Implications of Household Heterogeneity for the Effects of Monetary and Fiscal Policies on Carbon Emissions in Canada*

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Abstract

This paper aims to quantitatively assess the implications of heterogeneity for the conduct of monetary and fiscal policies in Canada in a context where these policies are constrained by the acceleration and mitigation of climate change. By modelling the Canadian economy, I consider the effects of monetary and fiscal policies, including via exchange rate fluctuations, tax and trade between Canada and the US. I formally test the importance of taking into account household heterogeneity, climate change, mitigation policies and forced savings when modelling the Canadian economy. Using Bayesian estimation techniques, I estimate two models that take these specificities into account, one with representative agents (RANK model), the other with heterogeneous agents that abstracts from idiosyncratic income risk (TANK model). Analysis of the results shows that household heterogeneity plays an important role in accounting for fluctuations in emissions and modifies the estimated effects of these policies. Fiscal policy plays an important role in attenuating fluctuations in emissions. Mitigation and climate transition policies force firms to decrease their level of pollution.

Keywords: Heterogeneity, RANK, Climate change, Monetary Policy, Fiscal Policy, New Keynesian Model

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1 Introduction

In Canada, as shown by the "Observatoire québécois des inégalités", income inequalities between low-income and high-income households have increased since the 1980s, both nationally and in each province. During the COVID period, this income gap widened even further, demonstrating the vulnerability of low-income households. According to the Government of Canada, climate change over the last 50 years has resulted in rising temperatures in most regions of Canada. Rising income inequality between households, the acceleration of climate change and severe recession in many of the world's currency areas are posing a real challenge to central banks and government in terms of price stability and inequality reduction.

The aim of this article is to analyse the contribution of heterogeneity to the implementation of these policies in a context where, on the one hand, households contribute differently to climate damage and, on the other hand, where in certain periods fluctuations are significant, such as that of COVID.

This article analyses these three phenomena and their implications for central bank policy-making and inequality reduction. Monetary policy can play a role in certain cyclical fluctuations in income and wealth inequalities through a variation in key interest rates and a general equilibrium on the labour market. On the one hand, a reduction in the key interest rate leads to a redistribution of income from savers to borrowers insofar as there is a reduction in interest and loan payments. On the other hand, this reduction in the key interest rate increases asset prices, which increases the wealth of households that hold assets to the detriment of those that do not. On the labour market, this cut in the key rate boosts employment and wages, but this depends on the household's position in the income distribution. As for the acceleration of climate change, as noted by Sahuc et al. (2024), central banks are faced with a delicate balancing act to maintain price stability while supporting economic activity. They have to take account of the inflationary effects of both climate change and the

transition to a low-carbon economy. Periods of severe recession like that of COVID 19 constrain central banks in their role of price stability: the bank cannot reduce the interest rate beyond a certain point (ELB), in the event of negative economic shocks.

This article explores these three phenomena and their implications for central bank and government policies. We develop and estimate two New Keynesian models (RANK and TANK) of a small economy open to the rest of the world. The RANK model is a model with homogeneous agents and the TANK model is a model with heterogeneous agents. The use of the TANK model to analyse heterogeneity is supported by the work of Debortoli and Galí (2024) which shows that in the absence of idiosyncratic income risk the TANK model can reasonably well capture the implications of household heterogeneity and the main channels through which it operates. Each of these models incorporates climate change mitigation and damage reduction policies, and they also incorporate shocks specific to COVID19 , which allow us to take account of periods of strong recession Cardani et al. (2022) and Cardani et al. (2023).

Our first contribution is to fill the gap in models for evaluating carbon emissions from a long-term perspective. Contrary to existing work that integrates climate change into the production process at the technological level , we on the other hand integrate climate change as a production factor with a price and a quantity. This specification allows us to analyse the impact of the variation in price (cost of climate change) and quantities (carbon stock) on the main macroeconomic variables and on policies.

The second contribution consists of using a resolution method that takes account of structural change and stochastic fluctuations. The former comes from changes in carbon emissions, income inequality and COVID 19, the latter from stochastic fluctuations due to exogenous shocks in the economy. Thus, in the state-space framework we construct a linear solution of the dynamic equilibrium of the economy Guerrieri and Iacoviello (2015), the model does not explicitly include a stochastic trend. Adopting this method allows us to solve dynamic models with occasionally binding constraints such as New Keynesian models subject to the zero lower bound on nominal interest rates, it also adequately captures key properties of the models and is applicable to models with a large number of state variables. For the estimations we use Bayesian

techniques as in Smets and Wouters (2003), we estimate the structural parameters using macroeconomic and climatic time series for Canada and the US, from 1980q1 to 2022q2. These estimates follow two steps as in the work of Neal (2003) and Cardani et al. (2023), we first estimate the model parameters using data from 1980q1 to 2019q4, then, in the second step, we estimate the variances of the COVID-specific heteroscedasticity shocks using data for 2020q1 to 2022q4 while keeping all other parameters unchanged and initializing the system state and covariance matrix to their estimates from the first step.

The third contribution is to use the estimated RANK and TANK models to assess the importance of heterogeneity in a context where, in certain periods, there are strong recessions and also constraints imposed by the acceleration of climate change. In the RANK model, the economy does not take account of income inequalities, there is a representative household, households have the same behavioural function and can buy or sell assets on the market. In the TANK model, we consider an economy that takes account of income inequalities between constrained and unconstrained households. Unconstrained households participate actively in the market and can smooth their behaviour function by buying or selling assets. On the other hand, constrained households cannot rely solely on their income to smooth their behaviour function. These two models incorporate the strong fluctuations linked to COVID 19 and the government's desire to reduce CO2 emissions through a carbon tax. We use these models to explore the impact of heterogeneity, current and future efforts to reduce climate damage and their implications for the main macroeconomic variables, monetary and fiscal policies responses.

Our research contributes to the literature that focuses on heterogeneity, periods of strong fluctuations and climate change through structural macroeconomic models that take into account micro foundations. Campbell (1989); Bilbiie and Straub (2013); Werning (2015); Kaplan et al. (2016); Debortoli and Galí (2017); Bilbiie et al. (2017); Farhi and Werning (2019); Broer et al. (2020); Bilbiie and Ragot (2021); Bilbiie (2024); Debortoli and Galí (2024) are among the authors who have included heterogeneity (constrained and unconstrained households) in New Keynesian models. They show

the key role of heterogeneity in profits and their distribution for the transmission of policies and the amplification of aggregate demand. These models also make it possible to better analyse the impact of monetary policy but trigger a larger liquidity trap. Debortoli and Galí (2024) compare different versions of heterogeneous agent models and numerically solve a consistent redistribution programme. They show that these different models can provide similar equilibrium responses to monetary policies and other shocks, for comparable redistribution schemes. The use of deterministic heteroscedasticity to take account of periods of strong fluctuations in the estimation of structural models was carried out by Lenza and Primiceri (2022), Cardani et al. (2022) and Cardani et al. (2023). This method makes it possible to take account of periods of strong fluctuations and to estimate the global model over the whole study period. Among the authors who have introduced carbon emissions into structural models are Diluiso et al. (2021); Gibson and Heutel (2023); Shapiro and Metcalf (2023); Nakov and Thomas (2023); Del Negro et al. (2023); Coenen et al. (2024); Sahuc et al. (2024); Ferrari and Nispi Landi (2024). While these studies separately provide interesting insights into the role of heterogeneity, the linear and non-linear nature of carbon accumulation, and the inclusion of the COVID 19 period. However, we will consider these factors in a single model, which makes our analytical framework interesting for analysing the implication of heterogeneity, climate change and the COVID period on policies responses.

The rest of the paper is organized as follows: In section 2 we present the related literature, Section 3 we construct the Canadian open economy model. In Section 4, we describe the econometric approach solution methods and estimation. In Section 5, we use the model to analyze the implications of household heterogeneity of Canada economy. In Section 6, we analyse the shock decomposition of GDP growth rate in these two models . Finally, in Section 7, we conclude.

2 Related literature

This article offers a comparative perspective on the macroeconomic impact of agent heterogeneity before, during and after the COVID 19 period in a context where climate damages are increasingly important. This work focuses on four strands of the literature.

The first concerns the class of structural open economy models. One of the first works is that of Smets and Wouters (2002) which analyses the implications of the exchange rate on monetary policy in the euro zone using a DSGE model. They conclude that if a central bank wishes to minimise the resource costs of staggered price setting, it will seek to minimise a weighted average of domestic and import price inflation.

The second strand of the literature concerns New Keynesian models with heterogeneous agents, more specifically the TANK model. Based on the work of Kaplan et al. (2016) who shows that the total effect[1] of monetary policy is determined by inter-temporal substitution in a RANK model, Bilbiie et al. (2017) using a TANK model show that the share of the indirect effect increases over time, but at a higher rate than in the RANK model. The key point here is that the TANK model is more powerful than the RANK model but it also triggers a larger liquidity trap. Debortoli and Galí (2017), Debortoli and Galí (2024) compare different versions of heterogeneous agent models and numerically solve a consistent redistribution program. They show the key role of fiscal policy in the stability of global activity.

The third concerns the consideration of the impact of COVID 19 on dynamic structural models and their implications for the main macroeconomic variables and the policies responses. Among these authors we have those who study the impact of COVID on overall markets Eichenbaum et al. (2021), on demand Guerrieri et al. (2022), and on prices Benigno and Eggertsson (2023). Given the strong recession during this COVID period, some authors have developed models which take this factor into account in a context where the level of interest rates cannot reach a lower limit (ELB), these include Chen et al. (2020); Cardani et al. (2022); Cardani et al. (2023).

[1] The indirect effect is measured by inter-temporal substitution and the direct effect is measured by the marginal propensity to consume

The fourth is the acceleration of climate change. Fischer and Springborn (2011) are the first authors to introduce climate change into dynamic structural models. For them, the main source of the acceleration of climate change is the increase in CO2 in the atmosphere due to the excessive increase in industrial production. The main consequence is a deterioration in agents' well-being and productivity. Some like Annicchiarico and Di Dio (2015); Dissou and Karnizova (2016); Annicchiarico and Di Dio (2017); Gibson and Heutel (2020); Barrage (2020); Diluiso et al. (2021); Gibson and Heutel (2023); Shapiro and Metcalf (2023); Nakov and Thomas (2023); Del Negro et al. (2023); Sahuc et al. (2024); Ferrari and Nispi Landi (2024) have developed recent models to analyse the impact of accelerating climate change on macroeconomic variables and policies responses.

This paper makes a contribution to the literature by providing a comparative perspective of models with and without heterogeneities of Canadian macroeconomic dynamics during and after the COVID 19 crisis in a climate of accelerating climate change. The rich structure of the models (RANK and TANK) with many observable variables implies that recent macroeconomic evolutions must take into account income inequalities, a strong recession period and climate change mitigation.

3 Canadian open economy model

Our analysis focuses on an estimated DSGE model of the Canadian economy that trades with the rest of the world[2]. Our model has a structure and shocks that make it possible to quantitatively analyse the macroeconomic dynamics of the Canadian economy, given the number of observable series in the estimates. We extend the Lepetyuk et al. (2020) model in four directions.

Firstly, we improve the modelling at household level by incorporating heterogeneity. Specifically, in the TANK model we consider on the one hand households that buy and sell assets on the markets and on the other hand those that cannot do so. This distinction establishes the difference between the RANK model and the TANK

[2]The rest of the world consists solely of the US

model. Secondly, as in the work of Cardani et al. (2023) we take account of short-time working during the pandemic by introducing a transitory labour hoarding shock on the one hand and a temporary forced saving shock during the COVID period on the other. Third, we introduce the variables that measure the acceleration of climate change at the level of the production function as a factor of production. The marginal climate cost is the sum of the CO2 emission cost and the technological abatement cost. This specification will allow us to analyse the impact of climate variables on the main macroeconomic variables of the Canadian economy. Fourthly, we include the block of public authorities that collect taxes and participate in the economy through public spending.

Each model (RANK and TANK) includes the household sector, firms operating on the domestic market or in the import-export sector, a central bank and public authorities. We introduce temporary COVID shocks at the household and labour level due to a strong recession during this period. The Canadian central bank conducts monetary policy according to the Taylor rule and is confronted with an effective lower bound (ELB) on the nominal short-term interest rate. The import-export sector consists of exporting domestic consumer goods and commodities and importing foreign goods for domestic production. The public sector includes taxation and public spending. The two economies are linked by trade in goods and internationally traded financial assets.

