Low Interest Rates, Debt Heterogeneity, and Firm Dynamics*

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Abstract

This paper develops a quantitative model of firm dynamics with debt heterogeneity to explore the implications of changes in real interest rates for firm size distribution and firm dynamics. The analysis demonstrates that the decline in long-term real interest rates since the early 1980s can explain significant shifts in employment shares, firm size distribution, and debt composition observed in the U.S. over the same period. In the model, firms endogenously choose debt covenants under either earnings-based (EBC) or asset-based (ABC) borrowing constraints by deciding whether to pledge their earnings. A decline in real interest rates disproportionately benefits firms with EBC due to their relatively less reliance on accumulated wealth and the large debt capacity generated by earnings-based financing. As firms with higher earnings optimally choose earnings-based lending, the decline in real interest rates shifts employment shares toward larger firms. Furthermore, the expansion of large firms crowds out smaller firms through general equilibrium effects. The paper empirically tests this mechanism using data on U.S. firms and finds a strong association between the decline in real interest rates and employment changes, particularly in firms with more earnings-based financing.

Keywords: Real Interest Rate, Earnings-Based Borrowing Constraints, Firm Dynamics, Firm Size Distribution, Debt Heterogeneity

JEL Codes: E22, E24, E44, G31

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1 Introduction

Over the past few decades, the real interest rate in the U.S. has declined sharply, falling from about 6 percent in the early 1980s to nearly zero by 2019. This significant decline raises an important question: how have falling real interest rates affected firm size distribution and dynamics in the U.S. economy? Addressing this question is crucial as the period of declining interest rates has coincided with notable shifts in the U.S. corporate landscape: (i) employment has shifted from smaller to larger firms, increasing the employment shares of large firms; (ii) the distribution of employment size has become more dispersed; (iii) the number of firms per capita and the firm entry rate have steadily declined; and (iv) large firms have increasingly dominated corporate credit.

This paper argues that the decline in long-term real interest rates can account for these changes in firm dynamics through its asymmetric effects on small and large firms. The source of these asymmetric effects is debt heterogeneity by firm size. Recent literature has documented that firms face different types of borrowing limits (Lian and Ma, 2021). Specifically, large public firms issue debt against their earnings (earnings-based borrowing limits), while other firms tend to borrow against their physical assets or collateral, also known as asset-based or collateral-based borrowing limits.

In particular, this paper studies the effects of long-term real interest rates on the firm size distribution and firm dynamics within a general equilibrium model, following the approach of Hopenhayn (1992). A key deviation from the Hopenhayn model is that firms must finance the capital used in production by endogenously choosing between two types of financing: earnings-based and asset-based financing. In particular, while all firms have access to asset-based financing, they can decide to pledge and borrow against their earnings, similar to Diamond et al. (2022). In equilibrium, these differences result in heterogeneous debt financing by firm size, and the difference in pledgeability, in particular, leads to the coexistence of earnings-based and asset-based borrowing limits.

The equilibrium results of the model demonstrate asymmetric effects: the decline in real interest rates can significantly shift employment shares toward large firms and, meanwhile, generate

crowding-out effects for smaller firms. Quantitatively, the decline in real interest rates can explain about 80% of the changes in employment shares, about 27% of the decline in firms per capita, and more than 12% of the decline in firm entry rate since the early 1980s. Additionally, the model can explain about 27% of the change in U.S. debt composition over the same period, particularly the increase in earnings-based financing.

Debt heterogeneity is crucial for understanding these impacts. The results suggest that the secular trends in firm size distribution and dynamics are closely linked to the decline in real interest rates. This linkage is novel, and the findings are particularly noteworthy because the decline in real interest rates, conventionally considered a positive financial development, can actually generate pervasive and significant negative effects on the employment and profitability of small firms.

Why can the decline in real interest rates generate asymmetric effects? Ultimately, earnings-based firms find it beneficial to expand their size more in a low-rate environment. Firms internalize the impact of production expansions on relaxing financial conditions when determining their optimal size. Asset-based financing relies more heavily on accumulated wealth, of which the accumulation becomes costlier, particularly in an environment with a lower rate of return. In contrast, earnings-based financing depends less on accumulated wealth and, importantly, is often associated with a sufficiently large debt capacity, rooted in its close connection to firms' going-concern values.[1] A higher debt capacity implies that firms utilizing earnings-based financing will find it beneficial to expand their size relatively more, as they can relax borrowing limits through additional profits generated by the expansion, so that they can issue more debt at a lower interest rate, provided that the marginal revenue from such expansion is not too low.[2] Additionally, when the interest coverage ratio, a common metric used in earnings-based borrowing, is applied, a lower rate further amplifies the debt capacity of earnings-based financing through decreased interest expenses. In equilibrium, the expansion of highly productive firms using earnings-based financing increases competition, generating crowding-out effects for other firms.

[1] Debt capacity is defined here as the maximum amount of debt that a given firm can borrow.

[2] The existence of depreciation rates and/or intermediary costs, or more generally, interest spreads, guarantees this condition.

The model economy consists of three types of agents: entrepreneurs who run the firms, financial intermediaries, and the representative household. Each period, an exogenous number of potential entrants draw their initial productivity and asset levels. After observing these levels, they decide whether to enter. Incumbent firms face an exogenous exit shock at the beginning of each period. Surviving incumbent firms then choose whether to continue operating or exit. All firms can own physical assets and use labor and capital as inputs to production. Firms finance capital by raising debt from financial intermediaries. Entrepreneurs who exit the market retain their assets.[3]

The model features two types of debt financing offered by financial intermediaries from which firms can borrow: earnings-based and asset-based financing. As in Diamond et al. (2022), I assume that all firms have access to asset-based financing; however, they have the option to access earnings-based financing by making their earnings pledgeable, which incurs a cost. The pledgability of earnings, particularly future earnings in a dynamic setting, can not only generate the reduced-form earnings-based borrowing limits observed in the real world but also rationalize the sufficient debt capacity of such financing, which is one of the key drivers of the model's results. In the model, differences in cost structures and potential borrowing capacity lead to sorting: firms with greater profits and greater capital needs will endogenously choose earnings-based lending, while others will opt for asset-based debt.

Finally, there is a representative household that resides in the economy. Each period, the household provides labor, earns wages, consumes goods, and saves through financial intermediaries. I assume that the household is sufficiently patient to act as the lender, while entrepreneurs are the borrowers in the economy. Changes in household preferences will drive changes in the real interest rate.[4]

Based on the model, I study the effects of the decline in real interest rates on the firm size distribution and firm dynamics. To explore the mechanism behind the heterogeneous effects of the

[3]This setting is similar to models of firm dynamics with financial frictions in the literature. See examples in Cooley and Quadrini (2001); Moll (2014); Kiyotaki et al. (2021), among others.

[4]This assumption can be interpreted as the decline in the real interest rate resulting from factors exogenous to firm owners, such as the global saving glut (Bernanke, 2005) or increasing wealth inequality among households (Dynan et al., 2004). Similar assumptions are commonly made in other studies (e.g., Liu et al., 2019; Chatterjee and Eyigungor, 2020; Hall, 2016).

decline in real interest rates on firms, I first analyze a simplified version of the model by abstracting from firm entry/exit and endogenous debt choices. This simplification allows for the examination of the determinants of a firm's optimal size and provides analytical solutions that elucidate the mechanism.

The simplified model reveals that a decline in real interest rates can have heterogeneous impacts on firms' optimal decisions. In the long run, a firm's optimal size decision involves a trade-off between the marginal benefits of expansion and the associated costs. Lower interest rates reduce the cost of financing, thereby incentivizing both types of firms to pursue expansion. In determining their optimal size, firms account for the impact of expansion on their borrowing constraints. For asset-based firms, a one-dollar investment in capital increases asset holdings, thereby increasing borrowing capacity. However, this necessitates the accumulation of additional wealth, which becomes increasingly costly in a low return environment. In contrast, earnings-based firms benefit from a one-dollar investment in capital by generating marginal revenues, which similarly relaxes their borrowing limits. The relatively lower reliance on accumulated wealth then, coupled with the ample debt capacity, positions these firms to expand their production more aggressively in response to declining interest rates, provided that the marginal revenue from such expansion remains adequate. Moreover, when an interest coverage ratio is applied, a lower interest rate, equivalently indicating a higher discounted value of future cash flows, further increases debt capacity and amplifies this advantage. In equilibrium, such expansion generates crowding-out effects, resulting in a shift in employment shares and changes in firm dynamics.

Reasonable parameterization ensures that these conditions hold: namely, a sufficiently large debt capacity through earnings-based financing and a sufficiently high marginal revenue from production. The debt capacity is rooted in the close connection between earnings-based borrowing limits and firms' going-concern values. The existence of depreciation and interest spreads prevents the marginal revenue from becoming too low.[5] Interestingly, in this simplified setting, changes in capital prices do not alter the results. While an increase in capital prices, for instance, raises the

[5]Note that depreciation also dampens the pledgeable portion of collateral, which reduces the debt capacity of asset-based financing .

value of collateral, it also indirectly increases the marginal revenue of capital endogenously due to the higher cost of capital, leading earnings-based firms, similar to their asset-based counterparts, to adjust production at the same rate when the capital price changes.

The simplified analysis provides a rich and intuitive demonstration of the underlying mechanism. However, the focus on the steady state and the assumption of exogenous debt types, while necessary for this long-term context, could overlook other important channels that may be at work. In particular, lower interest rates might prompt firms at the margin to switch their debt types endogenously. The larger debt capacity and reduced reliance on accumulated wealth associated with earnings-based financing could significantly facilitate the growth of highly productive firms along their transition paths, especially when the demand for debt increases in a lower interest rate environment.

Thus, I use the full model to perform quantitative experiments and analyze the impacts of the decline in long-term real interest rates on firm size distribution and firm dynamics. I calibrate the model to match a rich set of moments from U.S. firm dynamics, size distribution, and debt composition in the 1980s. The model provides an excellent fit for these moments. I then conduct quantitative experiments within the model, specifically subjecting the economy to a decline in real interest rates, as observed in the U.S. from 1980 to 2020. The experiment demonstrates that the calibrated model can explain a significant fraction of long-term changes in the U.S. economy in terms of stationary equilibrium. Specifically, the model explains about 80% of the shifts in employment shares and 27% of the changes in debt composition experienced in the U.S. over the past forty years. It can also account for more than 27% of the decline in firms per capita and 12% of the decline in the firm entry rate. A counterfactual analysis, based on an economy where only asset-based financing is available, shows that without debt heterogeneity, the decline in real interest rates cannot generate the patterns mentioned above, which demonstrates that debt heterogeneity plays a primary role in generating these effects.

Finally, I empirically test the model's mechanism using a comprehensive dataset of U.S. firms drawn from Compustat and Capital IQ, covering the period from 2000 to 2019. The analysis

investigates the relationship between firm-level interest rates and employment, focusing on the differential impact based on firms' reliance on earnings-based financing. By employing an identification strategy that leverages both time and industry or firm-specific fixed effects, I estimate how changes in interest rates influence employment across firms with varying degrees of reliance on earnings-based financing. The results demonstrate that firms with a higher reliance on earnings-based financing exhibit a greater response to interest rate changes, particularly in their employment decisions. Specifically, when interest rates decline, these firms show a more pronounced increase in employment compared to firms that depend more heavily on asset-based financing. These findings are consistent with the model's predictions and underscore the critical role that firm-specific financial structures play in mediating the effects of changes in interest rates on employment.

Related literature. This paper is related to several strands of literature, particularly those examining changes in firm size distribution and firm dynamics, the role of heterogeneous debt financing, and the impact of declining long-term real interest rates on concentration.

Firm size distribution and dynamics. Recent studies have documented significant shifts in the U.S. firm size distribution, characterized by a more dispersed distribution and a shift in employment shares from smaller firms to large and mature firms, alongside a decline in firm activities.[6] This trend has prompted a substantial body of literature exploring its causes and consequences. For example, Autor et al. (2020) document that shifts in economic activities toward the most productive firms, and in particular reallocation, can account for important secular trends such as the decline in the labor share. Poschke (2018) investigates the impact of skill-biased technological changes on global firm size distribution, including in the U.S., while Begenau et al. (2018) suggests that big data disproportionately benefits large firms, contributing to these shifts. Explanations for the decline in firm dynamics include demographic changes, such as population growth and aging (Hathaway and Litan, 2014; Pugsley and Şahin, 2019; Karahan et al., 2019; Bornstein et al., 2018), business consolidation (Hathaway and Litan, 2014), import competition (Pugsley and Şahin, 2019), and technological changes (Poschke, 2018; Salgado, 2020). Additionally, Akcigit

[6] First documented by Haltiwanger et al. (2011) and Haltiwanger et al. (2012).

and Ates (2021) highlights the decline in knowledge diffusion between frontier and laggard firms as a significant driver of these trends.

Heterogeneous debt financing. My work also intersects with studies on the causes and consequences of heterogeneous debt financing and its implications for firm dynamics. Lian and Ma (2021) show that firms vary in their borrowing limits, with large firms, particularly those borrowing from the credit , predominantly issuing debt under earnings-based borrowing limits, while smaller firms mainly rely on asset-based borrowing. This analysis was extended to small and medium-sized enterprises by Caglio et al. (2021). Subsequent research has focused on the implications of debt heterogeneity for short-run fluctuations (e.g., Greenwald et al., 2019; Drechsel, 2022). My contribution to this literature lies in theoretically demonstrating the importance of future earnings in distinguishing between these borrowing limits and integrating this framework into a firm dynamics model to explore its effects on secular trends in the U.S. economy. Unlike much of the existing literature, which often focuses on the short-run implications of financial frictions, this paper emphasizes the long-term aggregate effects of debt heterogeneity, highlighting its significant impact on firm size distribution and business dynamics.

Impact of declining long-term real interest rates. Another related strand of literature examines the impact of declining long-term real interest rates on market concentration. Liu et al. (2019) emphasize the strategic effects of lower rates on competition between market leaders and followers, while Chatterjee and Eyigungor (2020) focus on the relationship between firm size and leverage, suggesting that lower volatility in large firms' earnings enables greater leverage, leading to reduced entry rates and increased market concentration through acquisitions. Asriyan et al. (2021) discuss how lower interest rates can result in capital misallocation, favoring lower-productivity firms, and Kroen et al. (2021) provide empirical evidence that falling nominal rates disproportionately benefit industry leaders, particularly when initial rates are already low.

Overall, this paper complements these existing studies but differs in several important ways. It introduces a novel channel, debt financing heterogeneity, as a key mechanism through which the decline in long-term interest rates affects the economy. My work shows that the decline in real

interest rates can generate asymmetric effects even without the presence of specific competitive settings or market structures. Furthermore, this paper provides novel empirical validation, offering robust support for the proposed mechanisms and expanding the understanding of how debt heterogeneity influences long-term economic dynamics.

The paper is organized as follows. Section 2 documents the secular trends of the U.S. economy. Section 3 sets up a quantitative model of firm dynamics. Section 4 derives the analytical solutions of a simplified version of the model to provide intuitions. Section 5 reports the results from quantitative experiments. Section 6 provides the empirical evidence to validate the proposed mechanism. Section 7 concludes.

