# Dynamic Search in a Non-Stationary Search Environment: An Application to the Beijing Housing Market*

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**Abstract**

This paper studies how dynamics in the search environment affect consumers' search and purchase decisions. We develop a dynamic search model with a non-stationary search environment and estimate the model using data on the complete search record of consumers in the Beijing housing market. Our model yields reasonable estimates of search costs (in contrast to a static search model). We find that the dynamics in the search environment have significant effects on consumers' decisions and welfare. We also find that policies aimed at increasing housing supply benefit consumers mainly by influencing the dynamics of the search environment, which incentivizes consumers to search longer, visit more properties, and ultimately find properties that generate higher utility for them.

**JEL:** D8, L8, R3
**Keywords:** consumer search, non-stationary search environment, Beijing housing market

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# 1 Introduction

In many markets, consumers need to search before making a purchase decision. The extent to which a consumer searches thus determines the choice set for the purchase. Consequently, search frictions hinder search and limit the choice set. Examples abound in various sectors: the automobile market, the travel accommodation market, the real estate market, the mortgage market, and many more. Furthermore, in many scenarios, the search environment changes over time as prices change, new products enter the market, or existing products exit the market. For example, consumers searching for a car during the COVID-19 pandemic faced rapid price increases and volatile dealer inventories. Homebuyers in 2022–2023 saw significant increases in mortgage rates. Consumers planning vacations during peak seasons may experience rapid changes in the availability of accommodations and travel services. The real estate market is also dynamic, with property availability and prices changing over time.

In this paper, we study how the dynamics of the search environment affect consumers' search and purchase decisions. Consider a scenario where product availability changes over time due to product entry and exit. On the one hand, if a consumer searches longer, she can learn about more products, expand her choice set and possibly find a more suitable product. On the other hand, if she searches longer, her most preferred product in her previously searched set may become unavailable. In addition, if the prices of new products increase over time, then expanding her choice set by searching more may not provide much benefit, resulting in less incentive to search. Overall, the dynamics of the search environment affect the consumer's search and purchase decisions.

To study such effects, we develop a dynamic search model with a non-stationary search environment with two features. First, consumers make both search and purchase decisions, and both decisions are dynamic. In our model, consumers observe a set of product characteristics before searching, learn about product match values through searching, and choose a product among the products they have searched for and are available at the time of purchase. The purchase decision is dynamic because consumers can choose to buy immediately or wait for another period. The search decision is also dynamic because more searches in a period lead to a larger choice set and a higher value of buying now, while leading to a smaller set of products to search for in future periods and a lower value of waiting. Second, the search environment is non-stationary. In particular, existing products may exit the market, new products may enter the market, and prices may change over time. In both search and purchase decisions, consumers take into account the changing market environment.

To the best of our knowledge, this paper is the first to incorporate a non-stationary environment into a dynamic search model. Although our model is developed in the context of housing search, it can be used to study other settings where consumers make dynamic search and purchase decisions in a changing environment. In a changing environment, consumer search is affected not only by search costs but also by changes in the search environment. A search model ignoring the latter leads to biased estimates of search costs. For example, in scenarios where the search environment deteriorates over time, both search costs and the change in the environment contribute to limiting consumer search. Ignoring the changes in the environment would lead to an overestimation of search costs. Similarly, using models that ignore the dynamics of environment to conduct policy evaluations can also give incorrect results, especially if the policy of interest may change the dynamics of the environment.

Our empirical setting is the Beijing housing market between August 1, 2015 and July 31, 2016. This setting is ideal for studying how the dynamics of the search environment affect consumers' search and purchase for two reasons. First, the sample period is characterized by rapid price increase, with an annual increase of more than 30%. Second, the dataset we use for our analysis is novel in that it contains complete and detailed information on consumers' search activities. Our data come from the largest real estate agency in Beijing, Lianjia. For each consumer in our sample, the dataset provides a complete record of the consumer's search and purchase behavior. We observe when the consumer starts searching, when she stops searching, how many and which properties she visits in each period, and which property she eventually purchases.

Estimating our dynamic search and purchase model is challenging for two reasons. First, our model allows for product characteristics known to economic agents but unobservable to researchers. This model feature is important because it is often difficult for researchers to obtain the data on all product characteristics observable to consumers and relevant for their decisions (see, for example, Berry, Levinsohn and Pakes (1995) and many papers following the literature). At the same time, this feature implies an endogeneity problem: the price of a product is likely to be correlated with unobservable product characteristics. Therefore, we need to address this endogeneity issue in estimating our dynamic model. Second, the vector of state variables has a very high dimension because it includes characteristics of both searched and unsearched products as well as match values of searched products.

We address these two challenges with a combination of two ideas. First, before estimating the dynamic model, we back out the mean utility of each property by matching

the observed share of visits that the property receives.[1] This step requires an extension of the standard invertibility result because, in our setting, consumers typically visit *a set of* properties in each period. To this end, we derive the probability that a set is sampled and extend the contraction mapping result in Berry, Levinsohn and Pakes (1995). This step allows us to estimate parameters embedded in the mean utility before estimating the dynamic model, which means that we can include a large number of fixed effects in our utility function specification without significantly increasing the computational burden and can further address the price endogeneity issue using an instrumental variable approach. Moreover, this step allows us to reduce the number of state variables by replacing a vector of characteristics with the scalar mean utility for each property in estimating the dynamic model. Second, following many papers in the literature that approximate a value function of a large number of state variables with one of lower-dimensional statistics, we further reduce the number of state variables by assuming that consumers track several highest mean utilities, the average of the mean utilities of the other properties, and the number of properties, rather than tracking the mean utilities of all properties.

Our estimation yields intuitive results. According to our estimates, consumers prefer larger and newer properties with more living rooms and bedrooms, located on higher floors and close to subway stations. The estimated standard deviation of match values is equivalent to a value of CN¥100,593, which is about 3% of the average list price and more than 150% of the per capita annual disposal income in Beijing in 2016, indicating a significant benefit of searching to learn match values. We also find that on average, consumers incur a search cost of CN¥1,184, which amounts to an average search cost of CN¥176 per visit.

In contrast, a static search model estimated using the same dataset yields an unreasonably large search cost, highlighting the importance of considering dynamics in our setting. A static search model gives an estimated search cost of about CN¥46,000, or $7,000 per visit, which is more than two magnitudes higher than our estimate. This is because a static search model ignores the dynamics in the search environment and thus has to rely on very large search costs to explain the observed number of searches.

Based on the estimated dynamic search model, we quantify how the dynamics of the environment affect consumers' search and purchase behavior and consumer welfare through counterfactual simulations. We consider changes in both prices and product availability (i.e., entry of new listings and exit of existing listings). We find that increasing the arrival

[1] Similar sequential estimation procedures are used to estimate multi-stage static models (e.g., Eizenberg, 2014; Fan and Yang, 2020, 2024) and dynamic models (e.g., Hendel and Nevo, 2006; Chatterjee, Fan and Mohapatra, 2024; Bodéré, 2023; Elliott, 2024).

rate of new listings, reducing the price increase speed for new listings, or reducing the exit rate of existing listings leads to longer searching, more total visits before purchase, and purchases of properties that generate higher utility. Considering the trade-off between higher search and waiting costs (resulting from more property visits and longer searching) and higher utility from finding a more desirable property (again, resulting from more searches), the average net gain per consumer is CN¥124,316 when consumers face half of the price increase speed, CN¥109,743 when the new listing arrival rate is doubled, and CN¥18,094 when the exit rate is halved.[2] In comparison, the per-capita annual disposable income in Beijing in 2016 is CN¥57,275.[3]

To quantify the relative importance of traditional search frictions such as search costs versus dynamics in the search environment, we conduct a counterfactual simulation in which we halve the search cost per visit. Unsurprisingly, consumers search longer, visit more properties before buying and buy properties with higher utility. The average per-consumer net gain is CN¥117,511, which is of a similar magnitude to per-consumer net gains from varying the dynamics of the search environment studied above (i.e., reducing the speed of price increase and increasing the new listing arrival rate).

We also study the impact of housing policies designed to boost housing supply and find that they can benefit consumers significantly, primarily through altering the dynamics of the search environment, which in turn incentivizes consumers to search longer, view more properties, and ultimately find properties that generate greater utility. Housing policies such as taxing vacant properties can encourage new listings and slow down price increases. We find that with higher new listing arrival rates and lower price trends, consumers search longer and visit more properties, leading to higher search costs and waiting costs but better purchase outcomes. The net effect is positive. Specifically, doubling the new listing arrival rate and halving the price increase results in an average net gain of CN¥249,200, or about 7% of the average transaction price. A decomposition shows that about 75% of the consumer gain comes from searching longer and visiting more properties, and only 25% is mechanically due to the slower price increase.

This paper contributes to the empirical consumer search literature. Examples of this literature include Honka (2014) and Murry and Zhou (2020) for simultaneous search models, Hodgson and Lewis (2023) and Moraga-Gonzalez, Sándor and Wildenbeest (2023) for sequential search models, and Santos, Hortaçsu and Wildenbeest (2012) for testing

[2]In the counterfactual scenario where the price increase is halved, consumer welfare increases mechanically due to lower prices. To remove such a mechanical effect and to isolate the effect of inducing more searches, we use the actual prices (instead of the counterfactual ones) to compute net gains.

[3]See "the 2017 Beijing Statistical Yearbook, 9-14 Basic Data on Urban Households" (https://nj.tjj.beijing.gov.cn/nj/main/2017-tjnj/zk/e/indexeh.htm).

simultaneous versus sequential models. Other examples include Kim, Albuquerque and Bronnenberg (2010), Allen, Clark and Houde (2019) and Brown and Jeon (2024). While the existing papers consider stable search environments where both the set of products and their prices are fixed over time, our paper studies consumer search in changing search environments. Here, both product availability and prices can vary over time. Our results indicate that these changes have significant effects on consumers' search and purchase behavior as well as consumer welfare.

This paper also contributes to the literature on empirical search in general. A large branch of this literature focuses on job search. See Eckstein and Van den Berg (2007) and French and Taber (2011) for surveys of papers in the literature. This literature also mainly considers stationary search environment. An exception is Arcidiacono, Gyetvai, Maurel and Jardim (2022), who estimate a continuous-time non-stationary search model where the offer arrival rate, the wage offer distribution, and the flow payoff of unemployment vary with the duration of the unemployment spell. Moreover, again similar to the consumer search literature, the job search literature also assumes no unobservable job characteristics and thus no wage endogeneity. In contrast, we allow for and address price endogeneity.

The remainder of the paper is organized as follows: Section 2 describes the setting and the data. Section 3 develops the dynamic search and purchase model. Section 4 explains the estimation procedure. Section 5 presents the estimation results. Section 6 compares our estimation results with those of a static search model. Section 7 quantifies the effects of changes in the search environment and the effects of search costs while Section 8 the effects of housing supply policies. A final Section 9 concludes the paper.

# 2 Data

## 2.1 Data Description

Our data come from Lianjia, the largest brokerage company in the second-hand residential housing market in Beijing, and provide information on both properties and consumers.[4] The dataset provides information on 225,608 properties listed for sale on Lianjia between August 1, 2015 and July 31, 2016 and located in the six urban districts of Bei-

[4]The market share of Lianjian, as measured by the share of total second-hand residential property transactions in Beijing, was 54% in the first half of 2016. In contrast, the market shares of the second, third and fourth real estate companies were 13%, 5% and 4%, respectively (https://m.sohu.com/n/458280155/).

Using publicly available data on Lianjia.com, Habib, Peng, Wang and Wang (2023) and Peng (2023) study how macroeconomic conditions affect the Chinese housing market.

jing.[5] These six urban districts include two core districts (Dongcheng and Xicheng, which occupy the area inside the old walled city) and four surrounding urban districts (Haidian, Chaoyang, Fengtai, and Shijingshan). The dataset also includes the complete records of home visits and purchase choices of 455,774 consumers who registered on Lianjia and were actively searching during our sample period.

To construct our sample, we exclude properties with a list price higher than CN¥10 million or less than CN¥1 million, and properties with a size of less than 25 square meters. These properties are likely to belong to a separate market. Accordingly, we drop consumers who visited these properties. We also drop consumers who searched across multiple districts. The districts in Beijing are quite large. For example, Chaoyang District covers 470.8 square kilometers (181.8 square miles). In contrast, Manhattan Island is 59 square kilometers (22.7 square miles). The vast majority (more than 93%) of consumers searched within one district. At the same time, including the small share of consumers searched across multiple districts increases the computational burden exponentially.[6] Finally, we drop 4% of the properties in the data that were never visited by any consumer in our sample. In the end, our sample consists of 202,845 properties and 414,166 consumers.

For each property in the sample, we observe its list price, address, year of construction, floor level, property size, number of living rooms, and number of bedrooms. We define 221 exclusive segments based on the neighborhood and the list price range and assign each property to a segment.[7] In addition to these property characteristics, we also observe the transaction date and price if a property was sold before the end of the sample period.

For each consumer in the sample, we have a complete record of all her property visits until she purchases a property or until the end of the sample period. The search record is complete because all consumers in the sample sign a sole agency agreement with Lianjia.[8] For each property visit, we observe the date of the visit and the identity of the property. In addition, if a consumer purchased a property during the sample period, we also observe which property she purchased and when she purchased it.

[5] Beijing Municipality consists of six urban districts, eight suburban districts and two rural districts.

[6] As will be explained in Section 4, we carry out the estimation in two steps. In the first step, we invert the mean utilities district by district. If we were to include consumers who searched in multiple districts, we would have to do the inversion for all districts at once, which increases the memory requirement exponentially.

[7] Lianjia defines the neighborhood of a property based on its location. Neighborhoods differ in terms of transportation, amenities, etc. We consider four price ranges: lower than 3 million, between 3 and 4.5 million, between 4.5 and 6 million, and higher than 6 million.

[8] Moreover, there are no open houses in China. Consumers must work with a realtor agent to visit properties.

## 2.2 Summary Statistics

In this section, we present summary statistics on properties, show how the search environment changes over time, and report data patterns regarding consumer search and purchase behavior.

### 2.2.1 Properties

Table 1 presents the summary statistics of the properties in the sample. The average list price is CN¥4 million ($604,000). In the Chinese housing market, the most salient price is the price per square meter, which averages CN¥49,302 ($18,029) per square meter for the list price. The average property is 18 years old, with a size of 84 square meters, 2 bedrooms and 1 living room. About 80% of the properties are located within 1 km of a subway station, a criterion we use to define the indicator variable of whether a property is close to a subway station. More than 30% of them are located on the 10th floor or above.[9]

Of the 202,845 properties in our sample, 85,696 (42%) were sold during our sample period. On average, a property stays on the market for approximately 8 weeks. The average transaction price is CN¥3.7 million ($555,000) with a unit transaction price of CN¥48,367 ($17,501) per square meter.

Table 1: Summary Statistics of Properties

| | Mean | SD |
|---|---|---|
| List price (million CN¥) | 4.024 | 1.962 |
| List price per $m^2$ (CN¥) | 49,302 | 18,029 |
| Property size ($m^2$) | 83.707 | 35.903 |
| Property age (year) | 18.151 | 8.994 |
| Bedrooms | 1.997 | 0.777 |
| Living rooms | 1.142 | 0.547 |
| Indicator of above 10th floor | 0.316 | 0.465 |
| Close to subway stations | 0.797 | 0.402 |
| Indicator of being sold | 0.422 | 0.494 |
| Weeks on market | 8.193 | 7.402 |
| Transaction price (million CN¥) | 3.702 | 1.760 |
| Transaction price per $m^2$ (CN¥) | 48,367 | 17,501 |

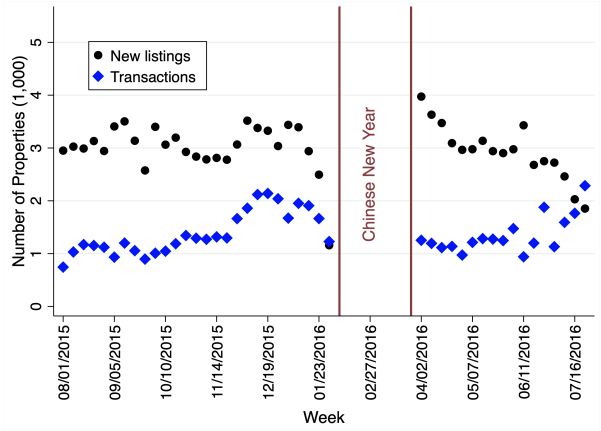
[9]Residential homes are typically apartments in high-rise buildings in Beijing.

### 2.2.2 Dynamic Search Environment

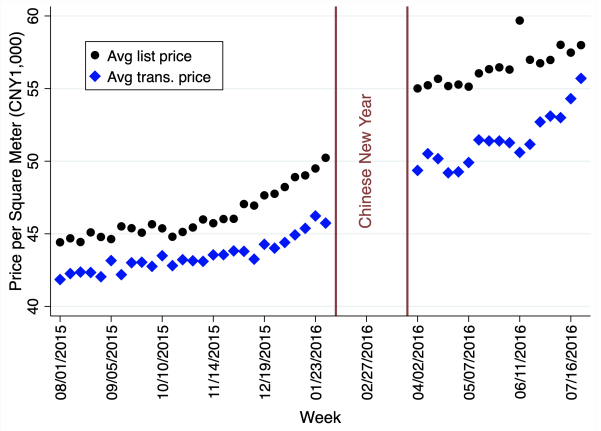
Both the number of new listings and the number of transactions are relatively stable over the sample period, as shown in Figure 1(a), which plots these two numbers by week. We omit the eight weeks around the Chinese New Year in early 2016. Many economic activities are put on hold every year during the Chinese New Year, when many Chinese return to their hometowns and resume economic activities only afterward. For example, 2.9 billion passenger trips were made during the 2016 holiday (Zhou (2016)).[10]

Figure 1: New Listings, Transactions, and Prices by Week

(a) New Listings and Transactions




(b) List Prices and Transaction Prices




However, both list and transaction prices increased rapidly during the sample period, as shown in Figure 1(b). For each week in our sample, we compute the average list price across all new listings in that week and the average transaction price across all transacted properties in that week. Since the most salient price in the Chinese housing market is the unit price per square meter, we consider prices in CN¥ per square meter in calculating these averages, and plot them in Figure 1(b). The figure shows a clear upward trend in both list and transaction prices. Specifically, the average list price increased from CN¥44,417 to CN¥57,993 per square meter, representing an annual price increase of approximately 30% and an average weekly increase of CN¥261 per square meter during the sample year. For an average-sized property (of 84 square meters), this weekly increase is equivalent to CN¥21,924, about 40% of the annual per-capita disposal income in Beijing in 2016. The average transaction price closely tracked the evolution of the average list price with a stable gap of around CN¥3,000 per square meter.

[10]In Supplemental Appendix SA, we plot the number of new listings and transactions during the eight weeks around the Chinese New Year in early 2016. It shows that during these eight weeks, the number of new listings and transactions dropped to zero and then rose rapidly to about double the pre-holiday level.

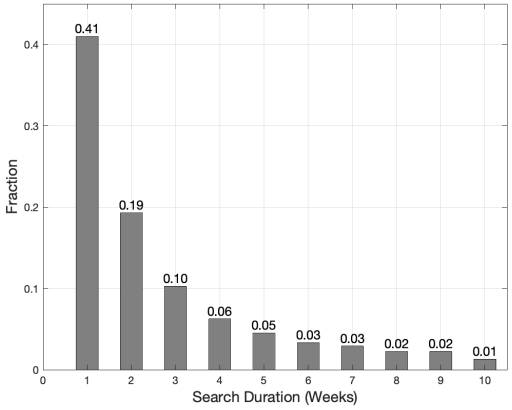
### 2.2.3 Consumer Search and Purchase

In our sample, 39,500 consumers started a search and made a purchase during the sample period. Among them, 26,543 consumers either ended their search before the Chinese New Year holiday or started their search after the holiday. For these consumers, we observe their complete search record from the beginning to the end of their search.

