Asset Pricing Dynamics in Economies with Limited Market Participation: A Continuous-Time Approach

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Abstract

We develop a continuous-time stochastic dynamic general equilibrium model featuring three agents with limited participation in capital markets. This model is designed to address economies characterized by significant financial frictions. We solve the 2-dimensional macro-finance model using approximation and numerical methods. Our findings reveal nuanced asset pricing dynamics. Increasing a household's share of total wealth continuously reduces asset prices. Conversely, an increase in the corporate sectors wealth share initially raises asset prices before causing a decline. Simulations of Iran's justice policy demonstrate that when stock market participation surpasses a critical threshold, redistributing wealth from the corporate sector to households results in higher asset prices. This simulation helps us answer the question for the first time why the Tehran stock market experienced stunning growth in 2020. These results provide valuable insights into the interaction between financial frictions and asset pricing, with important implications for policy design in economies with limited financial inclusion.

Keywords: Financial frictions, Asset pricing, Macro-finance model, Wealth structure, Market Participation

JEL Codes: E13, E44, G12, G20

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1 Introduction

Emerging economies grow over time, and their markets expand. Simultaneously, the financial markets are gradually deepening, a process that requires patience and understanding, especially in the face of initial frictions. For example, the significant access to liquidity in the financial markets can influence asset prices. One of the most critical frictions in economies with deep financial markets, including the U.S., is the limitation of participation in the capital market. This limitation is a key factor affecting asset prices, and one of the aims of this article is to investigate its effect.

Our goal in this article is to show the effect of increasing participation in the capital market according to the characteristics of a capital market in a developing country like Iran. This mechanism will be expressed through a theoretical model. From the theoretical perspective, how will the increase in household participation in the capital market affect asset prices? Does the structure of wealth distribution affect asset prices? We need a macro-finance model to answer these questions.

The main feature of our model is limited participation in the capital market. Financial theory states that regardless of risk aversion, each household should participate in the stock market as long as the equity premium is positive. However, empirical evidence shows that this is not the case. According to Campbell (2006), there are different ways to show limited participation. One way is to use a non-standard utility function as in Epstein and Zin (1989). In the Epstein and Wang (1994) model, there is a type of ambiguity aversion that causes non-participation in the capital market. Another reasonable way is to have a fixed fee for trading in the capital market. Vissing-Jorgensen (2003) shows that the constant cost of participating in the capital market has caused many American households not to participate in the stock market.

Limited participation in this research exists exogenously but is sustained endogenously. There are two types of households; one has the ability to participate in the market, and the other does not have this ability. Each has a share of the total wealth, which is positive for the

limited household in the equilibrium. This means that there is always a portion of the wealth held by households that cannot afford to participate in the capital market. The reason for this is the difference between the patience of these two types of households. According to Kaustia et al. (2023), one of the factors influencing participation in the stock market is cognitive factors, including patience. The third agent in the model is the corporate person. In Iran, a large part of big companies are owned by the government or sovereign institutions such as state-owned banks. All of these sectors are grouped under the title of corporate person in our model.

Limited participation in the capital market has different implications. Guvenen (2009) addressed the equity premium puzzle and asset pricing in a macroeconomic model with limited capital market participation. However, the results of changes in the amount of this limit on asset prices have yet to be investigated. In He and Krishnamurthy (2012) and He and Krishnamurthy (2013), there are two agents, one of which cannot participate in the stock market. In Brunnermeier and Sannikov (2014) model, two agents cooperate in the stock market, but their productivity is different. In our model, these two frameworks are mixed to obtain new meanings. We have an economy with three agents, two in the stock market with different productivity, and one is limited. He and Krishnamurthy (2013) shows that as the unconstrained agent's share of wealth increases, the risk premium decreases, and when it enters the unconstrained region, the risk premium remains almost constant. This article focuses more on the effect of capital constraints. Brunnermeier and Sannikov (2014)'s conclusion is that with the increase in the share of the wealth of the more productive agent, the price of the asset increases at first and then declines. However, in this framework, the effect of limited participation cannot be calculated.

In our paper, by combining these two frameworks, an economic environment has been created in which the effect of participation in the capital market can be investigated. Also, in Brunnermeier and Sannikov (2014), in the period when the wealth share of the more productive agent is considerable, the household does not appear in the capital market, which is slightly different from the real world, but in our new framework, the household is a member

of the market in the entire state space, and has shares.

This paper investigates the effect of wealth structure and capital market participation on asset prices in an economy with financial frictions. Due to the limitations that DSGE[1] models (for example, see Blanchard (2018)) have in creating nonlinear dynamics and global solutions, the framework of stochastic continuous time models has been used. In this model, there are three agents, the problem of stochastic control is solved for each one, and then the general equilibrium is obtained by solving market clearing equations. Solving these equations is done both approximately and numerically, and it has been shown that these two solutions are close. Also, to show the limitation of participation in the capital market, we assume that one of the agents is not able to play a role in the capital market. The origin of this framework requires an examination of evidence related to Iran's capital market.

The capital market of Iran in 2020 was confronted with the most fantastic bubble that occurred from the first day of opening this market. This super shock affected many households and created many financial issues for Iranian households. Before this, Trump returned the sanctions in a big way. People lined up before brokerages to open stock market accountslong queues formed in front of the brokers. In 2020, Iran experienced a financial bubble that was notably influenced by a confluence of economic, geopolitical, and domestic factors. The primary catalyst for this financial bubble was the severe impact of economic sanctions imposed by the United States. These sanctions, particularly restrictions on Iran's oil exports, weakened the country's economy, leading to a sharp decline in government revenue and exacerbating pre-existing economic challenges. The COVID-19 pandemic further strained Iran's economic resilience, affecting various sectors and contributing to a downturn.

The critical and noteworthy point is that Iran suffered such a financial bubble during the first wave of sanctions in 2012, but its size was much smaller than the recent bubble. We are faced with the same pattern at both times: after the imposition of sanctions, the oil revenues and the exports of the companies decreased, and as a result, the foreign currency supply is

[1] Dynamic Stochastic General Equilibrium

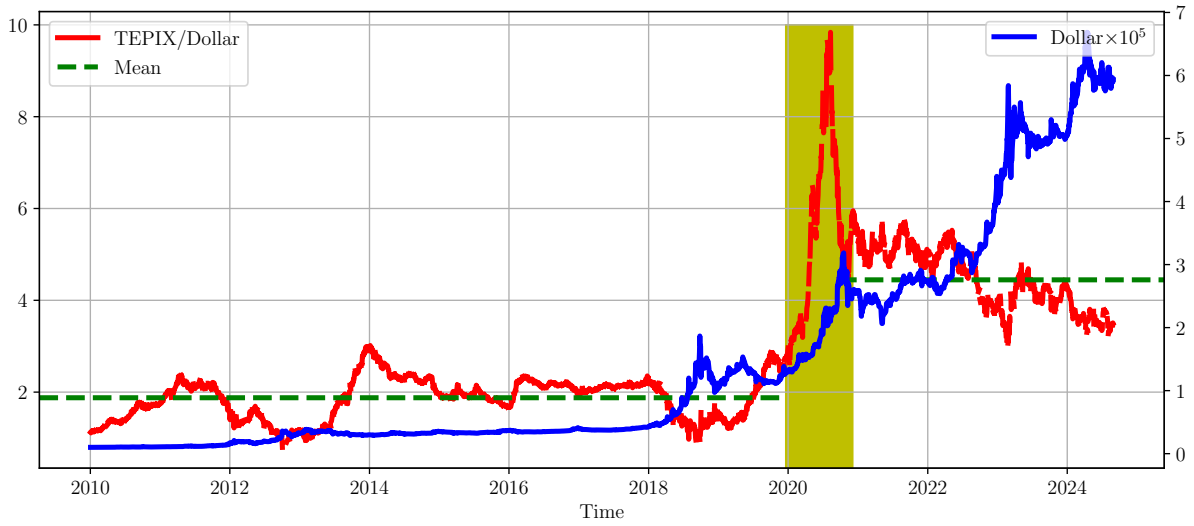


Figure 1: The change in level of the asset price

facing a shortage. On the other hand, the fixed exchange rate policy, which was made possible by the abundance of oil revenues and prevented the effect of inflation on the increase in the price of foreign currency, is no longer feasible, and the currency crisis begins. At first, there is a gap between the exchange rate and the financial market, but then this crisis enters the financial sector in the next stage and causes an increase in the price index of the Tehran Stock Exchange (TEPIX). Figure 1 shows a ratio of the price index of the Tehran Stock Exchange to the dollar rate in the open market from 2010 to 2023. The reason for displaying the index against the dollar is to remove the effect of high inflation in Iran during the last decade. According to this figure, after the price bubble, the price index level of the Tehran Stock Exchange has reached from less than 2 to an average of 4 compared to the value of the foreign currency. The strangest issue is that no Iranian economist has raised this issue, and therefore, there is no answer to it. Similarly, Figure 2 shows the annual growth rate of the Tehran Stock Exchange index without adjusting the exchange rate, which is more than 150% growth in 2019 and 2020.

What happened in 2020 that led to a change in the capital price level? To answer this question, we must return to the state of the capital market in Iran. In Iran, the capital market has a relatively small volume, resulting in various financial frictions. One of the most essential

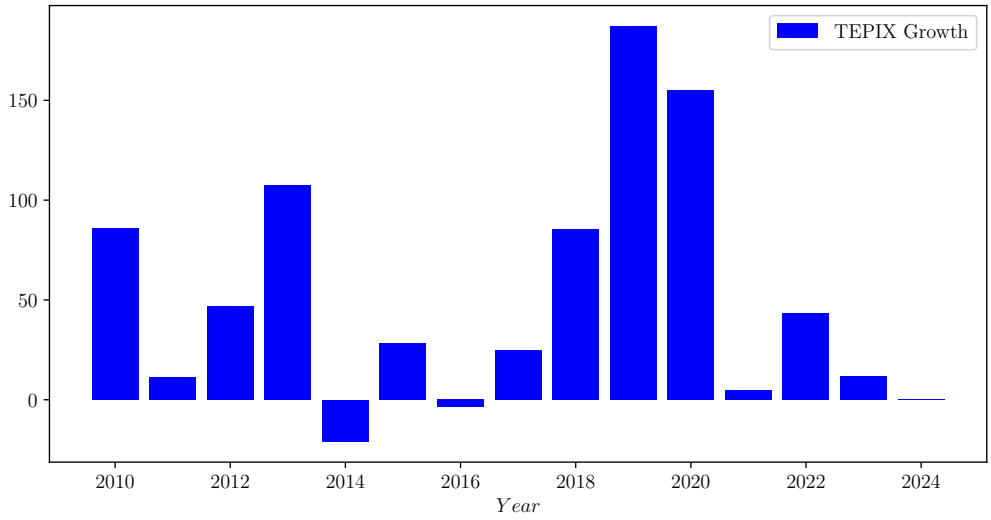


Figure 2: annual growth in Tehran Stock Exchange index

frictions that has a significant impact on the market is the level of people's participation in the market. Until 2019, the average number of brokerage accounts opened was 100,000 accounts per year. For example, in 2012, due to the rush to the capital market, this number reached nearly 200,000 accounts. However, only from 2019 to 2020 did the number of brokerage accounts increase from 10.3 million to 37.5 million.[2] The reason for this issue was the justice policy by the government. A decade before that, the government gave the shares of large companies to a large part of the people, but the people did not have the option to trade them until, in 2019, people found the right to trade their shares. This permission from the government in Iran was called the **justice policy**.

Another of the most critical financial frictions in the Tehran Stock Exchange is the existence of two types of natural and corporate persons, each of which behaves differently. By examining 30 large companies on the Tehran Stock Exchange, we see a correlation between changes in ownership from a corporate person to a natural person and changes in the company's price. For example, two of these 30 companies are given in Figure 3, and the charts of other companies are given in Appendix A. Of course, natural persons sell part of their shares to corporate persons

[2] see www.irna.ir/xjJ6qV

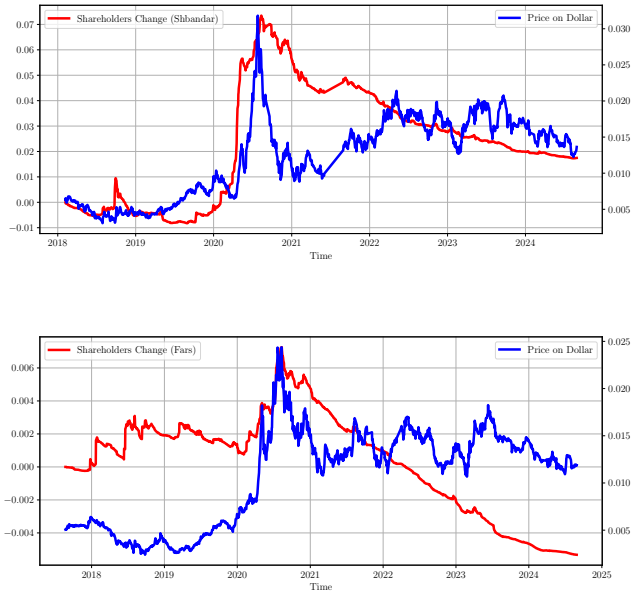


Figure 3: The change in ownership of the asset

over time. This feature is true for almost all companies in the Tehran Stock Exchange.

In Figure 4, in 30 large companies of the Tehran Stock Exchange, the relationship between the daily return of the stock price and the percentage change of ownership of the company from a corporate person to a natural person has been examined. Except in 3 companies with a slight negative correlation, in other companies, there is a high correlation between price return and change of ownership. This correlation may represent a causal relationship between these two variables.

Another statistical witness in this field is the relationship between the monthly returns of these 30 shares and the percentage of changes in their ownership. As Figure 5 shows, these two variables also have a strong correlation. This observation raises the question of whether the relative increase in wealth is effective on the amount of capital retention and, thus, on the price of capital.

One of the most detailed discussions in corporate ownership is the effect of capital structure on company value. Capital structure means how much of the company's assets are financed by issuing shares and how much of it is financed by issuing debt. One of the goals of this paper

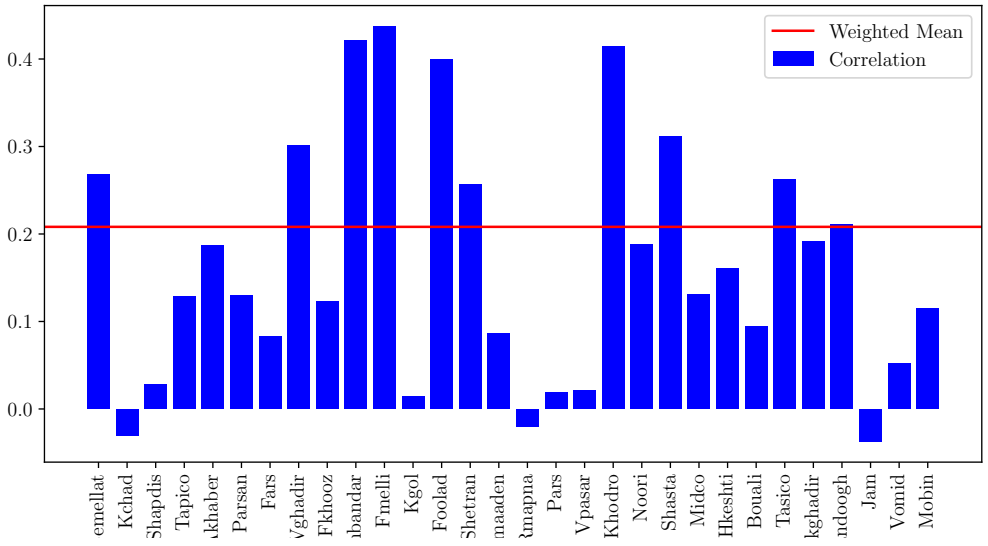


Figure 4: Correlation between daily change ownership and daily return

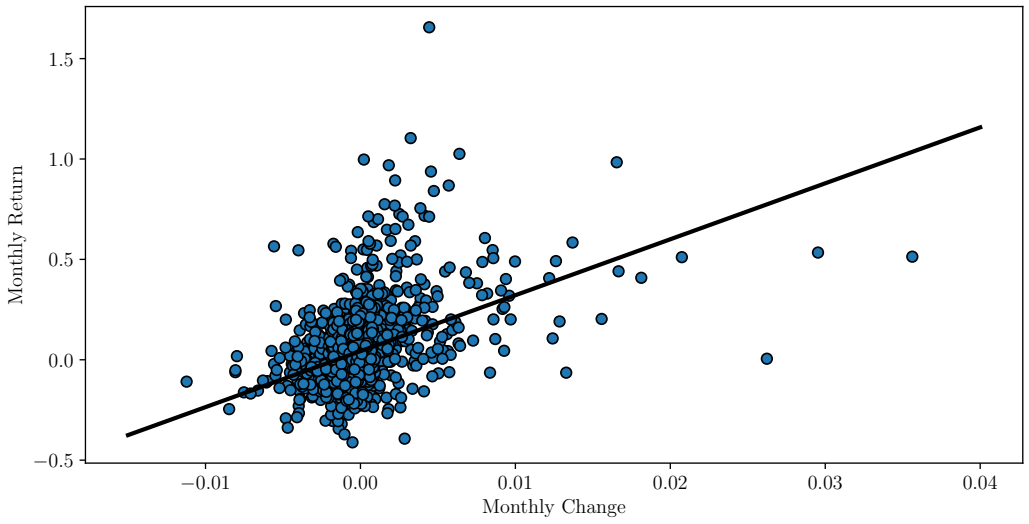


Figure 5: Correlation between daily change ownership and daily return

is to focus on the effect of wealth structure -rather than capital structure- on asset prices. The wealth structure shows how much wealth each group in the economy has. In fact, in an economy with financial frictions, which factor has a larger portion of the asset, it will have a severe impact on its price. In the next section of the literature, this topic will be examined. In

section 3, we will explain the model and its solution. Section 4 is related to discussions about model results and empirical facts, and the last section is dedicated to conclusions.

