A Macro-Finance Model with Ambiguity as Partial-Identification Problems

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Introduction

A large **macro-finance** literature aims at developing general equilibrium models featuring

- rational expectations eqm: true data generating process aligns with agents' subjective belief
- countercyclical risk premium and acyclical dividend growth expectations
- policy prescriptions involving actual implementations during market downturns
  He and Krishnamurthy (2012, 2013); Brunnermeier and Sanikov (2014); Di-Tella (2017)

New empirical findings from **survey expectations measures of market participants**

- acyclical subjective risk premium and countercyclical dividend growth expectations from survey
  De la O and Myer (2021); Nagel and Xu (2023)

Policy announcements without involving actual implementations alter asset valuations

- Mario Draghi "Whatever it takes" during market downturns of sovereign bonds in Euro area
  Haddad, Moreira, Muir (2024)

This Paper

Develop a new **non-rational expectations equilibrium framework with financial frictions**

- agents' uncertainty is represented by **a set of beliefs (models)** i.e. **ambiguity**.
- discipline the set of models based on observable data and knowledge of economic structure
- the model is partially identified and produces model uncertainty.
- agents make cautious decision under a model with adverse consequences from the set.
- decisions have equilibrium implications that feed back to observable data.

Finding: model rationalizes discrepancy of expectations based on survey and econometricians observe.

Policy promises to **affect the partially-identified set of models agents consider as plausible**.

- these policies may not require actual implementations.
- structural framework allows to assess the efficacy of alternative promises.

Literature

Technology, Markets, Agents

Endowment economy, infinite horizon, continuous-time in spirit of He and Krishnamurthy (2013)
Exogenous aggregate dividend hit by aggregate shock:

$$\frac{dD_t}{D_t} = gdt + \sigma dZ_t.$$

Two assets:

- risky asset is claim on aggregate dividend with a fixed unit supply
- risk-free asset with zero net supply

Agents: unit measure of intermediaries and identical households

Economic Structures: Households' Behavior and Financial Frictions

Each household j can invest

- $H_{j,t}$ in a randomly matched intermediary subject to margin constraint:

$$H_{j,t} \leq \underbrace{m_i}_{\text{inverse tightness of constraint}} \times \underbrace{w_{i,t}}_{\text{intermediary's wealth}}$$

- $H_{j,t}$ is invested in the levered risky asset portfolio managed by the intermediary.
- rest of wealth in risk-free asset without frictions

Economic structures affecting asset prices:

1. smaller aggregate intermediary wealth share $x_t = w_t/P_t$ or lower $\{m_i\}_i$ leads to
   - constraint binds and household fund flows to risky asset market are restricted
   - temporal asset price decline and higher expected return as intermediaries accumulate wealth
2. permanent decline of dividend growth g leads to lower risky asset price.
   - permanently lower asset prices and lower expected return

Intermediaries' Preferences: Maximization Part

Given a subjective belief S, an intermediary $i \in [0,1]$ solves a standard Merton-type problem:

$$\max_{\{c_{i,t},\alpha_{i,t}\}} \underbrace{\mathbb{E}^S}_{\text{expectations under a subjective belief } S} \left[\int_0^\infty e^{-\rho t} \frac{c_{i,t}^{1-\gamma}}{1-\gamma} dt \right]$$

s.t.

$$\frac{dw_{i,t}}{w_{i,t}} = \alpha_{i,t}(dR_t - r_t dt) + r_t dt - \frac{c_{i,t}}{w_{i,t}} dt.$$

Optimal portfolio choice crucially depends on subjective expected returns:

$$\alpha_{i,t} = \frac{1}{\gamma} \frac{\mathbb{E}_t^S(\overbrace{dR_t - r_t dt}^{\text{excess return for risky asset over risk-free rate}})}{\underbrace{\sigma_{R,t}^2}_{\text{excess return variance}}} + \textit{intertemporal hedging}.$$

Intermediaries are confronted with uncertainty about which model to use to forecast future return.

