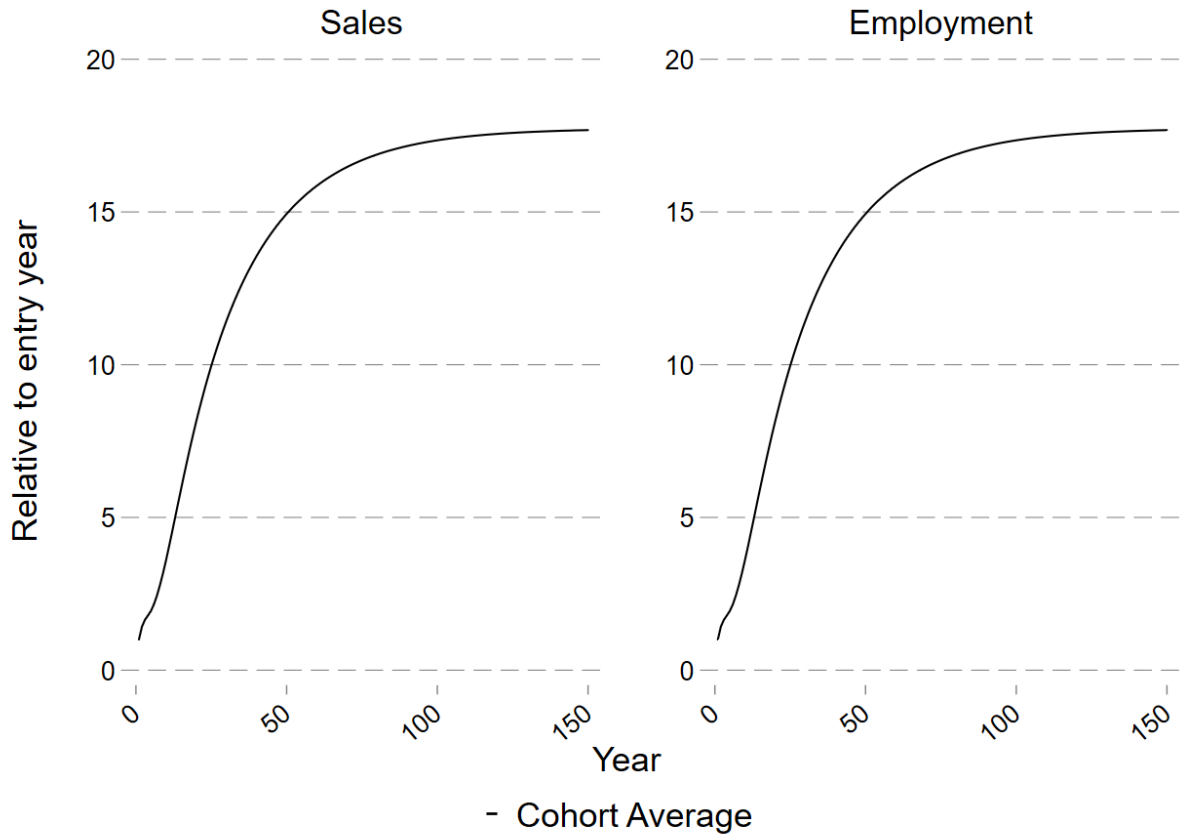
Figure 4: Comparison of model to untargeted size-distribution of firms



Note: The solid black bars show the distribution of firms over employment obtained using BDS data for the period 1980-2006. The grey patterned bars show the distribution of firms over employment for the non-stochastic steady-state of my model, with the proportion of firms in each bin obtained following the procedure described in Jo (2021) to determine the proportion of employment accounted for each bin, using the cumulative distribution of employment to define each bin within the model, and then calculating the proportion of firms that are within each such-defined bin.

Table 3 shows the parameter values chosen for my model (all expressed in levels). As the period of my model is one year, the household rate of time preference β is set to 0.96 to obtain a risk-free real interest rate of 4% (a period in my model is equivalent to one year), while the depreciation rate of capital is set at 6.9% following Khan and Thomas (2013); these are the only parameters that are set externally. Note that, as mentioned in Section 3, a firm's returns to scale with respect to inputs is given by $\alpha\frac{\theta-1}{\theta} + \gamma\frac{\theta-1}{\theta}$; hence, even though $\alpha + \gamma > 1$, each firm still has decreasing total returns to scale as $\frac{\theta-1}{\theta} = 0.73$ (such that the overall returns to scale are 0.77.

Figure 5: Growth of a cohort of entrants



Note: Each line shows the average employment or sales (defined as price times output times number of products) per firm for a simulated cohort of entrants where the level of consumption and output remains at the steady-state level throughout the simulation. Each line is relative to the value it takes during the year of the cohort's entry.

4.2. Business cycle

I assume that the aggregate productivity shock z a continuous process that follows an AR(1) process with a zero mean (in logs) with persistence ρ_z and a disturbance standard error of σ_z. In other words, the process for aggregate productivity is given by $\log z' = \rho_z \log z + \varepsilon_z$ where $\varepsilon_z \sim N(0, \sigma_z^2)$. I estimate the persistence ρ_z and standard deviation of the shock σ_z parameters from the Solow Residual measured using Hodrick-Prescott filtered data on real annual US GDP and private capital along with total civilian employment hours created by Cociuba, Prescott and Ueberfeldt (2018) over the period 1954-2007. This process obtains a persistence $\rho_z = 0.744$ and standard deviation $\sigma_z = 0.0128$. I discretise the process across three gridpoints using the Rouwenhorst algorithm to obtain its values and transition probabilities.

Table 3: Parameter values

Parameter	Description	Value	Parameter	Description	Value
β	Household rate of time preference	0.96	ρ_r	Persistence of idiosyncratic research productivity	0.85
δ_k	Depreciation rate for capital	0.069	v_r	Standard deviation of idiosyncratic research productivity shock	0.0075
γ	Cobb-Douglas Exponent on labour	0.82	ρ_o	Persistence of idiosyncratic output productivity	0.95
α	Cobb-Douglas Exponent on capital	0.225	v_o	Standard deviation of idiosyncratic output productivity shock	0.012
B	Household disutility of working	2.6	ξ_e	Fixed entry cost	0.01
θ	Elasticity of substitution between products	3.759	$m_{e,low}$	Lowest value of m_e	0.0095
δ_N	Depreciation rate for intermediate products	0.175	$m_{e,high}$	Highest value of m_e	0.45
χ	Logit probability parameter	550.0	Ξ	Pareto shape of m_e	-0.25
ξ_r	Fixed cost of making a positive R&D investment	0.06	π_d	Exogenous probability of death	0.106

I solve my model using the (modified) Krusell-Smith algorithm as per Khan and Thomas (2013) and approximate the distribution of firms using the aggregate number of goods $\bar{N}$ and the aggregate capital stock $\bar{K}$. More details regarding the solution algorithm are presented in Appendix A.2. Note that I repeat this procedure for each separate version of the model for which I present results in Section 5. In other words, I repeat this algorithm separately (and obtain distinct forecast rules) for each of: 1) my full model in which firms' R&D and entry decisions can vary over time; 2) the "exogenous entry" specification in which the entry (and initial investment) decisions by potential entrants are fixed at their non-stochastic steady-state level, but incumbents' R&D decisions can vary over time; 3) the "exogenous R&D" specification in which incumbents' R&D decision rule is fixed at the ones that prevail in the non-stochastic steady-state but potential entrants' entry and initial investment decisions can vary over time; and 4) the "exogenous entry and R&D" specification in which both incumbents' R&D decisions and potential entrants' entry and initial investment decisions are fixed at the steady-state level.

Simulating my model for 1,500 periods and then examining the economy from the five-hundredth period onward results in the business cycle statistics as presented in Table 4. The first row shows the mean of each series, while the second shows their coefficient of variation. The third row shows the volatility of each series relative to that of output. As one would expect, consumption is less volatile than output due to households' desire to smooth consumption. Similarly, the number of products $\bar{N}$ is more volatile than output even though R&D spending substantially less volatile than output. Moreover, the aggregate stock of capital $\bar{K}$ is more volatile than output due to the fact that firms can freely sell off their stock of capital in the face of adverse shocks (i.e. there are no capital adjustment frictions in this model).