2 Literature Review

The impact of financial markets on macroeconomics has long been the concern of economists and has led to the creation of the branch of macro-finance, which can be referred to as Fisher (1933), Keynes (1937), Gurley and Shaw (1955), Minsky (1957), and Kindleberger et al. (2005) pointed out. These articles show how the financial crisis spills over into the real economy.

Many articles relate to our paper in different ways. On the one hand, they discuss the importance of financial frictions. The foundational work of Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke (1999) shed light on several crucial mechanisms through which financial frictions influence the broader macroeconomy. These seminal contributions reveal that financial frictions are not merely a peripheral concern but play a central role in shaping economic outcomes. One of the key insights is that temporary shocks to the economy can have long-lasting effects, particularly because they impact the net worth of agents who are highly leveraged. When these agents experience a shock, their reduced net worth hampers their ability to borrow, invest, and contribute to economic activity. Since rebuilding net worth is a slow process, the adverse effects of the shock can persist over an extended period, leading to prolonged economic weakness. Refer to Brunnermeier et al. (2013) for a comprehensive survey of the literature on economies characterized by financial frictions.

In their groundbreaking work of 1958, Modigliani and Miller (1958) presented a fundamental proposition in corporate finance, asserting that under a set of idealized conditions, the capital structure of a firm is irrelevant to its overall value. This principle posits that in a perfect marketcharacterized by the absence of taxes, bankruptcy costs, and asymmetric information- the way a firm finances itself, whether through debt or equity, has no impact on its market

value. According to this view, the firm's value is determined solely by its underlying assets and the income they generate, independent of how these assets are financed.

However, the real world is far from the idealized conditions assumed by MM, and subsequent research has explored how deviations from these assumptionsreferred to as financial frictionssignificantly influence a firm's capital structure and, by extension, its value. Sannikov (2013) provides a comprehensive survey of the capital structure implications arising from these financial frictions. His work underscores that when financial markets are imperfect, with elements such as transaction costs, agency problems, and informational asymmetries, the choice between debt and equity financing can have profound effects on a firm's operations and its overall value.

Scheinkman and Weiss (1986) delve into this issue by examining how the distribution of wealth between just two agents can significantly affect overall economic activity. Their analysis highlights that when wealth is unevenly distributed, it can lead to disparities in investment opportunities and consumption patterns, which in turn influence aggregate demand and economic growth. In scenarios where one agent controls a disproportionate share of wealth, the economy may experience suboptimal levels of investment and output, as the wealthier agent may not fully utilize available productive resources, while the less wealthy agent may be constrained in their ability to invest.

Another significant issue in financial markets is the phenomenon of limited market participation. Contrary to the expectations of classical finance theory, most investors do not diversify their portfolios across a broad range of asset classes. This tendency is highlighted by King and Leape (1998), who examined data from a 1978 survey involving 6,010 U.S. households with an average wealth of nearly $\$250,000$. They categorized assets into 36 distinct classes and discovered that the median household owned assets in only eight of these classes, indicating a relatively narrow range of investment diversification.

Further research by Mankiw and Zeldes (1991) reveals that only a small fraction of consumers hold stocks, which is particularly surprising given the potential benefits of equity invest-

ment in wealth accumulation. Even among those with substantial liquid wealth, stock ownership remains relatively low. For example, among individuals with more than $100,000 in other liquid assets, only 47.7% held stocks, demonstrating that a significant portion of wealthy investors still choose not to engage in equity markets.

Guiso (2002) expand on this analysis by documenting the direct and indirect ownership of stocks across various countries. Their findings underscore the pervasive nature of limited market participation on a global scale, reflecting a broader trend where many investors, even those with substantial financial resources, do not fully capitalize on the potential diversification and returns offered by the stock market. This lack of diversification and limited engagement with different asset classes has important implications for both individual wealth management and broader market dynamics.

Allen and Gale (1994) demonstrate that when the decision to participate in the market is treated as an endogenous factor, the fundamental characteristics of the market undergo significant changes. Firstly, limited market participation has the potential to magnify the impact of liquidity trading compared to scenarios where there is full participation. In fact, in certain situations, even a very small aggregate liquidity shock can lead to substantial price volatility. Secondly, the model reveals the existence of multiple equilibria, each associated with different levels of market participation and varying degrees of asset-price volatility. Thirdly, under realistic conditions, these equilibria can be ranked in terms of Pareto efficiency, with the Pareto-preferred equilibrium featuring higher market participation and reduced volatility.

From a methodological standpoint, our paper aligns with the emerging and rapidly expanding body of literature on continuous-time macrofinancial dynamics, as exemplified by works such as Brunnermeier and Sannikov (2014), He and Krishnamurthy (2012), and He and Krishnamurthy (2013). This literature aims to explore the macroeconomic consequences of financial frictions by concentrating on the intricate relationships between agents' net worth, asset prices, and financing constraints.

He and Krishnamurthy (2012) and He and Krishnamurthy (2013) particularly emphasize

the role of capital constraints in their models. They illustrate how the combination of low net worth among more productive agents and binding capital constraints can give rise to endogenous risk, which subsequently inflates the premium on risky assets. However, their findings suggest that the economy tends to recover relatively swiftly from recessions, leading to long-term dynamics that do not significantly deviate from predictions based on steady-state analysis.

In contrast, Brunnermeier and Sannikov (2014) introduce a leverage constraint that limits agents' borrowing capacity to the market value of their assets. Unlike the models by He and Krishnamurthy (2012), endogenous risk is a persistent feature in Brunnermeier and Sannikov's framework and can drive the economy far from its steady-state equilibrium. In their model, endogenous risk creates a negative externality through agents' risk management decisions, which are influenced by financial frictions. Specifically, when the net worth of more productive agents declines, these agents mitigate their exposure to aggregate shocks by selling assets to less productive agents. Such fire sales exert downward pressure on asset prices, triggering complex feedback loops that significantly influence macroeconomic dynamics.

Klimenko et al. (2017) focus on a rental market for land, where farmers do not own the land but lease it from landlords. This streamlined framework allows for the derivation of explicit dynamic relationships while retaining key general equilibrium effects characteristic of the newer macroeconomic models in this genre. Their approach facilitates a clearer understanding of the essential mechanisms at play while maintaining the theoretical richness necessary to capture the broader economic implications of financial frictions.

3 Model

Similar to the model presented by Basak and Cuoco (1998), our model also has the limitation of investing in the stock market. Our model is a continuous time model with an infinite period and consists of three agents: hand-to-mouth household (HtM), simple household (HH), and corporate person (CP). The hand-to-hand household cannot participate in the capital market,

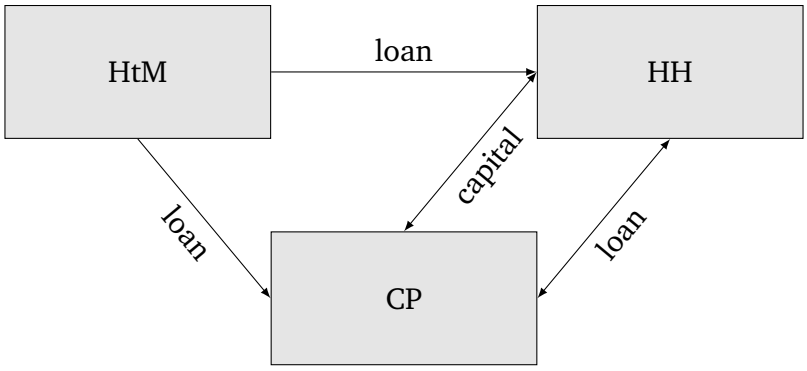


Figure 6: Financial relationships in economy

but the simple household and the corporate person can buy and sell capital together. The financial relationships of these three agents are shown in Figure 6.

We denote these three agents by $i \in \{h, s, c\}$ and $t \in [0, \infty)$ is time. Capital accumulation is as follows.

$$\frac{\mathrm{d}k_t^i}{k_t^i} = \left(\Phi\left(\iota_t^i\right) - \delta\right)\mathrm{d}t + \sigma \mathrm{d}Z_t, \qquad i = c, s,$$

where Z_t is a Brownian motion with mean 0 and variance t on the probability space $(\Omega, \mathscr{F}, P)$ and dZ_t and represents a systematic risk and directly affects capital. The investment function Φ satisfies

$$\Phi(0) = 0, \Phi'(.) > 0 \text{ and } \Phi''(.) < 0.$$

The investment function shows the adjustment cost in the investment, which means investing a unit of consumer goods does not lead to the creation of a unit of capital. This adjustment cost is similar to Bernanke (1999) but is not the same. We denote the investment rate by ι_t, and the depreciation rate by δ, and σ is the standard deviation of the shock.

The production function has CRS form

$$y_t^i = a^i k_t^i, \qquad i = c, s.$$

We note that the productivity of agents is not equal. Production enters consumption or invest-

ment. Only one asset exists in the economy, and that is the capital, defined as

$$K_t = k_t^c + k_t^s.$$

For the price of the capital at the time t, defined as q_t, the economy's total wealth is equal to $q_t K_t$. We assume that the asset price satisfies

$$\frac{\mathrm{d}q_t}{q_t} = \mu_t^q \mathrm{d}t + \sigma_t^q \mathrm{d}Z_t.$$

The aggregate net worth of any agent can be shown by

$$N_t^i = n_t^i, \qquad i = c, s, h.$$

Due to the existence of one unit of any agent in the economy, the wealth share of agents is

$$\eta_t^i = \frac{N_t^i}{N_t^c + N_t^s + N_t^h}, \qquad i = c, s, h.$$

The return of capital include two components: net product of capital and change in value of the capital. We can calculate the capital return, $\mathrm{d}r_t^K\left(\iota_t^i\right)$, with Ito's lemma

$$\begin{aligned} \mathrm{d}r_t^K\left(\iota_t^i\right) &= \overbrace{\frac{a^i - \iota_t^i}{q_t}\mathrm{d}t}^{\text{Dividend Yield}} + \overbrace{\frac{\mathrm{d}\left(q_t k_t^i\right)}{q_t k_t^i}}^{\text{Capital Gain}} \\ &= \left[\frac{a^i - \iota_t^i}{q_t} + \Phi\left(\iota_t^i\right) - \delta + \mu_t^q + \sigma\sigma_t^q\right]\mathrm{d}t + \left(\sigma + \sigma_t^q\right)\mathrm{d}Z_t. \end{aligned} \tag{1}$$

Now, we discuss the problem of each of the agents:

Simple Household has the possibility to participate in the capital market and can issue or buy bonds or capital shares. The simple household has no restrictions on participating in the capital market. We can indicate the optimization problem of simple household as below

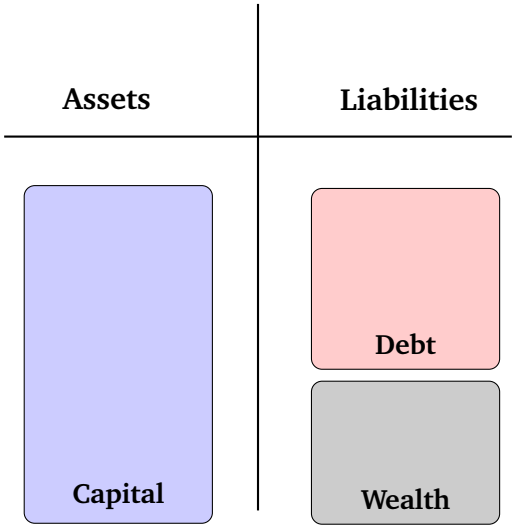


Figure 7: Balance Sheets of Simple Household

$$\begin{aligned} \max_{\{\iota_t^s,\theta_t^s,c_t^s\}_{t=0}^{\infty}} \quad & \mathrm{E}\left[\int_0^{\infty} e^{-\rho^s t}\log\left(c_t^s\right)\mathrm{d}t\right] \\ \text{s.t.} \quad & \frac{\mathrm{d}n_t^s}{n_t^s} = -\frac{c_t^s}{n_t^s}\mathrm{d}t + \left(1-\theta_t^s\right)\mathrm{d}r_t^K\left(\iota_t^s\right) + \theta_t^s r_t \mathrm{d}t, \end{aligned}$$

where θ_t^s is the share of risk-free asset in the total wealth of simple household.

Hand-to-Mouth Household is limited from entering the stock market. Therefore, it can only have bonds on its balance sheet. The hand-to-mouth household is a credit provider for capital market participants to invest. According to the definition of Kaplan et al. (2014), a hand-to-mouth household is a household that has little liquid assets. In our model, it is assumed that the hand-to-mouth household cannot participate in the capital market. We also have the optimization problem of hand-to-mouth household as follows.

$$\begin{aligned} \max_{\{c_t^h\}_{t=0}^{\infty}} \quad & \mathrm{E}\left[\int_0^{\infty} e^{-\rho^h t}\log\left(c_t^h\right)\mathrm{d}t\right] \\ \text{s.t.} \quad & \frac{\mathrm{d}n_t^h}{n_t^h} = -\frac{c_t^h}{n_t^h}\mathrm{d}t + r_t\mathrm{d}t. \end{aligned}$$

Corporate person is the issuer of bonds and can buy or sell in the stock market. As a result, the corporate person spends the debt created in the bonds to purchase the capital. The

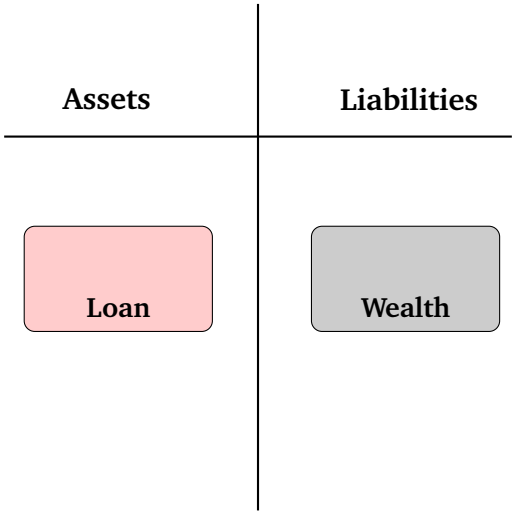


Figure 8: Balance Sheets of Hand-to-Mouth Household

corporate person is more productive than a simple household due to having inside information and special access. As the model in Kiyotaki and Moore (1997), the more productive agent is assumed to be less patient. The corporate person should optimize his utility

$$\begin{aligned} \max_{\{\iota_t^c,\theta_t^c,c_t^c\}_{t=0}^{\infty}} \quad & \mathrm{E}\left[\int_0^{\infty} e^{-\rho^c t}\log\left(c_t^c\right)\mathrm{d}t\right] \\ \text{s.t.} \quad & \frac{\mathrm{d}n_t^c}{n_t^c} = -\frac{c_t^c}{n_t^c}\mathrm{d}t + \left(1-\theta_t^c\right)\mathrm{d}r_t^K\left(\iota_t^c\right) + \theta_t^c r_t \mathrm{d}t. \end{aligned}$$