Intermediaries' Preferences: Minimization Part

Uncertain averse intermediary $i \in [0,1]$ uses the worst-case model to forecast the future:

$$\min_{S \in \Xi} \max_{\{c_{i,t}, \alpha_{i,t}\}} \underbrace{\mathbb{E}^S}_{\text{expectations under a subjective belief } S} \left[\int_0^\infty e^{-\rho t} \frac{c_{i,t}^{1-\gamma}}{1-\gamma} dt \right]$$

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This paper: the set of models Ξ viewed as plausible is endogenous.

- minimization chooses the belief leading to the lowest expected excess return $\pi_{R,t}^S \equiv \mathbb{E}^S(dR_t - r_t dt)$

How Intermediaries Form a Set of Models

Intermediaries want to predict the expected excess return $\pi_{R,t}$

- understand the mapping from parameters and states to $\pi_{R,t}$, i.e., economic structure in eqm
- but not know the true values of parameters and state

$$\Big(\underbrace{x_t}_{\text{aggregate intermediary wealth share}} , \underbrace{m}_{\text{inverse tightness of margin constraint}} , \underbrace{g}_{\text{long-run dividend growth rate}} \Big)$$

Use understanding of economic structure and information from asset markets to infer current $(x, m.g)$.

$$\underbrace{\sigma_{R,t}}_{\text{current observation}} = \underbrace{\sigma_R(x, m, g)}_{\text{model}};$$

$$\underbrace{r_t}_{\text{current observation}} = \underbrace{r(x, m, g)}_{\text{model}}.$$

Partial identification problem: many combinations of (x, m, g) are consistent.

- denote the set of those combinations as Ξ_t, partially-identified set
- each combination has different implication for $\pi_{R,t}(x, m, g)$.

Illustration of Economic Structure: Return Volatility

Return volatility

$$\sigma_R = \frac{p'}{p} \underbrace{\sigma_x}_{\text{Wealth share volatility}} + \sigma$$

Volatility of wealth share of intermediaries

$$\sigma_x = \frac{\overbrace{x}^{\text{Size of intermediaries}} \overbrace{(\alpha - 1)\sigma}^{\text{Portfolio heterogeneity}}}{\underbrace{1 - x(\alpha - 1)\sigma}_{\text{Amplification}}},$$

α: portfolio share of aggregate intermediaries on risky asset

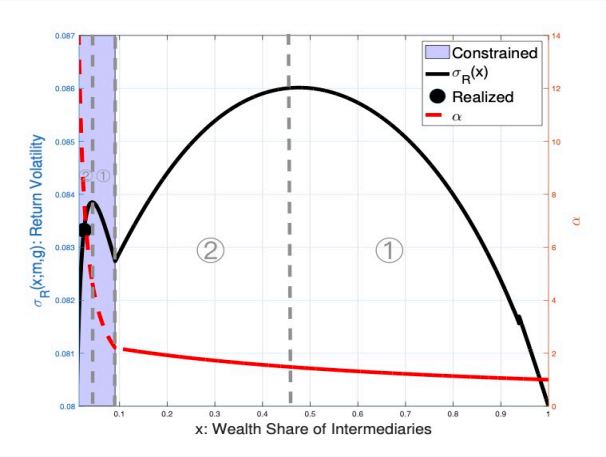
Negative aggregate shock hits: levered intermediaries with $\alpha > 1$ lose wealth share more than hhs.

- risky asset demand declines but to clear mkt, asset price must go down to induce more leverage

Pricing impact depends on leverage α and wealth share x

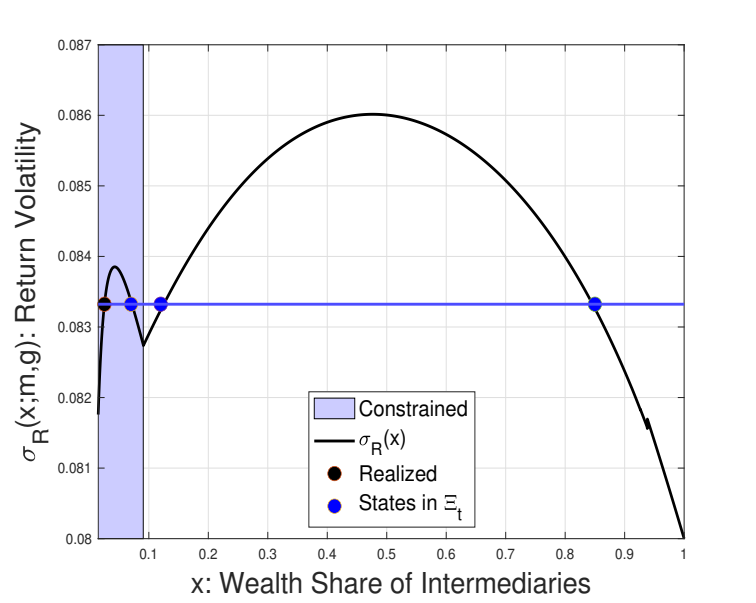
1. higher leverage requires more risk taking and much lower price
2. but pricing impact is limited if intermediaries wealth share is small