3.1 Households

3.1.1 RANK model

In the model with homogeneous agents, the utility function U_t depends on the consumption of goods C_t, the temporary shock of forced savings during the COVID period ε^{tC} and wage income L_{ht}^{paid}[3] of type h

$$U_t = \frac{\mu}{\mu - 1}\left(C_t - \varepsilon_t^{tC} - \xi\left(\bar{C}_{t-1} - \varepsilon_{t-1}^{tC}\right)\right)^{\frac{\mu-1}{\mu}} exp\left(\frac{\eta(1-\mu)}{\mu(1+\eta)}\int_0^1 (L_{ht}^{paid})^{\frac{\eta+1}{\eta}} dh\right) \quad (1)$$

[3] As in the work of Cardani et al. (2023), we introduce a difference between hours paid L_{ht}^{paid} and hours worked L_{ht} due to the restriction on labour supply during COVID 19

$$\left(\frac{\bar{\lambda}_t}{P_t}\sum_Q B_t^Q(\alpha^Q - \varepsilon_t^Q)\right)\varepsilon_t^C$$

where ε_t^C is the consumption demand shock that follows an AR(1) process such that $\log \varepsilon_t^C = \rho_C \log \varepsilon_{t-1}^C + \xi_t^C$, with ξ_t^C an iid variable and ρ_C its autocorrelation coefficient. ε_t^{tC}[4] is the forced savings shock which follows an AR(1) process such that $\log \varepsilon_t^{tC} = \rho_{tC} \log \varepsilon_{t-1}^{tC} + \xi_t^{tC}$, with ξ_t^C an iid variable and ρ_{tC} its autocorrelation coefficient. We also define ξ the consumption pattern, μ the elasticity of substitution of consumption and η the wage elasticity of labour supply. Following Cardani et al. (2023), we introduce asset-specific risk premium shocks ε_t^Q, with $Q \in [rf, B, G, S]$ ε_t^B is the international asset risk premium shock that determines the interest rate parity condition between Canada and the US, ε_t^S is the shock to the risk premium on private investment, ε_t^G is the shock to the risk premium on public investment, ε_t^{rf} is the shock to the risk premium on domestic bonds. $alpha^Q$ capture the stable state of the risk premium.

The representative household of type h maximizes its utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t \tag{2}$$

under budget constraint

$$P_t C_t + \sum_Q B_{t+1}^Q = \int_0^1 W_{ht} L_{ht}^{paid} dh + \sum_Q R_t^Q B_t^Q + \Pi_t + T_t \tag{3}$$

where P_t is the CPI, Π_t the profit paid by firms, R_t^Q the return on assets held by the household. Net income from transfers and taxes is such that

$$T_t = TN_t - tax_t - \tau^w W_{ht} L_{ht}^{paid} - \tau^C P_t C_t \tag{4}$$

where TN, tax_t, τ^w, τ^C represent transfers, flat-rate taxes, payroll tax and consumption tax respectively. At the optimum, we have :

$$E_t\left[\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \left(R_t^Q + \varepsilon_t^Q - \alpha^Q\right)\right] = 1 \tag{5}$$

[4]It should be noted here that this shock is zero for periods before 2019Q4 and non-zero during and after that date

$$\lambda_t = \left(C_t - \varepsilon_t^{tC} - \xi\left(C_{t-1} - \varepsilon_{t-1}^{tC}\right)\right)^{\frac{-1}{\mu}} exp\left(\frac{\eta(1-\mu)}{\mu(1+\eta)} \int_0^1 (L_{ht}^{paid})^{\frac{\eta+1}{\eta}} dh\right) \varepsilon_t^C \quad (6)$$

where $\alpha^{rf} = 0$ and $R_t^{rf} = R_t$.

We approximate uncovered interest rate parity as in Lepetyuk et al. (2020)

$$\frac{e_t}{E_t(e_{t+1})} R_t = R_t^f (1 + \kappa_t^f) \quad (7)$$

where R_t^f represents the interest rate on foreign bonds, κ_t^f the risk premium on the foreign interest rate and e_t the exchange rate.

3.1.2 TANK model

In this model we have two groups of households, 'savers' with a proportion (w_s) and 'hand-to-mouths' with a proportion ($1 - w_s$). Savers participate in the asset market and can buy or sell assets to adjust their behavioural function, whereas hand-to-mouth households do not participate in the asset market because they do not hold assets and rely solely on their income to adjust their behavioural function.

Savers

The budget constraint is

$$P_t C_t^s + \sum_Q B_{t+1}^Q = \int_0^1 W_{ht} L_{ht}^{s,paid} dh + \sum_Q R_t^Q B_t^Q + \Pi_t + T_t^s \quad (8)$$

Net income is

$$T_t^s = TN_t^s - tax_t^s - \tau^w W_{ht} L_{ht}^{s,paid} - \tau^C P_t C_t^s \quad (9)$$

and at the optimum we have :

$$E_t\left[\beta \frac{\lambda_{t+1}^s}{\lambda_t^s} \frac{P_t}{P_{t+1}} \left(R_t^Q + \varepsilon_t^Q - \alpha^Q\right)\right] = 1 \quad (10)$$

$$\lambda_t^s = \left(C_t^s - \varepsilon_t^{tC} - \xi\left(C_{t-1}^s - \varepsilon_{t-1}^{tC}\right)\right)^{\frac{-1}{\mu}} exp\left(\frac{\eta(1-\mu)}{\mu(1+\eta)} \int_0^1 (L_{ht}^{s,paid})^{\frac{\eta+1}{\eta}} dh\right) \varepsilon_t^C \quad (11)$$

Hand-to-mouth

This group of households does not face any borrowing constraints and their consumption depends on wages and transfer income. As mentioned, this group of

households during the COVID period saved heavily to cope with the post COVID period.

$$P_t C_t^h = \int_0^1 W_{ht} L_{ht}^{h,paid} dh + T_t^h - P_t \left(\varepsilon_t^{tC} - \frac{1}{6} \sum_{i=8}^{13} \varepsilon_{t-i}^{tC} \right) \tag{12}$$

Aggregate consumption and labour supply are given by $C_t = w_s C_t^s + (1 - w_s) C_t^h$ and $L_{ht} = w_s L_{ht}^s + (1 - w_s) L_{ht}^h$ [5] respectively.

3.2 Climate

Firms producing goods seek to maximise their profit by reducing their CO2 emissions [6]. Formally we have :

$$E_t \left[\sum_{j=0}^{\infty} \left(\beta\theta_e\right)^j \lambda_{t+j} \left(\frac{\prod_{k=1}^{j} \bar{\pi}_{t+k}}{\prod_{k=1}^{j} \pi_{t+k}^e} \right)^{-\varepsilon_e} me_{t+j} \left(\frac{\prod_{k=1}^{j} \bar{\pi}_{t+k}}{\prod_{k=1}^{j} \pi_{t+k}^e} \right) \frac{P_t^{*e}}{P_t} - \frac{\varepsilon_e}{\varepsilon_e - 1} rmce_{t+j} \right] = 0 \tag{13}$$

with $rmce_t$ the real marginal cost of CO2 abatement and emissions as in Sahuc et al. (2024)

$$rmce_t = \left(\theta_{1,t}^e \mu_t^{e,\theta_2} + \tau^e (1 - \mu_t^e) \varepsilon_t^e \right) Y_t \tag{14}$$

at the optimum we have :

$$\frac{P_t^{*e}}{P_t} = \frac{F_{1t}^e}{F_{2t}^e} \tag{15}$$

$$F_{1t}^e = \lambda_t me_t \frac{\varepsilon_e}{\varepsilon_e - 1} rmce_t + \beta\theta_e E_t \left[\left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}^e} \right)^{-\varepsilon_e} F_{1t+1}^e \right] \tag{16}$$

$$F_{2t}^e = \lambda_t me_t + \beta\theta_e E_t \left[\left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}^e} \right)^{-\varepsilon_e} F_{2t+1}^e \right] \tag{17}$$

the aggregate price of climate change is

$$\theta_e \left(\frac{\bar{\pi}_t}{\pi_t^e} \right)^{1-\varepsilon} + (1 - \theta_e) \omega_e \left(\frac{(\pi_{t-1}^e)^{\gamma_e} (\bar{\pi}_t)^{1-\gamma_e}}{\pi_t^e} \right)^{1-\varepsilon_e} + (1 - \theta_e)(1 - \omega_e) \left(\frac{P_t^{*e}}{P_t} \right)^{1-\varepsilon_e} = 1 \tag{18}$$

where $\theta_{1,2,t}^e$, μ_t^e, τ^e, ε_t^e, γ_{em} represent respectively the abatement cost, the abatement, the carbon tax, the carbon shock, the elasticity of climate damage. θ_e, ω_e, ε_e, π_t^e, γ_e, Y_t

[5] Acknowledging a temporary forced savings shock during the COVID period, $L_{ht}^{paid} = \frac{L_{ht}}{1 - \varepsilon_t^{lu}} . \varepsilon_t^{lu}$ follows an AR(1) process

[6] as Golosov et al. (2014) notes; Sahuc et al. (2024) climate damage follows an exponential function such that $me_t = \exp(-\gamma_{em}(m_t - m_{1700}))$ and $m_t - m_{1700} = (1 - \delta_e) m_{t-1} - m_{1700} + \xi_e (1 - \mu_t^e) \varepsilon_t^e) Y_t$

represent respectively the probability of indexation, the share of forward-looking firms, the elasticity of substitution, inflation due to climate change, the share of indexation to past inflation and aggregate demand.

3.3 Wage setting

Households offer a variety of services and labour on the labour market. In a situation of monopolistic competition, forward-looking workers seek a wage W_t^* that maximises their utility.

$$E_t \sum_{j=0}^{infty} \left(\beta\theta_w\right)^j U_{t+j} \tag{19}$$

under constraints

$$L_{ht}^{paid} = \left(\frac{\prod_{k=1}^{j} \bar{\pi}_{t+k} W_t^*}{W_{t+j}}\right)^{-\varepsilon_w} L_{t+j} \tag{20}$$

$$P_{t+j}C_{t+j} = \prod_{k=1}^{j} \bar{\pi}_{t+k} W_t^* L_{h,t+j}^{paid} dh + \Theta_{t+j} \tag{21}$$

where Θ_{t+j} includes all other elements of the budget constraint other than consumption C_{t+j} and labour $L_{h,t+j}^{paid}$. As shown in appendix C at the optimum we have :

$$\left(W_t^*\right)^{1+\frac{\varepsilon_w}{\eta}} \left(w_t\right)^{-\frac{\varepsilon_w}{\eta}} = \frac{F_{1t}^w}{F_{2t}^w} \tag{22}$$

with

$$F_{1t}^w = \lambda_t \frac{\varepsilon_w}{\varepsilon_w - 1}\left(C_t - \varepsilon_t^{tC} - \xi\left(C_{t-1} - \varepsilon_{t-1}^{tC}\right)\right)\left(L_t\right)^{\frac{1+\eta}{\eta}} + \beta\theta_w E_t\left[\left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}^w}\right)^{\frac{-\varepsilon_w(1-\eta)}{\eta}} F_{1,t+1}^w\right] \tag{23}$$

and

$$F_{2t}^w = \lambda_t L_t + \beta\theta_w E_t\left[\frac{\bar{\pi}_{t+1}}{\pi_{t+1}}\left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}^w}\right)^{-\varepsilon_w} F_{2,t+1}^w\right] \tag{24}$$

the aggregate wage is

$$\theta_w\left(\frac{\bar{\pi}_t}{\pi_t^w}\right)^{1-\varepsilon_w} + (1-\theta_w)\omega_w\left(\frac{(\pi_{t-1}^w)^{\gamma_w}(\bar{\pi}_t)^{1-\gamma_w}}{\pi_t^w}\right)^{1-\varepsilon_w} + (1-\theta_w)(1-\omega_w)\left(\frac{W_t^*}{w_t}\right)^{1-\varepsilon_w} = 1 \tag{25}$$

where $w_t = (1-\tau^w)W_t$ is the real wage, τ_w the tax on the wage. θ_w, ω_w, ε_w, π_t^w, γ_w represent the probability of indexation, the share of firms looking to the future, the elasticity of substitution, wage inflation, and the share of indexation to past inflation respectively.

3.4 Firms

Monopolistically competitive firms are divided into two groups in the production process. The first group produces intermediate goods using labour, capital, commodities, imports and climate change as inputs. The firms in the second group, in monopolistic competition, use the inputs of intermediate goods to produce final goods.