3.1 Solution of The Model

For solving the model, we should form the Hamilton-Jacobi-Bellman (HJB) equation for any agents. First, we rearrange the budget constraint of every agent

$$\begin{aligned} \frac{\mathrm{d}n_t^i}{n_t^i} &= -\frac{c_t^i}{n_t^i}\mathrm{d}t + \left(1-\theta_t^i\right)\mathrm{d}r_t^K\left(\iota_t^i\right) + \theta_t^i r_t\,\mathrm{d}t \\ &= \left[-\frac{c_t^i}{n_t^i} + \left(1-\theta_t^i\right)\left(\frac{a^i-\iota_t^i}{q_t} + \Phi\left(\iota_t^i\right) - \delta + \mu_t^q + \sigma\sigma_t^q\right) + \theta_t^i r_t\right]\mathrm{d}t \\ &\quad + \left(1-\theta_t^i\right)\left(\sigma+\sigma_t^q\right)\mathrm{d}Z_t\,, \qquad i=c,s. \end{aligned}$$

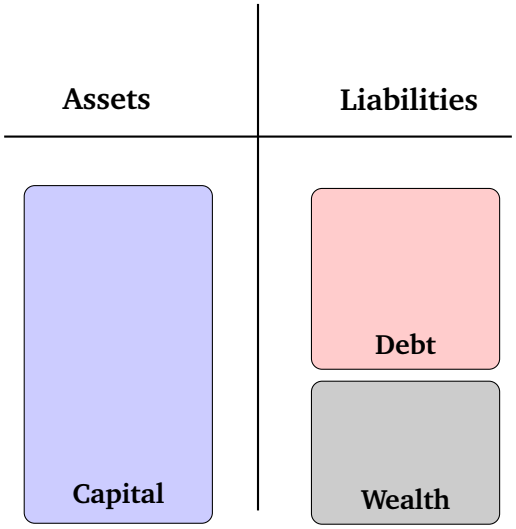


Figure 9: Balance Sheets of Corporate Person

We define the drift of the stochastic differential equation of wealth as

$$\frac{\mathrm{d}n_t^i}{n_t^i} = \mu_t^{n^i}\,\mathrm{d}t + \sigma_t^{n^i}\,\mathrm{d}Z_t\,, \qquad i = c,s.$$

There are two state variables, wealth share of the simple household and wealth share of the corporate person, that can be noted with $\eta_t = \left(\eta_t^c, \eta_t^s\right)$. So, the HJB equation of the simple household or corporate person can be written as

$$\rho^i V^i\left(n_t^i;\eta_t^i\right) = \max_{\iota_t^i,\theta_t^i,c_t^i}\Big\{\log\left(c_t^i\right) + \left(n_t^i\mu_t^{n^i}\right)\partial_n V_t^i + \left(\eta_t^i\mu_t^{\eta^i}\right)\partial_\eta V_t^i + \frac{1}{2}\left(n_t^i\sigma_t^{n^i}\right)^2\partial_{nn}V_t^i + \frac{1}{2}\left(\eta_t^i\sigma_t^{\eta^i}\right)^2\partial_{\eta\eta}V_t^i + \left(\eta_t^i\sigma_t^{n^i}\eta_t^i\sigma_t^{\eta^i}\right)^2\partial_{n\eta}V_t^i\Big\}, \qquad i = c,s,$$

that $V^i\left(n_t^i;\eta_t^i\right)$ is the value function of agent i. The control variables are $\iota_t^i,\theta_t^i,c_t^i$, for $i=c,s$, and we compute the first-order conditions (FOC) with respect to these variables

$$\begin{aligned}
\left[c_t^i\right]&:\frac{1}{c_t^s}=\partial_n V_t^i,\\
\left[\theta_t^i\right]&:\frac{a^i-\iota_t^i}{q_t}+\Phi\left(\iota_t^i\right)-\delta+\mu_t^q+\sigma\sigma_t^q-r_t\\
&\quad=-\left[\frac{\partial_{nn}V_t^i}{\partial_n V_t^i}\left(n_t^i\sigma_t^{n^i}\right)+\frac{\partial_{n\eta}V_t^i}{\partial_n V_t^i}\left(n\eta_t^i\sigma_t^{n\eta^i}\right)\right]\left(\sigma+\sigma_t^q\right),\\
\left[\iota_t^i\right]&:\Phi'\left(\iota_t^i\right)=\frac{1}{q_t}.
\end{aligned}$$

The functional form of adjustment cost that will be used here is the same as the adjustment cost of Brunnermeier and Sannikov (2016)

$$\Phi(\iota)=\frac{1}{\phi}\log(\phi\iota+1).$$

So the first order condition of ι_t^i is as follows.

$$\phi\iota_t^i=q_t-1\Rightarrow\iota_t^i=\frac{q_t-1}{\phi},\qquad i=c,s.$$

Lemma 1 *The rate of investment in the simple household and the corporate person are equal to*

$$\iota_t=\iota_t^c=\iota_t^s.$$

Proof. The above first-order condition simply implies that. ■

Another assumption is about the form of stochastic discount factor (SDF)

$$\frac{\mathrm{d}\xi_t^i}{\xi_t^i}=-r_t\mathrm{d}t-\varsigma_t\mathrm{d}Z_t.$$

As stated in Back (2017), a sufficient condition for a perfect market is that the marginal rate

of substitution of consumption is an SDF. Therefore, SDF is defined as follows.

$$\xi_t^i = e^{-\rho^i t} u'(c_t^i), \qquad i = c, s.$$

The martingale approach, with connection to the proof in Brunnermeier and Sannikov (2016) result that for two assets A, B, the difference between expected returns is equal to

$$\mu_t^A - \mu_t^B = \varsigma_t \left(\sigma_t^A - \sigma_t^B\right).$$

In our environment asset A is the capital and asset B is the bond, then

$$\mu_t^{r^K} - r_t = \varsigma_t^s \sigma_t^{r^K}.$$

By putting the equation 1 in the above equation, the result consists of

$$\frac{a^i - \iota_t^i}{q_t} + \Phi\left(\iota_t^i\right) - \delta + \mu_t^q + \sigma\sigma_t^q - r_t = \varsigma_t^s \left(\sigma + \sigma_t^q\right), \qquad i = c, s. \tag{2}$$

In this model, agents try to maximize their utility subject to budget constraints, and prices will be determined in the markets. Then, we can define the equilibrium as:
Given any initial allocation of capital among the agents, an equilibrium is a map from histories $\{Z_s | s \in [0, t]\}$ to price q_t, investment rate ι_t^i, price of risk ς_t^i and simple household portfolio choice θ_t^s, such that

1. all agents choose portfolios and consumption rates to maximize utility,

2. all markets, for capital and consumption goods, clear.

The total wealth of simple households or corporate persons is

$$\begin{aligned}\frac{\mathrm{d}N_t^i}{N_t^i}=\frac{\mathrm{d}n_t^i}{n_t^i}&=-\frac{c_t^i}{n_t^i}\,\mathrm{d}t+r_t\,\mathrm{d}t+\left(1-\theta_t^i\right)\left[\mathrm{d}r_t^K\left(\iota_t^i\right)-r_t\,\mathrm{d}t\right]\\&=-\frac{c_t^i}{n_t^i}\,\mathrm{d}t+r_t\,\mathrm{d}t+\left(1-\theta_t^i\right)\left\{\left[\frac{a^i-\iota_t^i}{q_t}+\Phi\left(\iota_t^i\right)-\delta+\mu_t^q+\sigma\sigma_t^q-r_t\right]\mathrm{d}t+\left(\sigma+\sigma_t^q\right)\mathrm{d}Z_t\right\}.\end{aligned}$$

with many calculations we have

$$\frac{\mathrm{d}N_t^i}{N_t^i}=-\frac{c_t^i}{n_t^i}\,\mathrm{d}t+r_t\,\mathrm{d}t+\left(1-\theta_t^i\right)\left\{\varsigma_t^i(\sigma+\sigma_t^q)dt+\left(\sigma+\sigma_t^q\right)dZ_t\right\}$$

Lemma 2 *The aggregate capital evolution has the below form*

$$\frac{\mathrm{d}K_t}{K_t}=\left(\Phi\left(\iota_t\right)-\delta\right)\mathrm{d}t+\sigma\,\mathrm{d}Z_t\,.$$

Proof.

$$\begin{aligned}\mathrm{d}K_t&=\mathrm{d}k_t^c+\mathrm{d}k_t^s\\&=\left(\Phi\left(\iota_t^c\right)-\delta\right)k_t^c\,\mathrm{d}t+\sigma k_t^c\,\mathrm{d}Z_t+\left(\Phi\left(\iota_t^c\right)-\delta\right)k_t^s\,\mathrm{d}t+\sigma k_t^s\,\mathrm{d}Z_t\\&=\left(\Phi\left(\iota_t\right)-\delta\right)\left(k_t^c+k_t^s\right)\mathrm{d}t+\sigma\left(k_t^c+k_t^s\right)\mathrm{d}Z_t\\&=\left(\Phi\left(\iota_t\right)-\delta\right)K_t\,\mathrm{d}t+\sigma K_t\mathrm{d}Z_t.\end{aligned}$$

The third equation was obtained concerning lemma 1. ■

Another essential dynamic belongs to the capital value

$$\begin{aligned}\frac{\mathrm{d}q_tK_t}{q_tK_t}&=\left[\mu_t^q+\Phi(\iota_t^i)-\delta+\sigma\sigma_t^q\right]\mathrm{d}t+(\sigma+\sigma_t^q)\,\mathrm{d}Z_t\\&=\left[r_t-\frac{a^i-\iota_t^i}{q_t}+\varsigma_t^i(\sigma+\sigma_t^q)\right]\mathrm{d}t+(\sigma+\sigma_t^q)\,\mathrm{d}Z_t\,,\qquad i=c,s.\end{aligned}$$

Totally, with the aid of the definition of wealth share of agents, the dynamic of two state variables can be obtained

$$\frac{\mathrm{d}\eta_t^c}{\eta_t^c} = \left[-\frac{c_t^c}{n_t^c} + \frac{a^c - \iota_t^c}{q_t} - \theta_t^c(\sigma + \sigma_t^q)\left(\varsigma_t^c - (\sigma + \sigma_t^q)\right) \right] \mathrm{d}t - \theta_t^c(\sigma + \sigma_t^q)\,\mathrm{d}Z_t\,,$$
$$\frac{\mathrm{d}\eta_t^s}{\eta_t^s} = \left[-\frac{c_t^s}{n_t^s} + \frac{a^s - \iota_t^s}{q_t} - \theta_t^s(\sigma + \sigma_t^q)\left(\varsigma_t^s - (\sigma + \sigma_t^q)\right) \right] \mathrm{d}t - \theta_t^s(\sigma + \sigma_t^q)\,\mathrm{d}Z_t\,.$$

Ito's quotient rule is applied on N_t^i and $q_t K_t$, then the above dynamic results.

3.2 Guess the Value Function Form

In the logarithmic utility or, generally, CRRA utility, if the wealth multiplies to a constant, then the optimal consumption multiplies to the same constant. So, the form of the value function will be guessed

$$V(n_t^i) = \frac{u(\omega_t^i n_t^i)}{\rho}, \qquad i = c, s, h.$$

According to the FOC of consumption, we have

$$V'(n_t^i) = u'(c_t^i) \Rightarrow \frac{c_t^i}{n_t^i} = \rho.$$

Hence, ω_t^i has no effect on the logarithmic utility. Now, we assume the dynamic of the consumption and wealth for any agents

$$\frac{\mathrm{d}c_t^i}{c_t^i} = \mu_t^{c^i}\,\mathrm{d}t + \sigma_t^{c^i}\,\mathrm{d}Z_t\,, \qquad i = c, s.$$

Proposition 1 *The volatility of the growth of wealth and the volatility of the growth of consumption for any agents are the same.*

Proof. If the ratio of consumption and wealth is constant, then volatilities should be equal to

$$\frac{c_t^i}{n_t^i} = \text{constant} \Rightarrow \sigma_t^{c^i} = \sigma_t^{n^i}, \qquad i = c, s, h.$$

■

Proposition 2 *The price of risk is equal to the volatility of the growth of wealth*

$$\varsigma_t^i = \sigma_t^{c^i} = \sigma_t^{n^i} = (1 - \theta_t^s)(\sigma + \sigma_t^q), \qquad i = c, s.$$

Proof. From Ito's lemma we obtain

$$\frac{\mathrm{d}\left(c_t^i\right)^{-1}}{(c_t^i)^{-1}} = \left[-\mu_t^{c^i} + \left(\sigma_t^{c^i}\right)^2\right]\mathrm{d}t - \sigma_t^{c^i}\,\mathrm{d}Z_t\,, \qquad i = c, s,$$

and SDF process is:

$$\xi_t^i = e^{-\rho^i t} u'(c_t^i) = e^{-\rho^i t}(c_t^i)^{-1}, \qquad i = c, s.$$

By Ito's chain rule the dynamic of SDF be founded

$$\frac{\mathrm{d}\xi_t^i}{\xi_t^i} = -\rho^i\,\mathrm{d}t + \frac{\mathrm{d}\left(c_t^i\right)^{-1}}{(c_t^i)^{-1}} = \left[-\rho - \mu_t^{c^i} + \left(\sigma_t^{c^i}\right)^2\right]\mathrm{d}t - \sigma_t^{c^i}\,\mathrm{d}Z_t\,, \qquad i = c, s.$$

Finally, the price of risk is

$$\varsigma_t^i = \sigma_t^{c^i} = \sigma_t^{n^i} = (1 - \theta_t^s)(\sigma + \sigma_t^q), \qquad i = c, s.$$

■

With Proposition 2 the dynamic of state variables are simplified

$$\begin{aligned}
\frac{\mathrm{d}\eta_t^c}{\eta_t^c} &= \left[\frac{a^c - \iota_t^c}{q_t} - \rho + (\theta_t^c)^2(\sigma + \sigma_t^q)^2\right]\mathrm{d}t - \theta_t^c(\sigma + \sigma_t^q)\,\mathrm{d}Z_t\,, \\
\frac{\mathrm{d}\eta_t^s}{\eta_t^s} &= \left[\frac{a^s - \iota_t^s}{q_t} - \rho + (\theta_t^s)^2(\sigma + \sigma_t^q)^2\right]\mathrm{d}t - \theta_t^s(\sigma + \sigma_t^q)\,\mathrm{d}Z_t\,.
\end{aligned}$$

3.3 Market Clearing

The economy has three markets: consumption goods, bonds, and capital. According to Walras's Law, if we have n markets and $n-1$ markets cleared, the nth market will also be cleared. Therefore, we only check the market clearing conditions of consumption goods and capital.