Table 4: Simulated business cycle moments for full model

	Y	C	Number of firms	$\bar{N}$	Total R&D Spending	Number of entrants	L	$\bar{K}$
Mean	1.19	0.84	4.91	9.89	0.02	0.52	0.33	2.05
Coefficient of Variation	2.6%	2.4%	0.5%	1.2%	1.91%	1.1%	0.5%	2.2%
$\frac{\sigma_X}{\sigma_Y}$	(0.031)	0.65	0.85	3.77	0.01	0.18	0.06	1.42
Correlation with GDP	1	0.98	0.61	0.48	0.89	0.76	0.50	0.65
Auto-correlation	0.79	0.86	0.98	0.99	0.93	0.73	0.61	0.97

Note: These moments are obtained from a simulation of 1,500 periods of the full model, dropping the first 500 periods of that simulation. These moments have not been filtered or smoothed. The first two rows show, respectively, the mean and coefficient of variation of each series. The third row shows the standard deviation of each series relative to that of output, except for the first column, which shows the actual standard deviation of output. The fourth row shows each series' contemporaneous correlation with output, while the last row contains each series 1-period auto-correlation. All numbers are expressed in levels.

Furthermore, all series have the expected sign for their correlation with output; in particular, the number of products, total number of firms, and the number of entrants are strongly pro-cyclical. In particular, the correlation between the total number of firms and GDP of 0.61 is close to the (untargeted) 0.78 observed in the data, while those for the number of entrants are 0.76 in the model and 0.65 in the data. The fact that R&D spending is more correlated with GDP in the model than in the data (0.89 in the model compared to 0.36 in the data) means that my model might be slightly biased towards finding that R&D is more important for business cycles than in reality; in other words, my model might overstate the relative importance of R&D and, as such, is conservative in any results relating to the relative contribution of entry.

I use the same approach to simulate the specification in which endogenous entry is

shut down, using different forecast rules from the full model). In this specification, the decisions of potential entrants each period are fixed at the choices that potential entrants make in the steady-state. Table 5 shows the same business cycle moments for this version of the model. As one would expect, output is slightly less volatile when entry is exogenous as the number of entrants does not change over time. This latter point also means that the number of firms does not vary over time. As a result, each series other than consumption is also less volatile relative to output than in the model when entry is endogenous. Fixing the distribution of entrants each period also makes all series more negatively correlated with output, with the most noticeable being that hours worked is now almost zero. This near-zero correlation between labour and output in this specification of the model is because the mass of firms in this specification does not change with gdp; this means that booms do not cause more firms to enter and increase labour hiring. Instead, the main effect on labour comes through the effect of the real wage and labour productivity: the real wage is higher during booms but the real wage is also higher, such that the two cancel each other out and labour does not change substantially over the business cycle.

Table 5: Simulated business cycle moments for specification with exogenous entry

	Y	C	Number of firms	$\tilde{N}$	Total R&D Spending	Number of entrants	L	$\tilde{K}$
Mean	1.19	0.84	4.92	9.90	0.02	0.52	0.33	2.05
Coefficient of Variation	2.2%	2.2%	0.0%	0.3%	0.65%	0.0%	0.3%	1.6%
$\frac{\sigma_X}{\sigma_Y}$	(0.026)	0.72	0.00	1.27	0.01	0.00	0.04	1.25
Correlation with GDP	1.00	0.99	-0.02	0.37	0.85	0.00	-0.03	0.56
Auto-correlation	0.77	0.82	-0.02	0.98	0.94	1.00	0.84	0.97

Note: These moments are obtained from a simulation of 1,500 periods of the model where the decisions and outcomes for potential entrants are the same as they are in the steady-state, dropping the first 500 periods of that simulation. These moments have not been filtered or smoothed. The first two rows show, respectively, the mean and coefficient of variation of each series. The third row shows the standard deviation of each series relative to that of output, except for the first column, which shows the actual standard deviation of output. The fourth row shows each series' contemporaneous correlation with output, while the last row contains each series 1-period auto-correlation. All numbers are expressed in levels.

Finally, I simulate the specification in which incumbent R&D is fixed at the steady-state level (again using different forecast rules from the full model). In this specification, the R&D investment decisions of incumbents each period are fixed at the choices that incumbents make in the steady-state (and their capital investment decisions adjust, where necessary, to ensure that the non-negative dividend constraint is not violated).

Table 6 shows the same business cycle moments for this version of the model.

Foreshadowing the relative importance of the effect of R&D (compared to the effect of entry) on the recovery from a recession, fixing incumbent firms' R&D decisions at their steady-state level has a much smaller effect on the economy over the business cycle than does fixing entrants' decisions. For example, fixing R&D increases the auto-correlation of consumption from 0.86 to 0.87, as compared to the reduction to 0.82 obtained by fixing entry. Similarly, the correlation between GDP and the number of products falls only by 0.01 when fixing incumbents' R&D decisions, but drops by more than ten times that amount (by 0.11) when fixing entrants' decisions. In other words, fixing incumbent R&D decisions appears to have a much smaller impact on the fluctuations of output and consumption over the business cycle than does fixing entry.

Table 6: Simulated business cycle moments for specification with exogenous R&D

	Y	C	Number of firms	$\tilde{N}$	Total R&D Spending	Number of entrants	L	$\tilde{K}$
Mean	1.19	0.84	4.91	9.88	0.02	0.52	0.33	2.05
Coefficient of Variation	2.6%	2.4%	0.5%	1.2%	1.85%	1.1%	0.5%	2.2%
$\frac{\sigma_X}{\sigma_Y}$	(0.031)	0.65	0.85	3.85	0.01	0.18	0.06	1.41
Correlation with GDP	1.00	0.98	0.61	0.47	0.84	0.76	0.50	0.66
Auto-correlation	0.79	0.87	0.98	0.99	0.95	0.73	0.60	0.97

Note: These moments are obtained from a simulation of 1,500 periods of the model where the decisions and outcomes for incumbent firms are the same as they are in the steady-state, dropping the first 500 periods of that simulation. These moments have not been filtered or smoothed. The first two rows show, respectively, the mean and coefficient of variation of each series. The third row shows the standard deviation of each series relative to that of output, except for the first column, which shows the actual standard deviation of output. The fourth row shows each series' contemporaneous correlation with output, while the last row contains each series 1-period auto-correlation. All numbers are expressed in levels.

5. Results

In this Section I set out the results of simulating a recession caused by a reduction in aggregate TFP z and compare these results between the different specifications of the model. First, I present the recovery from a recession caused by a 1% drop in aggregate TFP that gradually returns to its steady-state level for the full model (i.e. allowing incumbents' and entrants' decisions to vary over time). Second, I show how this recovery differs across groups of firms differentiated by their number of products.

Finally, I examine the impact of fixing firms' R&D or entry decisions to investigate the importance of each channel in determining the size of the recession and the timing and path of the recovery. In each analysis, the economy is set to run for 150 periods at the median level of TFP before TFP drops by 1% in the 151st period and then gradually returns to its median value as governed by the persistence parameter ρ_z. In all of the results presented below, one period of the simulation is equivalent to a year.

I find that the TFP shock initially results in a small increase in the average price per intermediate product due to a reduction in the amount of capital that firms hold. This is a result of the distribution shifting after the shock: even though the price per product for each combination of capital stock and idiosyncratic productivity follows the same path relative to the steady-state in response to the shock, firms with lower levels of capital stock and output productivity have higher price levels. Hence, as firms reduce their capital stock in response to the shock, the distribution of firms to the left, leading to a slight increases in the average price of intermediate goods. This reduction in prices also means that output falls by more than the drop in TFP (as intermediate goods firms have even less incentive to produce output), with the result that firms have less to invest in R&D despite substantially cutting back their investment in physical capital. Similarly, potential entrants are less willing and able to enter, so the number of firms also drops. The combination of the reduction in investment in R&D and physical capital (the former leading to a smaller number of products being produced) along with the decline in entry means that the economy takes roughly 30 years to return to the steady-state level.