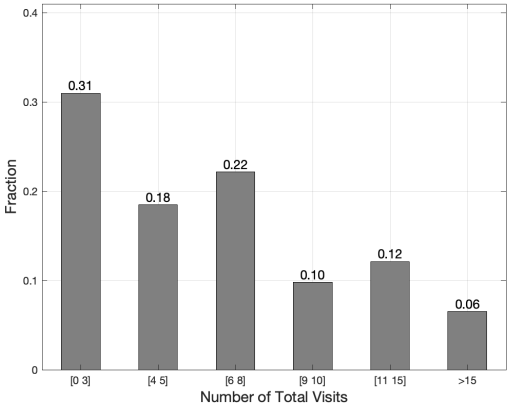
Specifically, we observe these consumers' search duration and total visits. Figure 2(a) shows that 41% of them searched for one week, 20% two weeks, 10% three weeks and the remaining 29% searched for more than three weeks. Figure 2(b) shows that 51% of them visited five or fewer properties, 32% of them visited more than five but no more than ten properties, and the remaining 18% visited more than ten properties before making their purchase. The average numbers of search weeks and visits are 3.5 weeks and 6.6 visits. In comparison, according to the National Association of Realtor's survey of 2,372 home buyers from 1987–2007, these numbers are 8.2 weeks and 10 visits in the US (Genesove and Han (2012)).

Figure 2: Search Duration and Total Visits

(a) Search Duration



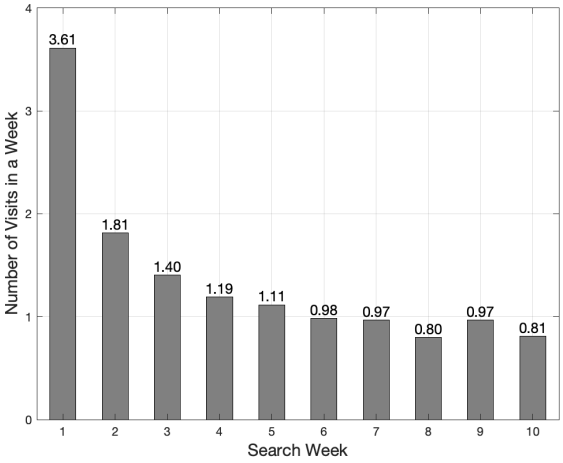

(b) Total Visits




A unique and important feature of our data is that we observe not only the total number of visits, but also the dynamic characteristics of consumers' searches (i.e., search duration and search intensity in each week). Figure 3 shows the average number of visits by search week. "Search week" refers to the week since a consumer's first property visit. For each search week $t$, we compute the number of visits a consumer makes in that search week averaged over consumers who search for $t$ or more weeks. From the figure, we can see that search intensity decreases over search weeks. The average search intensity starts with 3.61 visits in the first search week, quickly drops to 1.81 visits in the second search week,

and continues to drop to less than 1 visit after five weeks of searching.

Figure 3: Number of Visits by Search Week




In addition to a complete search record, we also observe when and which property a consumer bought. In our sample, some consumers bought a property that they searched for in the current period, while others buy a property that they searched for in a previous period. We call such an action a recall. The share of recalls in our sample is 17.6%.

We end this section with a description of consumers' search markets. For each consumer, we define her search market as the union of the segments in which she searched. On average, a consumer searches in two segments and her search market consists of 654 properties in a given week. On average, a consumer's search market has 29 new listings per week and experiences an average weekly exit rate of 12%.

# 3 Dynamic Search Model

In this section, we develop a model to describe consumers' search and purchase decisions. Each consumer $i$ arrives exogenously on the platform. To simplify the notation, we use $t$ (without the subscript $i$) to denote the week since a consumer's arrival on Lianjia, which we refer to as the "search week." We use $j$ to denote a property and $m(j)$ to denote the segment that property $j$ belongs to.

In each period $t$, consumer $i$ first decides on the number of properties to visit in the current period and then, after visiting these properties, decides whether to buy one (thus ending the search) and, if so, which one to buy. In what follows, we first describe the primitives of the model and then explain these decisions backwards.

## 3.1 Primitives

### 3.1.1 Utility

The utility that consumer $i$ gets from property $j$ is as follows:

$$u_{ijt} = \boldsymbol{x}_j\boldsymbol{\beta} + \alpha p_{jt} + \xi_j + v_{ij}, \tag{1}$$

where the vector $\boldsymbol{x}_j$ represents observable property characteristics such as the property size and the number of bedrooms. The term $\xi_j$ captures unobservable property characteristics that are known to consumers (but unobservable to researchers). For example, while consumers can infer whether a property gets a lot of natural light from photos posted on the platform, such information is difficult for researchers to obtain.

The price $p_{jt}$ in (1) is the sum of an expected transaction price ($p_j^e$) and a shock ($\eta_{jt}$). The expected transaction price is known to consumers before searching. It is the list price plus an expectation of the transaction and list price difference. Specifically, $p_j^e = p_j^l + g_{m(j)}x_{1j}$ where $p_j^l$ is the list price, $g_{m(j)}$ is the average price difference per square meter in segment $m(j)$, and $x_{1j}$ is the property size. The price shock $\eta_{jt}$ is unknown to consumers before search. It is assumed to be i.i.d. across properties and time and follows a normal distribution with mean $0$ and a variance that equals to the empirical variance.[11].

The idiosyncratic term $v_{ij}$ in (1) captures the match value of property $j$ for consumer $i$ that consumer $i$ learns after visiting property $j$. It includes, for example, consumer $i$'s tastes for a particular aspect of the property's floor plan or a particular amenity in the neighborhood. We assume that $v_{ij}$ is i.i.d. and follows a normal distribution with mean 0 and variance $\sigma_v^2$.

We rewrite the utility as

$$u_{ijt} = \delta_j + \alpha\eta_{jt} + v_{ij}, \tag{2}$$

by collecting the terms known to consumers before searching in

$$\delta_j \;\; = \;\; \boldsymbol{x}_j\boldsymbol{\beta} + \alpha p_j^e + \xi_j, \tag{3}$$

which we label as the "mean utility".

[11]We compute the empirical variance of $\eta_{jt}$ across all transactions in the sample where the observed price shock $\eta_{jt}$ is the difference between the realized transaction price and the expected transaction price.

### 3.1.2 Search Costs and Waiting Costs

We assume that the cost of searching $n$ properties at time $t$ is $C_{it}(n) - \vartheta_{itn}$, where the search cost shock $\vartheta_{itn}$ is i.i.d. and follows a type-1 extreme value distribution with location parameter $0$ and scale parameter $\kappa$. The deterministic component $C_{it}(n)$ is

$$C_{it}(n) = \left(\gamma_0 + (\gamma_1 + \gamma_2 m_{it})n + \gamma_3 n^2\right) \mathbb{1}(n > 0). \tag{4}$$

In this specification, we allow the marginal search cost to depend on the cumulative number of searches before time $t$ (denoted by $m_{it}$). We include the quadratic term $n^2$ in (4) to capture potential nonlinearity in search costs.

Consumer $i$ also incurs a waiting cost $w_{it}$ in each period in which she does not make a purchase. An example of the waiting cost is the psychological stress caused by not having a home for the newlyweds before they get married. In China, it is a cultural norm that the groom's family is expected to buy a home, or at least find a home and contribute to the down payment for the newlyweds before marriage. We assume that $w_{it}$ is i.i.d. and follows a normal distribution with mean $w$ and variance $\sigma_w^2$.

### 3.1.3 Search Set Conditional on the Number of Visits

In each period $t$, consumer $i$ optimally decides on the number of properties to visit (denoted by $n_{it}$). Conditional on $n_{it}$, the exact set of properties she visits in that period is exogenously drawn. This assumption is similar to that in Hortaçsu and Syverson (2004) and implies that a consumer's decision is the number, rather than the set, of properties to visit. The sampling probability we specify below is such that a property with a higher mean utility has a higher probability of being sampled. Therefore, this exogenous assumption only means that a consumer and her agent do not have the full control over the set of properties they can visit at a particular time and that there are exogenous reasons why a consumer may or may not be able to visit a property at a given time.[12]

Specifically, let $\mathcal{A}_{it}$ denote the set of properties in consumer $i$'s search market that she has not visited by time $t$ (here, $\mathcal{A}$ stands for "Available for search") and $\mathcal{C}^n(\mathcal{A}_{it})$ be the collection of all $n$-element subsets of $\mathcal{A}_{it}$. For a given number of visits $n$, we assume that

[12]This assumption simplifies both the consumer's decision problem and our estimation. First, if a consumer were to choose the set of properties to visit, she would have $C_{A_{it}}^{n_{it}}$ such sets to choose from, where $A_{it}$ is the number of properties available for consumer $i$ to search. The cardinality $C_{A_{it}}^{n_{it}}$ can be very large. For example, if $A_{it} = 15$ and $n_{it} = 8$, then consumer $i$ has $C_{A_{it}}^{n_{it}} = 6,435$ choices in period $t$. It seems too demanding for our model to explain why consumer $i$ chooses a particular set of properties out of the 6,435 possible sets. Second, as will be explained in Appendix B, this assumption also allows us to estimate model parameters in two steps, which in turn facilitates addressing a price endogeneity issue and estimating a large number of fixed effect parameters.

the probability of a particular set $\mathcal{N} \in \mathcal{C}^n(\mathcal{A}_{it})$ being sampled depends on the mean utility, i.e., $\{\delta_j : j \in \mathcal{A}_{it}\}$, as follows:

$$\Pr(\mathcal{N}|\mathcal{A}_{it}, n) = \sum_{k=1}^{n} \left[ (-1)^{k-1} \sum_{\mathcal{B} \in \mathcal{C}^k(\mathcal{N})} \frac{\sum_{l \in \mathcal{B}} exp(\delta_l)}{\sum_{l \in (\mathcal{A} \setminus \mathcal{N}) \cup \mathcal{B}} exp(\delta_l)} \right]. \tag{5}$$

This sampling probability has the following desirable features. First, it is consistent with an extended Logit model. In Appendix A, we extend a discrete choice model from a setting where a consumer chooses a single option to a setting where a consumer chooses $n$ options. Though seemingly complicated, this expression in (5) is a direct application of the analytic expression of the choice probability in a standard Logit model and the inclusion-exclusion principle.

Second, as we show in Appendix A, this sampling probability for *a set of properties* implies that the following sampling probability for *a particular property* $j \in \mathcal{A}_{it}$:

$$\begin{aligned} &\Pr(j|\mathcal{A}_{it}, n) \\ &= \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = A_{it} \\ \sum_{k=0}^{n-1} \left[ (-1)^k \sum_{\mathcal{B} \in \mathcal{C}^{A-n+k}(\mathcal{A} \setminus j)} \left( \frac{C^k_{A-n+k-1} exp(\delta_j)}{\sum_{l \in \mathcal{B}} exp(\delta_l) + exp(\delta_j)} \right) \right] & \text{if } 0 < n < A_{it}, \end{cases} \end{aligned} \tag{6}$$

where $A_{it} = \#\mathcal{A}_{it}$. This probability itself has the following three intuitive features: (i) it is increasing in $\delta_j$ and decreasing in $\delta_{j'}$ for $j' \neq j$, i.e., property $j$ is more likely to be sampled when its own mean utility increases or other properties' mean utilities decrease; (ii) it becomes the choice probability in a standard Logit model, i.e., $\frac{exp(\delta_j)}{\sum_{l \in \mathcal{A}_{it}} exp(\delta_l)}$, when $n = 1$; and (iii) the sum of the sampling probabilities over $j$ in $\mathcal{A}_{it}$ is $n$, i.e., $\sum_{j \in \mathcal{A}_{it}} Pr(j|\mathcal{A}_{it}, n) = n$.

### 3.1.4 Timing

Let $\mathcal{R}_{it}$ denote the set of properties that consumer $i$ has visited before period $t$ and that are still available in period $t$ (where $\mathcal{R}$ stands for "Recall"). In each period, a consumer makes two decisions: a search decision and a purchase decision. The timing is as follows:

- At the beginning of the period, consumer $i$ observes $\delta_j$ for properties in both $\mathcal{R}_{it}$ and $\mathcal{A}_{it}$ and the price shock $\eta_{jt}$ and the match value $v_{ij}$ for properties in her recall set $\mathcal{R}_{it}$. Consumer $i$ also observes the search cost shocks $\vartheta_{itn}$ for $n = 0, ..., \bar{n}$, where $\bar{n}$ is the maximum number of searches in a period. She decides how many properties

to search in time $t$. We denote the number of searches by $n_{it}$, which is an integer between 0 and $\bar{n}$.

- A search set $\mathcal{N}_{it} \in \mathcal{C}^{n_{it}}(\mathcal{A}_{it})$ is sampled according to the probability in (5). Here, $\mathcal{N}$ stands for "Newly searched".

- After visiting the properties in $\mathcal{N}_{it}$, consumer $i$ now observes the price shock $\eta_{jt}$ and the match value $v_{ij}$ for $j$ in $\mathcal{R}_{it} \cup \mathcal{N}_{it}$. She now also observes her waiting cost $w_{it}$. She decides whether to buy a property in $\mathcal{R}_{it} \cup \mathcal{N}_{it}$ or to continue searching. We denote this decision by $y_{it}$.

### 3.1.5 Transition of the Environment

From time $t$ to time $t+1$, there are three changes in consumer $i$'s search market. Because consumers differ in their search markets as defined in Section 2, the three changes in consumer $i$'s search market are also $i$-specific.

First, some properties may exit the market. We assume that the exit rate in consumer $i$'s search market is $\chi_i$ and use $\mathcal{EXIT}_{it}$ to denote the set of exited properties in consumer $i$'s search market at time $t$.

Second, new properties may enter the market. We assume that the number of newly listed properties in consumer $i$'s search market follows a Poisson distribution with an arrival rate $\lambda_i$ and use $\mathcal{NEW}_{it+1}$ to denote the set of newly listed properties in her search market at the beginning of time $t+1$. We assume that the mean utility of a newly listed property in consumer $i$'s search market follows a normal distribution $N(\mu_{it}^{new}, (\sigma_i^{new})^2)$.

Third, prices of new listings increase. We assume that the trend in the list price of new properties in consumer $i$'s search market is $\rho_i$. Therefore, in forming an expectation about the next period, consumer $i$ considers the transition of $\mu_{it}^{new}$ to be $\mu_{it+1}^{new} = \mu_{it}^{new} + \alpha\rho_i$.

The above three changes determine the transition of consumer $i$'s information set over time. Specifically, at the beginning of $t$, consumer $i$'s information set is $(\{\delta_j\}_{j\in\mathcal{A}_{it}}, \{\delta_j, \eta_{jt}, v_{ij}\}_{j\in\mathcal{R}_{it}}, \mu_{it}^{new}, m_{it}, \vartheta_{itn}, w_{it})$. Among them, $(\eta_{jt}, \vartheta_{itn}, w_{it})$ are shocks independent over time while $\Omega_{it} = (\{\delta_j\}_{j\in\mathcal{A}_{it}}, \{\delta_j, v_{ij}\}_{j\in\mathcal{R}_{it}}, \mu_{it}^{new}, m_{it})$ follows a transition determined by the following: the available-to-search set $\mathcal{A}_{it+1} = \mathcal{A}_{it} \backslash \mathcal{N}_{it} \backslash \mathcal{EXIT}_{it} \cup \mathcal{NEW}_{it+1}$, the recall set $\mathcal{R}_{it+1} = \mathcal{R}_{it} \cup \mathcal{N}_{it} \backslash \mathcal{EXIT}_{it}$, the average mean utility of new listings $\mu_{it+1}^{new} = \mu_{it}^{new} + \alpha\rho_i$, and the cumulative number of searches $m_{it+1} = m_{it} + n_{it}$.

## 3.2 Consumer Decisions

Having described the model primitives, we now describe how consumers make decisions. We describe consumer $i$'s problem in each period backwards: her purchase decision after searching and then her search intensity decision.

### 3.2.1 Purchase Decision

After visiting properties in $\mathcal{N}_{it}$ in period $t$, consumer $i$ observes the match values of the newly searched properties $\{v_{ij}\}_{j\in\mathcal{N}_{it}}$ in addition to the match values of properties in her recall set (i.e., $\{v_{ij}\}_{j\in\mathcal{R}_{it}}$ in $\Omega_{it}$). She also observes the price shocks of all properties in her choice set, i.e., $\{\eta_{jt}\}_{j\in\mathcal{R}_{it}\cup\mathcal{N}_{it}}$, and her waiting cost $w_{it}$. Consumer $i$ chooses among recalling from the previously searched set $\mathcal{R}_{it}$, buying from the newly searched set $\mathcal{N}_{it}$, or continuing to search. In other words, her optimization problem at the purchase-decision stage is as follows:

$$
\begin{aligned}
& \Gamma_i(\Omega_{it}, \{v_{ij}\}_{j\in\mathcal{N}_{it}}, \{\eta_{jt}\}_{j\in\mathcal{R}_{it}\cup\mathcal{N}_{it}}, w_{it}) \qquad (7)\\
= \; & \max\{ \underbrace{\max_{j\in\mathcal{R}_{it}} \delta_j + \alpha\eta_{jt} + v_{ij}}_{\text{recall}}, \; \underbrace{\max_{j\in\mathcal{N}_{it}} \delta_j + \alpha\eta_{jt} + v_{ij}}_{\text{buy a newly searched property}}, \; \underbrace{E_i\left[V_i(\Omega_{it+1})|\Omega_{it}, \{v_{ij}\}_{j\in\mathcal{N}_{it}}\right] - w_{it}}_{\text{wait}} \},
\end{aligned}
$$

where $E_i\left[V_i(\Omega_{it+1})|\Omega_{it}, \{v_{ij}\}_{j\in\mathcal{N}_{it}}\right]$ denotes the expected value of continuing the search. This expectation depends on the information set at the beginning of period $t$, $\Omega_{it}$, and the match values of the newly searched properties, $\{v_{ij}\}_{j\in\mathcal{N}_{it}}$.[13] A consumer's purchase decision (denoted by $y_{it}$) is the optimizer of the above optimization problem.

### 3.2.2 Search Decision

A consumer's optimal search intensity in each period, i.e., the number of properties to search ($n_{it}$), depends on the comparison between the benefits and costs of searching. We have specified the search costs in Section 3.1. We now explain the search benefits. The benefit of searching $n_{it}$ properties is that consumer $i$ learns about the $n_{it}$ newly searched properties and expands her choice set at the purchase stage from the recall set ($\mathcal{R}_{it}$) to the union of the recall set and the newly searched set (i.e., $\mathcal{R}_{it}\cup\mathcal{N}_{it}$).

Formally, the expected benefit of searching $n$ properties conditional on the information

[13] The expectation is consumer $i$ specific because the transition of $\Omega_{it}$ may be different in different consumers' search markets.

set $\Omega_{it}$ is

$$\begin{aligned} & EB_i(n|\Omega_{it}) \\ = & \sum_{\mathcal{N}\in\mathcal{C}^n(\mathcal{A}_{it})} E_{\left(\{v_{ij}\}_{j\in\mathcal{N}},\{\eta_{jt}\}_{j\in\mathcal{R}_{it}\cup\mathcal{N}_{it}},w_{it}\right)}\left[\Gamma_i(\Omega_{it},\{v_{ij}\}_{j\in\mathcal{N}},\{\eta_{jt}\}_{j\in\mathcal{R}_{it}\cup\mathcal{N}_{it}},w_{it})\right]\times\Pr(\mathcal{N}|\Omega_{it},n), \end{aligned} \tag{8}$$

where the first term $E_{...}\left[\Gamma_i(\Omega_{it},...)\right]$ is the expected value of searching a sampled set $\mathcal{N}$ and the second term $\Pr(\mathcal{N}|\Omega_{it},n)$ is the probability that the set $\mathcal{N}$ is sampled conditional on information set $\Omega_{it}$ and the number of searches $n$. Here, with a slight abuse of notation, we rewrite $\Pr(\mathcal{N}|\mathcal{A}_{it},n)$ in (5) as $\Pr(\mathcal{N}|\Omega_{it},n)$ to reflect its dependence on $\{\delta_j\}_{j\in\mathcal{A}_{it}}$, which is a subset of the information in $\Omega_{it}$.

Consumer $i$ chooses her search intensity $n_{it}$ to maximize her net gain from searching. That is, the optimal search intensity $n_{it}$ is the solution to the following optimization problem:

$$\max_{0\le n\le\bar{n}}\{EB_i(n|\Omega_{it})-C(n|\Omega_{it})+\vartheta_{itn}\}, \tag{9}$$

where, with a slight abuse of notation, we now use $C(n|\Omega_{it})$ to denote the search cost function $C_{it}(n)$ in (4) to reflect its dependence on $m_{it}$ in $\Omega_{it}$.