For simplicity, define $S(q)$ as the sum of the volatility of the capital and price

$$S(q) = \sigma + \sigma^q.$$

The capital market clearing condition shows that the sum of capital that is at the disposal of the simple household and the corporate person is equal to the total capital

$$(1-\theta_t^c)N_t^c + (1-\theta_t^s)N_t^s = q_t K_t \Rightarrow (1-\theta_t^c)\eta_t^c + (1-\theta_t^s)\eta_t^s = 1. \tag{3}$$

From 2 and Proposition 2, we conclude

$$\frac{a^i - \iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q - r_t = \left(1-\theta_t^i\right)\left(\sigma + \sigma_t^q\right)^2, \qquad i = c, s.$$

Therefore, the share of capital in the balance sheet of the simple household and the corporate person are equal to

$$1-\theta_t^i = \frac{1}{S^2}\left[\frac{a^i - \iota_t}{q_t} + \Phi(\iota_t) - \delta - r_t\right], \qquad i = c, s. \tag{4}$$

Substitute the share of capital in the market clearing condition of the capital market into 3 to drive

$$\left[\frac{a^c - \iota_t}{q_t} + \Phi(\iota_t) - \delta - r_t\right]\eta_t^c + \left[\frac{a^s - \iota_t}{q_t} + \Phi(\iota_t) - \delta - r_t\right]\eta_t^s = S^2. \tag{5}$$

Eventually, the risk-free interest rate should be equal to[3]

$$r = \frac{a^c - \iota}{q}\frac{\eta^c}{\eta^c + \eta^s} + \frac{a^s - \iota}{q}\frac{\eta^s}{\eta^c + \eta^s} + \Phi(\iota) - \delta - \frac{S^2}{\eta^c + \eta^s}.$$

Replace r in 4 with using 5 to conclude

$$\begin{aligned} 1 - \theta^c &= \frac{a^c - a^s}{S^2 q}\frac{\eta^s}{\eta^c + \eta^s} + \frac{1}{\eta^c + \eta^s}, \\ 1 - \theta^s &= \frac{a^s - a^c}{S^2 q}\frac{\eta^c}{\eta^c + \eta^s} + \frac{1}{\eta^c + \eta^s}, \end{aligned}$$

and then

$$\begin{aligned} \theta^c &= 1 - \frac{1}{\eta^c + \eta^s} - \frac{a^c - a^s}{S^2 q}\frac{\eta^s}{\eta^c + \eta^s}, \\ \theta^s &= 1 - \frac{1}{\eta^c + \eta^s} + \frac{a^c - a^s}{S^2 q}\frac{\eta^c}{\eta^c + \eta^s}. \end{aligned}$$

Therefore, the dynamic of the corporate person η^c is as follows.

$$\begin{aligned} \frac{\mathrm{d}\eta^c}{\eta^c} = &\left[\frac{a^c - \iota}{q} - \rho^c + \left(1 - \frac{1}{\eta^c + \eta^s} - \frac{a^c - a^s}{S^2 q}\frac{\eta^s}{\eta^c + \eta^s}\right)^2 (\sigma + \sigma^q)^2\right]\mathrm{d}t \\ &- \left[1 - \frac{1}{\eta^c + \eta^s} - \frac{a^c - a^s}{S^2 q}\frac{\eta^s}{\eta^c + \eta^s}\right](\sigma + \sigma^q)\,\mathrm{d}Z_t, \end{aligned}$$

and similarly, the dynamic of the simple household's wealth share is

$$\begin{aligned} \frac{\mathrm{d}\eta^s}{\eta^s} = &\left[\frac{a^s - \iota}{q} - \rho^s + \left(1 - \frac{1}{\eta^c + \eta^s} - \frac{a^s - a^c}{S^2 q}\frac{\eta^c}{\eta^c + \eta^s}\right)^2 (\sigma + \sigma^q)^2\right]\mathrm{d}t \\ &- \left[1 - \frac{1}{\eta^c + \eta^s} + \frac{a^c - a^s}{S^2 q}\frac{\eta^c}{\eta^c + \eta^s}\right](\sigma + \sigma^q)\,\mathrm{d}Z_t. \end{aligned}$$

[3] from here due to reduction of complexity we do not use time index t.

The dynamic of the hand-to-mouth household is obtained, but it is not applicable.

$$\frac{\mathrm{d}\eta^h}{\eta^h} = \left[\frac{a^c-\iota}{q} - \rho^s + \left(1 - \frac{1}{\eta^c+\eta^s} - \frac{a^c-a^s}{S^2 q}\frac{\eta^s}{\eta^c+\eta^s}\right)(\sigma+\sigma^q)^2\right]\mathrm{d}t$$
$$-(\sigma+\sigma^q)\,\mathrm{d}Z_t.$$

All of the variables in these dynamics are a function of q. So, we should find the functionality of the capital price on the state variables.To do this, we define aggregate productivity as a weighted mean of the productivity of capital owners

$$\hat{a} := \hat{a}\left(\eta^c,\eta^s\right) = \left(1-\theta^c\right)\eta^c a^c + \left(1-\theta^s\right)\eta^s a^s = \frac{\eta^c a^c + \eta^s a^s}{\eta^c+\eta^s} + \frac{\eta^c\eta^s}{\eta^c+\eta^s}\frac{\left(a^c-a^s\right)^2}{S^2 q}.$$

Also, the aggregate discount factor defines as the weighted mean of the discount factor of every agent

$$\hat{\rho} := \hat{\rho}\left(\eta^c,\eta^s\right) = \eta^s\rho^s + \left(1-\eta^c-\eta^s\right)\rho^h + \eta^c\rho^c.$$

The aggregate demand for the consumption goods is

$$C^d = \rho^s N^s + \rho^h N^h + \rho^c N^c = \left[\eta^s\rho^s + \left(1-\eta^c-\eta^s\right)\rho^h + \eta^c\rho^c\right]qK = \hat{\rho}qK.$$

The aggregate supply of the consumption good is equal to a share of output that is not invested

$$C^s = (\hat{a}-\iota)K.$$

In the equilibrium case,

$$\hat{\rho}qK = (\hat{a}-\iota)K.$$

Then, the equilibrium price can be derived as follows.

$$\hat{\rho}q = \hat{a} - \frac{q-1}{\phi} \Rightarrow \phi\hat{\rho}q = \phi\hat{a} - q + 1.$$

$$q := q(\eta^c, \eta^s) = \frac{1+\phi\hat{a}(\eta^c,\eta^s)}{1+\phi\hat{\rho}(\eta^c,\eta^s)}. \tag{6}$$

To describe q from the above equation, from the definition of $\hat{a}$ and $\hat{\rho}$, we should solve the following quadratic equation

$$(1+\phi\hat{\rho})q^2 - \left(1+\phi\frac{\eta^c a^c+\eta^s a^s}{\eta^c+\eta^s}\right)q - \frac{\phi\eta^c\eta^s}{\eta^c+\eta^s}\frac{(a^c-a^s)^2}{S^2} = 0.$$

but this quadratic equation is analytically unsolvable because σ^q is related to q. The alternative solution is an approximation way.

Proposition 3 *If $a^c - a^s$ is small enough, we have*

$$\hat{a} \approx \frac{\eta^c a^c+\eta^s a^s}{\eta^c+\eta^s}.$$

Proof. The second term of $\hat{a}$ is composed of the product of 2 shares and has a degree greater than the first term. Therefore, if $a^c - a^s$ is sufficiently small, then the second term is much smaller than the other. Additional explanations are given in Appendix B. ■

By using Proposition 3, the differential of q will be obtained

$$\begin{aligned} \mathrm{d}q &= \frac{\phi(1+\phi\hat{\rho})\mathrm{d}\hat{a} - \phi(1+\phi\hat{a})\mathrm{d}\hat{\rho}}{(1+\phi\hat{\rho})^2} + \frac{1}{2}\frac{\partial^2 q}{\partial\hat{a}^2}(\mathrm{d}\hat{a})^2 + \frac{1}{2}\frac{\partial^2 q}{\partial\hat{\rho}^2}(\mathrm{d}\hat{\rho})^2 + \frac{\partial^2 q}{\partial\hat{a}\partial\hat{\rho}}(\mathrm{d}\hat{a})(\mathrm{d}\hat{\rho}) \\ &= \frac{\phi}{1+\phi\hat{\rho}}(\mathrm{d}\hat{a} - q\mathrm{d}\hat{\rho}) + \frac{1}{2}\frac{\partial^2 q}{\partial\hat{a}^2}(\mathrm{d}\hat{a})^2 + \frac{1}{2}\frac{\partial^2 q}{\partial\hat{\rho}^2}(\mathrm{d}\hat{\rho})^2 + \frac{\partial^2 q}{\partial\hat{a}\partial\hat{\rho}}(\mathrm{d}\hat{a})(\mathrm{d}\hat{\rho}), \end{aligned}$$

$$\frac{\mathrm{d}q}{q} = \frac{\phi}{1+\phi\hat{\rho}}\left(\frac{d\hat{a}}{q} - \mathrm{d}\hat{\rho}\right) + \frac{1}{2q}\frac{\partial^2 q}{\partial\hat{a}^2}(\mathrm{d}\hat{a})^2 + \frac{1}{2q}\frac{\partial^2 q}{\partial\hat{\rho}^2}(\mathrm{d}\hat{\rho})^2 + \frac{1}{q}\frac{\partial^2 q}{\partial\hat{a}\partial\hat{\rho}}(\mathrm{d}\hat{a})(\mathrm{d}\hat{\rho}).$$

From Ito's formula, the aggregate discount factor satisfies

$$\mathrm{d}\hat{\rho} = \left(\rho^c - \rho^h\right)\mathrm{d}\eta^c + \left(\rho^s - \rho^h\right)\mathrm{d}\eta^s. \tag{7}$$

and the aggregate productivity satisfies

$$\begin{aligned}
\mathrm{d}\hat{a} &= \frac{(a^c \mathrm{d}\eta^c + a^s \mathrm{d}\eta^s)(\eta^c + \eta^s) - (a^c \eta^c + a^s \eta^s)(\mathrm{d}\eta^c + \mathrm{d}\eta^s)}{(\eta^c + \eta^s)^2} \\
&\quad + \frac{1}{2}\frac{\partial^2 \hat{a}}{\partial \eta^{c2}}(\mathrm{d}\eta^c)^2 + \frac{1}{2}\frac{\partial^2 \hat{a}}{\partial \eta^{s2}}(\mathrm{d}\eta^s)^2 + \frac{\partial^2 \hat{a}}{\partial \eta^c \partial \eta^s}(\mathrm{d}\eta^c)(\mathrm{d}\eta^s) \\
&= \frac{1}{\eta^c + \eta^s}((a^c - \hat{a})\mathrm{d}\eta^c + (a^s - \hat{a})\mathrm{d}\eta^s) + \frac{1}{2}\frac{\partial^2 \hat{a}}{\partial \eta^{c2}}(\mathrm{d}\eta^c)^2 + \frac{1}{2}\frac{\partial^2 \hat{a}}{\partial \eta^{s2}}(\mathrm{d}\eta^s)^2 + \frac{\partial^2 \hat{a}}{\partial \eta^c \partial \eta^s}(\mathrm{d}\eta^c)(\mathrm{d}\eta^s) \\
&= \frac{a^c - a^s}{(\eta^c + \eta^s)^2}(\eta^s \mathrm{d}\eta^c - \eta^c \mathrm{d}\eta^s) + \frac{1}{2}\frac{\partial^2 \hat{a}}{\partial \eta^{c2}}(\mathrm{d}\eta^c)^2 + \frac{1}{2}\frac{\partial^2 \hat{a}}{\partial \eta^{s2}}(\mathrm{d}\eta^s)^2 + \frac{\partial^2 \hat{a}}{\partial \eta^c \partial \eta^s}(\mathrm{d}\eta^c)(\mathrm{d}\eta^s).
\end{aligned} \tag{8}$$

Proposition 4 *The volatility of the price is equal to*

$$\begin{aligned}
\sigma^q = \frac{\phi S}{1 + \phi\hat{\rho}}\Bigg[\left(\frac{a^c - a^s}{Sq}\right)^2 \frac{\eta^c \eta^s}{(\eta^c + \eta^s)^2} + \rho^c \eta^c + \rho^s \eta^s - \frac{\rho^c \eta^c + \rho^s \eta^s}{\eta^c + \eta^s} + \rho^h(1 - \eta^c - \eta^s) \\
- (\rho^c - \rho^s)\left(\frac{a^c - a^s}{S^2 q}\frac{\eta^c \eta^s}{\eta^c + \eta^s}\right)\Bigg].
\end{aligned} \tag{9}$$

Proof. The proof will be in appendix C. ■

Our approximation was only applied to q, and the price volatility is derived from that approximation.

Proposition 5 *The function of capital price has a form similar to*

$$q = \frac{-C \pm \sqrt{C^2 - 4A(BS^2 - S^2 + \sigma S)}}{2S(BS - S + \sigma)}.$$

Proof. The proof will be coming in the appendix D, and A, B, and C will be defined. ■

σ^q is related to q and q is related to σ^q. In the approximation method, we use equation 6 for solving q so that only σ^q is placed under q.

3.4 Calibration

We need to value the parameters to stabilize the model using these values. We tried to choose the parameters close to the Brunnermeier and Sannikov (2014) and Brunnermeier and Sannikov (2016) parameters as much as possible. As Cheng et al. (2022) explain, models with nonlinear dynamics are extraordinarily fragile and cannot be easily quantified. In most of the papers that have been done in this field, the model has one state variable, but in our model, there are two state variables. The presence of two state variables makes it difficult to stabilize the system. In the **supplemental appendix**, some problems in this path are briefly presented.

Parameter	Meaning	Value
ρ^c	Discount Factor of Corporate Person	0.07
ρ^s	Discount Factor of Simple Household	0.04
ρ^h	Discount Factor of Hand-to-Mouse Household	0.03
a^c	Productivity of Corporate Person	0.09
a^s	Productivity of Simple Household	0.08
δ	Depreciation Rate	0.035
ϕ	Technological friction	1
σ	Volatility of Capital	0.1

Table 1: Calibrated Parameters

As shown in Figure 10, both state variables have an interval with a stable equilibrium. Of course, if there are big shocks to the system, it is possible for η^s to leave the stable state. The steady-state equilibrium is formed in the absence of the uncertainty part at the following point

$$\eta^c_{ss} \approx 0.311, \qquad \eta^s_{ss} \approx 0.130, \qquad \eta^h_{ss} \approx 0.559.$$

4 Results

In all binary figures, we fix one state variable at its steady-state value and allow the other state variable to change. Simple households and corporate persons are agents that operate in the capital market. They can store their wealth in the form of capital or debt securities. If

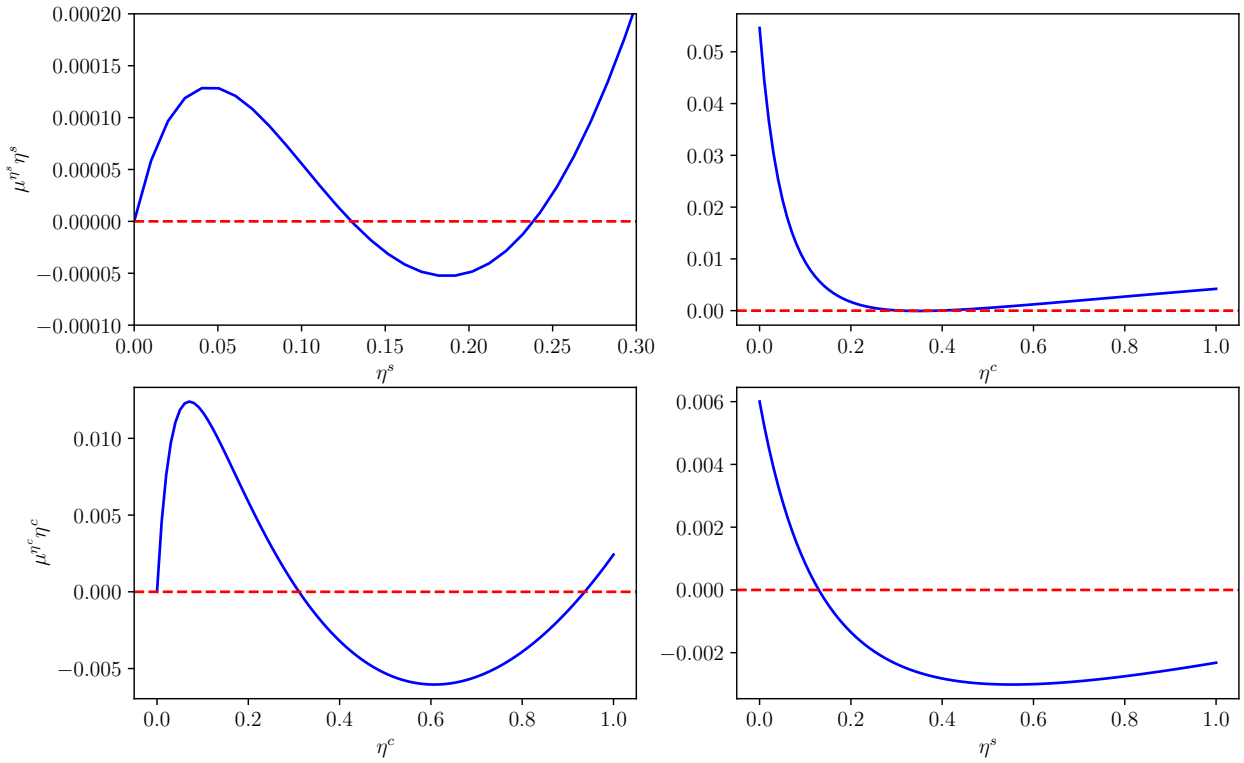


Figure 10: drifts of η^c and η^s

buying bonds is not optimal for either, they can issue bonds instead of buying bonds. Issuing bonds means using credit to purchase capital or investment. It is optimal for both agents to issue bonds, so the only credit suppliers are hand-to-mouth households. Hand-to-mouth households all the remaining wealth that they do not consume; they offer it as credit and buy bonds. Hand-to-mouth households do not bear any risk directly because bonds are not just any securities, they are risk-free, providing a sense of reassurance to the investors. However, the uncertainty that exists in simple households and corporate persons indirectly affects hand-to-mouth households.