There are substantial differences across firms during the economy's recovery. In particular, firms that already have a large number of products when the shock hits only reduce their investment in R&D and capital slightly, such that their output and number of products only falls by a relatively small amount. This is due to these firms already making positive dividends such that they can afford to maintain (almost) their steady-state R&D and capital investment levels. Conversely, smaller firms, already paying zero dividends when the TFP shock hits, must reduce their R&D and capital investments substantially. This means that their output and number of products fall by relatively large amounts. However, as the recovery develops, these smaller firms take advantage of the lower wage to hire more labour, increasing their output and investments in R&D and capital to a level above that of the steady-state, with this effect being more pronounced the smaller (in terms of the number of products they have) is the firm. As wages start to increase, labour becomes more expensive (from roughly 30

years after the initial shock), these smallest firms gradually reduce their labour, output, and investment back to steady-state. Nonetheless, the recovery path of the aggregate economy is driven by that of the firms with the highest number of goods since they account for the largest proportion of output, employment, and investment.

Comparing the relative importance of incumbents' R&D decisions to that of potential entrants' decisions in driving the economy's recovery, I find that entrants' decisions have a larger contribution to this recovery than do incumbents' R&D decisions.[24] In particular, entrants' decisions over the path of the recovery account for over 90% of the difference between the path of output in the simulated recovery observed in the full model and in a specification in which incumbents' and potential entrants' decisions are fixed at their steady-state level. In contrast, incumbents' R&D investment decisions account for only 20% of this difference, with the fact that the "total" combined contribution of both channels being above 100% indicating that the response of incumbents' ameliorates the response of potential entrants, and vice versa, over the course of the economy's recovery.

Each of these results is described in more detail below.

5.1. Aggregate response in the full model

At the outset it is useful to examine the path of prices for intermediate products in response to the shock to TFP, which is shown in Figure 6. The left panel of this figure shows the (distribution of) the path of the price of an intermediate product produced by a firm with physical capital k and idiosyncratic productivity ε_o relative to the steady-state price for that particular type of firm.

It is immediately clear that prices for products produced by firms with different combinations of capital and output productivity follow the same path relative to their steady-state level. In other words, even though different combinations of capital and output productivity result in a different price level for an intermediate product from firms with different combinations of these variables, the path of that level of price level relative to its steady-state level is the same across all combinations of capital and output productivity. This path is such that there is a slight delay in the price per product

[24] When I fix incumbents' R&D decisions, I allow their physical capital investment decisions to adjust where necessary to ensure that the non-negative dividend constraint is not violated.

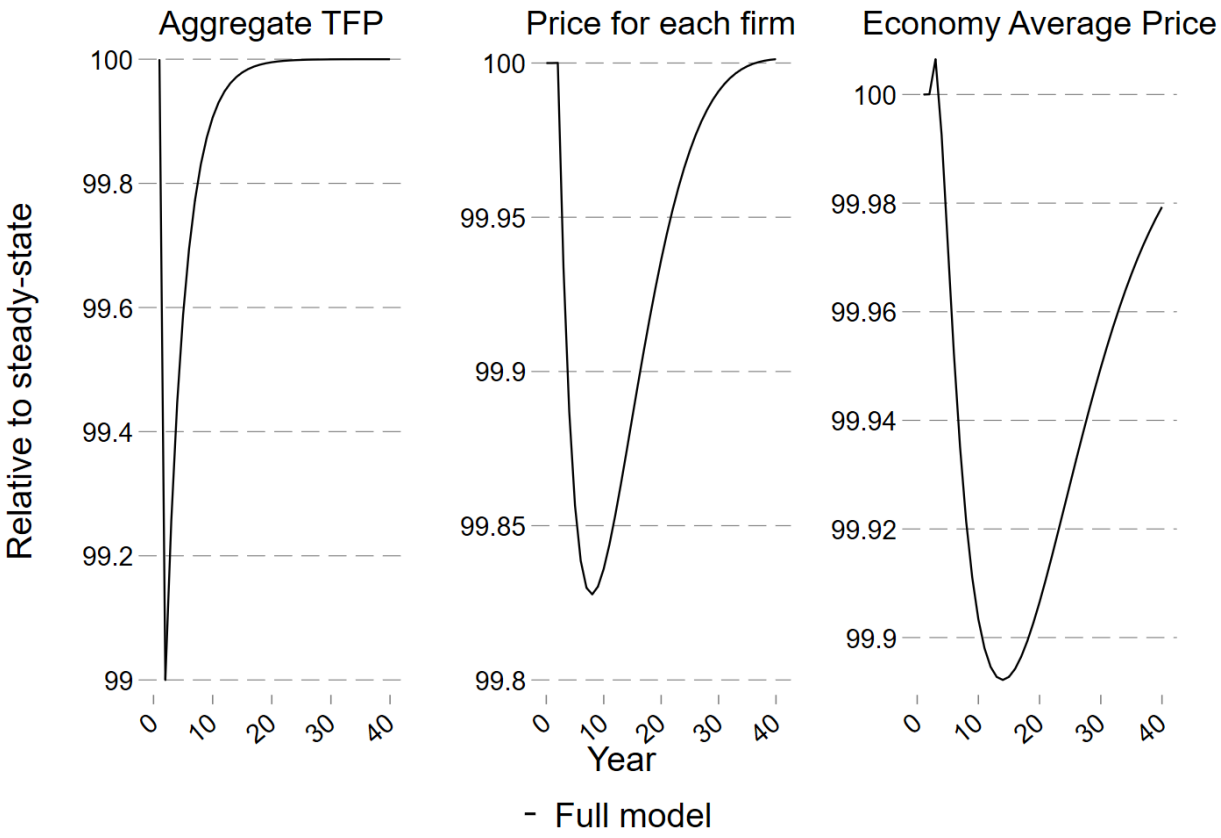
dropping after the TFP shock (due to aggregate output also falling such that the ratio of aggregate output to firm output is level) before dropping by just over 0.15% after 9 periods. From that period on, the price per product for each firm gradually recovers back to the steady-state level.

The picture is slightly different when looking at the average price per intermediate product that prevails across the economy, as shown in the right-hand panel. In particular, the average price of intermediate products increases initially. This is entirely due to the effect of the distribution shifting to the left: firms with lower capital stocks have higher prices, and the TFP shock leads to firms reducing their stock of capital. This means that there are more firms with lower levels of capital, and therefore output, such that their price level is higher. Hence, the average price across all intermediate products increases. After roughly 3 periods, this average price starts to fall below its steady-state level, but only reaches a trough of about 0.1% below the steady-state average price. As before, this is a result of firms reducing their capital stock, such that the distribution of firms shift slightly more towards those that charge a higher price. This is a notable difference from Bilbiie, Ghironi and Melitz (2012) that arises from incumbent firms being able to innovate and introduce new products in my model.

The response of output, consumption, aggregate R&D and the total number of goods to a 1% shock to TFP with a gradual return to its median level is shown in Figure 7. The initial 1% drop in TFP (which occurs in period 1 in the graph) has an immediate effect on output as output falls by more than 1%. This is due to the combined effect of 1) the lower level of aggregate productivity reducing the amount an intermediate goods producer can produce for the same amount of capital and labour; and 2) the lower aggregate output reducing the price per unit of output produced by intermediate goods firms via Equation (5). This reduces the ability and incentive of firms to produce in the initial period, which also reduces their profits and amount available to invest in R&D.