## 3.3 Bellman Equation and State Transition

We complete the description of the model by presenting the Bellman equation. We define the *ex ante* value function as the expectation of the maximum in (9) over the search cost shocks:

$$\begin{aligned} V_i(\Omega_{it}) =& E_{(\vartheta_{itn},n=0,...,\bar{n})}\{\max_{0\le n\le\bar{n}}[EB_i(n|\Omega_{it})-C(n|\Omega_{it})+\vartheta_{itn}]\} \\ =& \kappa ln\left(\sum_{n=0}^{\bar{n}} exp\left(\frac{EB_i(n|\Omega_{it})-C(n|\Omega_{it})}{\kappa}\right)\right)+\kappa\tau, \end{aligned} \tag{10}$$

where $\tau$ is the Euler constant. Therefore, plugging $\Gamma_i(\Omega_{it},\{v_{ij}\}_{j\in\mathcal{N}_{it}},\{\eta_{jt}\}_{j\in\mathcal{R}_{it}\cup\mathcal{N}_{it}},w_{it})$ in (7) into $EB_i(n|\Omega_{it})$ in (8) and then plugging $EB_i(n|\Omega_{it})$ into (10) give us the Bellman equation.

## 3.4 Discussions

We conclude this section with a discussion on three simplifications in the model. First, we assume that there is no correlated learning. In particular, we assume that visiting one property does not allow consumers to learn about other properties without visiting them.

In contrast, Hodgson and Lewis (2023) allow for correlations among the unobservable match values and allow the correlation to be stronger between products more similar in observable characteristics. As a result, consumer search tends to converge to the chosen product in the product characteristics space. We make the no-learning simplification because we do not observe clear evidence for such spatial learning search pattern in our setting. In Online Supplemental Appendix SA, we show that the share of visits that a consumer pays to properties in the residential complex of her final purchase does not increase as she approaches the end of her search. We assume away correlated learning, which implies dynamic learning, also because it would be computationally infeasible to include both a dynamic search environment and dynamic learning in our model. Given the focus of the paper (i.e., the effect of changing search environments), we keep the former and abstract away the latter.

Second, we do not model sellers' decisions. In reality, sellers also make strategic decisions. For example, Merlo, Ortalo-Magné and Rust (2015) study sellers' dynamic decisions regarding list prices, revising list prices, and accepting offers. Incorporating both sellers' and buyers' dynamic decisions in a non-stationary search environment would be challenging. Instead, we capture the effects of sellers in a somewhat reduced-form way. Regarding list prices, as will be explained in the estimation section, we deal with the endogeneity of list prices using an instrumental variable approach. Regarding revising list prices, this is extremely rare in our sample. Regarding a seller's decision on accepting offers, we capture its effect in the price shock $\eta_{jt}$. For example, if a seller is more willing to accept a low offer, the price shock may be negative. Conversely, if a seller is currently considering a higher offer, the price shock will be positive and large.

Third, we do not model buyer competition. In reality, several buyers may bid simultaneously for a property. However, we do not observe the number of bids and the amount of each bid that a property receives. In our model, the price shock $\eta_{jt}$ captures buyer competition to some extent: intensive buyer competition is captured by a large price shock.

# 4 Estimation

The estimation consists of two steps. In the first step, we estimate the parameters in the utility function $(\alpha, \boldsymbol{\beta})$ by matching the observed share of visits that each property receives. We call these parameters the static parameters. In this step of the estimation, we use all properties and consumers in our sample.

In the second step, we estimate the remaining parameters, which govern the dynamic search and purchase decisions. These parameters include: search cost parameters $(\boldsymbol{\gamma}, \kappa)$,

waiting cost parameters $(w, \sigma_w)$, and the standard deviation of match values $(\sigma_v)$. We call these parameters the dynamic parameters. In this estimation step, we use a random sample of 1,000 consumers whose entire search record falls within the sample period excluding the eight weeks around Chinese New Year.[14]

Appendix B provides a discussion on our two-step estimation procedure, highlighting its advantages (e.g., allowing for price endogeneity and a large set of fixed effects and addressing the issue of high dimensionality of the state space) and clarifying the assumptions needed. Supplemental Appendix SB provides details on the estimation.

## 4.1 Static parameters $(\alpha, \boldsymbol{\beta})$

We estimate the parameters in the utility function $(\alpha, \boldsymbol{\beta})$ by matching the observed share of visits that each property gets. Specifically, we first invert out the mean utility of each property based on the visit shares and then regress the mean utility on the property price and characteristics to obtain estimates of $(\alpha, \boldsymbol{\beta})$. In our sample, all consumers search within one district. Therefore, we partition the sample into six districts and do the inversion district by district. Let $\mathcal{J}_d$ and $\mathcal{I}_d$ represent the set of properties in district $d$ and the set of consumers searching in district $d$, respectively.

For a property $j$ in district $d$, its share of visits according to our model is

$$\tilde{s}_j(\boldsymbol{\delta}_d) = \frac{\sum_{i \in \mathcal{I}_d} \sum_{t=1}^{T_i} \Pr(j|\mathcal{A}_{it}, n_{it})}{\sum_{i \in \mathcal{I}_d} \sum_{t=1}^{T_i} n_{it}}, \tag{11}$$

where $\boldsymbol{\delta}_d = (\delta_j, j \in \mathcal{J}_d)$ represents mean utilities of properties in district $d$. In (11), $n_{it}$ is the observed number of properties that consumer $i$ visits in period $t$. $\Pr(j|\mathcal{A}_{it}, n_{it})$ is the probability that consumer $i$ visits property $j$ in period $t$, where $\mathcal{A}_{it}$ is the set of properties available for consumer $i$ to search. This probability is 0 for $j \notin \mathcal{A}_{it}$ and is given by (6) for $j \in \mathcal{A}_{it}$. The sum is taken over all consumers searching in district $d$ (indexed by $i \in \mathcal{I}_d$) and all periods during a consumer's search (indexed by $t = 1, ..., T_i$), where $T_i$ is the search duration of consumer $i$.

[14]We only consider consumers who started searching after the start of the sample because we need to know how many properties they visited before each period. However, strictly speaking, we do not have to restrict our sample to those who purchased before the end of the sample. We do so in the baseline estimation because without such a restriction, the sample may include consumers who were not seriously searching in the housing market. Nevertheless, as a robustness check, we repeat the estimation including consumers who did not purchase before the end of the sample period in Supplemental Appendix SC and show that our estimation results are robust.

The empirical counterpart of this share is

$$s_j = \frac{N_j}{\sum_{j \in \mathcal{J}_d} N_j}, \tag{12}$$

where $N_j$ is the number of visits that property $j$ receives in the sample. Note that the denominators in (11) and (12) are the same: both represent the sum of visits that properties in district $d$ receive.

We invert out mean utilities $\boldsymbol{\delta}_d$ by matching the model implication of the visit shares to their empirical counterparts, i.e.,

$$\tilde{s}_j(\boldsymbol{\delta}_d) = s_j, j \in \mathcal{J}_d. \tag{13}$$

In Appendix A, we extend the contraction mapping result in Berry, Levinsohn and Pakes (1995) for a single discrete choice model to our setting where a set of options is sampled and show that the system of equations in (13) has a unique solution. Since $\sum_{j \in \mathcal{J}_d} s_j = 1$, we normalize one dimension of $\boldsymbol{\delta}_d$ in each district $d$ to 0. Therefore, all inverted mean utilities are relative to that of the normalized property.

To estimate $(\alpha, \boldsymbol{\beta})$, we regress the inverted $\delta_j$ on the expected price $p_i^e$ and property characteristics $\boldsymbol{x}_j$ according to equation (3).[15] Because the price of a property is likely to be correlated with unobservable property characteristics, the expected price is endogenous. We address this endogeneity issue by using an instrumental variable approach. Specifically, we use the transaction price of properties in the same segment averaged across transactions occurred within the three weeks prior to property $j$'s listing as the instrumental variable. This instrumental variable is relevant because property owners and their agents are likely to choose list prices based on historical transaction prices in the same area and price range. At the same time, it is reasonable to assume that the transaction prices of properties sold in the last three weeks before a property is listed are uncorrelated with the unobservable characteristics of the property.

## 4.2 Dynamic Parameters $\theta = (\boldsymbol{\gamma}, \kappa, w, \sigma_w, \sigma_v)$

For each consumer $i$, we observe her search duration $T_i$. For each search week $t = 1, ..., T_i$, we observe her search intensity (i.e., the number of visits $n_{it}$) and her purchase decision (i.e., whether $y_{it} = recall$ – to buy a previously visited property, $y_{it} = j \in \mathcal{N}_{it}$ – to

[15] Since $\delta_j$ is relative to the mean utility of the normalized property, the regressors are $p_j^l - p_{j_0^d}^l$ and $\boldsymbol{x}_j - \boldsymbol{x}_{j_0^d}$ where $j_0^d$ is the normalized property in district $d$.

buy property $j$ which is newly visited by her in the current period, or $y_{it} = wait$ – to search longer).[16]

Letting $\theta = (\boldsymbol{\gamma}, \kappa, w, \sigma_w, \sigma_v)$ summarize the dynamic parameters, the likelihood of observing the search and purchase path $\{n_{it}, y_{it}\}_{t=1}^{T_i}$ for consumer $i$ is

$$l_i(\theta) = \int \left( \prod_{t=1}^{T_i} [\Pr_i(n_{it}|\Omega_{it};\theta) \cdot \Pr_i(y_{it}|\Omega_{it}, \{v_{ij}\}_{j\in\mathcal{N}_{it}};\theta)] \right) dF_v(\{\{v_{ij}\}_{j\in\mathcal{N}_{it}}\}_{t=1}^{T_i};\theta) \quad (14)$$

where $\Pr_i(n_{it}|\Omega_{it};\theta)$ is the probability of searching $n_{it}$ properties and $\Pr(y_{it}|\Omega_{it}, \{v_{ij}\}_{j\in\mathcal{A}_{it}};\theta)$ is the probability of making purchase decision $y_{it}$, both of which we derive below. In (14), $F_v(\{\{v_{ij}\}_{j\in\mathcal{N}_{it}}\}_{t=1}^{T_i};\theta)$ represents the distribution of the match value of all properties that consumer $i$ has ever visited.[17] We integrate out these match values because, after recovering $\delta_j$ in the first step of the estimation, we observe $\{\Omega_{it}\}_{j\in\mathcal{A}_{it}}$ and $\{\Omega_{it}\}_{j\in\mathcal{R}_{it}}$ in $\Omega_{it}$ and the only unobservable variables are $\{v_{ij}\}_{j\in\mathcal{R}_{it}}$ in $\Omega_{it}$ (i.e., match values of properties in the recall set) and $\{v_{ij}\}_{j\in\mathcal{N}_{it}}$ in $\Pr_i(y_{it}|\Omega_{it}, \{v_{ij}\}_{j\in\mathcal{N}_{it}};\theta)$ (i.e., match values of newly searched properties). Therefore, by integrating out match values of all properties that consumer $i$ has visited, we integrate out all unobservables.

To derive the probability of search intensity $\Pr_i(n_{it}|\Omega_{it};\theta)$, we note that at the beginning of a period, consumer $i$ observes $\Omega_{it}$ and search cost shocks $\vartheta_{itn}$ and chooses $n_{it}$ to maximize her net gain from searching according to the optimization problem in (9). Given that the search cost shock $\vartheta_{itn}$ follows the type-1 extreme value distribution with scale parameter $\kappa$, the probability that consumer $i$ searches $n_{it}$ is

$$\Pr_i(n_{it}|\Omega_{it};\theta) = \frac{exp\{[EB_i(n_{it}|\Omega_{it};\theta) - C(n_{it}|\Omega_{it};\theta)]/\kappa\}}{\sum_{n=0}^{\bar{n}} exp\{[EB_i(n|\Omega_{it};\theta) - C(n|\Omega_{it};\theta)]/\kappa\}}, \quad (15)$$

where we add the parameters $\theta$ to the expected search benefit $EB_i(n_{it}|\Omega_{it};\theta)$ and search costs $C(n_{it};\theta)$ to make their dependence on the parameters explicit.

To derive the probability of the purchase decision $\Pr(y_{it}|\Omega_{it}, \{v_{ij}\}_{j\in\mathcal{A}_{it}};\theta)$, we note that after visiting $n_{it}$ properties, consumer $i$ observes the original information set $\Omega_{it}$, the newly acquired information set $\{v_{ij}\}_{j\in\mathcal{N}_{it}}$, price shocks $\eta_{jt}$, and waiting cost $\omega_{it}$ and makes a

[16] We observe the identity of a consumer's purchased property regardless of whether it belongs to the recall set $\mathcal{R}_{it}$ or the newly searched set $\mathcal{N}_{it}$. However, our likelihood function captures whether a consumer recalls, rather than which property she recalls. This is because the probability of purchasing a particular property in the recall set depends on $\delta_j + v_{ij}$ for all $j \in \mathcal{R}_{it}$ (as well as $\{\delta_j\}_{j\in\mathcal{A}_{it}}$), resulting in high-dimensional state variables. However, the probability of recall depends on a summary statistic $\max_{j\in\mathcal{R}_{it}}(\delta_j + v_{ij})$ (as well as $\{\delta_j\}_{j\in\mathcal{A}_{it}}$). Therefore, for the computational reason, we do not use the information on which property that a consumer recalls for estimation.

[17] It is $\prod_{j\in\mathcal{N}_{it}} \prod_{t=1}^{T_i} \Phi(v_{ij}/\sigma_v)$.

purchase decision according to the optimization problem in (7). Therefore, the choice probabilities are:

$$\begin{aligned}
&\Pr_i(y_{it} = recall|\Omega_{it}, \{v_{ij}\}_{j\in\mathcal{N}_{it}}; \theta) \qquad (16)\\
=&E_{\{\eta_{jt}\}_{j\in\mathcal{R}_{it}\cup\mathcal{N}_{it}}}\Bigg\{\mathbb{1}\Bigg(\max_{j\in\mathcal{R}_{it}}\{\delta_j + \alpha\eta_{jt} + v_{ij}\} \geq \max_{j\in\mathcal{N}_{it}}\{\delta_j + \alpha\eta_{jt} + v_{ij}\}\Bigg)\\
&\times\Phi\Bigg(\Big\{\max_{j\in\mathcal{R}_{it}\cup\mathcal{N}_{it}}\{\delta_j + \alpha\eta_{jt} + v_{ij}\} - E_i\left[V_i(\Omega_{it+1};\theta)|\Omega_{it}, \{v_{ij}\}_{j\in\mathcal{N}_{it}}\right] + w\Big\}/\sigma_w\Bigg)\Bigg\},
\end{aligned}$$

$$\begin{aligned}
&\Pr_i(y_{it} = j \in \mathcal{N}_{it}|\Omega_{it}, \{v_{ij}\}_{j\in\mathcal{N}_{it}}; \theta) \qquad (17)\\
=&E_{\{\eta_{jt}\}_{j\in\mathcal{R}_{it}\cup\mathcal{N}_{it}}}\Bigg\{\mathbb{1}\Bigg(\delta_j + \alpha\eta_{jt} + v_{ij} \geq \max_{j\in\mathcal{R}_{it}\cup\mathcal{N}_{it}}\{\delta_j + \alpha\eta_{jt} + v_{ij}\}\Bigg)\\
&\times\Phi\Bigg(\Big\{\max_{j\in\mathcal{R}_{it}\cup\mathcal{N}_{it}}\{\delta_j + \alpha\eta_{jt} + v_{ij}\} - E_i\left[V_i(\Omega_{it+1};\theta)|\Omega_{it}, \{v_{ij}\}_{j\in\mathcal{N}_{it}}\right] + w\Big\}/\sigma_w\Bigg)\Bigg\},
\end{aligned}$$

$$\begin{aligned}
&\Pr_i(y_{it} = wait|\Omega_{it}, \{v_{ij}\}_{j\in\mathcal{N}_{it}}; \theta) \qquad (18)\\
=&E_{\{\eta_{jt}\}_{j\in\mathcal{R}_{it}\cup\mathcal{N}_{it}}}\Bigg\{\Phi\Bigg(\Big\{E_i\left[V_i(\Omega_{it+1};\theta)|\Omega_{it}, \{v_{ij}\}_{j\in\mathcal{N}_{it}}\right] - w - \max_{j\in\mathcal{R}_{it}\cup\mathcal{N}_{it}}\{\delta_j + \alpha\eta_{jt} + v_{ij}\}\Big\}/\sigma_w\Bigg)\Bigg\},
\end{aligned}$$

where $\Phi(\cdot)$ is the distribution function of a standard normal distribution.

We estimate $\theta$ using maximum likelihood estimation, where the log-likelihood function is $L(\theta) = \sum_{i=1}^{I} \ln l_i(\theta)$. To compute the likelihood function, we need to compute the value function $V_i(\Omega_{it};\theta)$ as a fixed point to the Bellman equation. The state variables $\Omega_{it} = (\{\delta_j\}_{j\in\mathcal{A}_{it}}, \{\delta_j, v_{ij}\}_{j\in\mathcal{R}_{it}}, \mu_{it}^{new}, m_{it})$ include the mean utilities of the properties available to search ($\{\delta_j\}_{j\in\mathcal{A}_{it}}$) as well as both the mean utilities and the match values of the properties available to recall ($\{\delta_j, v_{ij}\}_{j\in\mathcal{R}_{it}}$). Thus, the state space is large. Moreover, the dimension of $\Omega_{it}$ changes over time both exogenously and endogenously. This is because the sizes of both the available-to-search set $\mathcal{A}_{it}$ and the recall set $\mathcal{R}_{it}$ change over time due to entry of new listings and exiting of existing listings. The sizes are also determined by consumer $i$'s endogenous decision on the number of searches per period. Following much of the literature on dynamic estimation (e.g., Sweeting (2013); Hodgson (2024)), we solve this large state space problem by approximating the value function of high-dimensional state variables with one of lower-dimensional statistics. Furthermore, the state variables include both observable and unobservable variables. We simulate the unobservable variables in

estimation. We provide more details on dynamic estimation in Supplemental Appendix SB.

**Identification** The standard deviation of the match value ($\sigma_v$) is identified by both a static and a dynamic feature of consumer purchase patterns. Statically, we observe which property each consumer purchased. Because utility depends on both the mean utility and the match value, the gap between the mean utility of a consumer's purchased property and the highest mean utility in the consumer's choice set at the time of purchase informs us about the importance of match values. A larger gap implies a larger standard deviation of match values. Dynamically, we observe when each consumer's purchased property is visited by the consumer. If a consumer visited her purchased property in an earlier period and purchased it in a later period, such a recalling behavior means that the consumer decided to continue the search in the belief that there is a good chance that she gets a better draw of the match value. Therefore, a larger recall share also indicates a larger standard deviation of match values.

The search cost parameters $(\boldsymbol{\gamma}, \kappa)$ are identified by the mean and variance of the search intensity. Recall that the mean of the search cost is $C_{it}(n) = (\gamma_0 + (\gamma_1 + \gamma_2 m_{it})n + \gamma_3 n^2)\mathbb{1}(n > 0)$. While the fraction of observations with zero searches identifies $\gamma_0$, how the fraction of $n$ searches varies with $n$ helps us identify $\gamma_1$ and $\gamma_3$. Finally, how the search intensity varies with $m_{it}$ identifies $\gamma_2$. As for the scale parameter of the search cost shock $\kappa$, it is identified by the observed variance in the search intensity.

Similarly, the waiting cost parameters $(w, \sigma_w)$ are identified by the observed search duration. In particular, the overall level of search duration identifies the mean parameter $w$ and the variance of search duration across consumers identifies the variance of the waiting cost shock $\sigma_w^2$.

# 5 Estimation Results

In this section, we present our estimation results and evaluate the model fit.

## 5.1 Estimates of Static Parameters

We obtain the estimates of the parameters of the utility function $(\alpha, \boldsymbol{\beta})$ by regressing the inverted mean utility $\delta_j$ on the expected price and the property characteristics. In Table 2, we report the OLS results in column (I) and the IV regression results in column (II). Since properties that are more attractive to consumers in ways that are unobservable

to researchers may have higher prices, prices are potentially endogenous, leading to an upward bias for the price coefficient in the OLS regression. Consistent with this intuition, we find that the IV regression indeed yields a more negative price coefficient. In the following, we focus on the IV results.

Overall, the estimation yields intuitive results. Consumers like newer and larger properties with more bedrooms, more living rooms, located on the 10th floor or higher, and close to subway stations. For example, on average, an extra bedroom with an average size of 20 square meters is valued at 0.307 (=(0.344+0.050×20)/4.384) million CN¥. Similarly, an additional living room with an average size of 30 square meters is valued at 0.579 (=(1.040+0.050×30)/4.384) million CN¥. Locating on the 10th floor or above is worth CN¥35,400 more than locating below the 10th floor. As a robustness analysis, in Supplemental Appendix SC, we allow the price coefficient to be district specific and find mild heterogeneity in price sensitivity among consumers searching in different districts.