According to Campbell (2017), the wealth that belongs to people who can participate in the stock market makes the participation index in the capital market. In other words, only part of the participants' wealth that is in the capital market is not part of the index of participation in

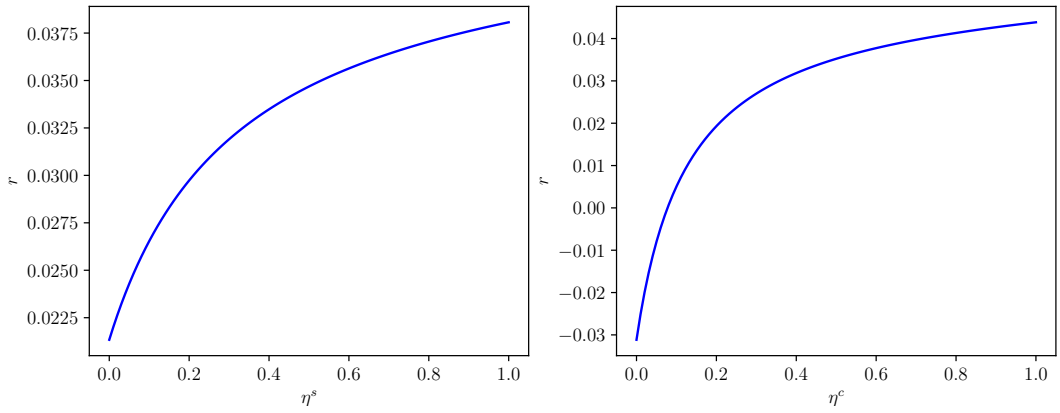


Figure 11: risk-free return

the capital market, but the rest of their wealth is also included in this index because other parts of their wealth can also be added to the capital market, unlike the wealth of non-participants, which cannot enter the stock market in any way. Therefore, increasing the share of participants' wealth means increasing the participation rate in the stock market.

An increase in the share of each of the capital market participants (simple household and corporate person) from the wealth of the entire economy causes the demand for credit to increase and decrease the supply of credit from hand-to-mouth households. As a result, the price of credit -the same interest rate without risk- will increase, as shown in Figure 11.

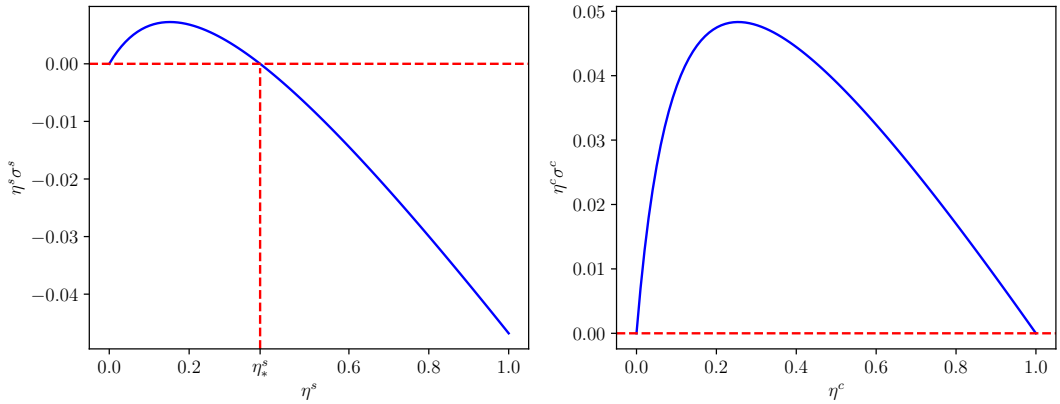


Figure 12: diffusions of η^c and η^s

The situation regarding the volatility of state variables is slightly different. In fact, the

volatility of a state variable means the impulse response of that state variable to a shock. The volatility of the corporate person's wealth share is always positive. In other words, when there is a positive shock to capital, the share of corporate wealth increases. Because the corporate person is the supplier of debt securities for the entire span. In the simple household, we face volatility that ranges from positive to negative. As shown in Figure 12, in the range of $\eta^s < \eta^s_*$, the positive shock increases the wealth share of the simple household. Because in this period, the simple household has leveraged, but after that, due to the interest rate increase, the household becomes a credit provider, shown in Figure 13. In the $\eta^s > \eta^s$, the only applicant for credit is the corporate person. As a general rule, when there is a positive shock to capital, the leveraged agent's share of wealth will increase, and the wealth share of the buyers of bonds will decrease.

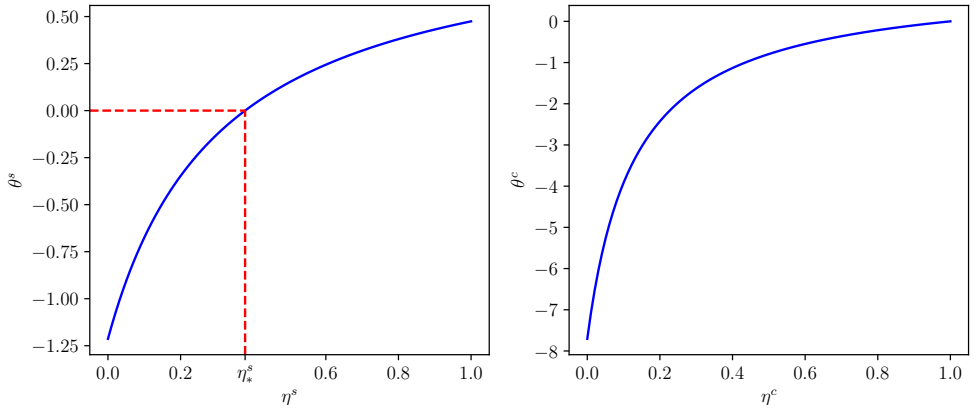


Figure 13: leverage of simple household and corporate person

As shown in Figure 13, with the increase in the share of the simple household or corporate person, due to the decrease in the share of the hand-to-mouth household, the credit provided decreases, and the risk-free interest rate increases. So, the demand for credit is reduced, and capital market participants are less leveraged. According to Brunnermeier and Pedersen (2009), this type of liquidity that capital market actors try to secure by pledging their assets is called "**funding liquidity**". A corporate person always acts as a borrower due to higher efficiency. But the simple household has been a borrower until it has a share less than η^s_*, but

after that, it keeps a part of its wealth in the form of bonds. As a rule, these bonds are issued by a corporate person. The threshold point is $\eta_*^s = 0.378$.

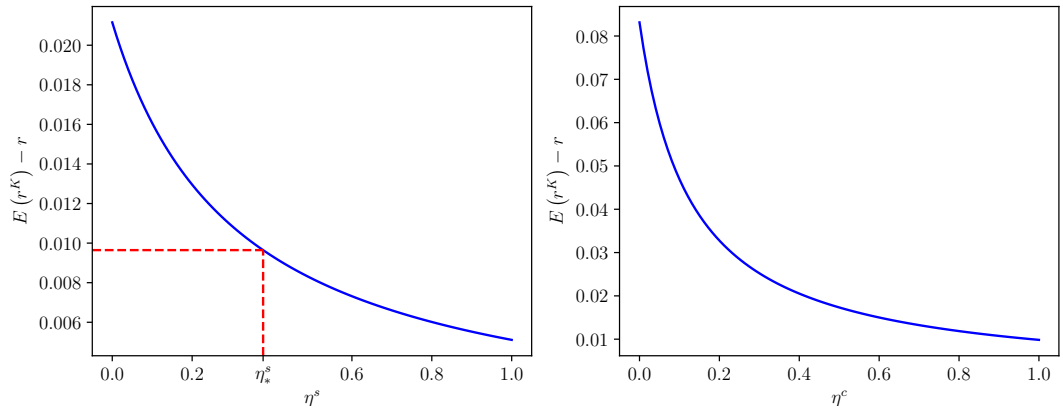


Figure 14: risk premium of simple household and corporate person

One of the critical variables to explain the mechanisms that occur in the economy is the range between the risk-free interest rate and the return rate of the risky asset, which can be called the **risk premium**. The importance of this variable will be more evident in asset pricing. When the share of a corporate person or a simple household acquires a larger share of wealth, it causes an increase in the risk-free interest rate, and as a result, the risk premium erodes. At the threshold point when the simple household decides to be the holder of the bonds instead of the issuer, the risk premium is equal to 0.96%. The simple household prefers the risk-free interest rate of 3.3% to the capital return rate of 4.26% due to its risk aversion.

Finally, we come to the most important analysis, which is asset price analysis. Before it, we define $\eta_q^c(\eta^s)$ as a solution of

$$\frac{\eta^s}{\left(\eta_q^c(\eta^s) + \eta^s\right)^2} = \frac{\rho^c - \rho^h}{a^c - a^s} q\left(\eta_q^c(\eta^s), \eta^s\right).$$

The following theorem expresses the relationship between the share of capital market and asset prices:

Theorem 1 *An increase in the wealth share of a simple household always lowers the asset price,*

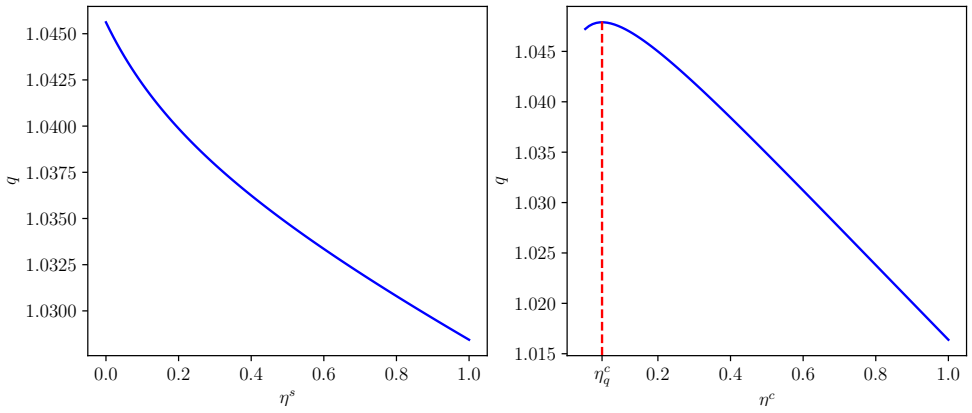


Figure 15: price of capital

but an increase in the wealth share of a corporate person:

1. *if* $\eta^c < \eta_q^c(\eta^s)$*, the price of capital increases.*

2. *if* $\eta^c = \eta_q^c(\eta^s)$*, the price of capital does not change.*

3. *if* $\eta^c > \eta_q^c(\eta^s)$*, the price of capital decreases.*

Proof. The proof will be in appendix E. ■

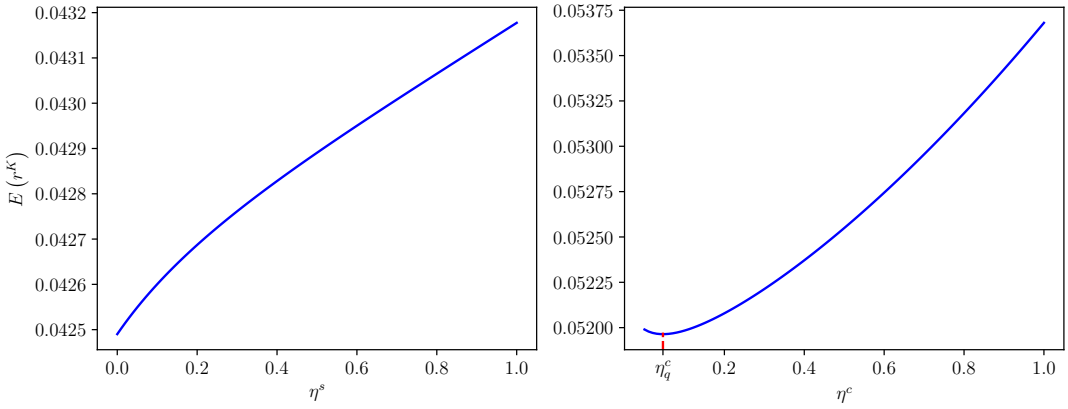


Figure 16: Capital Return

As seen in the right panel of Figure 15, with the corporate person's share increase from zero, the capital price will first increase and then decrease. An increase in the share of corporate persons will always increase the average productivity and, as a result, increase the supply

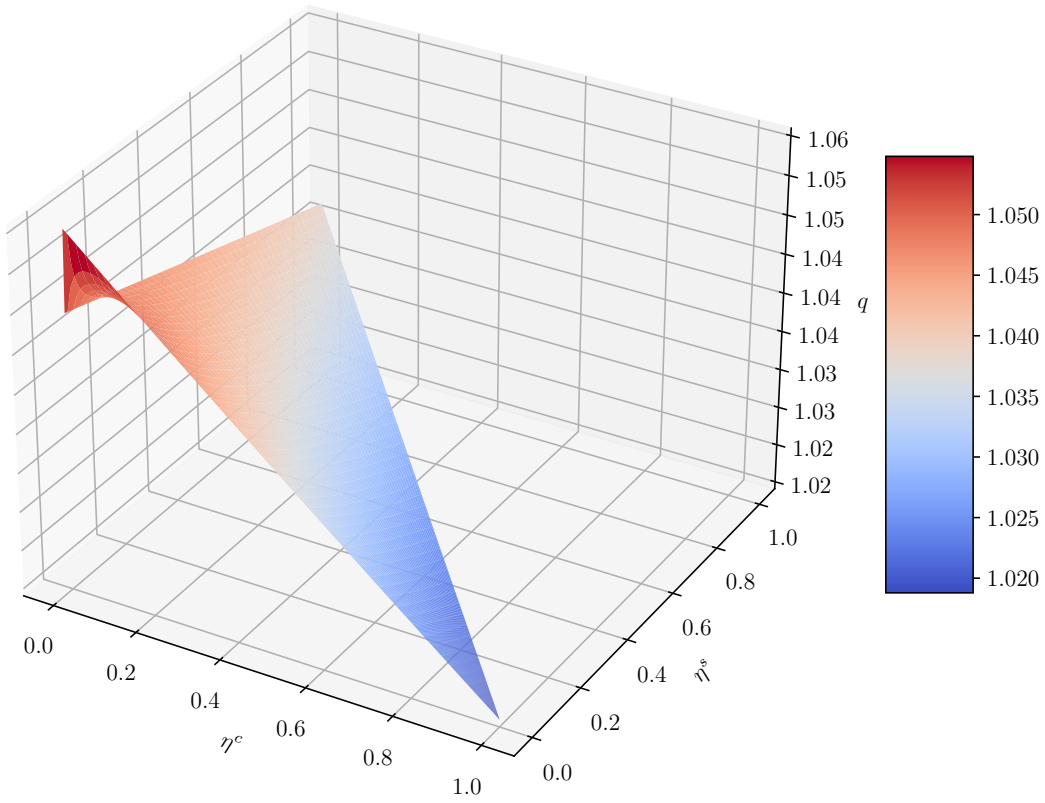


Figure 17: price of capital under whole of state space

of consumer goods. Besides that, it also increases the demand for consumer goods because corporate persons have the lowest discount rate. At first, the supply of consumer goods overcomes its demand and causes its price to decrease. A decrease in the price of consumer goods is equivalent to an increase in the price of capital. However, with the rise in demand, the price of consumer goods increases, and the price of capital goods decreases. The required return is also affected by the capital price situation.

In Figure 17, the entire state space $\eta^c + \eta^s \leq 1$ is covered, and it can be seen that except at

one region, the increase in η^c increases the price, in other areas, the increase in the share of participants in the capital market causes the capital price to decrease.