2 Secular Trends of the U.S. Economy

In this section, I present several stylized facts regarding secular trends in the U.S. economy. Specifically, I examine the changes in the long-term real interest rate in the U.S. and provide evidence of heterogeneous debt financing at both the firm and aggregate levels. Additionally, I document the shifts in the U.S. firm size distribution and firm dynamics over time.

2.1 The decline in Real Interest Rates and Debt Heterogeneity

The decline in real interest rates. Figure 1 illustrates the changes in long-term real interest rates in the U.S. over the past 40 years. This figure is based on Federal Reserve Economic Data (FRED), calculated using the 10-year Treasury rate minus the 10-year inflation expectation. It reveals a substantial decrease in the real interest rate, which has fallen from around 6% to below 0% since the early 1980s.

It is helpful to consider some potential explanations for this trend. The debates over the causes of the decline are extensive and remain contentious. Proposed explanations include changes in economic growth, demographic factors, government savings at both national and global levels, and increasing demand for safe assets, particularly in emerging-market economies. Kiley (2020)

and Gamber (2020) provide comprehensive reviews of the empirical evidence and potential causes. For the purposes of this paper, I assume that the decline in the real interest rate is driven solely by changes in lender preferences or foreign factors, rendering the changes in the real interest rate exogenous to the firms in question. Theoretically, the proposed mechanism operates through the real interest rate channel regardless of its underlying causes.[7]

Debt heterogeneity. The existing literature has demonstrated that firms are heterogeneous in terms of their debt types. Colla et al. (2020) provide a comprehensive review of the literature on debt structure, presenting both empirical evidence and potential economic mechanisms that explain the discrepancies between firm debt types and financing costs. As they highlight, most firms tend to specialize in one type of debt. Following the dichotomy proposed by Lian and Ma (2021), debt types can be broadly categorized based on their borrowing limits into asset-based and earnings-based financing.

Figure 2 presents the fraction of different loan types used by Compustat firms, alongside the aggregate composition of debt in the U.S. economy. The left panel replicates the analysis of Lian and Ma (2021), showing the fraction of earnings-based loans used by each Compustat company. I classify large firms as those with employment levels above the median within each 2-digit sector, while the remainder are categorized as small firms. Consistent with Lian and Ma (2021), large firms rely heavily on earnings-based loans, with these loans constituting more than three-quarters of their total debt on average. In contrast, small firms, on average, use only approximately 30% earnings-based loans.

Regarding the aggregate composition, the right panel utilizes data from the Federal Reserve Board to categorize U.S. non-financial corporate debt into earnings-based and asset-based debt. This analysis reveals the heterogeneity of debt types within the U.S. economy, with earnings-based loans comprising more than 40% of total corporate debt. Additionally, the data indicate that firms

[7] This assumption is made for both empirical and theoretical reasons. Empirically, as noted by Krueger and Ludwig (2007) and Kiley (2020), global factors, particularly those originating from foreign countries, play a crucial role in the decline of U.S. real interest rates. Furthermore, Del Negro et al. (2017) show that the premium for safety and liquidity, linked to the saving glut in emerging markets first proposed by Bernanke (2005), dominates other factors. Theoretically, this assumption allows for changes in the real interest rate without altering other parameters, thereby isolating the impact of debt heterogeneity from other factors.

are increasingly borrowing against earnings, reflecting a shift in debt composition over time.

2.2 The U.S. Firm Size Distribution and Firm Dynamics over the Years

Shifts in employment shares. The U.S. firm size distribution has changed dramatically over the past four decades, becoming increasingly dispersed, as documented in previous studies (see, for example, Poschke, 2018). As illustrated in the left panel of Figure 3, employment shares have shifted toward larger firms. The share of firms employing at least 500 workers has increased from 45% to over 50%. Similar trends are observed for firms with at least 1,000 or 2,500 employees, each category experiencing a net increase of approximately 5% in employment share over the past four decades. In contrast, small firms exhibit a distinct pattern. The right panel of Figure 3 shows the employment shares of small firms. Firms with fewer than 10, 20, or 100 employees have seen a decline in their employment share. For instance, in the early 1980s, firms with fewer than 20 employees accounted for approximately 22.5% of total employment. By 2018, this share had declined to below 17.5%.

Changes in the employment size distribution. In addition, consistent with the above patterns, the employment sizes of firms at the top of the distribution grow faster than those of other firms. Since (large) public firms primarily use earnings-based debt, the left-hand side of Figure 4 plots the ratio of the average size of Compustat firms to the average size of all firms over the years. The graph clearly shows that public firms, particularly large public firms, have outgrown other firms on average. While the size of all firms has generally increased over the past years, public firms have grown almost twice as fast as the average firm, with those possessing assets above the sector median growing even faster. For comparison, the right-hand side of the figure shows the ratio of the average size of small firms to the average size of all firms. The increase in firm size dispersion is evident, as the size growth patterns for large and small firms differ significantly.

The change in average firm size and firm dynamics. The left panel of Figure 5 shows that the average firm size has increased dramatically since 1980. Conversely, the number of firms per capita—defined as the total number of firms divided by the total working-age population or the

total number of employees—has declined by about 10% to 15% from its peak around the 1980s, as shown in the middle panel of Figure 5. The firm entry rate, measured as the fraction of new firms among all firms, has also declined steadily. These patterns indicate a decline in new business activities, with fewer firms being created and fewer firms engaged in operating businesses.

3 A Model of Firm Dynamics with Debt Heterogeneity

This section develops a quantitative model of firm dynamics with endogenous firm entry/exit and firm size distribution, building on the framework of Hopenhayn (1992). The model incorporates financial frictions similar to those in Cooley and Quadrini (2001) and Moll (2014). The key distinction of this model, compared to previous literature, is the introduction of earnings-based financing and endogenous choices of debt types.

In the following subsections, I will detail the economic environment, outline the model timing, and describe the problems faced by firms, financial intermediaries, and households. Finally, I will define a stationary equilibrium for the aggregate economy.

3.1 Environment

This subsection describes the model environment. Time is discrete, and the horizon is infinite. The economy consists of three types of agents: entrepreneurs, households, and financial intermediaries. The economy operates along a balanced growth path.

Entrepreneurs (firms). Entrepreneurs own firms. Each period, there is an exogenous measure of $m_0(1+\eta)^t$ potential entrants and an endogenous measure m_t of incumbent firms. Consistent with the literature, firms are assumed to be heterogeneous in their productivity, which comprises a permanent component z and a stochastic component s_t. Firms operate a decreasing-returns-to-scale (DRS) production technology, $y(z, s_t, k_t, l_t)$, using labor l_t and capital k_t as inputs. The output is sold in a competitive, homogeneous final goods market, with the price of the final goods serving as the numéraire of the economy.

All potential entrants receive an initial injection of equity a_0 from an exogenous asset distribution $\Gamma_{a_0}(a_0)$. They then draw a value of z from the initial distribution $\Gamma_z(z)$ and a value of s_0 from the initial distribution $\Gamma_{s_0}(s_0)$. Based on this draw, they decide whether to enter the market and become incumbents by paying the entry cost c_e.

For each period t, incumbent firms may exit either exogenously or endogenously. With probability ξ, a destruction shock occurs, forcing an incumbent firm to exit. Surviving firms observe their new values of s_t, which are drawn from the conditional distribution $\Gamma_s(ds', s_{t-1})$, and then decide whether to exit or continue production. In the case of either exogenous or endogenous exit, the firm distributes its positive net worth to the entrepreneur.

Firms that continue to produce will determine the optimal levels of labor and capital inputs, as well as select a source for financing their capital. All firms operate within a perfectly competitive and homogeneous labor market. Additionally, firms have the option to accumulate assets, which can be used as collateral and to generate interest revenue. To finance their capital, firms borrow from financial intermediaries.

Financial intermediaries and debt choices. Financial intermediaries issue debt to firms, which can be structured under either earnings-based or asset-based constraints, depending on the pledgeability of the firm's earnings. The differences in the pledgeability of earnings lead to two distinct borrowing constraints, arising from the limited commitment of debt contracts. Firms endogenously decide whether to make their earnings pledgeable. Specifically, firms can borrow against collateral, incurring a lower fixed cost per period. In contrast, to borrow against earnings, firms may opt to pay a higher per-period cost to make their earnings pledgeable. As will be shown, the debt capacity under earnings-based borrowing limits is greater than that under asset-based limits. Consequently, firms must trade off higher debt capacity against the different cost structures associated with these financing options.

Households. There is a measure of $H(1+\eta)^t$ representative households, each consisting of $\bar{L}$ family members. Households are risk-neutral and have a discount factor $\sigma_h \in (0, 1)$. They provide labor and receive wage payments from firms that employ their family members. Additionally,

households allocate their savings to financial intermediaries.

Before delving into the details of the firm's problem, it is helpful to outline the timing of events within the model. Each period unfolds in the following sequence: (i) realization of productivity shocks for incumbent firms; (ii) endogenous and exogenous exits of incumbents; (iii) realization of initial productivity and asset levels, followed by the entry decisions of potential entrants; (iv) financing decisions by all firms; (v) production and revenue generation from sales; (vi) payment of wage bills, costs of capital, hiring, and operating expenses; and (vii) saving decisions and household consumption.

3.2 Firm Problem

Exit. At the beginning of each period, incumbent firms face an exogenous shock that may force them to exit. After this shock, firms observe their new productivity levels and decide whether to continue operating and which type of debt to borrow. If a firm decides to exit, it pays out its current net worth. Let $x(z, s, a) \in 0, 1$ represent the exit decision. A firm with a permanent productivity component z, an idiosyncratic component s_t, and equity level a_t has a value V before the destruction shock, which is given by

$$V(z, s_t, a_t) = (1 - \xi) \cdot V^c(z, s_t, a_t) + \xi a_t(1 + r_t),$$

where V^c is the expected value of continuing operations at the end of the period. Firms then choose whether to exit endogenously, and they will do so only if the value offered by the outside option exceeds the value of remaining an incumbent. Thus, the value of continuing operations V^c satisfies

$$V^c(z, s_t, a_t) = \max\{V^i(z, s_t, a_t), a_t(1 + r_t)\},$$

where V^i represents the value of being an incumbent firm.

Entry. In each period, a measure of $m_0(1+\eta)^t$ potential entrants draw their productivity levels and receive equity injections, after which they decide whether to enter the market by paying an

entry cost c_e. Let $i_0(z, s, a) \in 0, 1$ denote the entry decision rule, which depends only on the initial productivity draw and equity injection. A firm will decide to enter if its expected value upon entry exceeds the value of its equity injection.

$$V^i(z, s_0, a_0 - c_e) \geq a_0(1 + r_t)$$

Debt financing choices. An incumbent firm with a permanent productivity component z, idiosyncratic component s_t, and equity level a_t must decide on the source of financing to use. The expected value V^i of the firm is determined by solving

$$V^i(z, s_t, a_t) = \max\{V^a(z, s_t, a_t), V^e(z, s_t, a_t)\},$$

where V^a and V^e are the value functions associated with borrowing under asset-based and earnings-based financing, respectively.

Firm problem. In the spirit of Cooley and Quadrini (2001) and Moll (2014), firms using debt financing of type j ($j \in a, e$) solve the following problem:

$$\begin{aligned} V^j(z, s_t, a_t) = & \max_{d_t, k_t, l_t, a_{t+1}} d_t + \beta_f \mathrm{E_t} V(z, s_{t+1}, a_{t+1}) \\ \text{s.t.} \qquad d_t + a_{t+1} = & f(k_t, l_t; z, s_t) - w_t l_t - \delta q_t k_t - r_{j,t} b_t - f_j + a_t \\ b_t = & q_t k_t - a_t \end{aligned}$$

Firms maximize their value using a discount factor β_f. They determine the next period's equity holding and current dividends by maximizing the revenues from production, net of interest on debt, wage payments, depreciation δ, and a fixed cost f_i associated with the chosen type of financing. The interest rate for financing is given by the real interest rate $r_{j,t}$, which equals the saving rate r_t plus a interest spread ρ_j for positive liabilities.

I further assume that both assets a_t and dividends d_t must be non-negative. Net borrowing,

defined as $b_t = q_t k_t - a_t$, is subject to the borrowing constraints specified below. As shown in Moll (2014), this setting is analogous to a scenario where entrepreneurs own and accumulate capital k and can trade in a risk-free bond. I assume perfect elasticity of transformation between capital goods k_t and consumption goods, as the model focuses on long-run relationships. In the next section, I discuss the implications of changes in the price of capital.

3.3 Financial Intermediaries

Asset-based financing. Competitive financial intermediaries issue intraperiod loans to firms to finance the purchase of capital. The debt issuance is subject to financial frictions, with the underlying friction being limited commitment. I assume that there is imperfect enforcement in the event of default, and firms cannot commit to avoiding renegotiation of debt repayment. Specifically, if a firm defaults, the financial intermediary can recover only up to a fraction ϕ_k of the undepreciated physical assets through a fire sale. This recovery constraint determines that the maximum amount of debt a firm can obtain will not exceed the value of its assets, as firms cannot commit to repaying debt if the loan amount surpasses this limit. In other words, the borrowing constraint is given by

$$b_t \leq \phi_k q_t k_t (1 - \delta)$$

This equation can be rewritten using $b_t = q_t k_t - a_t$ as follows:

$$q_t k_t \leq \frac{1}{1 - \phi_k (1 - \delta)} a_t$$

Earnings-based financing. In addition, firms can utilize earnings-based financing by endogenously making their earnings pledgeable through improvements in governance or accounting standards. I assume that firms are subject to two of the most common earnings-based borrowing limits: the debt-to-earnings ratio and the interest coverage ratio. These are expressed as follows.

$$b_t \leq \phi_e^{DE}[f(k_t, l_t; z, s_t) - w_t l_t - f_e]$$

, and

$$r_{j,t} b_t \leq \phi_e^{ICR}[f(k_t, l_t; z, s_t) - w_t l_t - f_e]$$

Here, the measure of earnings is EBITDA, earnings before interest, taxes, depreciation, and amortization.[8] As argued in Drechsel (2022) and Hartman-Glaser et al. (2022), these constraints are closely linked to the firm's going-concern value. In the next section, a microfoundation is provided to show that the above earnings-based borrowing limits at the steady state are the reduced-form representation of the pledgeable component of the firm's going-concern values under the limit commitment framework.

What matters in this paper are the implications of the linkage between earnings-based borrowing limits and firms' going-concern values for firms' debt capacity. As illustrated in the next section, the parameters, ϕ_e^{DE} and ϕ_e^{ICR}, that govern the tightness of borrowing limits essentially imply the magnitude of pledgeability of firms' going-concern values, which gives earnings-based firms significantly larger debt capacity without reliance on accumulated wealth.

This implication is crucial for understanding the mechanism behind the asymmetric effects. As demonstrated later, a lower interest rate not only enables large firms to leverage this advantage, but also amplifies the benefit due to less reliance on accumulated assets and reduced interest expenses. This, in turn, leads to the expansion of highly productive firms and generates crowding-out effects.

I further assume that, in general, $f_a < f_e$, so firms will trade off between cost structures and debt capacity. As a result, firms with greater financial needs will opt for earnings-based financing, while firms with sufficient self-financing or less need for external funding will choose asset-based financing. This simple mechanism effectively captures the trade-offs between the two different

[8] An alternative measure of earnings is EBIT. Our results are robust using this alternative measure.

types of financing. Earnings-based borrowing limits provide advantages in terms of greater borrowing capacities. To generate debt heterogeneity, the higher fixed cost broadly represents the expenses associated with raising pledgeability.