As with R&D spending, investment in physical capital drops as consumers wish to smooth consumption (which is why consumption does not fall by as much as output in response to the initial drop in TFP) and therefore do not use as much output for investment. This can be seen in Figure 8, which plots the responses of the aggregate capital stock, capital investment, the number of firms, and labour. In the initial period, the marginal productivity of labour is reduced via the decline in aggregate TFP and the lower price firms receive for their output, so hours worked also falls. The number of

Figure 6: Simulated path of price of intermediate products after a TFP recession in the full model

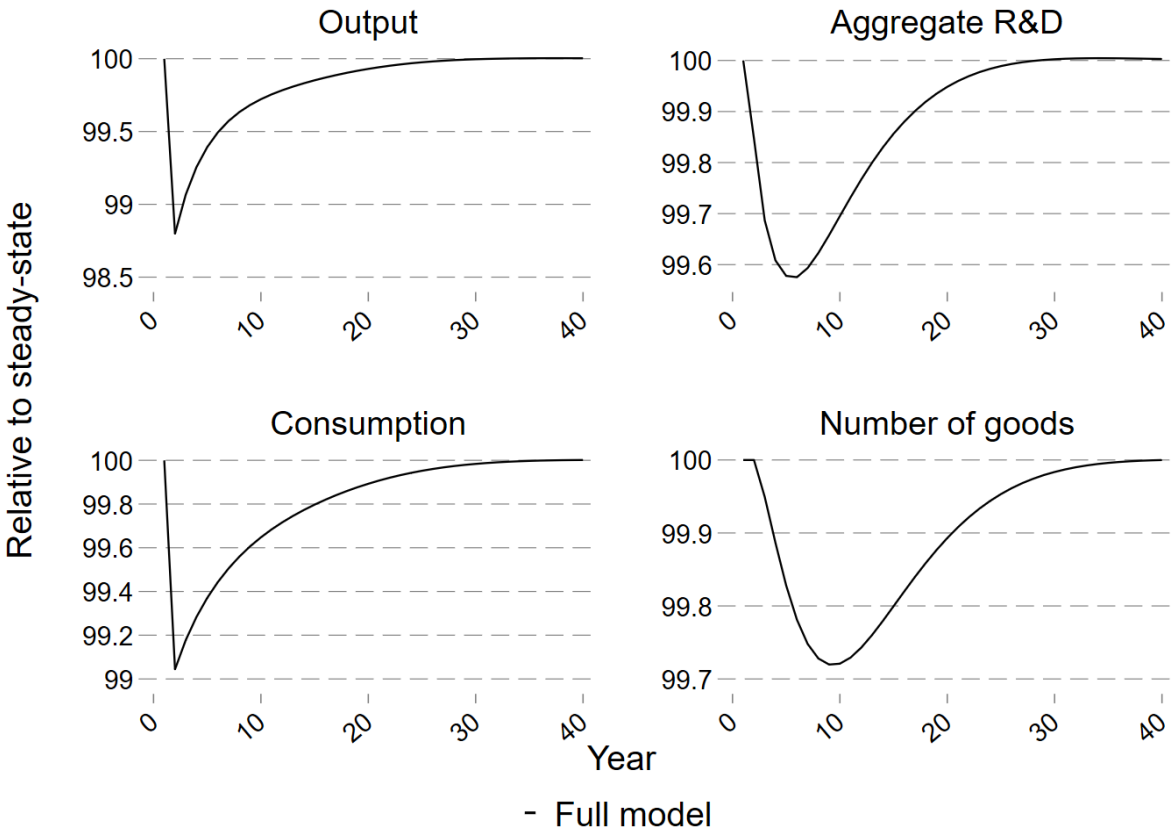


Note: The graphs show the simulated time path of the price of intermediate products in the full model economy. The economy is started at the steady-state distribution and runs for 150 periods with exogenous aggregate productivity at the median value of 1. In the 151st period aggregate TFP is subjected to a 1% reduction (i.e. is set to 0.99) before gradually recovering back to its median value of 1 at a rate determined by ρ_z. All series are expressed in terms of their percentage deviation from steady-state.

firms also falls as potential entrants now have less ability and incentive to try to enter and start producing (this also contributes to the decline in aggregate R&D and number of goods).

The drop in R&D and capital investment means that the stock of physical capital and the number of goods falls in the period after the shock hits. As R&D investment falls even further in the periods subsequent to the shock, the number of products also falls in subsequent periods. The trough of R&D investment and the number of goods is about 0.4% below the steady-state level, indicating that the elasticity of these variables in response to a decline in TFP is relatively high (roughly 40% at the trough). Likewise, as capital investment remains below what is necessary to cover depreciation of capital in

Figure 7: Simulated recovery of output, consumption, R&D and number of goods after a TFP recession in the full model

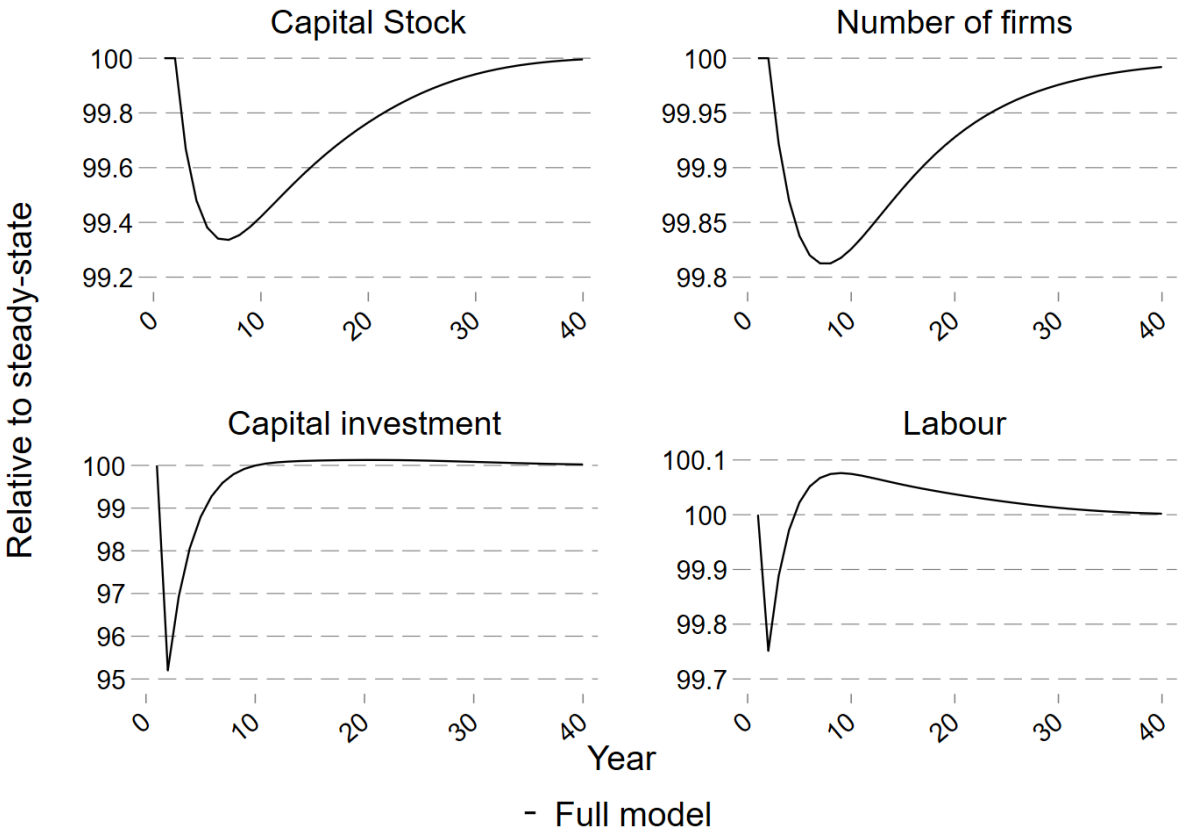


Note: The graphs show the simulated time path of aggregate variables in the full model economy. The economy is started at the steady-state distribution and runs for 150 periods with exogenous aggregate productivity at the median value of 1. In the 151st period aggregate TFP is subjected to a 1% reduction (i.e. is set to 0.99) before gradually recovering back to its median value of 1 at a rate determined by ρ_z. All series are expressed in terms of their percentage deviation from steady-state.

the periods after the shock, the capital stock also drops to a trough in period 7 of roughly 0.7% below the steady-state level. At this point, the number of firms is also at its lowest level relative to the steady-state as the number of successful entrants has continued to be below the exogenous exit rate of incumbent firms.

These troughs most closely follow the total (and average) R&D spending and capital investment for incumbents rather than entrants. Focusing on the path of R&D spending over the recovery, entrants' average R&D spending drops by roughly 0.2% points in the first period and then recovers gradually back to the steady state. Conversely, the u-shaped path of total R&D follows that of incumbents, simply because incumbents constitute nearly 90% of the mass of firms. Note that the troughs for R&D spending

Figure 8: Simulated recovery of capital stock, investment, number of firms, and hours worked after a TFP recession in the full model



Note: The graphs show the simulated time path of aggregate variables in the full model economy. The economy is started at the steady-state distribution and runs for 150 periods with exogenous aggregate productivity at the median value of 1. In the 151st period aggregate TFP is subjected to a 1% reduction (i.e. is set to 0.99) before gradually recovering back to its median value of 1 at a rate determined by ρ_z. All series are expressed in terms of their percentage deviation from steady-state.

and capital stock are more muted compared to those found in Bilbiie, Ghironi and Melitz (2012) predominantly due to my model allowing incumbent firms to innovate: allowing incumbent firms to innovate means that some firms are large enough in the steady-state such that they do not need to reduce their investment in R&D (and physical capital) by a large amount in order to maintain non-negative dividends in response to the shock. This is shown in more detail in Section 5.2.