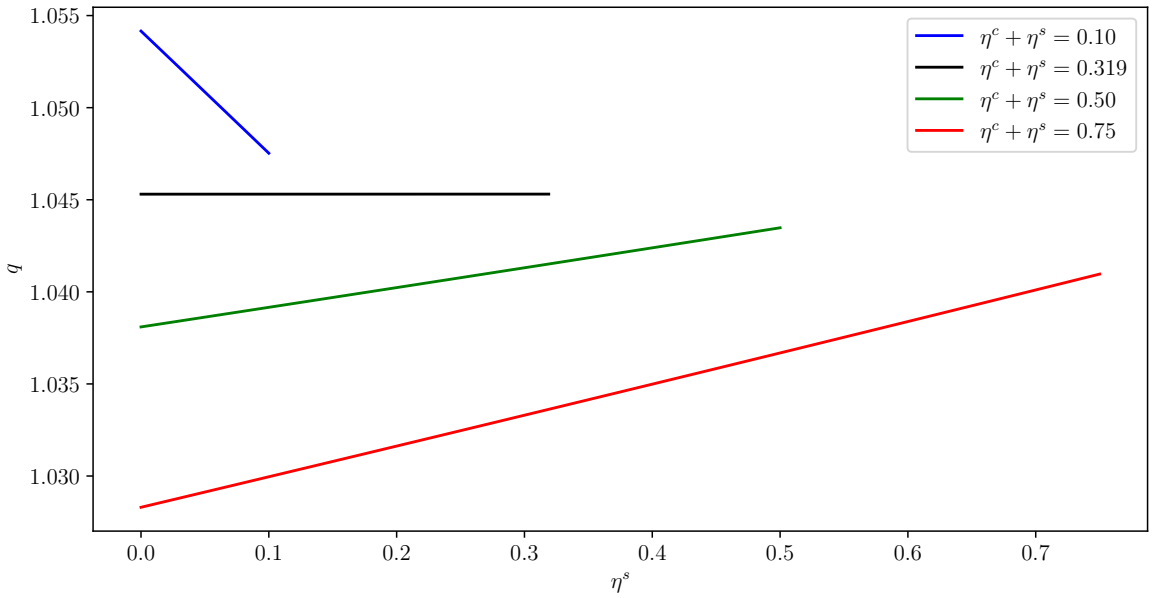


Figure 18: price of capital under transfer of wealth

What happened at the time of implementing justice policy was that the government, as a part of the corporate person, gave a part of its shares to the simple household. As a result, the total wealth of the corporate person and the simple household remained the same, but the share of the corporate person's wealth decreased, and the share of the simple household's wealth increased. In other words, in Theorem 2, an attempt has been made to simulate the transfer of assets that occurred by the government. The result of this simulation tells us that up to a certain amount of the sum of the shares of the simple household and the corporate person, the stock price will increase and then decrease. The threshold point is where the price does not change with the transfer of the share of the household and the corporate person.

Theorem 2 *If the sum of the shares of the simple household and the corporate person is equal to* $\frac{\Delta a\left(1+\phi\rho^h\right)}{\Delta\rho(1+\phi a^s)-\phi\Delta a\left(\rho^s-\rho^h\right)}$*, the transfer of the share of wealth does not affect the price of the capital.*

Proof. The proof will come in appendix F. ■

Therefore, by transferring wealth from the corporate person to the simple household:

1. if $\eta^c + \eta^s < \frac{\Delta a\left(1+\phi\rho^h\right)}{\Delta\rho(1+\phi a^s)-\phi\Delta a\left(\rho^s-\rho^h\right)}$, the price of capital decreases.
2. if $\eta^c + \eta^s = \frac{\Delta a\left(1+\phi\rho^h\right)}{\Delta\rho(1+\phi a^s)-\phi\Delta a\left(\rho^s-\rho^h\right)}$, the price of capital does not change.
3. if $\eta^c + \eta^s > \frac{\Delta a\left(1+\phi\rho^h\right)}{\Delta\rho(1+\phi a^s)-\phi\Delta a\left(\rho^s-\rho^h\right)}$, the price of capital increases.

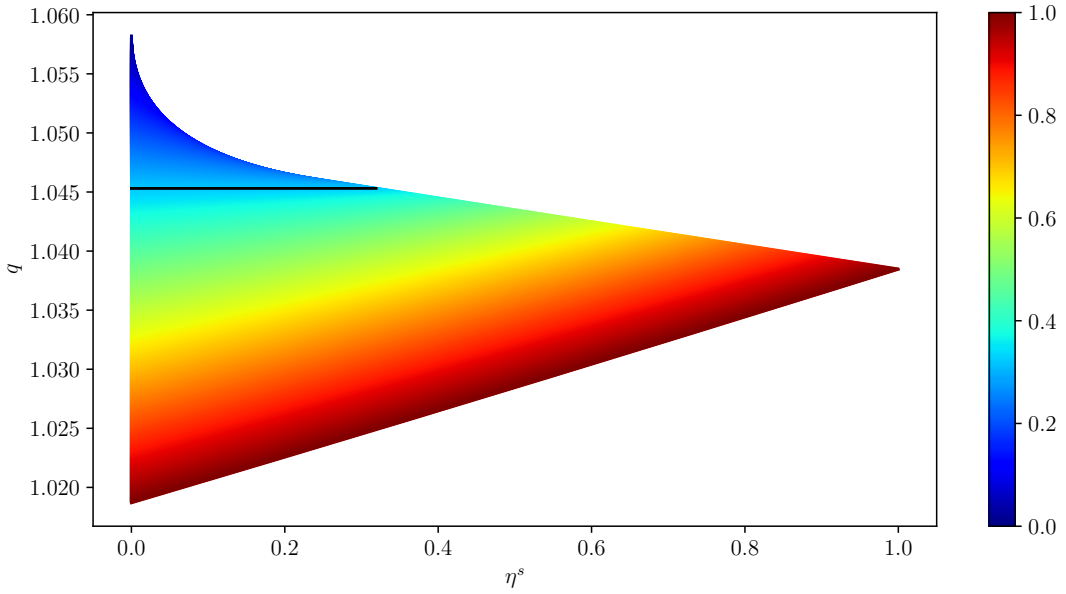


Figure 19: price of capital under whole of state space

When the threshold point is calculated according to the established parameters, we reach the approximated value of 0.319. Until this amount, the result of the transfer of wealth between the corporate person and the simple household is an increase in the capital price. More precisely, in this area, the decrease in the share of the corporate person and the increase in the share of the simple household have opposite effects. A reduction in the share of a corporate person causes an increase in the capital price, and an increase in the share of a simple household causes it to increase as long as the sum of their shares is less than this amount. First, the force of reducing by corporate person share has less effect, but then this effect will be more. The result is that if the share of the wealth of people participating in the capital market is less

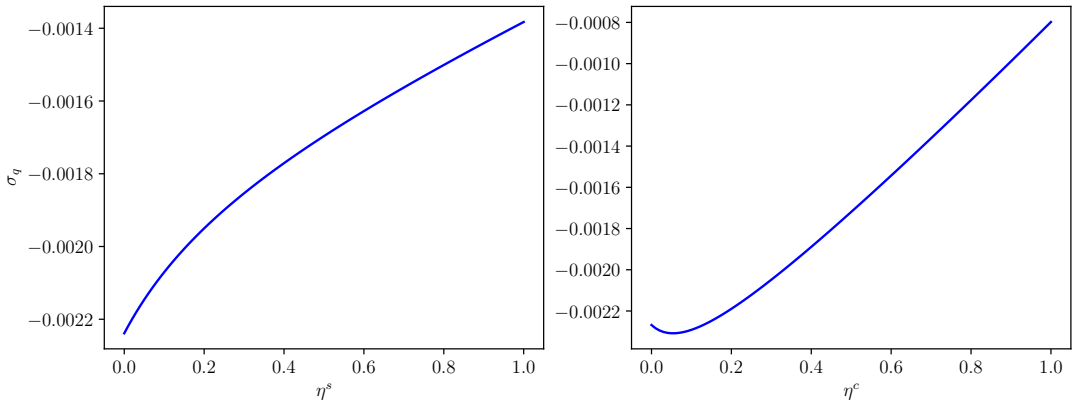


Figure 20: volatility of capital price

than 0.319, the change of wealth from a corporate person to a simple household will increase the price.

In other words, as long as the participation rate is tiny and the capital market has not reached enough depth, the increase in the wealth of participating households will lead to a rise in capital price. However, this issue can be completely reversed with the deepening of the market. The decrease in capital price by increasing the participation rate is one of the reasons that can destroy the motivation of entry of all households into the capital market and, in a way, cause **endogenous limited participation**. This mechanism happens not only in emerging economies but also in developed economies with deep financial markets, preventing households from fully participating in financial markets. The result of theorem 2 is shown in Figure 18 for some cases, and much completely in Figure 19.

5 Discussions

In this part, we will examine two important issues related to capital price and capital stock volatility.

5.1 Volatility of price in real world

The focus of this paper is not on asset price volatility, but there is an exciting result in the model that is relatively consistent with empirical evidence. Figure 20 contains many points. As is apparent, asset price volatility is negative. This result means that an adverse shock to the economy will cause an initial increase in asset prices. The second point is related to changes in asset price volatility. By increasing the share of a corporate person or a simple household, the absolute value of the asset price fluctuation decreases. Of course, the amount of asset fluctuation has a small value, and its changes are very small. We want to compare the results of the model with estimated volatility. Considering that it was assumed that the price of the asset follows a geometric Brownian movement, the price of the asset will be as follows.

$$q_t = q_0 \exp\left(\left(\mu_t^q - \frac{1}{2}\sigma_t^{q2}\right)t + \sigma_t^q B_t\right). \tag{10}$$

Therefore, the average and standard deviation of the price will be equal to:

$$\mathbb{E}(q_t) = q_0 \exp\left(\mu_t^q t\right),$$
$$\sigma(q_t) = q_0 \exp\left(\mu_t^q t\right)\left(\exp\left(\sigma_t^{q2} t\right) - 1\right).$$

We define the coefficient of variation index as follows.

$$CV(q_t) = \frac{\mathbb{E}(q_t)}{\sigma(q_t)} = \exp\left(\sigma_t^{q2} t\right) - 1. \tag{11}$$

Using this index, asset price volatility can be estimated:

$$\widehat{\sigma}^q = \sqrt{\frac{\ln\left(CV(q_t)^2 + 1\right)}{t}}. \tag{12}$$

For two periods before the occurrence of the asset price crisis and after its occurrence in the Tehran stock market, the above estimate for 50-day periods is shown in Figure 21. The mean

of the estimator before the crisis is 0.0072, and after, it is 0.0064.

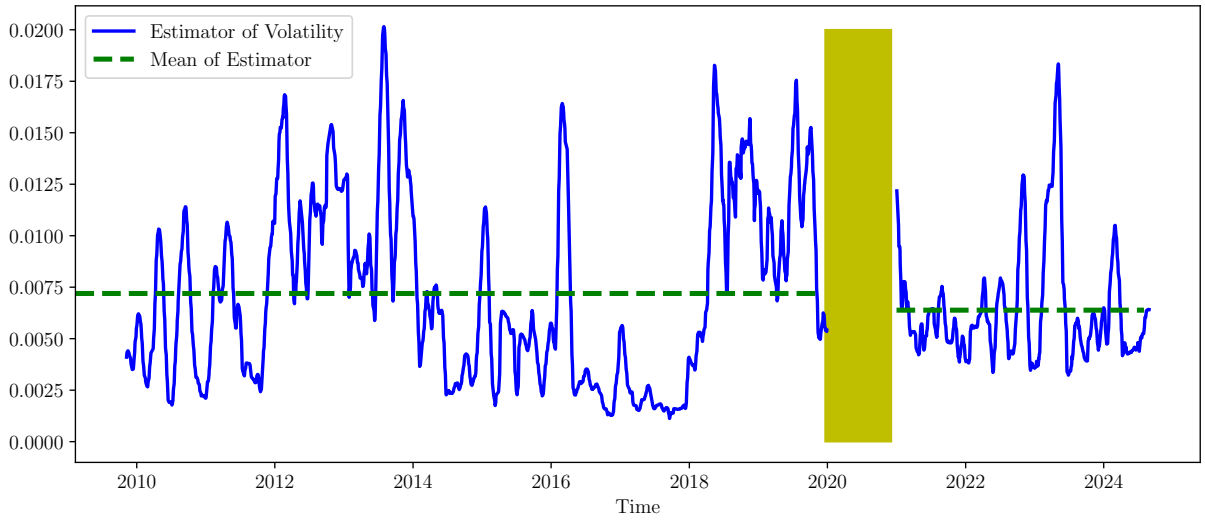


Figure 21: estimator of volatility of TEPIX

5.2 Systematic risk and price in steady state

In our model, σ represents the volatility of shocks that enter the economy, and $\sigma + \sigma^q$ shows systematic risk. A larger σ means the economy is more volatile. Our model can answer an important question: the effect of the economy's volatility on asset prices. Using the comparative static method, it is possible to measure the impact of the magnitude of σ on the price and volatility of assets.

Unlike models showing endogenous risk, in this model, the asset has the ability to damp part of the risk. With the increase of sigma, the volatility of the asset price also takes on a more negative value, which causes the volatility of the yield of the risky asset to be slightly adjusted and absorbs all the shocks to the economy. This critical feature of the asset makes the asset resistant to systematic shocks, and therefore, the higher the volatility of the shocks, the higher the value of the asset and the higher its price, which can be seen in Figure 22.

The last question we want to answer is why the steady-state capital price is still high. The

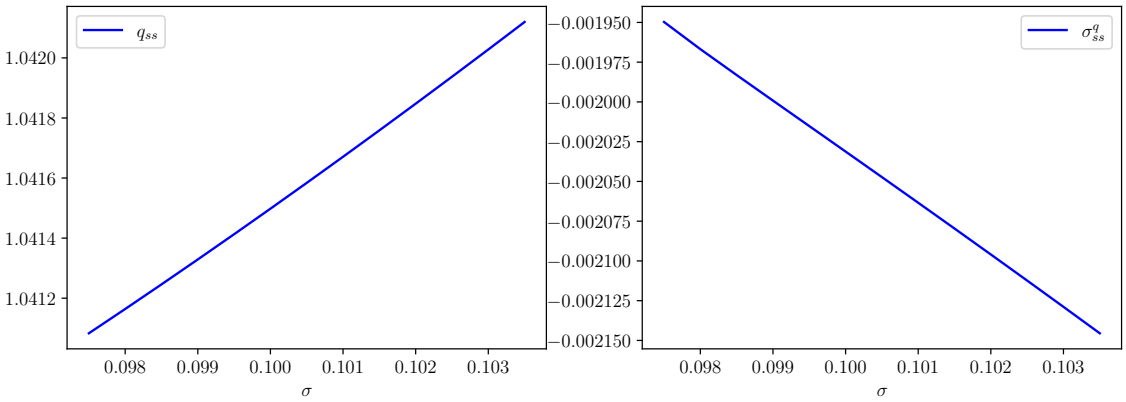


Figure 22: price and volatility of price in steady state

severity of sanctions in 2019 caused the systematic risk in Iran to increase sharply. A capital asset acts as a shield against systematic risk. But as Figure 23 shows, the simple households reduce their share in the stock market due to lower productivity and exit the asset market to some extent. This feature is consistent with data from 30 large companies on the Tehran Stock Exchange. After the high growth of the capital market, households gradually left it, but the price still remains at a high level, which shows that the other factor has kept the capital price high.

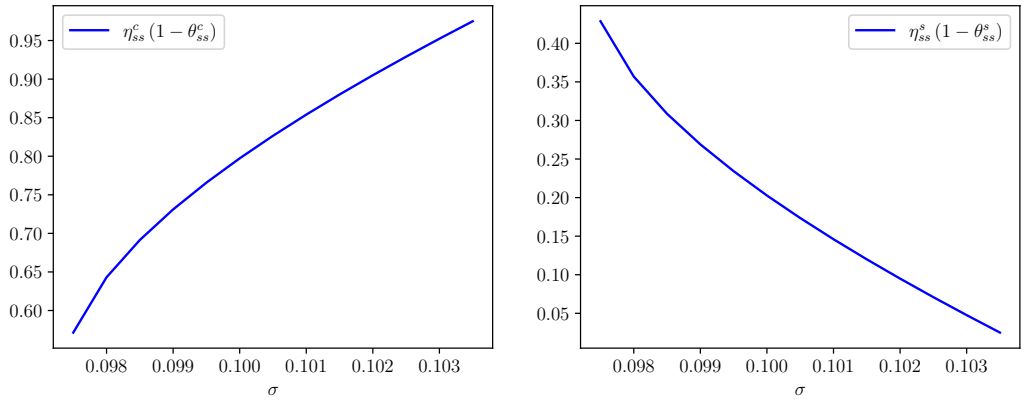


Figure 23: share in capital market in steady state

6 Conclusion

The participation rate in the capital market is significant in the asset pricing process. In this paper, with the help of a continuous-time stochastic general equilibrium model and by solving the agents' stochastic control problems, we showed that increasing the capital market participation rate has two different effects on the more productive and unproductive agents. Using this point, we simulated the justice policy that happened in Iran, and the simulation showed that if the participation rate in the capital market is higher than a specific limit, the change in the share of wealth will increase the price of assets. To put it more clearly, without any other changes, only which part of the economy has some wealth will cause a change in the price of the asset. Also, the heterogeneity of agents allows us to limit participation in the capital market in the model endogenously. This feature shows why, even though financial markets have deepened and diversified, the feature of limited participation in the capital market has remained stable.