Table 2: Estimates of Parameters in Mean Utility

| | (I) OLS | | (II) IV | |
|---|---|---|---|---|
| | Est | SE | Est | SE |
| List price (million CN¥) | -0.193*** | (0.006) | -4.384*** | (0.133) |
| Property age (year) | -0.031*** | (0.001) | -0.009*** | (0.001) |
| # Bedrooms | 0.206*** | (0.007) | 0.344*** | (0.015) |
| # Living rooms | 0.530*** | (0.008) | 1.040*** | (0.023) |
| Property size ($m^2$) | 0.005*** | (0.000) | 0.050*** | (0.002) |
| Above 10th floor | 0.089*** | (0.007) | 0.155*** | (0.016) |
| Close to subway stations | -0.029*** | (0.010) | 0.199*** | (0.022) |
| Neighborhood FE | yes | | yes | |

*** $p < 0.01$

## 5.2 Estimates of Dynamic Parameters

Table 3 reports the estimation results for the dynamic parameters, including the standard deviation of match values ($\sigma_v$), the waiting cost parameters ($w, \sigma_w$), and the search cost parameters ($\boldsymbol{\gamma}, \kappa$).

We find a significant benefit of searching to learn about the match value. The estimated standard deviation of the match value is 0.441. Based on the estimated price coefficient ($\hat{\alpha}$=-4.384), this estimated standard deviation corresponds to a value of CN¥100,593 (=0.441/4.384×$10^6$), about 3% of the average list price and 176% of the per-capita disposal income in Beijing in 2016.

The waiting cost is on average 0.051, which is equivalent to CN¥11,633 (=0.051/4.384$\times 10^6$) per week. This high waiting cost is consistent with the observation that consumers search for 3.5 weeks on average before purchasing a property. Because the main motivation for buying a property in China is often marriage, the large waiting cost may be explained by the cultural norm in China that the groom's family is expected to purchase a home for the newlyweds before marriage (Wei and Zhang (2011)). That said, in relative terms, this average waiting cost is only 0.28% of the average list price.

The estimated search cost parameters indicate that there is a baseline search cost that a consumer incurs as long as she searches in a period and that the search cost increases with the number of searches. Specifically, the baseline search cost ($\hat{\gamma}_0$) is CN¥703 (=3.080$\times$0.001/4.384$\times 10^6$). As for the marginal search cost, it increases with the cumulative number of searches a consumer has done in the previous weeks ($\hat{\gamma}_2 > 0$). It also increases with the current number of searches ($\hat{\gamma}_3 > 0$), implying that the search cost is convex in the number of visits. On average, consumers pay a total search cost of CN¥1,184. Given that an average consumer visits 6.6 properties in total, this implies an average search cost of CN¥176 per visit.

Table 3: Estimates of Dynamic Parameters

| | | Est | SE |
|---|---|---|---|
| SD of match value | ($\sigma_v$) | 0.441*** | (0.029) |
| Mean waiting cost | ($w$) | 0.051*** | (0.016) |
| SD of waiting cost shock | ($\sigma_w$) | 0.049* | (0.027) |
| Search cost: (0.001) | | | |
| const. | ($\gamma_0$) | 3.080*** | (0.713) |
| $n$ | ($\gamma_1$) | 1.039** | (0.442) |
| (past searches)$\times n$ | ($\gamma_2$) | 4.231*** | (0.887) |
| $n^2$ | ($\gamma_3$) | 0.202** | (0.089) |
| scale parameter | ($\kappa$) | 3.258*** | (0.599) |

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

## 5.3 Model Fit

To assess how well our estimated model fits the data, we simulate for each consumer in the data her decision "path", which describes how long a consumer searches, the number of visits in each week during her search, and which property she purchases. We simulate 50 such paths for each consumer. Supplemental Appendix SD provides details on the simulation.

Table 4 reports summary statistics on search and purchase outcomes according to the data and according to our simulations. The first two columns summarize the observed data, where the summary statistics are taken across consumers. The last two columns summarize the simulated results, taking summary statistics across consumers and simulations. The first three rows report summary statistics on search duration, total visits, and recall share (i.e., the share of consumers who purchase a previously searched property). The last row reports the utility of the purchased property. For this variable, we report the average of $\delta_{j(i)}$ across consumers as the mean according to the data (in the first column), where $j(i)$ indicates the property purchased by consumer $i$ in the data. We report the standard deviation of $\delta_{j(i)} + v_{ij(i)}$, which is the square root of $var(\delta_{j(i)}) + \hat{\sigma}_v^2$, as the standard deviation according to the data (in the second column). In the last two columns where we report summary statistics according to our simulations, we report the mean and the standard deviation of $\delta_{j^m(i)} + v_{ij^m(i)}$, where $j^m(i)$ indicates the purchased property of consumer $i$ in a simulation indexed by $m$.

From Table 4, we can see that the estimated model fits the data fairly well. For example, the simulated average search duration and total visits are 3.627 weeks and 6.732 visits while their observed counterparts are 3.448 weeks and 6.710 visits.[18] Similarly, the simulated recall share is 16.1% while the observed recall share is 15.5%. The mean and standard deviation of the utility of the purchased property are (2.861, 1.935) according to simulations and (2.845, 1.944) in the data.

Table 4: Model Fit: Summary Statistics – Data vs. Simulation

| | Data | | Model Simulation | |
|---|---|---|---|---|
| | Mean | SD | Mean | SD |
| Search duration (weeks) | 3.448 | 3.900 | 3.627 | 3.044 |
| Total visits | 6.710 | 5.116 | 6.732 | 4.677 |
| Recall share | 0.155 | | 0.161 | |
| Utility of the purchased property | 2.845 | 1.944 | 2.861 | 1.935 |

Our estimated model also successfully fits the dynamic patterns of search. Figure 4 depicts how the number of searches varies over search weeks. Specifically, Figure 4(a) shows how search intensity changes over time according to the data and our simulations. According to the data, consumers visit on average about 4 properties in the first week, the number drops sharply in the second week, and it continues to drop as consumers search longer.[19] Our simulation based on the estimated model tracks this dynamic pattern well.

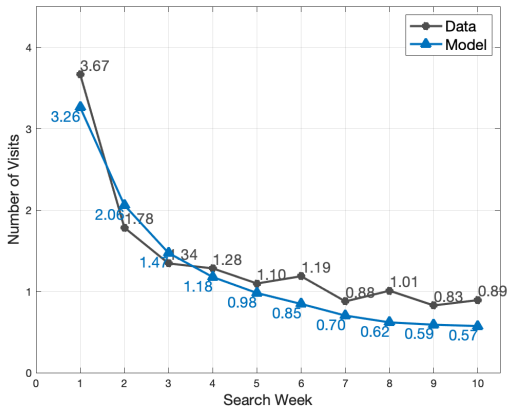
[18] These summary statistics (based on 1,000 randomly sampled consumers) are very close but not identical to those described in Section 2 (based on 26,543 consumers in the sample).

[19] The average number of visits in the $n^{th}$ week is taken over consumers who search for $n$ weeks or longer.

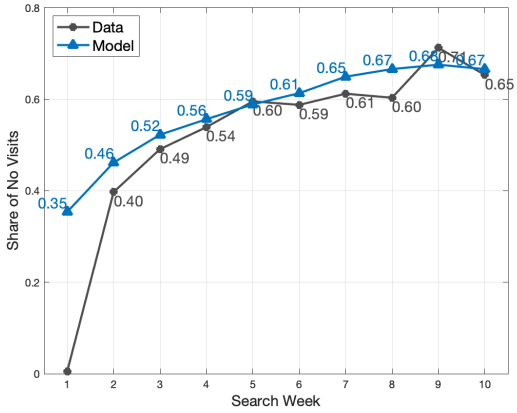
Figure 4(b) isolates the extensive margin: how the share of zero visits changes over time. It shows that the longer consumers search, the more likely they are to have zero visits in a week. Except for the first search week, our estimated model fits the share of zero visits well.[20]

Figure 4: Model Fit: Visits by Search Week – Data vs. Simulation

(a) Number of Visits




(b) Share of Zero Visits




Overall, our estimated dynamic search model fits the data well in both the static and the dynamic aspects of the data.

# 6 Comparison to a Static Search Model

In this section, we estimate a static search model and show that ignoring the dynamics leads to unreasonably large estimates of search costs. A model ignoring dynamics typically endogenizes the total number of searches and the property purchased, but ignores the number of searches in each period and the timing of purchases. It also ignores the dynamics in the search environment. There are two types of models with these features in the literature: a simultaneous search model and a sequential search model. In a simultaneous search model, a consumer searches a set of properties at once and then purchases one from the searched set. In a sequential search model, a consumer searches one property at a time and decides whether to continue the search after each visit. As pointed out by Santos, Hortaçsu and Wildenbeest (2012), in a classic sequential search model, a consumer purchases the last property visited and never recalls (unless she visits all properties). Since

[20]We do not fit the share of zero visits in the first search week well because the number of visits in the data is non-zero by construction (we define the first search week for a consumer as the first week with searches) but can be zero in our model, especially when the search cost shock is large.

more than 15% consumers in our data recall, we consider a simultaneous search model here.

## 6.1 A Static Search Model

The static model is similar to our dynamic search model except for two key differences. First, consumers in the static model choose the total number of visits instead of the number of visits in each period and the duration of the search. Second, the mean search cost in (4) is now $c(n) = \gamma_1 n + \gamma_2 n^2$. We no longer include a constant term $\gamma_0 \mathbb{1}(n > 0)$. This is because all consumers in the sample choose $n > 0$, where $n$ now represents the total number of visits instead of the number of visits in a period. We also do not include the covariate $m_{it}$, the cumulative number of searches prior to time $t$, because it is undefined in the static model.

The timing of the static model is the same as in the stage game of the dynamic model. First, consumer $i$ observes the information set $\Omega_i$ and the search cost shocks $(\vartheta_{in}, n = 0, ..., \bar{n})$ and decides the number of visits $n_i$. Recall that in the dynamic model, we distinguish between properties available for search at time $t$ ($\mathcal{A}_{it}$) and those searched before time $t$ ($\mathcal{R}_{it}$). There, the information set $\Omega_{it}$ consists of the mean utility $\delta_j$ for $j \in \mathcal{A}_{it}$ and both $\delta_j$ and the match value $v_{ij}$ for $j \in \mathcal{R}_{it}$. Here, in the static model, the information set $\Omega_i$ is simply $\{\delta_j\}_{j \in \mathcal{A}_i}$, i.e., the mean utilities for all properties in consumer $i$'s search market denoted by $\mathcal{A}_i$. Then, a search set $\mathcal{N}_i \in \mathcal{C}^{n_i}(\mathcal{A}_i)$ is sampled according to the probability given in (5) except that the subscript $t$ is dropped for the static model. After visiting the properties in the search set $\mathcal{N}_i$, consumer $i$ observes the match value of the searched set, i.e., $\{v_{ij}\}_{j \in \mathcal{N}_i}$. Finally, given the information $\Omega_i = \{\delta_j\}_{j \in \mathcal{A}_i}$ and $\{v_{ij}\}_{j \in \mathcal{N}_i}$, consumer $i$ decides which property to buy. This decision is denoted by $y_i$, where $y_i = j$ means that consumer $i$ chooses property $j \in \mathcal{N}_i$.

We estimate the search cost parameters $(\gamma_1, \gamma_2, \kappa)$ and the standard deviation of match values $(\sigma_v)$ using MLE. Let $\theta$ represent these parameters. The likelihood of observing that consumer $i$ searches $n_i$ properties and buys property $j$ out of her searched properties in $\mathcal{N}_i$ is given by:

$$l_i(\theta) = \Pr(n_i | \{\delta_j\}_{j \in \mathcal{A}_i}; \theta) \cdot \Pr(y_i = j | \{\delta_j\}_{j \in \mathcal{N}_i}; \theta), \tag{19}$$

where the probability that consumer $i$ purchases property $j$ is

$$\Pr(y_i = j | \{\delta_j\}_{j \in \mathcal{N}_i}; \theta) = \Pr(\delta_j + v_{ij} \geq \max_{j' \in \mathcal{N}_i} \delta_{j'} + v_{ij'}).$$

and the probability that consumer $i$ searches $n_i$ properties is similar to that in (15):

$$\Pr(n_i|\{\delta_j\}_{j\in\mathcal{A}_i};\theta) = \frac{exp\left\{\left[EB(n_i|\{\delta_j\}_{j\in\mathcal{A}_i};\theta) - C(n_i;\theta)\right]/\kappa\right\}}{\sum_{n=0}^{\bar{n}} exp\left\{\left[EB(n|\{\delta_j\}_{j\in\mathcal{A}_i};\theta) - C(n;\theta)\right]/\kappa\right\}},$$

where the expected benefit of searching $n$ properties is

$$EB(n|\{\delta_j\}_{j\in\mathcal{A}_i};\theta) = \sum_{\mathcal{N}\in C^n(\mathcal{A}_i)} E_{\{v_{ij}\}_{j\in\mathcal{N}}}(\max_{j\in\mathcal{N}}\ \delta_j + v_{ij}) \times \Pr(\mathcal{N}|\mathcal{A}_i, n).$$

## 6.2 Estimation Results Based on the Static Search Model

Table 5 reports the estimation results of the static search model. We have two findings from the comparison of these results with the estimation results of the dynamic model.

The first finding is that the static model yields a much smaller estimate of the standard deviation of the match value, implying a much smaller benefit from searching for a given set of available properties. The estimated standard deviation of match values is 0.235 compared to 0.441 in the dynamic model. These estimates correspond to a value of CN¥53,604 and CN¥100,593, respectively. Since the variance of the match value and the number and characteristics of available properties determine the benefits of searching (specifically, the larger the variance, the larger the benefits), the benefits of searching according to the estimated static search model are smaller than those according to the estimated dynamic search model for a given set of available properties.

Table 5: Estimates of the Static Search Model

| | | Est | SE |
|---|---|---|---|
| SD of match values | ($\sigma_v$) | 0.235*** | (0.014) |
| Search cost: (0.001) | | | |
| $n$ | ($\gamma_1$) | 301.994*** | (3.672) |
| $n^2$ | ($\gamma_2$) | -9.489*** | (0.086) |
| scale parameter | ($\kappa$) | 53.192*** | (0.449) |

*** $p < 0.01$

The static model yields a smaller estimate of $\sigma_v$ than the dynamic model because the dynamic model exploits more variation in the data to identify $\sigma_v$. In the static model, $\sigma_v$ is identified by comparing the mean utility of the purchased property to the highest mean utility among all searched properties. In the dynamic model, the parameter $\sigma_v$ is identified not only by such a static comparison (i.e., which property is purchased), but also by a dynamic feature of the data (i.e., when the purchased property is visited). If

a consumer purchases a property that she visited in an earlier period, we say that this consumer recalls. As explained in Section 4, a larger recall share implies a larger variance of the match value. In our estimation sample, the recall share is larger than 15%. As a result, our dynamic model yields a larger estimate of the variance of match values than the static model ignoring this dynamic aspect of the data.

The second and perhaps more important finding is that despite the smaller estimate of search benefits, the static model yields a much larger estimate of search costs, in fact, an unreasonably high estimate of search costs in our view. According to the estimated static model, a consumer pays an average search cost of CN¥308,177, more than two orders of magnitude higher than in the dynamic model (CN¥1,184). Given that a consumer in our estimation sample visits an average of 6.62 properties before buying, the average search cost for each property is approximately CN¥46,000, or $6,900 according to the estimated static model. We find this estimate unreasonably high.

The static model gives unreasonably high search costs because it does not take into account the dynamics in the search environment. It pools properties on the market across time, ignoring the possibility of exits and the fact that some properties enter the market only later. Because it mistakenly considers all these properties to be available to search, it overstates the benefits of searching (despite the underestimation of the variance of match values). As a results, it requires an overestimation of the search costs to explain the observed number of searches.

# 7 Effects of Changes in Search Environment and Effects of Search Costs

Having established the importance of considering dynamics in our setting, we now quantify how changes in the search environment affect consumers' search and purchase decisions. There are three changes to the search environment: price increases, new listings entry, and existing listings exit. We consider the effects of each change through three counterfactual simulations (CF1, CF2, CF3). Consumers' search and purchase behavior is also affected by search frictions such as search costs. To quantify the relative importance of the traditional search friction such as search costs versus the effects of the dynamics in the search environment, we consider another counterfactual scenario in which consumers' search costs are reduced by half (CF4) and compare the results with those in CF1–3.

In each counterfactual simulation, we simulate 50 decision paths for each consumer and report the mean and standard deviation of outcome variables that capture consumer

search behavior (search duration and total visits), purchase choice (utility of purchased property), and costs (search cost and waiting cost). Details of the simulation are in Appendix SD.

## 7.1 Effects of Changing Search Environment

To examine the effects of price increases, we consider a counterfactual scenario where the rate at which the list price of new properties increases over time is reduced by half. Recall that the final price is the list price minus a (transaction - list) price gap plus a price shock. Therefore, the transaction price is also reduced.

We find that as the price change becomes slower, consumers search longer, visit more properties before buying, and buy properties that generate higher utilities. Table 6 reports summary statistics of the main endogenous outcomes in CF1. For comparison, we also include the outcomes under the actual price change in column (I). From this table, we can see that the average search duration increases from 3.448 with the observed price increase rate to 3.851 with half the actual price increase rate (Row (1)). Similarly, the average total number of visits increases from 6.710 to 6.842 (Row (2)). As consumers search longer and more, they end up buying a property that generates a higher utility. Specifically, Row (3) shows that the average utility of the purchased property increases from 2.845 to 3.663, corresponding to an increase in value of CN¥186,588 ($=(3.663\text{-}2.845)/4.384\times10^6$). Part of the increase in utility comes mechanically from a lower price. To remove such a mechanical change that contributes to the utility increase, or equivalently, to isolate the effect of longer and more searches on utility, we also report in Row (3') what the utility of the purchased property would be at the observed price. Under the actual price change, Rows (3) and (3') are, of course, identical. When the price increase rate is half, the utility at the observed (higher) price in row (3') is, unsurprisingly, smaller than that in Row (3). Nevertheless, even ignoring the increase in utility due to lower prices, there is an increase in utility equivalent to CN¥124,316 ($=(3.390\text{-}2.845)/4.384\times10^6$) according to Row (3'). In other words, searching longer and more contributes to 67% of the increase in utility when the price change is halved.

While searching longer and more leads to an increase in utility, it results in higher search and waiting costs at the same time. The sum of search costs and waiting costs increases by an average of CN¥3,751 (Rows (4) and (5)), which is about 3% of the gain due to longer and more searches (CN¥124,316, Row (3')). Therefore, on balance, consumers are better off with a slower price change even ignoring the mechanical gain from lowered prices.

Table 6: Counterfactual Simulation Results

| | (I) Actual | | (II) CF1 Half Price Increase Rate | | (III) CF2 Double Arrival Rate | | (IV) CF3 Half Exit Rate | | (V) CF4 Half Search Costs | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | SD | Mean | SD | Mean | SD | Mean | SD | Mean | SD |
| (1) Search duration (week) | 3.448 | 3.900 | 3.851 | 3.266 | 4.265 | 3.617 | 3.723 | 3.145 | 3.760 | 3.247 |
| (2) Total visits | 6.710 | 5.116 | 6.842 | 4.800 | 7.761 | 4.735 | 6.653 | 4.737 | 9.093 | 6.528 |
| (3) Utility | 2.845 | 1.944 | 3.663 | 1.974 | 3.376 | 2.403 | 2.931 | 2.075 | 3.371 | 2.305 |
| (3') at the observed price | | | 3.390 | 1.973 | | | | | | |
| (4) Search cost (CN¥) | 1,184 | 1,796 | 1,448 | 1,965 | 1,965 | 2,890 | 1,220 | 1,803 | 1,302 | 2,039 |
| (5) Waiting cost (CN¥) | 26,259 | 12,929 | 29,746 | 14,231 | 36,885 | 19,655 | 27,753 | 13,464 | 28,612 | 16,851 |

To quantify how the entry of new listings and the exit of existing listings affect consumers' search and purchase decisions, we consider a counterfactual scenario in which the entry rate (i.e., the arrival rate of new listings) is doubled in CF2 and a scenario in which the exit rate (i.e., the probability that an existing listing will exit) is halved in CF3. In both counterfactual scenarios, consumers have more incentives to search longer compared to the actual search environment because consumers either expect more new listings in the future or face a lower probability that her previously visited properties will exit the market.