From period 7 on, the price of intermediate goods starts to increase from its trough. This means that hours worked increases above the steady-state level as the marginal product of labour improves (and the wage is still below the steady-state level). The combination of increasing prices for intermediate products and labour hiring (alongside

aggregate exogenous TFP continuing its gradual return to its steady-state level) results in a relatively large increase in output from its trough back towards the steady-state. As a result, firms are now able and willing (via higher operating profits per product) to invest more in R&D and capital. Indeed, capital and R&D investment actually exceed the steady-state level for a number of periods (after period 15 for capital, period 25 for R&D) to enable the return of the capital stock and the number of goods to their steady-state level.

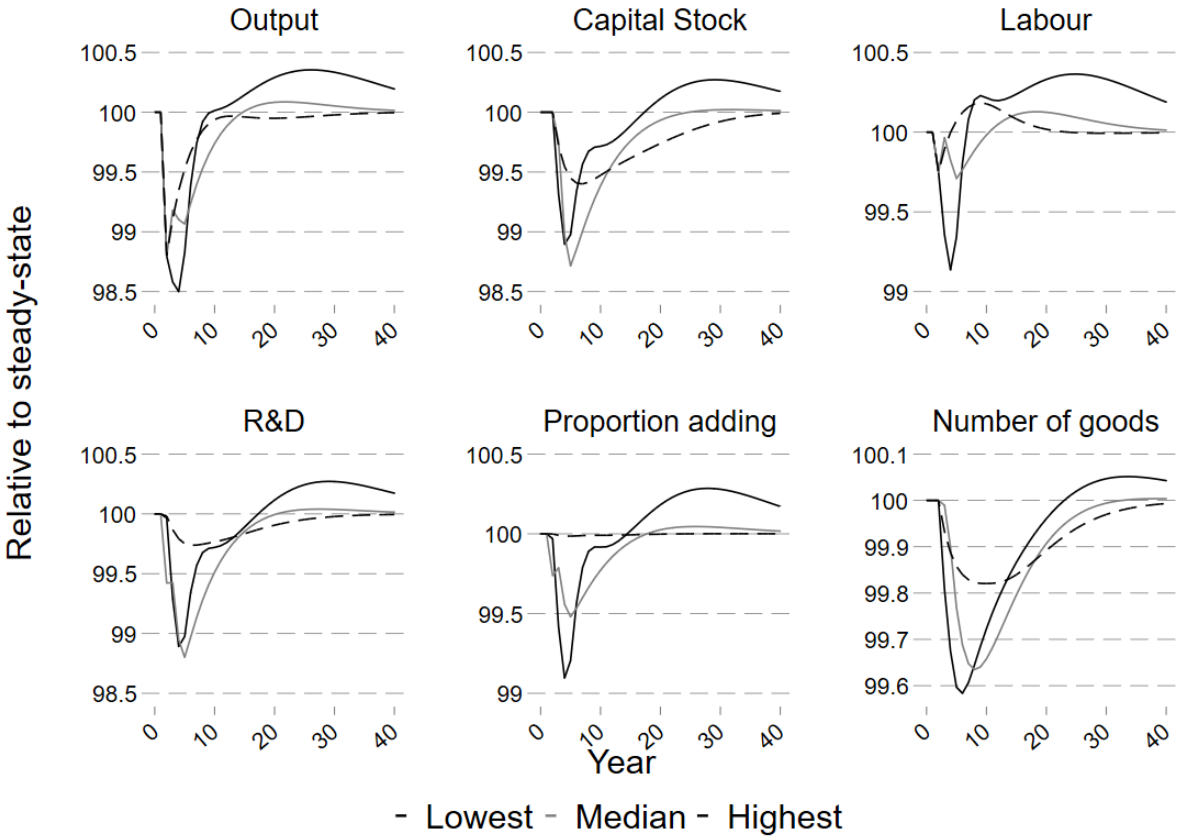
Finally, the number of firms gradually returns to the steady-state level as the number of entrants increases due to higher incentives and abilities of potential entrants to pay the entry cost and invest more in R&D, giving them a higher probability of developing a new product and entering successfully in the next period. Output and consumption are then back at (or very close to) the steady-state level around period 30 (i.e. after roughly 30 years); this long recovery time for output and consumption is due to the fact that the number of firms, R&D and capital investment take until around period 40 to recover to their respective steady-state levels. This long recovery period is due to the interlinking between aggregate output and the price per product each intermediate goods firm receives: low aggregate output means the price per product is low, which means firms have a lower incentive and ability to invest in R&D and innovate new products while incumbents have a lower incentive to enter. These two factors mean that capital investment, the introduction of new products, and the entry of new firms is slow to recover, which itself means that aggregate output is slow to recover, thereby slowing down the recovery of the price per product in a vicious circle.

5.2. Responses across the distribution of firms in the full model

It is also possible to examine how the path of these variables differ across the distribution of firms. Focusing on differences in terms of the number of products a firm has, Figure 9 shows the path of totals within certain quintiles of this distribution. In particular, the solid black line shows the path for the lowest quintile in terms of number of products; the solid grey line for the middle quintile; and the dashed line for the top quintile.

The paths of the variables for firms in the highest quintile (the dashed line) show that firms with the highest number of products are the least affected by the shock to aggregate TFP. This is because these larger firms have sufficient number of products already that

Figure 9: Simulated recovery of aggregate variables, split by quintile of products owned



Note: The graphs show the simulated time path of aggregate variables in the model economies, with separate series for each of three different quintiles of the number of products owned by a firm. In this graph, the level of number of products owned by the firm that denote each quintile changes as the distribution of firms changes over time. The solid black lines shows the path for all of those firms in the bottom quintile; the solid grey lines for all firms in the middle quintile; and the dashed lines for those firms in the highest quintile. The economy is started at the steady-state distribution and runs for 150 periods with exogenous aggregate productivity at the median value of 1. In the 151st period aggregate TFP is subjected to a 1% reduction (i.e. is set to 0.99) before gradually recovering back to its median value of 1 at a rate determined by ρ_z. All series are expressed in terms of their percentage deviation from steady-state.

they can produce enough output and profits such that the reductions in aggregate TFP and the price per product that these firms receive means that even with a fall in their output and operating profits, they do not need to reduce R&D investment or capital investment substantially in order to ensure their dividends remain non-negative. As such, these larger firms continue to invest in R&D and physical capital a relatively similar amount compared to their steady-state levels, such that they only experience a small drop in their number of products. This means their labour hiring only drops slightly in response to the shock to TFP, before all series gradually return to steady-state (with labour exceeding it for some periods to take advantage of the lower wage due to lower

aggregate consumption).

When looking at smaller firms (the solid black and grey lines), a different pattern emerges. Specifically, these firms are already paying zero (or close to zero) dividends in the steady-state, so any fall in their operating profit resulting from a drop in aggregate TFP must lead to a reduction in R&D investment and capital stock. This means that they are less able to add new products, leading to a relatively large decrease in the number of products they have, particularly compared to firms in the top quintile of the distribution; the decrease in the number of goods owned by firms in the lowest quintile is more than twice that of firms in the highest quintile. Note that despite the capital stocks and R&D investments of the smallest and median firms falling by roughly the same amount relative to steady-state, the number of products for the smallest firms falls by more (and more quickly) than the number of products for the median firm due to the fact that the marginal probability of a successful investment is decreasing in R&D investment. Hence, smaller firms, which already have relatively lower absolute levels of R&D investment in the steady-state, experience a larger drop in the probability of adding a new product when they cut their R&D expenditure by the same amount as the median firms.