3.4.1 Firms producing intermediate goods

Firms producing intermediate goods use a constant elasticity of substitution (CES) production function:

$$Z_t^g = \left[\delta_l\left(\varepsilon_t^A L_t\right)^{\frac{\sigma-1}{\sigma}} + \delta_k\left(\mu_t K_t\right)^{\frac{\sigma-1}{\sigma}} + \delta_{com}\left(COM_t^d\right)^{\frac{\sigma-1}{\sigma}} + \delta_m\left(M_t\right)^{\frac{\sigma-1}{\sigma}} + \delta_e\left(me_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{26}$$

where L_t, K_t, M_t, COM_t^d, me_t are labour, capital, imports, commodities and climate damage from the acceleration of climate change in the production process respectively. μ_t the use of capital, δ_{com}, δ_e, δ_k, δ_l, δ_m the share of commodities, climate damage, capital, labour and imports in the production function. The elasticity of substitution is σ, ε_t^A the level of total factor productivity following an AR(1) process. The law of motion of capital is $K_{t+1} = (1 - d_t)K_t + I_t$, where I_t is investment, d_t is the rate of depreciation of capital such that $d_t = d_0 + \bar{d}\exp(\rho(\mu_t - 1))$. d_0, $\bar{d}$, ρ represent the fixed rate of depreciation, the variable rate of semi-elastic depreciation respectively.

Firms producing intermediate goods face a quadratic adjustment cost on the level of investment. Net output is

$$Z_t^n = Z_t^g - \frac{\chi_i}{2}\left(\frac{I_t}{I_{t-1}}\right)^2 I_t \tag{27}$$

where χ_i is the coefficient of the adjustment cost.

The firm's objective is to choose a level L_t, K_t, M_t, COM_t^d, me_t which maximises its profit.

$$E_0 \sum_{\tau=0}^{\infty} \beta^\tau \frac{\lambda_{t+\tau}}{\lambda_t} \frac{P_t}{P_{t+\tau}} \left(P_{t+\tau}^z Z_{t+\tau}^n - W_{t+\tau} L_{t+\tau} - P_{t+\tau}^i I_{t+\tau} - P_{t+\tau}^{com} COM_{t+\tau}^d - P_{t+\tau}^m M_{t+\tau} - P_{t+\tau}^e me_{t+\tau}\right) \tag{28}$$

under constraint

$$Z_t^g = \left[\delta_l\left(\varepsilon_t^A L_t\right)^{\frac{\sigma-1}{\sigma}} + \delta_k\left(\mu_t K_t\right)^{\frac{\sigma-1}{\sigma}} + \delta_{com}\left(COM_t^d\right)^{\frac{\sigma-1}{\sigma}} + \delta_m\left(M_t\right)^{\frac{\sigma-1}{\sigma}} + \delta_e\left(me_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{29}$$

$$K_{t+1} = (1-d_t)K_t + I_t \tag{30}$$

$$Z_t^n = Z_t^g - \frac{\chi}{2}\left(\frac{I_t}{I_{t-1}}\right)^2 I_t \tag{31}$$

where λ_t represents the marginal utility of household consumption, P_t the price of the final good, β the discount factor. As shown in the appendix A we obtain the following optimal conditions:

$$W_t = P_t^z\left(Z_t^g\right)^{\frac{1}{\sigma}}\delta_l\left(\varepsilon_t^A\right)^{\frac{\sigma-1}{\sigma}}\left(L_t\right)^{\frac{-1}{\sigma}} \tag{32}$$

$$Q_t = \frac{1}{R_t^k}E_t\left[P_{t+1}^z\left(Z_{t+1}^g\right)^{\frac{1}{\sigma}}\delta_k\left(\mu_{t+1}\right)^{\frac{\sigma-1}{\sigma}}\left(K_{t+1}\right)^{\frac{-1}{\sigma}} + Q_{t+1}\left(1-d_{t+1}\right)\right] \tag{33}$$

$$P_t^i = Q_t - P_t^z\frac{\chi_i}{2}\left(\frac{I_t}{I_{t-1}-1}\right)\left(\frac{3I_t}{I_{t-1}-1}\right) + \frac{1}{R_t^k}E_t\left[P_{t+1}^z\chi_i\left(\frac{I_{t+1}}{I_t-1}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \tag{34}$$

$$P_t^{com} = P_t^z\left(Z_t^g\right)^{\frac{1}{\sigma}}\delta_{com}\left(COM_t^d\right)^{\frac{-1}{\sigma}} \tag{35}$$

$$P_t^m = P_t^z\left(Z_t^g\right)^{\frac{1}{\sigma}}\delta_m\left(M_t\right)^{\frac{-1}{\sigma}} \tag{36}$$

$$P_t^{me} = P_t^z\left(Z_t^g\right)^{\frac{1}{\sigma}}\delta_{me}\left(me_t\right)^{\frac{-1}{\sigma}} \tag{37}$$

$$Q_t\bar{d}\rho\exp(\rho(\mu_t-1)) = P_t^z\left(Z_t^g\right)^{\frac{1}{\sigma}}\delta_k\left(\mu_t\right)^{\frac{-1}{\sigma}}\left(K_t\right)^{\frac{-1}{\sigma}} \tag{38}$$

where Q_t is the Lagrange multiplier of the evolution of capital[7].

The nominal dividends in period t of firms producing intermediate goods are given by :

$$\frac{div_t}{P_t} = (1-\tau_k)\left(\frac{P_{i,t}}{P_t}Y_{i,t} - \frac{W_t}{P_t}L_{ht}^{paid}\right) + \tau_k d_0\frac{P_t^I}{P_t}K_{i,t-1} - \frac{P_t^I}{P_t}I_{i,t} - \varepsilon\frac{\gamma_p}{2}Y_t\left[\frac{P_{i,t}}{P_{i,t-1}} - \exp(\bar{\pi})\right]^2 \tag{39}$$

where τ_k, Y_t, γ_p are respectively corporation tax, aggregate demand and the parameter that captures the degree of the price adjustment cost.

[7]It should be noted that dividing the various variables by the price of final goods P_t gives real variables

3.4.2 Production of final goods

The firms producing the final goods use the products of the intermediate firms Z_{it}^{n} and the manufactured products Z_{it}^{mi}. The production technology Z_{it} is perfectly complementary such that

$$Z_{it} = \min\left(\frac{Z_{it}^{n}}{1-s_m}, \frac{Z_{it}^{mi}}{s_m}\right) \tag{40}$$

where s_m is the Leontief parameter.

The demand function for different goods that minimises costs is

$$Z_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Z_t \tag{41}$$

where ε, Z_t and P_t are the elasticity of consumption, output and prices respectively such that $Z_t = \left(\int_0^{\infty} Z_{it}^{frac\varepsilon-1\varepsilon} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$ et $P_t = \left(\int_0^{\infty} P_{it}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$. In the production process of final goods we distinguish two types of firms which are in monopolistic competition, the first type is that which is forward-looking with a probability of $(1-\omega)$ and the second type is that of the reference firms with a probability of ω[8].

Forward-looking firms [9] with probability $(1-\theta)$ choose a price P_t^* that maximises their profits

$$\max_{P_t^*} E_t \sum_{j=0}^{\infty} \theta^j R_{t,t+j}\left(\prod_{k=1}^{j} \bar{\pi_{t+k}} P_t^* Z_{i,t+j} - (1-s_m) P_{t+j}^{z} Z_{i,t+j} - s_m P_{t+j} Z_{i,t+j}\right) \tag{42}$$

under constraint

$$Z_{i,t+j} = \left(\frac{\prod_{k=1}^{j} \bar{\pi}_{t+k} P_t^*}{P_{t+j}}\right)^{-\varepsilon} Z_{t+j} \tag{43}$$

As shown in the appendix B the recursive optimal conditions give

$$\frac{P_t^*}{P_t} = \frac{F_{1t}}{F_{2t}} \tag{44}$$

[8] Within the two types of firms, with a probability θ certain firms can index their prices to the inflation target $\bar{\pi_t}$ as follows: $P_{it} = \bar{pi_t} P_{i,t-1}$. On the other hand, within the reference companies, some with a probability $(1-\theta)$ can partially fix their take on lagged inflation and the inflation target $P_{it} = (\pi_{t-1})^{\gamma} (\bar{\pi_t})^{1-\gamma} P_{i,t-1}$.

[9] As mentioned in Lepetyuk et al. (2020) forward-looking firms improve the Phillips curve forward-looking puzzle

$$F_{1t} = \lambda_t Z_t \frac{\varepsilon}{\varepsilon - 1} Rmc_t + \beta\theta E_t \left[\left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}} \right)^{-\varepsilon} F_{1t+1} \right] \tag{45}$$

with

$$Rmc_t = (1 - s_m) \frac{P_t^z}{P_t} + s_m \tag{46}$$

real marginal cost

$$F_{2t} = \lambda_t Z_t + \beta\theta E_t \left[\left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}} \right)^{-\varepsilon} F_{2t+1} \right] \tag{47}$$

the aggregate price give

$$\theta \left(\frac{\bar{\pi}_t}{\pi_t} \right)^{1-\varepsilon} + (1-\theta)\omega \left(\frac{(\pi_{t-1})^{\gamma} (\bar{\pi}_t)^{1-\gamma}}{\pi_t} \right)^{1-\varepsilon} + (1-\theta)(1-\omega) \left(\frac{P_t^*}{P_t} \right)^{1-\varepsilon} = 1 \tag{48}$$

3.5 Commodities

Firms producing the basic products used to produce final goods use a CES production function

$$COM_t = \left(Z_t^{com} \right)^{s_z} \left(A_t F \right)^{1-s_z} - \frac{\chi_{com}}{2} \left(\frac{Z_t^{com}}{Z_{t-1}^{com}} - 1 \right)^2 Z_t^{com} \tag{49}$$

where Z_t^{com} is the input of commodities in the production of the final good, F is the land factor considered to be fixed, χ_{com} is the adjustment cost of investment in the production of commodities. Trading with the rest of the world, commodities come on the one hand from the domestic economy (COM_t^d) and on the other hand from the rest of the world (X_t^{com}):

$$COM_t = COM_t^d + X_t^{com} \tag{50}$$

At the optimum we have :

$$P_t = P_t^{com} \frac{s_z COM_t}{Z_t^{com}} - P_t^{com} \frac{\chi_{com}}{2} \left(\frac{Z_t^{com}}{Z_{t-1}^{com}} - 1 \right) \left(\frac{3Z_t^{com}}{Z_{t-1}^{com}} - 1 \right) + \frac{1}{R_t} E_t \left[P_t^{com} \chi_{com} \left(\frac{Z_{t+1}^{com}}{Z_t^{com}} - 1 \right) \left(\frac{Z_{t+1}^{com}}{Z_t^{com}} \right)^2 \right] \tag{51}$$

where P_t^{com}, s_z represent the commodity prices, the elasticity in commodity production respectively.

3.6 Imports

Importing companies choose a price P_t^{m*} that maximises their profit. Their program is as follows

$$\max_{P_t^{m*}} E_t \sum_{j=0}^{\infty} \left(\theta_m\right)^j R_{t,t+j} \left(\prod_{k=1}^{j} \bar{\pi}_{t+k} P_t^{m*} M_{i,t+j} - e_{t+j} P_t^{mf} M_{i,t+j} \right) \tag{52}$$

under constraint

$$M_{i,t+j} = \left(\frac{\prod_{k=1}^{j} \bar{\pi}_{t+k} P_t^{m*}}{P_t^m} \right)^{-\varepsilon_m} M_{t+j} \tag{53}$$

At the optimum, we have :

$$\frac{P_t^{m*}}{P_t} = \frac{F_{1t}^m}{F_{2t}^m} \tag{54}$$

$$F_{1t}^m = \lambda_t M_t \frac{\varepsilon_m}{\varepsilon_m - 1} s_t P_t^{mf} + \beta \theta_m E_t \left[\left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}^m} \right)^{-\varepsilon_m} F_{1t+1}^m \right] \tag{55}$$

$$F_{2t}^m = \lambda_t M_t + \beta \theta_m E_t \left[\left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}^m} \right)^{-\varepsilon_m} F_{2t+1}^m \right] \tag{56}$$

aggregate price give

$$\theta_m \left(\frac{\bar{\pi}_t}{\pi_t^m} \right)^{1-\varepsilon_m} + (1-\theta_m)\omega_m \left(\frac{(\pi_{t-1}^m)^{\gamma_m} (\bar{\pi}_t)^{1-\gamma_m}}{\pi_t^m} \right)^{1-\varepsilon_m} + (1-\theta_m)(1-\omega_m) \left(\frac{P_t^{m*}}{P_t^m} \right)^{1-\varepsilon_m} = 1 \tag{57}$$

where s_t, P_t^m, P_t^{*m}, θ_m, ω_m, ε_m, π_t^m, γ_m, P_t^{mf} represent respectively the real exchange rate, domestic import prices, the optimal import price, the probability of indexation, the share of forward-looking firms, the elasticity of substitution, import inflation, the share of indexation to past inflation and import prices from the rest of the world.