The comparison of column (I) and CF2 in column (III) of Table 6 shows that, when consumers anticipate more new listings in the future, they indeed search longer, visit a greater number of properties before making a purchase, and ultimately purchase properties that generate higher utilities. As a result, their utility from the purchased property increases by 0.531, which is equivalent to an increase in value of CN¥121,150. Meanwhile, the average search cost increases by CN¥781 and the average waiting cost increases by CN¥10,626. Overall, consumers are better off.

Similarly, the comparison of column (I) and CF3 in column (IV) shows that, in anticipation of a slower exit of existing listings, consumers search for more weeks and purchase a more desirable property. Overall, the average increase in utility is equivalent to CN¥19,617, which more than offsets the average increase in search costs of CN¥36 and waiting costs of CN¥1,494.

## 7.2 Effects of Search Costs

While Section 7.1 quantifies the effects of changes in the search environment on consumers' search and purchase behavior, we now quantify the effects of the traditional search friction, i.e., search costs. To this end, we simulate a counterfactual scenario where we reduce the search cost function $C_{it}(n)$ by half in CF4.

The comparison of column (I) and CF4 in column (V) of Table 6 shows that when con-

sumers face a lower search cost per visit, they unsurprisingly extend their search duration by 0.312 weeks and visit 2.383 more properties on average. With more searches, consumers find properties that generate higher utility for them. The average increased utility is equivalent to CN¥119,982. Interestingly, despite the reduced search cost per visit, consumers end up incurring higher search costs due to more searches. However, the increased search costs (by CN¥118) and the increased waiting costs (by CN¥2,353) are dominated by the increased utility.

Table 7 summarizes the average net gain per consumer. To compute the average net gain per consumer, we first compute changes in the average utility, search costs, and waiting costs from the actual scenario (column (I) in Table 6) to a counterfactual scenario (columns (II)–(V) in Table 6) and then calculate (change in utility)/$\hat{\alpha}-$(change in search cost and waiting cost). The average net gain for consumers due to a reduction in search costs by half is CN¥117,511. This compares to an average net gain of CN¥124,316 when consumers face half the price increase speed (where utility is calculated based on actual prices), CN¥109,743 when the new listing arrival rate is doubled, and CN¥18,094 when the exit rate is halved.

Table 7: Net Gains

| | CF1: (II)-(I) | CF2: (III)-(I) | CF3: (IV)-(I) | CF4: (V)-(I) |
|---|---|---|---|---|
| Average Net Gain | 124,316 | 109,743 | 18,094 | 117,511 |

Overall, these results show that while traditional search frictions such as search costs affect consumers' search and purchase decisions, changes of the search environment also have significant effects on consumers' search and purchase behavior and consumer welfare. At least for the same percentage change (halving or doubling), varying the dynamics of the search environment, especially in terms of the price increase speed and the new listing arrival rate, has a comparable impact as varying search costs.

# 8 Effects of Housing Supply Policies

Many cities around the world, particularly those experiencing a housing crisis, have policies to encourage new listings in order to increase the supply of housing and address their housing shortages. For example, Melbourne's Vacant Residential Property Tax, Oakland's Vacant Property Tax, Toronto's Vacant Home Tax, and Vancouver's Empty Homes Tax all aim to increase the supply of housing. These policies are likely to both increase the number of new listings and reduce the upward price trend. In this section we simulate the

effects of such policies by simulating the outcomes under different new listing arrival rates and price trends. Specifically, we scale the observed new listing arrival rate up by $\phi_1 \geq 1$ and the price trend down by $\phi_2 \leq 1$. In other words, we consider scenarios where, for each consumer $i$, the arrival rate of new listings is $\phi_1 \lambda_i$, and the weekly increase in the list price of new properties is $\phi_2 \rho_i$.

Figure 5 presents the simulation results in terms of search duration and total visits and shows that as $\phi_1$ increases and $\phi_2$ decreases, consumers on average search longer and visit more properties before buying. Panel (a) shows that when the price increase trend is slowed from the original rate ($\phi_2 = 1$) to half the original rate ($\phi_2 = 0.5$), the average search duration increases from 3.68 to 3.85 (top row of panel (a)). Similarly, when the new listing arrival rate is doubled (from $\phi_1 = 1$ to $\phi_1 = 2$), the average search duration increases from 3.68 to 4.26 (rightmost column of panel (a)). A combination of an increase in the new listing arrival rate and a decrease in the price trend can lead to an increase in the average search duration to 4.59 (when $\phi_1 = 2$ and $\phi_2 = 0.5$). Moreover, these two changes reinforce each other: the effect of increasing the new listing arrival rate is greater when the price increase is slower and the effect of reducing the speed of the upward price trend is greater when the new listing arrival rate is higher. For example, the average search duration increases by 0.58 weeks (from 3.68 to 4.26) when $\phi_2 = 1$ and by 0.75 weeks (from 3.85 to 4.59) when $\phi_2 = 0.5$. Turning to the total number of visits, Panel (b) shows that consumers visit more properties before buying as $\phi_1$ increases and $\phi_2$ decreases. This is largely because they search for a longer period of time (panel (a)), rather than searching more in each period. The average number of visits per week is rather stable and varies between 2.25 and 2.4.

Figure 5: Effects of Housing Supply Policies: Search Duration and Total Visits

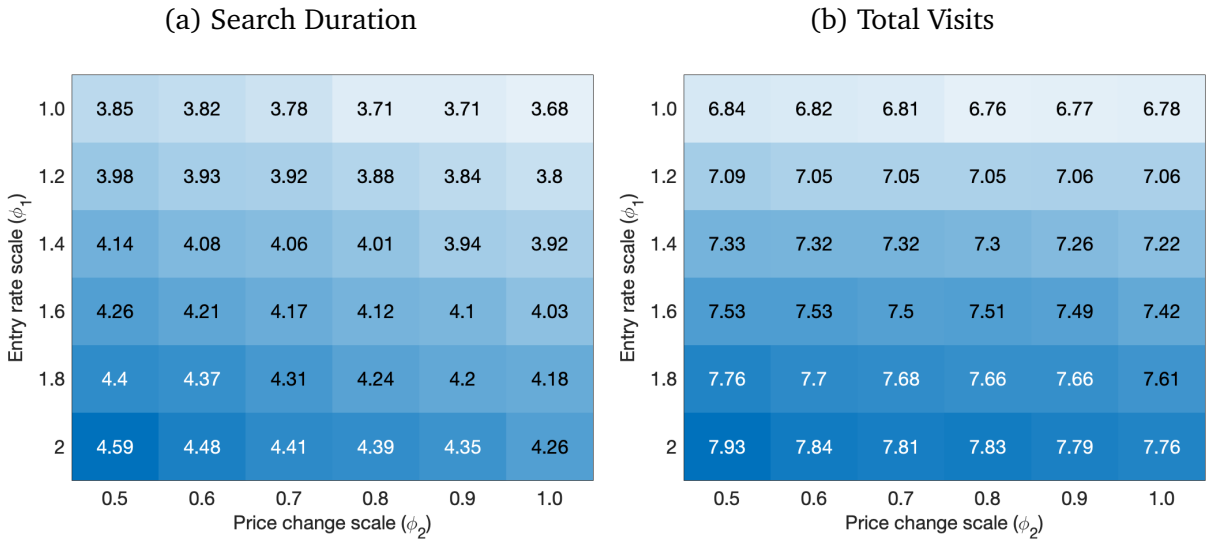
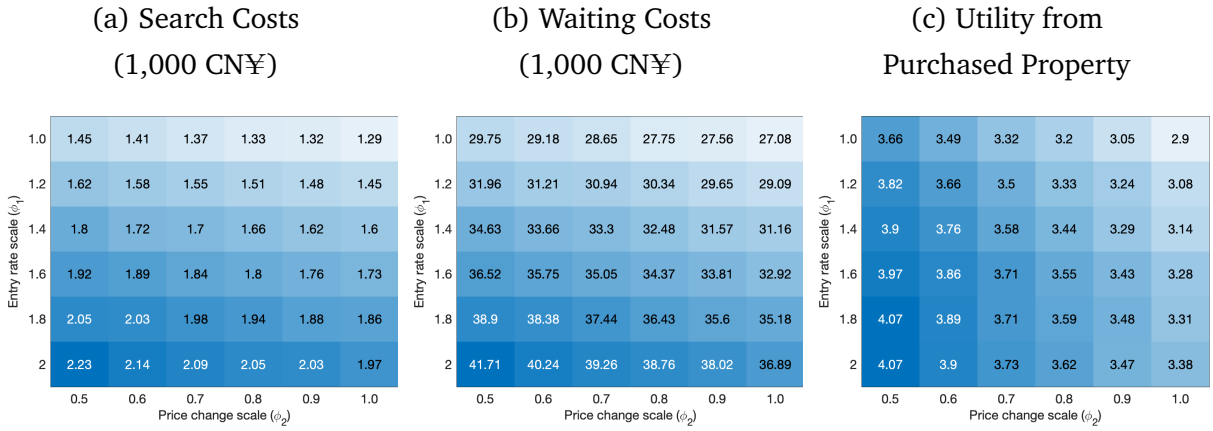


Figure 6 shows that as $\phi_1$ increases and $\phi_2$ decreases, the increased search duration and total visits lead to higher search costs (panel (a)) and waiting costs (panel (b)) but better search outcomes (panel (c)). In particular, the average search costs increase from CN¥1,290 to CN¥2,230 and the average waiting costs increase from CN¥27,080 to CN¥41,710 when the new listing arrival rate is doubled and the price trend is halved. At the same time, consumers end up buying properties that generate higher utility.

Figure 6: Effects of Housing Supply Policies: Search Costs, Waiting Costs, and Utility




As for the net effects (taking into account the better search outcomes as well as the increased search and waiting costs), panel (a) of Figure 7 shows that the net effects of housing supply policies are positive. In this panel, we present the average net gain per consumer as defined in Table 7. We can see that the gains from searching longer and visiting more properties outweigh the increased search and waiting costs. The effect of such a policy becomes stronger when it increases the arrival rate of new listings and decelerates the price more by a bigger margin. If the arrival rate is doubled and the price increase is halved, the average net gains per consumer can be as high as CN¥249,200, or about 7% of the average transaction price.

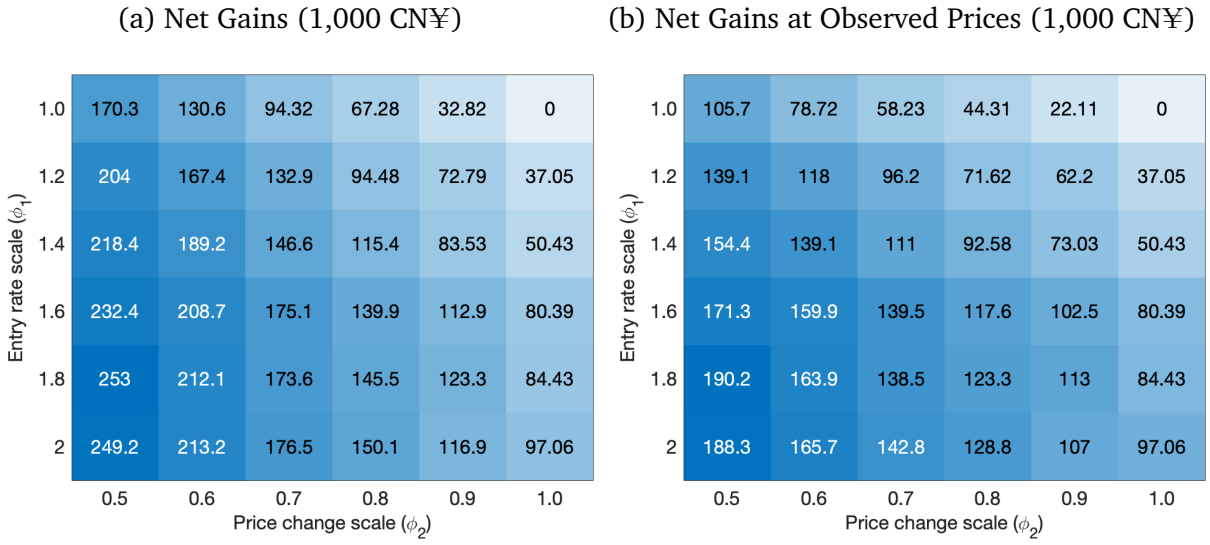
There are potentially three channels through which consumers benefit from such policies. First, consumers are better off mechanically because they have more properties to choose from. Second, consumers are better off because of lower prices. Third, consumers are better off due to changes in consumer search behavior in response to changes in the dynamics of the search environment. In our setting, consumers only choose among properties they have visited, and thus more listings do not mechanically increase consumer welfare without consumers changing their search behavior. To assess the importance of the second channel relative to the third, we recompute the utility using the prices in the

data in panel (b) of Figure 7.[21]

The comparison of panels (a) and (b) of Figure 7 shows that consumers' gains come mainly from the third channel, i.e., from searching longer and visiting more properties. For example, when $(\phi_1 = 0.5, \phi_2 = 2)$, the average net gains per consumer at the observed prices are CN¥188,300, which accounts for 75.5% of the total net gains considering changes in both prices and consumer search behavior.

Overall, these simulations suggest that housing supply policies can significantly benefit consumers if they encourage new supply and lower prices. This is mainly because they influence the dynamics of the search environment, which incentivizes consumers to search longer, visit more properties, and ultimately find properties that generate more utility for them.

Figure 7: Effects of Housing Supply Policies: Net Gains




# 9 Conclusion

We study how dynamics in the search environment affect consumers' search and purchase decisions and welfare. We present a dynamic model in which consumers make search and purchase decisions knowing that product prices and availability both change over time. We develop a feasible estimation routine to estimate our dynamic search model. We apply our model and estimation approach to the Beijing housing market between 2015 and

[21] For a simulated property $j$ that is not in the data (for example, in a counterfactual simulation with a higher arrival rate of new listings), we adjust its price as follows. Suppose consumer $i$ buys property $j$ at time $t$, we adjust this property's price by subtracting $(1-\phi_2)\rho_i(\text{EntryWeek}_j - \text{SampleStartWeek})$ to its simulated price, where $(1-\phi_2)\rho_i$ is the change in the weekly increase in the list price.

2016, and quantify the importance of dynamics in the search environment. We find that changes in product prices and availability have a significant effect on consumers' search and purchase decisions and welfare. We also find that a static search model would yield an unreasonably high estimate of search costs. Finally, we find that housing supply policies that increase new listings and slow down upward price trend can benefit consumers and that the benefits are mainly driven by consumers' response to changes in the search environment.

While our model is developed in the context of the housing market and the estimation approach is used to study the Beijing housing market, the model and the estimation approach are potentially applicable to many settings where consumers make purchase decisions after searching and the search environment changes over time. Moreover, the estimation procedure involves an extension of a discrete choice model from a single-option choice to a set-of-options choice, which is also generalizable to other settings where consumers choose a set of products instead of a single product.

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# Appendices

## A Micro Foundation for the Sampling Probabilities

We have three goals in this section. First, we show that the sampling probability $\Pr(\mathcal{N}|\mathcal{A}_{it}, n)$ in (5) can be derived from an extension of a discrete choice model allowing for a single-option choice to allowing for a set-of-options choice. Second, we show that this sample probability implies a probability for each option $j$, i.e., $\Pr(j|\mathcal{A}_{it}, n)$ in (6). Third, we show that there is a unique vector of mean utilities that solve the search share equations in (11). In fact, we show that the mapping used to invert out the mean utilities in Berry, Levinsohn and Pakes (1995) is a contraction mapping even in our extended model. We suppress subscripts $i$ and $t$ in Sections A.1 and A.2 and for simplicity.

### A.1 Micro Foundation for the Sampling Probability $\Pr(\mathcal{N}|\mathcal{A}, n)$

In this section, we show that the sampling probability $\Pr(\mathcal{N}|\mathcal{A}, n)$ in (5) is consistent with an extension of a Logit model from choosing a single option to choosing $n$ options.[22] In this probability, $\mathcal{A}$ represents the set of all options and $\mathcal{N} \subseteq \mathcal{A}$ represents a subset of $\mathcal{A}$ with $n$ elements. Let $\mathcal{C}^n(\mathcal{A})$ be the collection of $n$-element subsets of $\mathcal{A}$. Then, $\mathcal{N} \in \mathcal{C}^n(\mathcal{A})$.

We assume that the value associated with an option $j$ in $\mathcal{A}$ is $\delta_j + \epsilon_j$, where $\epsilon_j$ is i.i.d. and follows a type-1 extreme value distribution. We further assume that the $n$ highest valued options are sampled. In other words, $\mathcal{N} \in \mathcal{C}^n(\mathcal{A})$ is sampled if and only if $\min_{j \in \mathcal{N}} \delta_j + \epsilon_j \geq \max_{h \in \mathcal{A} \setminus \mathcal{N}} \delta_h + \epsilon_h$. Therefore,

$$\Pr(\mathcal{N}|\mathcal{A}, n) = \Pr(\delta_j + \epsilon_j \geq \max_{h \in \mathcal{A} \setminus \mathcal{N}} (\delta_h + \epsilon_h),\ \forall j \in \mathcal{N}).$$

This model is an extension of a standard Logit model from restricting $n = 1$ to any $n \geq 1$. When $n = 1$, it becomes a standard Logit model. We derive $\Pr(\mathcal{N}|\mathcal{A}, n)$ in (5) based on the inclusion-exclusion principle and the choice probability in a standard Logit model in Supplemental Appendix SE.

[22] An alternative expression of this probability is given in Moraga-Gonzalez, Sándor and Wildenbeest (2024), which can be derived from a model where a consumer chooses a sampled set $\mathcal{N}$ to maximize ("inclusive value of set $\mathcal{N}$"$-LogitError_{i\mathcal{N}}$) , where the Logit error term is i.i.d. and varies at the set level. We choose our model because we think Logit errors corresponding to overlapping sets are unlikely to be independent and those corresponding to sets of different sizes are unlikely to be identically distributed.

## A.2 Implied $\Pr(j|\mathcal{A},n)$ and Its Properties

### A.2.1 The Analytic Expression for $\Pr(j|\mathcal{A},n)$

The probability of sampling a set $\Pr(\mathcal{N}|\mathcal{A},n)$ implies the probability that a single option is sampled conditional because $\Pr(j|\mathcal{A},n)=\sum_{\mathcal{N}\in\mathcal{C}^n(\mathcal{A}):j\in\mathcal{N}}\Pr(\mathcal{N}|\mathcal{A},n)$. When $n=0$, $\Pr(j|\mathcal{A},n)=0$. When $n=A=\#\mathcal{A}$, $\Pr(j|\mathcal{A},n)=1$. When $0<n<A$, we show in Supplemental Appendix SE that

$$\Pr(j|\mathcal{A},n)=\sum_{k=0}^{n-1}\left[(-1)^k\sum_{\mathcal{B}\in\mathcal{C}^{A-n+k}(\mathcal{A}\backslash j)}\left(\frac{C_{A-n+k-1}^{k}exp(\delta_j)}{\sum_{l\in\mathcal{B}}exp(\delta_l)+exp(\delta_j)}\right)\right]. \tag{A.1}$$

The intuition for this analytic expression relies on two points. First, option $j$ is sampled if it's rank, according to value $\delta_j+\epsilon_j$, is no more than $n$ among all options in $\mathcal{A}$. In other words, $\Pr(j|\mathcal{A},n)=\sum_{k=0}^{n-1}\Pr(j\text{'s rank is } n-k)$. Second, the probability that $j$ has a certain rank is related to the probability that $j$ is the best in a subset of options. In (A.1), $\frac{exp(\delta_j)}{\sum_{l\in\mathcal{B}}exp(\delta_l)+exp(\delta_j)}$ is the probability that $j$ is the best in the subset $\mathcal{B}\cup\{j\}$. In Supplemental Appendix SE, we use a simple example to illustrate these two points.