Illustration of Economic Structure: Return Volatility



- In region 1, higher leverage effect dominates.
- In region 2, lower wealth share dampens pricing impact.

Multiple x's Consistent with Observations



Partially Identified Set during Market Downtown

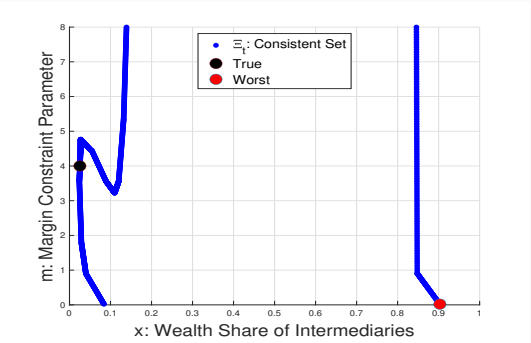
Observations when aggregate wealth share of intermediaries x is actually lower

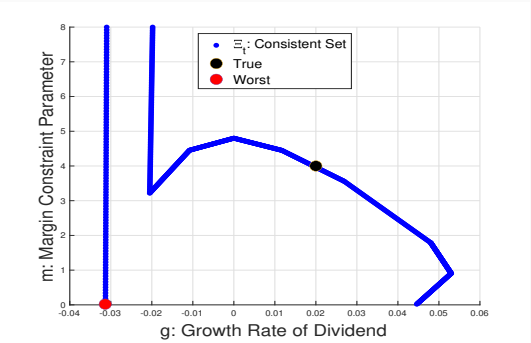
- return volatility σ_R is high
- risk-free rate is low

Individual intermediaries are uncertain about which observationally-equivalent scenario is realized:

1. low x causes higher return volatility and low risk-free rate due to higher precautionary savings
2. high x and low g cause high return volatility and low risk-free rate due to slower economic growth

The latter is more adverse due to permanent asset price decline and lower expected return π_R.





Calibration

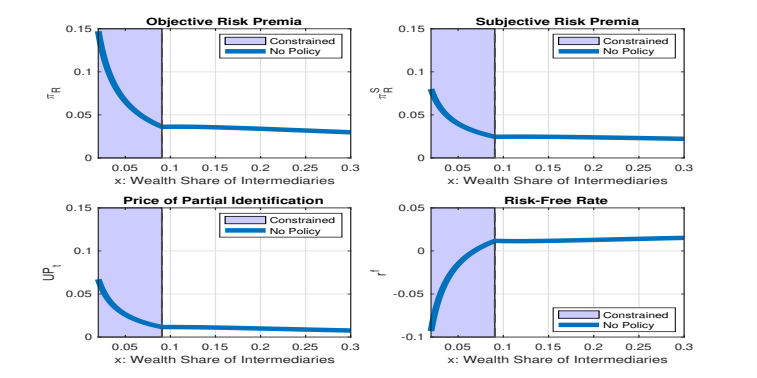
Parameter	Description	Value	Target	Target value	Model
γ	Relative Risk Aversion	1.8	Average expected excess return of MBS	3.4	3.8
ρ	Discount Rate	0.08	Average risk-free rate	1	0.78
σ	Dividend Volatility	0.08	Return volatility of MBS	0.81	0.83
λ	Debt Household Share	0.6	Average Debt-to-Asset ratio in 2007	0.52	0.55
l	Labor Income Ratio	1.84	Share of Labor Income in Total Income	0.66	0.64

Table 1: Matched Moments and Internally Calibrated Parameters

Parameter	Description	Value	Source
m	Intermediation multiplier	4	HK (Share of managers compensation in intermediarys' profit)
g	Dividend Growth	2%	HK (Average real output growth in the U.S.)