3.7 Monetary and fiscal policy

The monetary authorities set a nominal short-term interest rate in response to the deviation of the current inflation rate from its stable level and the deviation of current output from its potential level.

$$Q_t = \rho^r R_{t-1} + (1-\rho^r)\left[\bar{R} + \rho^\pi (\pi_t - \bar{\pi}_t) + \rho^y \left(\log Y_t - \log \bar{Y}_t \right)\right] + \varepsilon_t^r \tag{58}$$

where ρ^r, $\bar{R}$, ρ^π, $\bar{\pi}_t$, ρ^y, $\bar{Y}_t$, ε_t^r represent respectively the interest rate smoothing parameter, the nominal long-term interest rate, the response to the inflation gap, the

inflation target, the response to the output gap, the potential level of output, and the interest rate shock which follows an AR(1) process.

In a context where the interest rate must not fall below a certain value (R_t^{elb}) the interest rate is determined as the maximum between Q_t and R_t^{elb}.

$$R_t = \max\left(Q_t, R_t^{elb}\right) \tag{59}$$

The tax authorities collect a linear tax on consumption τ^c, wages τ^w, companies τ^k, labour τ^{Lu}, CO2 emissions τ^e and production tax_t. The budget constraint is :

$$B_t^G = (1 + R_t^G)B_{t-1}^G - Y_t^G + P_t^G G_t + P_t^{IG} I_t^G + P_t T_t \tag{60}$$

the government's nominal income Y_t^G is

$$Y_t^G = \tau^k\left(P_t Y_t - W_t L_{ht}^{paid} - P_t^I d_0 K_{t-1}\right) + \tau^w W_t L_{ht}^{paid} + \tau^c C_t P_t + tax_t P_t Y_t + \tau^e P_t^e me_t + \tau^{Lu} W_t\left(L_{ht}^{paid} - L_t\right) \tag{61}$$

lump-sum taxes are defined by

$$tax_t = \rho^\tau tax_{t-1} + \eta^d\left(\frac{\Delta B_{t-1}^G}{P_{t-1}Y_{t-1}} - \bar{def}\right) + \eta^B\left(\frac{B_{t-1}^G}{P_{t-1}Y_{t-1}} - \bar{BG}\right) + \varepsilon_t^{tax} \tag{62}$$

where $\bar{def}$, ε_t^{tax}, $\bar{BG}$, η^d, η^B, ρ^τ respectively represent the public deficit target, the tax shock, the debt level target, the deficit coefficient, the debt coefficient and the smoothing coefficient. B_t^G, Y_t^G, G_t, I_t^G represent, respectively, bonds, public revenues, public expenditure, and public investment.

3.8 Rest of the world economy

The rest of the world is specified by three shocks, namely the shock to foreign activities in the domestic economy ε_t^{Zf}, the shock to the foreign interest rate ε_t^{Rf} and the shock to the price of foreign commodities ε_t^{Pcomf}. These shocks follow an AR(1) process.

3.9 Market clearing conditions

The resource constraint for the Canadian economy is

$$p_t^y Y_t = P_t C_t + P_t^i I_t + P_t^{nc} X_t^{nc} + P_t^{com} X_t^{com} - P_t^m M_t + \upsilon_y p_t^y Y_t + P_t^G G_t + P_t^{IG} I_t^G \quad (63)$$

4 Econometric approach

This section presents the econometric approach to estimating the two models of a small open economy and exogenous shocks. section 4.1 presents the data, section 4.2 the model solution and the filtering. Section 4.3 summarises the calibrated and estimated parameters.

4.1 Data

To estimate the models we use nine Canadian quarterly series from 1980Q1 to 2022Q3 (Government Final Consumption, Consumer Price Index, Domestic Bonds, International Debt Securities, Interest Rates, Real Households Final Consumption Expenditure, CO2 emissions, Real Gross Domestic Product) and three US quarterly series (Real gross domestic product, Federal Funds Effective Rate, Producer Price Index by Commodity) from the rest of the world, appendix D provides additional details on the data sources. To obtain the per capita variable we divide the series with Working-Age Population Total from 15 to 64 Years for Canada.

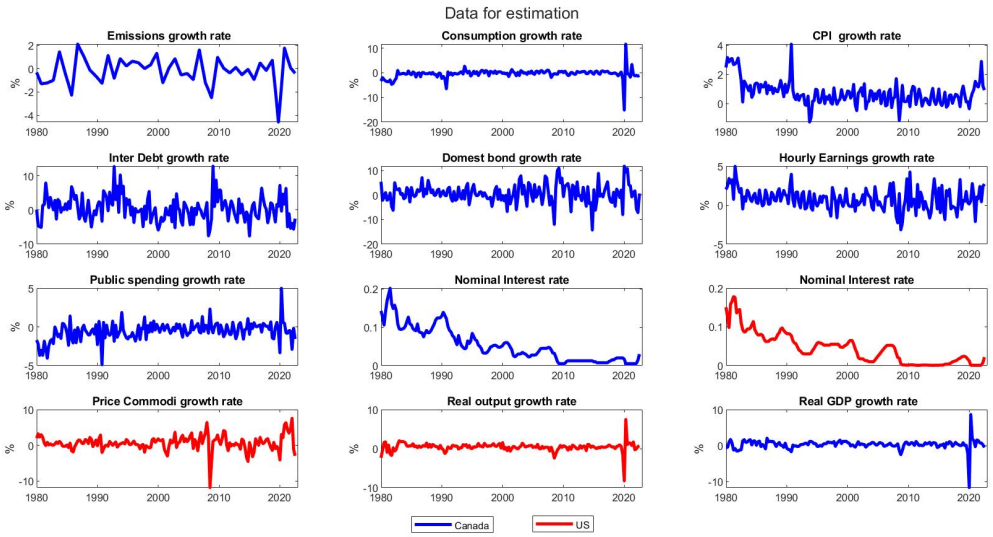


Fig. 1: Data for estimation

The measurement equations mapping our model to the twelve observable macroeconomic time series are given by:

$$\begin{bmatrix} Emissions - growth - rate \\ Consumption - //- \\ CPI - //- \\ International - Debt - //- \\ Domestique - bond - //- \\ Hourly - Earnings - //- \\ Public - spending - //- \\ Nominal - Interest - rate \\ Real - GDP - growth - rate \\ Nominal - Interest - rate - US \\ Price - Commodities - growth - rate - US \\ Real - output - //- \end{bmatrix} = \begin{bmatrix} \Delta \log me_t \\ \Delta \log C_t \\ \Delta \log P_t \\ \Delta \log B_t^G \\ \Delta \log G_t \\ \Delta \log w_t \\ \Delta \log Y_t^G \\ R_t \\ \Delta \log Y_t \\ rf_t \\ \Delta \log(P_t^{comf}) \\ \Delta \log Z_t^f \end{bmatrix}$$

4.2 Model solution

In the context of state space representation, we construct a linkage solution from a non-linear model. Contrary to the work of Cardani et al. (2023), we do not explicitly integrate the growth rate into our models in order to facilitate estimation. Let us define Θ the vector parameters of our dynamic models, the system of observable variables and the equation of states is given by :

$$Y_t^{data} = \Xi_1(\Theta) S_t, \tag{64}$$

$$S_t = \Phi_1(\Theta) S_{t-1} + \Phi_\varepsilon(\Theta)\varepsilon_t \tag{65}$$

$$\varepsilon_t \sim N\big(0, Q_t I\big)$$

The observation equation 64 links the vector of observed variables Y_t^{data} at time t to the model variables S_t through the coefficient matrix $\Xi_1(\Theta)$. The state equation 65 describes the transition of the system's state variables S_t using the coefficient matrices $\Phi_1(\Theta)$ and $\Phi_\varepsilon(\Theta)$. The model shocks ε_t follow a normal distribution with time-varying covariance matrix Q_t.

As in the work of Lenza and Primiceri (2022), Cardani et al. (2022) and Cardani et al. (2023), this specification allows us to take into account the periods before, during and after COVID. In fact, for the dates $\leq$ to 2019q4 the variance covariance matrix of COVID shocks is zero and for the dates $>$ to 2019Q4 this variance covariance matrix of COVID shocks is non-zero. The estimation framework always follows their work, first we determine a priori the variance of the heteroscedasticity specific to the COVID shocks from 2019q1 to 2022q3 and then we integrate this block into the model [10] and estimate all the parameters using all the data (1980q1 to 2022q3). Markov chain Monte Carlo methods are used with the DYNARE software Adjemian et al. (2022) to obtain a posteriori distributions of parameters and shocks. Considering endogenous ELB periods is done using the OccBin approach Guerrieri and Iacoviello (2015).

[10] The heteroscedasticity block in our programme contains two shocks, the forced savings shock ε_t^{tC} and the employment shock ε_t^{lu}. For the periods from 1980 to 2019 the variance of these shocks equals zero, after this date these shocks are equal to their variance determined from the consumption and labour series.

4.3 Model parameters

Calibration. Tables 1 and 2 show the parameters calibrated in the two models. ω^s was determined using Canadian statistics on households with high incomes and assets. τ^k, τ^C, τ^w, τ^{lu} on average taxes in all regions of Canada. The steady state block was calibrated according to the work of Lepetyuk et al. (2020), the household utility block to Lepetyuk et al. (2020) and Cardani et al. (2023), the production block to Lepetyuk et al. (2020) , the price and other block to Lepetyuk et al. (2020), the public sector to Cardani et al. (2023) and the climate parameters according to Sahuc et al. (2024).

Table 3 shows the comparison of models without heterogeneity (RANK) and with heterogeneity (TANK), based on the Laplace approximation. The two models were given the same weighting through identical Prior values (0.5). The results show that the marginal density in the RANK model is 3841.40 and in the TANK model it is 2076.195179, which explains why the TANK model reasonably captures the aggregate implications of household heterogeneity. This result is confirmed by the global (Bayes Ratio) and individual (Posterior Model Probability) analysis of the models, as their values are equal to unity. The considerable difference in marginal densities between the two models is confirmed by the work of Debortoli and Galí (2024), because when monetary policy is focused on inflation targeting, the gap between aggregate output and natural output, as in our models, plays a very large role in determining aggregate variable. Our analysis shows the role of heterogeneity in shaping economic fluctuations.

Posterior estimates. Overall, the parameter estimates in RANK model are similar to the ones in Lepetyuk et al. (2020). Table 4 shows the estimated parameters of the two models. These parameters can be divided into two main groups: those affected by heterogeneity and those not affected by heterogeneity. Among those that are affected, we have the parameters that enter into their production adjustment process, namely, the adjustment cost of prices γ_p and investments χ_i and share of reference firms ω. In the RANK model the value of the investment adjustment cost is 20.043 in the TANK model it is 17.795, the price adjustment cost is 20.034 in the RANK model and 21.892 in the TANK model, the share of reference firms is 0.888 in the RANK model and 0.595, the indexation of production γ in the RANK model is 0.049 and 0.059 in the

TANK model. These results show that firms can easily invest and cannot easily change their price in the model with heterogeneity because the adjustment cost is low for investments and high for prices. This phenomenon may be at the origin of the fluctuations, in fact, when faced with a structural change, companies will be able to quickly modify their investments compared to the model with homogeneous agents. We also have the smoothing parameter of the output gap in the Taylor rule, in the RANK model it is 0.117 and in the TANK model it is 0.309. Thus, we can say that taking into account household heterogeneity reduces the gap between aggregate output and potential output, i.e. production capacity is used to its full potential compared to the model with homogeneous agents. Among those not affected by heterogeneity are inflation smoothing parameters, import sector parameters and climate change parameters. One interpretation of this result is that these parameters do not enter directly into the household behaviour function.

Tables 5 and 6 show the smoothing and shock parameters respectively. The smoothing parameters in the RANK model are on average higher than those in the TANK model. The shocks in the TANK model are greater than those in the RANK model. This shows that heterogeneity or income inequalities between households are sources of fluctuations.

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5 Impulse response functions

This section examines the impulse response functions (IRFs) for productivity shock ξ_t^A, preference shock ξ_t^C, government spending shock ξ_t^G, the US interest rate shock ξ_t^{rf} and the Canadian interest rate shock ξ_t^{Rcor} in order to understand the dynamics of the model given the parameter estimates. We consider an interest rate constraint (ELB) given its importance in crisis periods as highlighted in the literature.

As shown in the figures 2, 4, 3, 6 and 5, we present two different solutions, one for the RANK model and one for the TANK model. It can be seen that the two solutions are different for the different shocks, which allows us to conclude that heterogeneity significantly affects fluctuations.

Preference shock ξ_t^C figure 2: A 2% increase in domestic consumption raises the level of production and lowers the level of exports of commodities and other products in the Canadian economy. The increase in labour enables companies to increase their investment. Increased production increases climate damage, which in turn increases the cost of reducing CO2 emissions. Following the rise in inflation, the monetary authorities increase the interest rate to stabilise the economy. This restriction is more important in the model with heterogeneity because inflation is also higher.