### A.2.2 Three Properties of $\Pr(j|\mathcal{A},n)$

We now show the three intuitive properties of $\Pr(j|\mathcal{A},n)$

**Property 1. $\Pr(j|\mathcal{A},n)$ is increasing in $\delta_j$ and decreasing in $\delta_k$ for $k\neq j$.** Option $j$ is sampled if and only if $\delta_j+\epsilon_j$ is among the top $n$ highest values in $\{\delta_{j'}+\epsilon_{j'}\}_{j'\in\mathcal{A}}$. Since an increase in $\delta_j$ leads to an increase in the probability that option $j$ is among the top-$n$ options, $\Pr(j|\mathcal{A},n)$ is increasing in $\delta_j$. Similarly, since an increase in $\delta_k$ for $k\neq j$ leads to a decrease in the probability that option $j$ is among the top-$n$ options, $\Pr(j|\mathcal{A},n)$ is decreasing in $\delta_k$ for $k\neq j$.

**Property 2. $\Pr(j|\mathcal{A},n)$ becomes the choice probability in a standard Logit model when $n=1$.** When $n=1$, $\Pr(j|\mathcal{A},n)$ in (A.1) becomes

$$\Pr(j|\mathcal{A},1)=\sum_{\mathcal{B}\in\mathcal{C}^{A-1}(\mathcal{A}\backslash j)}\left(\frac{exp(\delta_j)}{\sum_{l\in\mathcal{B}}exp(\delta_l)+exp(\delta_j)}\right)=\frac{exp(\delta_j)}{\sum_{l\in\mathcal{A}}exp(\delta_l)},$$

because $\mathcal{C}^{A-1}(\mathcal{A}\backslash j)$ has a singleton element, i.e., $\mathcal{A}\setminus j$.

**Property 3. The sum of $\Pr(j|\mathcal{A},n)$ across $j$ in $\mathcal{A}$ is $n$, i.e., $\sum_{j\in\mathcal{A}} Pr(j|\mathcal{A},n)=n$.** When $n=A$, $\Pr(j|\mathcal{A},A)=1$ and, therefore, $\sum_{j\in\mathcal{A}}\Pr(j|\mathcal{A},A)=A$. We now consider the case where $n<A$.

$$\begin{aligned}\sum_{j\in\mathcal{A}}\Pr(j|\mathcal{A},n) &= \sum_{j\in\mathcal{A}}\left\{\sum_{k=0}^{n-1}\left[(-1)^k\sum_{\mathcal{B}\in\mathcal{C}^{A-n+k}(\mathcal{A}\setminus j)}\left(\frac{C^k_{A-n+k-1}exp(\delta_j)}{\sum_{l\in\mathcal{B}}exp(\delta_l)+exp(\delta_j)}\right)\right]\right\}\\ &= \sum_{k=0}^{n-1}(-1)^k\left[\sum_{j\in\mathcal{A}}\left\{\sum_{\mathcal{B}\in\mathcal{C}^{A-n+k}(\mathcal{A}\setminus j)}\left(\frac{C^k_{A-n+k-1}exp(\delta_j)}{\sum_{l\in\mathcal{B}}exp(\delta_l)+exp(\delta_j)}\right)\right\}\right].\end{aligned}$$

Let $\mathcal{D}$ represent $\mathcal{B}\cup\{j\}$. Since $\mathcal{F}\in\mathcal{C}^{A-n+k}(\mathcal{A}\setminus j)$ and $\{j\}$ have no intersect, we can rewrite the above equation as

$$\sum_{j\in\mathcal{A}}\Pr(j|\mathcal{A},n)=\sum_{k=0}^{n-1}(-1)^k\left[\sum_{\mathcal{D}\in\mathcal{C}^{A-n+k+1}(\mathcal{A})}\left\{\sum_{j\in\mathcal{D}}\left(\frac{C^k_{A-n+k-1}exp(\delta_j)}{\sum_{l\in\mathcal{D}}exp(\delta_l)}\right)\right\}\right],$$

which can be further simplified to be:

$$\begin{aligned}\sum_{j\in\mathcal{A}}\Pr(j|\mathcal{A},n) &= \sum_{k=0}^{n-1}(-1)^k\left[\sum_{\mathcal{D}\in\mathcal{C}^{A-n+k+1}(\mathcal{A})}C^k_{A-n+k-1}\right] \qquad \text{(A.2)}\\ &= \sum_{k=0}^{n-1}(-1)^k C_A^{A-n+k+1}C^k_{A-n+k-1}\\ &= n.\end{aligned}$$

We provide a proof for the last equality, i.e., $\sum_{k=0}^{n-1}(-1)^k C_A^{A-n+k+1}C^{A-n-1}_{A-n+k-1}=n$ in Supplemental Appendix E.4.

## A.3 The System $\tilde{s}_j(\boldsymbol{\delta}_d)=s_j, j\in\mathcal{J}_d$ Has a Unique Solution

In this section, we show that, under certain regularity conditions, the system of equations $(\tilde{s}_j(\boldsymbol{\delta}_d)=s_j, j\in\mathcal{J}_d\setminus\{1\})$ has a unique solution $\boldsymbol{\delta}_d=(\delta_j, j\in\mathcal{J}_d)$ such that $\delta_1=0$. Recall that we normalize the mean utility of the first property in a district $d$ to 0 and that the search share function is

$$\tilde{s}_j(\boldsymbol{\delta}_d)=\frac{\sum_{i\in\mathcal{I}_d}\sum_{t=1}^{T_i}\Pr(j|\mathcal{A}_{it},n_{it})}{\sum_{i\in\mathcal{I}_d}\sum_{t=1}^{T_i}n_{it}}. \qquad \text{(A.3)}$$

The regularity conditions are:

1. $0 < s_j < \frac{\sum_{i\in\mathcal{I}}\sum_{t=1}^{T_i}\mathbb{1}(j\in\mathcal{A}_{it})}{\sum_{i\in\mathcal{I}}\sum_{t=1}^{T_i} n_{it}}$.

2. For any $it$, $\#\{\delta_j : j \in \mathcal{A}_{it}, \delta_j = \infty\} \leq n_{it}$.

3. For any $it$, $\#\{\delta_j : j \in \mathcal{A}_{it}, \delta_j = -\infty\} \leq A_{it} - n_{it}$.

Condition 1 imposes a constraint on the observed search share $s_j$. It means that each property is visited at least once ($s_j > 0$) but not to the extent that it is visited whenever it is in a consumer's available-to-search set ($s_j < \frac{\sum_{i\in\mathcal{I}}\sum_{t=1}^{T_i}\mathbb{1}(j\in\mathcal{A}_{it})}{\sum_{i\in\mathcal{I}}\sum_{t=1}^{T_i} n_{it}}$). This condition is an extension to the condition $0 < s_j < 1$ in a single discrete choice model.

Conditions 2 and 3 impose constraints on $n_{it}$ in the data. Condition 2 implies that if a consumer's available-to-search set includes properties with a mean utility of $\infty$, the consumer visits these properties. Condition 3 implies that if a consumer's available-to-search set includes properties with a mean utility of $-\infty$, the consumer does not visit these properties.

In what follows, we suppress the subscript $d$ for simplicity, use $\mathcal{J}$ to represent $\mathcal{J}_d\backslash\{1\}$, and use $J$ to denote the cardinality of $\mathcal{J}$. Recall that we normalize $\delta_1$ to 0. Following Berry, Levinsohn and Pakes (1995), we define a mapping $f : R^J \to R^J$ as

$$f_j(\boldsymbol{\delta}) = \delta_j + \ln s_j - \ln \tilde{s}_j(\boldsymbol{\delta}) \tag{A.4}$$

for $j \in \mathcal{J}$. In Supplemental Appendix SE, we show that this mapping is a contraction mapping. Therefore, its fixed point is the unique solution to the system of equations $\tilde{s}_j(\boldsymbol{\delta}) = s_j, j \in \mathcal{J}$.

# B Discussion of the Two-Step Estimation Procedure

In this two-step estimation procedure, we back out the mean utility and estimate the static parameters in the utility function before estimating the dynamic parameters.

For this two-step procedure to work, we need to be able to write down the model implication of search shares without solving the dynamic search model. We can do this because we assume while consumers choose the number of properties to search in each period, the set of properties they visit is exogenous. A similar exogeneity assumption is also made, for example, in Hortaçsu and Syverson (2004). We extend their specification of the sampling probability for a product to the sampling probability for a set of products. According to our sampling probability specification, a property with a higher mean utility has a higher probability of being sampled. The exogenous assumption only means that a

consumer and her agent do not have full control over the set of properties they can visit in a given period, certain random factors also play a role in determining the set of sampled properties, and these random factors are unknown to the consumer and thus exogenous. We believe that this exogeneity assumption is justified.

The advantage of the two-step estimation procedure is fourfold. First, following such a two-step procedure, we can allow for unobservable property heterogeneity (i.e., $\xi_j$ in the utility function) and allow list prices to be endogenous. If we were to estimate the utility parameters along with the dynamic parameters using maximum likelihood estimation, we would either have to assume that there is no unobservable property heterogeneity (so that there is no endogeneity in prices) or model how list prices are correlated with the unobservable property heterogeneity (to control for the endogeneity of prices).[23]

Second, this procedure allows us to include a large set of neighborhood fixed effects in the utility function without significantly increasing the computational burden because these parameters are estimated in the first stage without solving the dynamic model. While we have data on a set of observable property characteristics, it would be difficult to collect data on neighborhood amenity. Including neighborhood fixed effects thus allows us to control for important property heterogeneity at the neighborhood level.

Third, this procedure helps us to address the issue of high dimensionality of state variables. In our model, there are a large number of state variables, which include the characteristics of all unsearched products as well as the characteristics and match values of all searched products. The dynamic demand literature (e.g., Gowrisankaran and Rysman, 2012) addresses this challenge by reducing high-dimensional state variables to a one-dimensional inclusive value and assuming that the inclusive value evolves according to a stationary Markov process. However, unlike a dynamic demand model where state variables describe the choice set and the choice set evolves exogenously, in our model with both dynamic search and dynamic demand, state variables describes two sets (the searched and unsearched sets) and both sets evolve endogenously. Thus, it is inappropriate to assume that the inclusive value of each set evolves according to a stationary Markov process. We tackle the issue of high dimensionality by estimating the mean utilities of properties "offline", i.e., before estimating the dynamic parameters, and considering state variables to include the mean utilities of the top properties, the average mean utility of the remaining properties, and the number of properties.

Fourth, we back out the mean utilities of all properties in the sample in the first step,

[23]This is because, in this case, consumers' search and purchase decision would depend on $\xi_j$'s. Therefore, the likelihood function would be an integral over the distribution of $\xi_j$'s *conditional on the observed list prices*. We would need a model to describe such a conditional distribution.

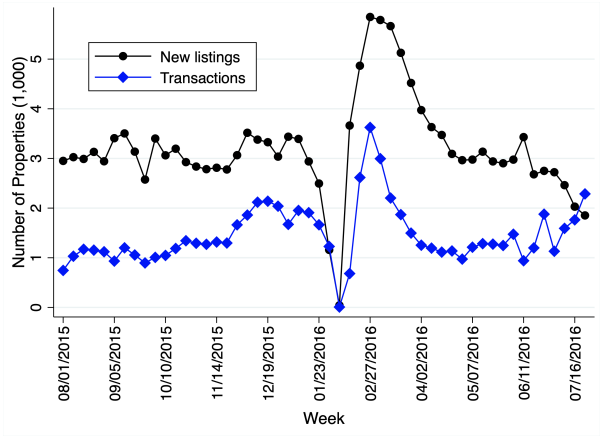
which allows us to estimate the transition of the search environment before estimating the dynamic model. For example, the mean utility of new listings follows a normal distribution $N(\mu_{it}^{new}, (\sigma_i^{new})^2)$. We estimate $\mu_{it}^{new}$ and $\sigma_i^{new}$ based on the inverted $\delta_j$ for new listings in the sample.

# SA Additional Figures

Figure SA.1 shows the number of new listings, the number of transactions, the average list price, and the average transaction prices by week, including the eight weeks during the Chinese New Year.

Figure SA.1: New Listings, Transactions, and Prices by Week

(a) New Listings and Transactions




(b) List Prices and Transaction Prices

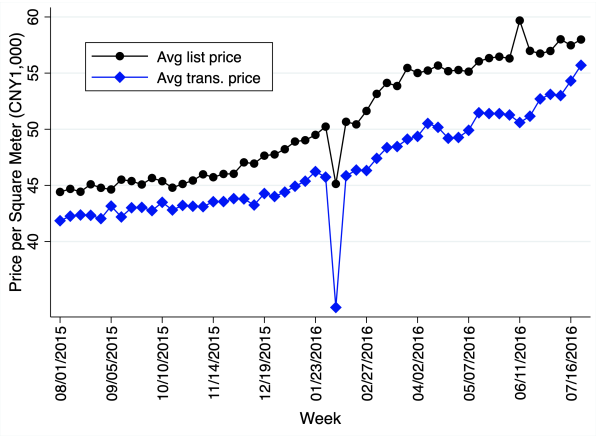


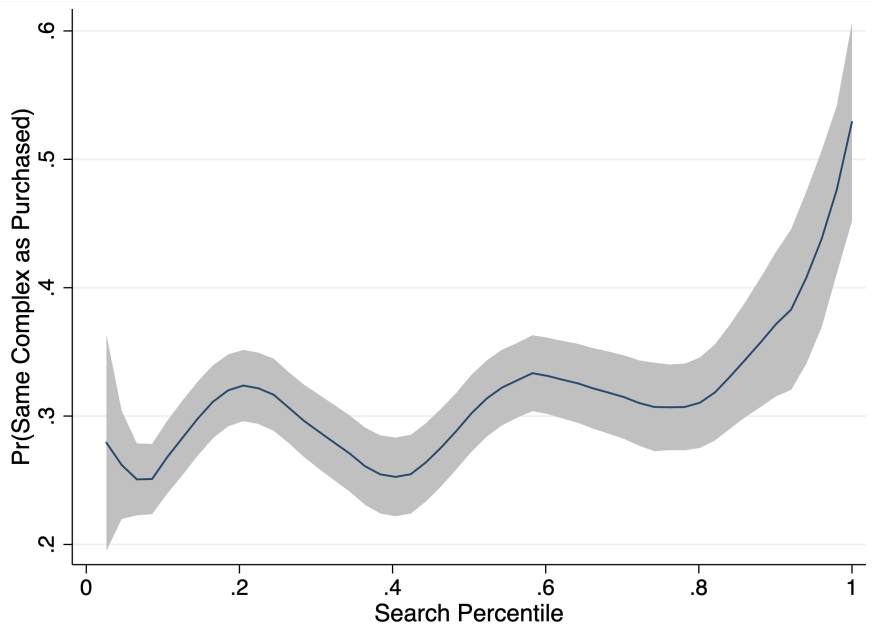

Figure SA.2 plots the probability of searching a property in the same residential complex as the purchased property against the search percentile. Specifically, for the x-axis, we define the search percentile as follows. Suppose consumer $i$ searches for $T_i$ weeks and visits $(n_{i1}, n_{i2}, ..., n_{iT_i})$ properties in each week. If she visits property $j$ in search week $t$, then the search percentile of property $j$ for consumer $i$ is $(\text{Search Percentile})_{ij} = (\sum_{\tau \leq t} n_{i\tau})/(\sum_{\tau \leq T_i} n_{i\tau})$. For the y-axis, we first define a dummy variable $\mathbb{1}(\text{Same Complex as Purchased})_{ij}$, which takes the value 1 if property $j$ is in the same residential complex as the property that consumer $i$ purchases, and 0 otherwise.

We run a kernel regression of $\mathbb{1}(\text{Same Complex as Purchased})_{ij}$ on $(\text{Search Percentile})_{ij}$ and plot the smoothed values in the solid line and the 95% confidence interval using the shaded band in Figure SA.2. The sample consists of $(i, j)$ combinations where consumer $i$ searched property $j$. In this regression, we exclude the purchased property for each consumer as well as properties visited by the consumer in a week when there is no available property in the same complex as the purchased property.

This figure shows the probability that a searched property is in the same complex as the purchased property goes up and down as a consumer approaches the end of her search. For example, this probability is roughly the same when the search percentile is 20%, 60%, or 85%. And this probably is lower than 40% even at the 90% search percentile. Overall,

this figure does not indicate a clear learning pattern.

Figure SA.2: How Pr(Same Complex as Purchased) Varies During the Search Process




# SB Estimation Details

## B.1 Details on Each Consumer's Search Environment

Consumer $i$'s search environment is characterized by the new listing arrival rate $\lambda_i$, the listing exit rate $\chi_i$, the trend in the list price of new properties $\rho_i$, the mean and standard deviation of a new listing's mean utility $(\mu_{it}^{new}, \sigma_i^{new})$ as well as the standard deviation of the price shock $\sigma_\eta$.

To define $(\lambda_i, \chi_i, \rho_i, \mu_{it}^{new}, \sigma_i^{new})$, we first define them at the segment level and then take a weighted average across segments within consumer $i$'s search market where the weight is consumer $i$'s search share in each segment. We define the segment-level new listing arrival rate $\lambda_m$ as the average number of new listings in segment $m$ per week. Similarly, the segment-level exit rate in segment $m$, $\chi_m$, is defined by the exit rate in that segment in a week (as the ratio of the number of exits to the number of listings), averaged across weeks.[24] We define the trend in a new listing's list price in segment $m$, $\rho_m$, as the weekly change in the average list prices of new listings in that segment in a week, averaged across weeks. We define the segment-level mean for a new listing's mean utility, $\mu_{mt}^{new}$, as the

[24]A property is considered to have exited in a particular week if it is either sold in that week or has not been visited since two weeks prior to that week.

average of $\delta_j$ across new listings in segment $m$ at time $t$. The standard deviation $\sigma_m^{new}$ is the standard deviation of mean utilities of all new listings in segment $m$.

## B.2 Value Function Approximation

As mentioned in the main body of the paper, the vector of state variables $\Omega_{it} = (\{\delta_j\}_{j\in\mathcal{A}_{it}}, \{\delta_j, v_{ij}\}_{j\in\mathcal{R}_{it}}, \mu_{it}^{new}, m_{it})$ is high dimensional, and its dimension changes over time both exogenously and endogenously. We approximate the value function $V_i(\Omega_{it};\theta)$ by a function of fewer state variables as follows.

First, we define $u_{it}^* = \max_{j\in\mathcal{R}_{it}} \delta_j + v_{ij}$ to be the maximum utility among properties in the recall set and replace $\{\delta_j, v_{ij}\}_{j\in\mathcal{R}_{it}}$ by $u_{it}^*$.

Second, we assume that instead of tracking $\delta_j$ for all properties in $\mathcal{A}_{it}$, a consumer tracks the mean utilities of the top $K$ properties and the average mean utility of the remaining properties. In other words, we replace $\{\delta_j\}_{j\in\mathcal{A}_{it}}$ by $\{\delta_{it}^{(1)}, ..., \delta_{it}^{(K)}, \bar{\delta}_{it}^{(K+1)}, A_{it}\}$, where $\delta_{it}^{(1)}, ..., \delta_{it}^{(K)}$ represent the highest $K$ mean utilities among $\{\delta_j\}_{j\in\mathcal{A}_{it}}$; $\bar{\delta}_{it}^{(K+1)}$ denotes the average of $\delta_j$ for properties outside the top $K$ in the set $\mathcal{A}_{it}$, and $A_{it} = \#\mathcal{A}_{it}$ is the number of properties available for search. In practice, we set $K = 5$.

Let $\tilde{\Omega}_{it} = \{\delta_{it}^{(1)}, ..., \delta_{it}^{(K)}, \bar{\delta}_{it}^{(K+1)}, A_{it}, u_{it}^*, \mu_{it}^{new}, m_{it}\}$ denote the reduced state variables. We randomly pick a subset of grid points for $\tilde{\Omega}_{it}$ and solve the value function at these grid points. Then, we interpolate the value function at other points with a polynomial of $\tilde{\Omega}_{it}$. See, for example, Keane and Wolpin (1994), Crawford and Shum (2005), and Sweeting (2013).

State variables $\tilde{\Omega}_{it}$ include observable variables $(\delta_{it}^{(1)}, ..., \delta_{it}^{(K)}, \bar{\delta}_{it}^{(K+1)}, A_{it}, \mu_{it}^{new}, m_{it})$ as well as the unobservable variable $u_{it}^*$, which depends on the unobservable match values $v_{ij}$. In estimation, we simulate $v_{ij}$ for all properties $j$ visited by consumer $i$, i.e., $j \in \mathcal{N}_{it}, t = 1, ..., T_i$.

# SC Robustness

In this section, we report the results from two robustness analyses.

In the first robustness analysis, we allow the price coefficient to be different for consumers who search in different districts. Table SC.1 reports the results from regressing the mean utility of a property on its characteristics where the price coefficient is district specific. We find some heterogeneity in the price coefficients. Parameters common in this robustness specification and the baseline specification (see Table 2) are close.