Raising pledgeability, in a narrow sense, might involve improving accounting quality or establishing a stronger board to prevent entrepreneurs from diverting project cash flows for personal use. More broadly, as noted by Rajan (2012), firms can enhance pledgeability through standardization—by simplifying corporate organizational structures and increasing transparency—or by implementing better governance structures, such as appointing a more expert and independent board. Such standardization can not only grant financiers more effective rights over the going-concern surplus through direct control but also make firms more attractive for sale in the secondary market, as discussed by Hartman-Glaser et al. (2022).

It is important to note that the limited commitment framework is not the only approach capable of generating the distinction between asset-based and earnings-based financing. Other commonly used frameworks, such as costly state verification and the presence of second-best users, can also create this distinction through endogenously determined pledgeability.[9]

Therefore, the model here emphasizes the importance and implications of pledgeability in future cash flows rather than focusing on specific underlying frictions. In the quantitative analysis, I will employ both reduced-form borrowing limits. Beyond computational convenience, these reduced-form borrowing limits reflect the actual constraints faced by firms. Previous studies have shown that the tightness of these limits is stable over time, which can be rationalized by the proposition discussed in the following section.

[9]See examples in Clementi and Hopenhayn (2006) and Hartman-Glaser et al. (2022), among others. Asset- and earnings-based covenants can also be viewed as signals or tripwires for firm performance. Here, as discussed in the following chapter, I emphasize the differing debt capacities implied by various types of financing, where pledgeability provides a clearer interpretation of this distinction.

3.4 Household Problem

At time t, the representative household, with a measure of $H(1+\eta)^t$, provides a total of $\bar{L}H(1+\eta)^t$ units of labor and receives wages

$$\begin{aligned} W(B_t) &= \max_{C_t, B_{t+1}} C_t + \beta_h(1+\eta)W(B_{t+1}) \\ \text{s.t.} \quad C_t + B_{t+1} &= w_t\bar{L} + \frac{(1+r_t)}{1+\eta}B_t, \end{aligned}$$

where C_t is household consumption, $\bar{L}$ is the labor supply per household unit, and B_t represents savings deposited with financial intermediaries.

3.5 Stationary Equilibrium

Let $m_t(Z, S, A)$ represent the measure of incumbent firms with permanent productivity $z \in Z$, idiosyncratic productivity $s \in S$, and previous equity holding $a \in A$ at the beginning of period t, following the draw of firm-level productivity but before the exogenous exit shock.

Let $\bar{m_t} = \frac{m_t}{H(1+\eta)^t}$ and $\bar{m_0} = \frac{m_0(1+\eta)^t}{H(1+\eta)^t}$ denote these measures on a per capita basis. Then, the law of motion of the distribution is given by:

$$\begin{aligned} \bar{m_{t+1}}(Z, S, A) = &\iint \frac{(1-\xi)}{1+\eta} Q(z, s_{t+1}, a_{t+1}|z, s_t, a_t)(1 - X(z, a_t, s_t))\bar{m}_t(dz, ds, da) \\ &+ \bar{m_0} \iiint Q_0(z, s_{t+1}, a_{t+1}|z, s_t, a_t)I(z, s_t, a_t)d\Gamma_{a_0}(a_0)d\Gamma_z(z)d\Gamma_{s_0}(s_0), \end{aligned}$$

where $Q_0(z, s_{t+1}, a_{t+1} \mid z, s_t, a_t)$ and $Q(z, s_{t+1}, a_{t+1} \mid z, s_t, a_t)$ are the resulting transition matrices based on the selection of equity holdings and the evolution of productivity shocks for entry and incumbent firms, respectively.

Closing the Model. Finally, the labor market clears such that

$$\bar{L} = \iiint l(z, s_t, a_t)(1 - X(z, a_t, s_t))\bar{m}_t(dz, ds, da) + \iiint l(z, a_t, s_t)I_0(z, s_t, a_t)\bar{m}_0(dz, ds, da),$$

where $l(z, s_t, a_t)$ is the policy function for labor demand for firms with state variables (z, s_t, a_t).

4 Lessons from a Stylized Example

This section employs a simplified version of the full model to illustrate the mechanism linking the decline in the real interest rate to changes in the firm size distribution and firm dynamics.

4.1 A Stylized Example

To keep the model tractable and facilitate analysis, I assume, only for this section, that there is no entry or exit and no balanced growth. Additionally, instead of allowing for endogenous choices of debt types, I assume that the economy consists of two types of firms, which are exogenously determined as either earnings-based or asset-based financing firms. Firms operate with Cobb-Douglas production function. There is no productivity difference between these two types of firms.[10]

Therefore, in the economy, an exogenous measure of ν_e earnings-based firms solves the following firm problem:

$$\begin{aligned} &\max_{d_t, k_t, l_t, a_{t+1}} \sum_{t=0}^{\infty} \beta_f^t d_t \\ \text{s.t.} \quad & d_t + a_{t+1} = k_t^{\alpha} l_t^{\gamma} - w_t l_t - r_{e,t} b_t - \delta q_t k_t + a_t \\ & r_{e,t} b_t \leq \phi_e \cdot [k_t^{\alpha} l_t^{\gamma} - w_t l_t] \\ & 0 \leq a_{t+1}, \end{aligned}$$

where firms aim to maximize each period's dividends, subject to budget and borrowing constraints. Note that the focus here is on the interest coverage ratio, which provides a more comprehensive view of all possible channels. As shown in the appendix and discussed in the following section, similar results can be derived if the debt-to-earnings ratio is used instead. Here, I slightly abuse the notation for simplicity and use ϕ_e to denote the tightness of the borrowing limit.

[10] It is easy to show that that the existence of productivity differences does not alter the results.

At the beginning of each period, firms start with an asset level a_t. They produce goods using a Cobb-Douglas production function, $k_t^\alpha l_t^\gamma$, where $\alpha+\gamma < 1$ is assumed to ensure decreasing returns to scale. Firms hire labor l_t and pay a wage w_t. Additionally, firms decide how much to allocate to assets, a_t, and how much to borrow from debt issuers for capital, k_t. After covering the net financing cost $r_t b_t$, firms distribute retained earnings either as current dividends or as savings for the next period, both of which must be non-negative. Net borrowing, b_t, is defined as $b_t = q_t k_t - a_t$.

In contrast, there is a measure of ν_a firms that use asset-based financing and face the following firm problem:

$$\begin{aligned} &\max_{d_t, k_t, l_t, a_{t+1}} \sum_{t=0}^{\infty} \beta_f^t d_t \\ \text{s.t.} \quad & d_t + a_{t+1} = k_t^\alpha l_t^\gamma - w_t l_t - r_{a,t} b_t - \delta q_t k_t + a_t \\ & q_t k_t \leq \frac{1}{1-\phi_k(1-\delta)} a_t \\ & 0 \leq a_{t+1} \end{aligned}$$

Similar to earnings-based firms, these firms aim to maximize each period's dividends while adhering to budget and borrowing constraints. However, unlike earnings-based firms, asset-based firms are subject to an asset-based borrowing constraint, as shown in the second equation, where the debt level k_t cannot exceed a fraction, $\frac{1}{1-\phi_k(1-\delta)}$, of their savings a_t.

For simplicity, the measures of both types of firms, ν_e and ν_a, are given exogenously, and I focus on an equilibrium in which both types of firms earn positive profits. Additionally, the economy has an inelastic labor supply of $\bar{L}$ units. The market-clearing condition for the labor market at time t is given by

$$\bar{L} = \nu_e \cdot l_{e,t} + \nu_a \cdot l_{a,t},$$

where l_t^e and l_t^a represent the equilibrium labor demand for earnings-based firms and asset-based firms, respectively.

4.2 The Effects of Decline in the Real Interest Rates

First, a statistic should be introduced to assess the effects of the decline in the real interest rate. Using simple algebra, the labor market-clearing condition can be rewritten as:

$$\bar{L} = l_{a,t} \cdot (\nu_e \cdot \frac{l_{e,t}}{l_{a,t}} + \nu_a)$$

In this framework, it is useful to examine the ratio $\frac{l_{e,t}}{l_{a,t}}$ (hereafter referred to as the labor ratio) to gain insights into firm size distribution and dynamics. The labor ratio is directly proportional to employment shares; for instance, if $\frac{l_{e,t}}{l_{a,t}}$ increases, it indicates a shift in employment shares toward earnings-based firms. Simultaneously, the labor demand for asset-based firms will decrease due to the fixed firm measures and inelastic labor supply. Additionally, since labor shares are indicative of firms' profitability, the labor ratio can also be employed to indicate changes in firm dynamics.

Therefore, I will focus on the impact of the decline in real interest rates on the labor ratio in the steady state to reveal their connections and potential mechanisms in the long run. I denote the long-run steady-state labor ratio as $\frac{l_e}{l_a}$. Also, to focus on the implications of borrowing limits, I set $r_t^e = r_t^a = r_t$, and assume the borrowing limits are binding.

To avoid unnecessary complexity, I further assume that earnings-based firms will have positive asset holdings, $a > 0$ when r is close to $\frac{1}{\beta_f} - 1$. This basically means the constraint is tight enough so that firms need some fraction of their own equity to fund capital. Then, the effects of the decline in long-run real interest rates through heterogeneous borrowing limits are summarized in the following proposition.

Proposition 1 *If $\frac{1}{\beta_f} - 1 > r_t$, then:*

- *If $\phi_e \left(1 + \frac{\delta}{1/\beta_f - 1}\right) \geq \phi_k(1-\delta)$, the labor ratio $\frac{l_e}{l_a}$ increases monotonically as r decreases.*
- *If $\phi_e \left(1 + \frac{\delta}{1/\beta_f - 1}\right) < \phi_k(1-\delta)$, the labor ratio $\frac{l_e}{l_a}$ and r exhibit a "U-shaped" relationship: $\exists r^* \in (0, \frac{1}{\beta_f} - 1)$ such that when $r > r^*$, the labor ratio decreases as r decreases, and when $r < r^*$, the labor ratio increases as r decreases.*

Proof. See the appendix. ■

The proposition states that under the interest coverage ratio, the effects of lower interest rates depend on three sets of parameters: (i) ϕ_e and ϕ_k, the debt capacities of the two types of financing, (ii) δ, the depreciation rate, and (iii) β_f, the discount factor.

The intuition behind these results is as follows. In the long term, a firm's optimal size is determined by balancing the marginal benefits and costs of expansion or contraction. A firm considers not only its marginal revenue and cost from such changes, but also internalizes the effects on its borrowing limits. When the interest rate decreases, the lower cost of capital incentivizes the firm to expand. A firm will increase its size more aggressively if the change has a greater positive effect on its borrowing limit.

For an asset-based firm, a one-dollar increase in capital value results in an increase in the debt limit by $\phi_k(1-\delta)$, which is invariant to r_t. In contrast, for an earnings-based firm, a one-dollar increase in capital value increases the debt limit by $\frac{\phi_e}{r_t}$ multiplied by marginal revenue. When r_t decreases, the debt capacity $\frac{\phi_e}{r_t}$ increases, but the expansion causes marginal revenue to decline. When $\delta = 0$, the debt capacity and marginal revenue change at the same rate but in opposite directions, leading to the following corollary.

Corollary 1 *If $\frac{1}{\beta_f} - 1 > r_t$ and $\delta = 0$, then:*

- *If $\phi_e \geq \phi_k$, the labor ratio $\frac{l_e}{l_a}$ increases monotonically as r decreases.*
- *If $\phi_e < \phi_k$, the labor ratio $\frac{l_e}{l_a}$ and r are "U-shaped": $\exists r^* \in (0, \frac{1}{\beta_f} - 1)$ such that when $r > r^*$, the labor ratio decreases as r decreases, and when $r < r^*$, the labor ratio increases as r decreases.*
- *As $r_t \to 0$, the labor ratio $\frac{l_e}{l_a} \to \infty$.*

Proof. See the appendix. ■

If $\delta = 0$, then the change in the labor ratio with respect to r depends solely on ϕ_k and ϕ_e, which govern the leverage ratios of the two types of firms. If $\phi_k \leq \phi_e$, meaning the effective recovery rate

of collateral assets is lower than that of earnings, a decline in real interest rates will cause the labor ratio to increase monotonically. In other words, equilibrium employment shares will shift toward firms with earnings-based financing. However, if asset-based firms can achieve sufficient leverage, meaning ϕ_k is higher, the relationship will be "U-shaped." At high real interest rates, a decline in the rate will cause the labor ratio to decrease, while at lower real interest rates, this relationship will reverse.

The "U-shaped" relationship, or more precisely, the reversal at lower interest rates, reflects the advantage of using earnings-based borrowing limits in a low-rate environment—namely, reduced reliance on accumulated assets. Accumulating assets is costly because the return on holding such assets is low, which also drives firms to prefer borrowing over saving. The lower the interest rate, the greater the benefit that earnings-based financing provides by relying less on asset accumulation. Therefore, when $\phi_k > \phi_e$, asset-based financing may facilitate expansion when r_t is relatively high, but this advantage reverses when r_t is lower. Indeed, as demonstrated by Proposition 1, the lower the discount factor β_f, the higher the opportunity cost of accumulating wealth, which strengthens the effect of earnings-based borrowing limits. This is because less reliance on assets increasingly benefits earnings-based financing at lower rates. Finally, without depreciation, as r_t approaches zero, the borrowing constraint becomes effectively non-binding, causing the labor ratio to approach infinity.

When $0 < \delta < 1$, the relationship can become monotonic for relatively smaller values of ϕ_e. In other words, the presence of depreciation benefits earnings-based firms when the interest rate declines. There are two primary reasons for this. First, depreciation effectively reduces the debt capacity of asset-based financing by diminishing the value of capital, as demonstrated by the right-hand side of the inequality. More importantly, while marginal revenue decreases with expansion, the presence of δ slows the pace of this decline, preventing marginal revenue from being too low. In fact, the existence of positive interest spreads, $\rho > 0$, can generate the same impact. As a result, earnings-based firms find it more effective to relax their constraints using earnings. The same logic suggests that other factors, such as interest spreads, will generate similar effects.

The analysis here focuses solely on the interest coverage ratio. As shown in the appendix, the same mechanism and similar conditions for monotonicity are found when the earnings-based constraint is expressed as the debt-to-earnings ratio, with the exception that lower rates do not directly relax the limits, so the labor ratio will not reach infinity. The following sections perform a sophisticated calibration and quantitative exercises using the full model, confirming that lower interest rates indeed shift employment shares to earnings-based firms.