Productivity shock ξ_t^A figure 3: A 2% rise in the TFP increases the level of production in both models. This rise leads to an improvement in the terms of trade, with the level of employment falling during the first five quarters before turning positive thereafter. Despite the rise in commodity prices, the number of raw materials extracted is still high, showing that Canada is rich in raw materials. The increase in production is leading to a rise in CO2 emissions, which in turn is increasing abatement costs. It can also be seen that CO2 abatement costs are higher in the model with homogeneity, since the level of production results in higher CO2 emissions.

Public spending shock ξ_t^G figure 4: On the one hand, the differences in IRFs between the RANK and TANK models are greater for the productivity shock than for the other shocks and, on the other hand, the fluctuations in the TANK model are also smaller than for the other shocks. This shows the key role of public spending in heterogeneity. This result is confirmed by the work of Debortoli and Galí (2024) public spending shocks are attenuated in the TANK model (compared with RANK) in the context of a balanced budget rule, but amplified in the case of deficit financing.

Canadian monetary policy shock ξ_t^{Rcor} figure 5: A 2% increase in the interest rate contracts economic activity by reducing all macro variables relative to the steady state. A general remark we can make concerning the Canadian monetary policy shock is that the gap between the variables in the two models is small compared to other shocks. The explanation we can give is that the stabilisation of inflation implies the stabilisation of other macroeconomic variables. As we can see, the stabilisation of inflation from the 9th quarter onwards leads to a stabilisation of the other variables over the same period, which is confirmed by the work of Debortoli and Galí (2024).

Foreign shock ξ_t^{rf} figure 6: A 2% rise in US interest rates increases the production level of the Canadian economy in both models, but reduces consumption. Investment falls in the TANK model, which reduces capital. The increase in production increases climate damage, but the latter is greater in the model with homogeneous agents, which leads to a higher cost of reducing emissions. Faced with rising inflation, the monetary authorities increase the level of interest rates to stabilise the economy.

6 Shock decompositions

This section highlights the decomposition of the shocks that divide the dynamics of the various endogenous variables of the models according to the exogenous shocks. They shed light on the contribution of shocks to the deviation of the various variables from their steady state. For example, we compare the decompositions of quarterly shocks to real GDP growth in the Canadian economy from 1980q1 to 2022q3 in the two models.

Figure 7 shows the decompositions of quarterly real GDP growth in Canada using the RANK model (a) and the TANK model (b) for 1980q1 to 2022q3. We observe that before the COVID period in both models the growth and recession periods are driven by domestic supply, domestic demand and monetary policy shocks. During the COVID period there is a contraction in economic activity and a recovery immediately afterwards, this contraction being driven by the domestic demand shock in both models. This result shows the important role of the fall in domestic demand in the pandemic recession, which is characterised by forced savings during this period. This recession is stronger in the TANK model than in the RANK model. Indeed, as shown by the estimate of the forced savings shock ξ_t^{tC}, its value is 0.125 in the RANK model and 0.544 in the TANK model.

7 Conclusion

We have built two DSGE models, one with representative agents (RANK model) and one with heterogeneous agents (TANK model) to discuss the conduct of monetary and fiscal policies in Canadian open economy. Extensions to the models allow us to include

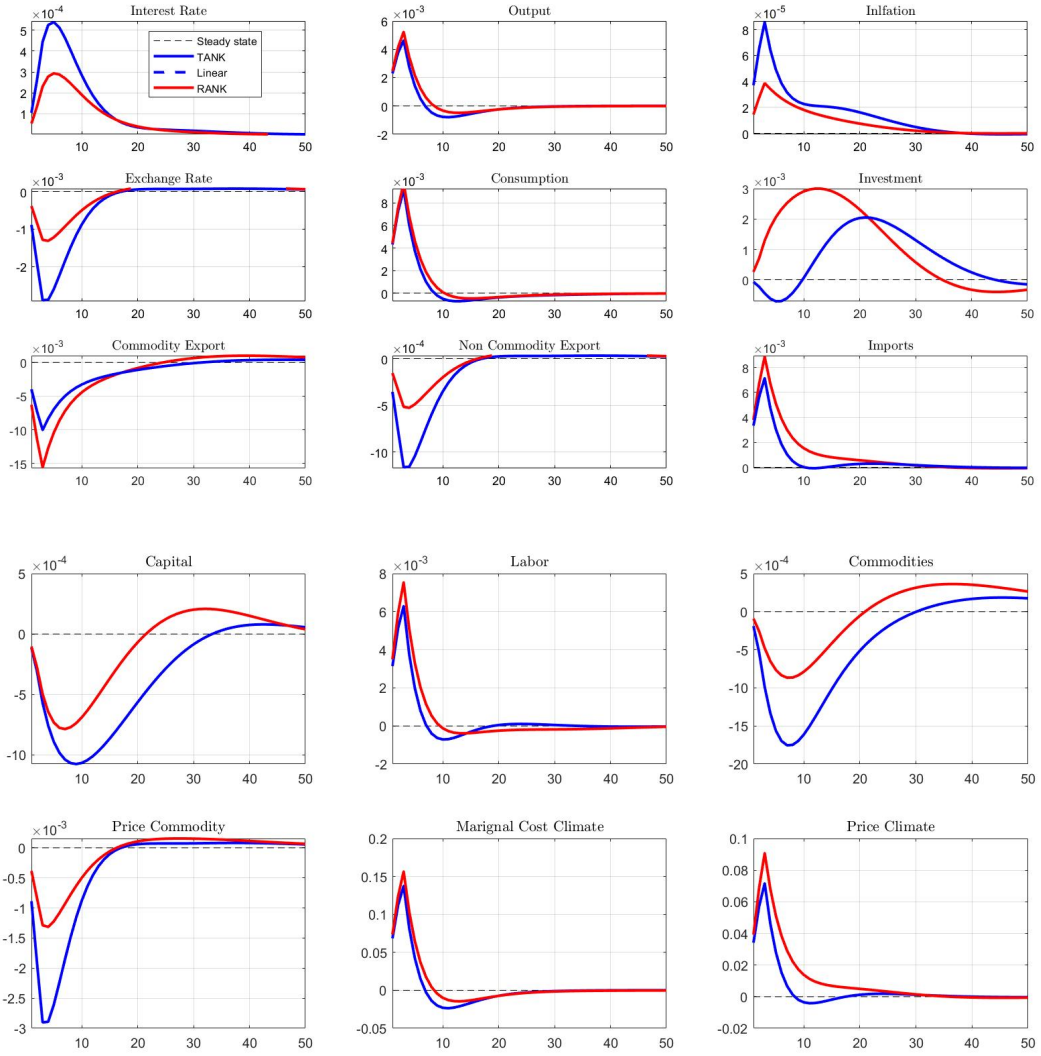


Fig. 2: Preference Shock. Notes : 2 percent deviation from consumption steady state

climate damage; mitigation policies; the COVID 19 period in the data set; and whether or not liquid and illiquid assets are held in the dynamics of the Canadian economy.

Bayesian estimates show us that household heterogeneity modifies inferences about different types of shocks in different ways and is a source of fluctuation. Indeed, as shown by the comparison of the two models through the Laplace approximation, the

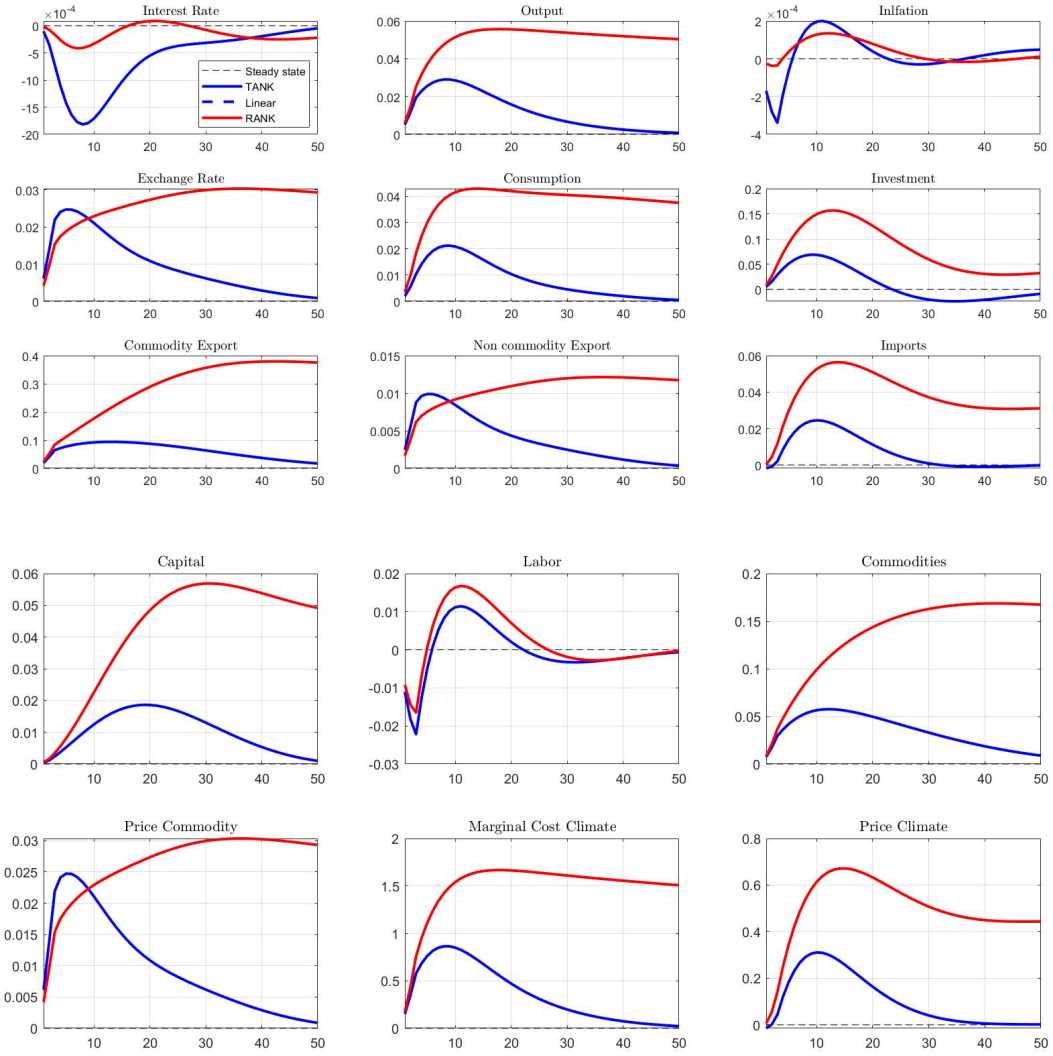


Fig. 3: Productivity Shock. Notes : 2 percent deviation from GDP steady state

value of the Bayes ratio in the TANK model is lower than that of the RANK model. The analysis of the irfs reveals the key role of fiscal policy in the stability of global activity, as shown in the literature Debortoli and Galí (2024). The decomposition of shocks reveals that domestic demand plays a primordial role in the periods of recession, recovery during and after COVID. Indeed, as noted by Cardani et al. (2023), the

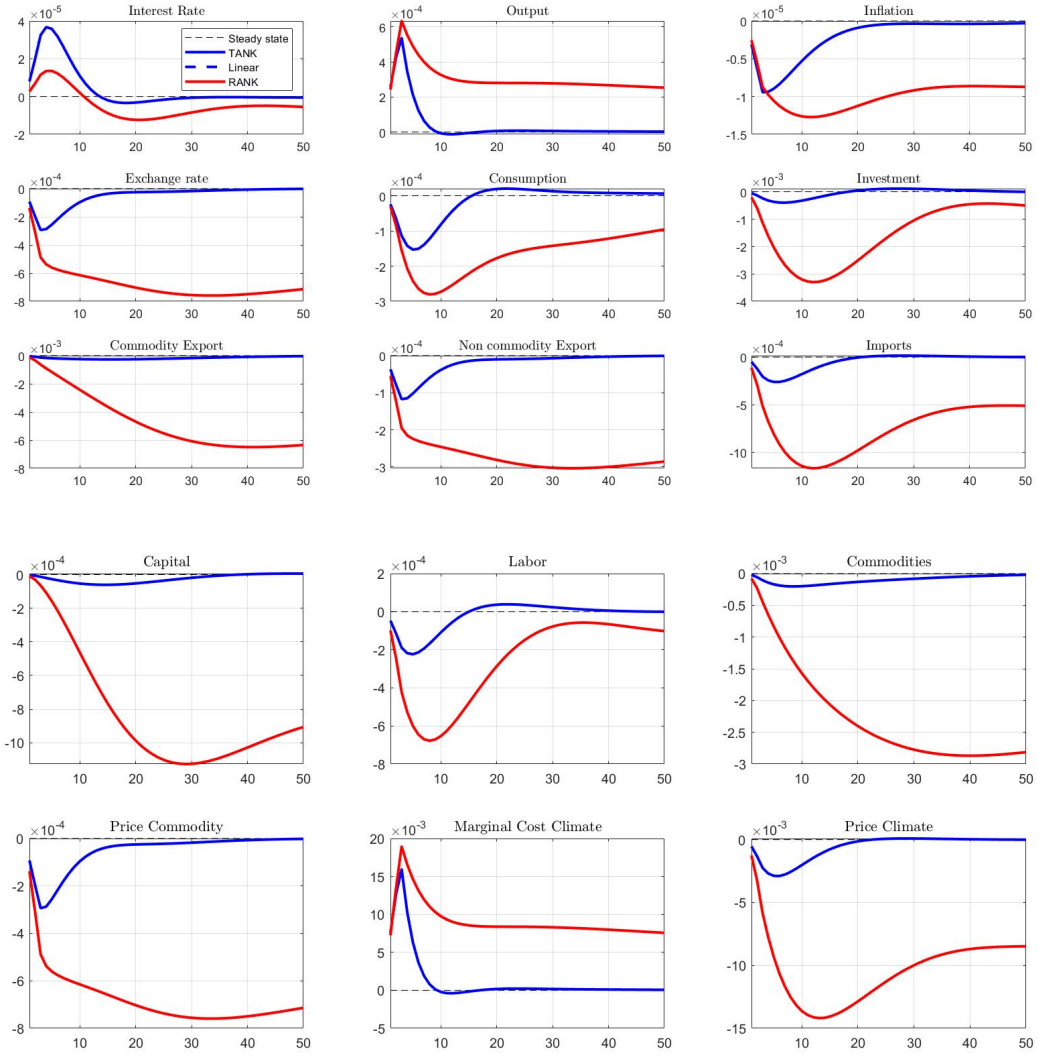


Fig. 4: Public Spending Shock. Notes : 2 percent deviation from Public spending steady state

forced savings of households during the COVID period contributed to reduce domestic demand, after this period households strongly increased their demand.