However, this research provides many opportunities for future research. This model has several assumptions, and relaxing them opens the way for further investigation. For example, in this model, the intertemporal elasticity of substitution is assumed to be constant for all agents, but using empirical evidence, Vissing-Jorgensen (2002) shows that the EIS for shareholders is between 0.3 and 0.4 and for bondholders between 0.8 and 1. Entering this feature into the model may change some results. In addition, it should be noted that the continuous-time stochastic general equilibrium framework is a framework that faces technical difficulties in solving, and it is still not easy to work on this class of models, unlike DSGE models. However, the presented framework can be considered for examining emerging economies with different financial frictions.

Acknowledgment

Attending the Princeton University Macro-finance summer school effectively built solid capabilities, and we thank Markus K. Brunnermeier and his assistants for organizing it. Also, we are grateful to Giuseppe Bertola, Galo Nuno, Gregory Phelan, Mehdi Khorram, Mehdi Feizi, Zhouzhou Gu, Navid Akbaripour, Alireza Azampour, and Mojtaba Hayati for insightful comments and helpful suggestions.

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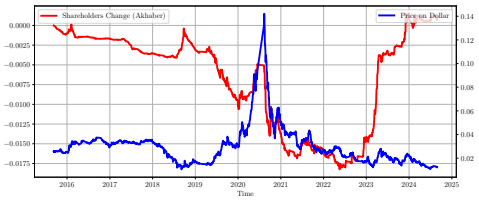
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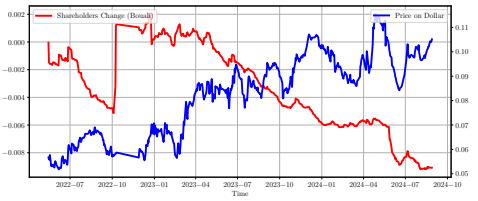
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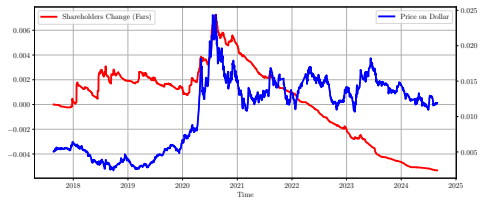
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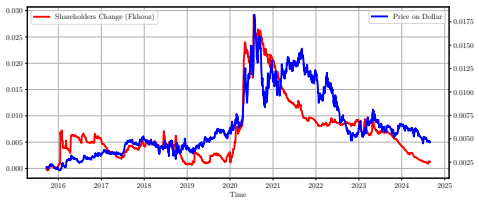
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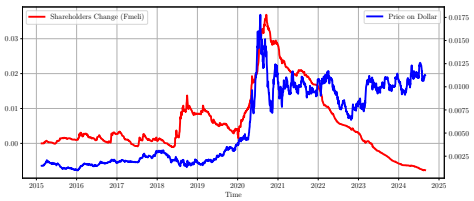
Appendix A Shareholders' Change and Asset Pricing

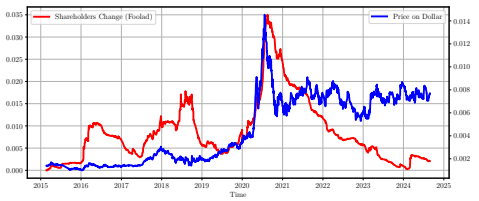


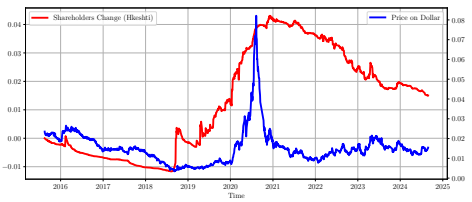


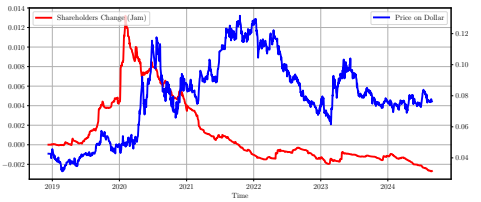


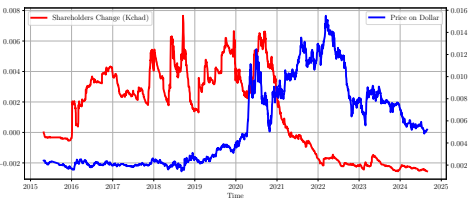


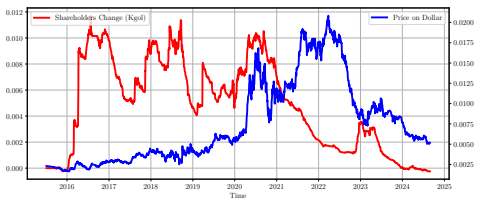


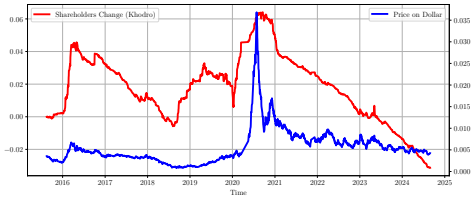


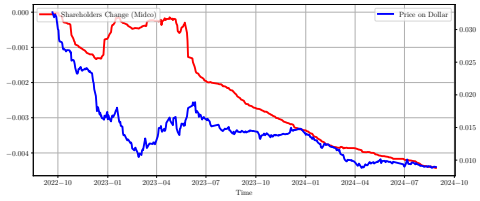


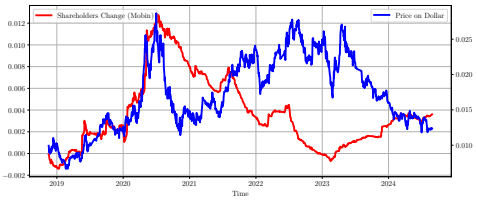


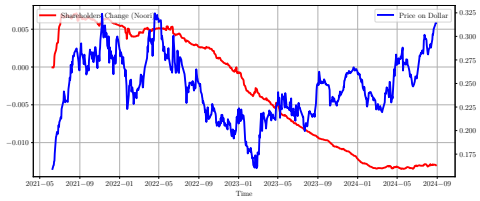


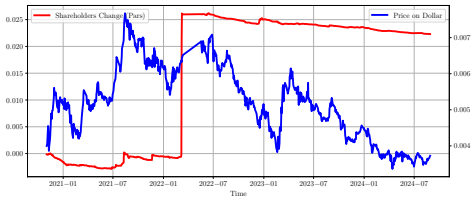


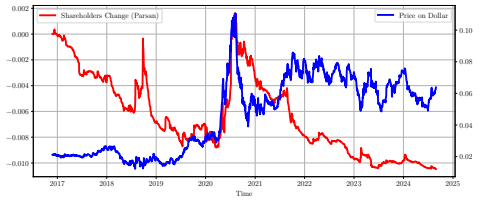

Shareholders Change (Khodro)
Price on Dollar
Time

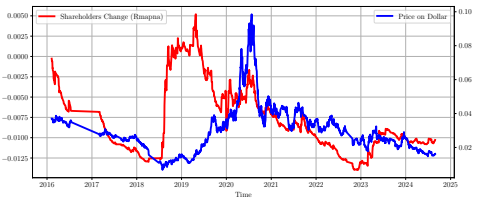

Shareholders Change (Midco)
Price on Dollar
Time

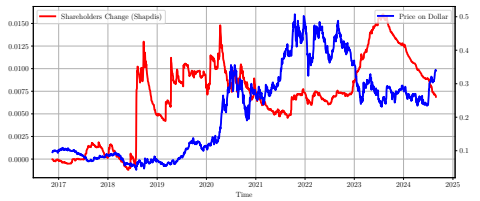

Shareholders Change (Mobin)
Price on Dollar
Time

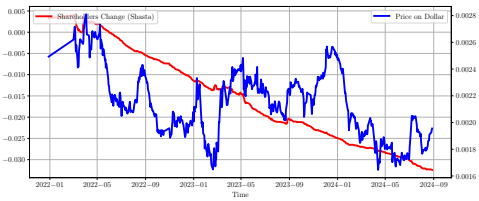

Shareholders Change (Noori)
Price on Dollar
Time

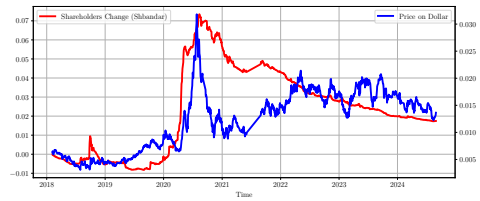

Shareholders Change (Para)
Price on Dollar
Time

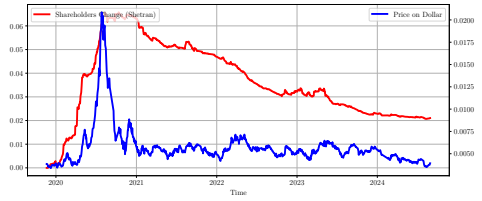

Shareholders Change (Parsan)
Price on Dollar
Time

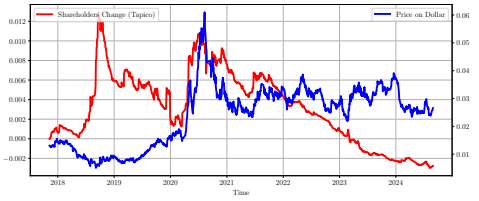

Shareholders Change (Rmapna)
Price on Dollar
Time

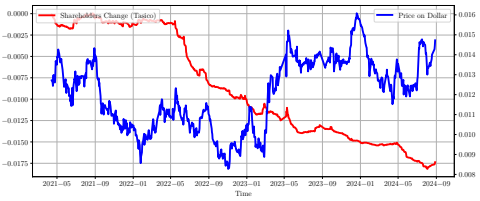

Shareholders Change (Shapdis)
Price on Dollar
Time

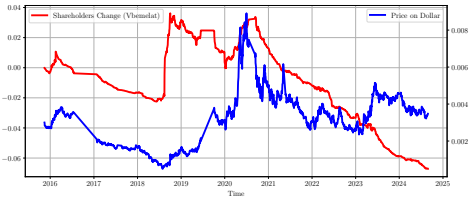

Shareholders Change (Shasta)
Price on Dollar
Time

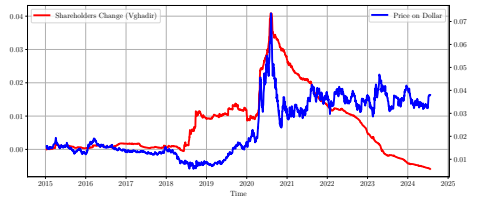

Shareholders Change (Shbandar)
Price on Dollar
Time


Shareholders Change (Shetran)
Price on Dollar
Time


Shareholders Change (Tapico)
Price on Dollar
Time

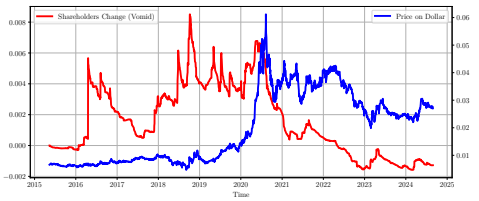

Shareholders Change (Tasico)
Price on Dollar
Time

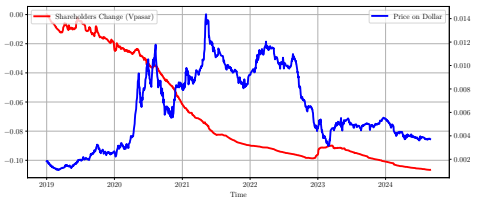

Shareholders Change (Vbemelat)
Price on Dollar
Time

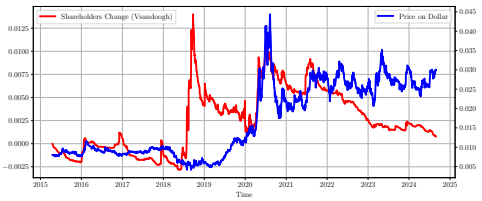

Shareholders Change (Vghadir)
Price on Dollar
Time


Shareholders Change (Vkavir)
Price on Dollar
Time


Shareholders Change (Vmasden)
Price on Dollar
Time


Shareholders Change (Vomid)
Price on Dollar
Time


Shareholders Change (Vpasar)
Price on Dollar
Time


Shareholders Change (Vsandoogh)
Price on Dollar
Time

Appendix B Approximate vs Numerical Solution

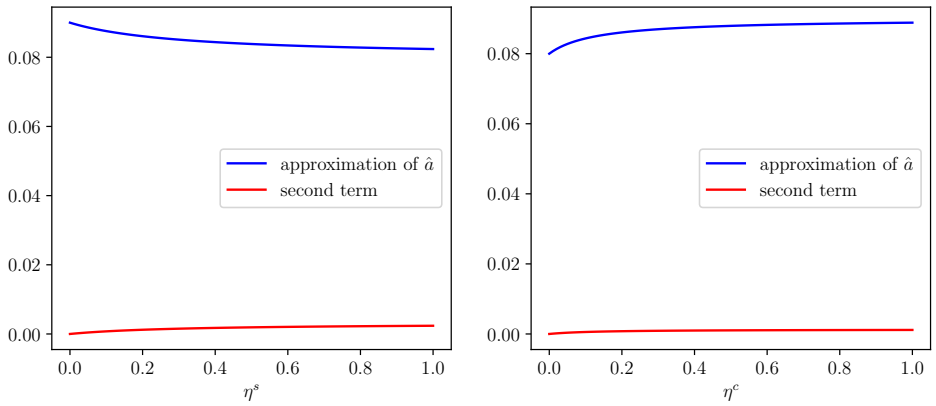


Figure 24: Approximation of $\hat{a}$ and second term

In Figure 24, it is clear that the difference between the approximate value of $\hat{a}$ and the second expression is very large, and $\hat{a}$'s approximation is a good approximation. But for higher accuracy, in Figure 25, the ratio between the second term and the approximate value of $\hat{a}$ is shown. The lowest ratio between these two is 34.7 in η^s change mode and 77.4 in η^c change mode. So it can be concluded that the value of the second term has a small effect on the value of the hot. For more confidence, instead of approximating, a numerical method can be

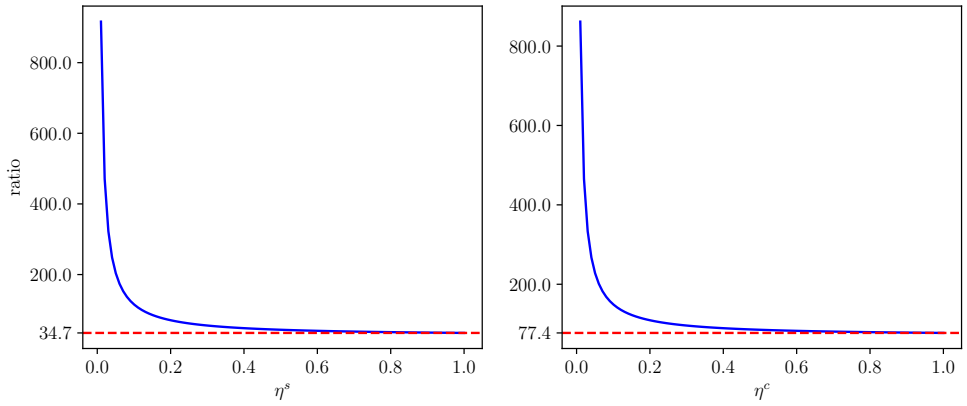


Figure 25: ratio between the second term and the approximate value of $\hat{a}$

used to make sure whether this approximation is a good approximation or not. By performing numerical solution iterations, the answers converge quickly in 3 iterations. As Figure 26 shows, the value of the price steady state difference and price fluctuation in each iteration compared to the value of the previous iteration quickly approaches zero.