Why should one expect the tightness of two types of constraints, namely ϕ_e and ϕ_k, of similar magnitude? While ϕ_k can be rationalized by the recovery rate in a fire sale, how should one think about ϕ_e? The linkage between earnings-based borrowing limits and firms' going-concern values provides an answer. In particular, if one assumes that debt repayment has limited commitment. With limited commitment, the debt amount cannot exceed the maximum that intermediaries can recover in default. Specifically, this is given by:

$$b_t \leq \psi V_t^{end},$$

where V^{end} represents the firm's going-concern value. The following proposition demonstrates that the tightness of earnings-based borrowing limits can be rationalized by the relationship between the two constraints and the pledgeability of a firm's going-concern value in the steady state. For now, I assume no depreciation.[11]

Proposition 2 *At the steady-state equilibrium, if* $b \leq \psi V^{end}$ *and* $\phi_e^{DE} = \frac{\psi}{1-\beta_f+r\psi}$, *then the debt capacity satisfies*

$$b_t \leq \phi_e^{DE}[f(k,l;z,s) - wl]$$

[11] If one considers depreciation, the measure of earnings become earnings before interest and taxes (EBIT), and the results remain unchanged. In practice, both measures are commonly used, and EBITDA is often used to measures earnings without the effects of financing or accounting decisions.

Furthermore, if $r_{j,t} = \frac{1}{\beta_f} - 1$ *and* $\phi_e^{ICR} = \frac{\psi}{\beta_f + \psi}$*, then*

$$rb \leq \phi_e^{ICR}[f(k, l; z, s) - wl]$$

Proof. See the appendix. ■

Simple numerical examples can illustrate this point. For interest coverage ratio, $\phi_e^{ICR} = 0.3$, for instance, means firms can borrow up to roughly 40% of firm's going-concern value. Drechsel (2022) documented that the average earnings-to-debt ratio is about 4.6. With $r = 0.05$, this result implies a pledgeability parameter $\psi = 0.36$. This is roughly consistent with empirical findings in Kermani and Ma (2020).[12] In other words, a typical earnings-based borrowing limit implies that the associated firm can borrow up to 35% to 40% of its steady-state going-concern value without reliance on accumulated wealth. This is in a similar, if not larger, magnitude to the liquidation value of fixed assets, or ϕ_k, estimated by Kermani and Ma (2020). In fact, as Kermani and Ma (2023) point out, due to the potential asset specificity, companies often have limited liquidation values.

Therefore, the connection between earnings-based financing and firms' going-concern values shapes the interpretation of this paper's results. Being able to borrow against going-concern values gives earnings-based firms advantages, namely a higher debt capacity. The debt capacity advantage grants a comparable tightness of borrowing constraints. More importantly, such an advantage is amplified when a lower interest rate environment triggers a higher demand for credit and when accumulating wealth becomes more costly. When the interest coverage ratio is used in particular, the decline in real interest rates further amplifies such an advantage through the change in the discounted value of the pledgeable component of future cash flows. Firms endogenously choose to make earnings pledgeable (make it general as in Rajan (2012)) to overcome reliance on physical assets, which have limited recovery rates due to irreversibility or asset specificity (Kermani and Ma, 2023).

[12] Kermani and Ma (2020), shows the total debt level is on average about 22% of total book assets, where the going-concern value is on average similar to the value of the total book assets.

Finally, how can the price of capital affect the results? In fact, if perfect elasticity of capital supply is assumed, then the steady-state price q will not change with respect to r, and therefore it will not affect the labor ratio. However, the proposition below demonstrates that in this simple model, even without assuming perfect elasticity of capital supply, the steady-state price of capital q does not influence the labor ratio in the steady state.

Proposition 3 *For a given interest rate r, variations in the capital price q do not influence the steady-state equilibrium labor ratio $\frac{l_e}{l_a}$.*

Proof. See the appendix. ■

In other words, the steady-state equilibrium labor ratio $\frac{l_e}{l_a}$ is independent of the capital price q. The reasoning is as follows: an increase in q raises the cost of capital, causing both types of firms to contract relative to the scenario where q remains unchanged. This change benefits asset-based financing by increasing the collateral value proportionally to q. For firms using earnings-based financing, while they do not directly pledge collateral, the higher capital cost necessitates a proportional increase in marginal revenue to maintain profitability. This increase in marginal revenue, driven by the rise in q, allows these firms to more effectively relax their borrowing limits, as the borrowing capacity is tied to the marginal earnings, which also scales with q. Consequently, a change in the capital price affects the borrowing capacity of both types of firms equally, leading them to adjust their sizes proportionately. As a result, the equilibrium labor ratio remains unaffected by changes in q.

It is important to note that the above analyses are based on the long-run steady-state equilibrium. The short-run equilibrium may differ, as the choices of both types of firms are influenced by the levels of their existing assets, which cannot adjust freely. In the long run, however, firms have the flexibility to adjust and accumulate wealth and capital in response to changes in real interest rates.

In addition to borrowing limits, endogenous debt choices, absent in this simple model, can also amplify these effects. As real interest rates decline, marginal firms that previously relied on asset-

based financing during periods of higher interest rates may switch to earnings-based financing, enabling them to borrow more and produce more efficiently.

More importantly, when these effects are sufficiently large, the profits of firms using asset-based financing may also decline. Consequently, the equilibrium rate of entrepreneurship may decline.

Another important insight that can be shown from the derivation is that, in the absence of heterogeneity in debt financing, the labor ratio and equilibrium labor demand would remain unaffected by changes in the real interest rate. Put it differently, debt heterogeneity is the key driver of the asymmetric effects.

5 Quantitative Experiments

This section presents the results of the quantitative experiments. I calibrate the model to match a comprehensive set of statistics from the U.S. economy. Then, I examine the effects of exogenous changes in real interest rates on firm dynamics and the firm size distribution at the stationary equilibrium. Finally, I compare the benchmark economy with a counterfactual scenario that lacks debt heterogeneity, highlighting the role that debt heterogeneity plays.

5.1 Parameterization

Externally calibrated parameters. I begin with the subset of parameters that are calibrated externally. Table 1 presents these parameters. The model is calibrated at an annual frequency. I set both β_f and β_h to 0.943 to replicate the annualized risk-free rate of 6% at the beginning of the 1980s. Since the measure of potential entrants, $\bar{m}_0$, scales $\bar{\mu}_t$, I choose $\bar{m}_0$ to normalize the total measure of incumbent firms to one. The parameters governing the tightness of earnings-based financing, ϕ_e^{ICR} and ϕ_e^{DE}, are calibrated based on values documented in the literature. Notably, the ratio is calibrated relatively conservatively compared with the literature to capture the idea of precautionary motives of avoiding covenant violations (Greenwald et al., 2019) .

Consistent with empirical evidence and the literature, I calibrate the magnitude of decreasing returns to scale to 0.90 and set the ratio of the output elasticity of capital to labor at 1/2, resulting in values of $\alpha = 0.30$ and $\gamma = 0.60$ (Poschke, 2018). Additionally, I calibrate the interest spread for both types of financing, setting $\rho_e = \rho_a = 0.020$ to match the average cost of financial intermediation.

Finally, the depreciation rate is calibrated to 10%, consistent with values commonly used in the literature. The population growth rate, η, is set at 1.9% to match the U.S. population growth rate, as in Karahan et al. (2019).

Internally calibrated parameters. The remaining parameters are calibrated internally. Table 2 lists these parameters, which are set by minimizing the weighted distance between an equal number of empirical moments and their equilibrium counterparts in the model. In the following section, I will discuss the specific moments used to calibrate the model and the rationale behind their selection.

The first set of parameters primarily relates to the initial productivity and equity distributions, the productivity shock process, and the shock to exit. The parameter σ_{z_0} governs the initial distribution of permanent productivity z, which is assumed to follow a log-normal distribution with $z_0 \sim N(-\sigma_{z_0}^2/2, \sigma_{z_0})$. The parameters μ_{s_0} and σ_{s_0} govern the initial distribution of the stochastic component of productivity s_0, which is also assumed to follow a log-normal distribution with $s_0 \sim N(\mu_{s_0}, \sigma_{s_0})$. I assume that s_t evolves according to an AR(1) process such that $s_t = (1-\rho_s)s_{ss} + \rho_s s_{t-1} + \sigma_\epsilon \epsilon_t$, where s_{ss} is set so that $E(s_t) = 0$ and $\epsilon_t \sim N(0,1)$. The parameters μ_{a_0} and σ_{a_0} govern the initial distribution of the equity draw a_0, which is assumed to be log-normally distributed with $\log(a_0) \sim N(\mu_{a_0}, \sigma_{a_0})$. These initial distributions and the productivity process are crucial for determining the equilibrium firm size distribution and firm entry/exit dynamics. Additionally, the exogenous exit shock ξ plays a significant role in determining the exit rate of firms, particularly for older firms.

I calibrate these parameters to match key annual statistics of firm size distribution and firm dynamics, averaged over the 1982–1986 period whenever available. Specifically, for the firm size

distribution, the model is calibrated to match the firm shares and employment shares for firms employing more than 500, 1,000, and 2,500 workers. Additionally, the overall exit rate and firm entry rate are targeted. To better capture firm dynamics, I also match the exit rate by firm age. Furthermore, the model is calibrated to match the standard deviation of employment growth. I additionally target the average firm size to be 19.76, consistent with the Business Dynamics Statistics (BDS) data for the period 1982–1986.

Another set of parameters is primarily related to firms' debt choices. The fixed costs, f_e and f_a, play a crucial role in determining firms' financing selections and their characteristics, particularly sizes, within each financing option. The parameter ϕ_k governs the ability of assets to serve as leverage. Given that the majority of firms in the economy rely on asset-based loans, this parameter is calibrated to match the debt-to-output ratio. Additionally, I match the ratio of earnings-based debt to asset-based debt using data based on the Board of Governors' statistical release.[13] Furthermore, the average size of earnings-based firms is targeted to match those firms with assets above the median in Compustat, as these firms predominantly use earnings-based financing.

Together, there are 16 moments used to identify 12 parameters. Table 3 presents some of these moments, while Figure 6 displays the rest. Overall, the moments are well-matched, and the model generates a good fit to the data. Additionally, Figure 7 illustrates the moments of the firm age $\times$ size distribution, which were not targeted in the calibration. The figure shows that these non-targeted moments also fit the data well.

5.2 Equilibrium Effects of the Decline in Real Interest Rates

This subsection presents the stationary equilibrium outcomes of the economy for different values of the real interest rate. I use the calibrated model to evaluate the effects of declining interest rates on the economy, with a particular focus on the asymmetric effects driven by debt heterogeneity. The results indicate that the decline in the real interest rate can account for about 80% of the

[13]The data are from the Financial Accounts of the United States (Z.1). To classify earnings-based debt, I include levels of commercial paper and corporate bonds from nonfinancial corporate business sector. I classify mortgages and depository loans as asset-based debt.

changes in the shifts in employment shares and 27% of the changes in debt composition observed in the U.S. over the past forty years. The model also explains more than 27% of the decline in firms per capita as well as a 12% decline in the firm entry rate. Counterfactual analysis further demonstrates that debt heterogeneity plays a crucial role in generating these effects.

Regarding the employment shares of large firms, the left panel of Figure 8 displays the model-predicted employment shares of large firms. Since most firms using earnings-based financing are larger companies in the model, the employment shares of large firms increase. Consistent with empirical patterns, the right panel of Figure 8 shows how debt composition also shifts toward earnings-based debt.

Regarding changes in the employment size distribution, the left panel of Figure 9 shows that the average size of firms at the top of the distribution grows faster than the average size of all firms. In contrast, the average size of smaller firms—those with fewer than 10 or 20 employees—shrinks relative to the average size of all firms, as illustrated in the right panel of Figure 9.

Figure 10 illustrates changes in average firm size, the number of firms per capita, and the firm entry rate. As the interest rate r_t decreases, earnings-based firms—mainly large firms in the model—expand their employment demands. Simultaneously, some firms that initially use asset-based financing switch to earnings-based financing and increase their production. As a result, smaller firms, which primarily use asset-based financing, shrink and are crowded out. This leads to a lower number of firms per capita and a lower firm entry rate. Less profitable firms exit the market due to rising labor costs driven by the faster growth of earnings-based firms. Consequently, the average firm size increases over time.

Finally, Table 4 summarizes the comparison between the secular changes observed in the U.S. and the model's predictions. The results indicate that the proposed channel can explain a significant portion of these long-term trends. Specifically, the decline in the real interest rate can account for roughly 80% in employment shares on average, 27% of the changes in debt composition observed in the U.S. over the past forty years. Additionally, the model explains approximately 27% of the decline in firms per capita and a 12% decline in the firm entry rate.

5.3 The Role of Debt Heterogeneity

In this section, I compare the quantitative results from the benchmark model with those from a counterfactual economy. In the counterfactual scenario, I assume there is no debt heterogeneity—meaning all firms rely solely on asset-based financing. As shown in the appendix, I calibrate this model to match the same U.S. statistics as the benchmark model and conduct the same quantitative experiments. The purpose of this exercise is to study the role of debt heterogeneity in shaping the observed economic outcomes.

Figure 11 compares the changes in the firm size distribution between the benchmark model and the counterfactual model. The left panel illustrates that, without debt heterogeneity, the decline in real interest rates does not significantly shift employment shares. The middle panel shows that, in the counterfactual economy, firms at the top grow slightly slower than the average. In contrast, the right panel indicates that smaller firms grow faster than the average in the absence of debt heterogeneity.

Figure 12 compares the changes in average firm size and firm dynamics between the benchmark model and the counterfactual model. It reveals distinct patterns between the economy with and without debt heterogeneity. In the absence of debt heterogeneity, a decline in real interest rates leads to an increase in firms per capita. This underscores the critical role of debt heterogeneity in shaping the effects of real interest rates on firm dynamics. Additionally, the comparison of average firm sizes shows that, consistent with the firms per capita results, debt heterogeneity is crucial in explaining the differences in average firm size changes between the two economies.

Therefore, it is evident that debt heterogeneity plays a crucial role in linking the decline in long-term real interest rates to the observed changes in firm size distribution and firm dynamics.

6 Evaluating the Mechanism Based on US Firms

In this section, I empirically test the proposed mechanism using data from U.S. firms. The central prediction of the model is that firms with earnings-based financial constraints will experience a

more significant change in employment (in percentage) in response to a decrease in interest rates compared to asset-based firms. To investigate this, I develop an identification strategy to analyze the relationship at the firm level. Utilizing linked data from Compustat and Capital IQ, I estimate the relationship between firm-level interest rates and employment, conditional on the degree to which a firm relies on earnings-based financing. The model predicts that when interest rates decline, firms heavily dependent on earnings-based financing will show a more pronounced increase in employment.

6.1 Data and Sample Characteristics

For this analysis, I use a comprehensive dataset that includes financial information from Compustat and Capital IQ at annual frequency, covering the period from 2001 to 2019.

The linked dataset contains employment levels, firm-level interest rates, and various firm characteristics. Capital IQ contains loan-level information, which is used to categorize debt types as in Lian and Ma (2021) as well as the firm-level usage of each type of debt. These observations allow me to test how firm-level employment responds to a change in interest rates and how such changes differ for firms using different types of financing. The descriptive statistics for the sample are presented in Table 5.

6.2 Identification Strategy

The identification strategy is designed to test the effect of interest rate changes on employment, conditional on a firm's reliance on earnings-based financing. The following equation forms the basis of the empirical model:

$$y_{s,t} = \alpha_s + \gamma_t + i_{s,t} + ratio_{s,t} + i_{s,t} \times ratio_{s,t} + X_{s,t} + \epsilon_{s,t}, \tag{1}$$

where: $y_{s,t}$ represents the log employment level for firm s at time t, α_s captures firm-specific fixed effects, γ_t controls for time-specific effects, $i_{s,t}$ is the firm-level imputed interest rate, calculated by

dividing interest expenses by total debt. $ratio_{s,t}$ denotes the fraction of earnings-based financing, $i_{s,t} \times ratio_{s,t}$ is the interaction term of interest, $X_{s,t}$ are firm-level controls including debt, capital, and sales to asset ratio, and $\epsilon_{s,t}$ is the error term.