Monetary authorities use different monetary policy strategies to stabilise or stimulate the economy. More specifically, monetary policy is restrictive when prices are

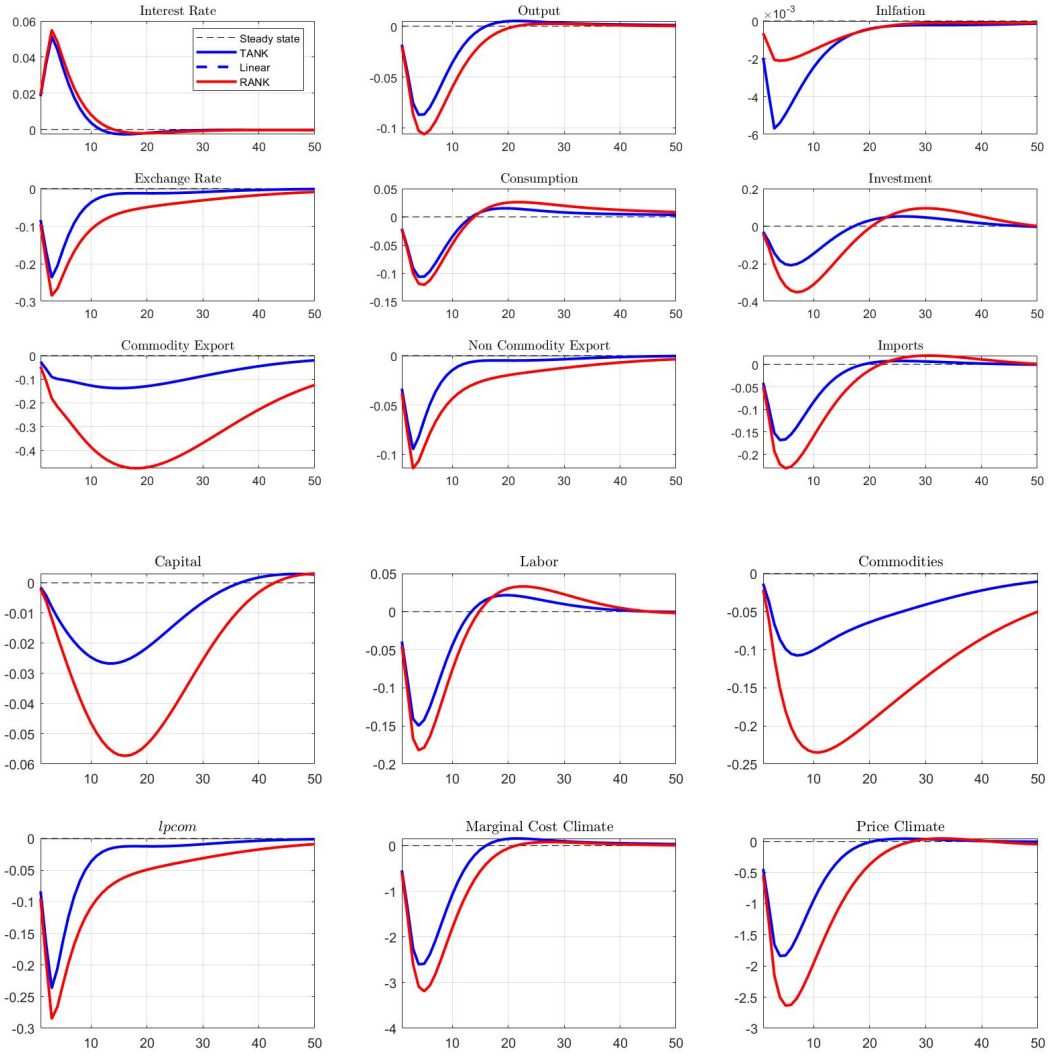


Fig. 5: Canada Monetary Policy Shock. Notes : 2 percent deviation from inflation-target steady state

rising and expansionary when prices are falling. Climate change mitigation policies force firms to increase their level of pollution. Indeed, as shown by the irfs, the marginal cost of climate change increases when the level of pollution increases.

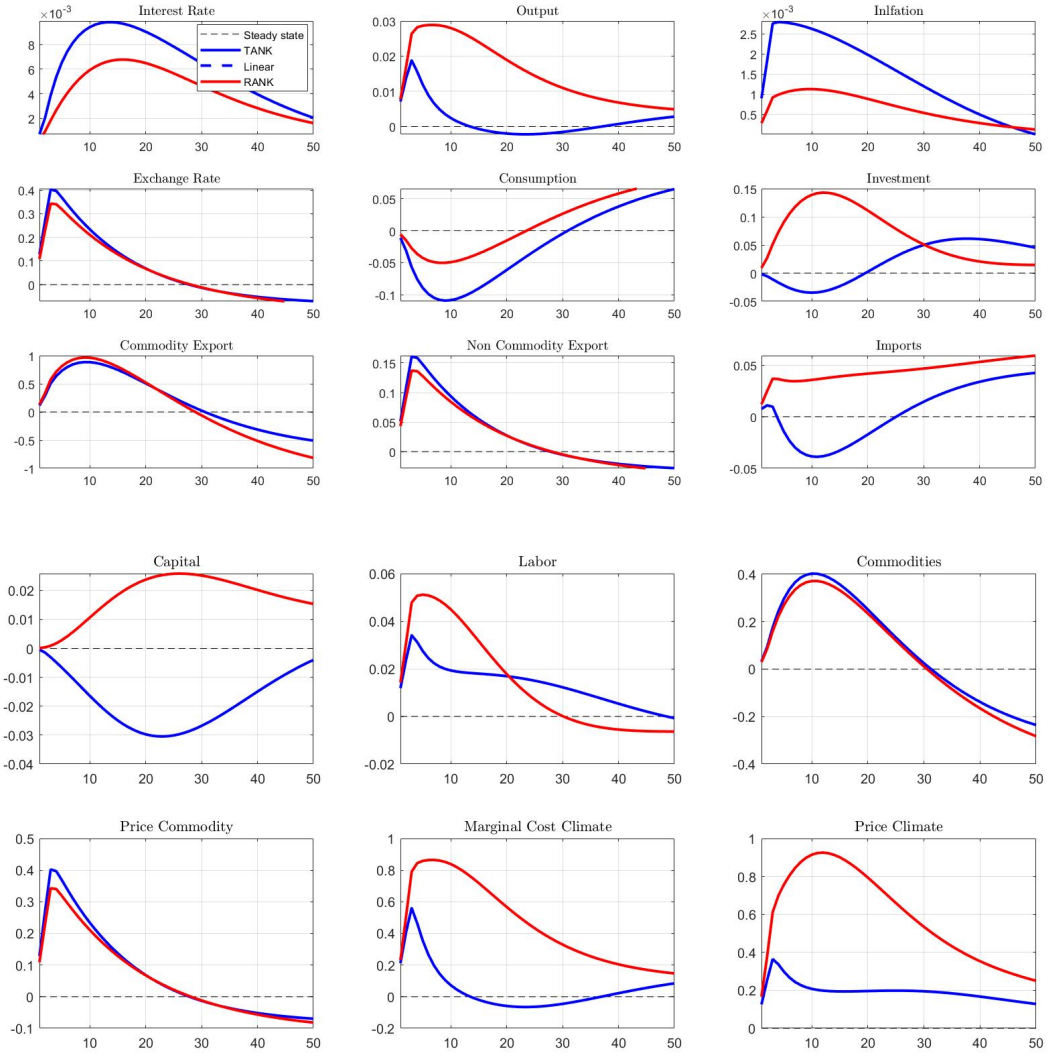
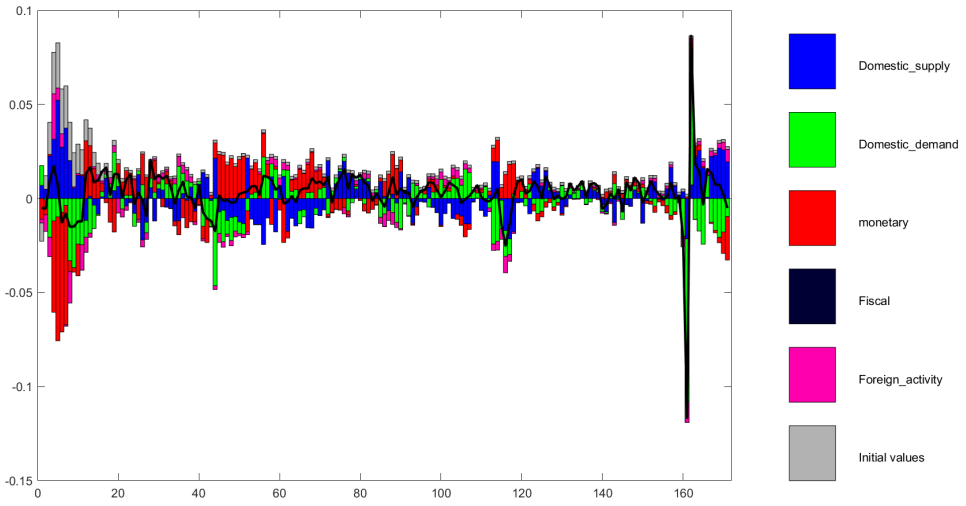
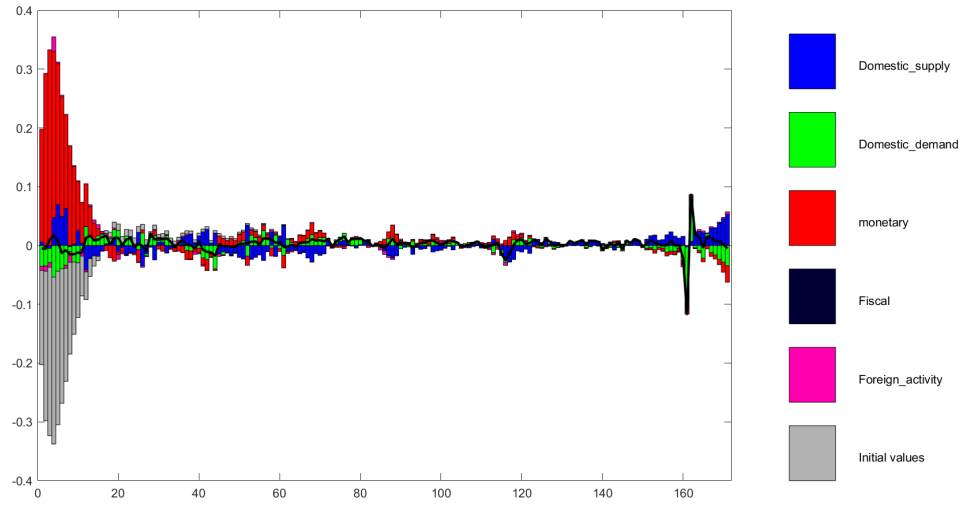


Fig. 6: US Monetary Policy Shock. Notes : 2 percent deviation from inflation-target steady state

Our models contribute to the open macroeconomic literature by highlighting the essential contribution of heterogeneity, climate change, mitigation policies and forced savings in accounting for both the transmission of shocks and the effect of monetary and fiscal policies decisions in Canada.