As the aggregate economy starts to return to the steady-state, the output by the smaller firms actually exceeds the steady-state level as these firms make use of the lower wage to hire more labour and invest in more R&D and capital. This results in a higher proportion of the smallest firms adding products compared to the steady-state, eventually exceeding the total number of products that they had in steady-state. From period 30 onward, as consumption continues to increase back to the steady-state, labour becomes more expensive, so labour hired by the smallest firms gradually decreases back down to the steady-state level, reducing their output and the funds they have available to invest in R&D and capital. Eventually, their number of goods also returns to the steady-state level. Note, however, that this means that the smallest firms take a much longer time to return to the steady-state than do larger firms.

Nonetheless, the path of the aggregate economy's recovery much more closely resembles that of larger firms because those larger firms account for a larger proportion of output, products, and investment. This is why the aggregate economy's response to a 1% TFP shock is more muted in my model compared to that of Bilbiie, Ghironi and Melitz (2012), thereby demonstrating the importance of taking into account the heterogeneity across firms.

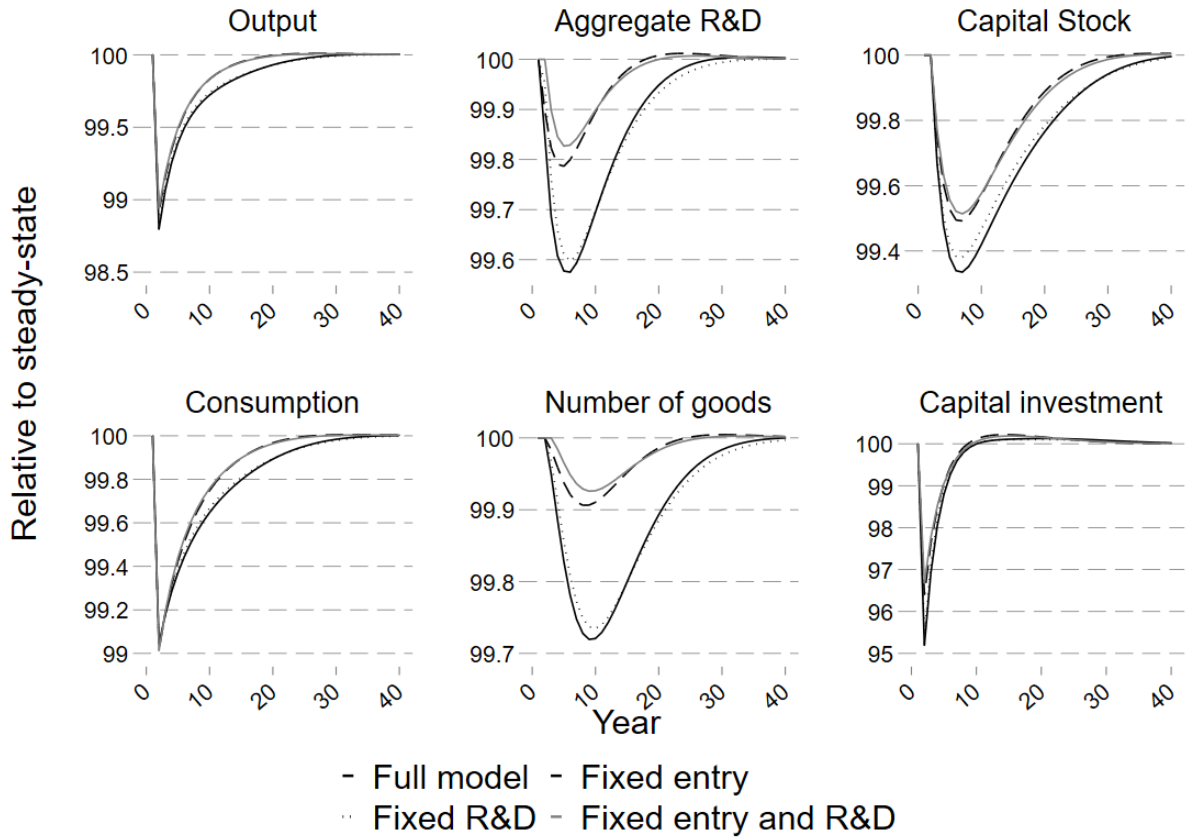
5.3. Relative importance of R&D and entry

To determine the relative importance of R&D and entry in determining the path of the recovery, I conduct a "horse race" between these two elements of the model, the results of which are shown in Figure 10. In particular, I simulate the recovery from a TFP recession for each of the following specifications: 1) the full model, shown by the solid black line; 2) a specification with just potential entrants' decisions fixed at their steady-state level throughout the recovery, shown by the dashed line; 3) a specification with just incumbents' R&D decisions fixed at their steady-state level throughout the recovery, shown by the dotted line; and 4) a specification with both entrants' initial decisions and incumbents' R&D decisions fixed at the steady-state level, shown by the solid grey line .[25] As mentioned in Section 4.2, my model might overstate the relative importance of R&D as R&D is more correlated with output in my model than it is in the data. As such, the models results regarding the relative contribution of entry (compared to R&D) are likely to be conservative. Note, also, that in specifications 3 and 4, in which incumbents' R&D decisions are fixed, incumbents capital investment decisions are adjusted where necessary to ensure that the non-negative dividend constraint is not violated.

Comparing first between the full model and the model with no endogenous responses by incumbents and entrants (i.e. between the solid black and grey lines), it is clear that entrants' and incumbents' responses to the drop in TFP both serve to increase the depth of the recession and the time it takes for the economy to recover. In particular, the drop in output is larger when incumbents and entrants can alter their decisions in response to the aggregate TFP shock. This is exactly as one would expect, as the time in which aggregate exogenous TFP is below its median level reduces the incentive and ability of potential entrants and incumbents to invest in R&D, thereby reducing how much output they produce and how much households can consume. This mechanism is absent when entrants' decisions and incumbents' R&D investments are fixed over time. Note, however, that there is still a decline in aggregate R&D investment (and, hence, the number of goods) even when incumbents' and entrants' R&D decisions are fixed: this is because incumbents' capital investment decisions adjust to ensure that the non-negative dividend constraint is not violated. Hence, firms will end up with different combinations

[25] For each of these specifications, I re-run the solution algorithm described in Section 4.2 and Appendix A.2 to obtain separate forecast rules (as well as value functions and decision rules) for each of the different specifications, which are then sued in the respective simulated recoveries.

Figure 10: Simulated recovery of output, consumption, R&D, number of goods, capital stock and investment after a TFP recession in each specification



Note: The graphs show the simulated time path of aggregate variables in the different specification. Each specification is started at the steady-state distribution and runs for 150 periods with exogenous aggregate productivity at the median value of 1. In the 151st period aggregate TFP is subjected to a 1% reduction (i.e. is set to 0.99) before gradually recovering back to its median value of 1 at a rate determined by ρ_z. The solid black line shows the results for the full model in which entry and R&D vary endogenously each period, while the dashed line shows the path of the economy in which entry is exogenously fixed at the distribution that is obtained in the steady-state. The dotted line shows the recovery for the specification in which incumbents' R&D (and physical capital) decisions are fixed at their steady-state level, while the solid grey line shows the recovery for the specification that fixes both entry and incumbent decision rules. All series are expressed in terms of their percentage deviation from steady-state.

of capital and number of goods compared to the full model, thus meaning that their R&D investment still changes during the recovery from the recession. The overall result is that allowing both entrants and incumbents to re-optimise in response to the path of TFP and the economy (as in the full model) delays the economy's return to its steady-state level by roughly 15 years.

The effect of fixing potential entrants' decisions to their steady-state level means that the recovery is quicker compared to when potential entrants' decisions vary over time: for each aggregate variable, the dashed line (fixing entrants' decisions) follows a similar

path to the solid black line (the full model), albeit shifted upwards. For example, after ten yeas, output is roughly 0.1% points closer to the steady-state level when entrants' decisions do not change over time compared to the full model in which potential entrants' decisions vary over time. This is due to the fact that fixing entrants' decisions means that there is no reduction in the number of entrants, nor in the size of entrants that do enter. Hence, the number of firms investing and producing remains the same throughout the recovery. Note, however, that there is still a small drop in aggregate R&D and the number of goods along the recovery path, and this drop is (slightly) larger than in the case when incumbents' R&D decisions are fixed alongside entrants' decisions. This is because fixing only entrants' decisions means that incumbents are still free to change their R&D investments as the recovery progresses, and the reduction in aggregate TFP and price means that incumbents have less incentive and ability to invest in R&D.