The interaction term $i_{s,t} \times ratio_{s,t}$ allows for the test of the hypothesis that firms with higher earnings-based financing ratios will show a more substantial employment response to interest rate changes.

6.3 Results

The model predicts that the coefficient on the interaction term $i_{s,t} \times ratio_{s,t}$ will be negative and significant. This would indicate that firms relying more on earnings-based financing experience a greater increase in employment when interest rates fall, consistent with theoretical predictions.

The regression analysis presented in Table 6 provides strong empirical support for the proposed relationship between firm-level interest rates, earnings-based financial constraints, and employment. The coefficient on the earnings-based financing ratio ($ratio_{s,t}$) is consistently positive and statistically significant at the 0.001 level across all specifications. This suggests that firms with a higher reliance on earnings-based financing tend to have higher levels of employment, indicating that larger firms rely more on earnings-based financing. The firm-level interest rate, as expected, exhibits a negative and statistically significant relationship with employment across all models. The size of the coefficient is substantial, demonstrating that higher interest rates are associated with lower employment levels within firms.

Crucially, the interaction term between the interest rate and the earnings-based financing ratio is also negative and statistically significant. This result is in line with the theoretical prediction that firms with a higher earnings-based financing ratio experience a more pronounced reduction in employment when interest rates increase. Conversely, these firms are more likely to increase employment when interest rates decline, relative to firms with lower reliance on earnings-based financing. The significance and direction of this interaction term emphasize the differential impact of interest rate changes on employment across firms with varying financial structures.

The robustness of these findings is further demonstrated by the consistency of the coefficients across different model specifications. In particular, the inclusion of firm fixed effects in columns (3) and (6) helps control for time-invariant unobserved heterogeneity, while the inclusion of industry fixed effects in columns (2) and (5) controls for industry-specific factors that could influence employment. Despite these additional controls, the coefficients for the ratio, interest rate, and their interaction remain significant.

Overall, these results provide supporting evidence that firms with earnings-based financial constraints are more responsive to changes in interest rates, particularly in their employment decisions.

7 Conclusion

Can the decline in real interest rates contribute to the shift in employment shares and the increasingly dispersed firm size distribution? The findings suggest that this decline can indeed produce asymmetric effects through debt heterogeneity, driving these secular changes. In particular, the model demonstrates that the decline in real interest rates can account for about 80% of the changes in employment share shifts and over 27% of the changes in debt composition observed in the U.S. over the past forty years. Additionally, it explains more than 27% of the decline in firms per capita and a 12% decline in the firm entry rate.

Debt heterogeneity emerges as a key mechanism in generating these effects. While much of the literature emphasizes the role of financial frictions in business cycle fluctuations, this paper highlights the importance of debt heterogeneity in shaping long-term and having asymmetric impacts on the aggregate economy. By disproportionately benefiting large firms, debt heterogeneity transforms lower interest rates—typically viewed as a positive financial development—into a factor that adversely affects small firms, illustrating the nuanced and often contrasting impacts of macroeconomic trends.

While contributing to the understanding of how long-term real interest rates affect firm size distribution and dynamics, my paper also aims to draw attention to the role of debt capacity shaped

by earnings-based debt financing. This perspective may have broader implications for macroeconomics and opens the door to further questions. Empirically, understanding what determines eligibility for earnings-based financing appears to be an important area for investigation. Theoretically, the link between earnings-based financing and pledgeable future cash flows suggests potential avenues for policy interventions. For instance, the model indicates that improving firm transparency and/or reducing the barriers and costs of restructuring bankruptcy might strengthen debt enforceability, allowing firms to borrow more efficiently. Exploring the empirical effects of such interventions could provide valuable insights for future research.

References

Akcigit, U. and Ates, S. T. (2021). Ten facts on declining business dynamism and lessons from endogenous growth theory. *American Economic Journal: Macroeconomics*, 13(1):257–298.

Asriyan, V., Laeven, L., Martin, A., Van der Ghote, A., and Vanasco, V. (2021). Falling interest rates and credit misallocation: Lessons from general equilibrium.

Autor, D., Dorn, D., Katz, L. F., Patterson, C., and Van Reenen, J. (2020). The fall of the labor share and the rise of superstar firms. *The Quarterly Journal of Economics*, 135(2):645–709.

Begenau, J., Farboodi, M., and Veldkamp, L. (2018). Big data in finance and the growth of large firms. *Journal of Monetary Economics*, 97:71–87.

Bernanke, B. (2005). The global saving glut and the us current account deficit. Technical report, Board of Governors of the Federal Reserve System (US).

Bornstein, G. et al. (2018). Entry and profits in an aging economy: The role of consumer inertia.

Caglio, C. R., Darst, R. M., and Kalemli-Özcan, 2021). Risk-taking and monetary policy transmission: Evidence from loans to smes and large firms. Technical report, National Bureau of Economic Research.

Chatterjee, S. and Eyigungor, B. (2020). The firm size and leverage relationship and its implications for entry and business concentration.

Clementi, G. L. and Hopenhayn, H. A. (2006). A theory of financing constraints and firm dynamics. *The Quarterly Journal of Economics*, 121(1):229–265.

Colla, P., Ippolito, F., and Li, K. (2020). Debt structure. *Annual Review of Financial Economics*, 12:193–215.

Cooley, T. F. and Quadrini, V. (2001). Financial markets and firm dynamics. *American economic review*, 91(5):1286–1310.

Del Negro, M., Giannone, D., Giannoni, M. P., and Tambalotti, A. (2017). Safety, liquidity, and the natural rate of interest. *Brookings Papers on Economic Activity*, 2017(1):235–316.

Diamond, D. W., Hu, Y., and Rajan, R. G. (2022). Liquidity, pledgeability, and the nature of lending. *Journal of Financial Economics*, 143(3):1275–1294.

Drechsel, T. (2022). Earnings-based borrowing constraints and macroeconomic fluctuations. *DP16975*.

Dynan, K. E., Skinner, J., and Zeldes, S. P. (2004). Do the rich save more? *Journal of political economy*, 112(2):397–444.

Gamber, E. (2020). *The Historical Decline in Real Interest Rates and Its Implications for CBO's Projections*. Congressional Budget Office.

Gavazza, A., Mongey, S., and Violante, G. L. (2018). Aggregate recruiting intensity. *American Economic Review*, 108(8):2088–2127.

Greenwald, D. et al. (2019). *Firm debt covenants and the macroeconomy: The interest coverage channel*. MIT Sloan School of Management.

Hall, R. E. (2016). The role of the growth of risk-averse wealth in the decline of the safe real interest rate. Technical report, National Bureau of Economic Research.

Haltiwanger, J., Jarmin, R., and Miranda, J. (2011). Historically large decline in job creation from startup and existing firms in the 2008–2009 recession. *Statistical Brief for the Kauffman Foundation, http://www. kauffman. org/uploadedFiles/bds_report_3-22-11. pdf*.

Haltiwanger, J., Jarmin, R., and Miranda, J. (2012). Where have all the young firms gone? *Business Dynamics Statistics Briefing*, 4.

Hartman-Glaser, B., Mayer, S., and Milbradt, K. (2022). A theory of asset-and cash flow-based financing. Technical report, National Bureau of Economic Research.

Hathaway, I. and Litan, R. E. (2014). What's driving the decline in the firm formation rate? a partial explanation. *The Brookings Institution*.

Hopenhayn, H. A. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica: Journal of the Econometric Society*, pages 1127–1150.

Karahan, F., Pugsley, B., and Şahin, A. (2019). Demographic origins of the startup deficit. Technical report, National Bureau of Economic Research.

Kermani, A. and Ma, Y. (2020). Two tales of debt. Technical report, National Bureau of Economic Research.

Kermani, A. and Ma, Y. (2023). Asset specificity of nonfinancial firms. *The Quarterly Journal of Economics*, 138(1):205–264.

Kiley, M. T. (2020). The global equilibrium real interest rate: concepts, estimates, and challenges. *Annual Review of Financial Economics*, 12:305–326.

Kiyotaki, N., Moore, J., and Zhang, S. (2021). Credit horizons. Technical report, National Bureau of Economic Research.

Kroen, T., Liu, E., Mian, A. R., and Sufi, A. (2021). Falling rates and rising superstars. Technical report, National Bureau of Economic Research.

Krueger, D. and Ludwig, A. (2007). On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare. *Journal of monetary Economics*, 54(1):49–87.

Lian, C. and Ma, Y. (2021). Anatomy of corporate borrowing constraints. *The Quarterly Journal of Economics*, 136(1):229–291.

Liu, E., Mian, A., and Sufi, A. (2019). Low interest rates, market power, and productivity growth. Technical report, National Bureau of Economic Research.

Moll, B. (2014). Productivity losses from financial frictions: Can self-financing undo capital misallocation? *American Economic Review*, 104(10):3186–3221.

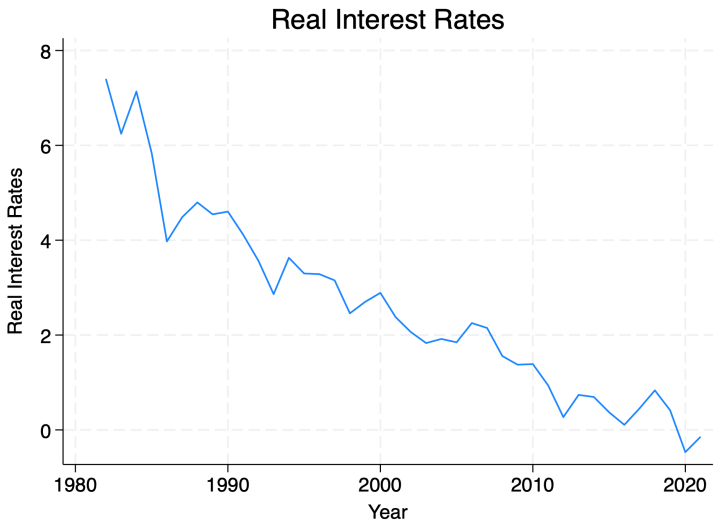
Philippon, T. (2015). Has the us finance industry become less efficient? on the theory and measurement of financial intermediation. *American Economic Review*, 105(4):1408–1438.

Poschke, M. (2018). The firm size distribution across countries and skill-biased change in en-

trepreneurial technology. *American Economic Journal: Macroeconomics*, 10(3):1–41.
Pugsley, B. W. and Şahin, A. (2019). Grown-up business cycles. *The Review of Financial Studies*, 32(3):1102–1147.
Rajan, R. G. (2012). Presidential address: The corporation in finance. *The Journal of finance*, 67(4):1173–1217.
Salgado, S. (2020). Technical change and entrepreneurship. *Available at SSRN 3616568*.

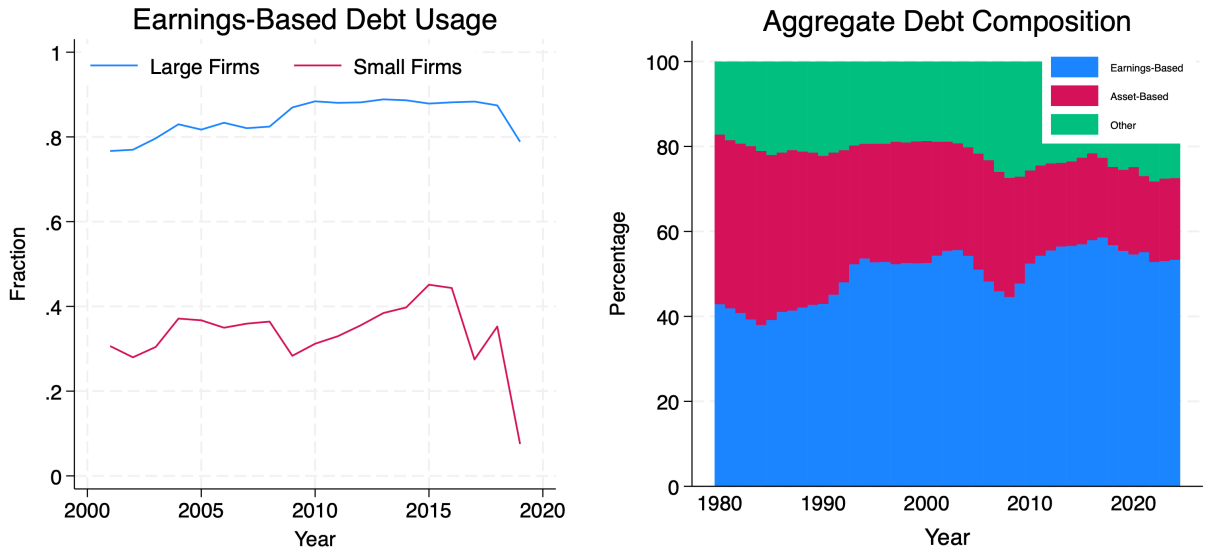
Main Figures and Tables

Figure 1: Real Interest Rates over Years



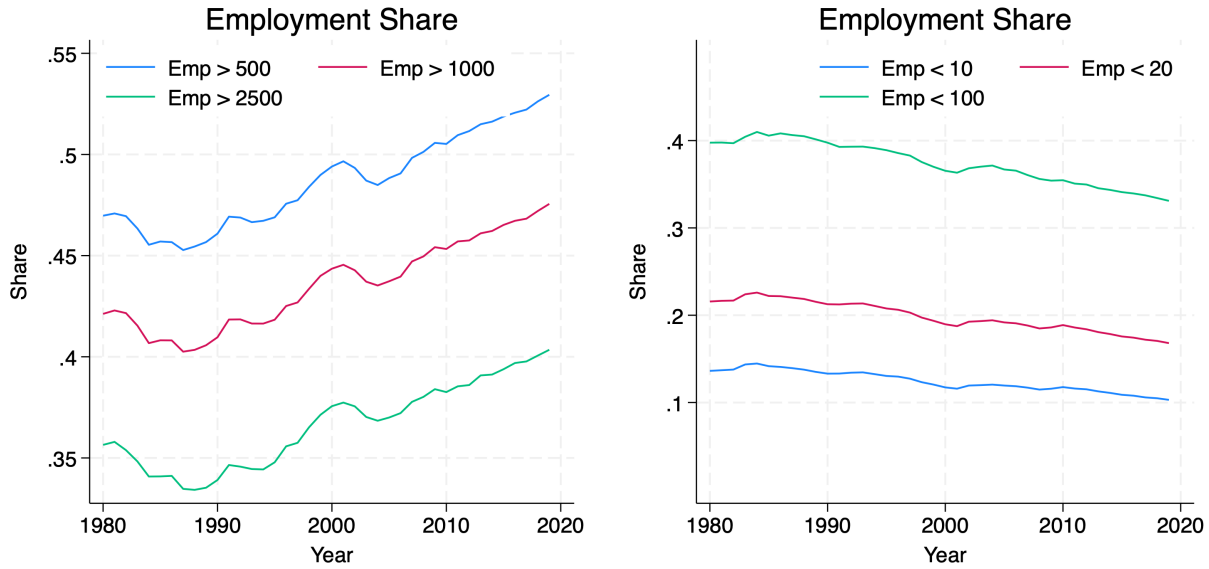
Note: This figure depicts the changes in the long-term real interest rate in the U.S. The plot is created based on Federal Reserve Economic Data (FRED) using the 10-year treasury rates minus the 10-year inflation expectation.