(a) Shock decomposition GDP growth rate RANK model



(b) Shock decomposition GDP growth rate TANK model

Fig. 7: Shock decomposition GDP. Notes : The solid black line represents the data. The bars below (above) the trend line indicate the negative (positive) contributions to the real growth rate of Canadian GDP. The set of structural shocks allows us to recover the observed time series of GDP growth (solid black line). We have grouped the estimated shocks into the following categories: (1) the domestic supply shock (blue line), which includes the productivity shock and the climate shock. (2) the Canadian monetary policy shock (red line). (3) the fiscal shock(black line). (4) the foreign shock (pink line), which includes the US monetary policy shock, the import price shock and the import shock. (5) the domestic demand shock (green line) including preference shock, cyclical saving shock, labour supply shock, transfer shock, domestic bonds shock, domestic firm shared shock, international trade asset shock, pubic investment shock and government consumption shock.

Appendix A Production of intermediate goods

The Lagrangian gives

$$E_0 \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \frac{P_t}{P_{t+\tau}} \Bigg(P^z_{t+\tau} \Bigg(\Big[\delta_l \big(\varepsilon^A_{t+\tau} L_{t+\tau}\big)^{\frac{\sigma-1}{\sigma}} + \delta_k \big(\mu_{t+\tau} K_{t+\tau}\big)^{\frac{\sigma-1}{\sigma}} + \delta_{com} \big(COM^d_{t+\tau}\big)^{\frac{\sigma-1}{\sigma}} +$$

$$\delta_m \big(M_{t+\tau}\big)^{\frac{\sigma-1}{\sigma}} + \delta_e \big(me_{t+\tau}\big)^{\frac{\sigma-1}{\sigma}} \Big]^{\frac{\sigma}{\sigma-1}} - \frac{\chi}{2} \Bigg(\frac{I_{t+\tau}}{I_{t+\tau-1}} \Bigg)^2 I_{t+\tau} \Bigg) - W_{t+\tau} L_{t+\tau} - P^i_{t+\tau} I_{t+\tau} -$$

$$P^{com}_{t+\tau} COM^d_{t+\tau} - P^m_{t+\tau} M_{t+\tau} - P^{em}_{t+\tau} me_{t+\tau} + Q_{t+\tau} \big((1-d) K_{t+\tau} + I_{t+\tau} - K_{t+\tau+1}\big) \Bigg) \quad \text{(A1)}$$

the first order condition gives

$$W_t = P^z_t \big(Z^g_t\big)^{\frac{1}{\sigma}} \delta_l \big(\varepsilon^A_t\big)^{\frac{\sigma-1}{\sigma}} \big(L_t\big)^{\frac{-1}{\sigma}} \quad \text{(A2)}$$

$$Q_t = \frac{1}{R^k_t} E_t \Big[P^z_{t+1} \big(Z^g_{t+1}\big)^{\frac{1}{\sigma}} \delta_k \big(\mu_{t+1}\big)^{\frac{\sigma-1}{\sigma}} \big(K_{t+1}\big)^{\frac{-1}{\sigma}} + Q_{t+1} \big(1 - d_{t+1}\big) \Big] \quad \text{(A3)}$$

$$P^i_t = Q_t - P^z_t \frac{\chi_i}{2} \Bigg(\frac{I_t}{I_{t-1} - 1} \Bigg) \Bigg(\frac{3 I_t}{I_{t-1} - 1} \Bigg) + \frac{1}{R^k_t} E_t \Bigg[P^z_{t+1} \chi_i \Bigg(\frac{I_{t+1}}{I_t - 1} \Bigg) \Bigg(\frac{I_{t+1}}{I_t} \Bigg)^2 \Bigg] \quad \text{(A4)}$$

$$P^{com}_t = P^z_t \big(Z^g_t\big)^{\frac{1}{\sigma}} \delta_{com} \big(COM^d_t\big)^{\frac{-1}{\sigma}} \quad \text{(A5)}$$

$$P^m_t = P^z_t \big(Z^g_t\big)^{\frac{1}{\sigma}} \delta_m \big(M_t\big)^{\frac{-1}{\sigma}} \quad \text{(A6)}$$

$$P^{me}_t = P^z_t \big(Z^g_t\big)^{\frac{1}{\sigma}} \delta_{me} \big(me_t\big)^{\frac{-1}{\sigma}} \quad \text{(A7)}$$

$$Q_t \bar{d} \rho \exp(\rho(\mu_t - 1)) = P^z_t \big(Z^g_t\big)^{\frac{1}{\sigma}} \delta_k \big(\mu_t\big)^{\frac{-1}{\sigma}} \big(K_t\big)^{\frac{-1}{\sigma}} \quad \text{(A8)}$$

Appendix B Production of final good

Forward-looking firms maximise profits

$$\max_{P^*_t} E_t \sum_{j=0}^{\infty} \theta^j R_{t,t+j} \Bigg(\prod_{k=1}^{j} \pi^{-}_{t+k} P^*_t Z_{i,t+j} - (1 - s_m) P^z_{t+j} Z_{i,t+j} - s_m P_{t+j} Z_{i,t+j} \Bigg) \quad \text{(B9)}$$

under constraint

$$Z_{i,t+j} = \Bigg(\frac{\prod_{k=1}^{j} \bar{\pi}_{t+k} P^*_t}{P_{t+j}} \Bigg)^{-\varepsilon} Z_{t+j} \quad \text{(B10)}$$

the first-order conditions combining equations B9 and B10 give

$$E_t \Bigg[\sum_{j=0}^{\infty} \theta^j R_{t,t+j} Z_{i,t+j} \Bigg(\prod_{k=1}^{j} \bar{\pi}_{t+k} (1 - \varepsilon) + \varepsilon \frac{(1 - s_m) P^z_{t+j} + s_m P_{t+j}}{P^*_t} \Bigg) \Bigg] = 0 \quad \text{(B11)}$$

$$\frac{P_t^*}{P_t} = \frac{F_{1t}}{F_{2t}} \tag{B12}$$

$$F_{1t} \equiv E_t\Big[\sum_{j=0}^{\infty}(\beta\theta)^j\lambda_{t+j}\Big(\frac{\prod_{k=1}^{j}\bar{\pi}_{t+k}}{\prod_{k=1}^{j}\pi_{t+k}}\Big)^{-\varepsilon}Z_{t+j}\frac{\varepsilon}{\varepsilon-1}Rmc_t\Big] \tag{B13}$$

$$F_{2t} \equiv E_t\Big[\sum_{j=0}^{\infty}(\beta\theta)^j\lambda_{t+j}\Big(\frac{\prod_{k=1}^{j}\bar{\pi}_{t+k}}{\prod_{k=1}^{j}\pi_{t+k}}\Big)^{-\varepsilon}Z_{t+j}\Big] \tag{B14}$$

Appendix C Wage setting

Households maximise

$$E_t\sum_{j=0}^{\infty}(\beta\theta_w)^j U_{t+j} \tag{C15}$$

under constraint

$$L_{ht}^{paid} = \Big(\frac{\prod_{k=1}^{j}\bar{\pi}_{t+k}W_t^*}{W_{t+j}}\Big)^{-\varepsilon_w}L_{t+j} \tag{C16}$$

$$P_{t+j}C_{t+j} = \prod_{k=1}^{j}\bar{\pi}_{t+k}W_t^*L_{h,t+j}^{paid}dh + \Theta_{t+j} \tag{C17}$$

where Θ_{t+j} includes all other elements of the budget constraint other than consumption C_{t+j} and labour $L_{h,t+j}^{paid}$.

The first-order conditions give

$$E_t\Big[\sum_{j=0}^{\infty}(\beta\theta_w)^j\Big(U_{C,t+j}\frac{\prod_{k=1}^{j}\bar{\pi}_{t+k}}{P_{t+j}}\Big(L_{h,t+j}+W_t^*\frac{\partial L_{h,t+j}}{\partial W_t^*}\Big)+U_{Lh,t+j}\frac{\partial L_{h,t+j}^{paid}}{\partial W_t^*}\Big)\Big]=0 \tag{C18}$$

where

$$U_{C,t} = \Big(C_t-\varepsilon_t^{tC}-\xi\big(C_{t-1}-\varepsilon_{t-1}^{tC}\big)\Big)^{\frac{-1}{\mu}}exp\Big(\frac{\eta(1-\mu)}{\mu(1+\eta)}\int_0^1(L_{ht}^{paid})^{\frac{\eta+1}{\eta}}dh\Big)\varepsilon_t^C \tag{C19}$$

$$U_{Lh,t} = -\Big(C_t-\varepsilon_t^{tC}-\xi\big(C_{t-1}-\varepsilon_{t-1}^{tC}\big)\Big)^{\frac{\mu-1}{\mu}}exp\Big(\frac{\eta(1-\mu)}{\mu(1+\eta)}\int_0^1(L_{ht}^{paid})^{\frac{\eta+1}{\eta}}dh\Big)\big(L_{ht}^{paid}\big)^{\frac{1}{\eta}}\varepsilon_t^C \tag{C20}$$

$$\frac{\partial L_{h,t+j}^{paid}}{\partial W_t^*} = \frac{-\varepsilon_w}{W_t^*}L_{h,t+j}^{paid} \tag{C21}$$

or

$$E_t\Big[\sum_{j=0}^{\infty}(\beta\theta_w)^j U_{C,t+j}L_{h,t+j}^{paid}\Big(\frac{\prod_{k=1}^{j}\bar{\pi}_{t+k}W_t^*}{\prod_{k=1}^{j}\pi_{t+k}P_t}-\frac{\varepsilon_w}{\varepsilon_w-1}\Big(\frac{-U_{Lh,t+t}}{U_{Lh,t+j}}\Big)\Big)\Big]=0 \tag{C22}$$

introducing wage taxes $w_t = (1-\tau^w)W_t$ we have

$$\left(W_t^*\right)^{1+\frac{\varepsilon_w}{\eta}}\left(w_t\right)^{-\frac{\varepsilon_w}{\eta}} = \frac{F_{1t}^w}{F_{2t}^w} \tag{C23}$$

with

$$F_{1t}^w = \lambda_t \frac{\varepsilon_w}{\varepsilon_w - 1}\left(C_t - \varepsilon_t^{tC} - \xi\left(C_{t-1} - \varepsilon_{t-1}^{tC}\right)\right)\left(L_t\right)^{\frac{1+\eta}{\eta}} + \beta\theta_w E_t\left[\left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}^w}\right)^{\frac{-\varepsilon_w(1-\eta)}{\eta}} F_{1,t+1}^w\right] \tag{C24}$$

and

$$F_{2t}^w = \lambda_t L_t + \beta\theta_w E_t\left[\frac{\bar{\pi}_{t+1}}{\pi_{t+1}}\left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}^w}\right)^{-\varepsilon_w} F_{2,t+1}^w\right] \tag{C25}$$

aggregate wage is

$$\theta_w\left(\frac{\bar{\pi}_t}{\pi_t^w}\right)^{1-\varepsilon_w} + (1-\theta_w)\omega_w\left(\frac{\left(\pi_{t-1}^w\right)^{\gamma_w}\left(\bar{\pi}_t\right)^{1-\gamma_w}}{\pi_t^w}\right)^{1-\varepsilon_w} + (1-\theta_w)(1-\omega_w)\left(\frac{W_t^*}{w_t}\right)^{1-\varepsilon_w} = 1 \tag{C26}$$

Appendix D Data Sources

Government Final Consumption Expenditure in Canada, Chained 2007 Canadian Dollars, Quarterly, Seasonally Adjusted. Consumer Price Index: All Items: City: Total for Canada, Index 2015=100, Quarterly, Not Seasonally Adjusted. Amount Outstanding of Domestic Bonds and Notes in General Government Sector, Long-Term at Original Maturity, Residence of Issuer in Canada, Millions of US Dollars, Quarterly, Not Seasonally Adjusted. Amount Outstanding of International Debt Securities for Issuers in General Government Sector, All Maturities, Residence of Issuer in Canada, Millions of US Dollars, Quarterly, Not Seasonally Adjusted. Interest Rates: Immediate Rates (¡ 24 Hours): Central Bank Rates: Total for Canada, Percent, Quarterly, Not Seasonally Adjusted. Hourly Earnings: Earnings: Economic Activity: Manufacturing: Total Economy for Canada, Index 2015=100, Quarterly, Not Seasonally Adjusted. Real Households Final Consumption Expenditure for Canada, Millions of Domestic Currency, Quarterly, Seasonally Adjusted. Real Gross Domestic Product for Canada, Millions of Domestic Currency, Quarterly, Seasonally Adjusted. CO2 emissions, which correspond to the emissions from the burning of fossil fuels for energy and cement

production. Real gross domestic product per capita, Chained 2017 Dollars, Quarterly, Seasonally Adjusted Annual Rate. In the US economy we have Federal Funds Effective Rate, Percent, Quarterly, Not Seasonally Adjusted. Producer Price Index by Commodity: All Commodities, Index 1982=100, Quarterly, Not Seasonally Adjusted. these data come form Federal Reserve Economic and World In.