Table SC.1: Estimates of Parameters in Mean Utility: District-Specific Price Coefficient

| | (I) OLS | | (II) IV | |
|---|---|---|---|---|
| | Est | SE | Est | SE |
| List price (million CN¥) | | | | |
| Dongcheng | -0.261*** | (0.021) | -5.497*** | (0.204) |
| Xicheng | -0.084*** | (0.016) | -4.827*** | (0.164) |
| Chaoyang | -0.245*** | (0.009) | -5.301*** | (0.164) |
| Haidian | -0.001*** | (0.012) | -3.661*** | (0.143) |
| Fengtai | -0.377*** | (0.015) | -4.770*** | (0.155) |
| Shijingshan | -0.400*** | (0.027) | -4.066*** | (0.248) |
| Property age (year) | -0.004*** | (0.001) | -0.009*** | (0.001) |
| # Bedrooms | 0.217*** | (0.007) | 0.367*** | (0.017) |
| # Living rooms | 0.527*** | (0.008) | 1.087*** | (0.025) |
| Property size ($m^2$) | 0.005*** | (0.000) | 0.054*** | (0.002) |
| Above 10th floor | 0.082*** | (0.007) | 0.171*** | (0.018) |
| Close to subway stations | -0.020** | (0.010) | 0.227*** | (0.025) |
| Neighborhood FE | yes | | yes | |
| R square | 0.575 | | 0.531 | |

*** $p < 0.01$, ** $p < 0.05$

In the second robustness analysis, we do not restrict the sample to consumers who purchased a property before the end of the sample in estimation. Specifically, we now include consumers who did not make a purchase during the sample period but visited at least one property during the last 4 weeks before the end of the sample.[25] Table SC.2 reports the estimation result using this larger sample. The comparison to Table 3 shows that our results are robust to including non-buyers.

[25]We add this restriction to rule out inactive consumers.

Table SC.2: Estimates of Dynamic Parameters of the Robustness Sample

| | | Est | SE |
|---|---|---|---|
| SD of match values | ($\sigma_v$) | 0.442*** | (0.014) |
| Mean waiting cost | ($w$) | 0.049*** | (0.015) |
| SD of waiting cost shock | ($\sigma_w$) | 0.050*** | (0.010) |
| Search cost: (0.001) | | | |
| const. | ($\gamma_0$) | 3.066*** | (0.218) |
| $n$ | ($\gamma_1$) | 1.051** | (0.485) |
| (past searches)$\times n$ | ($\gamma_2$) | 4.209*** | (0.186) |
| $n^2$ | ($\gamma_3$) | 0.200*** | (0.013) |
| scale parameter | ($\kappa$) | 3.396*** | (0.547) |

*** $p < 0.01$, ** $p < 0.05$

# SD Simulation Details

## D.1 Model Fit Simulation

To simulate a path (indexed by $r$) for each consumer, we first draw match values $v_{ij}^{(r)}$ for all properties in consumer $i$'s search market. We then conduct a forward simulation as follows. In the description below, a notation with a superscript $(r)$ indicates a simulated outcome and a notation without the superscript represents the observed outcome in the data.

At $t = 1$, the recall set is $\mathcal{R}_{i1} = \emptyset$, the number of past searches is $m_{i1} = 0$, and the vector of state variables is $\Omega_{i1} = (\{\delta_{j1}\}_{j\in\mathcal{A}_{i1}}, \mu_{i1}^{new}, m_{i1})$, where $\mathcal{A}_{i1}$ is the observed available-for-search set. We simulate the search intensity $n_{i1}^{(r)}$ and the purchase decision $y_{i1}^{(r)}$ in $t = 1$ in three steps: (1) We compute $\Pr_i(n|\Omega_{i1};\hat{\theta})$ according to (15) for $n = 0, ..., \bar{n}$ and simulate $n_{i1}^{(r)}$ based on these probabilities; (2) we draw $\mathcal{N}_{i1}^{(r)}$ based on $\Pr(\mathcal{N}|\mathcal{A}_{i1}, n_{i1}^{(r)})$ in (5); (3) we compute the probability of purchasing a newly searched property $\Pr_i(y_{i1} = j \in \mathcal{N}_{i1}^{(r)}|\Omega_{i1}, \{v_{ij}^{(r)}\}_{j\in\mathcal{N}_{i1}^{(r)}};\hat{\theta})$ according (17) and the probability of waiting $\Pr_i(y_{i1} = wait|\Omega_{i1}, \{v_{ij}^{(r)}\}_{j\in\mathcal{N}_{i1}^{(r)}};\hat{\theta})$ according to (18) and simulate $y_{i1}^{(r)}$ based on these probabilities. If $y_{i1}^{(r)} \neq wait$, the path ends. Otherwise, we continue to $t = 2$.

At $t = 2$, the recall set is updated to $\mathcal{R}_{i2}^{(r)} = \mathcal{R}_{i1} \cup \mathcal{N}_{i1}^{(r)} \backslash \mathcal{EXIT}_{i1}$, the available-for-search set is updated to $\mathcal{A}_{i2}^{(r)} = \mathcal{A}_{i1} \backslash \mathcal{N}_{i1}^{(r)} \backslash \mathcal{EXIT}_{i1} \cup \mathcal{NEW}_{i2}$, and the number of past searches is updated to $m_{i2}^{(r)} = n_{i1}^{(r)}$. The state variables are $\Omega_{i2}^{(r)} = (\{\delta_{j2}\}_{j\in\mathcal{A}_{i2}^{(r)}}, \{\delta_{j2}, v_{ij}^{(r)}\}_{j\in\mathcal{R}_{i2}^{(r)}}, \mu_{i2}^{new}, m_{i2}^{(r)})$. We simulate $(n_{i2}^{(r)}, y_{i2}^{(r)})$ following the same steps except that in step (3), we additionally

consider the probability of recall according to (16). If $y_{i2}^{(r)} \neq wait$, the path ends. Otherwise, we continue to $t = 3$ and repeat the process until $y_{it}^{(r)} \neq wait$.

## D.2 Counterfactual Simulations

The simulation procedure for the counterfactual simulations is similar to that for the model-fit simulation, with two differences. First, instead of using the estimated value function, we use backward induction to solve for the value function in a counterfactual scenario and compute $\Pr_i(n|\Omega_{i1};\hat{\theta})$ and $\Pr_i(y_{it}|\Omega_{it},\{v_{ij}^{(r)}\}_{j\in\mathcal{N}_{it}^{(r)}};\hat{\theta})$ based on the computed value function. Second, in counterfactual scenarios with different new listing arrival rates, we replace the observed $\mathcal{NEW}_{it}$ with the simulated $\mathcal{NEW}_{it}^{(r)}$. We simulate $\mathcal{NEW}_{it}^{(r)}$ as follows. Let $\lambda_i^{CF}$ denote the arrival rate of new listings in consumer $i$'s search market in a counterfactual scenario. We draw a number $new_{it}^{(r)}$ from a Poisson distribution with arrival rate $\lambda_i^{CF}$ and let $new_{it}^{(r)}$ be the number of new listings of consumer $i$'s search area at time $t$ in the counterfactual scenario. For each of the $new_{it}^{(r)}$ new listings, we draw $\delta_j^{(r)}$ from $N(\mu_{it}^{new},(\sigma_i^{new})^2)$.

# SE Proofs

Throughout this section, we suppress the subscripts $i$ and $t$ for simplicity.

## E.1 Proof for $\Pr(\mathcal{N}|\mathcal{A},n)$ in Equation (5)

We derive $\Pr(\mathcal{N}|\mathcal{A},n)$ based on the inclusion-exclusion principle and the choice probability in a standard Logit model.

We first apply the inclusion-exclusion principle. The probability that $\mathcal{N}$ is not chosen, i.e., $1-\Pr(\mathcal{N}|\mathcal{A},n)$, is the probability that there is at least one option $j\in\mathcal{N}$ such that $\max_{h\in\mathcal{A}\setminus\mathcal{N}}(\delta_h+\epsilon_h)>\delta_j+\epsilon_j$. This probability equals to the inclusion and exclusion of the following probabilities:

- include the probability that this inequality holds for one option in $\mathcal{N}$
- exclude the probabilities that this inequality holds for two options in $\mathcal{N}$
- include the probability that inequality holds for three options in $\mathcal{N}$
- so on and so forth

In other words,

$$1-\Pr(\mathcal{N}|\mathcal{A},n)=\sum_{k=1}^{n}\left[(-1)^{k-1}\sum_{\mathcal{B}\in\mathcal{C}^k(\mathcal{N})}\Pr(\max_{h\in\mathcal{A}\backslash\mathcal{N}}(\delta_h+\epsilon_h)>\delta_l+\epsilon_l,l\in\mathcal{B})\right].$$

We then derive the analytic expression for $\Pr(\max_{h\in\mathcal{A}\backslash\mathcal{N}}(\delta_h+\epsilon_h)>\delta_b+\epsilon_b,b\in\mathcal{B})$ using the choice probability in a standard Logit model as follows:

$$\begin{aligned}\Pr(\max_{h\in\mathcal{A}\backslash\mathcal{N}}(\delta_h+\epsilon_h)>\delta_l+\epsilon_l,l\in\mathcal{B})&=\sum_{h\in\mathcal{A}\backslash\mathcal{N}}\Pr(\delta_h+\epsilon_h>\delta_l+\epsilon_l,l\in(\mathcal{A}\backslash\mathcal{N})\cup\mathcal{B})\\&=\sum_{h\in\mathcal{A}\backslash\mathcal{N}}\frac{exp(\delta_h)}{\sum_{l\in(\mathcal{A}\backslash\mathcal{N})\cup\mathcal{B}}exp(\delta_l)}.\end{aligned}$$

Combining the above two steps yields

$$\Pr(\mathcal{N}|\mathcal{A},n)=1-\sum_{k=1}^{n}\left[(-1)^{k-1}\sum_{\mathcal{B}\in\mathcal{C}^k(\mathcal{N})}\frac{\sum_{h\in\mathcal{A}\backslash\mathcal{N}}exp(\delta_h)}{\sum_{l\in(\mathcal{A}\backslash\mathcal{N})\cup\mathcal{B}}exp(\delta_l)}\right],$$

which can be further simplified as

$$\begin{aligned}\Pr(\mathcal{N}|\mathcal{A},n)&=1-\sum_{k=1}^{n}\left[(-1)^{k-1}\sum_{\mathcal{B}\in\mathcal{C}^k(\mathcal{N})}\left(1-\frac{\sum_{l\in\mathcal{B}}exp(\delta_l)}{\sum_{l\in(\mathcal{A}\backslash\mathcal{N})\cup\mathcal{B}}exp(\delta_l)}\right)\right]\\&=1-\sum_{k=1}^{n}(-1)^{k-1}C_n^k+\sum_{k=1}^{n}\left[(-1)^{k-1}\sum_{\mathcal{B}\in\mathcal{C}^k(\mathcal{N})}\frac{\sum_{l\in\mathcal{B}}exp(\delta_l)}{\sum_{l\in(\mathcal{A}\backslash\mathcal{N})\cup\mathcal{B}}exp(\delta_l)}\right]\\&=\sum_{k=1}^{n}\left[(-1)^{k-1}\sum_{\mathcal{B}\in\mathcal{C}^k(\mathcal{N})}\frac{\sum_{l\in\mathcal{B}}exp(\delta_l)}{\sum_{l\in(\mathcal{A}\backslash\mathcal{N})\cup\mathcal{B}}exp(\delta_l)}\right],\end{aligned}\tag{SE.1}$$

where the last line holds because setting $x=-1$ in the binomial theorem $(1+x)^n=\sum_{k=0}^{n}C_n^kx^k$ yields $0=1-\sum_{k=1}^{n}C_n^k(-1)^{k-1}$.

## E.2 Proof for $\Pr(j|\mathcal{A},n)$ in Equation (6)

**Intuition** In Appendix A.2.1, we provide the intuition, which relies on two points, for the analytical expression for $\Pr(j|\mathcal{A},n)$ in (6). We now use a simple example where $j=1$, $\mathcal{A}=\{1,2,3,4\}$, and $n=2$ to illustrate the two points. In this example, the first point is

that

$$\Pr(1|\{1,2,3,4\},2) = \Pr(1\text{'s rank is } 1) + \Pr(1\text{'s rank is } 2).$$

To see the second point, note that

$$\Pr(1\text{'s rank is } 1) = \Pr(1 \text{ is the best in } \{1,2,3,4\}).$$

As for $\Pr(1\text{'s rank is } 2)$, note that

$$\begin{aligned}
&\Pr(1 \text{ is the best in } \{1,2,3\}) \\
&= \Pr(1 \text{ is the best in } \{1,2,3,4\}) \\
&+ \Pr(4 \text{ is the best in } \{1,2,3,4\} \text{ and } 1 \text{ is the best in } \{1,2,3\}),
\end{aligned}$$

$$\begin{aligned}
&\Pr(1 \text{ is the best in } \{1,3,4\}) \\
&= \Pr(1 \text{ is the best in } \{1,2,3,4\}) \\
&+ \Pr(2 \text{ is the best in } \{1,2,3,4\} \text{ and } 1 \text{ is the best in } \{1,3,4\}),
\end{aligned}$$

$$\begin{aligned}
&\Pr(1 \text{ is the best in } \{1,2,4\}) \\
&= \Pr(1 \text{ is the best in } \{1,2,3,4\}) \\
&+ \Pr(3 \text{ is the best in } \{1,2,3,4\} \text{ and } 1 \text{ is the best in } \{1,2,4\}).
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\Pr(1 \text{ is the best in } \{1,2,3\}) + \Pr(1 \text{ is the best in } \{1,3,4\}) + \Pr(1 \text{ is the best in } \{1,2,4\}) \\
=&3\Pr(1\text{'s rank is } 1) + \Pr(1\text{'s rank is } 2),
\end{aligned}$$

which counts $\Pr(1\text{'s rank is } 2)$ once but $\Pr(1\text{'s rank is } 1)$ three times.

In other words,

$$\Pr(1\text{'s rank is } 2) = \sum_{\mathcal{B}\in\mathcal{C}^2(\{2,3,4\})} \Pr(j \text{ is the best in } \mathcal{B}\cup\{j\}) - 3\Pr(1\text{'s rank is } 1).$$

These two points explain why the probability in (6) depends on $\frac{exp(\delta_j)}{\sum_{l\in\mathcal{B}} exp(\delta_l)+exp(\delta_j)}$, which is $\Pr(j$ is the best among $\mathcal{B}\cup\{j\})$, and constant $C^k_{A-n+k-1}$, which is used to adjust double counting.

**Proof** We derive the analytical expression for $\Pr(j|\mathcal{A},n)$ in (6) when $0<n<A$ based on $\Pr(\mathcal{N}|\mathcal{A},n)$ in (5).

Plugging $\Pr(\mathcal{N}|\mathcal{A},n)$ in (A.1) into $\Pr(j|\mathcal{A},n)=\sum_{\mathcal{N}\in\mathcal{C}^n(\mathcal{A}):j\in\mathcal{N}}\Pr(\mathcal{N}|\mathcal{A},n)$ yields

$$\Pr(j|\mathcal{A},n)=\sum_{\mathcal{N}\in\mathcal{C}^n(\mathcal{A}):j\in\mathcal{N}}\sum_{k=1}^{n}\left[(-1)^{k-1}\sum_{\mathcal{B}\in\mathcal{C}^k(\mathcal{N})}\frac{\sum_{l\in\mathcal{B}}exp(\delta_l)}{\sum_{l\in(\mathcal{A}\backslash\mathcal{N})\cup\mathcal{B}}exp(\delta_l)}\right]$$

$$=\sum_{k=1}^{n}\left[(-1)^{k-1}\sum_{\mathcal{N}\in\mathcal{C}^n(\mathcal{A}):j\in\mathcal{N}}\sum_{\mathcal{B}\in\mathcal{C}^k(\mathcal{N})}\frac{\sum_{l\in\mathcal{B}}exp(\delta_l)}{\sum_{l\in(\mathcal{A}\backslash\mathcal{N})\cup\mathcal{B}}exp(\delta_l)}\right] \tag{SE.2}$$

$$=\sum_{k=1}^{n}\left[(-1)^{k-1}\sum_{\mathcal{D}\in\mathcal{C}^{A-n}(\mathcal{A}\backslash j)}\sum_{\mathcal{B}'\in\mathcal{C}^{k-1}(\mathcal{A}\backslash\mathcal{D})}\frac{\sum_{l\in\mathcal{B}'}exp(\delta_l)+exp(\delta_j)}{\sum_{l\in\mathcal{D}\cup\mathcal{B}'}exp(\delta_l)+exp(\delta_j)}\right] \tag{SE.3}$$

$$+\sum_{k=1}^{n-1}\left[(-1)^{k-1}\sum_{\mathcal{D}\in\mathcal{C}^{A-n}(\mathcal{A}\backslash j)}\sum_{\mathcal{B}\in\mathcal{C}^{k}(\mathcal{A}\backslash\mathcal{D}\backslash j)}\frac{\sum_{l\in\mathcal{B}}exp(\delta_l)}{\sum_{l\in\mathcal{D}\cup\mathcal{B}}exp(\delta_l)}\right] \tag{SE.4}$$

$$=\sum_{k=1}^{n}\left[(-1)^{k-1}\sum_{\mathcal{F}\in\mathcal{C}^{A-n+k-1}(\mathcal{A}\backslash j)}\sum_{\mathcal{D}\in\mathcal{C}^{A-n}(\mathcal{F})}\frac{\sum_{l\in\mathcal{F}\backslash\mathcal{D}}exp(\delta_l)+exp(\delta_j)}{\sum_{l\in\mathcal{F}}exp(\delta_l)+exp(\delta_j)}\right] \tag{SE.5}$$

$$+\sum_{k=1}^{n-1}\left[(-1)^{k-1}\sum_{\mathcal{F}\in\mathcal{C}^{A-n+k}(\mathcal{A}\backslash j)}\sum_{\mathcal{B}\in\mathcal{C}^{k}(\mathcal{F})}\frac{\sum_{l\in\mathcal{B}}exp(\delta_l)}{\sum_{l\in\mathcal{F}}exp(\delta_l)}\right]. \tag{SE.6}$$

From (SE.2) to the sum of (SE.3) and (SE.4), we replace $\mathcal{A}\backslash\mathcal{N}$ by $\mathcal{D}$ and separately consider $\mathcal{B}\in\mathcal{C}^k(\mathcal{N})$ including $j$ in (SE.3) and those excluding $j$ in (SE.4). In (SE.3), we use $\mathcal{B}'$ to represent $\mathcal{B}\backslash j$. In (SE.4), the summation goes to $n-1$ instead of $n$ because when $k=n$, $\mathcal{B}\in\mathcal{C}^k(\mathcal{N})$ excluding $j$ does not exist.

From (SE.3) to (SE.5), we use $\mathcal{F}$ to represent $\mathcal{D}\cup\mathcal{B}'$. Because $\mathcal{D}$ and $\mathcal{B}'$ have no intersection, the double summation over $\mathcal{D}$ and $\mathcal{B}'$ in (SE.3) is equivalent to a double summation over $\mathcal{F}$ and subsets of $\mathcal{F}$ (i.e., $\mathcal{D}\in\mathcal{C}^{A-n}(\mathcal{F})$). We do the same to obtain line (SE.6) from line (SE.4).