Figure 2: Debt Heterogeneity



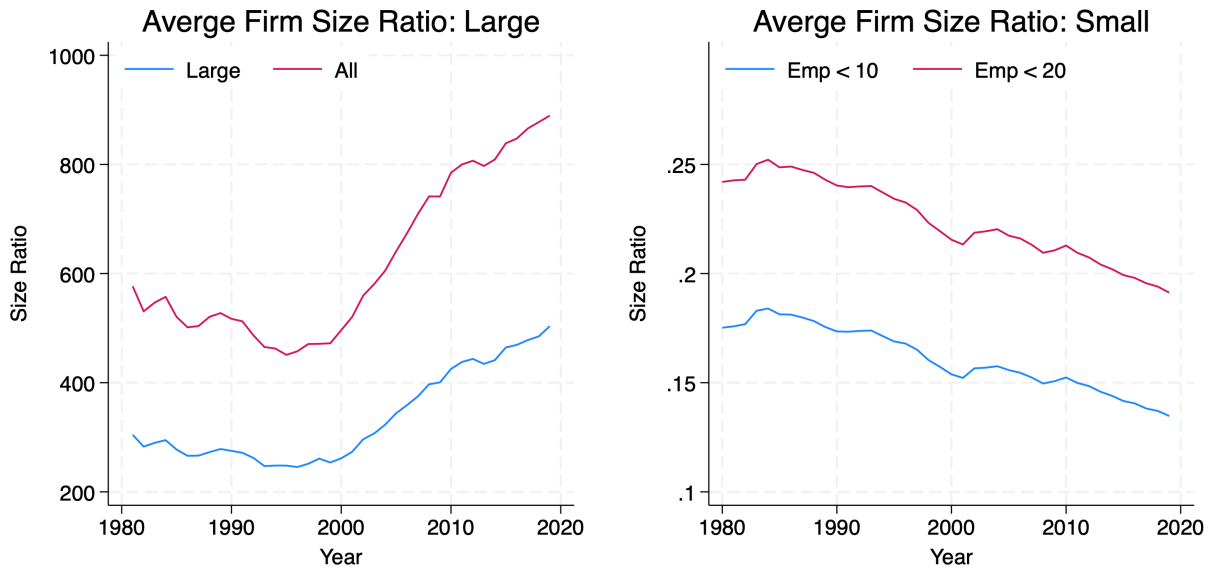
Note: This figure shows the fraction of different loan types used by Compustat firms and the aggregate composition. The left figure plots the median percentage of earnings-based loans of large and small Compustat firms. The percentages of earnings-based loans are calculated based on methods used by Lian and Ma (2021). The right panel categorizes the aggregate composition of different debt types. To keep conservative, I classify all U.S. bonds and commercial paper into earnings-based loans and all bank loans and mortgages into asset-based loans.

Figure 3: Employment Shares of Large and Small Firms over the Years



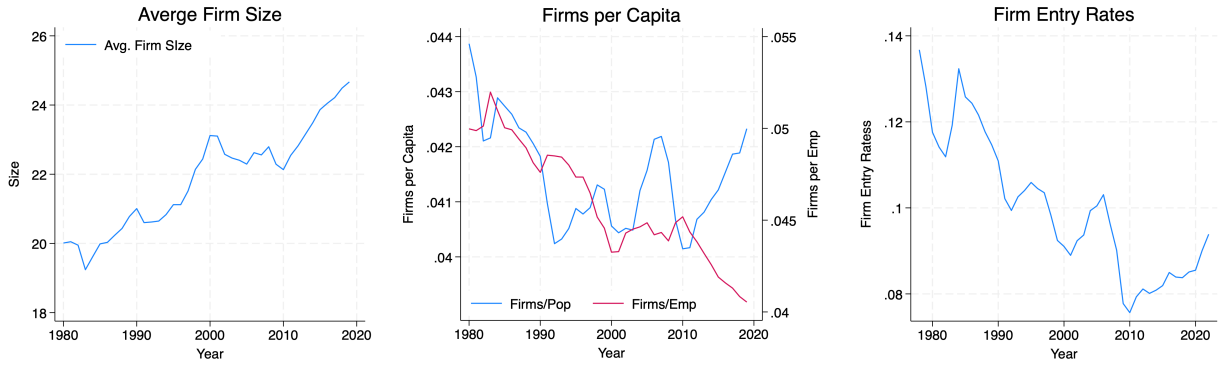
Note: This figure depicts long-term changes in the U.S. firm size distribution. The left panel shows the employment shares of firms with more than 500, 1000, and 2500 employees, respectively. The right panel shows the employment shares of firms with less than 10, 20, and 100 employees, respectively. All calculations are based on BDS data.

Figure 4: Average Firm Sizes on the Top and Bottom over the Years



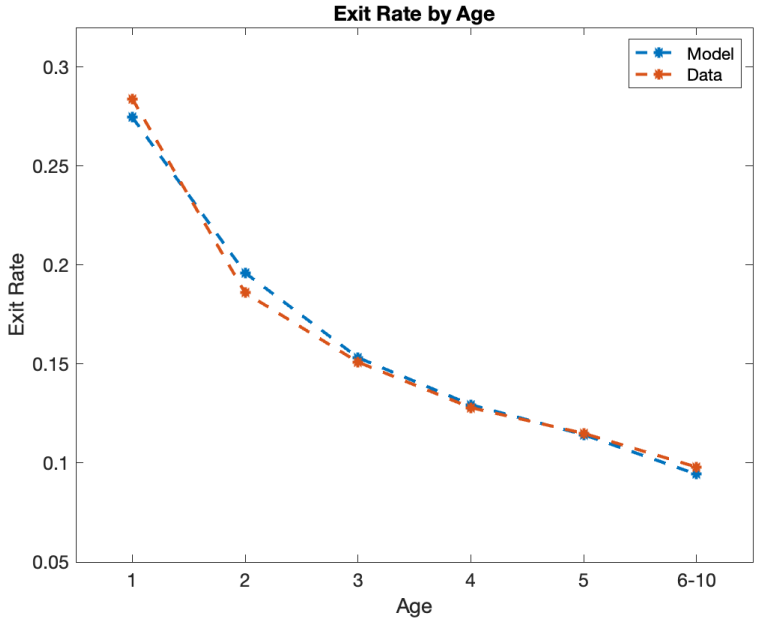
Note: This figure depicts long-term changes in the average size of firms at the top and bottom of the distribution in the U.S. The left panel shows the average employment of Compustat firms and those assets above the median compared with the average size of all firms. The right panel shows the average employment of firms with less than 10 and 20 employees, compared with the average size of all firms.

Figure 5: The Decline in Business Dynamics



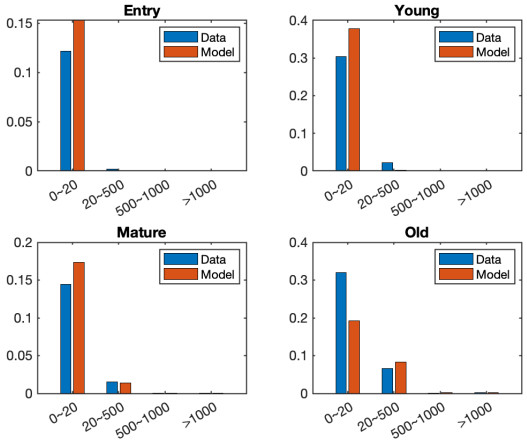
Note: This figure depicts long-term changes in business dynamics in the U.S. The left panel shows the increase in the average firm size calculated based on BDS data. The middle panel shows firms per capita, defined as the number of total firms divided by the total number of working age population 25 to 54 years old or the total number of employees, respectively. The right panel shows the firm entry rate, defined as the number of new firms divided by total firms.

Figure 6: Targeted Firm Exit Rates



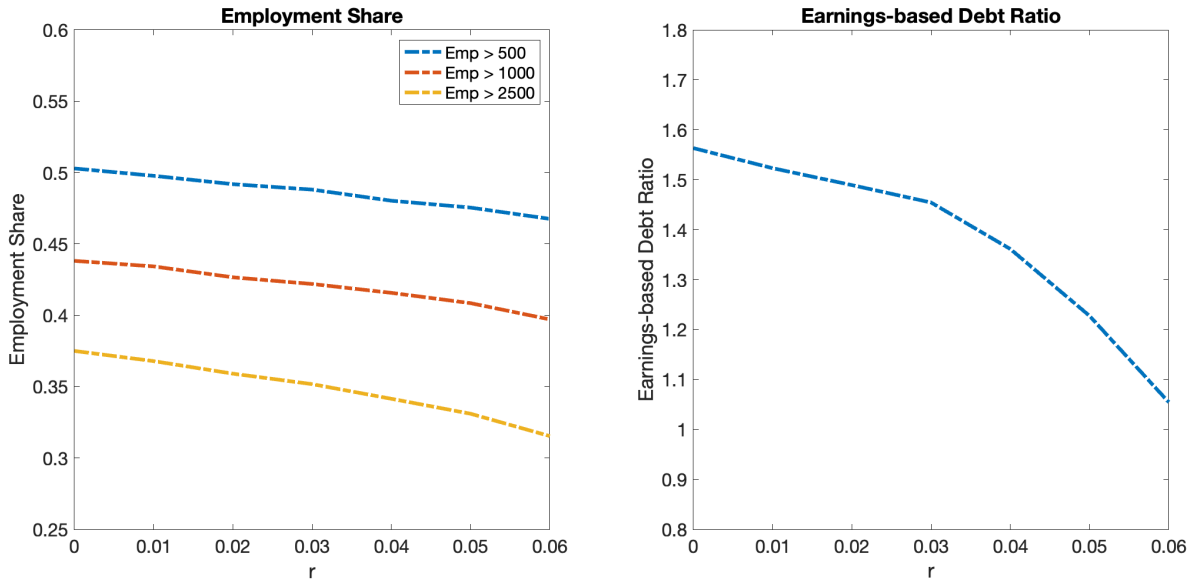
Note: This figure shows the targeted firm exit rates. The firm exit rates are calculated as the number of total exiting firms for each age divided by total incumbents at the same age.

Figure 7: Non-targeted Moments



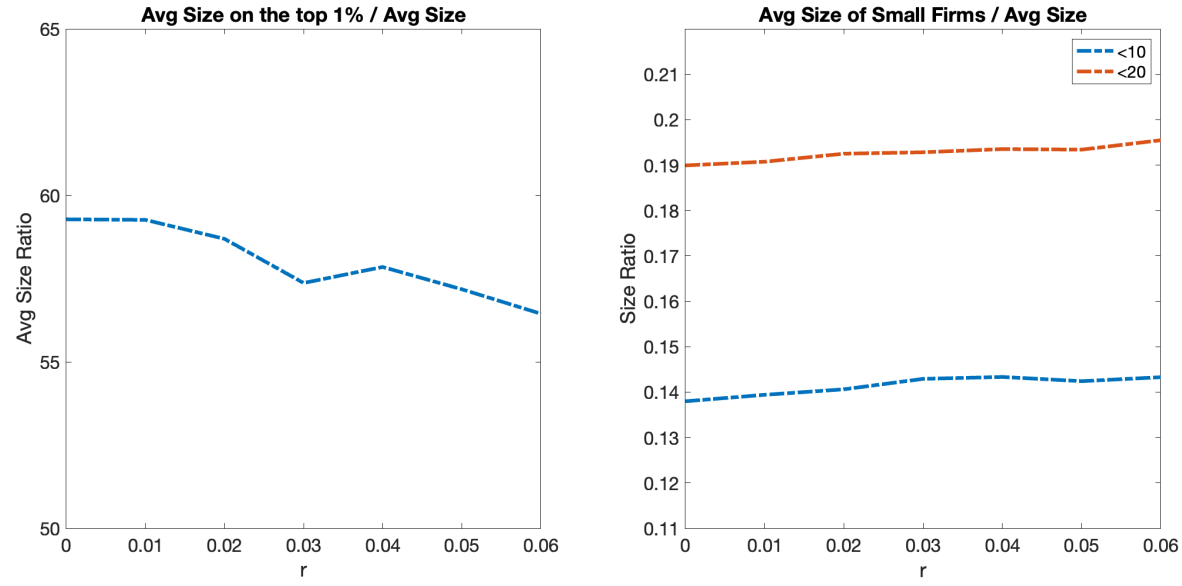
Note: This figure shows the non-targeted distributions of size $\times$ age bins. Entry is new firms. Young firms are those ages between 1 and 5. Mature firms are those ages between 6 and 10. Old firms are those with ages older than 10 years. All areas with the same color add up to one.

Figure 8: Shifts in Employment Shares and Debt Composition



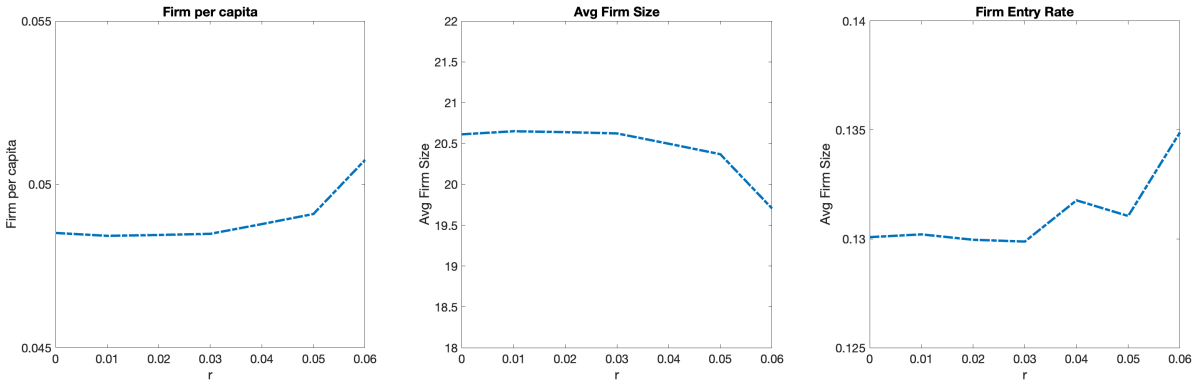
Note: This figure shows the model predicted employment shares and debt compositions at the stationary equilibrium for each level of r.The left panel shows the employment shares of firms with more than 500, 1000, and 2500 employees, respectively. The right panel shows the aggregate earnings-based to asset-based debt ratio.