Table 2: Fixed Parameters

Equilibrium Returns: Pessimism in Tail



$UP_t = \pi_{R,t} - \pi_R^S(x_t^{worst}, m_t^{worst}, g_t^{worst})$.

- compensation for uncertainty about expected returns

Around 40% from UP_t, amplified during crises survey evidence

Survey Evidence for Subjective Beliefs: Why Is Asset Price so volatile?

One-year ahead contribution of subjective expectations to price-dividend ratio:

$$\underbrace{\frac{cov(\mathbb{E}_t^S(\log D_{t+1}/D_t), \log P_t/D_t)}{var(\log P_t/D_t)}}_{CF_1\text{: dividend growth contribution}} + \underbrace{\frac{-cov(\mathbb{E}_t^S(R_{t+1} - R_t), \log P_t/D_t)}{var(\log P_t/D_t)}}_{DR_1\text{: discount rate contribution}} + \rho \frac{cov(\mathbb{E}_t^S(\log P_{t+1}/D_{t+1}), \log P_t/D_t)}{var(\log P_t/D_t)} = 1.$$

	Subjective (Model)	Survey data (De la O (2021))	Objective (Model)	Rational Model
CF_1	0.19	0.41	0	0
DR_1	-0.13	-0.05	0.06	0.12

Price-dividend ratio is more volatile in data, driven by

- countercyclical and volatile subjective expectations for dividend growth CF_1
- not countercyclical return expectations DR_1
  He and Krishnamurthy (2012, 2013); Brunnermeier and Sanikov (2014); Di-Tella (2017)

Survey Evidence for Subjective Beliefs: Lower Future Return during Downturns

Subjective risk premium is more acyclical than objective (what econometricians observe) in the model.

One-year ahead predictive regressions:

$$\mathbb{E}_t^S(R_{t+1} - R_t - \int_0^1 r_{t+\tau} d\tau) = \beta_0^S - 0.17 \times \log(P_t/D_t) + u_t^S$$

$$\mathbb{E}_t(R_{t+1} - R_t - \int_0^1 r_{t+\tau} d\tau) = \beta_0 - 0.34 \times \log(P_t/D_t) + u_t$$

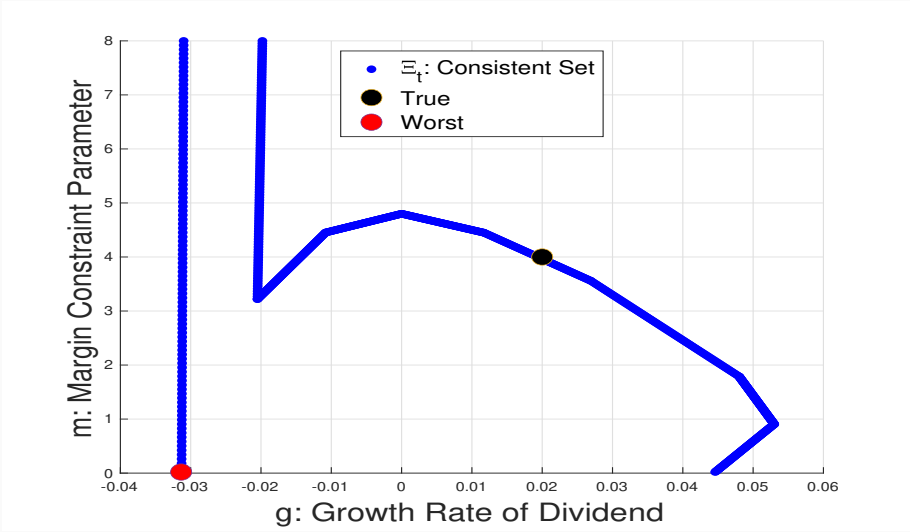
More consistent with survey evidence from Nagel and Xu (2023) than REE:

- slope coefficient for subjective is approximately 1/5 of that for objective
- cannot capture by rational expectations equilibrium

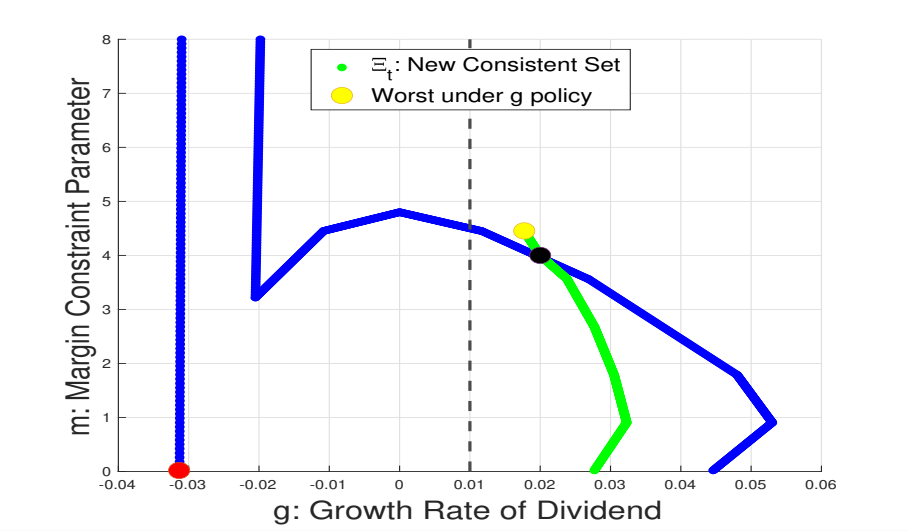
Subjective beliefs in the model is more consistent with measured beliefs than REEs.

Laboratory for belief management policies during market downturn

Partially Identified Set of (m, g) during Market DownTurns

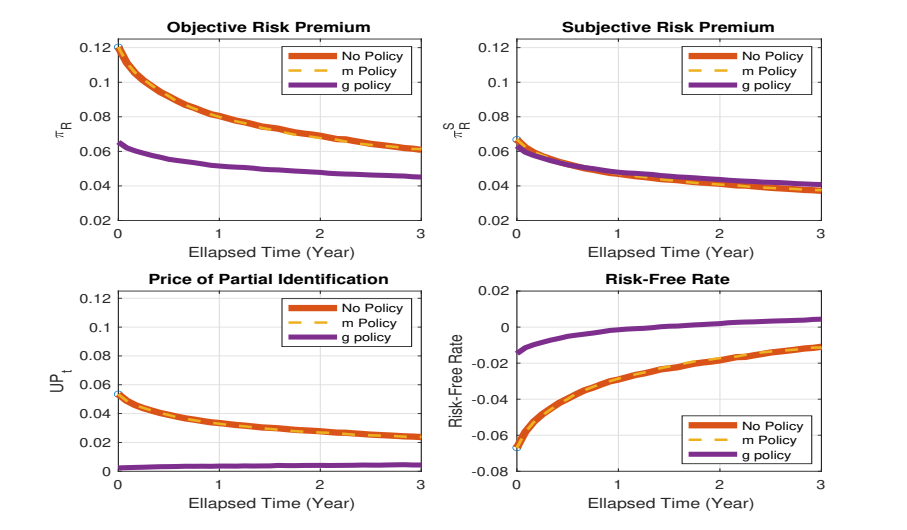


Partially Identified Set of (m, g): Policy Announcement



"Whatever it takes" guarantees the positive dividend growth

Mean Transition Dynamics: g Policy Reduces Risk Premia



Policy announcement reduces uncertainty due to partial identification.

Reduction in subjective and objective risk premia

Conclusion

Develop a general equilibrium asset pricing model with financial frictions

- incorporates novel endogenous formation of the set of models, i.e. ambiguity;
- rationalizes **survey evidence of expectations**;
- is a laboratory for **belief management policies**.

Work in progress: empirically disciplining the set of parameters and state

- option prices for financial sector stock prices are informative about investors' belief about x
- high frequencies, available for various horizons, and entire distributions