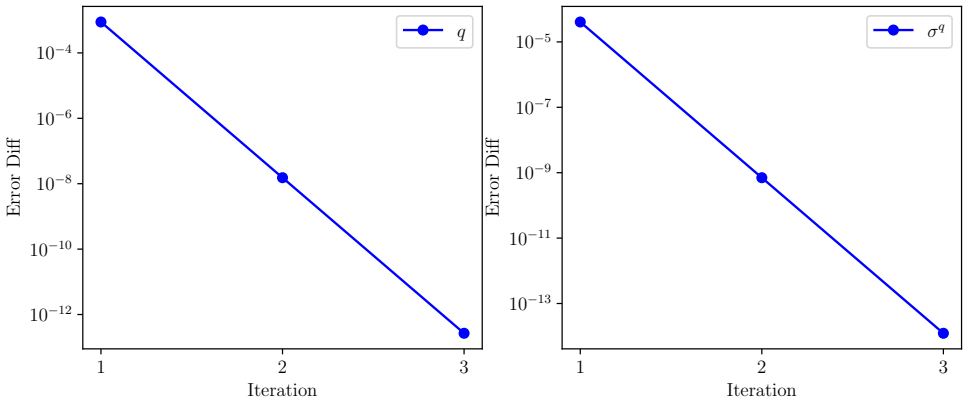


Figure 26: Difference between solution of iterations

By comparing the numerical and approximate response in Figure 27, it can be understood that the difference between the numerical and approximate response is very small with the change of η^s and η^c. This difference in price fluctuation is very small and in capital price, This difference is less than 0.005. In terms of ascending and descending form, both have the same form. Therefore, using a approximate solution instead of a numerical solution will not cause a problem.

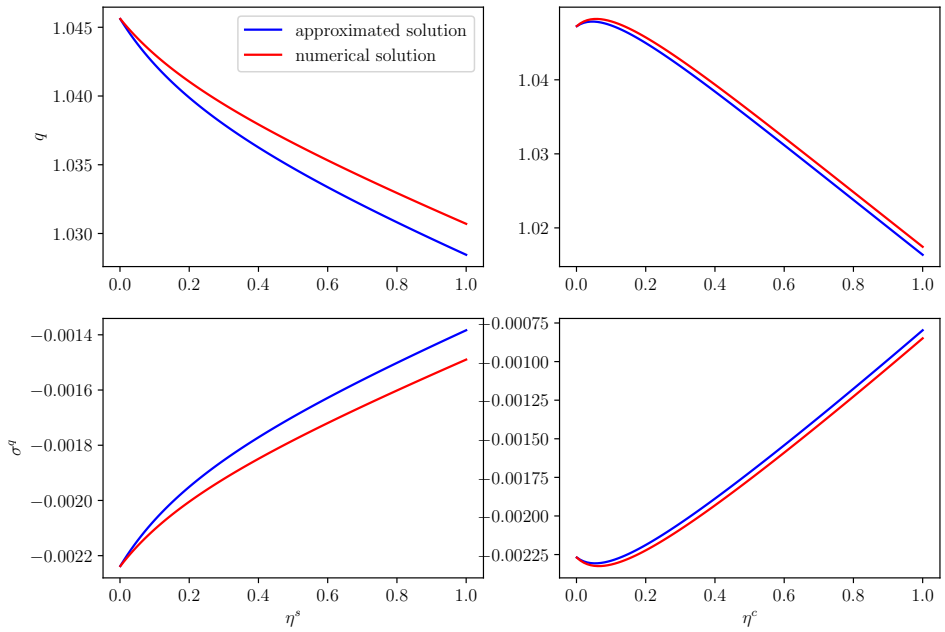


Figure 27: approximate vs numerical solution

Appendix C Proof of Volatility of Price Equation

We know that the volatility term of the dynamic of capital price is equal to the σ^q and the dynamic of capital price is:

$$\frac{\mathrm{d}q}{q} = \frac{\phi}{1+\phi\hat{\rho}}\left(\frac{\mathrm{d}\hat{a}}{q} - \mathrm{d}\hat{\rho}\right) + \frac{1}{2q}\frac{\partial^2 q}{\partial \hat{a}^2}(\mathrm{d}\hat{a})^2 + \frac{1}{2q}\frac{\partial^2 q}{\partial \hat{\rho}^2}(\mathrm{d}\hat{\rho})^2 + \frac{1}{q}\frac{\partial^2 q}{\partial \hat{a}\partial \hat{\rho}}(\mathrm{d}\hat{a})(\mathrm{d}\hat{\rho}).$$

So, from 7 and 8 the volatility of price is equal to:

$$\begin{aligned}\sigma^q = \frac{\phi S}{1+\phi\hat{\rho}}\Bigg[\frac{a^c-a^s}{(\eta^c+\eta^s)^2}\frac{\eta^c\eta^s}{q}\left(-1+\frac{1}{\eta^c+\eta^s}+\frac{a^c-a^s}{S^2q}\frac{\eta^s}{\eta^c+\eta^s}+1-\frac{1}{\eta^c+\eta^s}+\frac{a^c-a^s}{S^2q}\frac{\eta^c}{\eta^c+\eta^s}\right)\\ -\left(\rho^c-\rho^h\right)\left(-\eta^c+\frac{\eta^c}{\eta^c+\eta^s}+\frac{a^c-a^s}{S^2q}\frac{\eta^c\eta^s}{\eta^c+\eta^s}\right)\\ -\left(\rho^s-\rho^h\right)\left(-\eta^s+\frac{\eta^s}{\eta^c+\eta^s}+\frac{a^s-a^c}{S^2q}\frac{\eta^c\eta^s}{\eta^c+\eta^s}\right)\Bigg].\end{aligned}$$

$$\begin{aligned}\sigma^q = \frac{\phi S}{1+\phi\hat{\rho}}\Bigg[\left(\frac{a^c-a^s}{Sq}\right)^2\frac{\eta^c\eta^s}{(\eta^c+\eta^s)^2}-\left(\rho^c-\rho^h\right)\left(-\eta^c+\frac{\eta^c}{\eta^c+\eta^s}+\frac{a^c-a^s}{S^2q}\frac{\eta^c\eta^s}{\eta^c+\eta^s}\right)\\ -\left(\rho^s-\rho^h\right)\left(-\eta^s+\frac{\eta^s}{\eta^c+\eta^s}+\frac{a^s-a^c}{S^2q}\frac{\eta^c\eta^s}{\eta^c+\eta^s}\right)\Bigg].\end{aligned}$$

$$\begin{aligned}\sigma^q = \frac{\phi S}{1+\phi\hat{\rho}}\Bigg[\left(\frac{a^c-a^s}{Sq}\right)^2\frac{\eta^c\eta^s}{(\eta^c+\eta^s)^2}+\rho^c\eta^c+\rho^s\eta^s-\frac{\rho^c\eta^c+\rho^s\eta^s}{\eta^c+\eta^s}+\rho^h(1-\eta^c-\eta^s)\\ -\left(\rho^c-\rho^s\right)\left(\frac{a^c-a^s}{S^2q}\frac{\eta^c\eta^s}{\eta^c+\eta^s}\right)\Bigg].\end{aligned}$$

Appendix D Proof of Function of Capital Price

We can transform σ^q to with adding σ in two sides of the equation 9:

$$\begin{aligned}S = \frac{\phi S}{1+\phi\hat{\rho}}\Bigg[\left(\frac{a^c-a^s}{Sq}\right)^2\frac{\eta^c\eta^s}{(\eta^c+\eta^s)^2}+\rho^c\eta^c+\rho^s\eta^s-\frac{\rho^c\eta^c+\rho^s\eta^s}{\eta^c+\eta^s}+\rho^h(1-\eta^c-\eta^s)\\ -\left(\rho^c-\rho^s\right)\left(\frac{a^c-a^s}{S^2q}\frac{\eta^c\eta^s}{\eta^c+\eta^s}\right)\Bigg]+\sigma.\end{aligned}$$

then two sides of equation 9 is produced to Sq^2:

$$\begin{aligned}S^2q^2 = \frac{\phi}{1+\phi\hat{\rho}}\Bigg[(a^c-a^s)^2\frac{\eta^c\eta^s}{(\eta^c+\eta^s)^2}-\left(\rho^c-\rho^s\right)(a^c-a^s)\frac{\eta^c\eta^s}{\eta^c+\eta^s}q\\ +\left(\rho^c\eta^c+\rho^s\eta^s-\frac{\rho^c\eta^c+\rho^s\eta^s}{\eta^c+\eta^s}+\rho^h(1-\eta^c-\eta^s)\right)S^2q^2\Bigg]+\sigma Sq^2\end{aligned}$$

We name the long terms of equation for simple solving:

$$
\begin{aligned}
A &= \frac{\phi}{1+\phi\hat{\rho}}\left(a^c-a^s\right)^2\frac{\eta^c\eta^s}{\left(\eta^c+\eta^s\right)^2},\\
B &= \frac{\phi}{1+\phi\hat{\rho}}\left(\rho^c\eta^c+\rho^s\eta^s-\frac{\rho^c\eta^c+\rho^s\eta^s}{\eta^c+\eta^s}+\rho^h(1-\eta^c-\eta^s)\right),\\
C &= \frac{-\phi}{1+\phi\hat{\rho}}\left(\rho^c-\rho^s\right)\left(a^c-a^s\right)\frac{\eta^c\eta^s}{\eta^c+\eta^s}.
\end{aligned}
$$

Finally we arrive to a quadratic equation in order to q:

$$S^2q^2 = A + BS^2q^2 + Cq + \sigma Sq^2.$$

and the solution of the capital price:

$$q = \frac{-C \pm \sqrt{C^2-4A(BS^2-S^2+\sigma S)}}{2S(BS-S+\sigma)}.$$

Appendix E Proof of Wealth Structure Effect on asset pricing

We can derive partial derivative of $q\left(\eta^c,\eta^s\right)$ on η^s:

$$\frac{\partial q}{\partial \eta^s} = \frac{\phi\frac{\partial\hat{a}}{\partial\eta^s}\left(1+\phi\hat{\rho}\right)-\phi\frac{\partial\hat{\rho}}{\partial\eta^s}\left(1+\phi\hat{a}\right)}{\left(1+\phi\hat{\rho}\right)^2} = \frac{\phi}{1+\phi\hat{\rho}}\left[\frac{\partial\hat{a}}{\partial\eta^s}-\frac{\partial\hat{\rho}}{\partial\eta^s}q\right].$$

Also derivatives of $\hat{a}$ and $\hat{\rho}$ in order to η^s are equal to:

$$
\begin{aligned}
\frac{\partial\hat{a}}{\partial\eta^s} &= \frac{a^s\left(\eta^c+\eta^s\right)-\left(\eta^c a^c+\eta^s a^s\right)}{\left(\eta^c+\eta^s\right)^2} = \frac{\eta^c\left(a^s-a^c\right)}{\left(\eta^c+\eta^s\right)^2} < 0,\\
\frac{\partial\hat{\rho}}{\partial\eta^s} &= \rho^s-\rho^h > 0.
\end{aligned}
$$

The term $\frac{\phi}{1+\phi\hat{\rho}}$ is positive, So only second term determines the sign of derivative. Therefore the derivative is negative.

But the derivative of $q\left(\eta^c,\eta^s\right)$ on η^c is different:

$$\frac{\partial q}{\partial \eta^c} = \frac{\phi\frac{\partial\hat{a}}{\partial\eta^c}\left(1+\phi\hat{\rho}\right)-\phi\frac{\partial\hat{\rho}}{\partial\eta^c}\left(1+\phi\hat{a}\right)}{\left(1+\phi\hat{\rho}\right)^2} = \frac{\phi}{1+\phi\hat{\rho}}\left[\frac{\partial\hat{a}}{\partial\eta^c}-\frac{\partial\hat{\rho}}{\partial\eta^c}q\right].$$

The derivatives of $\hat{a}$ and $\hat{\rho}$ are:

$$\frac{\partial \hat{a}}{\partial \eta^c} = \frac{a^c(\eta^c+\eta^s)-(\eta^c a^c+\eta^s a^s)}{(\eta^c+\eta^s)^2} = \frac{\eta^s(a^c-a^s)}{(\eta^c+\eta^s)^2} > 0,$$
$$\frac{\partial \hat{\rho}}{\partial \eta^s} = \rho^c - \rho^h > 0.$$

So we have two terms that one is positive, and the other is negative, and the determinant of the sign of q's derivative is not obvious:

$$\frac{\partial \hat{a}}{\partial \eta^c} - \frac{\partial \hat{\rho}}{\partial \eta^c} q = \frac{\eta^s(a^c-a^s)}{(\eta^c+\eta^s)^2} - \left(\rho^c-\rho^h\right) q < 0,$$
$$\frac{\eta^s}{(\eta^c+\eta^s)^2} < \frac{\rho^c-\rho^h}{a^c-a^s} q.$$

We define previously $\eta_q^c(\eta^s)$ as a solution of

$$\frac{\eta^s}{\left(\eta_q^c(\eta^s)+\eta^s\right)^2} = \frac{\rho^c-\rho^h}{a^c-a^s} q\left(\eta_q^c(\eta^s), \eta^s\right).$$

So in region $\eta^c < \eta_q^c(\eta^s)$, the price of asset is increasing and after that the price stagnates.

Appendix F Proof of Wealth Transfer Effect

We define Δa and $\Delta\rho$ in the same way:

$$\Delta a \equiv a^c - a^s, \qquad \Delta\rho \equiv \rho^c - \rho^s.$$

The assumption of the theorem is that the sum of the shares of the simple household and the corporate person remains constant:

$$\eta^c + \eta^s = b.$$

In order for q to remain stable during trading, the following condition must be met:

$$\frac{\partial q}{\partial \eta^s} - \frac{\partial q}{\partial \eta^c} = 0 \Rightarrow \frac{\partial q}{\partial \eta^s} = \frac{\partial q}{\partial \eta^c}.$$

With a little calculation, we will have:

$$\frac{\phi}{1+\phi\hat{\rho}}\left[\frac{\eta^s(a^c-a^s)}{(\eta^c+\eta^s)^2}-\left(\rho^c-\rho^h\right)q\right]=\frac{\phi}{1+\phi\hat{\rho}}\left[\frac{\eta^c(a^s-a^c)}{(\eta^c+\eta^s)^2}-\left(\rho^s-\rho^h\right)q\right],$$

$$\frac{(a^s-a^c)(\eta^c+\eta^s)}{(\eta^c+\eta^s)^2}=(\rho^c-\rho^s)q,$$

$$q=\frac{a^c-a^s}{(\rho^c-\rho^s)(\eta^c+\eta^s)}=\frac{a^c-a^s}{b(\rho^c-\rho^s)}=\frac{\Delta a}{\Delta\rho b}.$$

We use the definition of q:

$$q=\frac{1+\phi\hat{a}}{1+\phi\hat{\rho}}=\frac{1+\phi\frac{a^sb+\Delta a\eta^c}{b}}{1+\phi\left((1-b)\rho^h+\rho^sb+\Delta\rho\eta^c\right)}$$

$$=\frac{\frac{b+\phi(a^sb+\Delta a\eta^c)}{b}}{1+\phi\left((1-b)\rho^h+\rho^sb+\Delta\rho\eta^c\right)}=\frac{b+\phi\left(a^sb+\Delta a\eta^c\right)}{b+\phi\left((1-b)b\rho^h+\rho^sb^2+\Delta\rho\eta^cb\right)}.$$

By combining the two equations obtained for q, the result is completed:

$$q=\frac{\Delta a}{\Delta\rho b}=\frac{b+\phi\left(a^sb+\Delta a\eta^c\right)}{b+\phi\left((1-b)b\rho^h+\rho^sb^2+\Delta\rho\eta^cb\right)},$$

$$\Delta\rho b^2+\phi a^s\Delta\rho b^2+\phi\Delta a\Delta\rho\eta^cb=\Delta ab+\phi\rho^h\Delta ab-\phi\rho^h\Delta ab^2+\phi\rho^s\Delta ab^2+\rho\Delta a\Delta\rho\eta^cb.$$

The solutions of the following quadratic equation give the constant value:

$$\left[\Delta\rho+\phi a^s\Delta\rho+\phi\rho^h\Delta a-\phi\rho^s\Delta a\right]b^2-\left[\Delta a+\phi\rho^h\Delta a\right]b=0.$$

But zero answer is not acceptable, so only one solution remains:

$$b^*=\frac{\Delta a\left(1+\phi\rho^h\right)}{\Delta\rho\left(1+\phi a^s\right)-\phi\Delta a\left(\rho^s-\rho^h\right)}.$$