Figure 9: Employment Size Distribution



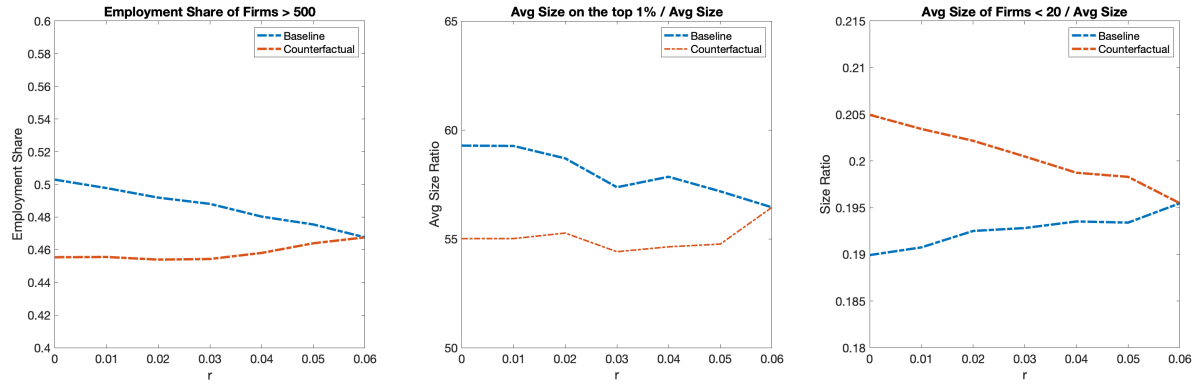
Note: This figure shows the model predicted changes in employment size distribution at the stationary equilibrium for each level of r. The left panel shows the ratio of the average employment size at 1% percentile to the average size of all firms. The right panel shows the ratio of average employment size with less than 10 or 20 employees to the average size of all firms.

Figure 10: The Decline in Firm Dynamics and the Average Firm Size



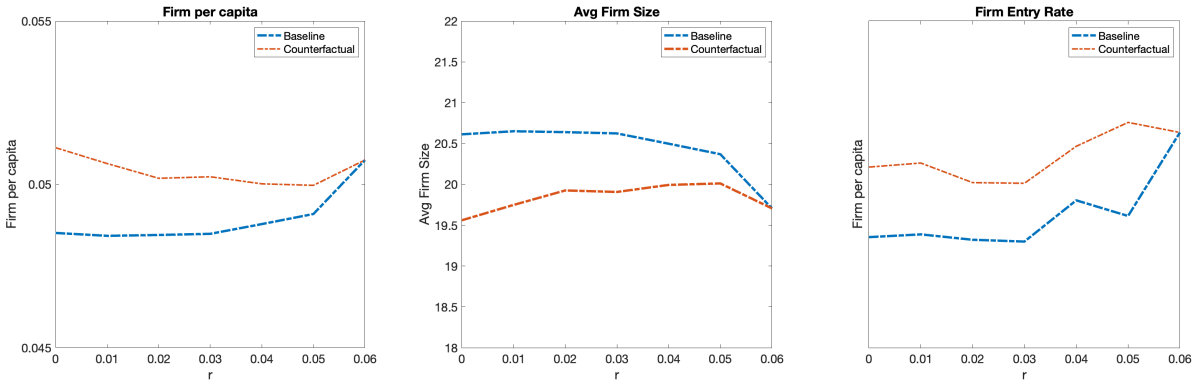
Note: This figure shows the model predicted levels of firms per capita, the average firm size, and the firm entry rate at the stationary equilibrium for each level of r. The left panel shows the levels of firms per capita, and the middle panel depicts the average firm size. The left panel shows the firm entry rate.

Figure 11: Changes in the Employment Size Distribution



Note: This figure compares the benchmark economy with a counterfactual economy without debt heterogeneity: all firms use asset-based financing. The results are the outcomes at the stationary equilibrium for each level of r. The left panel compares the shifts in employment shares, and the middle and right panels compare the average employment sizes at the top and the bottom of the distribution.

Figure 12: Changes in Firm Dynamics and the Average Firm Size



Note: This figure compares the benchmark economy with a counterfactual economy without debt heterogeneity: all firms use asset-based financing. The results are the outcomes at the stationary equilibrium for each level of r. The left panel compares firms per capita, and the middle panel compares the average firm size. The right panel compares the firm entry rate.

Table 1: Externally Calibrated Parameters

Parameter	Name	Value	Source
α	capital share	0.3	Poschke (2018)
γ	employment share	0.6	Poschke (2018)
β_f	discounting rate	0.943	Interest Rate at 1980s
δ	depreciation	0.10	Poschke (2018)
ρ_e	interest spread	0.020	Philippon (2015)
ρ_a	interest spread	0.020	Philippon (2015)
η	population growth	0.019	Karahan et al. (2019)
ϕ_e^{ICR}	interest coverage ratio	0.27	Lian and Ma (2021)
ϕ_e^{DE}	debt to earning ratio	3.5	Lian and Ma (2021)

Table 2: Internally Calibrated Parameters

Parameter	Name	Value
μ_{s_0}	mean of s_0	0.500
σ_{s_0}	std of log s_0	0.001
μ_{a_0}	mean of a_0	-2.257
σ_{a_0}	std of log a_0	1.756
σ_{z_0}	std of log z_0	0.087
σ_ϵ	std of shocks	0.058
ρ_s	persistence of shocks	0.985
f_a/y	fixed costs / output	0.018
c_e	entry cost	0.001
ξ	shock to exit	0.053
ϕ_k	leverage ratio of asset	0.331
f_m/y	fixed cost / output	0.016

Table 3: Some Targeted Moments

Moment	Model	Data	Source
Firm share ≥ 500	0.0038	0.0035	BDS
Employment share ≥ 500	0.468	0.460	BDS
Employment share ≥ 1000	0.397	0.412	BDS
Employment share ≥ 2500	0.315	0.345	BDS
Avg employment	19.71	19.76	BDS
Exit rate (Entry Rate)	0.121(0.134)	0.114(0.123)	BDS
S.T.D. of emp growth	0.301	0.420	Gavazza et al. (2018)
Avg. size for large firm	10372	10498	Compustat
Earnings/asset financing ratio	1.055	1.003	Financial Accounts
Debt-to-output ratio	0.367	0.350	Financial Accounts

Table 4: Effects of the Real Interest Rate on the Economy

This table compares the data moments and moments generated by calibrated model. I set the real interest rate to be 6% percent and 0% in the model to contrast the US economy in 1982 - 1986 and 2013 - 2017, respectively. The table compares employment shares for firms with more than 500, 1000, and 1500 employees, firms per capital (the number of firms divided by total employees), the firm entry rate, and earnings to asset debt ratio. o classify earnings-based debt, I include levels of commercial paper and corporate bonds from nonfinancial corporate business sector. I classify mortgages and depository loans as asset-based debt.

	Emp Share $\geq$ 500		Emp Share $\geq$ 1000		Emp Share $\geq$ 2500		Firms Per Capita		Firm Entry Rate		E/A Financing	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
1982 - 1986	0.460	0.468	0.412	0.397	0.345	0.315	0.0506	0.0507	0.123	0.135	1.00	1.05
2013 - 2017	0.519	0.503	0.465	0.438	0.394	0.375	0.0421	0.0484	0.082	0.130	2.89	1.56
Changes	0.059	0.035	0.053	0.041	0.049	0.060	-0.0085	-0.0023	0.041	0.005	1.89	0.51

Table 5: Summary Statistics

This table displays the summary statistics for Compustat and Captial IQ combined data from 2001 to 2019. Large (small) firms are defined as firms with assets above (below) the 2 digit sector median.

	Full Sample	Large Firms	Small Firms
Assets (millions)	5037.8	9755.5	355.9
Employment	11378	21347	1485
Earnings-base Debt (%)	56.2	70.0	42.4
Interest Rates	0.079	0.066	0.093
Physical Asset (% of Assets)	63.0	57.8	68.2
Debt-Assets Ratios	0.65	0.35	0.95
Sales to Assets Ratio	1.11	0.95	1.28
Observations	54,674	27,233	27,441

Table 6: Regression Results

This table displays the regression results. Standard errors are in parentheses. The dependent variable is log employment. $i_{s,t}$ is firm-level imputed interest rate, calculated by interest expenses (xint) divided by total debt (dltt + dlc), and ratio$_{s,t}$ is the fraction of earnings-based debt. Firm-level controls include debt to asset ratio, capital (ppegt) to asset ratio, and sales to asset ratio. $*$: p < 0.05, $**$: p < 0.01, $***$: p < 0.001

	Log Employment					
	(1)	(2)	(3)	(4)	(5)	(6)
$i_{s,t}$	-0.600***	-0.862***	-0.811***	-0.578**	-0.830***	-0.773***
	(0.101)	(0.106)	(0.108)	(0.098)	(0.105)	(0.105)
$ratio_{s,t}$	0.160***	0.178***	0.139***	0.158***	0.176***	0.130***
	(0.023)	(0.028)	(0.028)	(0.023)	(0.028)	(0.028)
$i_{s,t} \times ratio_{s,t}$	-0.870***	-0.998***	-0.766***	-0.859***	-0.987***	-0.752***
	(0.179)	(0.194)	(0.197)	(0.178)	(0.193)	(0.197)
Sample	Full	Full	Full	Full	Full	Full
Controls	N	N	N	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y
Industry FE	N	Y	N	N	Y	N
Firm FE	N	N	Y	N	N	Y
Observations	54,674	54,674	54,674	54,674	54,674	54,674

Appendix to Low Interest Rates, Debt Heterogeneity, and Firm Dynamics

Dongchen Zhao

January 21, 2025

Contents

A Proofs of Propositions **2**

A.1 Proof of Proposition 1 2

A.2 Proof of Proposition 2 3

A.3 Proof of Proposition 3 7

A.4 Proof of Propositions with the Debt to Earnings Ratio Constraint 7

B Computational Appendix **10**

C Calibration of Counterfactual Economy **11**

A Proofs of Propositions

A.1 Proof of Proposition 1

At the steady state, earnings-based firms' budget constraints become

$$d = f(k, l, z, s) - wl - rb$$

Then, $V^e = \sum_{s=0}^{\infty} \beta_f^s d$.

And,

$$V^e = \frac{1}{1 - \beta_f}[f(k, l, z, s) - wl - rb]$$

Therefore, with $b \leq \psi V^e$, we have

$$b \leq \frac{\psi}{1 - \beta_f}[f(k, l, z, s) - wl - rb]$$

A little algebra shows

$$b \leq \frac{\psi}{1 - \beta_f + r\psi}[f(k, l, z, s) - wl]$$

Further, if

$$1 = \beta_f(1 + r)$$

Then,

$$rb \leq \frac{\psi}{\beta_f + \psi}[f(k, l, z, s) - wl]$$

A.2 Proof of Proposition 2

Asset-based firms. The problem of asset-based firms is as follows:

$$\begin{aligned}
&\max_{d_t,k_t,l_t,a_{t+1}} \sum_{t=0}^{\infty} \beta_f^t d_t \\
\text{s.t.} \quad & d_t + a_{t+1} = k_t^\alpha l_t^\gamma - w_t l_t - r_t b_t - \delta q_t k_t + a_t \\
& q_t k_t \leq \frac{1}{1-\phi_k(1-\delta)} a_t \\
& 0 \leq a_{t+1}
\end{aligned}$$

The first-order conditions of the problem give:

$$\begin{aligned}
&[l_t] \quad \gamma k_t^\alpha l_t^{\gamma-1} - w_t = 0 \\
&[a_{t+1}] \quad \beta_f[1 + r_{t+1} + \frac{1}{1-\phi_k(1-\delta)}\mu_{a,t+1} + \eta_{a,t+1}] = 1 \\
&[k_t] \quad \alpha k_t^{\alpha-1} l_t^\gamma - r_t q_t = q_t \mu_{k,t}
\end{aligned}$$

Assume that the economy is at the steady-state equilibrium with interest rate r and wage w, then the labor demand for the asset-based firm l_a is

$$l_a = \gamma^{\frac{1}{1-\alpha-\gamma}} w^{-\frac{1-\alpha}{1-\alpha-\gamma}} \{\frac{\alpha}{\gamma} \frac{1}{rq + \mu_a q + \delta q}\}^{\frac{\alpha}{1-\alpha-\gamma}} \tag{1}$$

When $1/\beta_f - 1 > r$, it must be that $\eta_a = 0$. Otherwise, $a = k = 0$ so the firm will not produce.

Earnings-based firms. The problem of earnings-based firms is as follows:

$$\begin{aligned}
&\max_{d_t,k_t,l_t,a_{t+1}} \sum_{t=0}^{\infty} \beta_f^t d_t \\
\text{s.t.} \quad & d_t + a_{t+1} = k_t^\alpha l_t^\gamma - w_t l_t - r_t b_t - \delta q_t k_t + a_t \\
& r_t b_t \leq \phi_e \cdot [k_t^\alpha l_t^\gamma - w_t l_t] \\
& 0 \leq a_{t+1}
\end{aligned}$$

Replacing $b_t = q_t k_t - a_t$, the first-order conditions become:

$$[l_t] \quad (1 + \mu_{e,t}\phi_e) \cdot [\gamma k_t^{\alpha} l_t^{\gamma-1} - w_t] = 0$$

$$[a_{t+1}] \quad \beta_f[1 + r_{t+1} + r_{t+1}\mu_{e,t+1} + \eta_{e,t+1}] = 1$$

$$[k_t] \quad (1 + \phi_e\mu_{e,t}) \cdot \alpha k_t^{\alpha-1} l_t^{\gamma} = r_t q_t \mu_{e,t} + r_t q_t + \delta q_t$$

, where $\mu_{e,t}$ is the Lagrange multiplier of the borrowing constraint, and $\eta_{e,t}$ is the Lagrange multiplier of the non-negative equity holding constraint.

The case when the earnings-based firms are binding and have positive equity holdings: If I assume that the economy is at the steady-state equilibrium with interest rate r and wage w, the borrowing constraint is binding, and firms have positive equity holdings, then the labor demand for the earnings-based firm l_m is

$$l_e = \gamma^{\frac{1}{1-\alpha-\gamma}} w^{-\frac{1-\alpha}{1-\alpha-\gamma}} \left(\frac{\alpha}{\gamma} \frac{1 + \phi_e\mu_e}{rq\mu_e + rq + \delta q}\right)^{\frac{\alpha}{1-\alpha-\gamma}} \tag{2}$$

Then, the steady-state equilibrium labor ratio of the two is

$$\frac{l_e}{l_a} = \left(\frac{(r + \mu_a + \delta)(1 + \phi_e\mu_e)}{r + r\mu_e + \delta}\right)^{\frac{\alpha}{1-\alpha-\gamma}} \tag{3}$$

When $1/\beta - 1 > r$, as shown above, asset-based firms have positive equity so $\eta_a = 0$. Therefore, $\mu_a > 0$ and the borrowing constraint is always binding.