Line (SE.5) can be further simplified as

$$
\begin{aligned}
&\sum_{k=1}^{n}\left[(-1)^{k-1}\sum_{\mathcal{F}\in\mathcal{C}^{A-n+k-1}(\mathcal{A}\setminus j)}\sum_{\mathcal{D}\in\mathcal{C}^{A-n}(\mathcal{F})}\left(1-\frac{\sum_{l\in\mathcal{D}}exp(\delta_l)}{\sum_{l\in\mathcal{F}}exp(\delta_l)+exp(\delta_j)}\right)\right]\\
&=\sum_{k=1}^{n}\left[(-1)^{k-1}\sum_{\mathcal{F}\in\mathcal{C}^{A-n+k-1}(\mathcal{A}\setminus j)}\left(C_{A-n+k-1}^{A-n}-\sum_{\mathcal{D}\in\mathcal{C}^{A-n}(\mathcal{F})}\frac{\sum_{l\in\mathcal{D}}exp(\delta_l)}{\sum_{l\in\mathcal{F}}exp(\delta_l)+exp(\delta_j)}\right)\right]\\
&=\sum_{k=1}^{n}(-1)^{k-1}C_{A-1}^{A-n+k-1}C_{A-n+k-1}^{A-n}\\
&-\sum_{k=1}^{n}\left[(-1)^{k-1}\sum_{\mathcal{F}\in\mathcal{C}^{A-n+k-1}(\mathcal{A}\setminus j)}\frac{C_{A-n+k-2}^{A-n-1}\sum_{l\in\mathcal{F}}exp(\delta_l)}{\sum_{l\in\mathcal{F}}exp(\delta_l)+exp(\delta_j)}\right] \qquad \text{(SE.7)}\\
&=\sum_{k=1}^{n}(-1)^{k-1}C_{A-1}^{A-n+k-1}C_{A-n+k-1}^{A-n}\\
&-\sum_{k=1}^{n}\left[(-1)^{k-1}\sum_{\mathcal{F}\in\mathcal{C}^{A-n+k-1}(\mathcal{A}\setminus j)}\left\{C_{A-n+k-2}^{A-n-1}\left(1-\frac{exp(\delta_j)}{\sum_{l\in\mathcal{F}}exp(\delta_l)+exp(\delta_j)}\right)\right\}\right]\\
&=\sum_{k=1}^{n}(-1)^{k-1}C_{A-1}^{A-n+k-1}C_{A-n+k-1}^{A-n} \qquad \text{(SE.8)}\\
&-\sum_{k=1}^{n}(-1)^{k-1}C_{A-1}^{A-n+k-1}C_{A-n+k-2}^{A-n-1} \qquad \text{(SE.9)}\\
&+\sum_{k=1}^{n}\left[(-1)^{k-1}C_{A-n+k-2}^{A-n-1}\sum_{\mathcal{F}\in\mathcal{C}^{A-n+k-1}(\mathcal{A}\setminus j)}\left(\frac{exp(\delta_j)}{\sum_{l\in\mathcal{F}}exp(\delta_l)+exp(\delta_j)}\right)\right]
\end{aligned}
$$

Line (SE.7) holds because in the summation over $\mathcal{D}\in\mathcal{C}^{A-n}(\mathcal{F})$, each term $exp(\delta_l)$ for $l\in\mathcal{D}$ is repeated $C_{A-n+k-2}^{A-n-1}$ times.

Similarly, line (SE.6) can be further simplified as

$$
\begin{aligned}
&\sum_{k=1}^{n-1}\left[(-1)^{k-1}\sum_{\mathcal{F}\in\mathcal{C}^{A-n+k}(\mathcal{A}\setminus j)}C_{A-n+k-1}^{k-1}\right]\\
&=\sum_{k=1}^{n-1}\left[(-1)^{k-1}C_{A-1}^{A-n+k}C_{A-n+k-1}^{k-1}\right]. \qquad \text{(SE.10)}
\end{aligned}
$$

In Supplemental Appendix E.4, we show that line (SE.8) $=0$, line (SE.9) $=-1$, and line (SE.10) $=1$. Therefore, combining the above simplified expressions for (SE.5) and

(SE.6) yields

$$\Pr(j|\mathcal{A},n)=\sum_{k=1}^{n}\left[(-1)^{k-1}C_{A-n+k-2}^{A-n-1}\sum_{\mathcal{F}\in\mathcal{C}^{A-n+k-1}(\mathcal{A}\backslash j)}\left(\frac{exp(\delta_j)}{\sum_{l\in\mathcal{F}}exp(\delta_l)+exp(\delta_j)}\right)\right] \quad \text{(SE.11)}$$

$$=\sum_{k=0}^{n-1}\left[(-1)^{k}\sum_{\mathcal{F}\in\mathcal{C}^{A-n+k}(\mathcal{A}\backslash j)}\left(\frac{C_{A-n+k-1}^{k}exp(\delta_j)}{\sum_{l\in\mathcal{F}}exp(\delta_l)+exp(\delta_j)}\right)\right]. \quad \text{(SE.12)}$$

## E.3 Proof for the Contraction Mapping in (A.4)

In this section, we first show that the mapping in (A.4) has the following four features: (1) $\frac{\partial f_j}{\partial \delta_k}\geq 0$ for any $j,k$ ,/(2) $\sum_{k\in\mathcal{J}\backslash\{1\}}\frac{\partial f_j}{\partial \delta_k}<1$ for any $j$ ,/(3) $\min_{j\in\mathcal{J}}\inf_{\delta} f(\boldsymbol{\delta})>-\infty$, and (4) there is a value, $\bar{\delta}$, with such that if $\delta_j>\bar{\delta}$, then $f_j(\boldsymbol{\delta})<\delta_j$. We then show that these features imply that a truncated version of the mapping is a contraction mapping.

**(1)** $\frac{\partial f_j}{\partial \delta_k}\geq 0$ **for any** $j,k$

We prove this inequality in three steps.

**Step 1.** We show $\frac{\partial \Pr(j|\mathcal{A},n)}{\partial \delta_j}<\Pr(j|\mathcal{A},n)$ using induction. When $n=1$, $\Pr(j|\mathcal{A},1)$ is the choice probability in a standard multinomial Logit model. As a result, $\frac{\partial \Pr(j|\mathcal{A},1)}{\partial \delta_j}=\Pr(j|\mathcal{A},1)\left(1-\Pr(j|\mathcal{A},1)\right)<\Pr(j|\mathcal{A},1)$.

To show that $\frac{\partial \Pr(j|\mathcal{A},n-1)}{\partial \delta_j}<\Pr(j|\mathcal{A},n-1)$ implies $\frac{\partial \Pr(j|\mathcal{A},n)}{\partial \delta_j}<\Pr(j|\mathcal{A},n)$ for any $n\geq 2$, we first note that option $j$ is sampled if and only if its rank in terms $\delta_j+\epsilon_j$ is no more than $n$. Therefore,

$$\Pr(j|\mathcal{A},n)=\Pr(j|\mathcal{A},n-1)+\Pr(j\text{'s rank is } n).$$

where $\Pr(j\text{'s rank is } n)$ is the probability that some options $(j_1,\cdots,j_{n-1})$ are the top $n-1$ options while $j$ is the $n^{th}$ best option. In other words,

$$\begin{aligned}&\Pr(j\text{'s rank is } n) \quad \text{(SE.13)}\\ =\ &\sum_{\{(j_1,\cdots,j_{n-1}):j_l\neq j,l=1,\ldots,n-1\}}\ [\Pr(j_1|\mathcal{A},1)\times\cdots\times\Pr(j_{n-1}|\mathcal{A}\backslash\{j_1,\cdots,j_{n-2}\},1)\\ &\qquad\times\Pr(j|\mathcal{A}\backslash\{j_1,\cdots,j_{n-1}\},1)]\end{aligned}$$

The probabilities in (SE.13) are choice probabilities in a standard multinomial Logit model. We know that $\frac{\partial \Pr(j_l|\mathcal{A}\backslash\{j_1,\cdots,j_{l-1}\},1)}{\partial \delta_j}<0$ and $\frac{\partial \Pr(j|\mathcal{A}\backslash\{j_1,\cdots,j_{n-1}\},1)}{\partial \delta_j}<\Pr(j|\mathcal{A}\backslash\{j_1,\cdots,j_{n-1}\},1)$. Therefore, $\frac{\partial \Pr(j\text{'s rank is } n)}{\partial \delta_j}<\Pr(j\text{'s rank is } n)$. As a result, $\frac{\partial \Pr(j|\mathcal{A},n-1)}{\partial \delta_j}<\Pr(j|\mathcal{A},n-1)$ implies $\frac{\partial \Pr(j|\mathcal{A},n)}{\partial \delta_j}<\Pr(j|\mathcal{A},n)$.

**Step 2.** We show $\frac{\partial \Pr(j|\mathcal{A},n)}{\partial \delta_k} < 0$ for any $k \neq j$. Since $\Pr(j|\mathcal{A},n)$ is the probability that $\delta_j + \epsilon_j$ is among the top $n$ highest values in $\{\delta_{j'} + \epsilon_{j'}\}_{j' \in \mathcal{A}}$, it decreases as $\delta_k$ increases for any $k \neq j$.

**Step 3.** We show $\frac{\partial f_j}{\partial \delta_k} \geq 0$ using the results from Steps 1 and 2. For $k = j$, the result in Step 1 implies

$$\begin{aligned}
\frac{\partial f_j}{\partial \delta_j} &= 1 - \frac{1}{\tilde{s}_j(\boldsymbol{\delta})} \frac{\sum_{i \in \mathcal{I}} \sum_{t=1}^{T_i} \frac{\partial \Pr(j|\mathcal{A}_{it}, n_{it})}{\partial \delta_j}}{\sum_{i \in \mathcal{I}} \sum_{t=1}^{T_i} n_{it}} \\
&> 1 - \frac{1}{\tilde{s}_j(\boldsymbol{\delta})} \frac{\sum_{i \in \mathcal{I}} \sum_{t=1}^{T_i} \Pr(j|\mathcal{A}_{it}, n_{it})}{\sum_{i \in \mathcal{I}_d} \sum_{t=1}^{T_i} n_{it}} \\
&= 1 - \frac{1}{\tilde{s}_j(\boldsymbol{\delta})} \tilde{s}_j(\boldsymbol{\delta}) = 0.
\end{aligned}$$

For $k \neq j$, the result in Step 2 implies

$$\frac{\partial \tilde{s}_j(\boldsymbol{\delta})}{\partial \delta_k} = \frac{\sum_{i=1}^{I} \sum_{t=1}^{T_i} n_{it} \frac{\partial \Pr(j|\mathcal{A}_{it}, n_{it})}{\partial \delta_k}}{\sum_{i=1}^{I} \sum_{t=1}^{T_i} n_{it}} < 0.$$

Therefore,

$$\frac{\partial f_j}{\partial \delta_k} = -\frac{1}{\tilde{s}_j(\boldsymbol{\delta})} \frac{\partial \tilde{s}_j(\boldsymbol{\delta})}{\partial \delta_k} > 0.$$

These three steps complete the proof that $\frac{\partial f_j}{\partial \delta_k} \geq 0$ for any $j, k$.

**(2) $\sum_{k \in \mathcal{J}} \frac{\partial f_j}{\partial \delta_k} < 1$ for any $j$**

Because increasing the mean utility of every option (including $\delta_1$) does not change the sampling probabilities, $\frac{\partial \tilde{s}_j(\boldsymbol{\delta})}{\partial \delta_1} + \sum_{k \in \mathcal{J}} \frac{\partial \tilde{s}_j(\boldsymbol{\delta})}{\partial \delta_k} = 0$, which implies that $\sum_{k \in \mathcal{J}} \frac{\partial \tilde{s}_j(\boldsymbol{\delta})}{\partial \delta_k} = -\frac{\partial \tilde{s}_j(\boldsymbol{\delta})}{\partial \delta_1} > 0$. As a result, $\sum_{k \in \mathcal{J}} \frac{\partial f_j}{\partial \delta_k} = 1 - \frac{1}{\tilde{s}_j(\boldsymbol{\delta})} \sum_{k \in \mathcal{J}} \frac{\partial \tilde{s}_j(\boldsymbol{\delta})}{\partial \delta_k} < 1$.

**(3) There is a value $\underline{\delta}$ such that if $\delta_j < \underline{\delta}$ for any $j \in \mathcal{J}$, then $f_j(\boldsymbol{\delta}) > \delta_j$.**

Given Condition 2, $\delta_j = -\infty$ implies $\Pr(j|\mathcal{A}_{it}, n_{it}) = 0$ and thus $\tilde{s}_j(\boldsymbol{\delta}) = 0$. In other words, $\lim_{\delta_j \to -\infty} f_j(\boldsymbol{\delta}) = \infty$. By continuity of $f_j(\boldsymbol{\delta})$, there exists $\underline{\delta}_j$ such that $f_j(\boldsymbol{\delta}) > \delta_j$ for any $\boldsymbol{\delta}$ where $\delta_j < \underline{\delta}_j$. Let $\underline{\delta} = \min_j \underline{\delta}_j$.

**(4) There is a value $\bar{\delta}$ such that if $\delta_j > \bar{\delta}$ for any $j \in \mathcal{J}$, then $f_j(\boldsymbol{\delta}) < \delta_j$.**

Given Condition 3, $\delta_j = \infty$ implies $\Pr(j|\mathcal{A}_{it}, n_{it}) = 1$ and thus $\tilde{s}_j(\boldsymbol{\delta}) = \frac{\sum_{i=1}^{I}\sum_{t=1}^{T_i}\mathbb{1}(j\in\mathcal{A}_{it})}{\sum_{i=1}^{I}\sum_{t=1}^{T_i} n_{it}}$. According to Condition (1), $\frac{\sum_{i=1}^{I}\sum_{t=1}^{T_i}\mathbb{1}(j\in\mathcal{A}_{it})}{\sum_{i=1}^{I}\sum_{t=1}^{T_i} n_{it}} > s_j$. Therefore, $\lim_{\delta_j\to\infty} f_j(\boldsymbol{\delta}) < \delta_j$. By continuity of $f_j(\boldsymbol{\delta})$, there exists $\bar{\delta}_j$ such that $f_j(\boldsymbol{\delta}) < \delta_j$ for any $\boldsymbol{\delta}$ where $\delta_j > \bar{\delta}_j$. Let $\bar{\delta} = \max_j \bar{\delta}_j$.

**Features (1)–(4) imply that $f(\delta)$ is a contraction mapping.**

First, we show $\underline{\delta} < \bar{\delta}$ by contradiction. Suppose $\underline{\delta} = \bar{\delta}$. Then features (3) and (4) contradict each other. Suppose $\underline{\delta} > \bar{\delta}$. Then, feature (3) implies that $f_j(\bar{\delta}) > \delta_j$, which is contradiction to feature (4).

Then, we define another mapping $\hat{f}(\boldsymbol{\delta}) : [\underline{\delta}, \bar{\delta}]^J \to [\underline{\delta}, \bar{\delta}]^J$ as

$$\hat{f}_j(\boldsymbol{\delta}) = \max\{\underline{\delta},\ \min\{f_j(\boldsymbol{\delta}), \bar{\delta}\}\}.$$

For any $\boldsymbol{\delta}, \boldsymbol{\delta}' \in [\underline{\delta}, \bar{\delta}]^{J-1}$, we define $\vartheta = ||\boldsymbol{\delta} - \boldsymbol{\delta}'||$. We have

$$\hat{f}_j(\boldsymbol{\delta}') - \hat{f}_j(\boldsymbol{\delta}) \le \hat{f}_j(\boldsymbol{\delta} + \vartheta) - \hat{f}_j(\boldsymbol{\delta}) \le f_j(\boldsymbol{\delta} + \vartheta) - f_j(\boldsymbol{\delta}) = \int_0^{\vartheta} \sum_{k\in\mathcal{J}} \frac{\partial f_j(\boldsymbol{\delta} + z)}{\partial \delta_k} dz \le \varsigma\vartheta,$$

where $\varsigma = \max_j \max_{\boldsymbol{\delta}\in[\underline{\delta},\bar{\delta}]^{J-1}} \sum_{k\in\mathcal{J}} \frac{\partial f_j(\boldsymbol{\delta})}{\partial \delta_k}$. The first inequality holds because of feature (1). The second inequality holds because $\hat{f}$ is a truncated version of $f$.

By feature (2), $\varsigma < 1$. Therefore, $\hat{f}$ is a contraction mapping and has a unique fixed point. Since features (3) and (4) imply that the fixed point of $f$ is in $\hat{f}$'s domain, $f$ also has a unique fixed point.

## E.4 Proof of the Combinatorial Identities

In this section, we prove that for $A > n > 1$,[26]

$$\sum_{k=1}^{n}(-1)^{k-1}C_{A-1}^{A-n+k-1}C_{A-n+k-1}^{A-n} = 0 \text{ in line (SE.8)},$$
$$\sum_{k=1}^{n}(-1)^{k-1}C_{A-1}^{A-n+k-1}C_{A-n+k-2}^{A-n-1} = 1 \text{ in line (SE.9)},$$
$$\sum_{k=1}^{n-1}\left[(-1)^{k-1}C_{A-1}^{A-n+k}C_{A-n+k-1}^{k-1}\right] = 1 \text{ in line (SE.10)},$$
$$\sum_{k=0}^{n-1}(-1)^{k}C_{A}^{A-n+k+1}C_{A-n+k-1}^{A-n-1} = n \text{ in line (A.2)}.$$

First, we note that line (SE.10) can be rewritten as $\sum_{k=1}^{n-1}\left[(-1)^{k-1}C_{A-1}^{A-n+k}C_{A-n+k-1}^{A-n}\right]$ and, therefore, can be proved by replacing $n$ in line (SE.9) by $n-1$.

To prove the three identities in (SE.8), (SE.9), and (A.2), we rewrite them as

$$\sum_{k=r}^{m}(-1)^{h-r}C_{m}^{h}C_{h}^{r} = 0,$$
$$\sum_{k=r+1}^{m}(-1)^{h-r-1}C_{m}^{h}C_{h-1}^{r} = 1.$$
$$\sum_{k=r+2}^{m}(-1)^{h-r}C_{m}^{h}C_{h-2}^{r} = m-r-1,$$

using the change of variables ($m$=$A-1$, $r$=$A-n$, $h$=$A-n+k-1$) in the first identity, ($m$=$A-1$, $r$=$A-n-1$, $h$=$A-n+k-1$) in the second identity, and ($m$=$A$, $r$=$A-n-1$, $h$=$A-n+k+1$) in the third identity.

To prove the first identity, we take the $r^{th}$-order derivative of the equation $(1+x)^m = \sum_{h=0}^{m}C_{m}^{h}x^{h}$, which yields

$$C_{m}^{r}r!(1+x)^{n-r} = \sum_{h=r}^{m}C_{m}^{h}C_{h}^{r}r!x^{h-r}.$$

[26]We thank Pierre-Louis Blayac and Sergey Fomin at the University of Michigan for recommending "Tables of Combinatorial Identities Based on Seven Unpublished Manuscript Notebooks of Henry Gould" edited by Jocelyn Quaintance. The first identity in this section is an application of equation (1.23) in Table II (https://math.wvu.edu/~hgould/Vol.2.PDF). The proof of the other two identities is inspired by this equation.

Setting $x = -1$ leads to the first identity.

To prove the second identity, we take the $r^{th}$-order derivative of the equation $\frac{(1+x)^m}{x} = \frac{1}{x} + \sum_{h=1}^{m} C_m^h x^{h-1}$. The derivative of the LHS is the sum of terms like $a(1+x)^b x^{-c}$ where $b > 0$. Therefore, its value at $x = -1$ is $0$. The derivative of the RHS at $x = -1$ is

$$\begin{aligned}\frac{d^r}{dx^r}\left(\frac{1}{x} + \sum_{h=1}^{m} C_m^h x^{h-1}\right)\Bigg|_{x=-1} &= \left[r!(-1)^r x^{-1-r} + \sum_{h=r+1}^{m} C_m^h C_{h-1}^r r! x^{h-1-r}\right]_{x=-1} \\ &= r!\left[-1 + \sum_{h=r+1}^{m} C_m^h C_{h-1}^r (-1)^{h-1-r}\right].\end{aligned}$$

Therefore, $-1 + \sum_{h=r+1}^{m} C_m^h C_{h-1}^r (-1)^{h-1-r} = 0$, which implies the second identity.

To prove the third identity, we take the $r^{th}$-order derivative of the equation $\frac{(1+x)^m}{x^2} = \frac{1}{x^2} + \frac{m}{x} + \sum_{h=2}^{m} C_m^h x^{h-2}$. The derivative of the LHS evaluated at $x = -1$ is again $0$. The derivative of the RHS is

$$\begin{aligned}&\frac{d^r}{dx^r}\left(\frac{1}{x^2} + \frac{m}{x} + \sum_{h=2}^{m} C_m^h x^{h-2}\right)\Bigg|_{x=-1} \\ &= \left[(r+1)!(-1)^r x^{-2-r} + mr!(-1)^r x^{-1-r} + \sum_{h=r+2}^{m} C_m^h C_{h-2}^r r! x^{h-2-r}\right]_{x=-1}\end{aligned}$$

Its value at $x = -1$ is $r![(r+1) - m + \sum_{h=r+2}^{m} C_m^h C_{h-2}^r r!(-1)^{h-r}]$. Therefore,

$$(r+1) - m + \sum_{h=r+2}^{m} C_m^h C_{h-2}^r r!(-1)^{h-r} = 0,$$

which implies the third identity.