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Table 1: Selected calibrated parameters

Parameters	*Values*	*Denominations*
Steady state values		
rss	1.0065	real interest rate
beta	1/rss	discount factor
infqss	1.02^(1/4)	inflation target
Rss	rss*infqss	nominal interest rate
Rlimit	1.0076	lower bound on the interest rate
Household utility		
ω^s	0.40	Savers households share
ξ	0.73963	consumption habit
μ^h	0.87747	consumption elasticity of substitution
η^h	0.07038	wage elasticity of labor supply
α^G	-0.001	steady-state risk premia gov consump
α^S	0.005	steady-state risk premia domestic firm shares
α^B	0.10	steady-state risk premia internationally traded asset
Output production		
σ	0.5	CES elasticity of subtitution
δ_l	0.2490	CES labor share paramater
δ_k	0.5749	CES capital share parameter
δ_{com}	0.0014	CES commodity share parameter
δ_m	0.0286	CES import share parameter
d0	0.0053	fixed depreciation rate
$\bar{d}$	0.0221	variable depreciation rate
ρ	4.0931	depreciation semi elasticity
ι^I	1.2698	real investment price
ι^X	1.1429	real non commodity export price
Ass	100	labour productivity, normalization
Price setting parameters for consumption		
θ	0.75	probability of indexation
ε	11	elasticity of substitution of consumption goods
smc	0.6	Leontieff technology parameter
Price setting parameters for imports		
θ_m	0.8635	probability of indexation
ε_m	4.4	elasticity of substitution of imports
Price setting parameters for wages		
γ_w	0.1086	indexation to past inflation
ω_w	0.6895	RT share, ToTEM2 protw

Table 2: Selected calibrated parameters

Parameters	*Values*	*Denominations*
Other		
kprm	0.0674	capital premium, calibrated
auip	0.1584	exchange rate persistence parameter
pcomfss	1.6590	foreign commodity price
pmfss	1.2939	foreign import price
vsigma	0.0082	((define)), Schmitt-Grohe and Uribe (2003)
xgamma	18.3113	((define))
xphi	0.4	foreign demand elasticity, calibrated
sz	0.8	elasticity in commodity production
Land	0.1559	land
residZ	0.7650	((residual))
residY	0.311	((residual))
ρ^{Ypot}	0.75	persistence of potential GDP
Public sector parameters		
τ^{c}	0.11	Consumption tax
τ^{w}	0.2291	wage tax
τ^{k}	0.666	capital tax
Gss	0.204	Steady state gov cons
Tss	0.163	Steady state Transfer
$IGss$	0.031	Steady state gov Inv
τ^{lu}	0.2291	labour hoarding subsidy
η^{d}	0.3	Coefficient of gov deficit
$defss$	0.024	Target gov deficit
η^{B}	0.3	Coefficient of gov debt level
$BGss$	3.025	Target gov debt level
Climate parameters		
δ^{me}	0.00125	CO2 rate of transfer
ξ^{e}	0.27273	Marginal atmospheric retention
m^{1700}	545	pre industrial stock of carbon
μ^{qe}	0.05	low abatement CO2
γ^{em}	2.379*exp(-05)	climate change damage
$\theta^{e,1}$	0.30604	initial abatement cost
$\theta^{e,2}$	2.6	abatement cost
τ^{e}	0.3	climate tax
θ^{e}	0.75	probability of indexation climate

Table 3: Model Comparison (based on Laplace approximation)

Model	*RANK*	*TANK*
Priors	0.500000	0.500000
Log Marginal Density	3841.403049	2076.195179
Bayes Ratio	1.000000	0.000000
Posterior Model Probability	1.000000	0.000000

Source: Dynare outcomes

Table 4: Results from Metropolis-Hastings (parameters)

					RANK				TANK			
			Prior			Posterior				Posterior		
		Dist.	Mean	Stdev.	Mean	Stdev.	hp inf	hp sup	Mean	Stdev.	hp inf	hp sup
χ_i	Invest adjust cost	gamm	20.000	6.0000	20.043	0.0054	20.0353	20.0526	17.795	0.0522	17.7174	17.8682
γ	RT indexation prod	beta	0.050	0.0100	0.049	0.0078	0.0408	0.0611	0.059	0.0000	0.0586	0.0587
ω	RT share prod	beta	0.480	0.2000	0.888	0.0089	0.8758	0.8997	0.595	0.0018	0.5925	0.5981
γ_m	RT indexation imp	beta	0.730	0.2000	0.676	0.0041	0.6677	0.6818	0.559	0.0016	0.5565	0.5619
ω_m	RT share imp	beta	0.300	0.1000	0.297	0.0043	0.2895	0.3038	0.283	0.0005	0.2816	0.2833
γ_w	RT indexation wage	beta	0.100	0.0100	0.094	0.0033	0.0896	0.0999	0.103	0.0001	0.1030	0.1033
ω_w	RT share wage	beta	0.680	0.2000	0.477	0.0032	0.4719	0.4819	0.578	0.0010	0.5769	0.5800
ρ^r	Interest rate persistence	gamm	0.830	0.1250	0.849	0.0086	0.8348	0.8596	0.880	0.0003	0.8799	0.8809
ρ^π	Response to inflation gap	gamm	4.120	0.1250	4.097	0.0088	4.0867	4.1081	4.116	0.0019	4.1135	4.1189
ρ^y	Response to output gap	gamm	0.400	0.1250	0.117	0.0085	0.1044	0.1282	0.309	0.0016	0.3070	0.3115
χ_{com}	Como adjust cost	gamm	16.000	4.0000	15.763	0.0019	15.7601	15.7665	15.809	0.0106	15.7920	15.8262
γ_p	Price adjust cost	gamm	20.000	6.0000	20.034	0.0019	20.0315	20.0378	21.892	0.0173	21.8642	21.9182
γ_e	RT indexation climate	beta	0.050	0.0100	0.048	0.0043	0.0408	0.0543	0.051	0.0000	0.0505	0.0506
ω_e	RT share climate	beta	0.480	0.2000	0.471	0.0037	0.4647	0.4764	0.593	0.0006	0.5914	0.5935
ε^e	Elasticity of substitution	gamm	11.000	2.0000	11.064	0.0034	11.0570	11.0683	10.946	0.0190	10.9220	10.9793

Table 5: Results from Metropolis-Hastings (Persistence parameters)

					RANK				TANK			
			Prior		Posterior				Posterior			
		Dist.	Mean	Stdev.	Mean	Stdev.	hp inf	hp sup	Mean	Stdev.	hp inf	hp sup
ρ^{Ypot}	persis of potential GDP	beta	0.750	0.1500	0.693	0.0020	0.6895	0.6958	0.719	0.0017	0.7166	0.7216
ρ^{Rcor}	persis of interest rate	beta	0.750	0.1500	0.203	0.0084	0.1894	0.2135	0.200	0.0017	0.1977	0.2024
ρ^{A}	persis of producti	beta	0.750	0.1500	0.997	0.0018	0.9950	0.9997	0.918	0.0002	0.9180	0.9187
ρ^{Zf}	persis of for out	beta	0.750	0.1500	0.996	0.0028	0.9923	0.9999	0.987	0.0026	0.9834	0.9907
ρ^{pcomf}	persis of for comm price	beta	0.750	0.1500	0.993	0.0026	0.9890	0.9972	0.883	0.0008	0.8815	0.8839
ρ^{rf}	persis of for inter rate	beta	0.750	0.1500	0.999	0.0001	0.9996	1.0000	0.999	0.0002	0.9992	0.9999
ρ^{tC}	persis of cyclical saving	beta	0.750	0.1500	0.577	0.0058	0.5681	0.5863	0.863	0.0015	0.8614	0.8655
ρ^{me}	persis of climate change	beta	0.750	0.1500	0.994	0.0028	0.9907	0.9982	0.838	0.0004	0.8376	0.8387
ρ^{lu}	persis of Labour supply	beta	0.750	0.1500	0.998	0.0016	0.9954	0.9999	0.910	0.0004	0.9093	0.9105
ρ^{G}	persis of Gov consum	beta	0.750	0.1500	0.994	0.0031	0.9889	0.9987	0.739	0.0023	0.7365	0.7425
ρ^{RG}	persis of domestic bonds	beta	0.750	0.1500	0.999	0.0001	0.9996	1.0000	0.851	0.0007	0.8501	0.8522
ρ^{S}	persis of domes firm shar	beta	0.750	0.1500	0.872	0.0058	0.8646	0.8839	0.739	0.0008	0.7382	0.7405
ρ^{B}	persis of interna trad asset	beta	0.750	0.1500	0.603	0.0060	0.5933	0.6122	0.789	0.0005	0.7888	0.7905
ρ^{IG}	persis of Gov inves	beta	0.750	0.1500	0.990	0.0038	0.9837	0.9954	0.755	0.0011	0.7535	0.7570
ρ^{T}	persis of Transfert	beta	0.750	0.1500	0.999	0.0009	0.9974	0.9999	0.719	0.0006	0.7176	0.7196
ρ^{tax1}	persis Tax	beta	0.750	0.1500	0.754	0.0034	0.7499	0.7612	0.722	0.0002	0.7219	0.7227
ρ^{τ}	persis of Gov debt rule	beta	0.750	0.1500	0.503	0.0142	0.4833	0.5222	0.742	0.0008	0.7409	0.7434
ρ^{tax2}	persis of white noise	beta	0.750	0.1500	0.987	0.0069	0.9777	0.9977	0.798	0.0003	0.7979	0.7989

Table 6: Results from Metropolis-Hastings (standard deviation of structural shocks)

					RANK				TANK			
			Prior			Posterior				Posterior		
		Dist.	Mean	Stdev.	Mean	Stdev.	hp inf	hp sup	Mean	Stdev.	hp inf	hp sup
ξ^{Rcor}	shock interest rate	invg	0.100	2.0000	0.014	0.0009	0.0125	0.0152	0.279	0.0015	0.2766	0.2814
ξ^{A}	shock producti	invg	0.100	2.0000	0.035	0.0014	0.0325	0.0373	0.062	0.0012	0.0602	0.0638
ξ^{C}	shock prefere	invg	0.100	2.0000	0.240	0.0074	0.2302	0.2519	0.806	0.0081	0.7935	0.8192
ξ^{Zf}	shock for output	invg	0.100	2.0000	0.013	0.0004	0.0130	0.0141	0.014	0.0008	0.0128	0.0154
ξ^{pcomf}	shock for price	invg	0.100	2.0000	0.023	0.0008	0.0217	0.0243	0.045	0.0021	0.0422	0.0482
ξ^{rf}	shock for inter rate	invg	0.100	2.0000	0.011	0.0006	0.0101	0.0122	0.011	0.0002	0.0106	0.0112
ξ^{tC}	shock cyclical saving	invg	0.100	2.0000	0.135	0.0091	0.1218	0.1469	0.544	0.0056	0.5355	0.5533
ξ^{lu}	shock Labour supply	invg	0.100	2.0000	0.098	0.0036	0.0923	0.1042	0.153	0.0040	0.1465	0.1589
ξ^{G}	shock Gov consum	invg	0.100	2.0000	0.054	0.0038	0.0484	0.0594	0.453	0.0036	0.4477	0.4582
ξ^{RG}	shock domestic bonds	invg	0.100	2.0000	0.008	0.0003	0.0071	0.0082	0.120	0.0041	0.1131	0.1259
ξ^{S}	shock domes firm shar	invg	0.100	2.0000	0.491	0.0054	0.4840	0.5003	1.094	0.0061	1.0840	1.1029
ξ^{B}	shock interna trad asset	invg	0.100	2.0000	0.390	0.0096	0.3777	0.4012	0.054	0.0136	0.0346	0.0704
ξ^{IG}	shock Pub inve	invg	0.100	2.0000	0.054	0.0040	0.0485	0.0598	0.054	0.0087	0.0410	0.0662
ξ^{T}	shock transfer	invg	0.100	2.0000	0.116	0.0032	0.1103	0.1202	0.811	0.0272	0.7736	0.8588
ξ^{tax}	shock tax	invg	0.100	2.0000	0.040	0.0014	0.0379	0.0423	0.056	0.0024	0.0520	0.0595
ξ^{me}	shock climate	invg	0.100	2.0000	0.015	0.0012	0.0137	0.0173	0.033	0.0015	0.0305	0.0355