Nonetheless, removing the ability of incumbents to vary their R&D investments over time has a very small effect on the path of the recovery (seen by comparing the dotted line to the solid black line, or the dashed line to the solid grey line). Focusing on comparing the full model (solid black line) to the specification in which incumbents' R&D investment is fixed but entrants can vary their decisions (the dotted line), the recovery paths are almost identical. Indeed, the path of output and consumption are never more than 0.03% points different from each other in any single period. This is because incumbents' R&D decisions do not vary substantially in response to a TFP shock, such that the number of intermediate goods produced by incumbents changes only slightly when aggregate TFP falls. As R&D investment amounts to only about 2% of output and almost all incumbents spend less than 30% of their output on R&D, then it would require very large changes in aggregate TFP for firms to need or want to reduce their R&D investments by sufficient amounts to substantially decrease the number of goods they add each period.

Note, that although not shown here, fixing incumbents' R&D decisions at the steady-state level has no effect on the number of firms in each period, while fixing entrants' decisions at their steady-state level means that the number of firms does not change over time at all (compared to it reaching a trough roughly 0.2% below its steady-state level when entrants can change their decisions over time): in other words, the only difference in the number of firms each period is based on whether or not potential entrants' decisions can vary over time (so the number of firms follows the same path in the full model and when incumbents' R&D decisions are fixed at their steady-state level, while the number of firms

is constant over time when entrants' decisions are fixed over time). Hence, entrants' decisions appear to have a much larger effect on the recovery from a shock to aggregate TFP.

It is also possible to numerically decompose the effect of each of entry and R&D into their contribution to the difference between the full model and the specification with completely fixed entrants' and incumbents' decisions. This is shown in Table 7, in which the first column lists the variable being examined. The second column shows the total absolute difference (in terms of their deviations from the steady-state) between the full model and the specification with completely fixed entrants' and incumbents' decisions. This is calculated as the absolute difference between the full model's deviation from steady-state and the completely restricted specification's deviation from steady-state, summed over the entire period of the simulated recovery. For example, the total difference in output between the full model and the most restricted over the course of the recovery was 2.2% points.[26]

Table 7: Relative contributions of entry and R&D to the economy's recovery from a 1% drop in TFP

	Total Difference	% Contribution from entry	% Contribution from R&D	Total % Contributions
Output	0.022	99%	17%	116%
Consumption	0.021	97%	16%	113%
Number of products	0.032	95%	12%	107%
R&D Investment	0.033	93%	18%	111%
Capital stock	0.036	103%	22%	124%

Note: The second column of this table presents the total (percentage) deviation across the entire 150 period simulated recovery from a 1% drop in TFP comparing the full model (with endogenous entrants' and incumbents' R&D and capital decisions) against a model in which entrants' and incumbents' decisions are fixed at their steady-state level, for the set of variables listed in the first column. The third column obtains the contribution to these total deviations that result from letting entrants' decisions to vary over time and is calculated by comparing the full model with the specification in which entrants' decisions are fixed at their steady-state level and then dividing those deviations by the total deviations. The fourth column does the same exercise except for a comparison between the full model and the specification in which incumbents' decisions are fixed at their steady-state level. The final column is the sum of these third and fourth columns.

The third and fourth columns show, respectively, the contribution to this total difference that results from the endogenous changes in entrants' decisions and the endogenous

[26] It is also possible to conduct a similar exercise by comparing the troughs of each variable (relative to their steady-state levels) in each specification to focus on the severity of the recession, rather than comparing the total deviations which accounts for a combination of the severity and length of the recession. Such an exercise obtains very similar results to those for the total deviations over the entire recession.

change in incumbents' decisions. These are obtained by adding up the absolute difference between the full model's deviation from steady-state and the more restricted specification across the entire recovery and dividing that by the number in the first column. Hence, endogenous entry responses can account for almost the entirety (99%) of the path of output over the recovery, while incumbents' R&D responses account for 17% of the recovery path of output. The final column then shows the "total" contributions of each element, with the fact that these totals sum to more than 100% for each variable indicating that the endogenous responses of entrants and incumbents combined ameliorates the depth and amount of time the recovery takes. In particular, the drop in incumbents' R&D and capital investments means that the reduction in entrants' incentives to enter and invest themselves is ameliorated (and vice versa) during the recovery, such that entrants and incumbents do not alter their decisions by as much as they would have done without the endogenous changes from the other channel.

The response of entrants over time contributes the overwhelming majority (over 90%) to the path of each aggregate variable over the recovery. Hence, entrants' decisions appear to have a much larger effect on the recovery from a shock to aggregate TFP. These results reflect the indicative evidence presented in Section 2: namely, that the introduction of new products returning to its previous level within a few years after the Great Recession compared to the entry rate remaining at least one percentage point below its pre-recession level suggests that declines in entry are a stronger contributor to sluggish recoveries than are any changes in the introduction of new products. This means that, contrary to Bilbiie, Ghironi and Melitz (2012), it is new entry in and of itself, rather than the introduction of new products (by entrants and incumbents), that drives this channel of business cycles. This result suggests that a policymaker who wants to hasten the recovery from a recession might be advised to focus on encouraging more entry rather than trying to increase incumbent investment in R&D, although the question regarding the exact nature of the policies that would be most effective at doing so is left open.

6. Conclusion

Both entry and the introduction of new products by incumbents (driven by incumbents' investments in R&D) have an effect on an economy's path over the business cycle. I have developed a discrete time dynamic stochastic general equilibrium model with

heterogeneous multi-product firms that invest in R&D to gain the chance to innovate and introduce a new product to their existing portfolio of products. Potential entrants also make endogenous entry and initial investment decisions endogenously in my model. Aggregate uncertainty arises due to the presence of exogenous aggregate productivity.

To investigate the relative importance of entry compared to (incumbent) R&D, I conduct a "horse race" between the two different channels by turning off each one independently and then combined to examine just how much of an effect each channel has on the recovery of an economy from a 1% decrease to aggregate TFP. I show that the effect of entrants' decisions has a larger impact on those business cycles than do the R&D investment decisions of incumbents. In particular, entrants' decisions account for more than 90% of an economy's recovery from a recession caused by a shock to aggregate TFP, whereas incumbents' R&D investment decisions account for roughly 20% of the recovery. The fact that the combined contribution of both channels is greater than 100% indicates that the response over time of one channel ameliorates the effect of the recession and recovery on the other channel, and vice versa.

These results imply that if a policymaker wanted to speed up the recovery from a recession then they should try to stimulate entry by new firms rather than incentivising incumbents to increase their R&D spending. However, further work would be required to determine the exact nature of the policies that would be most effective at increasing entry so as to hasten the recovery from a recession. There are also additional potential avenues for future research: for example, incorporating firms being able to borrow subject to a collateral constraint would let one examine whether the relatively higher importance of R&D over entry is also true in response to financial shocks as it is for shocks to aggregate TFP.

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Appendix A. Solution algorithm

In this Appendix I first set out the solution algorithm I use to solve for the non-stochastic steady-state before describing the solution method I use to obtain my results with aggregate uncertainty in the form of shocks to aggregate TFP. Throughout, a firm's optimum labour-hiring is given by the solution to $\pi(k, \varepsilon_o, z, \mu) = \max_{\ell,p} p(k, \varepsilon_o, z, \mu) y(z, \mu) - w(z, \mu)\ell$ where $y = z\varepsilon_o k^{\alpha}\ell^{\gamma}$. This results in optimum labour hiring given by

$$l^*(k, \varepsilon_o, z, \mu) = \left[\frac{w(z, \mu)}{\gamma^{\frac{\theta-1}{\theta}} Y(z, \mu)^{\frac{1}{\theta}} (\varepsilon_o z)^{\frac{\theta-1}{\theta}}} k^{-\alpha\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\gamma\theta-\gamma-\theta}}$$

I then plug this back into Equation (7) when solving for firms' choices of capital k' and R&D R.

Appendix A.1. Steady-state

Individual firms' decisions of how much to produce depend on the wage (which itself depends on aggregate consumption as discussed in Section 3.1) and aggregate output. Therefore, the first step is to make guesses of aggregate output and consumption.