This is not the same for earnings-based firms. When earnings-based firms have positive assets, $\eta_e = 0$. When $\eta_e = 0$, since $1 - \alpha - \gamma > 0$, we need only look at the ratio

$$\frac{(r + \mu_a + \delta)(1 + \phi_e\mu_e)}{r + r\mu_e + \delta}$$

Since $\frac{1}{1-\phi_k(1-\delta)}\mu_a = \frac{1}{\beta_f} - 1 - r = r\mu_e$, replacing r and μ_a with μ_e, one can get

$$\frac{(\frac{1}{\beta_f} - 1 - \phi_k(1-\delta)\frac{\mu_e(\frac{1}{\beta_f}-1)}{1+\mu_e} + \delta)(1+\phi_e\mu_e)}{\frac{1}{\beta_f} - 1 + \delta}$$

Observe that μ_e increases monotonically as r decreases, it is sufficient to look at how the labor ratio changes with respect to μ_e. The derivative with respect to μ_e yields the following.

$$\frac{d\frac{l_e}{l_a}}{d\mu_e} = \frac{A\mu_e^2 + B\mu_e + C}{D}$$

$$A = \phi_e\left((1-\delta)\left((\beta_f - 1)\phi_c - \beta_f\right) + 1\right)$$

$$B = 2\phi_e\left((1-\delta)\left((\beta_f - 1)\phi_c - \beta_f\right) + 1\right)$$

$$C = \phi_e\left(1 - \beta_f\left(1-\delta\right)\right) - \left(1-\beta_f\right)\left(1-\delta\right)\phi_c$$

$$D = \left(\beta_f\left(\delta - 1\right) + 1\right)\left(\mu_e + 1\right)^2$$

It is easy to see that D > 0. Given that $\beta_f < 1$, $\phi_c < 1$, and $\delta \geq 0$, it is easy to see that A and B > 0. Therefore, the numerator is a quadratic function opening upwards with the vertex at $\mu < 0$. Then, if $C \geq 0$, that is,

$$\phi_e(1 - \beta_f\left(1-\delta\right)) \geq \left(1-\beta_f\right)\left(1-\delta\right)\phi_k,$$

The steady state labor ratio increases monotonically as r decreases. When this condition is violated, the relationship will not be monotonic and the labor ratio will decrease at the beginning and then increase monotonically afterwards.

The case when the earnings-based firms are binding and have zero equity holding. While I assume earning-based firms have positive asset holdings when r is close to $\frac{1}{\beta_f} - 1$, as r decreases, the firms might reduce asset holdings as the constraint becomes less tight due to lower interest expenses. Therefore, it might have zero asset holdings. When $\eta > 0$, earnings-based firms have zero

asset holding. If the borrowing constraint is binding, then the borrowing constraint for earnings-based firms gives

$$\begin{aligned} rqk_e &= \phi(k_e^{\alpha} l_e^{\gamma} - wl_e) \\ &= \phi(\frac{1}{\gamma} - 1)wl_e \end{aligned}$$

Asset-based firms have the following equation

$$\frac{\gamma q k_a}{\alpha} = \frac{wl_a}{r + \delta + \mu_a}$$

Therefore, a little algebra can show

$$\frac{l_e}{l_a} = [(\frac{\gamma\phi(\frac{1}{\gamma} - 1)}{\alpha})\frac{r + \delta + \mu_a}{r}]^{\frac{\alpha}{1-\alpha-\beta}} \tag{4}$$

, and it is trivial that the labor ratio increases monotonically as r decreases.

The case when the earnings-based firms are unbinding: Finally, with $\delta > 0$, it is possible that earnings-based firms can be unbinding. In particular, having an unbinding borrowing limit is equivalent to both $\eta = 0$ and $\mu_e = 0$, that is, $a = 0$.

The sufficient and necessary condition for this case is that

$$rk_e \leq \phi_e(\frac{1}{\gamma} - 1)\frac{\gamma}{\alpha}(r + \delta)k_e,$$

where the right-hand side combines the first-order conditions.

It is trivial that when r gets close to zero, the earnings-based firm will become unbinding as the left-hand side reaches zero but the right-hand side remains positive. It is again straightforward that the labor ratio will increase monotonically in such a scenario.

When $\delta = 0$, the constraint of the earnings-based firm can be either unbinding or binding. Regardless of the status of the constraint, from the equations shown above, the labor ratio goes to

infinity when r reaches zero.

A.3 Proof of Proposition 3

From the equations 2 and 4above, it is easy to see that q does not appear in either of them. In other words, when the borrowing constraint for earnings-based firms is binding, q does not affect the labor ratio. When the earnings-based firm is not binding, the labor ratio becomes

$$\frac{l_e}{l_a} = [\frac{r+\delta+\mu_a}{r}]^{\frac{\alpha}{1-\alpha-\beta}},$$

and it is trivial that q does not affect the labor ratio.

A.4 Proof of Propositions with the Debt to Earnings Ratio Constraint

Earnings-based firms. The problem of earnings-based firms is as follows:

$$\begin{aligned} &\max_{d_t,k_t,l_t,a_{t+1}} \sum_{t=0}^{\infty} \beta_f^t d_t \\ \text{s.t.} \quad & d_t + a_{t+1} = k_t^\alpha l_t^\gamma - w_t l_t - r_t b_t - \delta q_t k_t + a_t \\ & b_t \leq \phi_e \cdot [k_t^\alpha l_t^\gamma - w_t l_t] \\ & 0 \leq a_{t+1} \end{aligned}$$

First of all, I show that if the constraint is binding and firms have positive asset holdings when r is close to $\frac{1}{\beta_f} - 1$, then earnings-based firms must always have positive asset holdings for any $r \in [0, \frac{1}{\beta} - 1)$.

For the borrowing limit to hold, it must be that

$$k \leq a + \phi_e(k^\alpha l^\gamma - wl)$$

Applying first-order conditions, that is,

$$\alpha k^{\alpha-1} l^{\gamma} = \frac{r + \delta + \mu_e}{1 + \phi_e \mu_e},$$

and note also that

$$\gamma k^{\alpha} l^{\gamma-1} = w,$$

so we have

$$k_e(1 - \phi_e \frac{1-\gamma}{\alpha} \frac{r + \delta + \mu_e}{1 + \phi_e \mu_e}) \leq a$$

Also, that

$$1 = \beta_f(1 + r + \mu_e + \eta_e)$$

implies that

$$r + \mu_e \leq \frac{1}{\beta_f} - 1 \tag{5}$$

When r is close to $\frac{1}{\beta_f} - 1$, that the constraint is binding and $a > 0$ implies that

$$\begin{aligned} k_e &= \phi_e \frac{1-\gamma}{\alpha} \frac{r + \delta + \mu_e}{1 + \phi_e \mu_e} k_e + a \\ &\geq \phi_e \frac{1-\gamma}{\alpha} (\frac{1}{\beta_f} - 1 + \delta) k_e + a \\ &> \phi_e \frac{1-\gamma}{\alpha} (\frac{1}{\beta_f} - 1 + \delta) k_e \end{aligned}$$

Therefore,

$$1 \geq \phi_e \frac{1-\gamma}{\alpha} (\frac{1}{\beta_f} - 1 + \delta)$$

Then, for any r, we have

$$k_e(1 - \phi_e \frac{1-\gamma}{\alpha} \frac{r + \delta + \mu_e}{1 + \phi_e \mu_e}) \leq a,$$

and using equation 5 and $\mu_e > 0$ we have

$$k_e(1-\phi_e\frac{1-\gamma}{\alpha}(\frac{1}{\beta_f}-1+\delta)) \leq k_e(1-\phi_e\frac{1-\gamma}{\alpha}\frac{r+\delta+\mu_e}{1+\phi_e\mu_e}) \leq a$$

Since

$$1 \geq \phi_e\frac{1-\gamma}{\alpha}(\frac{1}{\beta_f}-1+\delta),$$

a must be greater than zero for k_e to have a positive value. Therefore, the earnings-based firm must have a positive holding of the asset. That is $\eta_e = 0$.

Therefore, the labor ratio is

$$\frac{l_m}{l_b} = (\frac{(r+\mu_c+\delta)(1+\phi\mu_e)}{r+\mu_e+\delta})^{\frac{\alpha}{1-\alpha-\gamma}} \tag{6}$$

Since $\frac{1}{1-\phi_c(1-\delta)}\mu_c = \frac{1}{\beta_f} - 1 - r = \mu_e$, replacing μ_e and μ_c with r, one can get

$$\frac{\{r+\delta+(\frac{1}{\beta_f}-1-r)[1-\phi_c(1-\delta)]\}(1+\phi_e(\frac{1}{\beta_f}-1-r))}{\frac{1}{\beta_f}-1+\delta}$$

The derivative with respect to r gives

$$\frac{\phi_e\left((\delta-1)\left(2\phi_c\left(\beta_f\left(r+1\right)-1\right)-\beta_f\right)-1\right)-\beta_f\left(\delta-1\right)\phi_c}{\beta_f\left(\delta-1\right)+1}$$

Therefore, the labor ratio is increasing monotonically when r decreases if

$$\frac{1}{\phi_e} \leq \frac{1}{\phi_c(1-\delta)}(\frac{1}{\beta_f}-1+\delta) - 2(\frac{1}{\beta_f}-1-r)$$

The labor ratio is monotonically decreasing for $r \in [0, \frac{1}{\beta_f}-1]$ if and only if

$$\frac{1}{\phi_e} \leq \frac{1}{\phi_c(1-\delta)}(\frac{1}{\beta_f}-1+\delta) - 2(\frac{1}{\beta_f}-1)$$

We reach a similar conclusion from the above equation, that is the labor ratio monotonically de-

creases when r increases when i) ϕ_e is sufficiently large, ii) ϕ_c is sufficiently small, iii) discount factor β_f is sufficiently small, or iv) δ is large enough. Same logic works here: debt capacity, reliance on asset accumulation, and marginal revenue from production matter for the effects.

B Computational Appendix

This section describes how I compute the solution to the balanced growth path. The model is solved globally over a discretized state space using the following procedure.

Step 1: Given the real interest rate and wage, I solve the firm value function over the grid points by value function iteration and find the associated policy function by golden section search. I approximate firm decisions over a grid for permanent productivity z, the stochastic component s, and the level of physical assets a. I use 10 grid points to discretize the distribution of permanent productivity $\Gamma_z(z)$. For z , we use the 0.005 and 0.995 percentiles of the distribution as the grid bounds. For the distribution of s, I choose 25 grid points and set the lowest and largest points to correspond to log productivity that are 5 standard deviations below and above the mean. For the distribution of assets, the lower bound of the grid is 0, and the upper bound is chosen so that the largest customer base in the solution of any version of the model is smaller than the bound. The grid points are chosen evenly across the log level of assets.

Step 2: To approximate the distribution of firms, we construct a finer grid with 10, 100, and 100 points in the z, s, and a direction, respectively. Then, I solve the firm's problem once on the new grid using the approximation to the value functions from the previous step. I use the grid method to find the stationary distribution. This method approximates the distribution of firms on a histogram based on the finer grid. Since both optimal policies are allowed to vary continuously, we assign values of a that do not fall on points in the grid by using the transition matrix to allocate firms in the histogram based on the proximity of the optimal policy to each point in the finer grid. Finally, the distribution of firms is obtained by iterating until convergence.

Step 3: Finally, I calculate the distance between aggregate labor demand and labor supply. If

the distance is small, stop. Otherwise, update wage with a Newton method and go to Step 1.

C Calibration of Counterfactual Economy

In the counterfactual economy, I assume all firms borrow from the asset-based financing. I keep externally calibrated parameters the same as the benchmark model. Also, I do not need to calibrate ρ_e and f_e as there is no earnings-based financing. In terms of targeted moments, I follow the benchmark economy as closely as possible.

Table 1 and Table 2 show the externally and internally calibrated parameters. Table 3 and Figure 1 show the targeted and non-targeted moments.

Table 1: Externally Calibrated Parameters: Counterfactual

Parameters	Name	Value	Source
α	capital share	0.3	Poschke (2018)
γ	employment share	0.6	Poschke (2018)
β_f	discounting rate	0.943	Interest Rate at 1980s
δ	depreciation	0.10	Poschke (2018)
ρ_a	interest spreads	0.020	Philippon (2015)
η	population growth	0.019	Karahan et al. (2019)

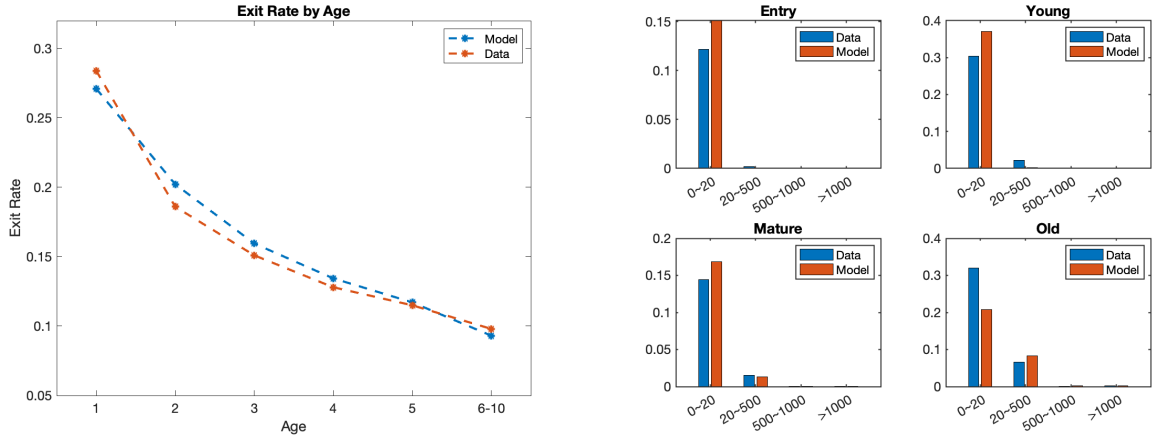
Table 2: Internally Calibrated Parameters: Counterfactual

Parameters	Name	Value
μ_{s_0}	mean of s_0	0.350
σ_{s_0}	std of log s_0	0.011
μ_{a_0}	mean of a_0	-3.656
σ_{a_0}	std of log a_0	2.032
σ_{z_0}	std of log z_0	0.100
σ_ϵ	std of shocks	0.059
ρ	persistence of shocks	0.984
f_a/y	fixed costs / output	0.015
c_e	entry costs	0.001
ϕ_k	leverage ratio of assets	0.494

Table 3: Targeted Moments: Counterfactual

	Model	Data	Source
Firm share ≥ 500	0.0045	0.0035	BDS
Employment share ≥ 500	0.466	0.460	BDS
Employment share ≥ 1000	0.386	0.412	BDS
Employment share ≥ 2500	0.272	0.345	BDS
Avg employment	19.70	19.76	BDS
Exit rate (Entry Rate)	0.119 (0.133)	0.114 (0.123)	BDS
S.T.D. of emp growth	0.333	0.420	Gavazza et al. (2018)
Debt to output ratio	0.329	0.350	Financial Accounts

Figure 1: Other Targeted and Non-targeted Moments: Counterfactual



Note: This Figure shows the targeted firm exit rates and non-targeted distribution of size $\times$ age bins. Firm exit rates are calculated as the number of total exiting firms for each age divided by total incumbents at the same age. For the right panel, entry is new firms. Young firms are those ages between 1 and 5. Mature firms are those ages between 6 and 10. Old firms are those with ages older than 10 years. All areas with the same color add up to one.

References

Gavazza, A., Mongey, S., and Violante, G. L. (2018). Aggregate recruiting intensity. *American Economic Review*, 108(8):2088–2127.

Karahan, F., Pugsley, B., and Şahin, A. (2019). Demographic origins of the startup deficit. Technical report, National Bureau of Economic Research.

Philippon, T. (2015). Has the us finance industry become less efficient? on the theory and measurement of financial intermediation. *American Economic Review*, 105(4):1408–1438.

Poschke, M. (2018). The firm size distribution across countries and skill-biased change in entrepreneurial technology. *American Economic Journal: Macroeconomics*, 10(3):1–41.