Next, for each starting combination of the number of products and output productivity, I obtain a firm's optimum choice of capital for the next period k' conditional on its choice of investment (assuming that the firm can afford both without paying negative dividends; i.e. that it is "unconstrained"). A firm's choice of capital for the next period is given by maximising Equation (7) with respect to k' and then applying the Benveniste-Shenkman result to obtain an expression for the derivative of the value function in the next period. This gives the optimum choice of capital for the next period, conditional on the choice of R&D investment as:

$$k' = \frac{\frac{q(z,\mu)}{\beta} - (1-\delta_k)\sum q(z', \mu')}{\frac{-\alpha(\theta-1)}{\gamma\theta-\gamma-\theta}\left[\Phi(R)((1-\delta_n)N+1)\sum q(z', \mu')\Omega + (1-\Phi(R))(1-\delta_n)N\sum q(z', \mu')\Omega\right]}$$

where

$$\Omega = Y^{\frac{1}{\theta}}(\varepsilon_o z)^{\frac{\theta-1}{\theta}}\left[\frac{w}{\gamma(\frac{\theta-1}{\theta})Y^{\frac{1}{\theta}}(\varepsilon_o z)^{\frac{\theta-1}{\theta}}}\right]^{\frac{\gamma(\theta-1)}{\gamma\theta-\gamma-\theta}} - w\left[\frac{w}{\gamma(\frac{\theta-1}{\theta})Y^{\frac{1}{\theta}}(\varepsilon_o z)^{\frac{\theta-1}{\theta}}}\right]^{\frac{\theta}{\gamma\theta-\gamma-\theta}}$$

Note that as the optimum unconstrained choice of k' depends on the probability of a successful innovation $\Phi(R)$, which itself is a function of R&D investment, the unconstrained choice of k' depends on the choice of R&D investment.

In the next step, I use this expression to solve for the decision rules via Value Function Iteration (VFI) for just "unconstrained firms using a single variable Golden Section Search over R&D investment R. In other words, I assume that all firms can afford the optimum choice of R&D and physical capital and search over their choice of R&D investment (obtaining the implied choice of k' via the expression above) and obtaining the value of that choice. Note that as this optimum choice of k' does not depend on a firm's starting capital, and this stage assumes that firms are unconstrained by the non-negative dividend condition, these value functions only need to obtain the optimum choices for starting number of products and output and research productivities. In other words, the VFI at this stage does not need to loop over starting capital stock for unconstrained firms.

Next, I identify which firms are unconstrained and those which are "constrained": the former are able to afford the unconstrained optimum choices of R&D and physical capital found in the previous step without violating the non-negative dividend constraint. Constrained firms, on the other hand, are not able to afford these choices without paying negative dividends. These firms, therefore, must pay zero dividends and are constrained to ensure that $N\pi(k, \varepsilon_o, z, \mu)_i - \frac{R}{\varepsilon_r} - \mathbb{I}_r\xi_r - k' + (1-\delta_k)k = 0$. In other words, a constrained firm's choice of k' depends on its investment in R&D such that $k' = N\pi(k, \varepsilon_o, z, \mu)_i - \frac{R}{\varepsilon_r} - \mathbb{I}_r\xi_r + (1-\delta_k)k$. This means that, even for constrained firms, their decision over the two choice variables R&D investment and physical capital stock for the next period can be reduced to a Golden Section Search over just R&D investment. Using this, I then conduct a second value function iteration (using the value function found in the previous step as the starting point) and find the decision rules for all constrained firms (leaving those for unconstrained firms unchanged) to get a value function for all firms. I then use this single value function to obtain the decisions (whether or not to try to enter, their initial R&D investment, and their initial physical capital investment) made by potential entrants.

Using these decision rules I iterate on the distribution until it converges and use that distribution along with firms' decision rules to obtain a new value for aggregate output and consumption. The initial guesses of these variables are then updating using Broyden's algorithm until the initial guess and the new values converge.

Appendix A.2. With aggregate uncertainty

The presence of aggregate uncertainty in the form of varying aggregate productivity z (which can take three values) means that I use the Krusell-Smith solution method to solve my model, adapted for heterogeneous firms as per Khan and Thomas (2013). This means that I approximate the distribution of firms μ using the summary statistic of the total number of products $\bar{N}$ and the aggregate capital stock $\bar{K}$ and use that in the forecasting rules that take the form

$$\log x = \beta_{0,i} + \beta_{1,i} \log \bar{N} + \beta_{2,i} \log \bar{K} + v_i$$

where x is the variable to be forecast, β_0, β_1, and β_2 are coefficients to be estimated, v is the error term, and i represents the level of aggregate productivity in the current period. Each forecast rule is obtained via regressions using the final 1,000 observations from a simulation of 1,500 periods (i.e. the first 500 periods of the simulation are dropped when estimating the forecast rules).

As mentioned in the main text, I require forecasting rules for aggregate output Y (as that affects the price intermediate firms charge for their output, thereby affecting the output of those intermediate firms) and the household's marginal utility of consumption q (to determine the marginal utility value of firm profits today, as firms are owned by the household) in the current period. I also need forecasting rules for the number of products next period $\bar{N}'$ and the aggregate stock of capital next period $\bar{K}'$.

The inner loop of my solution algorithm uses a guess of the forecasting rules for each of these four variables alongside the conditional optimum capital stock and value function iteration procedures described in Appendix A.1. In an outer loop, I simulate the economy for 1,500 periods, using Broyden's algorithm each period to obtain the marginal utility of consumption q and aggregate output Y that prevails in that period (and which firms use in their decision making). I then estimate new forecast rules dropping the first 500

observations of the simulated economy and update the guess of the forecasting rules. This iterative procedure continues until convergence.

The estimated coefficients, and some regression statistics for each of these forecast rules for the full model are shown in Table A.8. The first column indicates which variable is being forecast and the second column shows to which level of the aggregate productivity in the current period the forecast rule applies. Columns three, four, and five contain the estimated coefficient values, while column six shows the number of observations. The final three columns contain statistics that represent the accuracy of these forecast rules: column seven shows the standard error of the regression itself; column eight shows the R^2 of the forecast rules; and the final column shows the maximum Den Haan error from a simulation of 1,500 periods.[27]

Table A.8: Forecast rules for the full model

Forecast rule for	Aggregate productivity	β_0	β_1	β_2	Obs	SE	R^2	Max Den Haan error
$\bar{N}'$	z_1	0.431	0.778	0.104	278	5.35E-04	0.9977	0.0092
	z_2	0.517	0.736	0.122	511	5.44E-04	0.9976	0.0094
	z_3	0.620	0.683	0.149	212	4.36E-04	0.9983	0.0085
$\bar{K}'$	z_1	0.041	0.028	0.844	278	2.01E-04	0.9998	0.0096
	z_2	0.011	0.051	0.824	511	1.94E-04	0.9999	0.0100
	z_3	-0.025	0.076	0.804	212	1.75E-04	0.9999	0.0082
q	z_1	1.257	-0.373	-0.297	278	3.65E-04	0.9987	0.0111
	z_2	1.215	-0.365	-0.296	511	2.96E-04	0.9991	0.0083
	z_3	1.169	-0.352	-0.303	212	2.96E-04	0.9991	0.0059
Y	z_1	-0.847	0.414	0.060	278	4.08E-04	0.9956	0.0090
	z_2	-0.790	0.401	0.063	511	3.29E-04	0.9971	0.0049
	z_3	-0.731	0.384	0.076	212	3.27E-04	0.9970	0.0072

Note: SE is the standard error in each regression

In each forecasting rule, and for each value of the aggregate TFP shock z, the R^2 is above 0.99 and the maximum Den Haan error is only 1.1%, with the majority being below 1%. In

[27] This maximum Den Haan error is obtained by obtaining a multi-period ahead forecast using the forecasting rules and comparing it to the actual outcome of the economy simulated over that same number of periods.

other words, these forecasting rules are accurate in terms of agents in the model correctly forecasting the simulated economy's actual state. Although I only present forecasting rules for the full model here (i.e. the specification in which incumbent firms' and potential entrants' decisions can vary over time), I obtain new forecast rules for each specification of the economy by repeating the solution algorithm described above for each specification (not presented here) and each of those obtains similar levels of accuracy as the forecasting rules for the full model.